Title: Explorations in Numerical Relativity - Lecture 12

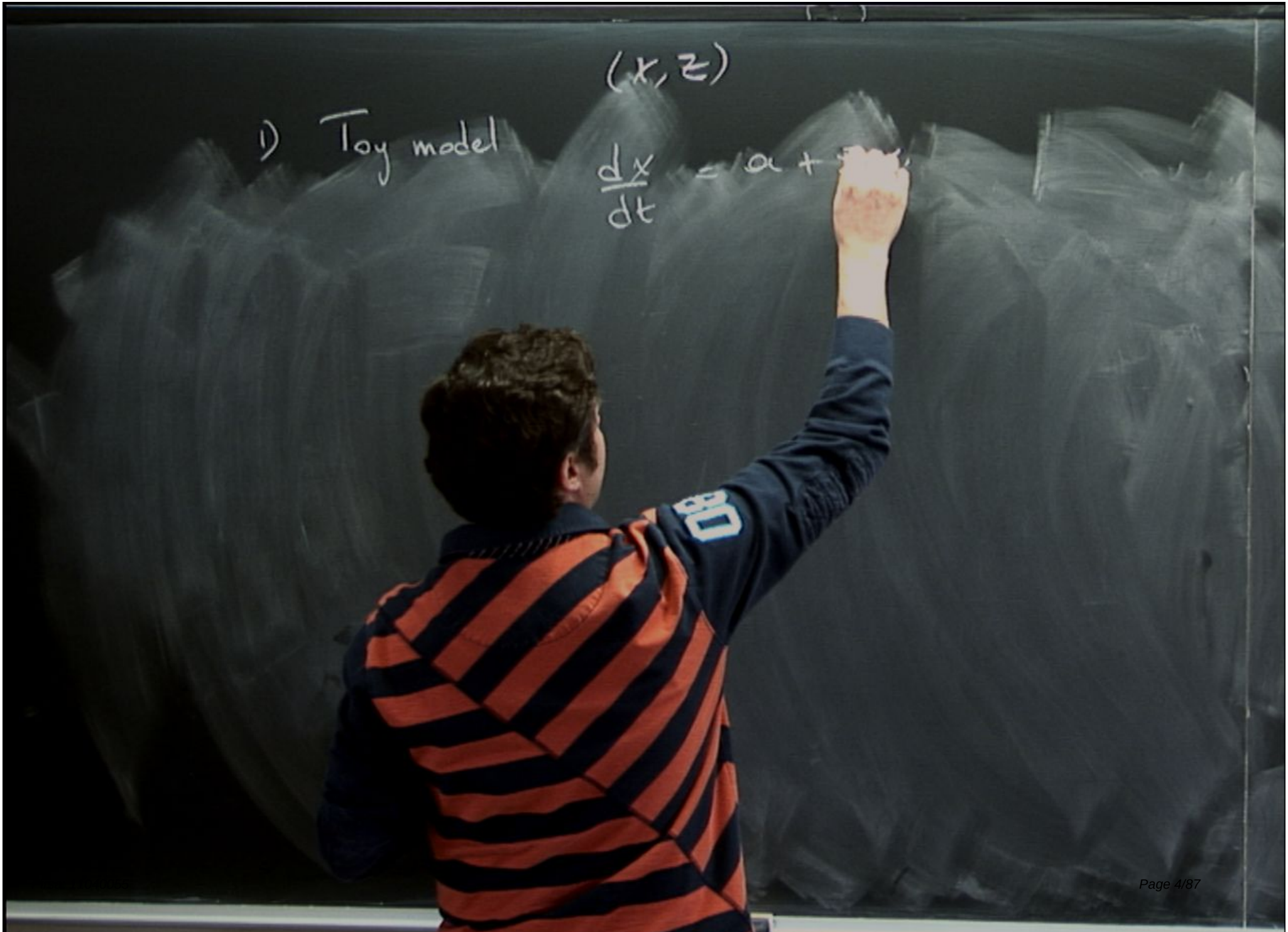
Date: Apr 19, 2011 11:30 AM

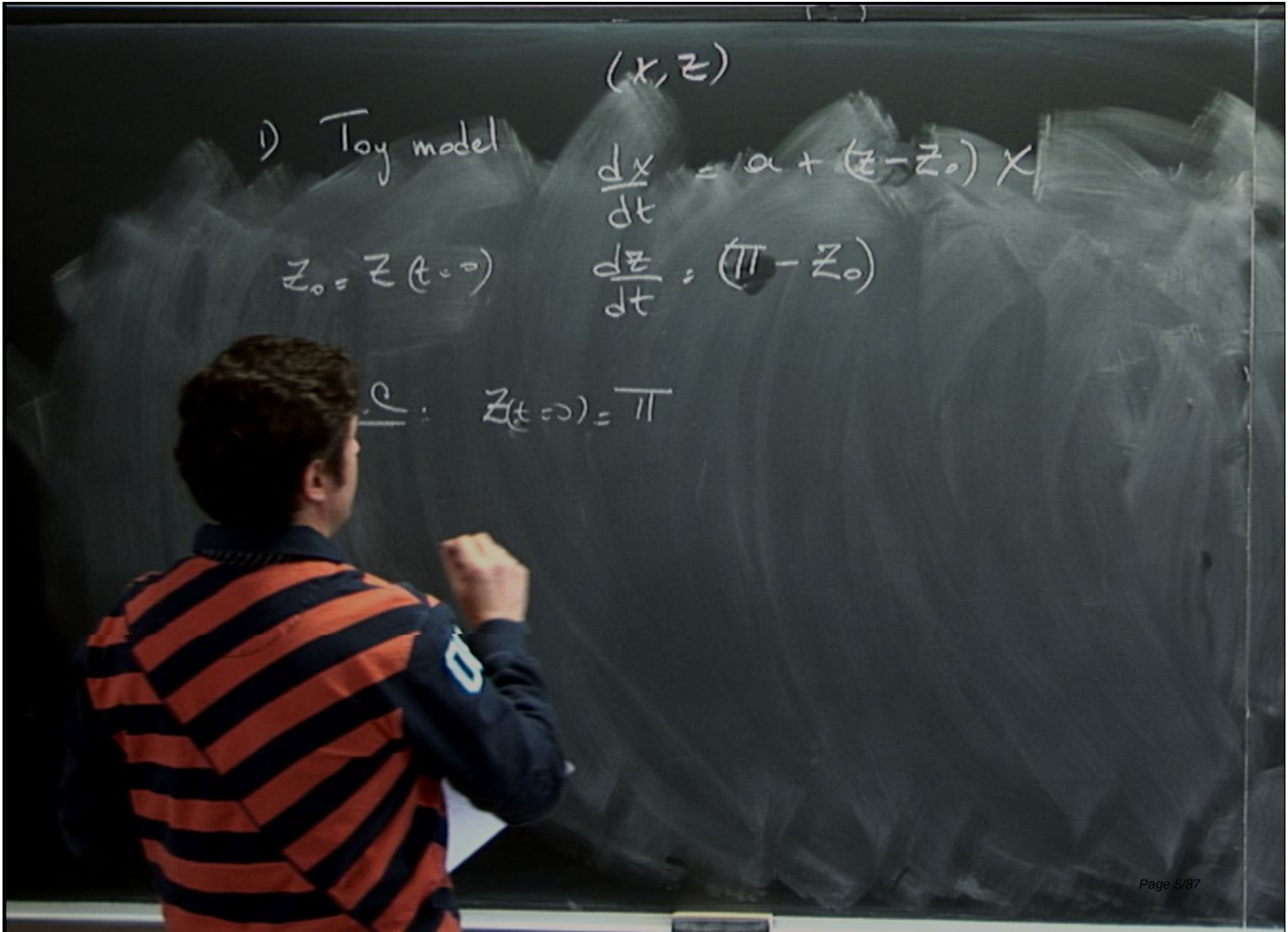
URL: http://pirsa.org/11040055

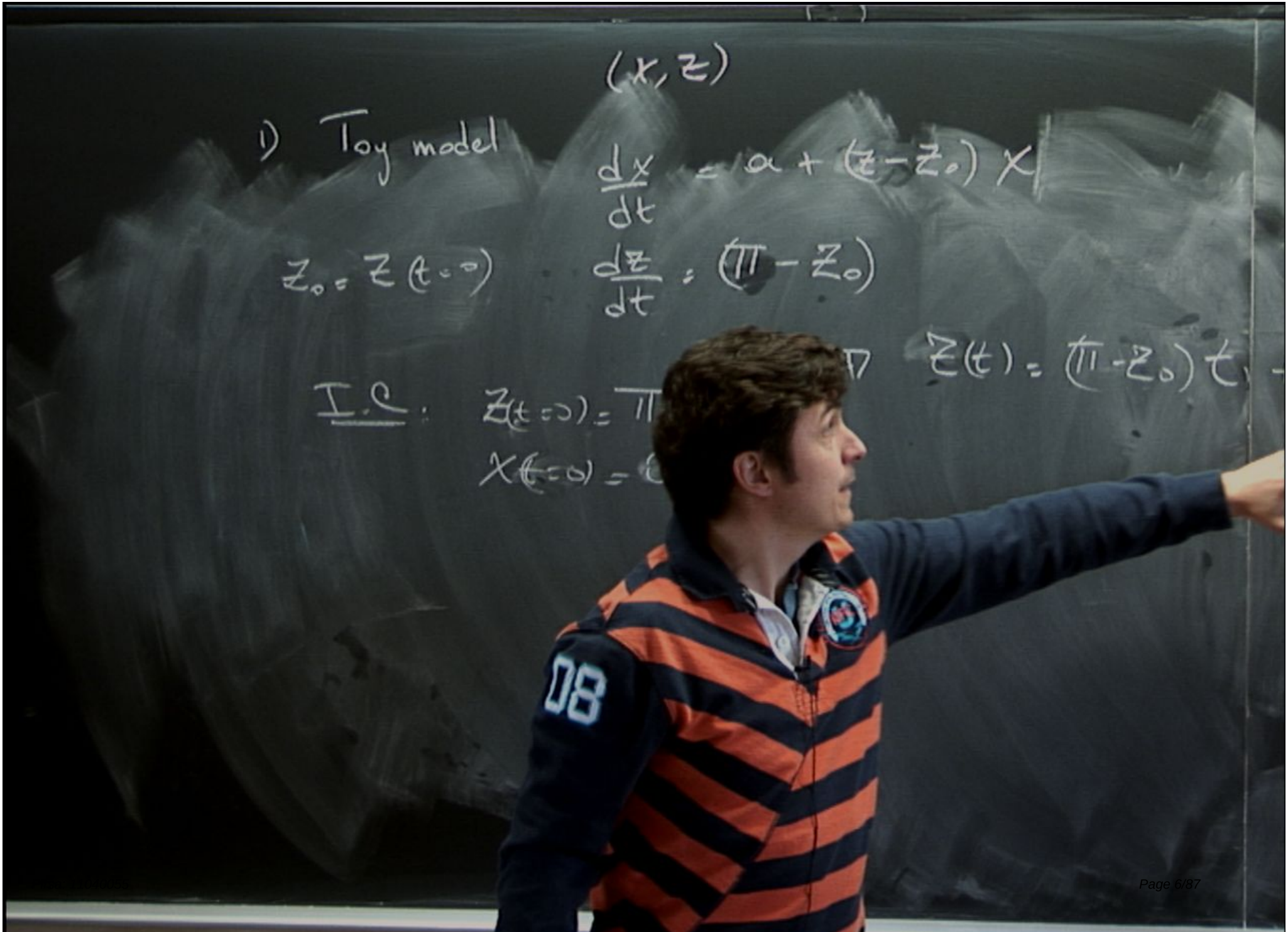
Abstract:

perimeter scholars
international

1) Toy model


(x, z)
1) Toy model
$\frac{dx}{dt} = a +$


(x, z)
1) Toy model
$\frac{dx}{dt} = a + (z - z_0)\,x$
$z_0 = z(t=0)$
$\frac{dz}{dt} = (\Pi - z_0)$
$z(t=0) = \Pi$


(x, z)
1) Toy model
dx/dt = a + (z − z₀) x
z₀ = z(t = 0)
dz/dt = (π − z₀)
z(t) = (π − z₀) t
I.C.
z(t = 0) = π
x(t = 0) =
08

(x, z)

1) Toy model

$$\frac{dx}{dt} = \alpha + (z - z_0)\, x$$

$$z_0 = z(t=0) \qquad \frac{dz}{dt} = (\Pi - z_0)$$

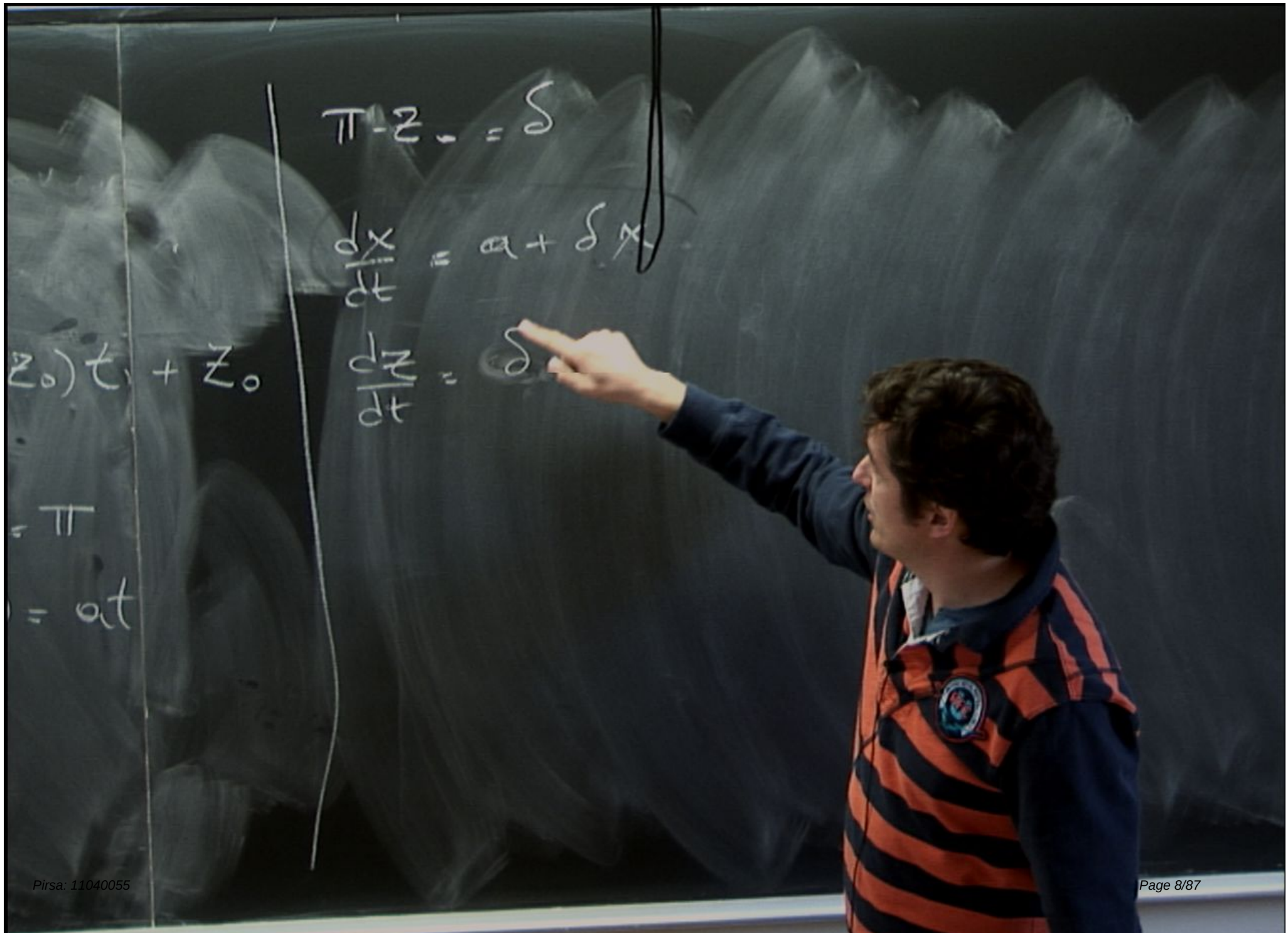
$$\Rightarrow z(t) = (\Pi - z_0)\, t + z_0$$

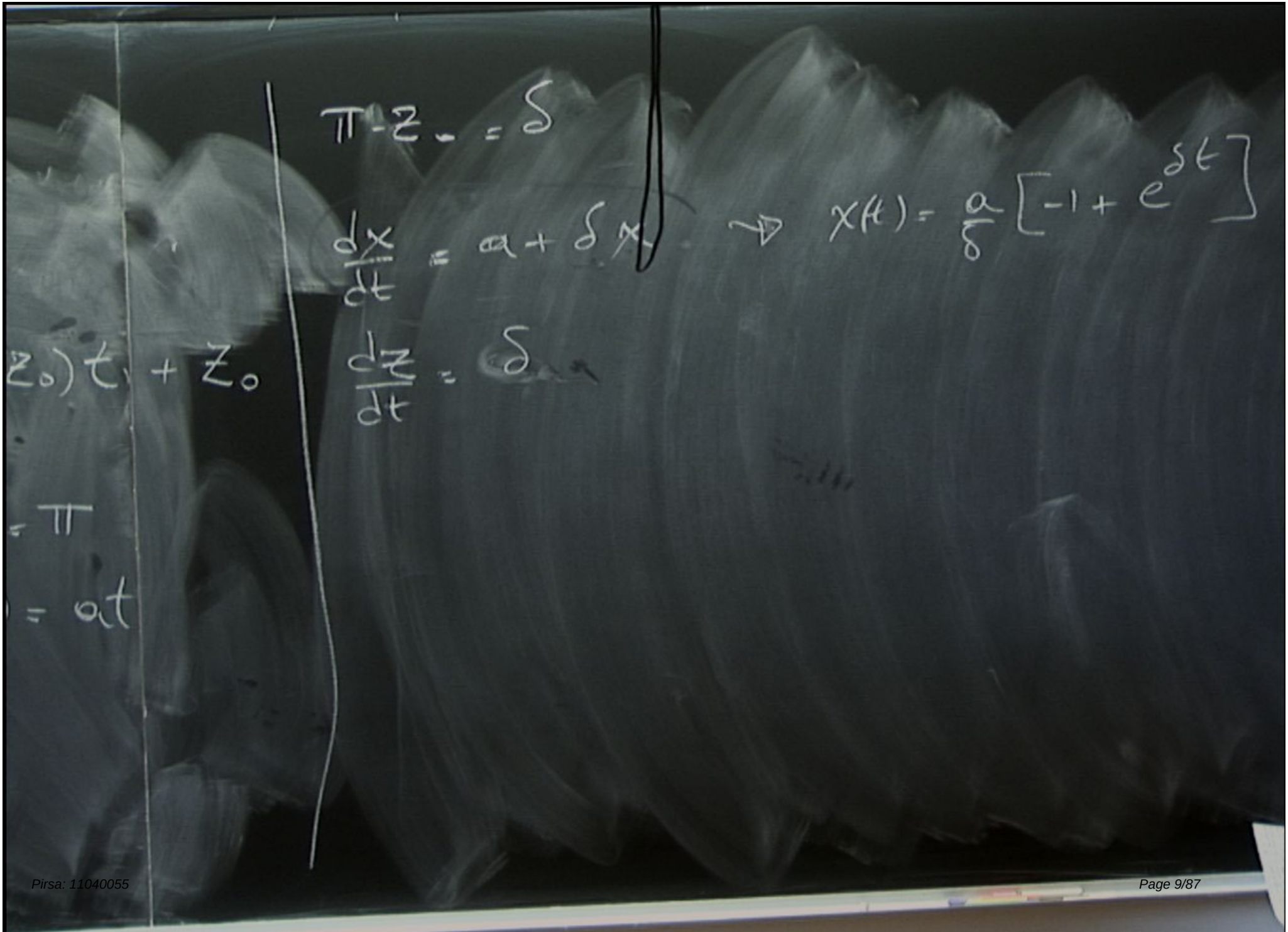
I.C.: $z(t=0) = \Pi$

$x(t=0) = 0$

Soln.: $z(t) = \Pi$

$x(t) = \alpha t$


$\pi - z_0 = \delta$
$\frac{dx}{dt} = a + \delta x$
$\frac{dz}{dt} =$
$z_0) t + z_0$
$= \pi$
$= at$


$\pi - z_0 = \delta$
$\frac{dx}{dt} = a + \delta x$
$\rightarrow x(t) = \frac{a}{\delta}\left[-1 + e^{\delta t}\right]$
$\frac{dz}{dt} = \delta$
$z_0)t + z_0$
$= \pi$
$= at$

$\pi - z_0 = \delta$

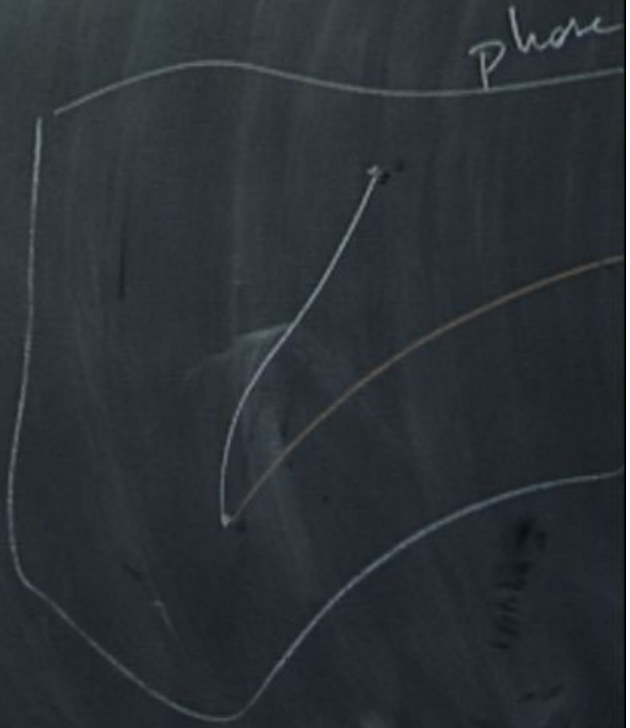
$\frac{dx}{dt} = a + \delta x \rightarrow x(t) = \frac{a}{\delta}\left[-1 + e^{\delta t}\right]$

$\frac{dz}{dt} = \delta$

$z_0)t + z_0$

$= \pi$

$= at$



$\pi - z_- = \delta$

$\frac{dx}{dt} = \alpha + \delta x \quad \Rightarrow \quad x(t) = \frac{\alpha}{\delta}\left[-1 + e^{\delta t}\right]$

$+ z_0$ $\quad \frac{dz}{dt} = \delta$

(x, z)

1) Toy model

$$\frac{dx}{dt} = a + (z - z_0)\, x$$

$$\frac{dz}{dt} = (\pi - z_0) - \beta (z - \pi)$$

$z_0 = z(t=0)$

$$\Rightarrow z(t) = (\pi - z_0)\, t + z_0$$

I.C.: $z(t=0) = \pi$

$x(t=0) = 0$

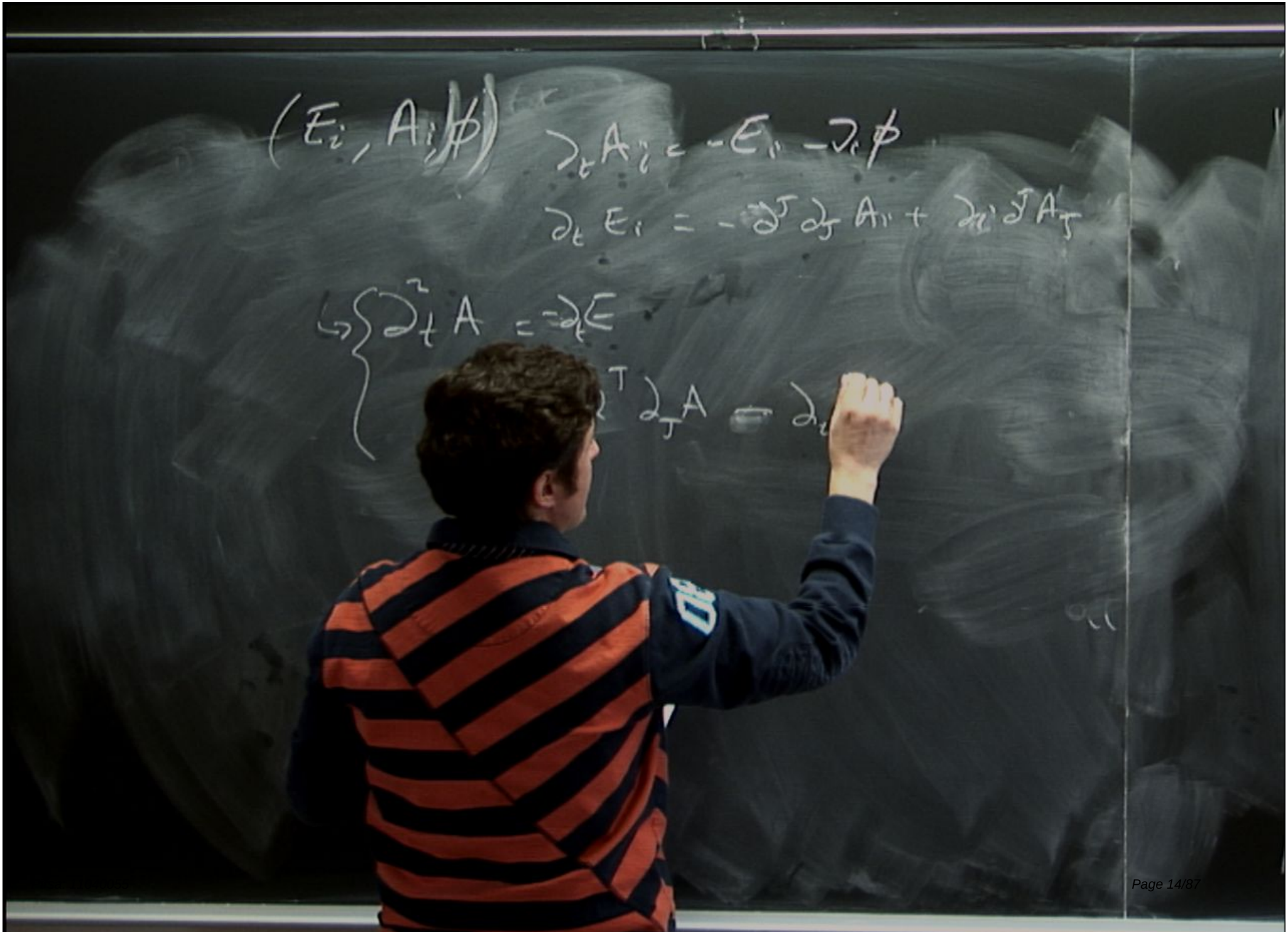
Soln.: $z(t) = \pi$

$x(t) = at$

(E_i, A_i, ϕ)

$\partial_t A_i = -E_i - \partial_i \phi$

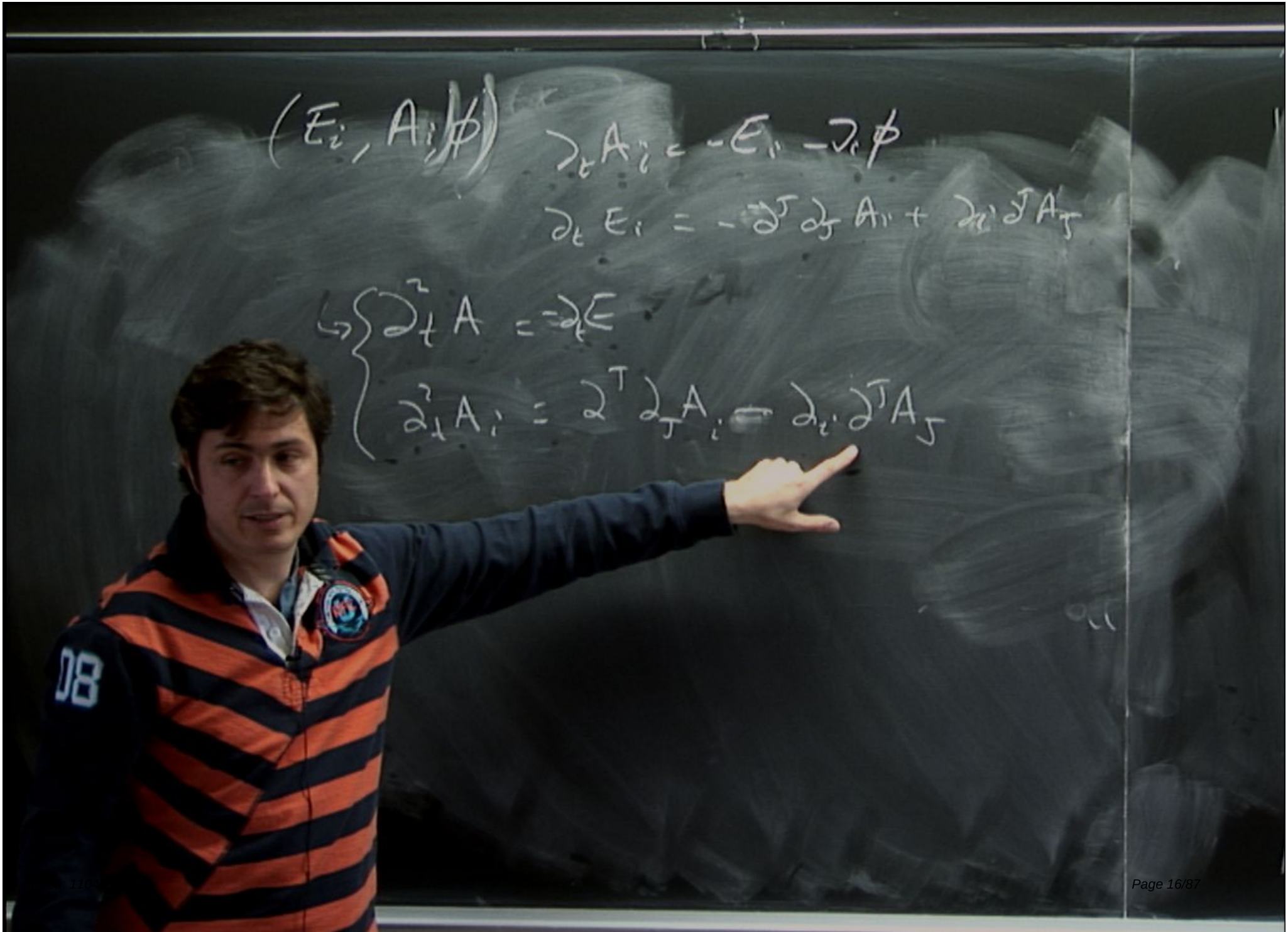
$\partial_t E_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$


(E_i, A_i, ϕ)
$\partial_t A_i = -E_i - \partial_i \phi$
$\partial_t E_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$
$\hookrightarrow \begin{cases} \partial_t^2 A = -\partial_t E \\ \quad \partial^T \partial_j A = \partial_t \end{cases}$

(E_i, A_i, ϕ) $\partial_t A_i = -E_i - \partial_i \phi$

$\partial_t E_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$

$\hookrightarrow \begin{cases} \partial_t^2 A = -\partial_t E \\ \partial_t^2 A_i = \partial^j \partial_j A_i - \partial_i \partial^j A_j \end{cases}$


(E_i, A_i, ϕ)
$\partial_t A_i = -E_i - \partial_i \phi$
$\partial_t E_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$
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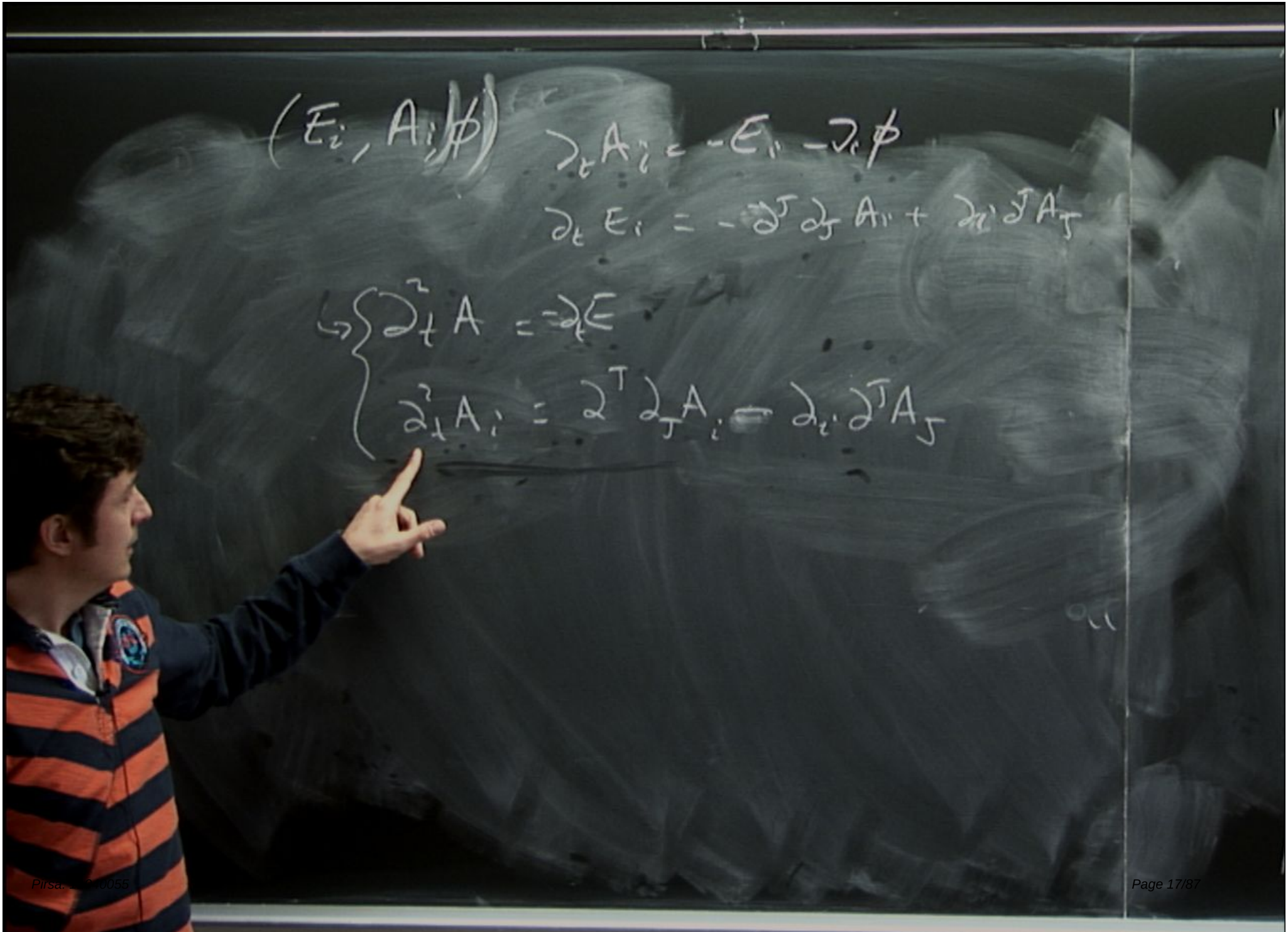

