Title: Electronic liquid crystal phases of strongly correlated systems

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Abstract: It is a generic feature of strongly correlated electronic systems that several mechanisms (broadly represented by interactions) compete with each other. This competition often leads to the phenomenon of frustration. In
strongly correlated systems such as doped Mott insulators kinetic energy and Coulomb repulsion frustrate the tendency of doped holes to phase separate. The result is the onset of a set of novel phases, which we
dubbed Electronic Liquid Crystal (ELC) states, with varying degrees of complexity. Much like classical liquid crystals, electronic liquid crystal phases break translation and rotational invariance to varying degrees.
In this talk I will focus on the experimental evidence and theory of two these phases, stripes and nematics, in several different systems including two-dimensional electron gases in large magnetic fields, ruthenate
oxides, heavy fermions, and cuprate and pnictide superconductors.

Collaborators

S. Kivelson, V. Emery, E. Berg, D. Barci, E.-A. Kim, M. Lawler, V. Oganesyan, J. Eisenstein, A. Kapitulnik, A. Mackenzie, J. Tranquada

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- In addition to their **charge** and **spin orders**, these phases may also be **superconducting**

How Liquid Crystals got an $\hbar$
or
Soft Quantum Matter

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- Are charge orders friends or foes of high T_c superconductivity?

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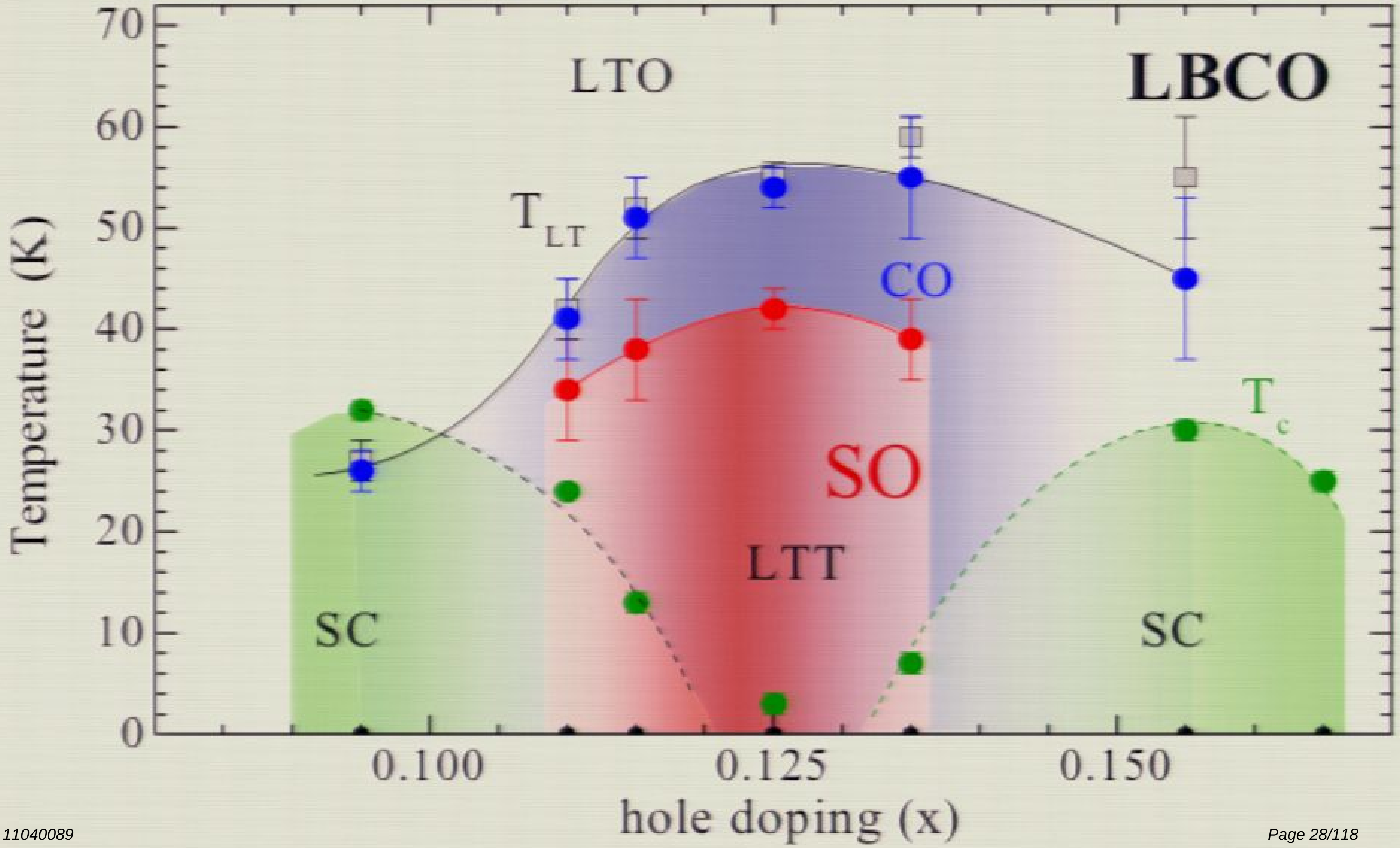
- Clearest example of the interconnection between charge and spin order with superconductivity
- Layer decoupling, long range charge and spin stripe order and superconductivity: a novel striped superconducting state, a ***Pair Density Wave***, in which charge, spin, and superconducting orders are intertwined!

