Title: Mass gap, topological molecules and large-N volume independence

Date: Apr 26, 2011 11:00 AM

URL: http://pirsa.org/11040108

Abstract: Mass, a concept familiar to all of us, is also one of the deepest mysteries in nature. Almost all of the mass in the visible universe, you, me and any other stuff that we see around us, emerges from QCD, a theory with a negligible microscopic mass content. How does QCD and the family of gauge theories it belongs to generate a mass? This class of non-perturbative problems remained largely elusive despite much effort over the years. Recently, new ideas based on compactification have been shown useful to address some of these. Two such inter-related ideas are circle compactifications, which avoid phase transitions and large-N volume independence. Through the first one, we realized the existence of a large-class of "topological molecules", e.g. magnetic bions, which generate mass gap in a class of compactified gauge theories. The inception of the second, the idea of large-N volume independence is old. The new progress is the realization of its first working examples. This property allows us to map a four dimensional gauge theory (including pure Yang-Mills) to a quantum mechanics at large-N.

Mass gap, topological molecules, and large-N volume independence

Mithat Ünsal, Physics Department, Stanford University

In part, based on work done in collaboration with
Larry Yaffe, Erich Poppitz

This talk is about nonperturbative gauge dynamics. Things that one would like to understand in any gauge theory:

- Does a mass gap exist? how? why?
- does it confine? how? why?
- does it break its (super) symmetries?
- is it conformal? Why?
- what are the spectrum, interactions...?

tough to address, in almost all theories but relevant.

g^2 and $\exp[-1/g^2]$

Consider a gauge theory with coupling g. In almost all gauge theories in which we understand some of these phenomena, they often appear as $\exp[-1/g^2]$ effects.

Recall classification of singularities in complex analysis: removable(not a singularity), pole(easy), essential (bad).

$\exp[-1/g^2]$ has an essential singularity at $g=0$, hence cannot appear at any order in perturbation theory. (Expanding around zero, one gets 0+0+0.....)

Loosely, there is some rationale to think of gauge theory as if it has two expansion parameters: $\exp[-1/g^2]$ and g^2.

Is there a way to systematically study $\exp[-1/g^2]$ effects?

“New” Tools

1) Circle compactification--pbc for fermions

2) Large-N volume independence

3) Center-stabilizing double-trace deformations

Tools that usefully apply to any gauge theory.

Yang-Mills, QCD-like, chiral theories
N=1 Supersymmetric Yang-Mills, SQCD, SUSY chiral theories
N=2 SYM
N=4 SYM

Outline

- Large-N volume independence (conceptual new progress.
- Zero temperature compactification and theories without phase transitions.
- Example I: Existence of Mass gap in a vector-like example on $R_3 \times S_1$: QCD(adj)
- Example II: Absence of mass gap in a chiral-susy gauge theory which appeared in the context of SUSY breaking. (If time permits.)

--The two examples are as distinct as possible to demonstrate the wide range of applicability of the new methods.

Disclaimer

- What I will describe today is a broad-brush overview and conceptual description of the progress in gauge theories that began around 2007.
- One can also give a technical talk about one particular aspect of it, and I would be happy to do so. But it would be less useful. My apologies to the folks who like more detailed and technical talks. I will be happy to discuss in person.

Toroidal or circle compactification

The general theme is about inferring properties of infinite-volume theory by studying (arbitrarily) small-volume dynamics. The small volume may be

$$\mathbb{R}^4 \longrightarrow \mathbb{R}^3 \times S^1, \quad \mathbb{R}^2 \times T^2, \quad \mathbb{R} \times T^3, \quad T^4$$

of characteristic size "L", while keeping the theory locally 4d.

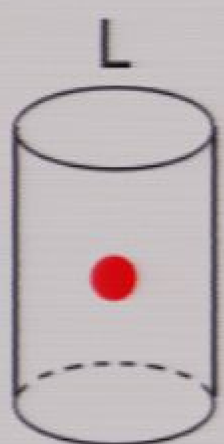
Can we use compactification as an expansion parameter to study non-perturbative dynamics?
Can we use small volume theory to describe infinite volume theory? (Sounds weird, if not crazy.)

What is volume (in)dependence?

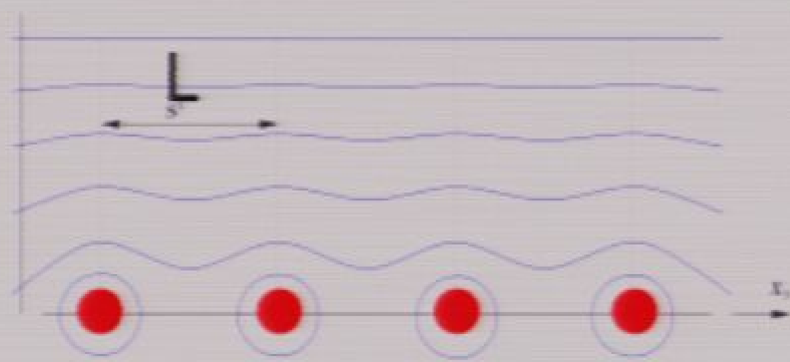
with simple electrostatic analogy

- Consider a point charge in R3. Its potential is 1/r.
- Now, compactify one of the dimensions to a circle with size L. Space is R2 x S1.

R2 x S1



by method of images



$r >> L \rightarrow \ln r.$

$r < L \rightarrow 1/r$

- The characteristic length at which the potential (interaction between charges) changes from 3d behavior to 2d behavior is L. Intuitive!

- By compactify more dimensions down to a space with size L, and using method of images, we obtain

The potential of a point charge in d-dimension: Gauss' law

$$3d: \quad \frac{1}{r} = \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

$$2d: \quad \log r = \log\sqrt{x_1^2 + x_2^2}$$

$$1d: \quad |r| = |x_1|$$

Whereas volume independence demands

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$$1d: \quad \frac{1}{r} = \frac{1}{\sqrt{x_1^2}}$$

Sounds outrageous.

Certainly wrong in electrodynamics, (or U(1) gauge theory), where our intuition is based on.

Large N volume independence or

"Eguchi-Kawai reduction" or "large-N reduction"

Theorem: $SU(N)$ gauge theory on toroidal compactifications of $\mathbb{R}^4$ to four-manifold $\mathbb{R}^{4-d} \times (S^1)^d$

No volume dependence in leading large N behavior of topologically trivial single-trace observables (or their connected correlators)

provided

there are no phase transitions as the volume of the space is shrunk. More technically, no spontaneous breaking of center symmetry or translation invariance

Proof: Comparison of large N loop equaions (Eguchi-Kawai 82) or N=∞ classical dynamics (Yaffe 82)

The only problem was that no-one was able to find any example of gauge theory in which **"provided"** holds.

Why should anybody care?

If true: Consider the Schrödinger equation for QFT in infinite space and in the theory where the space is reduced to a single point, i.e. ordinary quantum mechanics.

$$H^{\mathrm{YM}}_{\mathbb{R}^3}|\Psi_n\rangle = E_{\mathbb{R}^3}(n)|\Psi_n\rangle$$

$$H^{\mathrm{YM}}_{\bullet}|\Psi_n\rangle = E_{\bullet}(n)|\Psi_n\rangle$$

$$E(1) - E(0) = \Delta = \text{mass gap}$$

PROMISE: Spectrum of large-N gauge theory = Spectrum of quantum mechanics of large matrices.

- We would like to think (or we pretend) that we understand quantum mechanics better than quantum field theory.
- It may give us a new way to think about quantum field theories.