(E_i, A_i, ϕ)
$\partial_t A_i = -E_i - \partial_i \phi$
$\partial_t E_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$
$\hookrightarrow \begin{cases} \partial_t^2 A = -\partial_t E \\ \partial_t^2 A_i = \partial^j \partial_j A_i - \partial_i \partial^j A_j \end{cases}$

Assume $\rho = 0$; 2D

$\partial_x A_1 = a$ $\quad \partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\quad \partial_y A_2 = b$

Assume $\phi = 0$; 2D

$\partial_x A_1 = a$ $\quad \partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\quad \partial_y A_2 = b$

$\partial_t a = \partial_t \partial_x A_1$

$= \partial_x \partial_t A_1 = -$

Assume $\rho = 0$; 2D

$\partial_x A_1 = a$ $\partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\partial_y A_2 = b$

$\partial_t a = \partial_t \partial_x A_1$

$= \partial_x \partial_t A_1 = -\partial_x E_1$

$\partial_t \alpha = \partial_t \partial_y A_1$

$= \partial_y \partial_t A_1 = -\partial_y E_1$

$\partial_t \beta = \partial_x \partial_t A_2 = -\partial_x E_2$

$\partial_t \beta = -\partial_y \partial_t A_1 = -\partial_y E_2$

Assume $\rho = 0$; 2D

$\partial_x A_1 = a$ $\partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\partial_y A_2 = b$

$\partial_t A_x = -E_1$

$\partial_t E_1 = -$

$\partial_t a = \partial_t \partial_x A_1$

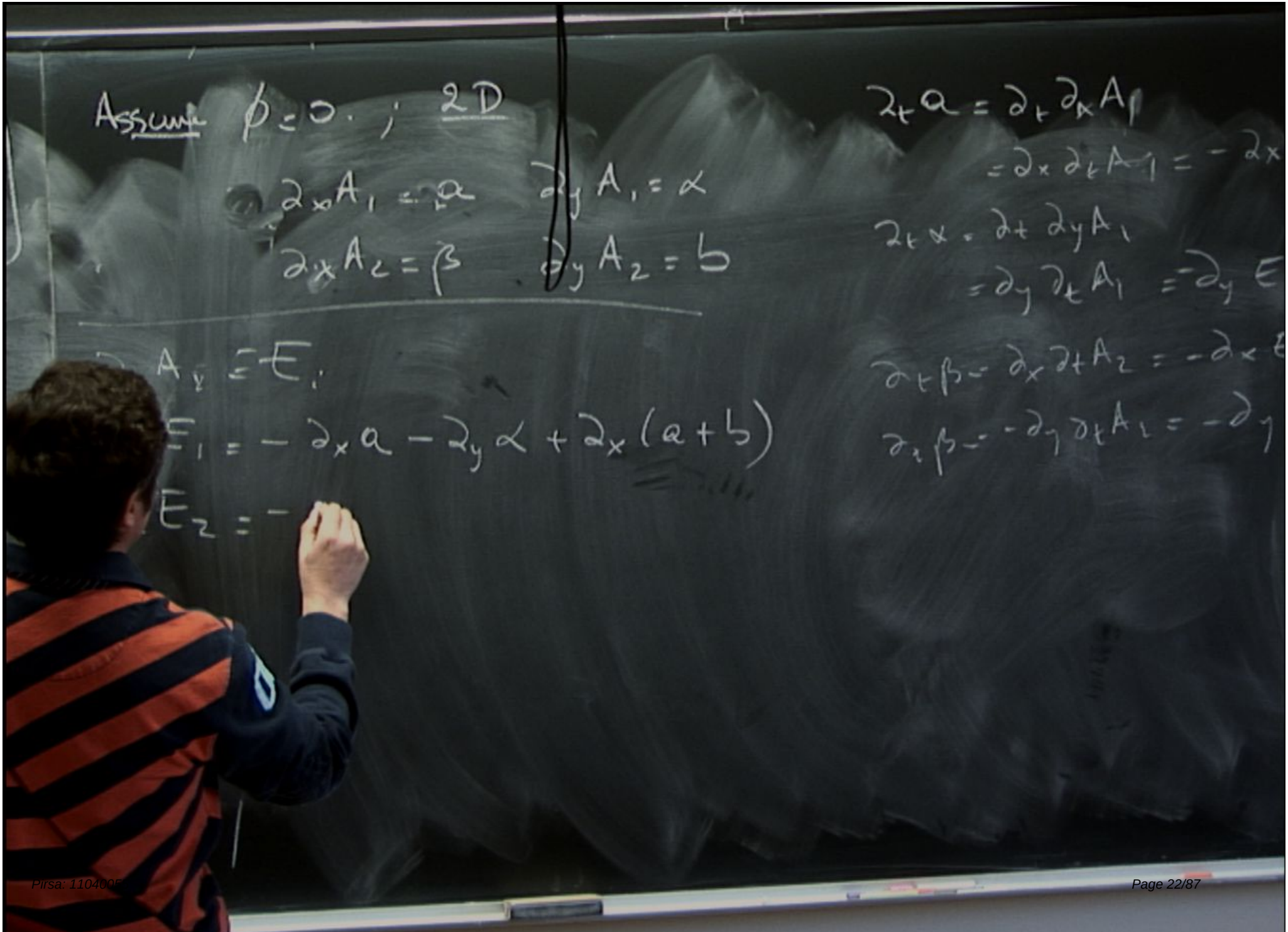
$= \partial_x \partial_t A_1 = -\partial_x E_1$

$\partial_t \alpha = \partial_t \partial_y A_1$

$= \partial_y \partial_t A_1 = -\partial_y E_1$

$\partial_t \beta = \partial_x \partial_t A_2 = -\partial_x E_2$

$\partial_t \beta = -\partial_y \partial_t A_2 = -\partial_y E_2$


Assume β = 0. ; 2D
∂x A1 = α
∂y A1 = α
∂x A2 = β
∂y A2 = b
∂t A1 = −E1
E1 = − ∂x a − ∂y α + ∂x (a + b)
E2 = −
∂t a = ∂t ∂x A1
= ∂x ∂t A1 = − ∂x
∂t α = ∂t ∂y A1
= ∂y ∂t A1 = −∂y E
∂t β = ∂x ∂t A2 = −∂x E
∂t β = − ∂y ∂t A2 = −∂y

Assume $\beta = 0$; 2D

$\partial_x A_1 = a$ $\quad \partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\quad \partial_y A_2 = b$

$\partial_t A_x = -E_i$

$\partial_t E_1 = -\partial_x a - \partial_y \alpha + \partial_x (a+b)$

$\partial_t E_2 = -\partial_x \beta - \partial_y b + \partial_y (a+b)$

$\partial_t a = \partial_t \partial_x A_1$

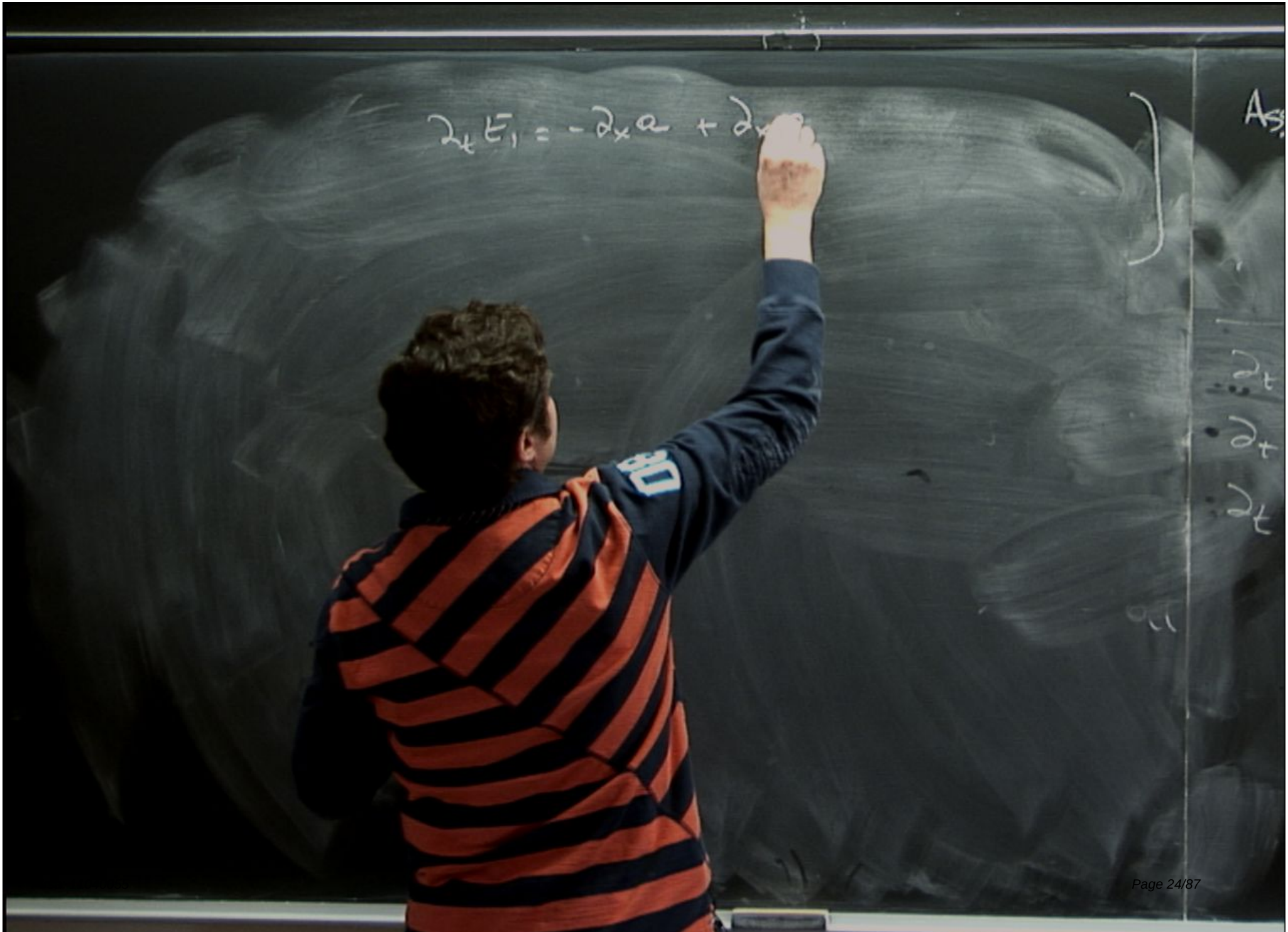
$= \partial_x \partial_t A_1 = -\partial_x$

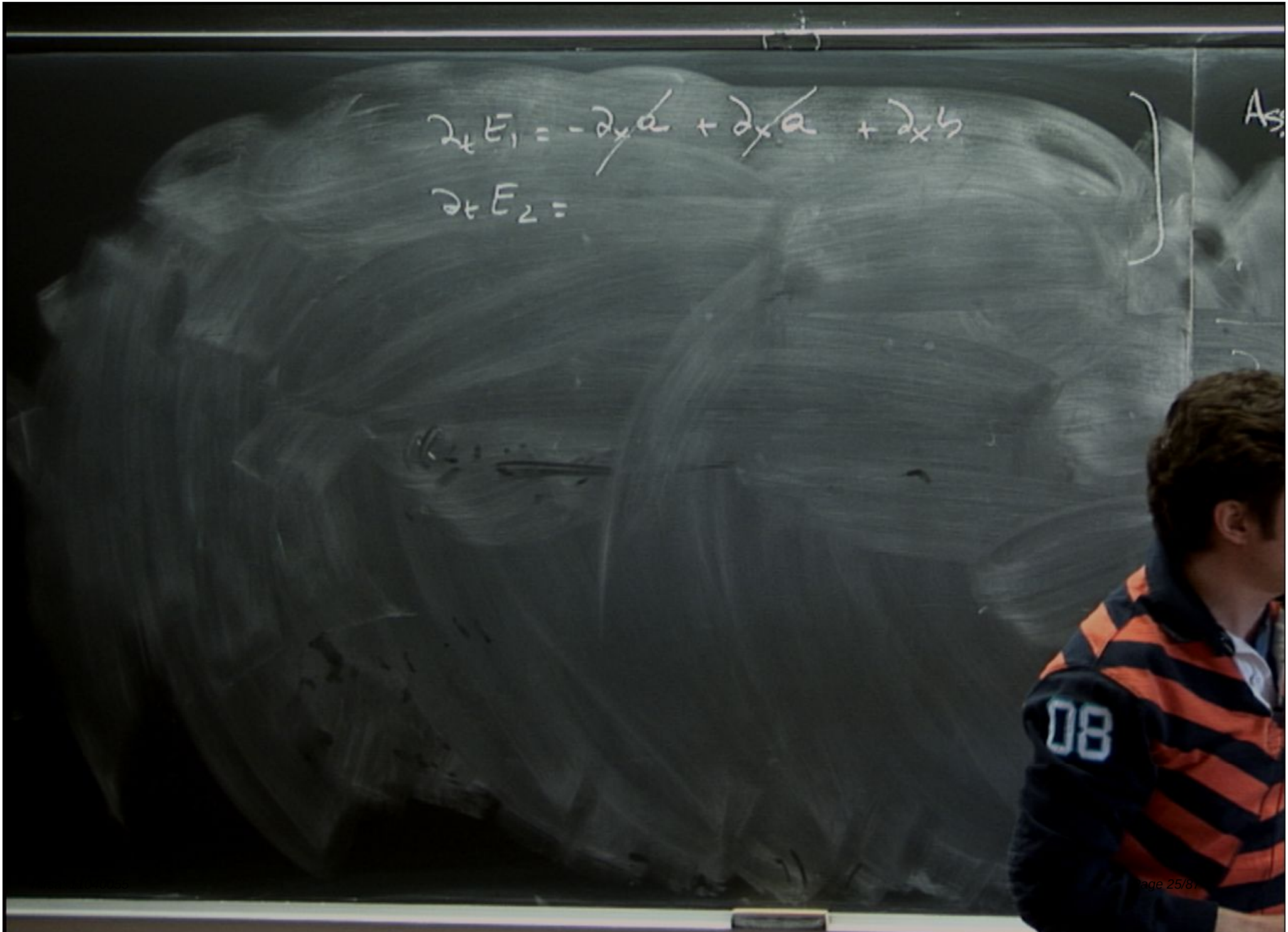
$\partial_t \alpha = \partial_t \partial_y A_1$

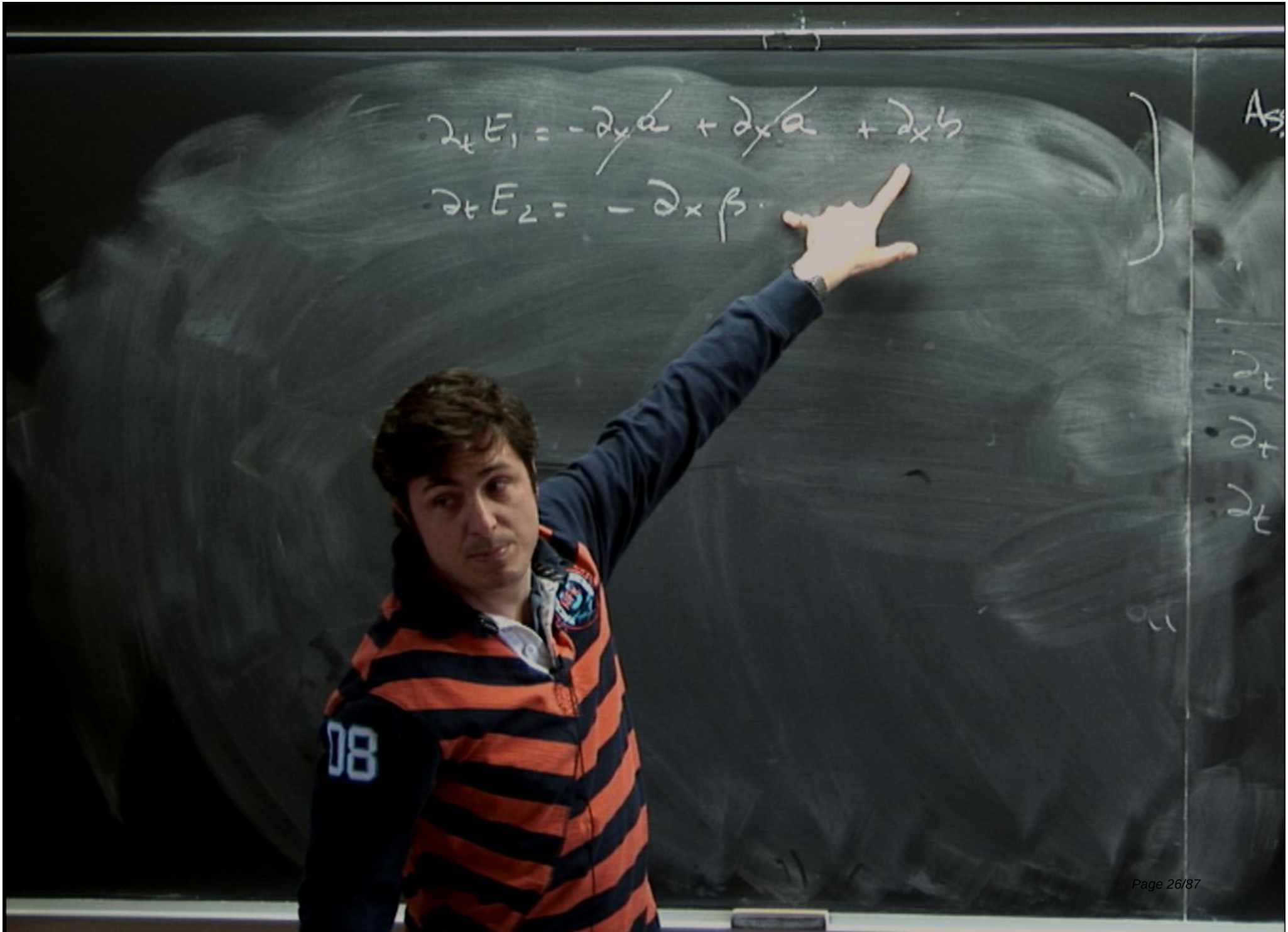
$= \partial_y \partial_t A_1 = -\partial_y E$

$\partial_t \beta = \partial_x \partial_t A_2 = -\partial_x E$

$\partial_t \beta = -\partial_y \partial_t A_2 = -\partial_y$


$\partial_t E_1 = -\partial_x a + \partial_x$
∂_t
∂_t
∂_t


$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$
$\partial_t E_2 =$
08

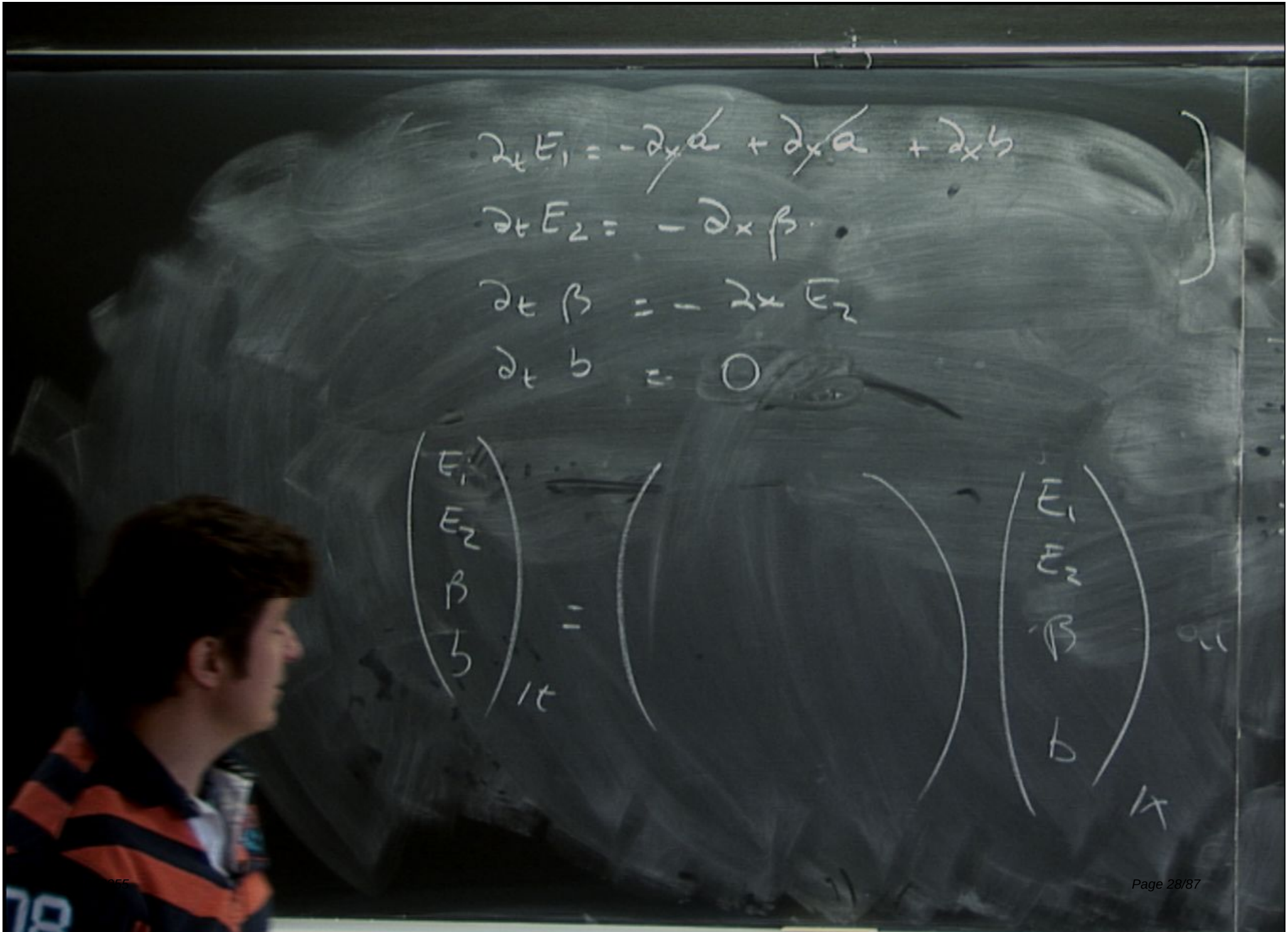

$\partial_t E_1 = -\partial_x \alpha + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
08

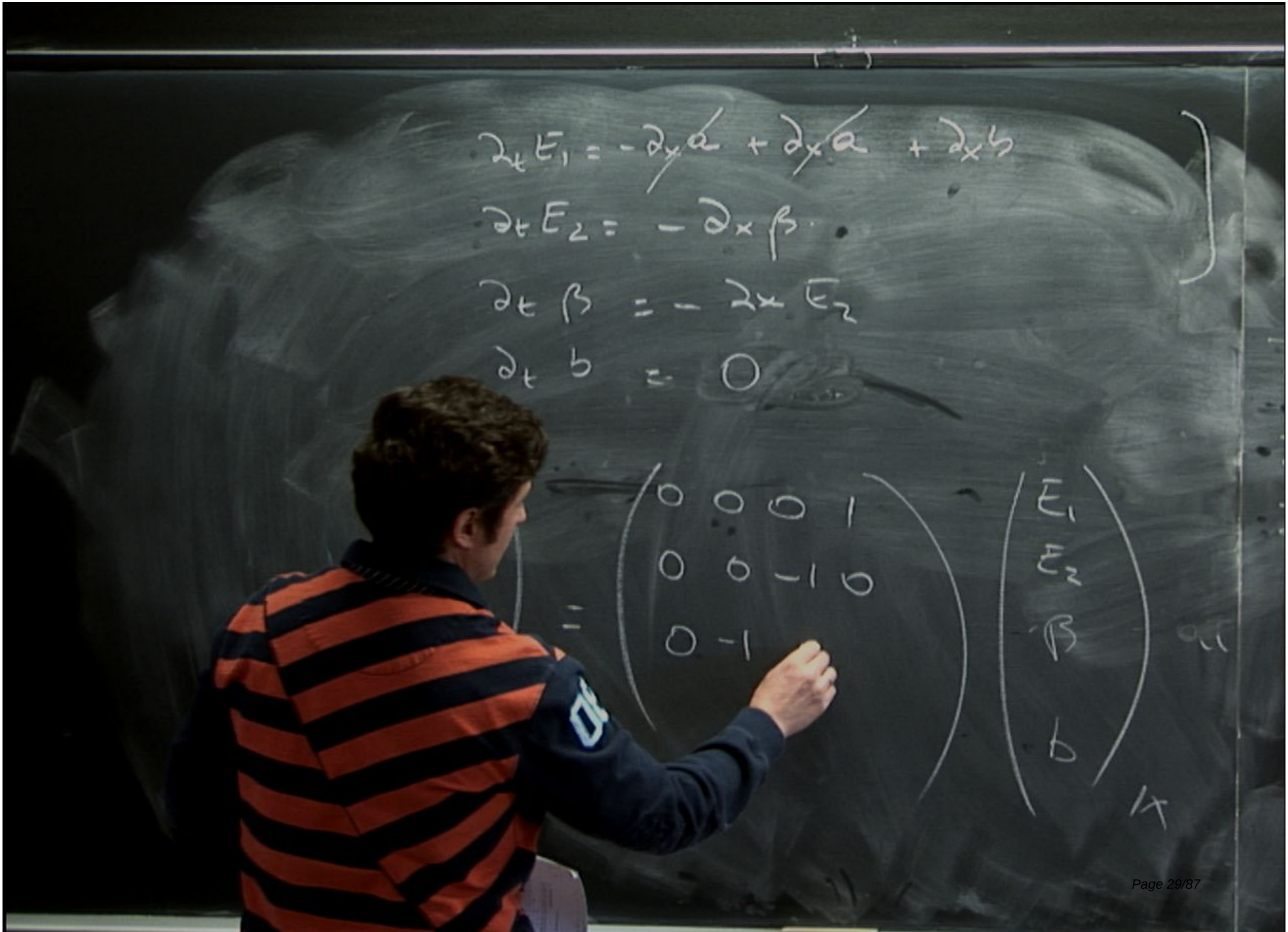
$$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$$

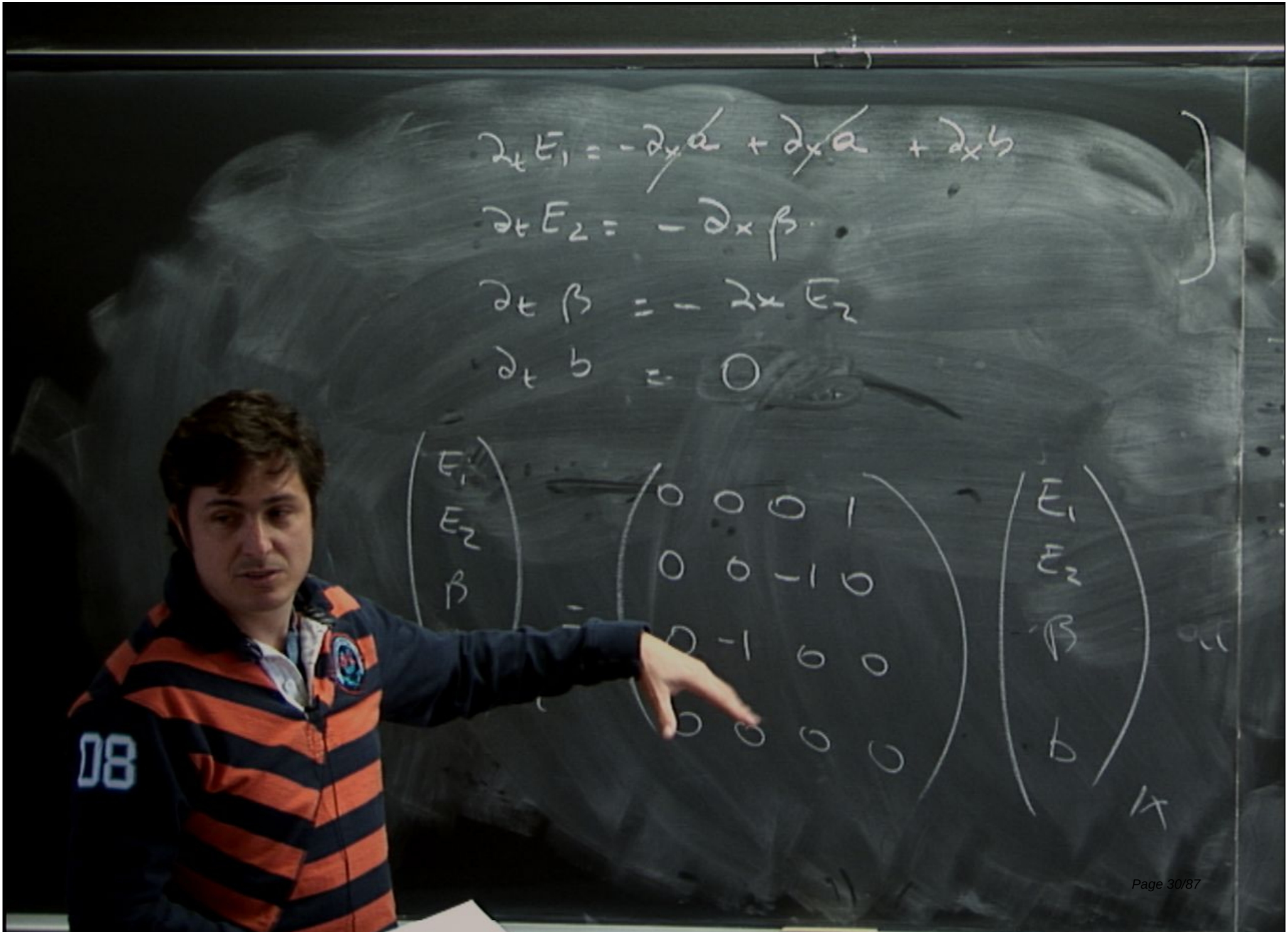
$$\partial_t E_2 = -\partial_x \beta$$

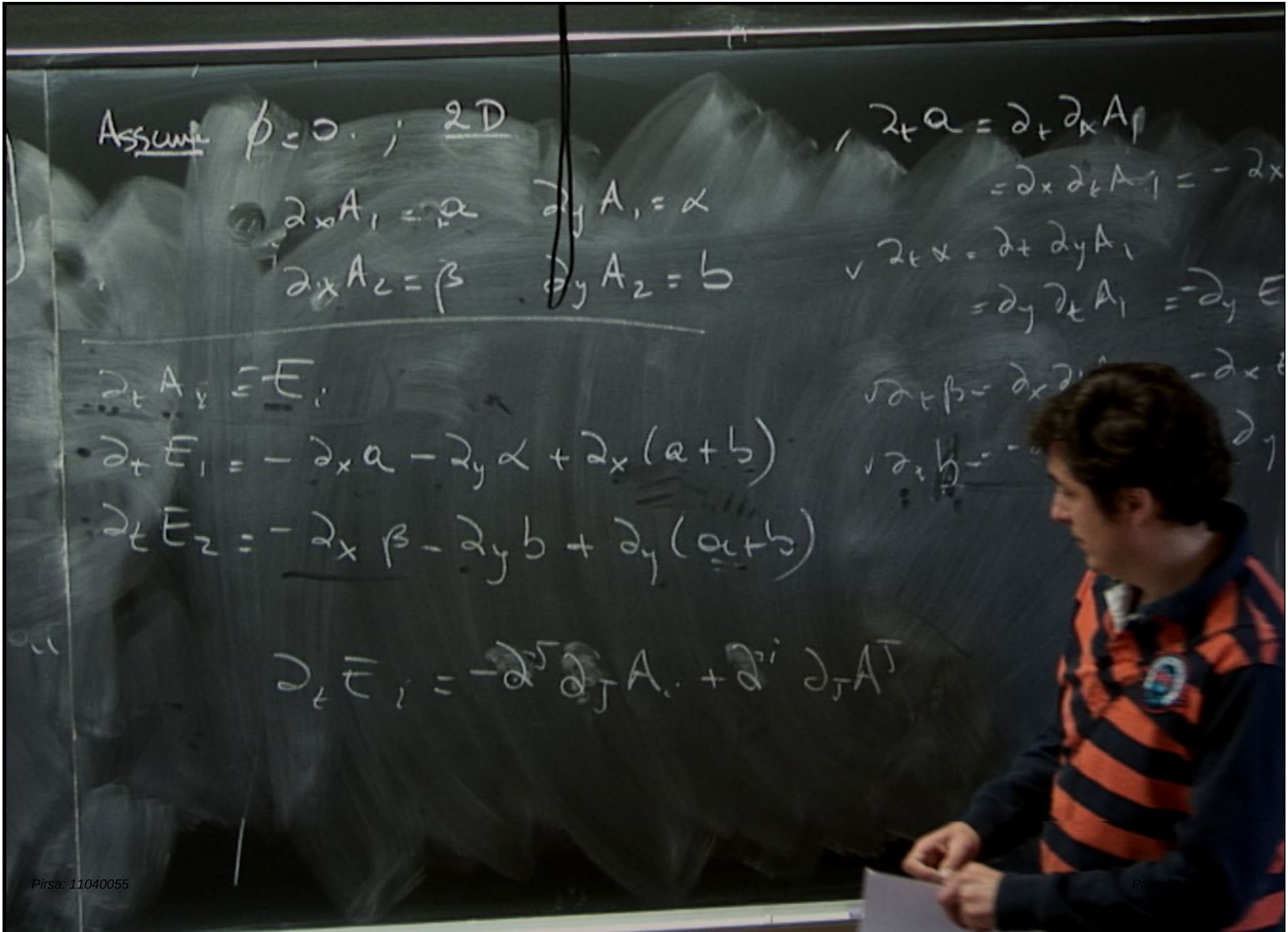
$$\partial_t \beta = -\partial_x E_2$$

$$\partial_t b = -\partial_x E_2$$


$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = 0$
E_1
E_2
β
b
$/t$
$=$
E_1
E_2
β
b
$/x$


