

Title: Explorations in Numerical Relativity - Lecture 13

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Abstract:



perimeter scholars
international

$$R_{ab} = 0$$

$$\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bde})$$

(3+1) ADM vs 4D.

$$\dot{g}_{ij} = \dots$$

$$\dot{K}_{ij} = \dots$$

Hamilton const

Moment const

$$\alpha; \beta^i$$

$$\ddot{g}_{ab} = \dots$$

(3+1) ADM vs. 4D.

$$\dot{g}_{ij} = \dots$$

$$\dot{K}_{ij} = \dots$$

Hamilton const

Moment const

$$\alpha; \beta^i$$

$$\ddot{g}_{ab} = \dots$$

$$Q_{ab} = \dot{g}_{ab}$$

$$R_{ab} = \partial_a \Gamma_b - \partial_b \Gamma_a$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$R_{ab} = 0$$

$$= -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \dots)$$

$$\nabla_a \Gamma_b = 0 \Rightarrow \partial X_b = 0$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \dots)$$

$$\nabla_a \Gamma_b = 0 \Rightarrow \square x_b = 0$$

$$\nabla_a \Gamma_b = H_b \Rightarrow \square x_b = H_b$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \underline{\nabla_a \Gamma_b} + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \dots)$$

$$1) \quad \Gamma_a = 0 \Rightarrow \square X_b = 0$$

$$2) \quad \nabla_a H_b \quad \Gamma_b = +H_b \Rightarrow \square X_b = +H_b$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \partial_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bde})$$

$$1) \quad \delta \quad \Gamma_a = 0 \Rightarrow \square X_a = 0$$

$$2) \quad \partial_a H_b \quad \Gamma_b = +H_b \Rightarrow \square X_b = +H_b$$

$$3) \quad \text{Dynamical } H_b \quad \underline{\text{Define}}$$

$$g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

$$1) \quad \Gamma_k = 0 \Rightarrow \square X_b = 0$$

$$2) \quad \nabla_a H_b \quad \Gamma_b = +H_b \Rightarrow \square X_b = H_b$$

$$3) \quad \text{Dynamical } H_b \quad \Gamma_b = H_b \Rightarrow \square X_b = H_b \sin(x)$$

$$+ g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

$$\Gamma_k = 0 \Rightarrow \square X_5 = 0$$

$$\Gamma_b = H_b \Rightarrow \square X_5 = H_b$$

$$\square H_b$$

define

$$\partial_t H_b = v_b \sin(x)$$

$$\partial_t H_b = g^{ab} v_a$$

$$; \partial_a g_{ab}$$

Coord condit

1) Ha

2) "Ge

co

$$\square H_b$$

$$+ g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

$$\Gamma_a = 0 \Rightarrow \square X_a = 0$$

$$\Gamma_b = H_b \Rightarrow \square X_b = H_b$$

cool H_b

define

$$\partial_t H_b = \zeta_b \sin(x)$$

$$\partial_t H_b = g^{ab} \zeta_a$$

$$g^{ab} \square \square H_b$$

$$\partial_a g_{ab}$$

Coord. condit

1) H_a

2) "G"

Pollen to
Evidence
for Atoms

How
Big Is A
Molecule?

$$\begin{matrix} \dot{q}_5 \\ \# \\ \{\alpha, \beta, \dot{q}_5\} \end{matrix}$$

(3+1) ADM .vs. 4D.

$$\begin{aligned} \dot{g}_{ij} &= \dots \\ K_{ij} &= \dots \\ \text{Hamil't const} \\ \text{Moment const} \\ \alpha, \beta \end{aligned}$$

$$\begin{aligned} \ddot{q}_5 &= \dots \\ Q_{\dot{q}_5} &= \dot{q}_5 \\ \{ \dot{q}_5 &= \dots \\ Q_{\dot{q}_5} &= \dots \\ \square x^5 &= \text{something} \end{aligned}$$

$$\partial_c g_{ab} = \partial_c g_{ab} - \Gamma_{acc} \Gamma_{bct}$$

$$\Rightarrow \square X_b = 0$$

$$\hookrightarrow \square X_b = H_b$$

Define $\partial_t H_b = \nabla_b \delta u(x)$

$$\partial_t H_b = g^{ab} \nabla_a \delta u ; \quad \partial_t g_{ab}$$

Coord condition

1) Harmonic coords. $\Gamma_b = 0 = \square X_b$

2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \square X_b$

$$g^{\alpha\beta} \square_\alpha \nabla_\beta H_b = \zeta \frac{(\alpha-1)}{2^p}$$

$$2\kappa \partial_t g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

$$1) \quad \checkmark \quad \Gamma_b = 0 \Rightarrow \square X_b = 0 \quad (-H_b) = \Gamma_b = (\square X_b)$$

$$2) \quad \nabla_a H_b \quad \Gamma_b = H_b \Rightarrow \square X_b = H_b$$

3) Dynamical H_b

Define

$$\partial_t H_b = \psi_b \sin(x)$$

$$\partial_t H_b = g^{ab} \psi_a$$

$$\psi_a ; \partial_a g_{ab}$$

Coord condit

1) Har

2) "Ge

co

$$g^{ab} \nabla_a \nabla_b H_b =$$

$g_{ab} \rightarrow \{\alpha, \beta^i, g_{ij}\}$ (3+1) ADM . vs . 4D.

