Title: Entanglement and quantum noise: Is it possible to measure entanglement entropies?

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Abstract: I will discuss the growth of entanglement under a quantum quench at point contacts of simple fractional quantum Hall fluids and its relation with the measurement of local observables. Recently Klich and Levitov recently proposed that, for a free fermion system, the noise generated from a local quantum quench provides a measure of the entanglement entropy. In this work, I will examine the validity of this proposal in the context of a strongly interacting system, the Laughlin FQH states. We find that local quenching in fractional quantum Hall junctions gives time dependent correlation functions that have universal behavior on sufficiently long time and length scales. The growth of entanglement entropy and the noise generated by the quench are generally unrelated quantities.

Entanglement and quantum noise: Is it possible to measure entanglement entropies?

Colloquium at the Perimeter Institute, Waterloo, Ontario (Canada), April 27, 2011

Eduardo Fradkin

Department of Physics
University of Illinois at Urbana Champaign

April 25, 2011

Collaborators and References

- Benjamin Hsu
- Eytan Grosfeld
- Benjamin Hsu, Eytan Grosfeld, and Eduardo Fradkin, *Quantum noise and entanglement generated by a local quantum quench*, Phys. Rev. B **80**, 235412 (2009), arXiv:0908.2622.
- Eduardo Fradkin, *Scaling of Entanglement Entropy at 2D quantum Lifshitz fixed points and topological fluids*, Journal of Physics A: Mathematical and Theoretical **42**, 504011 (2009), (special issue on Entanglement Entropy, P. Calabrese, J. Cardy and B. Doyon, editors); arXiv:0906.1569v1.

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Density Matrix and Entanglement Entropy

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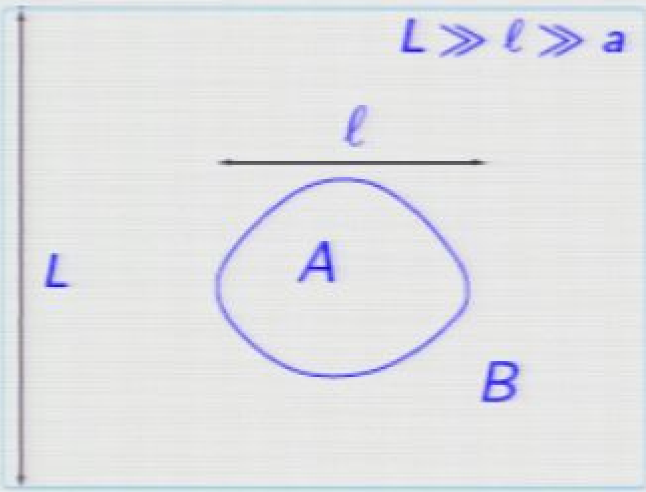

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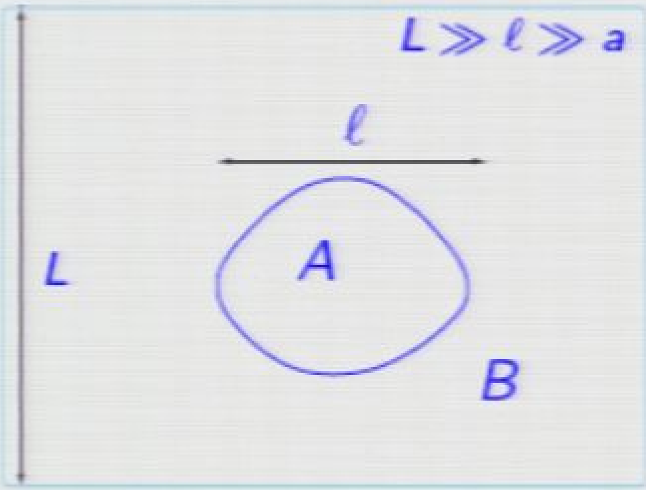

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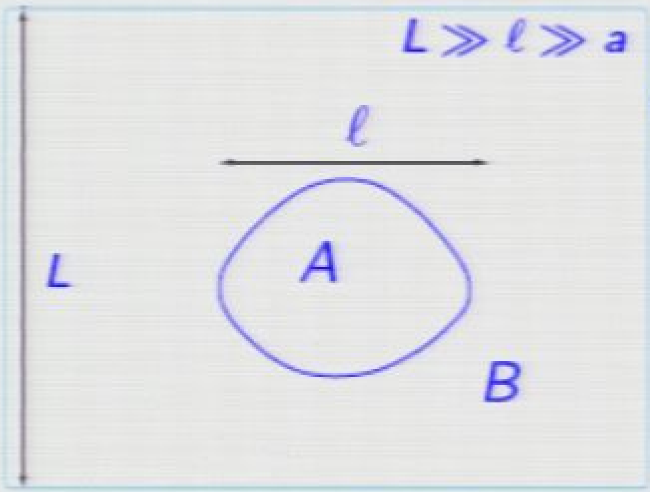

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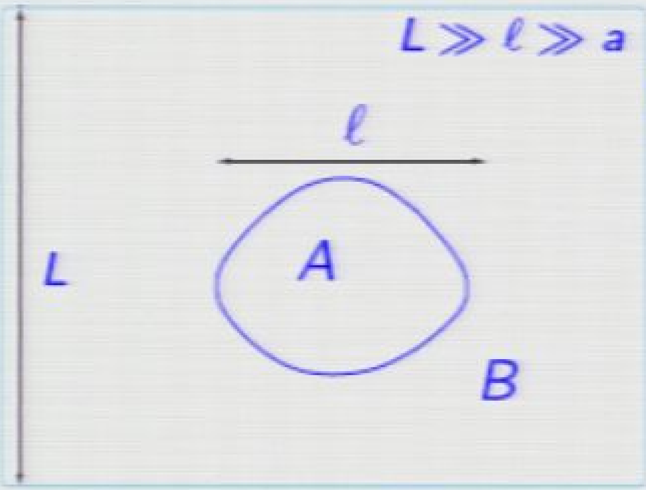