Basic intuition behind volume independence

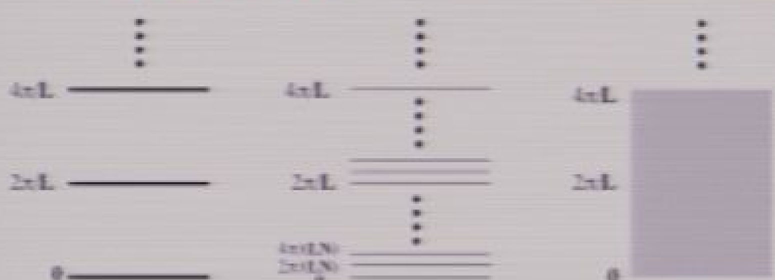
Consider the momentum modes in perturbation theory — Unsal, Yaffe 2010

Infinite space: Continuum
on circle with **size L**: Discrete $P = \frac{2\pi}{L}k, \quad k \in \mathbb{Z}$
on circle with **size L** if **"provided"** holds: Discrete, but much-finer $P = \frac{2\pi}{LN}k, \quad k \in \mathbb{Z}$

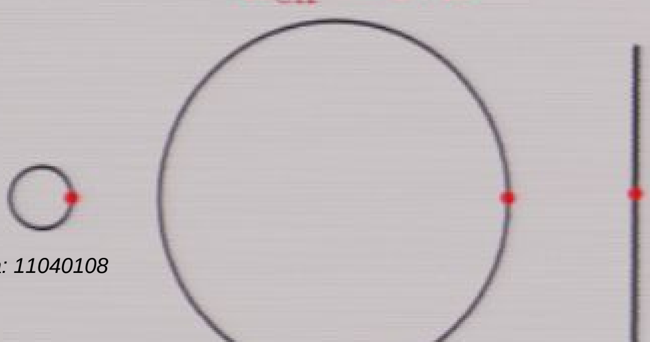
Decompactification: • discrete spectrum ⇒ continuum

• $L \to \infty$, N fixed (intuitive.)

if **"provided"** holds: • $N \to \infty$, L fixed (**surprising!**)



$L_{\text{eff}} = NL$

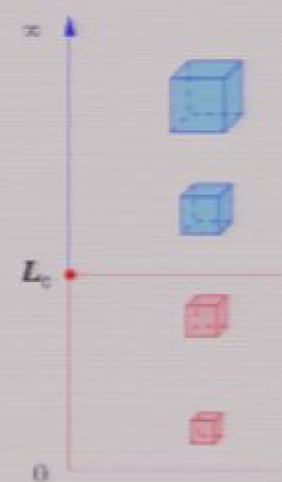


- The characteristic length at which the potential (interaction between charges) changes from 3d behavior to 2d behavior is LN. And LN→ ∞ as N → ∞. Interactions never become 2-dimensional. Counter-intuitive, but correct.

$$r < NL \Longrightarrow V(r) \sim 1/r$$
$$r \gg NL \Longrightarrow V(r) \sim \log r$$

Stumbling block

- Because of the attractiveness of the idea, much effort has been devoted. It was one of the hot subjects in mid-80's.
- However, there was always a phase transition when the space shrunk to small volume.
- Technically, an effective potential calculation in terms of Wilson lines (used to determine the phase of the small volume theory) gave a **negative** sign for **all** gauge theories. And we needed a positive sign! People gave up.



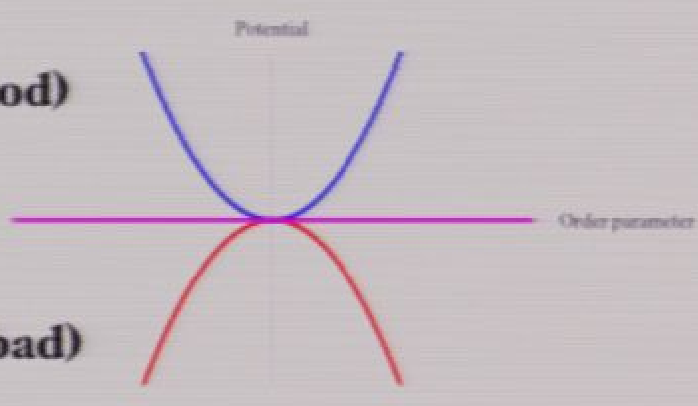
Instability

80's: EK, QEK, TEK.
Eguchi, Kawai, EK, Fails — 3rd ref in the list.
Gonzalez-Arroyo, Okawa, TEK, Fails — Teper, Vairinhos
Bhanot, Heller, Neuberger, QEK, Fails — Bringoltz, Sharpe
Gross, Kitazawa,
Yaffe,
Migdal, Kazakov,
Parisi et.al.
Das, Wadia, Kogut,
+ 500 papers.... , but no single working example!

Stability (good)

Marginal

Instability (bad)



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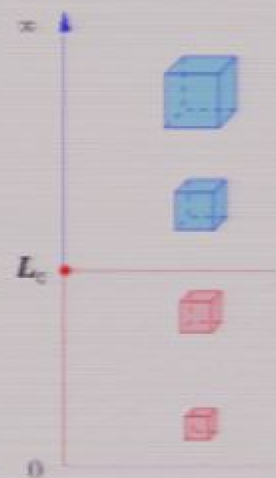

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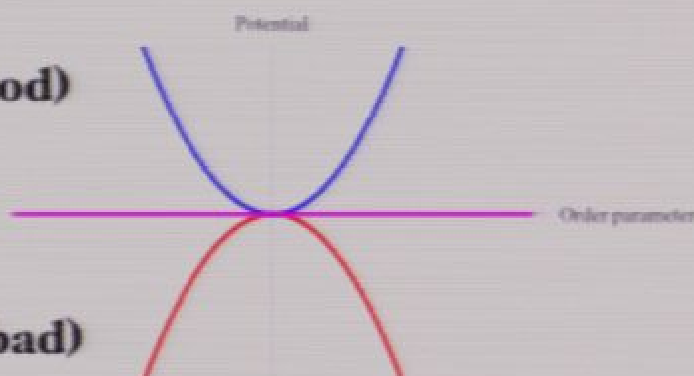


More technically, # Yang − Mills on $\mathbb{R}^3 \times S^1$

circumference L

- Z_N center symmetry, order parameter = Wilson line Ω

$g(x+L) = hg(x), \qquad h^N = 1$ Aperiodic gauge rotations, h $\in Z_N$ 't Hooft

$\mathrm{tr}\Omega(x, x+L) \to h\ \mathrm{tr}\Omega(x, x+L)$

- $L > L_c$: unbroken center symmetry

$$\langle \mathrm{tr}\ \Omega^n \rangle = 0$$

confined phase

- $L < L_c$: broken center symmetry

$$\langle \mathrm{tr}\ \Omega^n \rangle \neq 0$$

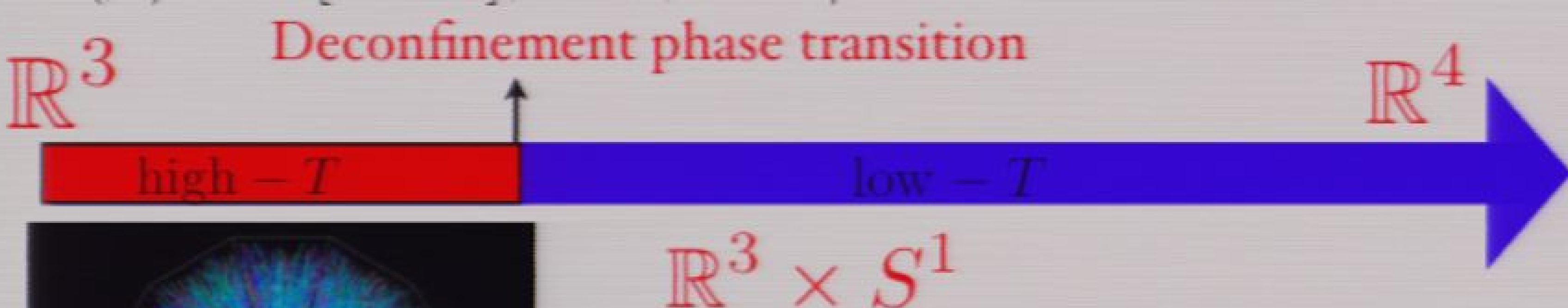
deconfined plasma phase failure of EK reduction

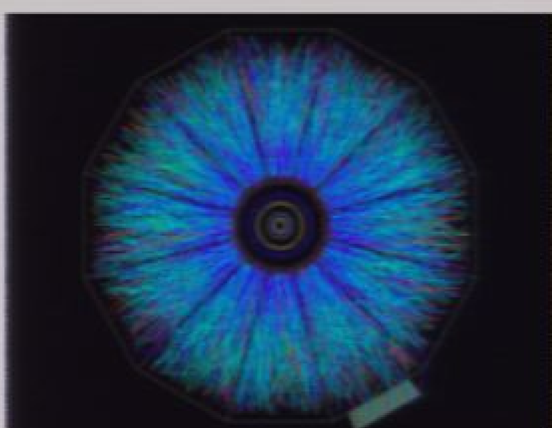
Thermal compactification

"provided" part from old point of view is actually impossible.

