Title: Thick-wall tunneling in a piecewise linear and quadratic potential

Date: Apr 19, 2011 02:00 PM

URL: http://pirsa.org/11040082

Abstract: After reviewing the basics of Coleman deLuccia tunneling, especially in the thin-wall limit, I discuss an (almost) exact tunneling solution in a piecewise linear and quadratic potential. A comparison with the exact solution for a piecewise linear potential demonstrates the dependence of the tunneling rate on the exact shape of the potential.
Finally, I will mention applications when determining initial conditions for inflation in the landscape. Based on arXiv:1102.4742 [hep-th].

Thick wall tunneling in a piecewise linear and quadratic potential

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in collaboration with
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arXiv:1102.4742 [hep-th]

April 19, 2011

Outline

Motivation

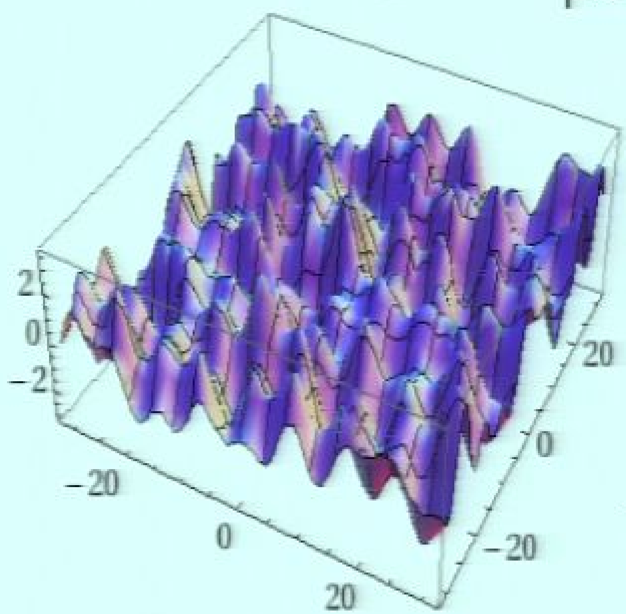
Review of CdL tunneling

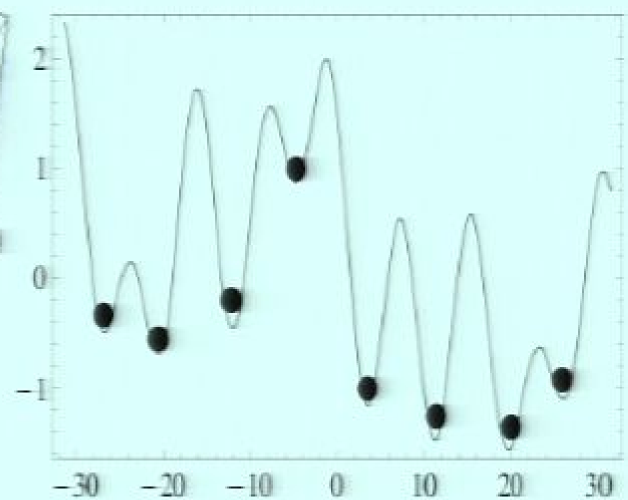
Piecewise linear and quadratic potential

Conclusions and Outlook

Motivation

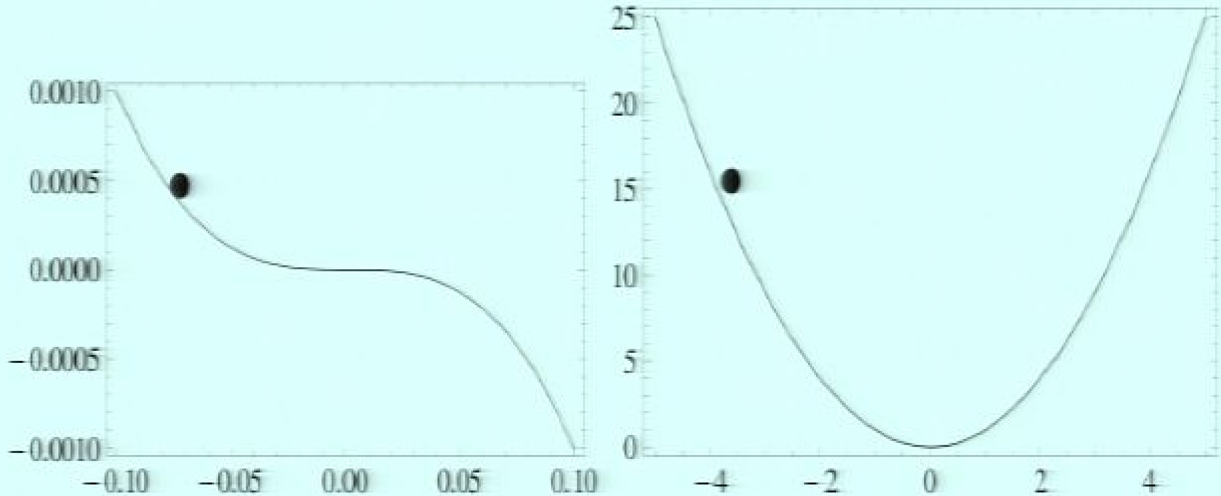
picture of CMBPol





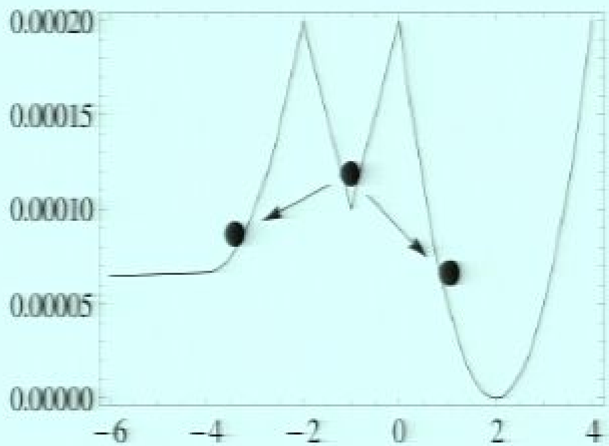
What does String Theory predict for r?

Motivation



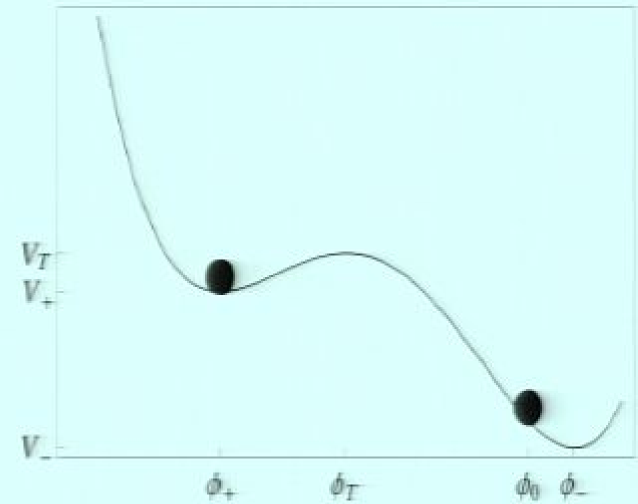
- large field models (detectable gravity waves)
- small field models (undetectable gravity waves)
- relative frequency of potentials of either shape
- populating the initial condition
- ignore measure problem
- best observations will be able to detect $r \approx 0.01$
- Lyth bound $\Delta\phi > \sqrt{r\Delta N}M_P$

Toy model



- initial velocity zero
- slowly accelerating
- entering the inflection part with finite speed
- sufficient inflation $\Delta N > 60$