Phase Diagram of LBCO



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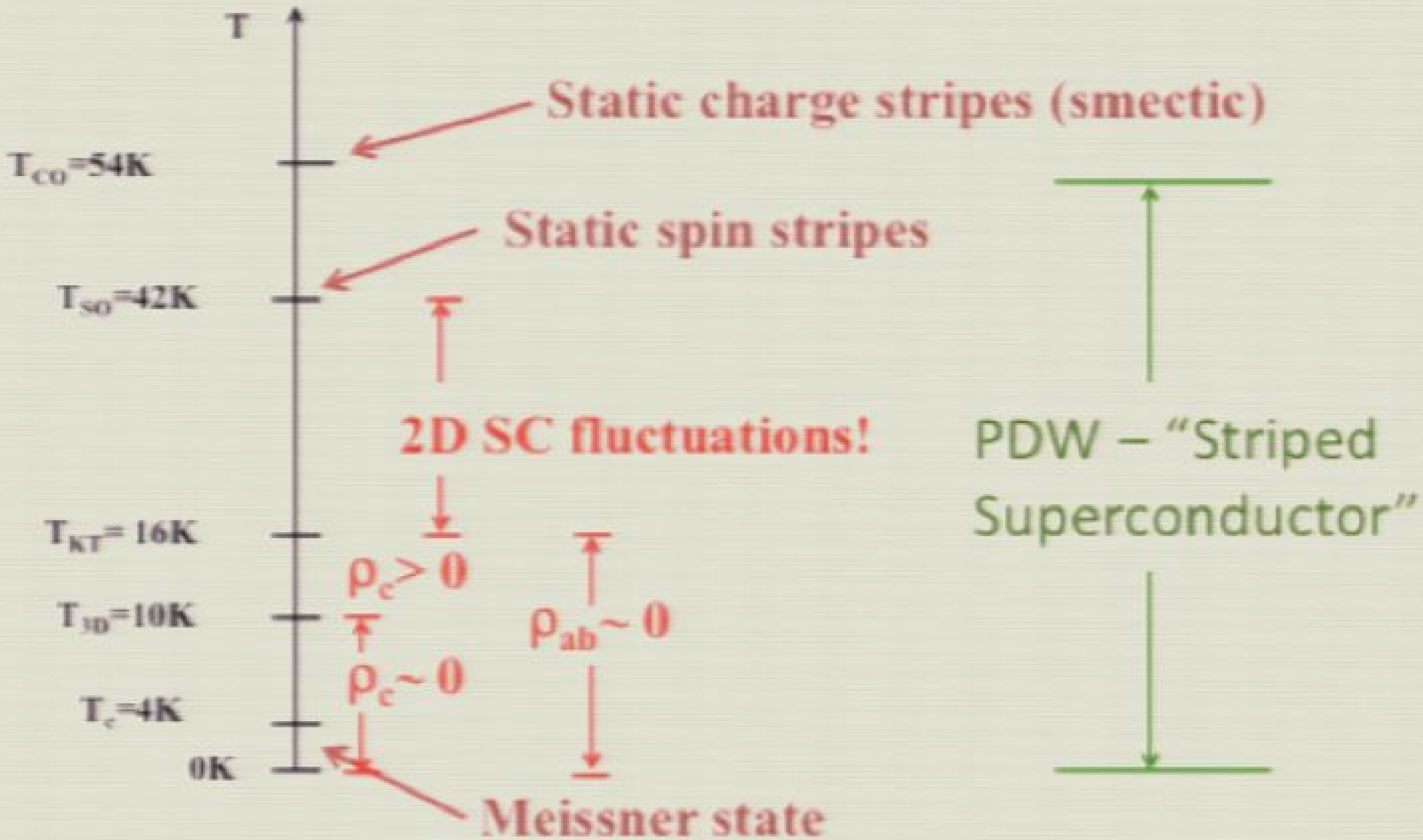
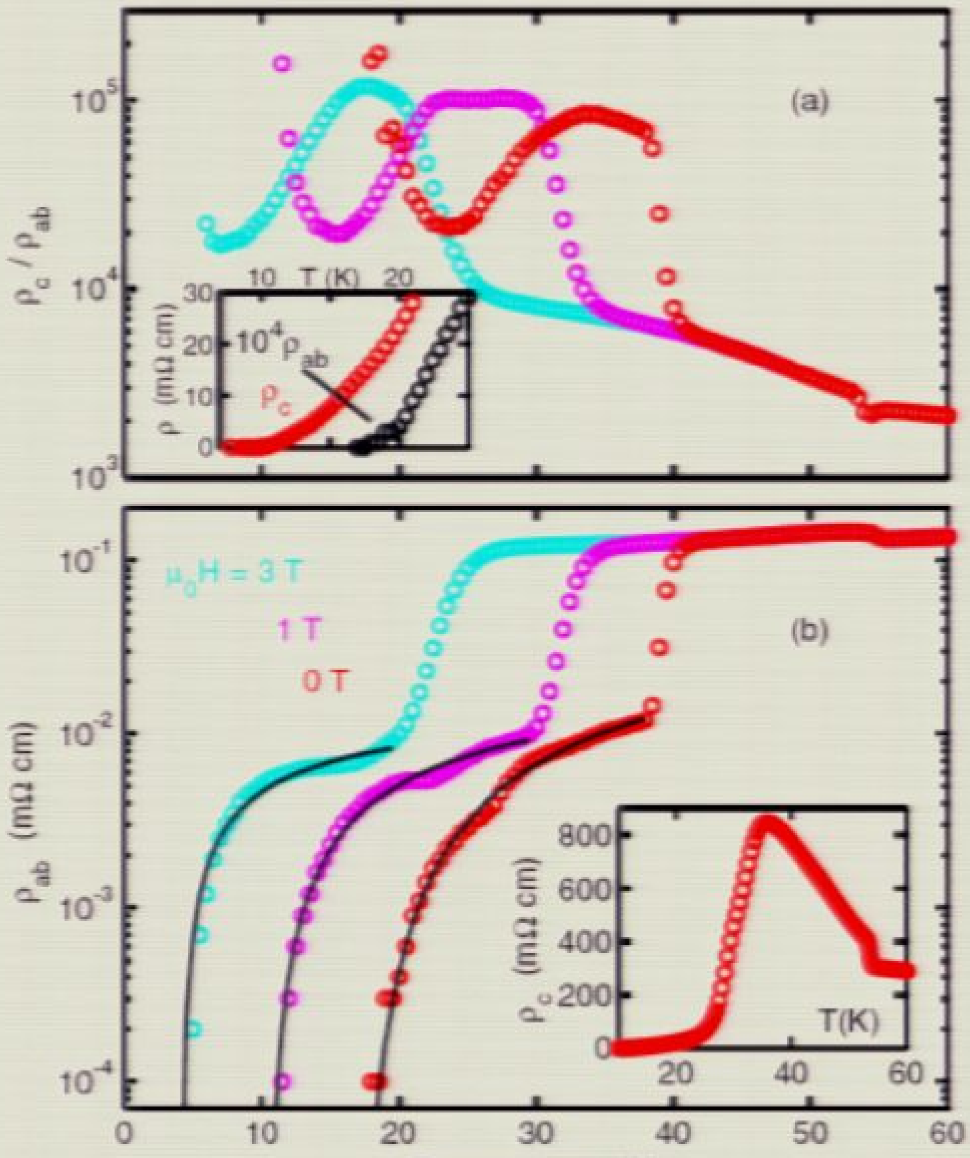
- Meissner state only below $T_c = 4$K

