Title: Ordering in the Spatially Anisotropic Heisenberg Model on a Triangular Lattice

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Abstract: The spin 1/2 Heisenberg model on a triangular lattice with interchain exchange, J', weaker than the intrachain exchange J, is a particularly well-studied frustrated magnet because of its relevance to Cs2CuCl4, which is thought to be in close proximity to a spin liquid phase. Although an incomensurate spiral state is stable for J'~J, a variet of theoretical studies find evidence for spin liquid behavior well before the decoupled chain limit, J'=0, is reached. However, a renormalization group approach found the surprising result that a collinear antiferromagnetic phase was stable for small J'/J. This talk will briefly review earlier studies and present new results on the relative stability of spiral, collinear and spin liquid phases.

Ordering in the Spatially Anisotropic Heisenberg Antiferromagnet on an Triangular Lattice

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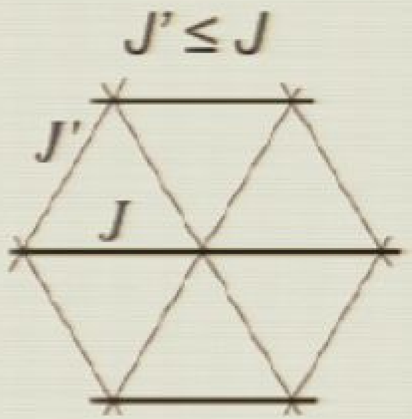
and Erik Sørensen, McMaster University

4 Corners, Perimeter, April 26, 2011

Spin ½ Heisenberg on Anisotropic Triangular Lattice

$$H = J\sum_{x,y}\vec{S}_{x,y}\cdot\vec{S}_{x+1,y} + J'\sum_{x,y}\vec{S}_{x,y}\cdot(\vec{S}_{x-1/2,y+1}+\vec{S}_{x+1/2,y+1})$$



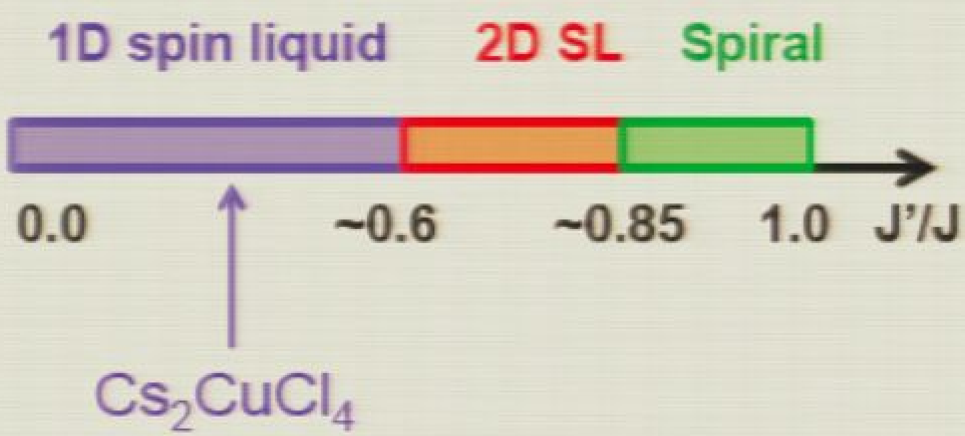
- Minimal model for Cs_2CuCl_4 (also DM)
- Classical gs is incommensurate spiral state with $q_x = \pi + 2\sin^{-1}(J'/2J) = \pi + \varepsilon$
- Spin wave theory works fine for J'~J

What is the ground state for small J'/J?

Spin ½ Heisenberg on Anisotropic Triangular Lattice

Spin liquid?

1d spin liquid for J'=0, but not likely candidate for 2d spin liquid – no macroscopic classical degeneracy or large magnetic ring exchange



Alicea, Motrunich, Fisher (2006)
Isakov, Senthil, Kim (2005)
Weng, Sheng, Weng, Bursill (2006)
Yunoki, Sorella (2006)
Heidarian, Sorella, Becca (2009)

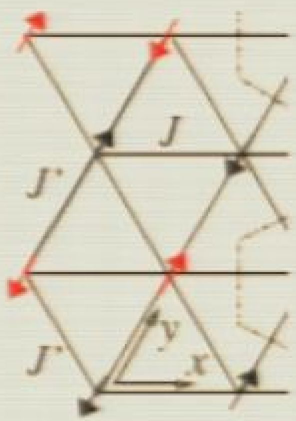
Incommensurate Spiral?

$$q_x - \pi = \varepsilon \sim e^{-a(J/J')^2}$$

Random Phase Approximation with weakly coupled chains: Bocquet, Essler, Tsvelik, Gogolin (2001)
Pardini, Singh (2008)

Collinear Antiferromagnet?

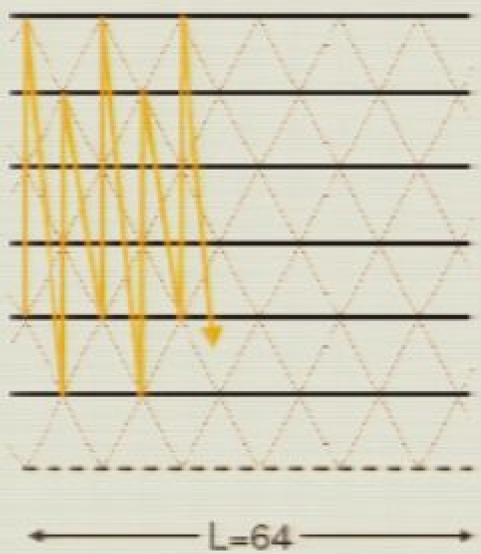
Starykh and Balents, PRL (2007)
Starykh, Katsura, Balents, PRB (2010)
Bishop (2009, 2010)



Renormalization group analysis of weakly coupled chains → collinear AF order selected at order $(J'/J)^4$

