Title: Explorations in Condensed Matter - Lecture 15

Date: Apr 01, 2011 10:15 AM

URL: http://pirsa.org/11040084

Abstract:

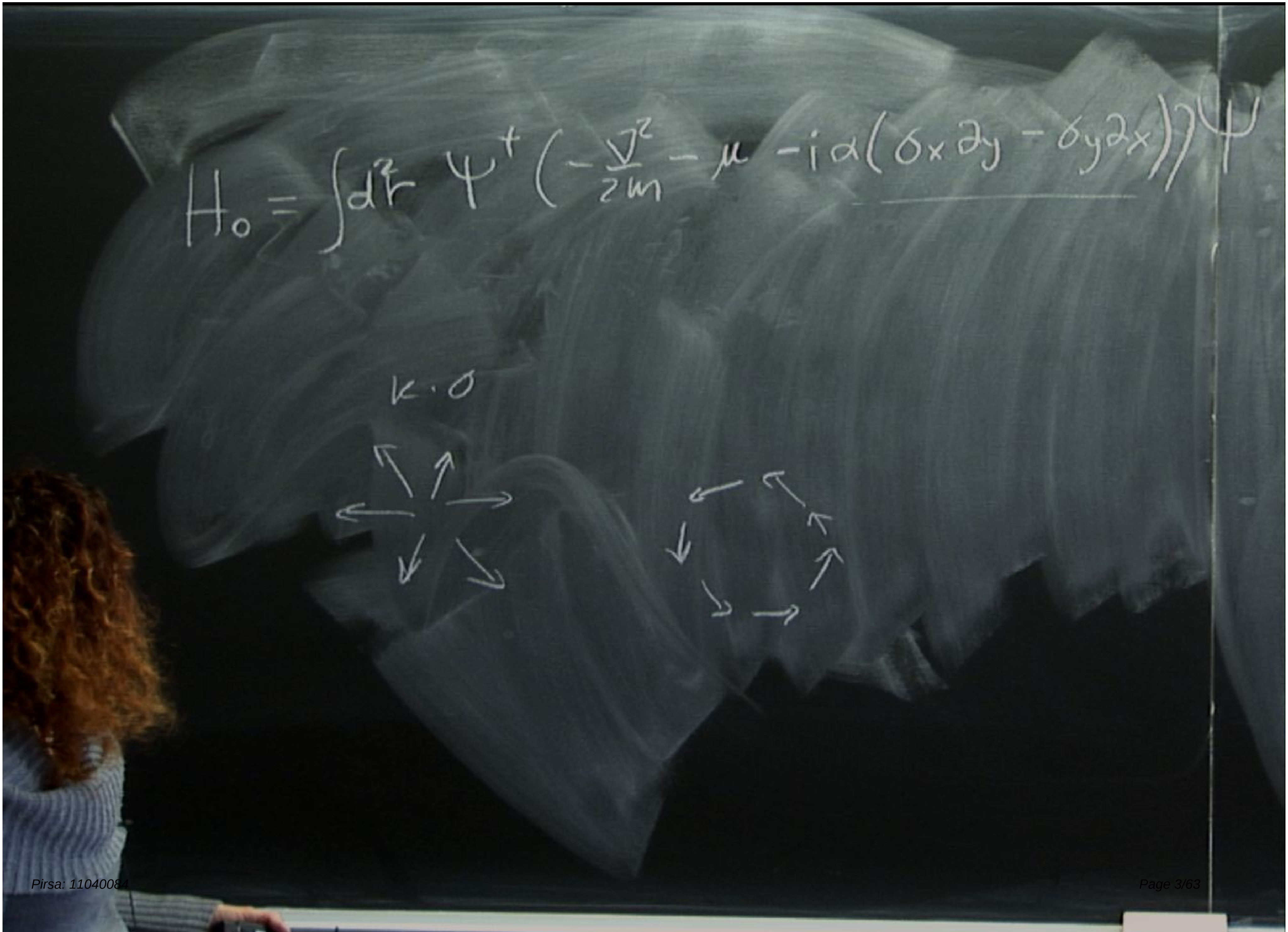
3D TI

$(\nu_0; \nu_1, \nu_2, \nu_3)$

$\downarrow$

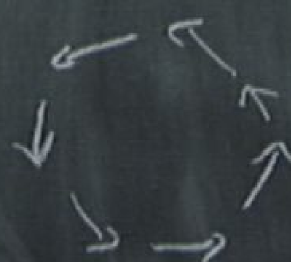
$\nu_0 = 0 \longrightarrow$ Weak TI

$\nu_0 = 1 \longrightarrow$ Strong TI


$H_0 = \int d^2r\, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x) \right) \psi$
$k \cdot \sigma$

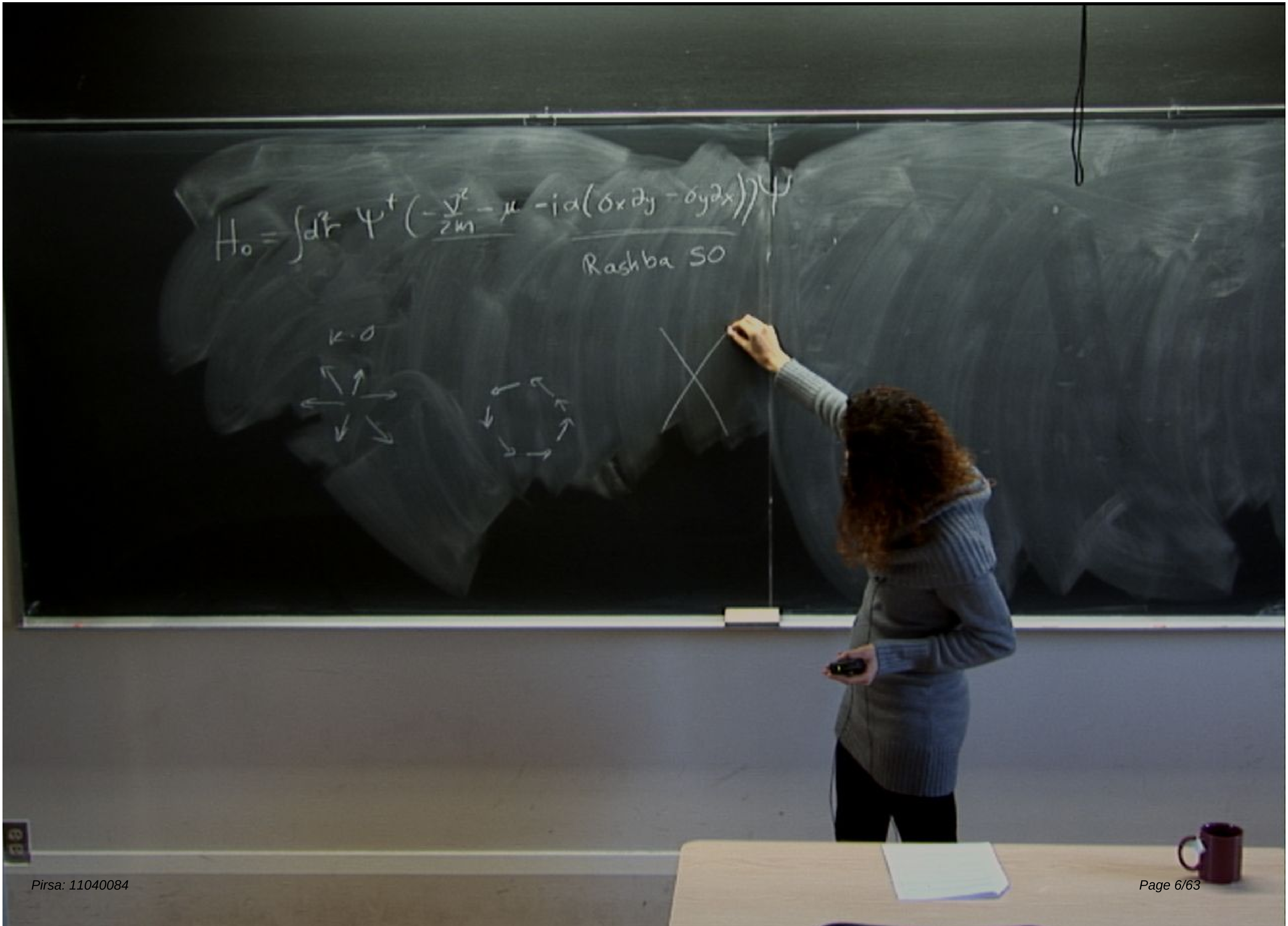
$$H_0 = \int d^2r \, \psi^\dagger \Big(\underbrace{-\frac{\nabla^2}{2m} - \mu}_{} \; \underbrace{- i\alpha(\sigma_x \partial_y - \sigma_y \partial_x)}_{\text{Rashba SO}} \Big) \psi$$

$k \cdot \sigma$

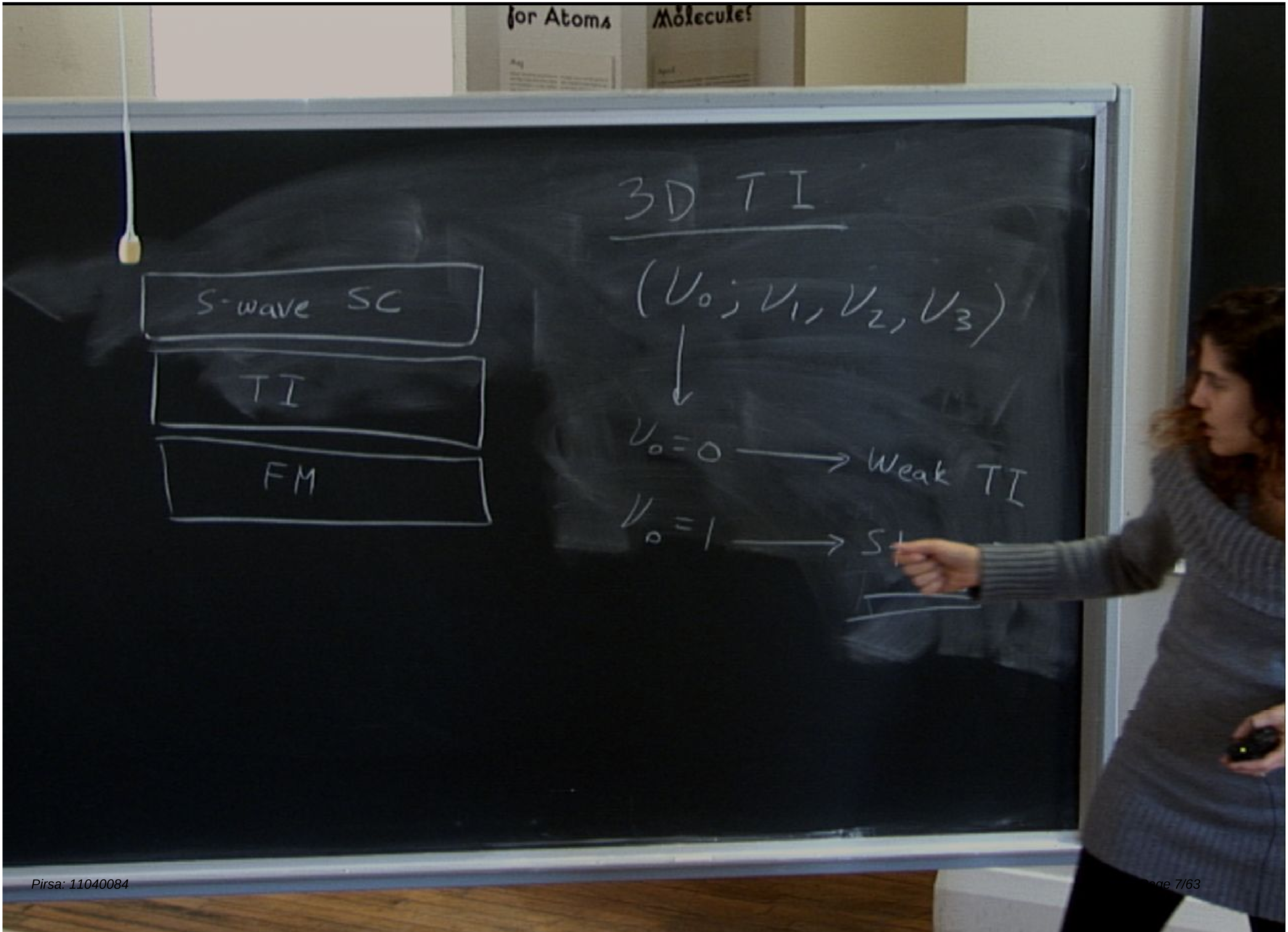


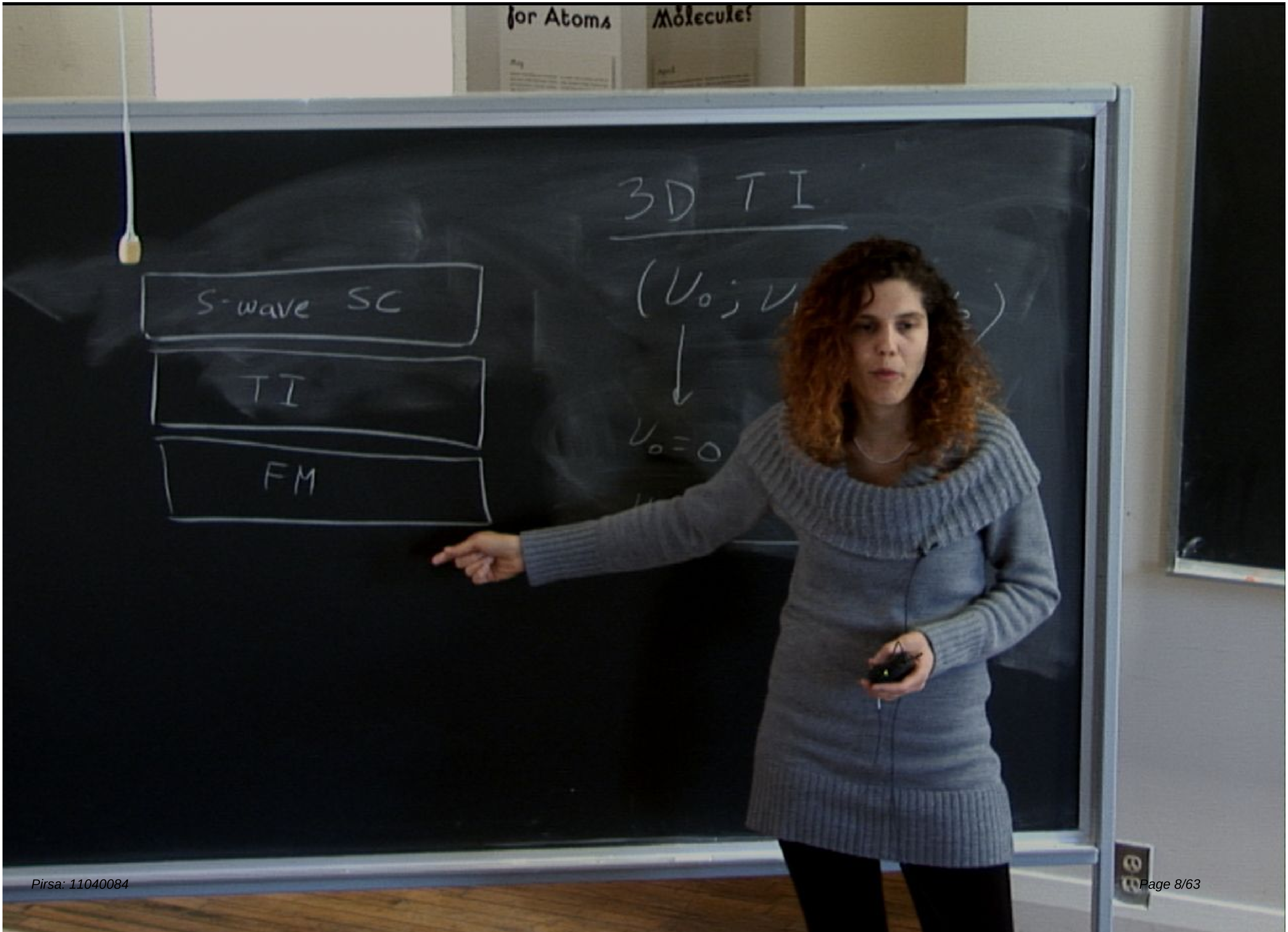
$$H_0 = \int d^2r\, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x)\right)\psi$$