Cascade of thermal transitions/crossovers in $La_{1.877}Ba_{.125}CuO_4$

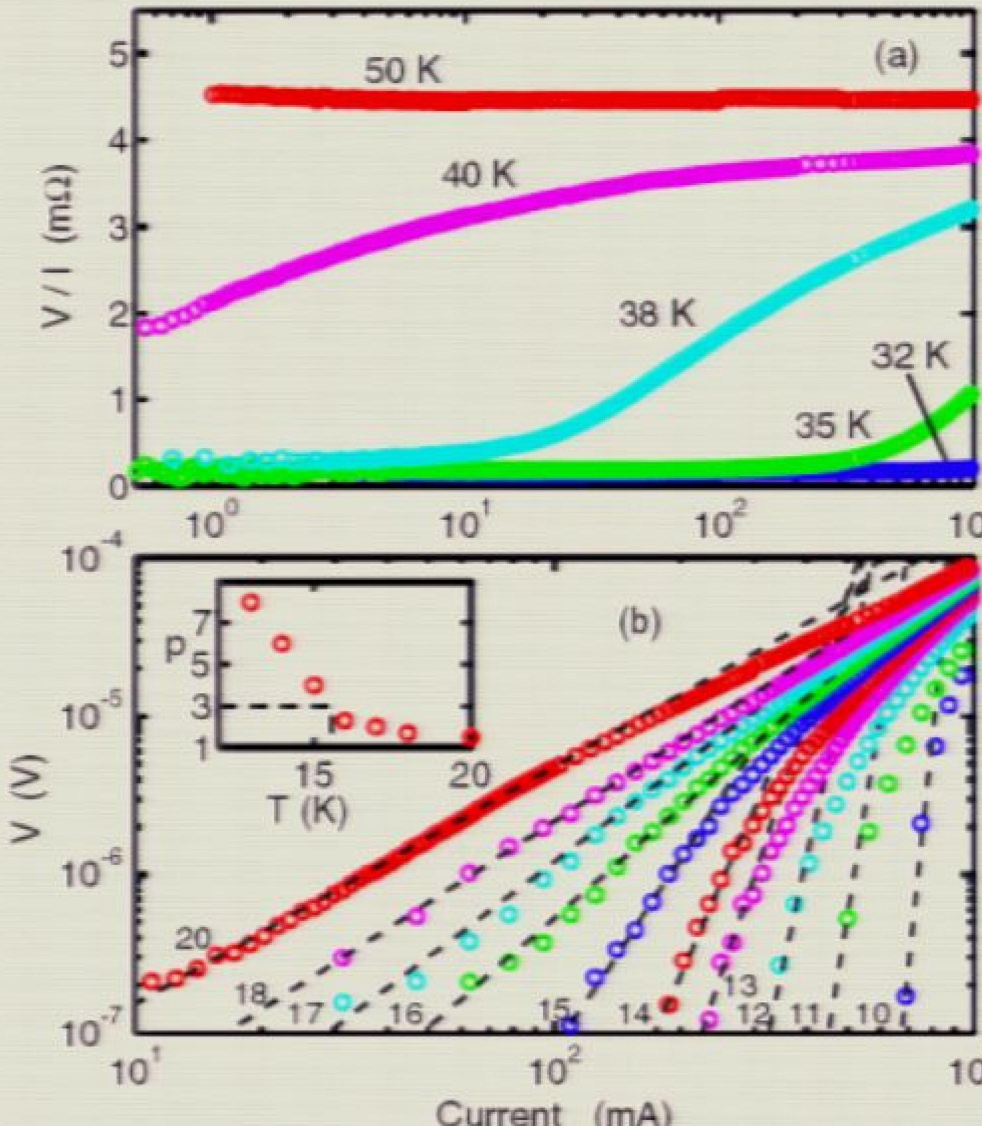


Anisotropic Transport Below the Charge Ordering transition



Li et al, 2007

The 2D Resistive State and 2D Superconductivity



Li, *et al.* 2007

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- Broad temperature range, $T_{3D} < T < T_{2D}$ with 2D superconductivity but not in 3D, as if there is not interlayer Josephson coupling
- In this regime there is both striped charge and spin order
- This can only happen if there is a special symmetry of the superconductor in the striped state that leads to an almost complete cancellation of the c-axis Josephson coupling.