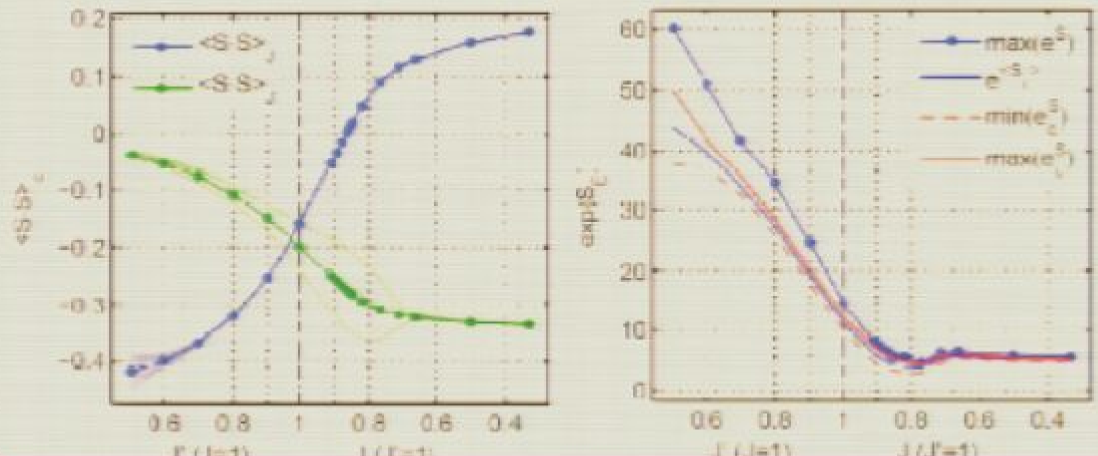
DMRG Study from A. Weichselbaum and S.R. White (in preparation)

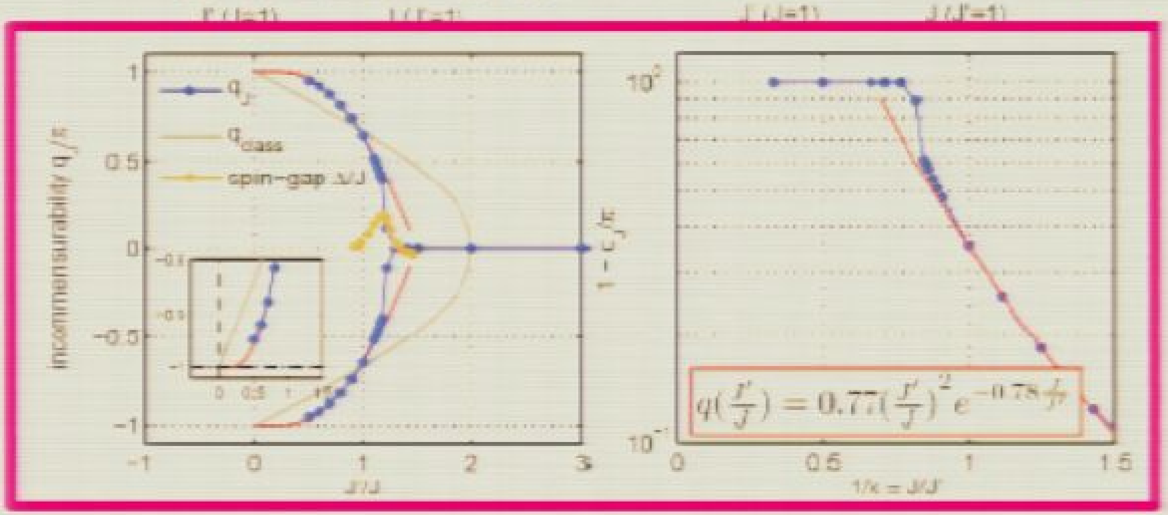
6-chain system (cylindrical BC, periodic in y-direction)



D≤3200, $\varepsilon=10^{-4}$, 24 sweeps

- dimerization reappears for 3 coupled zigzag ladders
- similar exponential fit of incommensurablitity for J'/J<1





T=0 linked cluster series expansion
T. Pardini & R.R.P. Singh, PRB (2008)

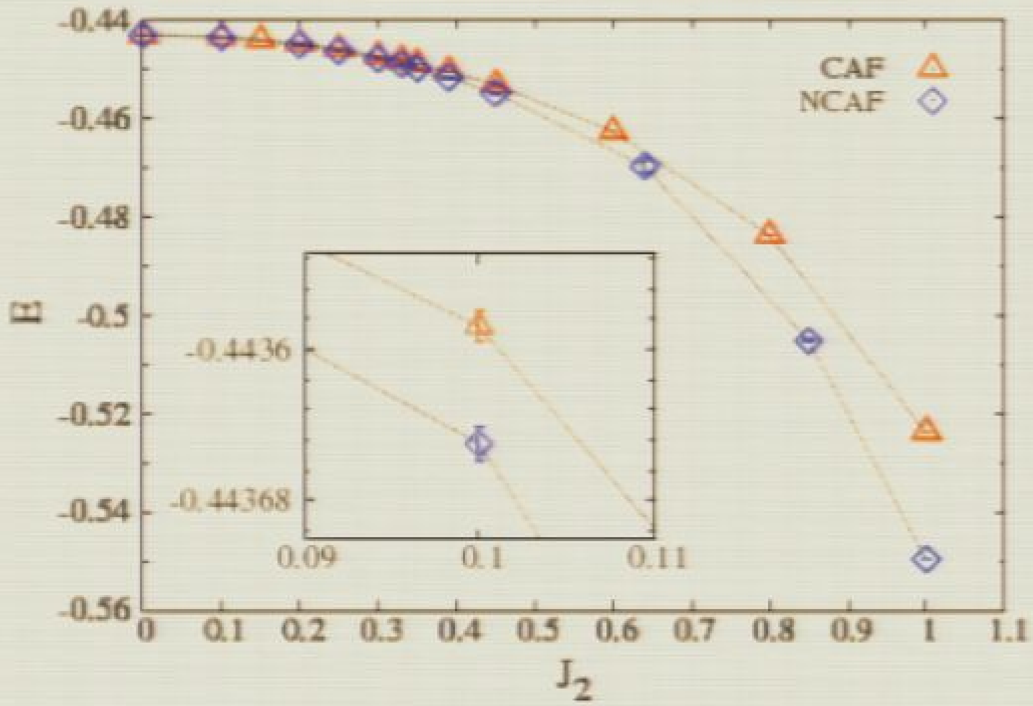


FIG. 4. (Color online) Energy for CAF (red triangles) and NCAF (blue squares) phases. The inset shows a zoom in of the region around $J_2=0.1$.