$$\dot{g}_{ij} = \dots$$

$$\dot{K}_{ij} = \dots$$

Hamilt const

Moment const

$$\alpha, \beta^i$$

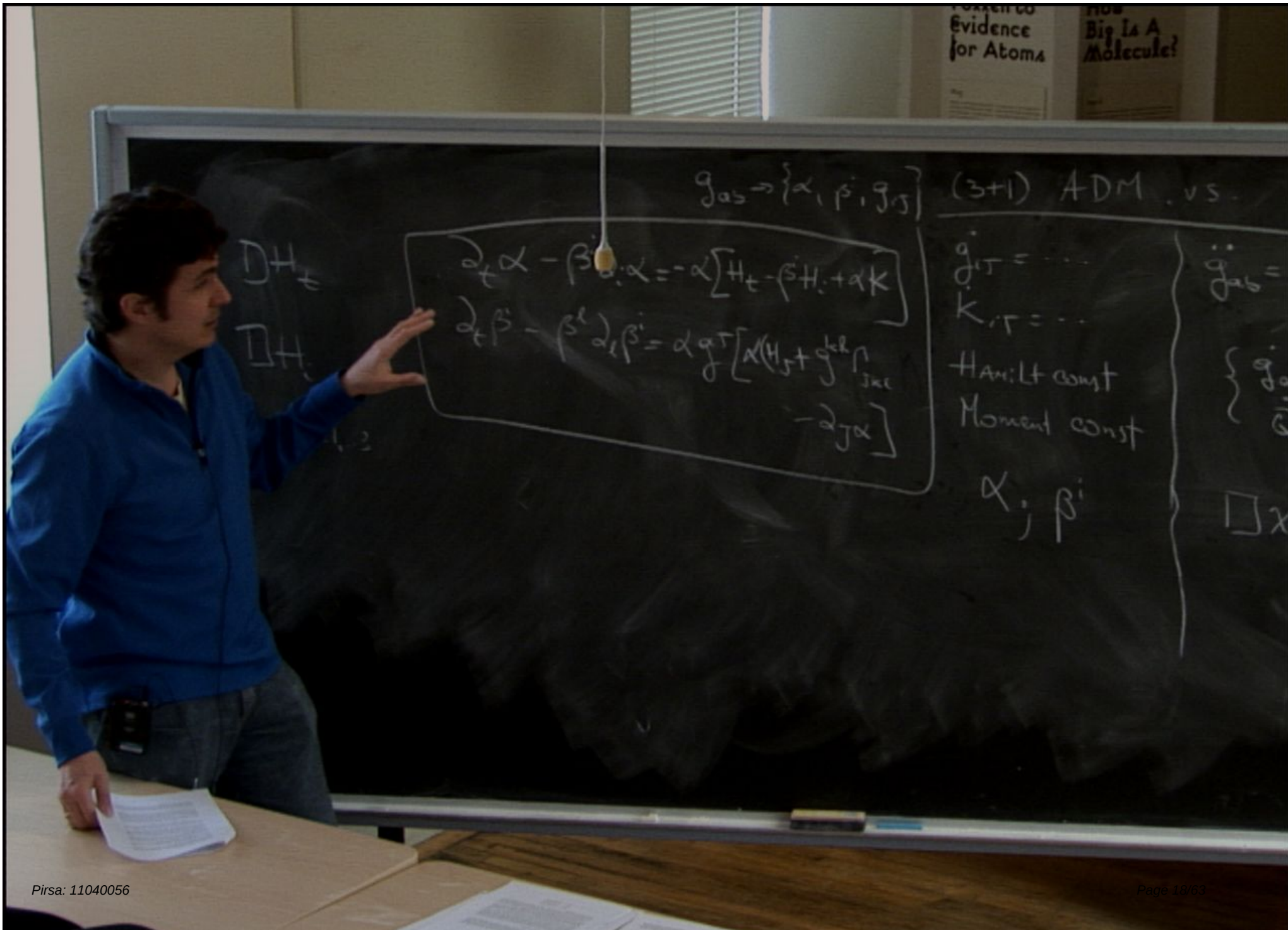
$$\ddot{g}_{ab} = \dots$$

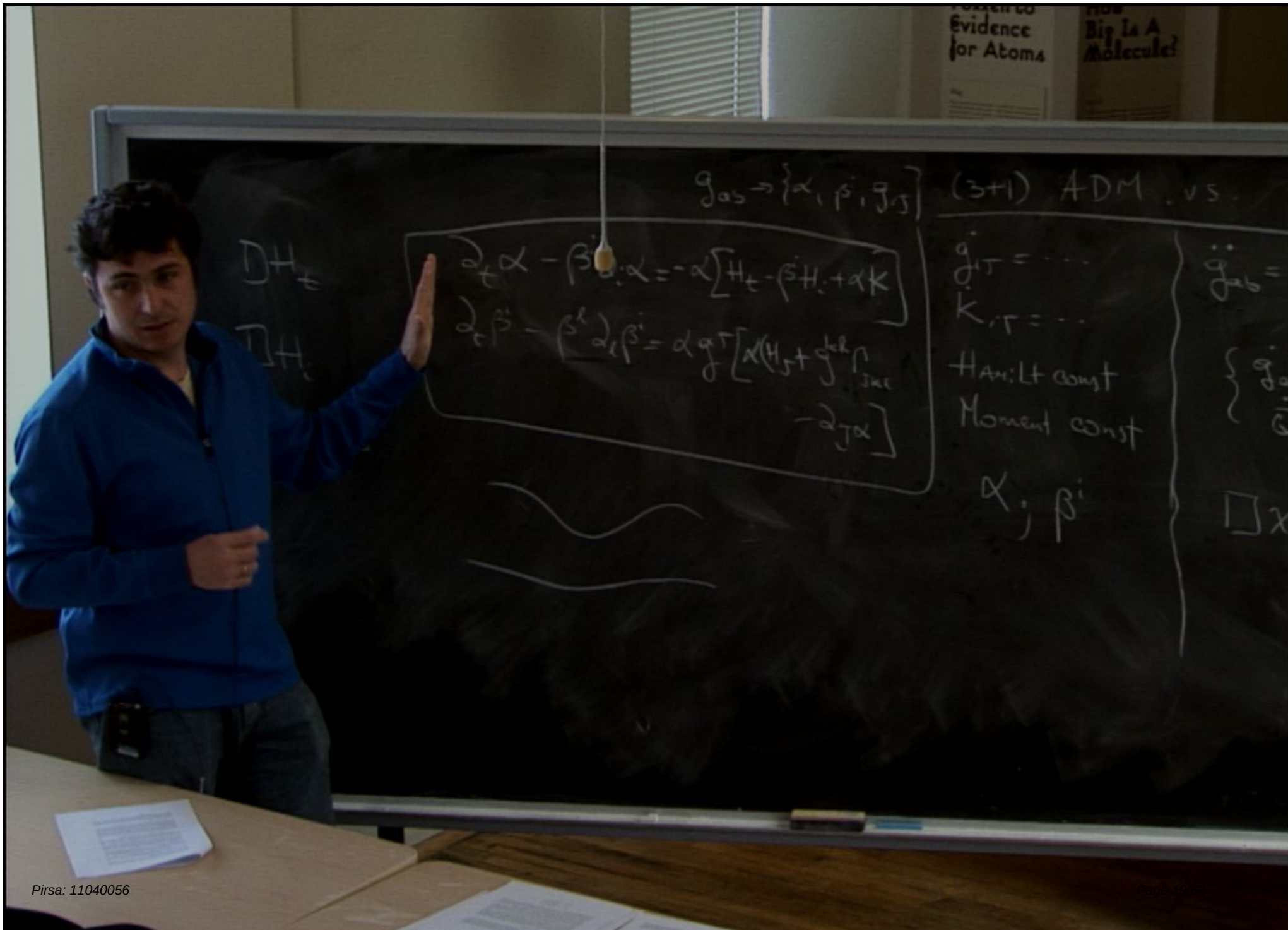
$$Q_{ab} = \dot{g}_{ab}$$

$$\left\{ \begin{array}{l} \dot{g}_{ab} = \dots \\ Q_{ab} = \dots \end{array} \right.$$

$$\ddot{Q}_{ab} = \dots$$

$$\square x^a = \text{something}$$





$$R_{ab} = 0$$

$$- \frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$



$$\square X_b = 0 \quad (+H_b)$$

$$\square X_b = H_b$$

free

$$\partial_t H_b$$

$$\partial_t H_b$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$



$$\begin{aligned} \square X_5 &= 0 \quad (+H_5) \\ \square X_5 &= H_5 \end{aligned}$$

$$\begin{aligned} \text{free: } \partial_t H_5 \\ \partial_t H_5 \end{aligned}$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b^a + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

