Title: Beyond bosons and fermions: how to detect and use anyons

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Abstract: One of the key features of the quantum Hall effect (QHE) is the fractional charge and statistics of quasiparticles. Fractionally charged anyons accumulate non-trivial phases when they encircle each other. In some QHE systems an unusual type of particles, called non-Abelian anyons, is expected to exist. When one non-Abelian particle makes a circle around another anyon this changes not only the phase but even the direction of the quantum-state vector in the Hilbert space. This property makes non-Abelian anyons promising for fault-tolerant quantum computation. Several experiments allowed an observation of fractional charges. Probing exchange statistics is more difficult and has not been accomplished for identical anyons so far. We will discuss how the statistics can be probed with Mach-Zehnder interferometry, tunneling experiments and far-from-equilibrium fluctuation-dissipation theorem.

Beyond Bosons and Fermions: How to Detect and Use Anyons

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Outline

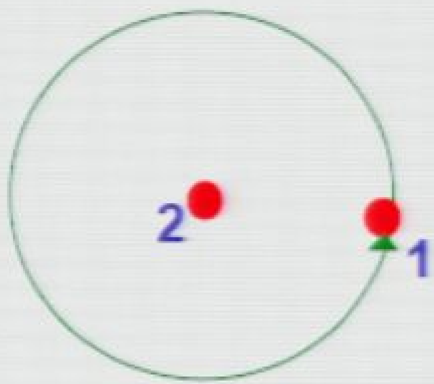
- Introduction
  - fractional statistics
  - quantum Hall effect
  - non-Abelian anyons

- Detecting non-Abelian anyons
  - electronic Mach-Zehnder interferometer
  - Tunneling in different geometries
  - Nonequilibrium fluctuation-dissipation theorem

Fractional Statistics

Bosons: $\psi(x_1, x_2) = \psi(x_2, x_1)$

Fermions: $\psi(x_1, x_2) = -\psi(x_2, x_1)$



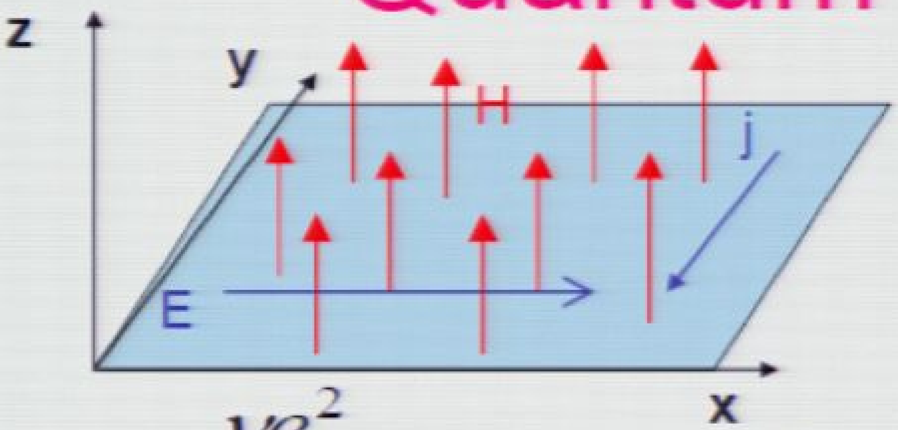
Statistical phase:

$$\psi \rightarrow \exp(i\theta)\psi$$

Anyons: $\theta \neq 2\pi n$

No anyons in 3D

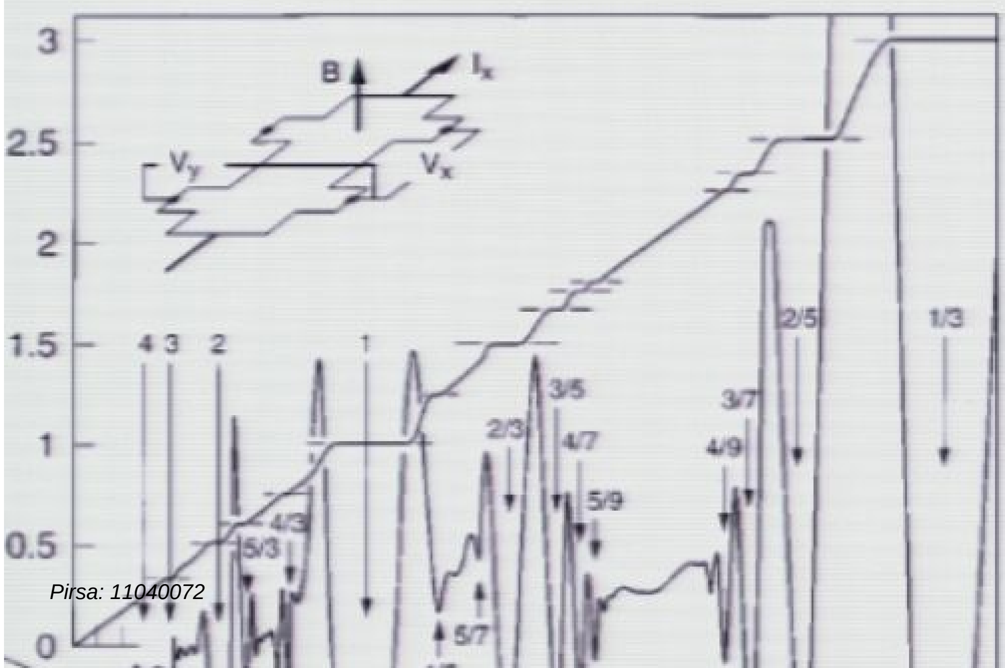
Quantum Hall Effect



$$I_x = \sigma_{xx} V = 0$$

$$I_y = \sigma_{xy} V$$

$$I = \frac{\nu e^2}{h} V; \quad \nu \text{ - rational number (filling factor)}$$



$$\frac{e^2}{h} = (25.8 k\Omega)^{-1}$$

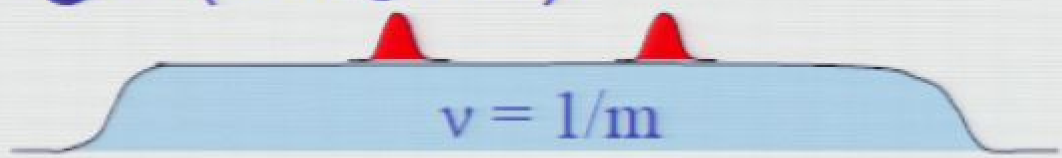
$$= 1\,\text{Klitzing}$$

Laughlin State

$$\nu = \frac{1}{2n+1};$$

Quasiparticle charge (Laughlin)

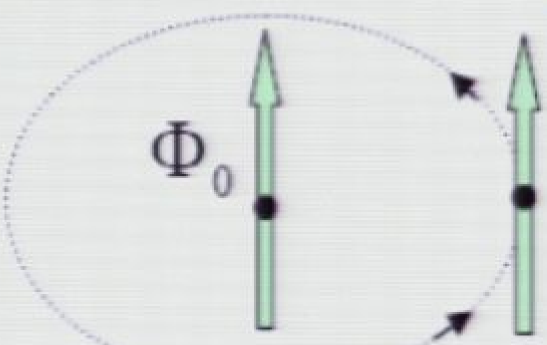
$$q = \nu e$$



Quasiparticle statistics

(Arovas, Schrieffer, Wilczek)

$$\theta = 2\pi\nu$$



$$\theta = \frac{q}{\hbar c}\oint \vec{A}\,d\vec{r}$$

Moore-Read (Pfaffian) state

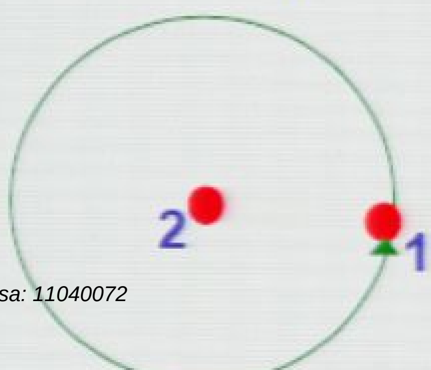
$$\nu = 5/2$$

$$q = e/4; \text{non-Abelian statistics}$$

$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

Several states at given quasiparticle positions

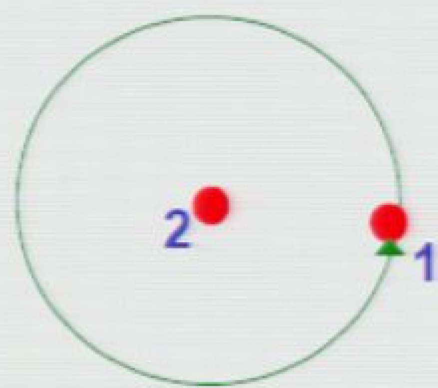
Vacuum superselection sector $\left|1\right\rangle$



$$\psi \rightarrow \psi$$

$$\theta = 0$$

Fermion sector $\left|\varepsilon\right\rangle$

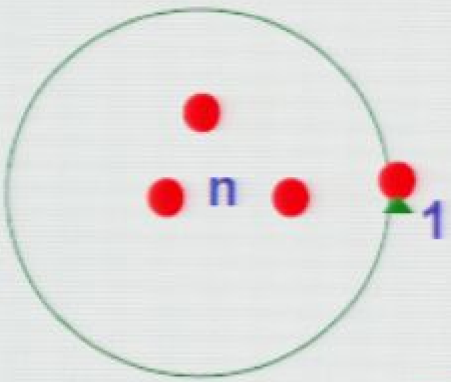


$$\psi \rightarrow -\psi$$

$$\theta = \pi$$

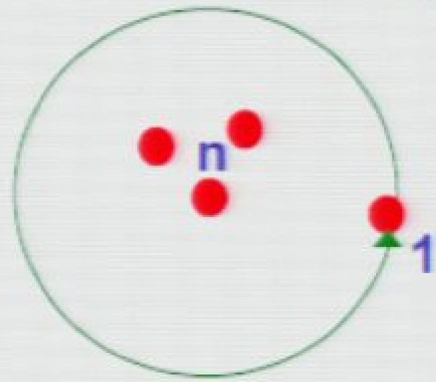
Non-Abelian statistics

Abelian anyons



$$\theta_n = n\theta$$

Non-Abelian anyons



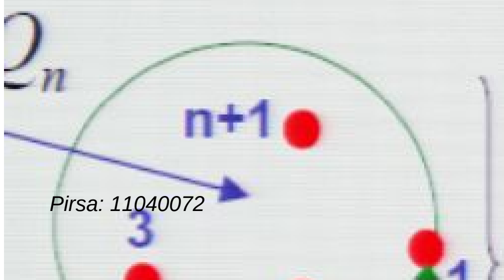
$$\theta_n \neq n\theta$$

Topological charge

even quasiparticle number : $Q = 1 \text{ or } \varepsilon$

odd quasiparticle number : $Q = \sigma$

Statistical phase



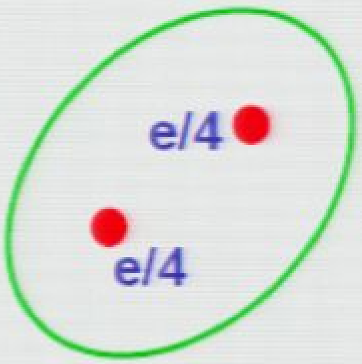
$$\theta = n\pi/4 + \varphi(Q_n, Q_{n+1})$$

$$\varphi(1,\sigma) = 0; \varphi(\sigma,1) = -\pi/4;$$

Fusion rules

$$1\times\sigma=\sigma;\quad \varepsilon\times\sigma=\sigma;\quad \sigma\times\sigma=1+\varepsilon$$