Rashba SO



$H_0 = \int d^2r\, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x) \right) \psi$
Rashba SO

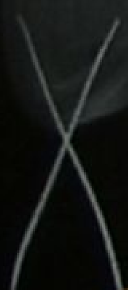

for Atoms
Molecules
S-wave SC
TI
FM
3D TI
(ν₀; ν₁, ν₂, ν₃)
ν₀ = 0 → Weak TI
ν₀ = 1 → S


for Atoms
Molecules
3D TI
S-wave SC
TI
FM

$$H_0 = \int d^2r\, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x) \right) \psi$$

Rashba SO

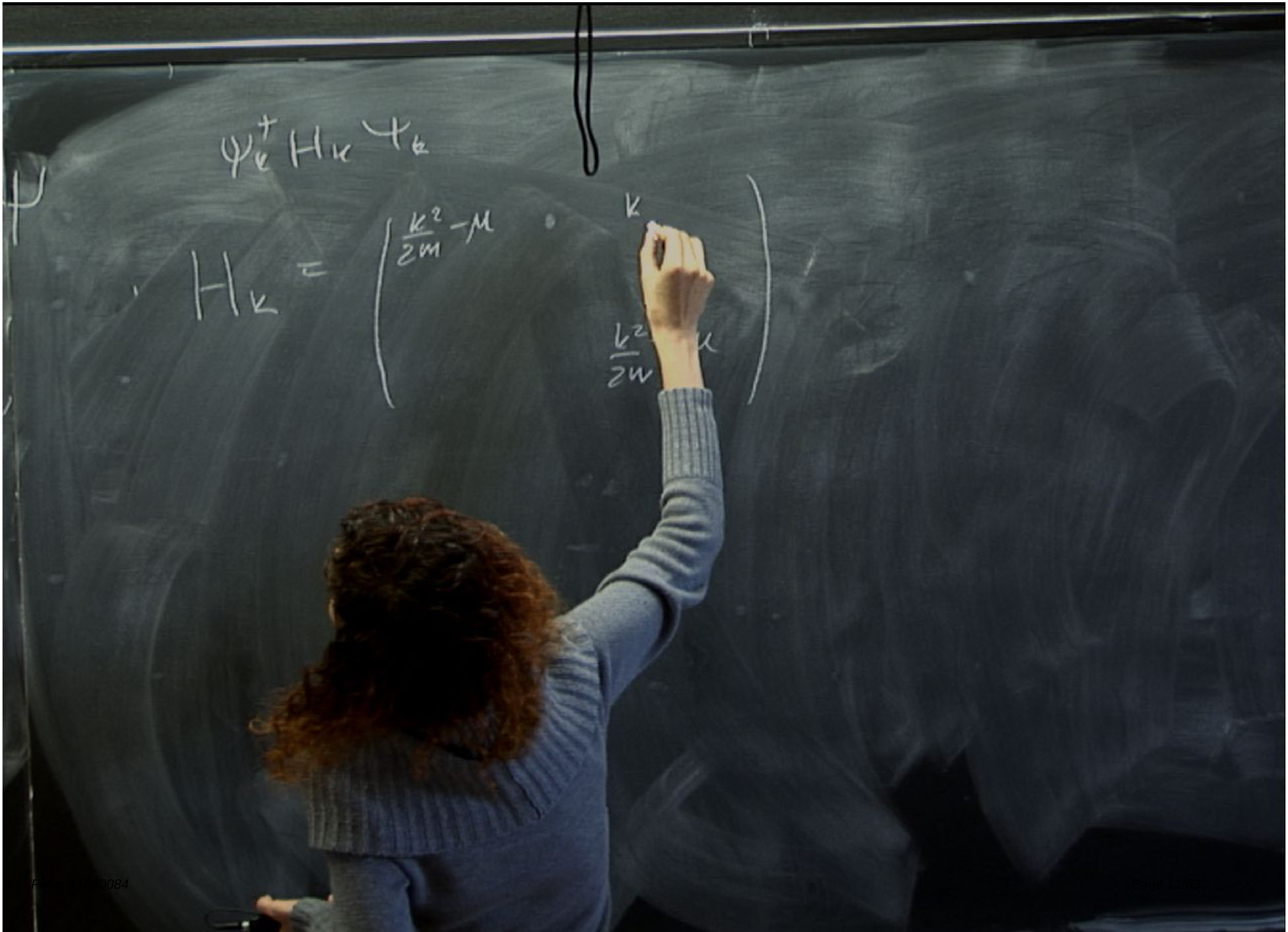
$$H_Z = \int d^2r\, \psi^\dagger \sigma_z V_Z \psi$$

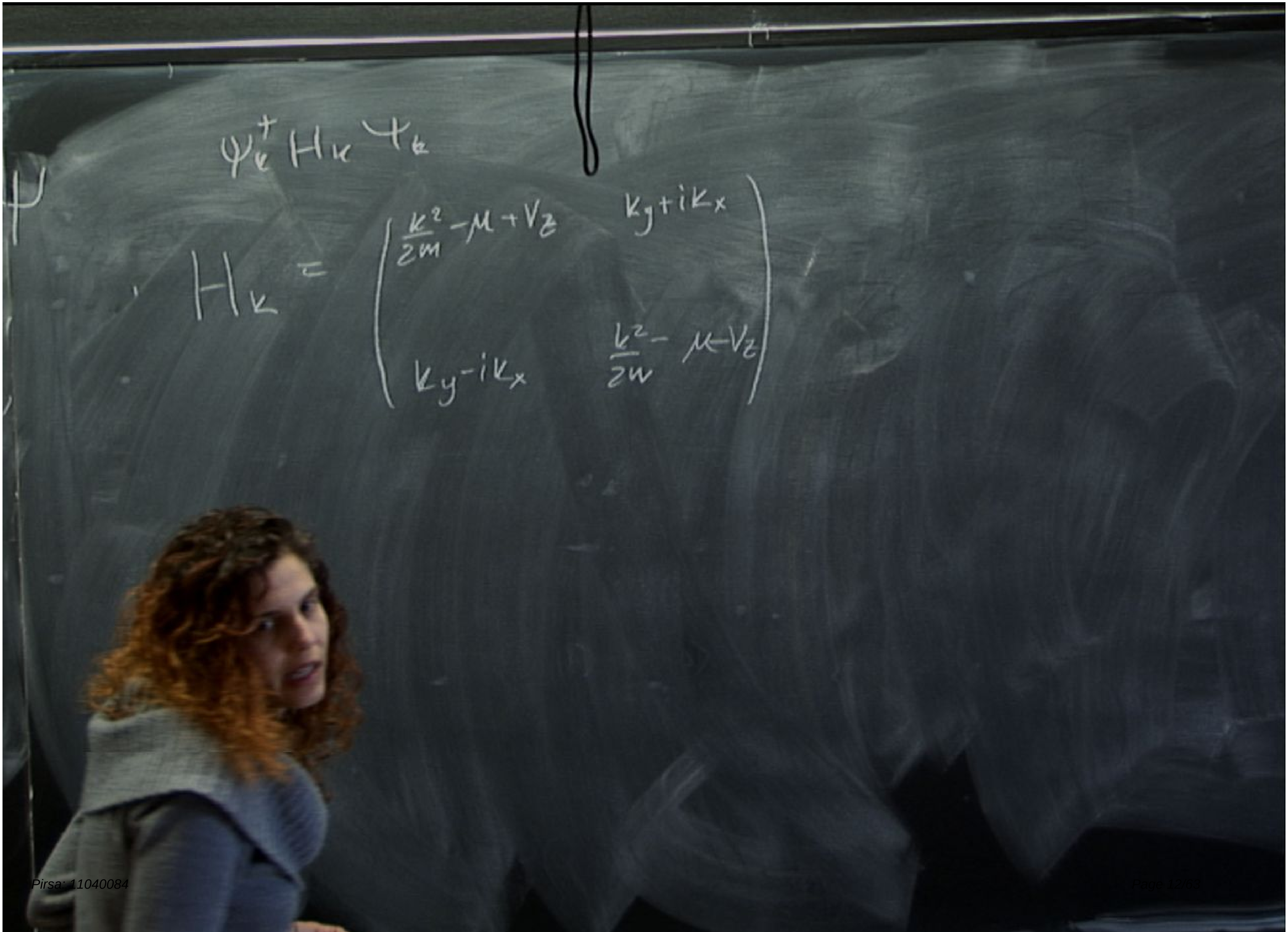


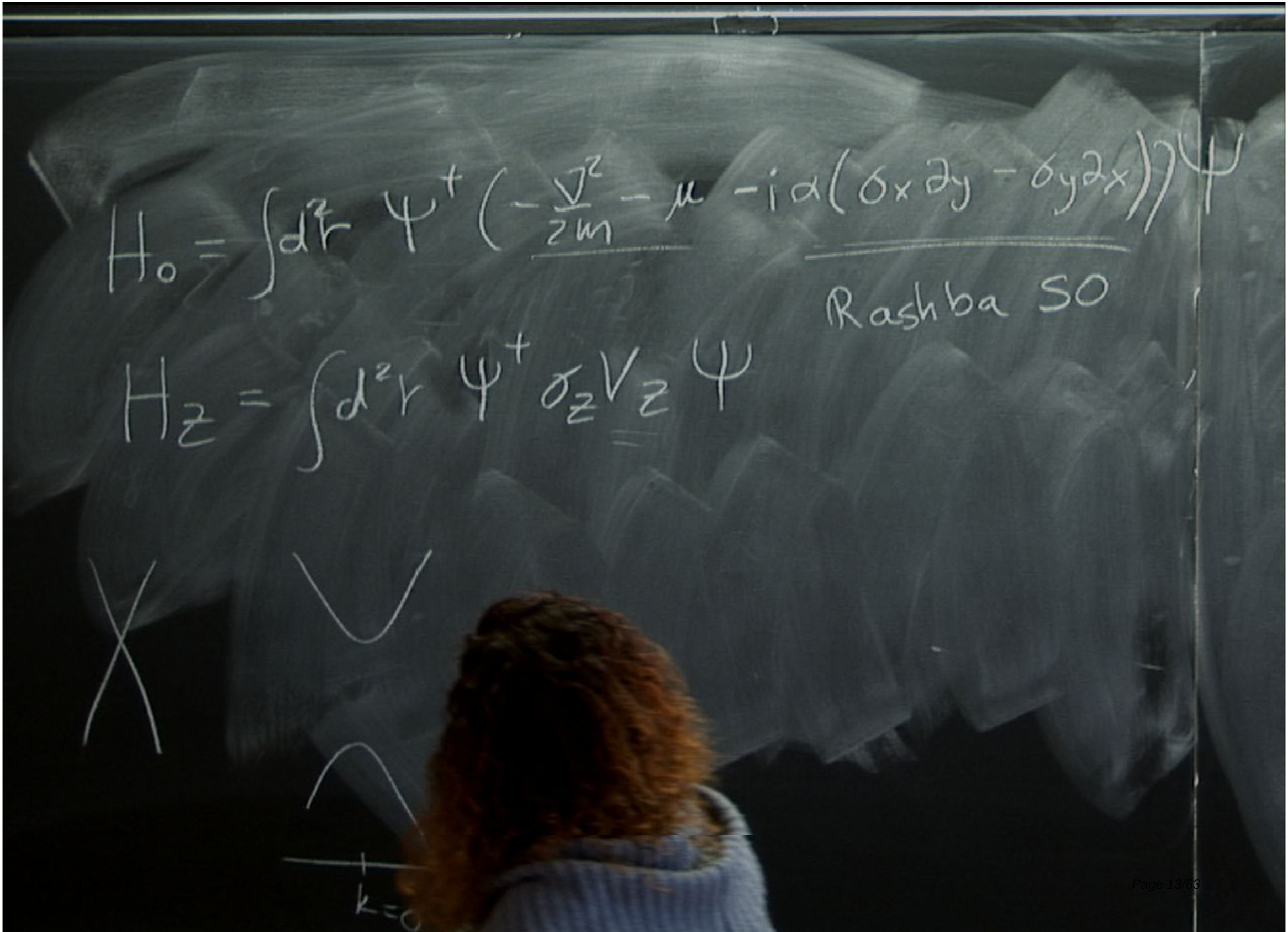
$$H_0 = \int d^2r \, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x) \right) \psi$$

