Title: Conformal Field Theories as Building Blocks of Nature

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Abstract: Conformal Field Theory is the language in which we often think about strong dynamics, be that in Condensed Matter, Quantum Gravity, or Beyond the Standard Model Physics. AdS/CFT led to significant advances of our understanding. What should come next?

PI Colloquium 13/04/2011

Conformal Field Theories

as

Building Blocks of Nature

Slava Rychkov

Université Pierre et Marie Curie

and

École Normale Supérieure, Paris

Back to the Bootstrap
(PI, 11-14/04/2011)

Back to the Bootstrap

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Baron von Münchhausen (1720-1797)

Sasha Polyakov

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Mud = Space of CFT data

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Hair = Conformal Block Decomposition

Scale Invariance

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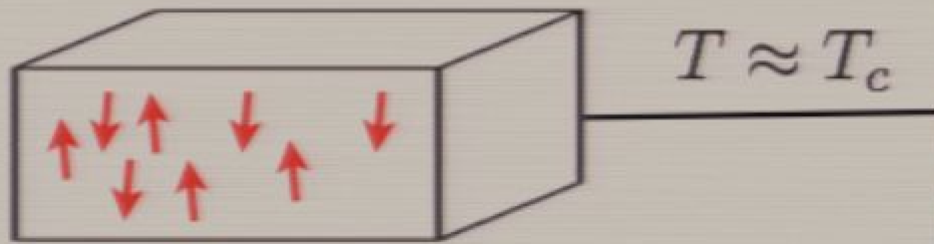
Ferromagnet (Ising Model)



$T \approx T_c$

Magnetization $\vec{M}(x)$

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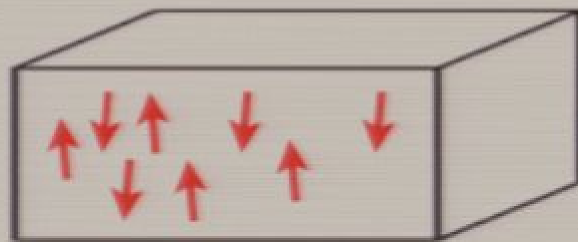

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High temperature
T>T$_c$

$$\langle M(x) \rangle = 0$$

$$\langle M(x) M(0) \rangle \sim \frac{1}{|x|^{2\Delta}} \times \exp(-|x|/\xi(T))$$

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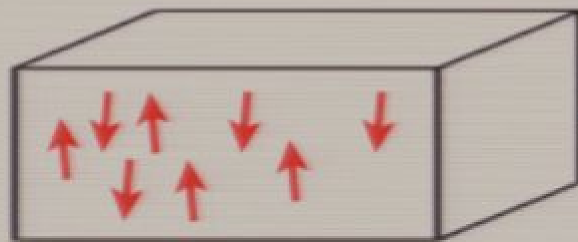

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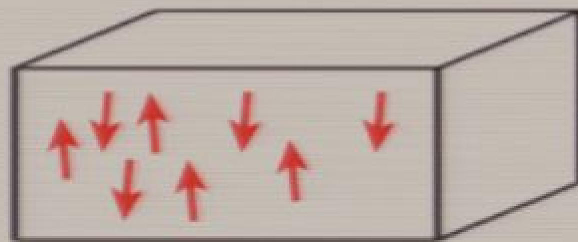

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Scale invariance $x \to \lambda x$

Conformal Invariance

- emergent at the critical point

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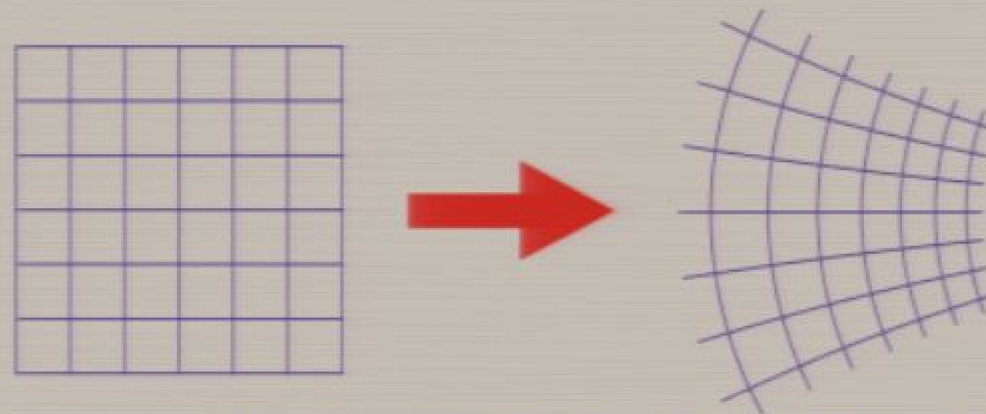
Conformal transformation