Non-Abelian qubit and quantum computing



Vacuum state = 1

Fermion state = 0

Stable with respect to local perturbations!

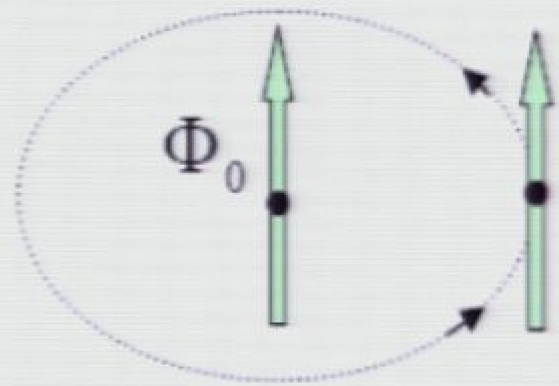
Operations $\longrightarrow$ Quasiparticle braiding

Universal quantum computation :

9 / 31 72.7% Find

But other 5/2 states might be possible

=8 state: Cooper pairs form Laughlin state with the filling factor /8 and quasiparticle charge e/4. belian statistics: $\theta = \pi / 4$



31 state: 2e/3 quasiparticles on top of the Laughlin /3 state condense into an Abelian state

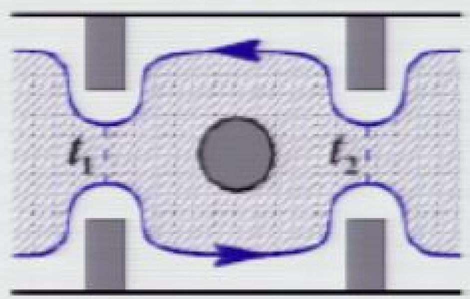
Non-Abelian states:

- Pfafian state
- Anti-Pfaffian state
- and others

Detecting fractional statistics

- Fractional charge + gauge invariance = fractional statistics; current vs. flux in the 1/3 state: F. E. Camino et al., PRL **98**, 076805 (2007).

- Theoretical proposals: P. Bonderson et al., PRL **96**, 016803 (2006) ; A. Stern and B. I. Halperin PRL **96**, 016802 (2006)

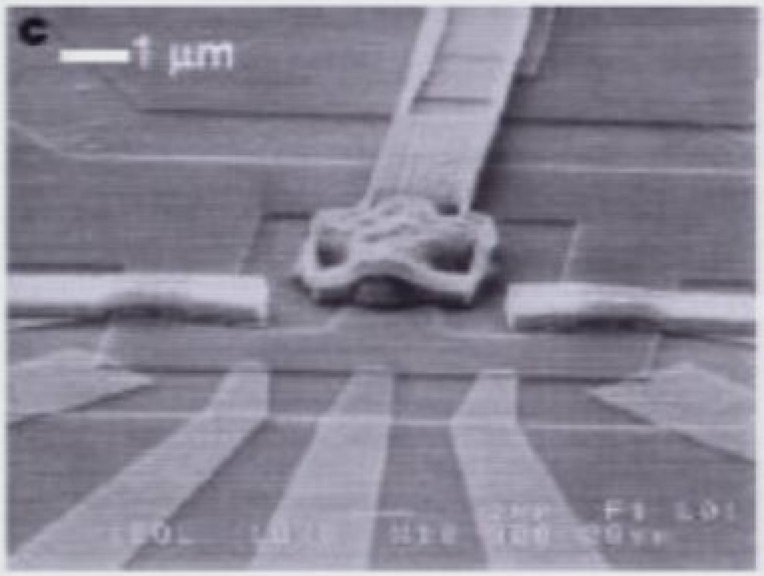


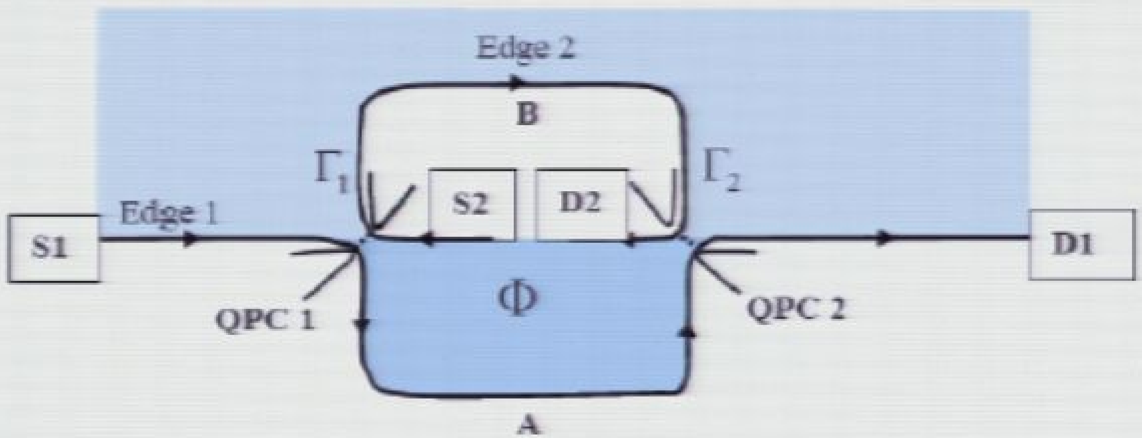
fluctuations of the number of the trapped quasiparticles; identical signatures of some Abelian and non-Abelian states

- Mach-Zehnder interferometer is free from these limitations!

11 / 31 72.7% Find

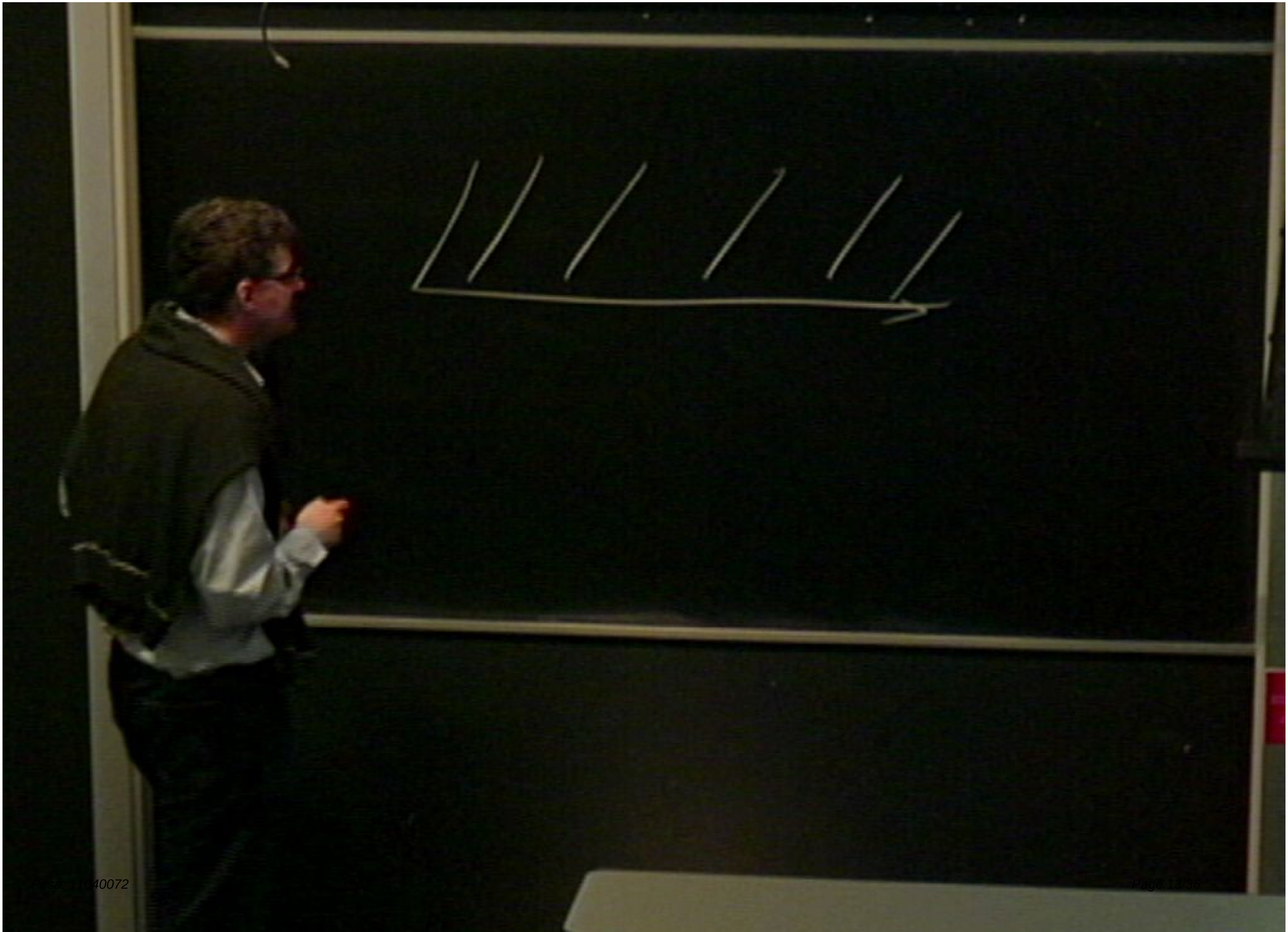
How to determine the right state: Electronic Mach-Zehnder interferometer