A Striped Textured Superconducting Phase

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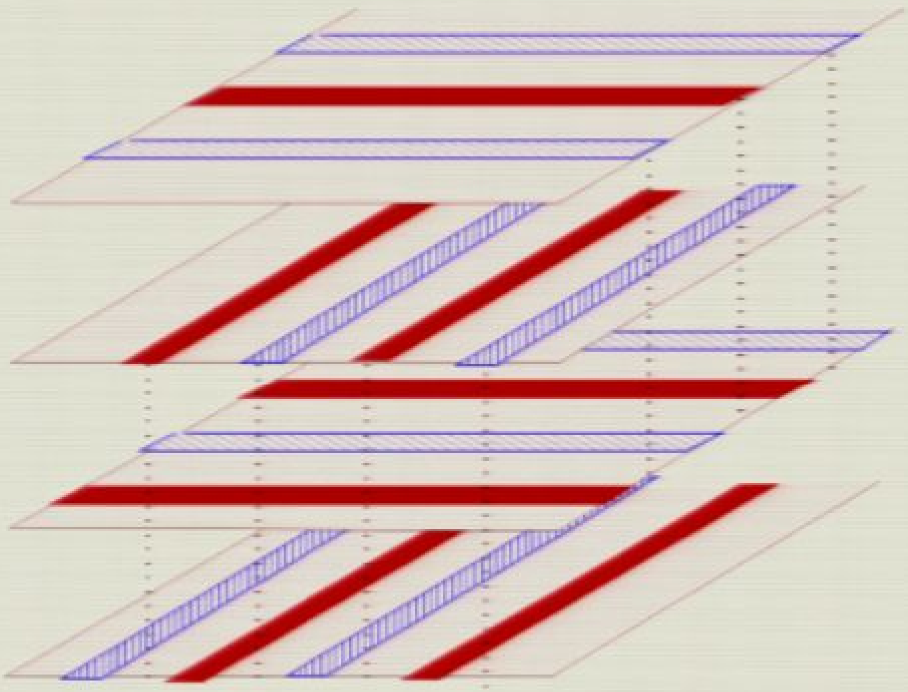
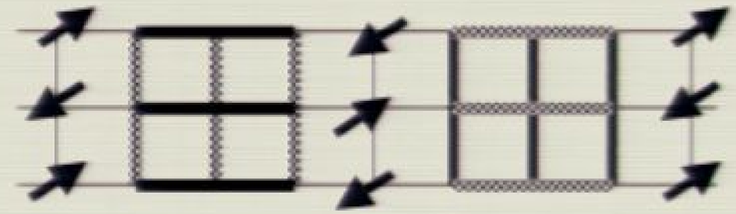
- The superconductivity resides in the spin gap regions and there is a π phase shift in the SC order across the AFM regions

Period 4 Striped Superconducting State

E. Berg et al, 2007





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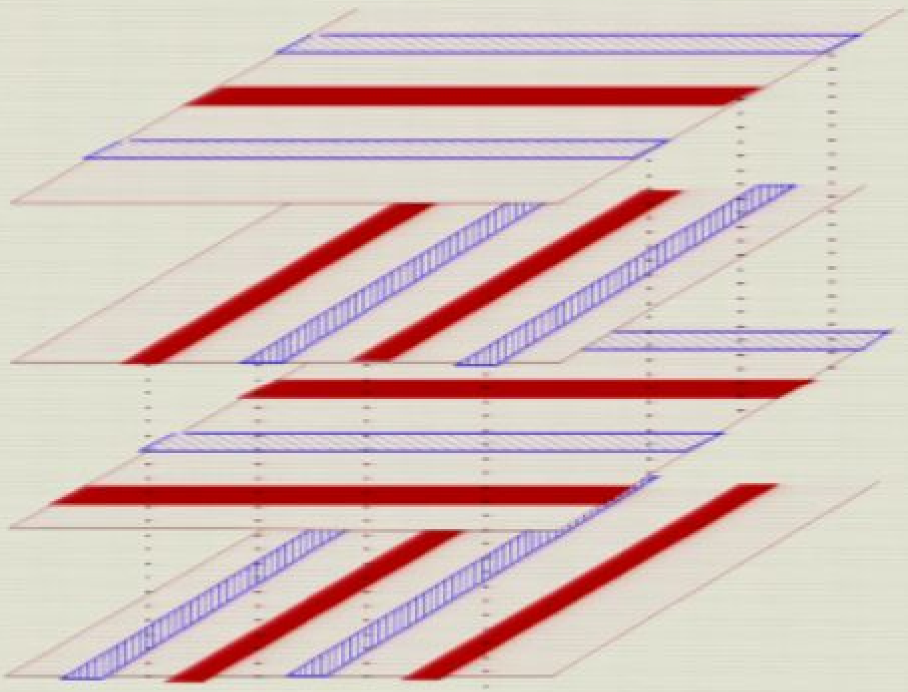

