Title: Explorations in Numerical Relativity - Lecture 14

Date: Apr 21, 2011 11:30 AM

URL: http://pirsa.org/11040057

Abstract:

1. Evolution eqns.

(4D) GH . BSSN . (3+1, ADM)

$\{\alpha, \beta^i, g_{ij}\}$

g_{ab}.

$\Box X_b = H_b$

$\partial_t \alpha - \beta^i \partial_i \alpha = -\alpha [H_t - \beta^i H_i + \alpha K]$

→ Evoln eqn. $\partial_t \partial_t g_{ab} = \cdots$

Initial Data

[Boundary data]

ADM formulation.

(ADM)

- $\gamma_{ij,t} = \ldots$
- $K_{ij,t} = \ldots$

GH . BSSN · (3+1, ADM)

$\{\alpha, \beta^i, g_{ij}\}$

$\partial_i \alpha = -\alpha [H_t - \beta^i H_i + \alpha K]$

$\partial_t \partial_t g_{ab} = \cdots$

(ADM)

- $\gamma_{ij,t} = \cdots$
- $K_{ij,t} = \cdots$

$\rho = T_{ab} n^a n^b$

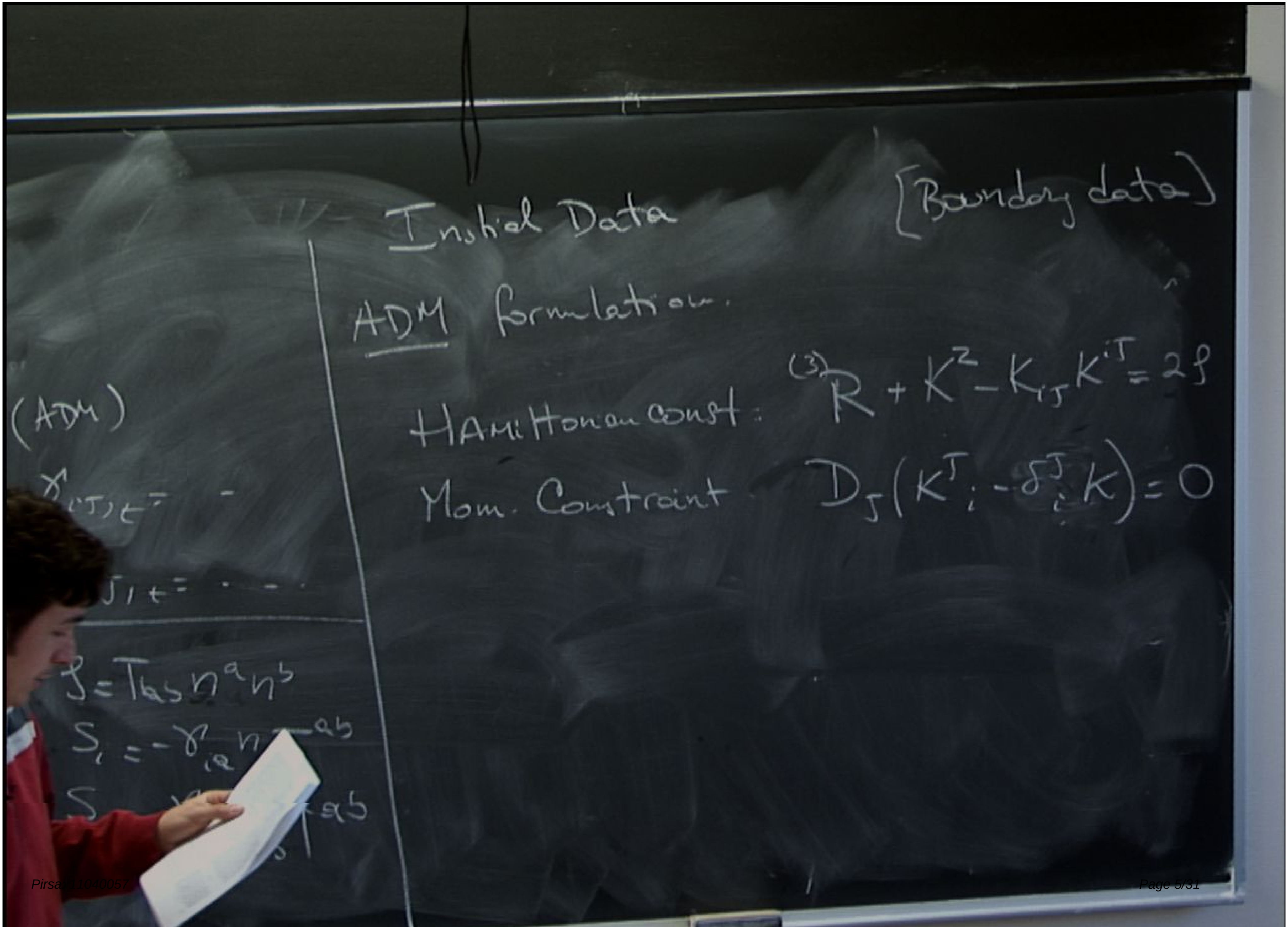
$S_i = -\gamma_{ia} n_b T^{ab}$

$S_{ij} = \gamma_{ia} \gamma_{jb} T^{ab}$

Initial Data

ADM formulation

Hamiltonian con


Initial Data
[Boundary data]
ADM formulation.
Hamiltonian const: $^{(3)}R + K^2 - K_{ij}K^{ij} = 2\rho$
Mom. Constraint $D_j(K^j{}_i - \delta^j_i K) = 0$
(ADM)
$\rho = T_{ab}n^a n^b$

(ADM)

$-\ \gamma_{ij,t} = \ldots$

$-\ K_{ij,t} = \ldots$

$$\rho = T_{ab} n^a n^b$$
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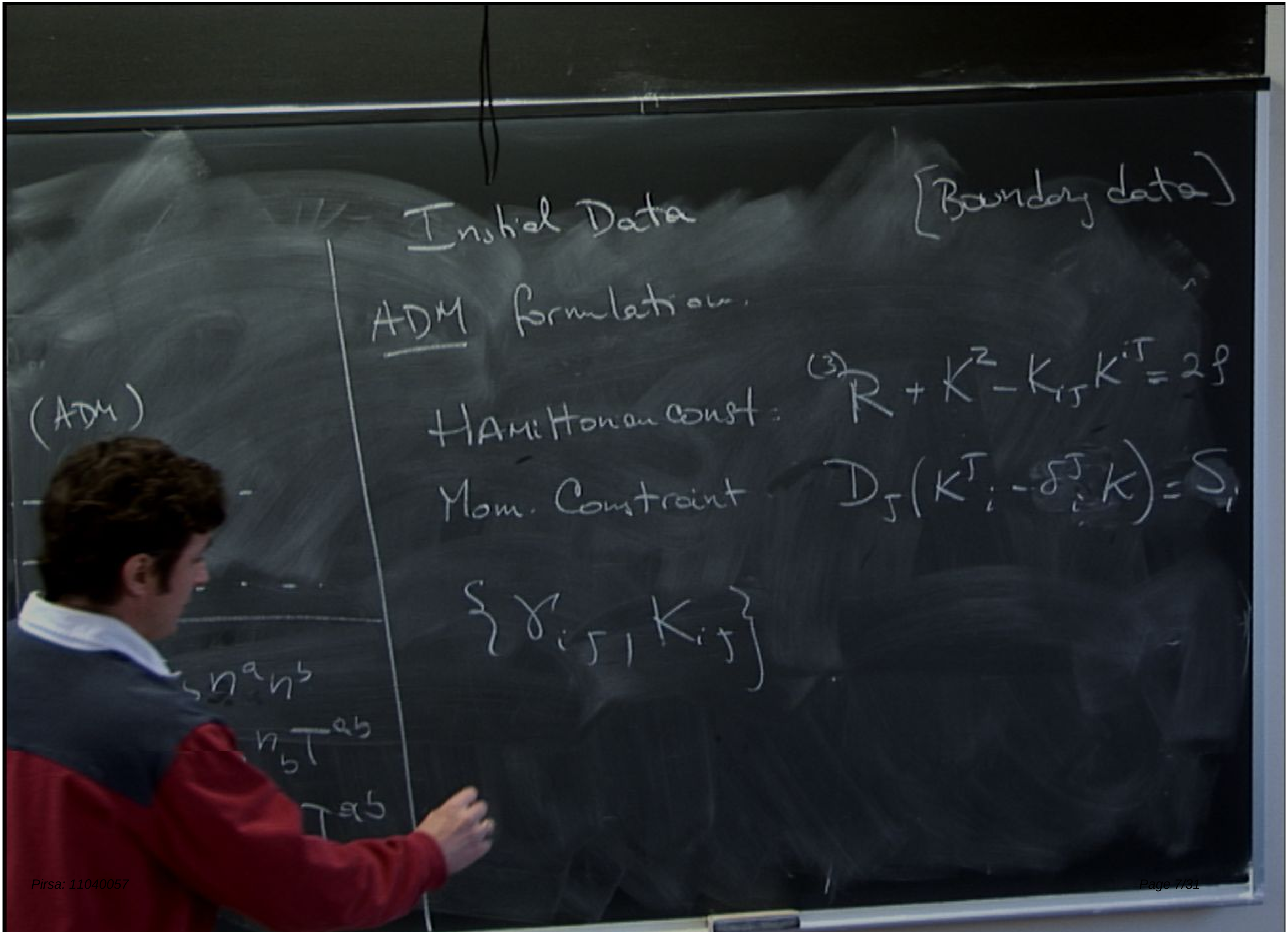
$$\{\gamma_{ij}, K_{ij}\}$$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$\{\gamma_{ij}, K_{ij}\}$

