Title: Elucidating the quantum measurement problem

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Abstract: Ideal measurements are described in quantum mechanics textbooks by two postulates: the collapse of the wave packet and BornÃ¢ÂÂ€ÂÂ™s rule for the probabilities of outcomes. The quantum evolution of a system then has two components: a unitary (Hamiltonian) evolution in between measurements and non-unitary one when a measurement is performed. This situation was considered to be unsatisfactory by many people, including Einstein, Bohr, de Broglie, von Neumann and Wigner, but has remained unsolved to date.
The quantum measurement problem, that is, understanding why a unique outcome is obtained in each individual run of an experiment, is tackled by solving a Hamiltonian model within standard quantum statistical mechanics. The model describes the measurement of the z-component of a spin through interaction with a magnetic memory. The latter apparatus is modeled by a CurieÃ¢ÂÂ€ÂÂ“Weiss magnet having N Ã¢ÂÂ‰Â« 1 spins weakly coupled to a phonon bath.
The Hamiltonian evolution exhibits several time scales. The reduction, a rapid decay of the off-diagonal blocks of the systemÃ¢ÂÂ€ÂÂ“apparatus density matrix, arises from the many degrees of freedom of the pointer (the magnetization). The registration occurs due to a phase transition from the initial metastable state to one of the final stable states triggered by the tested system. It yields a stationary state in which the apparatus and the system are correlated. Under proper conditions the process satisfies all the features of ideal measurements, including collapse and BornÃ¢ÂÂ€ÂÂ™s rule.
As usual, irreversibility is ensured by the macroscopic size of the apparatus, in particular by the large value of N. Nothing else than the usual quantum statistical mechanics and Schro ÃÂŒÂÂˆdinger equation is needed, and the results support a specified version of the statistical interpretation. The solution of the quantum measurement problem requires a combination of the reduction and the registration, the properties of which arise from the irreversible dynamics.

Elucidating the Quantum Measurement Problem

Theo M. Nieuwenhuizen
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Europhysics Letters 2003
arXiv 2004, 2005, 2006
Beyond the Quantum 2007
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Opus Magnum, in progress

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Setup

Statistical interpretation of QM

Text books on measurements

Problems & paradoxes + the big questions

The model: system S + apparatus A
spin-½ A = M + B = magnet + bath

Selection of collapse basis & fate of Schrodinger cats

Registration of the Q-measurement & classical measurement

Post measurement & the Born rule

The Q measurement problem elucidated

Summary

Statistical interpretation of QM

Density matrix $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ describes ensemble of identically prepared systems

Pure state $|\psi\rangle \mapsto \rho = |\psi\rangle\langle\psi|$ is limiting case; same meaning (no special role)

Ensemble can be real (many particles: bundle at LHC
one trapped ion in photon field, repeated excitation)
or virtual (as in classical statistical physics)

QM = tool for calculating averages from density matrix ρ

$$\langle A\rangle = \langle\psi|A|\psi\rangle \mapsto \mathrm{tr}(\rho A)$$

=> QM = about what we can measure, not about what is
epistomology ⇔ ontology

Quantum measurement theory describes ensemble of measurements on ensemble of systems

Q-measurements in text books

$|\psi\rangle = \sum_i \psi_i |s_i\rangle$ measurement => $|s_i\rangle$ collapse of the wave function
(reduction of wave packet)
pure state => mixed state

$$p_i = |\psi_i|^2$$

Born probability

projector on eigenstate *i* : $\hat{\Pi}_i = |s_i\rangle\langle s_i|$

collapse $\rho \mapsto \rho_i = \frac{1}{p_i}\hat{\Pi}_i \rho \hat{\Pi}_i$

Born $p_i = \mathrm{tr}(\hat{\Pi}_i \rho)$

No Schrödinger cats: no terms $\hat{\Pi}_i \rho \hat{\Pi}_j \quad (i \neq j)$ after measurement

Problems & paradoxes

Einstein, Bohr, de Broglie, von Neumann, Wigner, Bohm, Bell, Balian, van Kampen ...

Collapse = non-unitary

Small part of apparatus described by QM ⇔ why not measurement process?

Preferred basis paradox: on which basis will reduction take place?

When does collapse happen ? How long does it take?

What happens to Schrödinger cats?

The biggest of them all:
The quantum measurement problem:
QM = statistical theory, but in experiments we see individual outcomes
Solvable within QM ?? => probably not ...

Requirements for Q measurement models

simulate as much as possible real experiments

ensure unbiased and robust registration by the *macroscopic* pointer of A

apparatus initially in a metastable state to amplify the quantum signal
(not in pure state)

include a bath for dumping the free energy released by pointer

be solvable and exhibit the characteristic times

lead to final state devoid of “Schrödinger cats”

satisfy Born's rule and correlations between system & apparatus

solvable and flexible

Spin-spin interactions in magnet M

$$\hat{H}_{\rm M} = -\frac{J_2 N}{2}\hat{m}^2 - \frac{J_4 N}{4}\hat{m}^4$$
$$= -\frac{J_2}{2N}\sum_{m,n=1}^{N}\hat{\sigma}_z^{(m)}\hat{\sigma}_z^{(n)} - \frac{J_4}{4N^3}\sum_{k,l,m,n=1}^{N}\hat{\sigma}_z^{(k)}\hat{\sigma}_z^{(l)}\sigma_z^{(m)}\hat{\sigma}_z^{(n)}$$

- J_2 term: Curie-Weiss magnet
- J_4 term: allows first order phase transition

- Extreme case of first order transition: $J_2 = 0$, $J_4 = J$

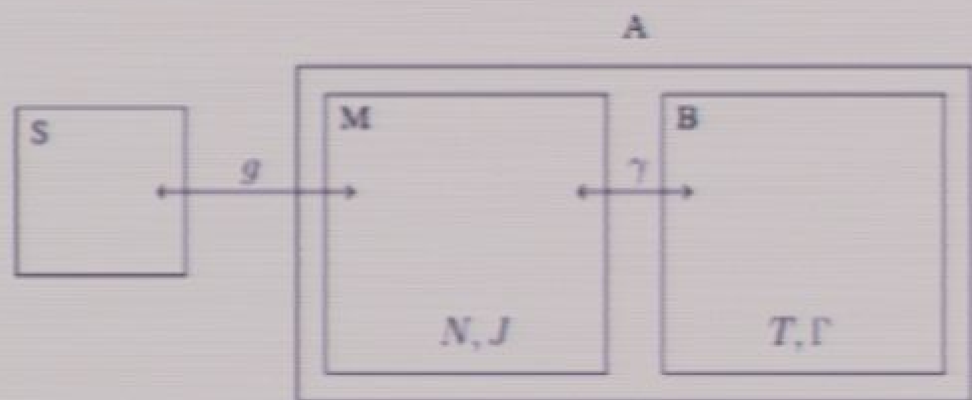
Curie-Weiss model for quantum measurements

System S: s = spin-½. For ideal measurement: $\hat{H}_{\rm S} = 0$

Apparatus = Magnet + Bath (A = M + B).

M = N spins-½ $\sigma^{(n)}$. Magnetization $\hat{m} = \frac{1}{N}\sum_{n=1}^{N} \hat{\sigma}_z^{(n)}$

B = phonon bath: harmonic oscillators



$$\hat{H} = \hat{H}_{\rm S} + \hat{H}_{\rm SA} + \hat{H}_{\rm A}$$

$$\hat{H}_{\rm S} = 0 \quad \hat{H}_{\rm SA} = -g\hat{s}_z \sum_{n=1}^{N} \hat{\sigma}_z^{(n)} = -Ng\hat{s}_z\hat{m}$$

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Bath Hamiltonian

Standard harmonic oscillator bath: $\hat{H}_{\rm B} = \sum_{n=1}^{N} \sum_{a=x,y,z} \sum_{k} \hbar\omega_k \hat{b}_{k,a}^{\dagger(n)} \hat{b}_{k,a}^{(n)}$

x,y,z components of the spins of M couple to their own harmonic oscillators

$$\hat{H}_{\rm MB} = \sqrt{\gamma} \sum_{n=1}^{N} \left(\hat{\sigma}_x^{(n)} \hat{B}_x^{(n)} + \hat{\sigma}_y^{(n)} \hat{B}_y^{(n)} + \hat{\sigma}_z^{(n)} \hat{B}_z^{(n)} \right) \equiv \sqrt{\gamma} \sum_{n=1}^{N} \sum_{a=x,y,z} \hat{\sigma}_a^{(n)} \hat{B}_a^{(n)}$$

$$\hat{B}_a^{(n)} = \sum_k \sqrt{c(\omega_k)} \left(\hat{b}_{k,a}^{(n)} + \hat{b}_{k,a}^{\dagger(n)} \right)$$

Bath characterized by $\mathrm{tr}_{\rm B}\left[\hat{R}_{\rm B}(0)\, \hat{B}_a^{(n)}(t)\, \hat{B}_b^{(p)}(t')\right] = \delta_{n,p}\delta_{a,b} K(t-t')$

Weak M-B coupling: $\gamma \ll 1$ allows to work at lowest order in γ

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$$R_M(0) = \Pi_{n=1}^{N} \rho^{(n)}(0), \qquad \rho^{(n)}(0) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Bath: Gibbs state (mixed state) $R_{\mathrm{B}}(0) = \frac{e^{-\beta \hat{H}_{\mathrm{B}}}}{Z_{\mathrm{B}}}$ $\beta = 1/T$

Selection of collapse basis

What selects collapse basis? *The interaction Hamiltonian !*

$$\hat{H}_{\mathrm{SA}} = -g\hat{s}_z \sum_{n=1}^{N} \hat{\sigma}_z^{(n)} = -Ng\hat{s}_z\hat{m}$$

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Trace out Apparatus (Magnet+Bath) in von Neumann eqn

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{D}} = [\hat{H}, \hat{\mathcal{D}}] \qquad \mathcal{D} = \begin{pmatrix} \mathcal{D}_{\uparrow\uparrow} & \mathcal{D}_{\uparrow\downarrow} \\ \mathcal{D}_{\downarrow\uparrow} & \mathcal{D}_{\downarrow\downarrow} \end{pmatrix}$$

The 4 sectors decouple; only interaction H_{SA} remains
Trace out A = M + B

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$$r_{ij}(t) = \mathrm{tr}_{\lambda = M+B}\, \mathcal{D}_{ij}(t)$$

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Diagonal terms of r(t) conserved ->= $r_{ii}(t) = r_{ii}(0)$: Born probabilities

Off-diagonal terms evolve => disappearence of Schrodinger cats

Fate of Schrodinger cat terms

Short times: spin-spin couplings in magnet are irrelevant

x,y components of spins of M rotate in magnetic field g of spin S

$$\hat{\rho}^{(n)}(t) = \frac{1}{2}\mathrm{diag}(e^{2igt/\hbar}, e^{-2igt/\hbar})e^{-g^2\gamma K t^4}$$