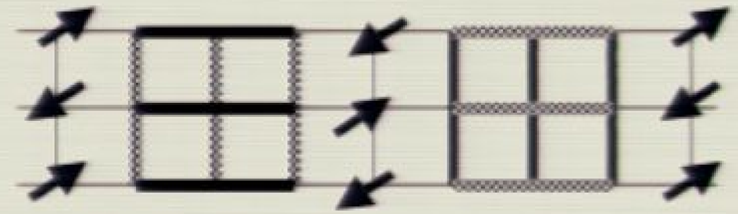

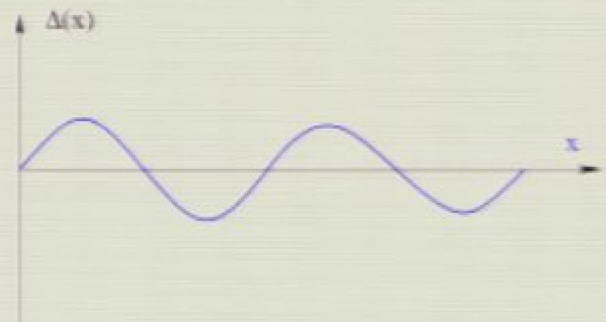

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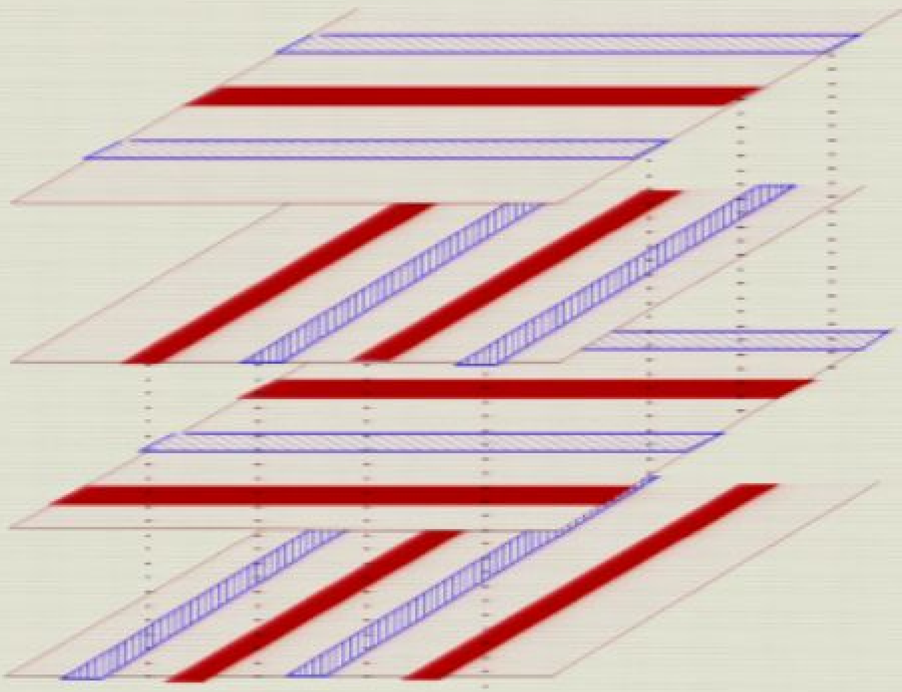

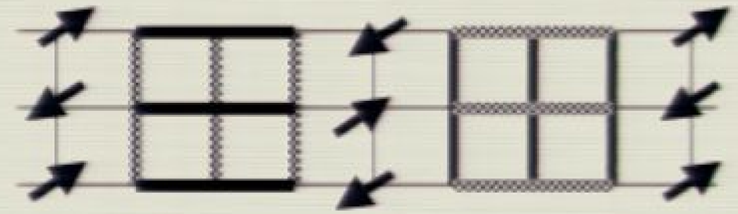

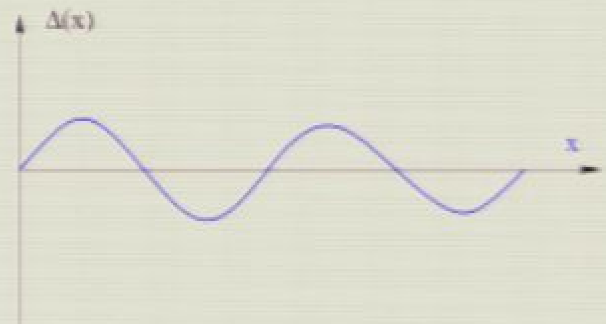

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- A state of this type was found in variational Monte Carlo (Ogata *et al* 2004) and MFT (Poilblanc *et al*

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- The first non-vanishing coupling J_3 occurs at four spacings. It is quite small and it is responsible for the non-zero but very low T_c
- Defects and/or discommensurations gives rise to small Josephson coupling J_0 neighboring planes

Are there other interactions?

- It is possible to have an inter-plane biquadratic coupling involving the product SC of the order parameters between neighboring planes $\Delta_1\ \Delta_2$ and the product of spin stripe order parameters also on neighboring planes $\mathbf{M}_1 . \mathbf{M}_2$

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- In a large enough perpendicular magnetic field it is possible (spin flop transition) to induce such a term and hence an effective Josephson coupling.
- Thus in this state there should be a strong suppression of the 3D SC T_c but not of the 2D SC T_c

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- Discommensurations are defects that induce a finite Josephson coupling between neighboring planes $J_1 \sim |x-1/8|^2$, leading to an increase of the 3D SC T_c away from 1/8
- Similar effects arise from disorder which also lead to a rise in the 3D SC T_c

Landau-Ginzburg Theory of the striped SC: Order Parameters

- Striped SC: $\Delta(\mathbf{r})=\Delta_{\mathbf{Q}}(\mathbf{r})\ e^{i\,\mathbf{Q}.\mathbf{r}}+\Delta_{-\mathbf{Q}}(\mathbf{r})\ e^{-i\mathbf{Q}.\mathbf{r}}$, complex charge 2e singlet pair condensate with wave vector, (*i.e.* an FFLO type state at zero magnetic field)
- Nematic: detects breaking of rotational symmetry: N, a real neutral pseudo-scalar order parameter
- Charge stripe: $\rho_{\mathbf{K}}$, unidirectional charge stripe with wave vector $\mathbf{K}$
- Spin stripe order parameter: $\mathbf{S}_{\mathbf{Q}}$, a neutral complex spin vector order parameter, $\mathbf{K}=2\mathbf{Q}$

Ginzburg-Landau Free Energy Functional

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- The small uniform component Δ_0 removes the sensitivity to quenched disorder of the PDW state

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- ϕ and θ_{-} are locked $\Rightarrow$ topological defects of ϕ and θ_{+}