(ADM)

$- \gamma_{ij,t} = \ldots$

$- K_{ij,t} = \ldots$

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Initial Data [Boundary data]

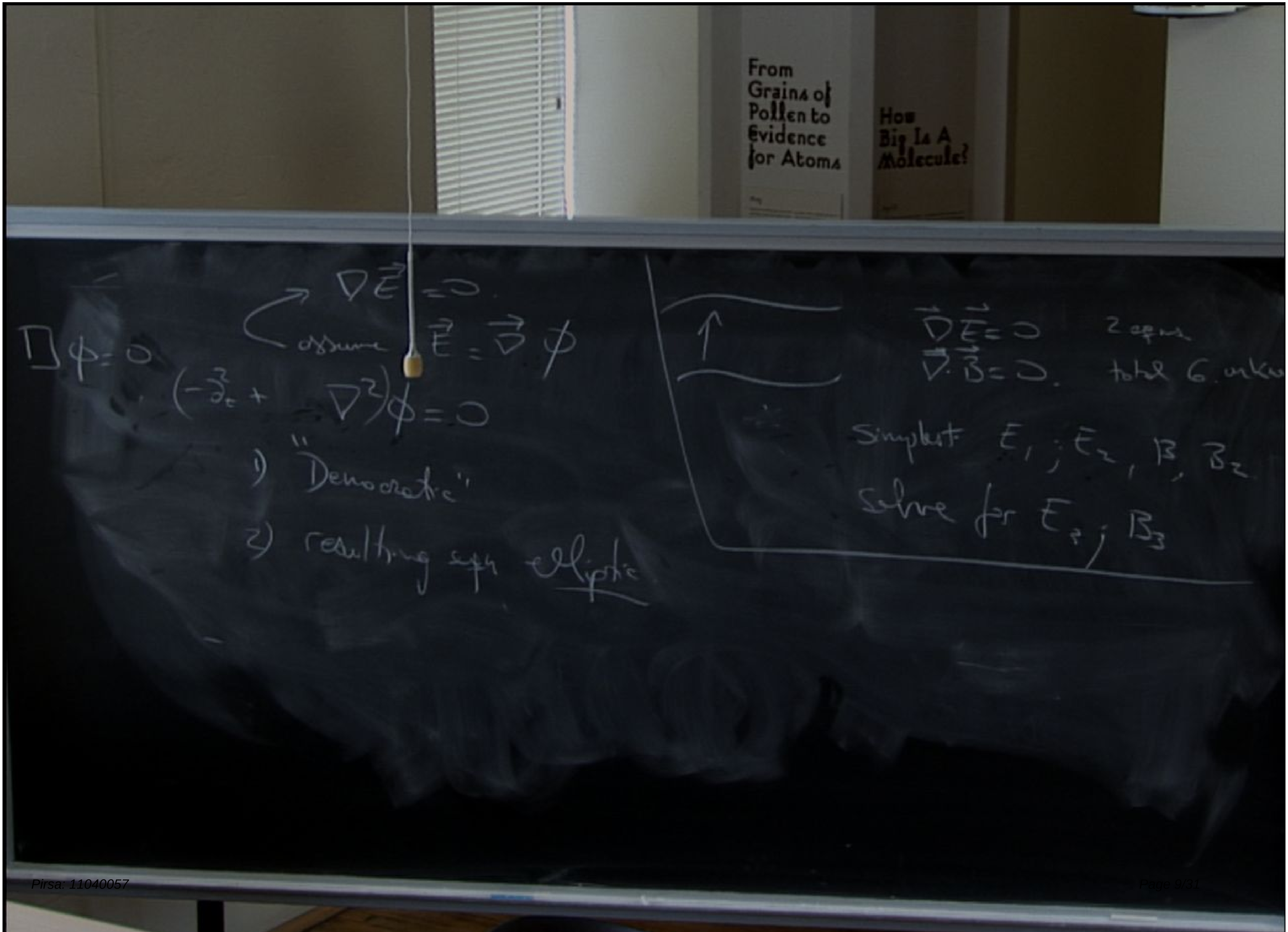
ADM formulation.

