Title: Recursion Relations for AdS/CFT Correlators

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URL: http://pirsa.org/11040087

Abstract: Correlation functions in the gauge-gravity correspondence (AdS/CFT) are dual to scattering amplitudes in anti-de Sitter space (AdS). In this talk, I will describe how techniques that were recently developed to study scattering amplitudes in flat space can be generalized to AdS leading to a new and efficient method of computing correlation functions in AdS/CFT.

References:

1) S. Raju, "Generalized Recursion Relations for Correlators in the Gauge Gravity Correspondence", Phys.Rev.Lett. 106 (2011) 091601. http://arxiv.org/abs/arXiv:1011.0780

2) S. Raju, "Recursion Relations for AdS/CFT Correlators",
http://arxiv.org/abs/arXiv:1102.4724

Recursion Relations for AdS/CFT Correlators

Suvrat Raju

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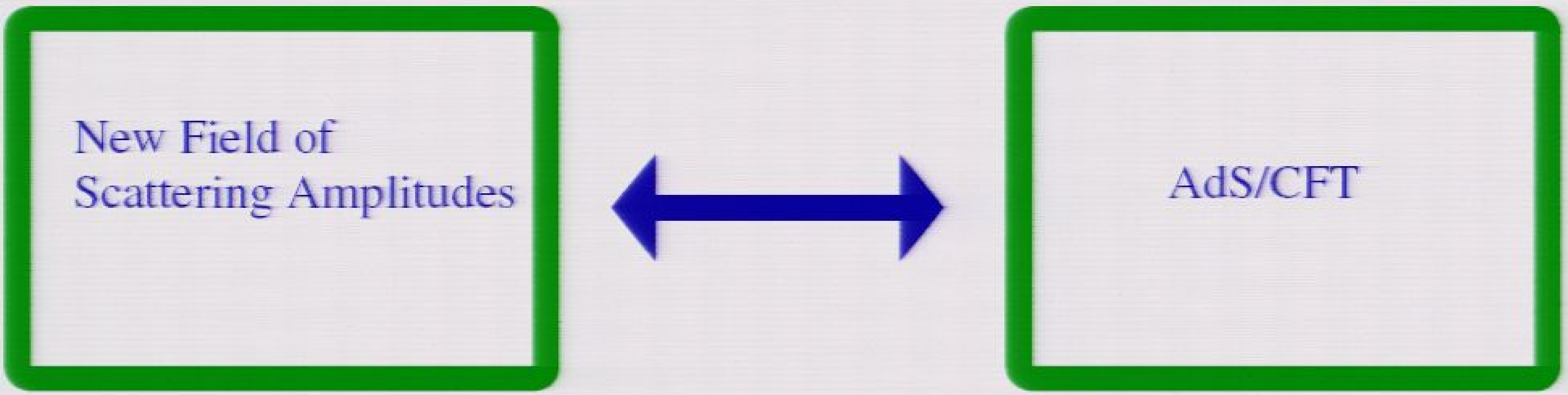
Perimeter Institute for Theoretical Physics
19 April 2011

References

This talk is based on

- S. Raju, BCFW for Witten Diagrams, Physical Review Letters (2011) [arXiv: 1011.0780]
- S. Raju, Recursion Relations for AdS/CFT Correlators, arXiv: 1102.4724.

Subject



- The study of scattering amplitudes in Quantum Field Theory, has been developing very rapidly in the past few years.
- This talk is about an application of techniques from this field to AdS/CFT.

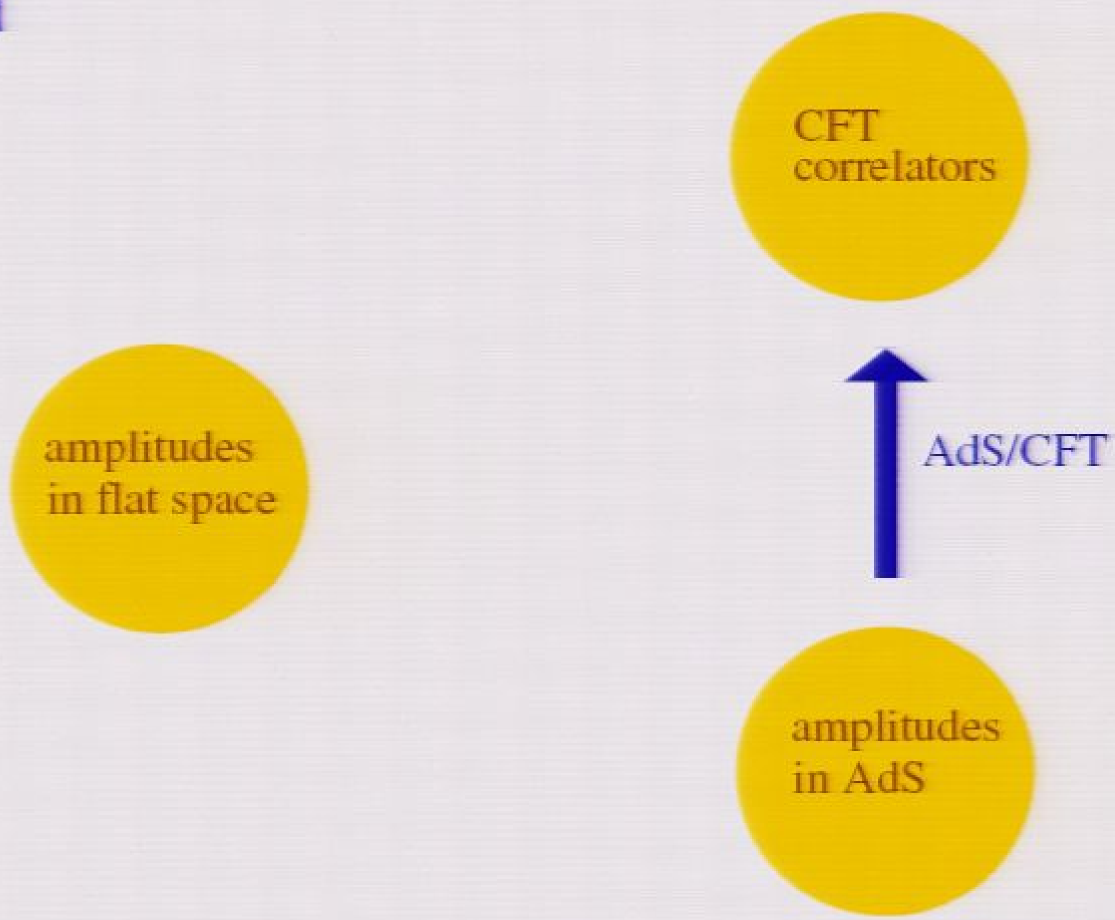
Setting: Studies of Scattering Amplitudes

- Final answers for amplitudes in Yang-Mills theory and gravity are much simpler than one would expect from Feynman diagrams. (Examples soon)
- These properties are studied for two reasons
  1. Practical: These simplifications are useful to compute amplitudes at the LHC.
  2. Formal: What is the physics behind these simplifications? Could they lead to a new perspective on quantum field theory?

Setting: Guage Gravity Duality

Scattering amplitudes in AdS give correlation functions in a dual conformal field theory.

Central Idea



- Previously believed that AdS amplitudes do not share the nice features of flat-space amplitudes $\Rightarrow$ these subjects are disjoint.

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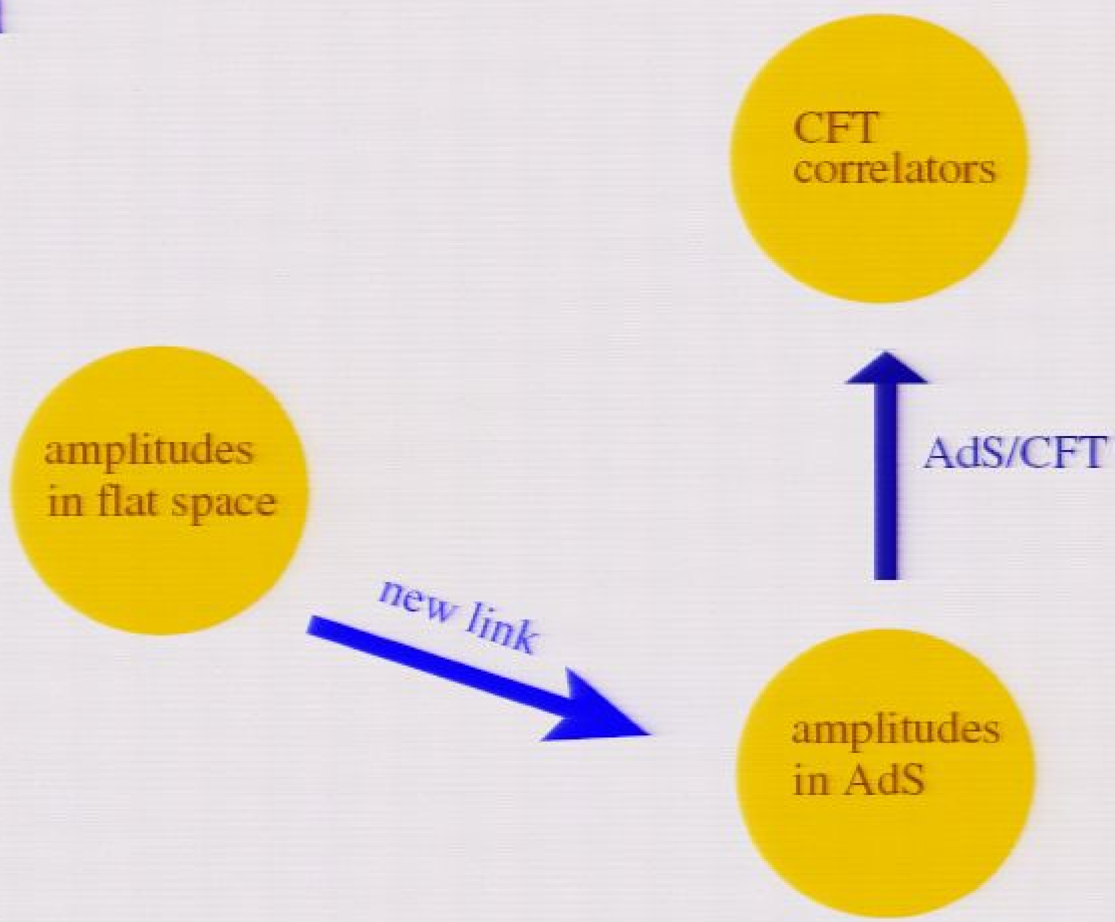


- Previously believed that AdS amplitudes do not share the nice features of flat-space amplitudes $\Rightarrow$ these subjects are disjoint.
- Message of this talk is that techniques developed to study flat-space amplitudes can be adapted to AdS with surprising consequences for AdS/CFT correlators.

Outline

Setting

Developments in Amplitudes

- Unexpected Simplifications
- BCFW Recursion Relations
- Formal Motivations
- Computational Motivations

AdS/CFT

- Witten Diagrams

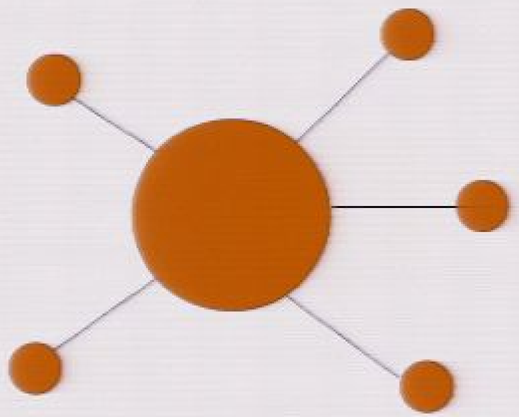
BCFW for Witten Diagrams

- Why this is surprising
- Physical Intuition
- Sketch of Derivation
- Applications of the new Recursion Relations

Extensions

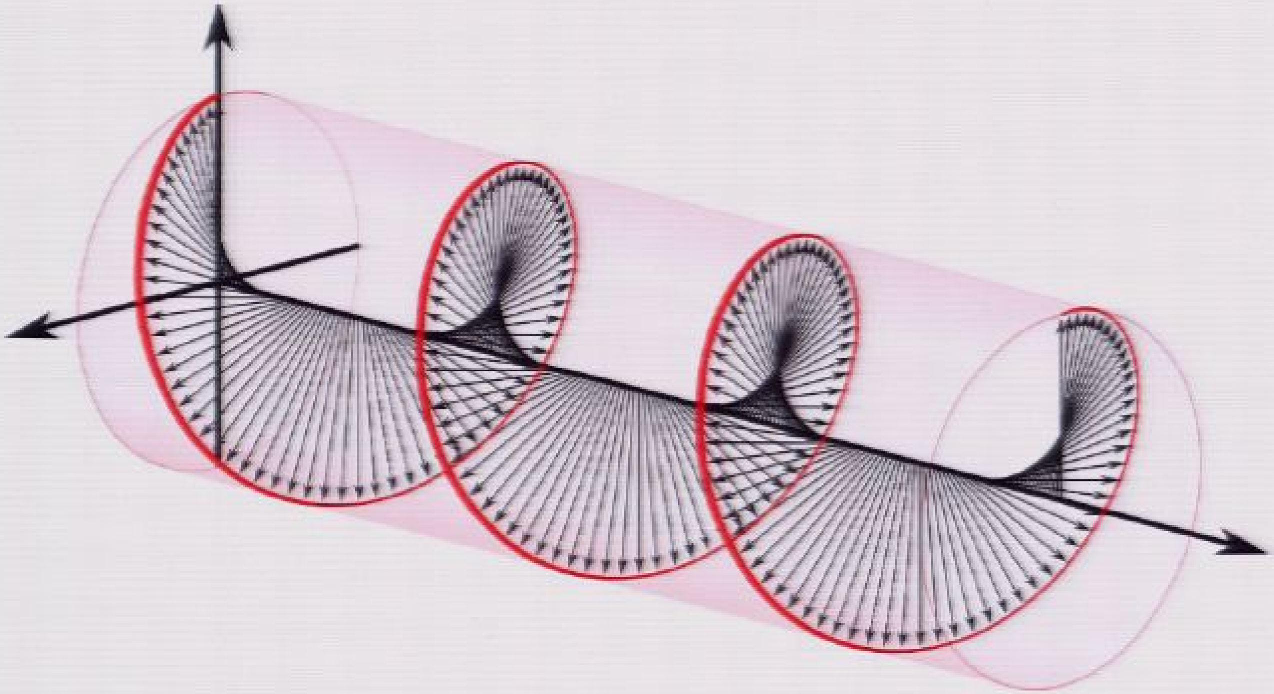
- Supersymmetric Theories
- Other Possible Extensions

Scattering Amplitudes



- Scattering amplitudes are distinct from correlation functions.
- Obtained by putting external legs on-shell and contracting with polarization vectors.
- Scattering amplitudes, but not correlation functions, of gravitons and gluons have nice properties.

Helicity Amplitudes



- Polarization vectors tells us about the state of the external particle.
- For example, in 4 dimensions vector bosons can be in one of two states: light can be right or left circularly polarized. So, we will speak of objects like

$$M(+,+,-,-,-,+,-,+,\ldots)$$

nteractions in Quantum Gravity

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\
&\qquad -P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],
\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\tau\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\tau\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\
&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\
&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\mu\eta^{\tau\lambda}\eta^{\nu\rho}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\
&-P_{12}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})-P_{12}(p^\rho p'^\lambda\eta^{\mu\iota}\eta^{\kappa\sigma}\eta^{\tau\nu})-P_{24}(p_\sigma p'^\mu\eta^{\tau\iota}\eta^{\kappa\rho}\eta^{\lambda\nu})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\
&+P_6(p\cdot p'\eta^{\sigma\rho}\eta^{\lambda\mu}\eta^{\nu\iota}\eta^{\kappa\tau})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\nu})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\kappa\nu})\\
&\qquad -P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].
\end{aligned}$$

If we expand metric fluctuations about a flat background, we get these 3 and 4-pt vertices. (These are written in highly condensed notation.) Actually 2850 terms in 4-pt vertex. Also, an infinite number of higher vertices!