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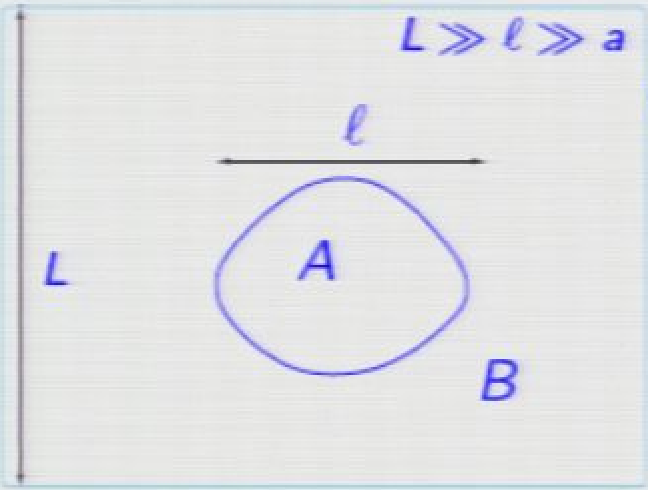

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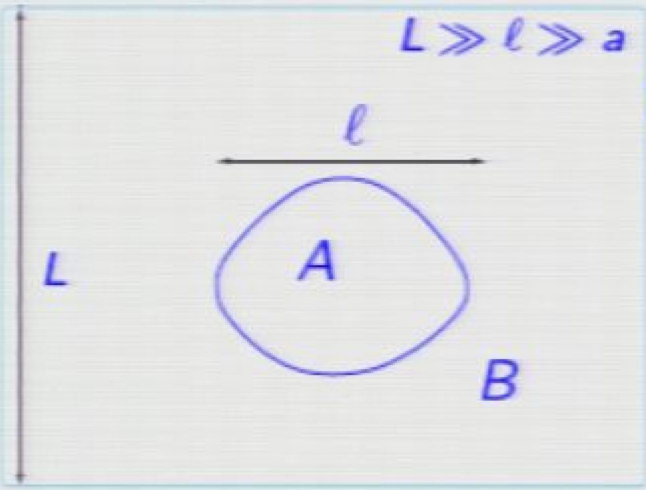

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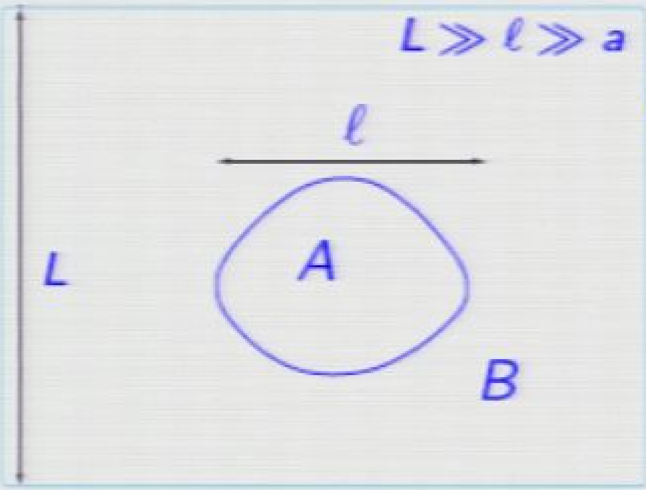

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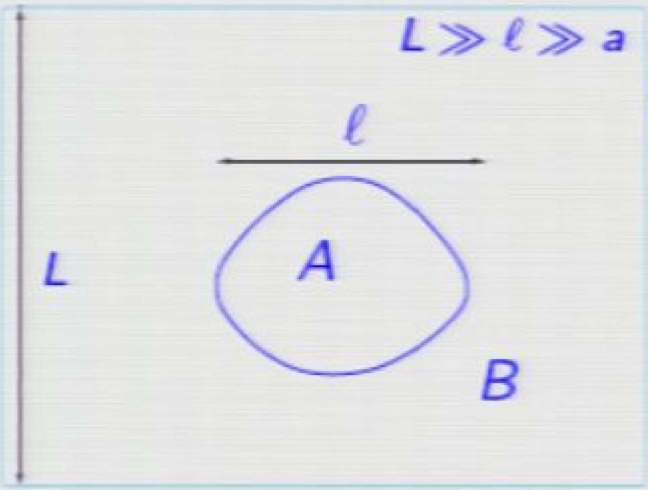

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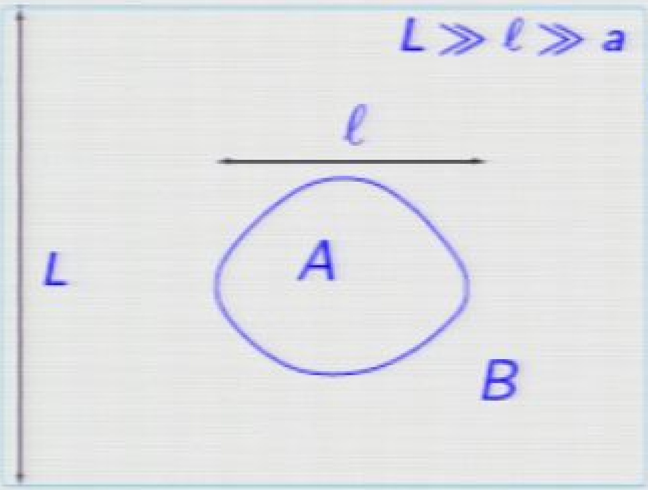