Spiral appears to have lower energy for all J'.

[Different result from Coupled Cluster Method (Bishop et al. 2010)]

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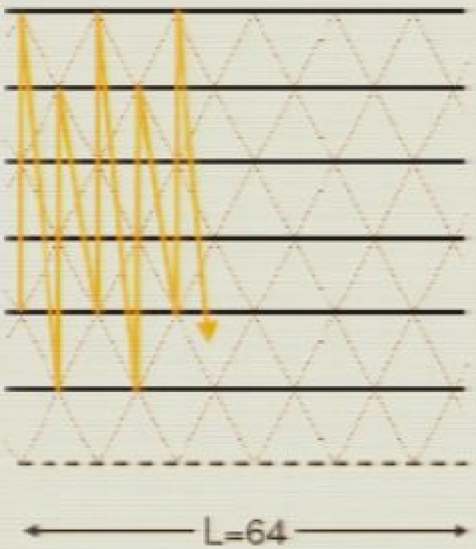

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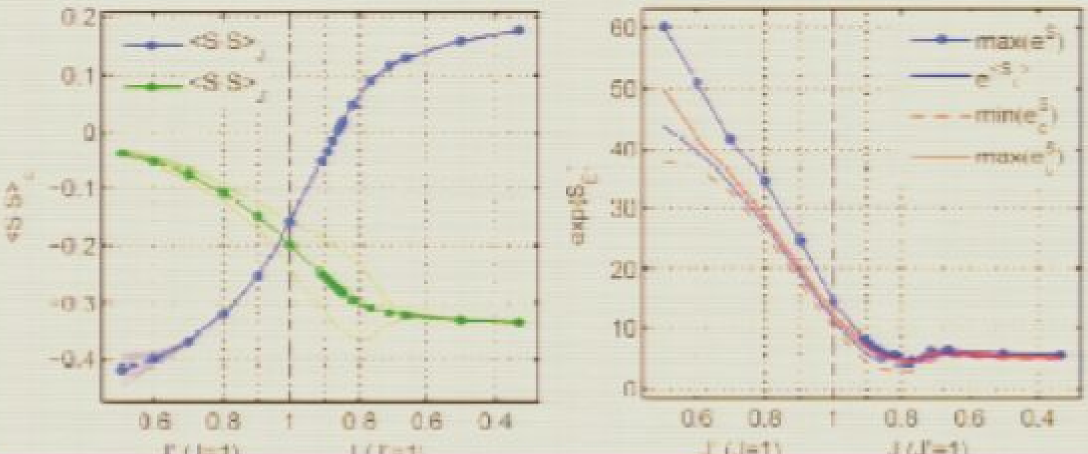