EW

$$\square X_5 = 0 \quad (+H_5)$$

$$\square X_5 = H_5$$

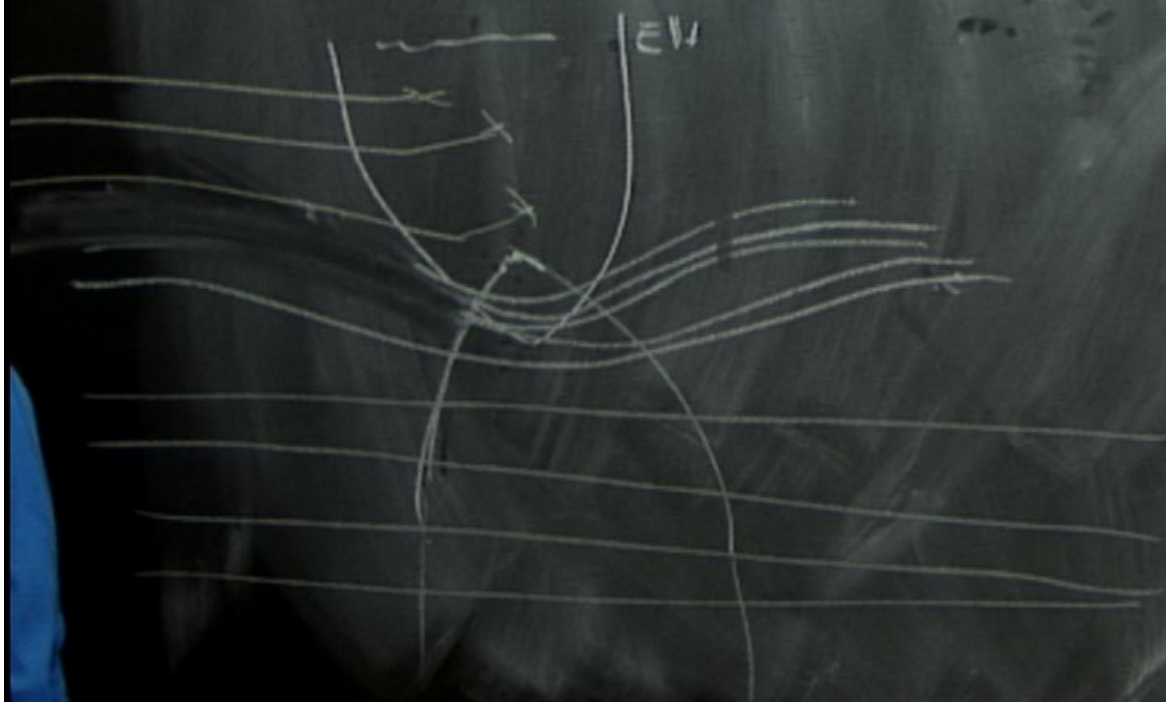
free $\partial_t H_5$

$$\partial_t H_5$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

Excision



$$\partial X_5 = 0 \quad (-th)$$

$$\partial X_5 = H_5$$

free $\partial_t H_5$
 $\partial_t H_5$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