Traditionally, in QCD or QCD-like theories, compactification to $\mathbb{R}^3 \times S^1$ is only used to study the theory at finite temperature.

$$Z(\beta) = \mathrm{tr}[e^{-\beta H}], \qquad \beta = 1/T = \text{radius of } S^1$$





Quark-Gluon Plasma phase (sQGP)

True for all confining gauge theories. This is the impasse for the volume independence as well as the stumbling block to use the radius as a expansion parameter. Singularity on the way.

Evading the stumbling block

In 2006, I realized that the analog of the effective potential calculation in a supersymmetric gauge theory gave zero. At the heart of the cancelation was following identity:

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$$-1 + 1 = 0$$

More precisely,

$$-1 \times (\text{stuff}) + 1 \times (\text{same stuff}) = 0$$

The crucial point: +1 appears due to the boundary conditions, and not supersymmetry!!

Immediately, we deduce:

$$-1 + N_f > 0 \qquad \text{for } N_f > 1$$

Our simple calculation was the first positive sign in such a calculation.
All earlier calculations were done for a specific (thermal) boundary condition.

This sign flip may give one of the most promising windows to non-perturbative QCD.

QCD(adj) on $\mathbb{R}^3 \times S^1$

$N_f \geq 1$ massless adjoint rep. fermions
periodic boundary conditions ➡ stabilized center symmetry

$$\widetilde{Z}(L) = \mathrm{tr}[e^{-LH}(-1)^F]$$

$$Z = Z_B + Z_F$$
$$\widetilde{Z} = Z_B - Z_F$$

Susy-theory: **Supersymmetric Witten Index, useful.**
Non-susy theory: **Twisted partition function, probably more useful!**

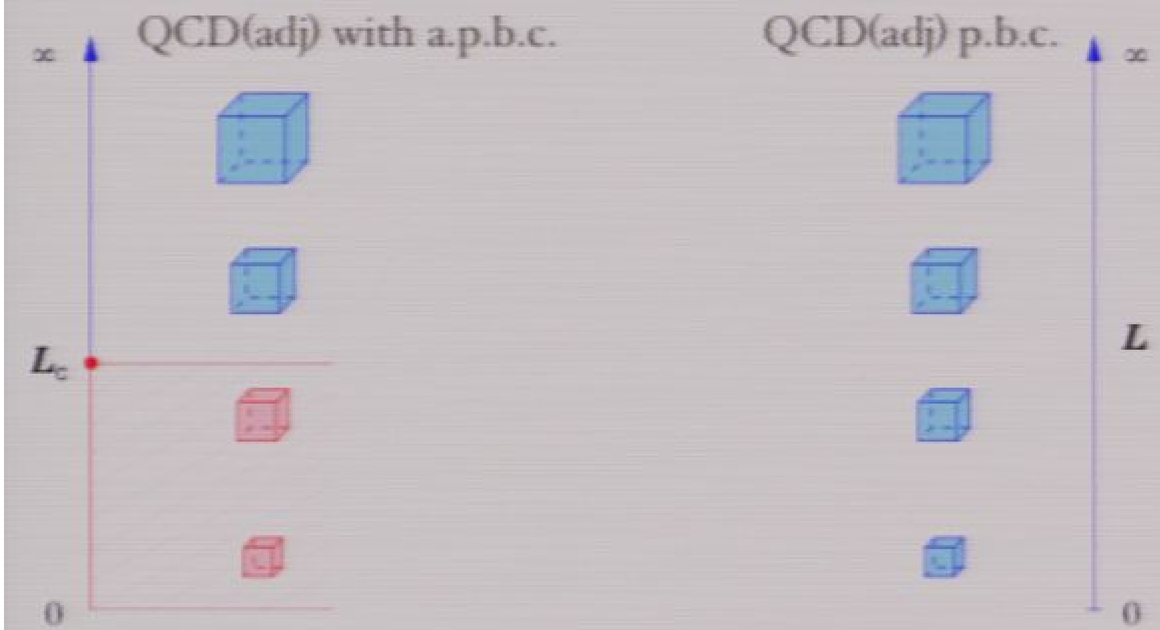
$$V_{1-\mathrm{loop}}[\Omega] = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^4}(-1+N_f)}_{m_n^2} |\mathrm{tr}\,\Omega^n|^2$$

$m_n^2 < 0$ instability, "calculations between 1980-2007"

$m_n^2 = 0$ Supersymmetric case, $N_f = 1$, marginal,

$m_n^2 > 0$ QCD(adj), $N_f > 1$, stability Kovtun, Unsal, Yaffe,07

Large-N Volume Independence



Two solutions:

SYM/QCD(adj): Kovtun, MU, Yaffe (2007)

Lattice tests: (Last two years)
Bringoltz, Sharpe,
Hanada, Azeyanagi, MU, Yacoby,
Narayanan, Hietanen,
Catterall, Galvez, MU,
Vairinhos, ...

Related works:
Ogilvie, Myers, Meisinger,
Bedaque, Cherman, Buchoff,
D'elia, Cossu,
Poppitz, MU,
Shifman,MU,
Veneziano, Wosiek.

First working examples, 25 years after the beautiful idea of Eguchi and Kawai

Can we now use quantum mechanics to solve 4d gauge theory?

Numerically, yes. **Analytic attempts proves to be hard. Work in progress.**
Needs next good idea!

Center-stabilized YM

- Unwanted symmetry breaking? Fix it!

- *Theorem:* double-trace deformation prevents symmetry breaking but has no effect on $N=\infty$ center symmetric dynamics! Unsal, Yaffe 2008

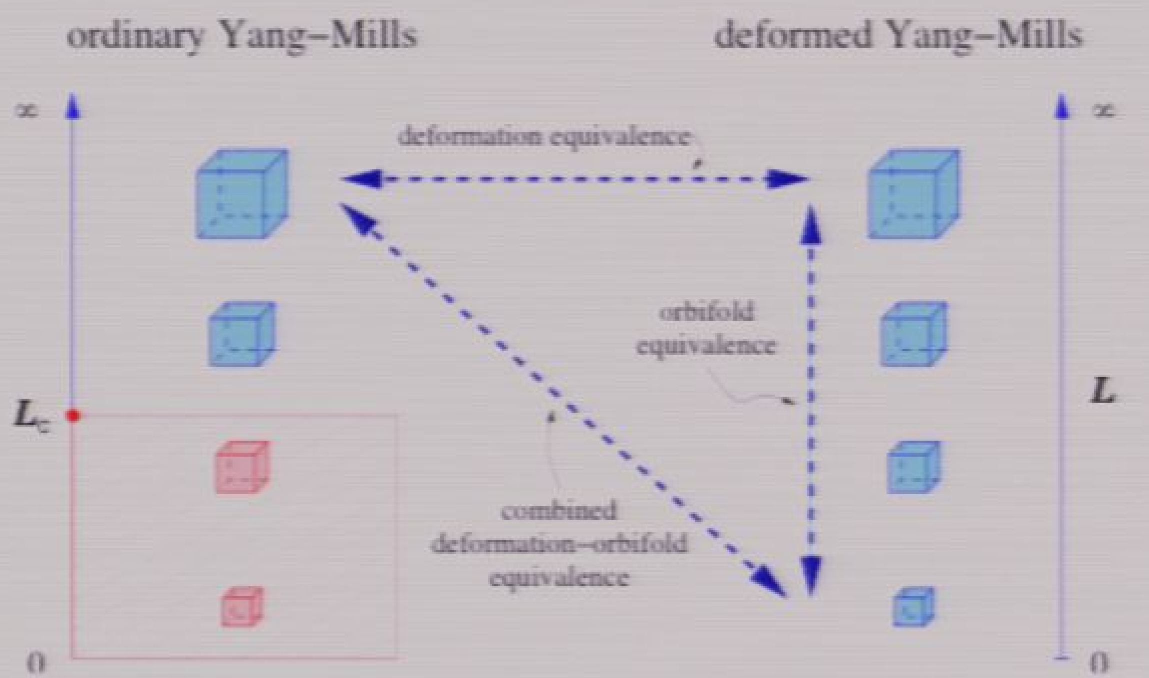
$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[\Omega(\mathbf{x})] \quad P[\Omega] = A \frac{2}{\pi^2 L^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left| \mathrm{tr}\,(\Omega^n) \right|^2$$

A sufficiently positive, $O(1)$ as N goes to ∞. Not a small perturbation, deformation is order N^2.