Perturbative Gravity

- To compute a 4-pt amplitude, by brute force, we would need to compute

$$3 \times 28 \times 28 + 2850 = 5,202 \text{ terms}$$

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- However, final answers for S-matrix elements are remarkably simple. For example

$$|M_{+-}|^2 = E_{\text{cm}}^2 \cos^8\left(\frac{\theta}{2}\right) \cot^4\left(\frac{\theta}{2}\right)$$

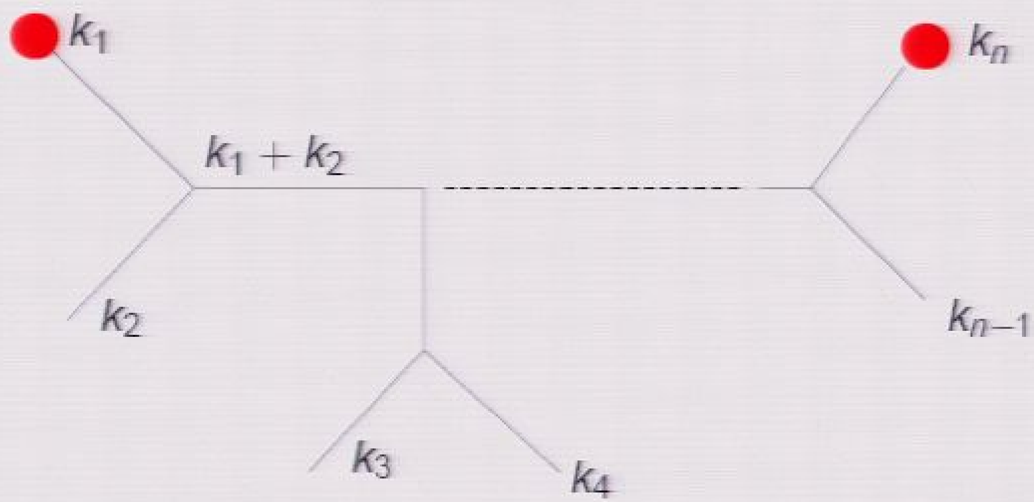
[DeWitt, 67]

- DeWitt, who first worked this out, remarked:

  "The tediousness of the algebra involved ... combined with the fact that the final results are ridiculously simple, leads one to believe that there must be an easier way."

BCFW Relations: The Easier Way

A few years ago, a remarkable technique was discovered that makes this simplicity manifest.

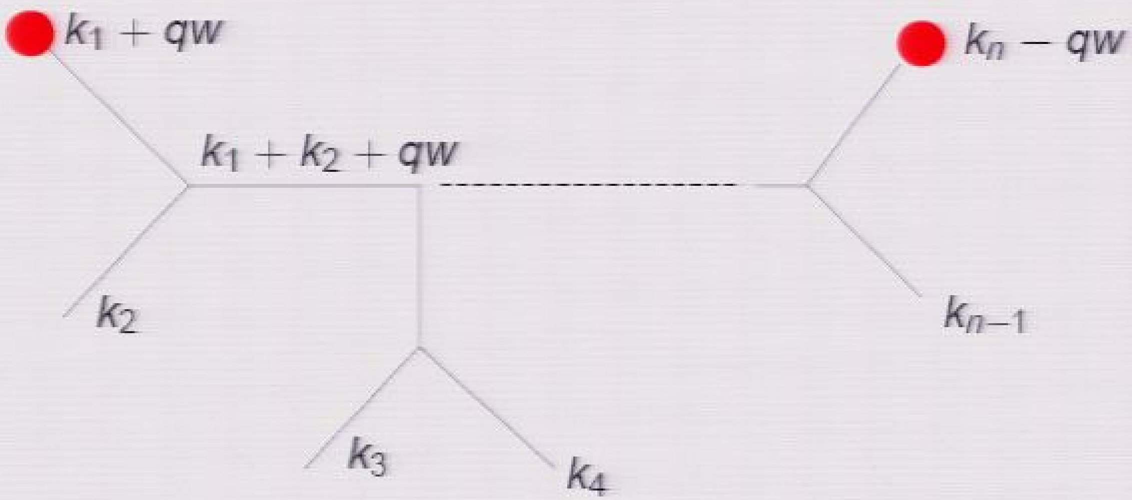


Say $k_1 = (1, 1, 0, 0)$, $k_n = (1, -1, 0, 0)$, $q = (0, 0, 1, i)$.

BCFW Extension: $k_1 \to (1, 1, 0, 0) + (0, 0, 1, i)w$;
$k_n \to (1, -1, 0, 0) - (0, 0, 1, i)w$.

Note that k_1 and k_n remain null and momentum is conserved

Analytic Properties



- The amplitude is a holomorphic function of w.
- It has simple poles when a propagator goes on shell.
- The residue at each pole is the product of two smaller amplitudes.
- Can we reconstruct the full amplitude from these residues?

Large w Behaviour

- This depends on whether there is a pole at $w = \infty$.
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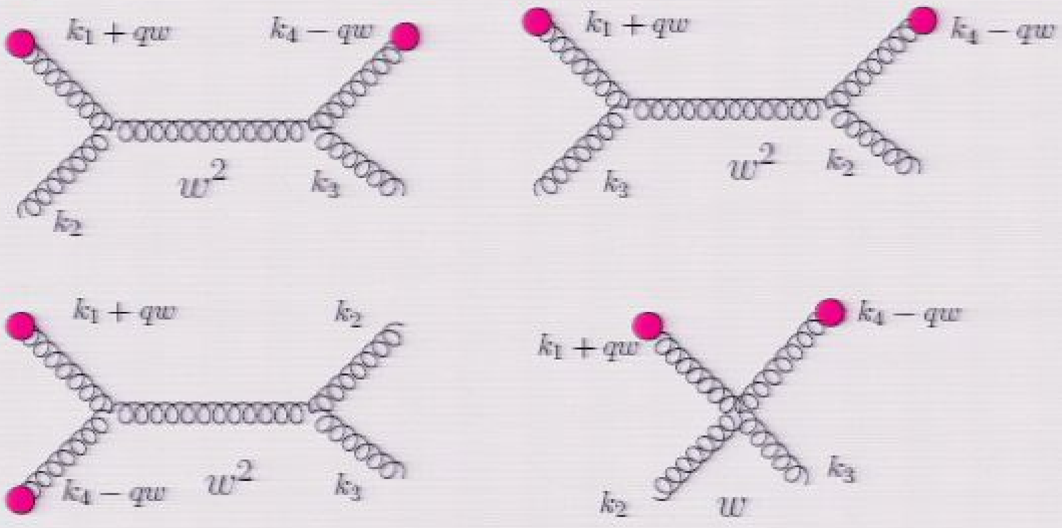
- Correct Answer: For 3 out of 4 possible polarizations:
  - $M \to O(1/w)$ for gauge theories.
  - $M \to O(1/w^2)$ for gravity.
- For 1 polarization, naive expectation is justified.

Amplitudes vs Feynman Diagrams



- This is a very surprising property of gauge and gravity theories that holds only for sum of all Feynman diagrams.

- In particular, individual Feynman diagrams have the naive scaling at large w, but there are cancellations when we add them together and dot with polarization vectors.

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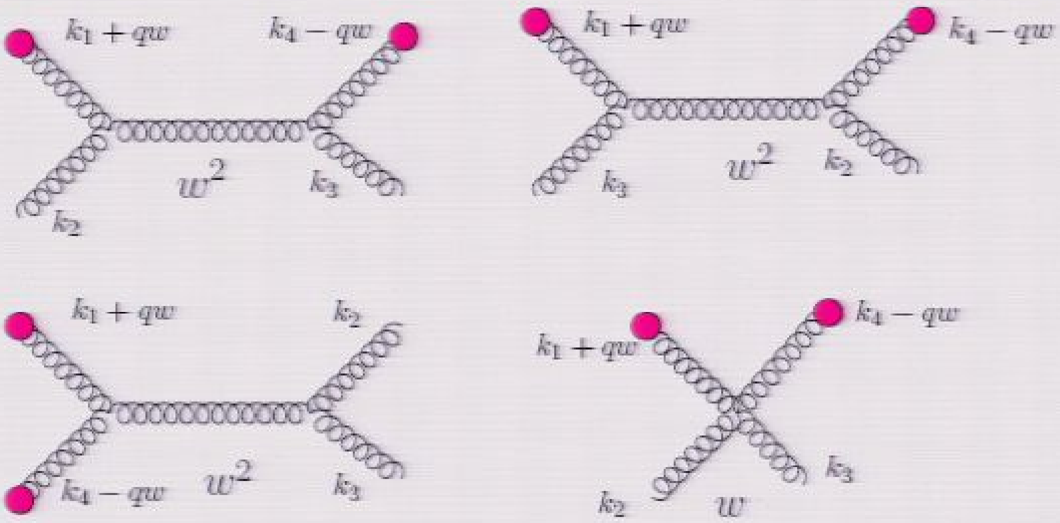

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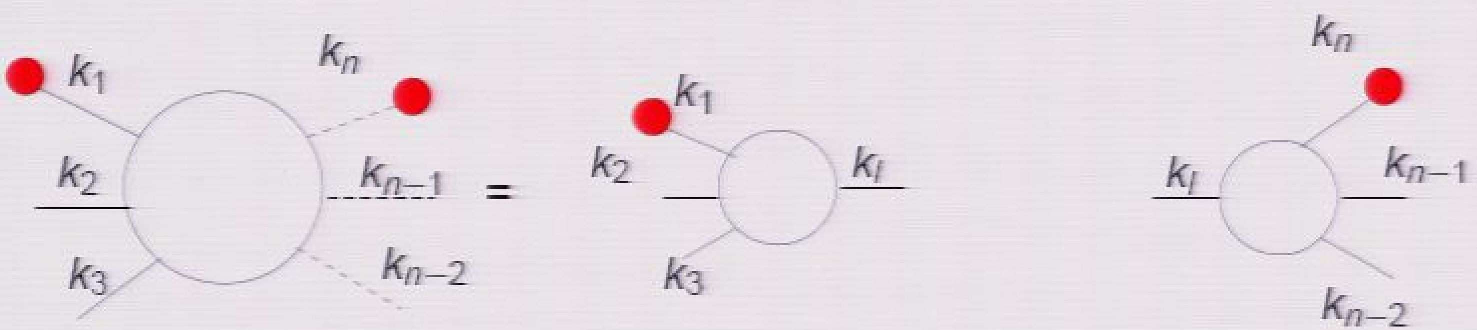
Recursion Relations



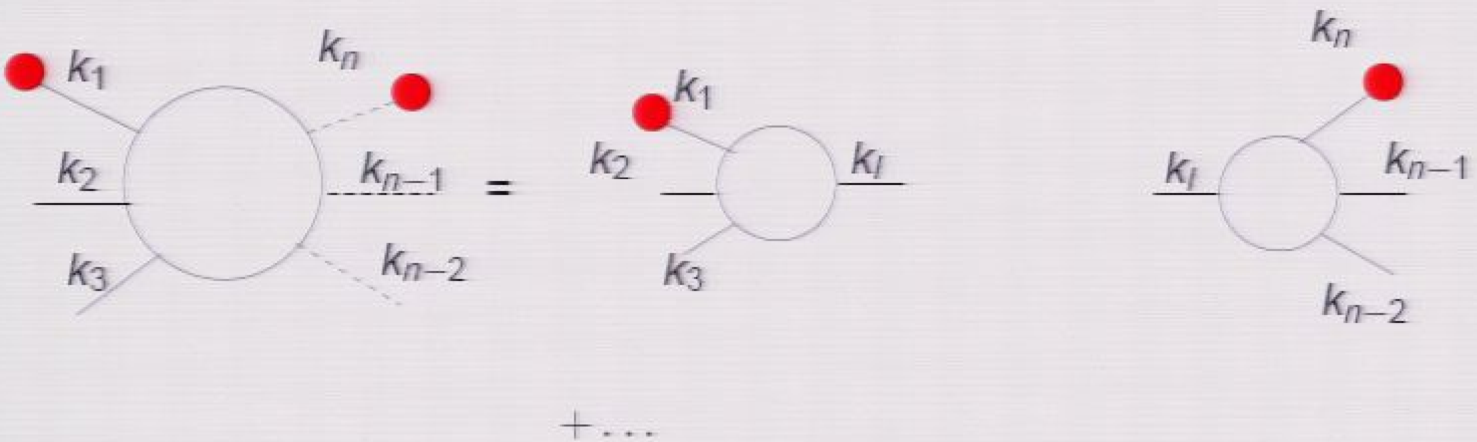
- This leads to powerful recursion relations

$$M \sim \sum_{\text{partitions}} M_{\text{left}} \frac{1}{K^2} M_{\text{right}}$$

[Britto, Cachazo, Feng, Witten, 2005]

- A big amplitude breaks into a sum over products of smaller amplitudes.

Recursion Relations II



- We can continue this process till we have only 3 particles left.
- So, BCFW recursion allows us to reconstruct all tree amplitudes from a knowledge of the 3-pt. amplitude!
- The 3-pt. amplitude is very simple, and this explains the remarkable simplicity of gauge and gravity amplitudes.

Locality vs Simplicity of Amplitudes

- QUESTION (Philosophical): Why is the gravity Lagrangian so complicated when graviton amplitudes are simple?

Locality vs Simplicity of Amplitudes

- QUESTION (Philosophical): Why is the gravity Lagrangian so complicated when graviton amplitudes are simple?
- The point is that there are two physical degrees of freedom, and their interactions are quite simple. (Completely encoded by a three-point function.)
- However, to write down a local Lagrangian description, we need to encode these degrees of freedom in a metric field.
- However, the metric field also contains other unphysical degrees of freedom.

Locality vs Simplicity of Amplitudes

- We now need to impose gauge-invariance to project out these degrees of freedom.
- This leads to an infinite number of interaction vertices.
- So, the reason gravity looks so complicated is that we insisted on writing down a manifestly local description.

Flat Space Holography?

VS

$\cos^8\left(\frac{\theta}{2}\right)\cot^4\left(\frac{\theta}{2}\right)$

Locality

Simplicity

Can we find a dual description that makes simplicity rather than locality manifest? Dual for flat-space quantum field theory?