$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = 0$
$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & & \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}$


$\partial_t E_1 = -\partial_x \alpha + \partial_x \alpha + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = 0$
$\begin{pmatrix} E_1 \\ E_2 \\ \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}$
08



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Assume $\rho = 0$; 2D

$\partial_x A_1 = a$ $\partial_y A_1 = \alpha$

$\partial_x A_2 = \beta$ $\partial_y A_2 = b$

$\partial_t A_i = -E_i$

$\partial_t E_1 = -\partial_x a - \partial_y \alpha + \partial_x (a + b)$

$\partial_t E_2 = -\partial_x \beta - \partial_y b + \partial_y (a + b)$

$\partial_t E_i = -\partial^j \partial_j A_i + \partial^i \partial_j A^j$

$\partial_t a = \partial_t \partial_x A_1$

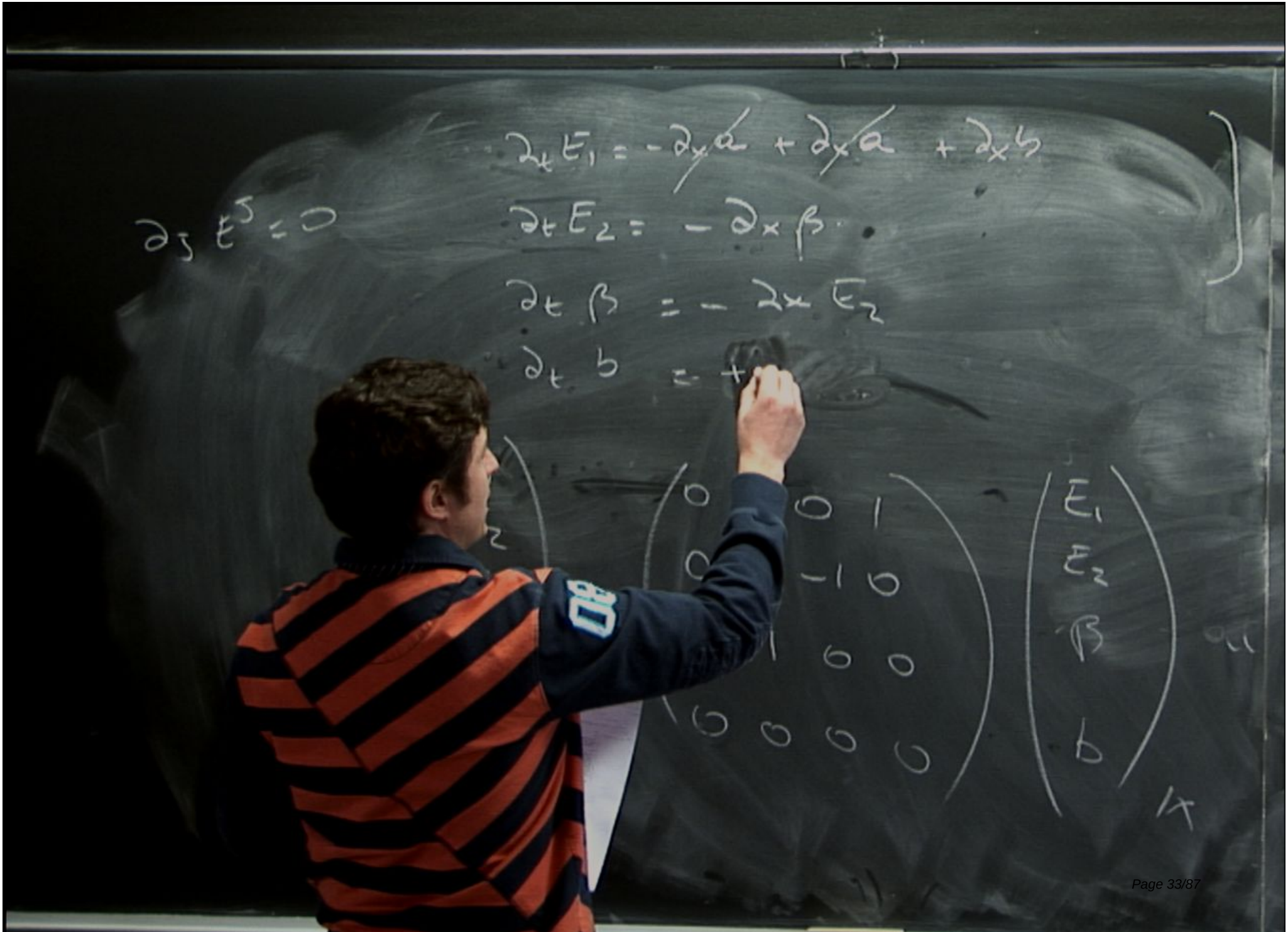
$= \partial_x \partial_t A_1 = -\partial$

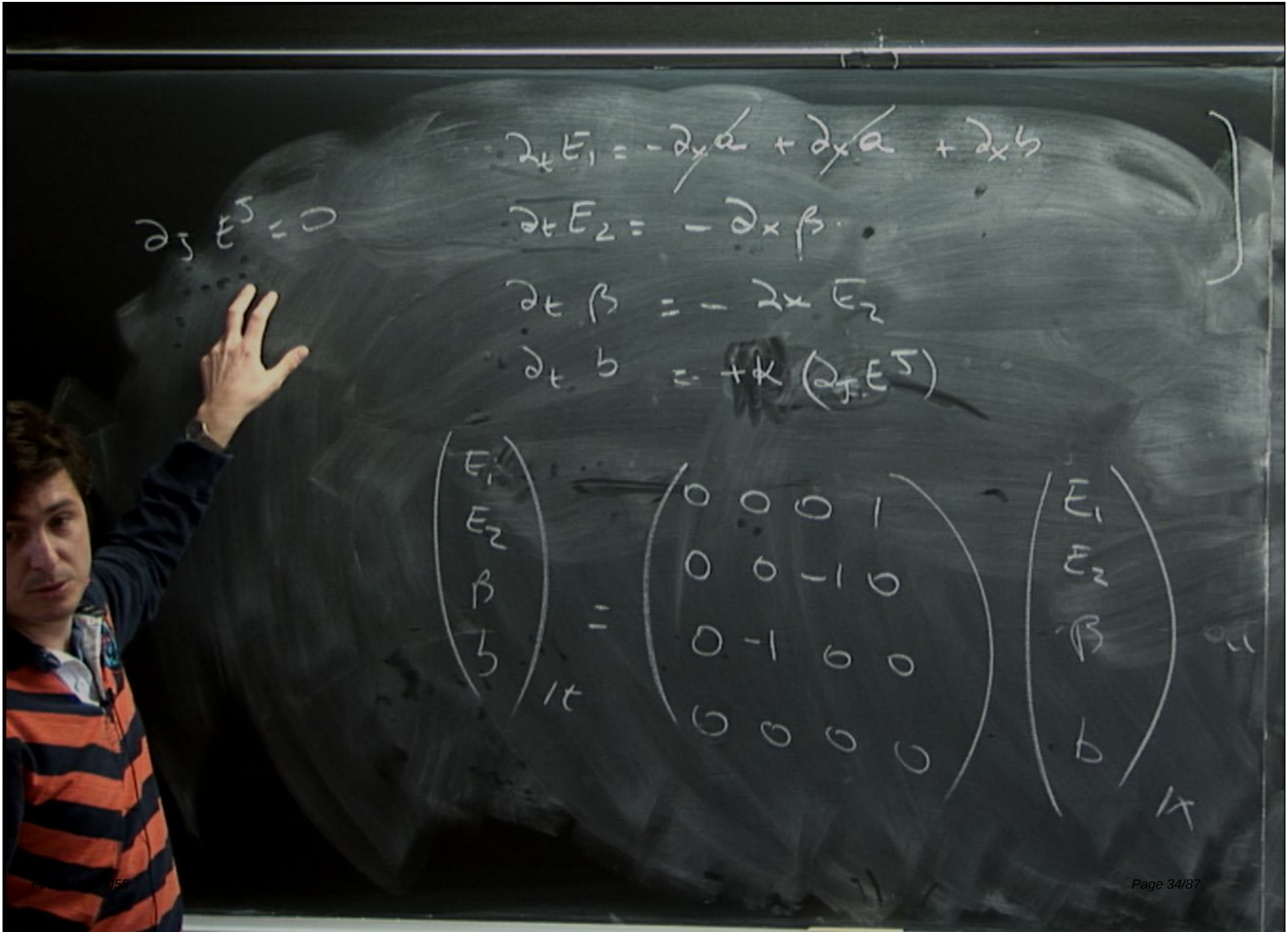
$\partial_t \alpha = \partial_t \partial_y A_1$

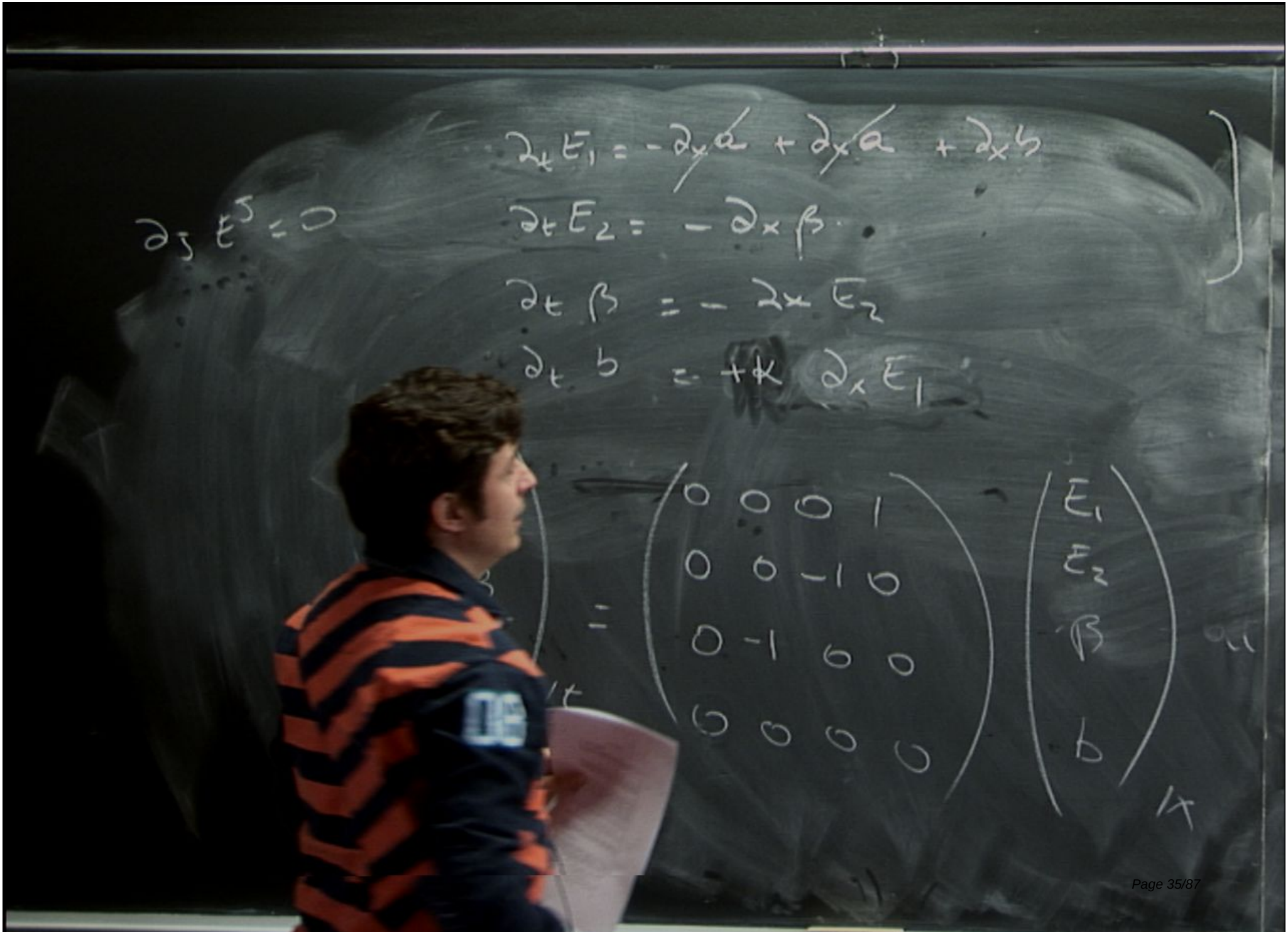
$= \partial_y \partial_t A_1 = -\partial_y$

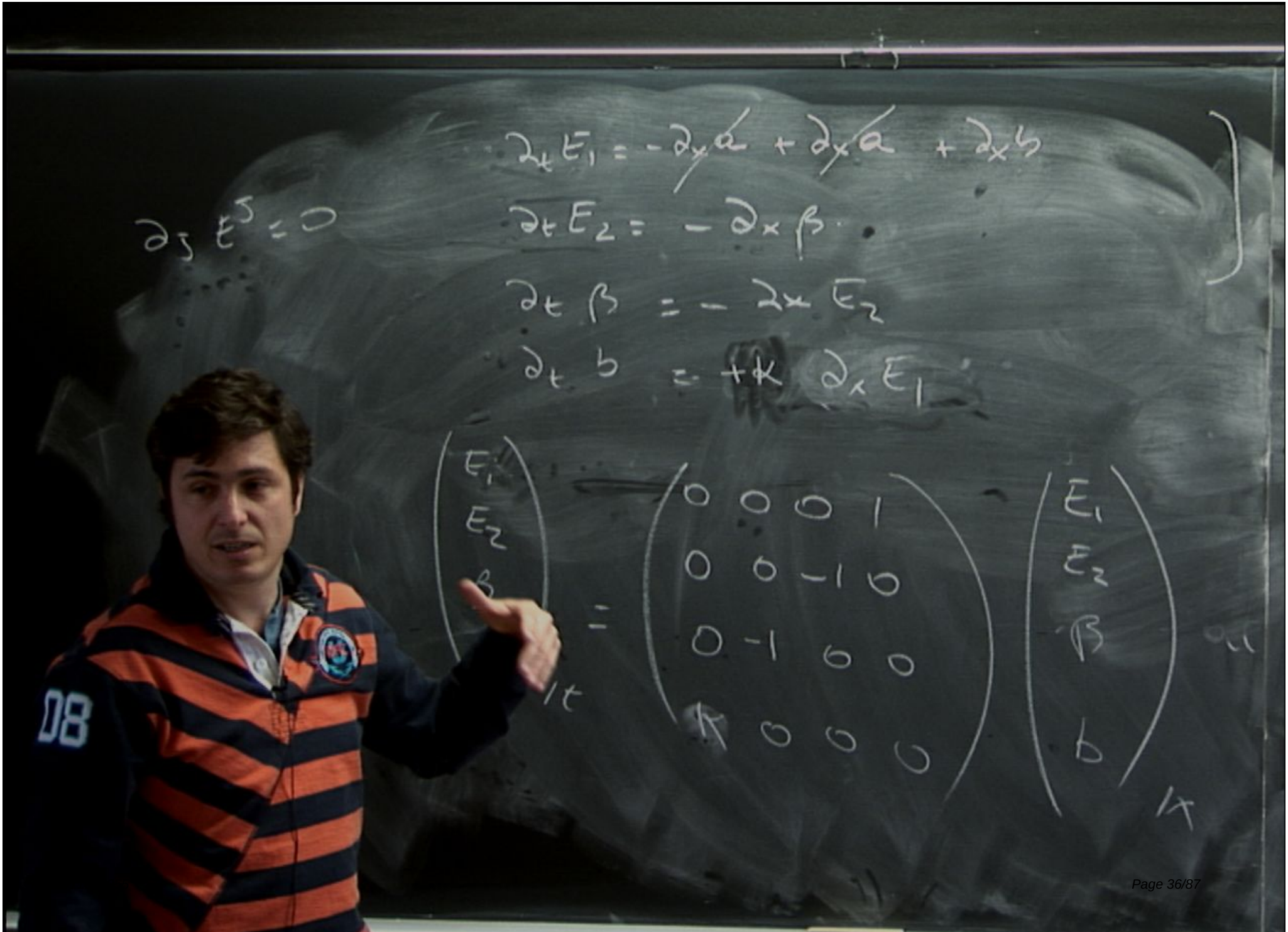
$\partial_t \beta = -\partial_x \partial_t A_2 = -\partial_x$

$\partial_t b = -\partial_y \partial_t A_2 = -\partial$


$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = +$
$\partial_j E^j = 0$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}$


$\partial_j E^j = 0$
$\partial_t E_1 = -\partial_x \alpha + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = +k(\partial_j E^j)$
$\begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}_t = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}_x$


$\partial_j E^j = 0$
$\partial_t E_1 = -\partial_x a + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = +k\, \partial_x E_1$
$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}$


$\partial_t E_1 = -\partial_x \alpha + \partial_x a + \partial_x b$
$\partial_t E_2 = -\partial_x \beta$
$\partial_t \beta = -\partial_x E_2$
$\partial_t b = +k\, \partial_x E_1$
$\partial_j E^j = 0$
$\begin{pmatrix} E_1 \\ E_2 \\ \beta \\ \end{pmatrix}_{/t} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ k & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}_{/x}$
08

$$\partial_j E^j = 0$$

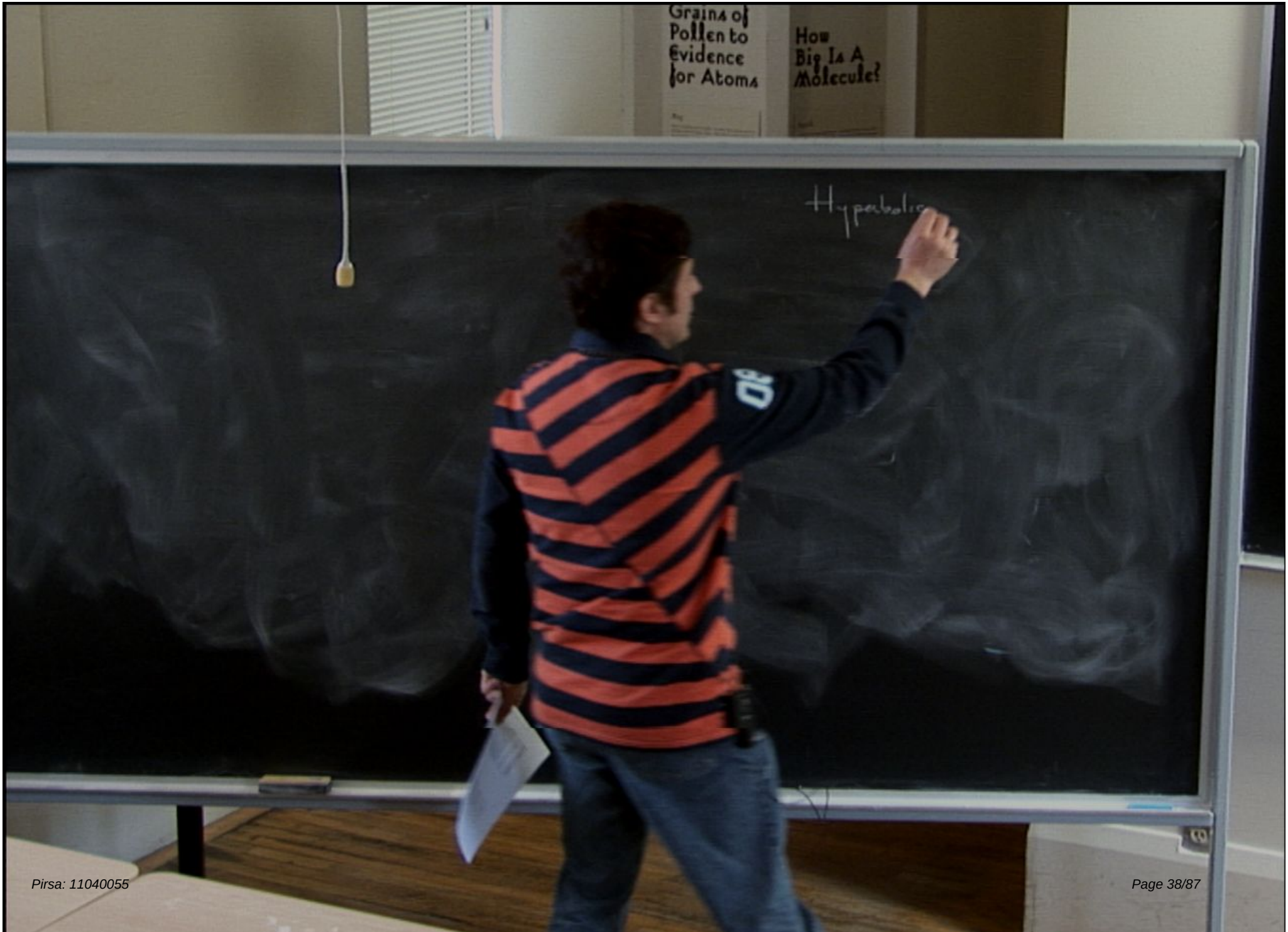
$$\partial_t E_1 = -\partial_x \alpha + \partial_x a + \partial_x b$$

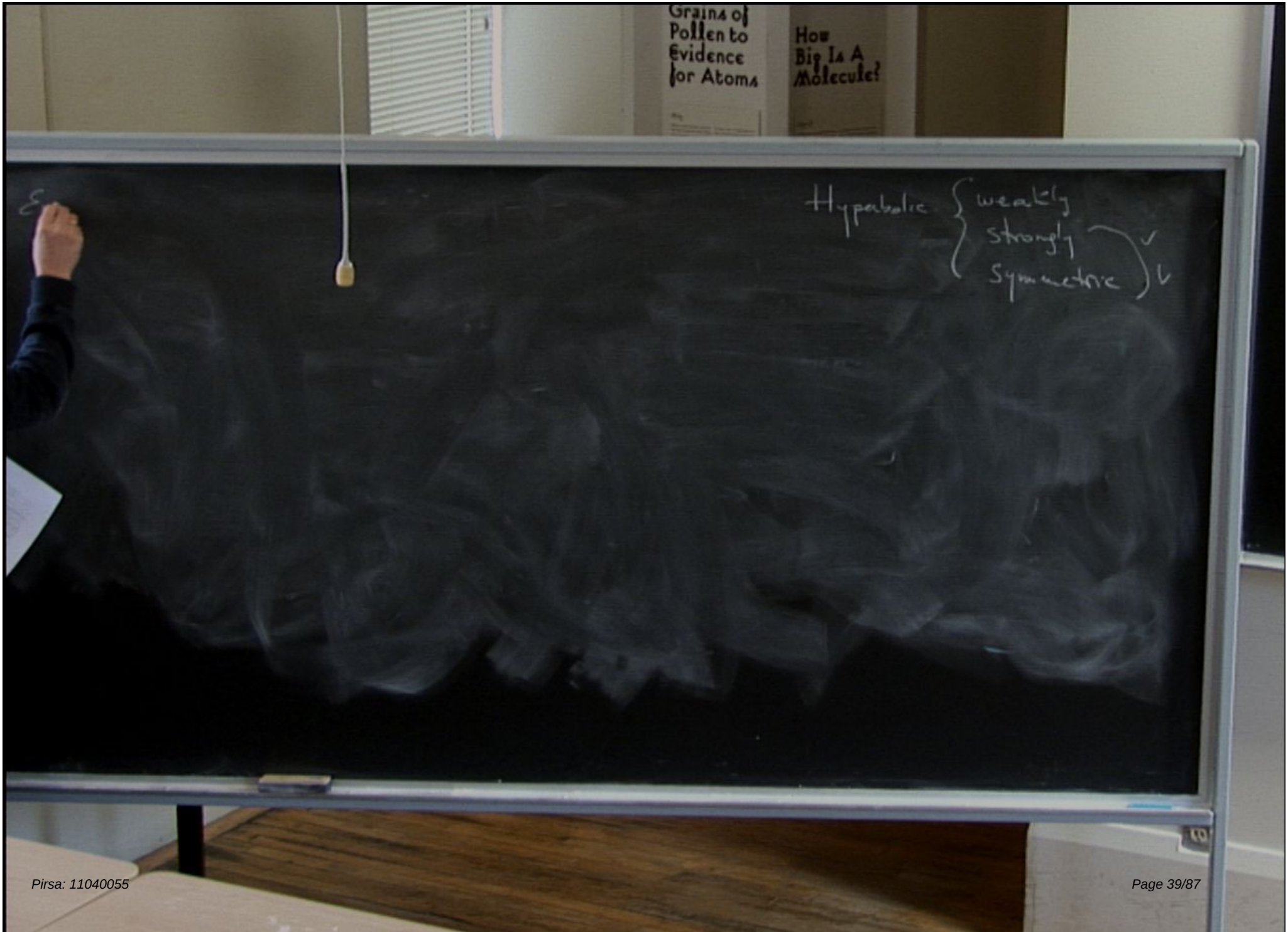
$$\partial_t E_2 = -\partial_x \beta$$

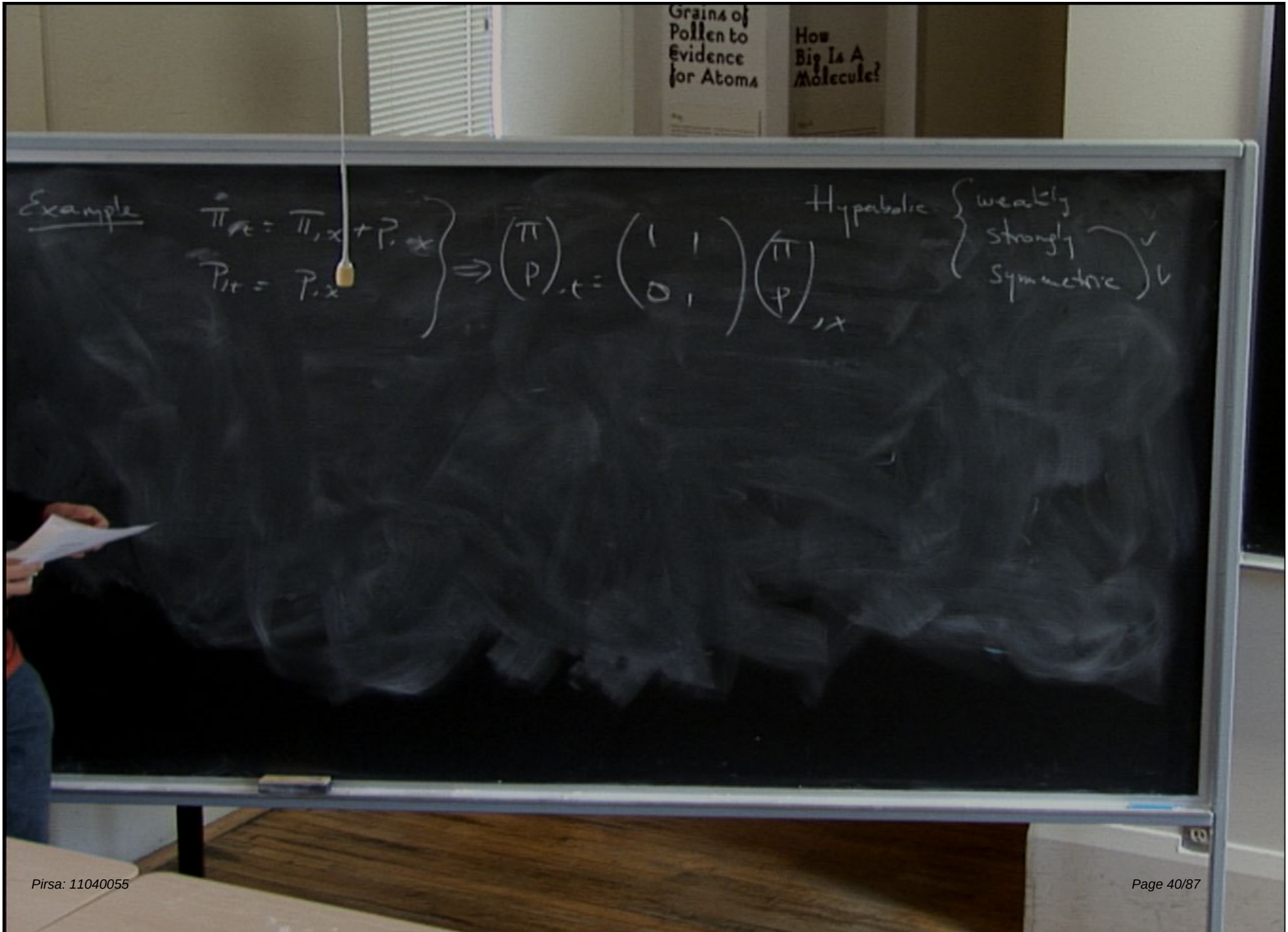
$$\partial_t \beta = -\partial_x E_2$$

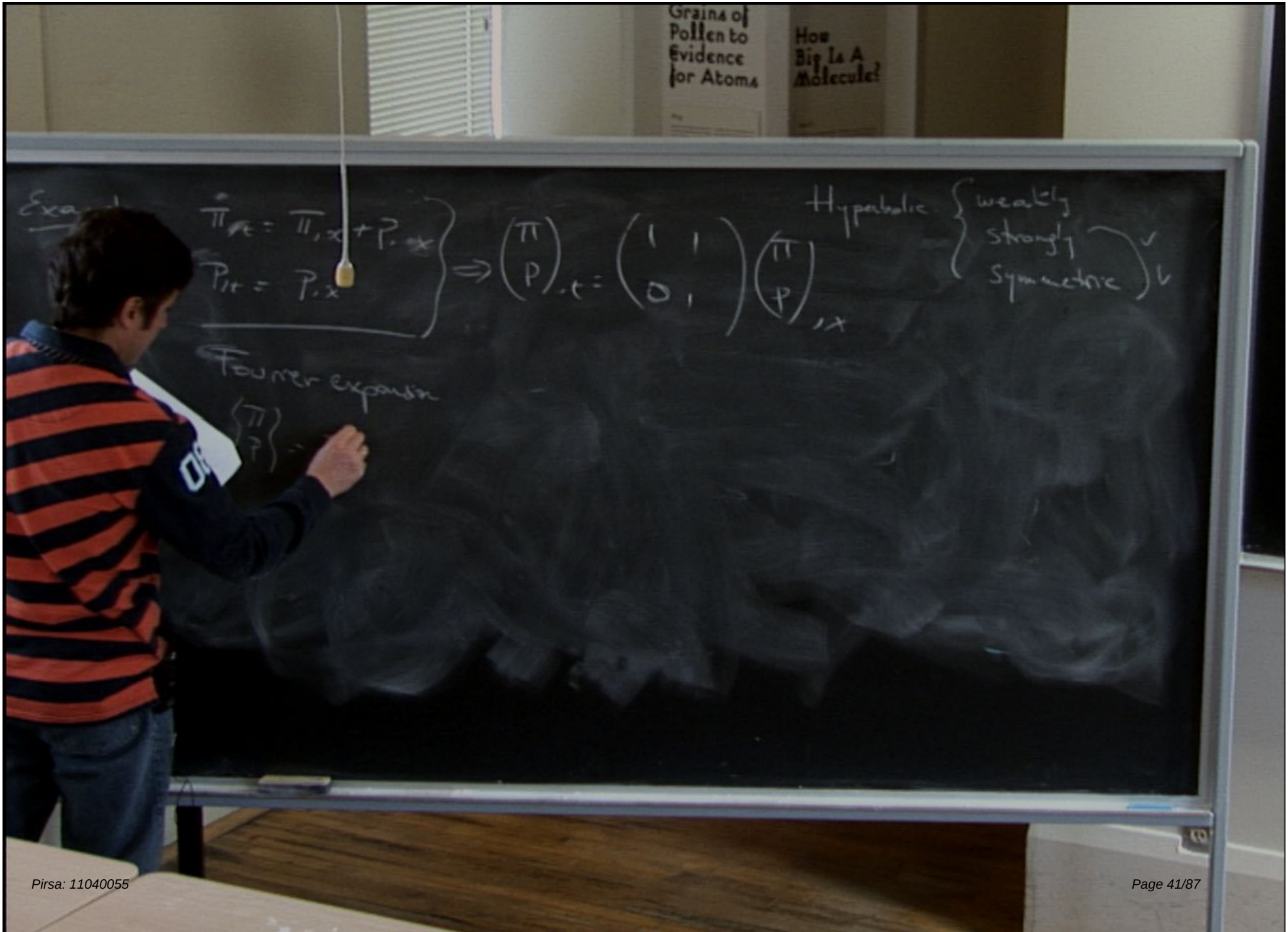
$$\partial_t b = +k\, \partial_x E_1$$

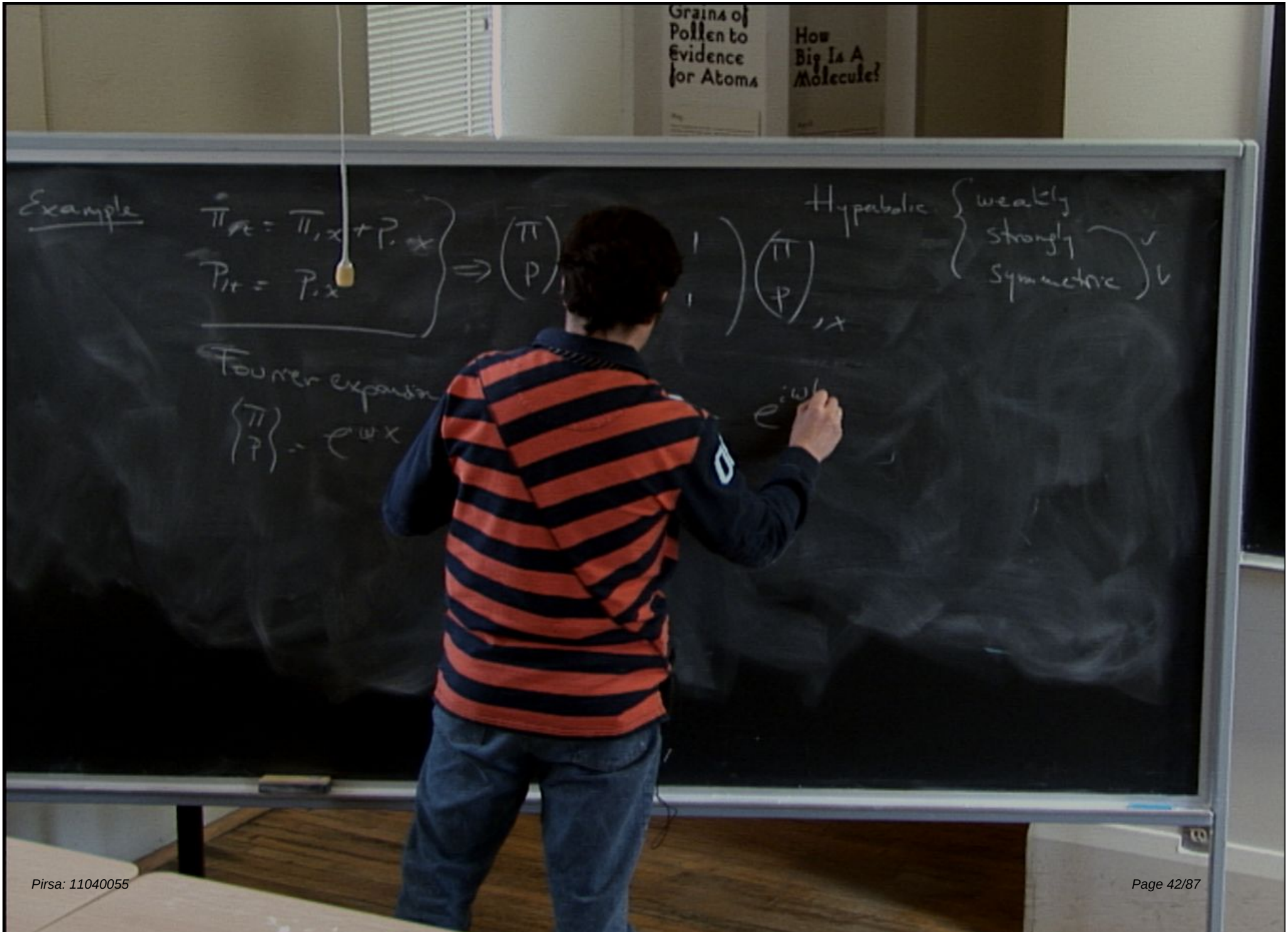
$$\begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}_{lt} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ k & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ \beta \\ b \end{pmatrix}_{lt}$$