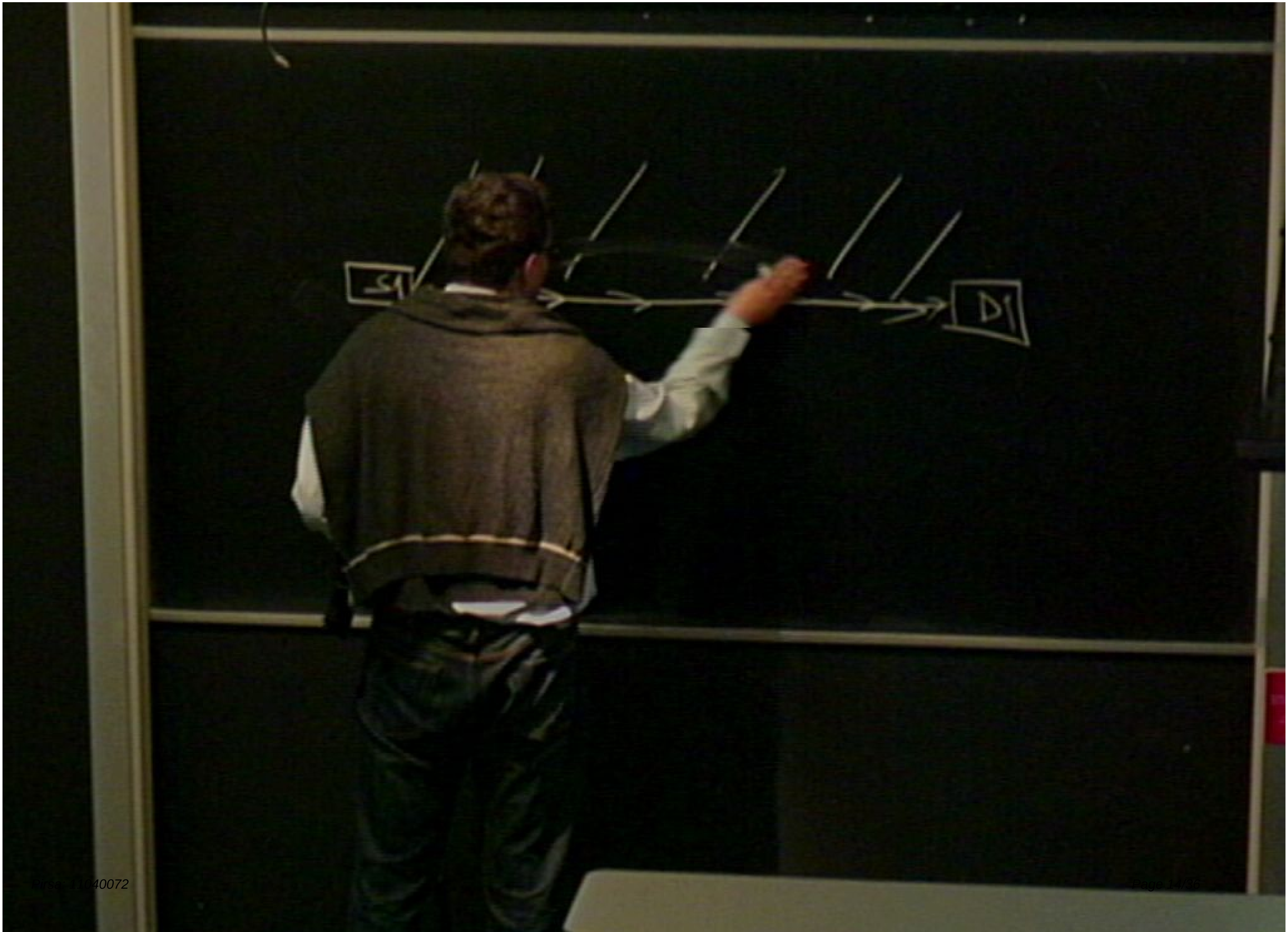


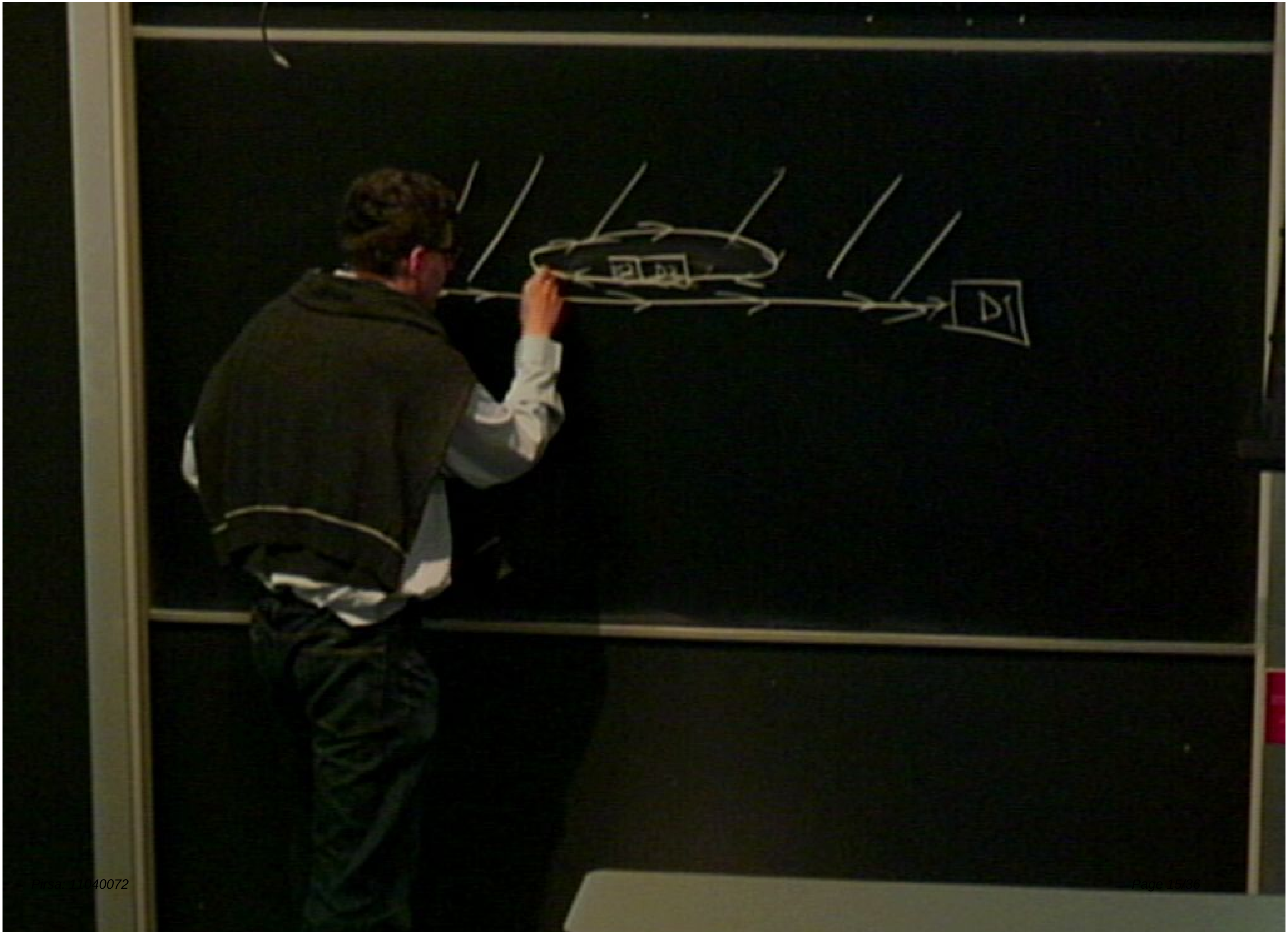


Integer quantum Hall effect

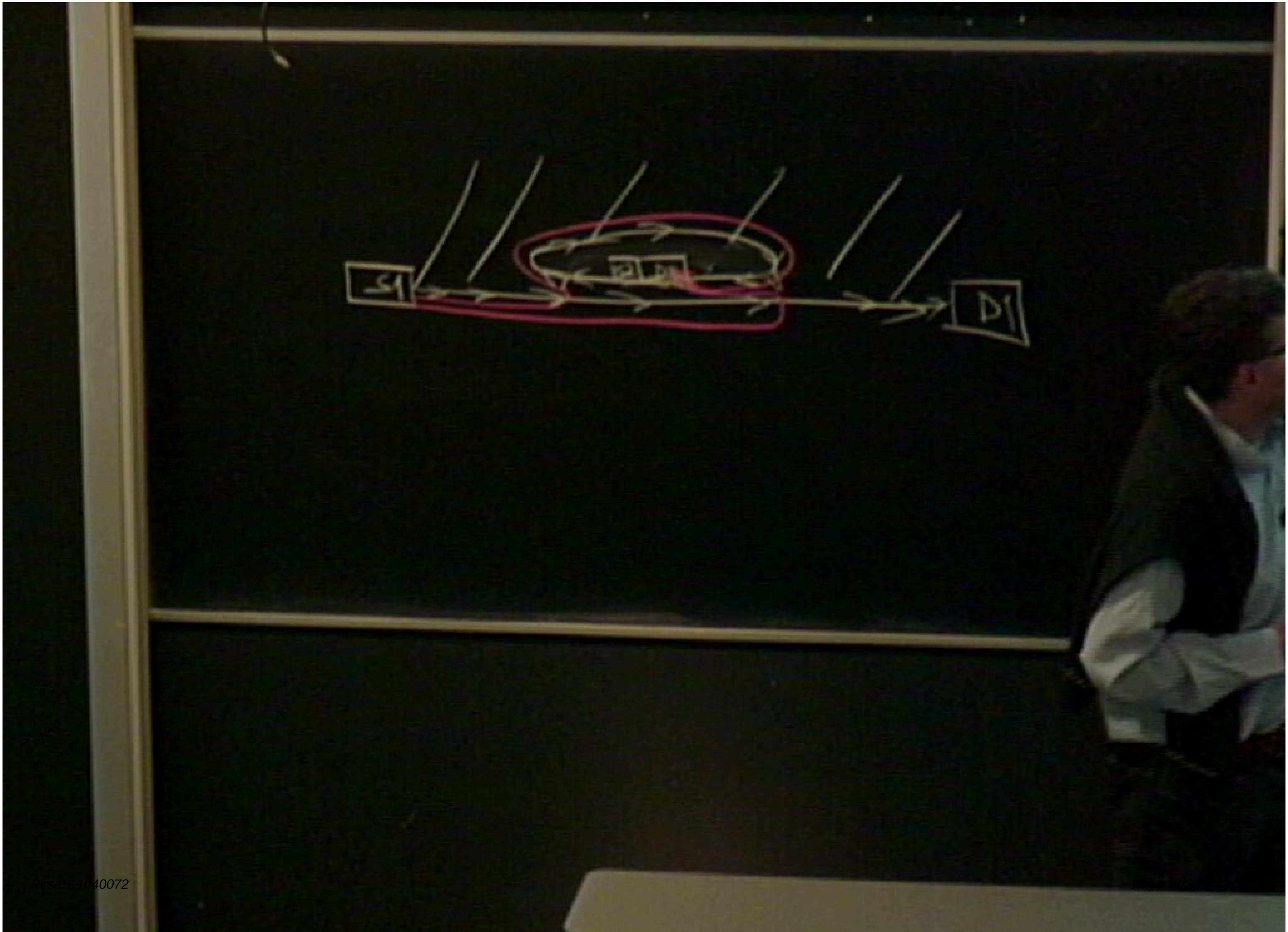
$$I = Vc_1[|\Gamma_1|^2 + |\Gamma_2|^2] + Vc_2[\Gamma_1\Gamma_2^* \exp(-2\pi i\Phi/\Phi_0) + c.c]$$




S1
D1


D1


S1
D1


S1
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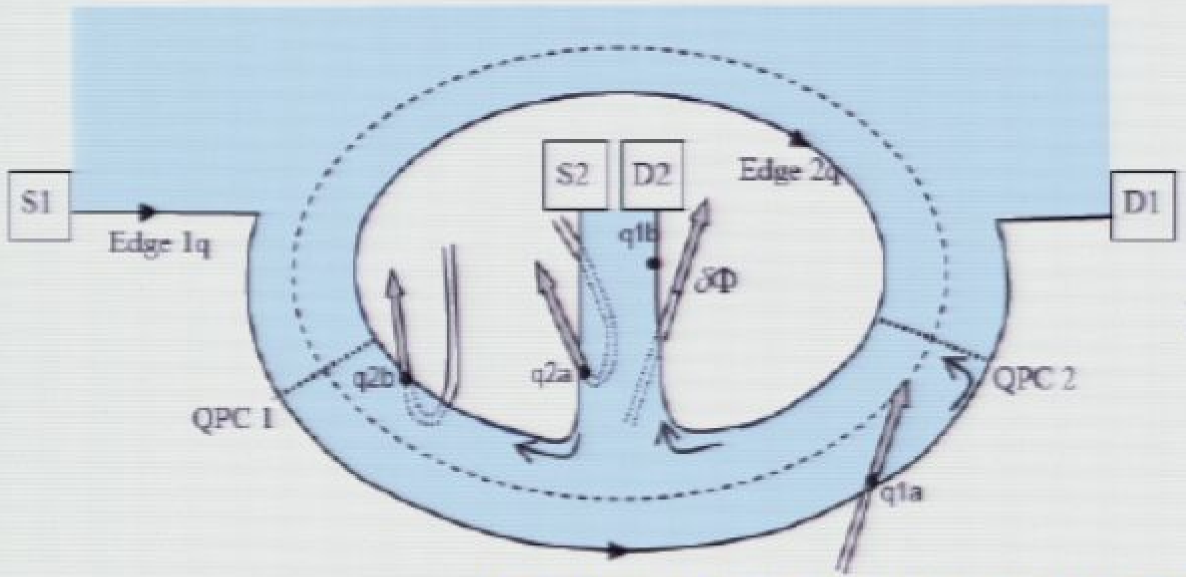