Calculations at the LHC

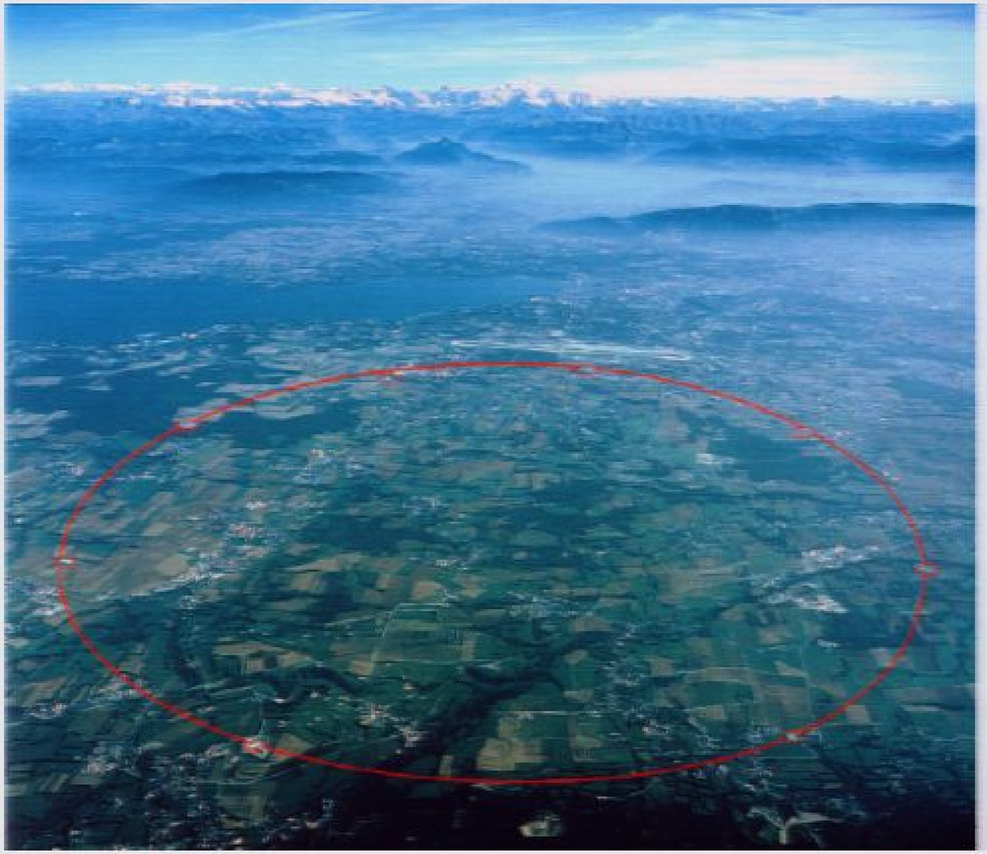


Figure: The Large Hadron Collider (image-credit: CERN)

These developments are not only of formal interest. They have applications for LHC calculations.

W + multi-jets at the LHC

- At the LHC, if we want to detect new physics, we need to subtract off the Standard Model background; this requires accurate next-to-leading order Standard Model predictions.
- An example is the production of W + 4-jets at the LHC. This provides a background to signatures that involve lepton + multi-jet + missing energy.
- There are many aspects to this calculation — going from the amplitude to what is seen at the detector is messy — but surprisingly, till recently, the bottleneck was the computation of the NLO amplitude.

Blackhat

- The problem is that the number of Feynman-diagrams grows so fast, that even computers can't handle this computation.
- By automating these on-shell techniques in a program called Blackhat, Berger et. al. were able to compute the amplitude for W + 4-jets recently. [Berger, Bern, Dixon et al., 2010]
- Important Breakthrough!

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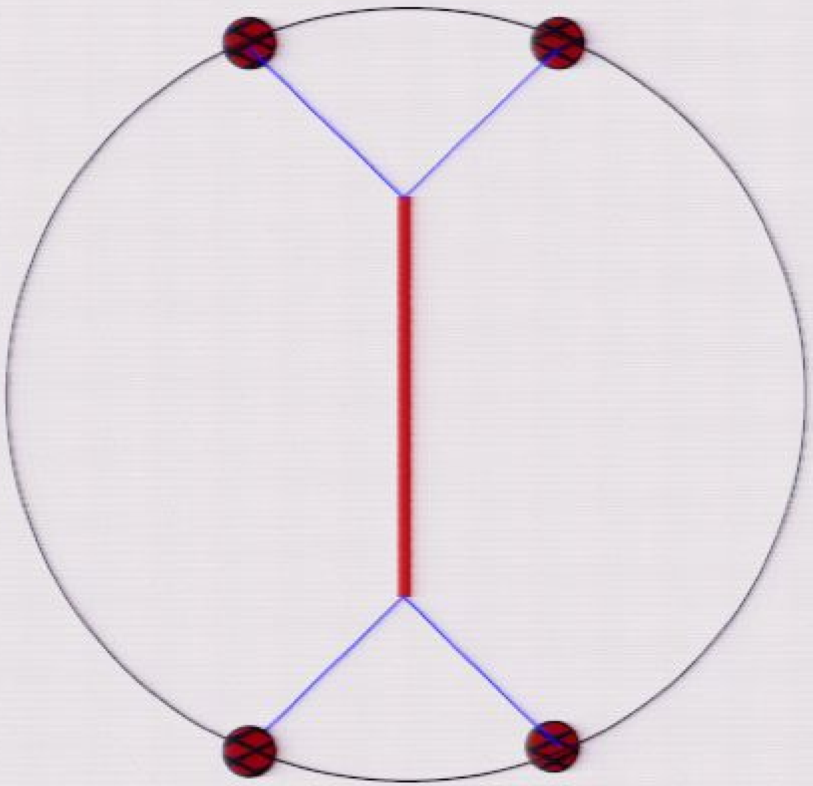
- So, these techniques are of interest, not just for formal reasons but for very practical reasons.

Witten Diagrams



CFT correlation functions are calculated by bulk "Witten Diagrams." These are the analogues of scattering amplitudes.

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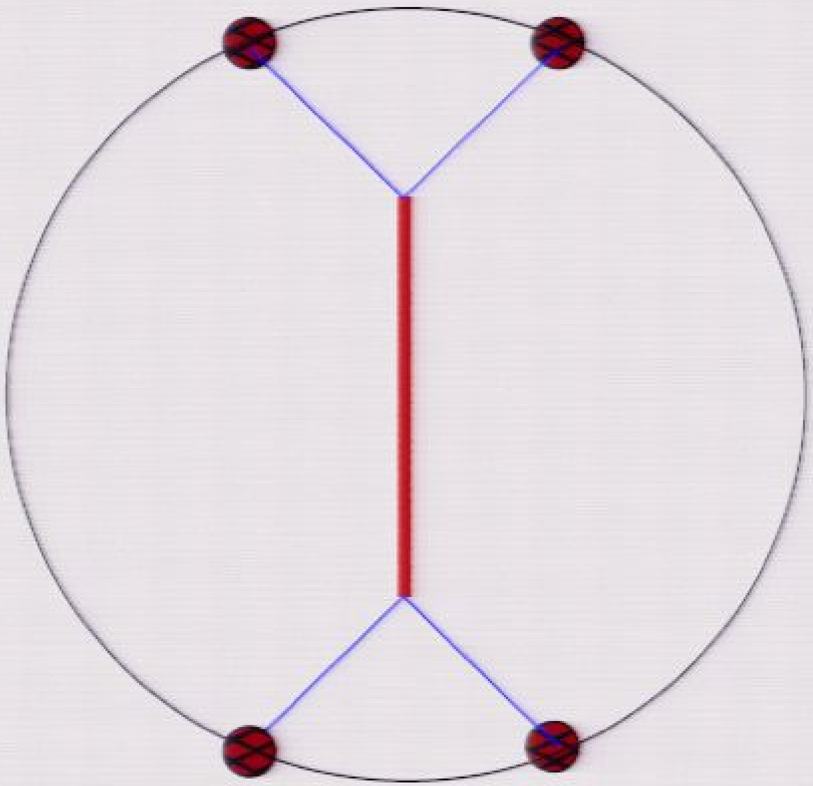


CFT correlation functions are calculated by bulk "Witten Diagrams." These are the analogues of scattering amplitudes.

Transition Amplitudes

- Usually, one computes vacuum-correlators in the CFT

$$T(k_1, \ldots k_n) = \langle 0 | O(k_1) \ldots O(k_n) | 0 \rangle.$$

- In our context, it is more natural to consider an enhanced set of correlators,

$$T(p_1, \ldots p_{n_1}, k_1, \ldots k_{n_2}, l_1, \ldots l_{n_3}) \\ = \langle p_1, \ldots p_{n_1} | O(k_1) \ldots O(k_{n_2}) | l_1, \ldots l_{n_3} \rangle.$$

- In bulk-perturbation theory, these are computed by replacing some bulk-boundary propagators by normalizable modes. We call them "transition amplitudes"

Outline of the rest of the talk

In the rest of this talk I will:

1. State the result for a generalization of the BCFW recursion relations to AdS.
2. Describe why the existence of such a result is surprising.
3. Describe the physical intuition behind the result.
4. Try and describe what this result may be used for.
5. Mention how it can be extended.

Recursion Relations for Transition Amplitudes

CENTRAL RESULT: Transition Amplitudes for gluons or gravitons in AdS are related to integrated products of lower-point amplitudes:

$$T(k_1, \epsilon_1, \ldots k_n, \epsilon_n) = \sum_{\{\pi\}, m, \epsilon'_m} \int \frac{-i\mathcal{T}^2}{2(p^2 + K^2)} dp^2,$$

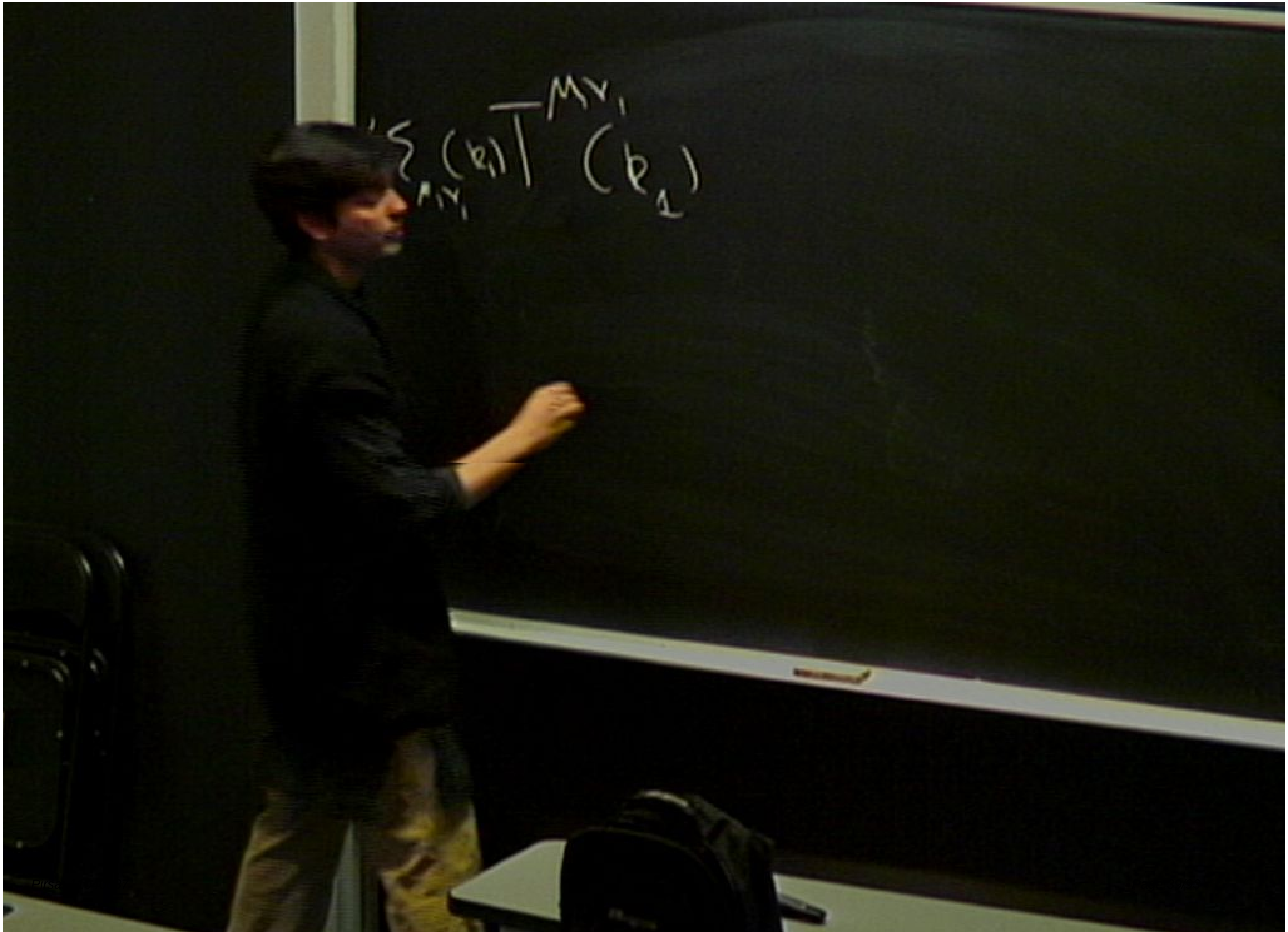
$$\mathcal{T}^2 \equiv T(k_1(p), \epsilon_1, \ldots k'_m, \epsilon'_m) T(-k'_m, \epsilon'_m, \ldots k_n(p), \epsilon_n),$$