Rashba SO

$$H_Z = \int d^2r \, \psi^\dagger \sigma_Z V_Z \psi$$

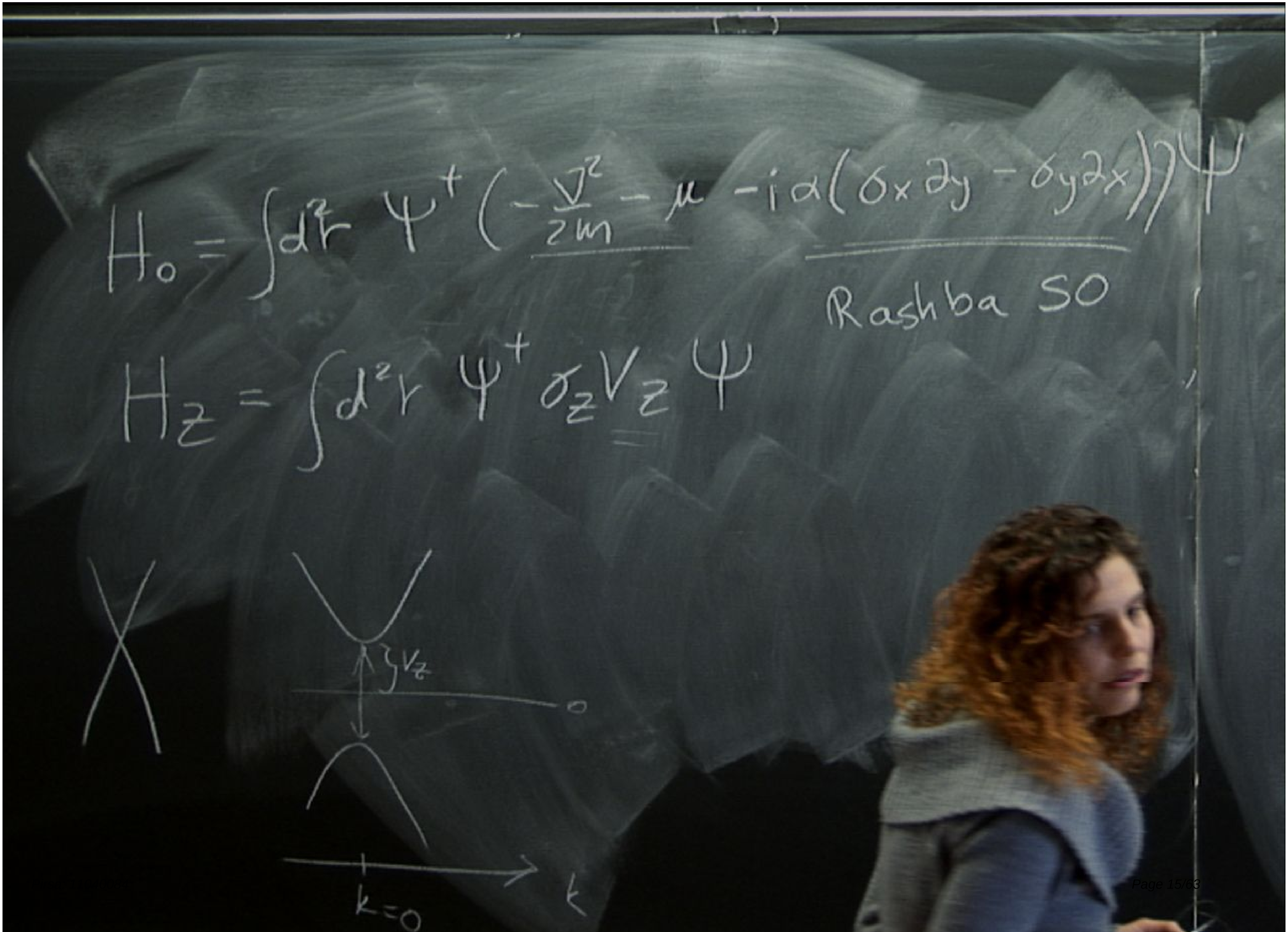

ψ_k† H_k ψ_k
H_k = (k²/2m − μ   k   k²/2m − μ)

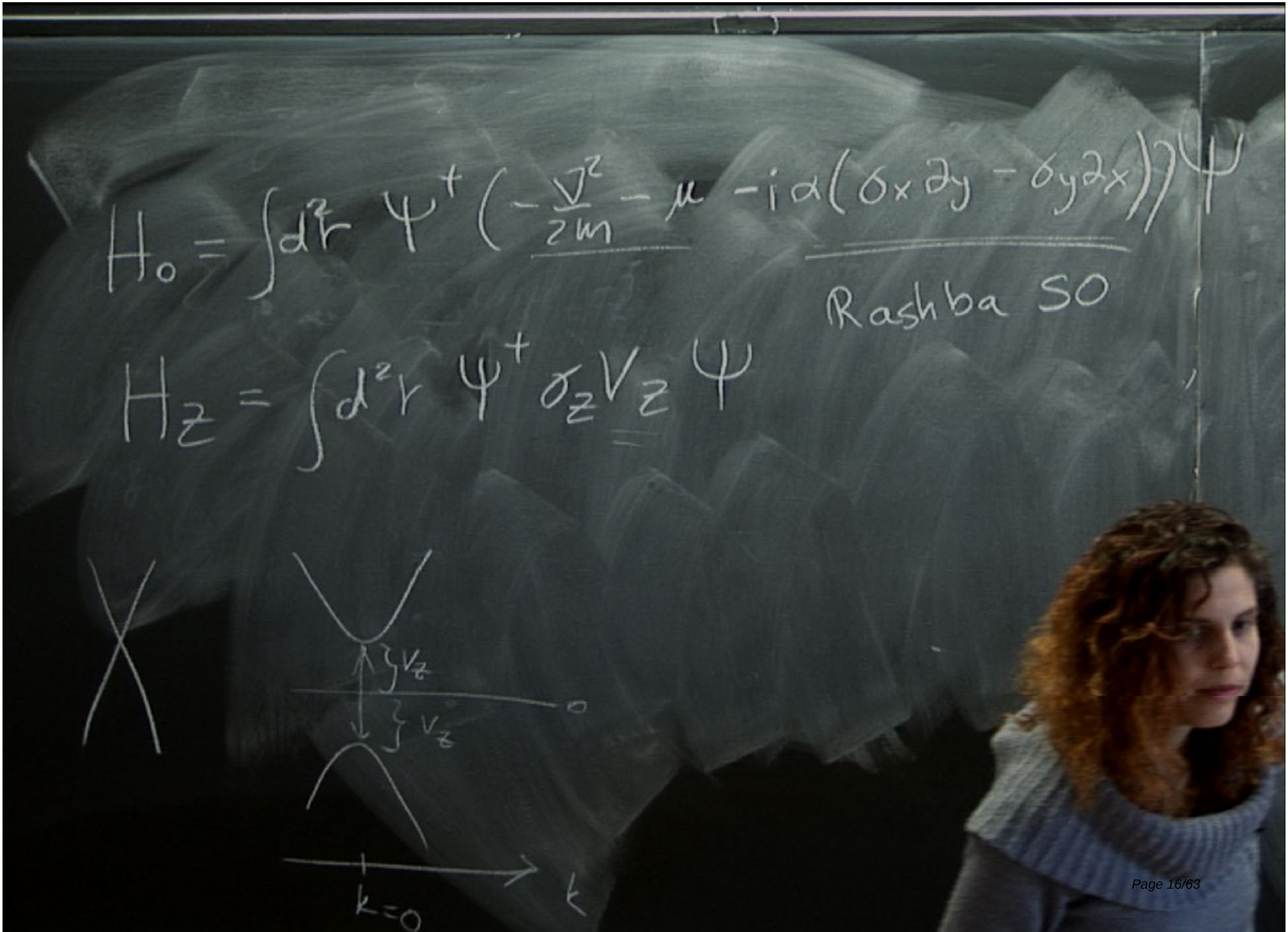

ψ_k^† H_k ψ_k
H_k = (k²/2m − μ + V_z   k_y + ik_x)
(k_y − ik_x   k²/2m − μ − V_z)


$H_0 = \int d^2r \, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x)\right)\psi$
Rashba SO
$H_Z = \int d^2r \, \psi^\dagger \sigma_Z V_Z \psi$
k=0

$$H_0 = \int d^2r \, \psi^\dagger \Big(\underbrace{-\frac{\nabla^2}{2m} - \mu}_{} \; \underbrace{-i\alpha(\sigma_x \partial_y - \sigma_y \partial_x)}_{\text{Rashba SO}} \Big) \psi$$

$$H_Z = \int d^2r \, \psi^\dagger \sigma_z V_z \psi$$


$H_0 = \int d^2r\, \Psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x \partial_y - \sigma_y \partial_x) \right) \Psi$
Rashba SO
$H_Z = \int d^2r\, \Psi^\dagger \sigma_Z V_Z \Psi$
V_Z
$k=0$
k


$H_0 = \int d^2r\, \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma_x\partial_y - \sigma_y\partial_x)\right)\psi$
Rashba SO
$H_Z = \int d^2r\, \psi^\dagger \sigma_z V_z \psi$
V_z
V_z
0
k=0
k

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & k_y + i k_x \\ k_y - i k_x & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

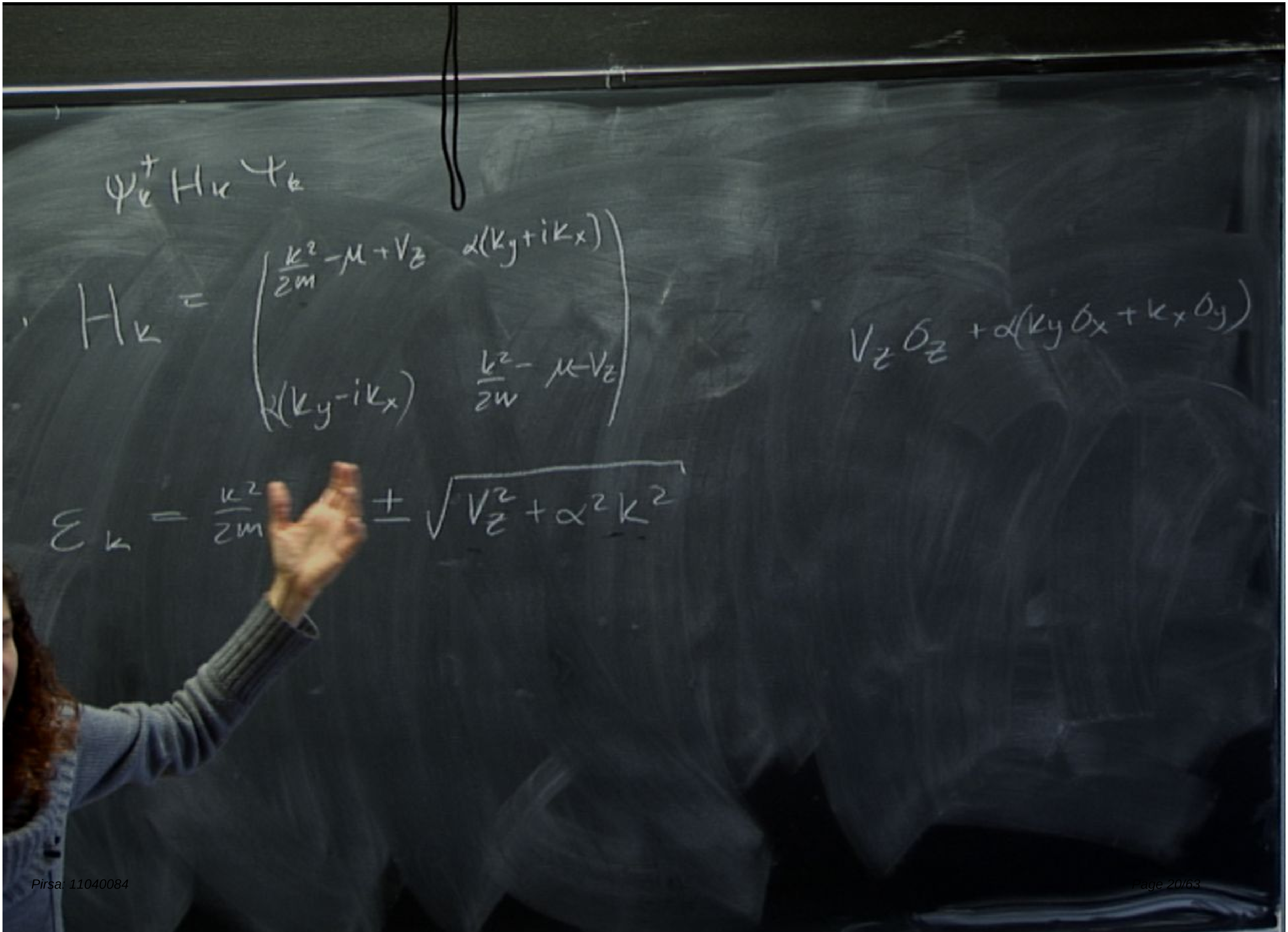
$$\varepsilon_k =$$

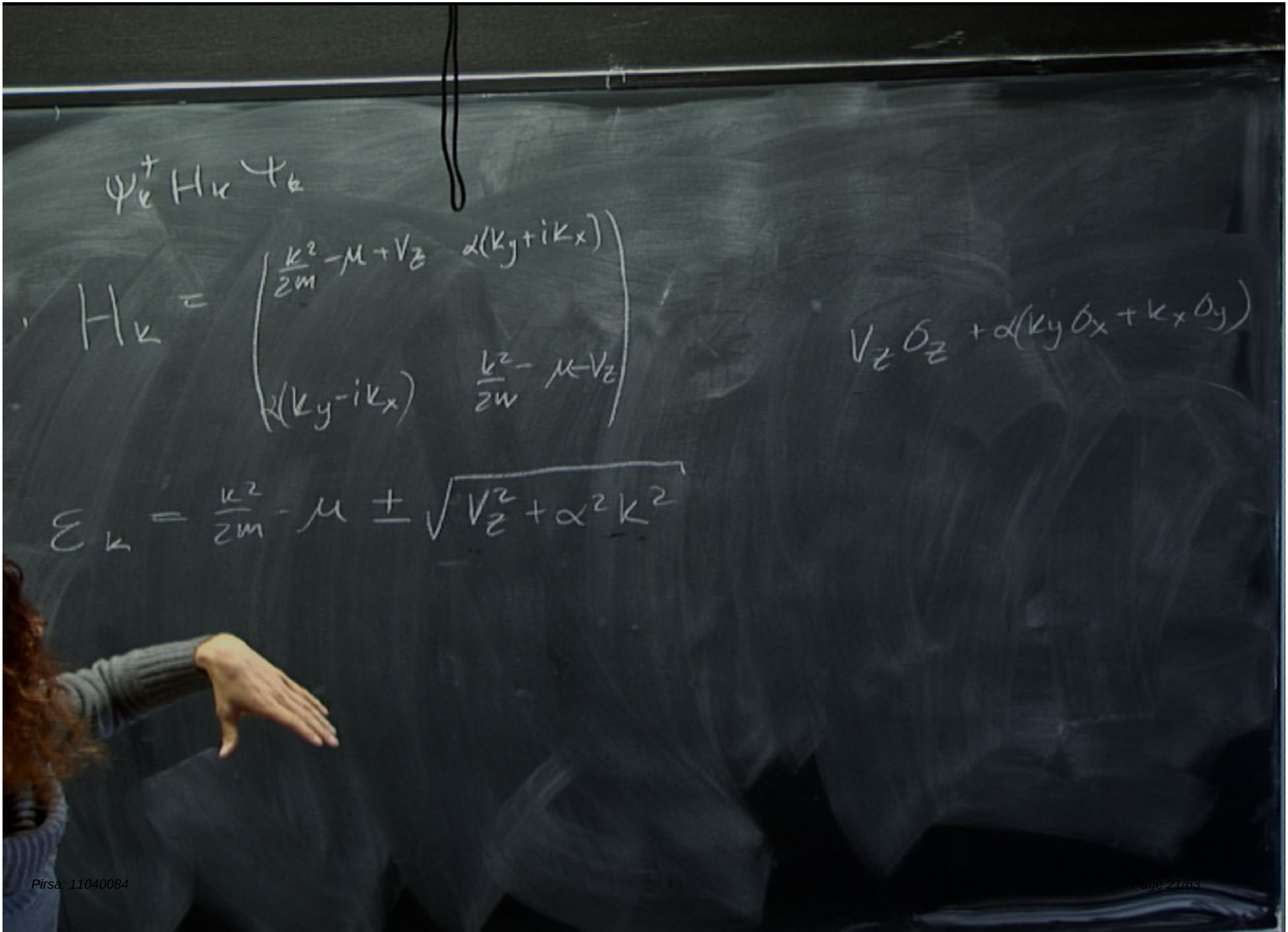
$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$V_z\sigma_z + \alpha(k_y\sigma_x + k_x\sigma_y)$$

$$\varepsilon_k = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2k^2}$$


$\psi_k^\dagger H_k \psi_k$
$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$
$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$
$E_k = \frac{k^2}{2m} \pm \sqrt{V_z^2 + \alpha^2 k^2}$