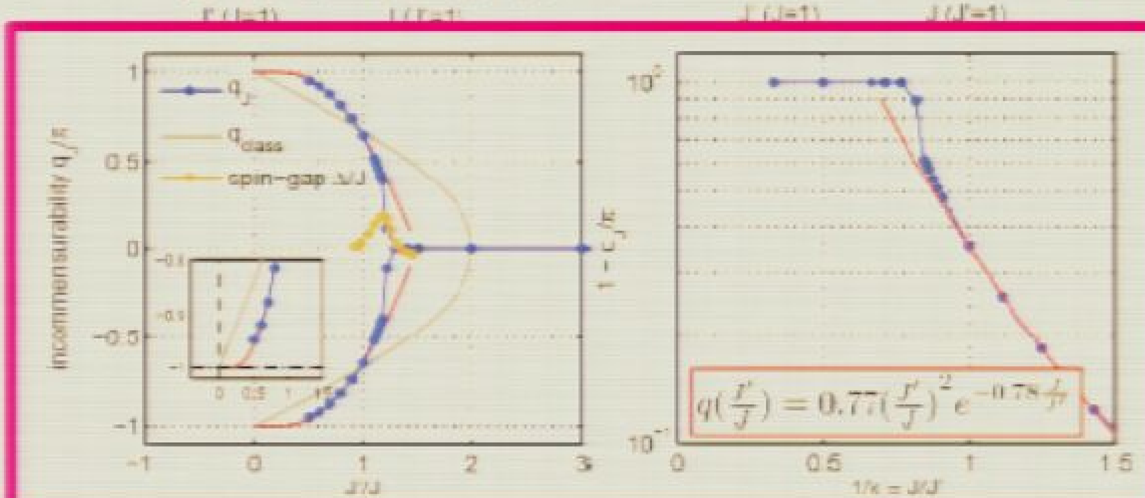

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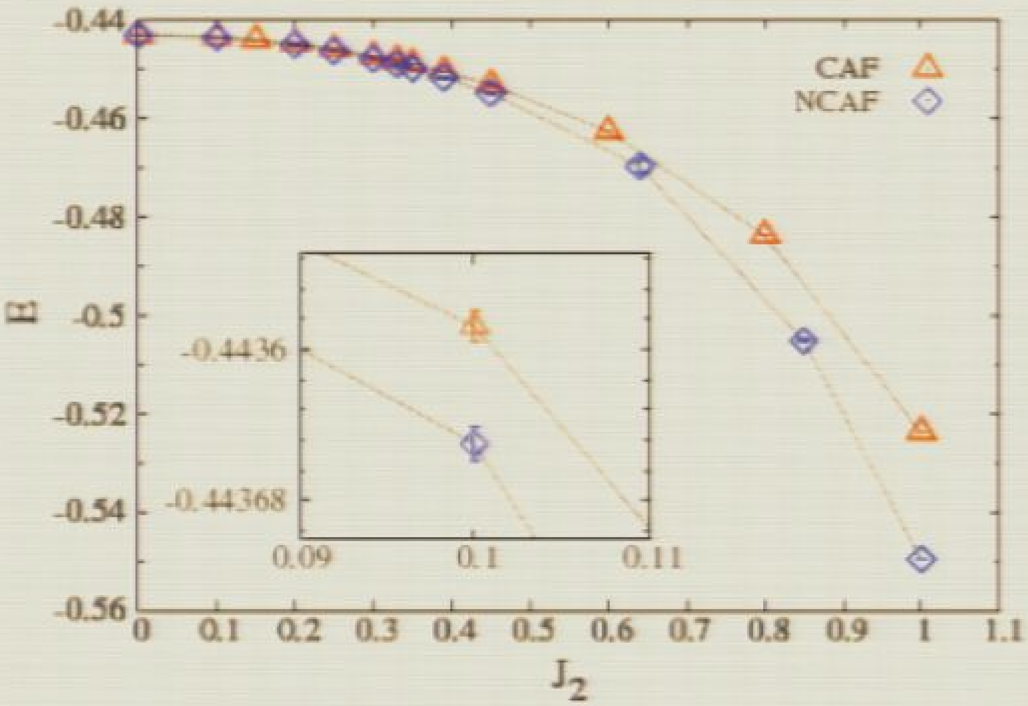

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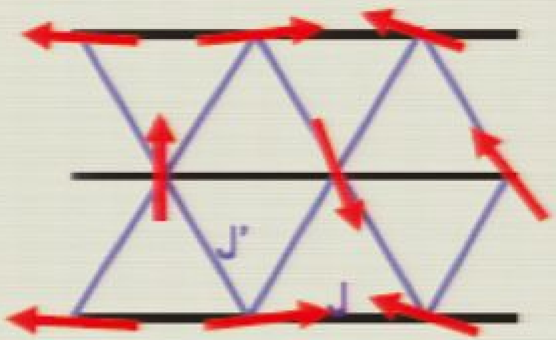
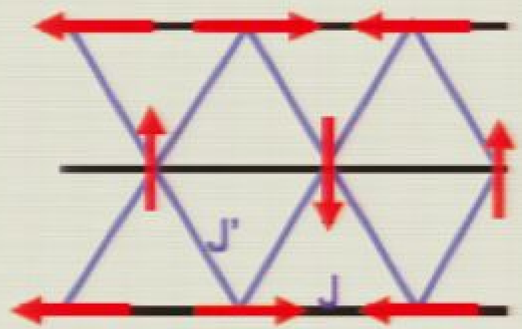


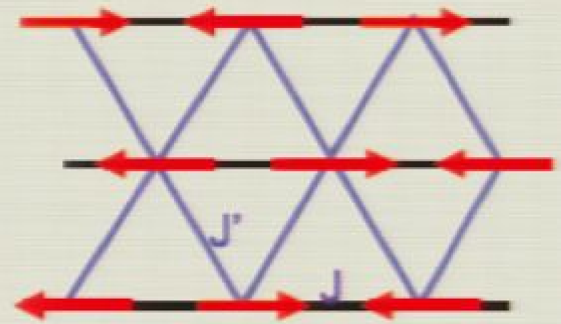
0 < J'/J < 1

Classical Spiral SDW



J'/J = 0

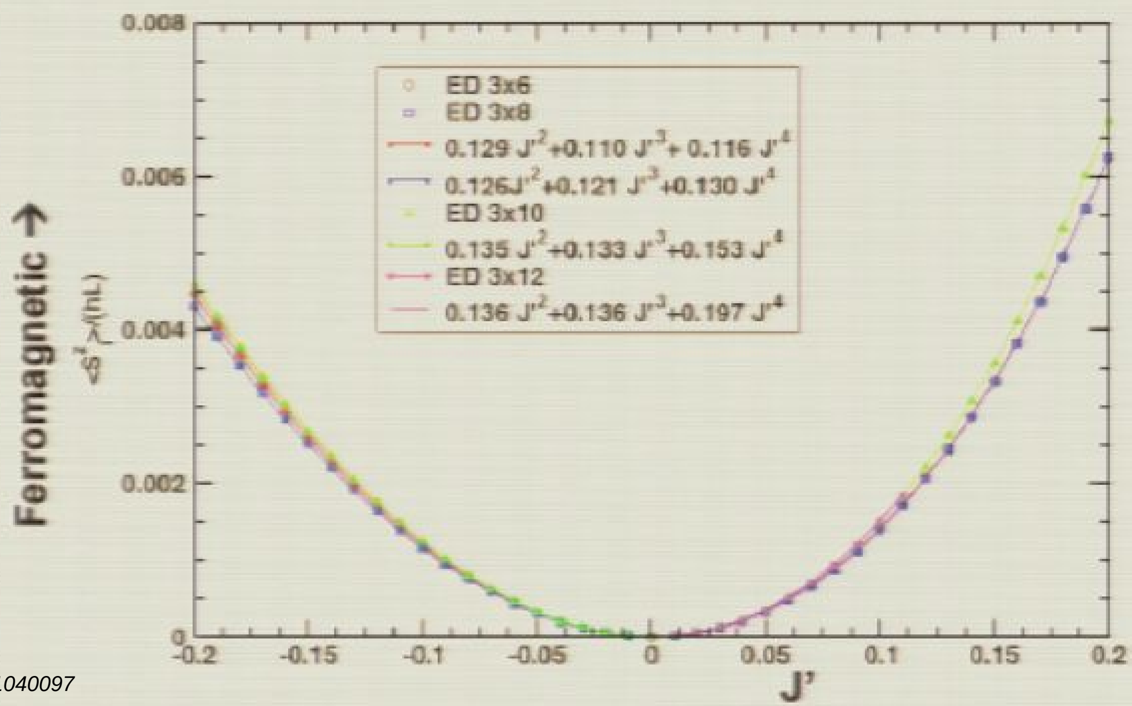
Spiral at J'→0



Small J'/J

Collinear AF

Selected at order $(J'/J)^4$



Staggered interchain susceptibility:

Ferromagnetic correlation between 2nd nn chains for small systems at order J'^2 and higher. What happens for large L?