HAMiltonian const: ${}^{(3)}R + K^2 - K_{ij}K^{ij} = 2\rho$

Mom. Constraint $D_j(K^j{}_i - \delta^j_i K) = S_i$

$\{\gamma_{ij}, K_{ij}\}$ for 4 eqns.

12 pieces of info.


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\nabla \vec{E} = 0$
$\Box \phi = 0$
assume $\vec{E} = \vec{\nabla} \phi$
$(-\partial_t^2 + \nabla^2)\phi = 0$
1) "Democratic"
2) resulting eqn elliptic
$\vec{\nabla} \vec{E} = 0$ 2 eqns.
$\vec{\nabla} \cdot \vec{B} = 0$. total 6 unkn
Simplest: E_1; E_2, B, B_2
Solve for E_3; B_3

stationarity $\Rightarrow K_{ij} = 0$

H.C. $\Rightarrow {}^{(3)}R = 0$

Assume $\gamma_{ij} = \phi^{2} \eta_{ij}$

Soln is conformally flat.

stationarity $\Longrightarrow K_{ij} = 0$

H.C. $\Rightarrow {}^{(3)}R = 0$

Assume: $\gamma_{ij} = \phi^2 \eta_{ij}$

soln is conformally flat.

$$\nabla^2 \phi = 0$$

stationarity $\Rightarrow K_{ij} = 0$

H.C. $\Rightarrow {}^{(3)}R = 0$

Assume: $\gamma_{ij} = \phi^2 \eta_{ij}$

Soln is conformally flat.

$\nabla^2 \phi = 0$

$\phi \to 1$

$O\phi$

Kerr BH

stationarity $\rightarrow K_{ij} = 0$

H.C. $\Rightarrow {}^{(3)}R = 0$

Assume: $\gamma_{ij} = \phi^2 \eta_{ij}$

soln is conformally flat.

$$\nabla^2 \phi = 0$$

$\phi \rightarrow 1$

$O\phi$

Kerr BH $\quad K_{ij} = 0 \;;\; \gamma_{ij} = \phi^2 \eta_{ij}$

"perturbed Kerr

$K_{ij} = 0$

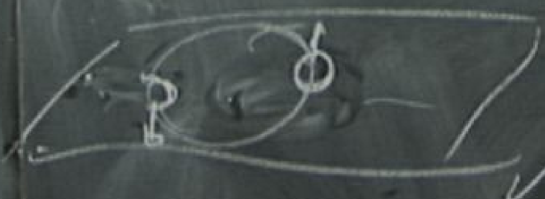
$R = 0$

$\gamma_{ij} = \phi^4 \eta_{ij}$

$\nabla^2 \phi = 0$

$K_{ij} = 0$; $\boxed{\gamma'_{ij} = \phi^2 \eta_{ij}}$

$\phi \to 1$



"perturbed Kerr"

Initial Data

ADM formulation.

Hamiltonian const:

Mom. Constraint

$\{\gamma_{ij}, K_{ij}\}$

12 pieces of in

known

$$K_{ij} = \tilde{K}_{ij} + D_{(i}\omega_{j)}$$

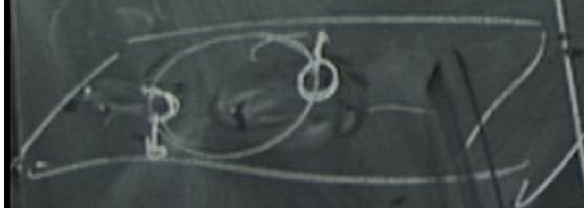
Mom. Const. $D^2 \omega_j = \ldots$

H. const $D^2 \phi = \ldots$

$$\gamma_{ij} = \phi^2 \eta_{ij}$$ known

$$D^2 \phi$$

$\phi \to 1$



"perturbed Kerr"