Larmor procession   damping by zero-point flucts of bath

Schrodinger cat:

$$r_{\uparrow\downarrow}(t) = r_{\uparrow\downarrow}(0)(\cos\frac{2gt}{\hbar})^N e^{-Ng^2\gamma K t^4}$$

$$= r_{\uparrow\downarrow}(0)e^{-(t/\tau_{\mathrm{red}})^2} e^{-(t/\tau_{\mathrm{dec}})^4}$$

dephasing as in NMR   decoherence (zpf of bath) comes in later

Recurrences eliminated by: decoherence by zero point flucts of bath

or even by a spread in g's

Creation of multi-particle correlations: weak, dying out

$$\langle(\hat{s}_x - i\hat{s}_y)\hat{m}^k(t)\rangle_c \sim \langle(\hat{s}_x - i\hat{s}_y)(0)\rangle i^k t^k e^{-(t/\tau_{\mathrm{red}})^2} \leq \left(\frac{1}{\sqrt{N}}\right)^k$$

Schrodinger cat terms die out quickly

Registration of the measurement

Solve Q-dynamics of diagonal elements to second order in the coupling to bath

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Free energy F=U-TS: minima are stable states of free energy

$$\frac{F}{N} = -gs_z m - \frac{Jm^4}{4} - T[\frac{1+m}{2}\ln\frac{2}{1+m} + \frac{1-m}{2}\ln\frac{2}{1-m}]$$

Free energy landscape: classical Curie-Weiss model

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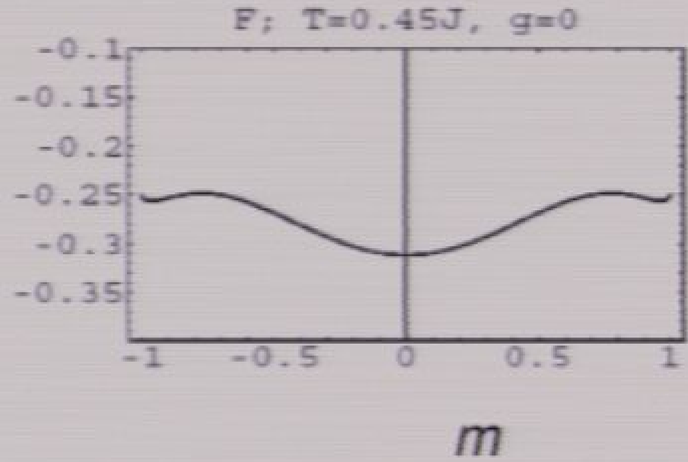
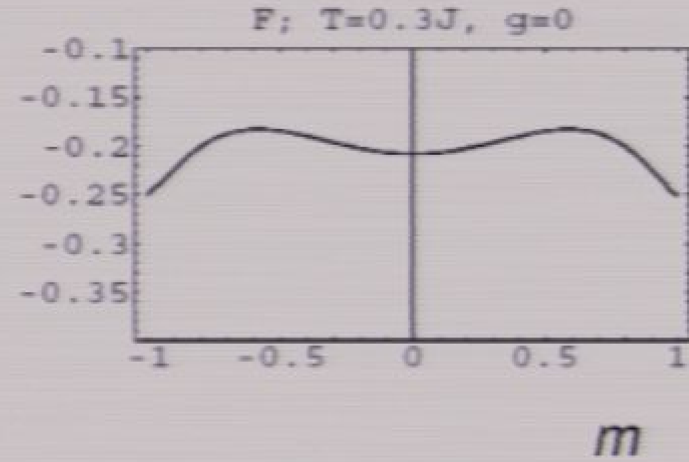
At g=0:

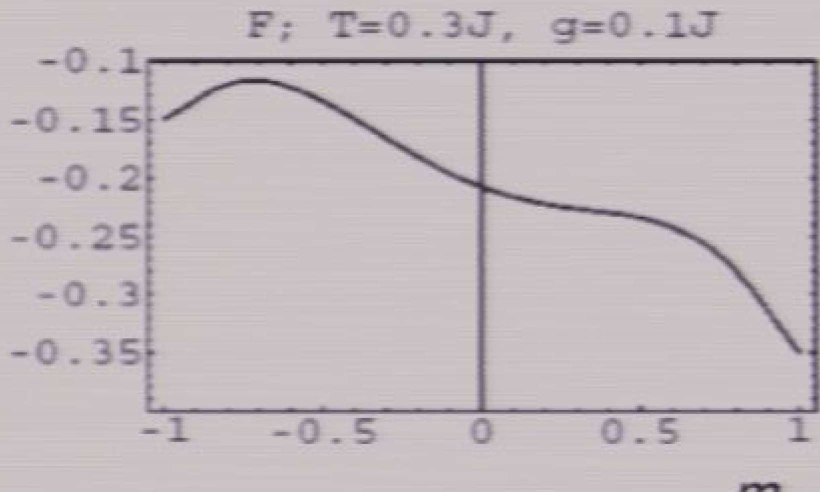


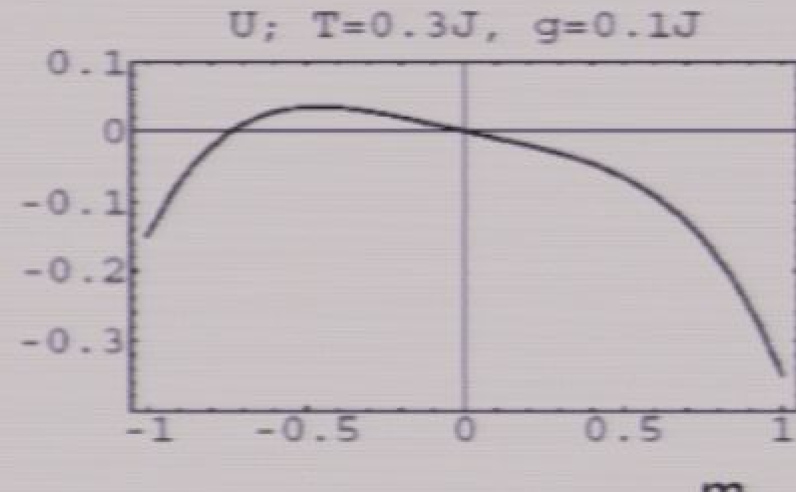


High T: paramagnet is stable

Low T:
paramagnetic initial state;
stable ferromagnetic states
can act as measuring apparatus

During measurement: turn on coupling ➡ field $h=g\ s_z$





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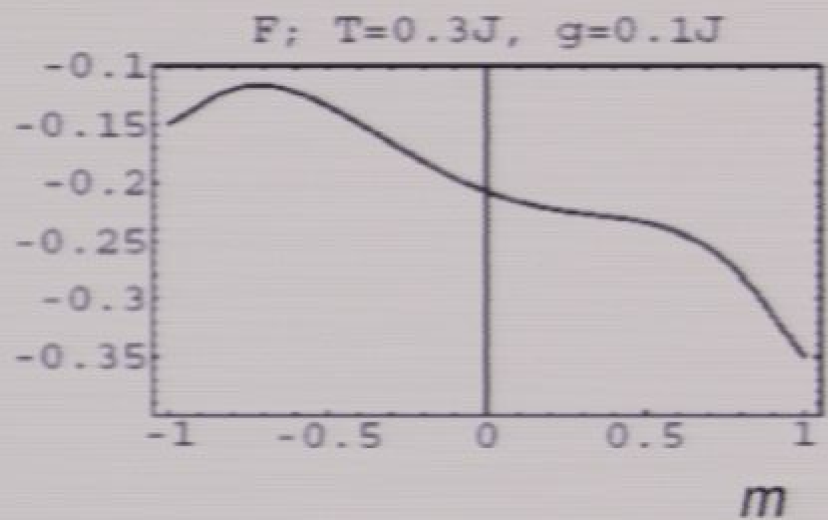

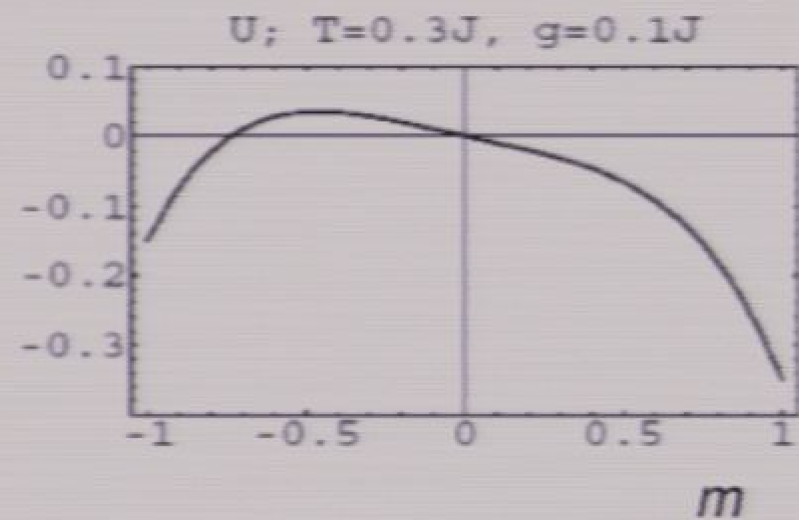

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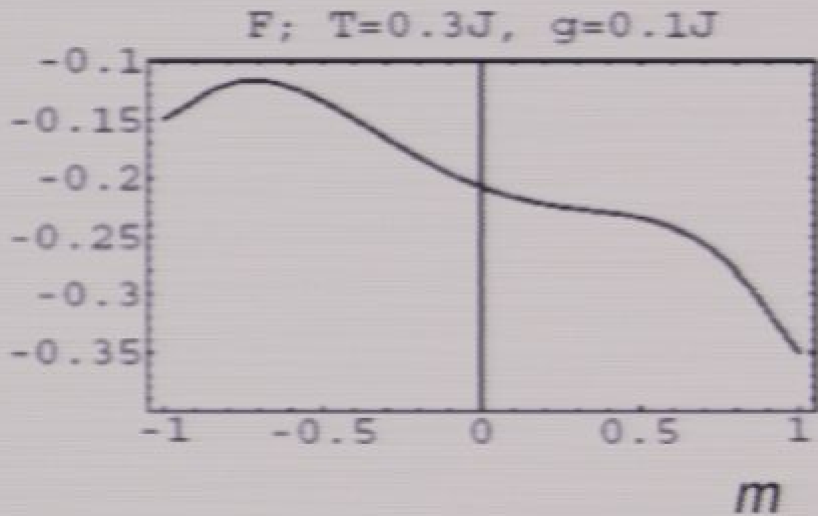