$$\delta_\kappa x_a = 2(\kappa \cdot x)x_a - x^2\kappa_a$$



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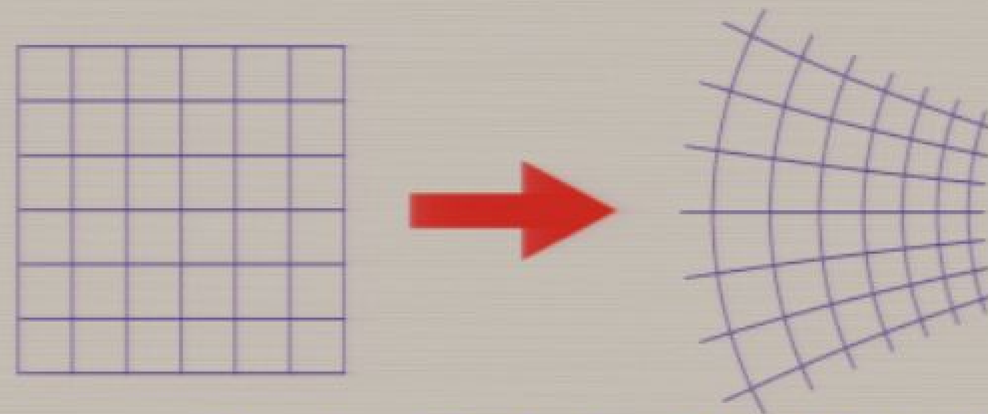

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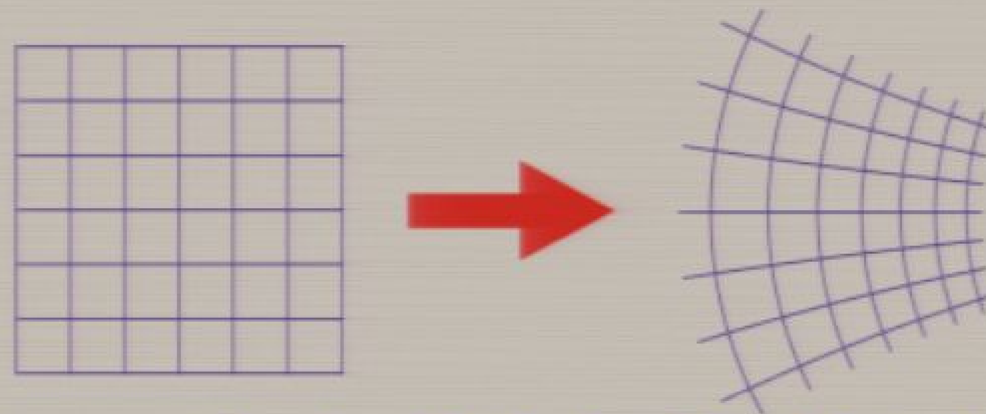

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Why this extra symmetry?

- not yet fully understood

Generically but *not always* true

Power of conformal symmetry

e.g. *constrains 3-point correlation functions*

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[Polyakov 1970]

A success story - 2D CFT

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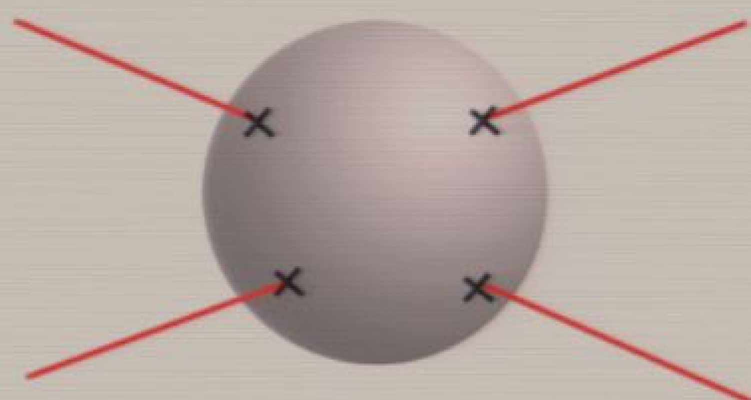
- Lots of exactly solvable models (2D Ising,...)
- Applications to worldsheet perturbative string theory



Reasons to think about CFTs in D$\geq$3

- Quantum criticality
- Quantum Gravity in AdS
- Hierarchies in particle physics

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anisotropic 2D
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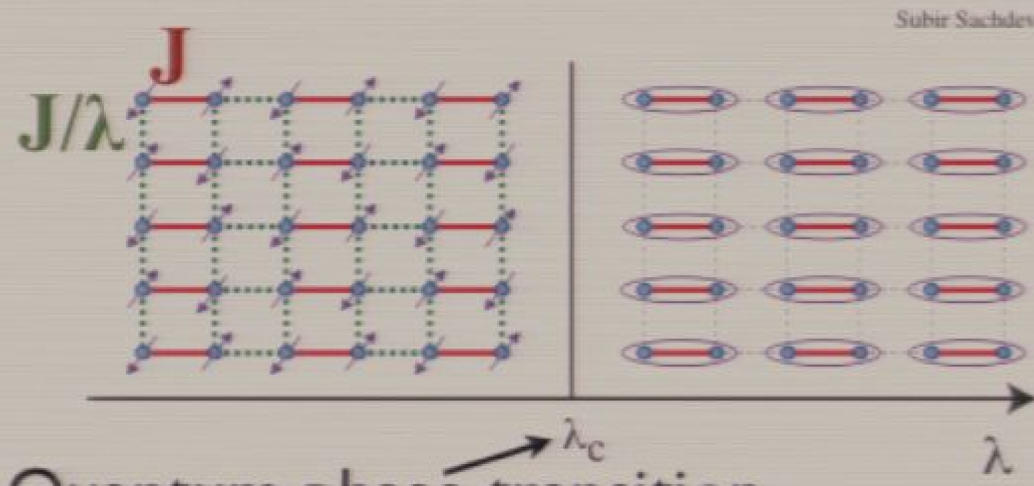