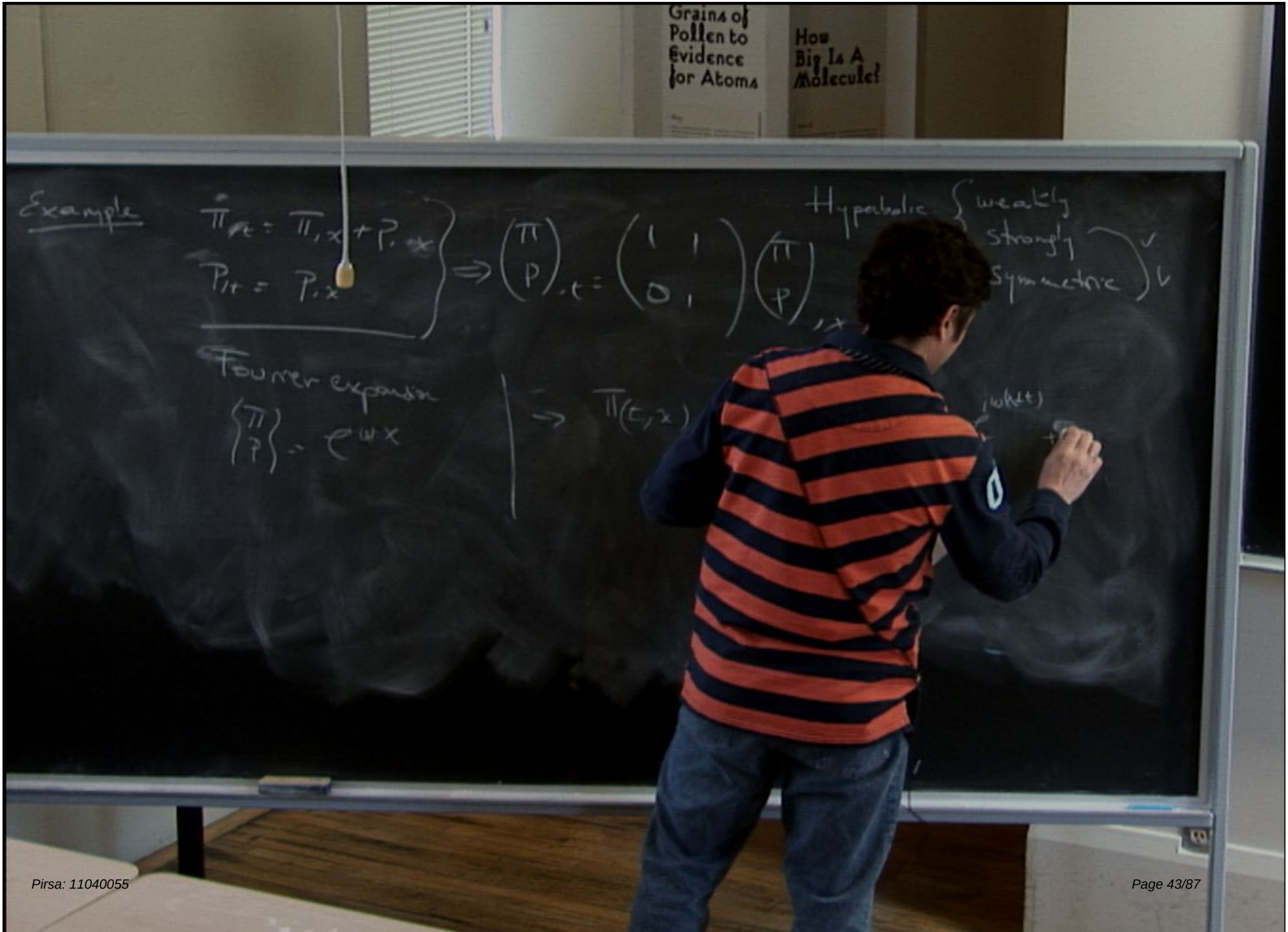

Grains of
Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Hyperbolic

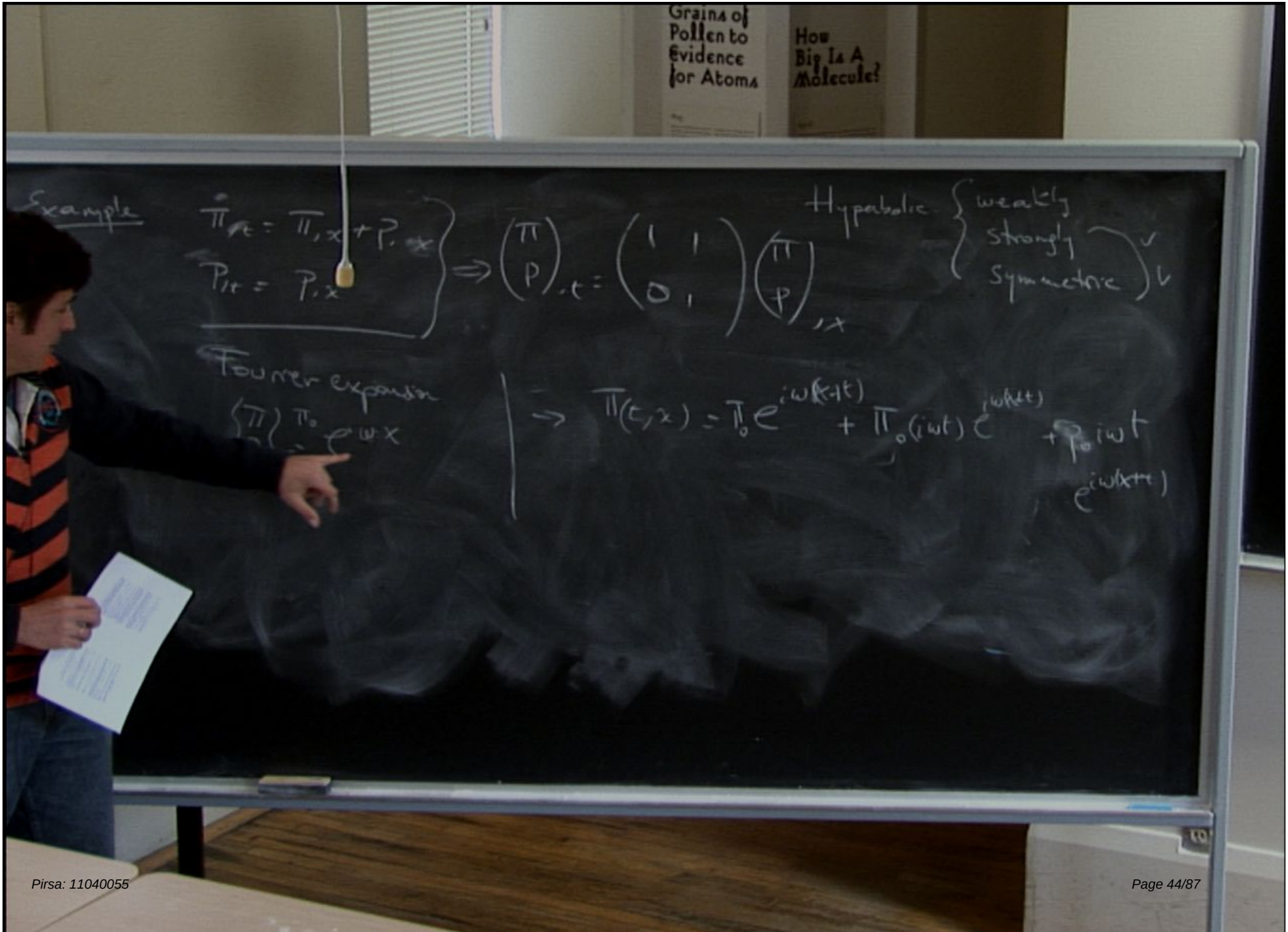

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Hyperbolic
weakly
strongly
Symmetric


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Example
$\dot{\pi}_{t+\epsilon} = \pi_{,x} + P_{,x}$
$P_{,t} = P_{,x}$
$\Rightarrow \begin{pmatrix} \pi \\ P \end{pmatrix}_{,t} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \pi \\ P \end{pmatrix}_{,x}$
Hyperbolic { weakly strongly Symmetric }


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Hyperbolic
weakly
strongly
symmetric
Fourier expansion


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Example
Hypebolic
weakly
strongly
Symmetric
Fourier expansion


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Example
Hypebolic
weakly
strongly
Symmetric
Fourier expansion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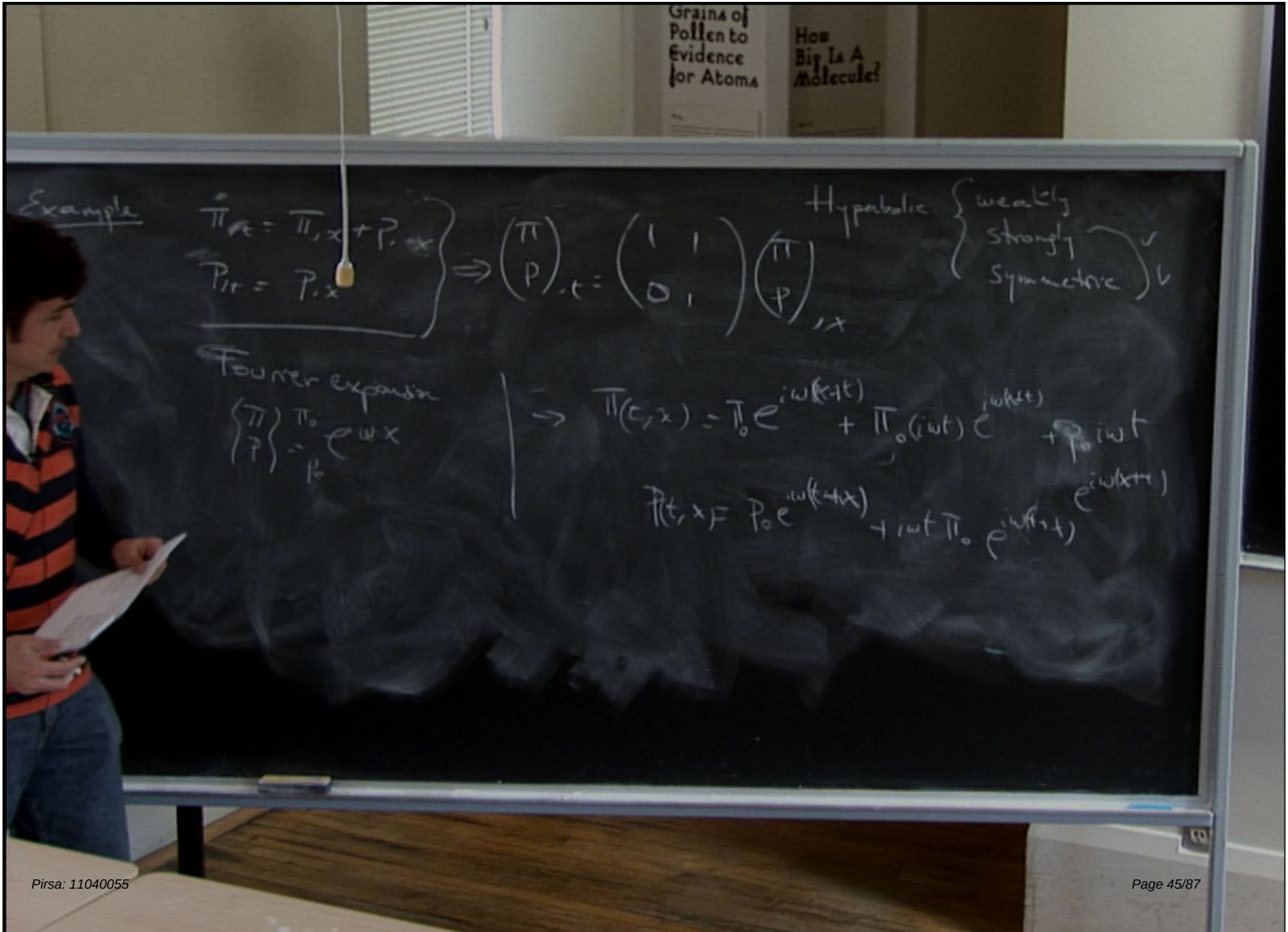
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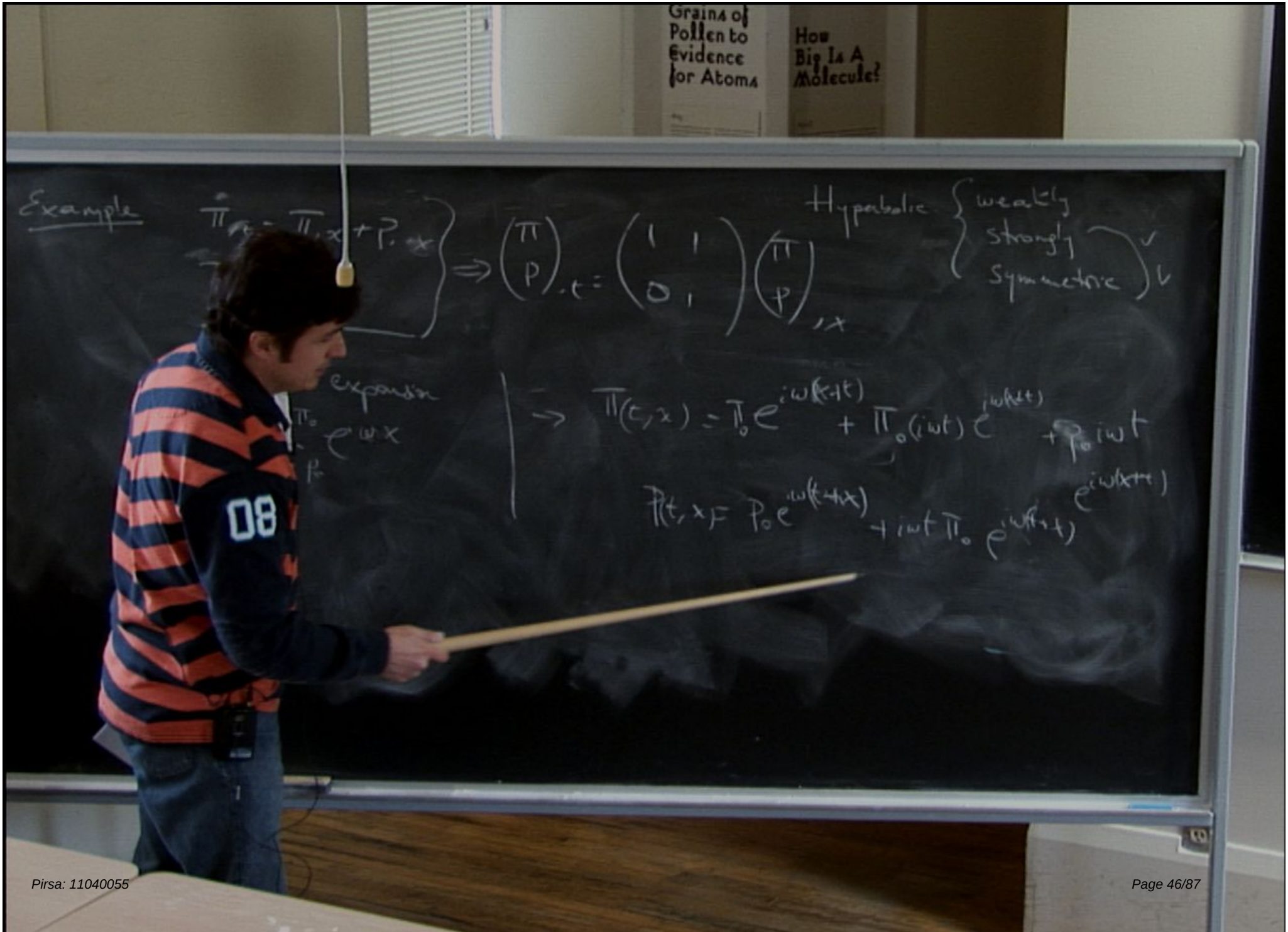
Grains of Pollen to Evidence for Atoms
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Hyperbolic
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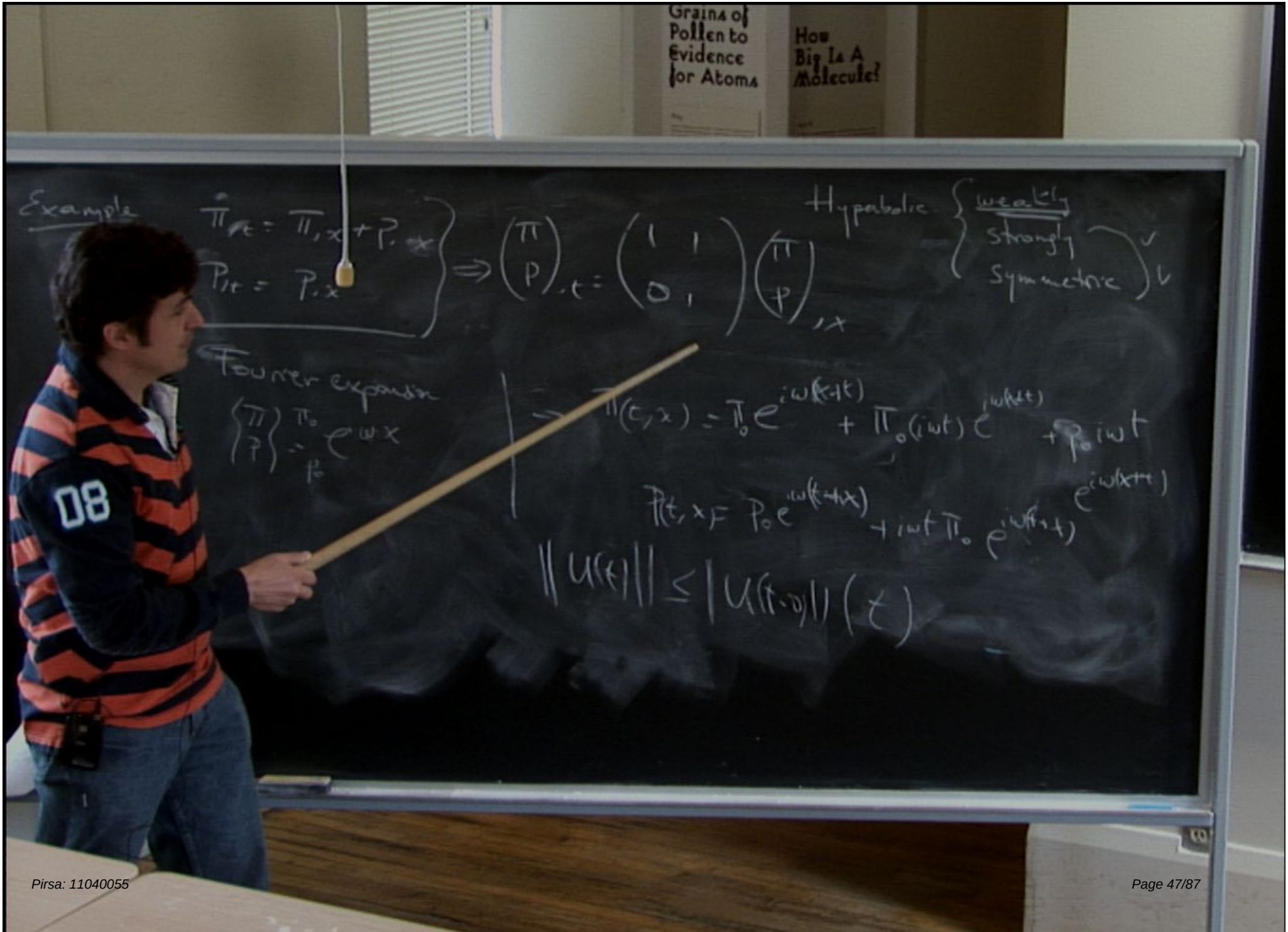
symmetric
expansion
08


Grains of Pollen to Evidence for Atoms
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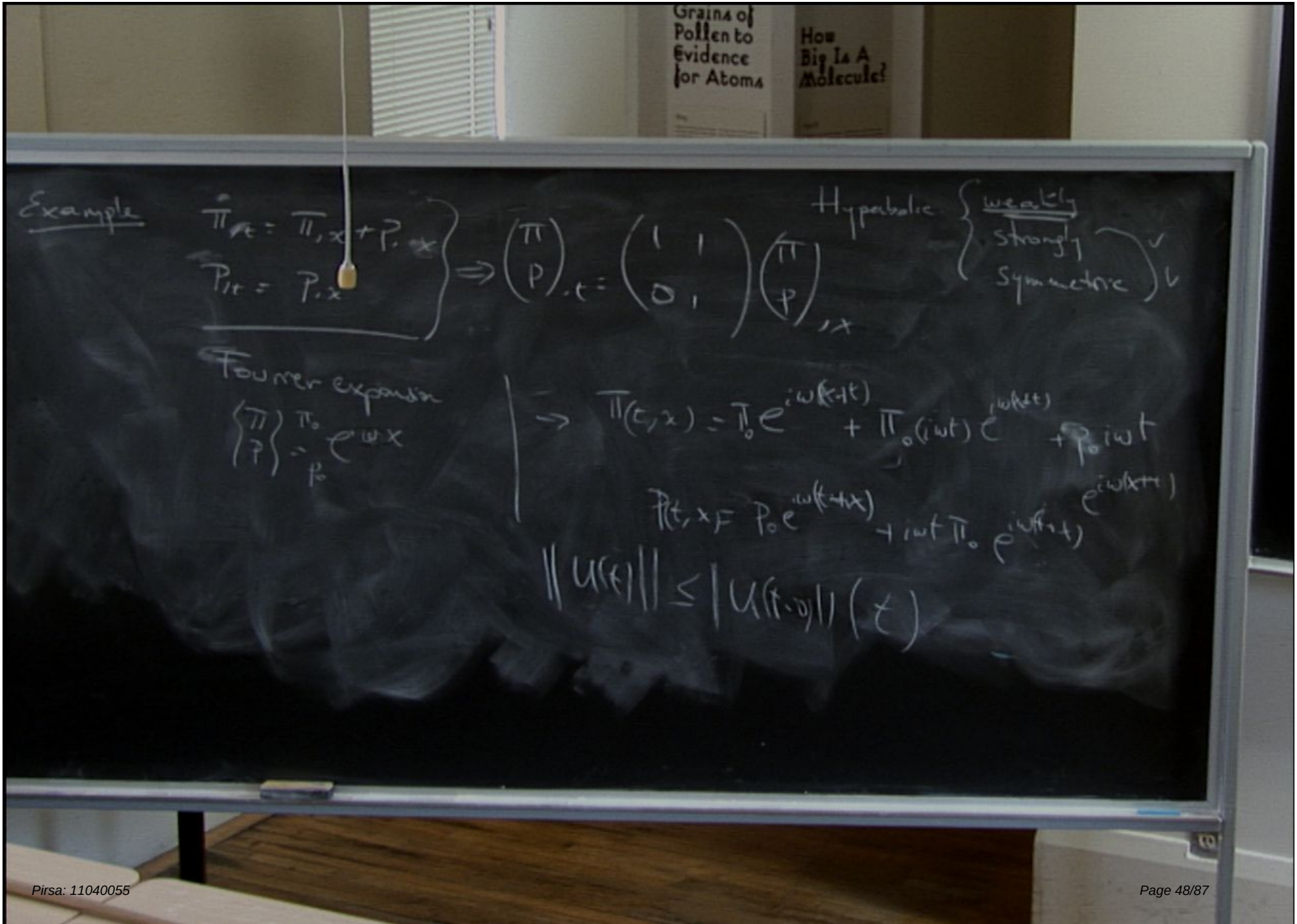
Grains of Pollen to Evidence for Atoms
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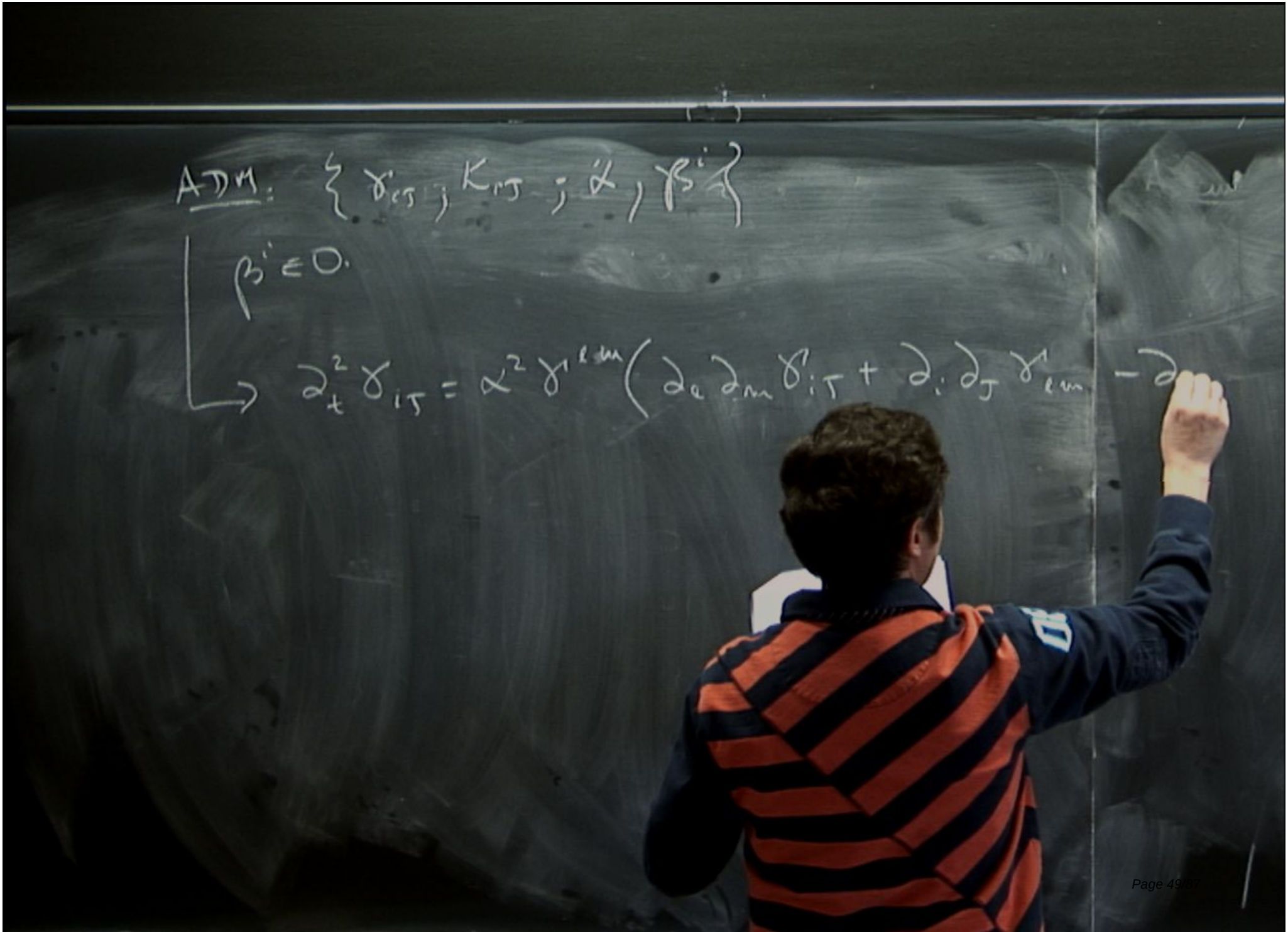
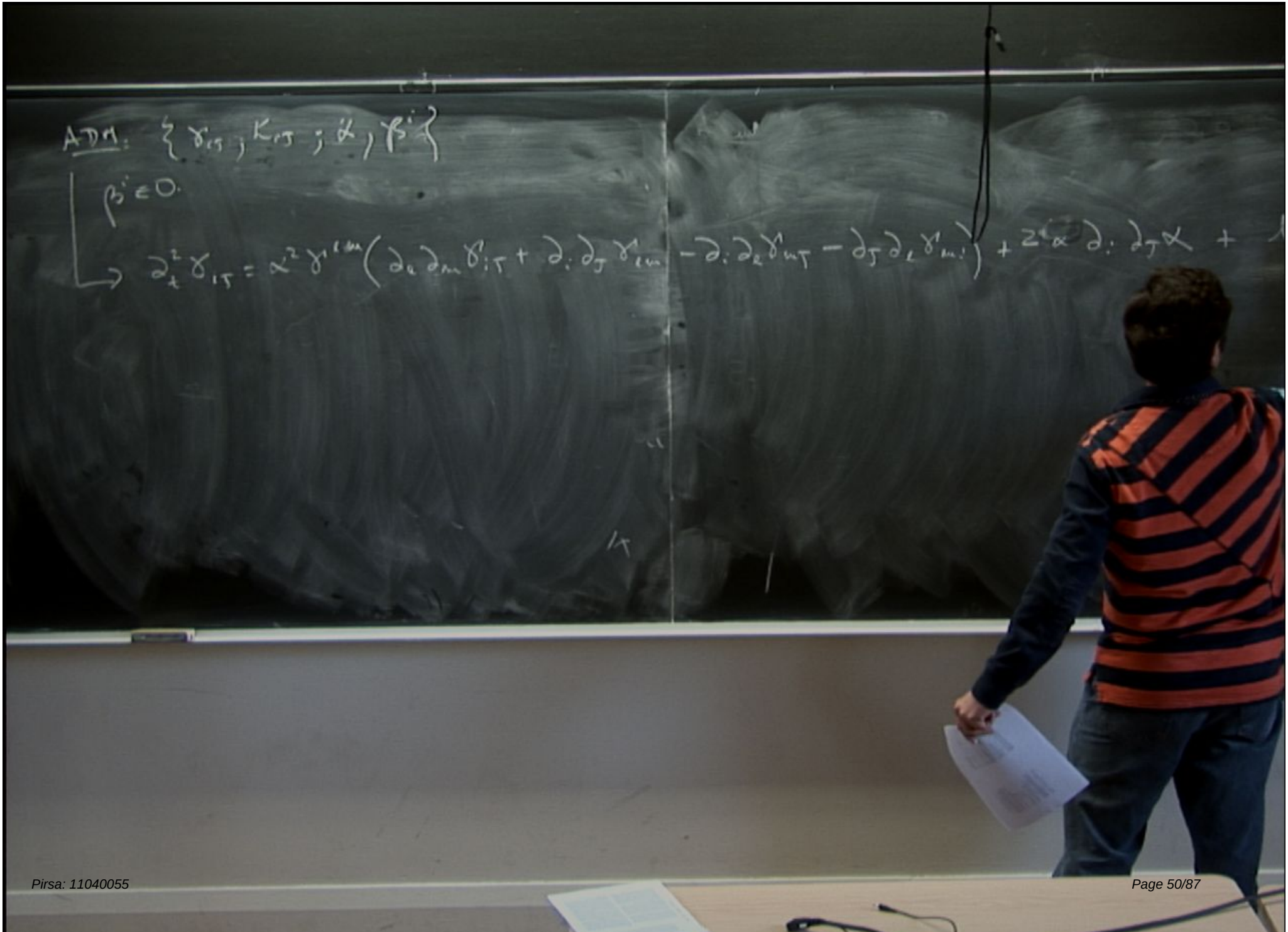
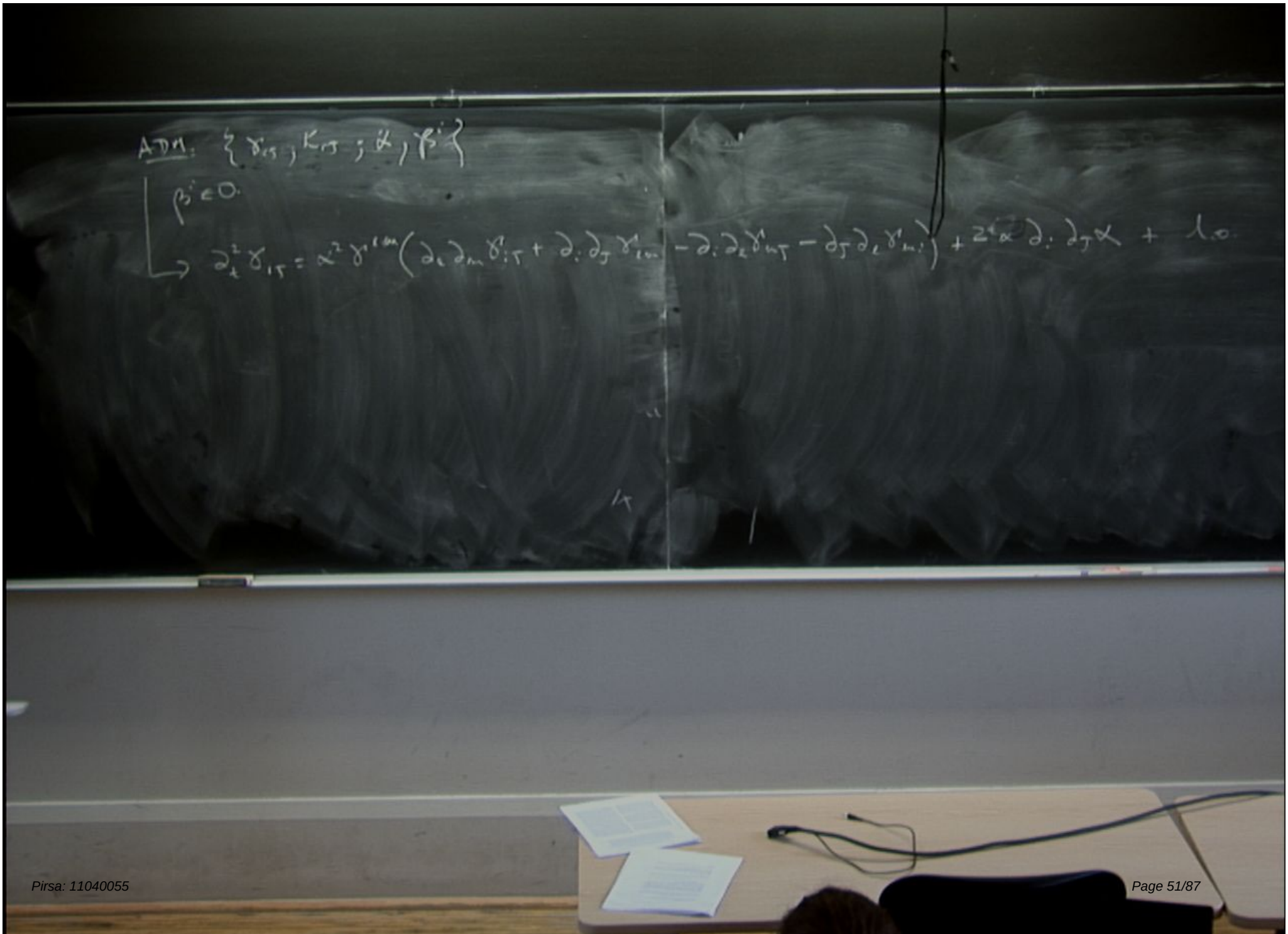