Excision



$$\partial X_5 = \emptyset \quad (+H_5)$$

$$\partial X_5 = H_5$$

free

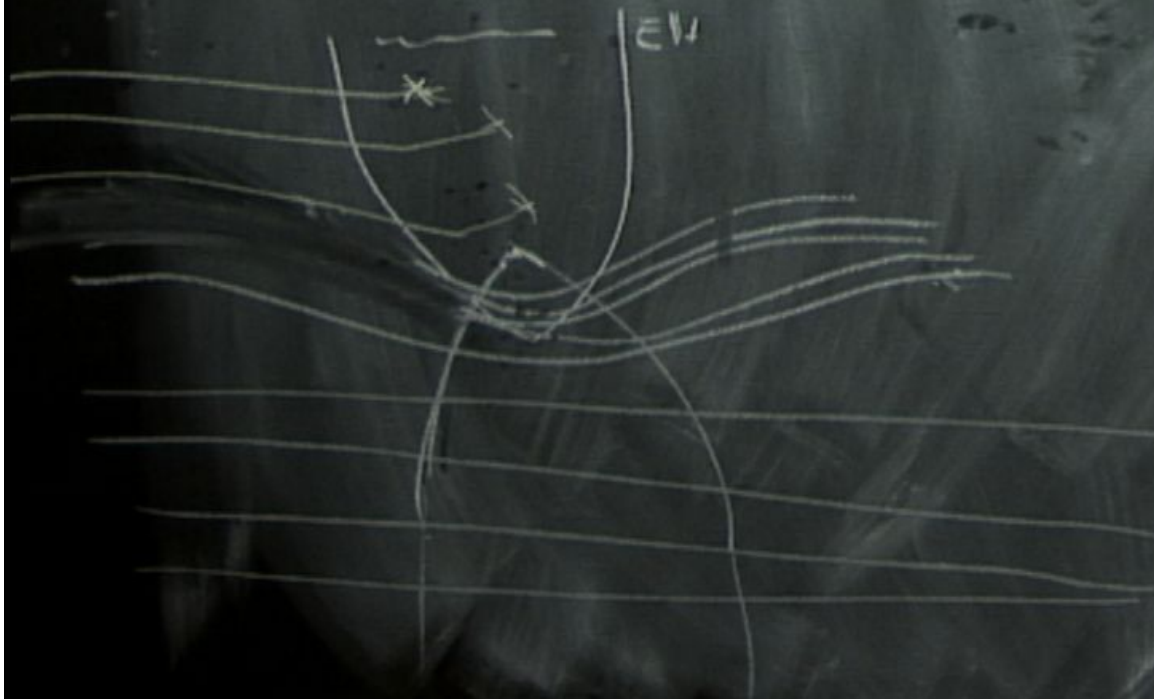
$$\partial_t H_b$$

$$\partial_t H_s$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

Excision



$$\begin{aligned} \partial X_5 &= \emptyset \quad (+H_5) \\ \partial X_5 &= H_5 \end{aligned}$$

$$\begin{aligned} \text{free} \quad \partial_t H_b \\ \partial_t H_b \end{aligned}$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

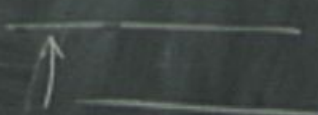
Excision

$$\vec{E} = -\nabla \Phi + \vec{E}_{\text{error}}$$

$$\vec{B} = -\nabla \chi + \vec{B}_{\text{error}}$$

$$\nabla \cdot \vec{E} = \rho$$

$$\nabla \cdot \vec{B} = 0$$



fine

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$C_b = \Gamma_b + H_b$$

$$\Rightarrow D X_b = 0$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$C_b = \Gamma_b + H_b = 0 \quad (0 \text{ B.S.}) \quad X_b = 0$$

$$\partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b =$$

fine ∂_α
 ∂_β

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$C_b = \Gamma_b + H_b = 0 \quad (0 \text{ or } \infty) \quad X_b = 0$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b =$$

$$\hookrightarrow R_{ab} - \nabla_a C_b = 0$$

fine

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Pi_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$C_b = \Pi_b + H_b = 0 \quad (D_B = 0) \quad X_b = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} - \nabla_a (\Pi_b + H_b) = 0 \quad X_b = 0$$

$$\hookrightarrow R(C_b) = 0$$

$$\nabla^\alpha \nabla_\alpha C_b + \text{free}$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \partial_c g_{eb} \partial_f g_{da})$$

$$C_b = \Gamma_b + H_b = 0 \quad (D_B = 0) \quad X_b = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b =$$

$$\hookrightarrow R_{ab} - \nabla_a C_b = 0$$

$$\Rightarrow \nabla^b \nabla_b C_a + \overset{\text{fine}}{C^b \nabla_a C_b} = 0 \quad \partial_c$$

$$\partial_\mu g_{\alpha\beta} - \Gamma_{\alpha\beta}^\mu \Gamma_{\mu\alpha\beta}$$

$$\partial_\mu X_b = 0 \quad \text{---} \quad \textcircled{+H_b} = \Gamma_b^\mu = \textcircled{\Gamma_b^\mu X_b}$$

$$X_b = +H_b$$

$$\partial_\mu C_a = 0 \quad \Rightarrow \quad \nabla^\mu \nabla_\mu C_a = 0$$

Coord. condition

- 1) Harmonic coords. $\Gamma_b^\mu = 0 = \Gamma_b^\mu X_b$
- 2) "Generalized" Harmonic conditions $\Gamma_b^\mu = +H_b^\mu = \Gamma_b^\mu X_b$

$$\partial_t g_{ab} = \Gamma_{ace} \Gamma_{bde} \dots$$

$$\partial_t X_b = 0 \quad \text{---} \quad \textcircled{+H_b} = \Gamma_b = \textcircled{\nabla X_b}$$

$$X_b = H_b$$

$$\nabla_a =$$

$$\nabla_a C_b = 0 \Rightarrow \nabla_a \nabla_b C_a = 0 \quad \text{if ID}$$

if ID

Coord. condition

1) Harmonic coords. $\Gamma_b = 0 = \nabla X_b$

2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \nabla X_b$

$$\left\{ \begin{array}{l} C_a = 0, \partial_t C_a = 0 \Rightarrow C_a = 0 \end{array} \right.$$

$$\partial_t g_{ab} - \Gamma_{ace} \Gamma_{bde}$$

$$\Gamma_b = 0 \quad \Gamma_b = \Gamma_b = \Gamma_b X_b$$

$$X_b =$$

$$\Gamma_b = 0$$

$$\Gamma_b = 0$$

if ID.