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- D>1:*Universal* $O(1)$ terms at QCP (Fradkin and Moore; Hsu, Mulligan, Fradkin and Kim; Metlitski and Sachdev; Hsu and Fradkin; Stéphane, Misguich and Pasquier)

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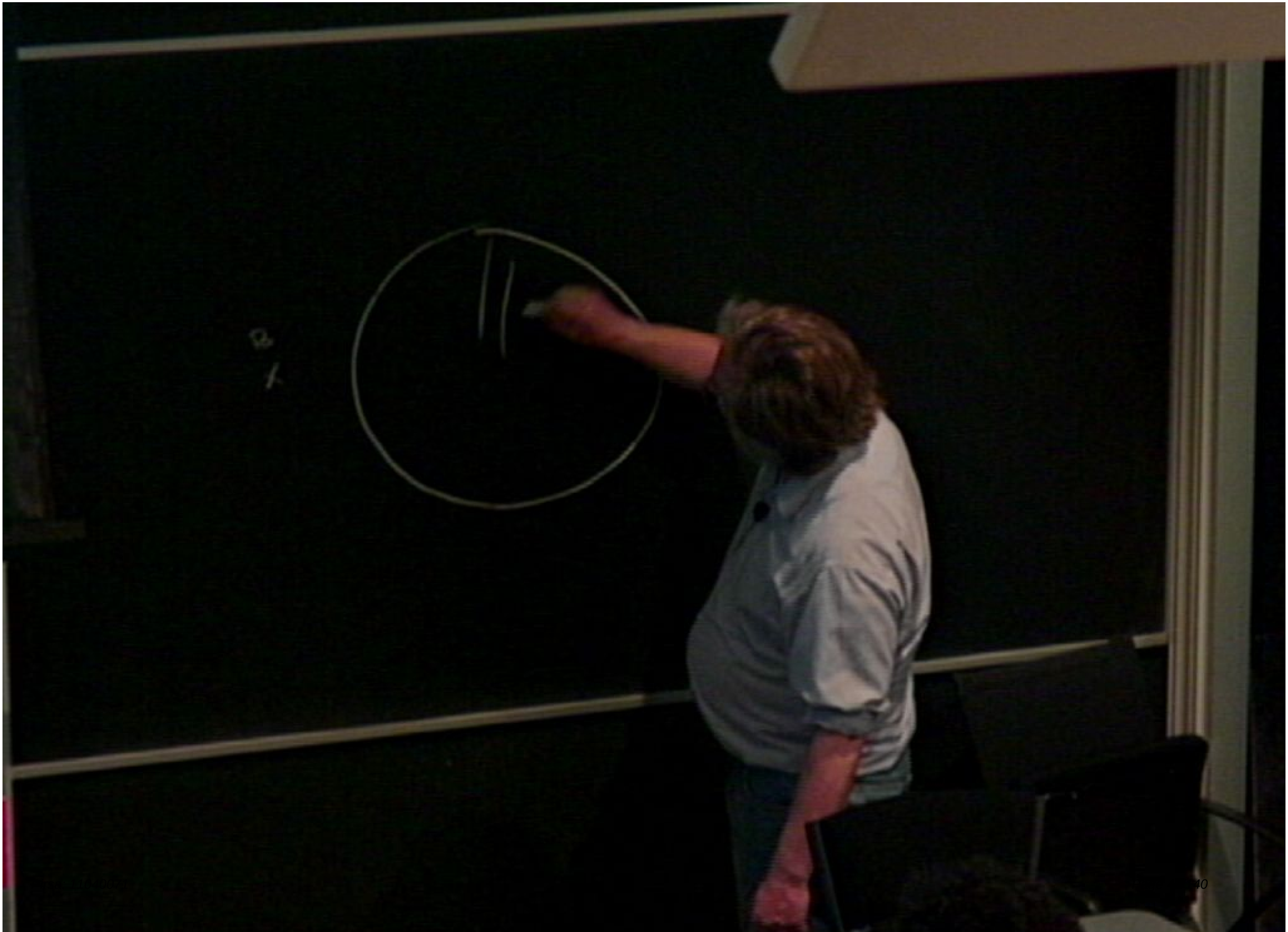