Double-trace operators are proposed in string theory, but to my view, its most important aspect is understood in field theory, and it is what I describe here.

Veneziano cleverly referred to this deformation as a good samaritan. It does the good deed and sequesters itself. Pictorially, here is what it does:

MU, Yaffe 2008



Volume independence is an example of large-N orbifold equivalence, in the modern language. The small and large volume theories are related by orbifold projections.

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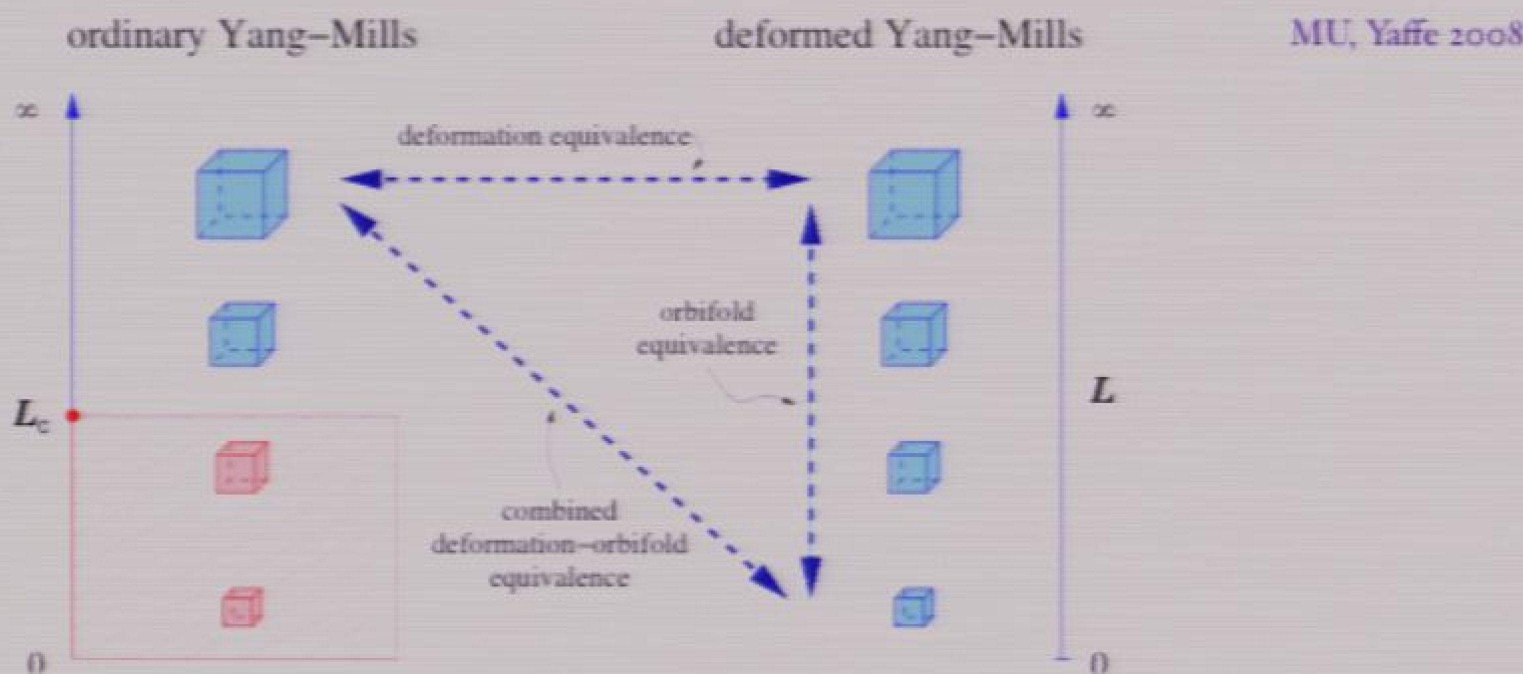

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Dimensional Reduction ?

- small L, asymptotic freedom, heavy, weakly coupled KK modes

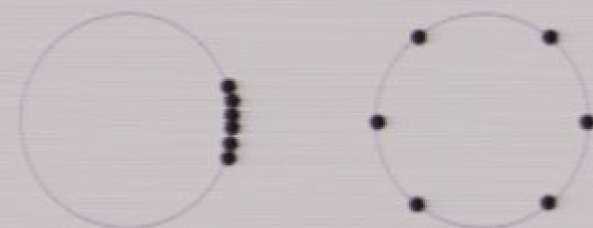
- usual case: broken center symmetry

  $\langle \mathrm{tr}\,\Omega \rangle \neq 0 \Leftrightarrow$ eigenvalues clump

  $mKK = 1/L,\ 2/L,\ \ldots,$

  perturbative control when $L\Lambda \ll 1$

  integrate out $\Rightarrow$ $3d$ effective theory, L-dependent



a) Attractive | b)repulsive

broken center | unbroken center

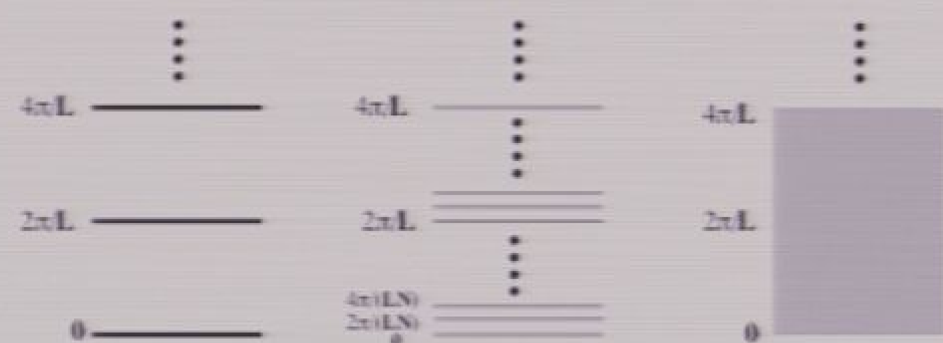
- center-symmetric case:

  $\langle \mathrm{tr}\,\Omega \rangle = 0 \Leftrightarrow$ eigenvalues repel

  $mKK = 1/NL,\ 2/NL,\ \ldots,$

  perturbative control when $NL\Lambda \ll 1$

topological defects (instantons), mass gap, confinement (Polyakov)



a)Center–broken finite or large N | b1)Center–symmetric finite N | b2)Center–symmetric large N

$$m_W = \frac{2\pi}{LN},\quad m_\gamma \sim m_W e^{-4\pi^2/(g^2(m_W)N)} \Longrightarrow \frac{m_\gamma}{m_W} \sim (LN\Lambda)^{11/6}$$

Mass gap and confinement

The theories with no phase transitions in the sense of confinement provide a new window to 4d dynamics.

