Title: Explorations in Numerical Relativity - Lecture 13

Date: Apr 20, 2011 11:30 AM

URL: http://pirsa.org/11040056

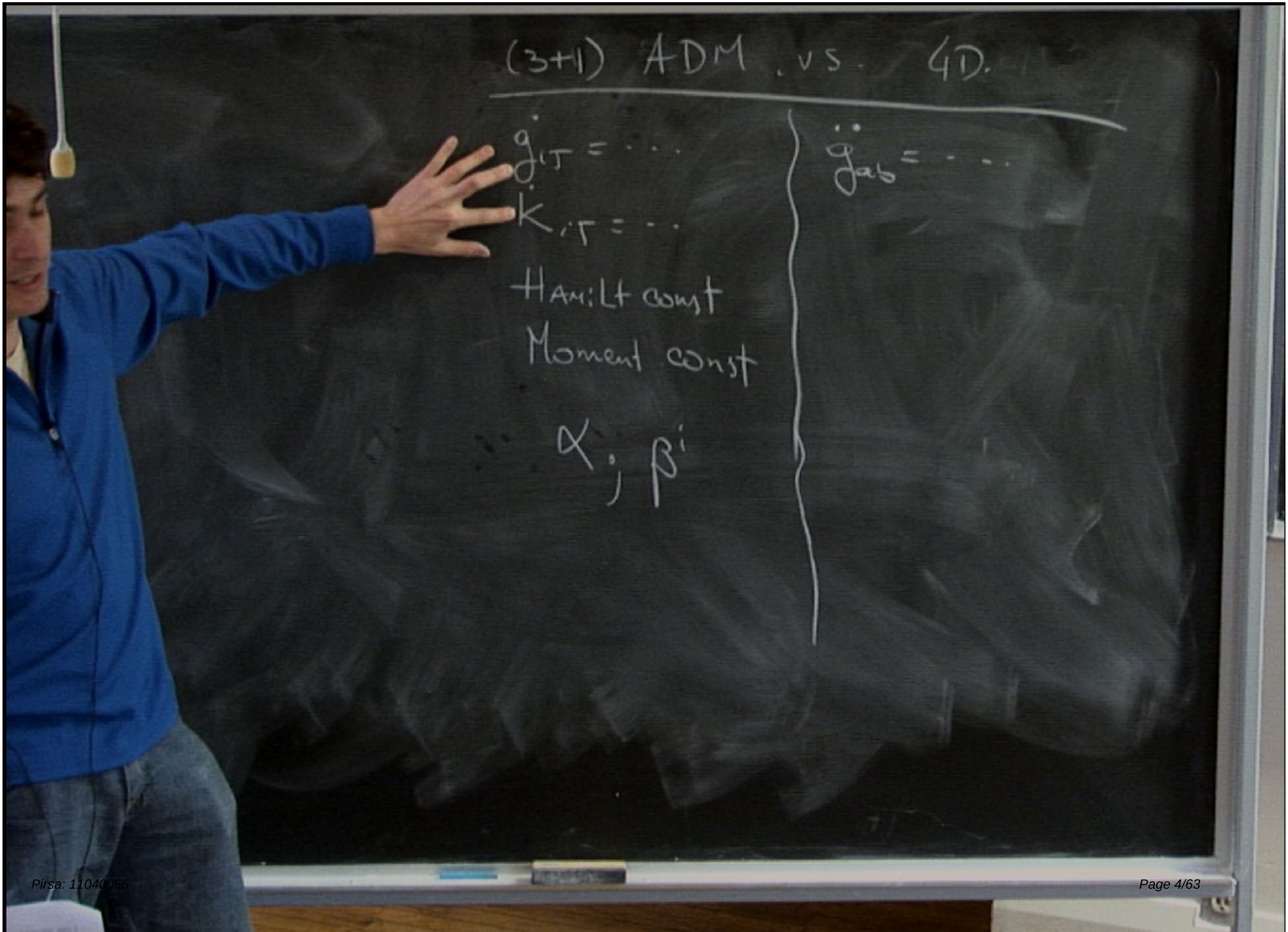
Abstract:

perimeter scholars
international

$$R_{ab} = 0$$

$$\parallel$$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a} \Gamma_{b)} + g^{cd} g^{ef} \left(\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf} \right)$$


(3+1) ADM . vs . 4D.
$\dot{g}_{ij} = \ldots$
$\dot{K}_{ij} = \ldots$
Hamilt const
Moment const
$\alpha ; \beta^i$
$\ddot{g}_{ab} = \ldots$

(3+1) ADM .vs. 4D.

$\dot{g}_{ij} = \dots$

$\dot{K}_{ij} = \dots$

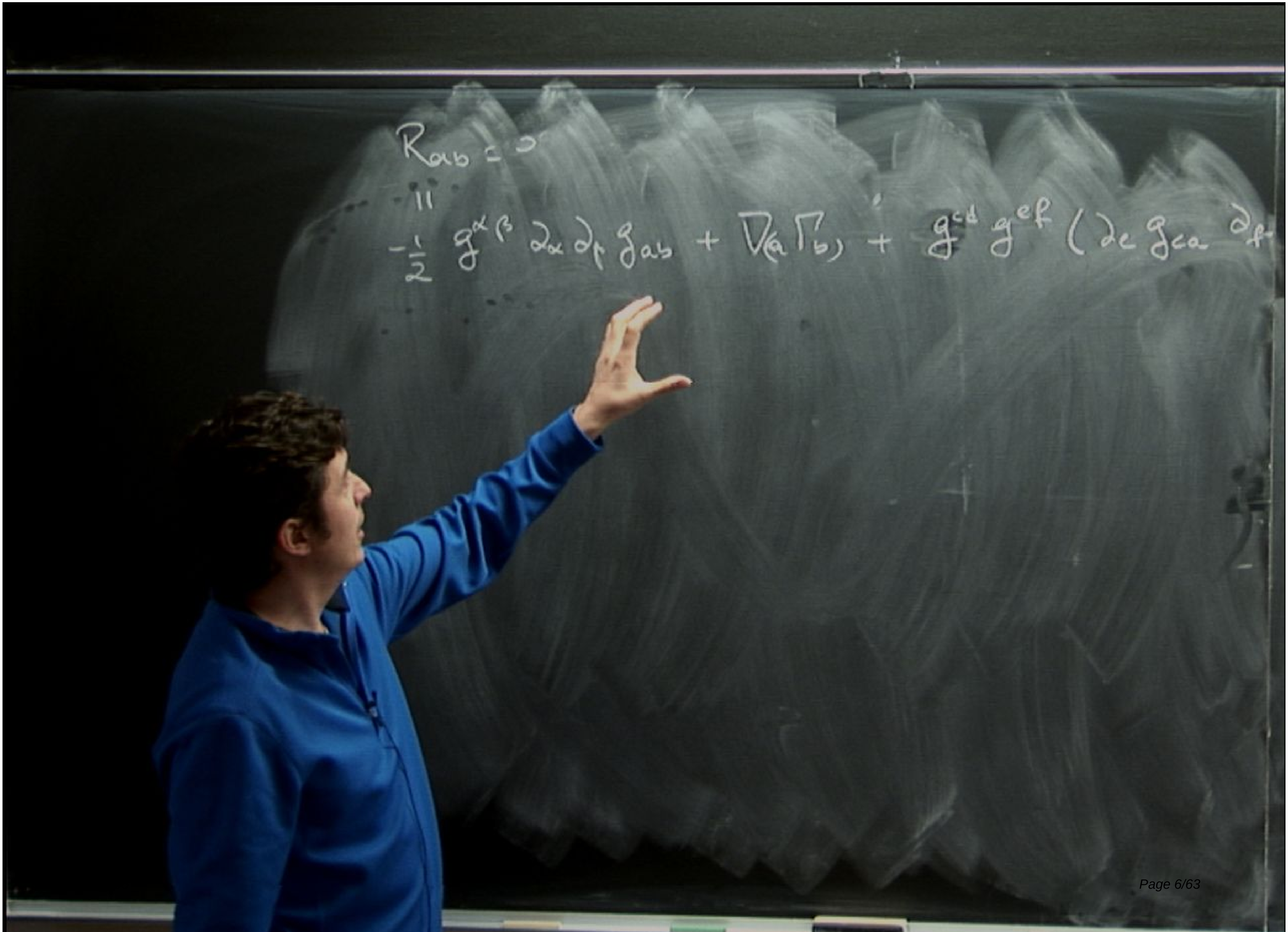
Hamilt const

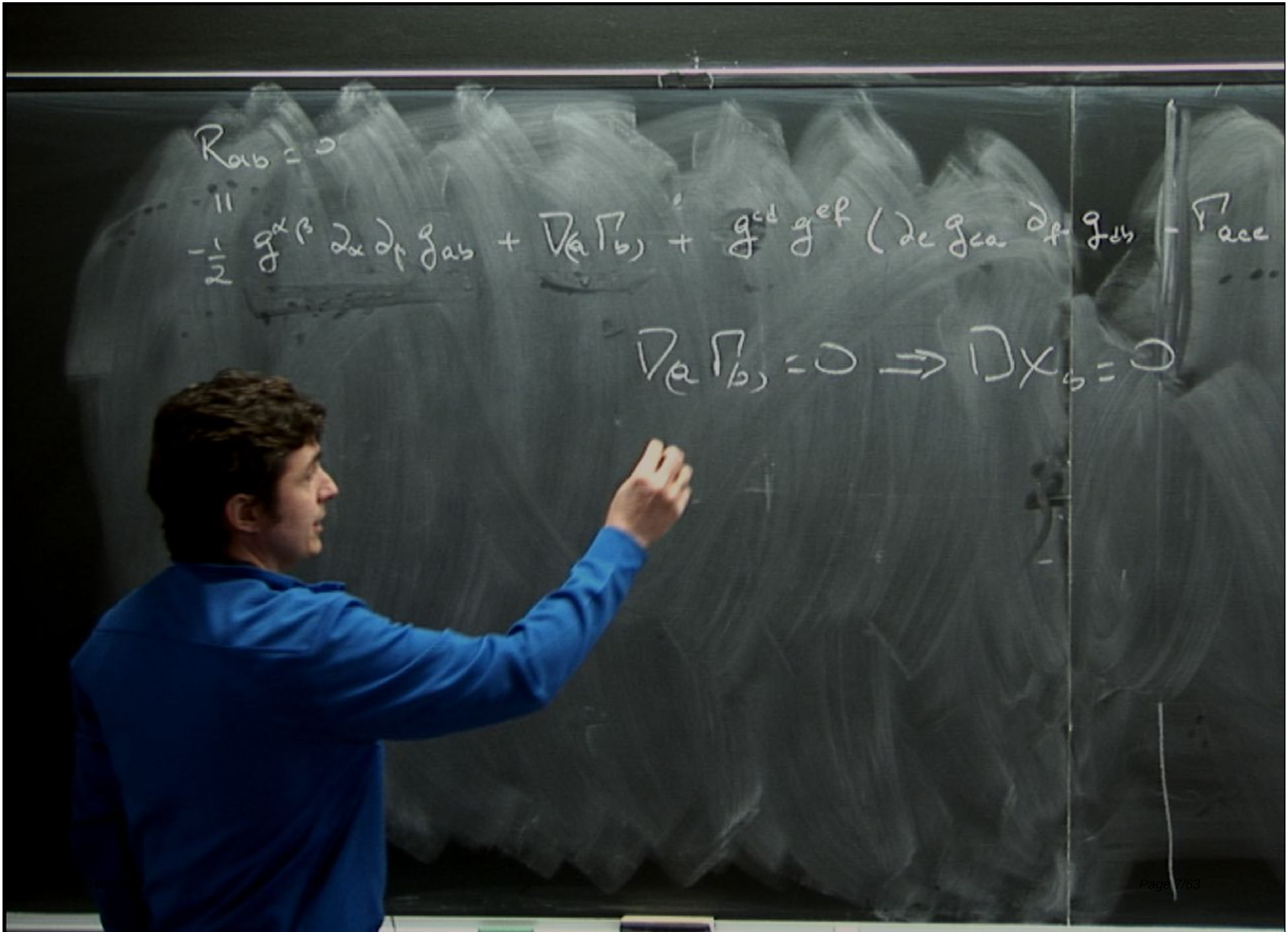
Moment const

α, β^i

$\ddot{g}_{ab} = \dots$

$Q_{ab} = \dot{g}_{ab}$


$R_{ab} = 0$
$\parallel$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a} \Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f$


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + D_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db} - \Gamma_{ace}$
$D_{(a}\Gamma_{b)} = 0 \Rightarrow \Box x_b = 0$

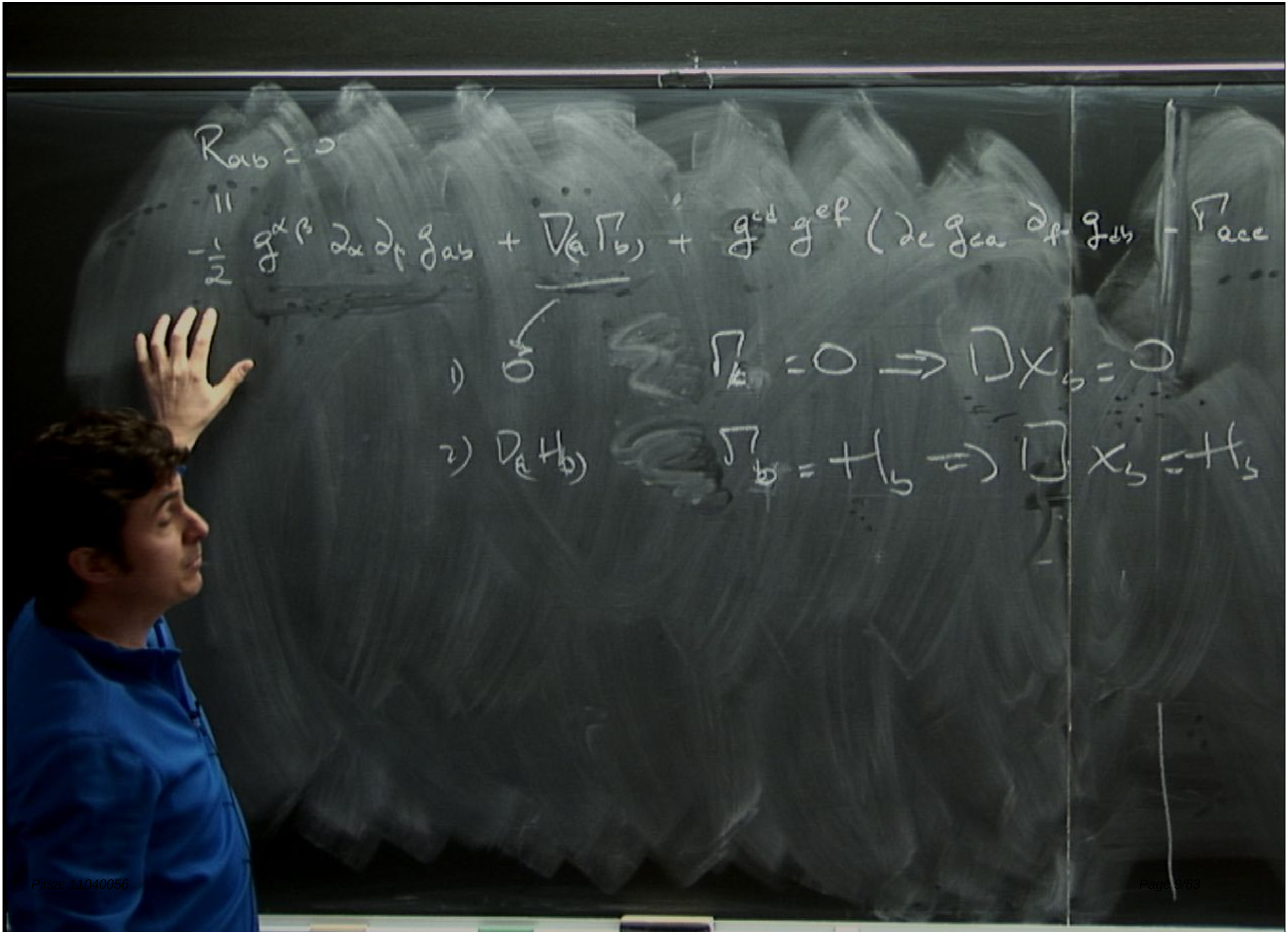
$R_{ab} = 0$

$\parallel$

$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a} \Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \; \partial_f g_{db} \; - \Gamma_{ace}$

$\nabla_{(a} \Gamma_{b)} = 0 \Rightarrow \Box x_b = 0$

$\nabla_{(a} \Gamma_{b)} = H_b \Rightarrow \Box x_b = H_b$


$R_{ab} = 0$
$\|$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + D_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db} - \Gamma_{ace}$
1) 0 $\Gamma_b = 0 \Rightarrow \Box x_b = 0$
2) $D_{(a} H_{b)}$ $\Gamma_b = H_b \Rightarrow \Box x_b = H_b$

$R_{ab} = 0$

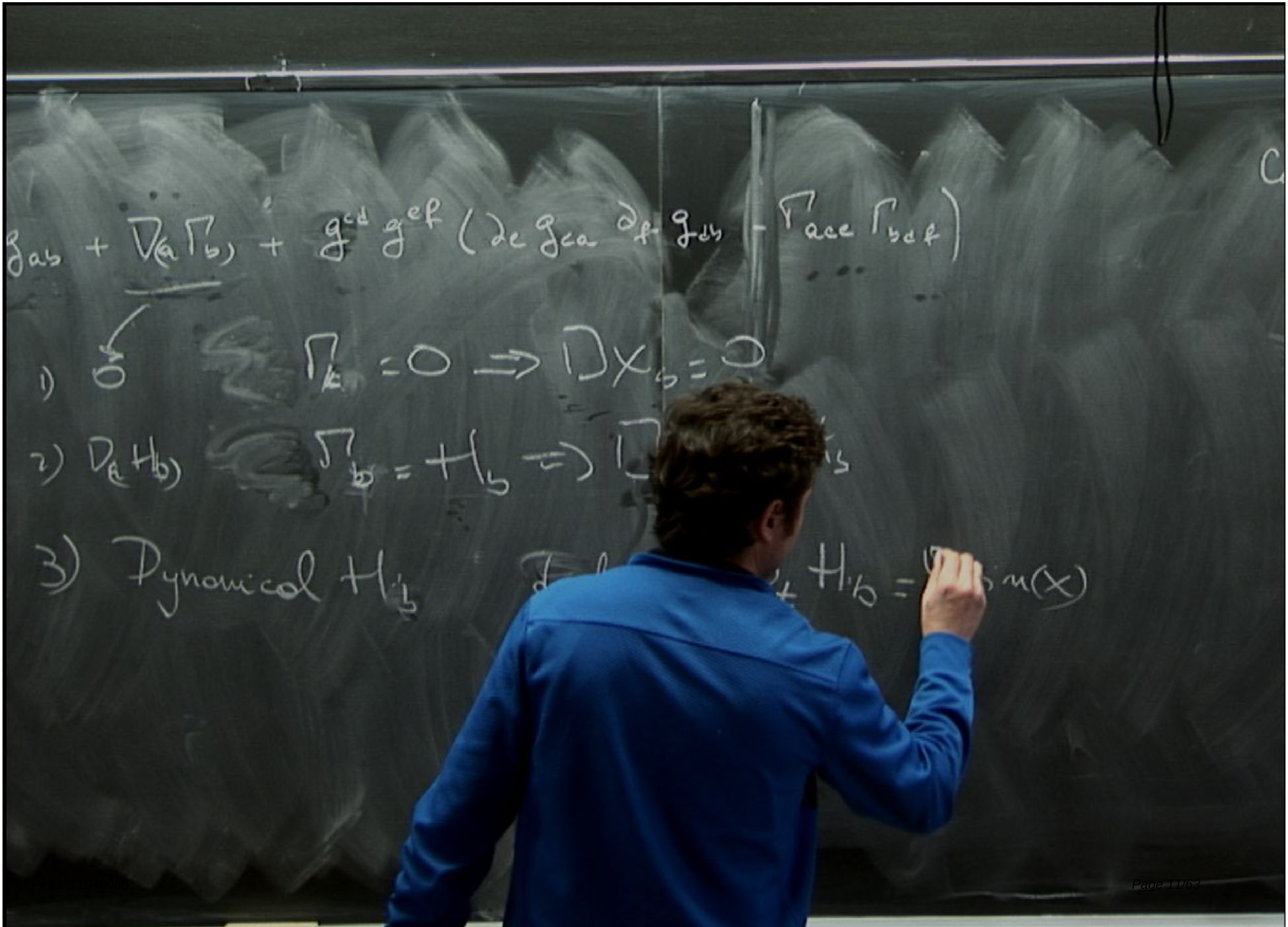
$\parallel$

$$-\frac{1}{2} g^{\alpha\beta}\partial_\alpha\partial_\beta g_{ab} + \underline{\nabla_{(a}\Gamma_{b)}} + g^{cd} g^{ef} \left(\partial_e g_{ca}\, \partial_f g_{db} - \Gamma_{ace} \right.$$

1) 0 $\qquad \Gamma_a = 0 \Rightarrow \Box X_b = 0$

2) $\nabla_{(a} H_{b)}$ $\qquad \Gamma_b = H_b \Rightarrow \Box X_b = H_b$

3) Dynamical H_b $\qquad$ Define


gab + D(aΓb) + gcd gef (∂e gca ∂f gdb − Γace Γbdf)
1) 0 Γb = 0 ⇒ □Xb = 0
2) D(aHb) Γb = Hb ⇒
3) Dynamical Hb
Hb = n(X)

$+ \; g^{cd} g^{ef} \left(\partial_e g_{ca} \, \partial_f \, g_{db} - \Gamma_{ace} \Gamma_{bdf} \right)$

Coord condit

1) H_a ...