$\psi_k^\dagger H_k \psi_k$
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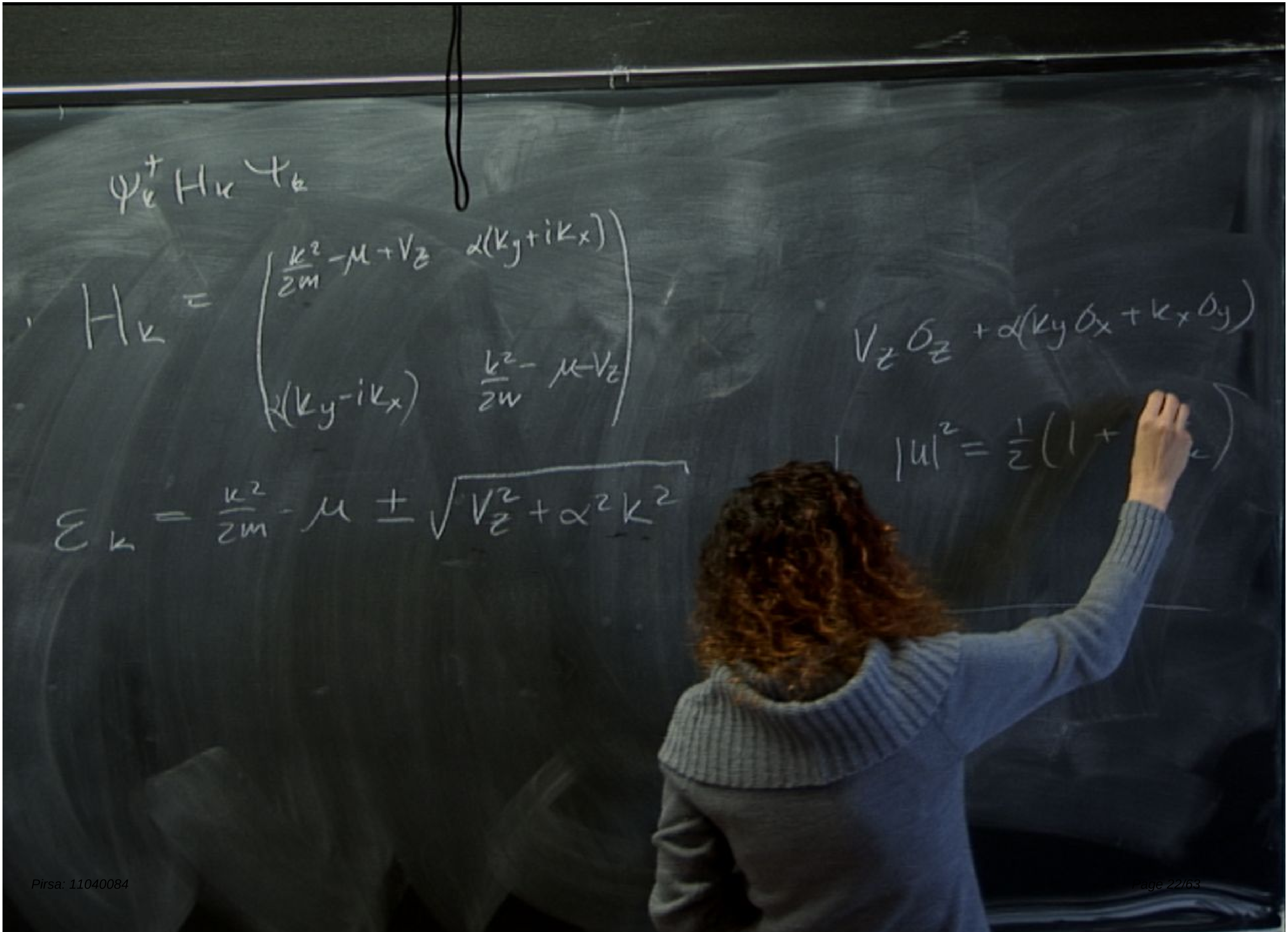
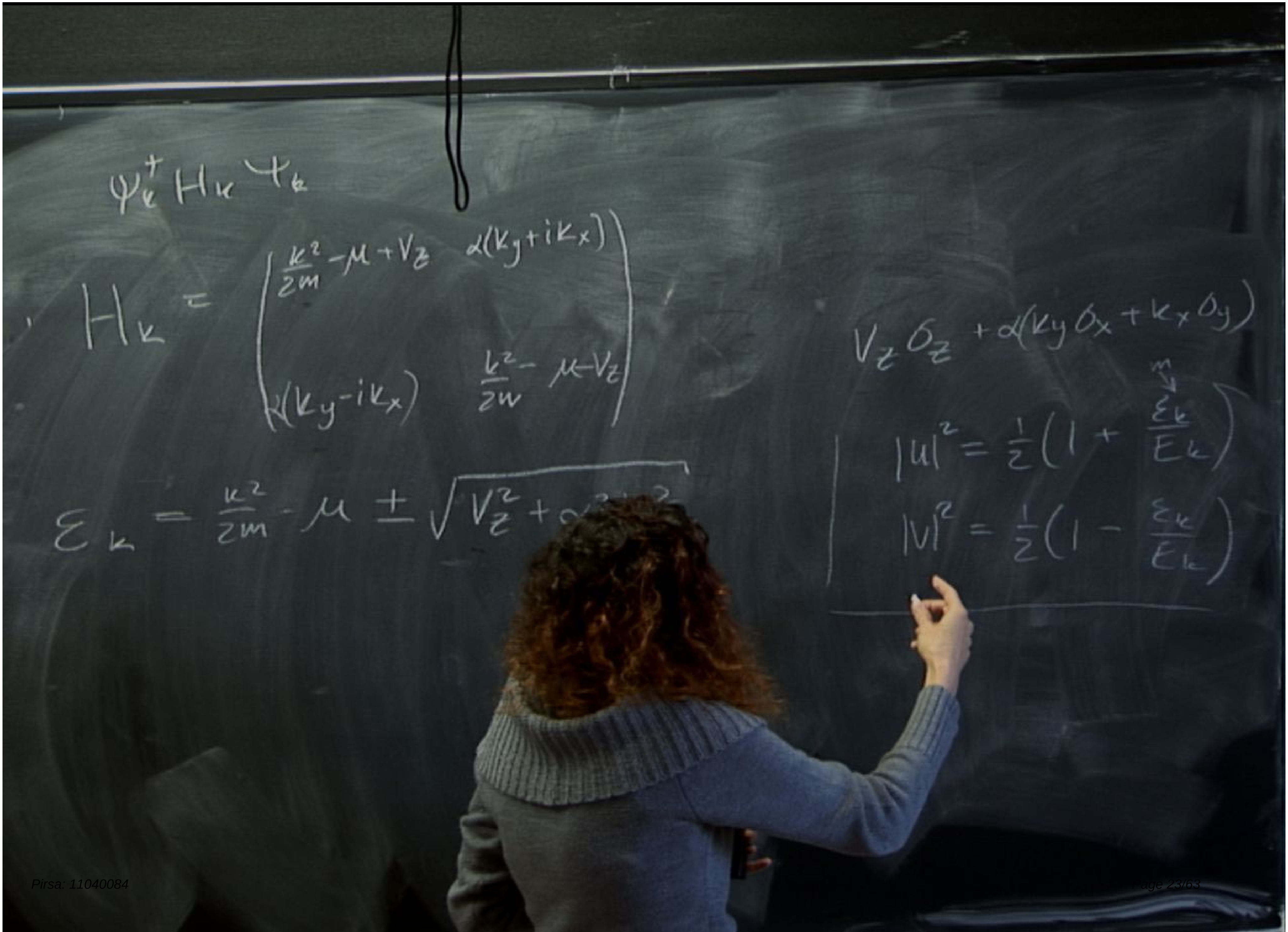


$\psi_k^\dagger H_k \Psi_k$
$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$
$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$
$|u|^2 = \frac{1}{2}(1 +$
$\varepsilon_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$



$\psi_k^\dagger H_k \psi_k$
$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$
$\varepsilon_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$
$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$
$|u|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_k}{E_k}\right)$
$|v|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k}{E_k}\right)$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_Z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_Z \end{pmatrix}$$

$$\varepsilon_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_Z^2 + \alpha^2 k^2}$$

$$\varphi_+ = \begin{pmatrix} A_\uparrow \\ A_\downarrow \left(\frac{k_y - i k_x}{k}\right) \end{pmatrix} \qquad \varphi_- = \begin{pmatrix} B_\uparrow \frac{k_y + i k_x}{k} \\ B_\downarrow \end{pmatrix}$$

$$\Delta_0 e^{i\gamma}$$

$$\varepsilon_k$$

$$\uparrow$$

$$V_Z \sigma_Z + \alpha(k_y \sigma_x + k_x \sigma_y)$$

$$m$$

$$|u|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_k}{E_k}\right)$$

$$|v|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k}{E_k}\right)$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\mathcal{E}_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$\varphi_+ = \begin{pmatrix} A_\uparrow \\ A_\downarrow \left(\frac{k_y - ik_x}{k}\right) \end{pmatrix} \qquad \varphi_- = \begin{pmatrix} B_\uparrow \frac{k_y + ik_x}{k} \\ B_\downarrow \end{pmatrix}$$

$$\Delta_0 e^{i\chi}$$

$$\overset{\xi_k \uparrow}{V_z \sigma_z} + \alpha(k_y \sigma_x + k_x \sigma_y)$$

$$|u|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{|E_k|}\right)$$

$$|v|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{|E_k|}\right)$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\Delta_0 e^{i\gamma}$$

$$\xi_k$$

$$\uparrow$$

$$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$$

$$\mathcal{E}_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$|u|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{|E_k|}\right)$$

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$$\varphi_+ = \begin{pmatrix} A_\uparrow \\ A_\downarrow \left(\frac{k_y - ik_x}{k}\right) \end{pmatrix} \qquad \varphi_- = \begin{pmatrix} B_\uparrow \frac{k_y + ik_x}{k} \\ B_\downarrow \end{pmatrix}$$

$$A_{\uparrow} = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}}$$

$\psi_k^\dagger H_k \psi_k$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\mathcal{E}_k = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$\Delta_0 e^{i\gamma}$

$\mathcal{E}_k$

$\uparrow$

$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$

$$|u|^2 = \frac{1}{2}\left(1 + \frac{\mathcal{E}_k}{|E_k|}\right)$$

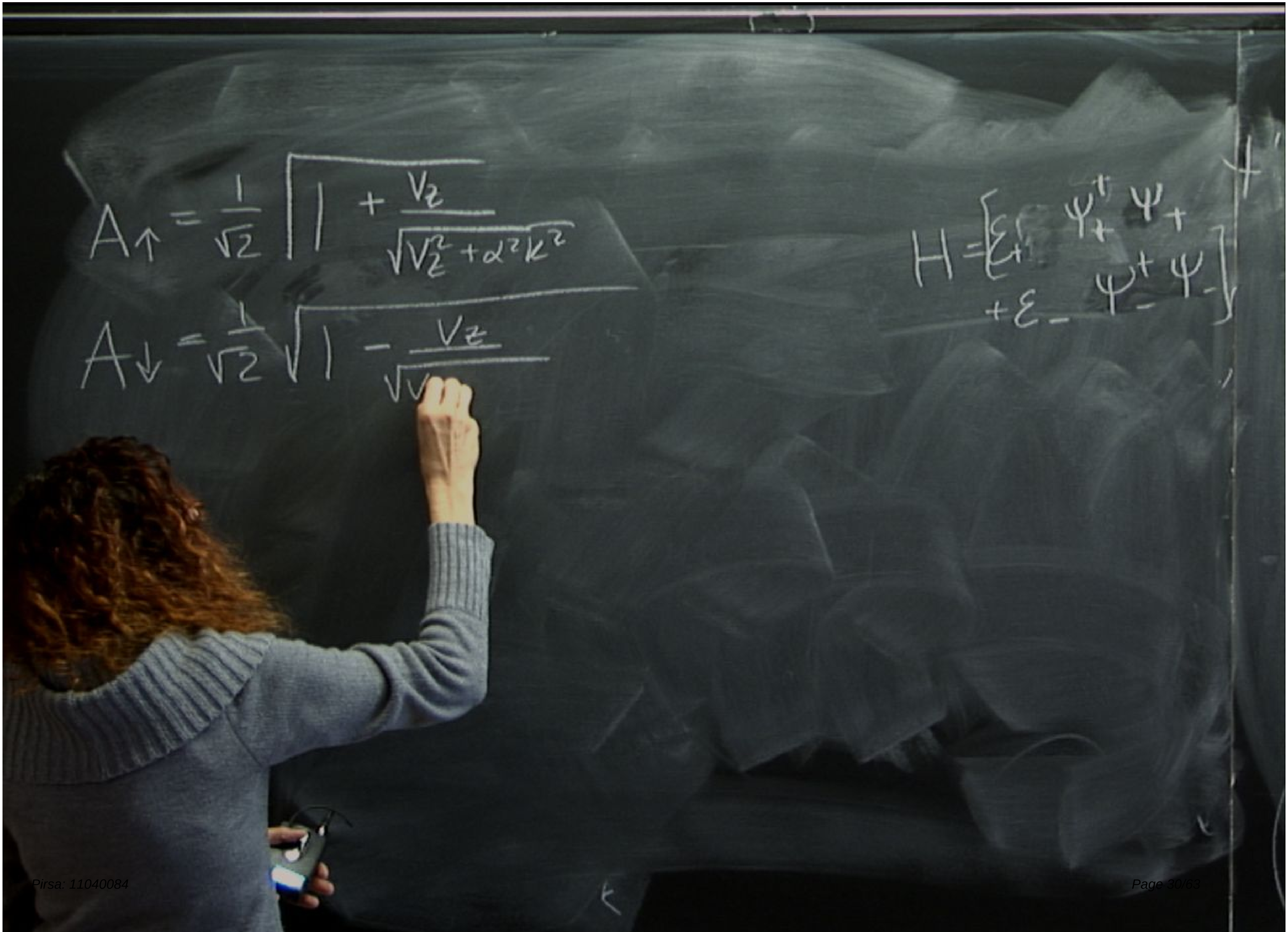
$$|v|^2 = \frac{1}{2}\left(1 - \frac{\mathcal{E}_k}{|E_k|}\right)$$

$$\varphi_+ = \begin{pmatrix} A_\uparrow \\ A_\downarrow \left(\frac{k_y - ik_x}{k}\right) \end{pmatrix} \qquad \varphi_- = \begin{pmatrix} B_\uparrow \frac{k_y + ik_x}{k} \\ B_\downarrow \end{pmatrix}$$

$$\begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix} = \begin{pmatrix} A_\uparrow \varphi_+ + B_\uparrow \varphi_- e^{i\gamma} \\ A_\downarrow \varphi_+ e^{i\gamma} + B_\downarrow \varphi_- \end{pmatrix}$$

$$A_\uparrow = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}}$$

$$H = \left[\varepsilon_+ \psi_+^\dagger \psi_+ + \varepsilon_- \psi_-^\dagger \psi_- \right]$$