Quantum phase transition
(Néel-dimer; 3D Ising universality class)

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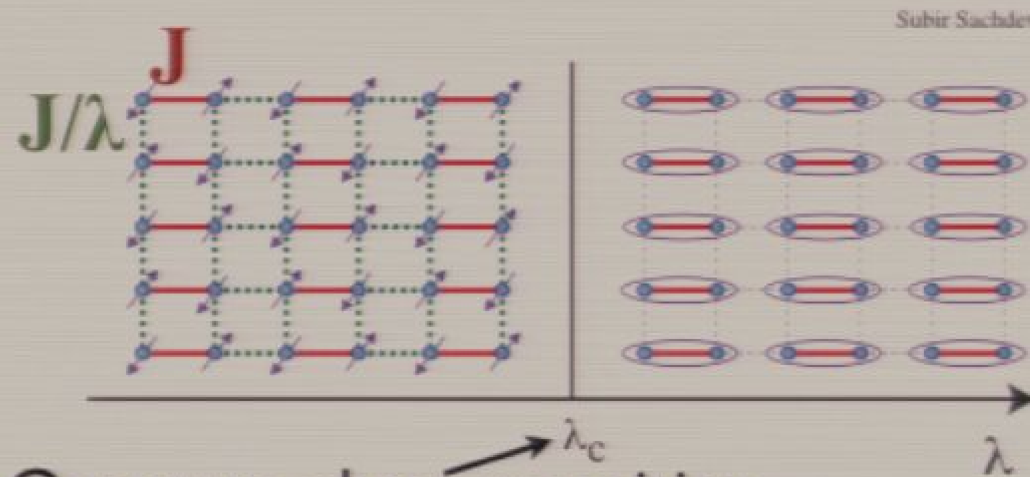

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In general:

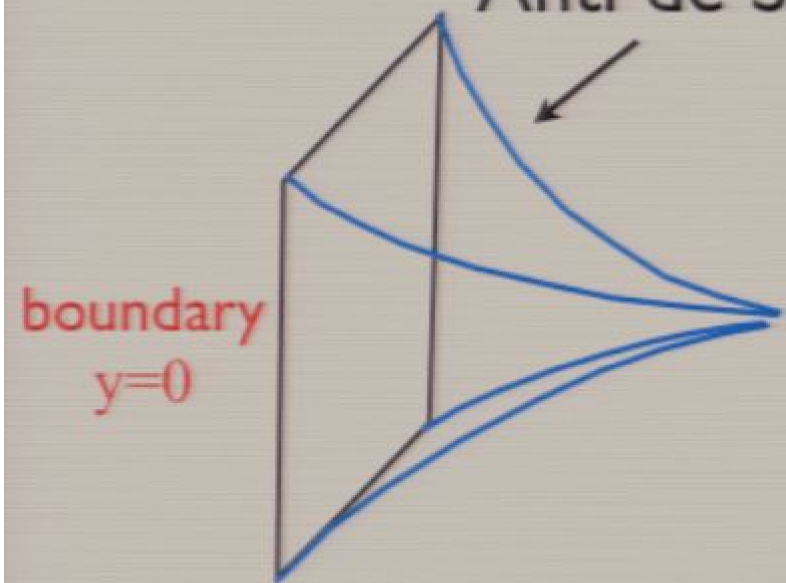
D-dim quantum system at T=0 ⇔ (D+1)-dim QFT

- Request for more D ≥ 3 CFTs
- Also with fermionic excitations

AdS/CFT

[Maldacena; Gubser,Klebanov, Polyakov; Witten]