$$\begin{cases} C_a = 0, \partial_t C_a = 0 \Rightarrow C_a = 0 \\ C_a = 0, \partial_x C_a = 0 \Rightarrow C_a = 0 \end{cases}$$

Coord. condition

- 1) Harmonic coords. $\Gamma_b = 0 = \Gamma_b X_b$
- 2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \Gamma_b X_b$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ac}^e \Gamma_{bd}^f)$$

$$C_b = \Gamma_b + H_b = 0 \quad (D.B. \neq 0) \quad X_b = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b = H_b$$

$$\hookrightarrow R_{ab} - \nabla_a C_b = 0$$

$$\Rightarrow \nabla^b \nabla_b C_a + C^b \nabla_a C_b = 0 \quad \approx 1$$

$$C_b = \Gamma_b + H_b = 0 \quad (0.8.7) \quad X_b = 0 \quad (-H_b) = \Gamma_b = (\Gamma_b X_b)$$

$$\partial_p g_{ab} + \nabla_a \Gamma_b = 0 \quad \nabla_a (\Gamma_b + H_b) = 0 \quad X_b = -H_b$$

$$\rightarrow R_{ab} - \nabla_a \nabla_b = 0 \quad \left[n_{ab} C_b - \frac{1}{2} g_{ab} n^c C_c \right] = 0$$

$$C_b = \Gamma_b + H_b = 0 \quad (\text{D.B.} \rightarrow) X_b = 0 \quad (-H_b) = \Gamma_b = (\Gamma_b) X_b$$

$$\partial_\mu g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b = H_b$$

$$\rightarrow R_{ab} - \nabla_a C_b + \gamma \left[n_{ia} C_b - \frac{1}{2} g_{ab} n^c C_c \right] = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b = 0 \quad (0.13.3)$$

$$C_b = \Gamma_b + H_b = 0 \quad (0.13.4) \quad X_b = 0 \quad (-H_b) = \Gamma_b =$$

$$1 \quad g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b - \nabla_b \Gamma_a = 0 \quad X_b = H_b$$

$$\hookrightarrow R_{ab} - \nabla_a C_b + \nabla_b C_a + \gamma \left[\eta_{ab} C_c - \frac{1}{2} g_{ab} \eta^c{}_c C_c \right] = 0$$

$$\Rightarrow \nabla^c \nabla_c C_a - 2\gamma \nabla^b (\eta_{ab} C_b) + C^b \nabla_b C_a - \frac{1}{2} \gamma \eta_a{}^b C^b C_a =$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$$

$$C_b = \Gamma_b + H_b = 0 \quad (0 \text{ B.T.}) \quad X_b = 0 \quad (-H_b) = \Gamma_b \equiv (\Gamma_b) X_b$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{ab} + \nabla_a \Gamma_b - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b = H_b$$

$$\hookrightarrow R_{ab} - \delta_{ab} \left[\eta_{cd} C_{ab} - \frac{1}{2} g_{ab} \eta^c{}_c C^c{}_c \right] = 0$$

$$\Rightarrow \nabla^c \nabla_c (\eta_{ab} C_{ab}) + C^b{}_b \nabla_a C^a{}_a - \frac{1}{2} \delta_{ab} \eta^c{}_c C^b{}_b = 0$$

$\frac{d}{dt} (\Gamma_{ace} \Gamma_{bdf})$

$$\frac{d^2 x}{dt^2} + 2\beta \frac{dx}{dt} + \omega^2 x = 0.$$

$= 0$

$$x_s = H_s$$

$$n^c C_c = 0$$

$$\square C_a - 2\gamma \nabla^b (n_b C_a) = 0$$

$$\square C_a - \gamma n_b \nabla^b C_a - \gamma n_a \nabla^b C_b = 0$$

$$a C_b - \frac{1}{2} \gamma n_a C^b C_b = 0$$

$\frac{d}{dt} (\Gamma_{ace} \Gamma_{bde})$

$$\frac{d^2 x}{dt^2} + 2\beta \frac{dx}{dt} + \omega^2 x = 0.$$

$= 0$

$$x_s = H_s$$

$$n^c C_c = 0$$

$$\square C_a - 2\pi \nabla^b (n_b C_a) = 0$$

$$\square C_a - \gamma n_b \nabla^b C_a - \gamma n_a \nabla^b C_b = 0$$

$$a(C_5) - \frac{1}{2} \gamma n_a C^5 C_5 = 0$$

$$R_{ab} = 0$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + g^{cd} g^{ef} (\partial_c g_{ea} \partial_f g_{db})$$

$$C_b = \Gamma_b + H_b = 0 \quad (\text{Dirac}) \quad X_b = 0$$

$$g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_a \Gamma_b + \dots - \nabla_a (\Gamma_b + H_b) = 0 \quad X_b = 0$$