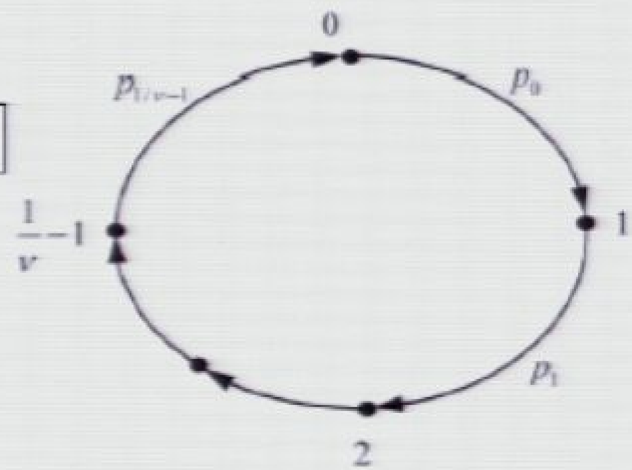
MZI: Laughlin state





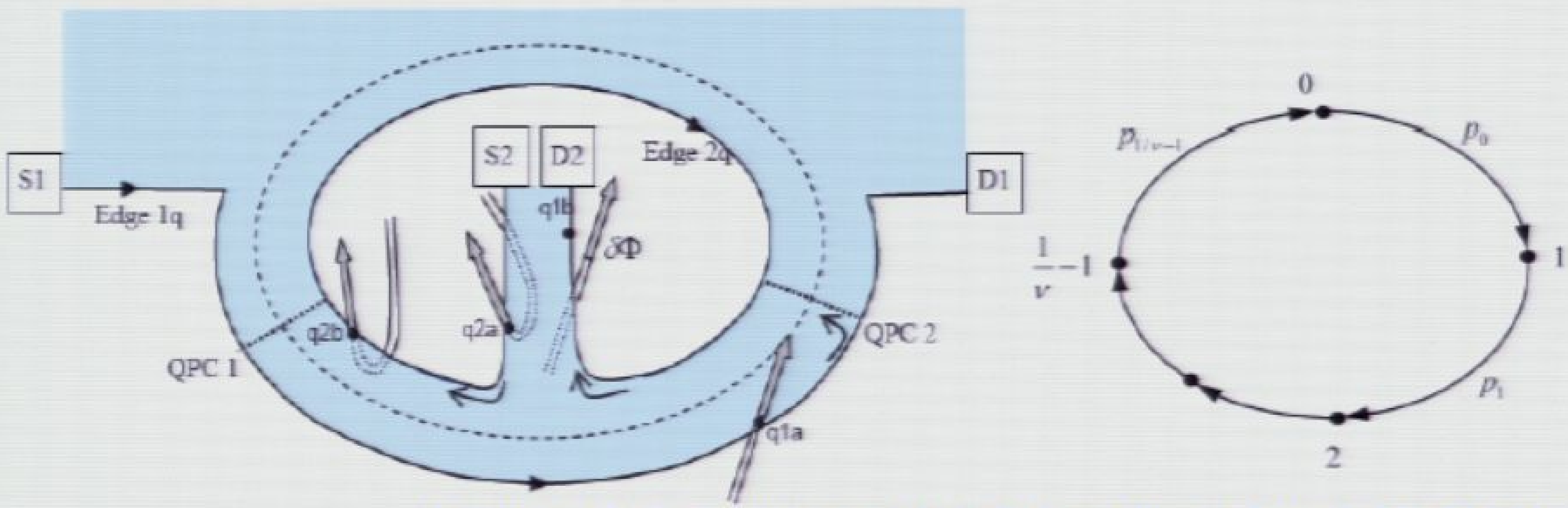
K.T. Law, D.E. Feldman and Y. Gefen, Phys. Rev. B **74**, 045319 (2006)

Tunneling probability:

$$p(\Phi) = c_1[|\Gamma_1|^2 + |\Gamma_2|^2] + c_2\Gamma_1\Gamma_2^* \exp(-2\pi\nu i\Phi/\Phi_0) + c.c.$$