2) "G...

c...

$\Gamma_b = 0 \Rightarrow \Box X_b = 0$

$\Gamma_b = H_b \Rightarrow \Box X_b = H_b$

... H_b Define $\partial_t H_b = \xi_b \sin(x)$

$\Box H_b$

$\partial_t H_b = g^{ab} \, v_a \; ; \; \partial_t g_{ab}$

$+ g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$

$\Gamma_b = 0 \Rightarrow \Box x_b = 0$

$\Gamma_b = H_b \Rightarrow \Box x_b = H_b$

ical H_b Define: $\partial_t H_b = \xi_b \sin(x)$

$\partial_t H_b = g^{ab} \; v_a \; ; \; \partial^a g_{ab}$

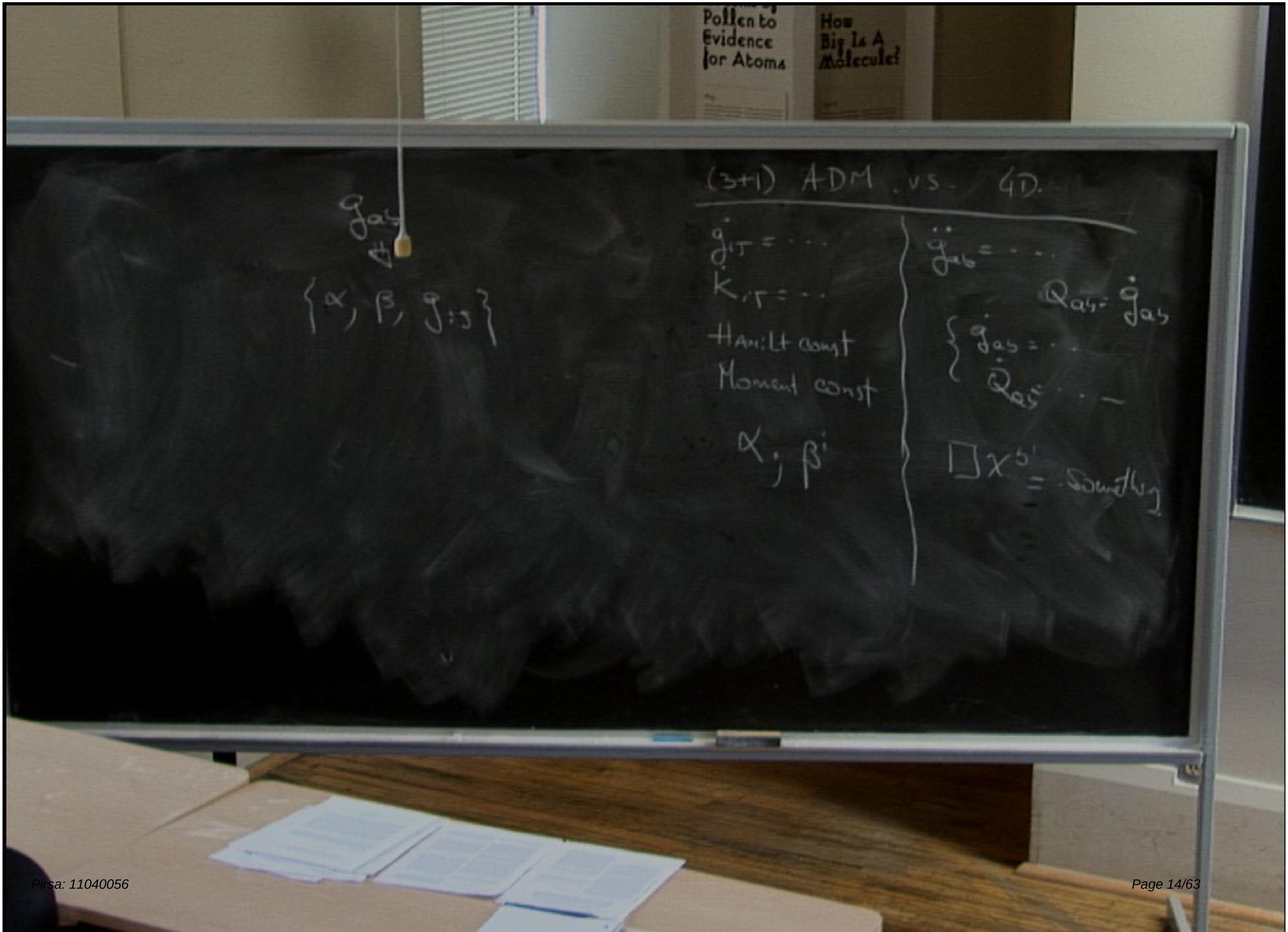
Coord condit

1) H_a

2) "Ge

co

$g^{\alpha\beta} \nabla_\alpha \nabla_\beta H_b$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
(3+1) ADM vs. 4D.
Hamilt const
Moment const
Something

Coord condition

1) Harmonic coords. $\Gamma_b = 0 = \Box X_b$

2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \Box X_b$

$\partial_e g_{ca}\, \partial_f g_{db} - \Gamma_{ace}\Gamma_{bdf})$

$\Rightarrow \Box X_b = 0$

$H_b \Rightarrow \Box X_b = H_b$

Define $\partial_t H_b = \xi_b \sin(x)$

$g^{\alpha\beta}\nabla_\alpha \nabla_\beta H_b = \xi \frac{(\alpha - 1)}{\alpha^p}$

$\partial_t H_b = g^{ab}\, v_a \; ; \; \partial_t g_{ab}$

Coord condit

$$\partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca}\, \partial_f g_{db} - \Gamma_{ace}\Gamma_{bdf})$$

1) Harm

2) "Gen co

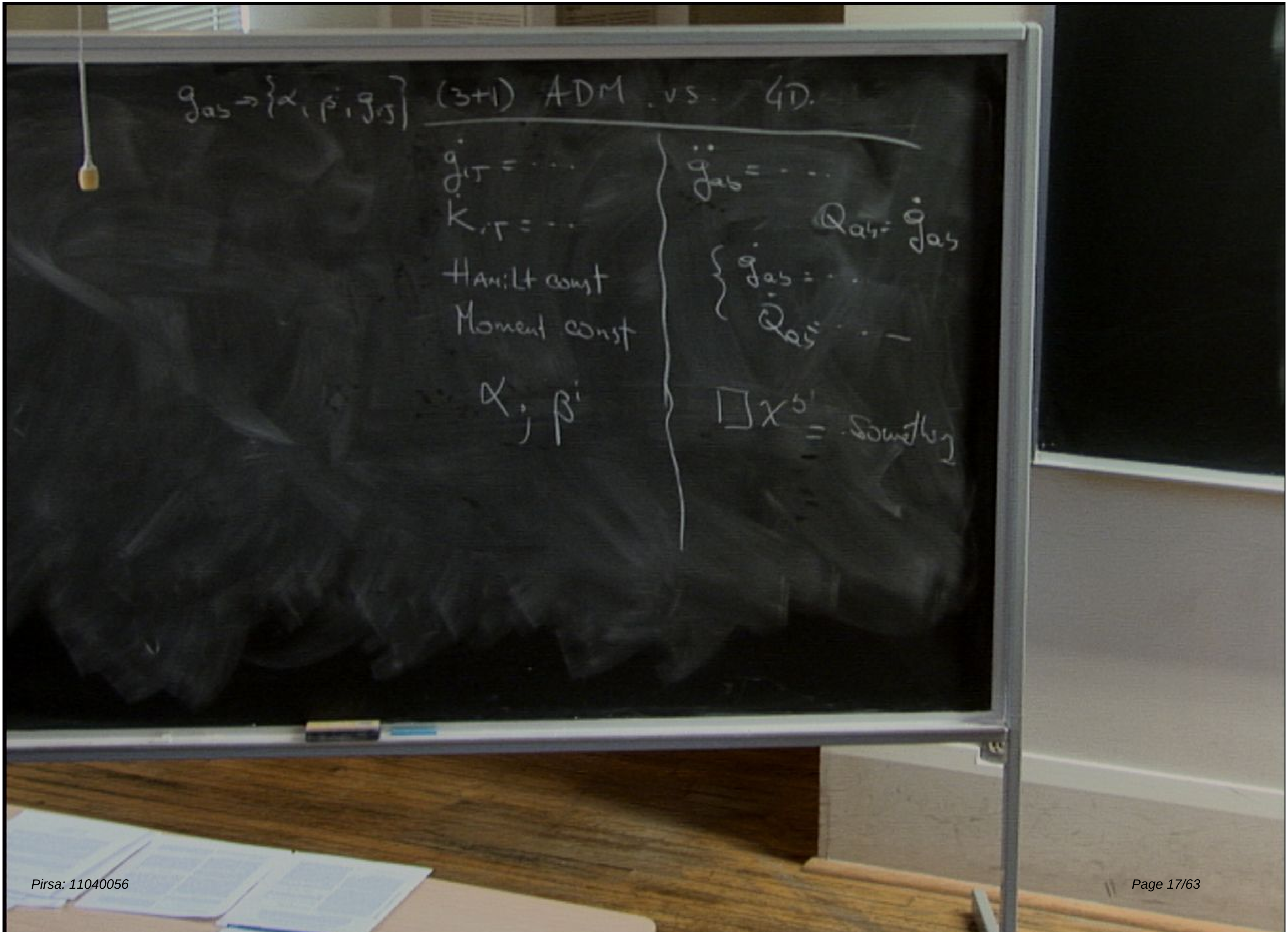
1) 0 $\quad \Gamma_b = 0 \Rightarrow \Box X_b = 0 \quad H_b = \Gamma_b \equiv \Box X_b$

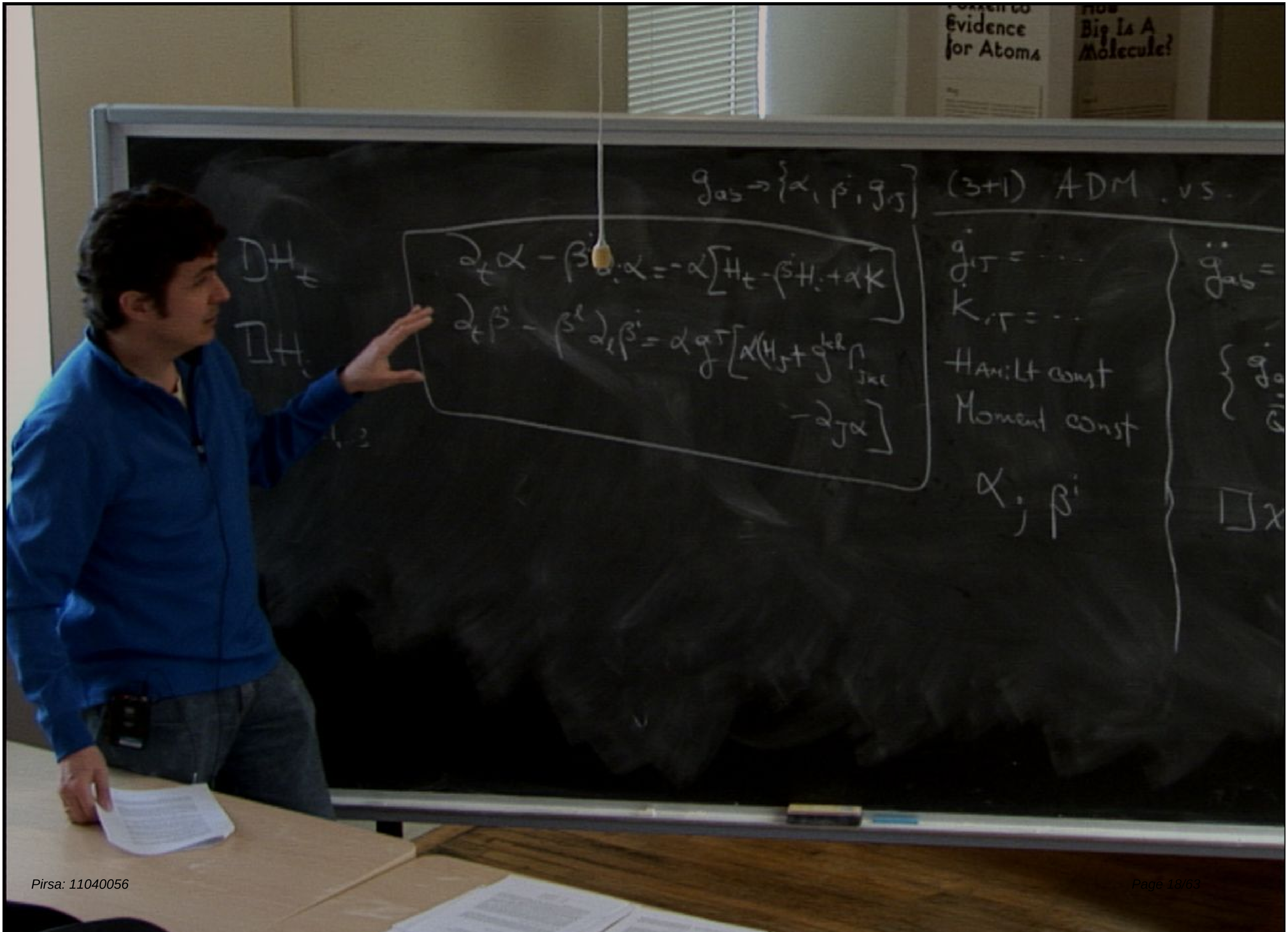
2) $\nabla_{(a} H_{b)}$ $\quad \Gamma_b = H_b \Rightarrow \Box X_b = H_b$

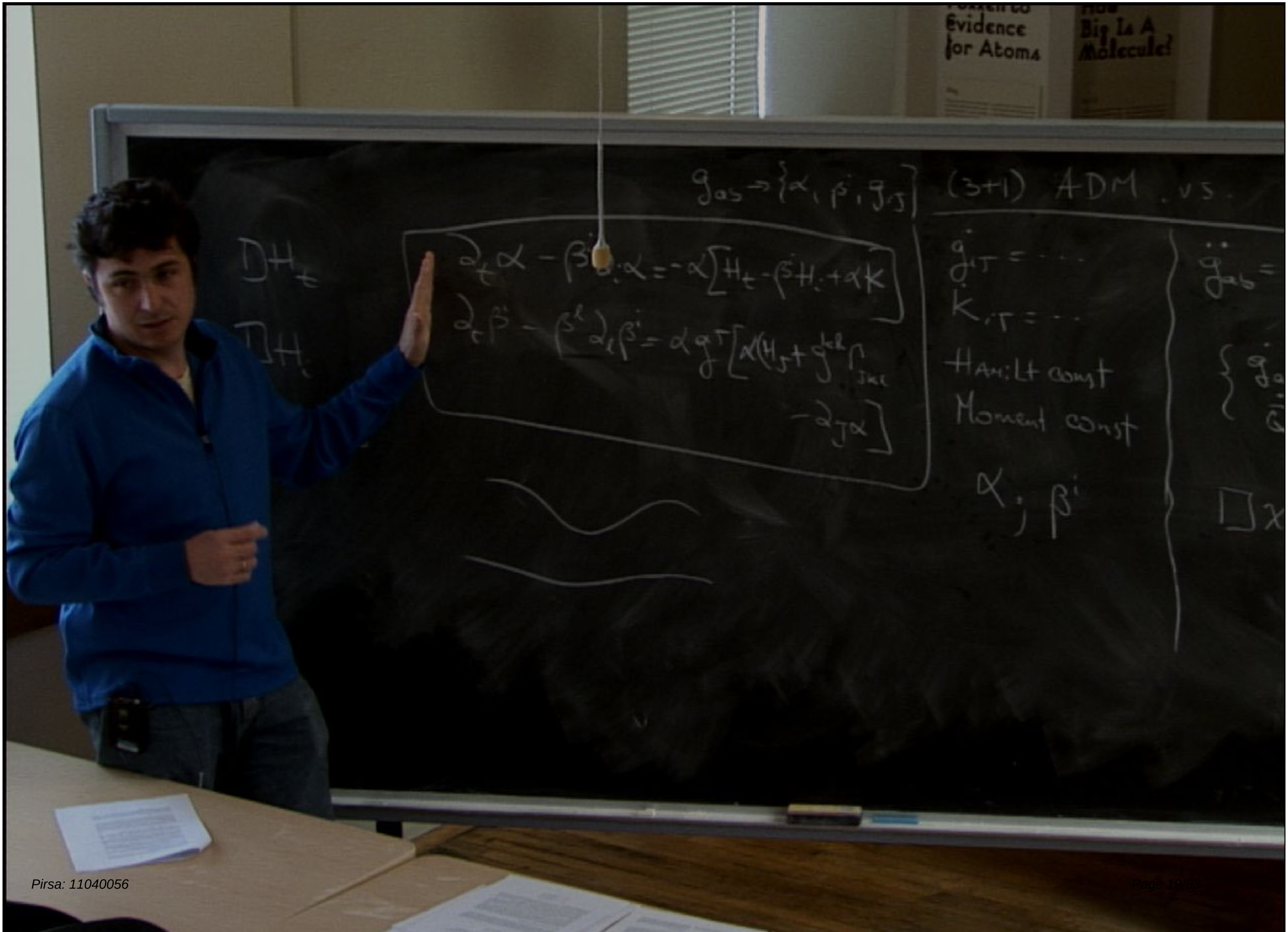
3) Dynamical H_b $\quad$ Define $\quad \partial_t H_b = f\, \sin(x)$

$$\partial_t H_b = g^{ab}\, v_a \; ; \; \partial_t g_{ab}$$

$$g^{\alpha\beta}\nabla_\alpha \nabla_\beta H_b =$$


$g_{ab} \to \{\alpha, \beta^i, g_{ij}\}$ (3+1) ADM . vs . 4D.
$\dot{g}_{ij} = \dots$
$\dot{K}_{ij} = \dots$
Hamilt const
Moment const
α, β^i
$\ddot{g}_{ab} = \dots$
$Q_{ab} = \dot{g}_{ab}$
$\dot{g}_{ab} = \dots$
$\dot{Q}_{ab} = \dots$
$\Box x^{b'} =$ Something


Evidence
for Atoms
Big Is A
Molecule?
(3+1) ADM vs
Hamilt const
Moment const


Evidence
for Atoms
Big Is A
Molecule?
(3+1) ADM vs
Hamilt const
Moment const

$$R_{ab} = 0$$

$$\| $$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma \ldots$$

$$\Rightarrow \Box X_b = 0$$

$$\Rightarrow \Box X_b = H_b$$

fine

$$\partial_t H_b$$

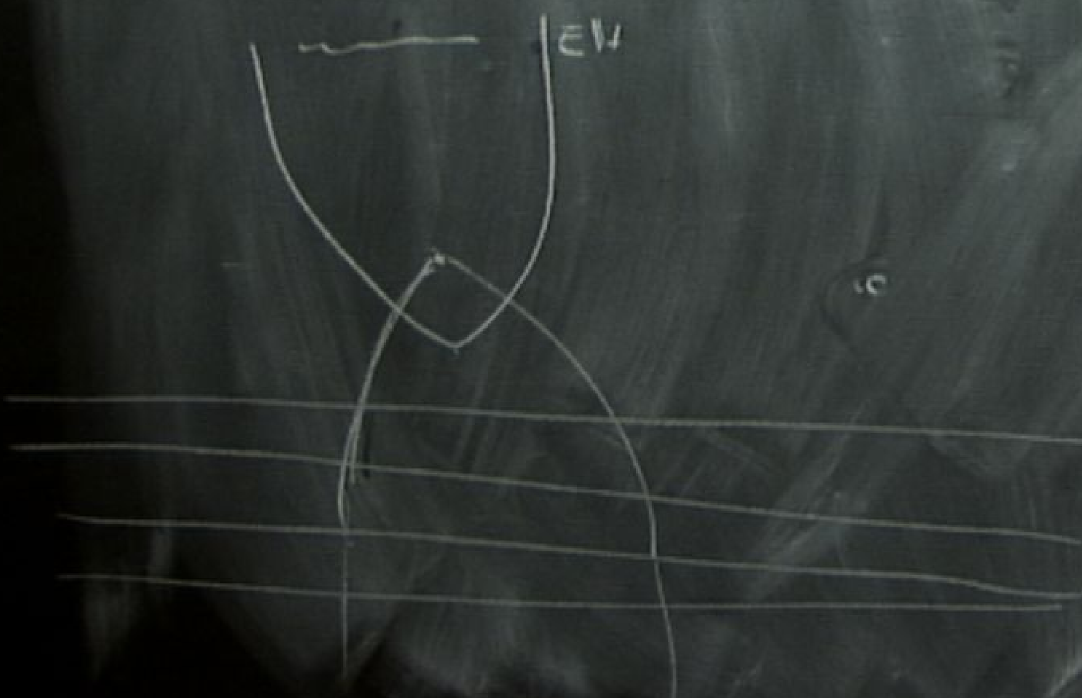
$$\partial_t H_b$$

$R_{ab} = 0$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \underline{\nabla_{(a} \Gamma_{b)}} + g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma \ldots$$