$\mathbb{R}^3$ $\qquad$ $\mathbb{R}^3 \times S^1$ $\qquad$ $\mathbb{R}^4$

A smooth journey

A complementary regime to that of volume independence - a calculable shadow of the dynamics of the 4 dimensional "real thing".

fix-N, take L-small: Semiclassical studies of confinement
Many new surprising phenomena
New composite topological excitations--**Topological Molecules**

MU 2007; QCD(adj)
--for vectorlike or chiral theories + classification + thermal QCD
with Yaffe 2008, Shifman 2008 + Poppitz 2008...., + Argyres 2010... + Cherkis 2011

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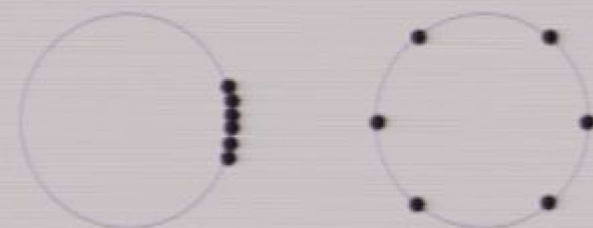

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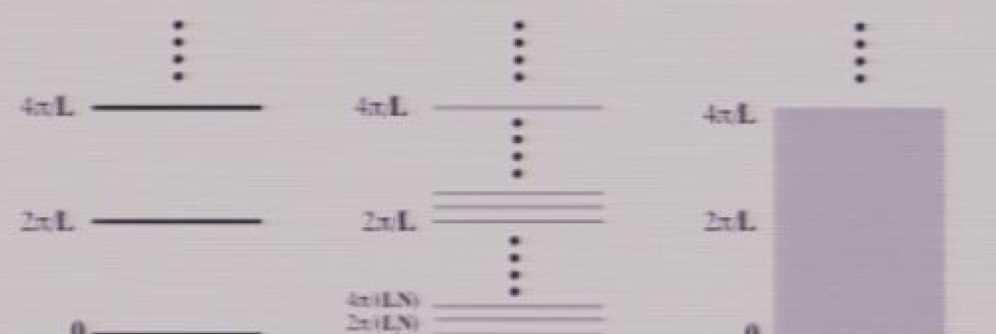

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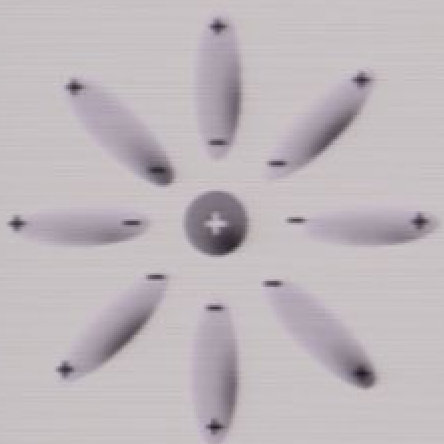
The essence of mass gap in Polyakov-mechanism in 3d

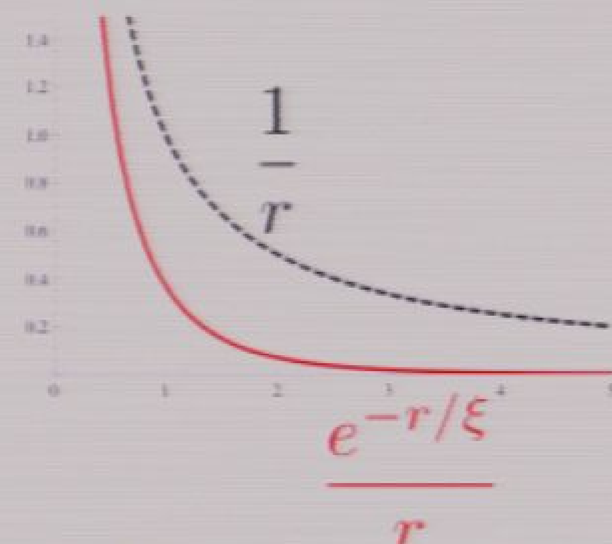
Polyakov 1977

't Hooft-Polyakov monopole solutions (instantons in 3d) in Georgi-Glashow model.
Partition function of gauge theory = The grand canonical ensemble of classical monopole plasma.
The field of external charge in a classical plasma decay exponentially. Debye-Hückel 1923.
Proliferation of monopole-instantons generates mass gap for gauge fluctuations.



Due to screening

$$\frac{1}{r} \longrightarrow \frac{e^{-r/\xi}}{r}$$



Finite magnetic screening length=mass for gauge fluctuations for U(1) photon=Confinement of electric charge (I will not show this part explicitly since I would like to emphasize mass gap. But the two are intimately related.)

Duality

In 2+1 d, photon has just one polarization, one degree of freedom. It is dual to a scalar.

$$B = \partial_0 \sigma$$
$$E_x = \partial_y \sigma$$
$$E_y = -\partial_x \sigma$$

$$L = \frac{1}{4} F_{\mu\nu}^2 \longleftrightarrow \frac{1}{2} (\partial_\mu \sigma)^2$$

Maxwell term

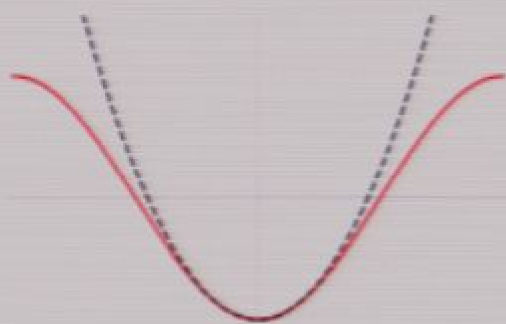
The proliferation of monopoles generate monopole operators in Lagrangian.

$$L = \frac{1}{2} (\partial \sigma)^2 - e^{-S_0} (e^{i\sigma} + e^{-i\sigma})$$

Monopole operators: Contribution of instanton amplitude to the effective lagrangian.

Expanding the cos potential to quadratic order,

$$L^{\text{small fluc.}} = \frac{1}{2} (\partial \sigma)^2 + e^{-S_0} \sigma^2$$



Inverse Debye length = mass gap

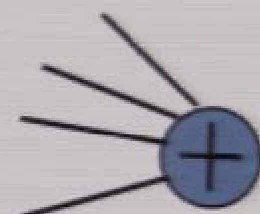
Massless fermions

Theories with massless fermions: take SU(2) QCD(adj)

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr}\left[\frac{1}{4}F_{MN}^2 + i\bar{\psi}^I \bar{\sigma}^M D_M \psi_I\right]$$

monopole operators have fermionic zero modes.

$$e^{-S_0} e^{i\sigma} \qquad \underbrace{\psi \ldots \psi}_{\text{fermion zero modes}}$$



Hence, unlike Polyakov mechanism, monopoles can no longer induce mass gap or confinement, instead a photon-fermion interaction. What is going on?

How many zero modes are there?

Is there a new mechanism of confinement?

Index theorems

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

APPENDIX A. ADIABATIC LIMITS OF η-INVARIANTS

$$\operatorname{ind}(D_{\mathbb{A}}^{+}) = \int_X \operatorname{ch}(\mathbb{E}) + \frac{1}{\mu_0}\sum_{\mu} \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \operatorname{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\lim} \quad (22)$$

Last formula in the paper. Following great tradition of translating mathematics to physics:

index theorems

Atiyah-M.I.Singer 1975

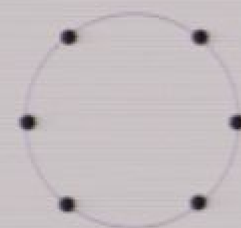
Callias 1978 → E. Weinberg 1980

Nye-A.M.Singer, 2000 → Poppitz, MU 2008: The one relevant for us!