Open BC to not frustrate spiral

Quasi-1d RG

Weakly coupled chains – make continuum approx along chain direction, x, keeping y discrete.

$$\vec{S}_{x,y} \rightarrow \vec{S}_y(x) = \vec{M}_y(x) + (-1)^x \vec{N}_y(x)$$

Heisenberg Hamiltonian becomes $H^{intra} + H^{inter}$

$$H^{\text{intra}} = \sum_y \{ H_y^{WZNW} + \gamma_{bs} \int dx J_{R,y} \cdot J_{L,y} \}$$

$$H^{\text{inter}} = \sum_y \int dx \left\{ \gamma_M M_y \cdot M_{y+1} + \gamma_{tw} \frac{(-1)^y}{2} (N_y \cdot \partial_x N_{y+1} - \partial_x N_y \cdot N_{y+1} \right\}$$

Initially J'

Relevant 2nd nn chain interaction generated:

$$H^{2nn} = \sum_y \int dx \{ g_N N_y \cdot N_{y+2} + g_\varepsilon \varepsilon_y \varepsilon_{y+2} \}$$

RG Equations

Key parts of RG flows:

$$\partial_l \gamma_{bs} = \gamma_{bs}^2$$

$$\partial_l \gamma_{tw} = -\frac{\gamma_{bs}}{2}\gamma_{tw} + \gamma_M \gamma_{tw} - 3\gamma_{tw} g_N$$

$$\partial_l g_N = (1 - \frac{\gamma_{bs}}{2}) g_N + \frac{\gamma_{tw}^2}{4} + g_M \xi_N$$

Only antiferromagnetism.
FM buried in nature of γ_{tw}^2.

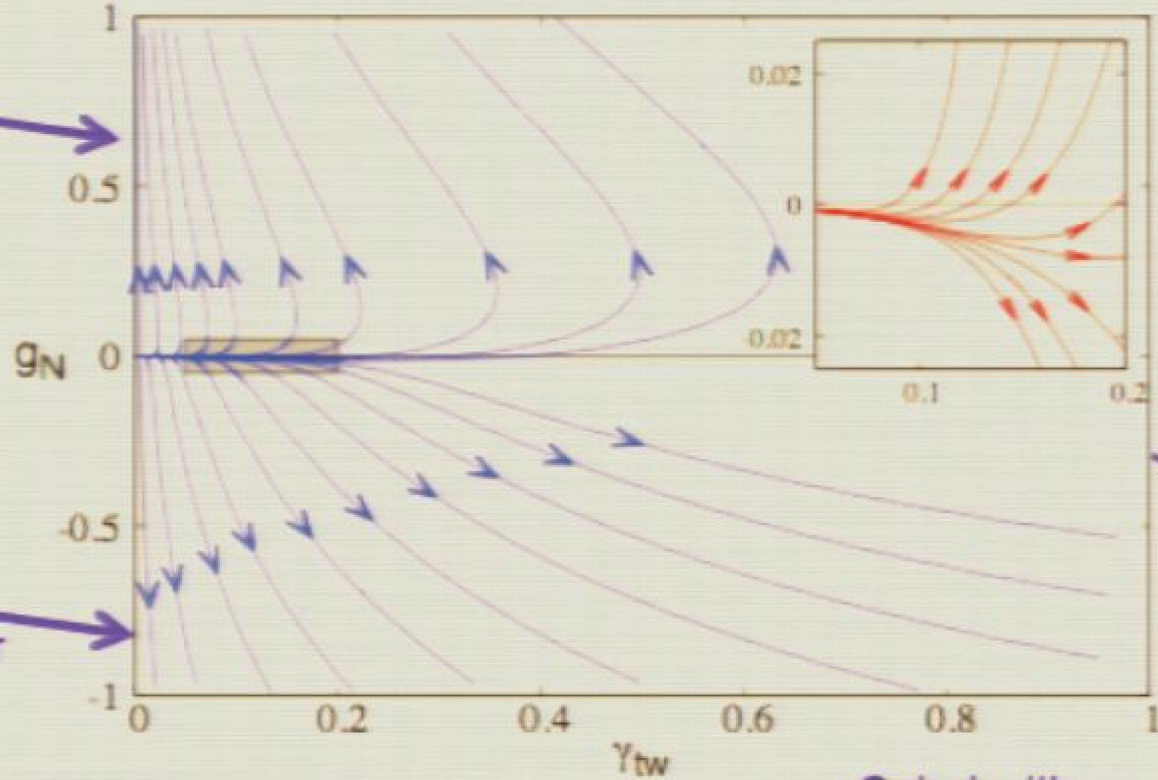
$$-\gamma_{tw}^2 \int dz dz' \left[\partial_x \partial_{x'} \frac{1}{|z - z'|} \right] N_{y-1}(z) N_{y+1}(z')$$

Ferromagnetic at short length scales, but this is eroded by antiferromagnetism at longer length scales.

RG eqs + initial conditions → fate of system

RG flows for different initial conditions

Collinear AF



Spiral with
$q-\pi \sim (J'/J)^{n+1}$
If g_N grows in order $(J'/J)^n$.

Spiral with
$q-\pi \sim \exp[-a(J/J')^2]$
if neither g_N nor g_ε can grow.

For weak AF initial conditions, spiral can become more stable as J' increases.

Staryk & Balents result for $L>(J/J')^2/b$ recovered for initial conditions where $g_N(0)$ is "tuned" except for small AF imbalance $b(J'/J)^4$.