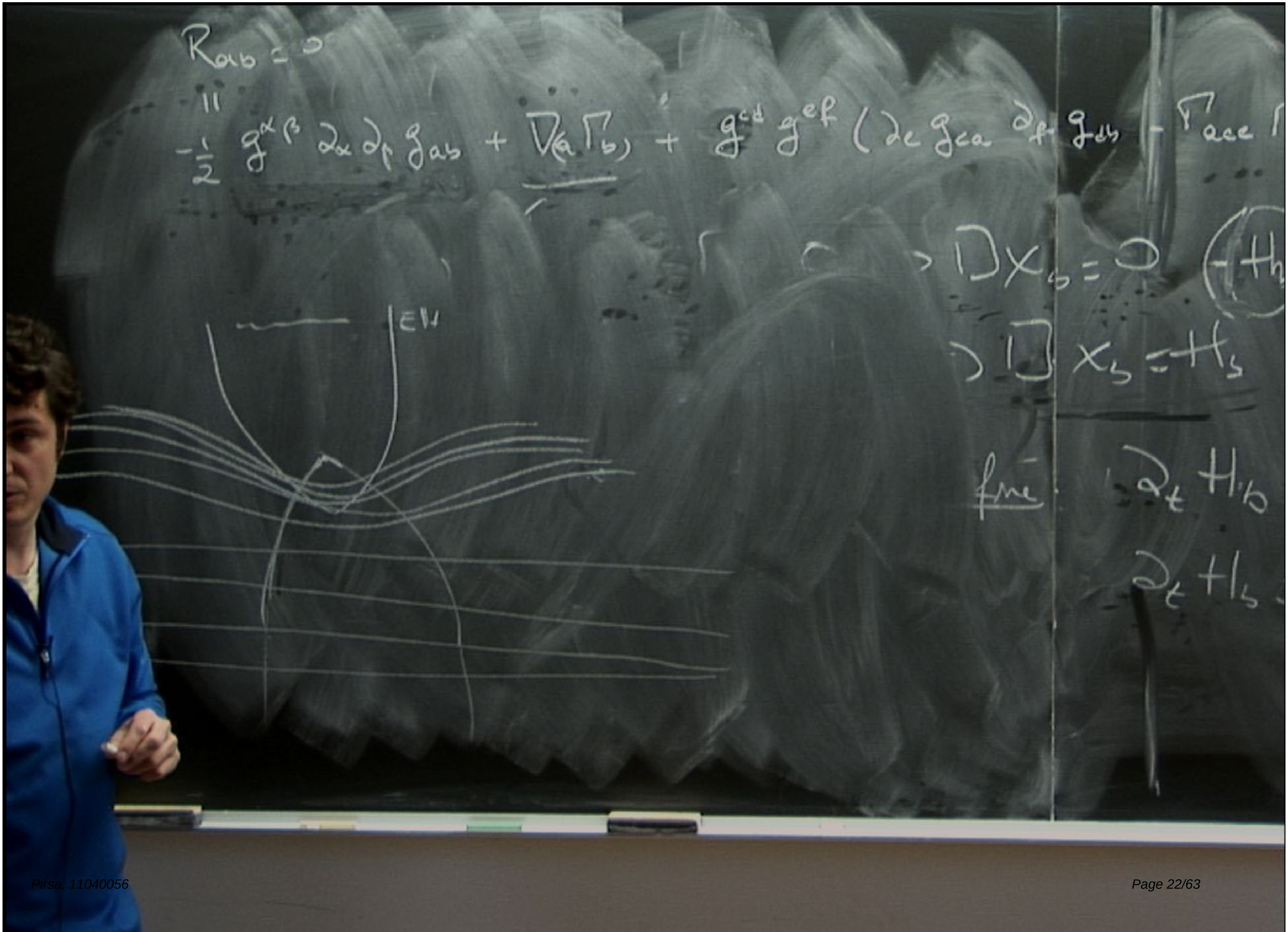
$\Rightarrow \Box x_b = 0 \quad (-H_b)$

$\Rightarrow \Box x_b = -H_b$



$\underline{fine}$: $\partial_t H_b$

$\partial_t H_b$


Rab = 0
-1/2 g^αβ ∂α ∂β gab + ∇(a Γb) + g^cd g^ef (∂e gca ∂f gdb - Γace
□ Xb = 0
□ Xb = Hb
free
∂t Hb
∂t Hb

$R_{ab} = 0$

$\parallel$

$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a} \Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma$

Excision

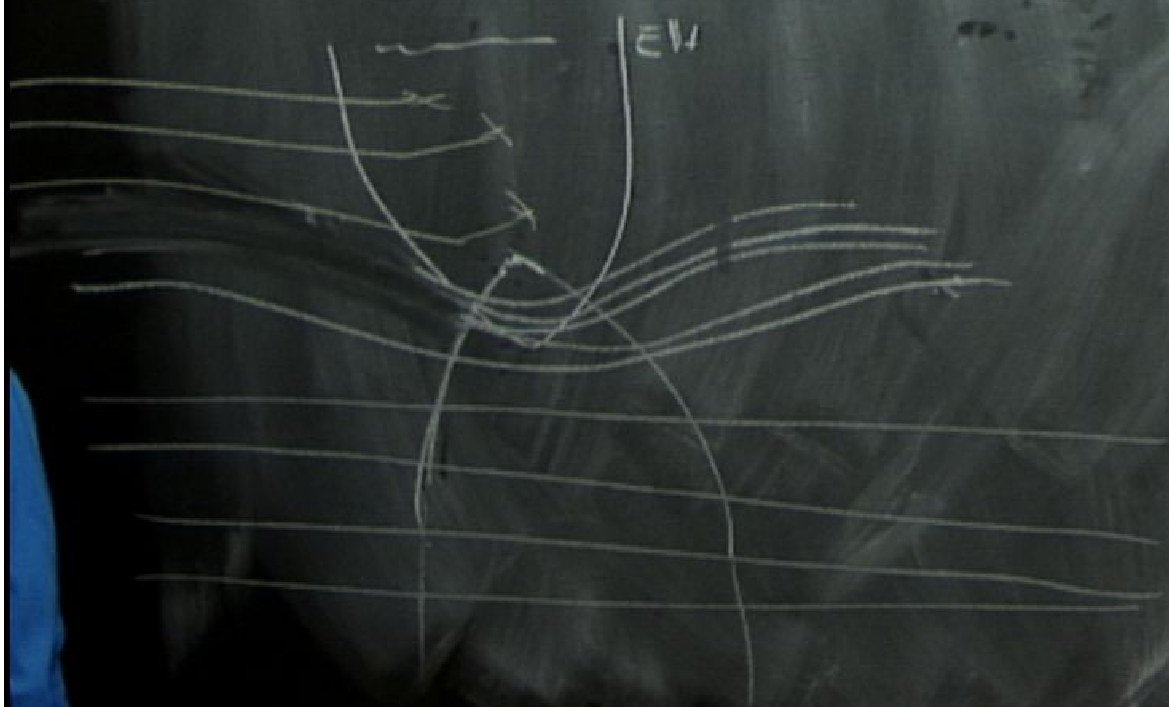
$\Box X_b = 0$

$\Box X_b = H_b$

fine

$\partial_t H_b$

$\partial_t H_b$



$R_{ab} = 0$

$\parallel$

$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \underline{\nabla_{(a}\Gamma_{b)}} + g^{cd} g^{ef} (\partial_e g_{ca} \; \partial_f g_{db} - \Gamma_{ace} \Gamma$

Excision

$\Gamma \;\; \rightarrow \Box X_b = 0$

$\rightarrow \Box X_b = H_b$

fine $\quad \partial_t H_b$

$\partial_t H_b$

$R_{ab} = 0$

$$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db} - \Gamma_{ace} \Gamma \ldots$$

Excision

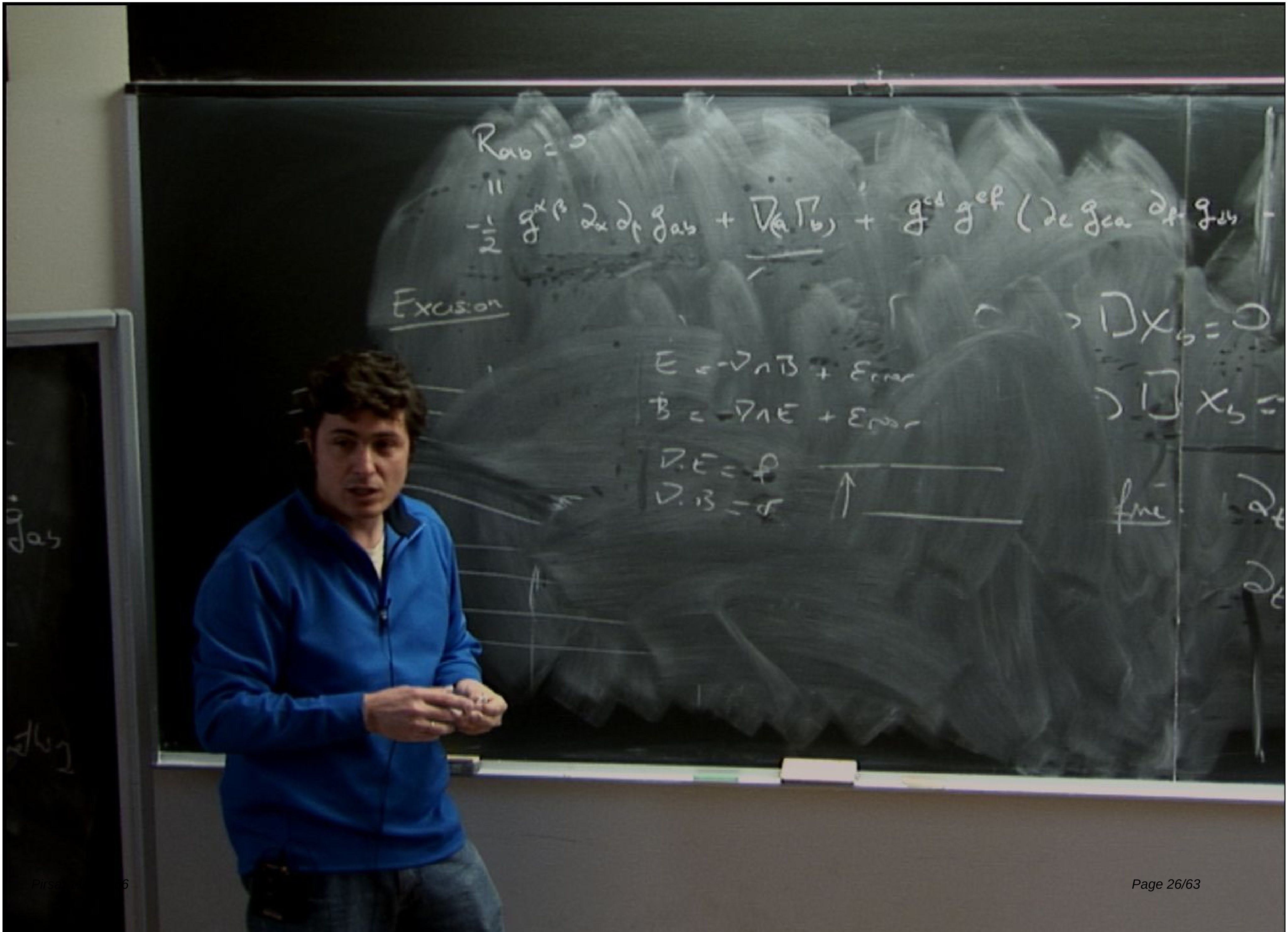
$\Box x_b = 0$

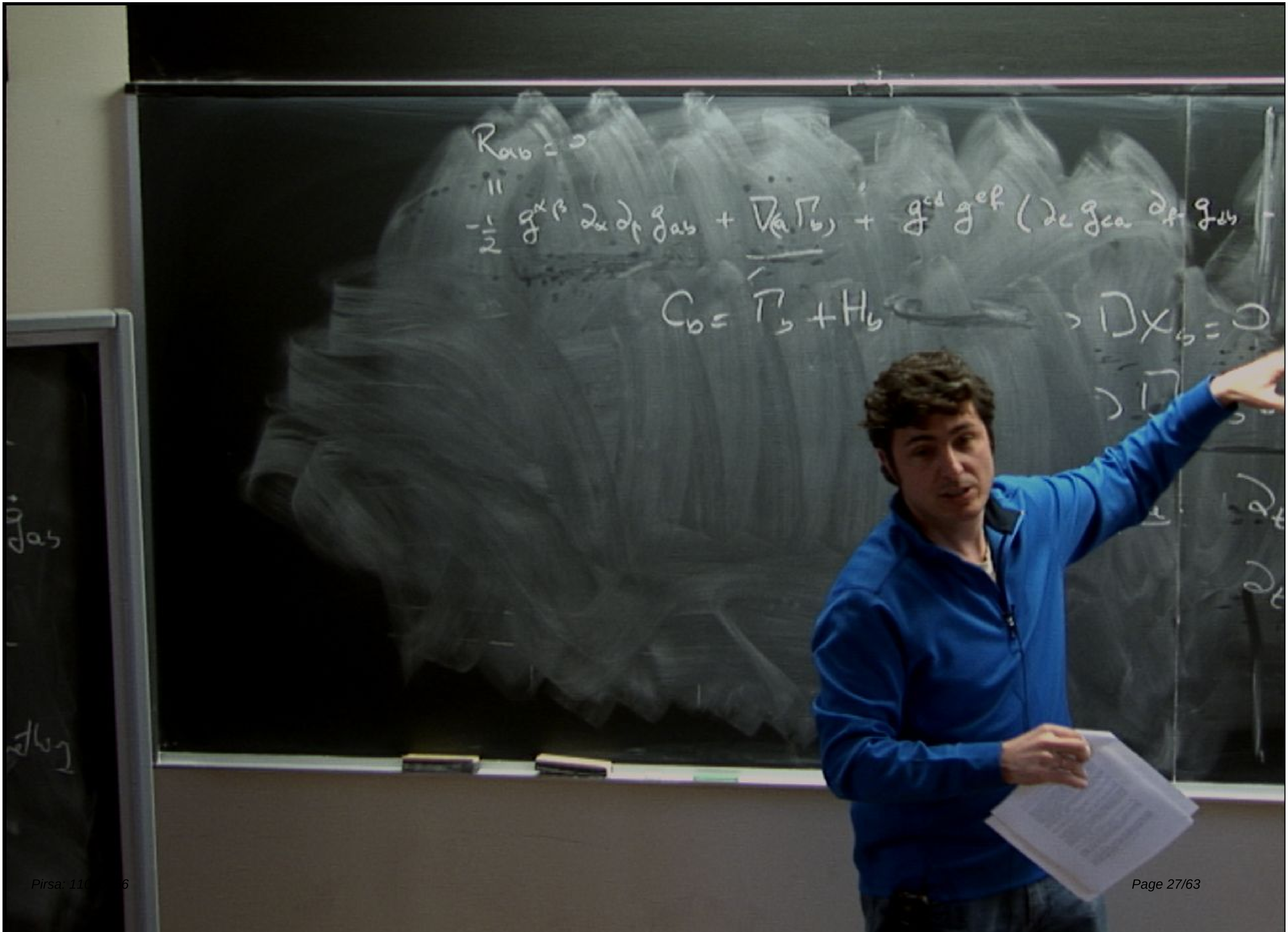
$\Box x_b = H_b$

free

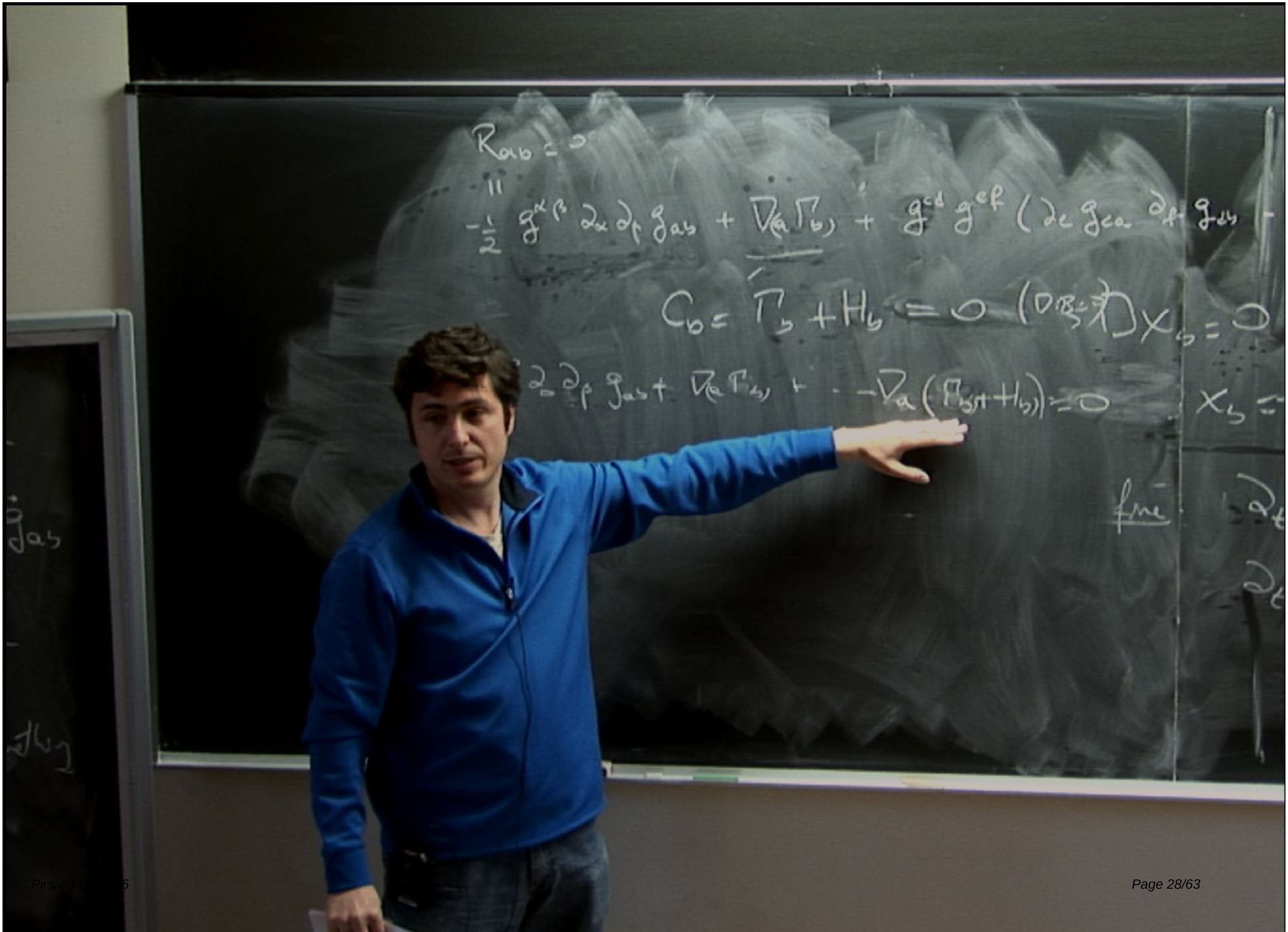
$\partial_t H_b$

$\partial_t H_b$


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \, \partial_f g_{db}$
Excision
$\dot{E} = -\nabla \wedge B + \varepsilon_{rror}$
$\dot{B} = \nabla \wedge E + \varepsilon_{rror}$
$\nabla \cdot E = \rho$
$\nabla \cdot B = 0$



$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db}$
$C_b = \Gamma_b + H_b$
$\Box X_b = 0$



$R_{ab} = 0$

$= -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \underline{\nabla_{(a}\Gamma_{b)}} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db}$