$$p_1 = p(\Phi+\Phi_0); p_2 = p(\Phi+2\Phi_0); \ldots; p_{1/\nu} = p(\Phi)$$

MZI: Laughlin state



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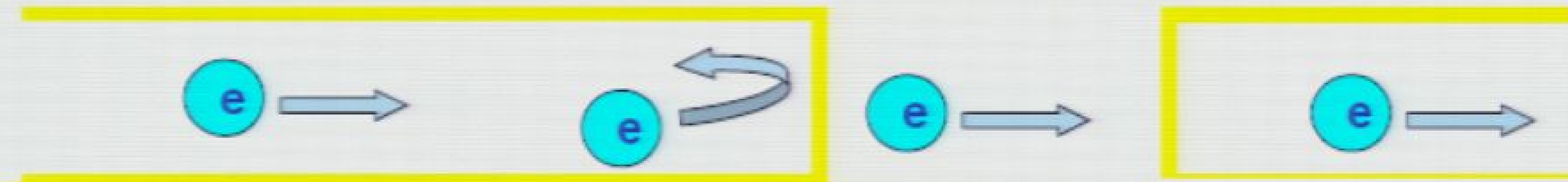
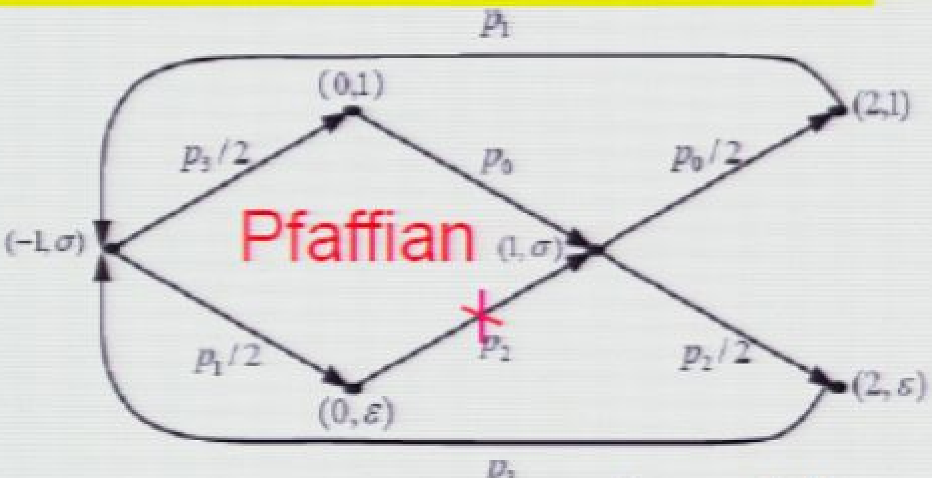
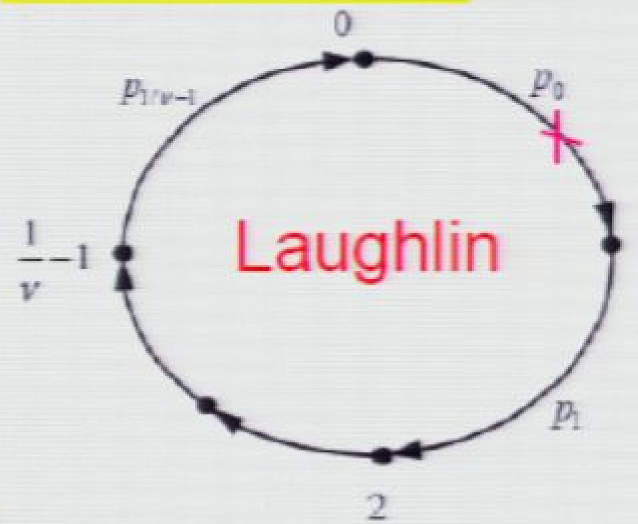
Shot Noise

$$S = 2\int [\langle I(0)I(t)\rangle - \langle I\rangle^2] dt = 2q\langle I\rangle$$

Walter Shottky





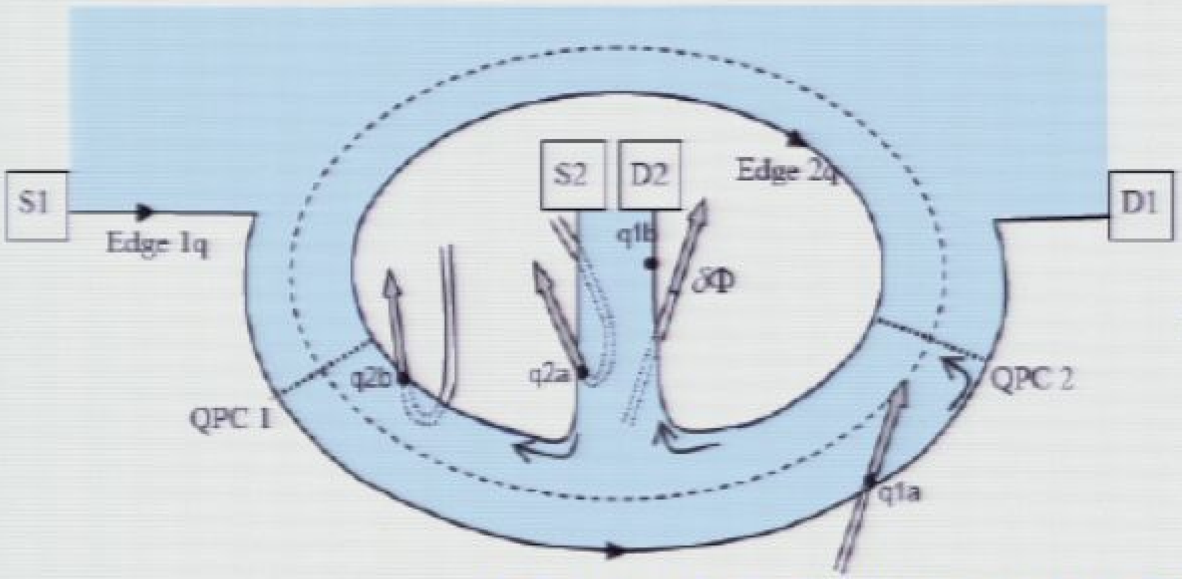


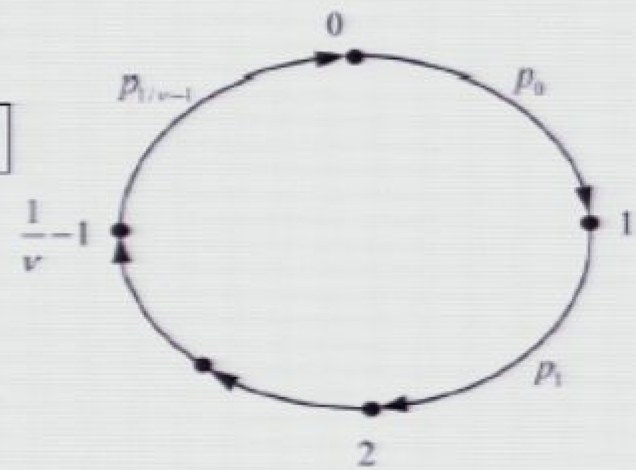
Laughlin states: $q^* < e$

331 state: $q^* < 2.3e$

Pfaffian state: $q^* < 3.2e$

MZI: Laughlin state





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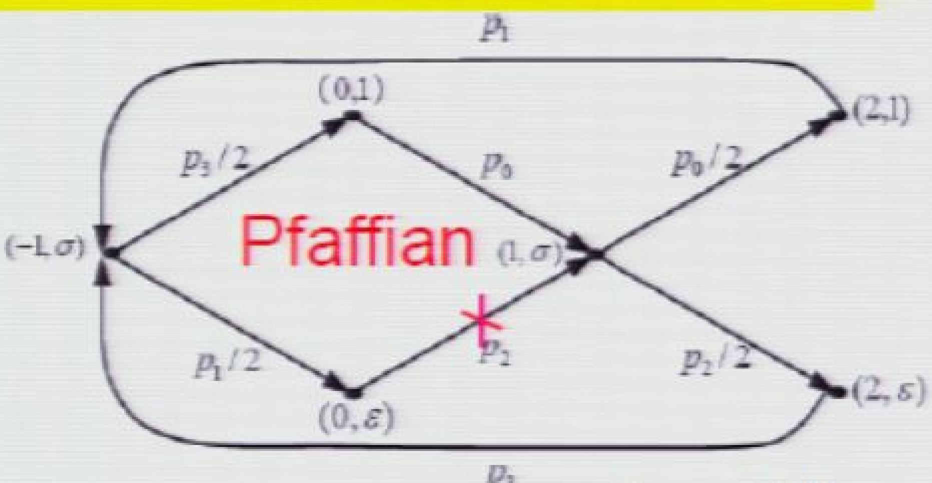

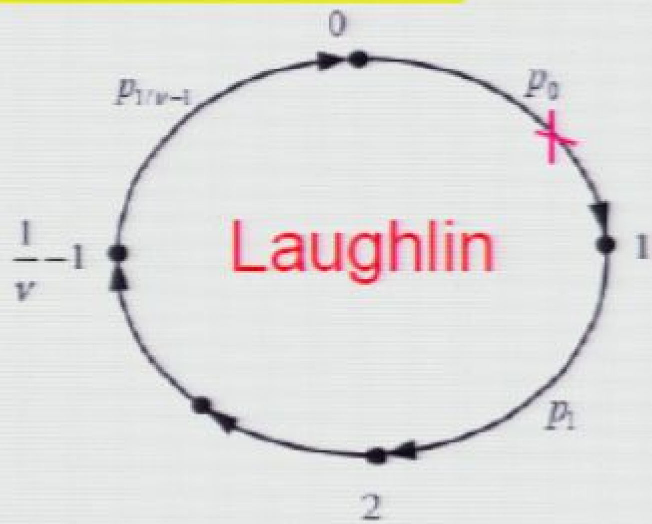

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331 state: $q^* < 2.3e$

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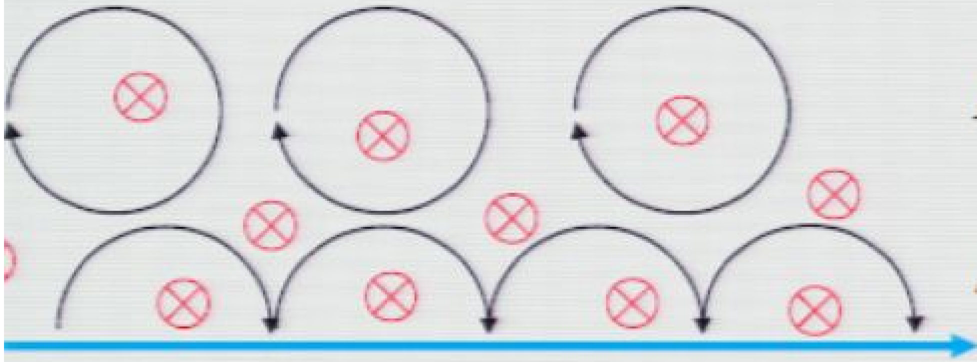
Probing anyons without interferometry



$$L = \frac{1}{4\pi}\int dt dx[\partial_t \varphi \partial_x \varphi - v(\partial_x \varphi)^2];$$

$$\rho = \partial_x \varphi / 2\pi$$

Non-Abelian states exhibit

charge-statistics separation

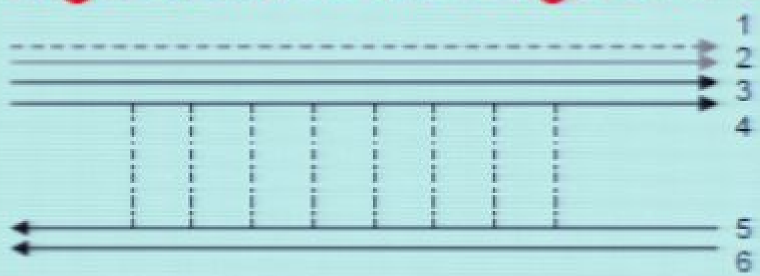
Anyons can be described by two or more fields:

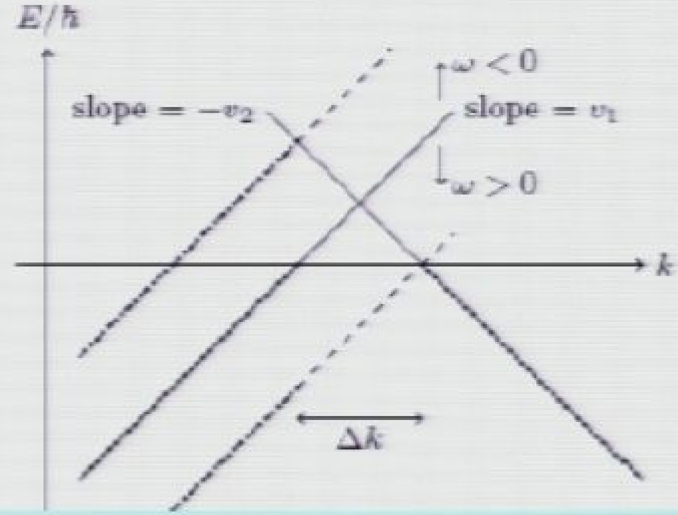
charged bosons + neutral fields which carry information about statistics.

For example, the Pfaffian state has a charged boson and a chiral Majorana fermion $\lambda = \lambda^+$

Momentum resolved tunneling

Electron Tunneling from the edge of 5/2 to the edge of v=2



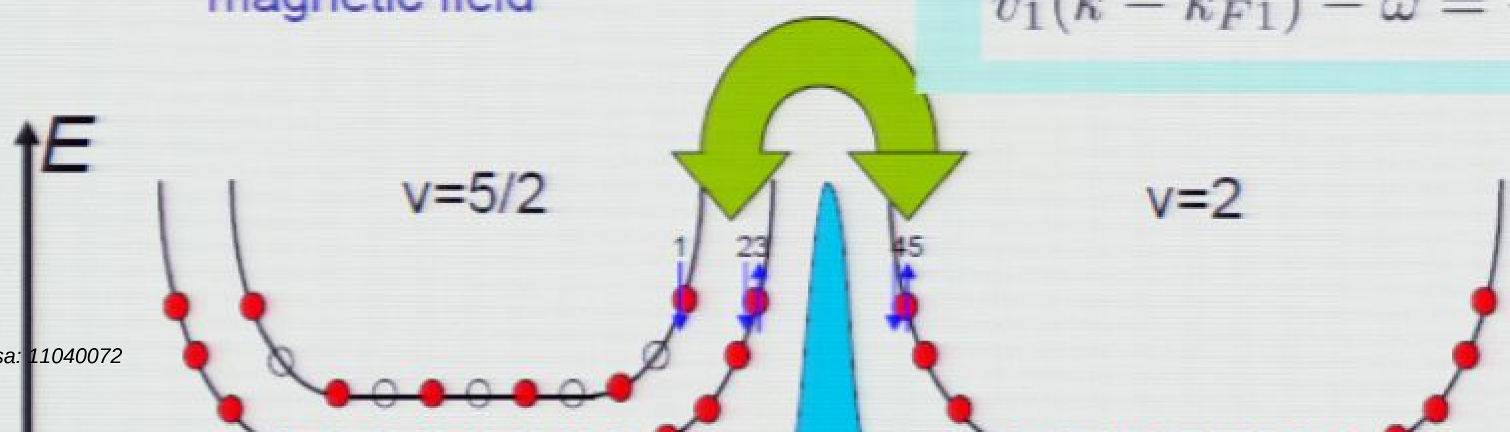


$$p \rightarrow p - \frac{e}{c} A$$

Tuning momentum difference by magnetic field

Tunneling between two integer modes. Momentum and energy conversation gives singularities at $V = v_1 \Delta k$ and $V = -v_2\Delta$

$$v_1(k - k_{F1}) - \omega = -v_2(k - k_{F2}),$$



Singularities of conductance for tunneling into boundary of 5/2 and 2 states

K=8 State: 2 singularities

331 State: 6 singularities

Pfaffian state: 3 singularities

Edge-reconstracted Pfaffian state: 10 singularities

Anti-Pfaffian state: 3 singularities


S1
S2
D2
D1


D1

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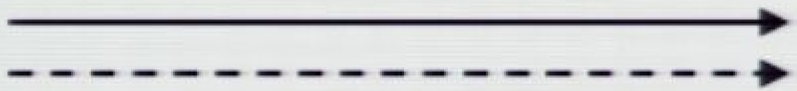
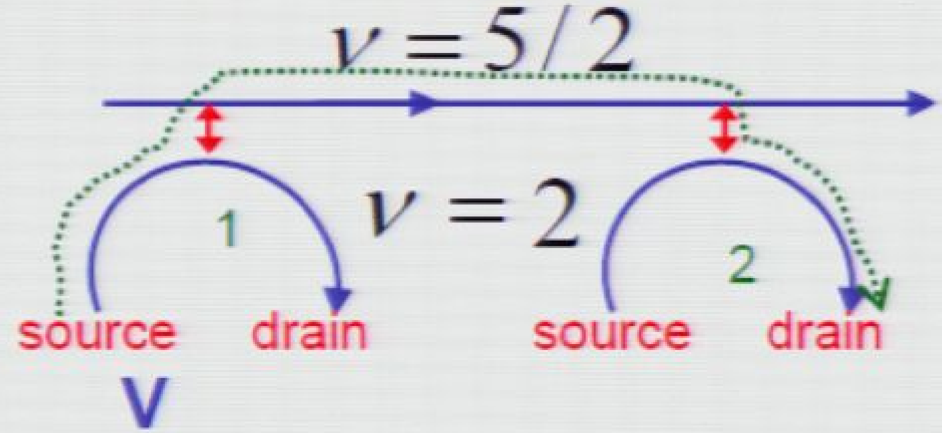
Pfaffian state:





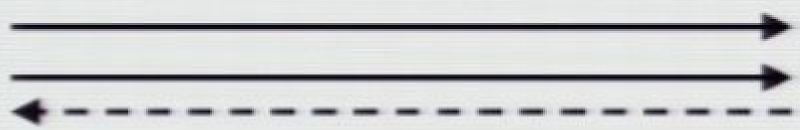
Charge mode φ

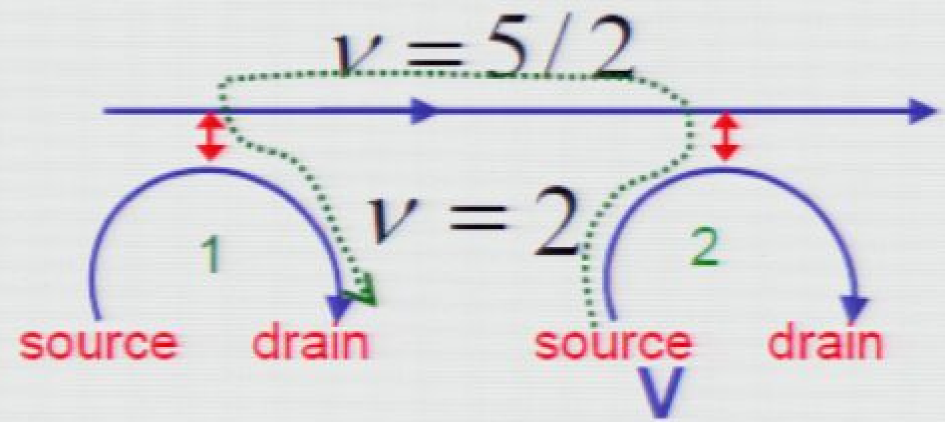
Majorana fermion $\lambda = \lambda^{+}$

$$\Psi_{el} = \lambda \exp(2i\varphi)$$

Excessive noise: $S = 5/4 \times 2eI$

Edge reconstructed Pfaffian:

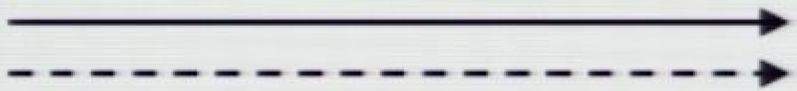


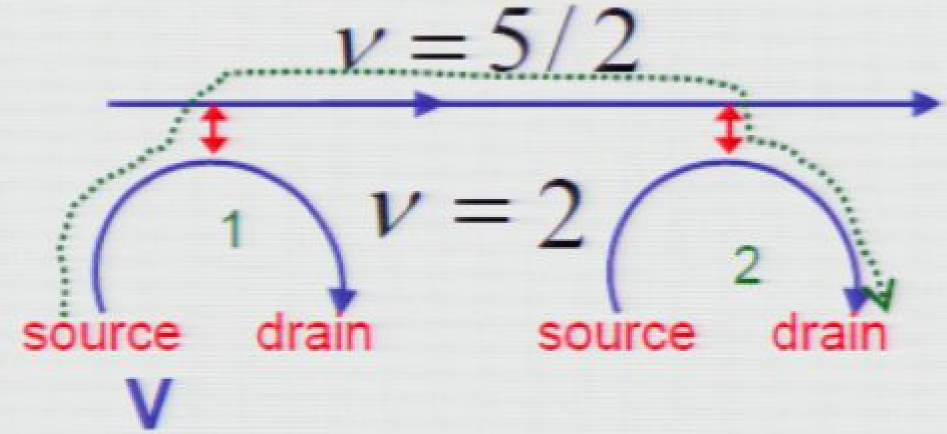


Majorana mode is left moving

$$S_1 = \frac{2\pi^3 e^2 \tau_c^8}{15\hbar^9} \Gamma_1^2 \Gamma_2^2 (eV_2)^5$$

Pfaffian state:





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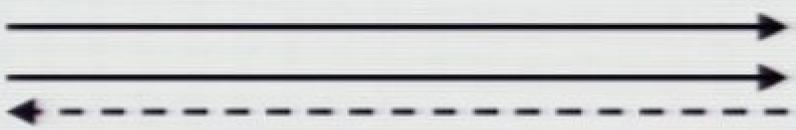

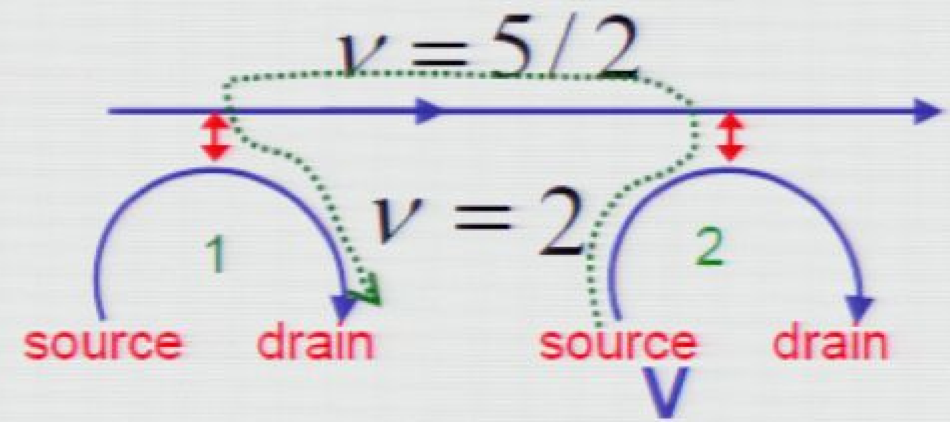

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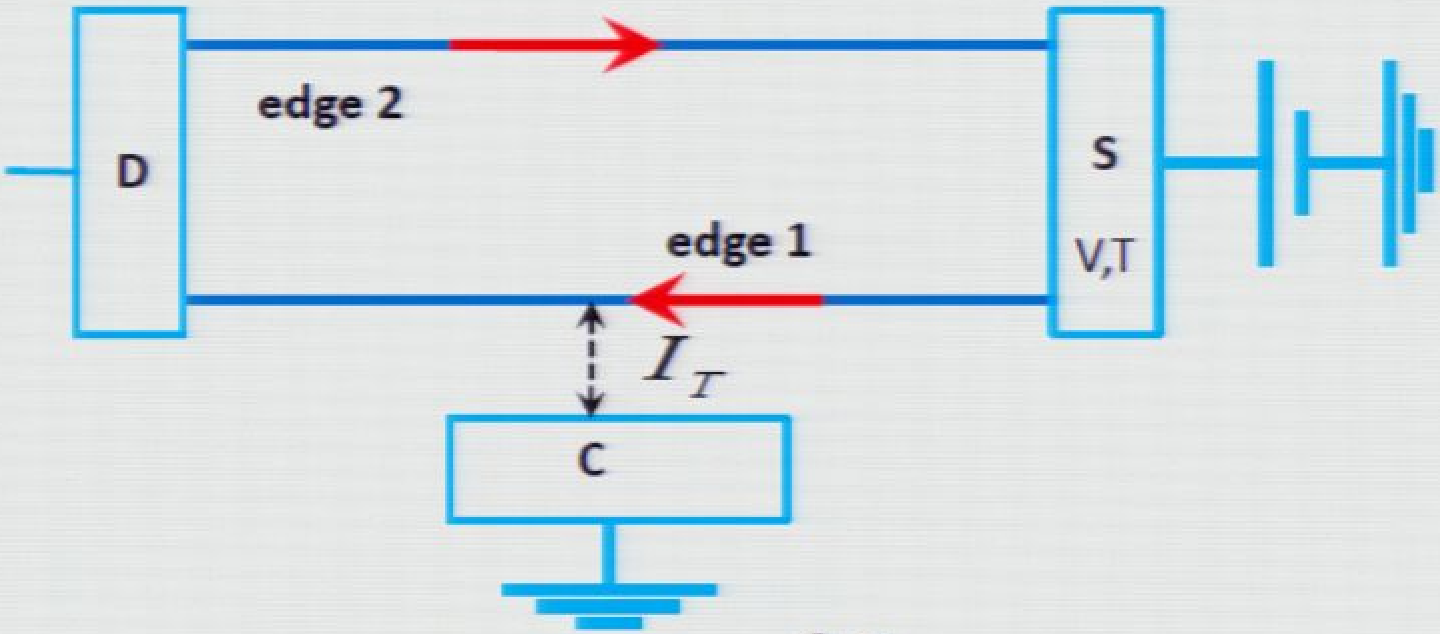
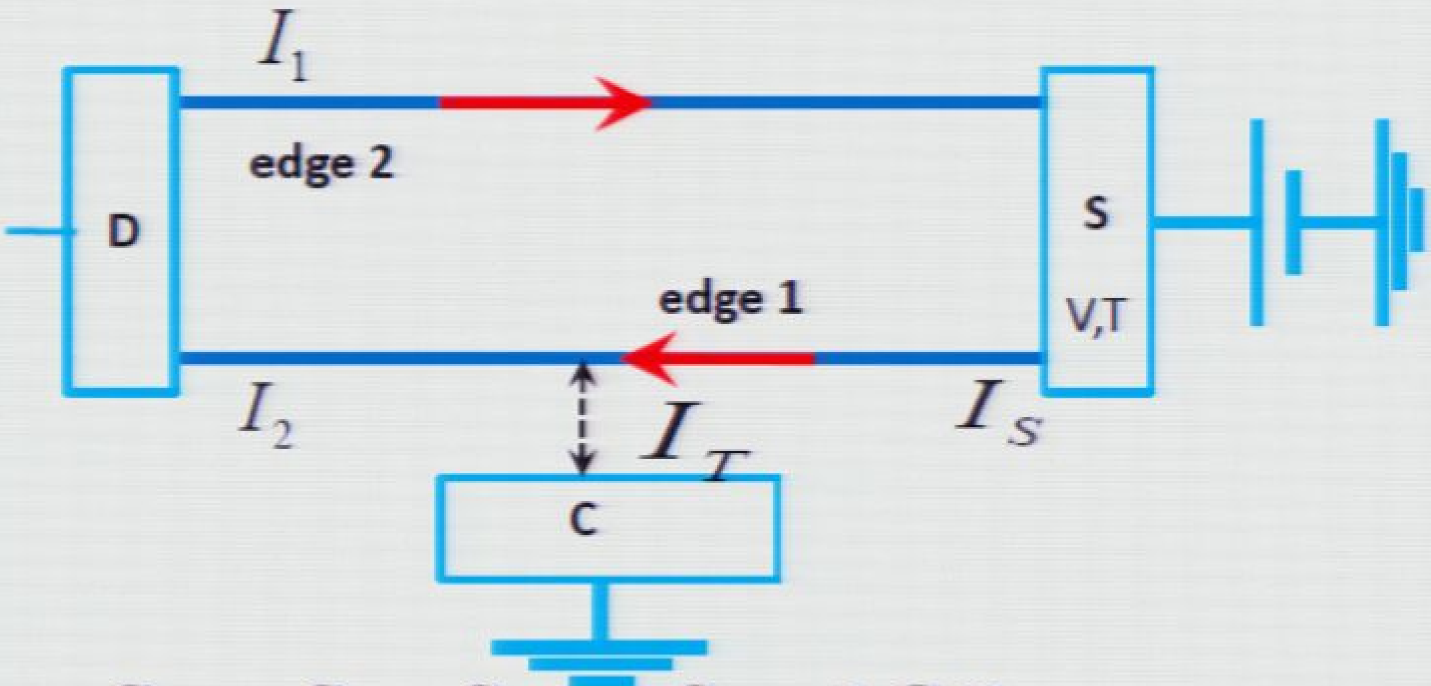
Far-from equilibrium FDT



$$S_D = S_C - 4T\frac{\partial I_T}{\partial V} + 4GT$$

C. J. Wang and D. E. Feldman, arXiv:11042878

elated result in an exactly solvable model in a different
eometry: C. L. Kane and Matthew P. A. Fisher, Phys. Rev. Lett