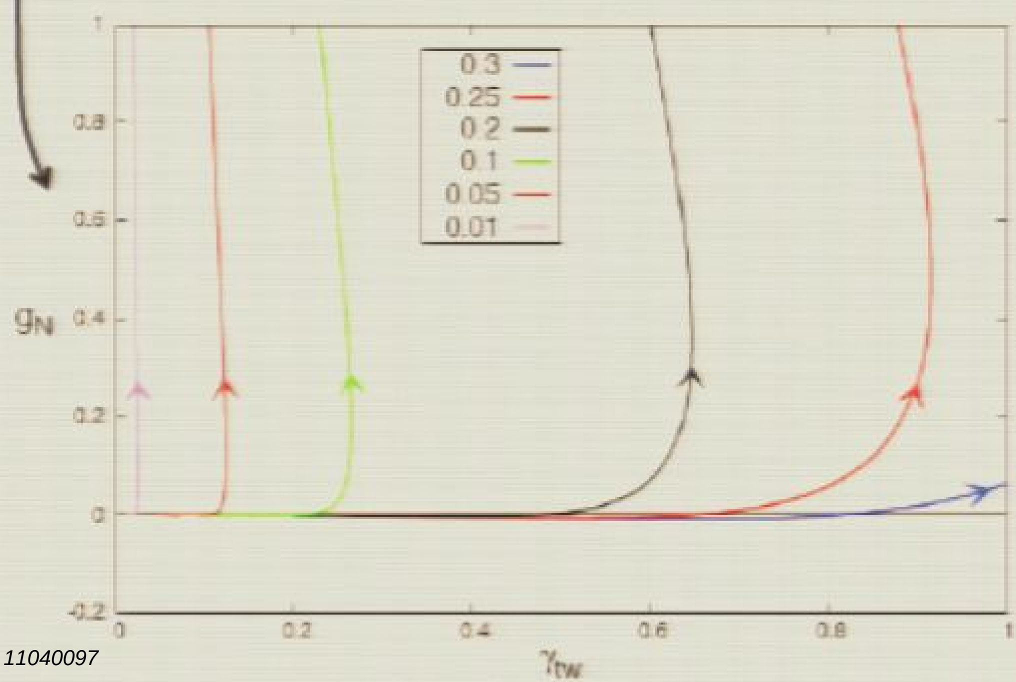
$$\partial_l \gamma_{tw} = -\frac{\gamma_{bs}}{2}\gamma_{tw} + \gamma_M \gamma_{tw} - 3\gamma_{tw} g_N$$

$$\partial_l g_N = \left(1-\frac{\gamma_{bs}}{2}\right) g_N + \frac{\gamma_{tw}^2}{4} + g_M \zeta_N$$

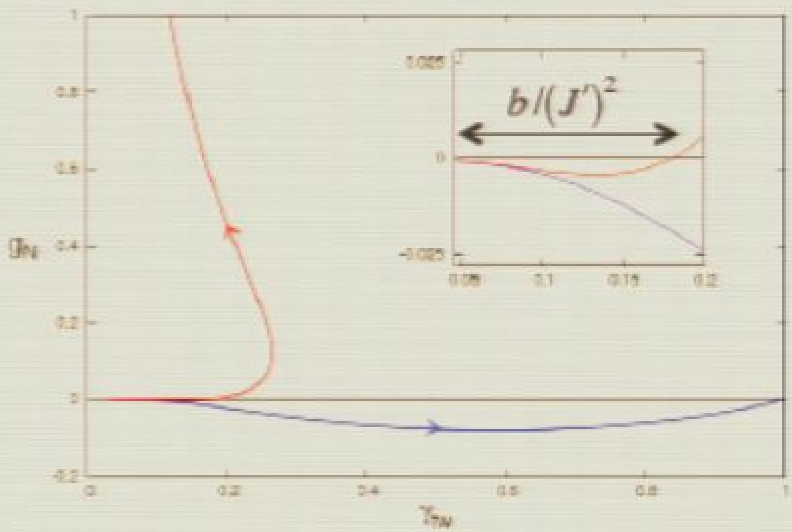
$$-\frac{g_N^c(0)}{\gamma_{tw}^2(0)} = e^{-\frac{1}{\gamma_{bs}(0)}} \frac{\sqrt{-\gamma_{bs}(0)}}{4} \Gamma_{\text{Upper}}\left(\frac{3}{2}, -\frac{1}{\gamma_{bs}(0)}\right)$$

$\rightarrow$ ¼ for $\gamma_{bs}(0)=0$

$$\Delta \mathbf{g}_N(0) = -C\gamma_{bs}\gamma_{tw}^2\gamma_M^2$$



At $J'_{cr} \approx 0.3J$
$\gamma_{tw} \rightarrow 1$ first



g_N is ferromagnetic for $L<b/(J')^2$ ($b \approx 20$) → numerics difficult, i.e. FM for $L<200a_0$ at $J'/J=0.3$

$$\langle (-1)^x \vec{S}_{x_0,y} \cdot \vec{S}_{x_0+x,y+2} \rangle \sim -(J')^4 + (J')^2 f(x)$$

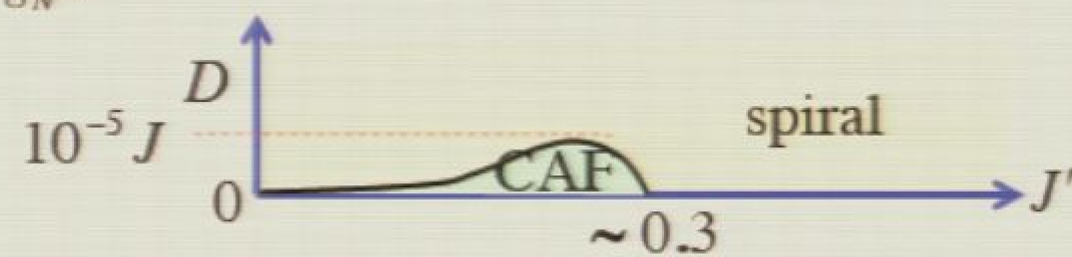
$$H_{DM} = D \sum_{x,y} \hat{z} \cdot \vec{S}_{x,y} \times \left(\vec{S}_{x+\frac{1}{2},y+1} - \vec{S}_{x-\frac{1}{2},y+1} \right)$$

The **Dzyaloshinskii-Moriya** interaction enhances (and is enhanced by) FM and tilts the balance toward the spiral state.