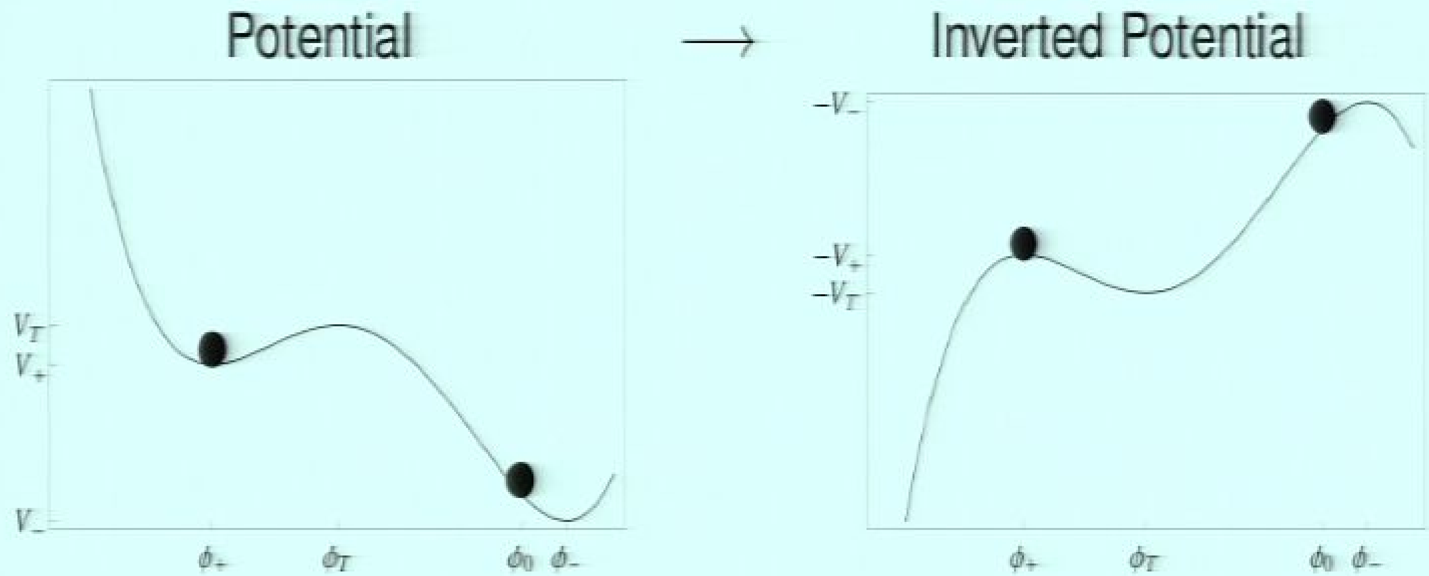
CdL tunneling Coleman (1977), Coleman, De Luccia (1980)



- Tunnel rate per unit volume: $\dfrac{\Gamma}{V} = Ae^{-B}$
- calculate $B = S_E(\phi_B) - S_E(\phi_+)$
- $O(4)$ symmetric bounce ϕ_B minimizes Euclidean action

$$S_E = 2\pi^2 \int_0^\infty dr\, r^3 \left(\frac{1}{2}\phi'^2 + V(\phi) \right)$$

CdL tunneling

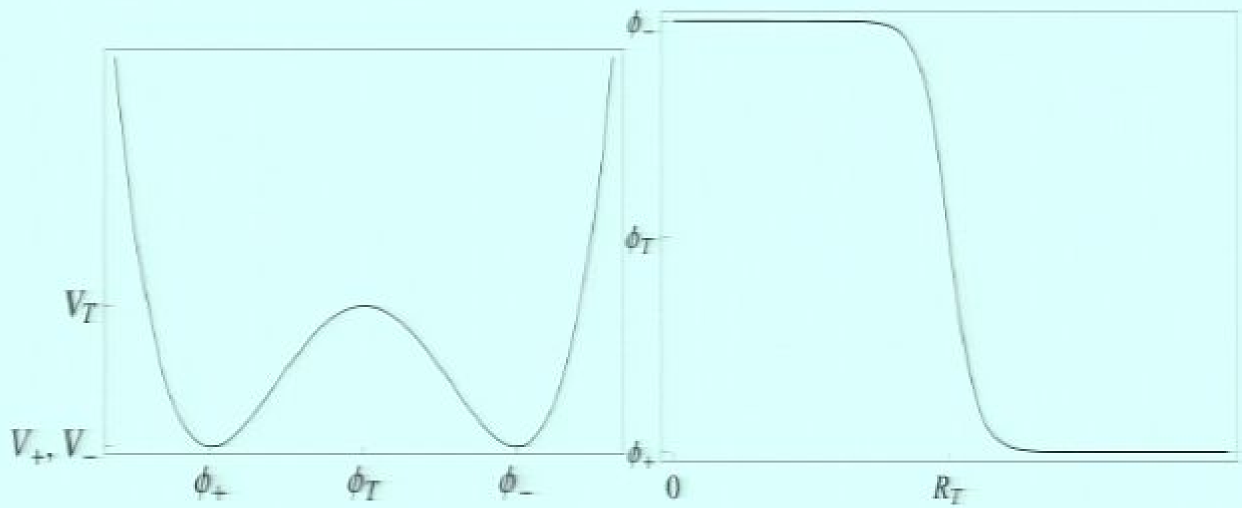


- Eom

$$\phi'' + \frac{3}{r}\phi' - \partial_\phi V = 0$$

- Exactly solvable for special case of linear potential (Duncan and Jensen, 1992)

Thin wall approximation

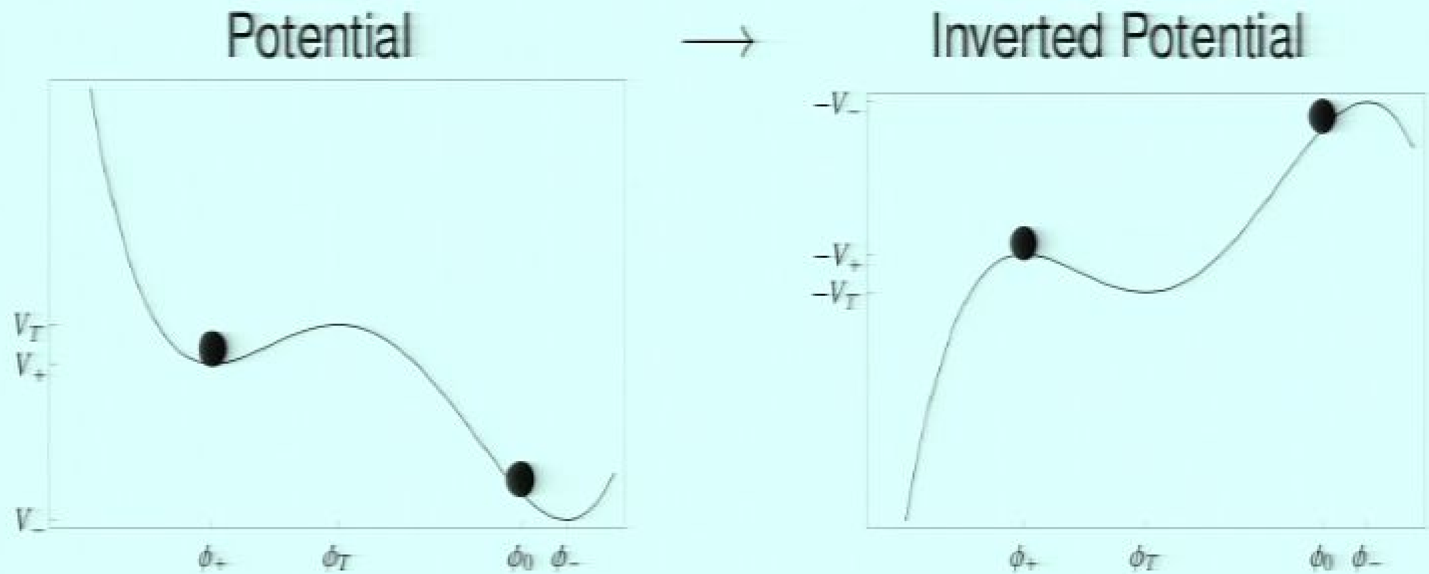


- In the limit of $\epsilon = V_- - V_+ \to 0$ no need to solve eom
- Instead

$$B = \frac{27\pi^2}{2}\frac{S_1^4}{\epsilon}, \quad S_1 = \int_{\phi_-}^{\phi_+} d\phi \sqrt{2(V(\phi) - V(\phi_+))}$$

- Tunnling process $\phi_+ \to \phi_0 \approx \phi_-$
- $\partial_r B|_{r=R_T} = 0 \Rightarrow$ bubble radius $R_T = 3\frac{S_1}{\epsilon}$