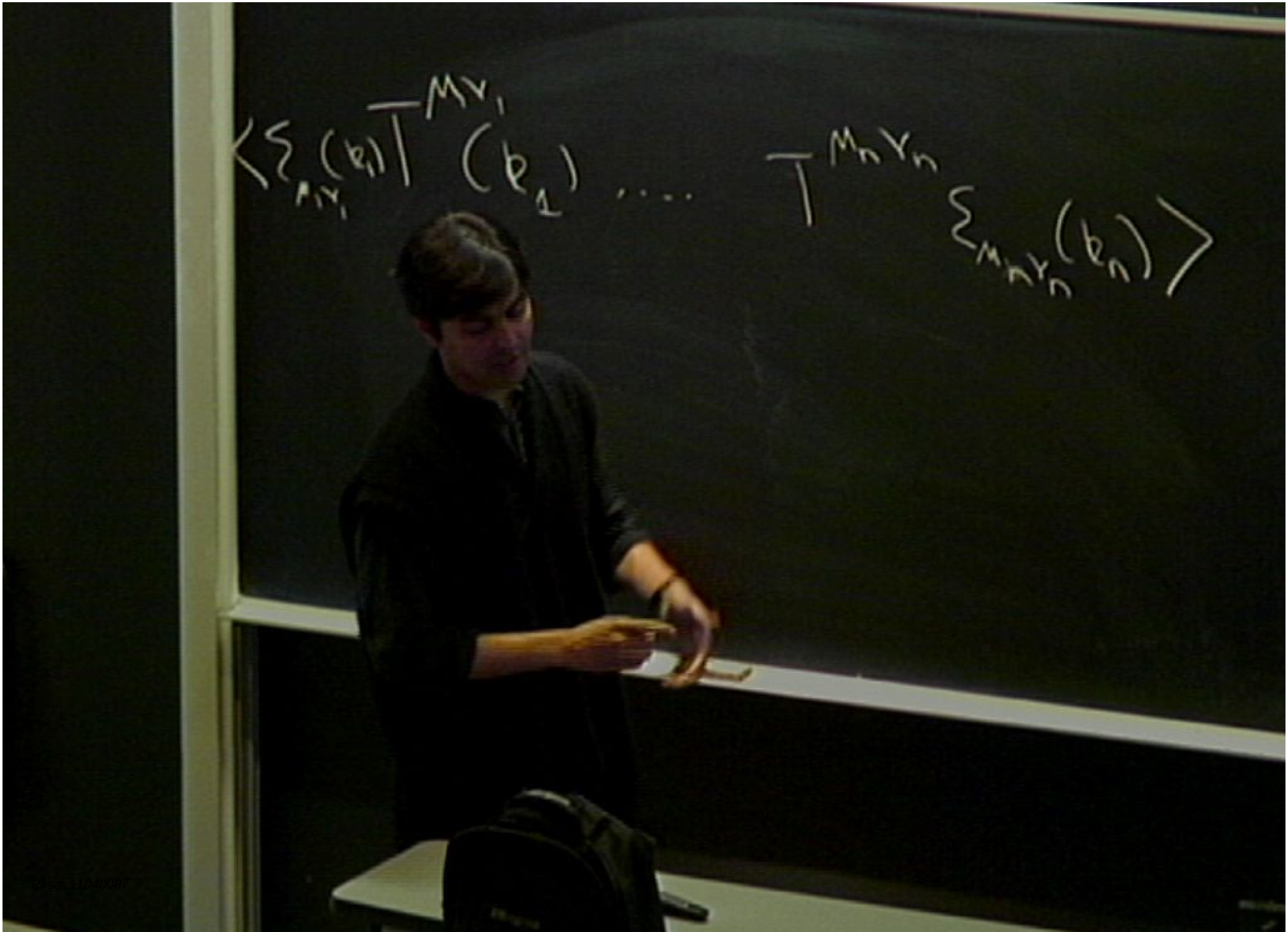
where

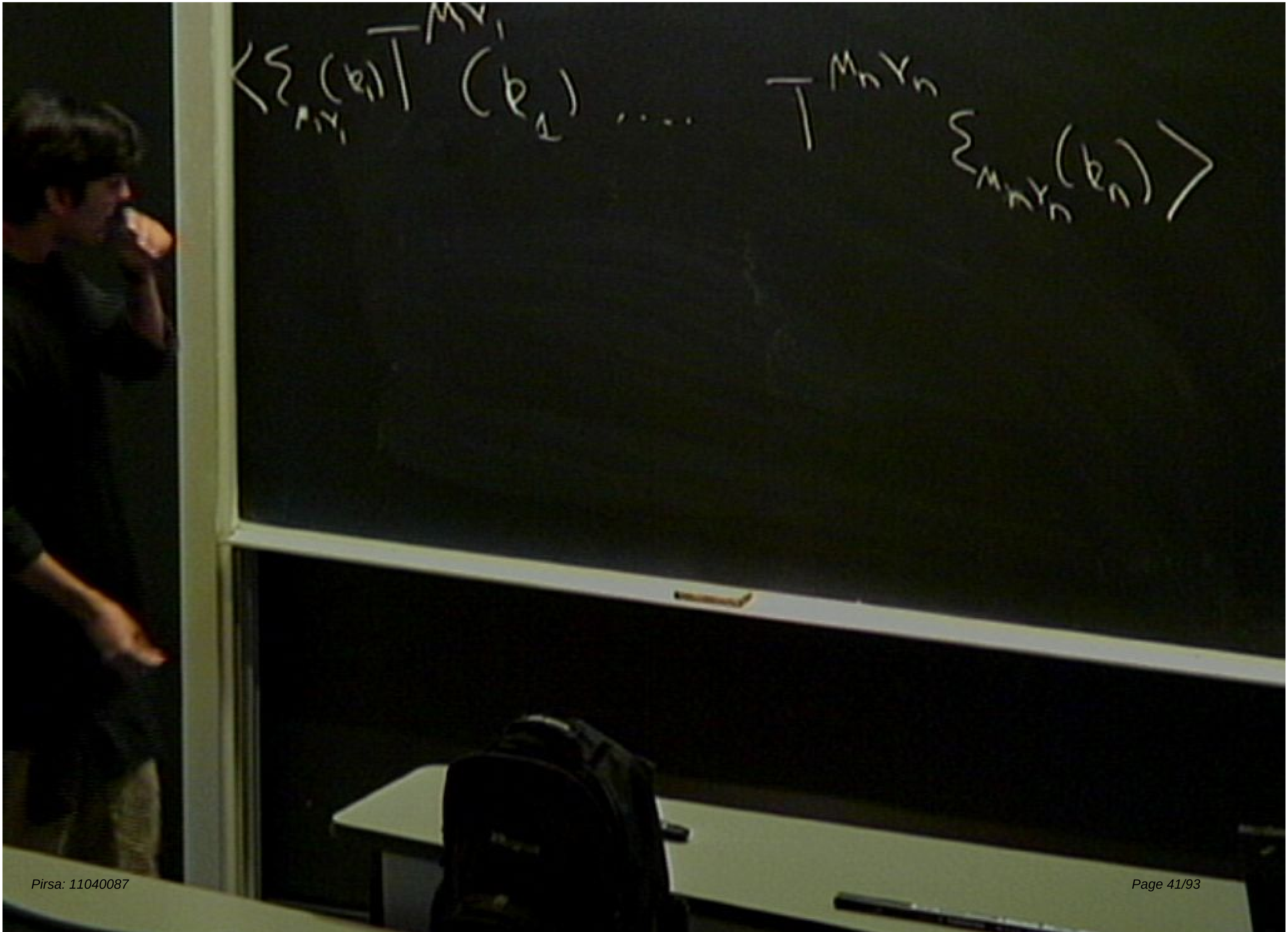
$$K = k_1 + \sum_{j=2}^{m} k_{\pi_j}; \quad w(p) = -(K^2 + p^2)/(2K \cdot q);$$

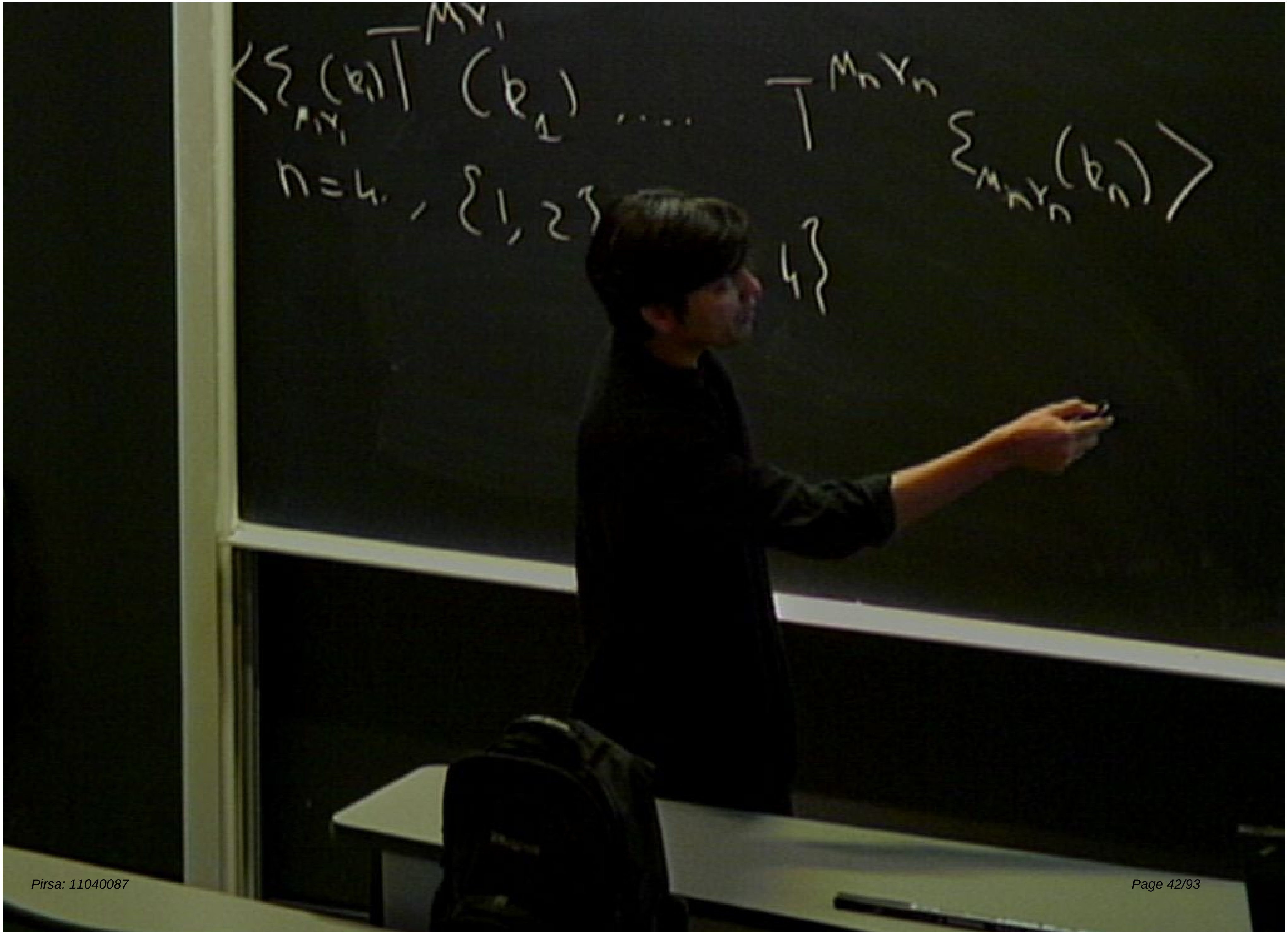
$$k_1(p) = k_1 + qw(p); k_n(p) = k_n - qw(p);$$