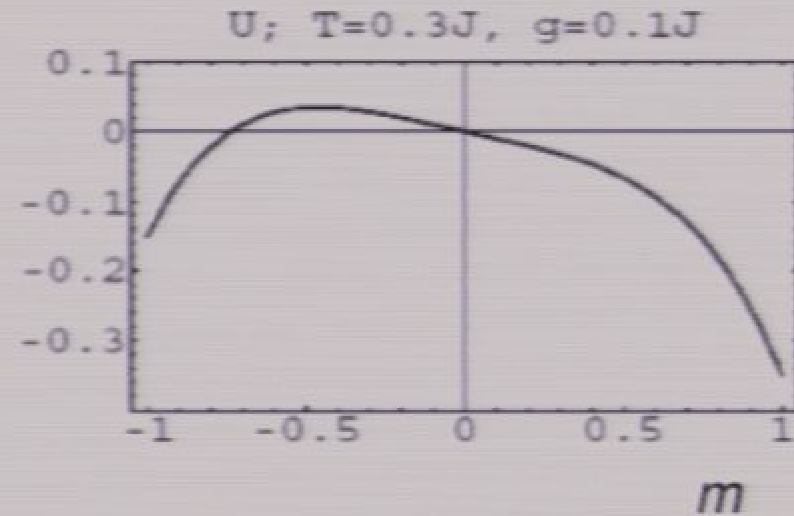

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Procedure:
Turn off coupling g after minimum is reached
Magnet goes to ferromagnetic minimum at g=0. Stays there a time exp(N)

(Result is stable and may be read off, or not)

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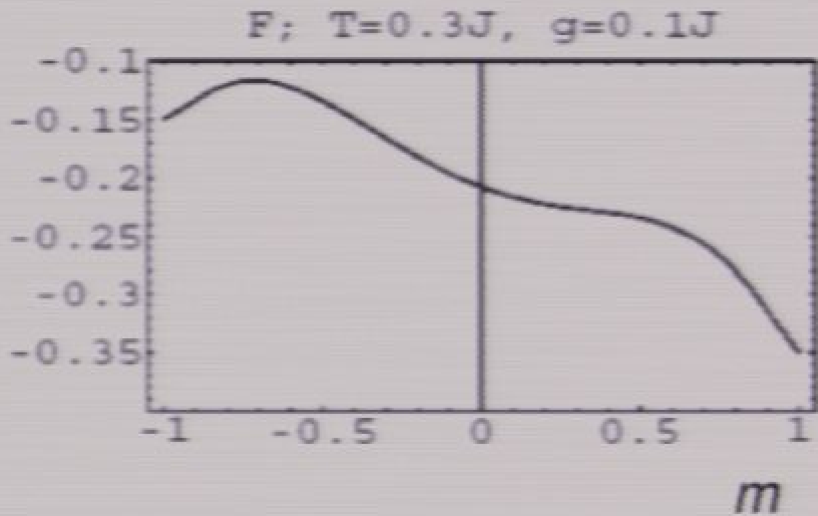

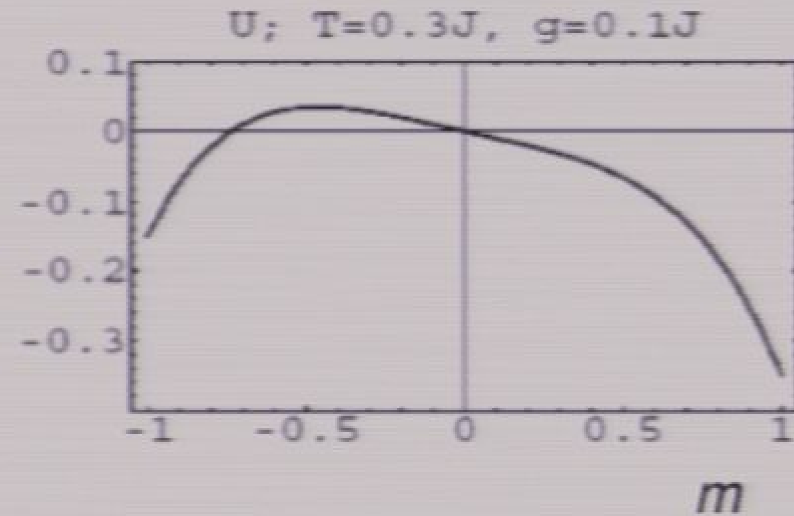

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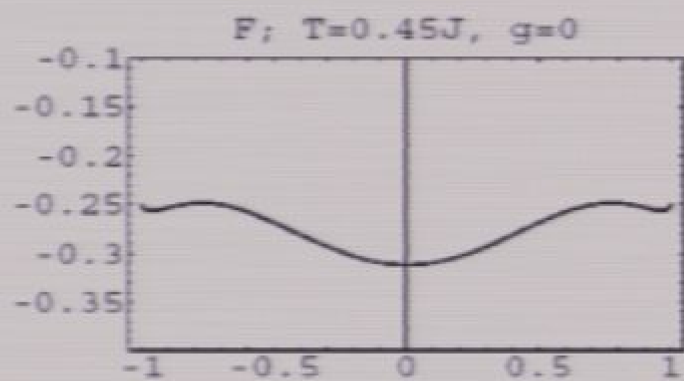

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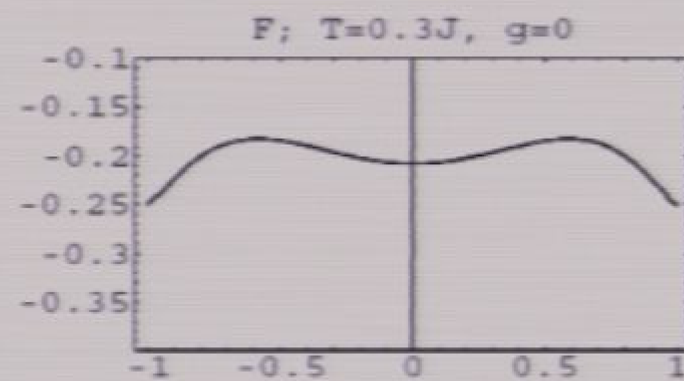

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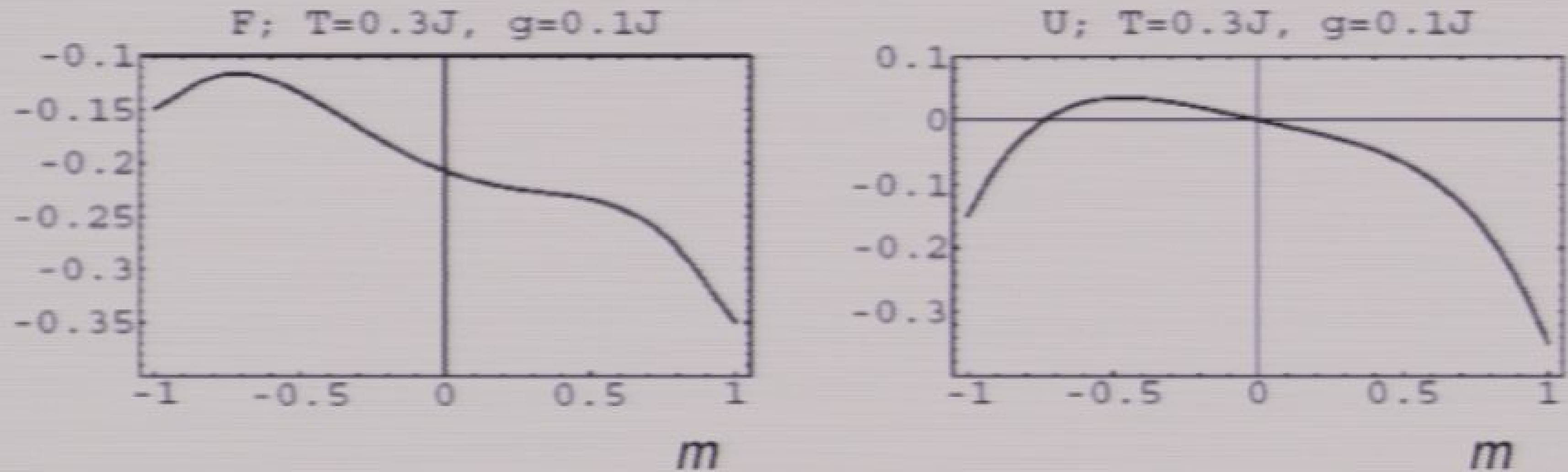

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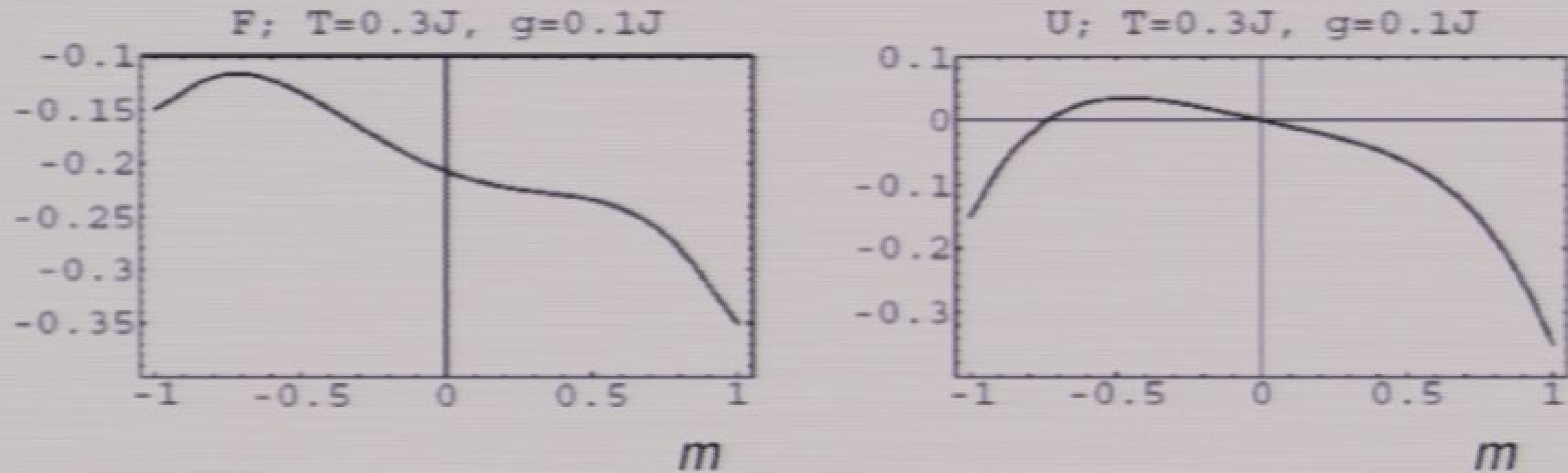