Anti de Sitter $ds^2 = \frac{R^2}{y^2}(dx_\mu^2 + dy^2)$



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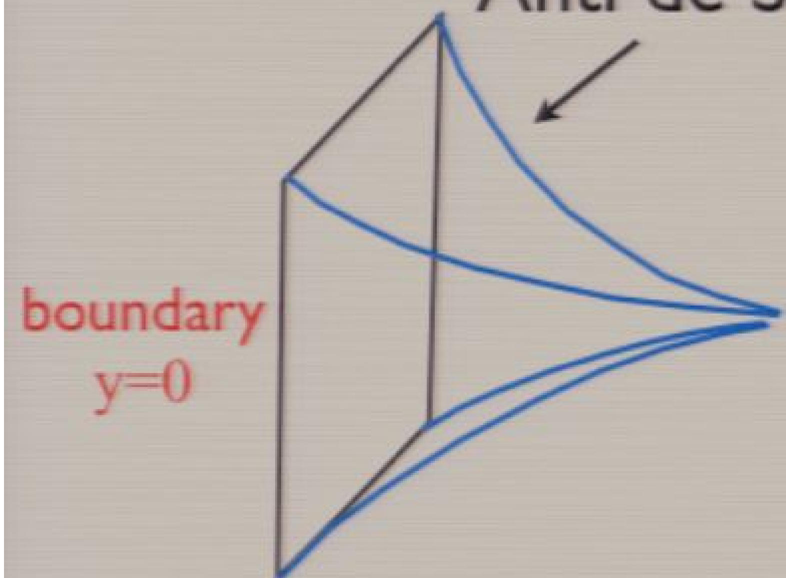
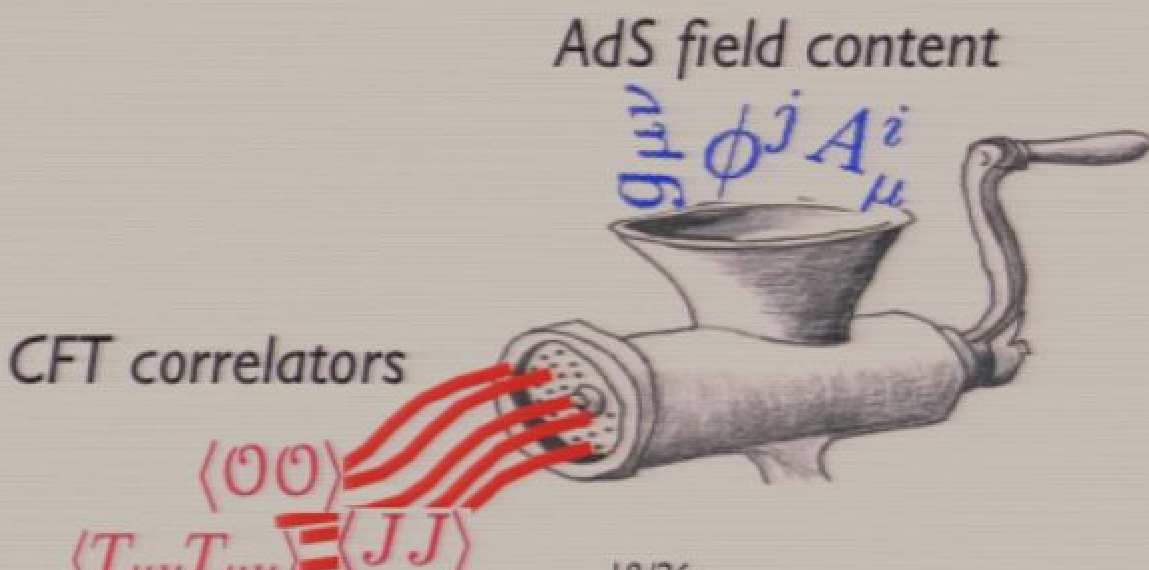


Gravity+other fields in AdS_{D+1}
$\Leftrightarrow$ CFT on D-dim boundary



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- Easy to go to T>0: put Black Hole in AdS

boundary
y=0

Black hole horizon

$$T = T_{\text{Hawking}}$$

$\Rightarrow$ can study transport properties

Limitations of AdS/CFT

- Factorization of operator dimensions

$$\mathcal{O}_1(x) \times \mathcal{O}_2(0) \to \mathcal{O}_1\mathcal{O}_2 \qquad \Delta \approx \Delta_1 + \Delta_2$$

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Cf. 3D Ising model:

$$M \times M \to \varepsilon$$

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Cf. 3D Ising model:

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⇒ not every CFT has an AdS dual

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Gravity theory in AdS is not UV complete

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'Effective CFT' describing operators up to $\Delta = \Delta_c \approx R M_{Pl} >> 1$

[Fitzpatrick, Katz, Poland, Simmons-Duffin '10]

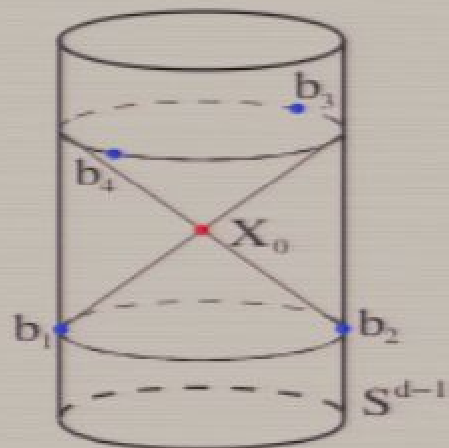
UV-completing theory of gravity in AdS ↔ Completing Effective CFT on the boundary

Quantum Gravity problem in AdS is mapped into a better-defined problem about boundary CFTs

UV-completing theory of gravity in AdS ↔ Completing Effective CFT on the boundary

Example 1

Graviton 2→2 S-matrix can be extracted from CFT $T_{\mu\nu}$ 4-point function *(if you know it)*



[Gary,Giddings,Penedones '09]

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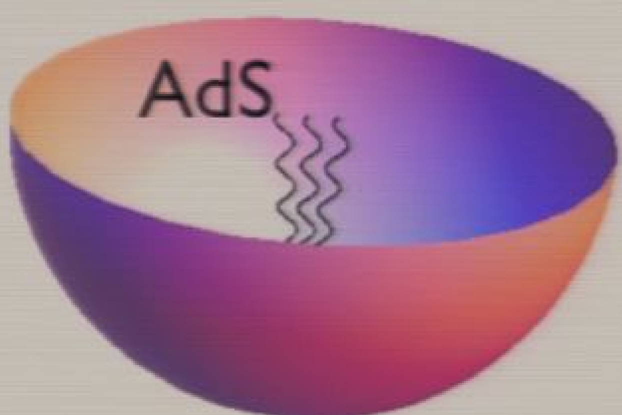