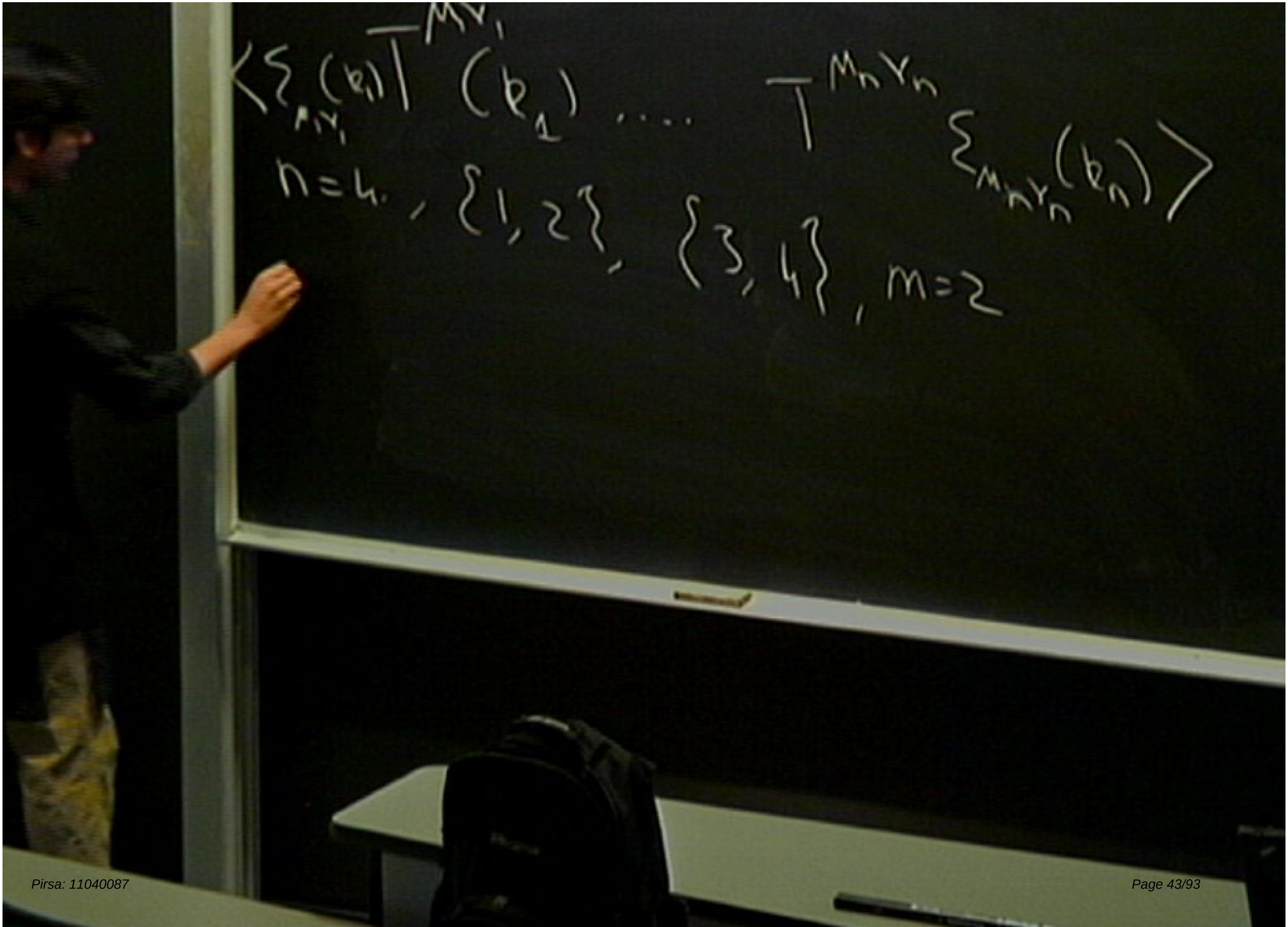
$$k'_m = -K - qw(p).$$

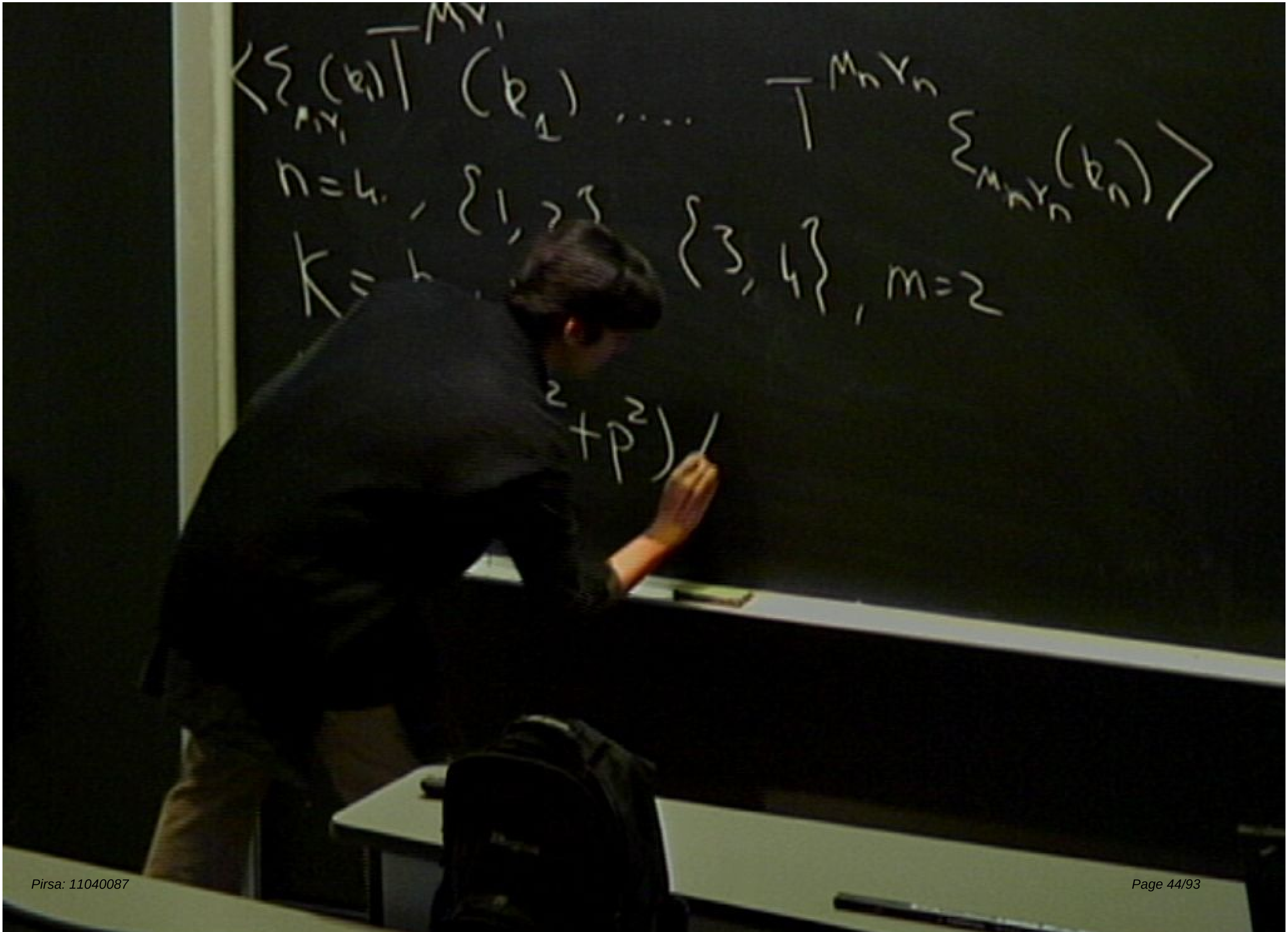


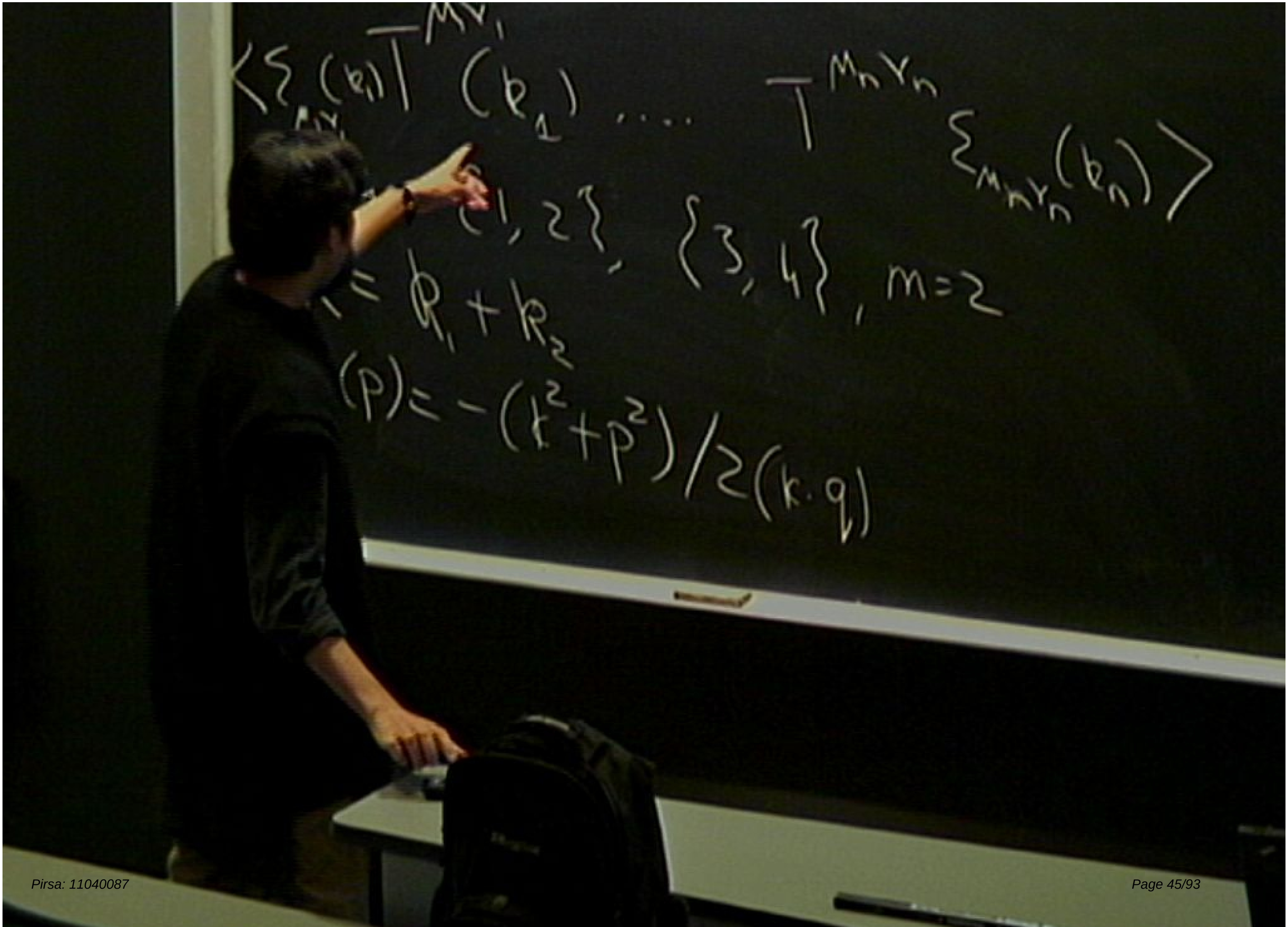

$\langle \Sigma_{\mu_1\nu_1}(k_1) T^{\mu_1\nu_1}(k_1) \ldots T^{\mu_n\nu_n} \Sigma_{\mu_n\nu_n}(k_n) \rangle$

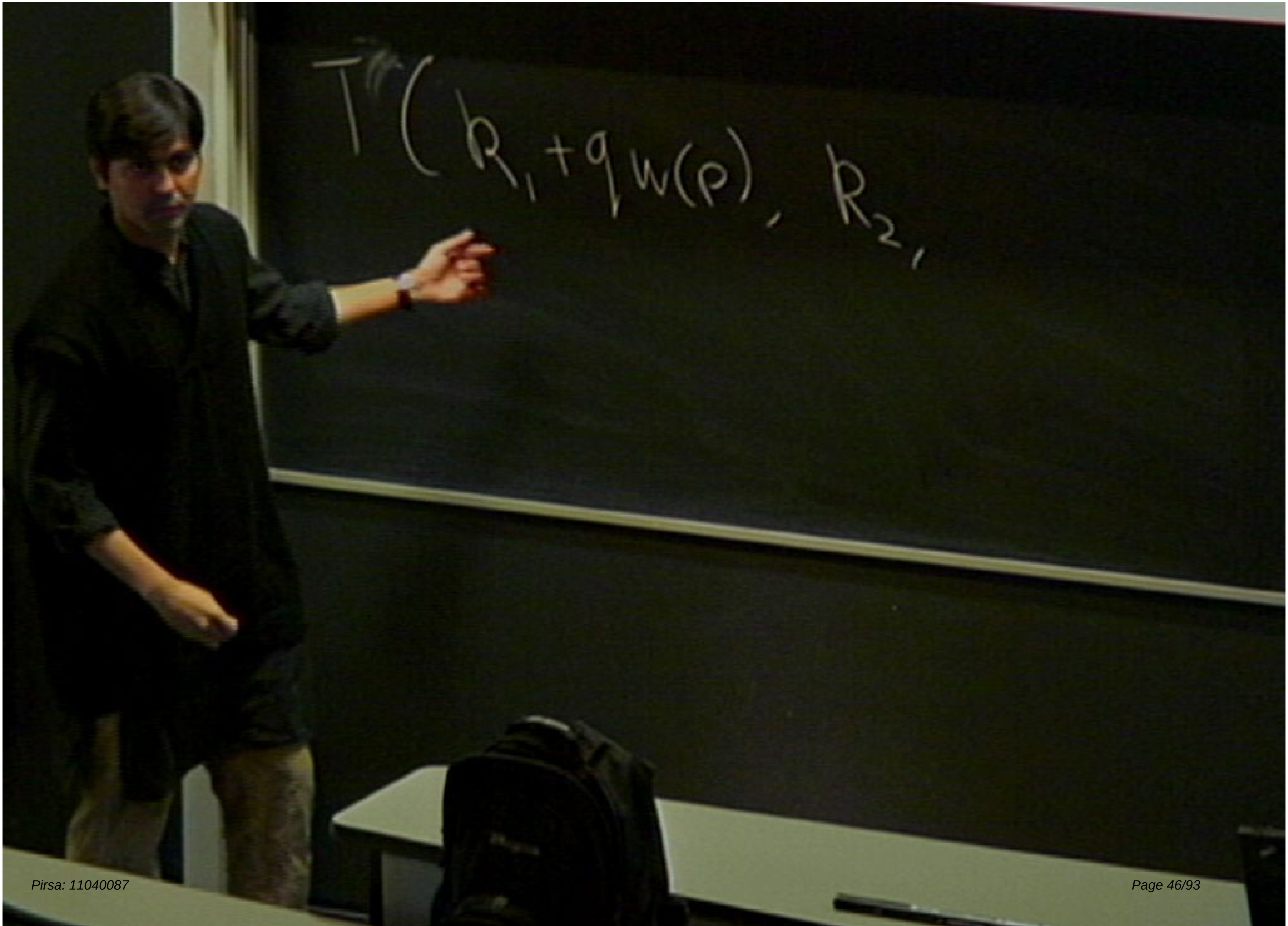

$\langle \Sigma_{\mu_1\nu_1}(k_1) T^{\mu_1\nu_1}(k_1) \ldots T^{\mu_n\nu_n} \Sigma_{\mu_n\nu_n}(k_n) \rangle$

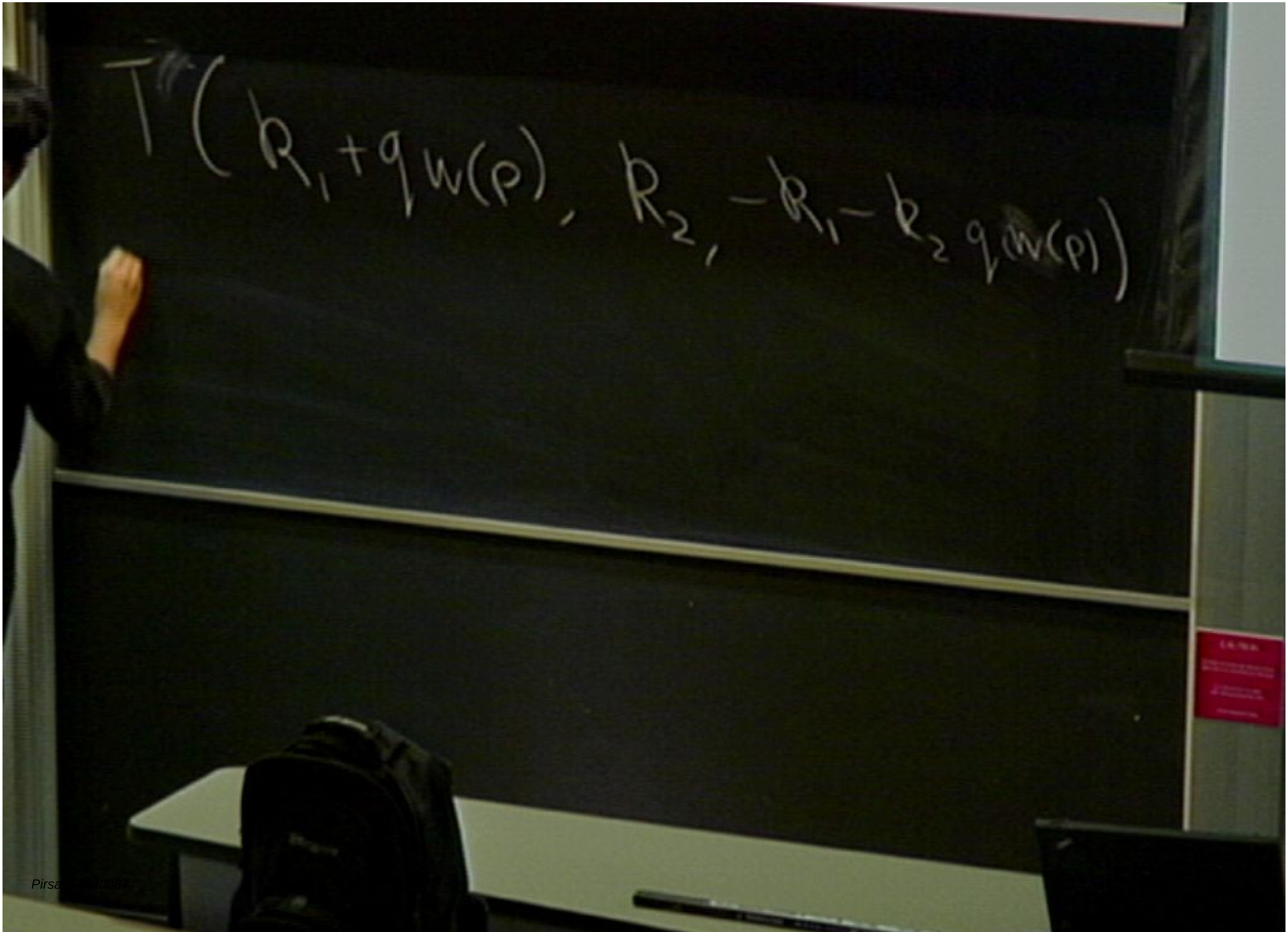


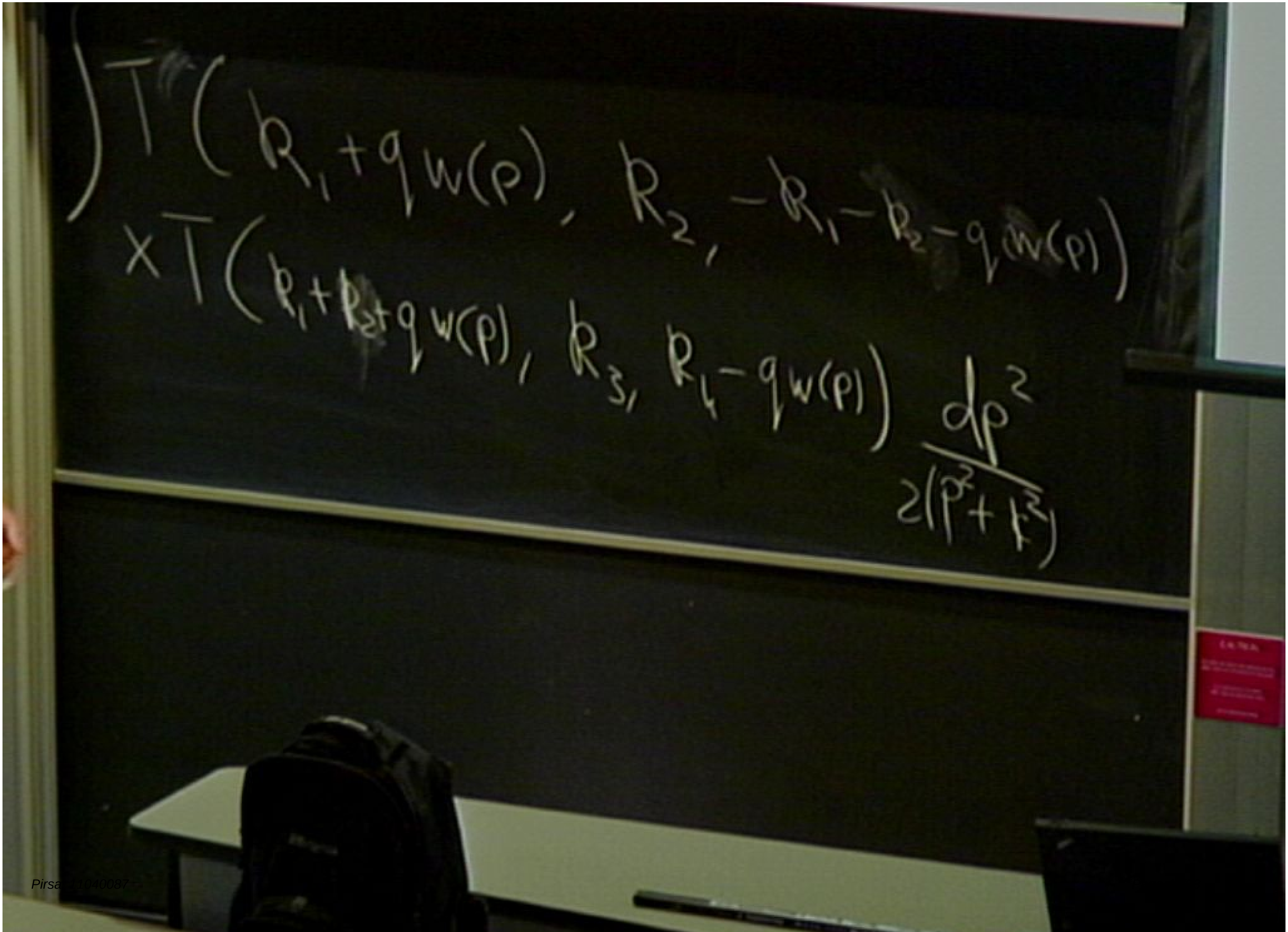

$\langle \xi_{\mu_1\nu_1}(k_1) T^{\mu_1\nu_1}(k_1) \ldots T^{\mu_n\nu_n} \xi_{\mu_n\nu_n}(k_n) \rangle$
$n=4$, $\{1,2\}$, $\{3,4\}$, $m=2$