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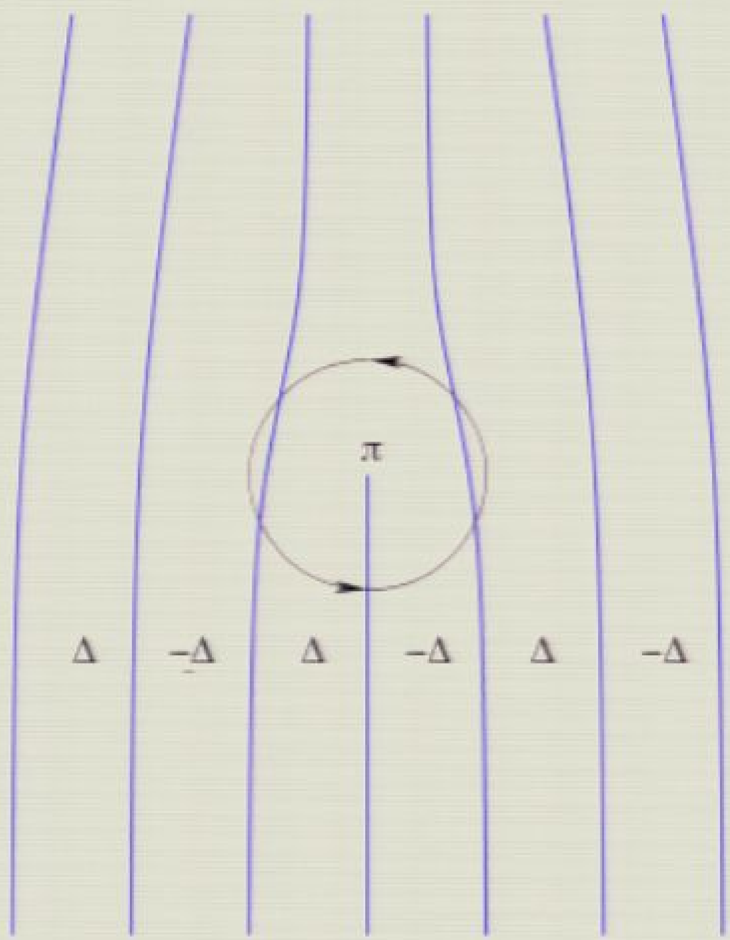
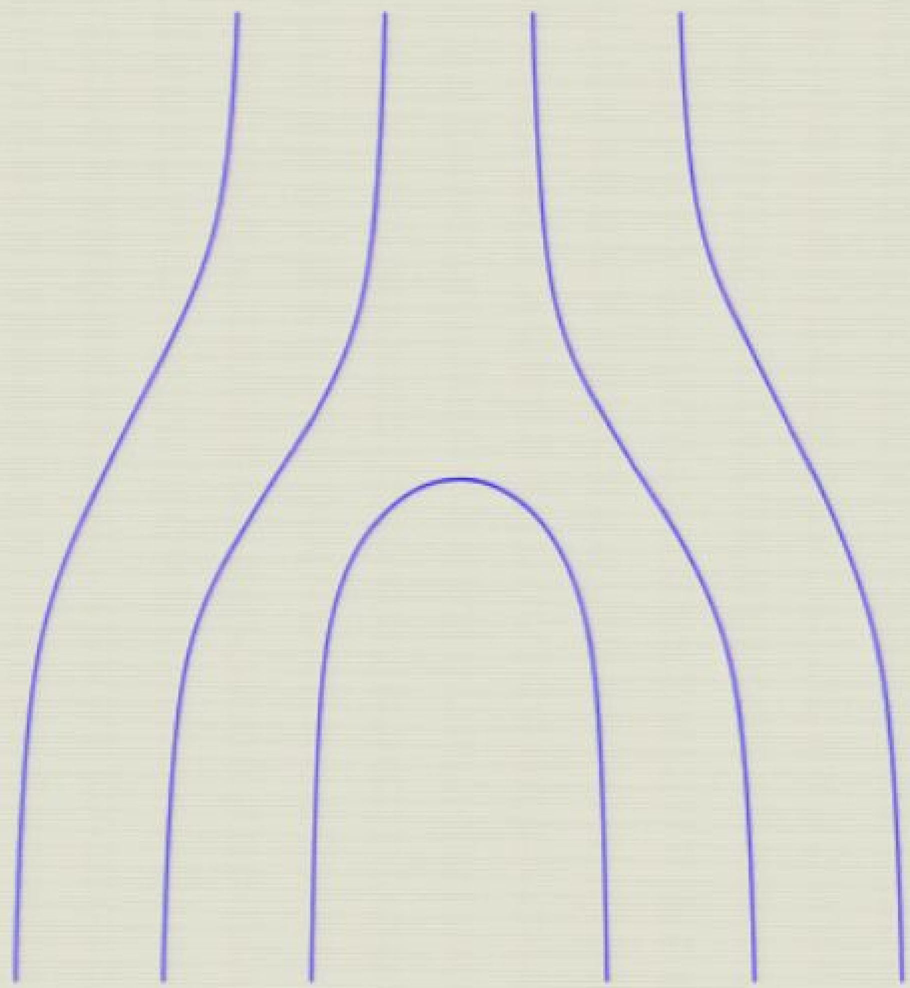
- All three topological defects have logarithmic interactions

Half-vortex and a Dislocation



Double Dislocation



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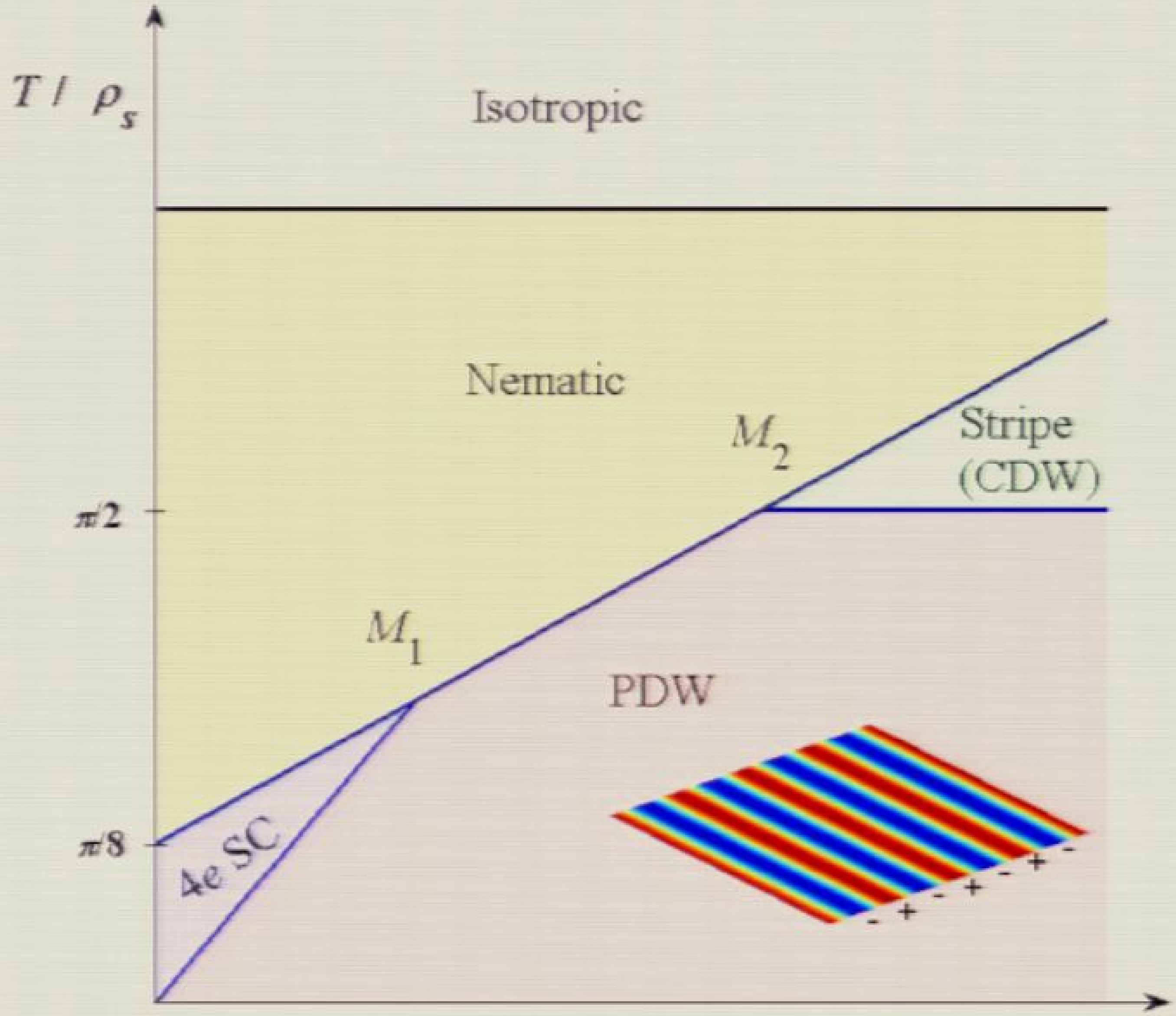
- Phases: PDW, Charge 4e SC, CDW, and normal (Ising nematic)