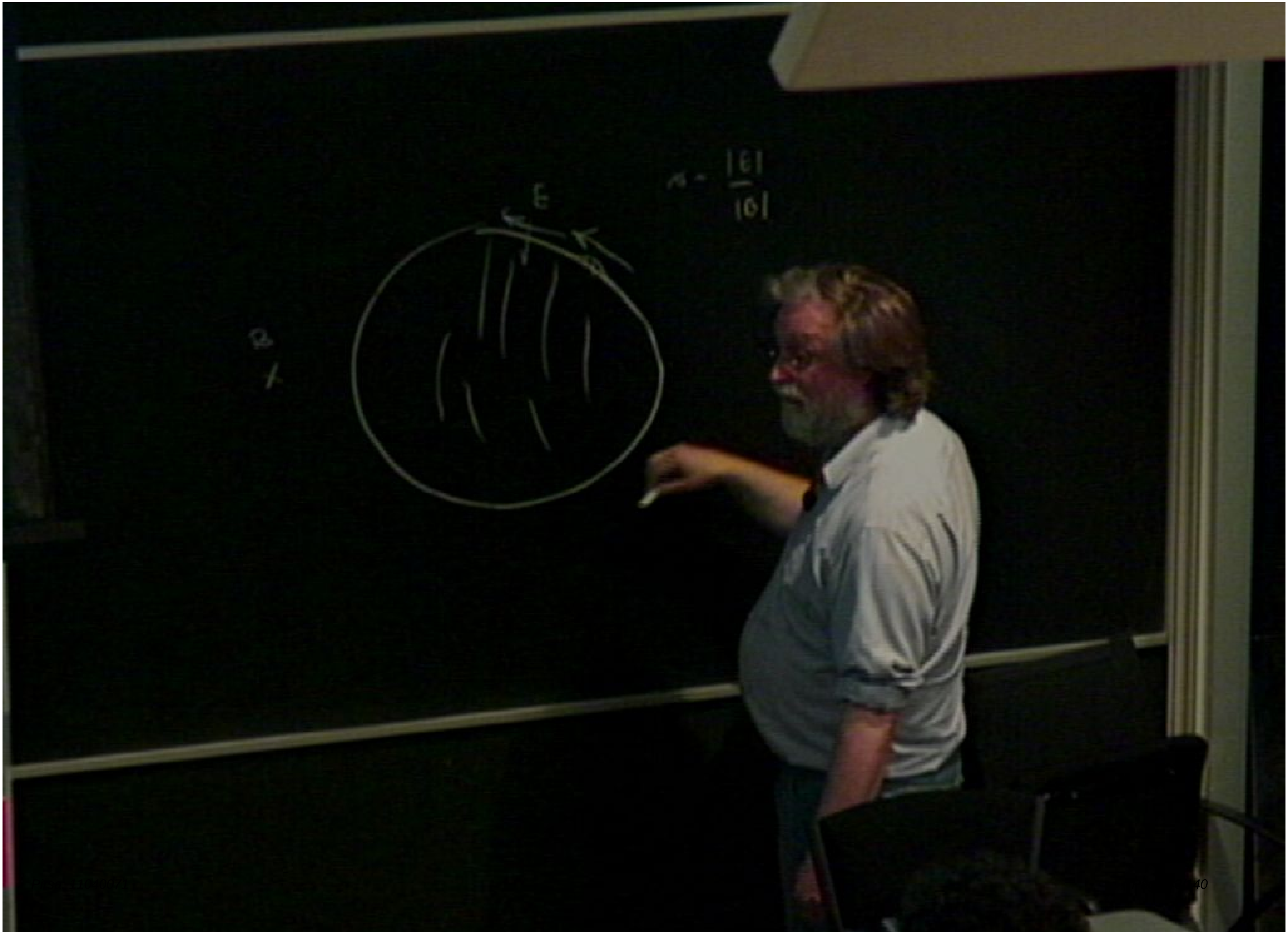
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The excitations are vortices with fractional charge $q = e/m$ and fractional (braid) statistics $\theta = \pi/m$.

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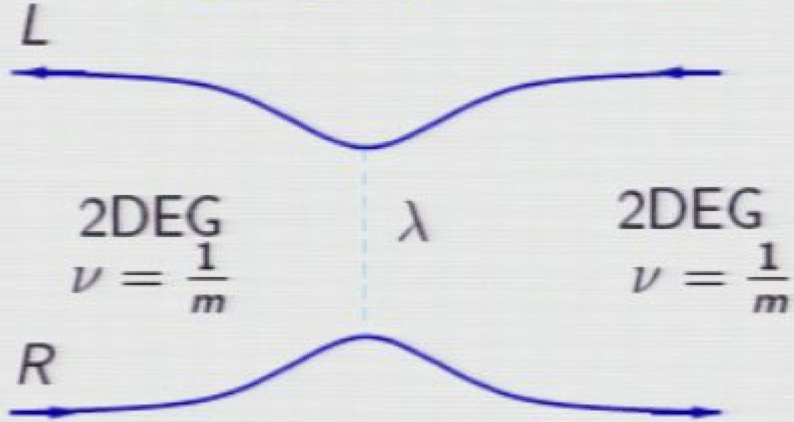

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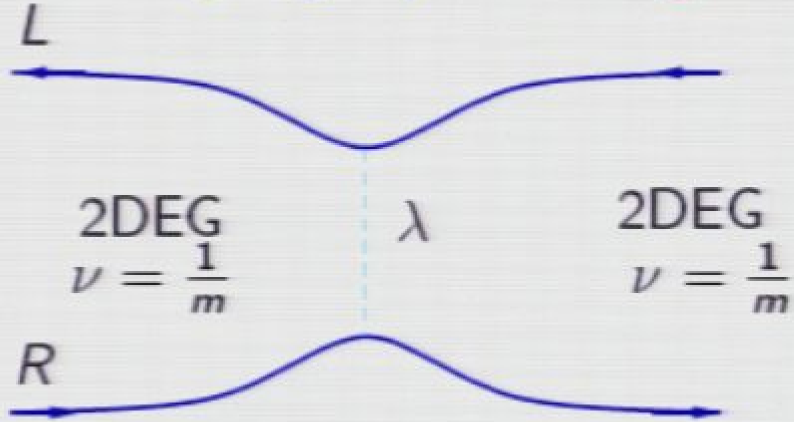