$A_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}}$
$A_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V}}}$
$H = [\varepsilon_+ \psi_+^\dagger \psi_+ + \varepsilon_- \psi_-^\dagger \psi_-]$

$$A_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}}$$

$$A_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}}$$

$$\Bigg\} \; \mathcal{E}_+$$

$$H = \left[\mathcal{E}_+ \psi_+^\dagger \psi_+ + \mathcal{E}_- \psi_-^\dagger \psi_- \right]$$

$\psi_k^\dagger H_k \psi_k$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$\varphi_+ = \begin{pmatrix} A_\uparrow \\ A_\downarrow \left(\frac{k_y - ik_x}{k}\right) \end{pmatrix} \qquad \varphi_- = \begin{pmatrix} B_\uparrow \left(\frac{k_y + ik_x}{k}\right) \\ B_\downarrow \end{pmatrix}$$

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} A_\uparrow \varphi_+ + B_\uparrow \varphi_- e^{i\gamma} \\ A_\downarrow \varphi_+ e^{i\gamma} + B_\downarrow \varphi_- \end{pmatrix}$$

$\Delta_0 e^{i\gamma}$

δ_z

$\uparrow$

$V_z \sigma_z + \alpha(k_y \sigma_x + k_x \sigma_y)$

$$|u|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_k}{|E_k|}\right)$$

$$|v|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k}{|E_k|}\right)$$

$$A_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

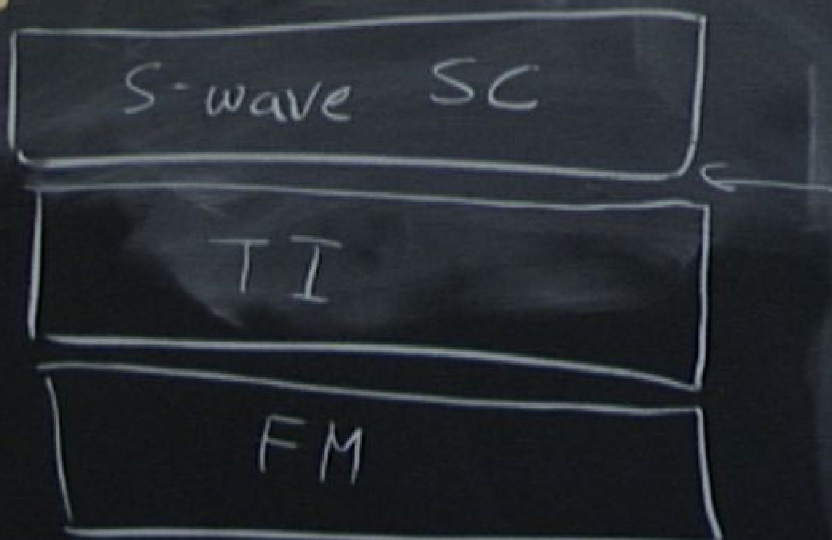
$$A_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$\Big\}\ \mathcal{E}_+$

$$H = \left[\mathcal{E}_+ \psi_+^\dagger \psi_+ + \mathcal{E}_- \psi_-^\dagger \psi_-\right]$$

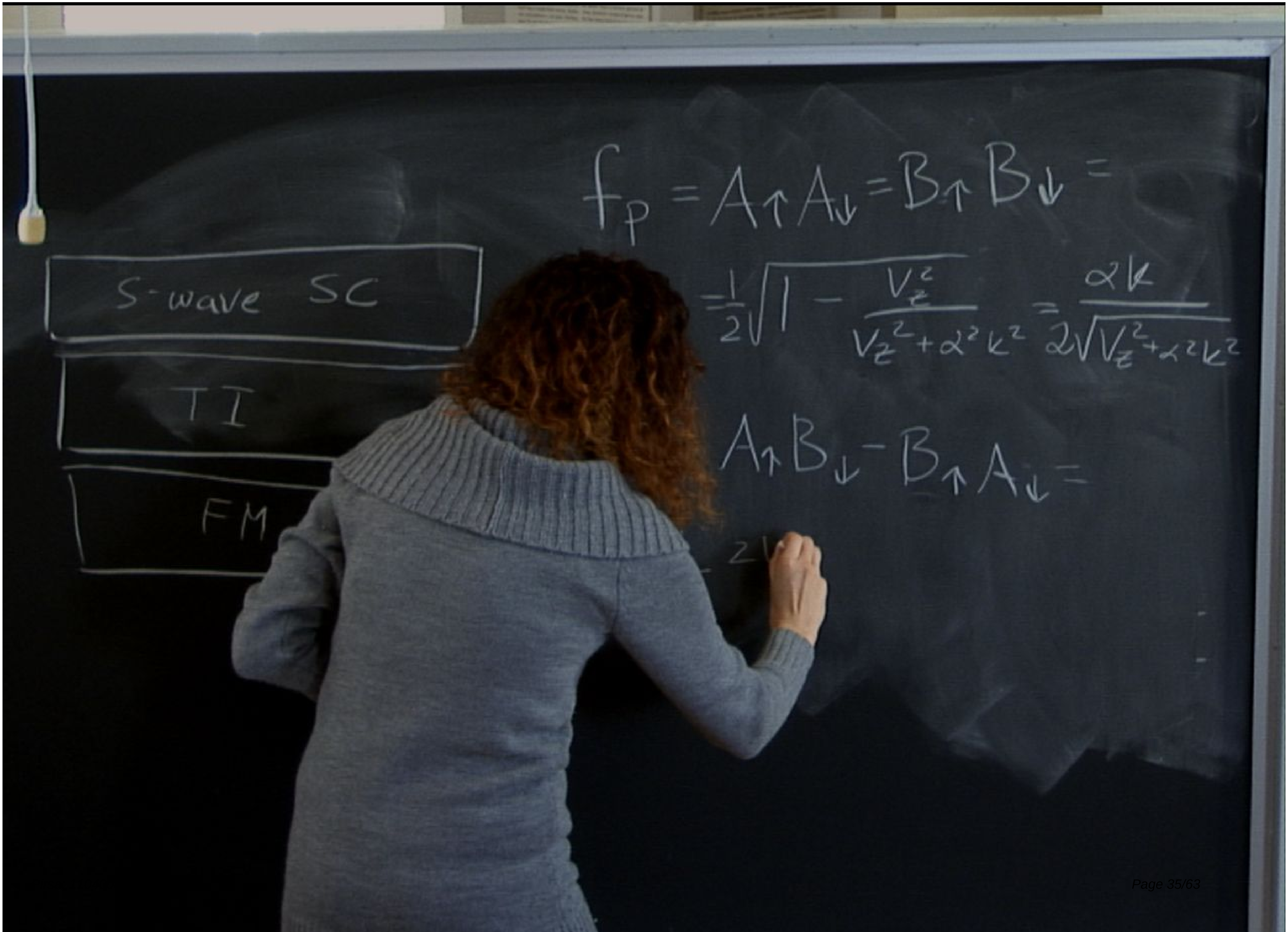
$$B_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$B_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2+\alpha^2 k^2}} = \frac{k}{\sqrt{V_z^2+\alpha^2k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow =$$



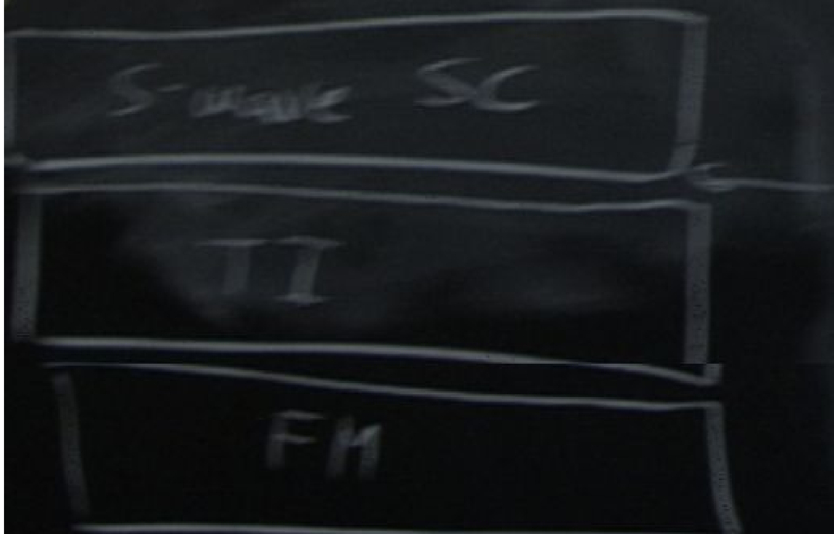
S-wave SC
TI
FM
$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow =$
$= \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2 + \alpha^2 k^2}} = \frac{\alpha k}{2\sqrt{V_z^2 + \alpha^2 k^2}}$
$A_\uparrow B_\downarrow - B_\uparrow A_\downarrow =$

$$\left.\begin{aligned} A_\uparrow &= \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}} \\ A_\downarrow &= \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}} \end{aligned}\right\} \mathcal{E}_+$$

$$H = \left[\mathcal{E}_+ \psi_+^\dagger \psi_+ + \mathcal{E}_- \psi_-^\dagger \psi_-\right]$$

$$B_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$B_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

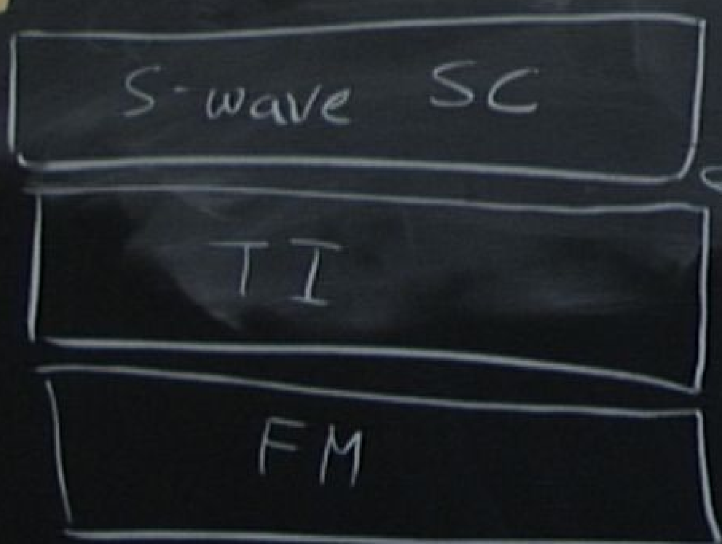


$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow =$$

$$= \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2 + \alpha^2 k^2}} = \frac{\alpha k}{2\sqrt{V_z^2 + \alpha^2 k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow =$$

$$= \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}$$



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2+\alpha^2 k^2}} = \frac{\alpha k}{2\sqrt{V_z^2+\alpha^2 k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2+\alpha^2 k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

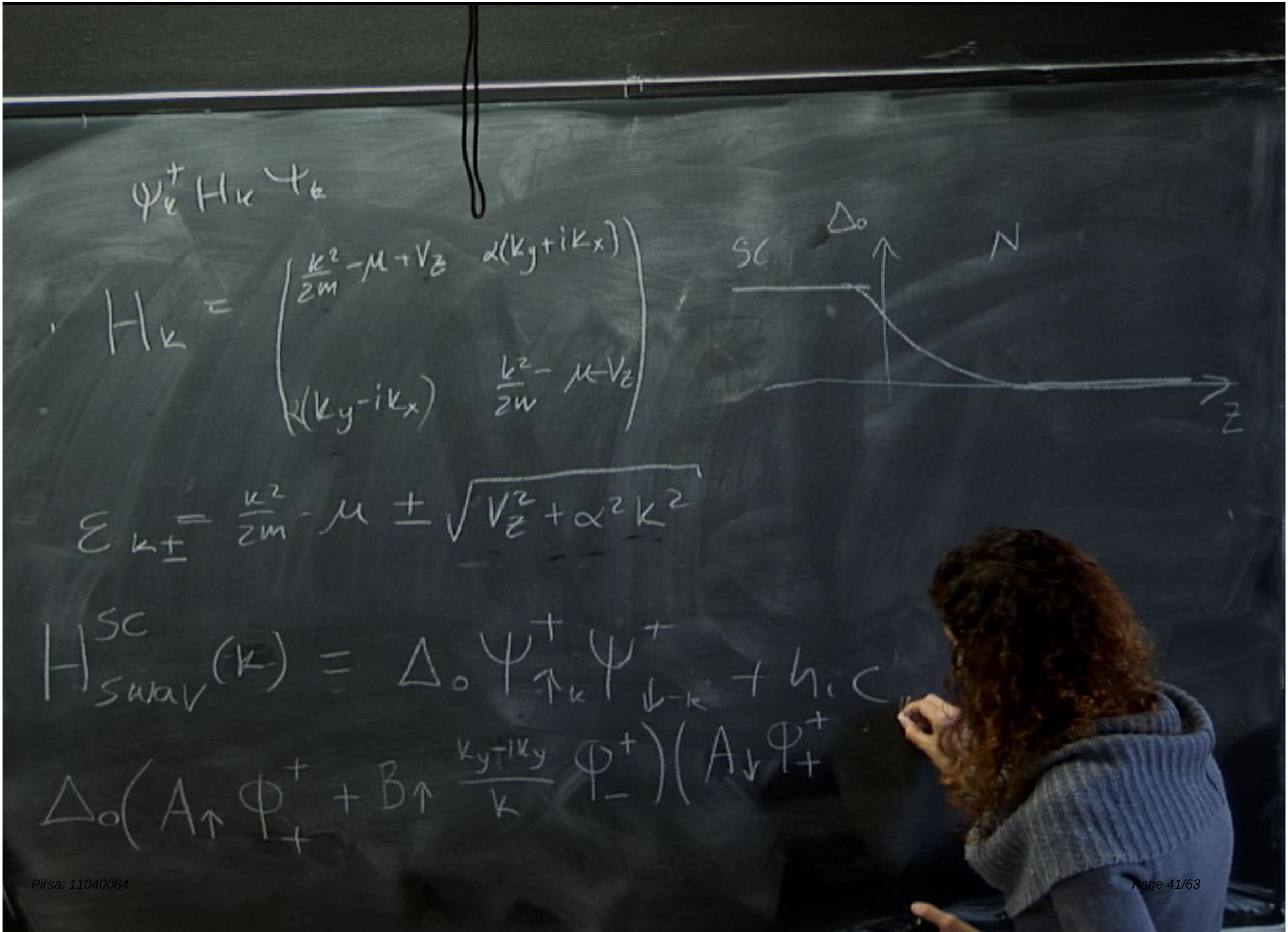
$$\varepsilon_{k\pm} = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$H^{SC}_{swav}(k) = \Delta_0 \psi^\dagger_{\uparrow k} \psi^\dagger_{\downarrow -k}$$