$$\hookrightarrow R_{ab} - \nabla_a C_b + \delta' [n_{ia} C_b - \frac{1}{2} g_{ab} n^c C_c]$$

$$\Rightarrow \nabla^c \nabla_c C_a - 2\delta' \nabla^b (n_{ab} C_a) + C^b \nabla_b C_a$$

$$d_t \equiv \partial_t - \mathcal{L}_\beta$$

$$d_t \gamma_{ij} = -2\alpha K_{ij}$$

$$d_j K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

$$R_{ij} =$$

$$R_{ij} = -\frac{1}{2} \partial_i \partial_m \partial_e \partial_m \delta_{ij} + \mathcal{P}_{ij}^{(4)} + \dots$$

$$d_t \equiv \alpha_t - \mathcal{L}_p$$

$$d_t \gamma_{i,j} = -2\alpha K_{i,j}$$

$$d_j K_{i,j} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

BSSN

$$g_{ab} \rightarrow \{\alpha, \beta, g_{ij}\}$$

(3+1) ADM vs 4D

$$\square H_t$$

$$\square H_i$$

$i=1,2,3$

$$\partial_t \alpha - \beta^i \partial_i \alpha = -\alpha [H_t - \beta^i H_i + \alpha K]$$

$$\partial_t \beta^i - \beta^j \partial_j \beta^i = \alpha g^{ij} [\alpha (H_j + g^{kl} \pi_{kl}) - \partial_j \alpha]$$

$$\dot{g}_{ij} = \dots$$

$$K_{ij} = \dots$$

Hamilton const

Momentum const

$$\alpha; \beta^i$$

$$\ddot{g}_{ab} = \dots$$

$$\begin{cases} \dot{g}_{ab} = \dots \\ \ddot{g}_{ab} = \dots \end{cases}$$

$$\square x^a = \dots$$

$$d_t \equiv \partial_t - \mathcal{L}_\beta$$

$$d_t \gamma_{ij} = -2\alpha K_{ij}$$

$$d_j K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

BSSN

$$1) \tilde{\gamma}_{ij} = \bar{e}^{cd} \gamma_{ij} \quad [\det \tilde{\gamma}_{ij} = 1]$$

$$2) \phi, K$$

$$R_{ij} = -\frac{1}{2} \gamma^{kl} \partial_i \partial_j \gamma_{kl}$$

$$d_t \equiv 2\epsilon - \mathcal{L}_\beta$$

$$d_t \gamma_{i,j} = -2\alpha K_{i,j}$$

$$d_j K_{i,j} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

ADM	BSSN
γ_{ij}	$\tilde{\gamma}_{ij}, \phi$
K_{ij}	$\tilde{K}, A_{ij}$

$$1) \tilde{\gamma}_{i,j} = e^{-4\phi} \gamma_{i,j} \quad [\det \tilde{\gamma}_{i,j} = 1]$$

$$2) \phi, K$$

$$3) A_{i,j} = K_{i,j} - \frac{1}{3} K \gamma_{i,j}$$

$$R_{ij} = -\frac{1}{2} \gamma^{klm} \partial_k \partial_l \gamma_{im} \partial_j \gamma_{kl}$$

$$R_{ij} = -\frac{1}{2} \gamma^{ab} \partial_a \partial_b \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{d}{dt} \gamma_{ij} = \dots + 0$$

Space harmonics

$$^{(3)}\tilde{\gamma}_{ij} = 0$$

$$R_{ij} = -\frac{1}{2} \gamma^{km} \partial_k \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + \mathcal{O}(\alpha^2)$$

Space harmonics

$${}^{(3)}\tilde{\gamma}_{ij} = 0$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\delta}_{ij,lm}$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$+ \tilde{\gamma}_{ik} (\partial_j \tilde{\gamma}^k{}_j + 1.0)$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\delta}_{ij,lm}$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$+ \tilde{\gamma}_{ik} (\tilde{\omega}^k_j - \tilde{\omega}^j_k) + 1.0$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} K = -\tilde{\gamma}^{ij} \tilde{D}_i \tilde{D}_j \alpha$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\delta}_{ij,lm}$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$+ \tilde{\gamma}_{kl} \partial_j \tilde{\gamma}^k_{i} + 0.0$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\delta}_{ij,lm}$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$+ \tilde{\gamma}_{kl} \partial_j \tilde{\gamma}^k_l + 1.0$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{d}{dt} K = -\gamma^{ij} D_i D_j \alpha + 1.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$\tilde{r}^k = H^k$$

$$R_{i\tau} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{i\tau} + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{i\tau} = -2\alpha \tilde{A}_{i\tau}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma_{i\tau} D_i D_\tau \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{i\tau} =$$

$$\tilde{R}_{i\tau} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{i\tau,lm} + \tilde{\gamma}_{k\tau} \partial_{j\tau} \tilde{r}^k + 1.0$$

$$\tilde{r}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$\frac{d}{dt} \tilde{r}^k =$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{kl} \partial_j \tilde{r}^k + 1.0$$

in def
11.11

$$\tilde{r}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\delta}_{ij,lm} + \tilde{\delta}_{kl} \partial_j \tilde{r}^k + 1.0$$

in der
1.1.1.