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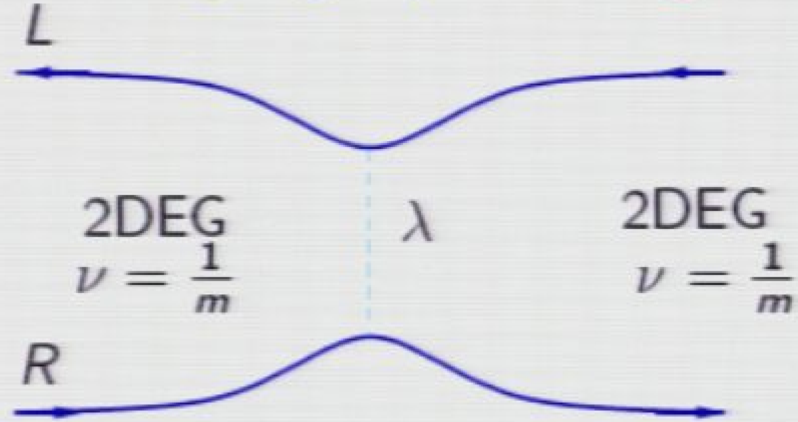

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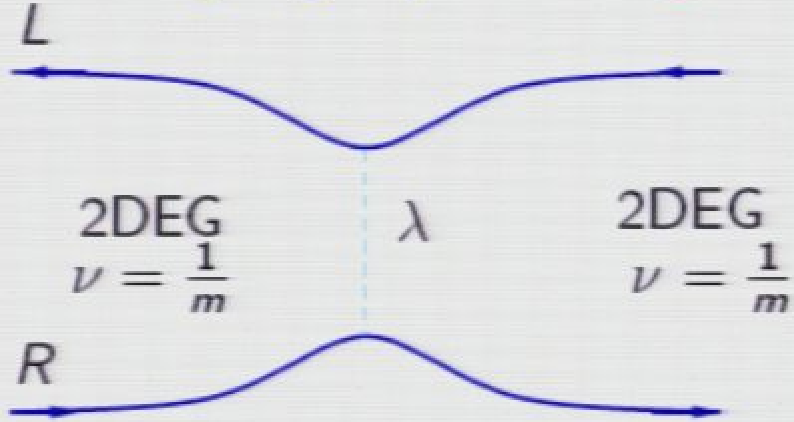

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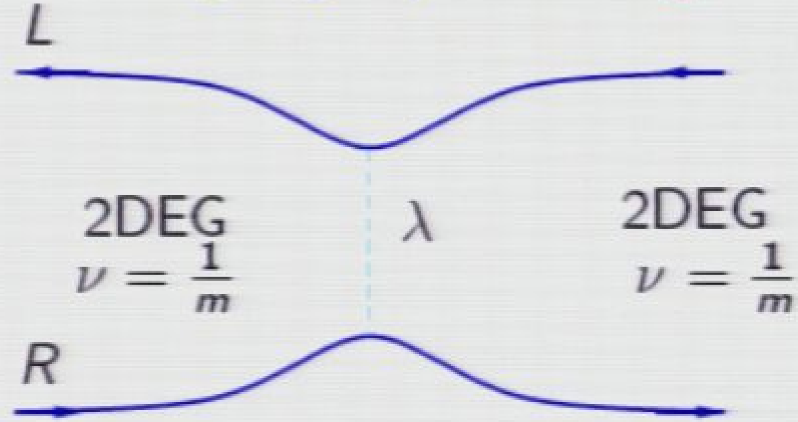

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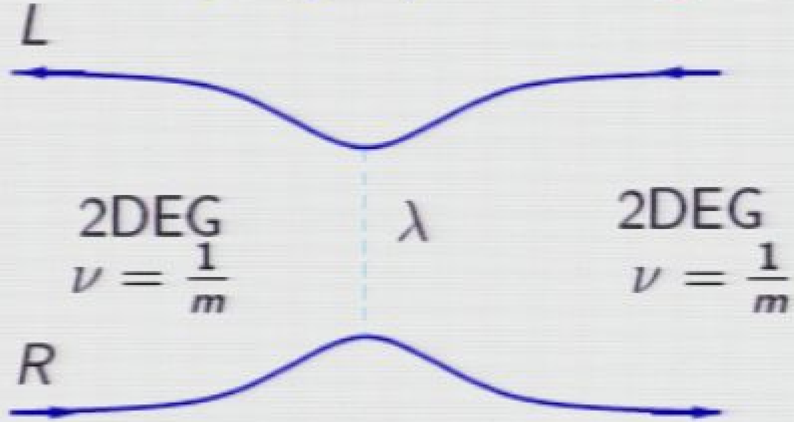

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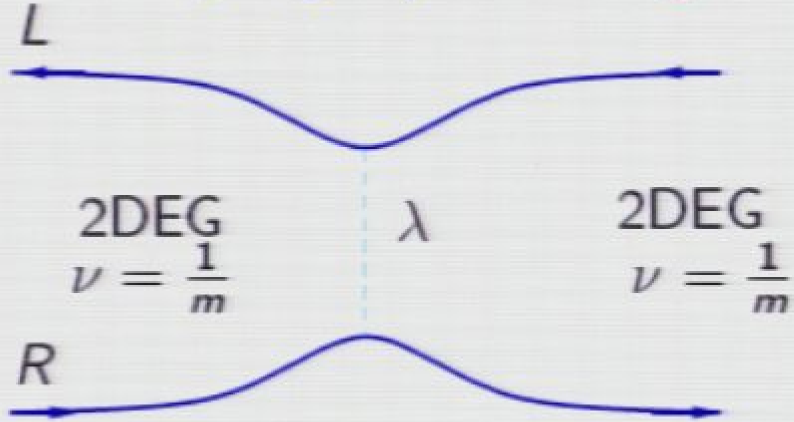