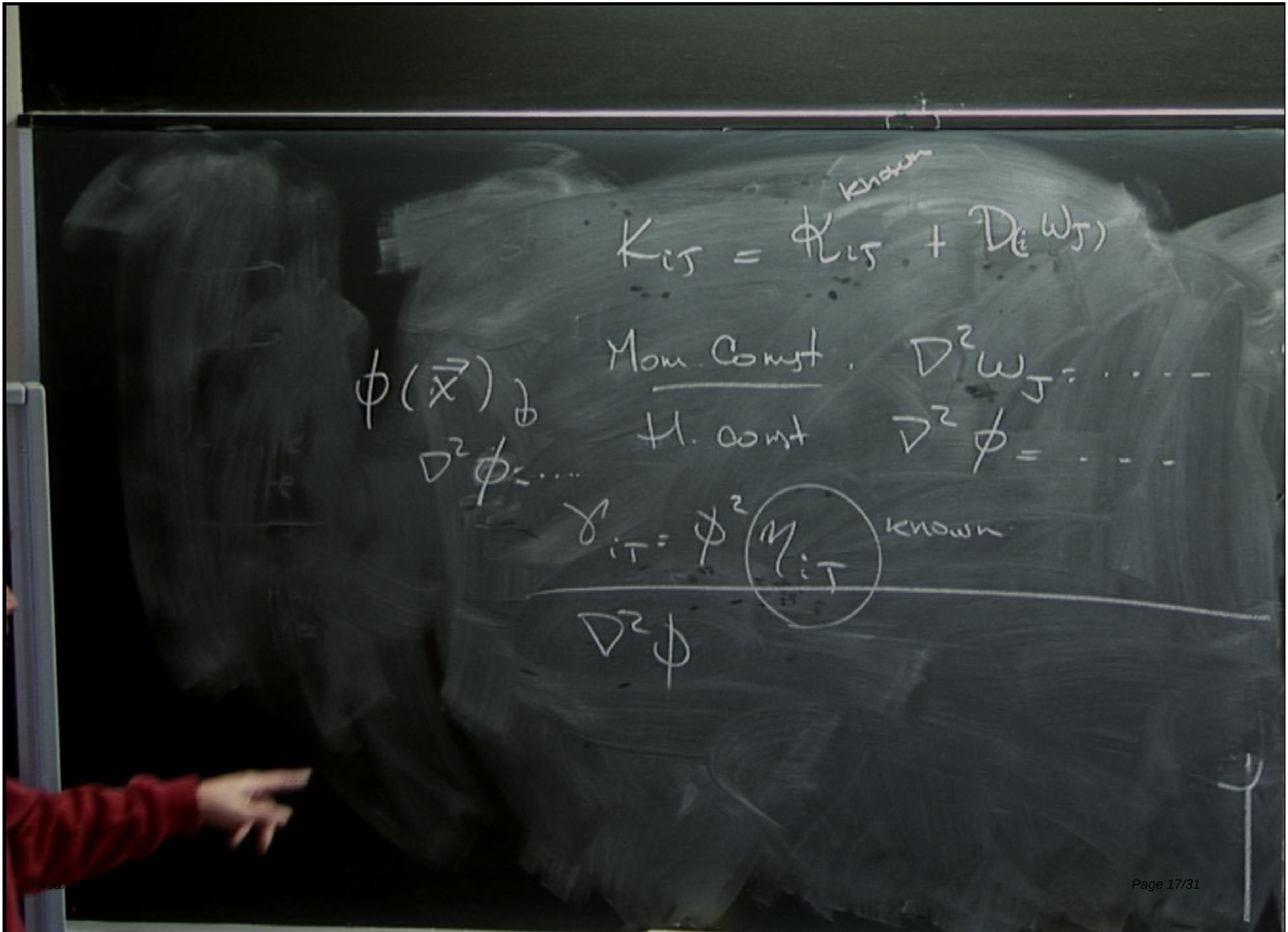
$$\partial_t \psi = \tilde{\kappa} \nabla^2 \psi \xrightarrow{1D} \tilde{\kappa}\, \partial_x^2 \psi$$

Fourier. $\psi = e^{st}\, e^{ikx}$

$$\partial_t\left(e^{st} e^{ikx}\right) = \tilde{\kappa}\, (ik)^2\, e^{st}\, e^{ikx}$$

$$\hookrightarrow \quad s = \tilde{\kappa}\,(-)k^2$$

$$\psi = e^{-\tilde{\kappa} k^2 t}\, e^{ikx}$$


known
$K_{ij} = \tilde{K}_{ij} + D_{(i}\omega_{j)}$
Mom. Const. $D^2\omega_j = \ldots$
H. const $D^2\phi = \ldots$
$\phi(\vec{x})$
$D^2\phi = \ldots$
$\gamma_{ij} = \phi^2 \eta_{ij}$ known
$D^2\phi$