ψ_k^† H_k Ψ_k
H_k = (k²/2m − μ + V_z   α(k_y + ik_x) ; α(k_y − ik_x)   k²/2m − μ − V_z)
Δ₀
SC
N
z
ε_k± = k²/2m − μ ± √(V_z² + α²k²)
H^SC_swav(k) = Δ₀ Ψ^†_↑k Ψ^†_↓−k + h.c.
Δ₀(A_↑ φ^†_+ + B_↑ (k_y − ik_y)/k φ^†_−)(A_↓ φ^†_+

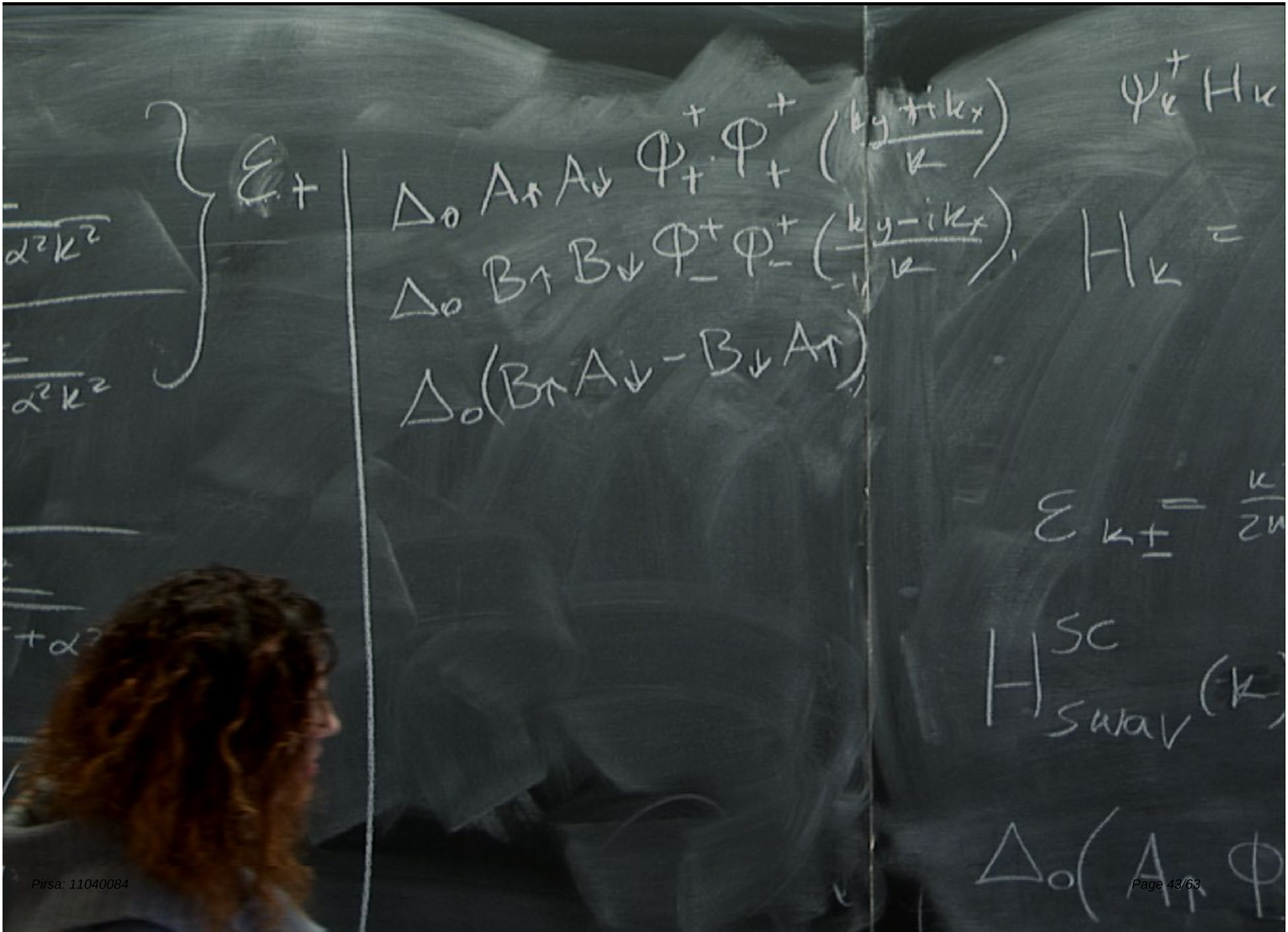
$$\psi_k^\dagger H_k \psi_k$$

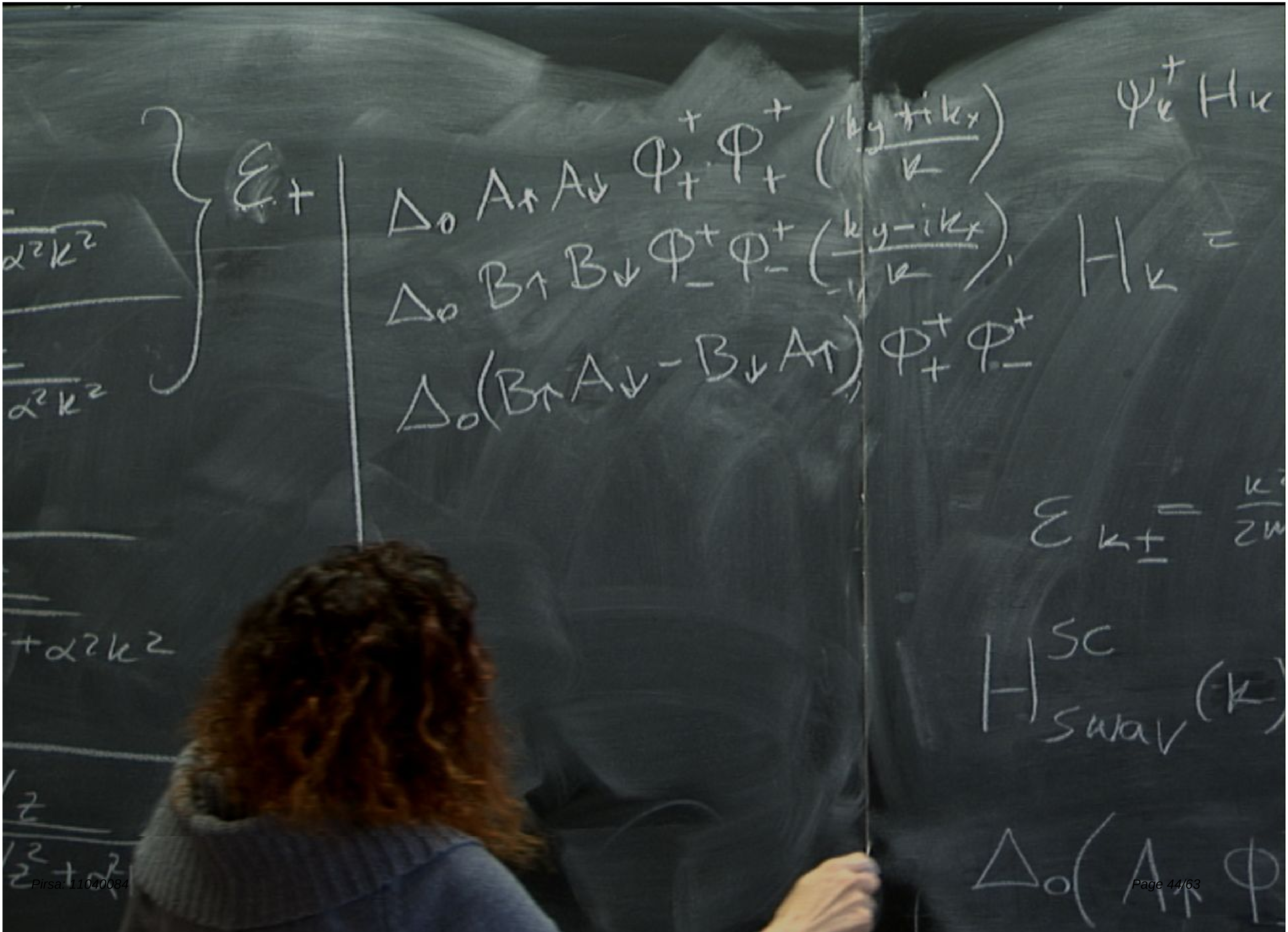
$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2 k^2}$$

$$H^{SC}_{swav}(k) = \Delta_0 \psi^\dagger_{\uparrow k}\psi^\dagger_{\downarrow -k} + h.c.$$

$$\Delta_0\left(A_\uparrow \phi_+^\dagger + B_\uparrow \frac{k_y - ik_y}{k}\phi_-^\dagger\right)\left(A_\downarrow \phi_+^\dagger \frac{k_y+ik_x}{k} + B_\downarrow \phi_-^\dagger\right)$$


ε_+
$\Delta_0 A_\uparrow A_\downarrow \varphi_+^+ \varphi_+^+ \left(\frac{k_y + i k_x}{k}\right)$
$\Delta_0 B_\uparrow B_\downarrow \varphi_-^+ \varphi_-^+ \left(\frac{k_y - i k_x}{k}\right)$
$\Delta_0 (B_\uparrow A_\downarrow - B_\downarrow A_\uparrow)$
$\Delta_0 (B_\uparrow A_\downarrow$
$\psi_k^+ H_k$
$H_k =$
$\varepsilon_{k\pm} =$
$H^{sc}_{swav}(k)$
$\Delta_0 (A_\uparrow \varphi$


Δ₀ A↑ A↓ φ₊⁺ φ₊⁺ ((k_y + i k_x)/k)
Δ₀ B↑ B↓ φ₋⁺ φ₋⁺ ((k_y − i k_x)/k)
Δ₀ (B↑ A↓ − B↓ A↑) φ₊⁺ φ₋⁺
H_k =
ε_{k±} =
H^{sc}_{swav}(k)

$$+\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$

$$-\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$

$\Big\}\ \varepsilon_+$

$$-\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$

$$\sqrt{1+\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$\Delta_0 A_\uparrow A_\downarrow \varphi_+^+ \varphi_+^+ \left(\frac{k_y+ik_x}{k}\right)$$

$$\Delta_0 B_\uparrow B_\downarrow \varphi_-^+ \varphi_-^+ \left(\frac{k_y-ik_x}{k}\right),$$

$$\Delta_0 (B_\uparrow A_\downarrow - B_\downarrow A_\uparrow)\, \varphi_+^+ \varphi_-^+$$

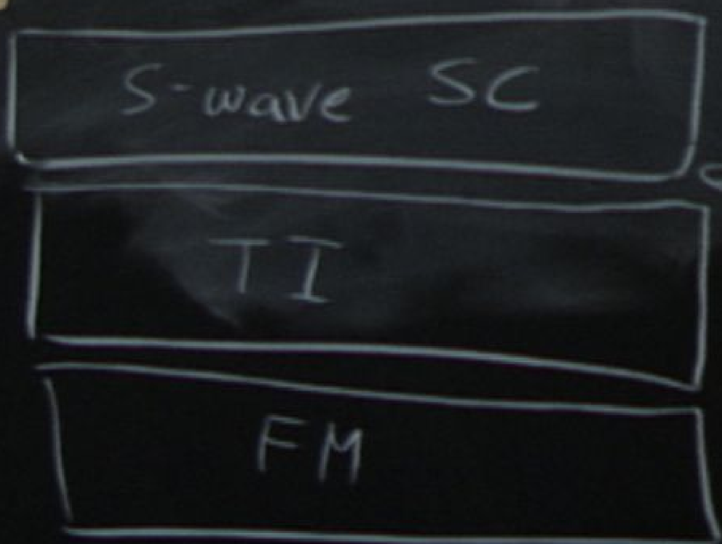
$$H_0 + \tilde{\Delta}$$

H

ε_k

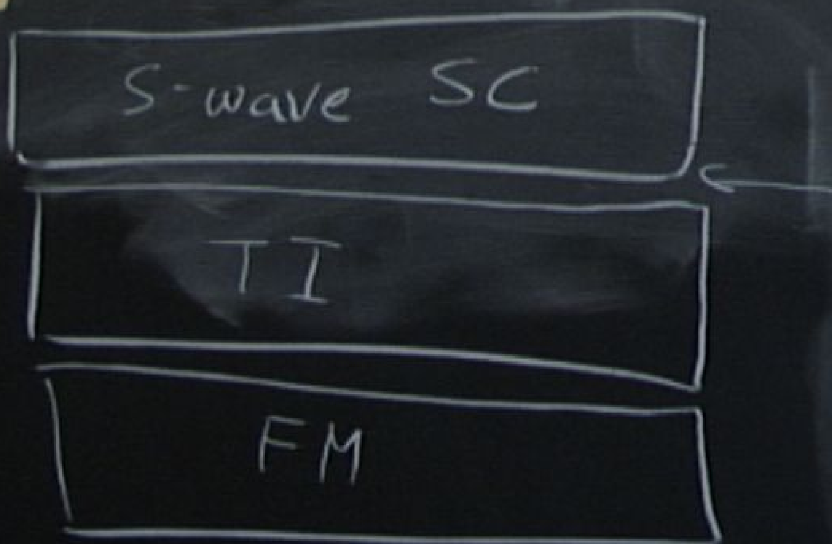
H^{SC}_{SM}

$\Delta_0($



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2 + \alpha^2 k^2}} = \frac{\alpha k}{2\sqrt{V_z^2 + \alpha^2 k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2 + \alpha^2 k^2}}$$