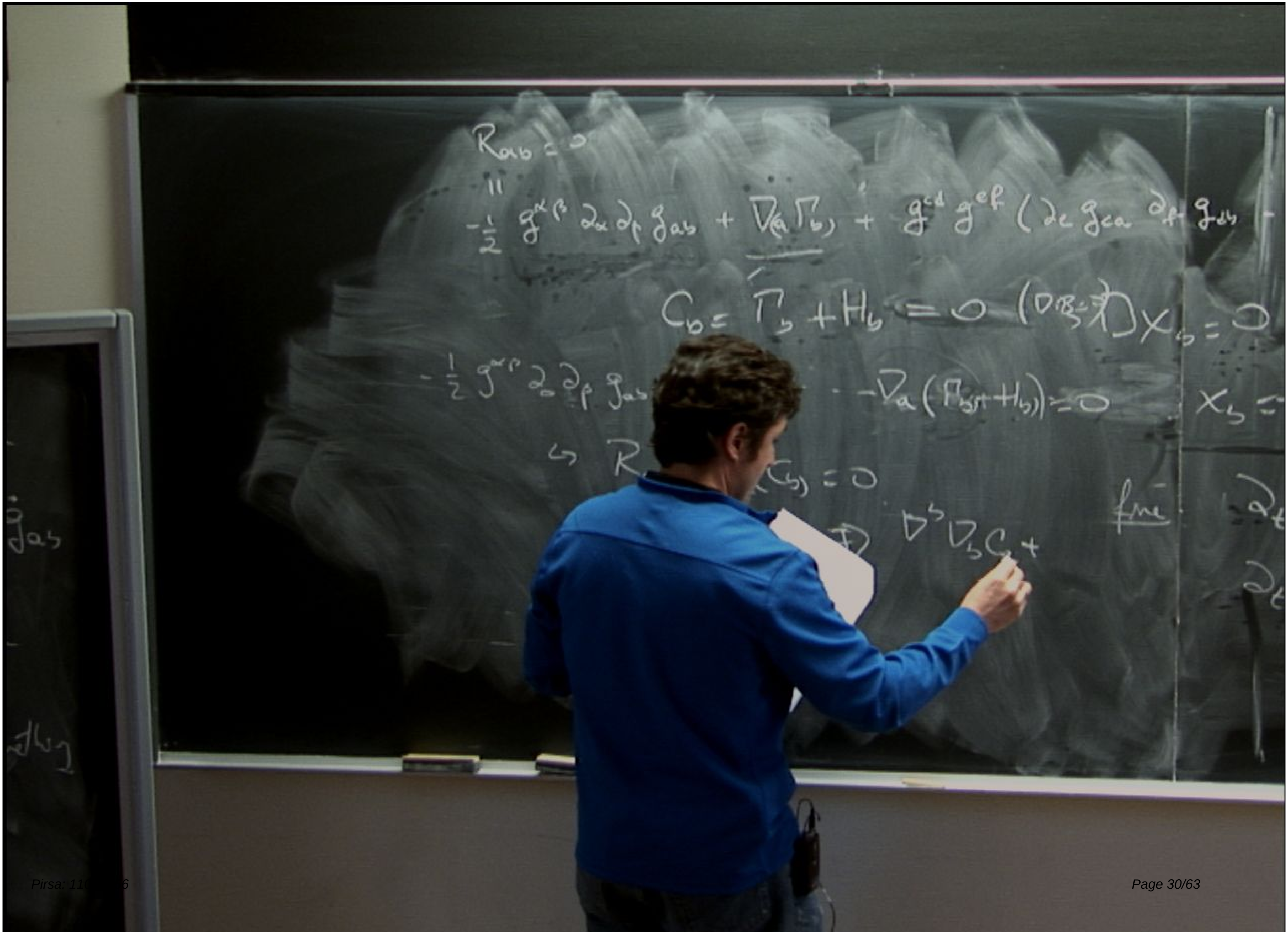
$C_b = \Gamma_b + H_b = 0$ $\quad (\Box \ldots) X_b = 0$

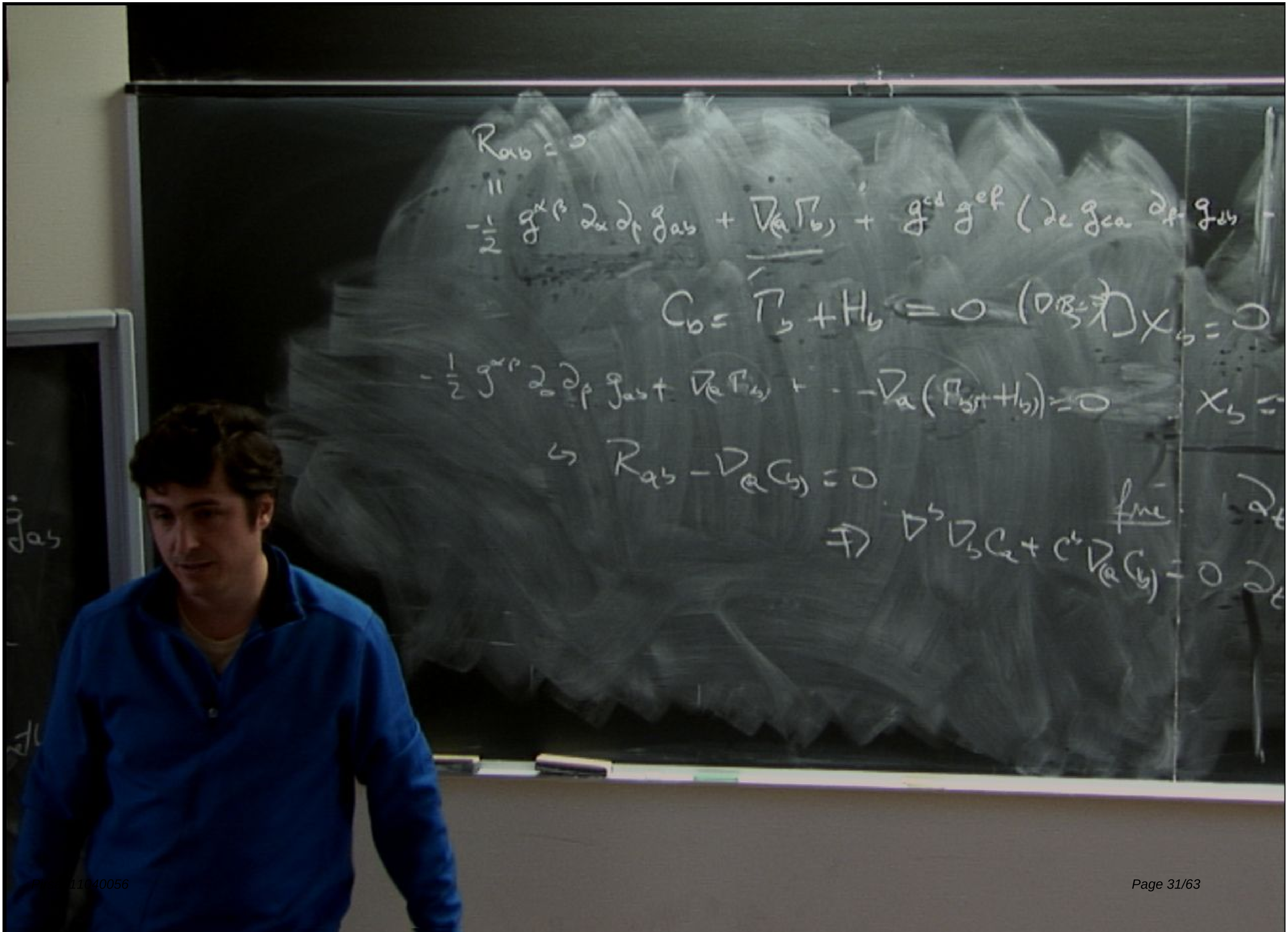
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + \ldots - \nabla_{(a}(\Gamma_{b)} + H_{b)}) = 0$ $\quad X_b =$

$\Leftrightarrow R_{ab} - \nabla_{(a} C_{b)} = 0$

$\underline{\text{fine}}$ $\quad \partial_t$

∂_t


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a} \Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db}$
$C_b = \Gamma_b + H_b = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab}$
$-\nabla_{(a} (\Gamma_{b)} + H_{b)}) = 0$
$\nabla^b \nabla_b C_a +$


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db}$
$C_b = \Gamma_b + H_b = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + \cdots - \nabla_{(a}(\Gamma_{b)} + H_{b)}) = 0$
$\hookrightarrow R_{ab} - \nabla_{(a}C_{b)} = 0$
$\Rightarrow \nabla^b \nabla_b C_a + C^b \nabla_{(a} C_{b)} = 0$
fine

$\partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$

$\Box X_b = 0 \quad -H_b = \Gamma_b \equiv \Box X_b$

$X_b = -H_b$

$\nabla_{(a} C_{b)} = 0 \rightsquigarrow \nabla^b \nabla_b C_a = 0$

Coord condition

1) Harmonic coords. $\Gamma_b = 0 = \Box X_b$

2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \Box X_b$

$\partial_f g_{db} - \Gamma_{ace}\Gamma_{bdf})$

$\Box X_b = 0 \quad H_b = \Gamma_b \equiv \Box X_b$

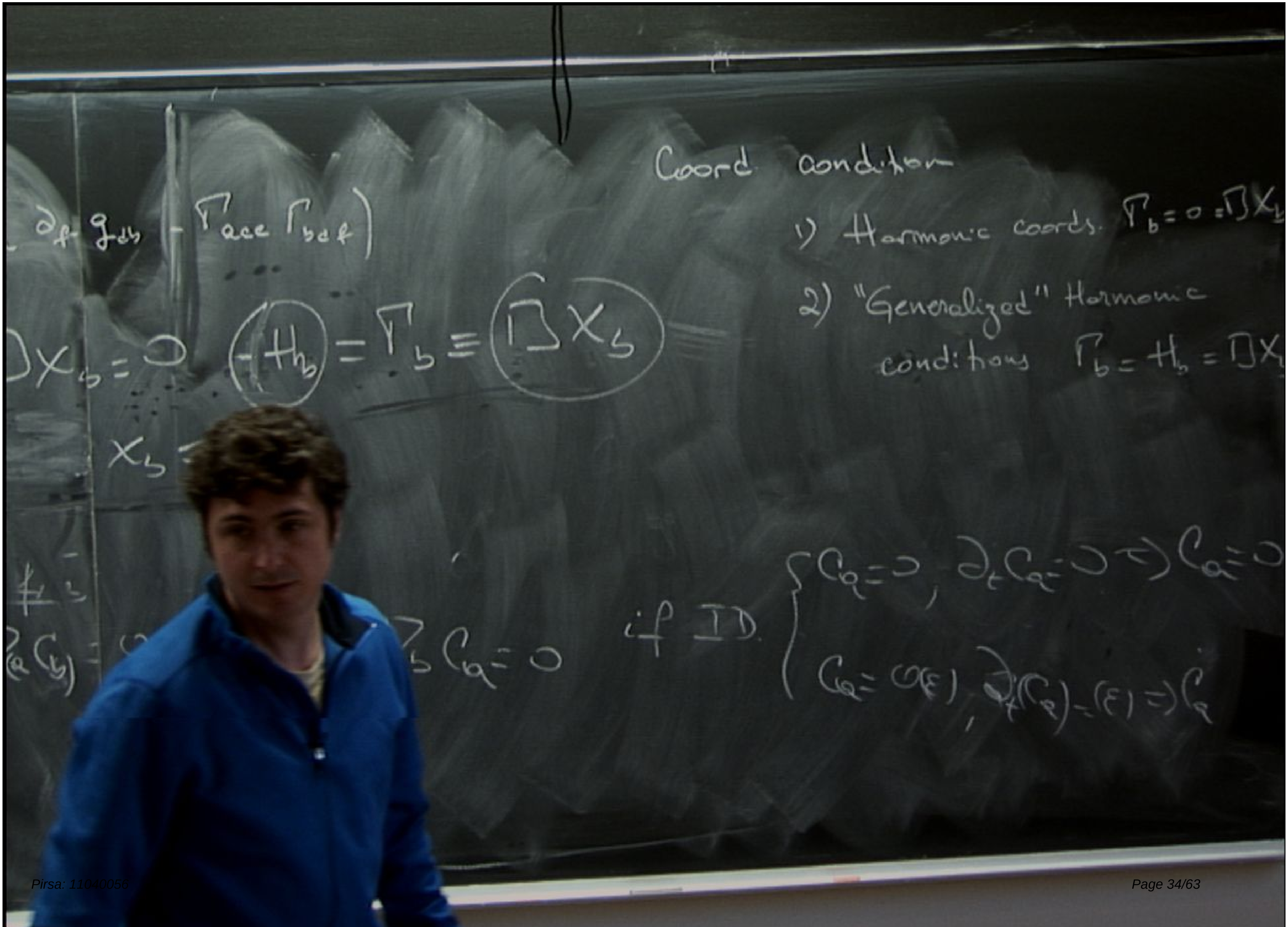
$X_b = H_b$

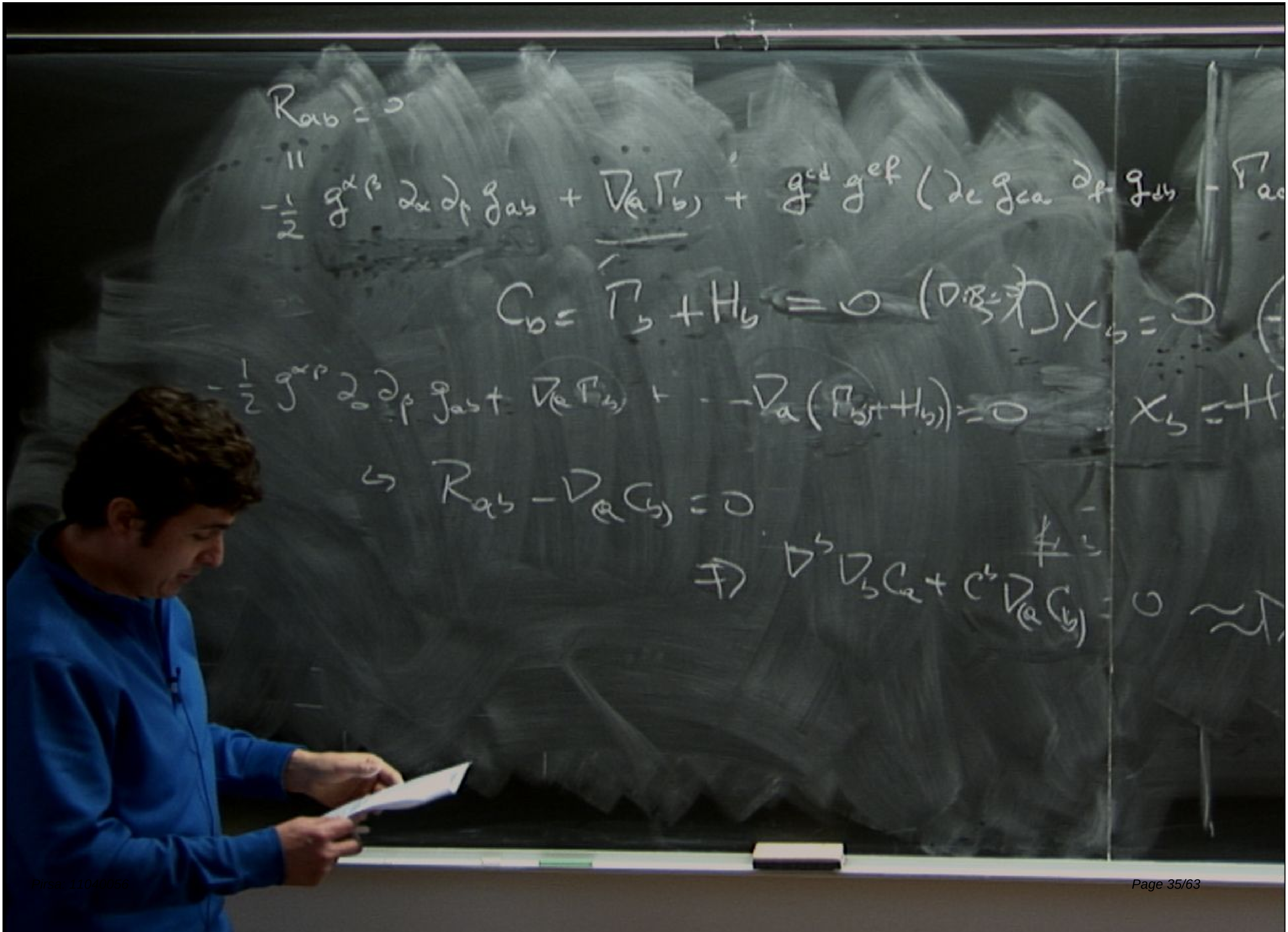
$\nabla_a C_b) = 0 \rightsquigarrow \nabla^b \nabla_b C_a = 0$ if ID $\{ C_a = 0,\ \partial_t C_a = 0 \Rightarrow C_a = 0$

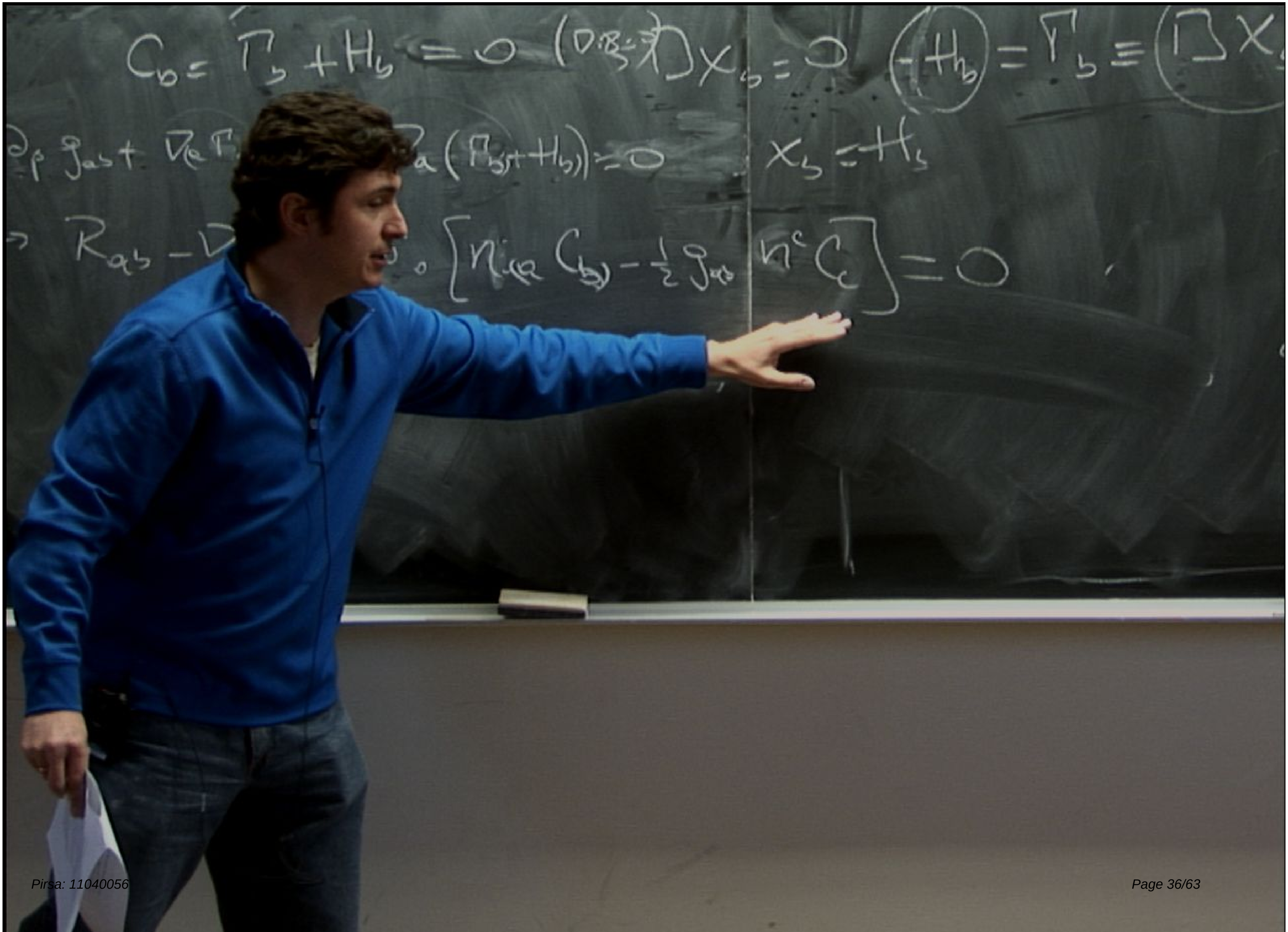
Coord condition

1) Harmonic coords. $\Gamma_b = 0 = \Box X_b$

2) "Generalized" Harmonic conditions $\Gamma_b = H_b = \Box X_b$


Coord condition
1) Harmonic coords. Γb = 0 = □Xb
2) "Generalized" Harmonic
conditions Γb = Hb = □Xb
H b = Γb ≡ □Xb
if ID
Ca = 0, ∂t Ca = 0 => Ca = 0
Ca = O(ε), ∂t(Ca) = O(ε) => Ca