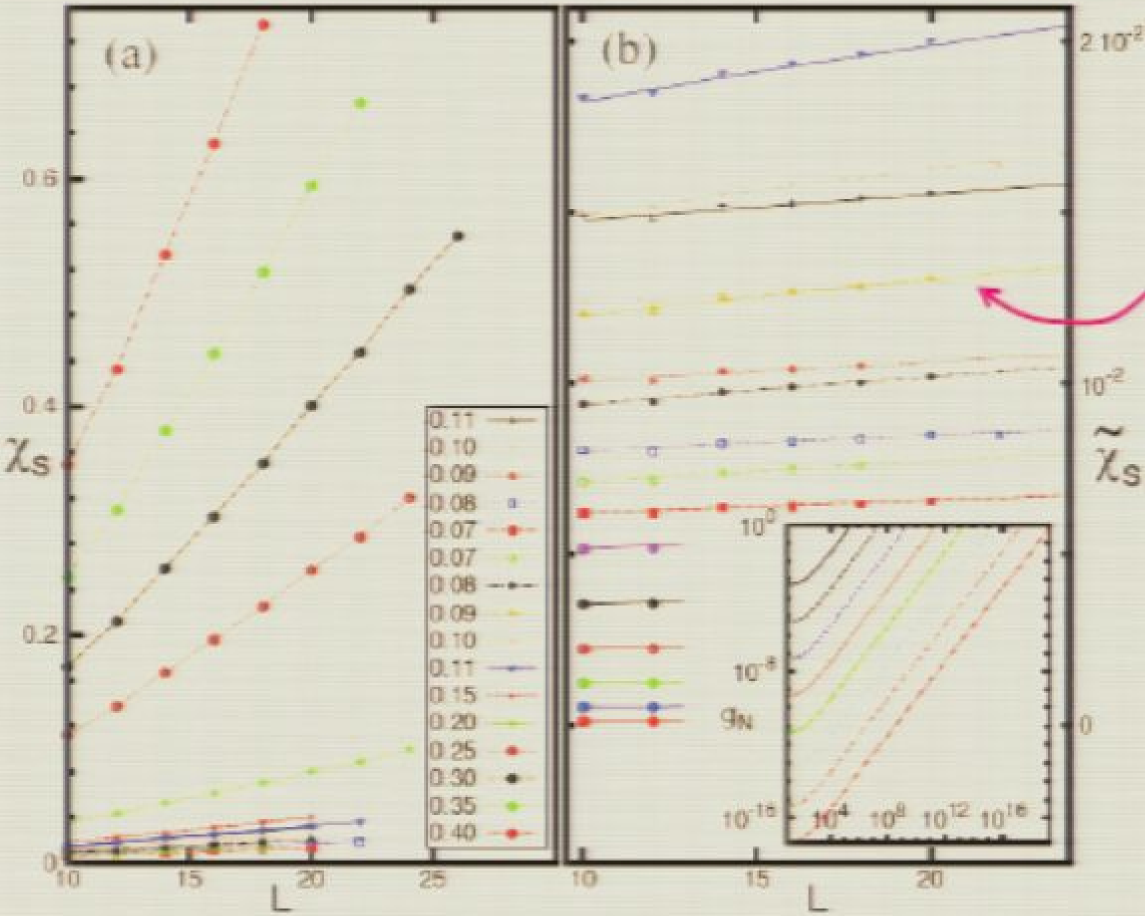
$$\boxed{D > \Delta g_N(0) \propto (J')^4 \Rightarrow \text{ spiral}}$$

$$\partial_l D = (1 - \frac{\gamma_{bx}}{2})D + \frac{1}{2}\gamma_M D - 4 g_N D$$

$$\partial_l g_N = \cdots -2D^2$$



Exact diagonalization and DMRG studies



Study the staggered interchain susceptibility for 24 ~ 100 spins (open BC). **Only ferro-magnetism seen for all J'.**

Fit susceptibility data for L≥10 to RG flows using initial conditions at L=10 as fitting parameters.

Find $g_N(L=10)=AJ'^2+BJ'^3$

A is 5% FM (consistent to zero within 1/L finite size effects)
B is 25-30% FM → spiral (also a 1/L effect?)

In agreement with DMRG on larger systems (J'≥0.5).

For small g_N (small J'):

$$\chi_s(L,J',L_0) \propto L[1-\gamma_{bs}(L_0)\ln(L/L_0)\}]^{\frac{1}{2}} g_N(L/L_0)$$

In conclusion

- Real space RG from short length scales shows competition between FM (short distances) and AF (long distances) and connects to numerics on finite size systems. Large size and extent of FM makes it difficult to see CAF in numerics.

- Transition between CAF and spiral at J'~0.3J. DM enhances spiral order relative to CAF, which does not survive for D/J>b $(J'/J)^4 \sim 10^{-5}$ at J'=0.3J.

- If collinear AF state is stable, $\langle N_y N_{y+2} \rangle$ must change sign as system size is increased. No sign of this with open BC. Interesting to study this as function of backscattering (add J_2).