n=4
{3, 4}
m=2


{3, 4} , m=2

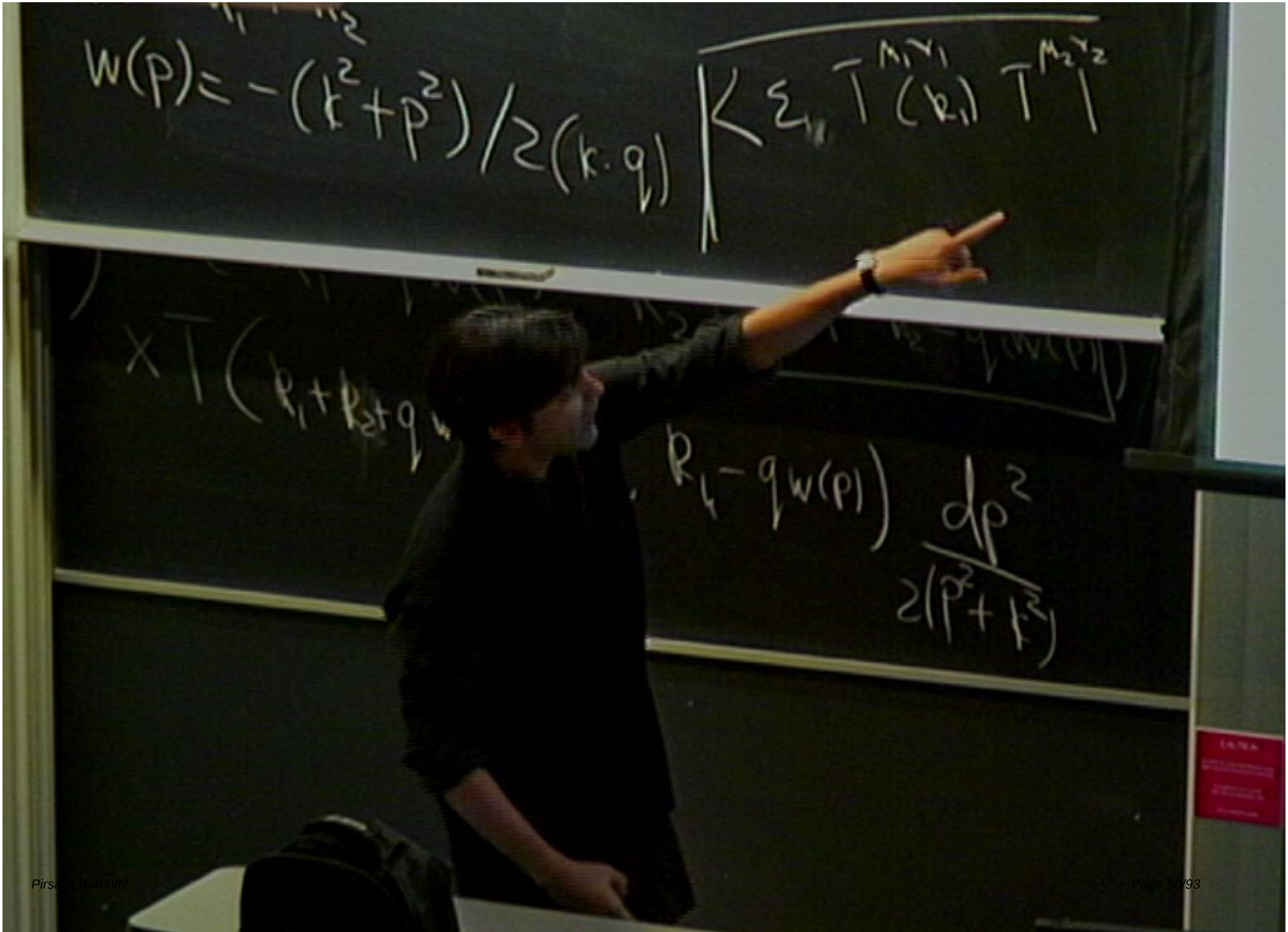

$T(R_1 + q w(p), R_2,$


$T^{\pi}(k_1 + q\,w(p),\ k_2,\ -k_1 - k_2\, q\, w(p))$

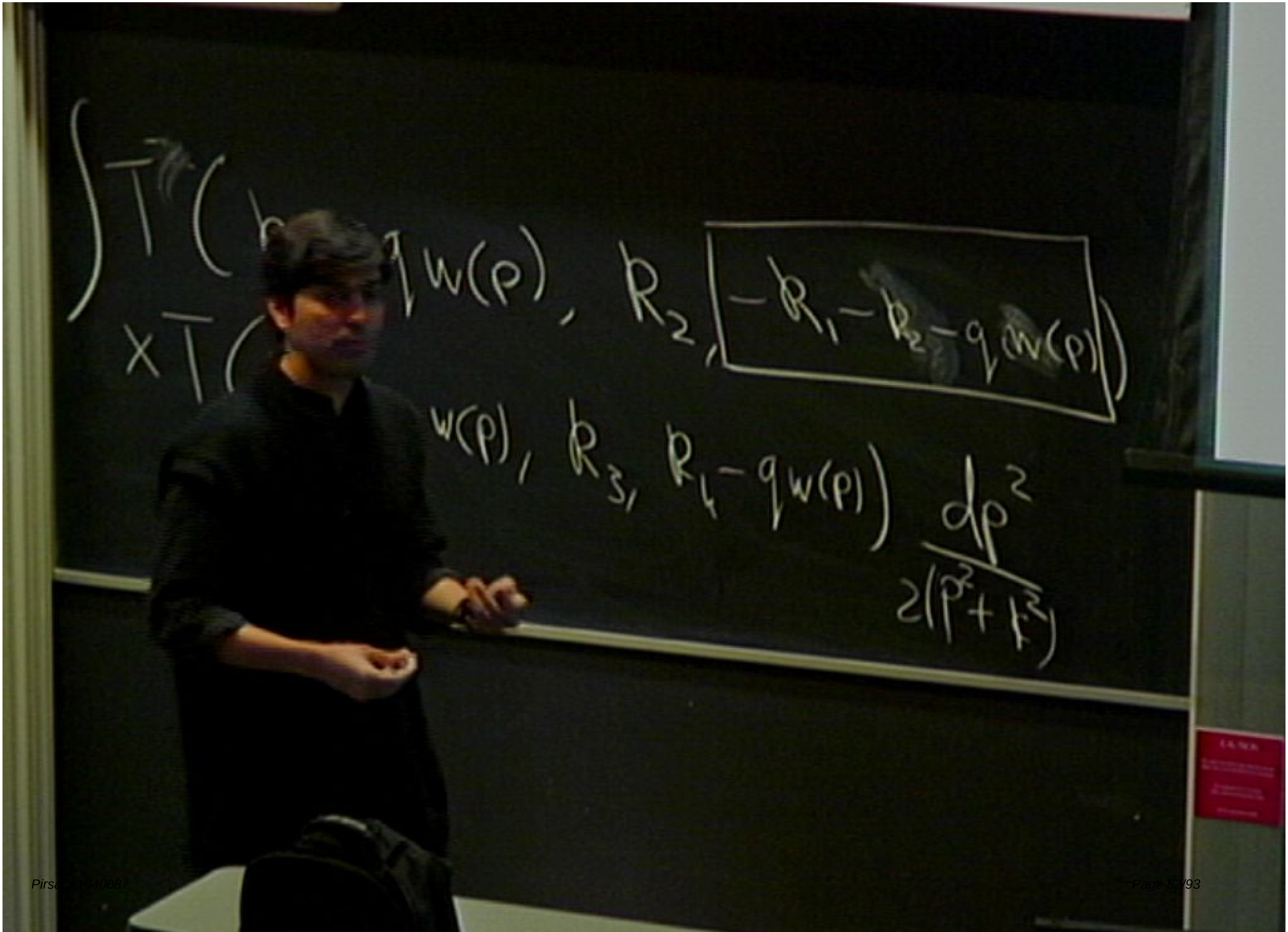

∫ T(k1 + q w(p), k2, −k1 − k2 − q w(p))
× T(k1 + k2 + q w(p), k3, k4 − q w(p)) dp² / 2(p² + k²)

$$\int T\left(k_1 + q\,w(p),\ k_2,\ -k_1 - k_2 - q\,w(p)\right)$$

$$\times\, T\left(k_1 + k_2 + q\,w(p),\ k_3,\ k_4 - q\,w(p)\right) \frac{dp^2}{2(p^2 + k^2)}$$


W(p) = -(k²+p²)/2(k·q)
dp²
2(p²+k²)

$$\int T\left(k_1 + q\,w(p),\ k_2,\ \boxed{-k_1 - k_2 - q\,w(p)}\right) T\left(k_1 + k_2 + q\,w(p),\ k_3,\ k_4 - q\,w(p)\right) \frac{dp^2}{2(p^2 + k^2)}$$