$\vec{x})$

$K_{ij} = \overset{\text{known}}{\mathcal{K}_{ij}} + D_{(i} \omega_{j)}$

$\nabla^2 \psi = 0$

Mom. Const. $\quad D^2 \omega_j = \dots$

H. const $\quad D^2 \phi = \dots$

$\phi = S$

$\phi \to 1$

Farrar

$= \phi^2 \eta_{ij}$ known

$\nabla^2 \phi - S') = \partial_\lambda \phi$

$d\lambda \sim dx$ (hyperb)

$\quad\quad dx^2$ (parabolic)

$\phi(\lambda, \vec{x})$

$\uparrow$ promote

$\phi(\vec{x})$

$\boxed{D^2 \phi = S}$

$$K_{ij} = \overset{\text{known}}{A_{ij}} + D_{(i} W_{j)}$$

Mom. Const. $D^2 W_j = \ldots$

H. const $D^2 \phi = \ldots$

$\gamma_{ij} = \phi^2 \eta_{ij}$ known

$$\left(D^2 \phi - S\right) = \partial_\lambda \phi \qquad d\lambda \sim dx, \; dx^2$$

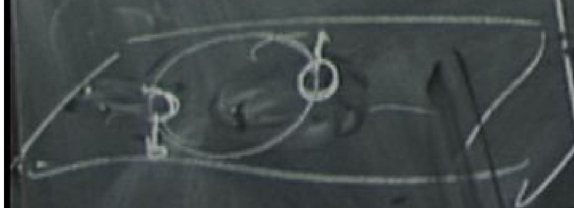
$\phi \to 1$

$$\nabla^2 \psi = 0$$

1D

$$\partial_t \psi = \tilde{k} \nabla^2 \psi \Rightarrow \tilde{k}\, \partial_x^2 \psi$$

$\phi \to 1$



<u>Fourier</u>. $\psi = e^{st}\, e^{ikx}$

$$\partial_t\left(e^{st} e^{ikx}\right) = \tilde{k}\, (ik)^2\, e^{st} e^{ikx}$$

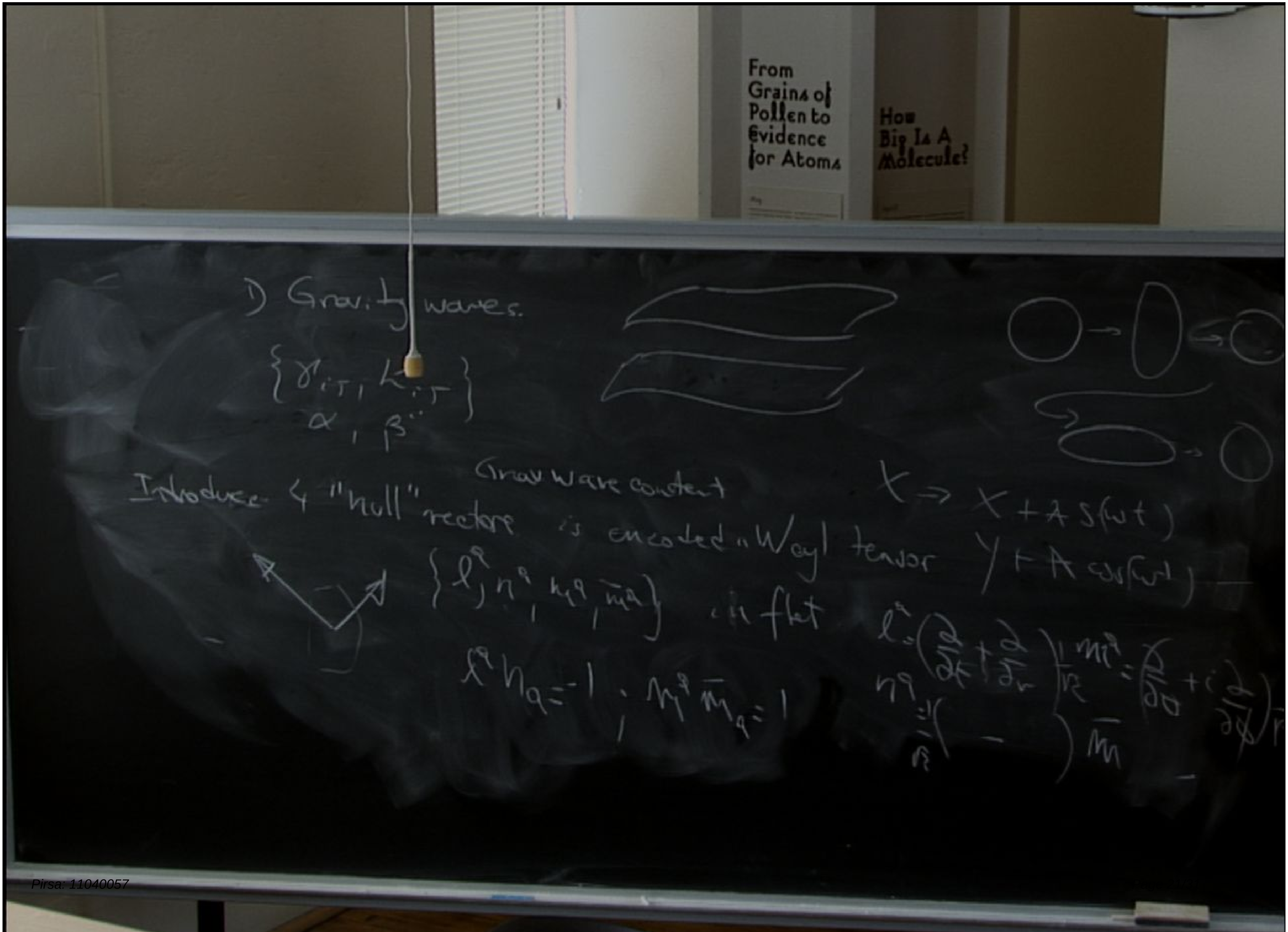
$\hookrightarrow \quad s = \tilde{k}\,(-1)k^2$ $\qquad$ $\underline{\tilde{k} > 0}$