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db}$
$C_b = \Gamma_b + H_b = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + \dots - \nabla_a(\Gamma_b + H_b)) = 0$
$\Leftrightarrow R_{ab} - \nabla_{(a} C_{b)} = 0$
$\Rightarrow \nabla^b \nabla_b C_a + C^b \nabla_{(a} C_{b)} = 0$

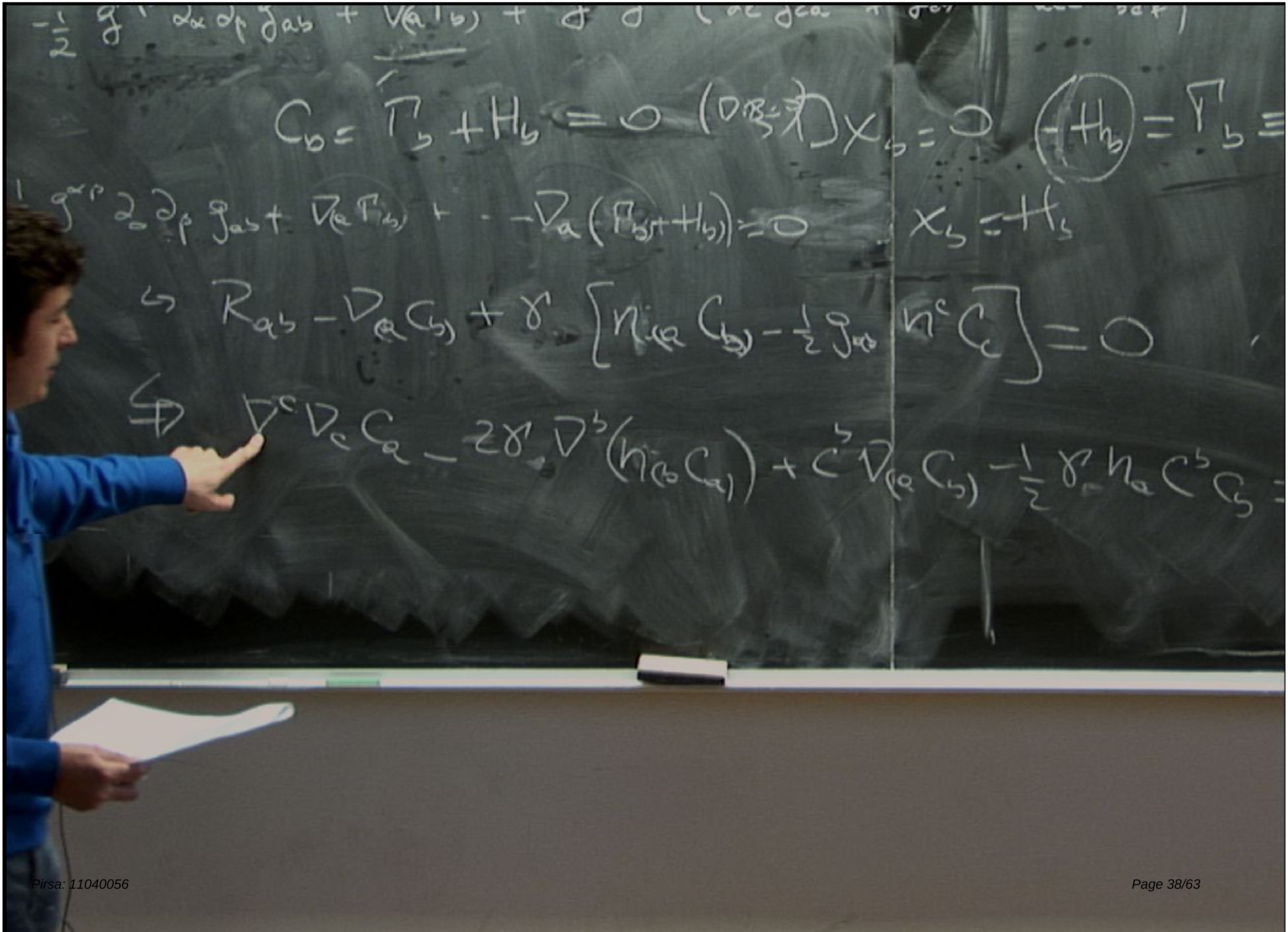


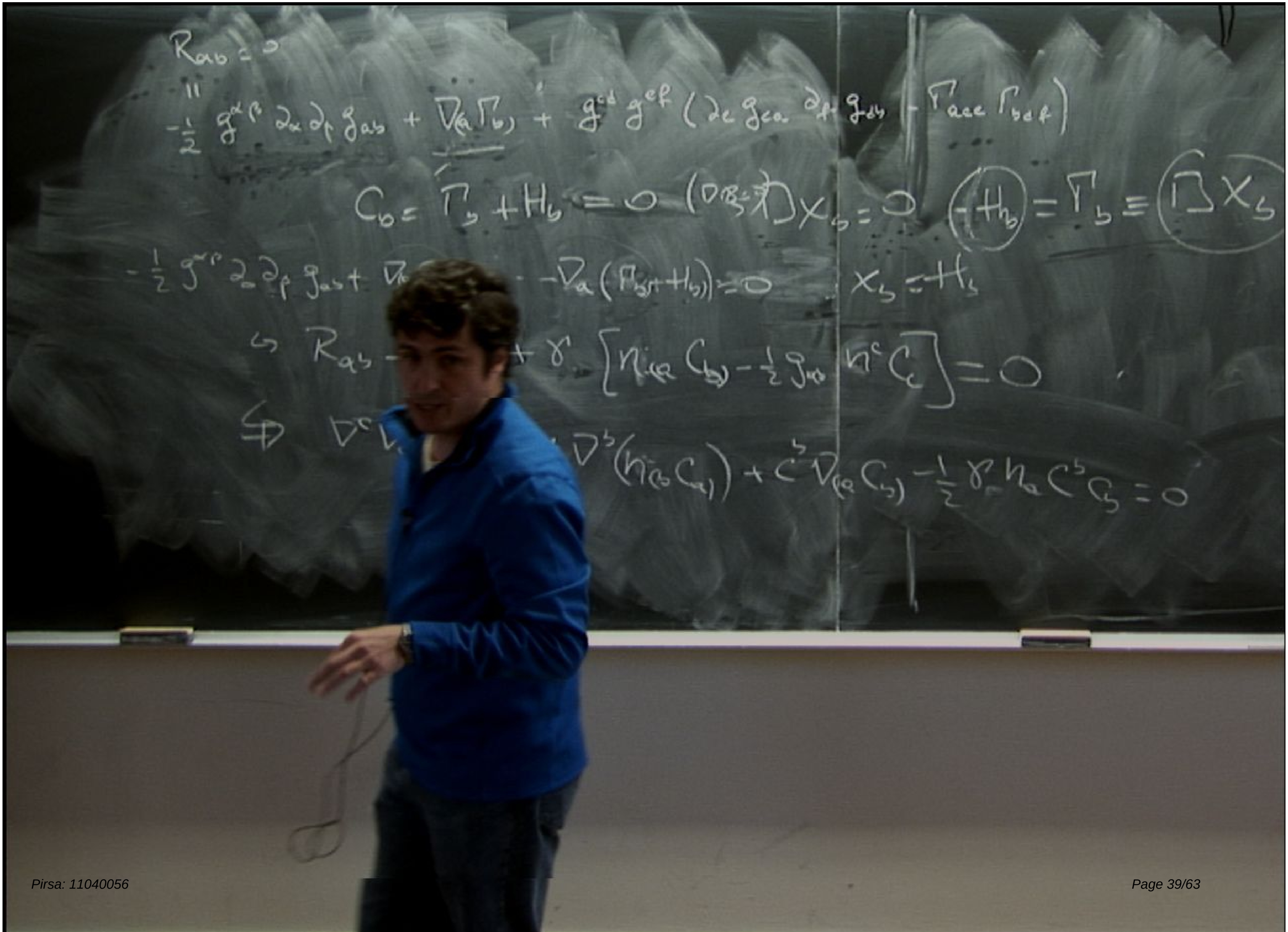
$C_b = \Gamma_b + H_b = 0$
$X_b = -H_b$

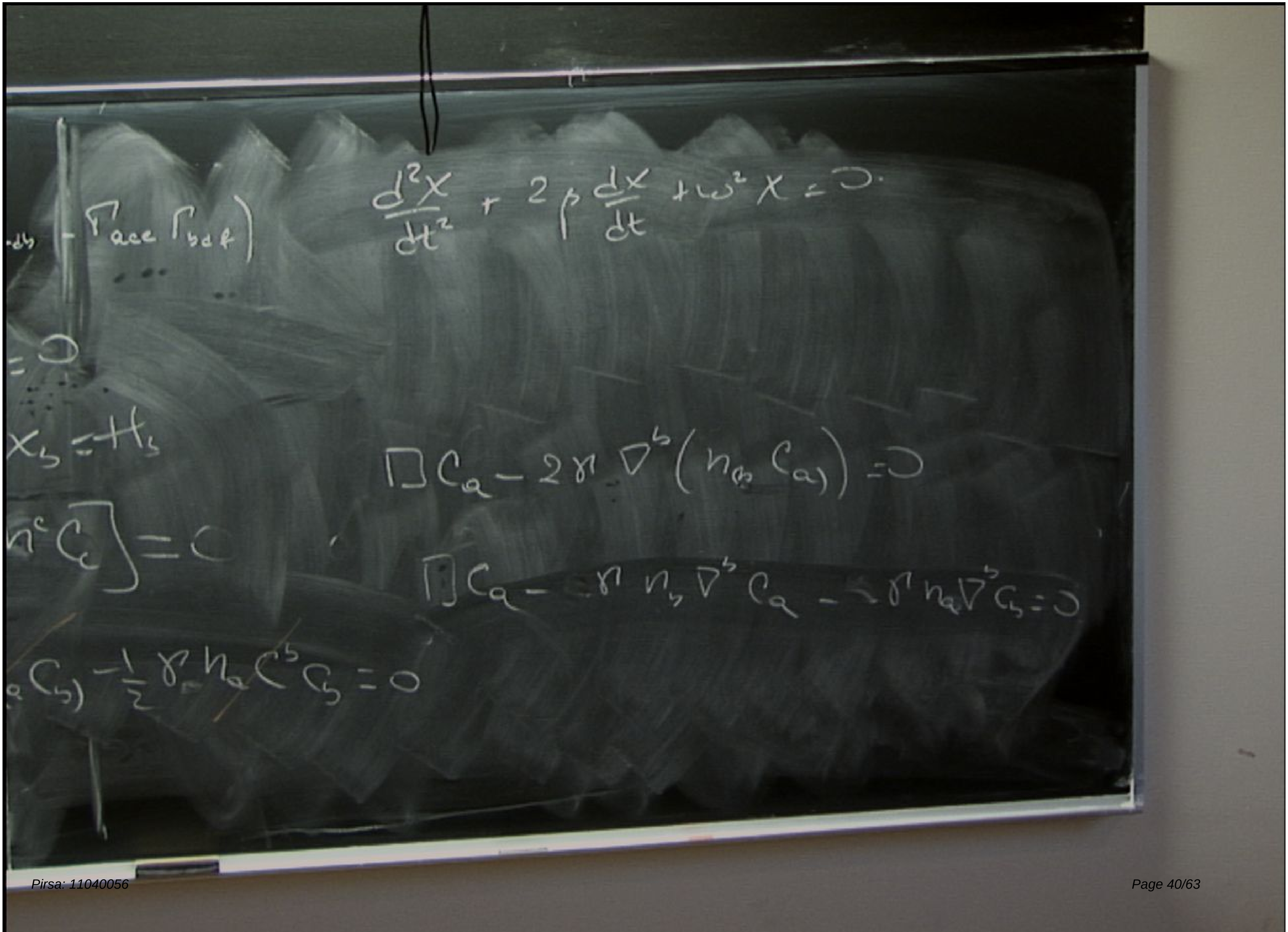
$$C_b = \Gamma_b + H_b = 0 \quad (\nabla\cdot\beta = ?) \quad \Box x_b = 0 \quad -H_b = \Gamma_b \equiv \Box x_b$$

$$\partial_t g_{ab} + \nabla_{(a}\Gamma_{b)} + \cdots - \nabla_{(a}(\Gamma_{b)} + H_{b)}) = 0 \qquad x_b = H_b$$

$$\Rightarrow R_{ab} - \nabla_{(a}C_{b)} + \gamma\left[n_{(a}C_{b)} - \tfrac{1}{2}g_{ab}\, n^c C_c\right] = 0$$


$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} +$
$C_b = \Gamma_b + H_b = 0$
$x_b = 0$
$H_b = \Gamma_b =$
$g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + \cdots - \nabla_a(\Gamma_{b)} + H_{b)}) = 0$
$x_b = H_b$
$R_{ab} - \nabla_{(a}C_{b)} + \gamma \left[n_{(a}C_{b)} - \frac{1}{2} g_{ab} n^c C_c \right] = 0$
$\nabla^c \nabla_c C_a - 2\gamma \nabla^b (n_{(b}C_{a)}) + C^b \nabla_{(a}C_{b)} - \frac{1}{2} \gamma n_a C^b C_b =$


$R_{ab} = 0$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd} g^{ef} (\partial_e g_{ca} \partial_f g_{db} - \Gamma_{ace} \Gamma_{bdf})$
$C_b = \Gamma_b + H_b = 0$
$\Box X_b = 0$
$-H_b = \Gamma_b \equiv \Box X_b$
$-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} +$
$- \nabla_{(a} (\Gamma_{b)} + H_{b)}) = 0$
$X_b = -H_b$
$\Leftrightarrow R_{ab}$
$+ \gamma \left[n_{(a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c \right] = 0$
$\Rightarrow \nabla^c$
$\nabla^b (n_{(b} C_{a)}) + C^b \nabla_{(a} C_{b)} - \frac{1}{2} \gamma n_a C^b C_b = 0$


$\frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega^2 x = 0.$
$\Gamma_{ace} \Gamma_{bdf})$
$= 0$
$X_b = H_b$
$n^c C_c] = 0$
$\Box C_a - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\Box C_a - \gamma n_b \nabla^b C_a - \gamma n_a \nabla^b C_b = 0$
$_a C_b) - \frac{1}{2} \gamma n_a \nabla^b C_b = 0$

$\left. - \Gamma_{ace}\Gamma_{bdf} \right)$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0.$$