CdL tunneling



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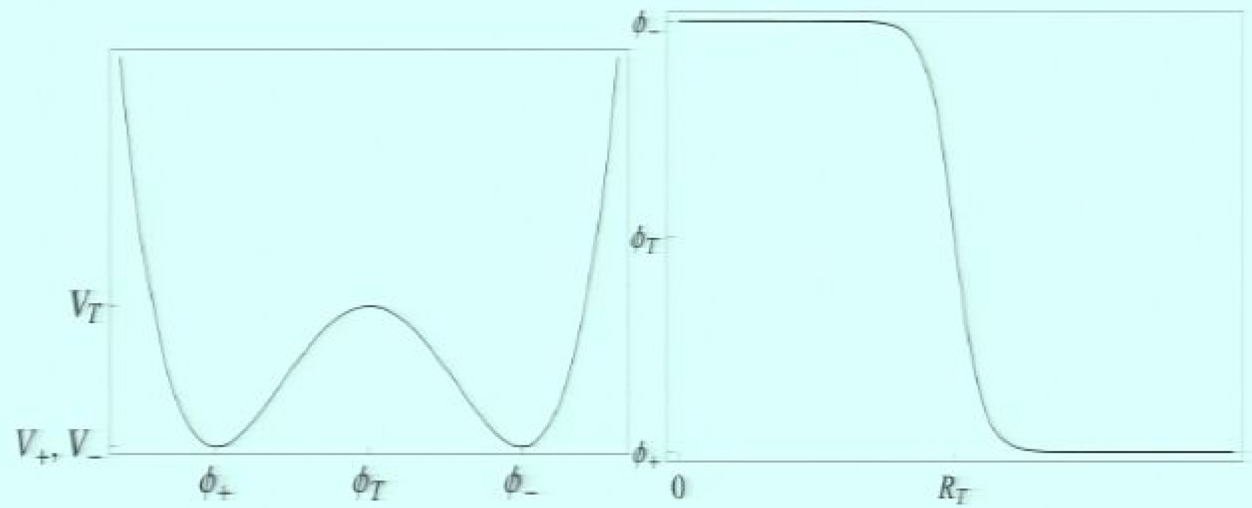

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Thin wall approximation

$$\epsilon = V_+ - V_-, \qquad V = V_0 + O(\epsilon)$$

$$\phi'' + \frac{3}{r}\phi' - \partial_\phi V = 0 \;\rightarrow\; \phi'' = \partial_\phi V_0$$

- Away from the wall: $\phi' = 0$
- At the wall: $r \gg \phi'$
- neglecting friction $\Rightarrow$ energy conservation

$$\frac{1}{2}\phi'^2 - V_0 = \text{const} = V_\pm$$

Thin wall approximation

- Inside the bubble

$$B_{\text{in}} = 2\pi^2 \int_0^{R_T} dr\, r^3 \left(V_- - V_+\right) = \frac{\pi^2}{2} R_T^4 \left(V_- - V_+\right) = -\frac{\pi^2}{2} R_T^4 \epsilon$$

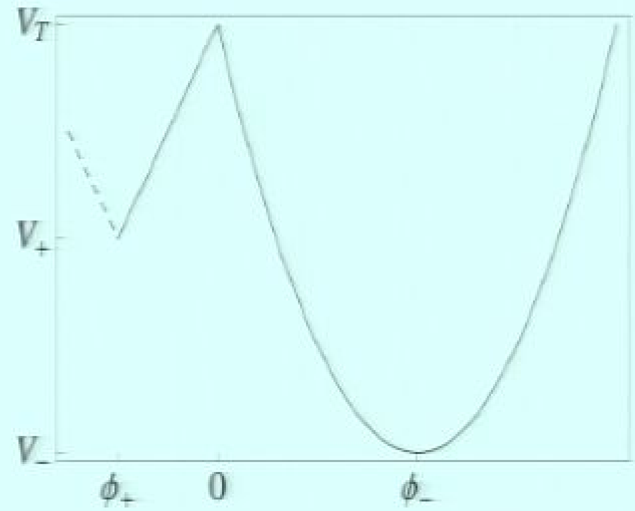
- Within the wall

$$\begin{aligned}
B_{\text{wall}} &= 2\pi^2 \int_{R_T-\Delta r}^{R_T+\Delta r} dr\, r^3 \left(\frac{1}{2}\phi'^2 + V_0(\phi) - V_0(\phi_+)\right) \\
&= 2\pi^2 R_T^3 \int_{\phi_-}^{\phi_+} d\phi\, \frac{1}{\phi'} 2\left(V_0 - V_0(\phi_+)\right) \\
&= 2\pi^2 R_T^3 \underbrace{\int_{\phi_-}^{\phi_+} d\phi\, \sqrt{2\left(V_0 - V_0(\phi_+)\right)}}_{S_1}
\end{aligned}$$

- Outside of the bubble $B_{\text{out}} = 0$

$\Rightarrow$ tunneling amplitude $B = 2\pi^2 R_T^3 S_1 - \frac{\pi^2}{2} R_T^4 \epsilon = \frac{27\pi^2}{2\epsilon^3} S_1^4$

Piecewise linear and quadratic potential



- Piecewise potential

$$V = \begin{cases} \frac{m^2}{2}\phi_-^2 + V_0 + \lambda_+\phi\,, & \phi < 0 \\ \frac{m^2}{2}(\phi - \phi_-)^2 + V_0\,, & \phi > 0 \end{cases}$$

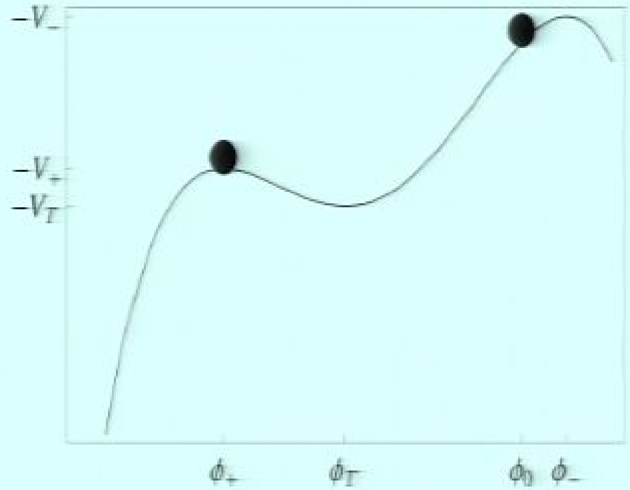
- boundary conditions

$\phi_R(0) = \phi_0 > 0, \phi_R'(0) = 0, \phi_L(R_+) = \phi_+, \phi_L'(R_+) = 0$

- eom

$$\phi'' + \frac{3}{r}\phi' - \partial_\phi V = 0$$

Comment on numerics



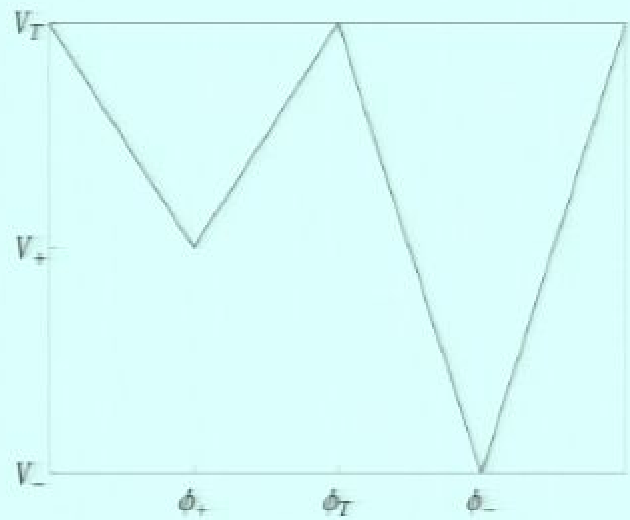
- ideally: start integrating inwards from ϕ_+, run until field stops
- but: limited numerical precision

$\Rightarrow$ integrate from ϕ_0, searching trajectory that ends at ϕ_+

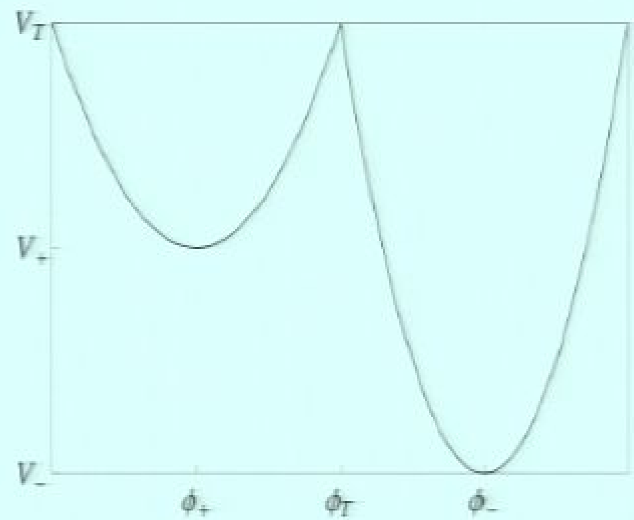
- Mathematica package

```
Needs['tunnel'];
tunnel'SetStartStop[0, 10^8];
tunnel'SetVprime[Vp];
tunnel'ComputeSolution[φm, φ+];
```