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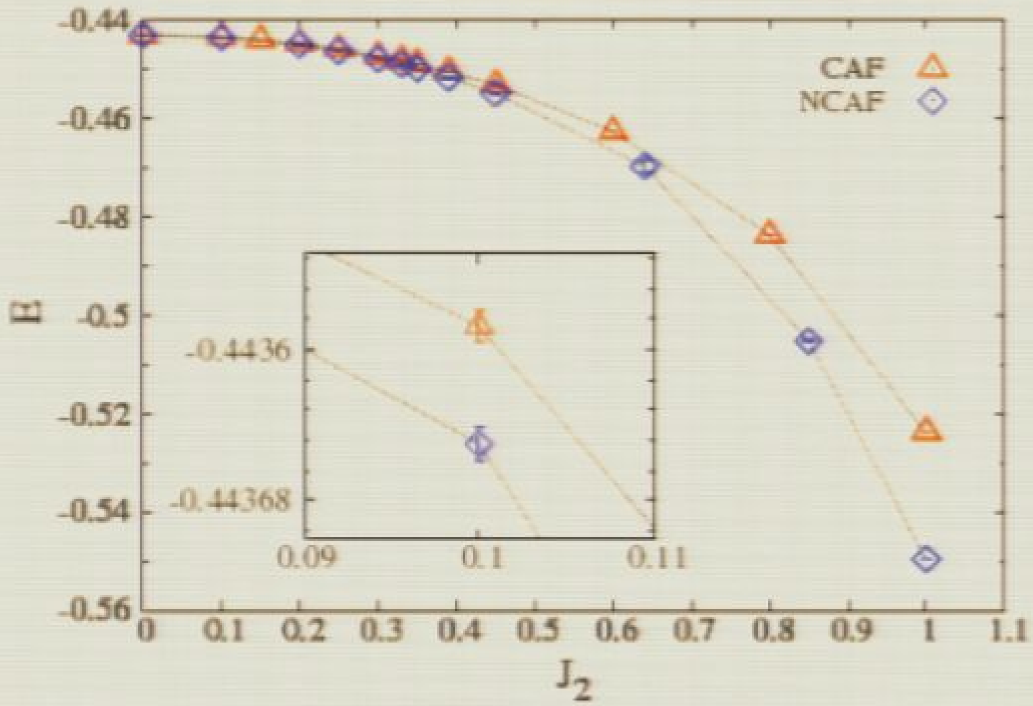

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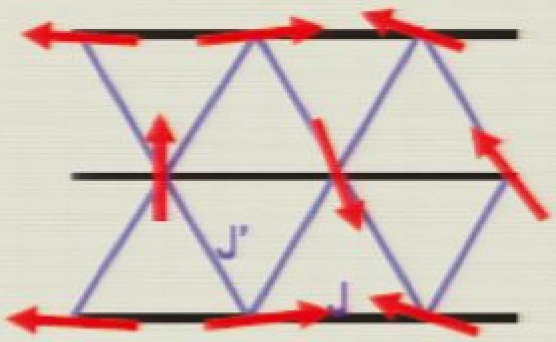

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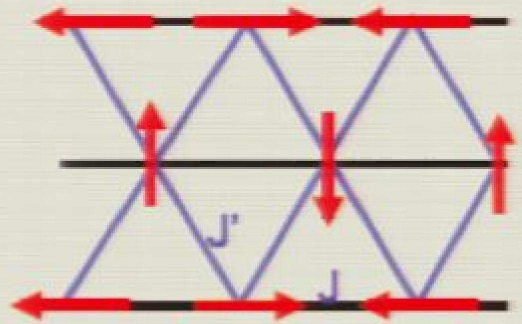
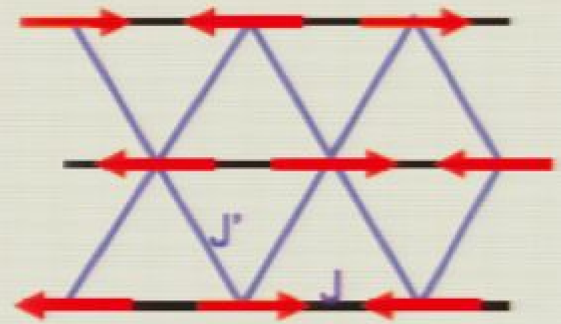


$J'/J = 0$

Spiral at $J' \to 0$



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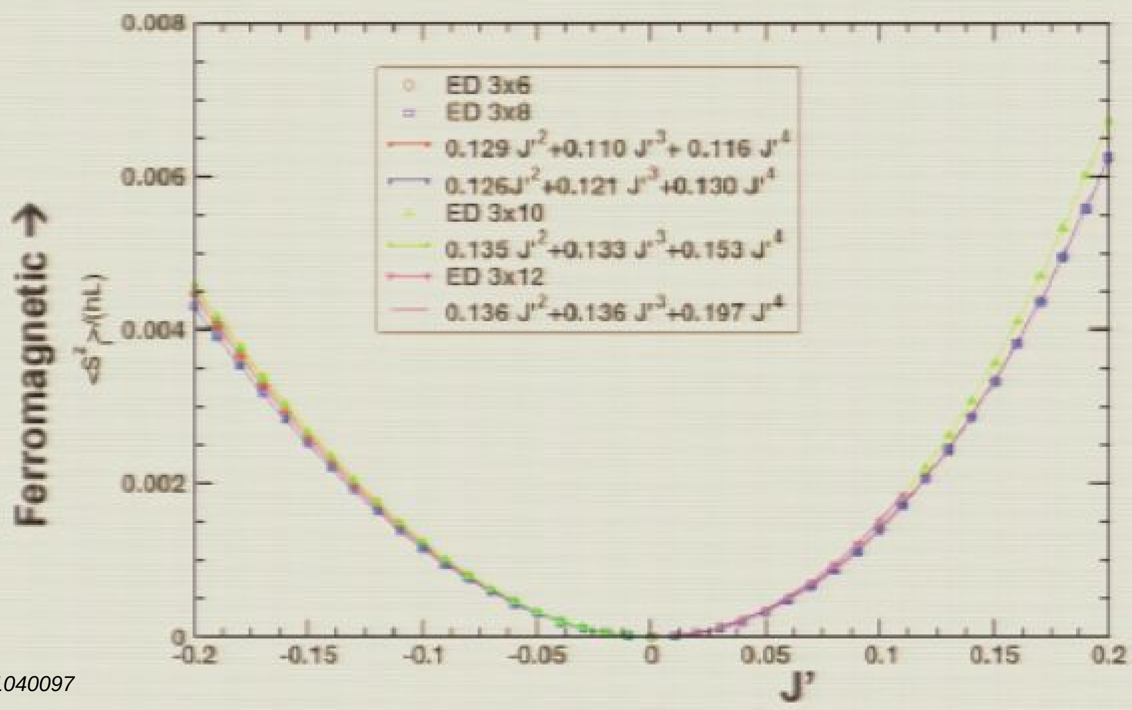

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