$$\psi = e^{-\tilde{k} k^2 t}\, e^{ikx}$$

dx (hyperb)

dx^2 (parabolic)

<u>Multigrid</u>



From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
1) Gravity waves.
{γ_ij, h_ij, α, β}
Grav wave content
Introduce 4 "null" vectors is encoded in Weyl tensor
{l^a, n^a, m^a, m̄^a} in flat
l^a n_a = -1 ; m^a m̄_a = 1
x → x + A sin(ωt)
y + A cos(ωt)
l^a = (∂/∂t + ∂/∂r) 1/√2
m^a = (∂/∂θ + i ∂/∂φ) 1/r
n^a = 1/√2 (−)
m̄

Newman Penrose

$$\Psi_0 = -C_{abcd}\, n^a m^b n^c m^d$$

$$\Psi_1 = -C_{abcd}\, n^a \ell^b n^c m^d \; \ldots$$

$$\vdots$$

$$\Psi_4 = -C_{abcd}\, \bar{m}^a \ell^b \bar{m}^c \ell^d$$

Newman Penrose

$$\Psi_0 \equiv -C_{abcd}\, n^a m^b n^c m^d$$

$$\Psi_1 = -C_{abcd}\, n^a \ell^b n^c m^d \ldots$$

$$\vdots$$

$$\Psi_4 = -C_{abcd}\, \bar{m}^a \ell^b \bar{m}^c \ell^d$$

$p = 0, 4$

$$\Psi_p = \mathcal{O}\left(\frac{1}{r^{-5+p}}\right)$$

$$\Psi_4 = \frac{1}{2}\left(\ddot{h}_+ - i\,\ddot{h}_\times\right) \rightarrow h_+ = \frac{1}{2}\left(h^{TT}_{\hat\theta\hat\theta} - h^{TT}_{\hat\phi\hat\phi}\right)$$

$$h_\times = h^{TT}_{\hat\theta\hat\phi}$$

$$\frac{dM}{dt}$$

$$g_{ab} = \eta_{ab} + h^{TT}_{ab}$$

$$\Box h^{TT}_{ab} = 0$$

$$\Psi_4 = \frac{1}{2}\left(\ddot{h}_+ - i\,\ddot{h}_\times\right) \to h_+ = \frac{1}{2}\left(h^{TT}_{\hat\theta\hat\theta} - h^{TT}_{\hat\phi\hat\phi}\right)$$

$$h_\times = h^{TT}_{\hat\theta\hat\phi}$$

$$\frac{dM}{dt} = \frac{1}{4\pi}\int_{\substack{\text{surface at}\\ r\to\infty}} \left|\int_0^t \Psi_4\, dt\right|^2 r^2\, d\Omega$$