Other exact solutions



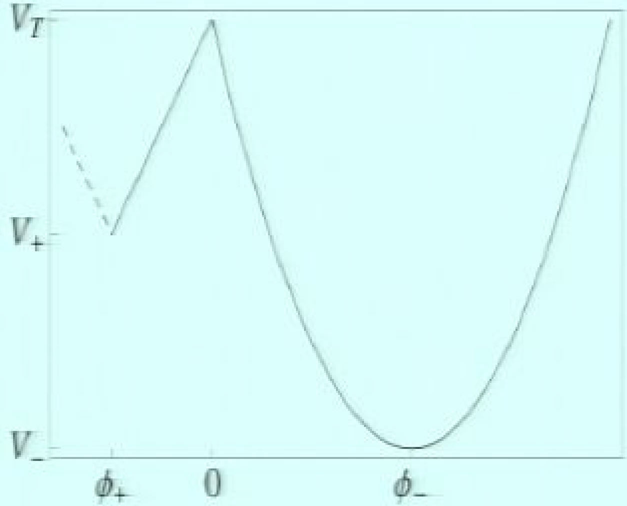
Duncan Jensen (1992)

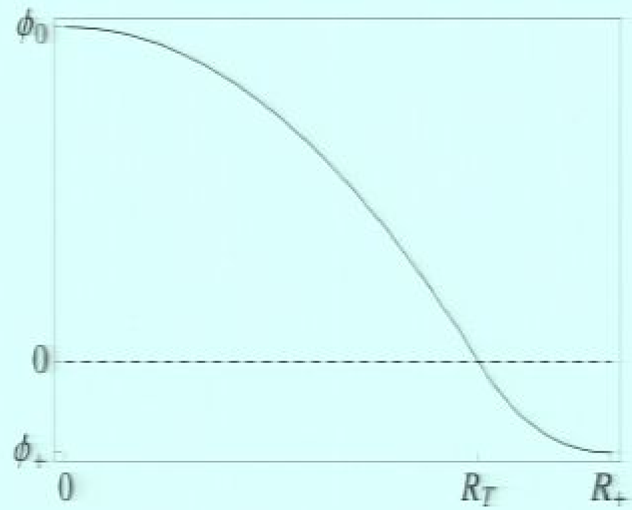


Hamazaki Sasaki Tanaka Yamamoto (1995), Pastras (2011)

- Duncan and Jensen: really exact, no approximations necessary
- Hamazaki et al, Pastras: B in terms of R_T

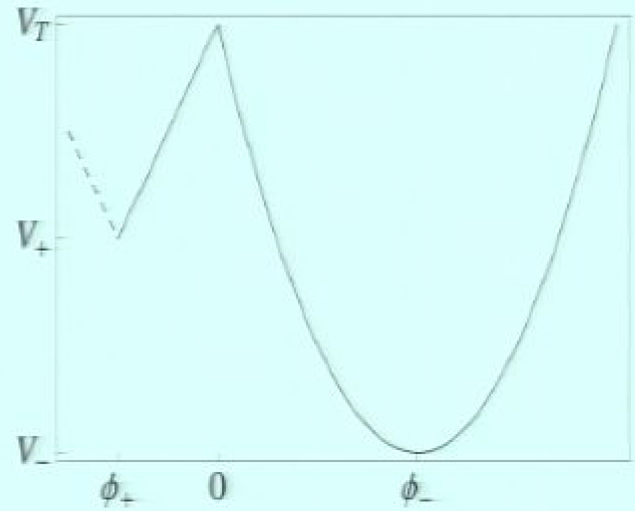
Strategy





- Solve eom for the left and right part of the potential
- Observe boundary conditions:
  - Bubble nucleates at $\phi_R(0) = \phi_0$ at rest $\phi'_R(0) = 0$ (subsequently starts rolling)
  - False Vacuum outside of the bubble $R_+ > R_T$, $\phi_L(r > R_+)$ =const, $\phi'_L(r > R_+) = 0$
- Match solutions at $\phi(R_T) = 0$
- Integrate the action for the solutions $\Rightarrow B$

Piecewise linear and quadratic potential



- Piecewise potential

$$V = \begin{cases} \frac{m^2}{2}\phi_-^2 + V_0 + \lambda_+\phi\,, & \phi < 0 \\ \frac{m^2}{2}(\phi - \phi_-)^2 + V_0\,, & \phi > 0 \end{cases}$$

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$\phi_R(0) = \phi_0 > 0, \phi_R'(0) = 0, \phi_L(R_+) = \phi_+, \phi_L'(R_+) = 0$

- eom

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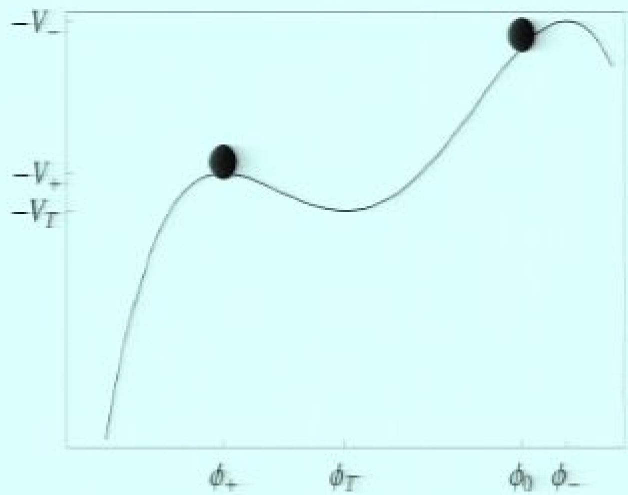
Comment on numerics



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$\Rightarrow$ integrate from ϕ_0, searching trajectory that ends at ϕ_+

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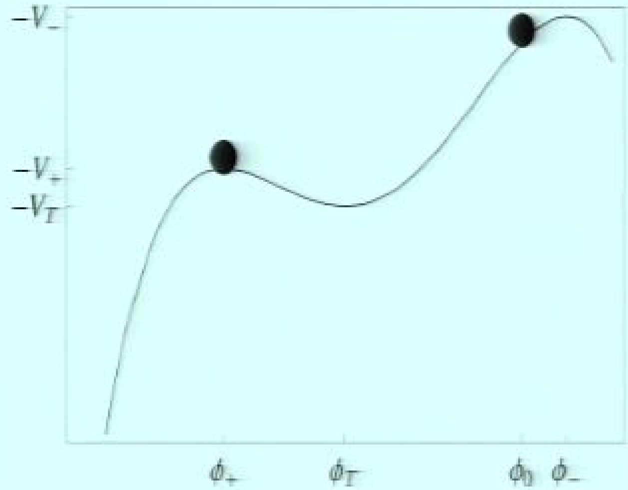

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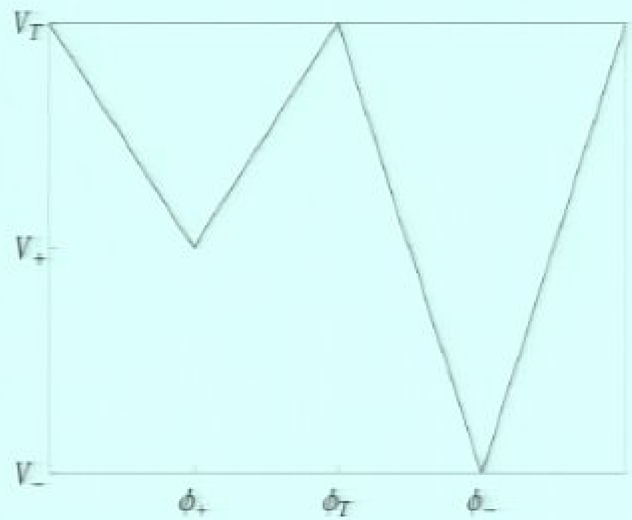