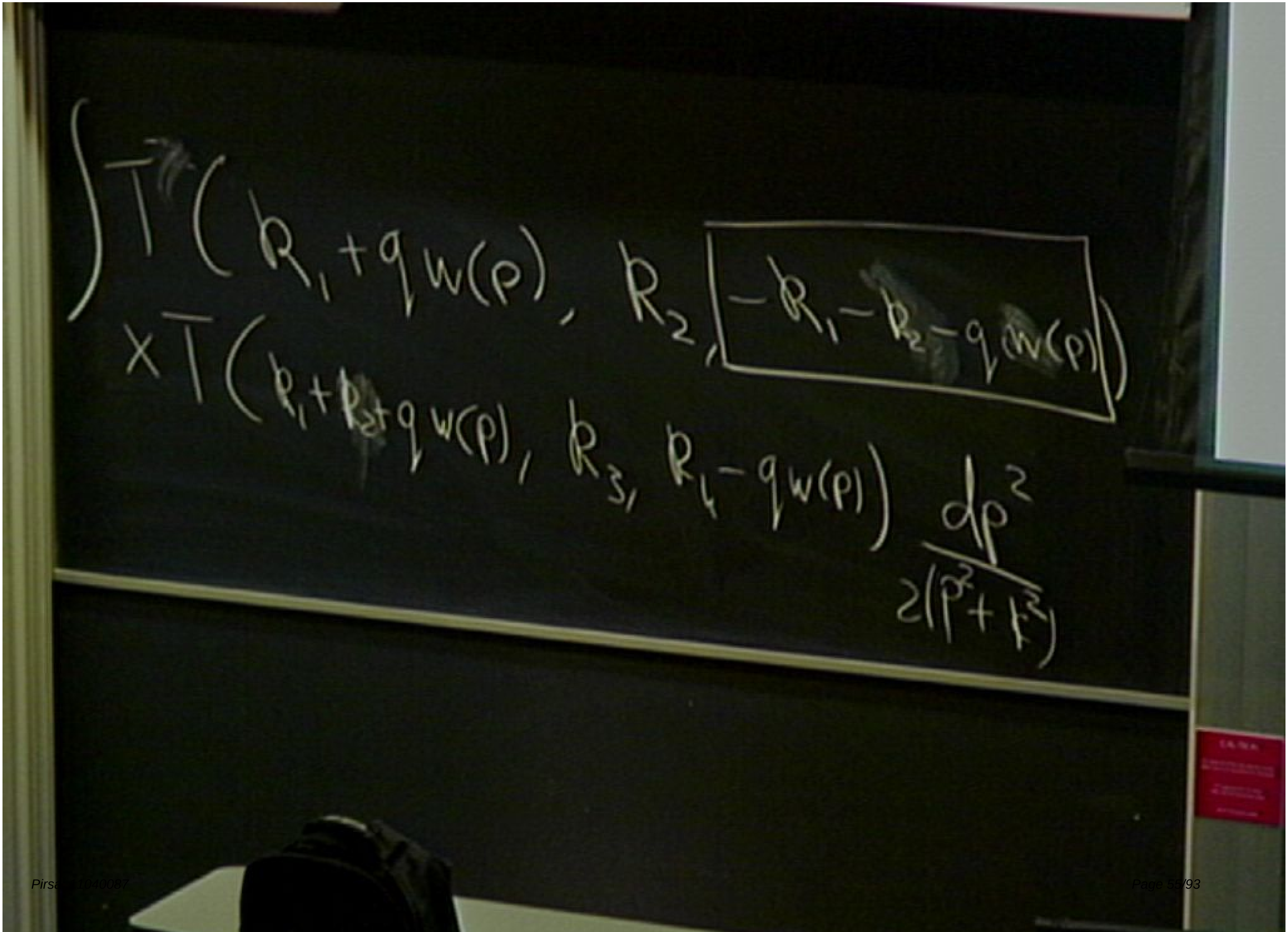


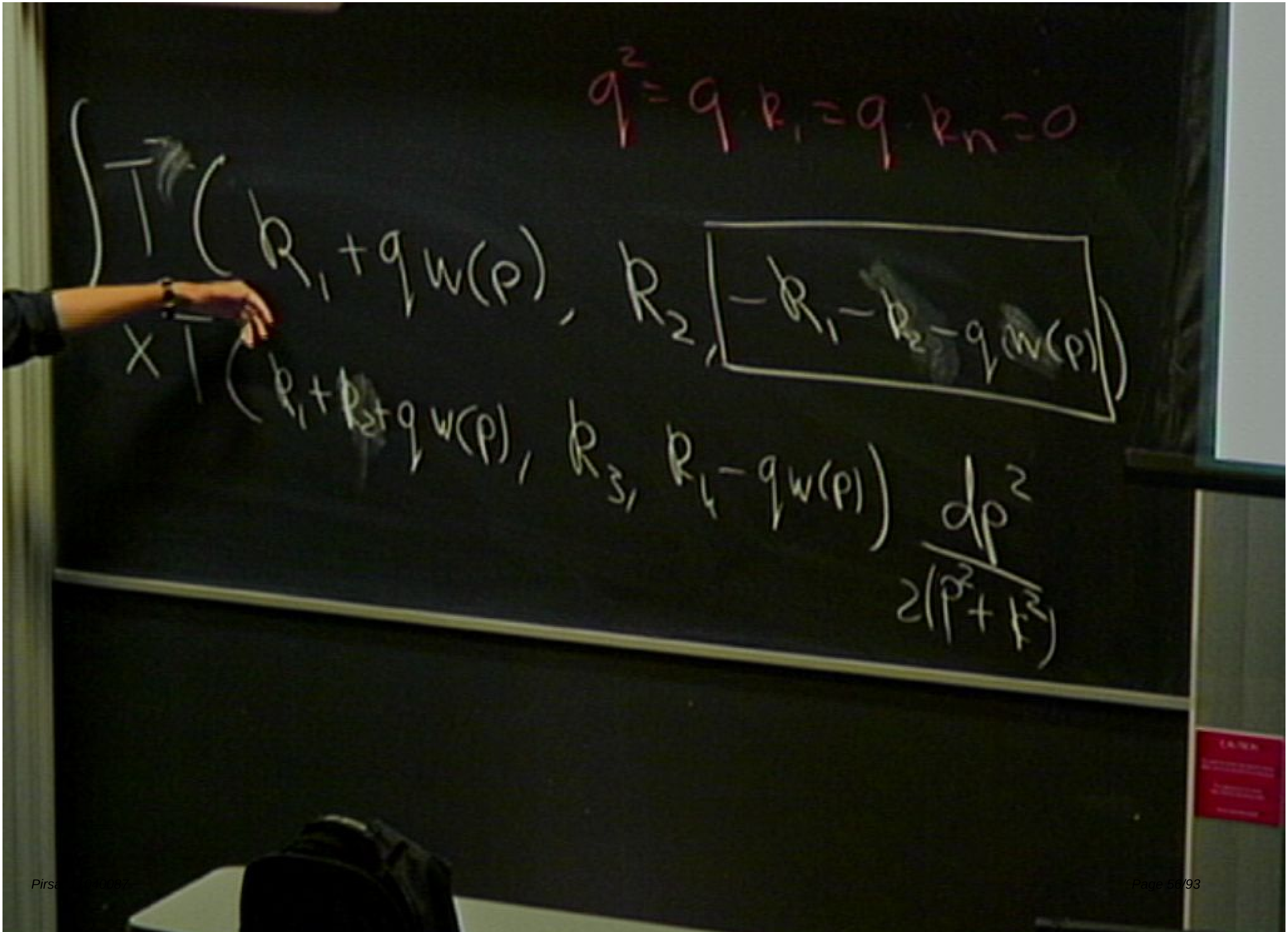
w(p), k2, −k1 − k2 − q w(p))
×T(
w(p), k3, k4 − q w(p)) dp^2 / 2(p^2 + k^2)

$$\int T\left(k_1 + q\,w(p),\ k_2,\ \boxed{-k_1 - k_2 - q\,w(p)}\right)$$

$$\times\, T\left(k_1 + k_2 + q\,w(p),\ k_3,\ k_4 - q\,w(p)\right) \frac{dp^2}{2(p^2 + k^2)}$$

$$\int T\left(k_1 + q\,w(p),\ k_2,\ \boxed{-k_1 - k_2 - q\,w(p)}\right) \times T\left(k_1 + k_2 + q\,w(p),\ k_3,\ k_4 - q\,w(p)\right) \frac{dp^2}{2(p^2 + k^2)}$$


$\int T(k_1 + q w(p), k_2, -k_1 - k_2 - q w(p))$
$\times T(k_1 + k_2 + q w(p), k_3, k_4 - q w(p)) \frac{dp^2}{2(p^2 + k^2)}$


$q^2 = q \cdot k_1 = q \cdot k_n = 0$
$\int T(k_1 + q\,w(p),\ k_2,\ -k_1 - k_2 - q\,w(p))$
$\times\, T(k_1 + k_2 + q\,w(p),\ k_3,\ k_4 - q\,w(p))\ \frac{dp^2}{2(p^2 + k^2)}$

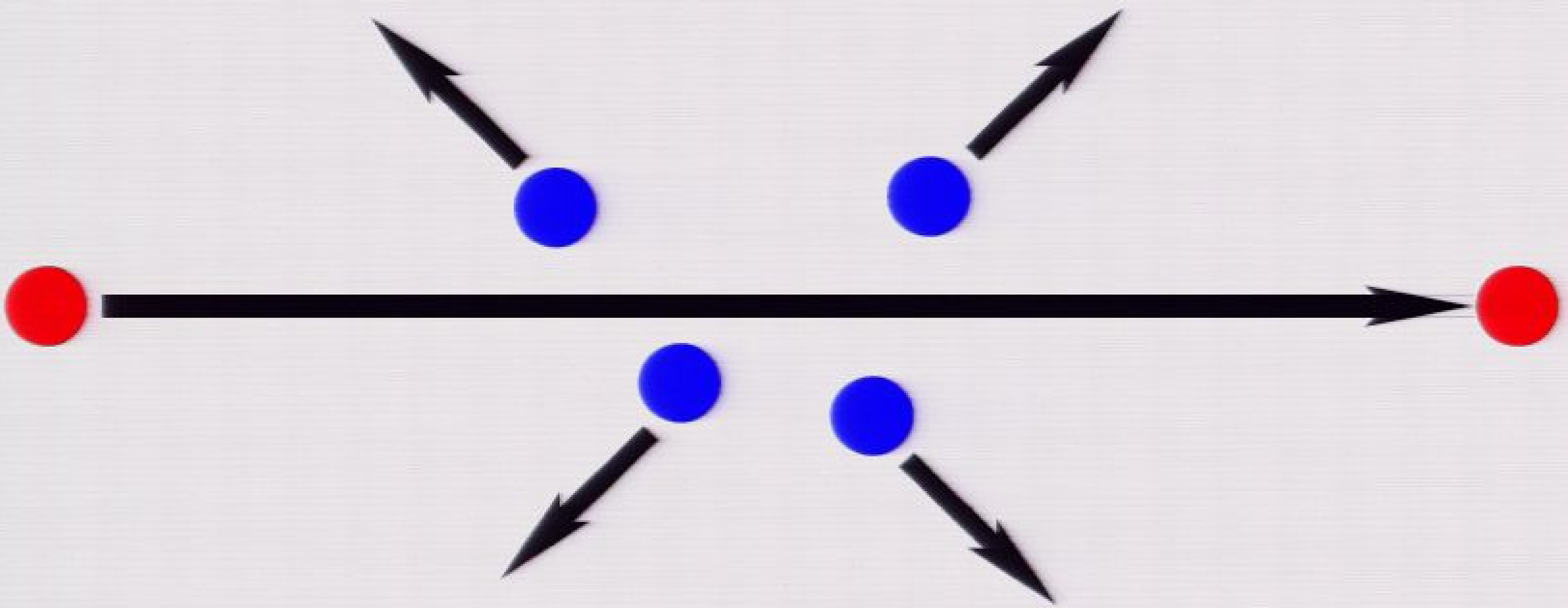
[redacted]

Surprising Result

- The usual BCFW recursion relations rely on the behaviour of tree-amplitudes under large complex deformations of the momenta.
- In AdS, amplitudes typically have essential singularities in the complex plane!
- Second, since AdS is like a box, particles in AdS are never infinitely separated.
- What we are calling amplitudes don't come from a conventional S-matrix. In fact, they compute correlation functions.

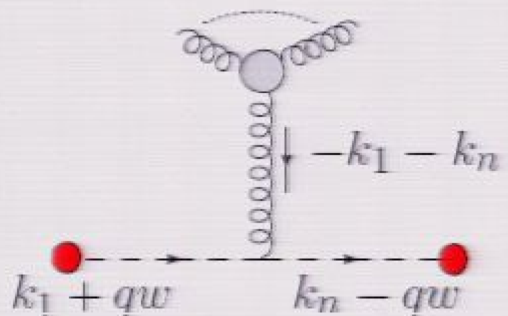
Underlying Physical Intuition

- The BCFW relations rely on extending two momenta to infinity in a complex direction.
- The amplitude involves one "highly boosted particle" interacting with a gas of soft particles.

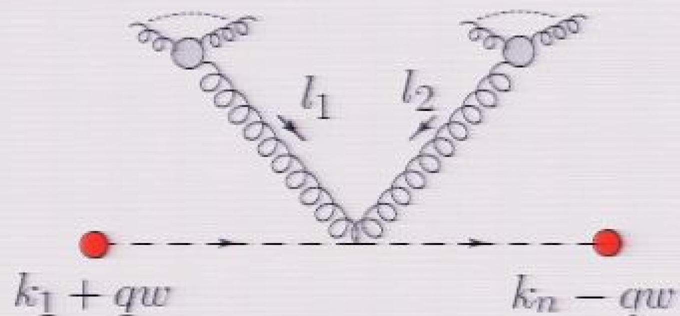


Physics of Large BCFW Extension

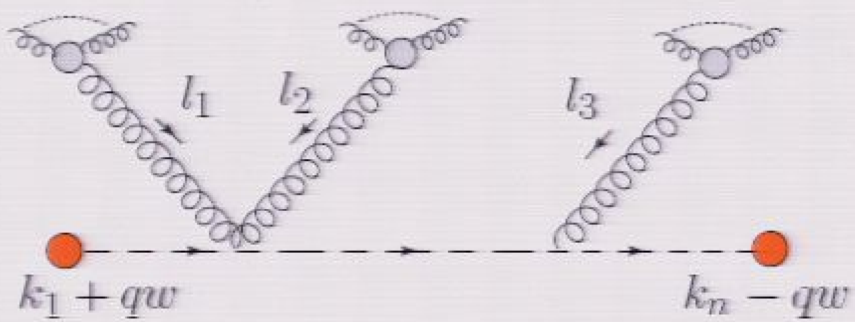
- Analyze this limit using background field gauge. eg. in Yang-Mills, consider a two point function in a classical background A_μ
- Choose q-lightcone gauge: $q \cdot A = 0$.



(a) O(w) Contribution



(b) O(1) contribution



(c) O($\frac{1}{\ }$) Contribution

Physical Intuition: Large w

- Amplitude is dominated by interactions between the boosted-particle and the soft-gas at a single point.

  [Arkani-Hamed, Kaplan, 2008]

- So, in this limit, curvature of background spacetime is not important.

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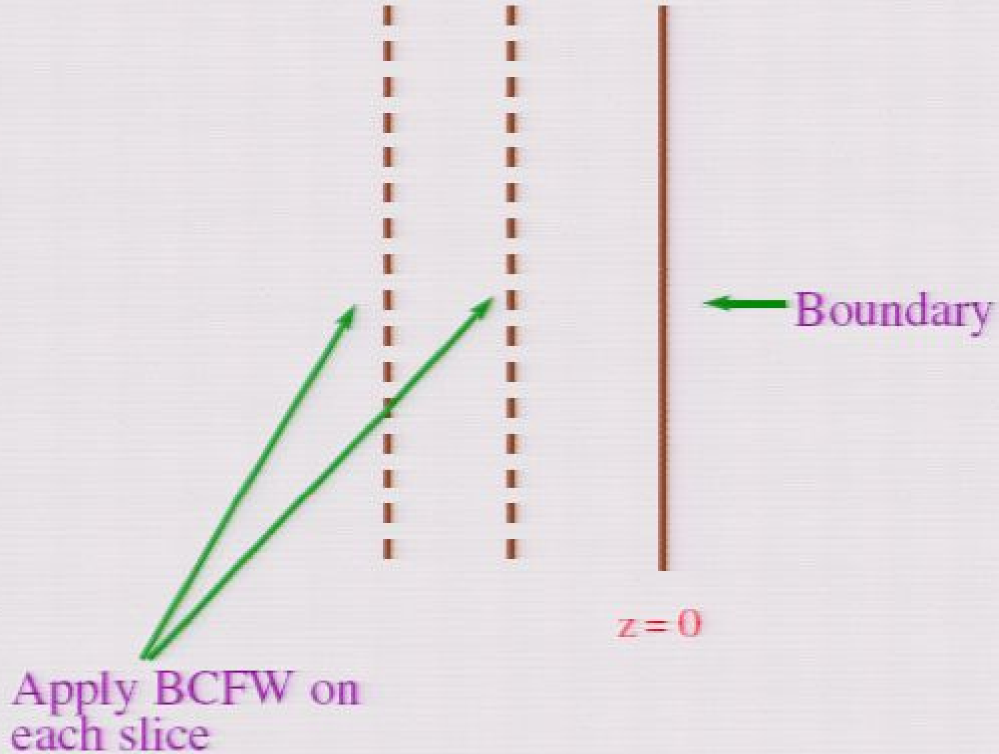
  [Arkani-Hamed, Kaplan, 2008]

- So, in this limit, curvature of background spacetime is not important.

- Caveat: In flat space, the position of the point is unimportant. In AdS, we need to integrate over all positions of this point.

Physical Intuition: Slicing AdS



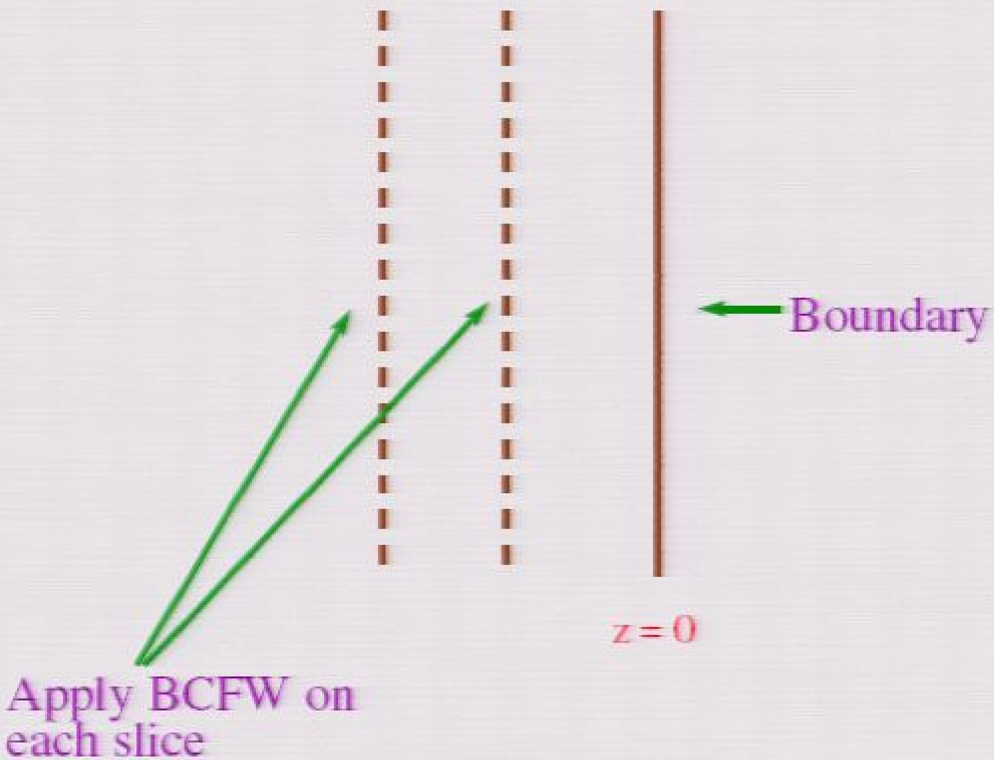
- The right procedure is to write the AdS amplitude as an average over AdS slices and apply BCFW on each slice.