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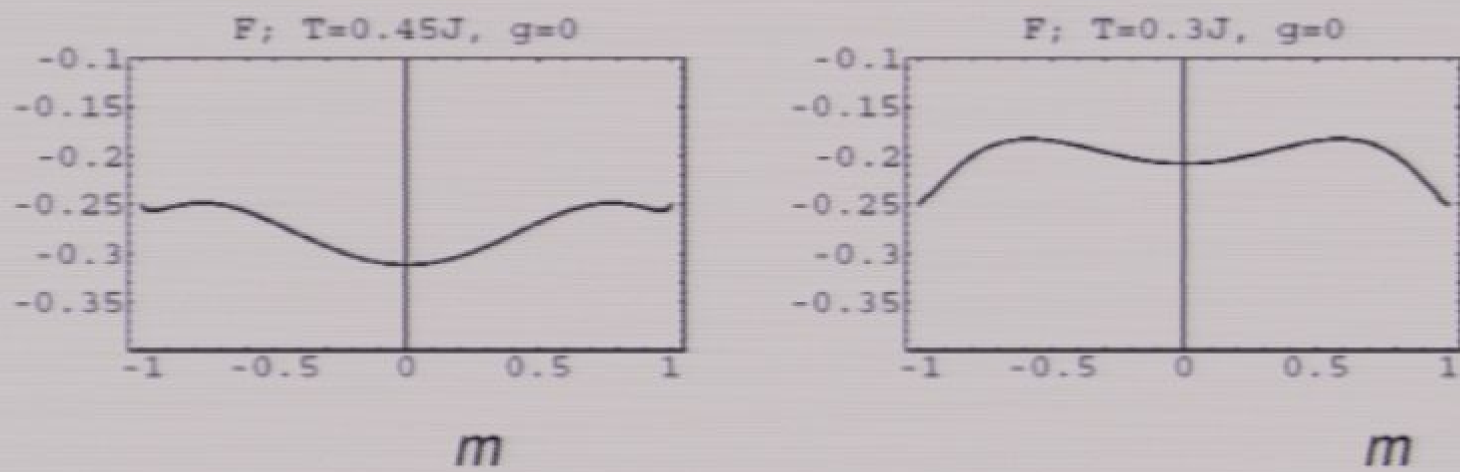

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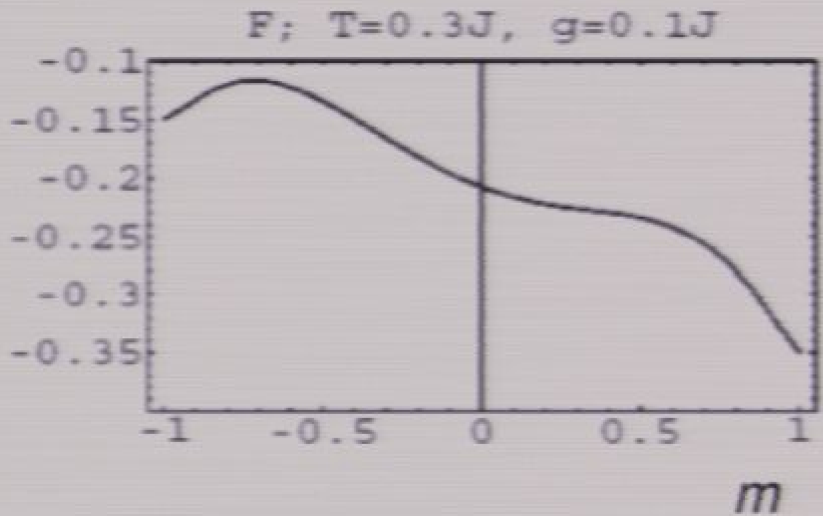
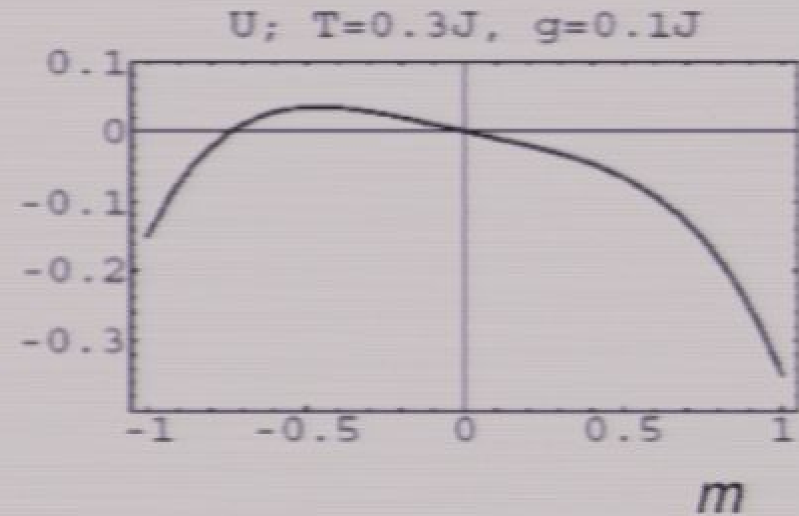
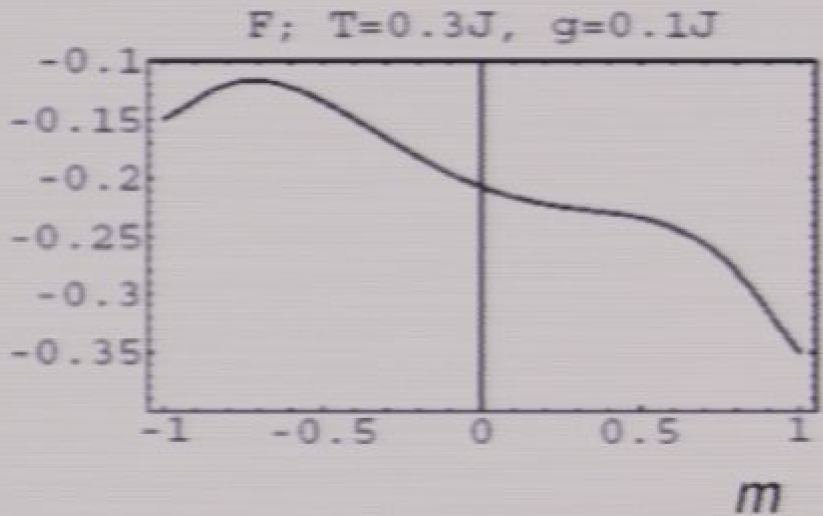
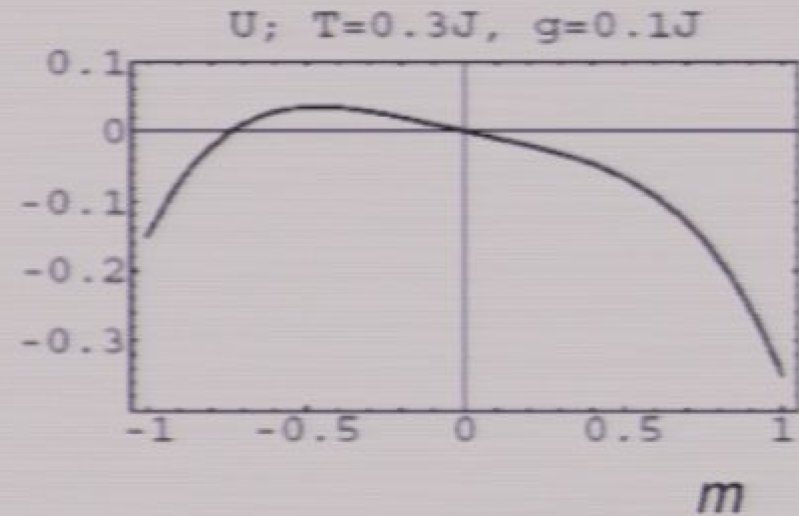




Bath is needed to dump the free energy

During measurement: turn on coupling ➡ field $h=g\,s_z$





Bath is needed to dump the free energy

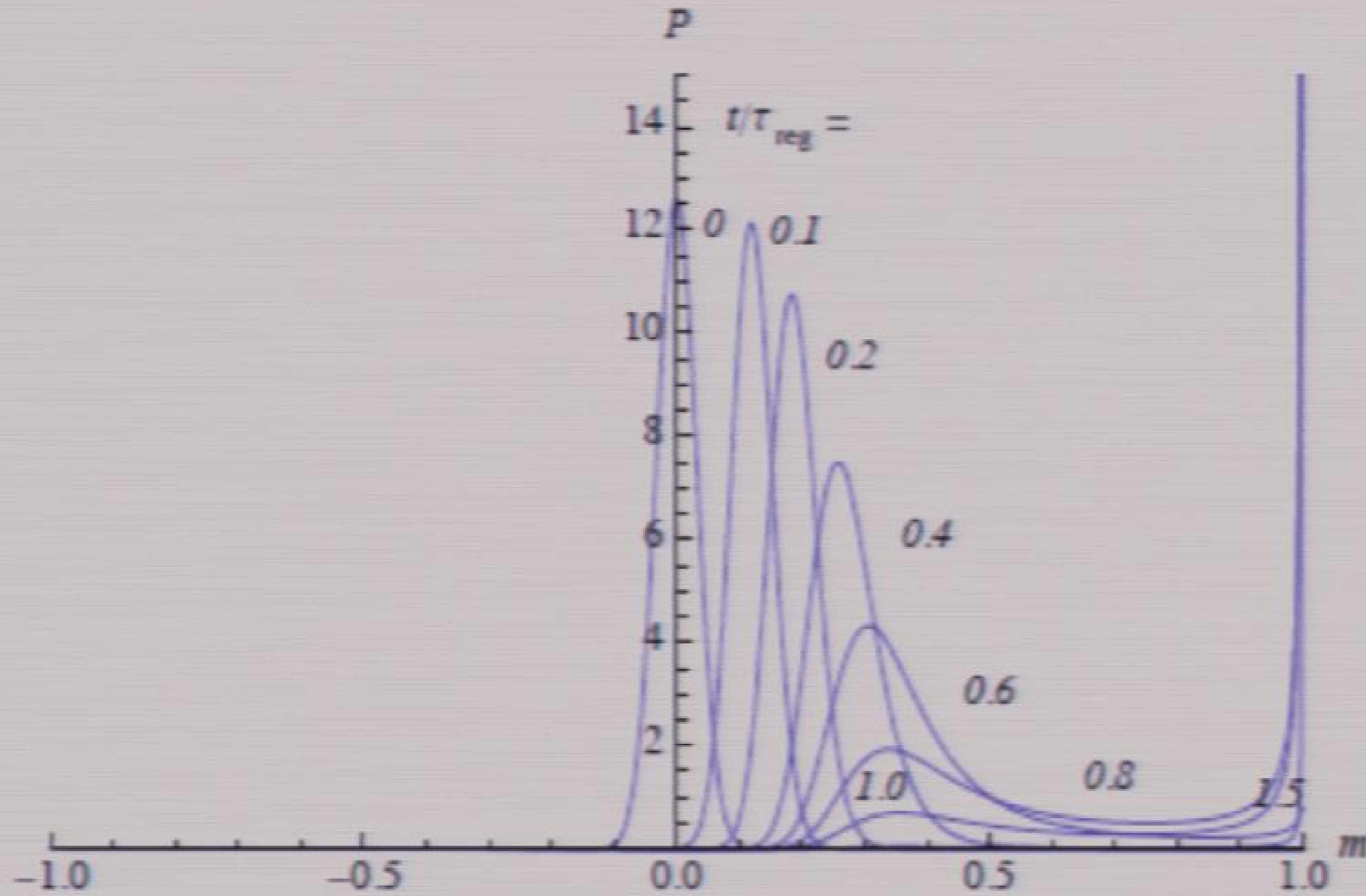
Procedure:

Turn off coupling g after minimum is reached

Magnet goes to ferromagnetic minimum at g=0. Stays there a time exp(N)

(Result is stable and may be read off, or not)

Real solution: QM widens the distribution P(m;t)



- Finally: sharp peak at $m=+m_F$ in sector $s_z=+1$
- $m=-m_F$ in sector $s_z=-1$

Post-measurement state

Magnet ends up in up/down ferromagnetic state
Sign of magnetization maximally correlated with sign spin S
Probabilities satisfy Born rule

No Schrodinger cat terms

Post-measurement state

$$\hat{\mathcal{D}}(t_{\rm f}) = r_{\uparrow\uparrow}(0)|\uparrow\rangle\langle\uparrow| \otimes \hat{R}_{{\rm A}\uparrow}(t_{\rm f}) + r_{\downarrow\downarrow}(0)|\downarrow\rangle\langle\downarrow| \otimes \hat{R}_{{\rm A}\downarrow}(t_{\rm f})$$

Magnet ends up in up/down ferromagnetic state
Sign of magnetization maximally correlated with sign spin S
Probabilities satisfy Born rule

No Schrodinger cat terms

On the Quantum Measurement Problem

The macroscopic magnet itself is well understood.
Initial paramagnet relaxes to stable ↑ or ↓ ferromagnet
Ensemble describes breaking of ergodicity
Individual ↑ and ↓ ferromaget is stable, Poincare time is very large

In Q measurement this transition is triggered by the tested spin S
Long life time of FM state ensures *uniqueness* of outcome for $m=\pm m_F$
and for the measured spin $s_z = \pm 1$ *that is fully correlated with it*

No survival of Schrodinger cat terms, that could spoil this view.

One may select individual outcomes with magnetization ↑
(possible because ferromagnetic ↑ state longlived)
This unambiguous subensemble is a pure ensemble of spins ↑ of S

Summary: the measurement problem elucidated

Collapse basis determined by interaction Hamiltonian

Measurement in two steps: cats die & registration of the result
very fast is slower
small multi-particle correlations

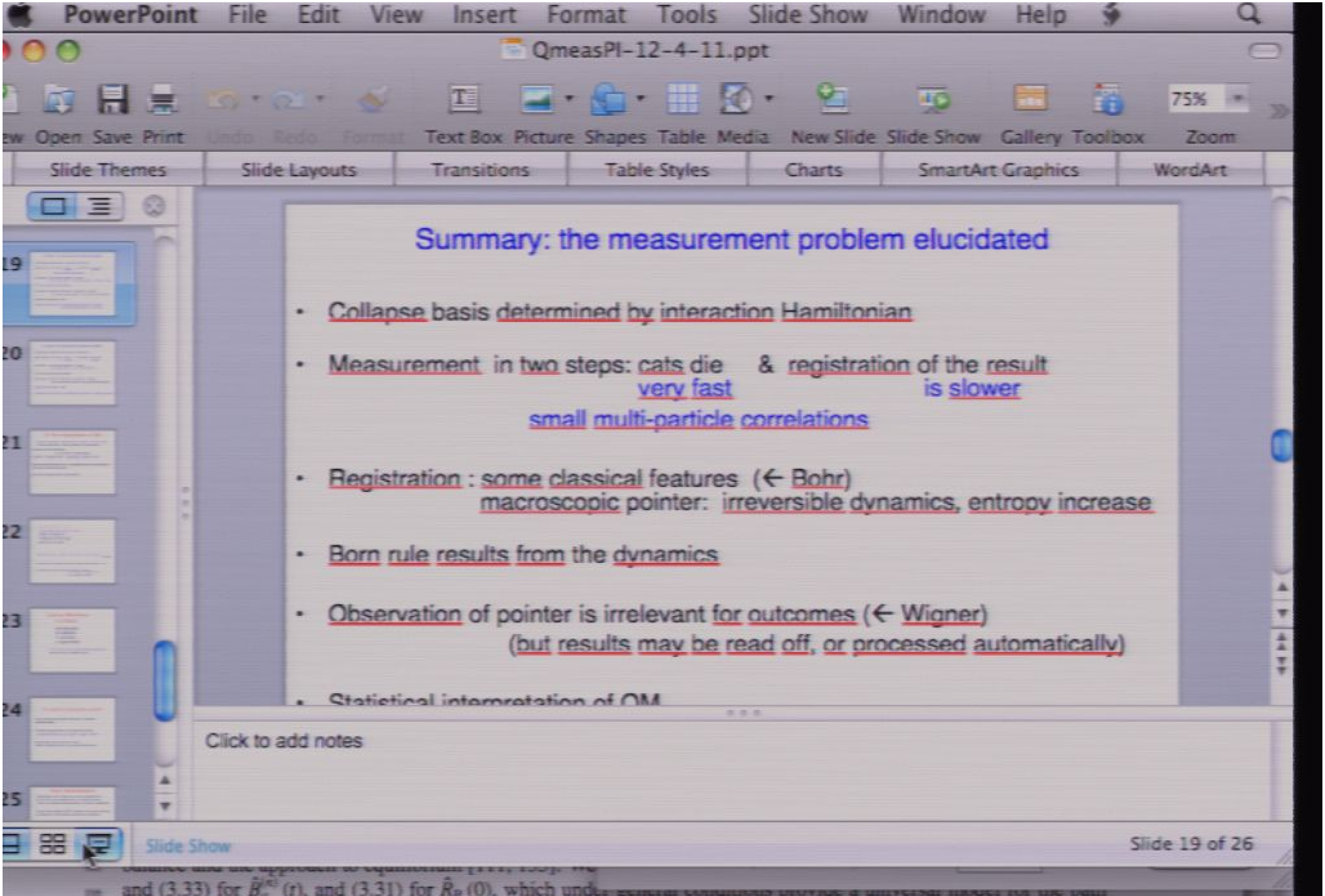
Registration : some classical features (← Bohr)
macroscopic pointer: irreversible dynamics, entropy increase

Born rule results from the dynamics

Observation of pointer is irrelevant for outcomes (← Wigner)
(but results may be read off, or processed automatically)

Statistical interpretation of QM

Individual outcomes due to irreversible phase transition in magnet
long lived pointer indication, no cat terms