ADM: $\{\gamma_{ij}, K_{ij}, \alpha, \beta^i\}$
$\beta^i = 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{kl} \gamma^{mn} (\partial_k \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{kl} - \partial$

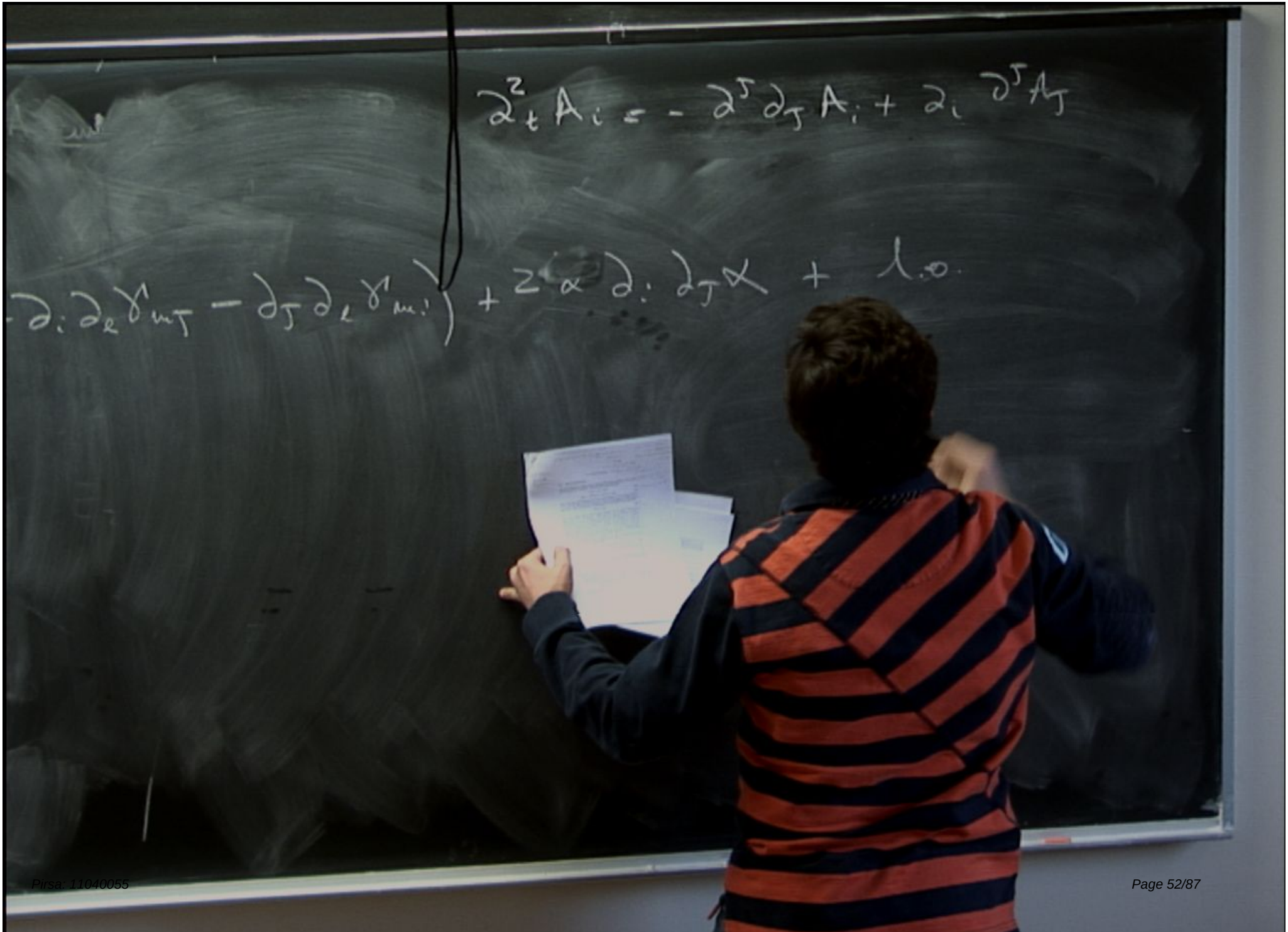

ADM: $\{\gamma_{ij}, K_{ij}; \alpha, \beta^i\}$
$\beta^i \in 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}\left(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}\right) + 2\alpha \partial_i \partial_j \alpha +$

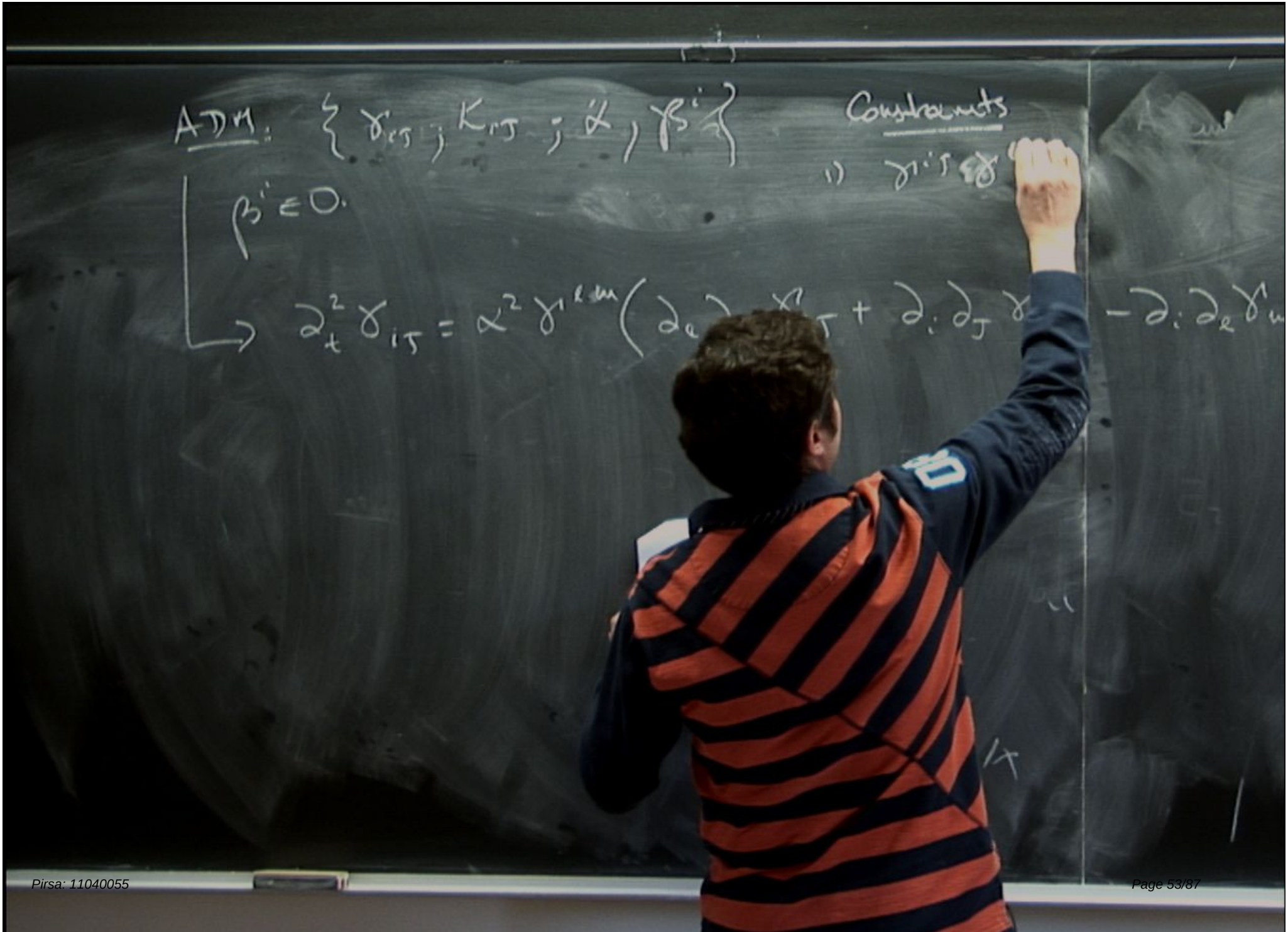
ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$

$\beta^i = 0$

$$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}\left(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}\right) + 2\alpha \partial_i \partial_j \alpha + \ldots$$



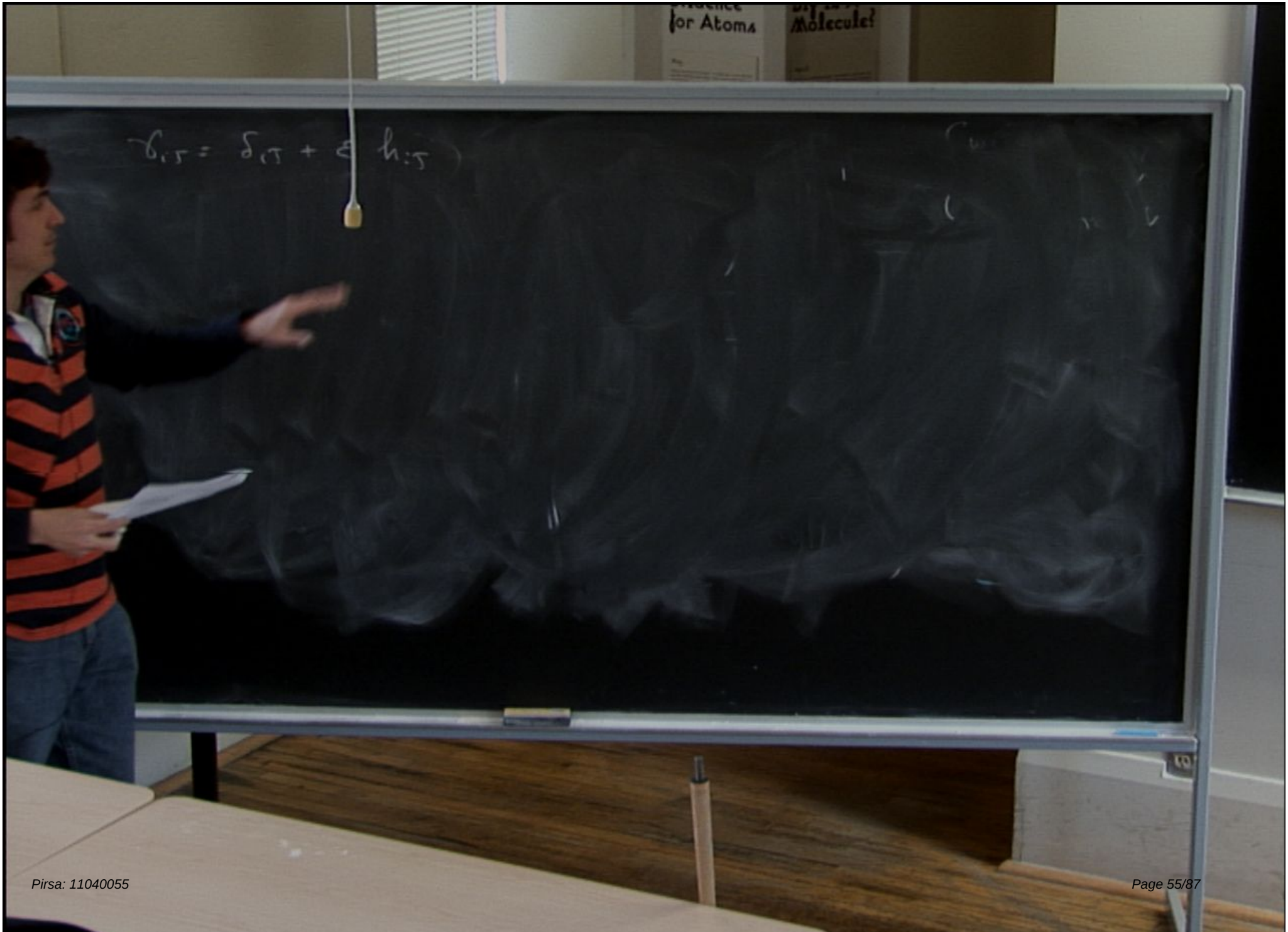

$\partial_t^2 A_i = - \partial^j \partial_j A_i + \partial_i \partial^j A_j$
$\partial_i \partial_k \gamma_{mj} - \partial_j \partial_k \gamma_{mi}) + 2 \alpha \partial_i \partial_j \alpha + \ldots$

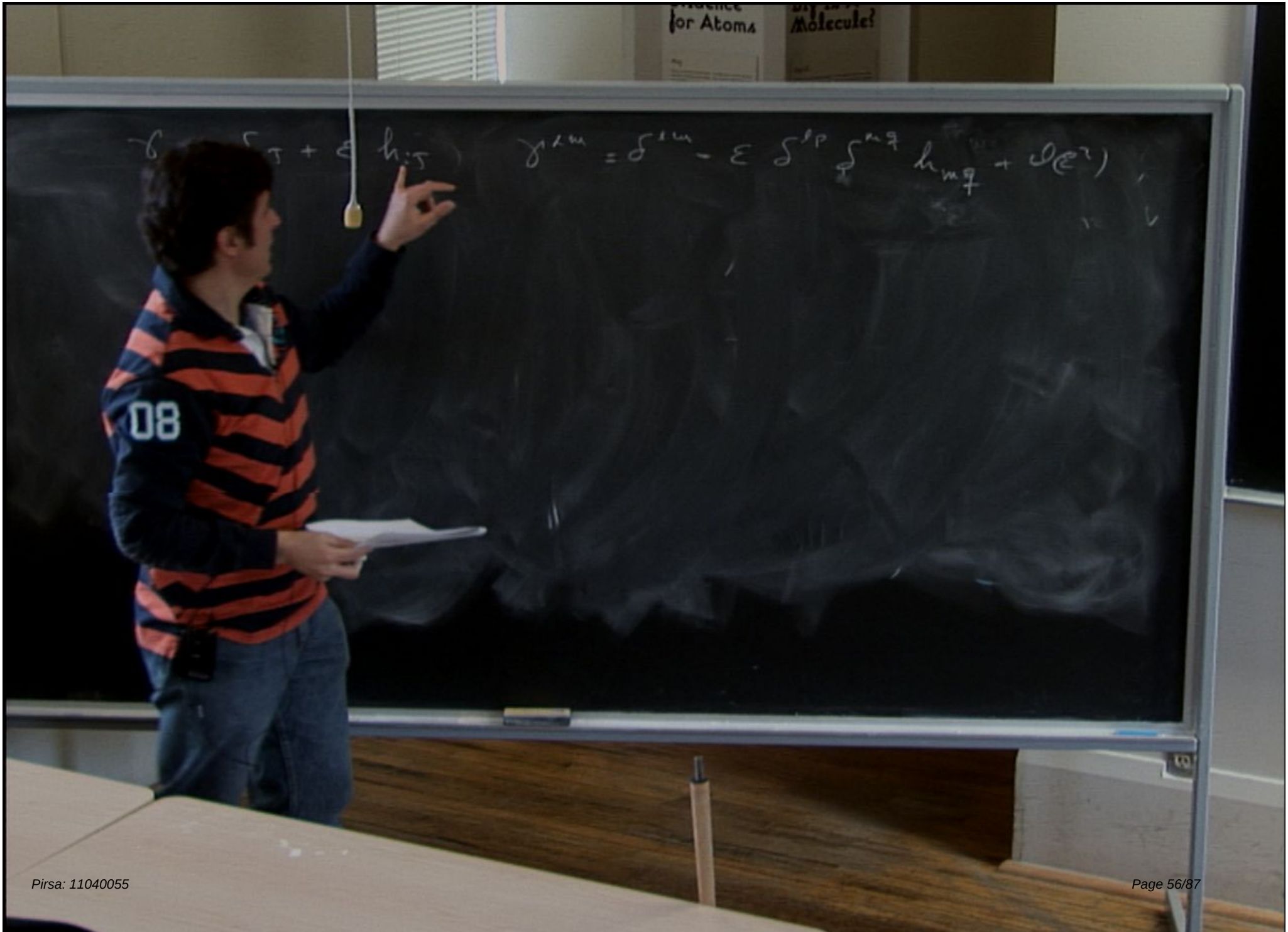

ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$
Constraints
$\beta^i = 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{kl} \gamma^{mn} \left(\partial_k \partial_l \gamma_{jn} + \partial_i \partial_j \gamma \right) - \partial_i \partial_k \gamma_{jn}$

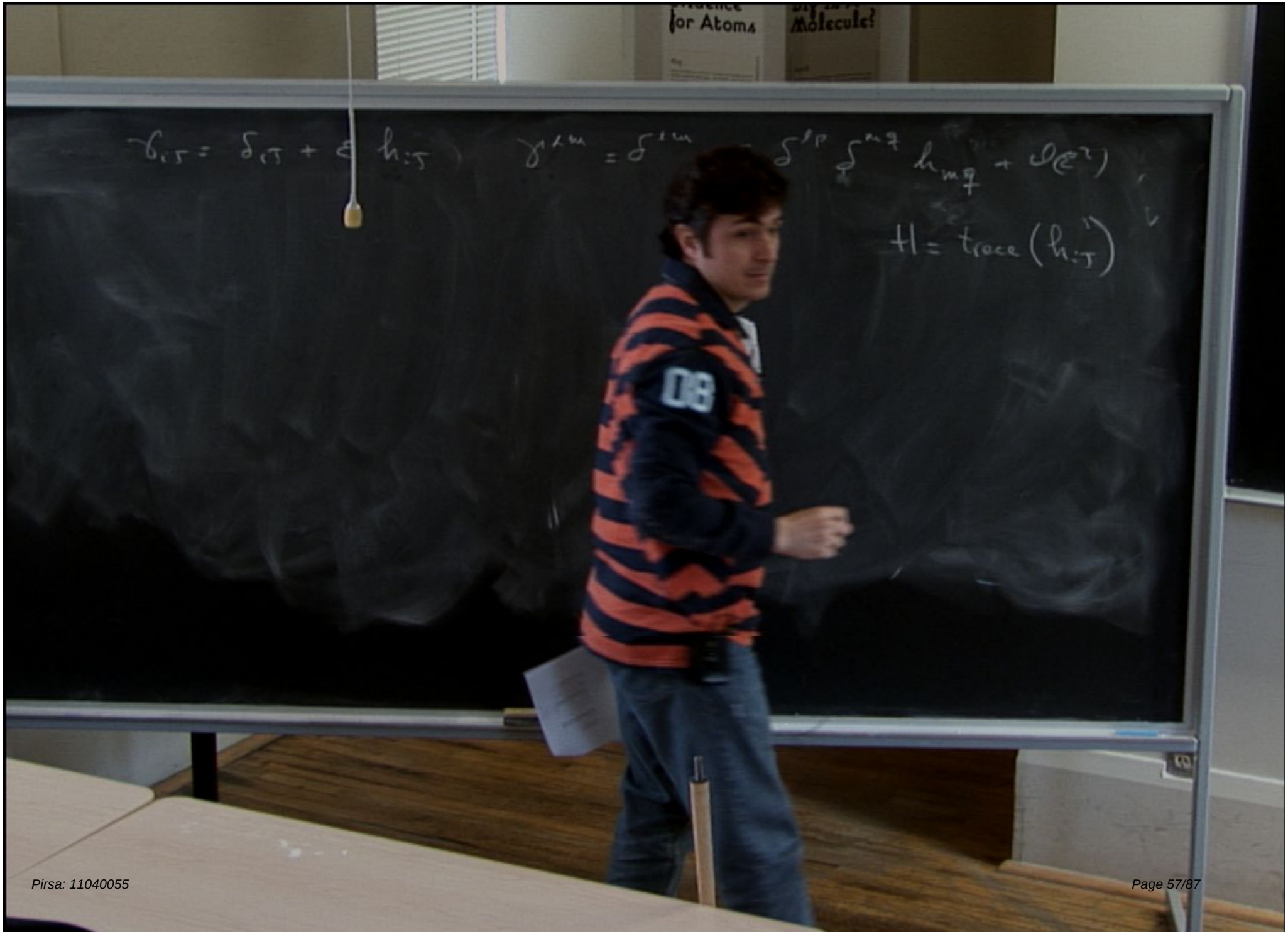
Constraints

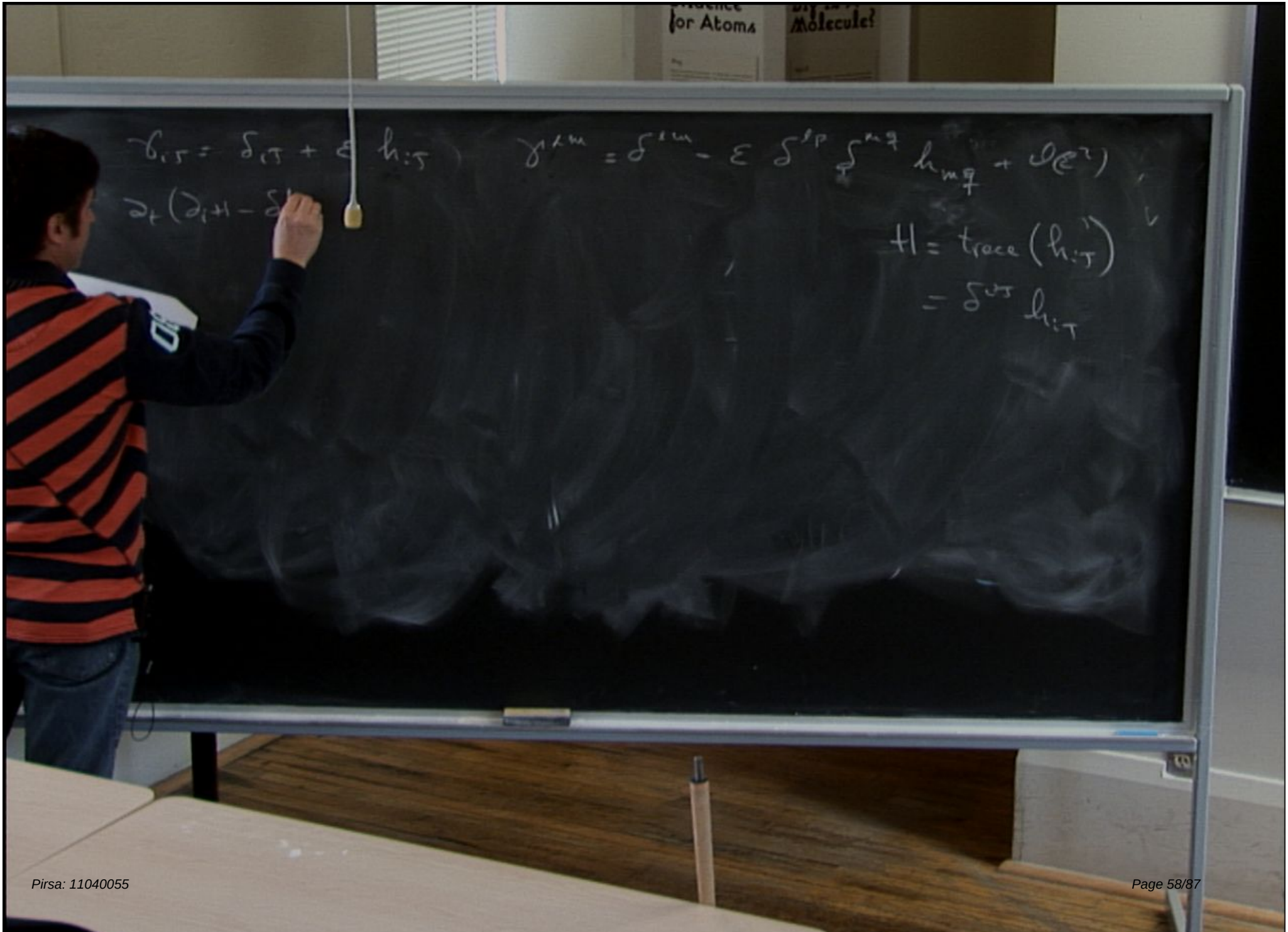
1) $\gamma^{ij}\gamma^{lm}\left(\partial_i \partial_l \gamma_{jm} - \partial_j \gamma_{il}\right) + l.o. = 0$

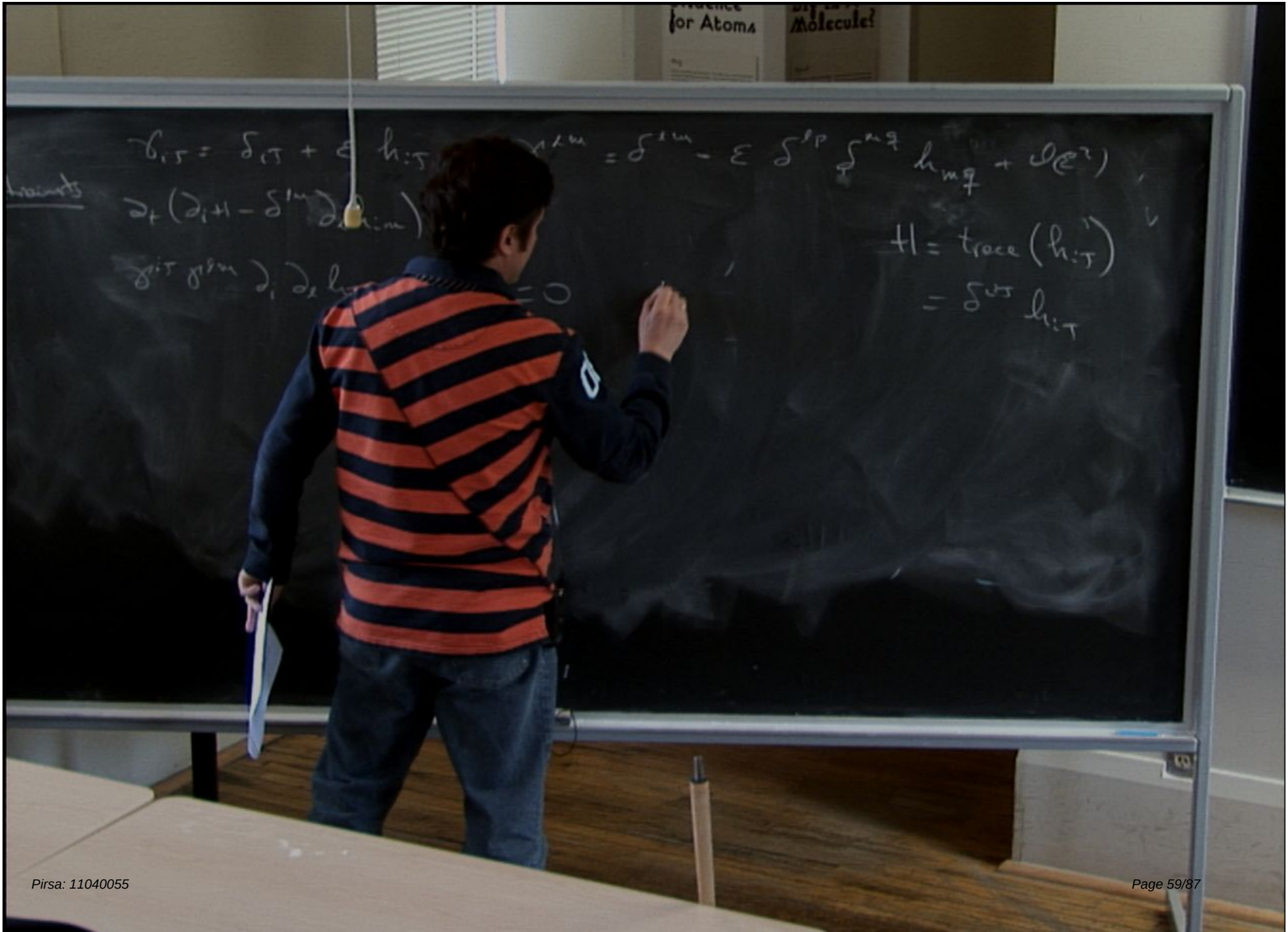
$\gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}\Big) +$

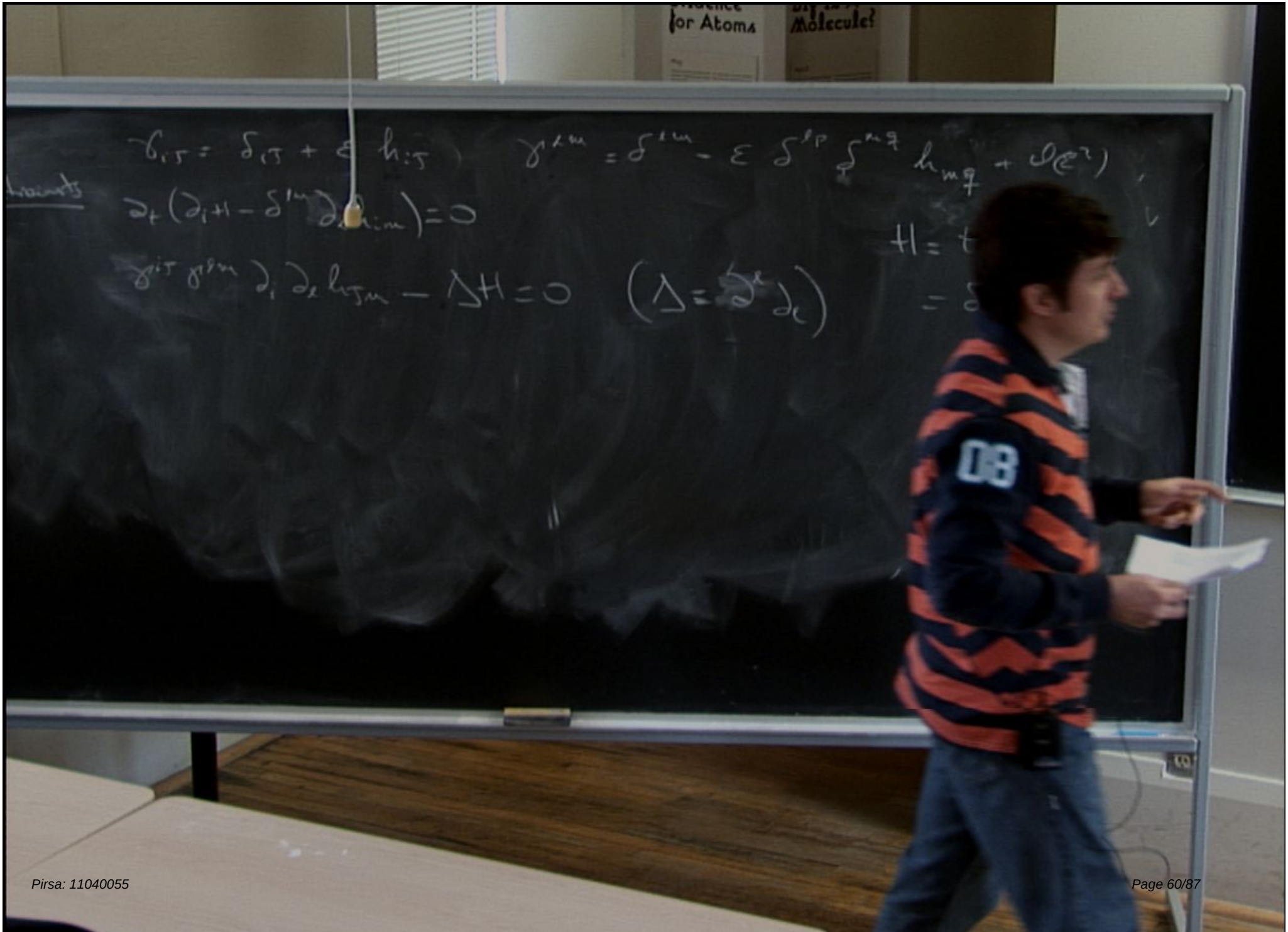

for Atoms
Molecule?