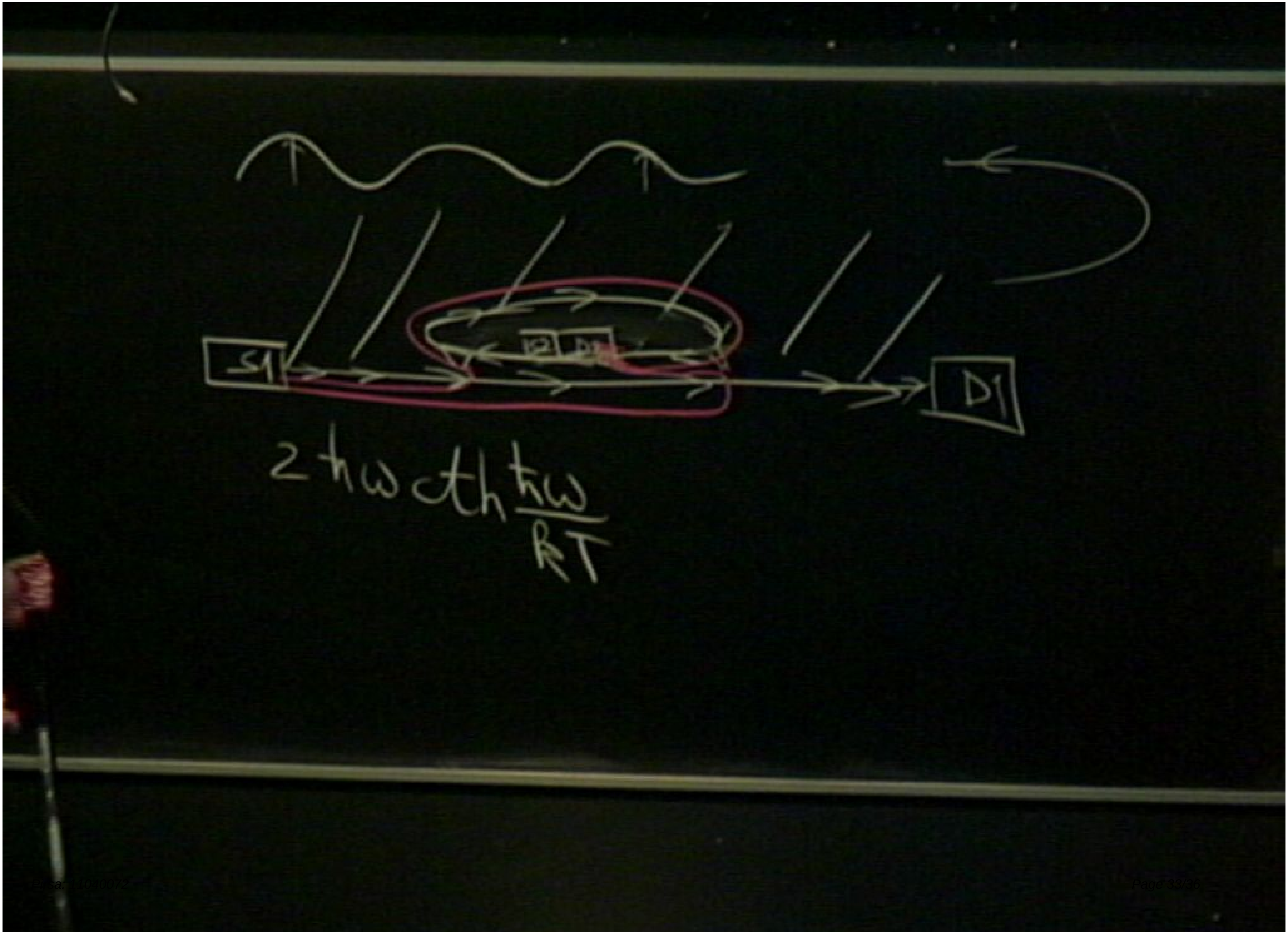
$$S_D = S_1 + S_2; \quad S_1 = 2GT;$$

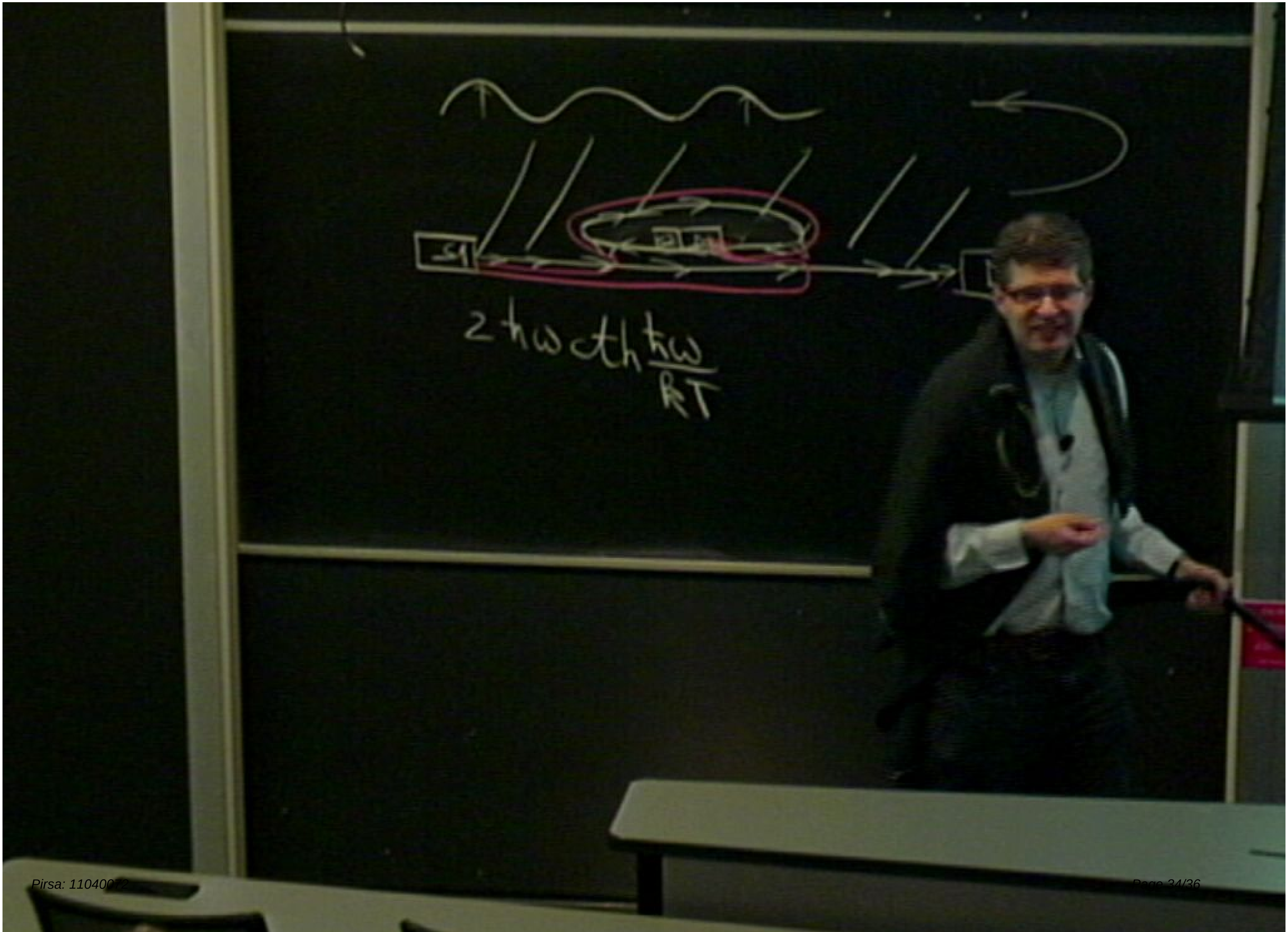
$$S_2 = \left\langle (I_S - I_T)^2 \right\rangle = S_S + S_C - 2\langle I_S I_T \rangle$$

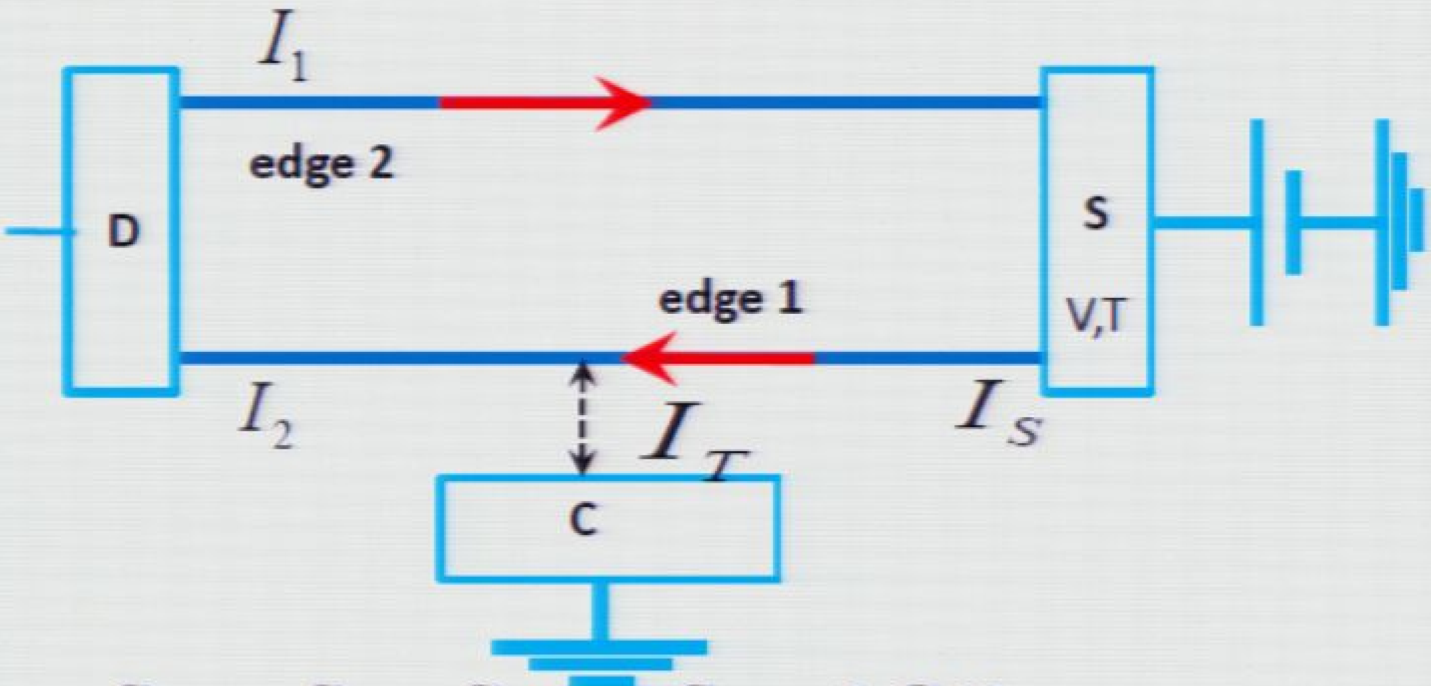
$$S_S = 2GT;$$

$$I_T = I(V, I_{S<}, I_{S>}) = I(V + I_{S<} / G, I_{S>}) \to$$

$$\to I(V + I_{S<} / G);$$


S1
D1
2 ħω cth ħω/kT


2 ħω cth ħω
kT



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$$S_2 = \left\langle (I_S - I_T)^2 \right\rangle = S_S + S_C - 2\langle I_S I_T \rangle$$

$$S_S = 2GT;$$

$$I_T = I(V, I_{S<}, I_{S>}) = I(V + I_{S<}/G, I_{S>}) \rightarrow$$

$$\rightarrow I(V + I_{S<}/G);$$

Summary

- Non-Abelian anyons may exists in the 5/2 QHE state
- Mach-Zehnder interferometer can distinguish different proposed states
- Charge-statistics separation makes it possible to probe states through tunneling
- Far-from-equilibrium FDT for chiral systems
- The smallest particles in the Universe!