$= 0$

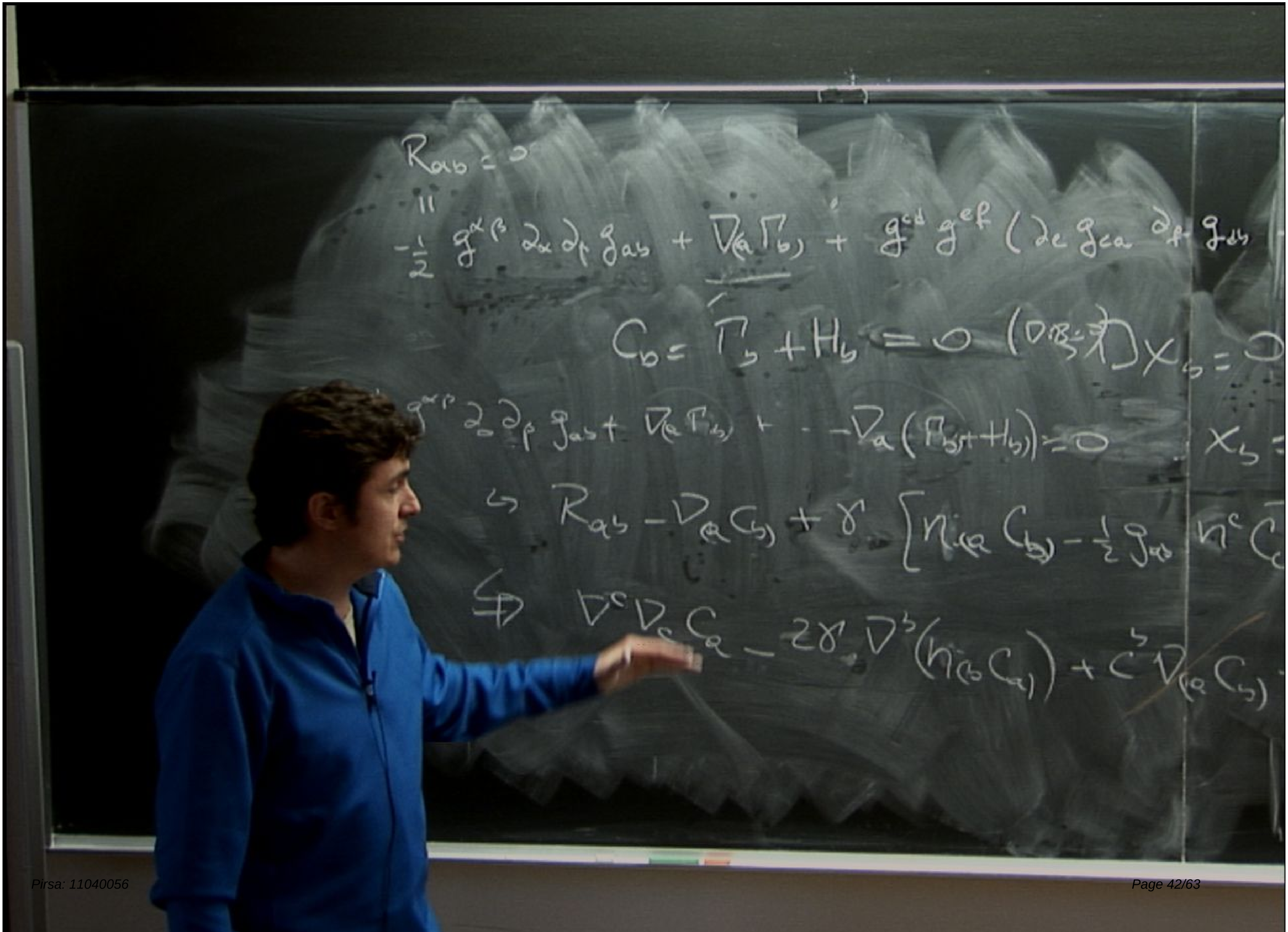
$X_b = H_b$

$$\Box C_a - 2\gamma \nabla^b \left(n_{(b} C_{a)} \right) = 0$$

$\left. n^c C_c \right] = 0$

$$\Box C_a - \gamma n_b \nabla^b C_a - \gamma n_a \nabla^b C_b = 0$$

$_{a} C_{b)} - \frac{1}{2}\gamma n_a \nabla^b C_b = 0$

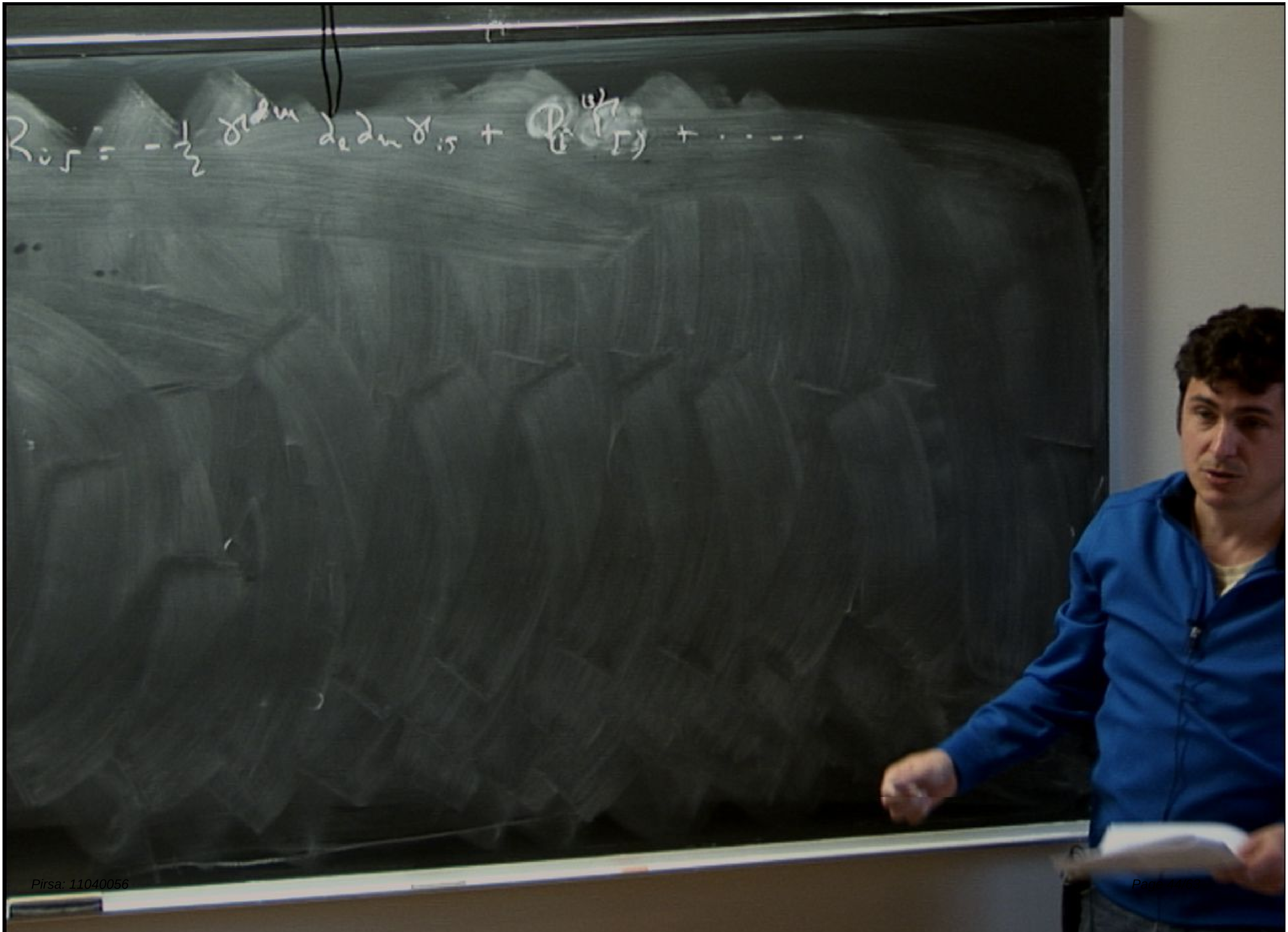

$R_{ab} = 0$
$C_b = \Gamma_b + H_b = 0$

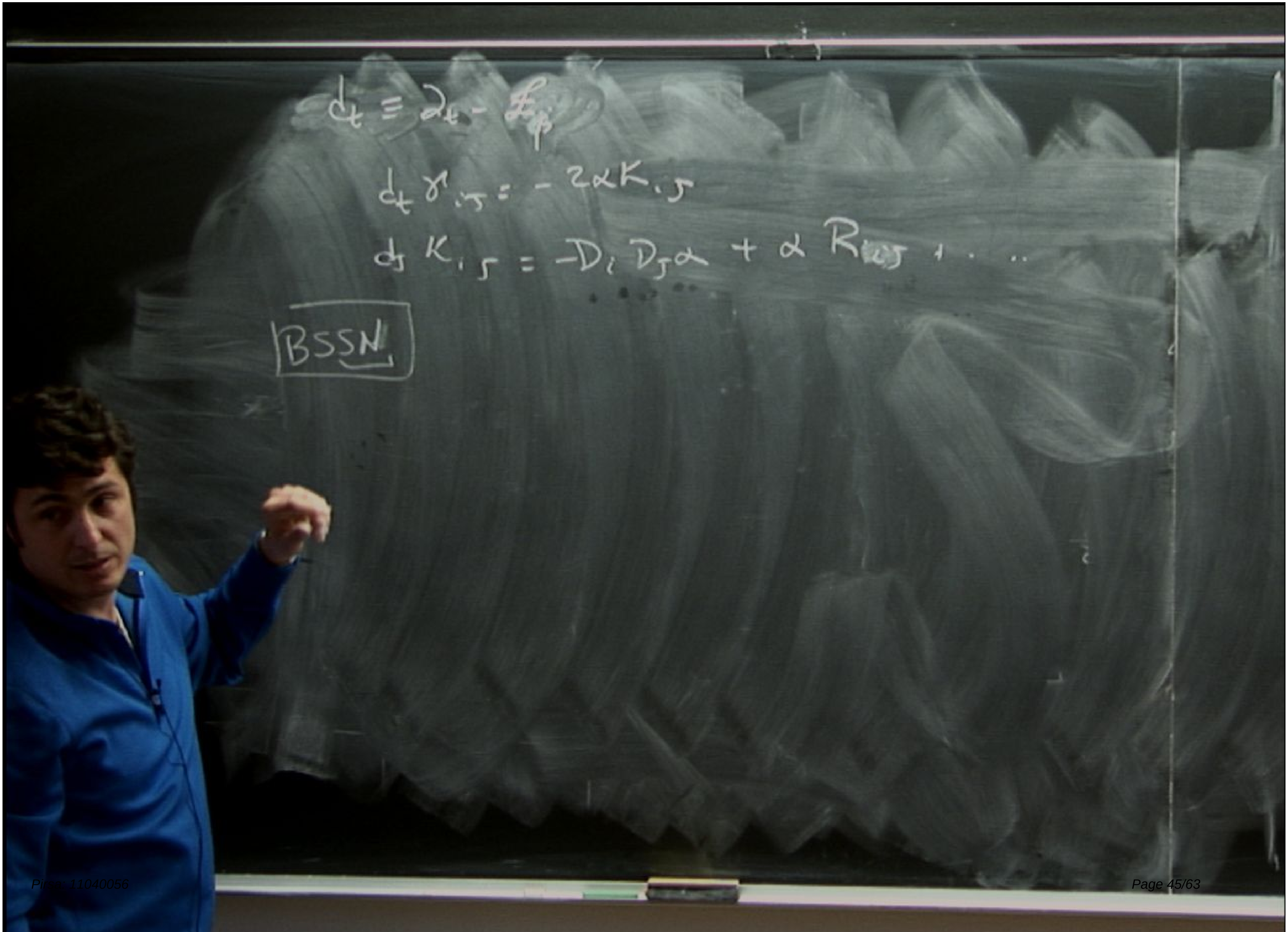
$$d_t \equiv \partial_t - \mathcal{L}_\beta$$

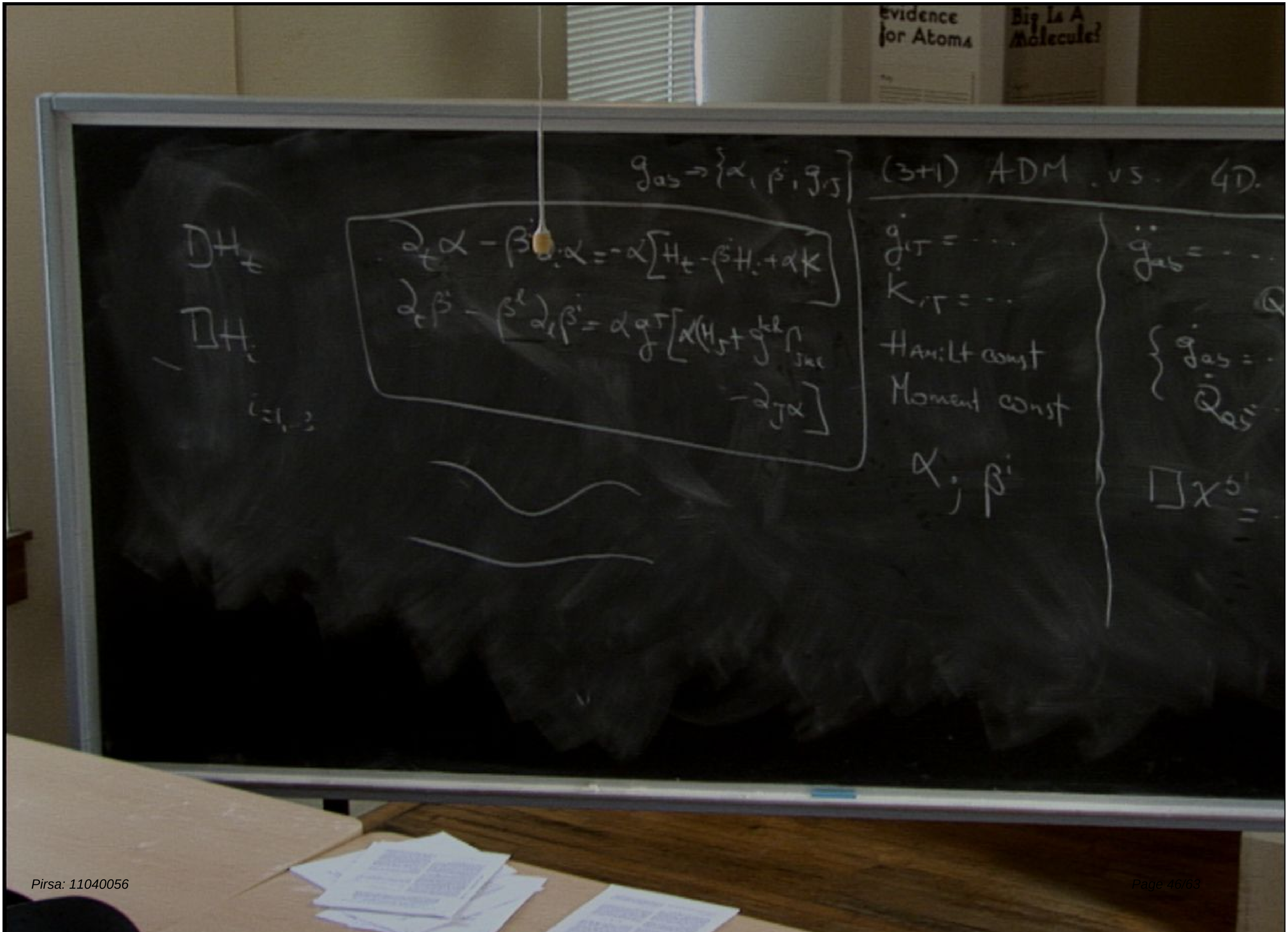
$$d_t \gamma_{ij} = -2\alpha K_{ij}$$

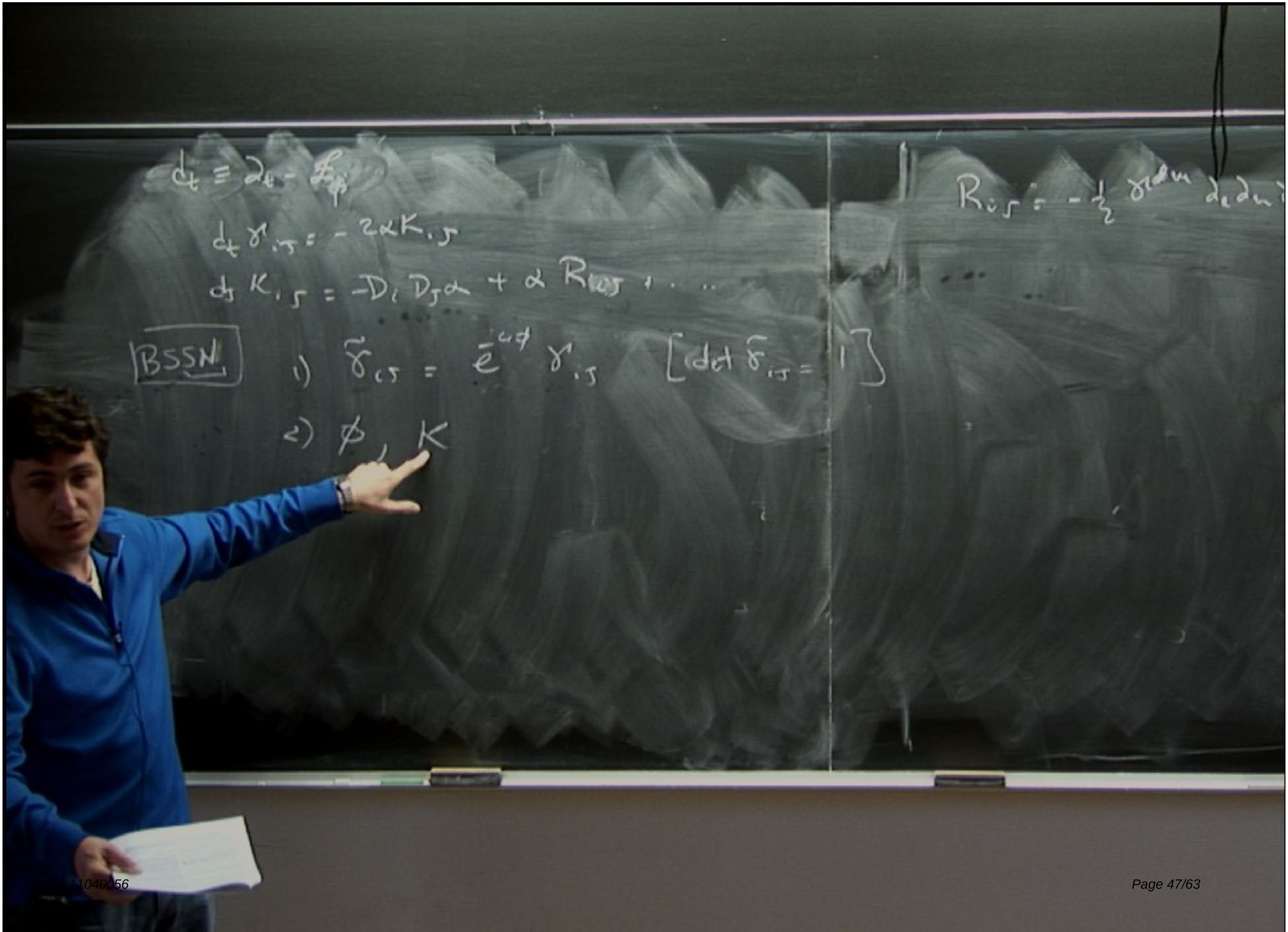
$$d_t K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

$R_{ij} =$


$R_{ij} = -\frac{1}{2}\gamma^{km}\partial_k\partial_m\gamma_{ij} + \dots + \dots$


$d_t \equiv \partial_t - \mathcal{L}_\beta$
$d_t \gamma_{ij} = -2\alpha K_{ij}$
$d_t K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \ldots$
BSSN


Evidence for Atoms
(3+1) ADM vs. 4D.
Hamilt const
Moment const


$d_t = \partial_t - \mathcal{L}_\beta$
$d_t \gamma_{ij} = -2\alpha K_{ij}$
$d_t K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \ldots$
BSSN
1) $\tilde{\gamma}_{ij} = e^{-4\phi} \gamma_{ij}$ $[\det \tilde{\gamma}_{ij} = 1]$
2) ϕ, K
$R_{ij} = -\frac{1}{2} \gamma^{lm} \partial_l \partial_m$

$$d_t \equiv \partial_t - \mathcal{L}_\beta$$

$$d_t \gamma_{ij} = -2\alpha K_{ij}$$

$$d_t K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \dots$$

$$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m \dots$$

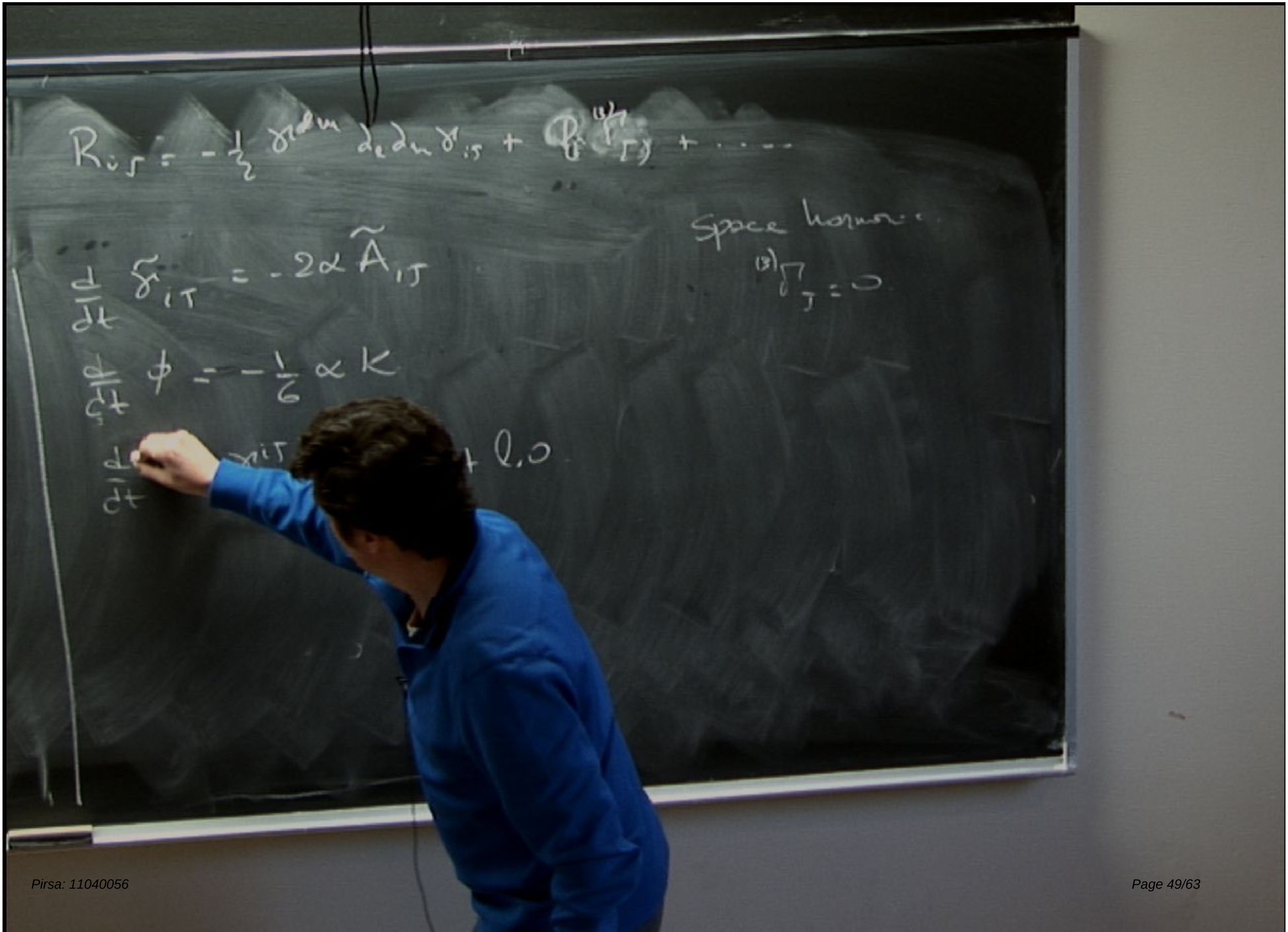
$\boxed{\text{BSSN}}$

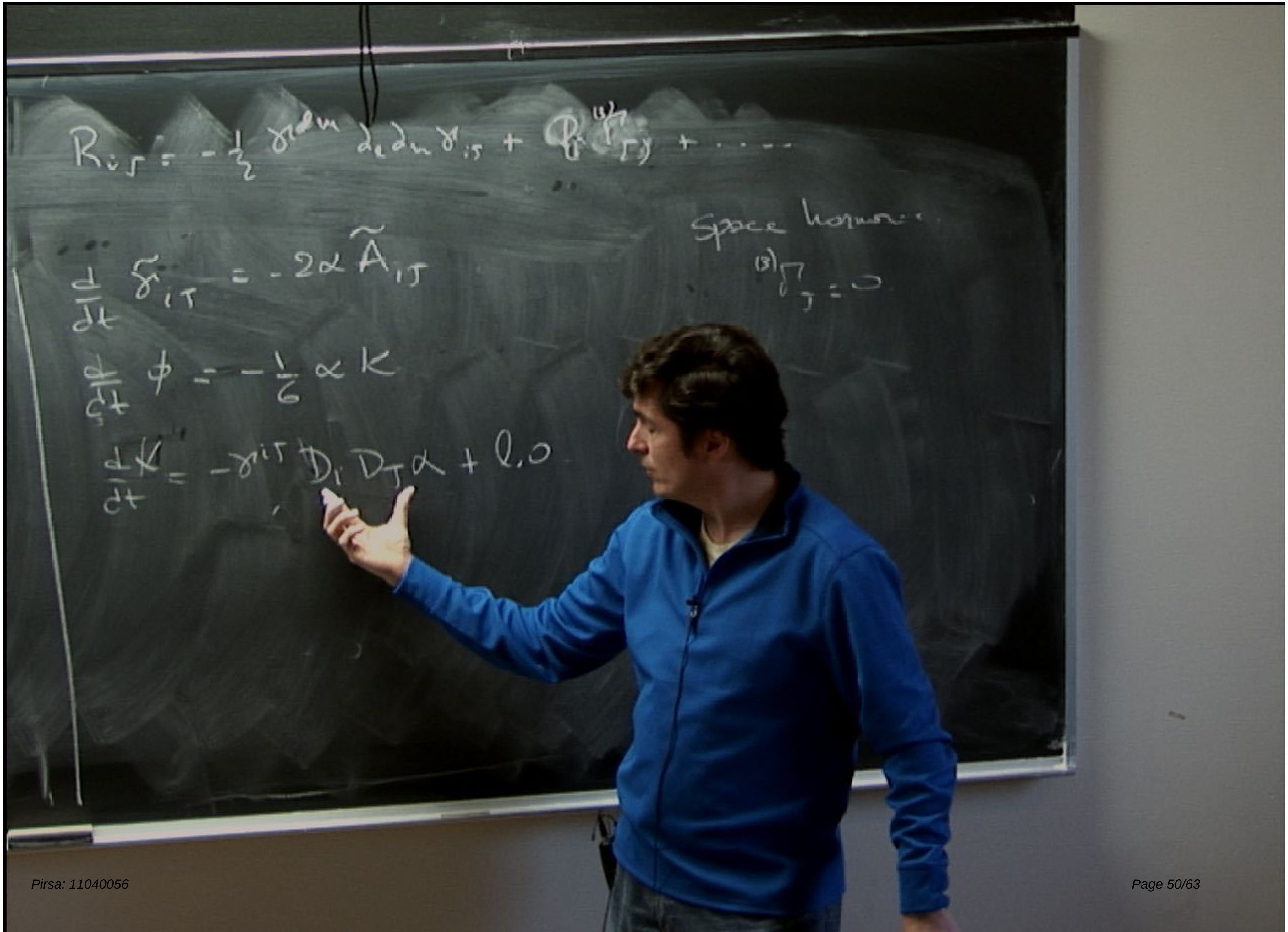
ADM	BSSN
γ_{ij}	$\tilde\gamma_{ij}$; ϕ
K_{ij}	K, A_{ij}