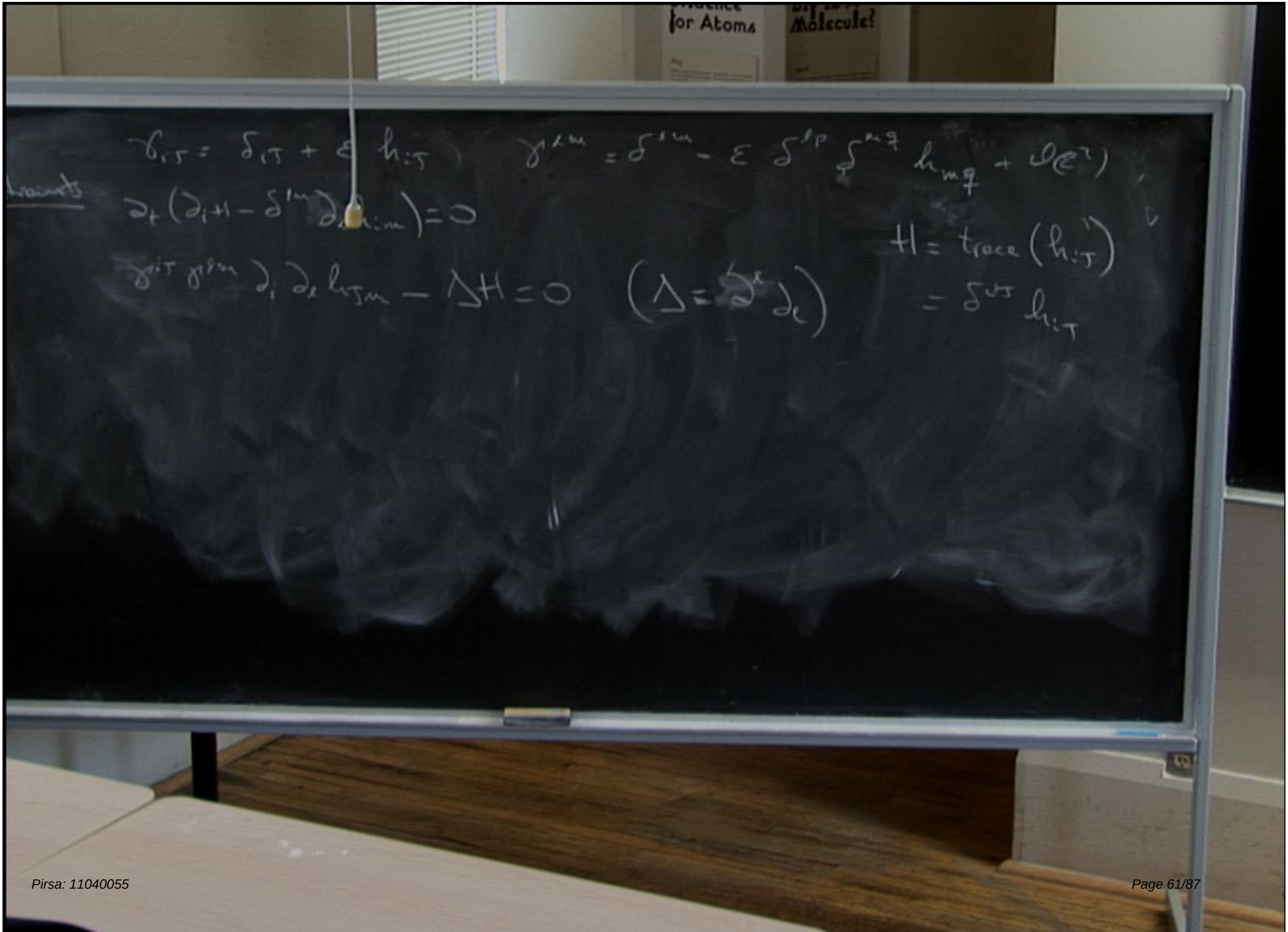

for Atoms
Molecule?
08


for Atoms
Molecule?
$h_{ij} = \delta_{ij} + \epsilon\, h_{ij}$
$\gamma^{ik} = \delta^{ik} - \epsilon\, \delta^{ip}\delta^{kq} h_{pq} + \mathcal{O}(\epsilon^2)$
$tr\, h = \mathrm{trace}\,(h_{ij})$


for Atoms
Molecule?
$g_{ij} = \delta_{ij} + \varepsilon h_{ij}$
$\gamma^{ik} = \delta^{ik} - \varepsilon \delta^{ip} \delta^{kq} h_{pq} + \mathcal{O}(\varepsilon^2)$
$\partial_t (\partial_{t+1} - \delta$
$+1 = \text{trace}(h_{ij})$
$= \delta^{ij} h_{ij}$


for Atoms
Molecule?
H = trace (h_ij)
= δ^ij h_ij


for Atoms
Molecule?
$\gamma_{ij} = \delta_{ij} + \varepsilon h_{ij}$
$\gamma^{lm} = \delta^{lm} - \varepsilon \delta^{lp} \delta^{mq} h_{mq} + \mathcal{O}(\varepsilon^2)$
$\partial_t (\partial_i H - \delta^{lm} \partial_l h_{im}) = 0$
$\gamma^{ij} \gamma^{lm} \partial_i \partial_l h_{jm} - \Delta H = 0$
$(\Delta = \partial^l \partial_l)$
08


for Atoms
Molecule?
$g_{ij} = \delta_{ij} + \varepsilon h_{ij}$
$g^{ij} = \delta^{ij} - \varepsilon \delta^{ip} \delta^{jq} h_{pq} + O(\varepsilon^2)$
straints
$\partial_t (\partial_i H - \delta^{lm} \partial_l h_{im}) = 0$
$H = \text{trace}(h_{ij}) = \delta^{ij} h_{ij}$
$g^{ij} g^{lm} \partial_i \partial_l h_{jm} - \Delta H = 0 \quad (\Delta = \partial^i \partial_i)$

Constraints

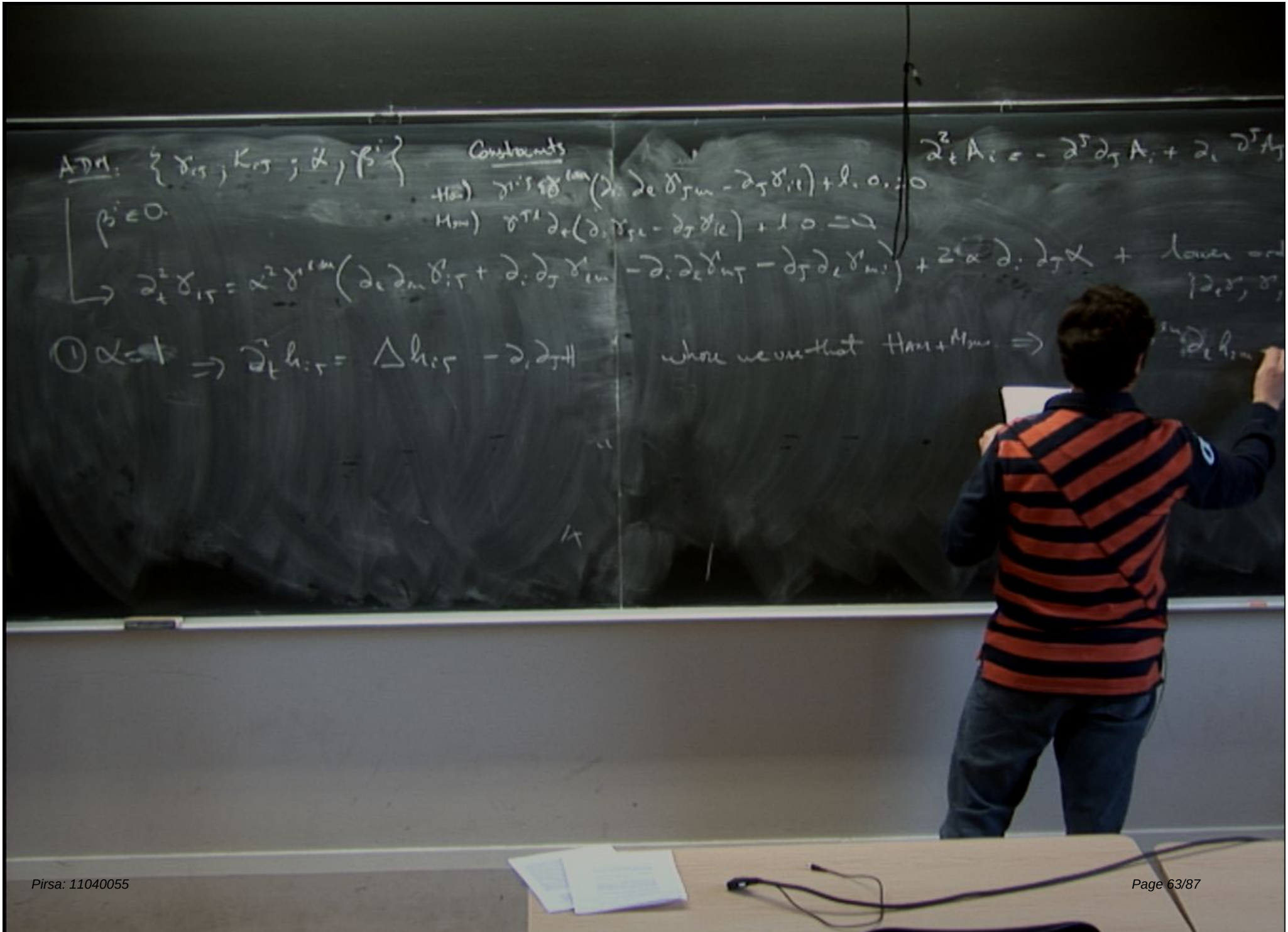
$$\partial_t^2 A_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$$

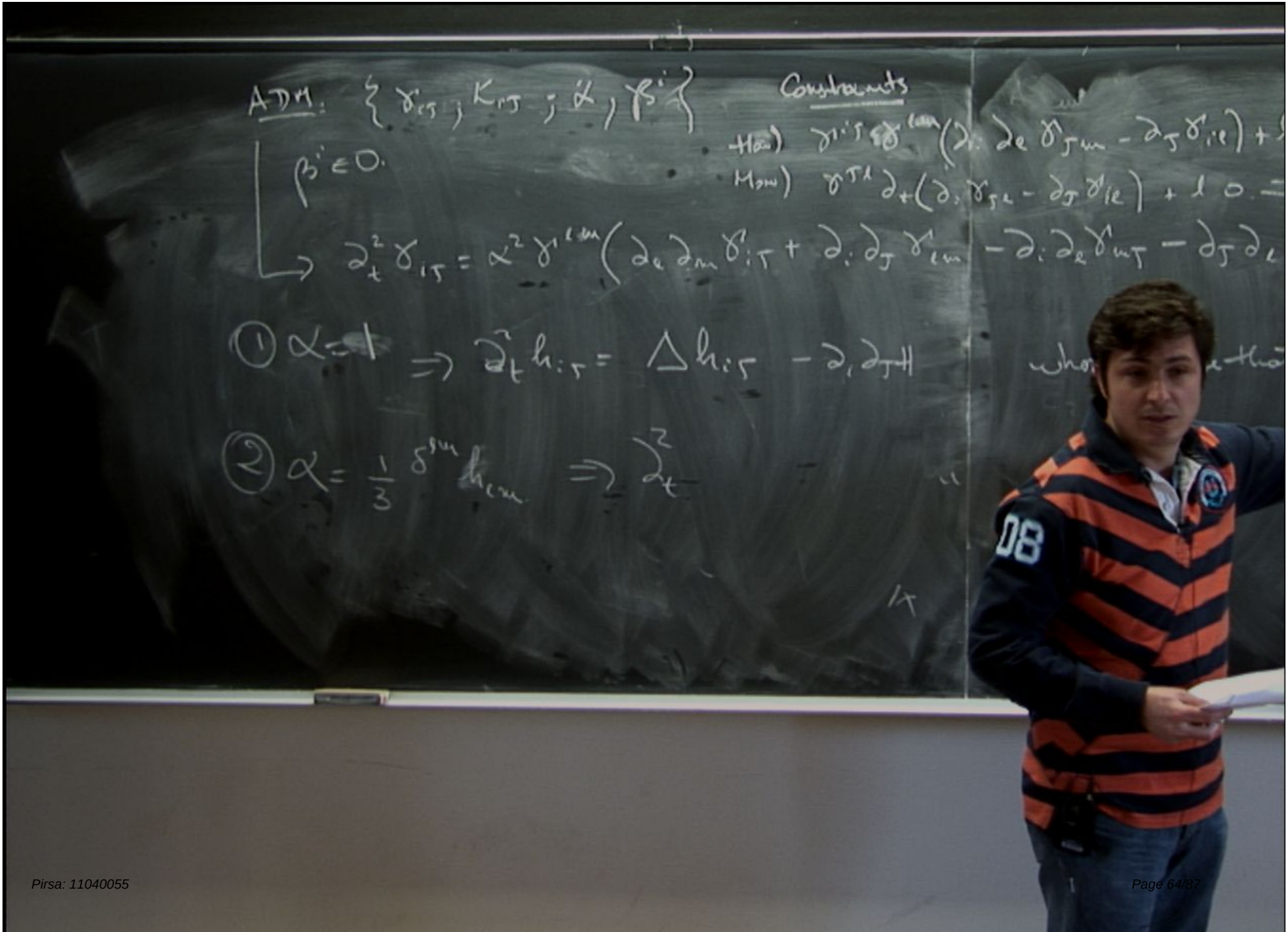
Ham) $\gamma^{ij}\gamma^{lm}(\partial_i \partial_l \gamma_{jm} - \partial_j \gamma_{il}) + l.o. = 0$

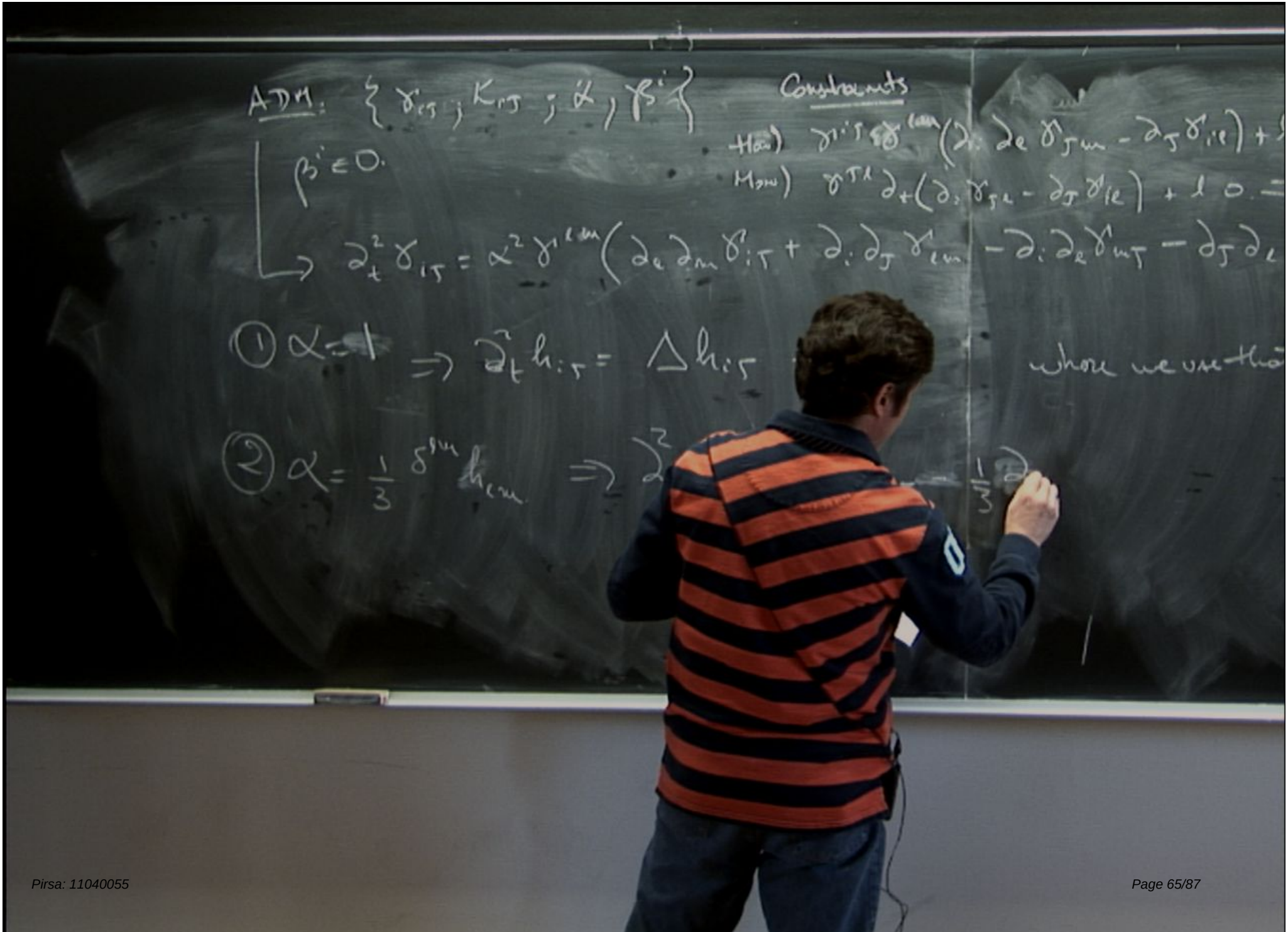
Mom) $\gamma^{jl}\partial_t(\partial_i \gamma_{jl} - \partial_j \gamma_{il}) + l.o. = 0$

$\partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}) + 2\alpha\, \partial_i \partial_j \alpha$ + lower order

$\{\partial_l \gamma, \gamma, \partial_i$


ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$
Constraints
$\beta^i \in 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}) + 2\alpha \partial_i \partial_j \alpha$ + lower
① $\alpha = 1 \Rightarrow \partial_t^2 h_{ij} = \Delta h_{ij} - \partial_i \partial_j H$
where we use that Ham + Mom $\Rightarrow$


ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$
Constraints
$\beta^i = 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l$
① $\alpha = 1 \Rightarrow \partial_t^2 h_{ij} = \Delta h_{ij} - \partial_i \partial_j H$
② $\alpha = \frac{1}{3} \delta^{lm} h_{lm} \Rightarrow \partial_t^2$


ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$
Constraints
$\beta^i = 0$
$\partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l$
① $\alpha = 1 \Rightarrow \partial_t^2 h_{ij} = \Delta h_{ij}$
② $\alpha = \frac{1}{3} \delta^{lm} h_{lm} \Rightarrow$
where we use

ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$

Constraints

Ham) $\gamma^{ij}\gamma^{lm}(\partial_i \partial_l \gamma_{jm} - \partial_j \gamma_{il}) + \ldots$

Mom) $\gamma^{jl}\partial_t(\partial_i \gamma_{jl} - \partial_j \gamma_{il}) + l\,0 = \ldots$

$\beta^i \in 0.$

$\hookrightarrow \partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \ldots$

① $\alpha = 1 \Rightarrow \partial_t^2 h_{ij} = \Delta h_{ij} - \partial_i \partial_j H$ where we use tha…

② $\alpha = \frac{1}{3}\delta^{lm} h_{lm} \Rightarrow \partial_t^2 h_{ij} = \left(\Delta h_{ij} - \frac{1}{3}\partial_i \partial_j H\right) =$

ADM: $\{\gamma_{ij}; K_{ij}; \alpha, \beta^i\}$

$\beta^i \in 0.$

Constraints

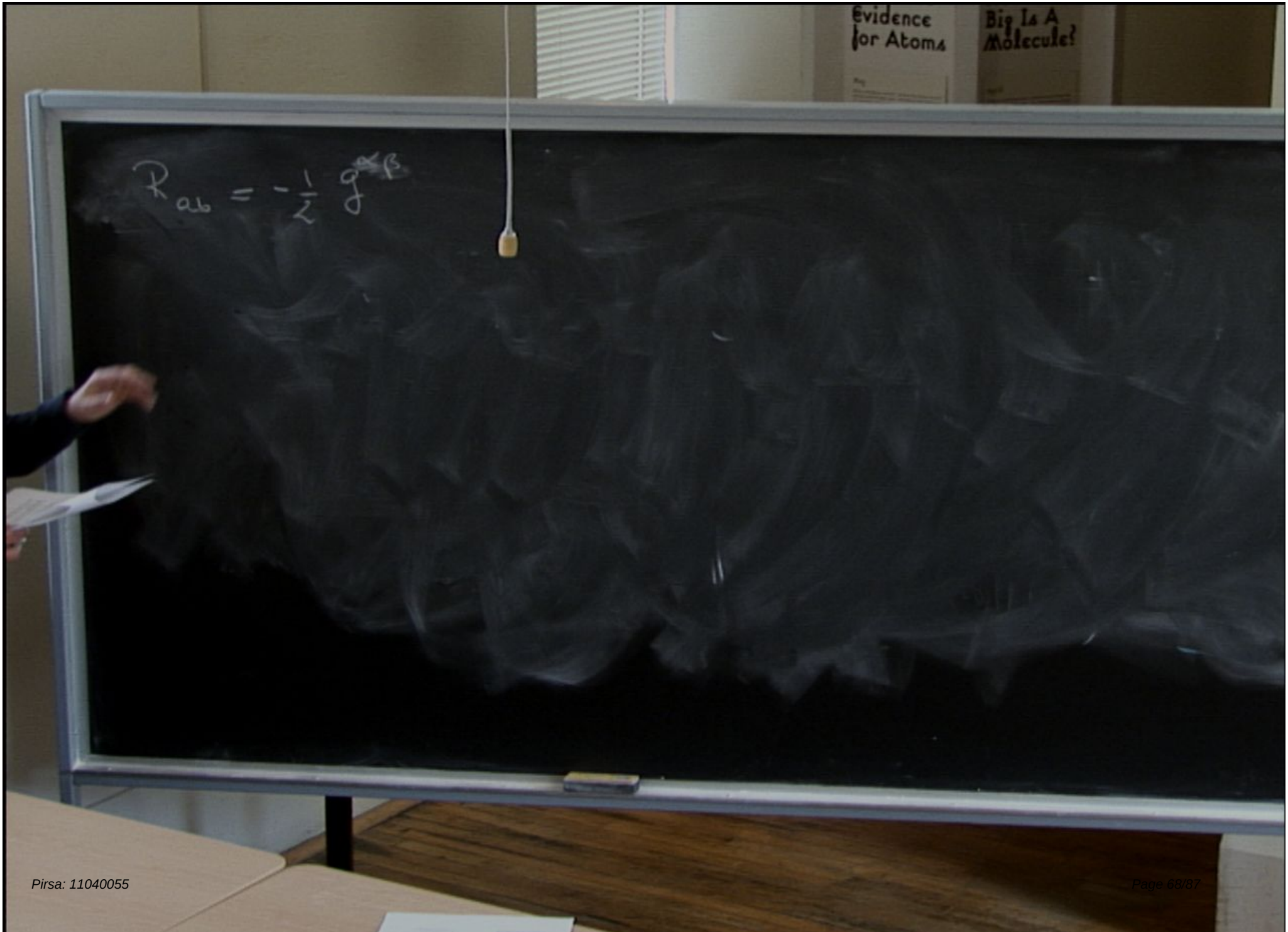
Ham) $\gamma^{ij}\gamma^{lm}(\partial_i \partial_l \gamma_{jm} - \partial_j \gamma_{il}) + \ldots$

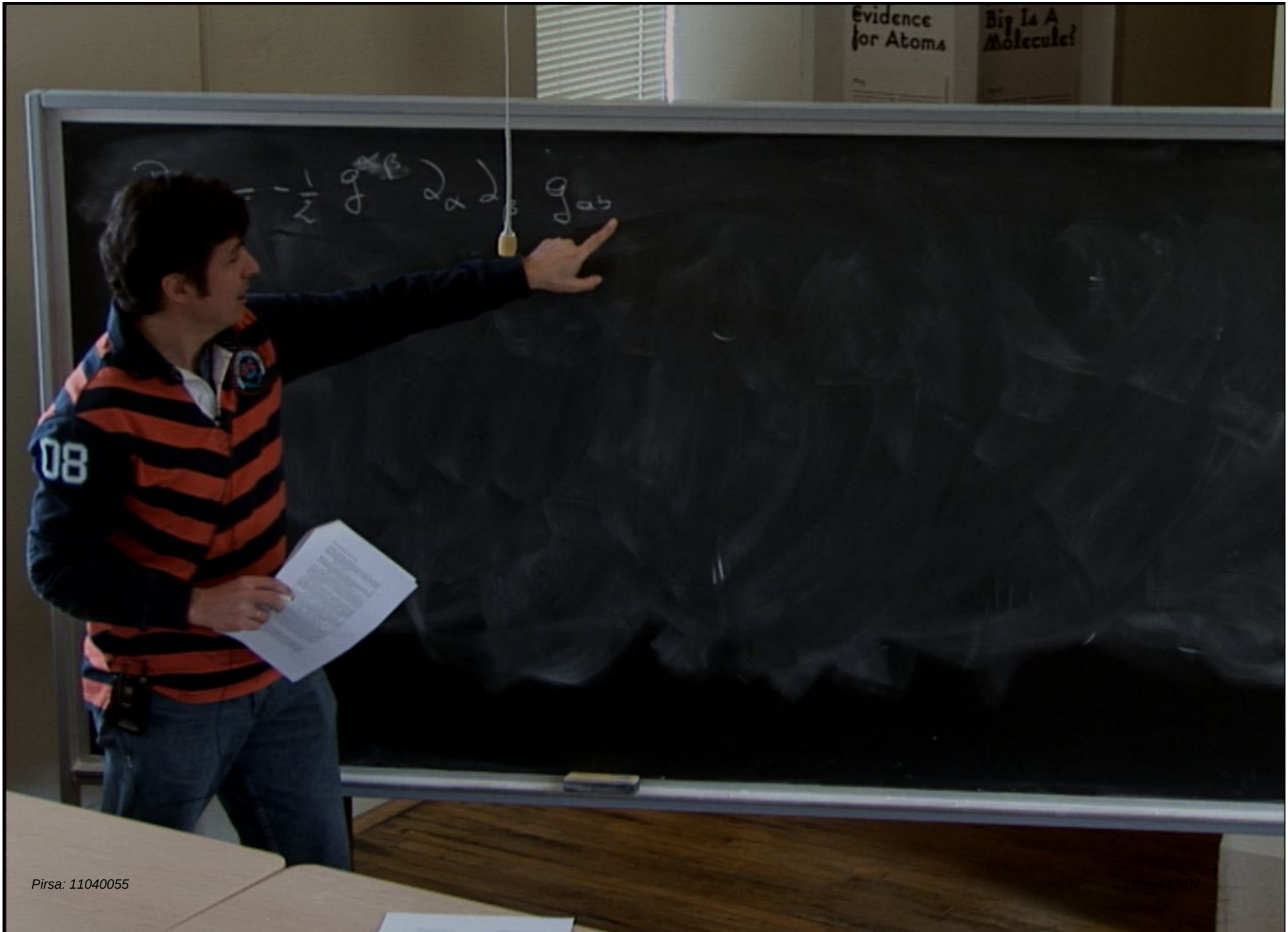
Mom) $\gamma^{jl}\partial_t(\partial_i \gamma_{jl} - \partial_j \gamma_{il}) + l\,0 = \ldots$

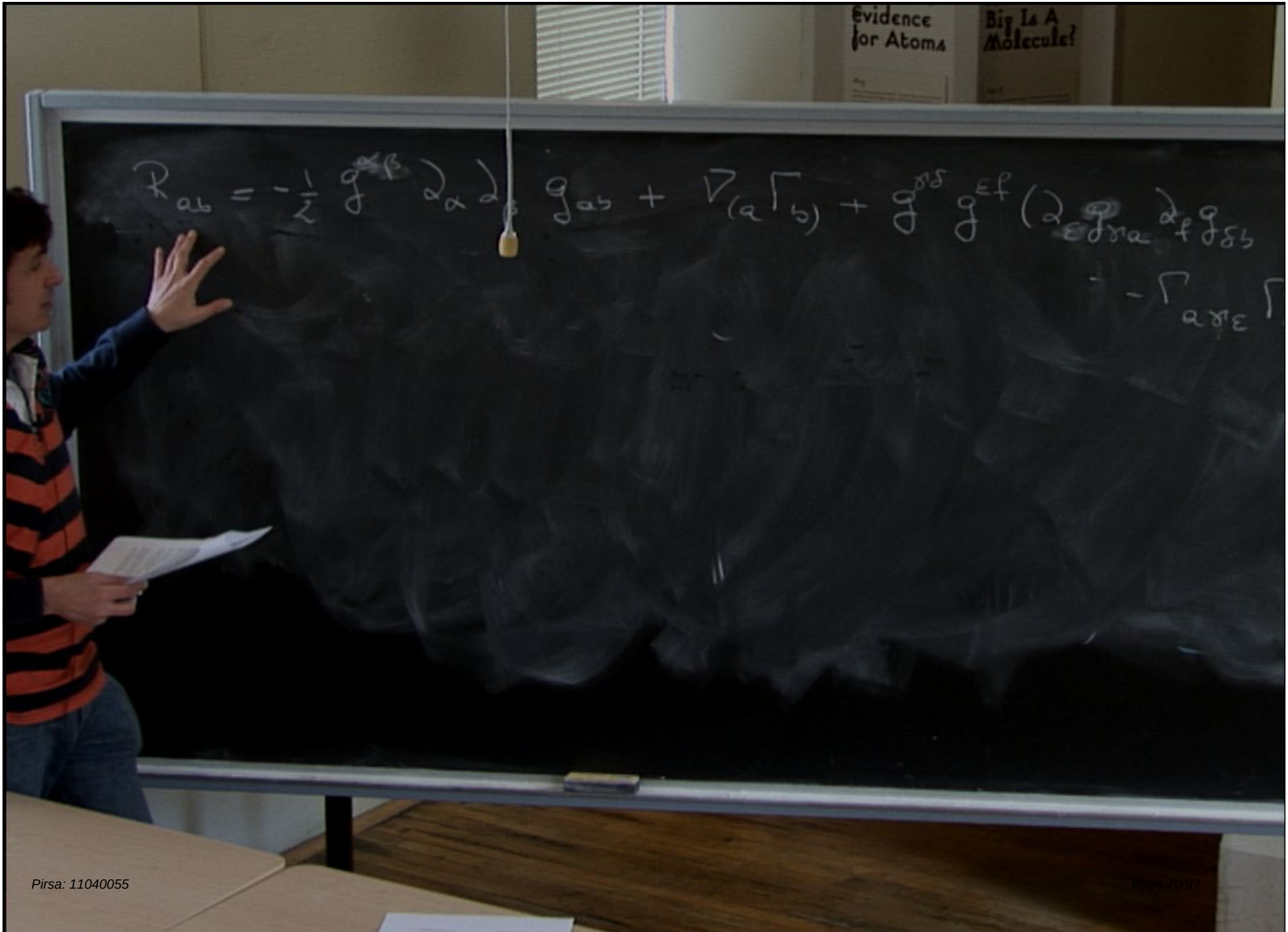
$\hookrightarrow \partial_t^2 \gamma_{ij} = \alpha^2 \gamma^{lm}(\partial_l \partial_m \gamma_{ij} + \partial_i \partial_j \gamma_{lm} - \partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \ldots$