$$g_{ab} = \eta_{ab} + h^{TT}_{ab}$$

$$\Box h^{TT}_{ab} = 0$$

$n^c m^d$

$m^d \ldots$

$\bar{m}^b \ell^d$

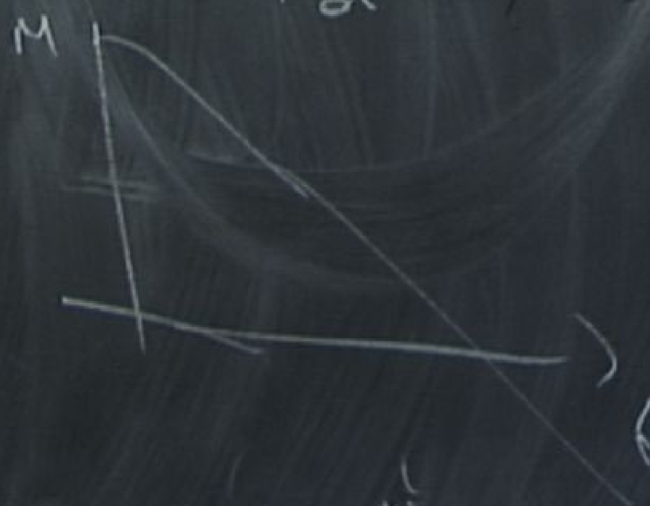
$$\Psi_4 = \frac{1}{2}\left(\ddot{h}_+ - i\,\ddot{h}_\times\right) \rightarrow h_+ = \frac{1}{2}($$

$$h_\times =$$

$$\frac{dM}{dt} = -\frac{1}{4\pi}\int_{\text{surface}\ r\to\infty} \left|\int_0^t \Psi_4\, dt\right|^2 r^2\, d\Omega$$

$$g_{ab} = \eta$$

$\Box\ell$



$u_{,t} = u_{,x}$

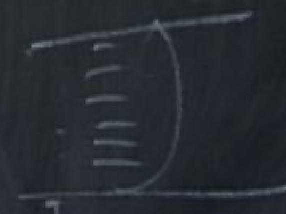
1) $u_{,x} \rightarrow \dfrac{u_{i+1} - u_{i-1}}{2\Delta x}$

Time Integ.

$u_{,t} = u_{,xx}$

Method of Lines

$$\frac{df}{dt} = F(f)$$



$u_{,t} = u_{,x}$

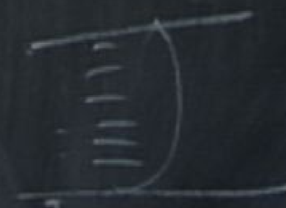
1) $u_{,x} \rightarrow \frac{u_{i+1} - u_{i-1}}{2\Delta x}$

Time Integ.

$t - u_{,xx}$

Method of Lines

$\frac{df}{dt} = F(f)$



$\partial_t u = (\partial_x u) f + u^2 \ldots$

$\partial_t u = F(u)$

 $e^{st} e^{ikx} = u$

$$\partial_t u = \partial_x u + \varepsilon(\Delta x)\,\partial_{xx} u$$

$$s = ik - \varepsilon k^2$$

$$e^{ik(t+x)} \cdot e^{-\varepsilon \Delta k^2 t}$$

Method of Lines

$$\frac{df}{dt} = F(f)$$

$$\partial_t u = (\partial_x u) f + u^2 \dots$$

$$\partial_t u = F(u)$$

$e^{st} e^{ikx} = u$

$$\partial_t u = \partial_x u + \varepsilon(\Delta x)\,\partial_{xx} u$$

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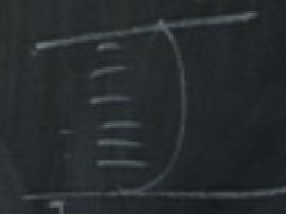
Method of Lines

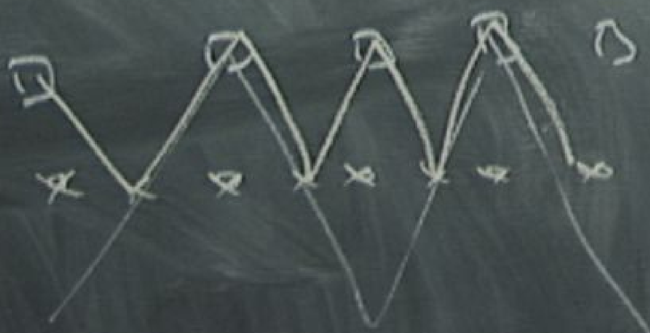
$$\frac{df}{dt} = F(f)$$



$$\partial_t u = (\partial_x u) f + u^2 \dots$$

$$\partial_t u = F(u)$$

$\det(\gamma_{\sigma\tau})$



$e^{st} e^{ikx} = u$

$\partial_t u = \partial_x u + \varepsilon (\Delta x)^6 \partial_{xxxxxx} u$

$s = ik - \varepsilon k^6$

$e^{ik(t+x)} \cdot e^{-\varepsilon \Delta k^6 t}$