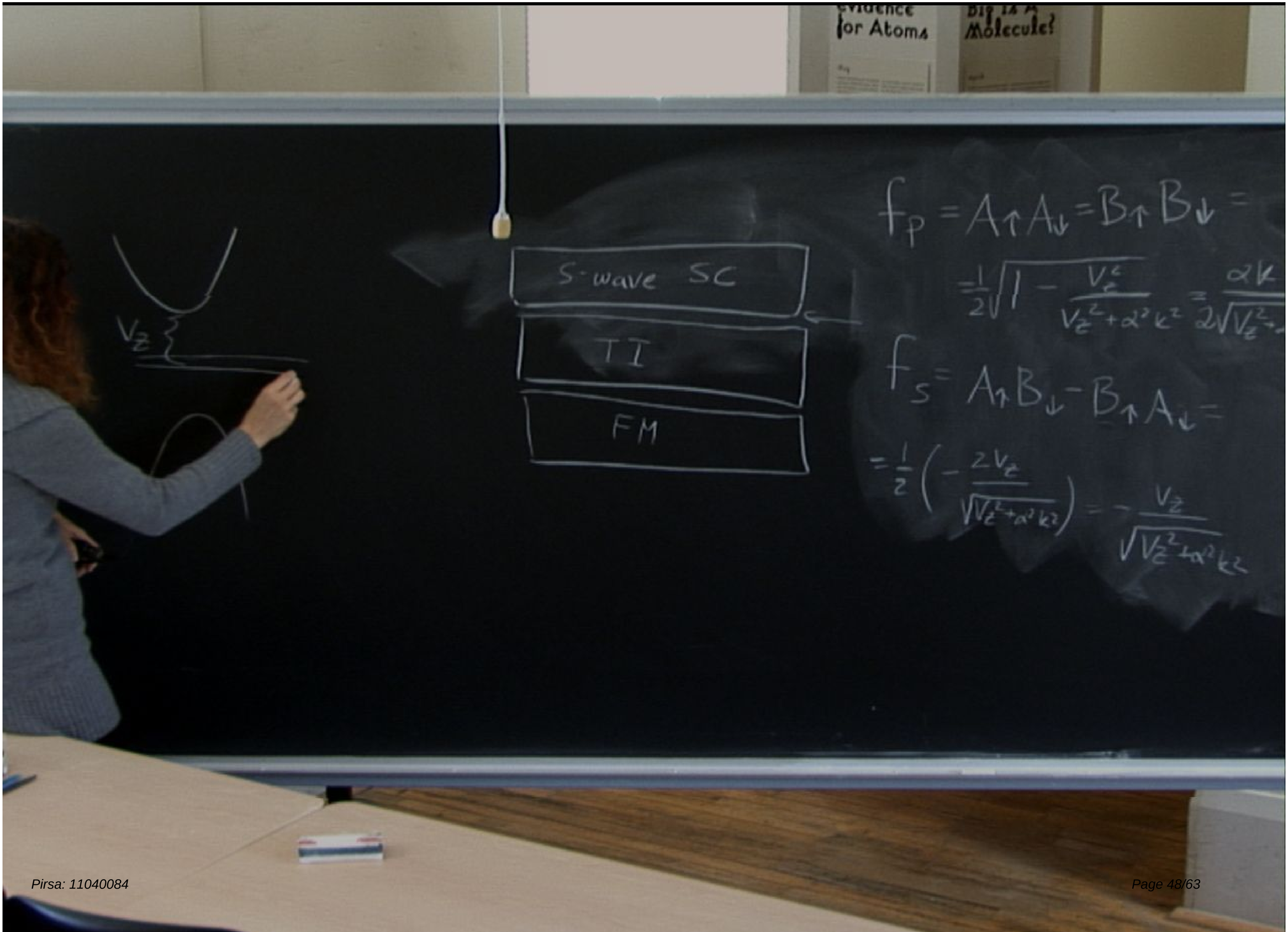


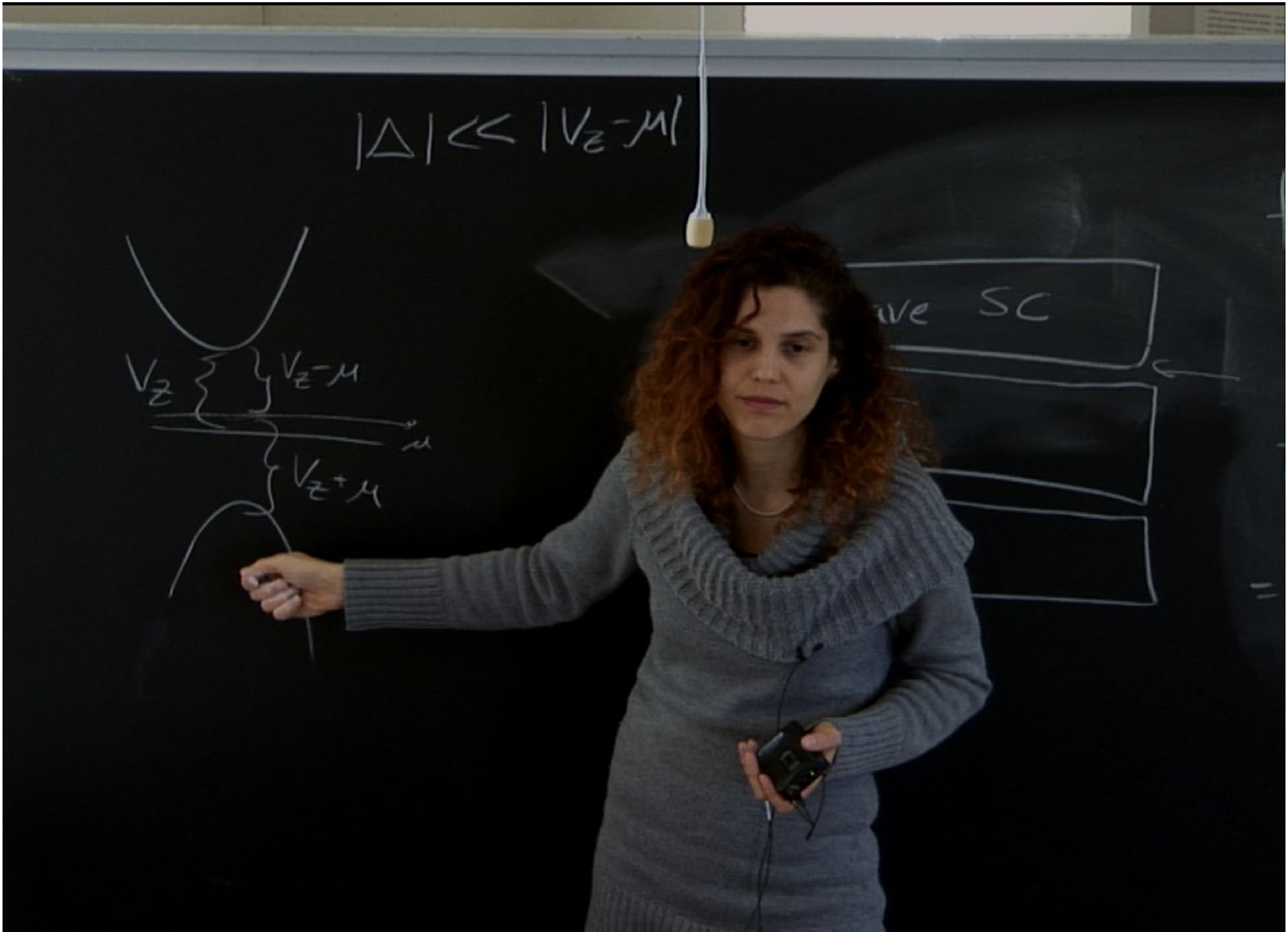
$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = $$

$$= \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2+\alpha^2 k^2}} = \frac{\alpha k}{2\sqrt{V_z^2+\alpha^2 k^2}}$$

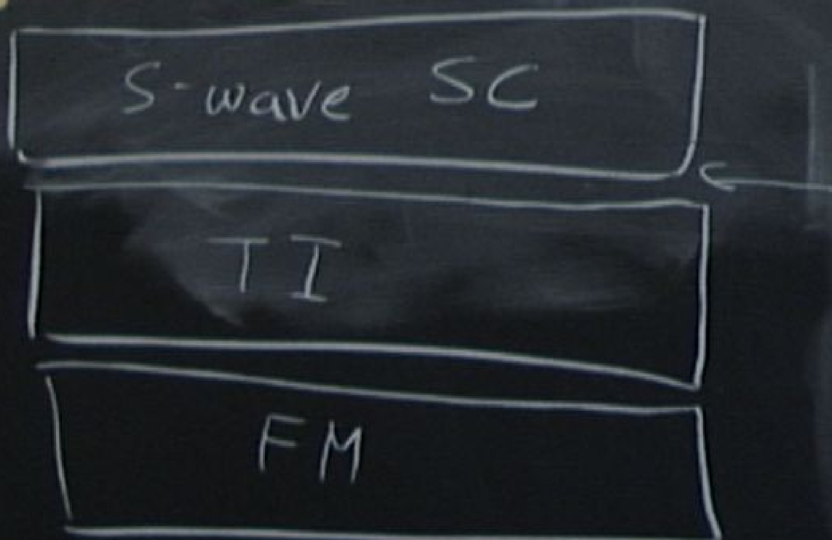
$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = $$

$$= \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2+\alpha^2 k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}$$


for Atoms
Molecule?
S-wave SC
TI
FM
Vz
fp = A↑A↓ = B↑B↓ =
fs = A↑B↓ − B↑A↓ =

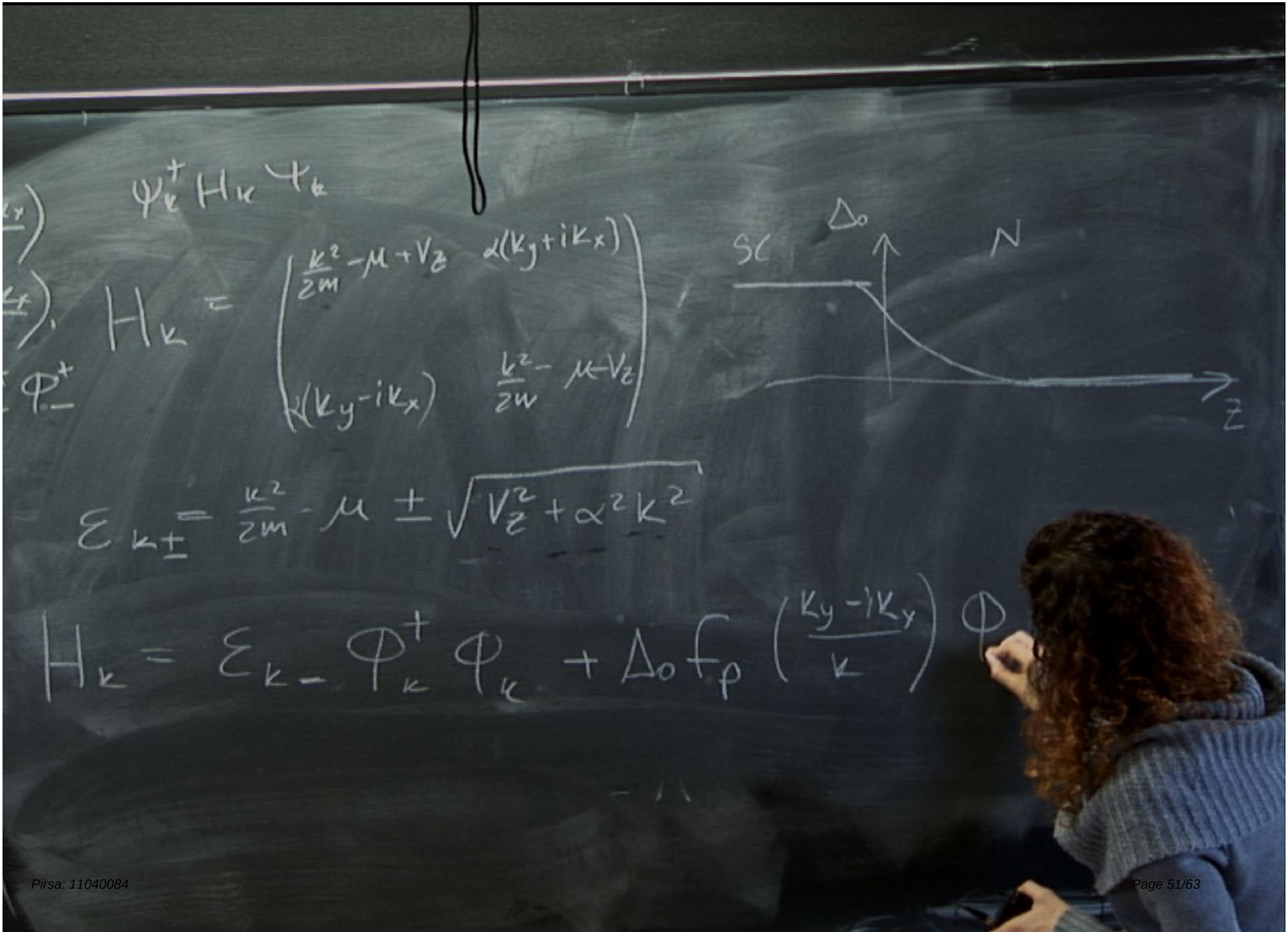


|Δ| << |V_Z − μ|
V_Z
V_Z − μ
μ
V_Z + μ
SC



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1-\frac{V_z^2}{V_z^2+\alpha^2k^2}} = \frac{\alpha|k|}{2\sqrt{V_z^2+\alpha^2k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2+\alpha^2k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$


ψ_k^† H_k Ψ_k
H_k = (k²/2m − μ + V_z   α(k_y + ik_x) ; α(k_y − ik_x)   k²/2m − μ − V_z)
Δ₀
SC
N
z
ε_k± = k²/2m − μ ± √(V_z² + α²k²)
H_k = ε_k− φ_k^† φ_k + Δ₀ f_p ((k_y − ik_x)/k) φ

$$\left.\right\} \varepsilon_+$$

$$\Delta_0 A_\uparrow A_\downarrow \varphi_+^+ \varphi_+^+ \left(\frac{k_y + i k_x}{k}\right)$$

$$\Delta_0 B_\uparrow B_\downarrow \varphi_-^+ \varphi_-^+ \left(\frac{k_y - i k_x}{k}\right),$$

$$\Delta_0 (B_\uparrow A_\downarrow - B_\downarrow A_\uparrow) \varphi_+^+ \varphi_-^+$$

$$H_0 + \tilde{\Delta}$$

$$P = |\varphi_-\rangle\langle\varphi_-|$$

$$\psi_k^+ H_k$$

$$H_k =$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}$$

$$H_k = \varepsilon_k$$

$$\alpha^2 k^2$$

$$\alpha^2 k^2$$

$$+\alpha^2 k^2$$

$$z^2 + \alpha^2 k^2$$

$$A_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}}$$

$$A_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}}$$

$\Big\}\ \mathcal{E}_+$

$$B_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}}$$

$$B_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2 k^2}}}$$

$$\Delta_0 A_\uparrow A_\downarrow \varphi_+^\dagger \varphi_+^\dagger \left(\frac{k_y + ik_x}{k}\right)$$

$$\Delta_0 B_\uparrow B_\downarrow \varphi_-^\dagger \varphi_-^\dagger \left(\frac{k_y - ik_x}{k}\right)$$