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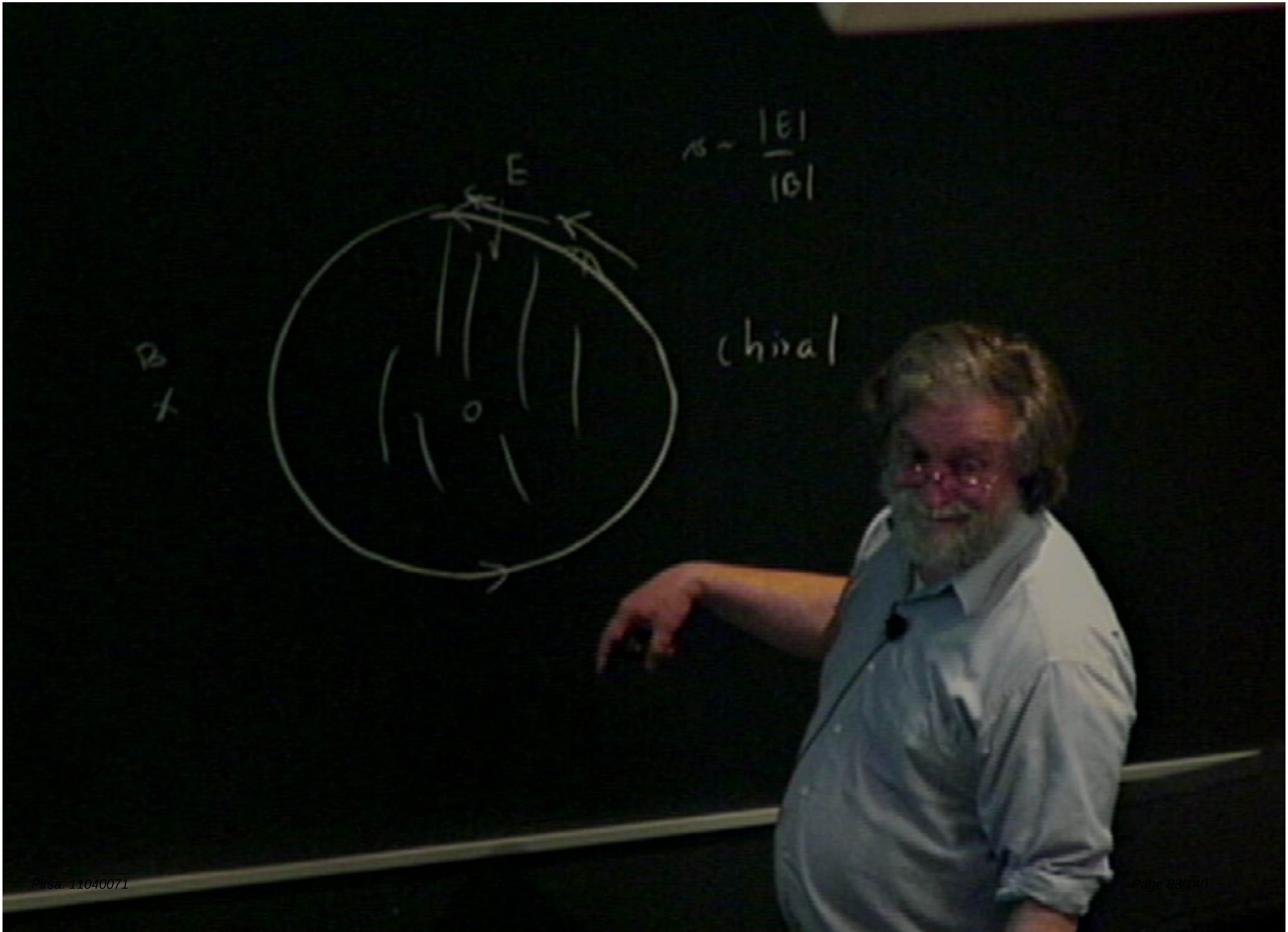
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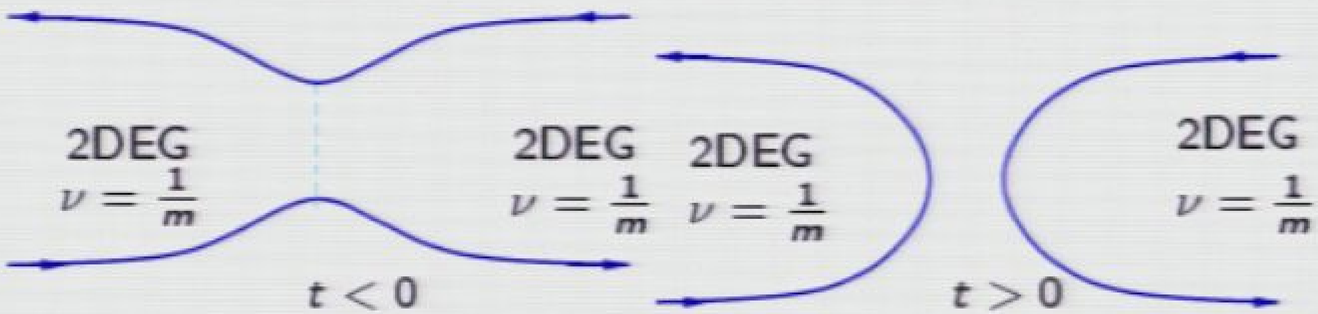