CFT boundary

any theory of quantum gravity must contain gravitons (dual to $T_{\mu\nu}$ in CFT)

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Example 2

Can get constraints on Quantum Gravity spectrum from CFT consistency [Hellerman '09]

- Can show that CFT must have more operators (not just $T_{\mu\nu}$)
- These are interpreted as dual to quantum black holes (mass $\sim M_{Pl}$)

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But at E >> 100 GeV (e.g. at LHC)
Standard Model ≈ CFT
(free theory perturbed by slowly running weak couplings)

Two types of CFTs

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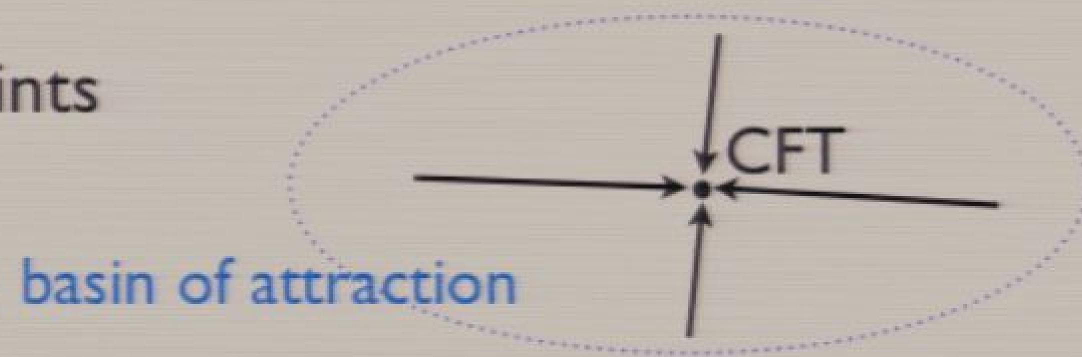
1) **Stable** IR fixed points of RG flows



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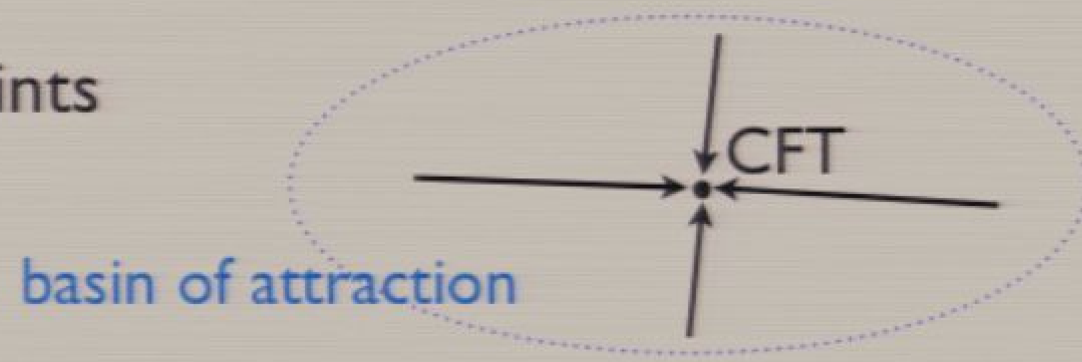

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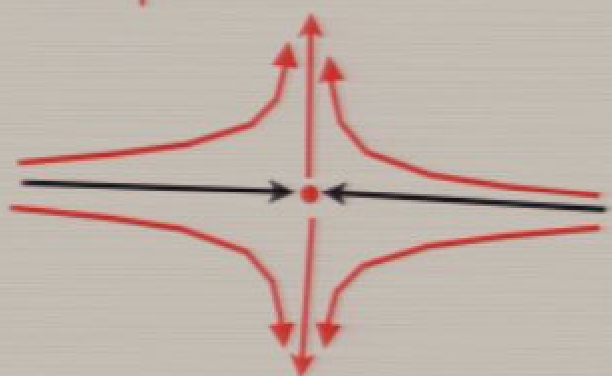
repulsive direction ⇔ scalar operator of dimension <D



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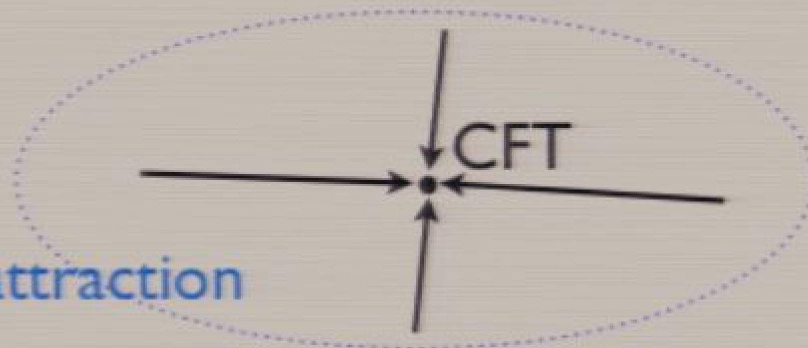