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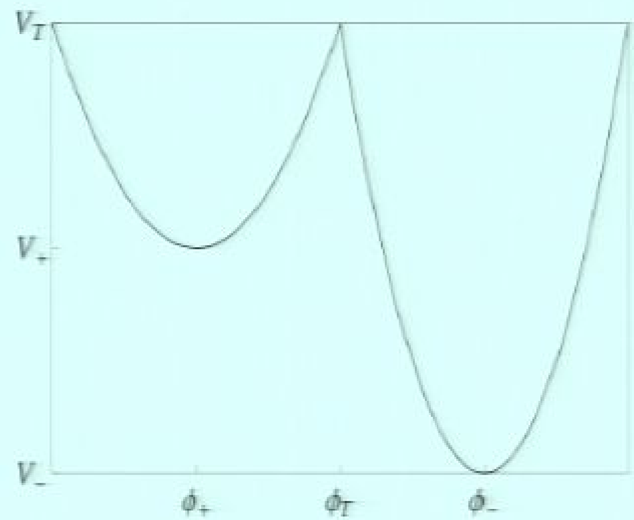

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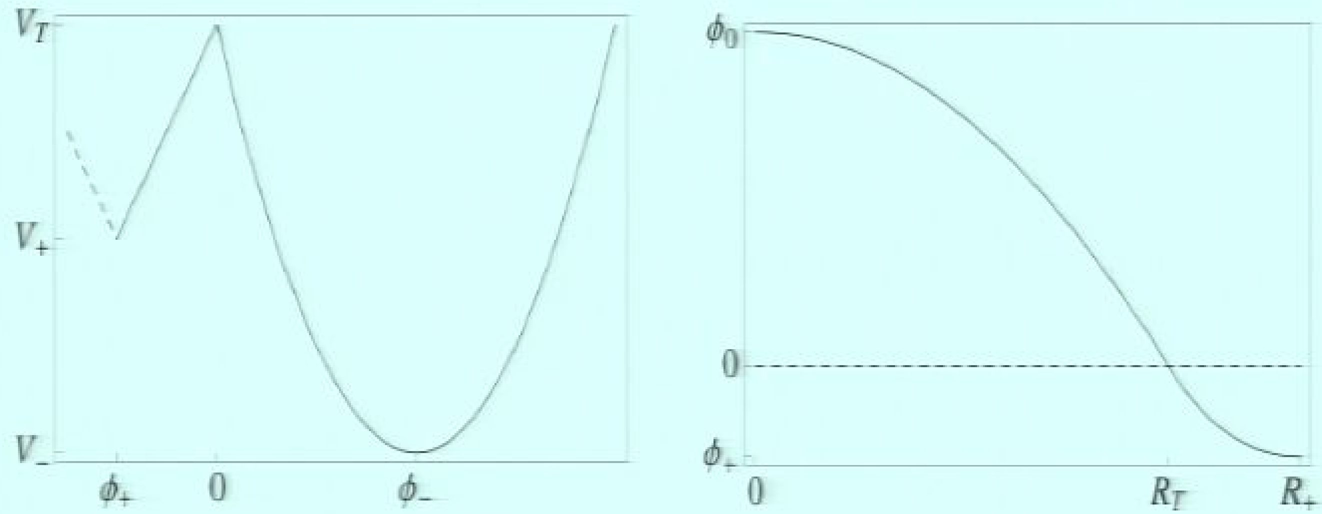

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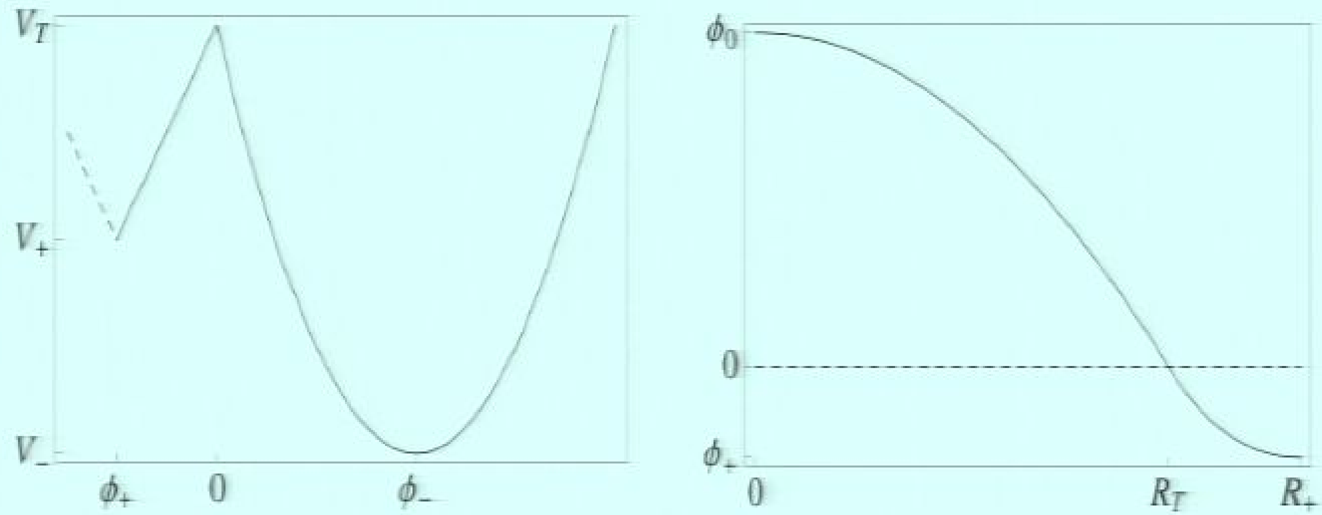
Piecewise linear and quadratic potential II



- can solve eom exactly for both pieces

$$\phi_R = \phi_- + 2(\phi_0 - \phi_-)\frac{I_1(mr)}{mr}, \quad \phi_L = \phi_+ + \frac{\lambda_+}{8r^2}(r^2 - R_+^2)^2$$

- r is $O(4)$ symmetric initial radius of the bubble
- need to determine R_+, ϕ_0 by matching conditions for $\phi_L(R_T) = \phi_R(R_T) = 0, \phi_L'(R_T) = \phi_R'(R_T)$

Action as a function of nucleation radius

- compute B as a function of the tunneling radius R_T exactly

$$B = 2\pi^2\phi_-^2 R_T^2\left[\alpha^2 + \frac{1}{2}\left(\frac{4}{3}\alpha\sqrt{\Delta} + \frac{I_2(mR_T)}{I_1(mR_T)}\right) mR_T - \frac{1-\Delta}{8} m^2 R_T^2\right]$$

with

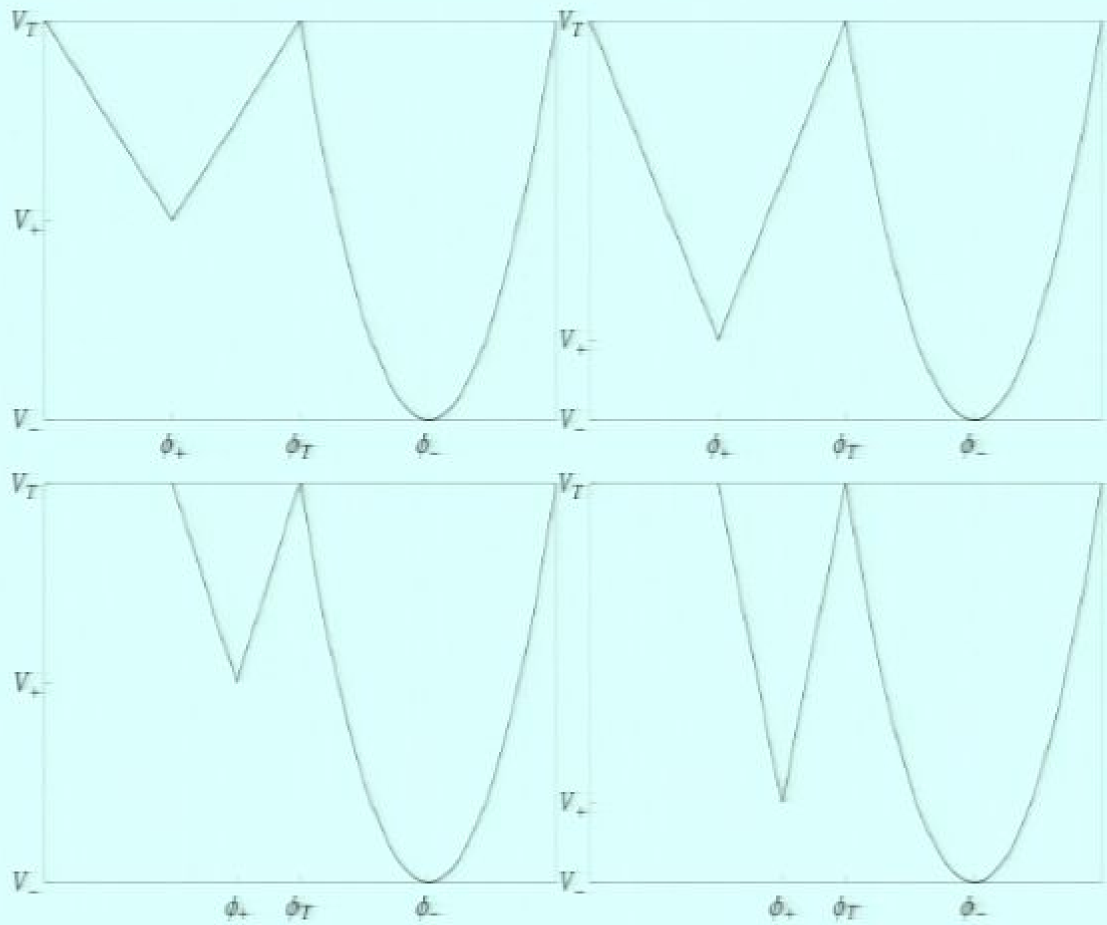
$$\alpha = -\frac{\phi_+}{\phi_-}, \Delta = \frac{-2\lambda\phi_+}{m^2\phi_-^2}$$