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$\partial_j \tilde{A}_{ij}$$

$$\tilde{\Gamma}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \Phi \gamma_{ij} + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\delta}^{lm} \tilde{\partial}_l \tilde{\partial}_m \gamma_{ij} \quad \text{in der$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + 0.0$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = \tilde{\partial}_j \tilde{A}^j_i$$

$$\tilde{\Gamma}^k = \partial_j \tilde{\Gamma}^k + 1.0$$

$$\partial_j (\tilde{A}^j_i)$$

$$\tilde{r}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\phi = -\frac{1}{6} \alpha K$$

$$= -\gamma^{ij} D_i D_j \alpha + \text{L.O.}$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{ij,lm} + \tilde{\gamma}^{kl} \partial_j \tilde{\gamma}_{kl} + \text{L.O.}$$

in det
1/4K!

$$\text{Matter Cou. } \partial_j (\tilde{A}^{ji}) \dots$$

$$\tilde{r}^i = \partial_j \tilde{A}^{ji}$$

$$\tilde{r}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{O}(\gamma^2) + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\frac{dK}{dt} = -\gamma^{ij} D_i D_j \alpha + \text{L.O.}$$

$$\frac{d}{dt} \tilde{A}_{ij} =$$

$$\frac{d}{dt} \tilde{r}_i = \frac{\partial}{\partial \tilde{A}^{ji}} \rightarrow \{ \partial K, \partial_j (\alpha, \beta) \}$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{kl} \partial_j \tilde{r}^k + \text{L.O.}$$

in def
11.11

Mon. Cons. $\partial_j (\tilde{A}^{ji}) \dots$

$$\tilde{r}^k = H^k$$

$$R_{ij} = -\frac{1}{2} \delta^{lm} \partial_l \partial_m \gamma_{ij} + \mathcal{D}_i \mathcal{D}_j \gamma + \dots$$

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} \quad \checkmark$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$$

$$\rightarrow \frac{dK}{dt} = -\gamma_{ij} \mathcal{D}_i \mathcal{D}_j \alpha + \text{L.O.}$$

$$\frac{d}{dt} \tilde{A}_{ij} = \quad \checkmark$$

$$\rightarrow \frac{d}{dt} \tilde{r}_i = \cancel{\partial_j \tilde{A}^{ji}} \rightarrow \{ \partial K, \partial_j (\alpha, \beta) \}$$

$$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{kl} \partial_j \tilde{r}^k + \text{L.O.}$$

in det
1/16

Mon. Cov. $\partial_j (\tilde{A}^{ji}) \dots$

$$d_t \equiv \lambda_t - \mathbb{E}_p$$

$$d_t \gamma_{i,t} = -2\alpha K_{i,t}$$

$$d_t K_{i,t} = -D_{i,t} \alpha$$



$$\tilde{R}_{i,t} \quad R_{i,t} \\ (\tilde{\gamma}, \tilde{\alpha}) \quad \gamma$$

$$\times R_{i,t} + \dots$$

$$R_{i,t} = -$$

$$\frac{d}{dt} \tilde{\gamma}_{i,t}$$

$$\frac{d}{dt} \phi =$$

$$\rightarrow \frac{d}{dt} K_{i,t} = -2$$

$$\frac{d}{dt} \tilde{\alpha}_{i,t} =$$

$$\frac{d}{dt} \tilde{\gamma}_{i,t} =$$

$$\frac{d}{dt} \beta \dots \quad 2\tilde{\gamma}$$

$$d_t \equiv \partial_t - \mathcal{L}_p$$

$$d_t \delta_{i,j} = -2\alpha K_{i,j} \quad \checkmark$$

$$d_t K_{i,j} = -D_{i,j} \alpha \times R_{i,j} + \dots$$

$$\begin{matrix} \tilde{R}_{i,j} & R_{i,j} \\ (\tilde{\delta}, \delta) & \delta \end{matrix}$$

$$R_{i,j} = -$$

$$\frac{d}{dt} \tilde{\delta}_{i,j}$$

$$\frac{d}{dt} \phi =$$

$$\rightarrow \frac{d}{dt} K_{i,j} = -2$$

$$\frac{d}{dt} A_{i,j} =$$

$$\frac{d}{dt} \tilde{\gamma}_{i,j} =$$

$$\frac{d}{dt} \beta \dots \quad 2\tilde{F}$$