Topological excitations in QCD(adj), SU(2), Nf=2

Magnetic Monopoles (3d instantons + a twisted 3d-instanton) with two quantum numbers: $\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$

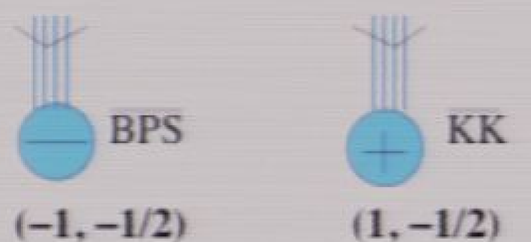
fermionic zero modes



b)repulsive

unbroken center

$$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$$



$$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$$

Duality + Index thm + Symmetry allow

$$e^{-2S_0}(e^{2i\sigma} + e^{-2i\sigma}) \quad (\mathbb{Z}_2)_*$$

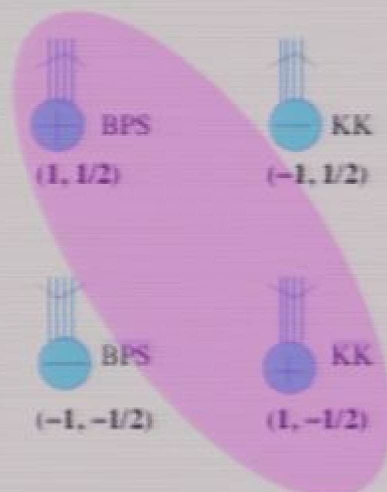
which generate a mass gap. What is the topological defect which leads to this?

Crucial earlier work: Realization of KK-monopole van Baal et al. and Lu, Yi, 97

Topological molecules

The quantum numbers associated with $e^{-2S_0}(e^{2i\sigma} + e^{-2i\sigma})$ are (2, 0) and (-2,0). Since (2,0) = (1, 1/2) + (1,-1/2), we may think of it as a molecule. We refer to it as magnetic bion.

How is a stable molecule possible? Same sign magnetic charge objects should repel each other due to Coulomb law.



Coulomb law: $1/r$ repulsion

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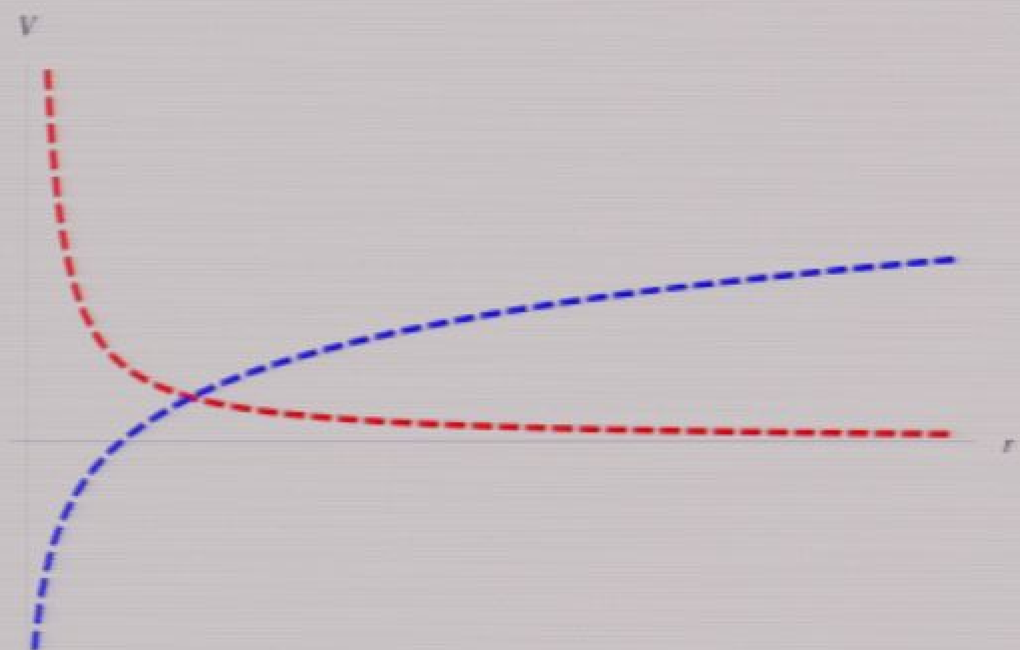

Fermion zero mode exchange: *log(r)* attraction.

Coulomb law: *1/r* repulsion

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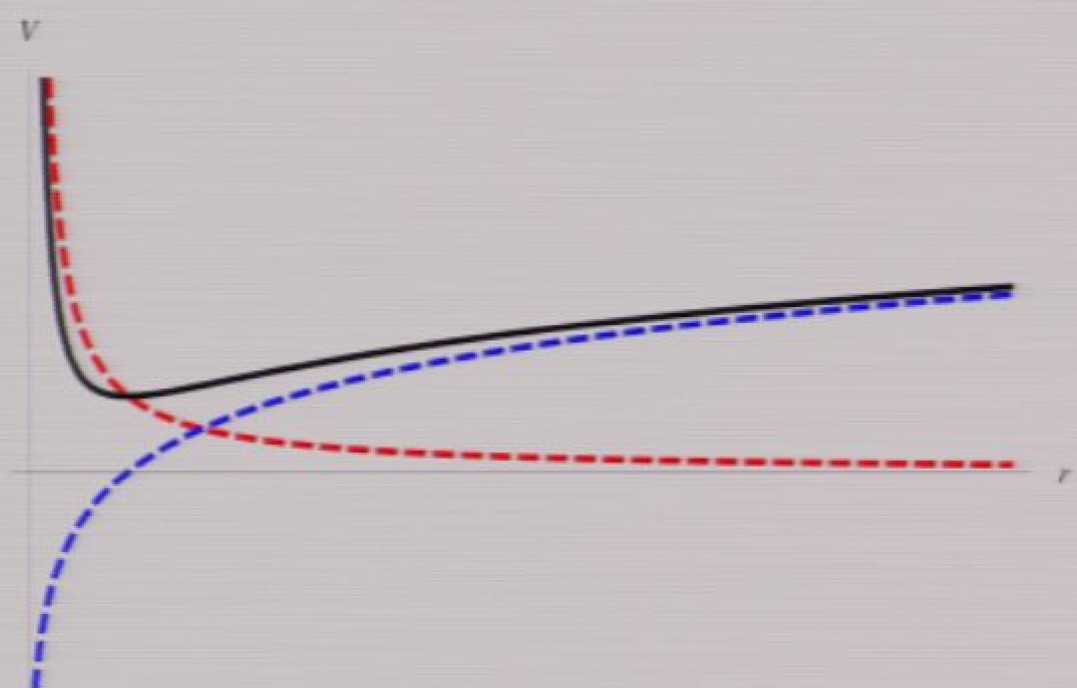


Fermion zero mode exchange: *log(r)* attraction.

Coulomb law: *1/r* repulsion

Sum has a unique minimum.

Stable molecules with sizes parametrically larger than monopoles!

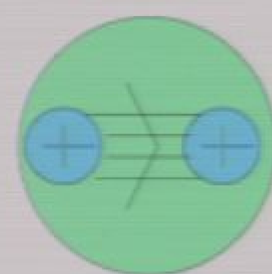
QCD(adj) vacuum is a plasma of magnetic bions

$$L^{\rm dQCD} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0}\cos 2\sigma + i\bar{\psi}^I\gamma_\mu\partial_\mu\psi_I + c\, e^{-S_0}\cos\sigma(\det_{I,J}\psi^I\psi^J + \text{c.c.})$$

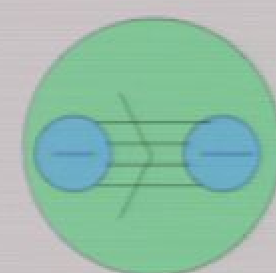
magnetic bions lead to mass gap!

magnetic monopoles

Alice with Tweedledum and Tweedledee, Through the Looking-Glass and what Alice found there (1871).



(2,0)



(−2, 0)

No net topological charge!!

Strongly-correlated pairs.

This is the reason why nobody attempted to look for these things.