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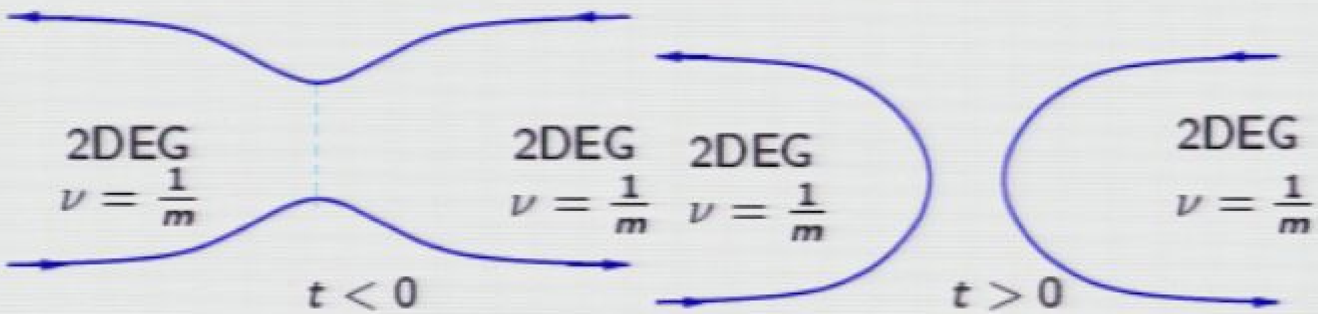

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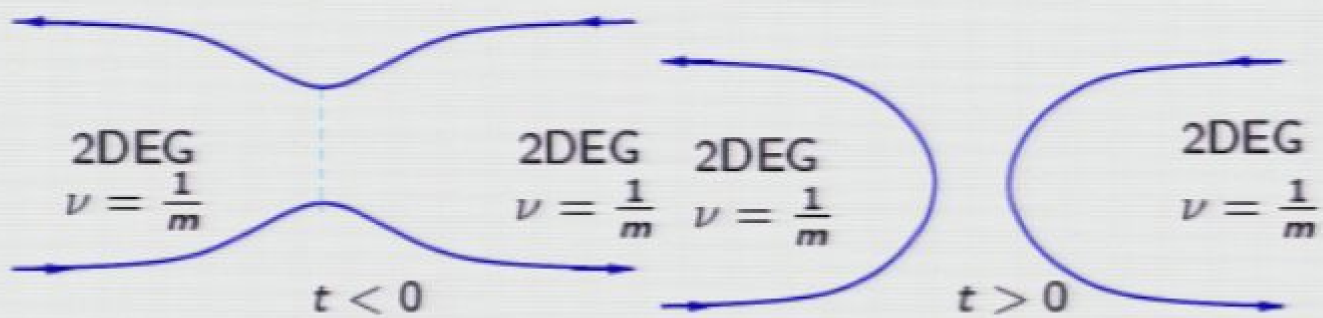

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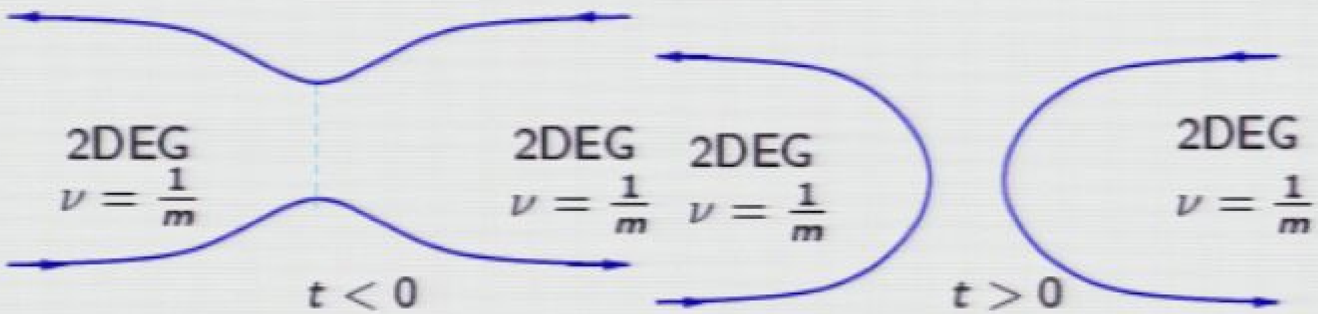

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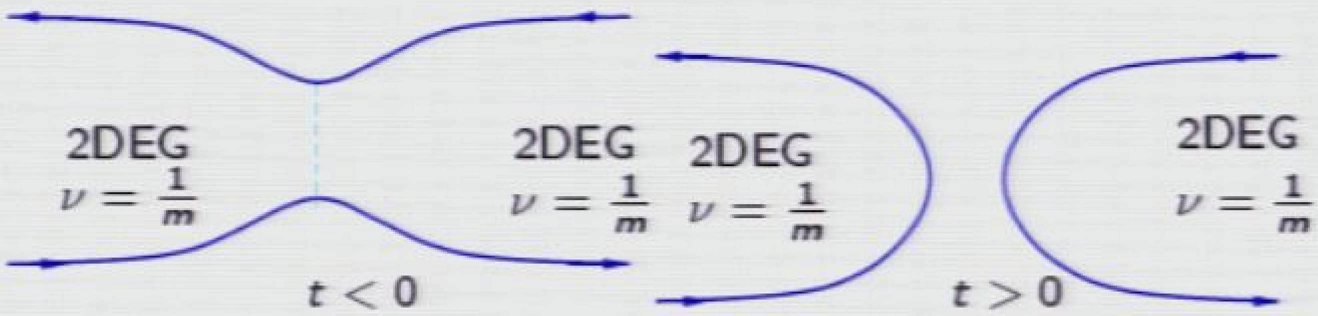