- R_T from

$$2\alpha + \sqrt{\Delta} m R_T = m R_T \frac{I_2(mR_T)}{I_1(mR_T)}$$

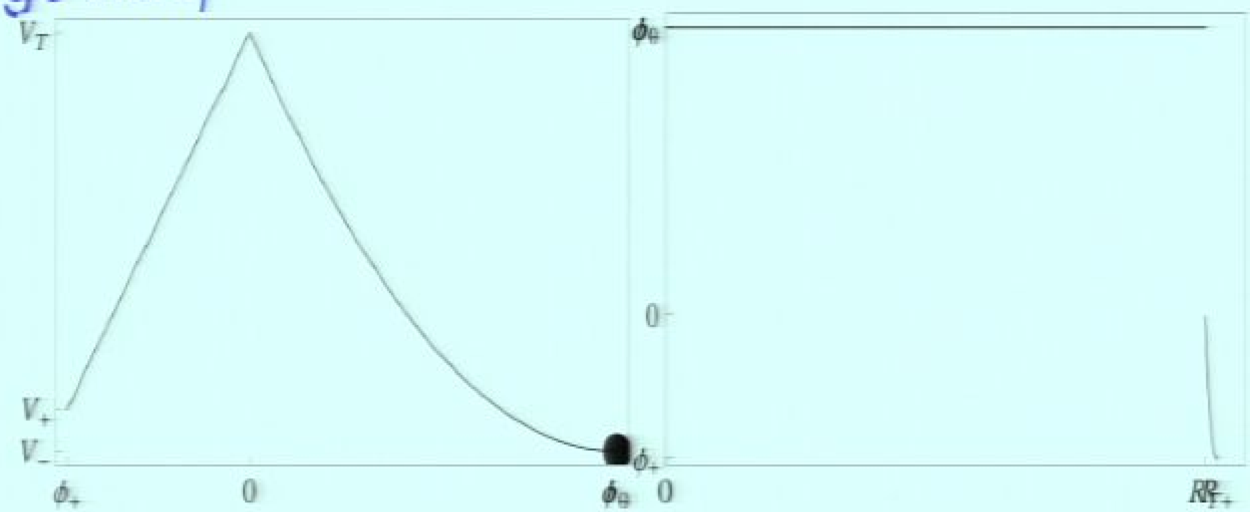
- R_T not from $\partial_r B|_{R_T} = 0$
- take limits $mR_T \gg 1, mR_T \ll 1$

What are α, Δ?



- variables: $\lambda_+ \Rightarrow \Delta$, $\phi_+ \Rightarrow \alpha$
- plus: m, ϕ_-
- for fixed m, ϕ_-: $\alpha \Rightarrow$ width (in field space) of the potential barrier, $\Delta \Rightarrow$ relative height of the minima

large mR_T



$\alpha = 0.5, \Delta = 0.9, mR_T = 49$

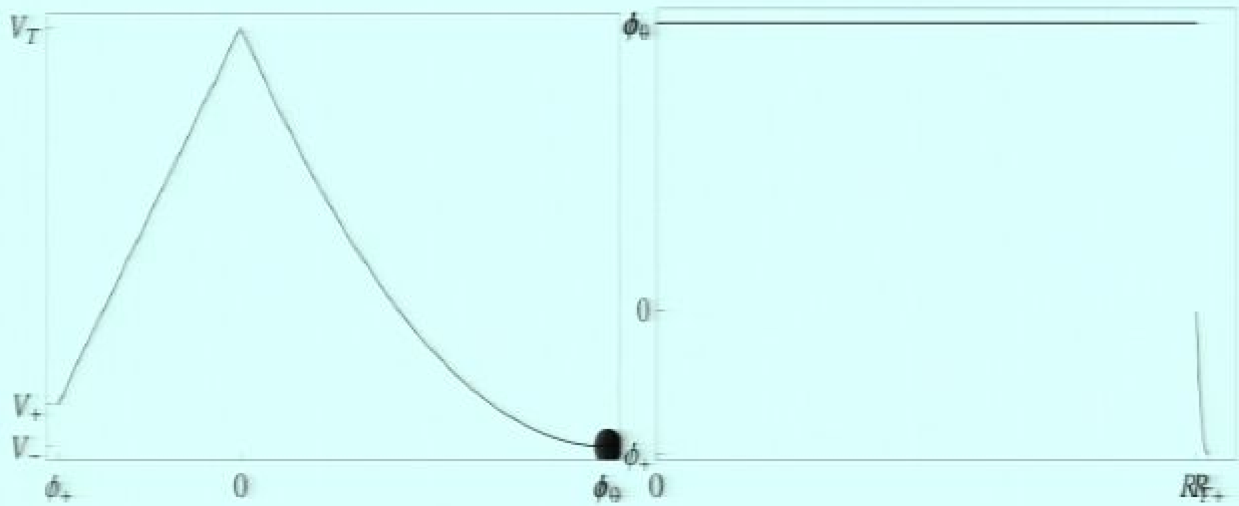
$$2\alpha + \sqrt{\Delta} mR_T = mR_T \frac{I_2(mR_T)}{I_1(mR_T)} \approx mR_T - \frac{3}{2}$$

$$mR_T \approx \frac{3+4\alpha}{2} \frac{1}{1-\sqrt{\Delta}} \xrightarrow{\epsilon \to 0} \frac{(3+4\alpha) m^2 \phi_-^2}{2\epsilon}$$

$$\phi_0 \approx \phi_- \left(1 - \sqrt{\frac{\pi}{2}} (mR_T)^{3/2} e^{-mR_T}\right)$$

$$B \approx \frac{\phi_-^2}{m^2} \times f(\alpha, \Delta) = \frac{\pi^2}{96\epsilon^3} m^4 \phi_-^8 (3+4\alpha)^4 = \frac{27\pi^2}{3\epsilon^3} S_1^4$$

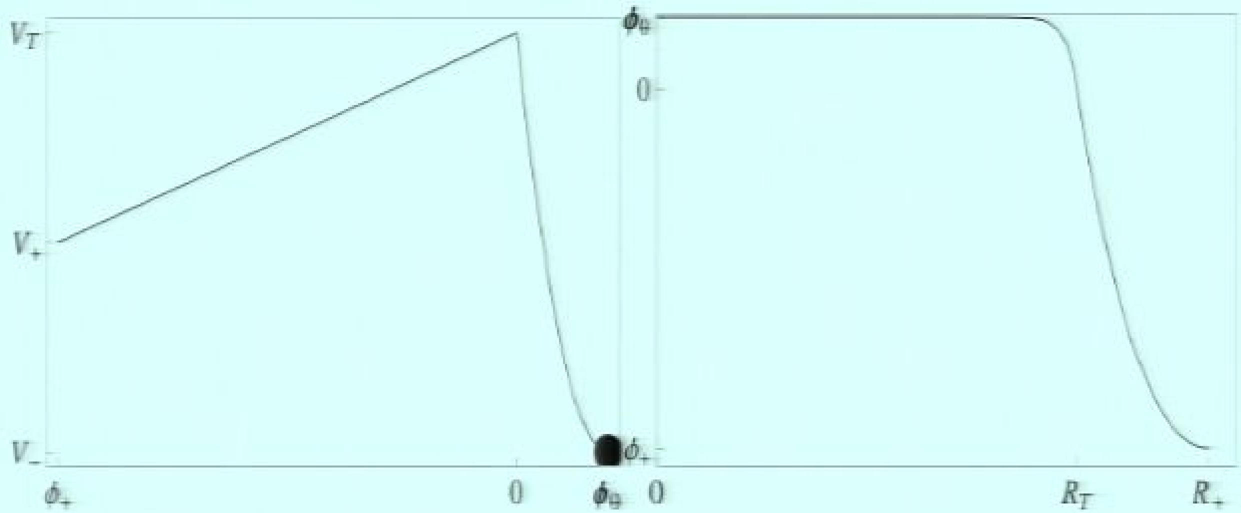
large mR_T



$\alpha = 0.5, \Delta = 0.9, mR_T = 49$

- $\frac{\phi_-}{m}$ sets the scale of B
- agreement with thin-wall if $\Delta \to 1$ or equivalently $\epsilon \to 0$
- Non-thin wall $\Delta \ll 1, \alpha \gg 0 \Rightarrow mR_T \gg 1$