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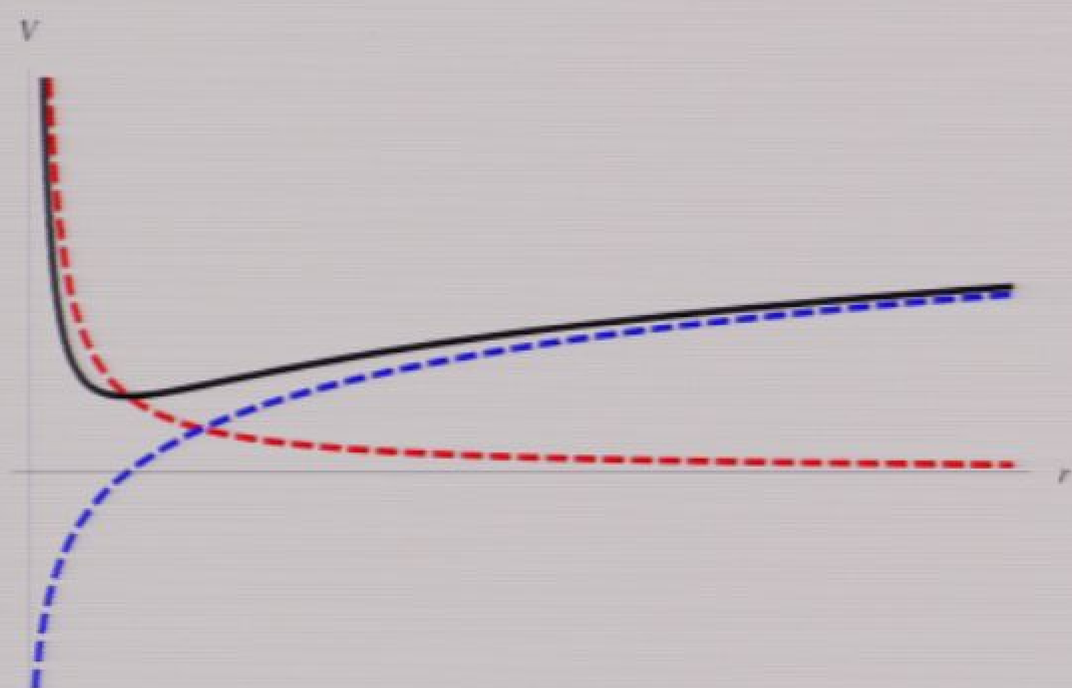

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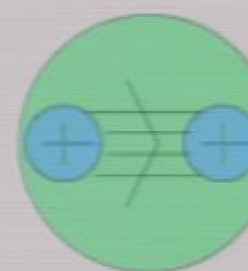
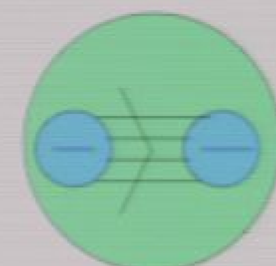


(2,0)



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Supersymmetric chiral SU(2)with I = 3/ 2 matter

Intriligator-Shenker-Seiberg: Proposal for a Simple Model of Dynamical SUSY Breaking, 94

Instanton operator: $I(x) = e^{-S_{\text{inst}}}\psi^{10}\lambda^4,\ U(1)_R$

$$[\lambda] = +1,$$

$$[Q] = \frac{3}{5}, \qquad [\psi] = -\frac{2}{5},$$

$u = Q^4$

$$[u] = \frac{12}{5}, \qquad [\psi_u] = [q^3\psi] = \frac{7}{5}\ .$$

W=0

If theory confines, with u - the single massless composite saturating 't Hooft (as is easily checked), adding W = u gives "simplest" susy breaking theory. (This is the reason why ISS suggests that it should be so.)

Does it? Hard to be sure. None of the usual SUSY deformations works!

Does circle deformation-the only available tool-say anything?

Index theorem and monopole operators

$$\mathcal{I}_1 = (4\psi, 2\lambda), \;\; \mathcal{I}_2 = (6\psi, 2\lambda), \;\; \mathcal{I}_{\text{inst}} = (10\psi, 4\lambda) \;.$$

$$\mathcal{M}_1 = e^{-S_0} e^{-\phi + i\sigma} \psi^4 \lambda^2, \;\; \overline{\mathcal{M}}_1 = e^{-S_0} e^{-\phi - i\sigma} \bar{\psi}^4 \bar{\lambda}^2,$$

$$\mathcal{M}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^6 \lambda^2, \;\; \overline{\mathcal{M}}_2 = e^{-S_0} e^{+\phi + i\sigma} \bar{\psi}^6 \bar{\lambda}^2 \;,$$

Compare with monopole operators in non-susy theory. One major difference, under U(1)_R:

$$\psi^4 \lambda^2 \to e^{i \frac{2\alpha}{5}} \psi^4 \lambda^2, \qquad \psi^6 \lambda^2 \to e^{-i \frac{2\alpha}{5}} \psi^6 \lambda^2 \;.$$

The invariance of monopole operator demands that the U(1)_R to intertwine with the topological continuous shift symmetry of the dual photon.)

$$\sigma \to \sigma - \frac{2}{5}\alpha, \qquad [Y] = -\frac{2}{5}$$

An explicit mass term (such as magnetic bion) for dual photon is forbidden.

More systematically, let us start in 3d, work our way "up" to 4d.

similar symmetry arguments in Aharony, Intriligator, Hanany, Seiberg, Strassler 97

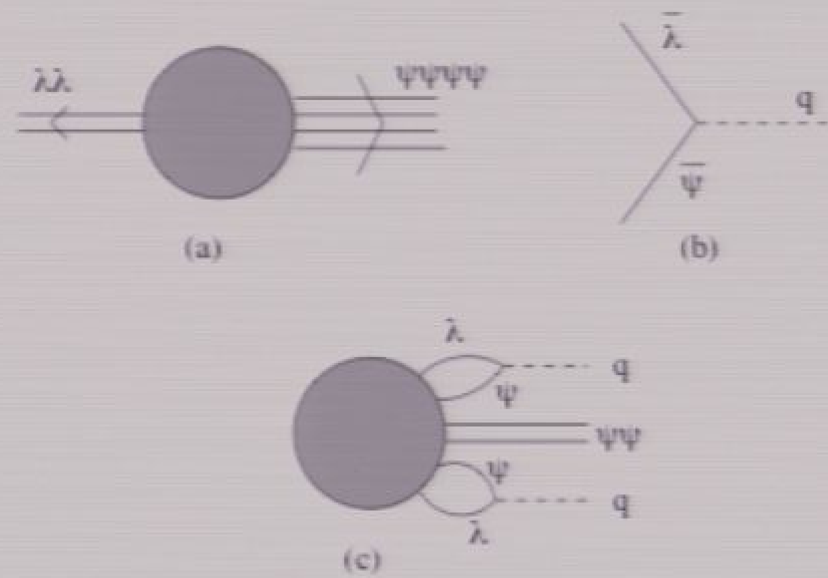
	$[U(1)_{R'}]_*$	$[U(1)_A]_*$
λ	1	0
ψ	-1	1
Q	0	1
Y	2	-4

$$Y \sim e^{-\phi+i\sigma}, \qquad u = Q^4$$

$W[Y,u] = b\,Y u$. Symmetry allows. Is it there?

Microscopic origin of superpotential and modified monopole operators

	$[U(1)_{R'}]_*$	$[U(1)_A]_*$
λ	1	0
ψ	-1	1
Q	0	1
Y	2	-4



$$Y \sim e^{-\phi+i\sigma}, \qquad u = Q^4$$

$$W[Y,u] = b\,Y u\ .$$

Yukawa lifting

$$e^{-S_0} e^{-\phi+i\sigma} \psi^4 \lambda^2(x) \left(\int d^3y\, q\bar{\lambda}\bar{\psi}(y) \right)^2 \longrightarrow \widetilde{M}_1 \equiv e^{-S_0} e^{-\phi+i\sigma} q^2 \psi^2\ .$$

$$W[Y,Q] \sim YQ^4, \qquad \widetilde{M}_1 = \frac{\partial^2 W}{\partial q^2} \psi\psi\ . \quad V_F(\phi, q) \sim e^{-2S_0} e^{-2\phi} q^6 (1 + \mathcal{O}(q^2))$$

Coulomb branch not lifted

No region in moduli space where both Y and u are both light.
Higgs branch: gauge multiplet is heavy, Coulomb branch: U is heavy. How about the origin?