QmeasPI-12-4-11.ppt
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Statistical interpretation of QM
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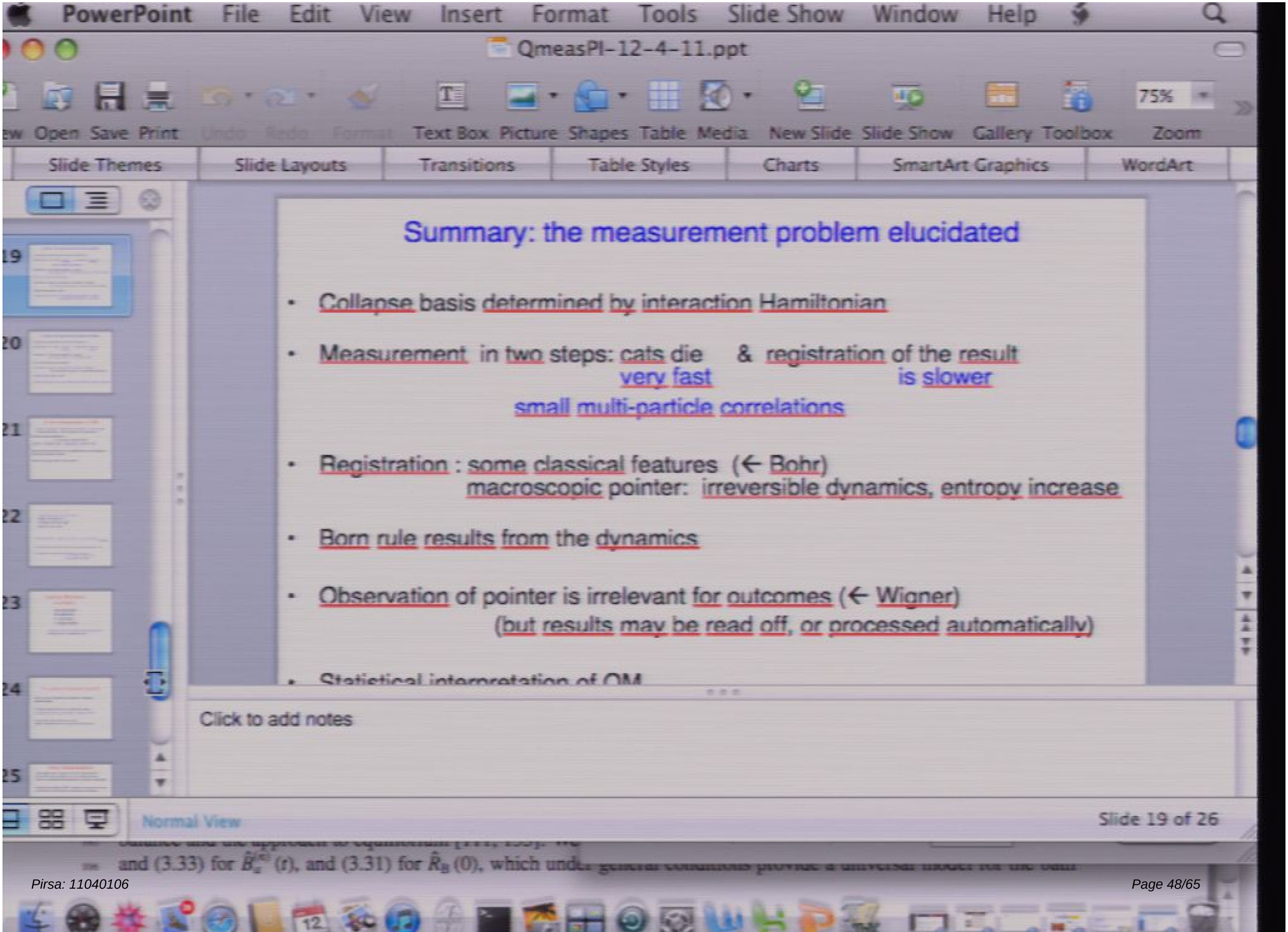
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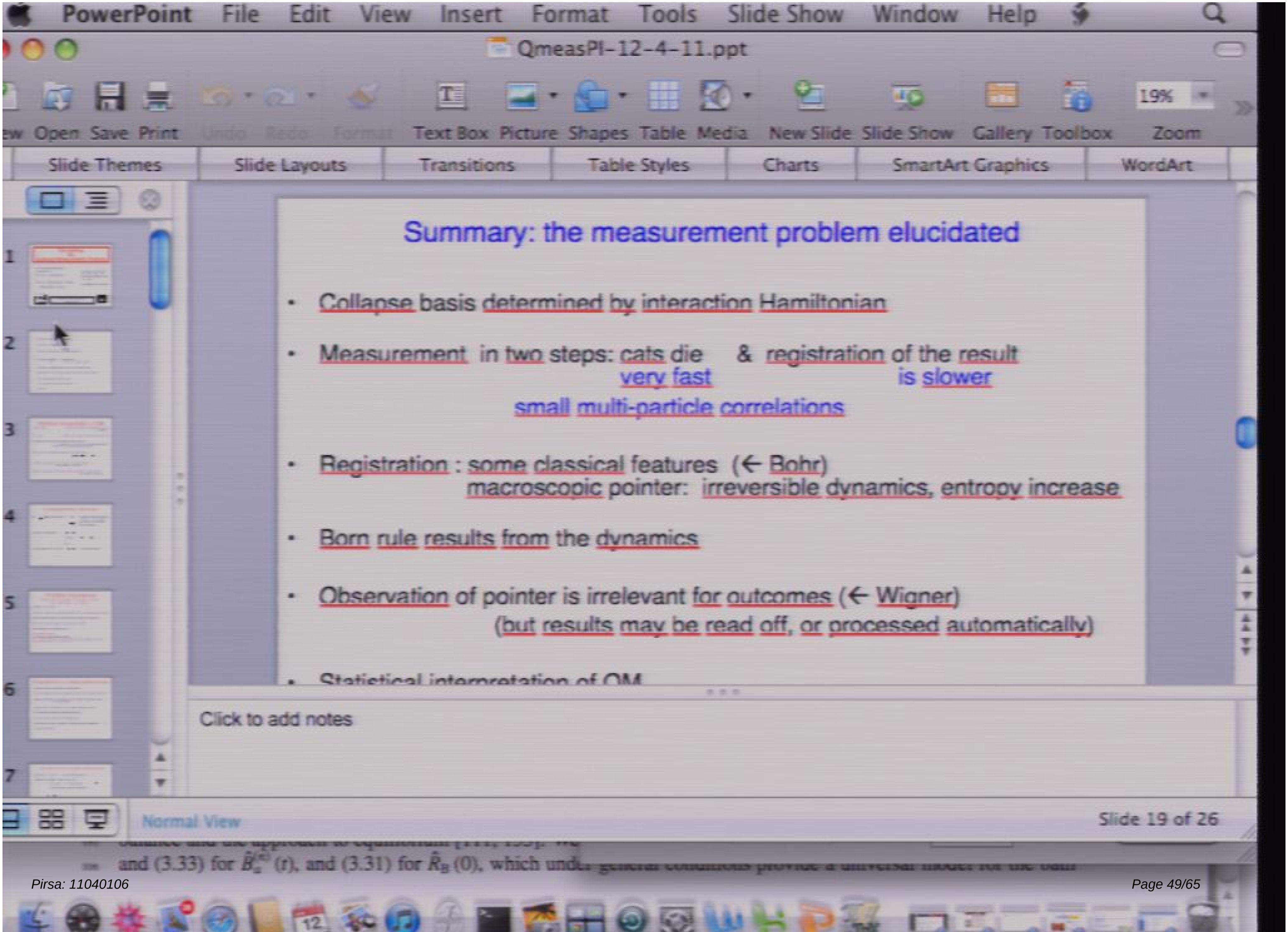
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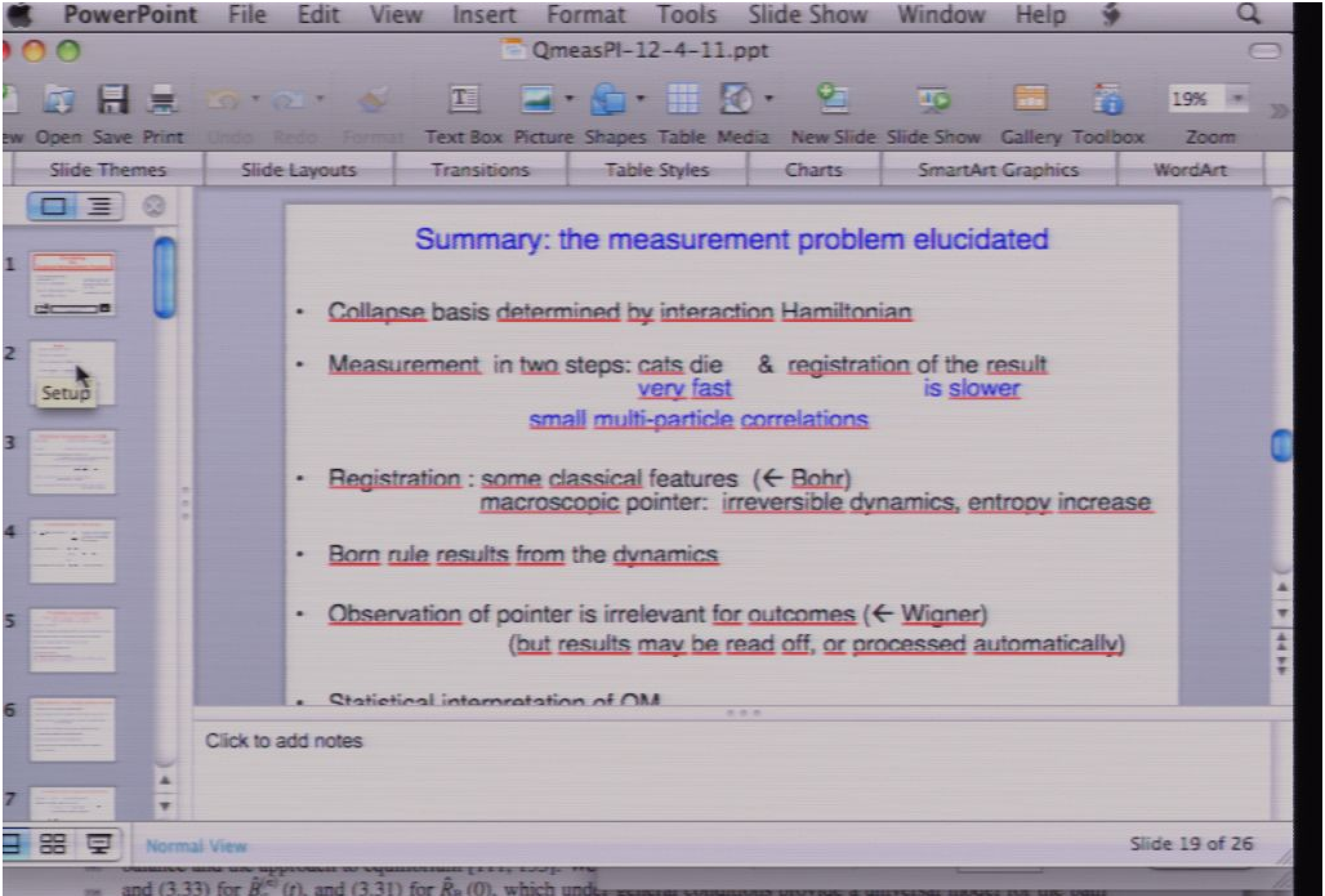
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Setup

Statistical interpretation of QM

Text books on measurements

Problems & paradoxes + the **big** questions

The model: system S + apparatus A
spin-½ A = M + B = magnet + bath

Selection of collapse basis & fate of Schrodinger cats

Registration of the Q-measurement & classical measurement

Statistical interpretation of QM

Density matrix $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ describes ensemble of identically prepared systems

Pure state $|\psi\rangle \mapsto \rho = |\psi\rangle\langle\psi|$ is limiting case; same meaning (no special role)

Ensemble can be real (many particles: bundle at LHC
one trapped ion in photon field, repeated excitation)
or virtual (as in classical statistical physics)

QM = tool for calculating averages from density matrix ρ

$$\langle A\rangle = \langle\psi|A|\psi\rangle \mapsto \mathrm{tr}(\rho A)$$

=> QM = about what we can measure, not about what is

Q-measurements in text books

$|\psi\rangle = \sum_i \psi_i |s_i\rangle$ measurement => $|s_i\rangle$ collapse of the wave function (reduction of wave packet)

$p_i = |\psi_i|^2$ pure state => mixed state
Born probability

projector on eigenstate i: $\hat{\Pi}_i = |s_i\rangle\langle s_i|$

collapse $\rho \mapsto \rho_i = \frac{1}{p_i}\hat{\Pi}_i \rho \hat{\Pi}_i$

Born $p_i = \mathrm{tr}(\hat{\Pi}_i \rho)$

Problems & paradoxes Einstein, Bohr, de Broglie...

...ger cats: no terms $\hat{\Pi}_i \rho \hat{\Pi}_j \quad (i \neq j)$ after measurement