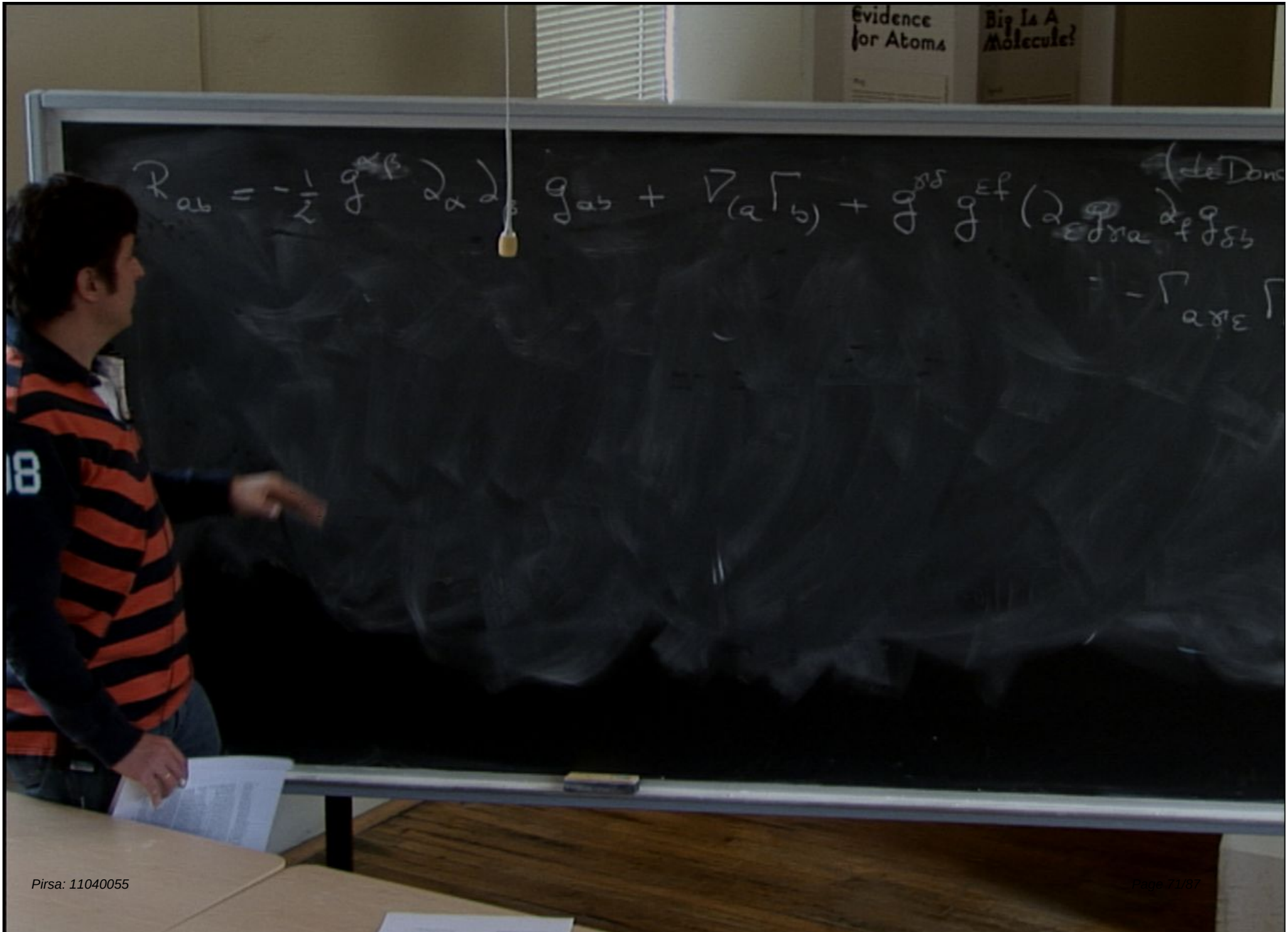
① $\alpha = 1 \Rightarrow \partial_t^2 h_{ij} = (\Delta h_{ij} - \partial_i \partial_j H)$ where we use tha...

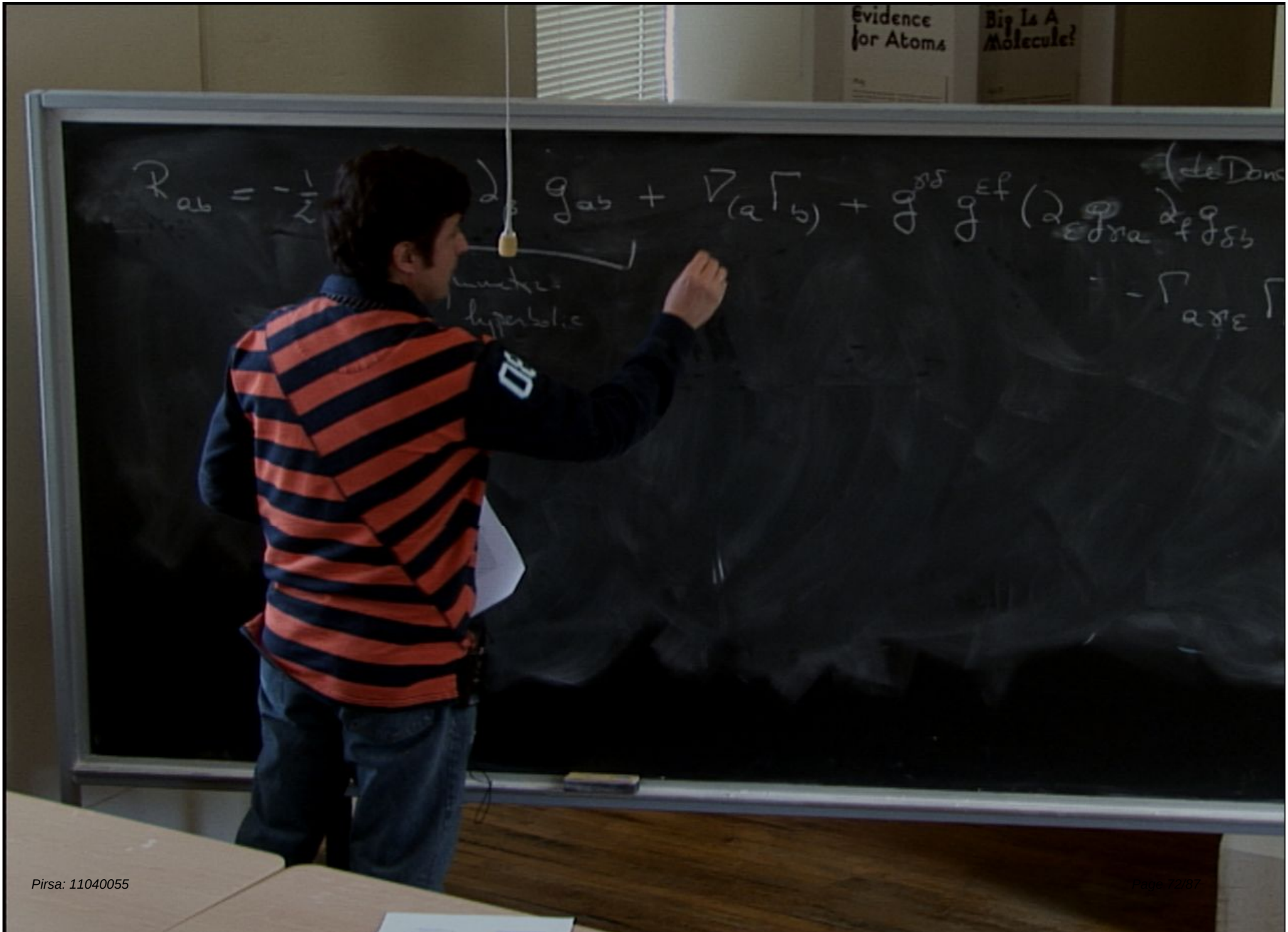
② $\alpha = \frac{1}{3}\delta^{lm} h_{lm} \Rightarrow \partial_t^2 h_{ij} = (\Delta h_{ij} - \frac{1}{3}\partial_i \partial_j H)$

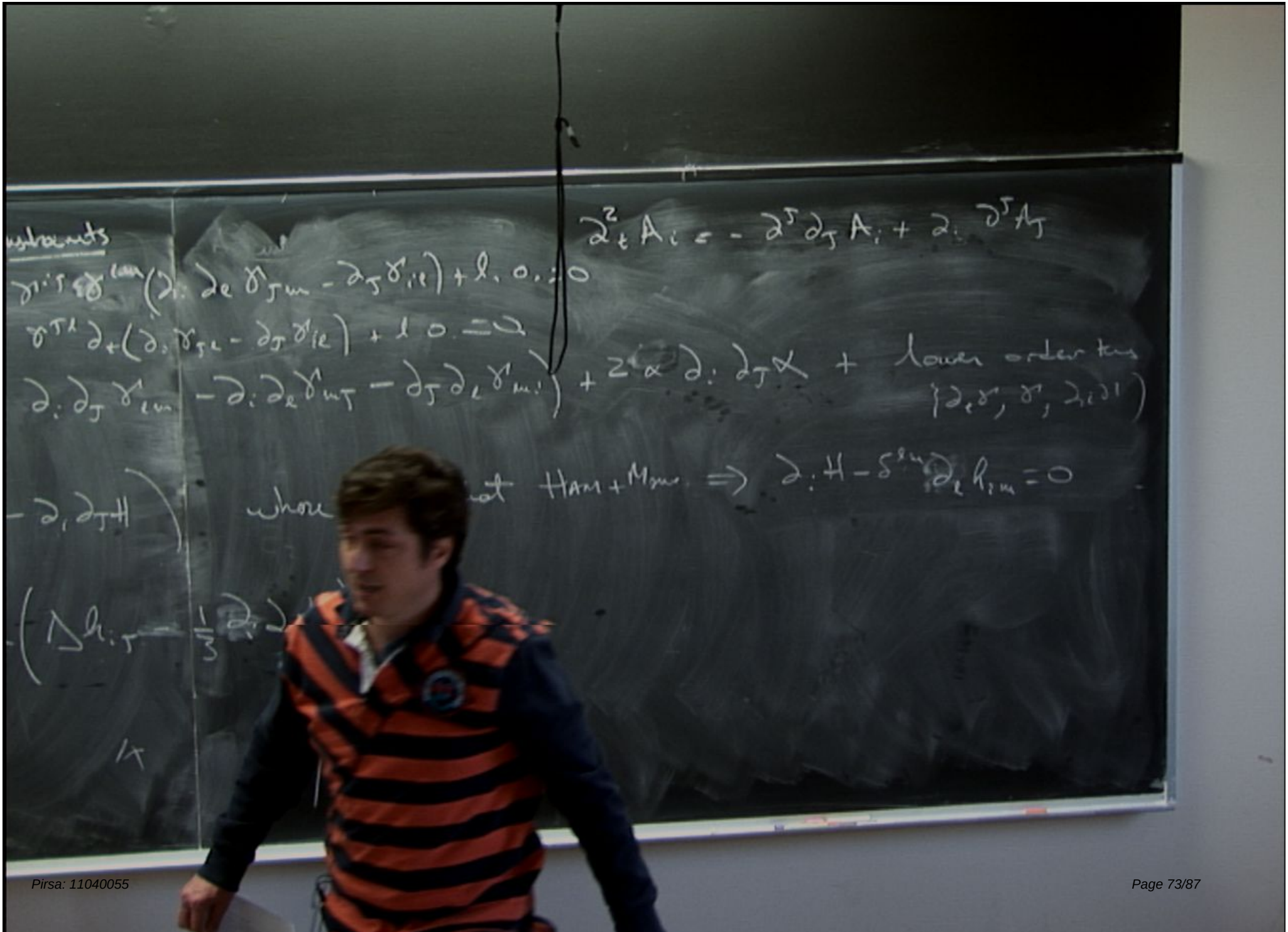

Evidence for Atoms
Big Is A Molecule?
$R_{ab} = -\frac{1}{2} g^{\alpha\beta}$

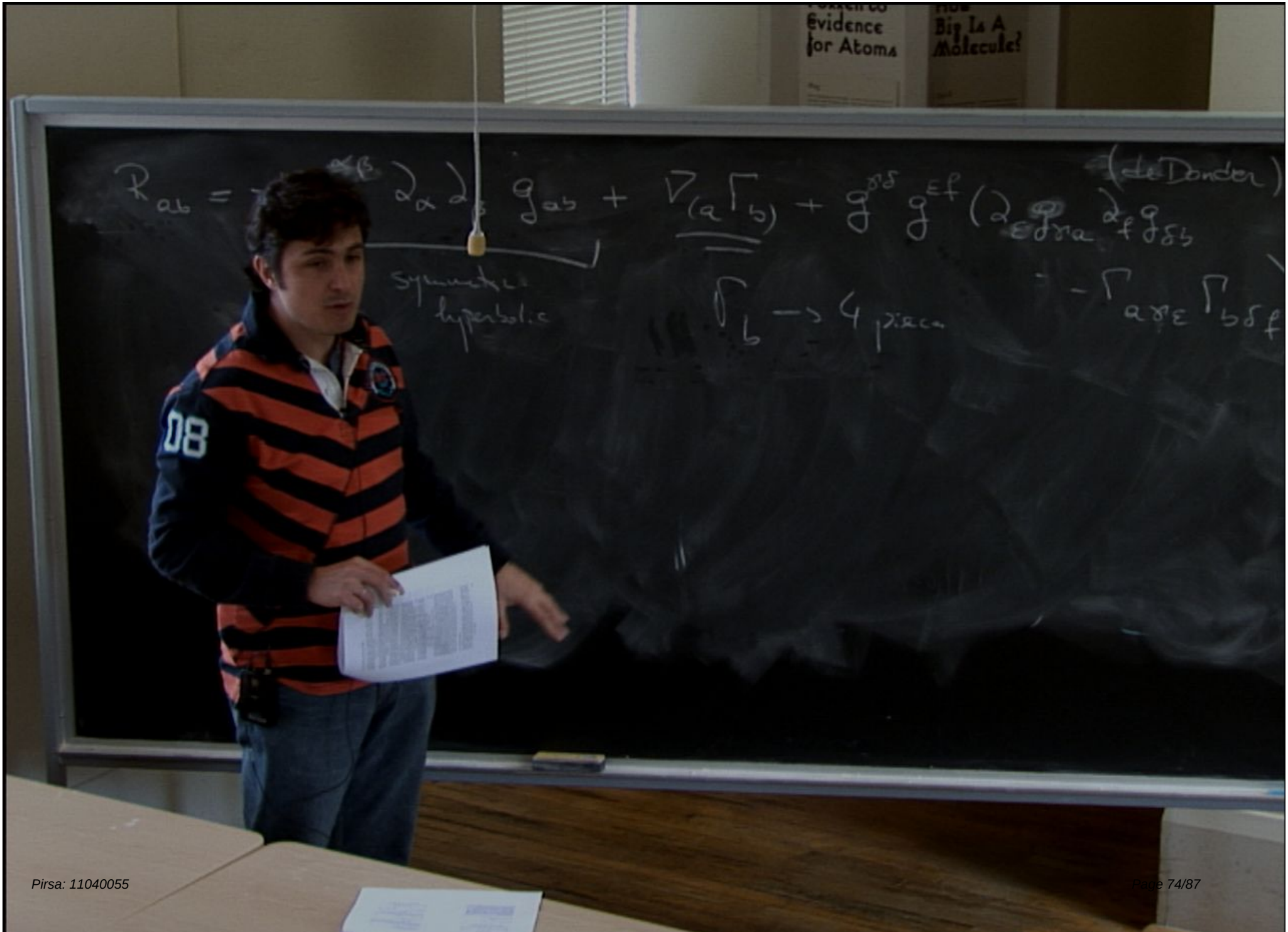

Evidence for Atoms
Big Is A Molecule?
08

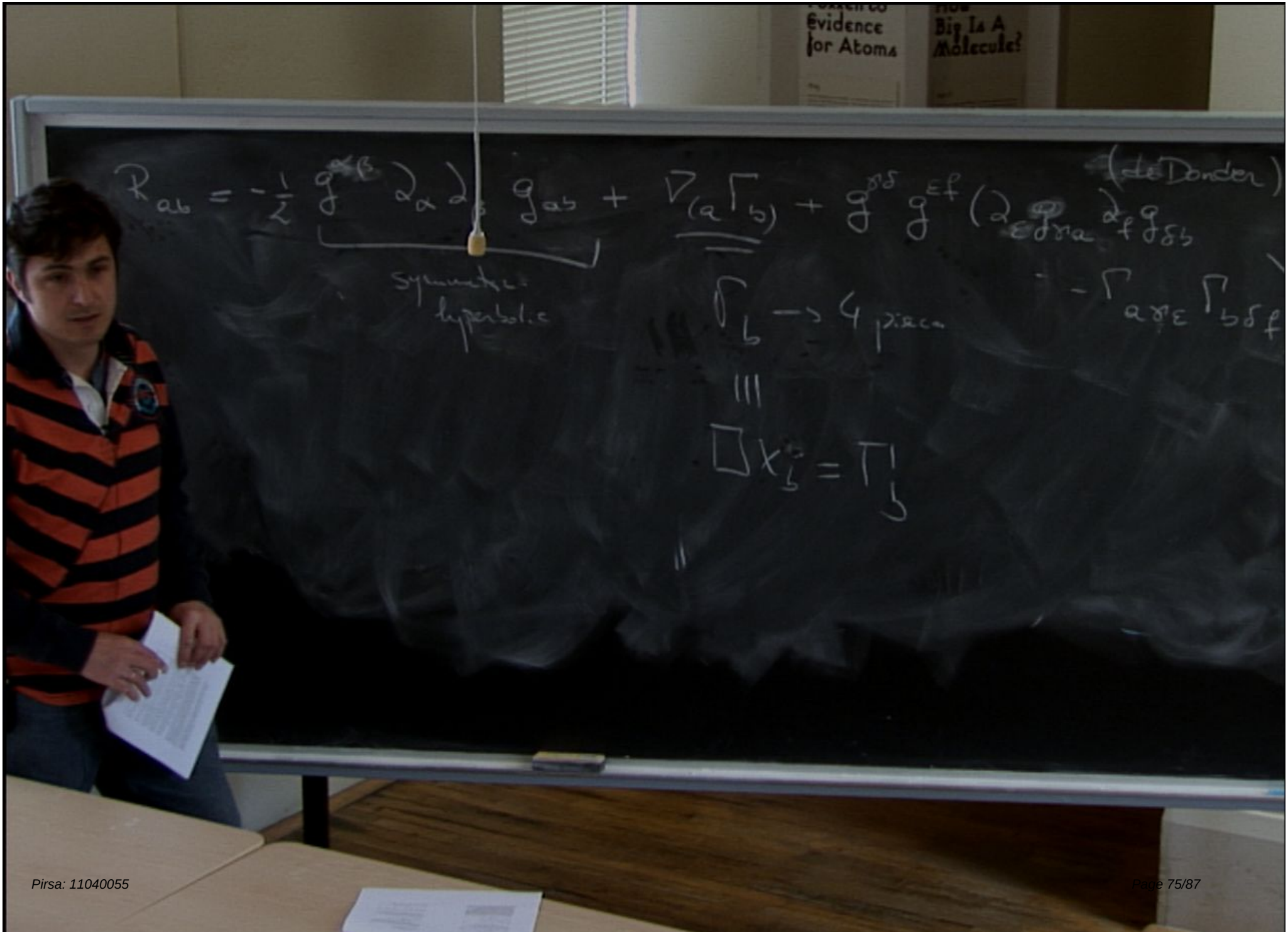

Evidence for Atoms
Big Is A Molecule?
$R_{ab} = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{\delta\delta} g^{\varepsilon f} (\partial_\varepsilon g_{\gamma a} \partial_f g_{\delta b}$
$- \Gamma_{a\gamma\varepsilon} \Gamma$

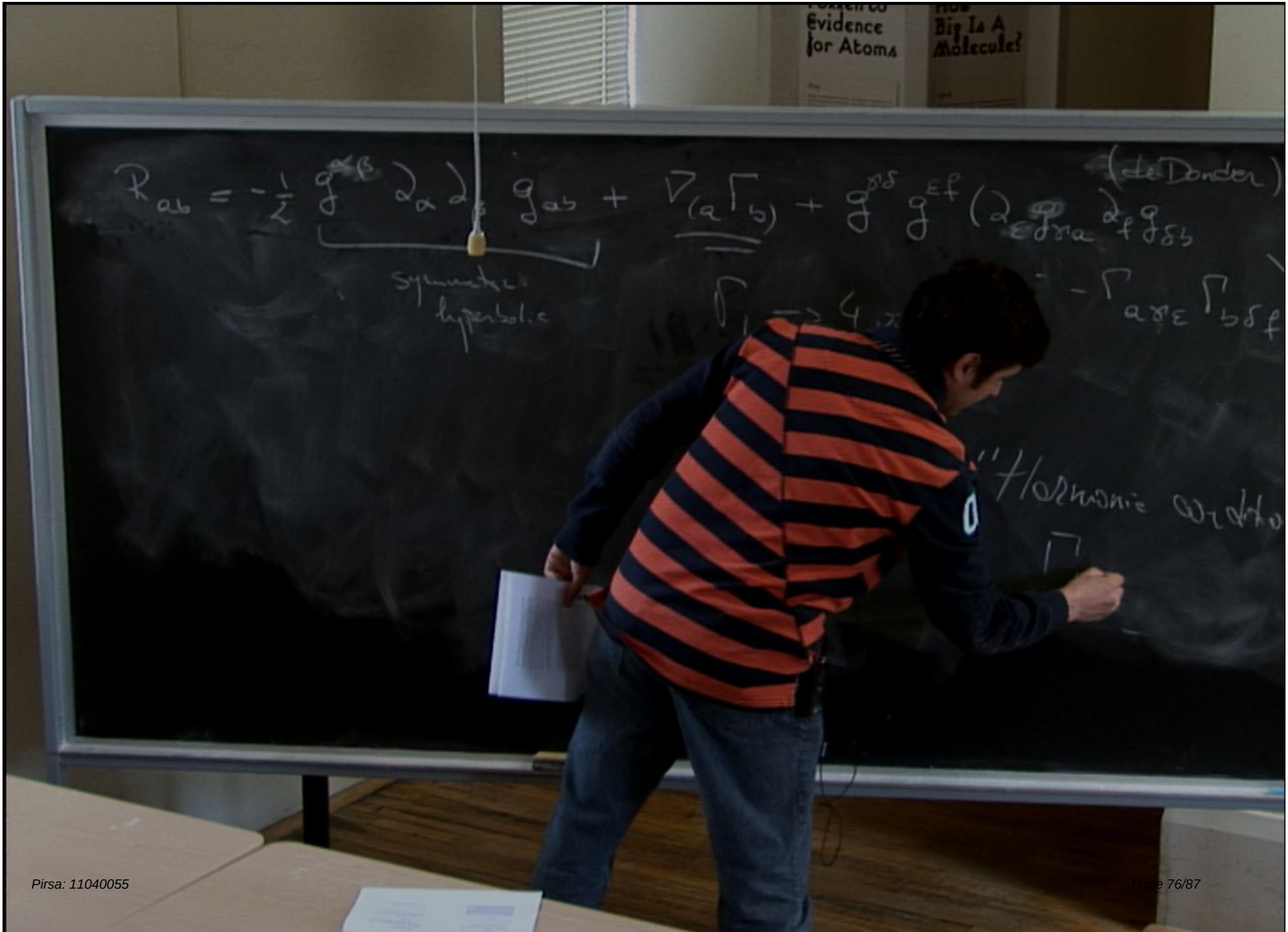

Evidence for Atoms
Big Is A Molecule?

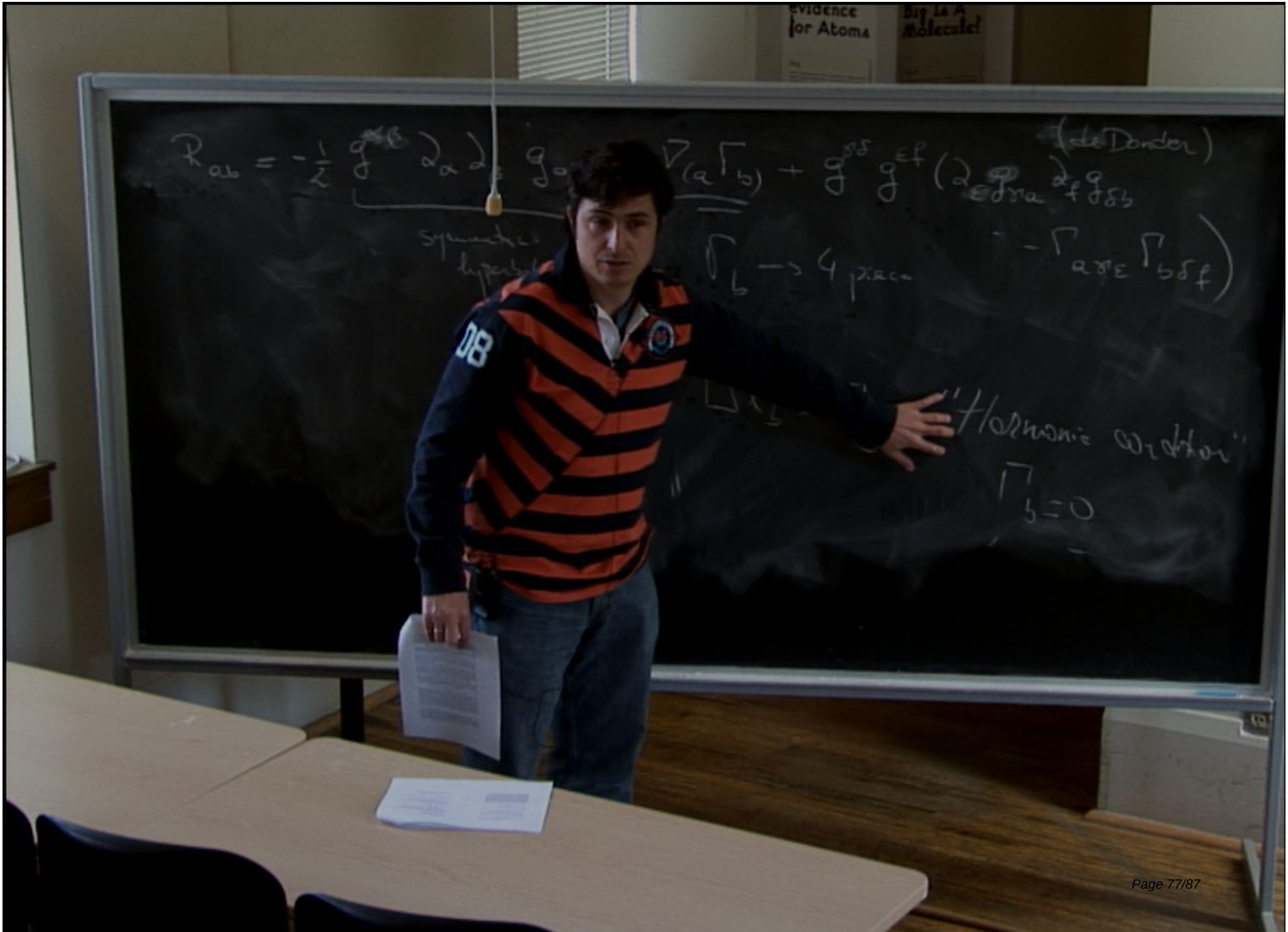

Evidence for Atoms
Big Is A Molecule?
hyperbolic

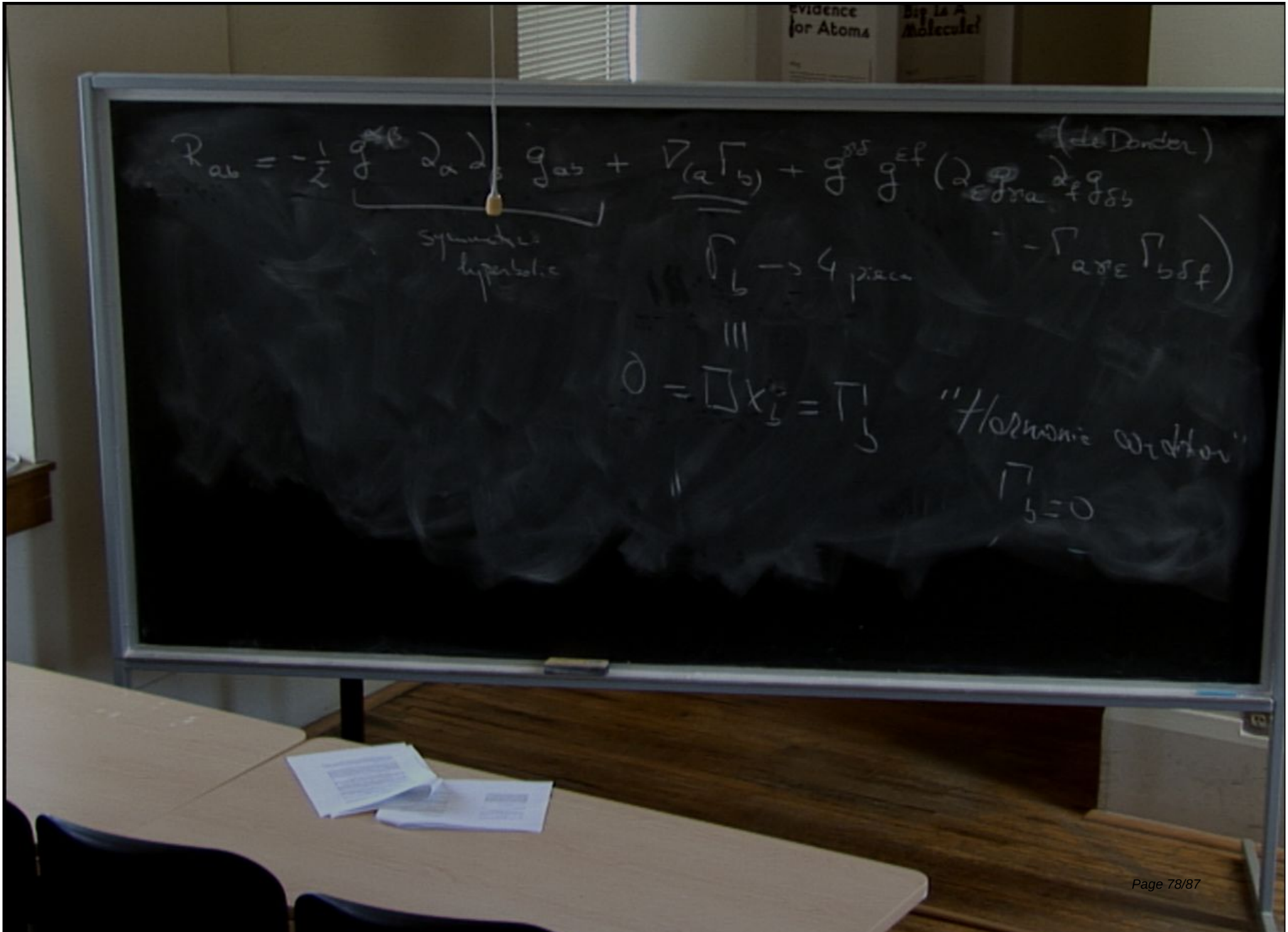

$\partial_t^2 A_i - \partial^j \partial_j A_i + \partial_i \partial^j A_j$
$\partial_t (\partial_i \gamma_{jk} - \partial_j \gamma_{ik}) + \dots = 0$
$\partial_i \partial_j \gamma_{km} - \partial_i \partial_k \gamma_{mj} - \partial_j \partial_k \gamma_{mi}) + 2\alpha \partial_i \partial_j \alpha +$ lower order terms
$\{\partial_t \gamma, \gamma, \partial_i \gamma\}$
$H_{ADM} + M_{SW} \Rightarrow \partial_i H - \delta^{kl} \partial_k h_{im} = 0$