Micro/macro discrete parity anomalies mismatch. (hence, b=0)

$$k_{R'R'} = \frac{1}{2}\left[3(1)^2 + 4(-1)^2\right] = \frac{7}{2} \in Z + \frac{1}{2}$$

$$k_{R'R'} = \frac{1}{2}\left[1(1)^2 + 1(-1)^2\right] = 1 \in Z$$

At the origin, need new degrees of freedom.
Most likely a CFT of strongly coupled quarks and gluons on $\mathbb{R}^3$

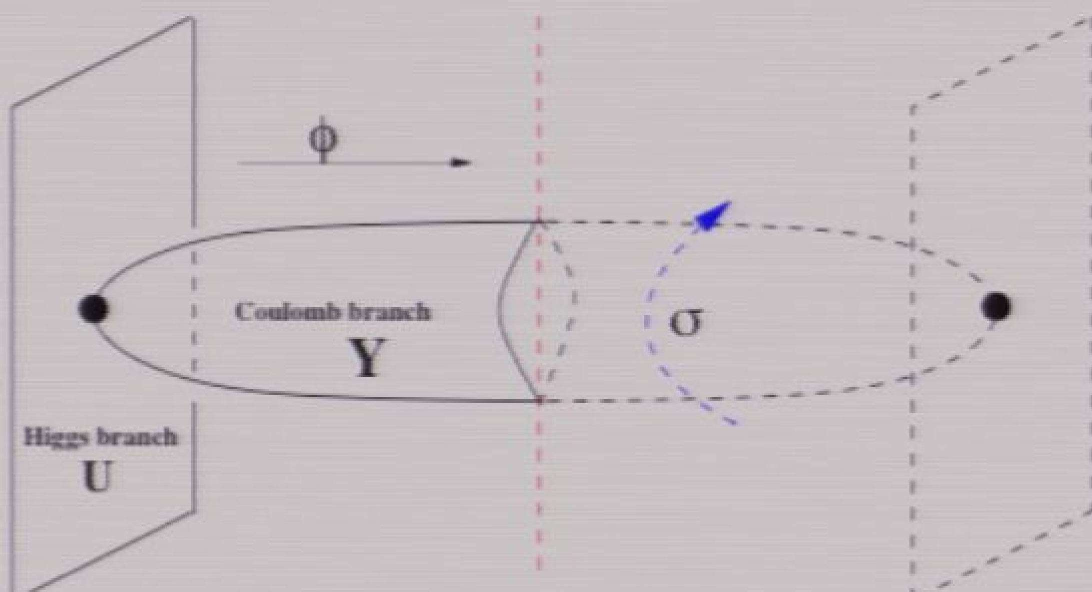
Chiral theory on $\mathbb{R}^3 \times S^1$

Yukawa lifting

$$\mathcal{M}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^6 \lambda^2 \longrightarrow \widetilde{\mathcal{M}}_2 = e^{-S_0} e^{+\phi - i\sigma} \psi^4 q^2,$$

too many zero modes to contribute to the superpotential.

$W_{\mathbb{R}^3 \times S^1}[u] = W_{\mathbb{R}^4}[u] = 0$ Moduli space on $\mathbb{R}^3 \times S^1$



Decompactification and ~~SUSY~~ ?

In supersymmetric theories, there is some lore (no theorem, though) about the absence of phase transitions, based on holomorphy and the ensuing fact that singularities of the superpotential and the holomorphic gauge coupling are of co-dimension two and therefore one can always ``go around" them. Seiberg, Witten:94, Intriligator, Seiberg:94

Currently, there is no known example of susy gauge theories with periodic spin connections undergoing a phase transition as a function of compactification radius. Thus, we believe, we have strong evidence which indicates that the theory on decompactification limit is as well a CFT.

The theory at the origin of moduli space does not confine. Hence, W=u is quite irrelevant and its addition does not alter the long distance dynamics. Hence, no mass gap for gauge fluctuations, and no SUSY breaking in ISS model.

Intriligator a-theorem:CFT and Vartanov, 2010:CFT

Chiral SU(2) with J=3/2

Well-defined, gauge and global (Witten) anomaly free.
No framework to address its dynamics until recently.

Instantons: $$I(x) = e^{-S_{\text{inst}}}\psi^{10}$$

Symmetry: $$\mathbb{Z}_{10} : \psi \to e^{i\frac{2\pi k}{10}}\psi,$$

Shifman, M.U. 08 for new techniques applied to chiral quiver gauge theories,
Poppitz, MU, relatively simpler applications.

Chiral SU(2) with J=3/2

relevant index theorem, Nye-Singer, 00
Poppitz, MU 08

Monopole operators

$$\mathcal{M}_1 = e^{-S_0} e^{i\sigma} \psi^4, \qquad \overline{\mathcal{M}}_1 = e^{-S_0} e^{-i\sigma} \bar{\psi}^4,$$

$$\mathcal{M}_2 = e^{-S_0} e^{-i\sigma} \psi^6, \qquad \overline{\mathcal{M}}_2 = e^{-S_0} e^{i\sigma} \bar{\psi}^6,$$

Topological shift symmetry intertwines with chiral symmetry.

$$\mathbb{Z}_5 : \qquad \psi^4 \to e^{i\frac{2\pi}{5}} \psi^4, \qquad \sigma \to \sigma - \frac{2\pi}{5}$$

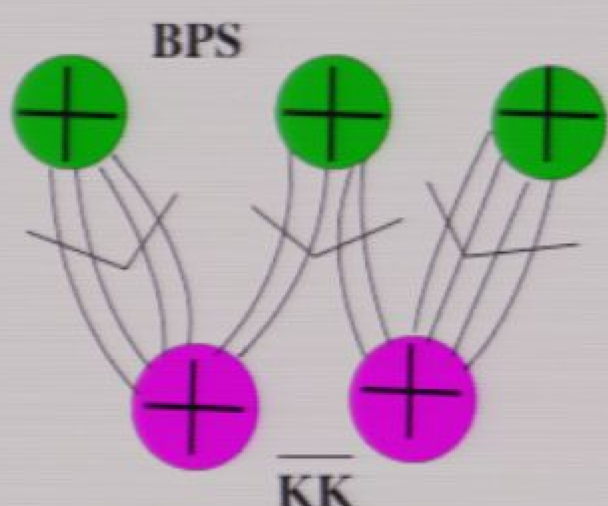
$$(\mathbb{Z}_5)_*$$

Mass gap magnetic quintet op. $\qquad e^{-5S_0} \cos 5\sigma$

In the absence of fermion zero modes, the constituents of the magnetic quintet interact repulsively.

$$[\mathcal{M}_1]^3[\overline{\mathcal{M}}_2]^2 \equiv [\mathrm{BPS}]^3[\overline{\mathrm{KK}}]^2$$

$$\left(\int_{S^2_\infty} B, \ \int F\tilde{F}\right) = \left(\pm 5, \ \pm\frac{1}{2}\right)$$



"The magnetic quintet" leading topological excitation that leads to confinement in non-susy chiral theory. (Testable on lattice)

Conclusions and prospects

- There are new ways to study non-perturbative aspects of 4d gauge theories by using circle compactifications.
- Large-N volume independence and its semi-classical avatar provide many new insights, both numeric and analytic. Useful for both supersymmetric and ordinary gauge theories.
- Topological molecules, new mechanism of confinement and mass gap. Magnetic bions are responsible for confinement at small circle for many gauge theories. Can we extend it to large circle and $\mathbb{R}^4$? (Yes in Seiberg-Witten theory. Work in progress, with Poppitz.)
- Possible applications to finite-temperature QCD and RHIC-physics. New insights to (QCD) phase transitions? (Poppitz, Argyres)
- Possible application to EWSB problem and BSM physics.