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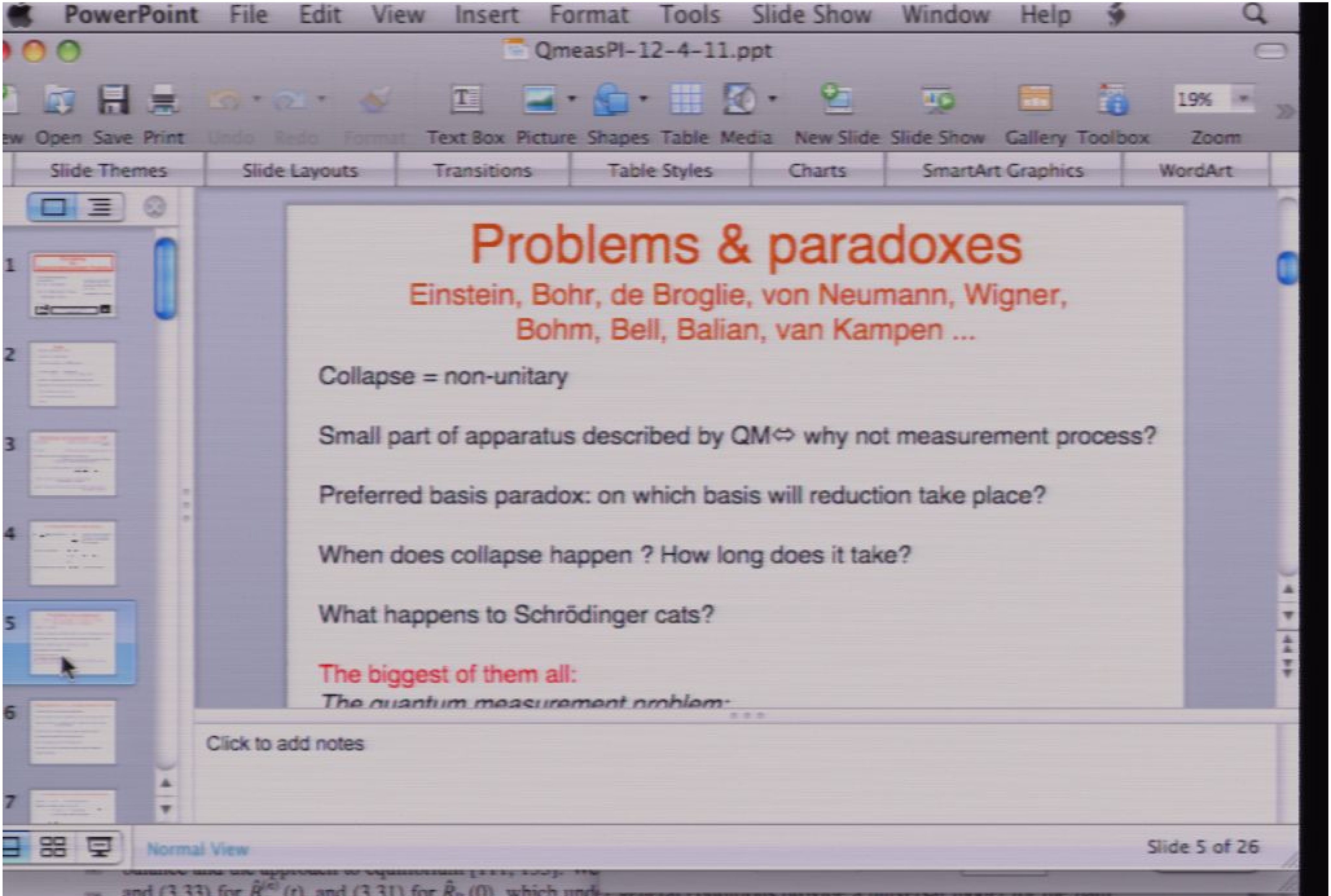
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Problems & paradoxes
Einstein, Bohr, de Broglie, von Neumann, Wigner,
Bohm, Bell, Balian, van Kampen ...
Collapse = non-unitary
Small part of apparatus described by QM⇔ why not measurement process?
Preferred basis paradox: on which basis will reduction take place?
When does collapse happen ? How long does it take?
What happens to Schrödinger cats?
The biggest of them all:
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Problems & paradoxes

Einstein, Bohr, de Broglie, von Neumann, Wigner, Bohm, Bell, Balian, van Kampen ...

Collapse = non-unitary

Small part of apparatus described by QM⇔ why not measurement process?

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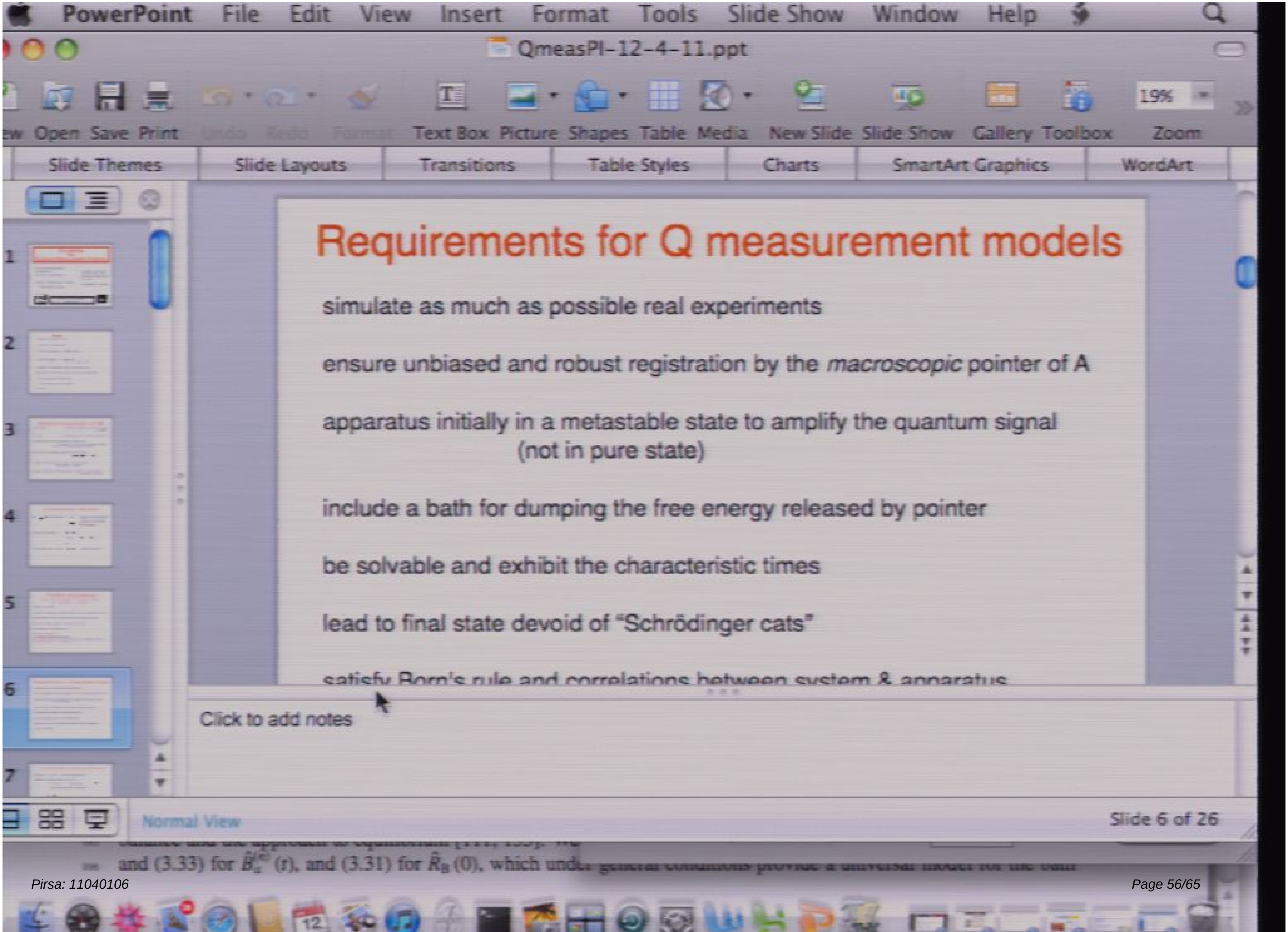
The biggest of them all:

The quantum measurement problem: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Requirements for Q measurement models
simulate as much as possible real experiments
ensure unbiased and robust registration by the macroscopic pointer of A
apparatus initially in a metastable state to amplify the quantum signal
(not in pure state)
include a bath for dumping the free energy released by pointer
be solvable and exhibit the characteristic times
lead to final state devoid of "Schrödinger cats"
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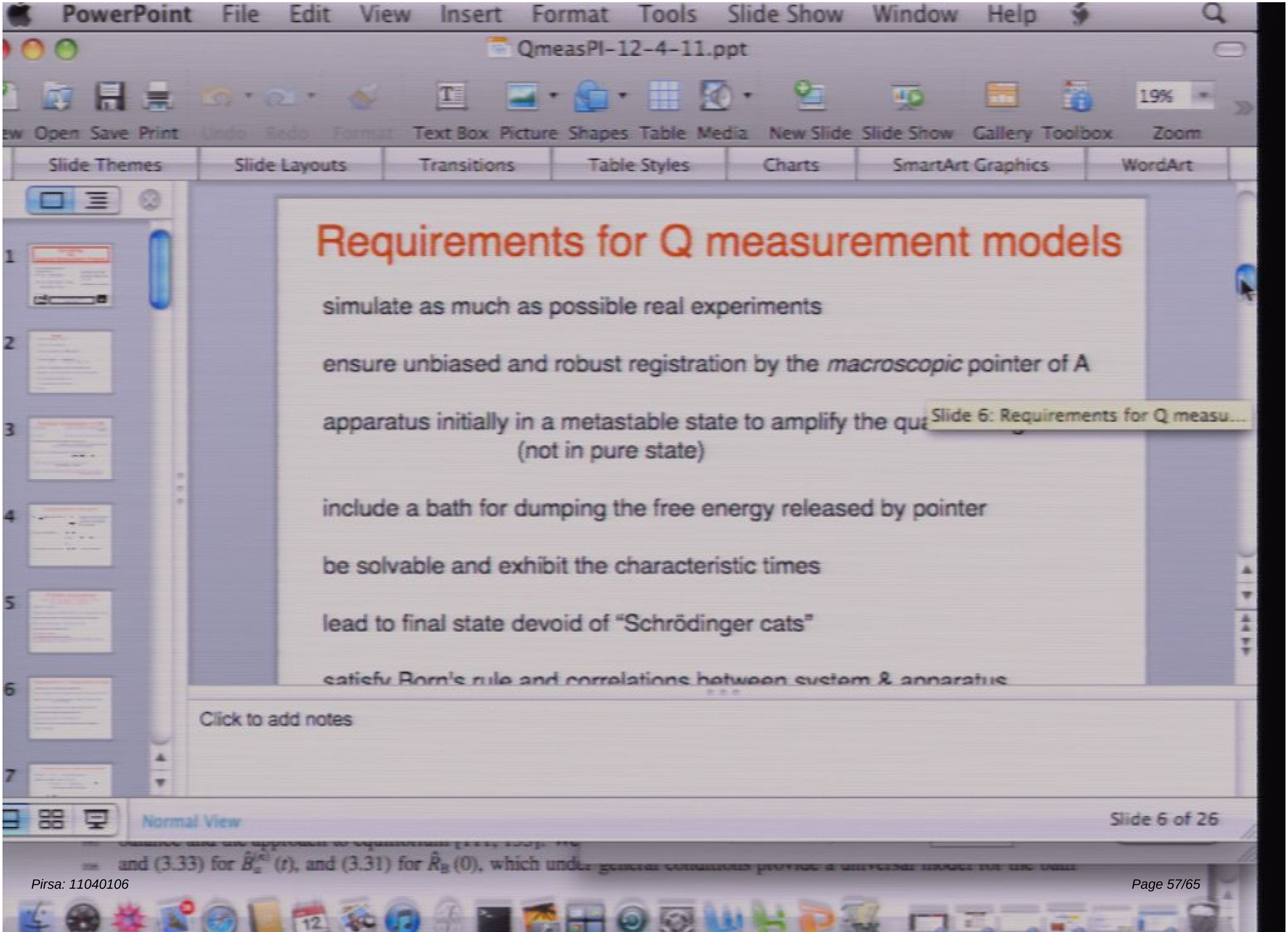
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Slide 6: Requirements for Q measu...
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Normal View
Slide 6 of 26

ensure unbiased and robust registration by the *macroscopic* pointer of A

apparatus initially in a metastable state to amplify the quantum signal
(not in pure state)

include a bath for dumping the free energy released by pointer

be solvable and exhibit the characteristic times

lead to final state devoid of "Schrödinger cats"

satisfy Born's rule and correlations between system & apparatus

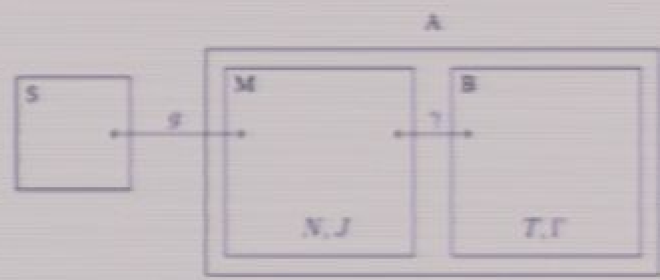
solvable and flexible

Click to add notes

Apparatus = Magnet + Bath (A = M + B).