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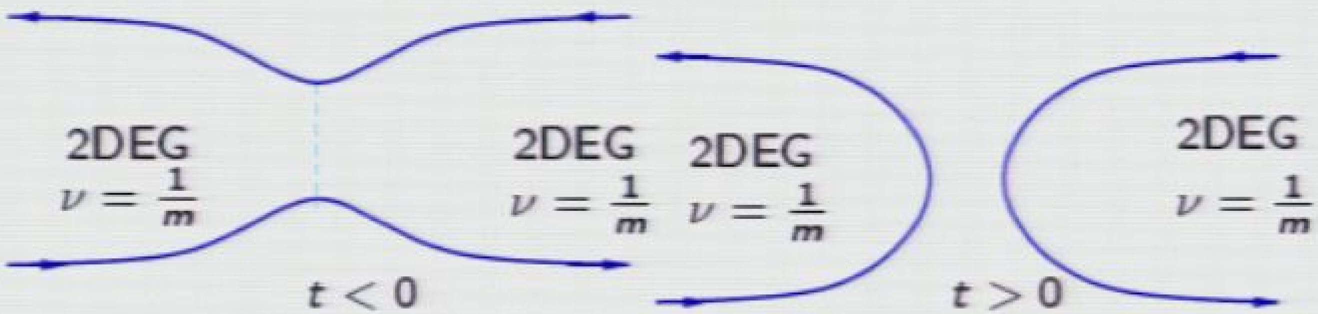

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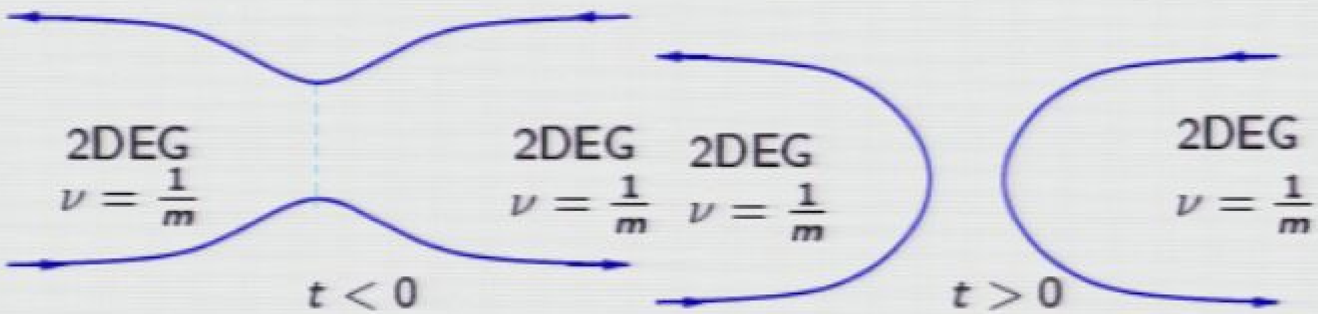

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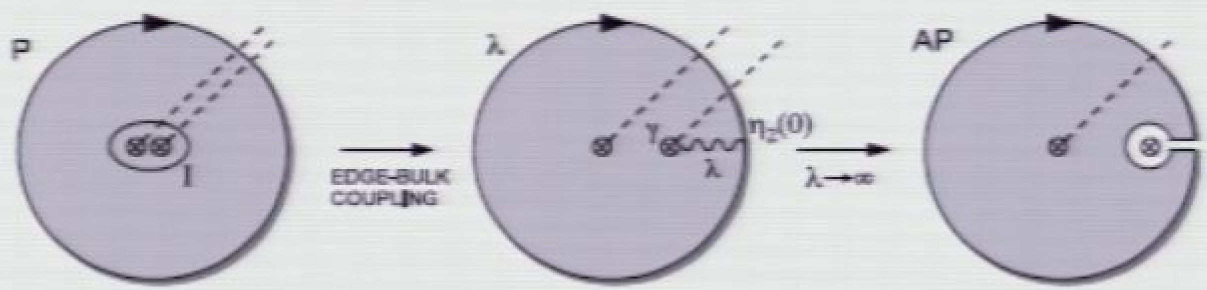

Two σ_2 operators are drawn from the vacuum. Tunneling (of strength λ) is introduced between one of the σ_2 operators and the edge. Finally, in the limit $\lambda \to \infty$, the edge circumvents the σ_2 operator.

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- We also discussed the time dependence of the current noise generated by the quantum quench
- The entanglement entropy computed here describes the evolution of excitations, not the entanglement of the ground state
- In the case of the noise this follows from the scaling dimension of the current (1) while in the entanglement entropy it does not (it follows from the conformal structure)
- Thus entanglement entropy and noise have the same (logarithmic) behavior but for very different reasons
- We checked this conclusion in the case of an (honest-to god!) Ising chain in which the entanglement still grows logarithmically but the noise (of the energy current) decays as a power law