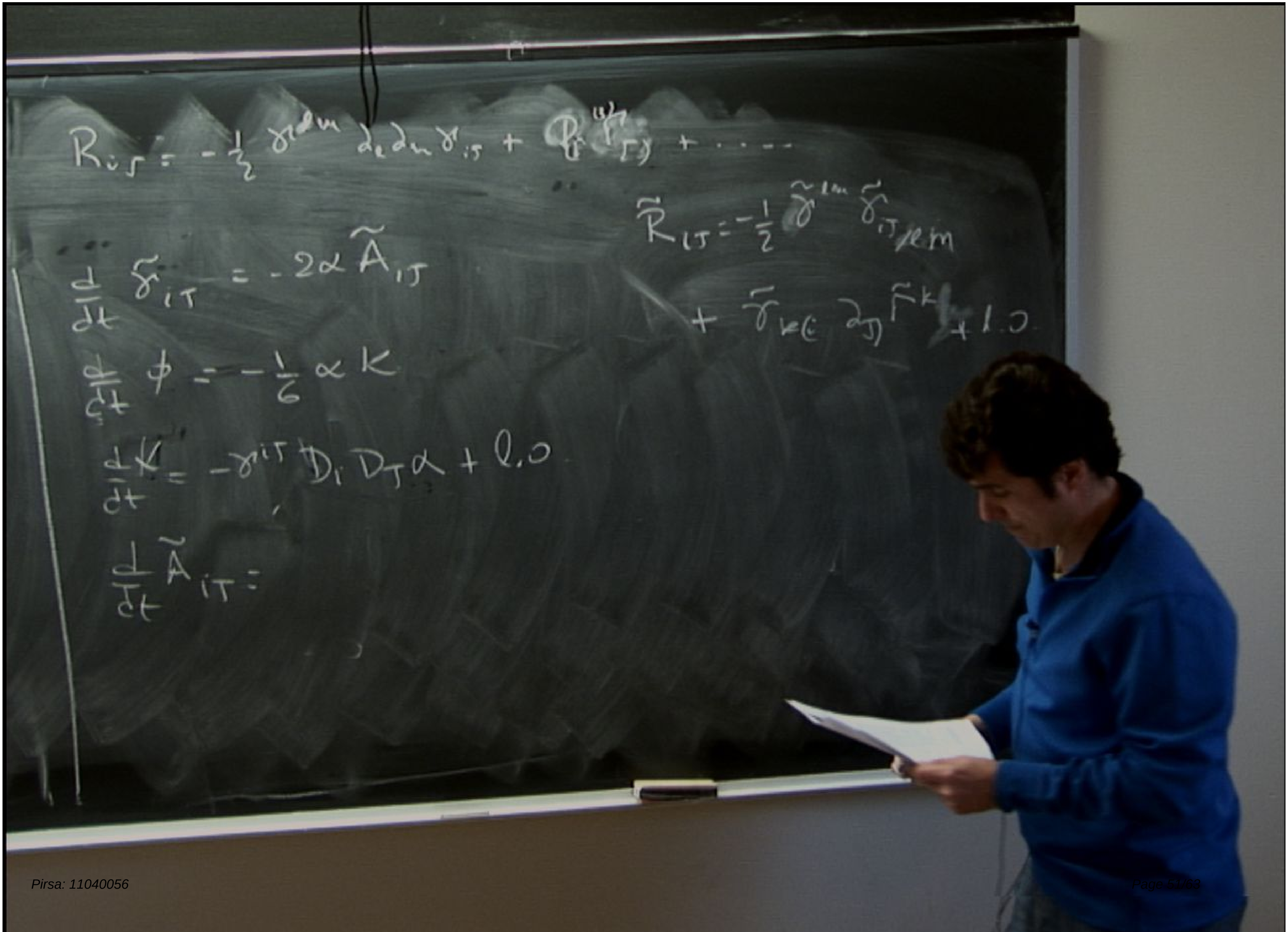
1) $\tilde\gamma_{ij} = e^{-4\phi}\gamma_{ij}$ $\quad [\det \tilde\gamma_{ij} = 1]$

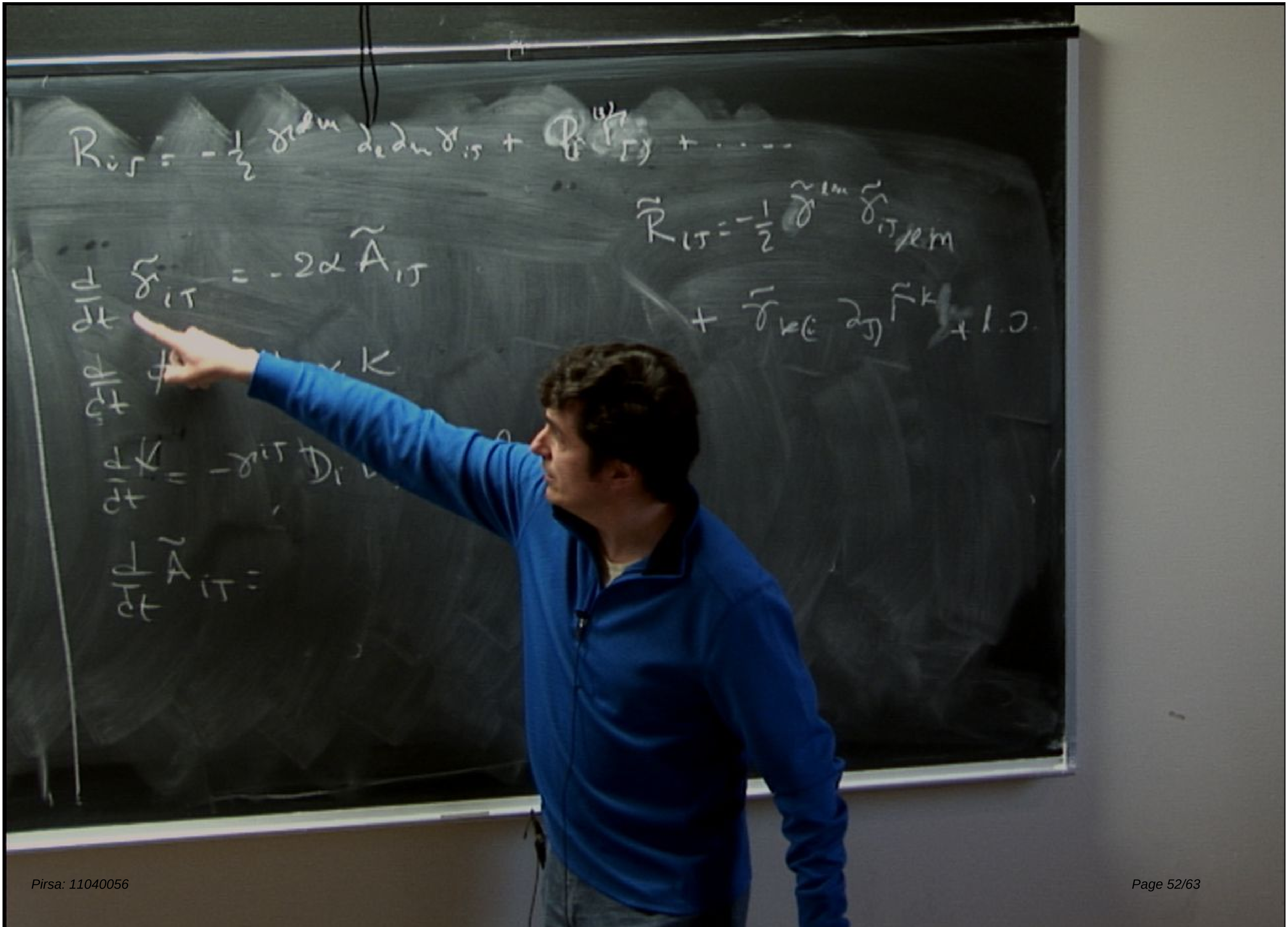
2) ϕ, K

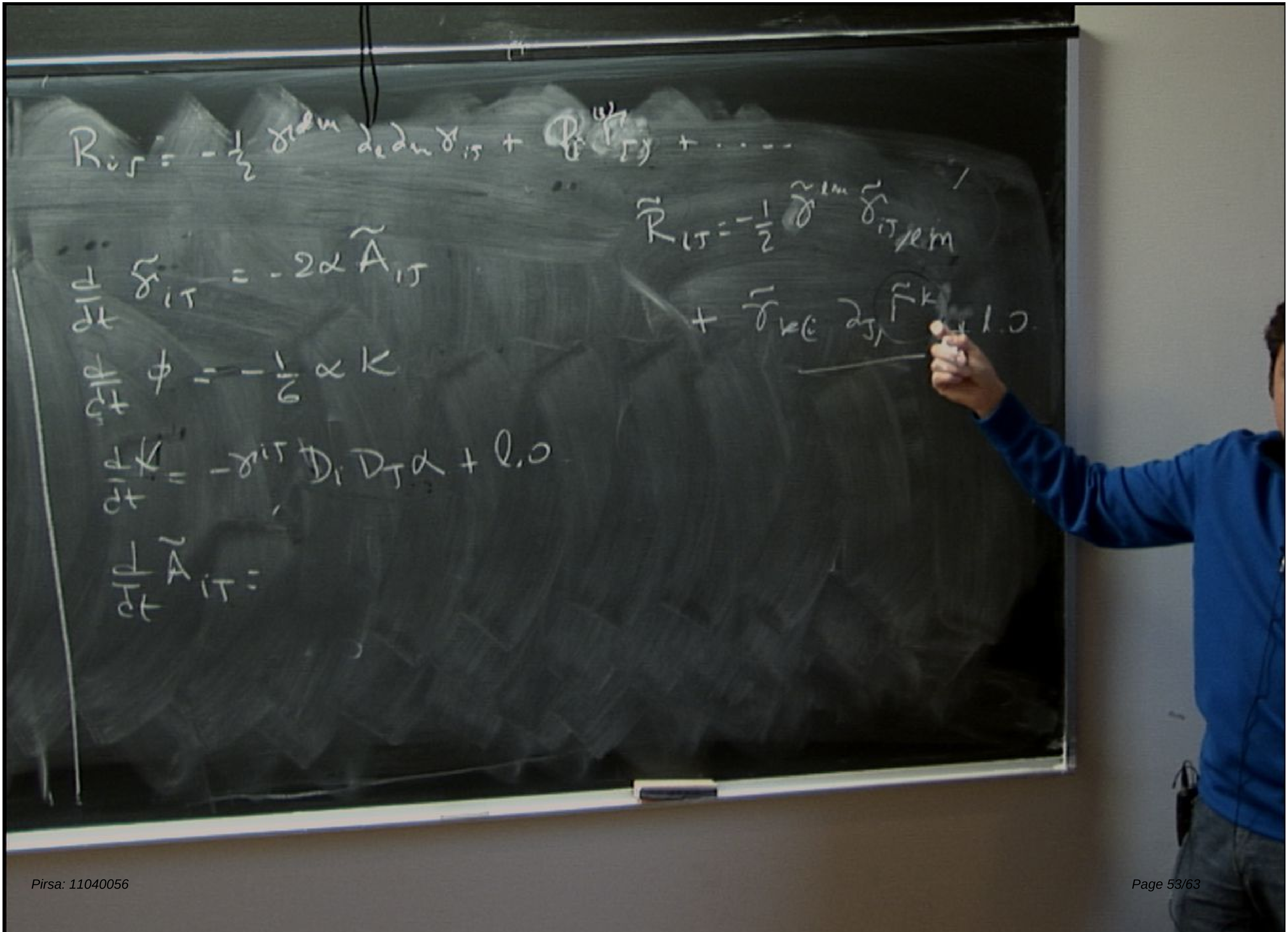
3) $A_{ij} = K_{ij} - \frac{1}{3}K\gamma_{ij}$

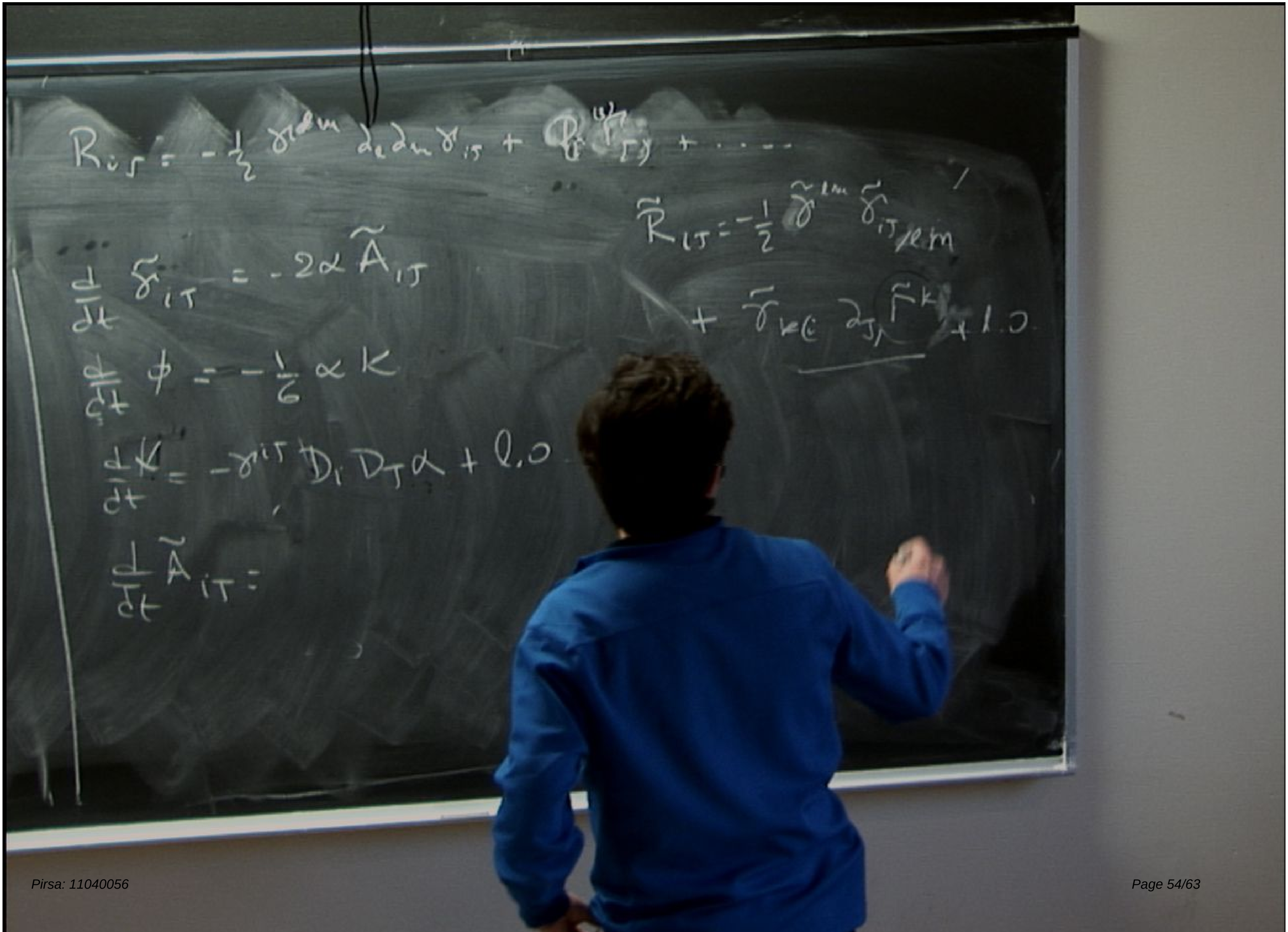

$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \ldots$
Space harmon
$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$+ \text{l.o}$


$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \partial_{(i}{}^{(3)}\Gamma_{j)} + \dots$
Space harmon
${}^{(3)}\Gamma_j = 0$
$\frac{d}{dt}\gamma_{ij} = -2\alpha\tilde{A}_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{dK}{dt} = -\gamma^{ij}D_iD_j\alpha + \text{l.o}$

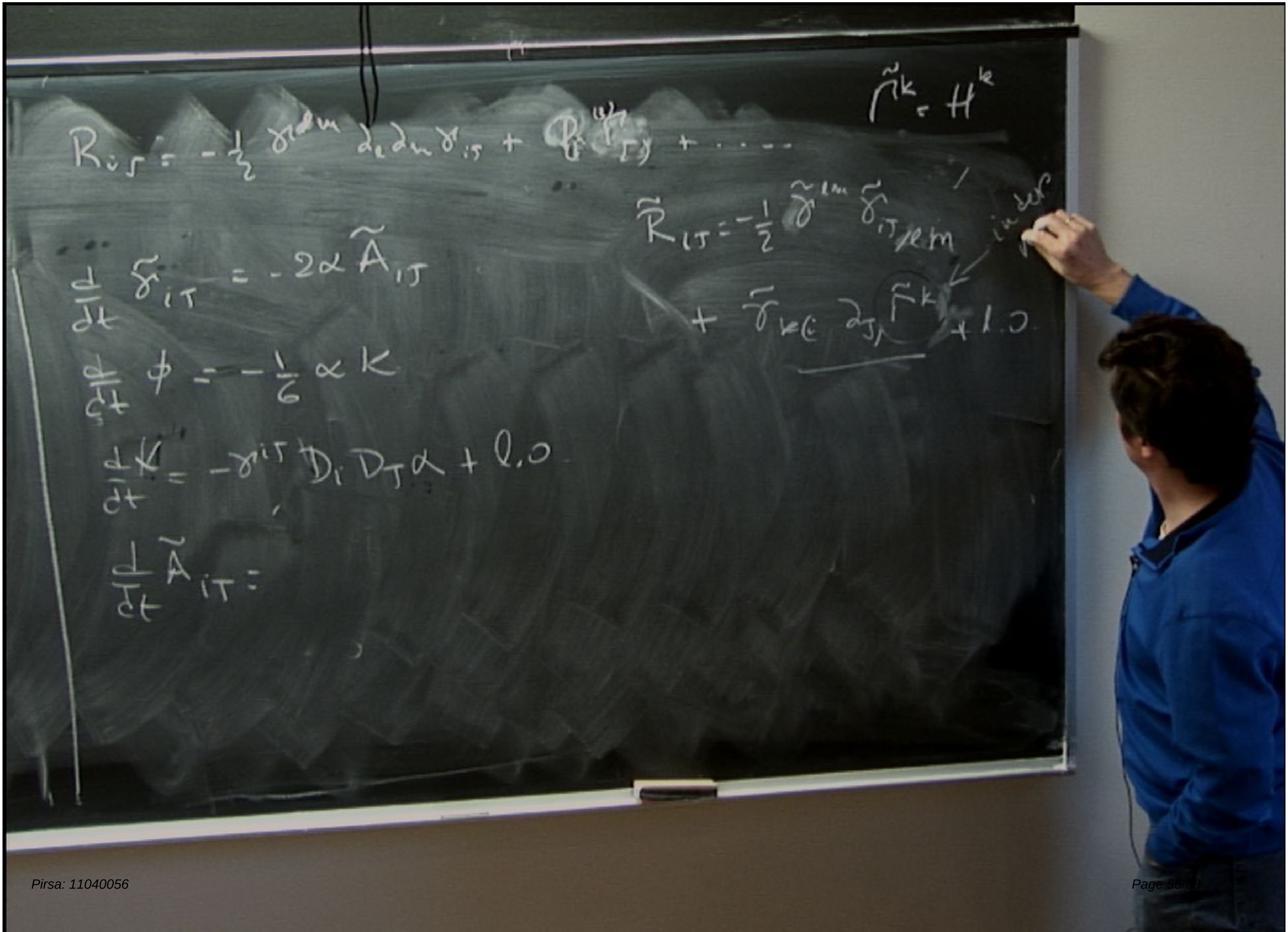

$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \dots$
$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm}$
$+ \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + l.o.$
$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{d}{dt}K = -\gamma^{ij}D_iD_j\alpha + l.o.$
$\frac{d}{dt}\tilde{A}_{ij} =$