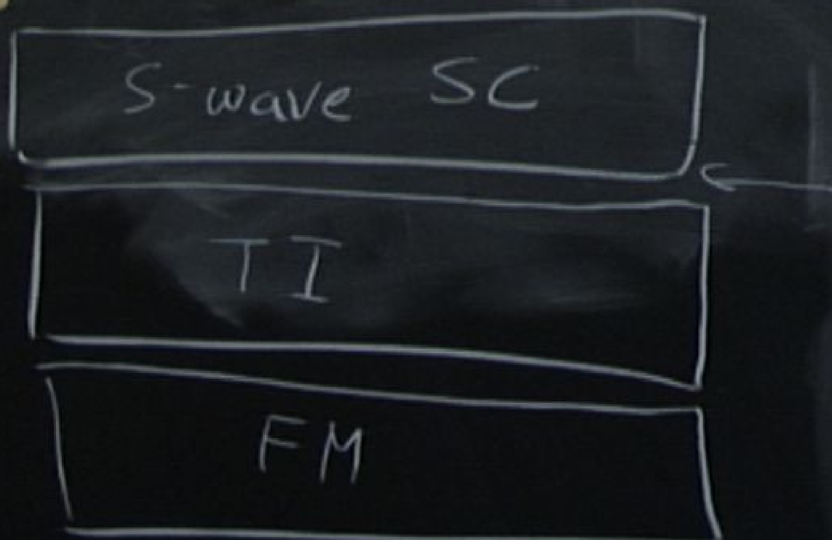
$$\Delta_0 (B_\uparrow A_\downarrow - B_\downarrow A_\uparrow) \varphi_+^\dagger \varphi_-^\dagger$$

$$H_0 + \tilde{\Delta}$$

$$P = |\varphi_-\rangle\langle\varphi_-|$$

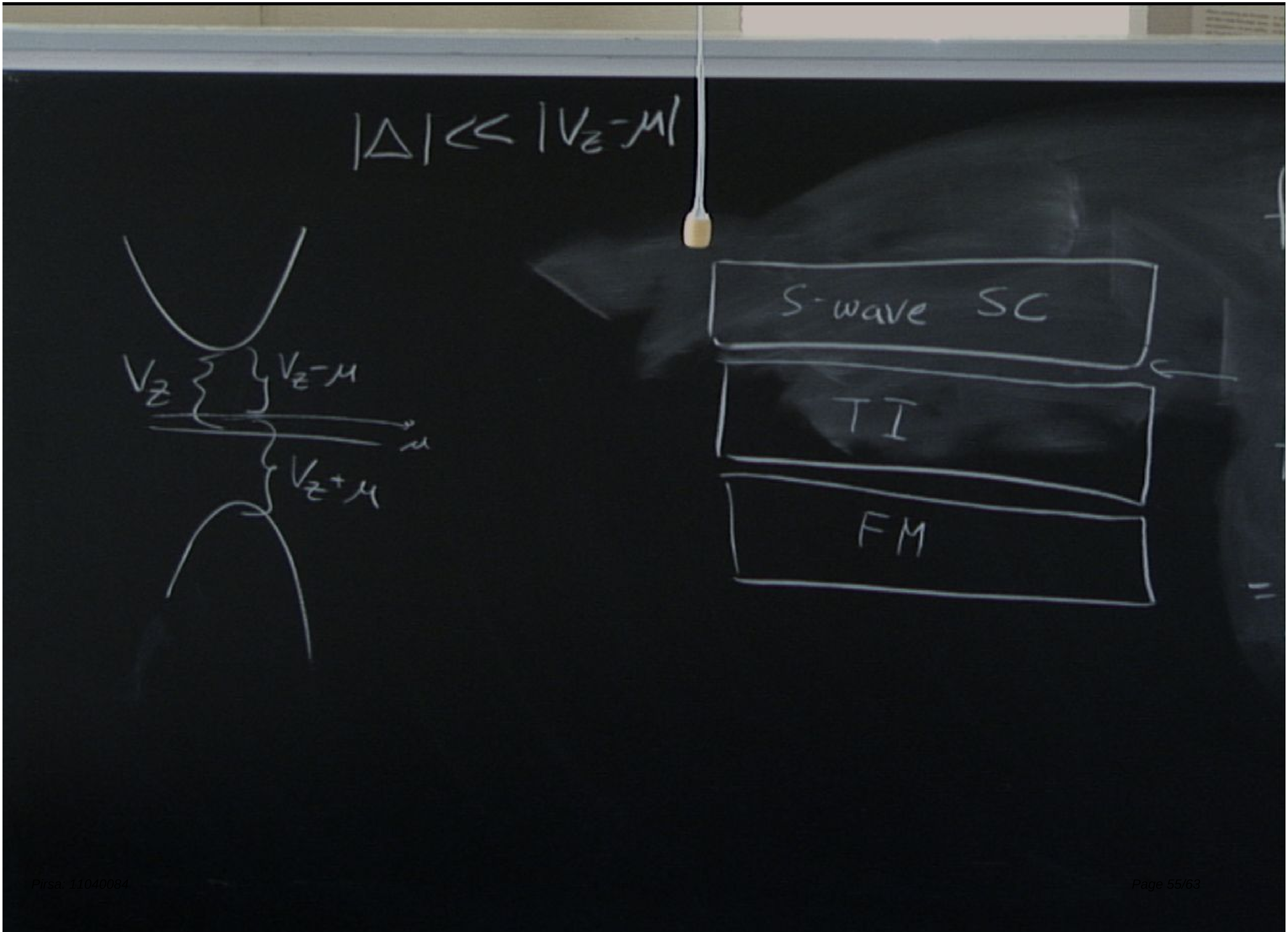
$\mathcal{E}$

H_k



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2+\alpha^2 k^2}} = \frac{\alpha|k|}{2\sqrt{V_z^2+\alpha^2k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2+\alpha^2k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$


|Δ| << |V_z − μ|
V_z
V_z − μ
μ
V_z + μ
S-wave SC
TI
FM

$$A_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$A_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$\Big\}\ \mathcal{E}_+$

$$B_\uparrow = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$B_\downarrow = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}}$$

$$\Delta_0 A_\uparrow A_\downarrow \varphi_+^\dagger \varphi_+^\dagger \left(\frac{k_y + ik_x}{k}\right)$$

$$\Delta_0 B_\uparrow B_\downarrow \varphi_-^\dagger \varphi_-^\dagger \left(-\frac{k_y - ik_x}{k}\right)$$

$$\Delta_0 (B_\uparrow A_\downarrow - B_\downarrow A_\uparrow)\, \varphi_+^\dagger \varphi_-^\dagger$$

$$H_0 + \tilde{\Delta}$$

$$P = |\phi_-\rangle\langle\phi_-|$$

$\mathcal{E}$

H_k

$$\psi_k^\dagger H_k \psi_k$$

$$\varphi_-^\dagger$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2 k^2}$$

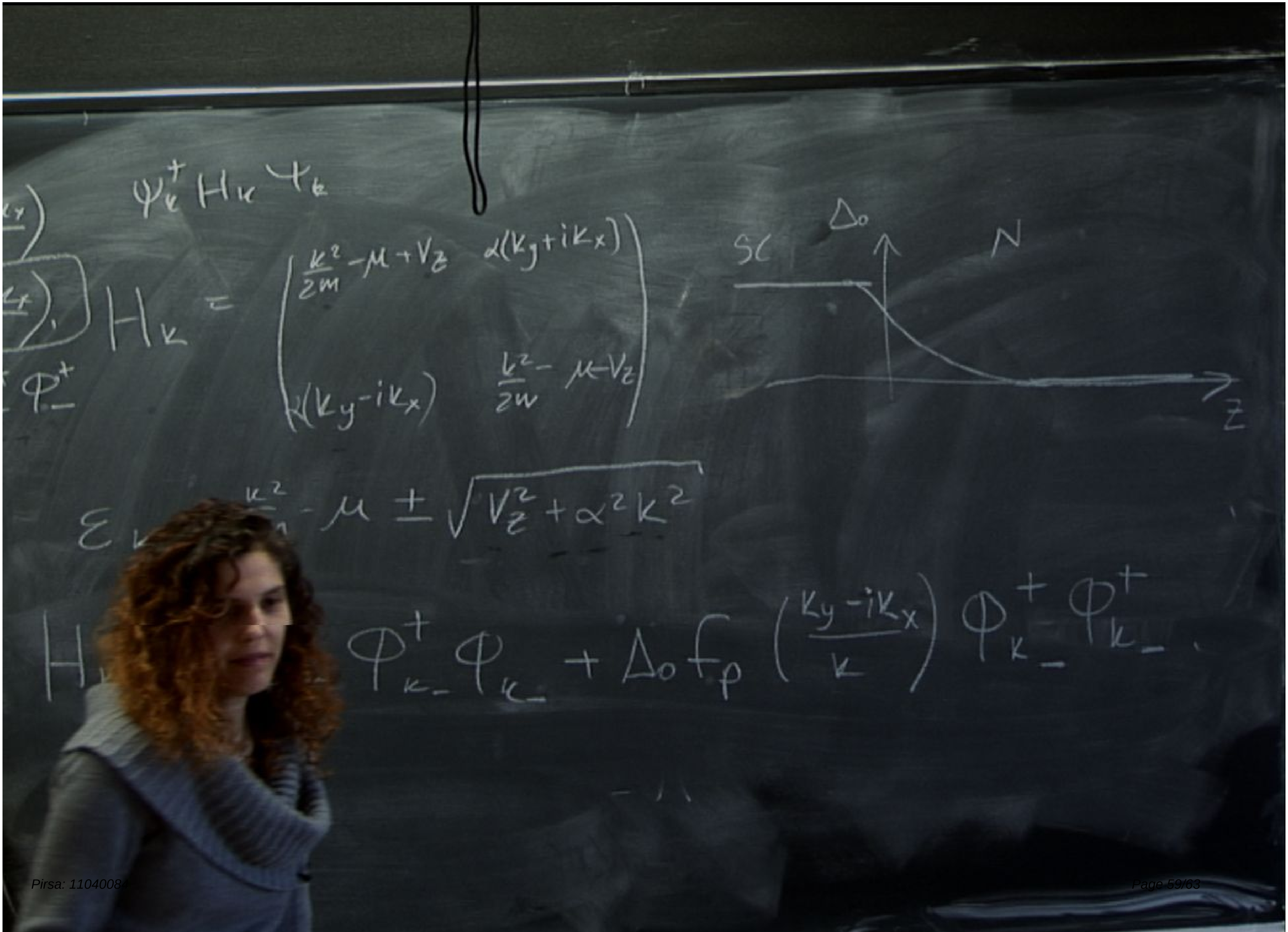
$$H_k = \varepsilon_{k-}\varphi_{k-}^\dagger\varphi_{k-} + \Delta_0 f_p \left(\frac{k_y-ik_x}{k}\right)\varphi_{k-}^\dagger\varphi_{k-}^\dagger$$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m} - \mu + V_z & \alpha(k_y + i k_x) \\ \alpha(k_y - i k_x) & \frac{k^2}{2m} - \mu - V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}$$

$$H_k = \varepsilon_{k-} \phi_{k-}^\dagger \phi_{k-} + \Delta_0 f_p \left(\frac{k_y - i k_x}{k} \right) \phi_{k-}^\dagger \phi_{k-}^\dagger .$$


ψ_k^† H_k Ψ_k
SC
Δ₀
N
z
H_k = (k²/2m − μ + V_z   α(k_y + ik_x) ; α(k_y − ik_x)   k²/2m − μ − V_z)
E_k = k²/2m − μ ± √(V_z² + α²k²)
φ_{k−}^† φ_{k−} + Δ₀ f_p ((k_y − ik_x)/k) φ_{k−}^† φ_{k−}^†

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2 k^2}$$

$$= \varepsilon_{k-}\phi^\dagger_{k-}\phi_{k-} + \Delta_0 f_p \left(\frac{k_y-ik_x}{k}\right)\phi^\dagger_{k-}\phi^\dagger_{k-}.$$

$$\varepsilon_k =$$

$\varphi_-^\dagger$

$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2 k^2}$$

$$H_k = \varepsilon_{k-}\varphi_{k-}^\dagger\varphi_{k-} + \Delta_0 f_p\left(\frac{k_y-ik_x}{k}\right)\varphi_{k-}^\dagger\varphi_{k-}^\dagger$$

$$\varepsilon_k = \frac{k^2}{2m}-\mu-\sqrt{V_z^2+\alpha^2 k^2}$$

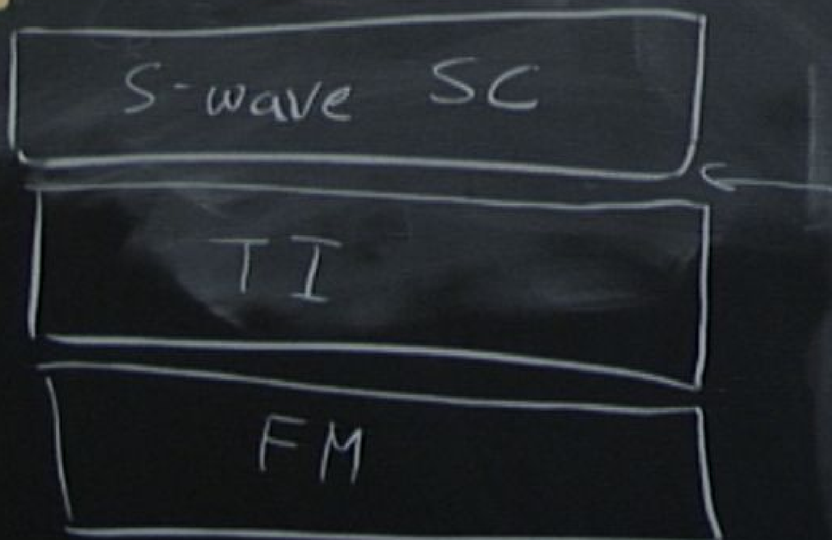
$$\psi_k^\dagger H_k \psi_k$$

$$H_k = \begin{pmatrix} \frac{k^2}{2m}-\mu+V_z & \alpha(k_y+ik_x) \\ \alpha(k_y-ik_x) & \frac{k^2}{2m}-\mu-V_z \end{pmatrix}$$

$$\varepsilon_{k\pm} = \frac{k^2}{2m}-\mu \pm \sqrt{V_z^2+\alpha^2k^2}$$

$$H_k = \varepsilon_{k-}\varphi_{k-}^\dagger\varphi_{k-} + \Delta_0 f_p\left(\frac{k_y-ik_x}{k}\right)\varphi_{k-}^\dagger\varphi_{k-}^\dagger$$

$$\varepsilon_k = \frac{k^2}{2m}-\mu-\sqrt{V_z^2+\alpha^2k^2}$$



$$f_p = A_\uparrow A_\downarrow = B_\uparrow B_\downarrow = \frac{1}{2}\sqrt{1 - \frac{V_z^2}{V_z^2+\alpha^2k^2}} = \frac{\alpha|k|}{2\sqrt{V_z^2+\alpha^2k^2}}$$

$$f_s = A_\uparrow B_\downarrow - B_\uparrow A_\downarrow = \frac{1}{2}\left(-\frac{2V_z}{\sqrt{V_z^2+\alpha^2k^2}}\right) = -\frac{V_z}{\sqrt{V_z^2+\alpha^2k^2}}$$