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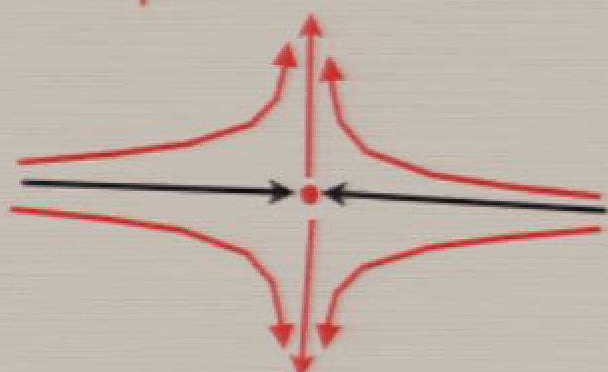

Life in such a fixed point needs an 'experimentalist' adjusting the control knobs

E.g. for 3D Ising $\Delta_\varepsilon=1.4<3$ ⇒ need temperature adjustment

In Standard Model, there is one such **repulsive** scalar operator

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Usually one considers extensions with a new symmetry (Supersymmetry or Goldstone symmetry)

Conformal Technicolor

[Holdom '81; Luty,Okui '04]

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At energies E>>TeV the Higgs sector
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How close?

Depends on assumptions about theory of flavor.

Δ_H<1.3 is OK (conservative)

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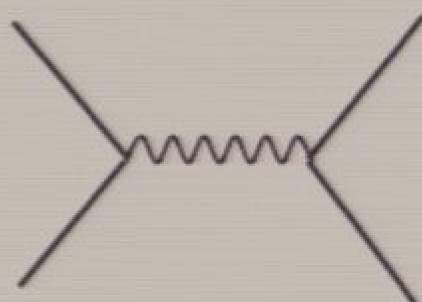
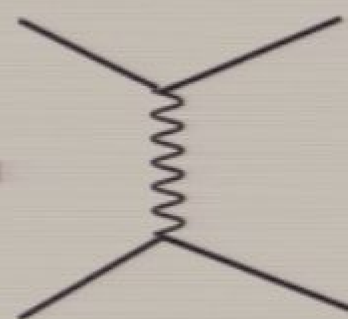
Yes: Crossing symmetry constraint

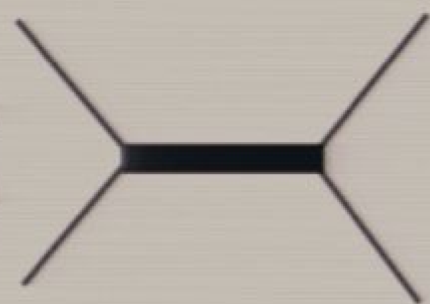
Scattering amplitudes in weakly coupled theory

(Feynman diagrams)

$$\mathcal{M}(e^+e^- \to e^+e^-) =$$  $+$ 

Correlation functions in CFT

(conformal block expansion)

$$\langle HH^\dagger HH^\dagger \rangle = \sum_{\mathcal{O}}$$ 

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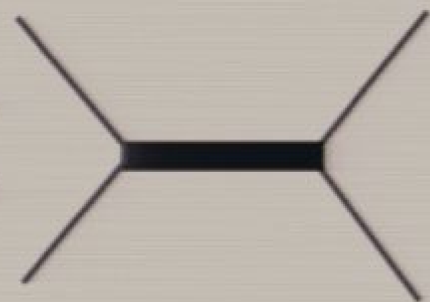

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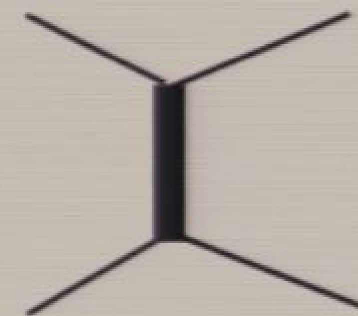

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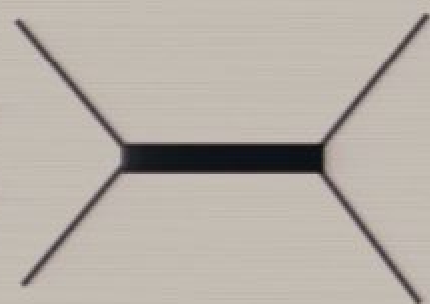

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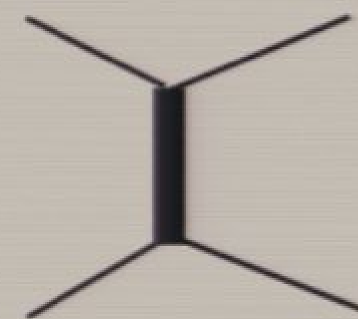

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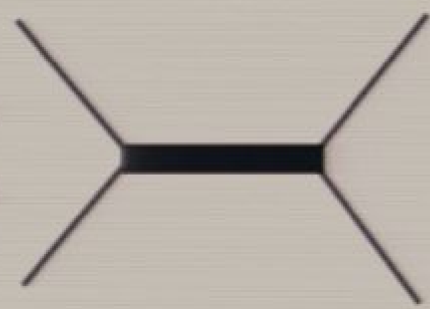
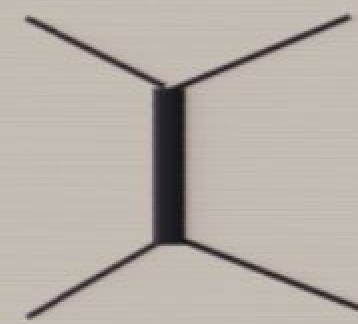
$$\langle H H^\dagger H H^\dagger \rangle = \sum_{\mathcal{O}}$$  $$= \sum_{\mathcal{O}}$$ 