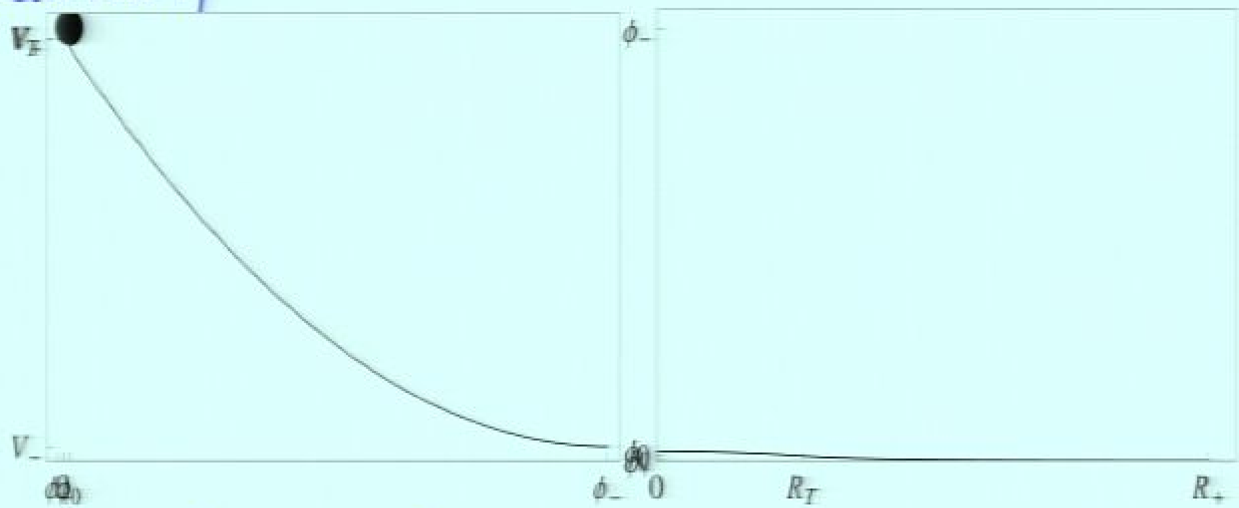
large mR_T – Non-Thin wall



$\alpha = 5, \Delta = 0.5, mR_T = 39$

- large R_T was crucial for thin-wall approximation
-

small mR_T



$\alpha = 10^{-2}, \Delta = 10^{-5}, mR_T = 0.3$

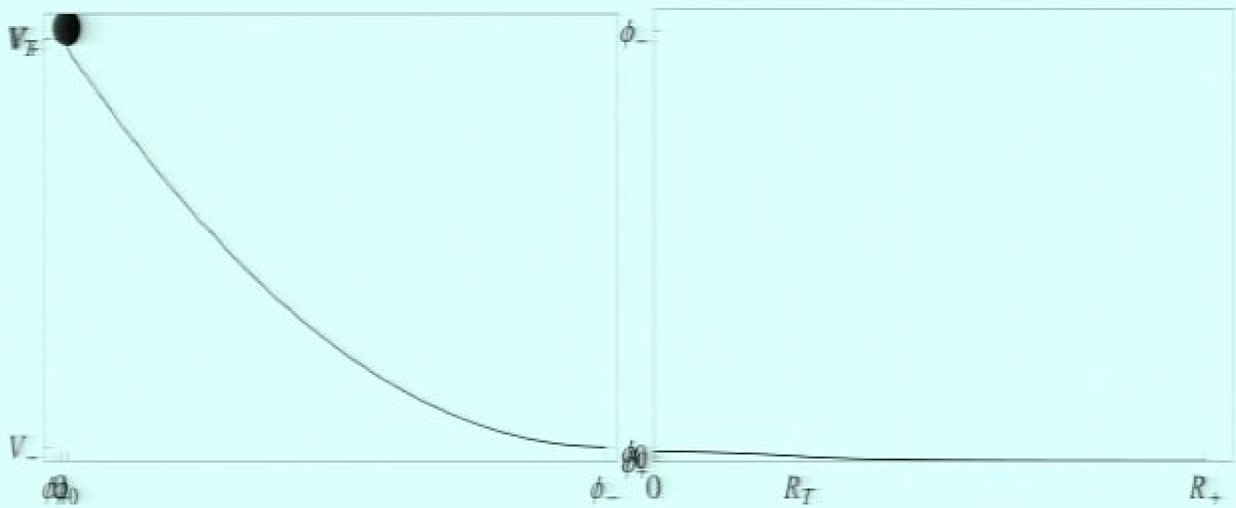
$$2\alpha + \sqrt{\Delta} mR_T = mR_T \frac{I_2(mR_T)}{I_1(mR_T)} \approx \frac{1}{4} m^2 R_T^2$$

$$mR_T \approx 2(\sqrt{\Delta} + \sqrt{2\alpha + \Delta}), \quad \phi_0 \approx \phi_- \left(1 + \frac{8}{mR_T}\right)^{-1}$$

$$B \approx \frac{16\pi^2}{3} \frac{\phi_-^2}{m^2} (2\alpha + \Delta)$$

$$\times \left[(\alpha + \Delta)^2 + \Delta^2 \left(1 + \frac{2\alpha}{\Delta}\right) \left(1 + \sqrt{1 + \frac{2\alpha}{\Delta}}\right) \right]$$

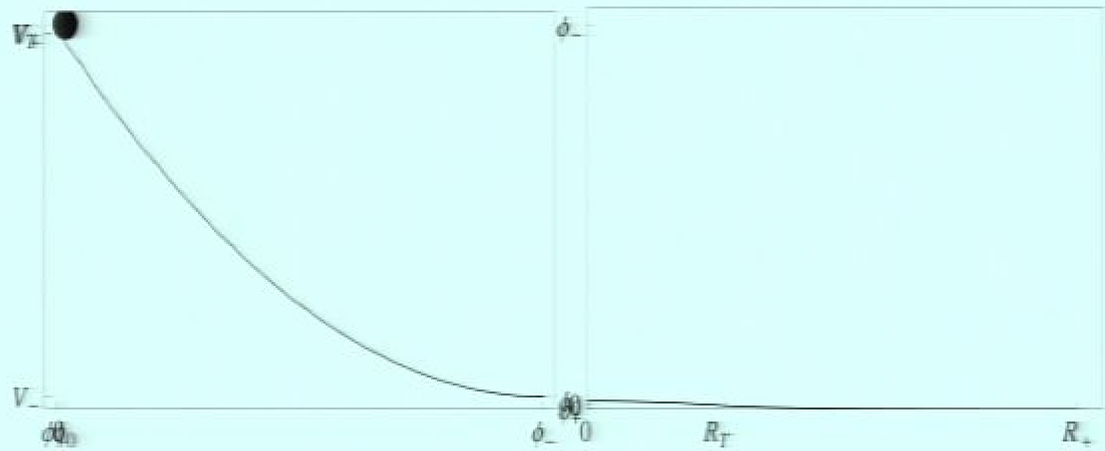
small mR_T



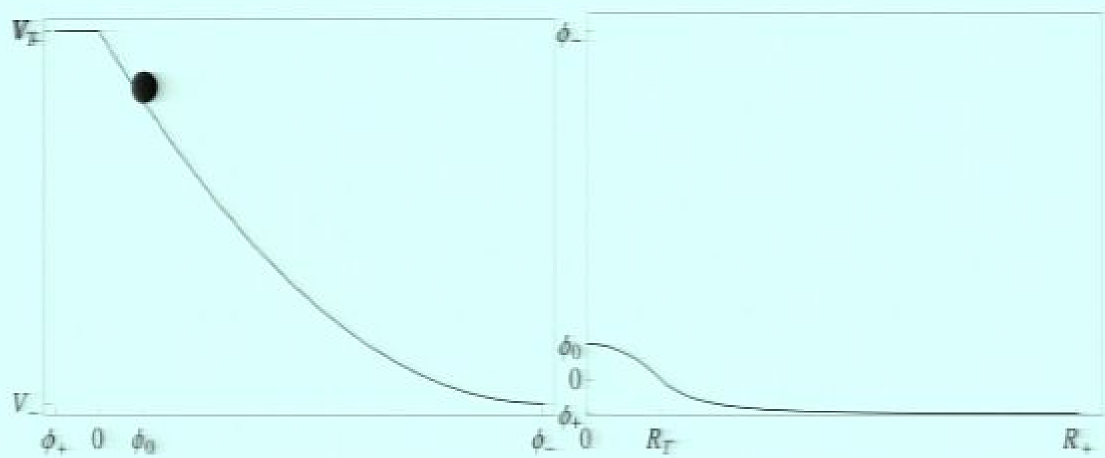
$\alpha = 10^{-2}, \Delta = 10^{-5}, mR_T = 0.3$