- So, a higher-point amplitude breaks down into the integrated product of lower-point amplitudes.

mplementing the Intuition

- In AdS, there is a way of writing Witten diagrams as integrals over rational functions of the BCFW parameter w.
- We can argue, extending the logic above that, under a BCFW extension, the integrand dies off at large w.
- So, it can be completely reconstructed by the residues at its poles at finite w.

Physical Intuition: Slicing AdS



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- So, it can be completely reconstructed by the residues at its poles at finite w.
- As we will see, these residues are the products of the integrands of lower-point transition amplitudes.

AdS: Notation and Conventions

- Metric: $ds^2 = \frac{1}{z^2}(dz^2 + \eta_{ij}dx^i dx^j)$.
- The equation $\Box\phi = 0$ has solutions:

$$\text{normalizable:} \quad \phi(z) = z^{\frac{d}{2}}\phi_0 J_{\frac{d}{2}}(|k|z),$$

$$\text{non-normalizable:} \quad \phi(z) = z^{\frac{d}{2}}\phi_0 H^{(1)}_{\frac{d}{2}}(|k|z),$$

- The non-normalizable solution is called the bulk-to-boundary propagator.
- The bulk-bulk propagator is given by

$$G_k(z_1, z_2) = \int \frac{-ip\, dp}{(k^2 + p^2 - i\epsilon)} z_1^{\frac{d}{2}} J_{\frac{d}{2}}(pz_1) J_{\frac{d}{2}}(pz_2)(z_2)^{\frac{d}{2}},$$

BCFW in AdS

- Momenta do not have to be on-shell in AdS.
- However, it is still useful to deform

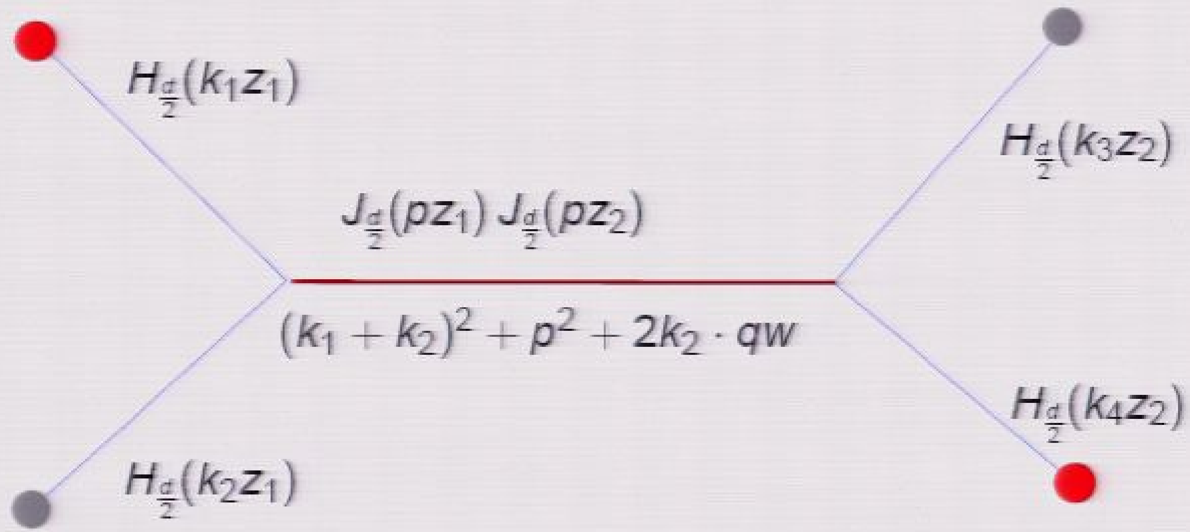
$$k_1 \to k_1 + qw$$
$$k_n \to k_n - qw$$

with

$$q^2 = q \cdot k_1 = q \cdot k_n = 0.$$

- This ensures that the k_1^2 and k_n^2 are unchanged $\Rightarrow$ Bessel functions never see w!

Anatomy of a Witten Diagram



Precisely at a pole,

$$w = -\frac{(k_1 + k_2)^2 + p^2}{2k_2 \cdot q},$$

the two Bessel functions in the bulk-bulk propagator can be combined with the left and right parts of the Witten diagram to give the product of two transition amplitudes.

Large w Behaviour

- For scalars, the integrand cannot be reconstructed from these residues, because the diagram where k_1 and k_n meet at a point goes to a constant at large w.
- For non-Abelian Yang-Mills in AdS, or General Relativity, one can show that the integrand dies off at large w.
- This leads to recursion relations: a higher-point correlator breaks up into an integrated product of transition amplitudes.

Comparison with Flat Space

FLAT SPACE:

$$M \sim \sum_{\text{partitions}} \frac{M_{\text{left}} M_{\text{right}}}{K^2}$$

AdS:

$$T \sim \sum_{\text{partitions}} \int \frac{T_{\text{left}}(p) T_{\text{right}}(p)}{p^2 + K^2} \frac{dp^2}{2}$$

- Integrating over the intermediate momentum is like integrating over the radial direction in AdS.
- The constraints on external polarizations are like those of massive flat-space theories.

Relevance of Result

This result is again relevant for two reasons:

1. Computational
2. Formal

Computational Difficulties in AdS

$$\xrightarrow[\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}]{\delta^3 S}$$

$$\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})$$
$$-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})$$
$$-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],$$

$$\xrightarrow[\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}]{\delta^4 S}$$

$$\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})$$
$$+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$$
$$+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$$
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$$-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\rho p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})$$
$$+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\sigma}\eta^{\tau\lambda}\eta^{\rho\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})$$
$$-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].$$

The difficulties with gravitational perturbation theory are exacerbated in AdS.

Severity of Computational Difficulties

- In AdS, this is so severe that even the four-graviton scattering amplitude has never been calculated.
- This amplitude is dual to the simplest nontrivial correlator of the stress-tensor.
- This is despite the fact that this correlator is of special interest because it is universal, i.e. it should be the same in any conformal field theory with a gravity dual.

Ameliorating Computational Difficulties

- These new recursion relations
  (a) suggest that the answer should be simple.
  (b) give a method of computing it by reducing it down to a product of three-point functions.

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- These new recursion relations
  (a) suggest that the answer should be simple.
  (b) give a method of computing it by reducing it down to a product of three-point functions.

- Disadvantage: Correlators in momentum space have divergences that need to be regulated.

Formal Motivations

- These recursion relations are very unexpected from the boundary point of view.
- No such (known) relations at weak coupling.
- Can we understand the origin of these relations on the boundary?

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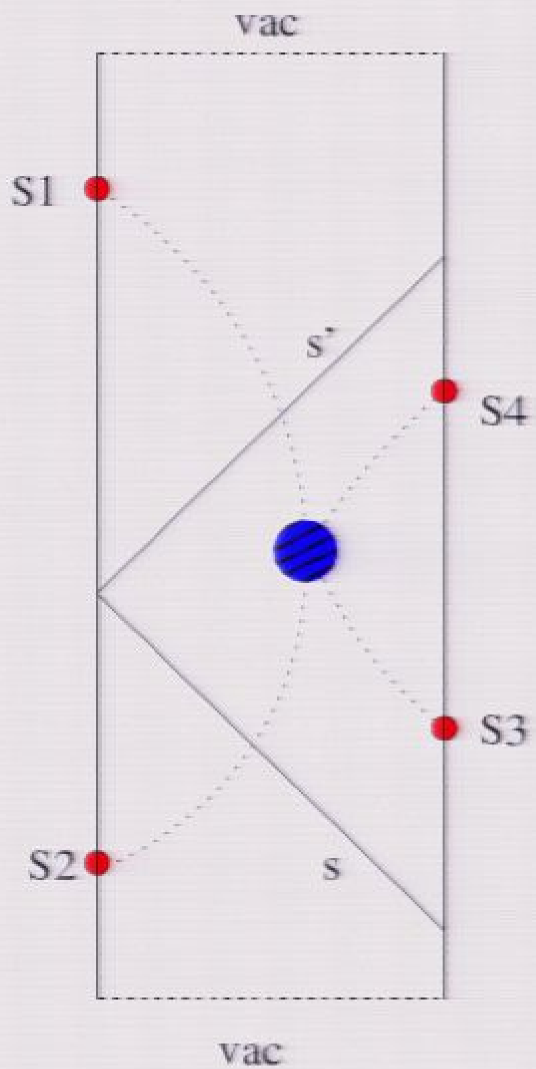
- These recursion relations are very unexpected from the boundary point of view.
- No such (known) relations at weak coupling.
- Can we understand the origin of these relations on the boundary?
- However, the new recursion relations also tell us other interesting things about the boundary.

Transition Amplitudes as Correlators



Transition Amplitudes on the Poincare patch can be interpreted as correlators in global AdS

Formal Motivations: Implications for the OPE

- This suggests that stress-tensor correlators can be written in terms of lower-point correlators: somewhat unexpected!
- The OPE of the stress-tensor is not closed; contains other operators including multi-trace operators.

$$T(x)T(0) = \sum_k C_k(x) O_k(0).$$

- So, if we break up a higher-point correlator into smaller correlators using the OPE, these smaller correlators will contain all these operators.
- Somehow, this is automatically accounted for by this closed set of recursion relations!

Extension to Supersymmetric Theories

- BCFW extensions of scalars and fermions are not well behaved; supersymmetric theories necessarily contain these particles.
- However, with sufficient supersymmetry, we can relate the scattering of other particles to that of gluons or gravitons.
- We now calculate the amplitude via the usual BCFW extension for gluons/gravitons.

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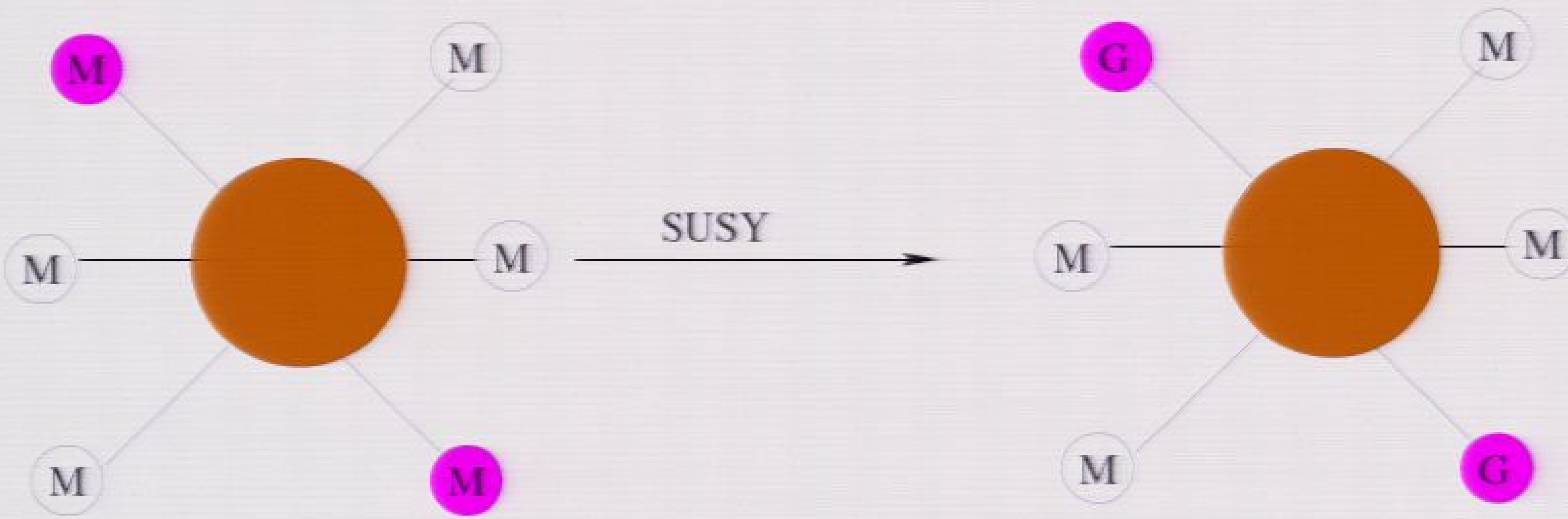
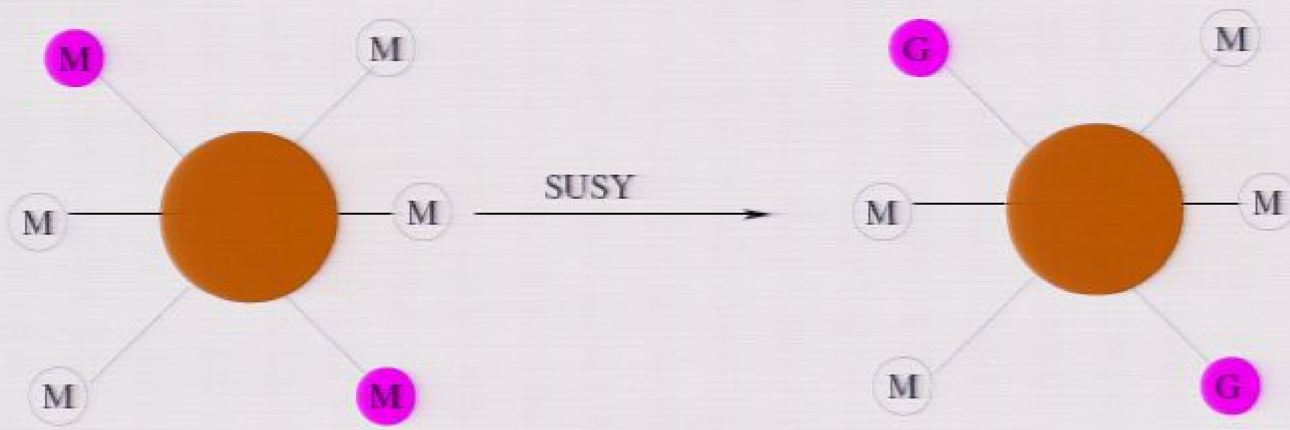
Supersymmetric Amplitudes



- If external particles live in a (1/2)-BPS multiplet, we can convert two particles to gluons/gravitons.

Complications With Susy In Ads



- However, we cannot control the polarizations of these gluons/gravitons.
- In flat-space, maximal Susy (as in $\mathcal{N} = 4$ SYM or $\mathcal{N} = 8$ SUGRA) is enough to compute all amplitudes.
- In AdS, we can only compute a subset of amplitudes, even with maximal Susy.

Extensions of these Results I

- We would like to include α' and $\frac{1}{N}$ corrections in the bulk.
- Incorporating $\frac{1}{N}$ corrections involves generalizing loop-level flat-space techniques to AdS.
- In flat space, a version of the BCFW relations works for string theory. Generalization to AdS?

  [Boels, Larsen, Obers, Vonk, 2008]
- Many other properties of flat-space amplitudes can be investigated: twistors, Grassmannian, KLT, ...

Extensions of these Results II

- The physical intuition given here, suggests that these calculations should go through in the presence of a black-hole in the bulk.
- This should give an easy way of calculating stress-tensor correlators at finite-temperature.
- Are there any phenomenological applications for heavy-ion physics or other systems? (Speculative!)

Summary

Setting

Developments in Amplitudes

- Unexpected Simplifications
- BCFW Recursion Relations
- Formal Motivations
- Computational Motivations

AdS/CFT

- Witten Diagrams

BCFW for Witten Diagrams

- Why this is surprising
- Physical Intuition
- Sketch of Derivation
- Applications of the new Recursion Relations

Extensions

- Supersymmetric Theories
- Other Possible Extensions

To conclude, I would just like to say that this is a very rich field and also not very difficult to enter. So, I hope that some of you will join!

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THANK YOU!