Schematic Phase Diagram



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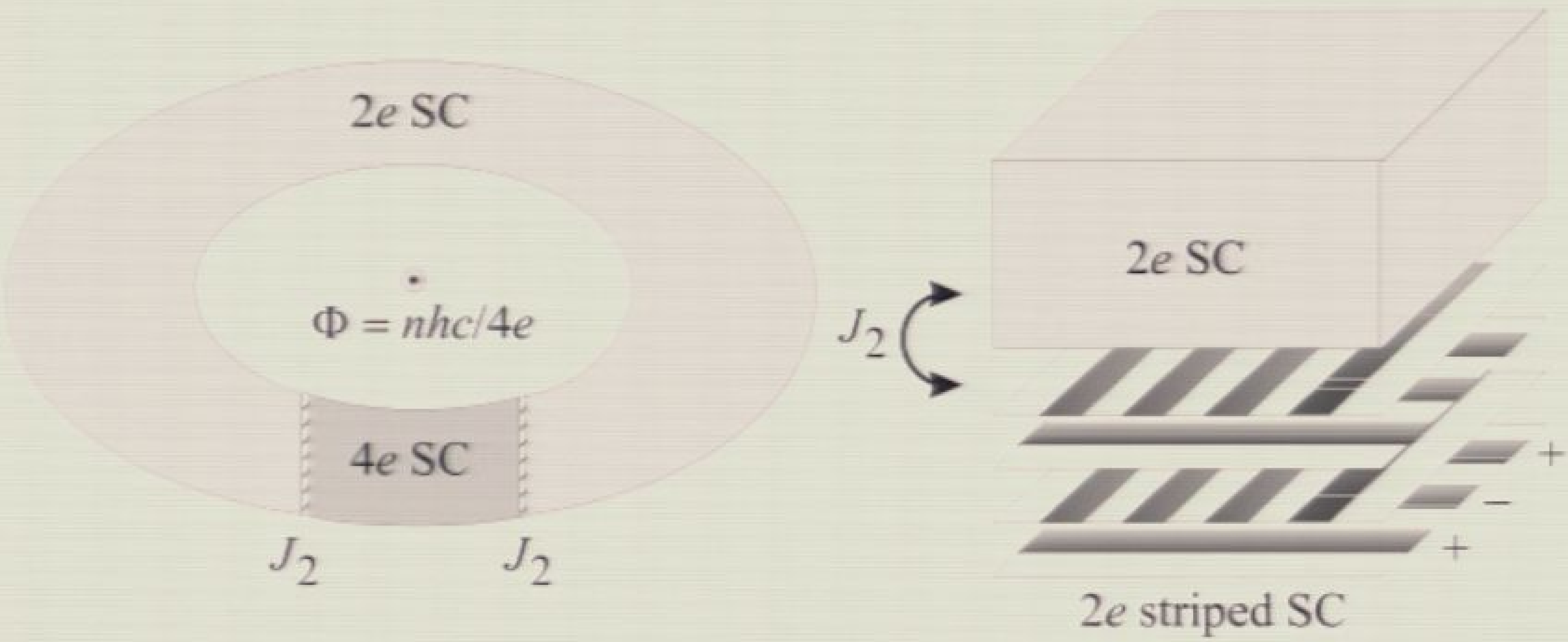
- Disorder induces dislocation defects in the stripe order
- Due to the coupling between stripe order and SC, $\pm\pi$ flux vortices are induced at the dislocation core.
- Strict layer decoupling only allows for a magnetic coupling between randomly distributed $\pm\pi$ flux vortices
- Novel glassy physics and "fractional" flux
- the charge 4e SC order is unaffected by the Bragg glass of the pinned CDW