R_ij = -½ γ^lm ∂_l ∂_m γ_ij + ...
R̃_ij = -½ γ̃^lm γ̃_ij,lm
+ γ̃_k(i ∂_j) Γ̃^k + l.o.
d/dt γ̃_ij = -2α Ã_ij
d/dt Ã_ij =


$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \gamma_{ki}\Gamma^{k}_{j} + \ldots$
$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm}$
$+ \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^{k} + l.o.$
$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{d}{dt}K = -\gamma^{ij}D_iD_j\alpha + l.o.$
$\frac{d}{dt}\tilde{A}_{ij} =$



$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \ldots$
$\tilde R_{ij} = -\frac{1}{2}\tilde\gamma^{lm}\tilde\gamma_{ij,lm}$
$+ \tilde\gamma_{k(i}\partial_{j)}\tilde\Gamma^k + l.o.$
$\frac{d}{dt}\tilde\gamma_{ij} = -2\alpha\tilde A_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{dK}{dt} = -\gamma^{ij}D_iD_j\alpha + l.o.$
$\frac{d}{dt}\tilde A_{ij} =$


$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \dots$
$\tilde{\Gamma}^k = H^k$
$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm}$
$+ \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \mathrm{l.o.}$
indep
$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{dK}{dt} = -\gamma^{ij}D_iD_j\alpha + \mathrm{l.o.}$
$\frac{d}{dt}\tilde{A}_{ij} =$

$$\tilde{\Gamma}^k = H^k$$

$$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \partial_{(i}\Gamma_{j)} + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \text{l.o.}$$

indep vars

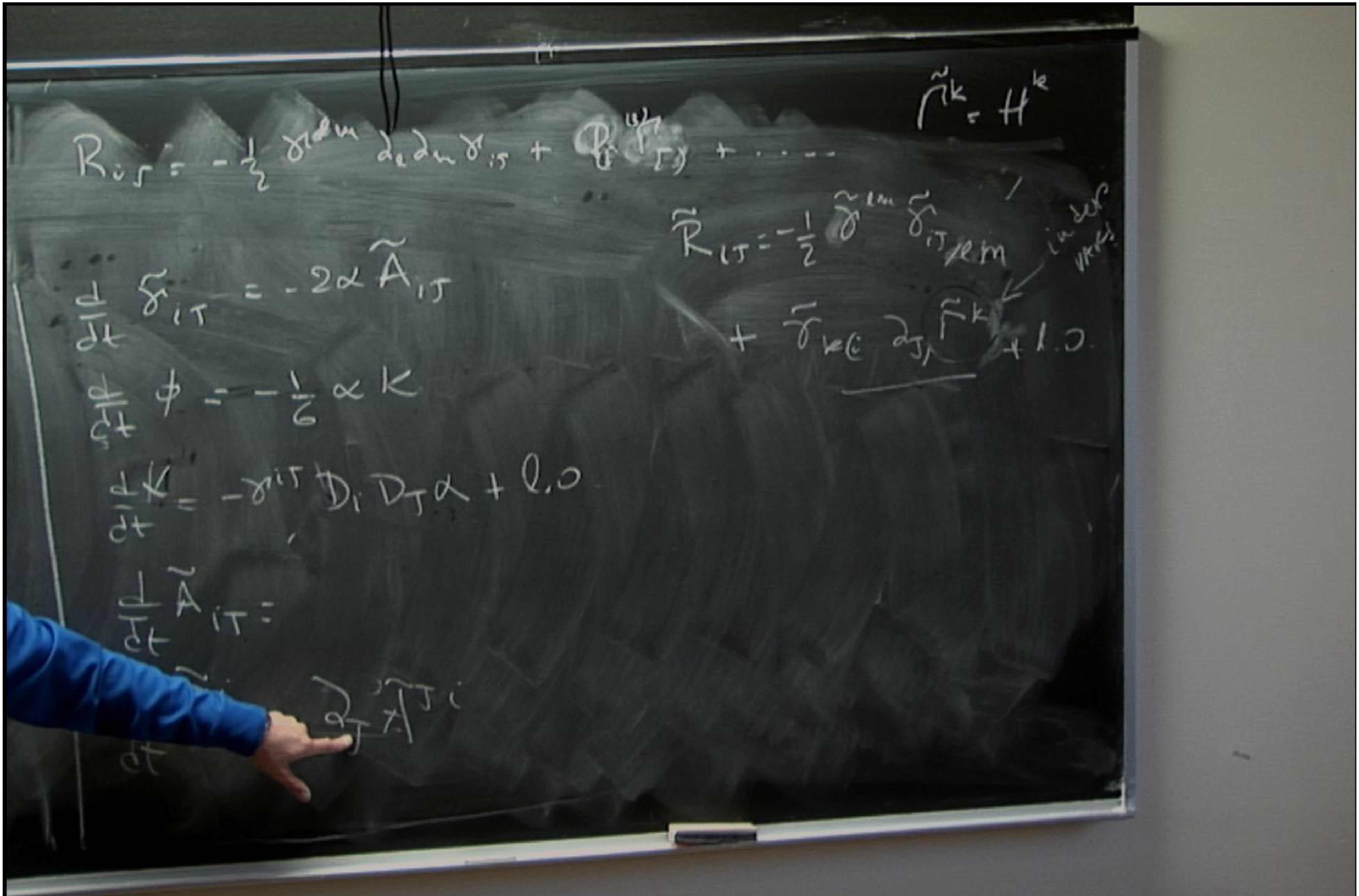
$$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$$

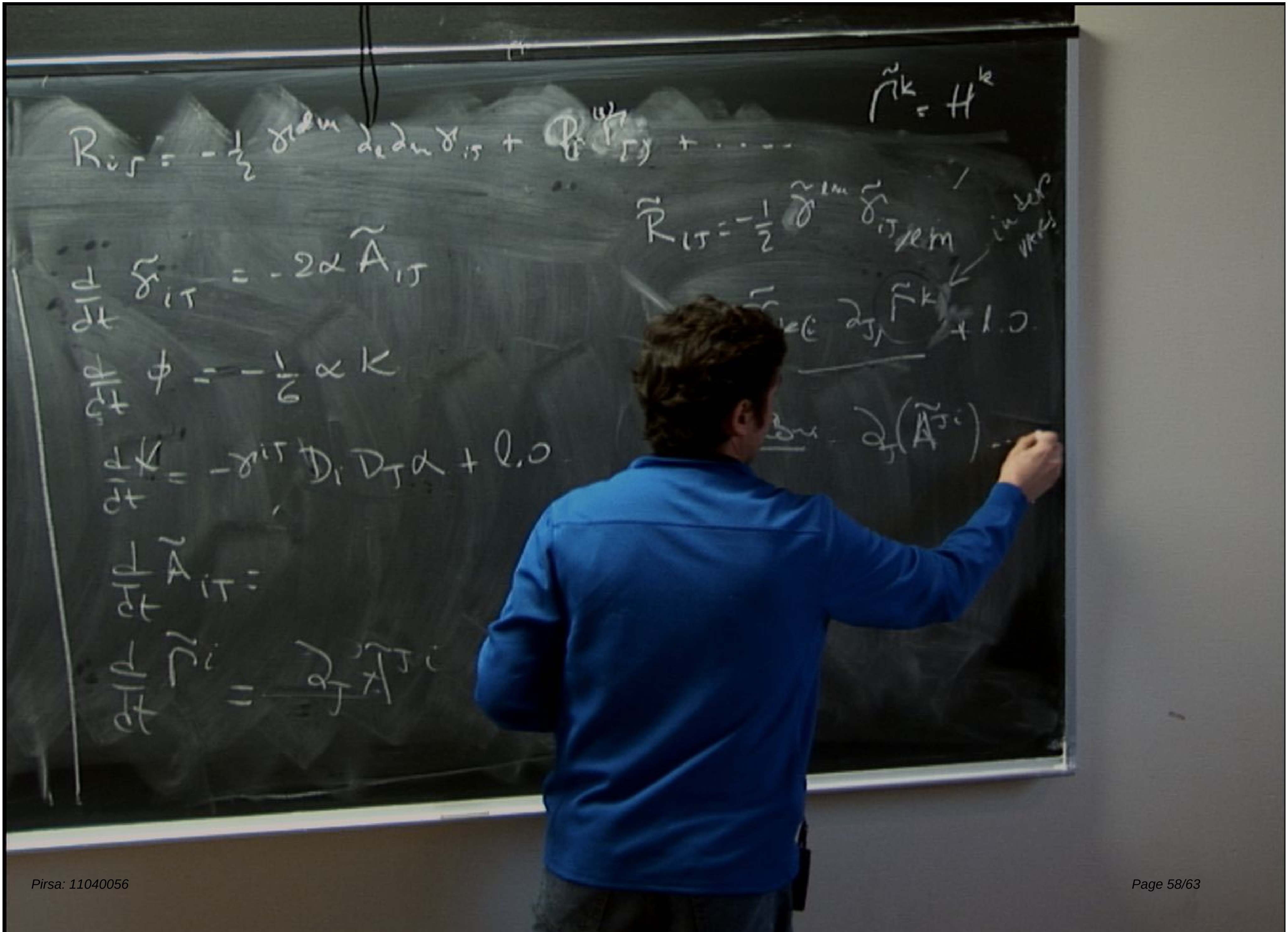
$$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$$

$$\frac{d}{dt}K = -\gamma^{ij}D_iD_j\alpha + \text{l.o.}$$

$$\frac{d}{dt}\tilde{A}_{ij} =$$

$$\frac{d}{dt}\tilde{\Gamma}^i =$$


$\tilde{\Gamma}^k = H^k$
$R_{ij} = -\frac{1}{2} \gamma^{lm} \partial_l \partial_m \gamma_{ij} + \ldots$
$\tilde{R}_{ij} = -\frac{1}{2} \tilde{\gamma}^{lm} \tilde{\gamma}_{ij,lm}$
$+ \tilde{\gamma}_{k(i} \partial_{j)} \tilde{\Gamma}^k + \text{l.o.}$
$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}$
$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K$
$\frac{d}{dt} K = -\gamma^{ij} D_i D_j \alpha + \text{l.o.}$
$\frac{d}{dt} \tilde{A}_{ij} =$


$\tilde{\Gamma}^k = H^k$
$R_{IJ} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{IJ} + \ldots$
$\tilde{R}_{IJ} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{IJ,lm}$
in dep. vars
$\frac{d}{dt}\tilde{\gamma}_{IJ} = -2\alpha\tilde{A}_{IJ}$
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$
$\frac{d}{dt}K = -\gamma^{IJ}D_I D_J\alpha + l.o.$
$\frac{d}{dt}\tilde{A}_{IJ} =$
$\frac{d}{dt}\tilde{\Gamma}^i = \partial_J\tilde{A}^{Ji}$
$\partial_J\tilde{\Gamma}^k + l.o.$
$\partial_J(\tilde{A}^{Ji})$

$$\tilde{\Gamma}^k = H^k$$

$$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \partial_{(i}\Gamma_{j)} + \dots$$

$$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + l.o.$$

indep vars

$$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$$

$$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$$

$$= -\gamma^{ij}D_iD_j\alpha + l.o.$$

Mom. Const. $\partial_j(\tilde{A}^{ji})\dots$

$$\frac{d}{dt}\tilde{\Gamma}^i = \partial_j\tilde{A}^{ji}$$

$$\tilde{\Gamma}^k = H^k$$

$$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \Gamma\,\Gamma_{jk} + \ldots$$

$$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm} + \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \text{l.o.}$$

indep. vars

$$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$$

$$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$$

$$\frac{d}{dt}K = -\gamma^{ij}D_iD_j\alpha + \text{l.o.}$$

$$\frac{d}{dt}\tilde{A}_{ij} =$$

$$\frac{d}{dt}\tilde{\Gamma}^i = \cancel{\partial_j\tilde{A}^{ji}} \to \{\partial K,\ \partial_j(\alpha,\beta)\}$$

Mom. Cons.: $\partial_j(\tilde{A}^{ji})\ldots$

$\tilde{\Gamma}^k = H^k$

$R_{ij} = -\frac{1}{2}\gamma^{lm}\partial_l\partial_m\gamma_{ij} + \partial_{(i}\Gamma^{(k)}_{j)} + \dots$

$\tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\tilde{\gamma}_{ij,lm}$

$+ \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \text{l.o.}$

indep vars

$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$ ✓

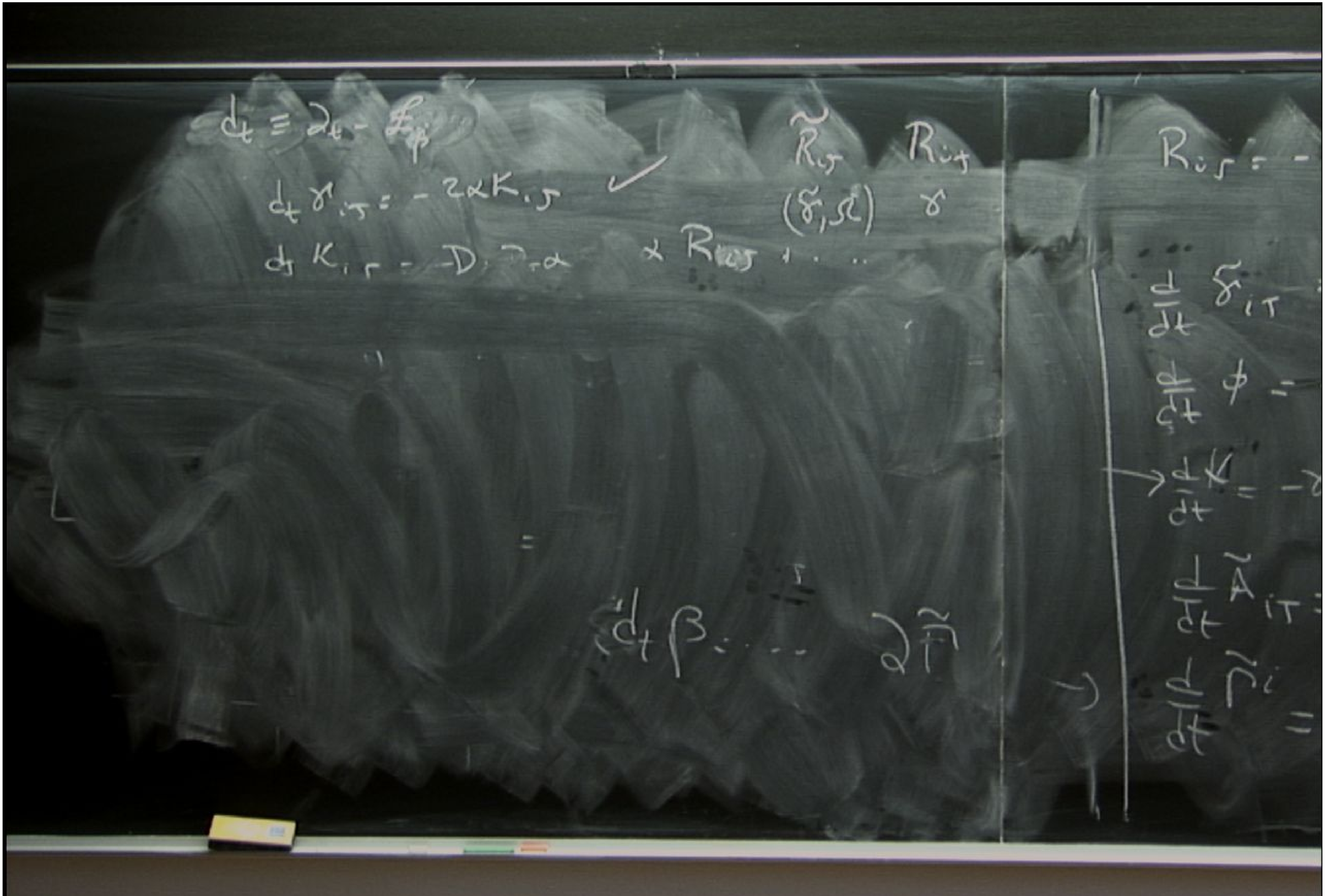
$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$

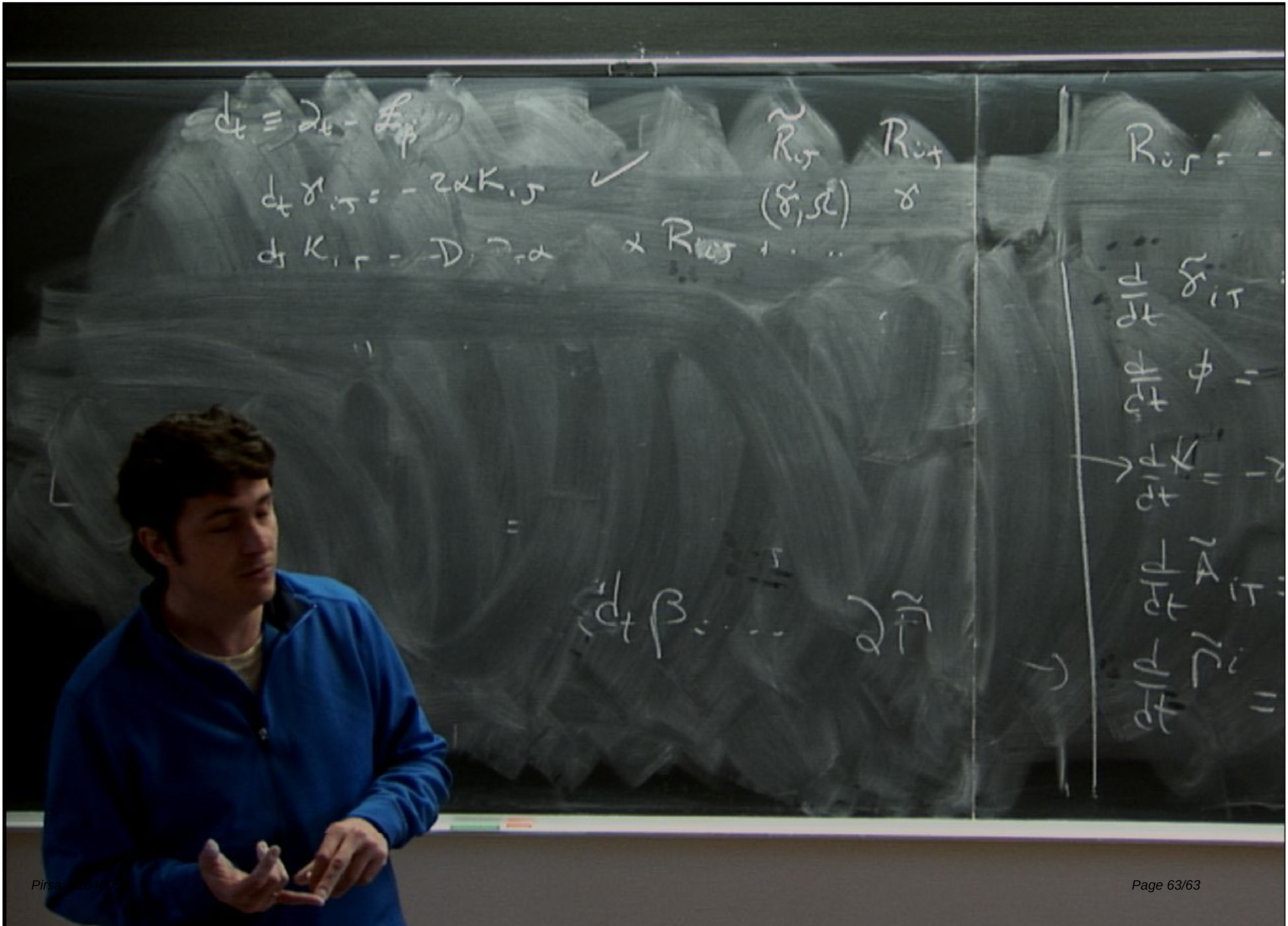
$\frac{dK}{dt} = -\gamma^{ij}D_iD_j\alpha + \text{l.o.}$

Mom. Con.: $\partial_j(\tilde{A}^{ji})\dots$

$\frac{d}{dt}\tilde{A}_{ij} =$ ✓

$\frac{d}{dt}\tilde{\Gamma}^i = \cancel{\partial_j\tilde{A}^{ji}} \to \{\partial K, \partial_j(\alpha, \beta)\}$


$d_t \equiv \partial_t - \mathcal{L}_\beta$
$d_t \gamma_{ij} = -2\alpha K_{ij}$
$d_t K_{ij} = -D_i D_j \alpha + \alpha R_{ij} + \ldots$
$\tilde{R}_{ij}$ R_{ij}
$(\tilde{\gamma}, \Omega)$ γ
$d_t \beta \ldots \partial \tilde{\Gamma}$
$R_{ij} = -$
$\frac{d}{dt} \tilde{\gamma}_{ij}$
$\frac{d}{dt} \phi =$
$\frac{d}{dt} K = -$
$\frac{d}{dt} \tilde{A}_{ij}$
$\frac{d}{dt} \tilde{\Gamma}^i =$


$d_t \equiv \partial_t - \mathcal{L}_\beta$
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