- agreement with DJ
  - $mR_T = 2(\sqrt{\Delta} + \sqrt{2\alpha + \Delta})$
  - B agrees to order 6 in R_T
- B scales as $\frac{\phi_-^2}{m^2}$ just like the large mR_T case

Intermediate mR_T I

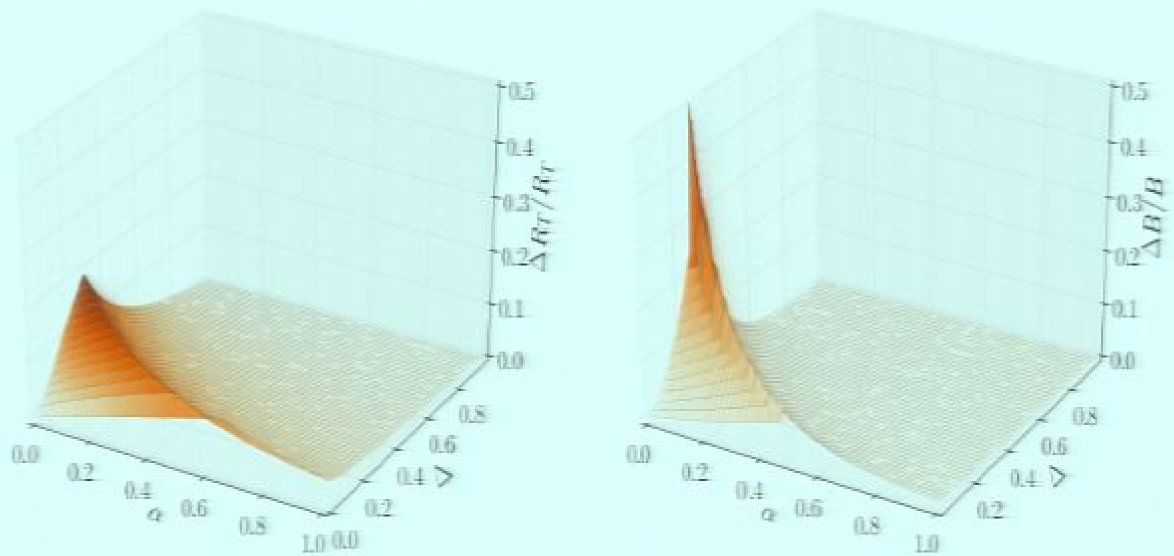


$\alpha = 0.01, \Delta = 10^{-5}, mR_T = 0.30, mR_T^{\text{small}} = 0.30, mR_T^{\text{large}} = 1.5$



$\alpha = 0.1, \Delta = 10^{-5}, mR_T = 0.93, mR_T^{\text{small}} = 0.91, mR_T^{\text{large}} = 1.7$

Intermediate mR_T II



$$
mR_T = \begin{cases} 2\sqrt{\Delta}\left(1+\sqrt{1+\frac{2\alpha}{\Delta}}\right), & \Delta < (0.8\alpha - 0.5)^2 \\ \frac{3+4\alpha}{2}\frac{1}{1-\sqrt{\Delta}}, & (0.8\alpha-0.5)^2 < \Delta \end{cases}
$$

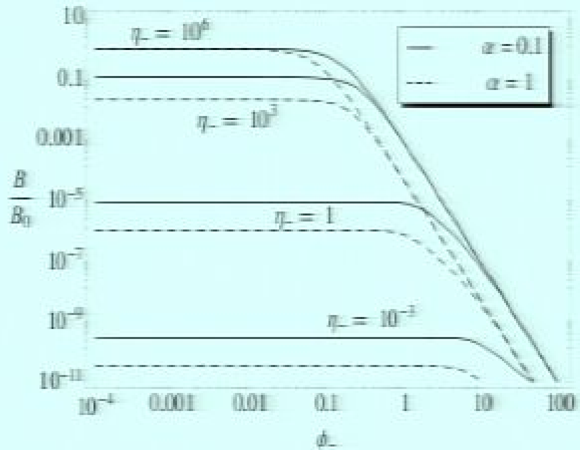
- numerical solution easy: algebraic equation instead of differential equation
- Combination of large and small mR_T limits is not too bad:

Comparsion with thin wall and DJ

	mR_T	B_{exact}	B_{DJ}	$B_{thin-wall}$
$\alpha = 0.01$	0.6	0.0023	0.0022	72.4
$\alpha = 0.1$	1.2	0.3	0.04	113.3
$\alpha = 0.5$	2.5	23.9	0.6	529.8

- All values for $\Delta = 0.01$
- Regions where neither thin wall nor the linear approximation holds

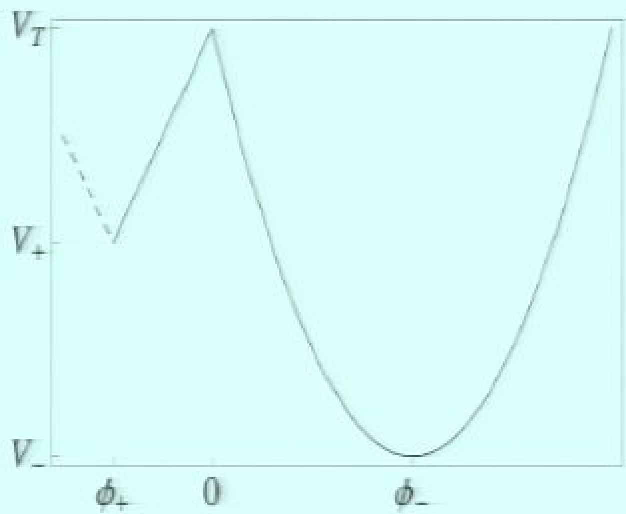
Inclusion of gravity

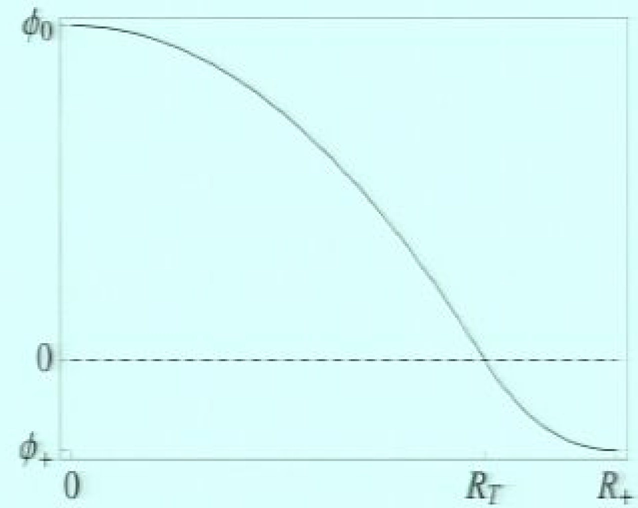


Parke (1981)

- $\eta_- = \frac{m^2}{V_-}$
- only in thin wall limit
- idea: pull of dS from outside vs pull of dS from inside ⇒ smaller B
- gravitational correction stronger for wider barrier in field space

Conclusions and Outlook





- exact tunneling amplitude B in terms of R_T
- R_T from
  - solving algebraic equation
  - analytic approximation (with $< 50\%$ errors for B)
- populating initial conditons for inflation
- TODO:
  - include gravity
  - determine the length of inflation
  - estimate the relative frequency of both types of potentials