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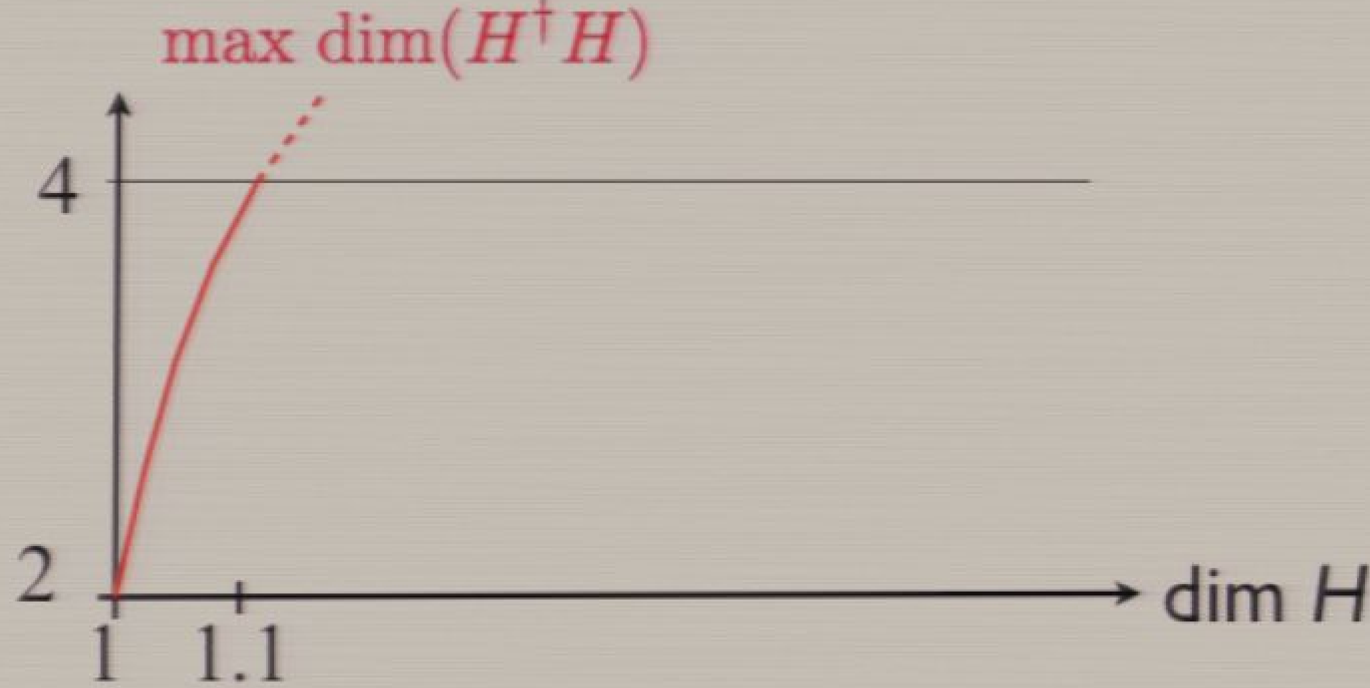
Bootstrap hypothesis : this equation should be enough to fix the CFT [Polyakov '74]

Spectrum constraints

[Rattazzi, S.R., Tonni, Vichi 2008, …]



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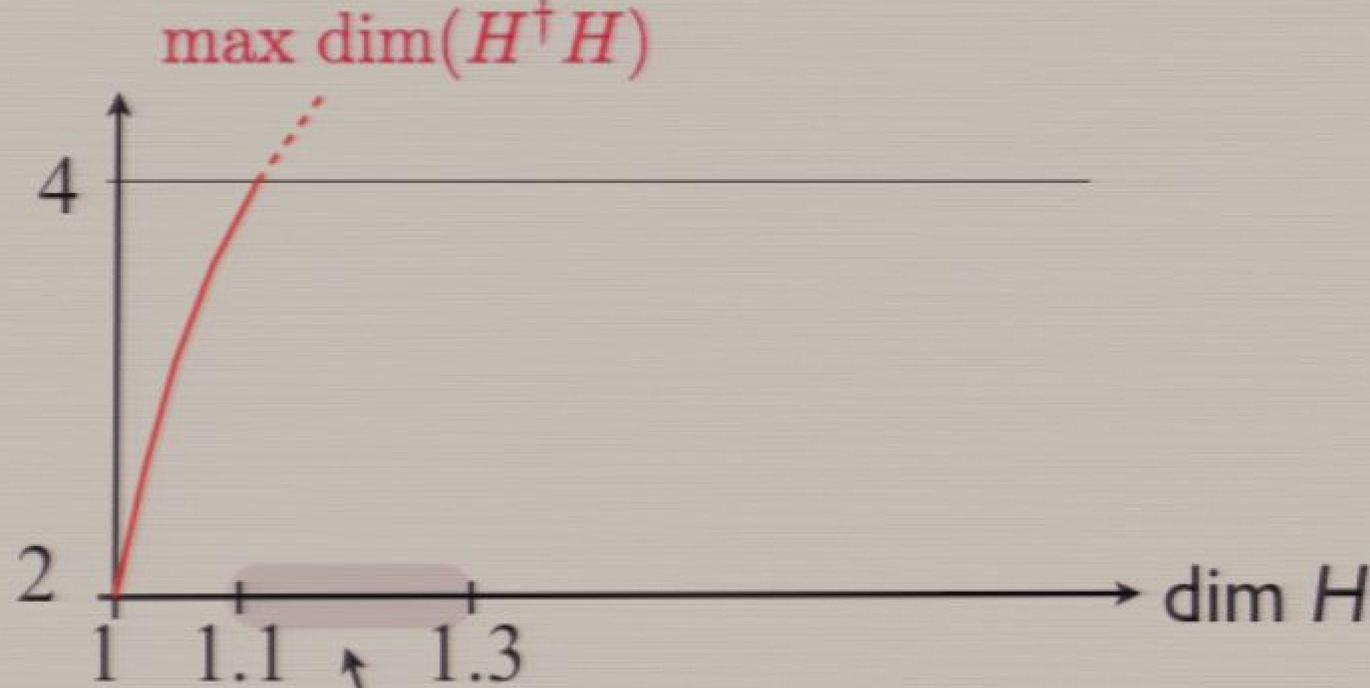