M = N spins-½ $\sigma^{(n)}$. Magnetization $\hat{m} = \frac{1}{N}\sum_{n=1}^{N}\hat{\sigma}_z^{(n)}$

B = phonon bath: harmonic oscillators



$$\hat{H} = \hat{H}_{\rm S} + \hat{H}_{\rm SA} + \hat{H}_{\rm A}$$

$$\hat{H}_{\rm S} = 0 \quad \hat{H}_{\rm SA} = -g\hat{s}_z\sum_{n=1}^{N}\hat{\sigma}_z^{(n)} = -Ng\hat{s}_z\hat{m}$$

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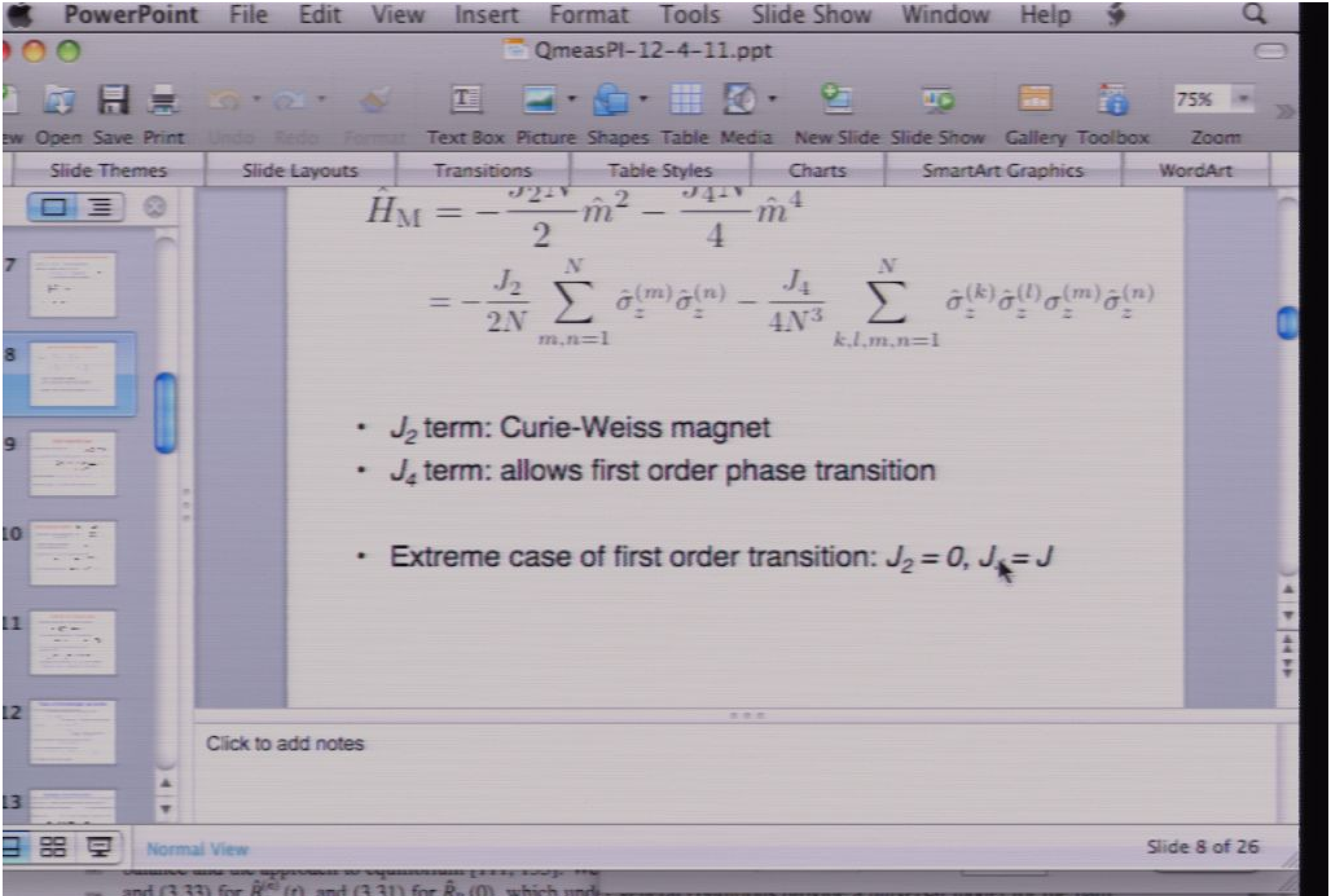
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$\hat{H}_{\mathrm{M}} = -\frac{J_2 N}{2}\hat{m}^2 - \frac{J_4 N}{4}\hat{m}^4$
$= -\frac{J_2}{2N}\sum_{m,n=1}^{N}\hat{\sigma}_z^{(m)}\hat{\sigma}_z^{(n)} - \frac{J_4}{4N^3}\sum_{k,l,m,n=1}^{N}\hat{\sigma}_z^{(k)}\hat{\sigma}_z^{(l)}\sigma_z^{(m)}\hat{\sigma}_z^{(n)}$
• J_2 term: Curie-Weiss magnet
• J_4 term: allows first order phase transition
• Extreme case of first order transition: $J_2 = 0$, $J_4 = J$
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Standard harmonic oscillator bath: $$\hat{H}_{\mathrm{B}} = \sum_{n=1}^{N} \sum_{a=x,y,z} \sum_{k} \hbar\omega_k \hat{b}^{\dagger(n)}_{k,a} \hat{b}^{(n)}_{k,a}$$

x,y,z components of the spins of M couple to their own harmonic oscillators

$$\hat{H}_{\mathrm{MB}} = \sqrt{\gamma} \sum_{n=1}^{N} \left(\hat{\sigma}_x^{(n)} \hat{B}_x^{(n)} + \hat{\sigma}_y^{(n)} \hat{B}_y^{(n)} + \hat{\sigma}_z^{(n)} \hat{B}_z^{(n)} \right) \equiv \sqrt{\gamma} \sum_{n=1}^{N} \sum_{a=x,y,z} \hat{\sigma}_a^{(n)} \hat{B}_a^{(n)}$$

$$\hat{B}_a^{(n)} = \sum_k \sqrt{c(\omega_k)} \left(\hat{b}^{(n)}_{k,a} + \hat{b}^{\dagger(n)}_{k,a} \right)$$

Bath characterized by $\mathrm{tr}_{\mathrm{B}} \left[\hat{R}_{\mathrm{B}}(0)\, \hat{B}_a^{(n)}(t)\, \hat{B}_b^{(p)}(t') \right] = \delta_{n,p} \delta_{a,b} K(t - t')$

Weak M-B coupling: $\gamma \ll 1$ allows to work at lowest order in γ

uncorrelated with A

Apparatus A starts in mixed state,
product of magnet M and bath B

$$\hat{R}_A(0) = \hat{R}_M(0) \otimes \hat{R}_B(0)$$

Magnet M: N spins ½, starts as paramagnet (mixed state)

$$R_M(0) = \Pi_{n=1}^{N} \rho^{(n)}(0), \qquad \rho^{(n)}(0) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Bath: Gibbs state (mixed state) $R_B(0) = \frac{e^{-\beta \hat{H}_B}}{Z_B}$ $\beta = 1/T$

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$$\hat{H}_{\mathrm{SA}} = -g\hat{s}_z \sum_{n=1}^{N} \hat{\sigma}_z^{(n)} = -Ng\hat{s}_z\hat{m}$$

Trace out Apparatus (Magnet+Bath) in von Neumann eqn

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{D}} = [\hat{H}, \hat{\mathcal{D}}] \qquad \mathcal{D} = \begin{pmatrix} \mathcal{D}_{\uparrow\uparrow} & \mathcal{D}_{\uparrow\downarrow} \\ \mathcal{D}_{\downarrow\uparrow} & \mathcal{D}_{\downarrow\downarrow} \end{pmatrix}$$

The 4 sectors decouple; only interaction H_{SA} remains

Trace out A = M + B

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} r_{ij} &\equiv \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{tr}_{\mathrm{M,B}}\, \mathcal{D}_{ij} = -gN(s_i - s_j)\mathrm{tr}_{\mathrm{M,B}}[\hat{m}, \mathcal{D}_{ij}] \\ &= 0 \qquad \text{iff } s_i = s_j \end{aligned}$$

Diagonal terms of r(t) conserved ->= $r_{ii}(t) = r_{ii}(0)$: Born probabilities

Off-diagonal terms evolve => disappearence of Schrodinger cats

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$$\hat{H}_{\mathrm{SA}} = -g\hat{s}_z \sum_{n=1}^{N} \hat{\sigma}_z^{(n)} = -Ng\hat{s}_z\hat{m}$$

Trace out Apparatus (Magnet+Bath) in von Neumann eqn

$$i\hbar\frac{d}{dt}\hat{\mathcal{D}} = [\hat{H}, \hat{\mathcal{D}}] \qquad \mathcal{D} = \begin{pmatrix} \mathcal{D}_{\uparrow\uparrow} & \mathcal{D}_{\uparrow\downarrow} \\ \mathcal{D}_{\downarrow\uparrow} & \mathcal{D}_{\downarrow\downarrow} \end{pmatrix}$$

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$$\hat{\rho}^{(n)}(t) = \frac{1}{2}\mathrm{diag}(e^{2igt/\hbar}, e^{-2igt/\hbar})e^{-g^2\gamma K t^4}$$

Larmor procession damping by zero-point flucts of bath

Schrodinger cat:

$$r_{\uparrow\downarrow}(t) = r_{\uparrow\downarrow}(0)(\cos\frac{2gt}{\hbar})^N e^{-Ng^2\gamma K t^4}$$

$$= r_{\uparrow\downarrow}(0) e^{-(t/\tau_{red})^2} e^{-(t/\tau_{dec})^4}$$

dephasing as in NMR — decoherence (zpf of bath) comes in later

Recurrences eliminated by: decoherence by zero point flucts of bath
or even by a spread in g's

Creation of multi-particle correlations: weak, dying out

$$\langle(\hat{s}_x - i\hat{s}_y)\hat{m}^k(t)\rangle_c \sim \langle(\hat{s}_x - i\hat{s}_y)(0)\rangle i^k t^k e^{-(t/\tau_{red})^2} \leq \left(\frac{1}{\sqrt{N}}\right)^k$$

Schrodinger cat terms die out quickly

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