Phase Sensitive Experiments



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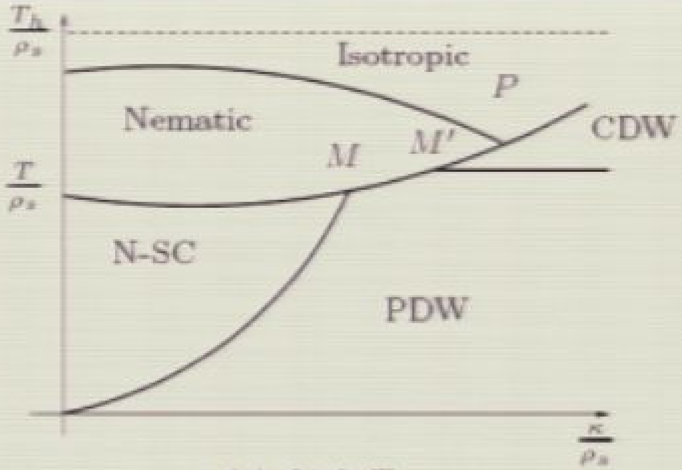
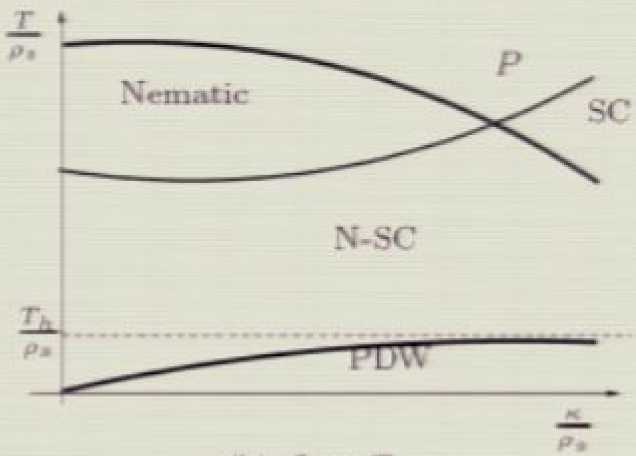
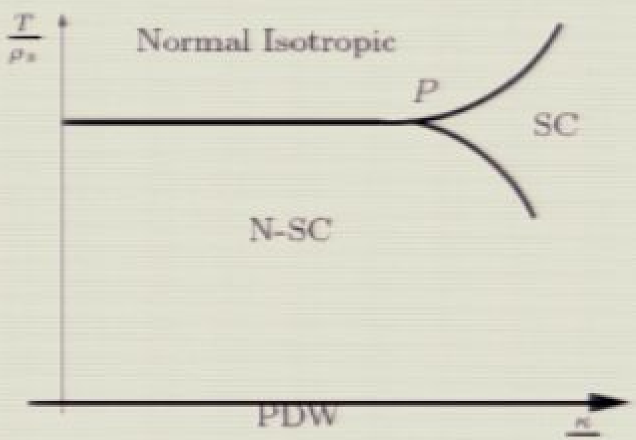
- Coupling to the lattice breaks continuous rotational invariance to the point group of the lattice
- For a square lattice the point group is C_4 and the nematic-isotropic transition is 2D Ising
- As the coupling to the lattice is weakened the structure of the phase diagram changes and the nematic transition is pushed to lower temperatures



(a) high T_h



(b) Low T_h



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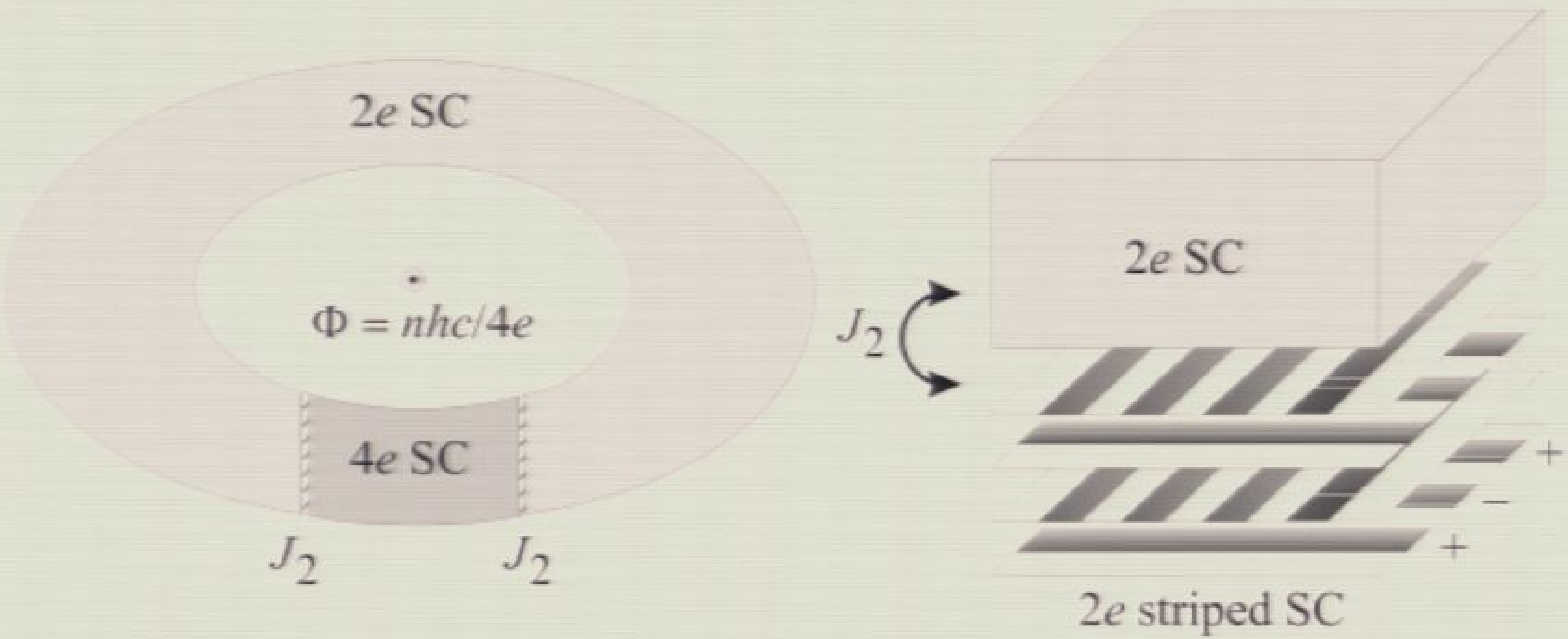

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