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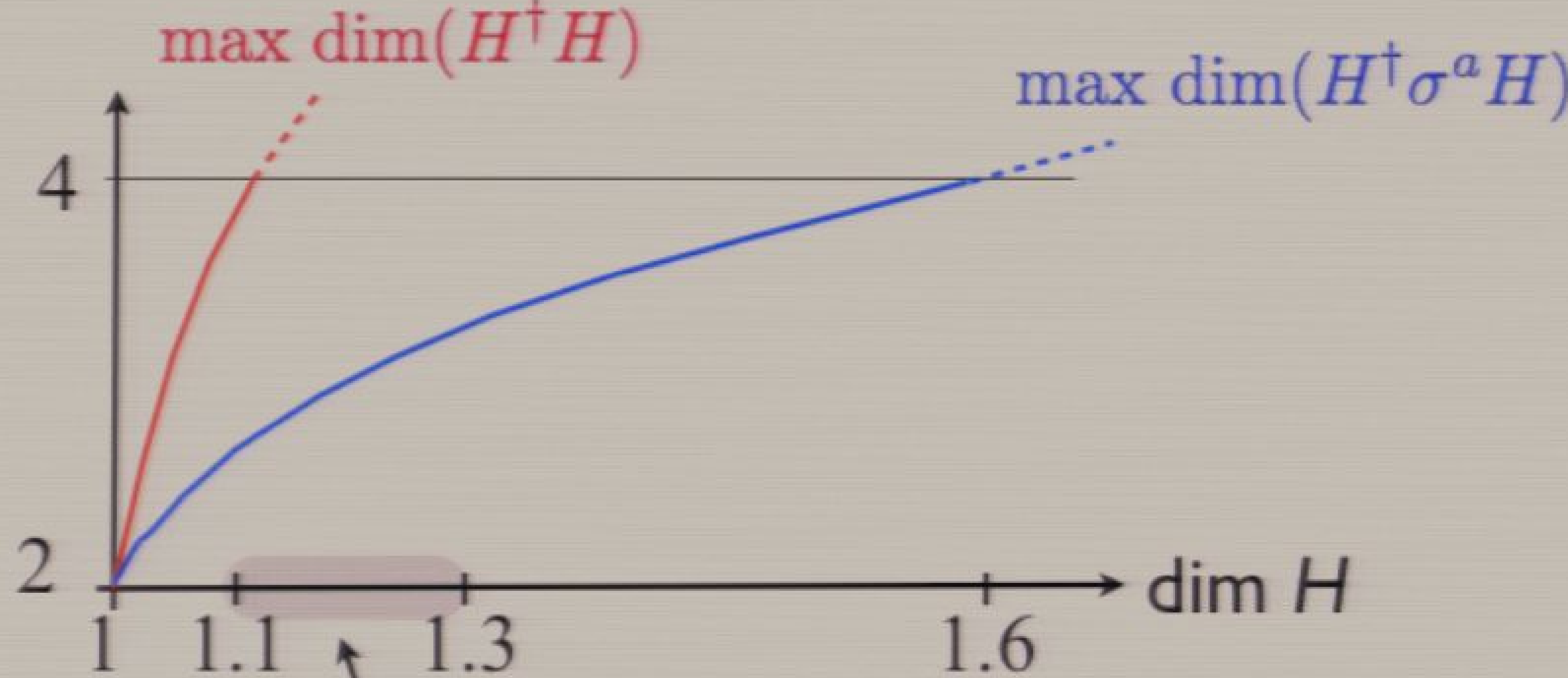

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Physics demands that we continue studying CFTs, especially in $D \geq 3$

AdS/CFT...

Recently, many general results about CFTs
just from prime principles
without any AdS unput.

Forward to the bootstrap!