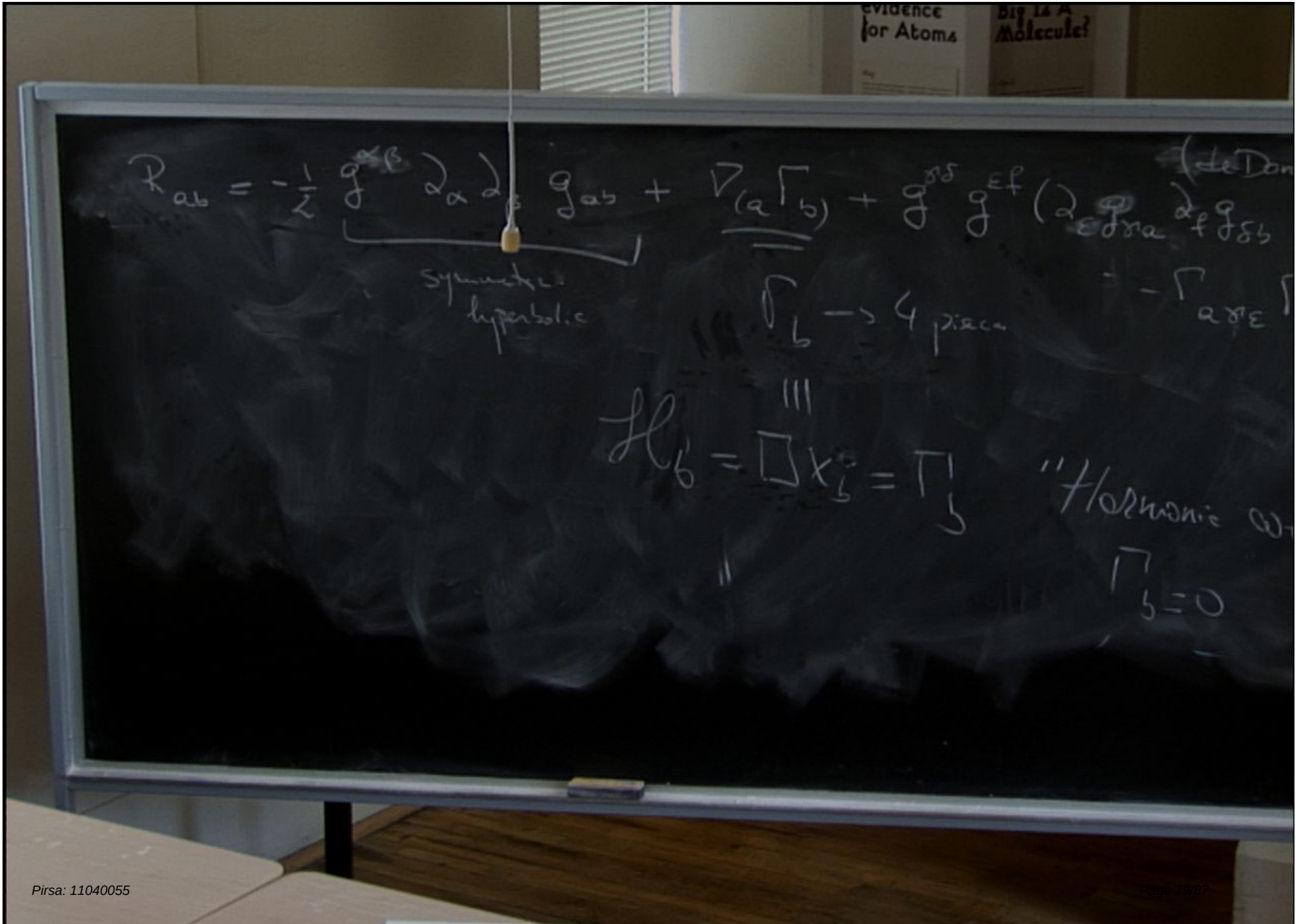

Evidence
for Atoms
Big Is A
Molecule?
(de Donder)
symmetric
hyperbolic
Γb → 4 pieces

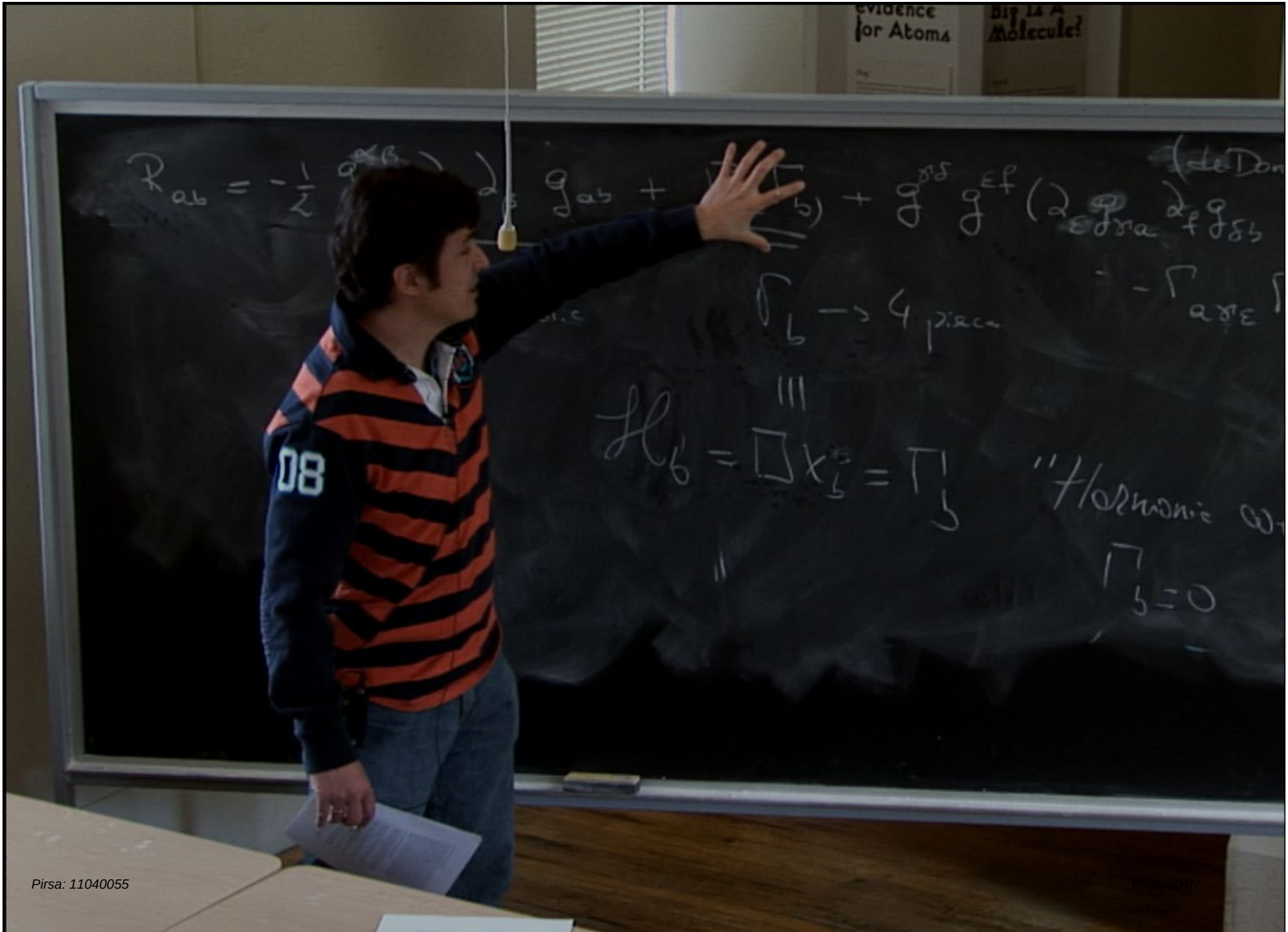

Evidence for Atoms
Big Is A Molecule?
$R_{ab} = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{\gamma\delta} g^{\varepsilon f} (\partial_\varepsilon g_{\gamma a} \partial_f g_{\delta b} - \Gamma_{a\gamma\varepsilon}\Gamma_{b\delta f})$
(de Donder)
symmetric hyperbolic
$\Gamma_b \to$ 4 piece
$\Box x^i_b = \Gamma^i_b$


Evidence
for Atoms
Big Is A
Molecule?
(de Donder)
symmetric
hyperbolic
"Harmonic


evidence
for Atoms
Big Is A
Molecule?
(de Donder)
"Harmonic condition"
$\Gamma_b = 0$


Evidence for Atoms
Big Is A Molecule?
(de Donder)
symmetric hyperbolic
Γb → 4 pieces
0 = □ xb = Γb
"Harmonic condition"
Γb = 0


Evidence for Atoms
Big Is A Molecule?
$R_{ab} = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{\delta\delta} g^{\varepsilon f} (\partial_\varepsilon g_{\delta a} \partial_f g_{\delta b}$
symmetric hyperbolic
$\Gamma_b \to$ 4 pieces
$\mathcal{H}_b = \Box x_b = \Gamma_b$
"Harmonic
$\Gamma_b = 0$


evidence
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Big Is A
Molecule?
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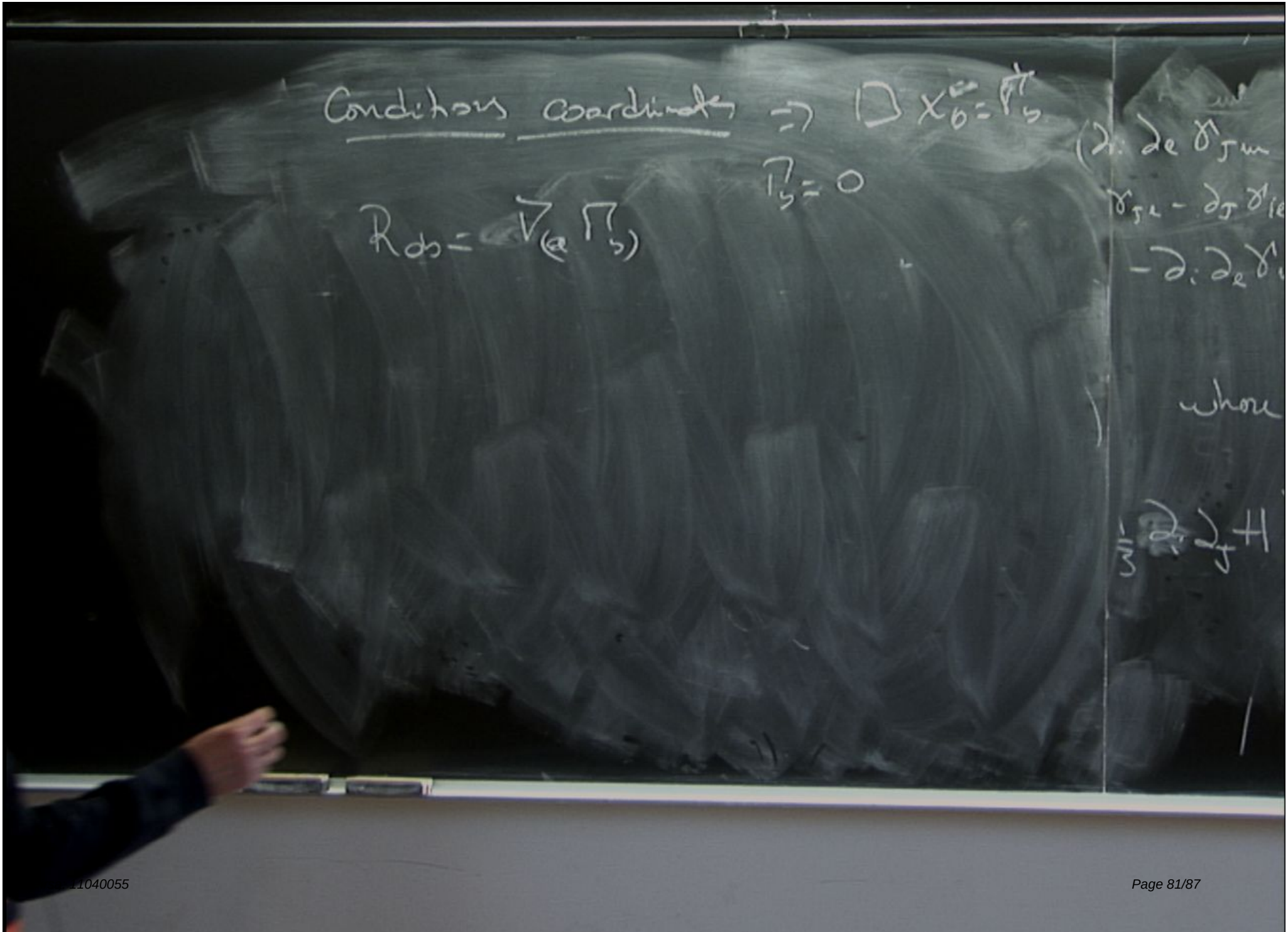
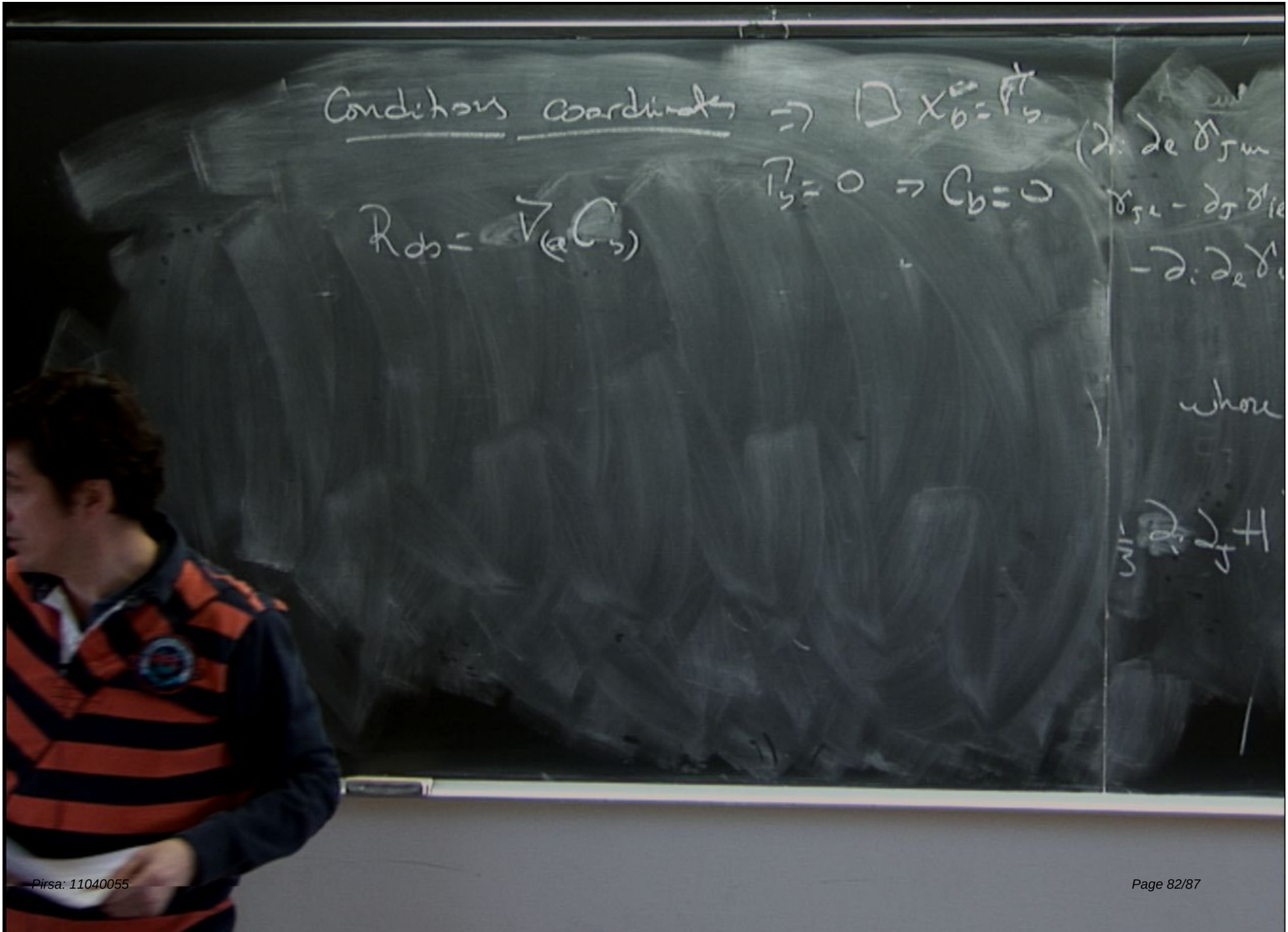
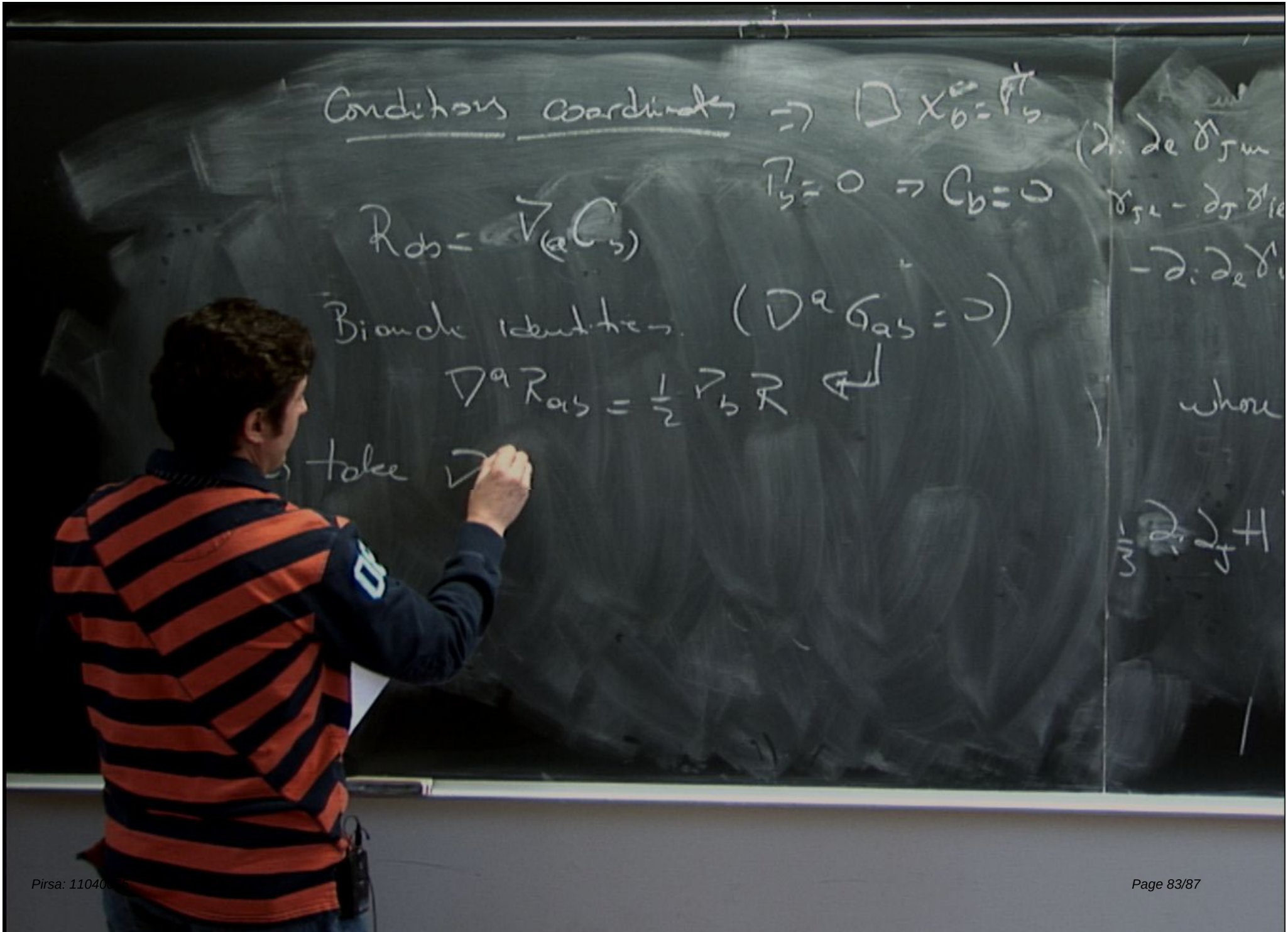

Conditions coordinates ⇒ □ $x_b^\mu = \Gamma_b$
$\Gamma_b = 0$
$R_{ab} = \nabla_{(a} \Gamma_{b)}$
$(\partial_i \partial_l \gamma_{jm}$
$\gamma_{jl} - \partial_j \gamma_{il}$
$- \partial_i \partial_l \gamma$
where
$\frac{1}{3} \partial_i \partial_j H$


Conditions coordinates
$\Rightarrow D X^{\mu}_{b} = \Gamma^{a}_{b}$
$\Gamma^{a}_{b} = 0 \Rightarrow C_b = 0$
$R_{ab} = \nabla_{(a} C_{b)}$
where


Conditions coordinates
$R_{ab} = \nabla_{(a} C_{b)}$
Bianchi identities. $(\nabla^a G_{ab} = 0)$
$\nabla^a R_{ab} = \frac{1}{2} \nabla_b R$
take
where

Conditions coordinates $\Rightarrow \Box X^{\mu}_{b} = \Gamma^{\mu}_{b}$

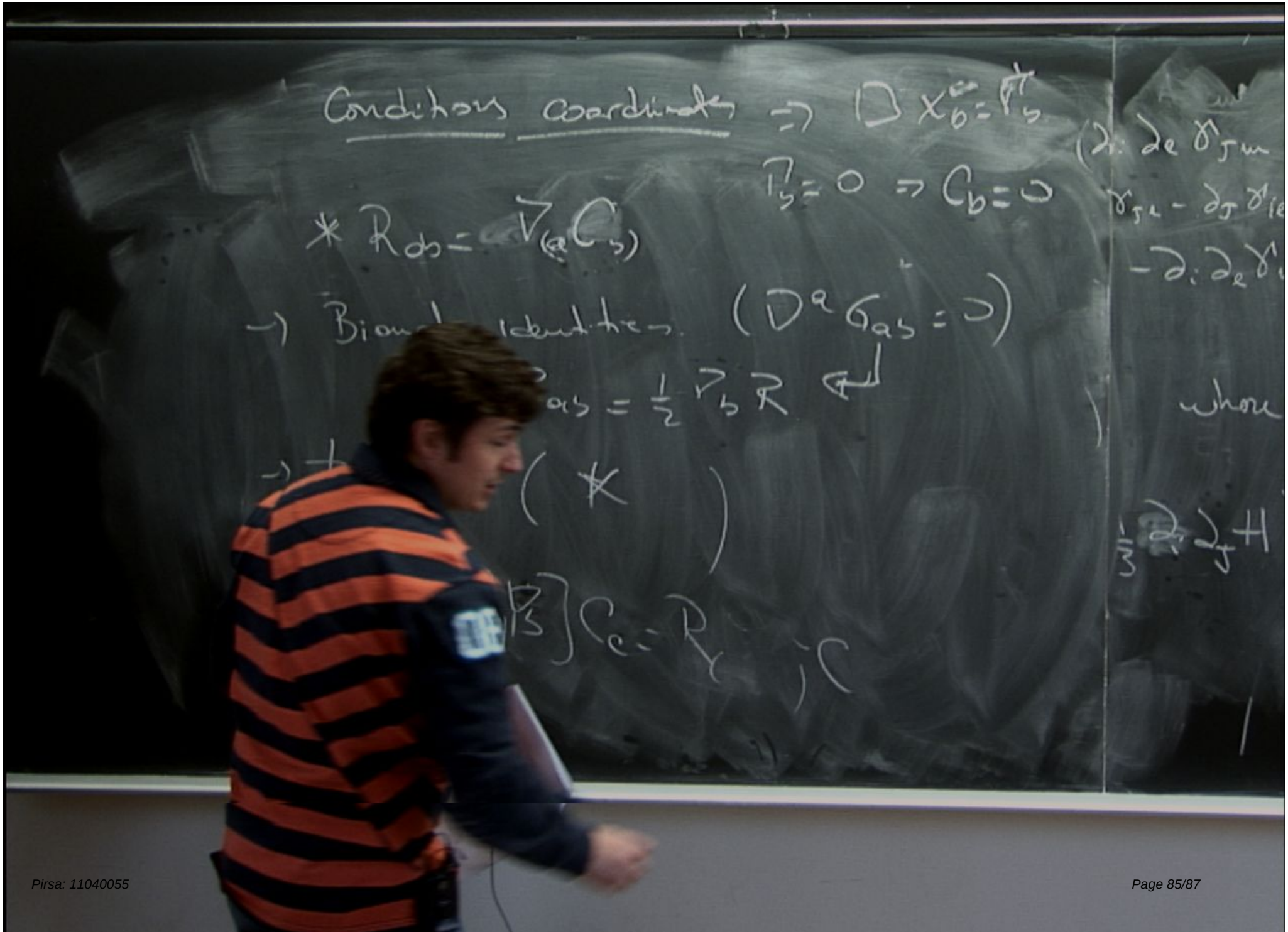
$\Gamma^{\mu}_{b} = 0 \Rightarrow C_b = 0$

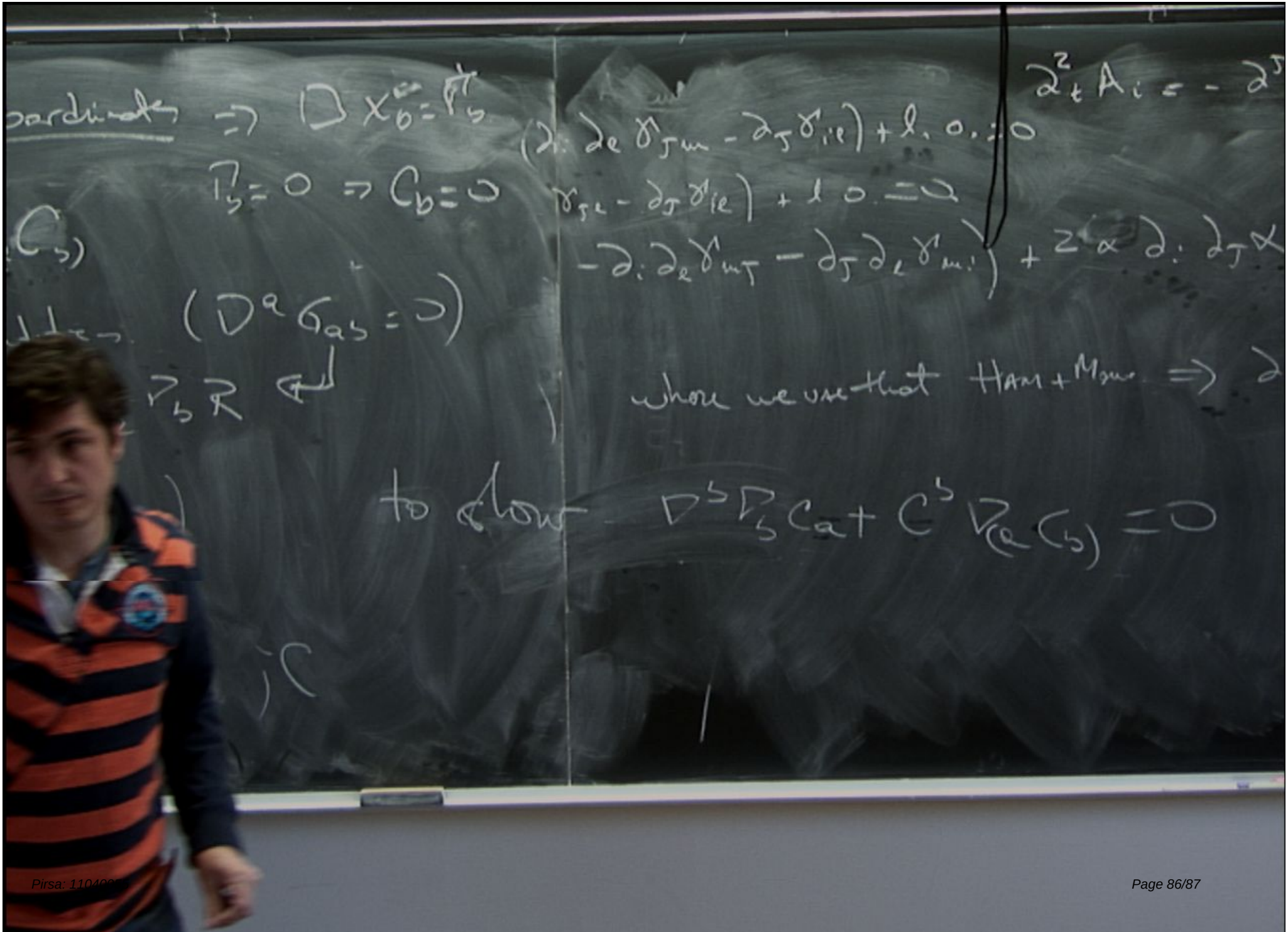
$* R_{ab} = \nabla_{(a} C_{b)}$

$\rightarrow$ Bianchi identities. $(\nabla^a G_{ab} = 0)$

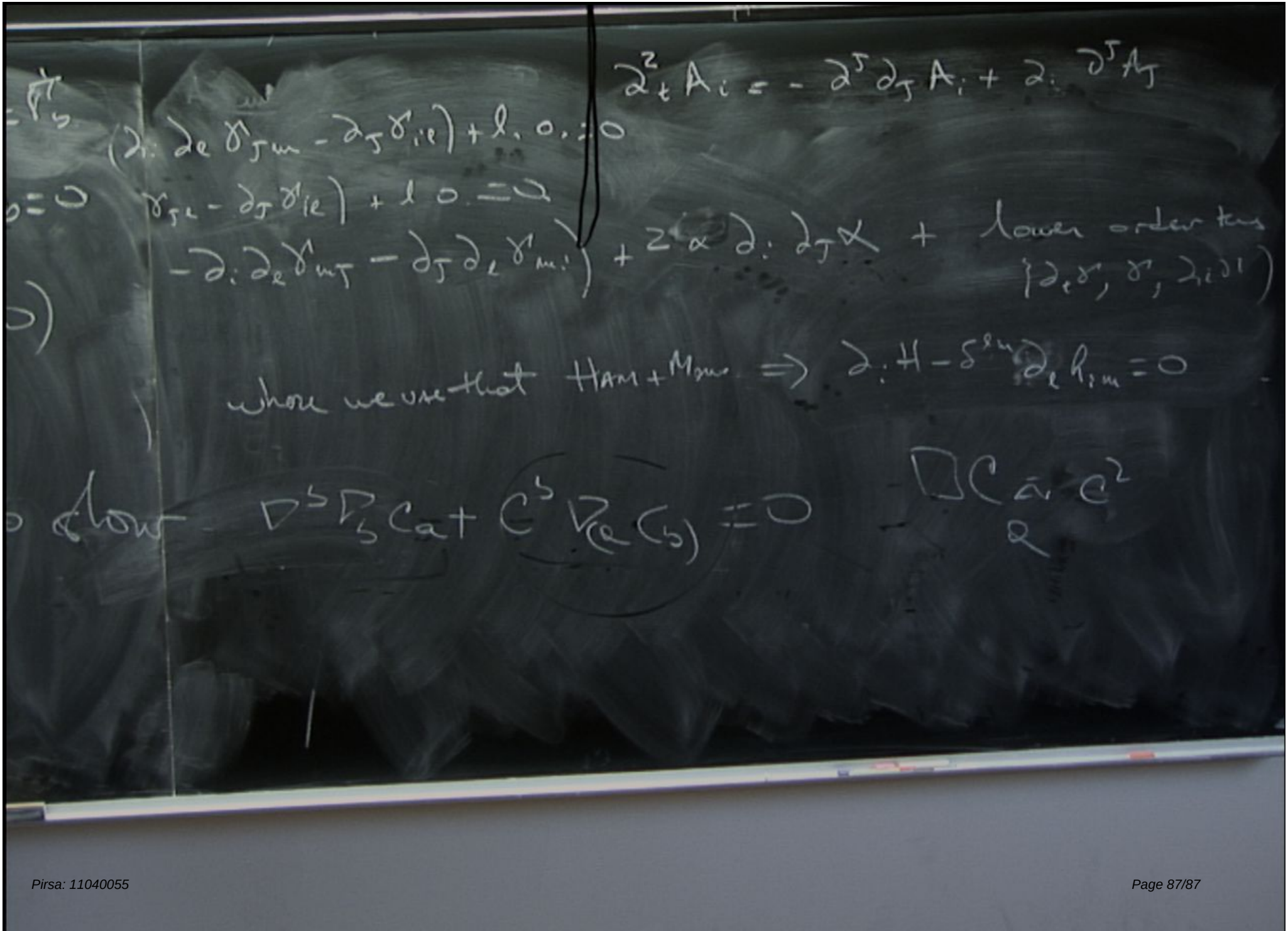
$\nabla^a R_{ab} = \frac{1}{2} \nabla_b R$

$\rightarrow$ take $\nabla^a (* \quad)$


Conditions coordinates
Bianchi identities
where



$\Rightarrow \Box X_b = \Gamma_b$
$\Gamma_b = 0 \Rightarrow C_b = 0$
$(\partial_i \partial_l \gamma_{jm} - \partial_j \gamma_{il}) + l.o. = 0$
$\gamma_{jl} - \partial_j \gamma_{il}) + l.o. = 0$
$-\partial_i \partial_l \gamma_{mj} - \partial_j \partial_l \gamma_{mi}) + 2\alpha \partial_i \partial_j \alpha$
$\partial_t^2 A_i = -\partial^j$
$(D^a G_{ab} = 0)$
$\nabla_b R$
where we use that Ham + Mom $\Rightarrow$
to show $\nabla^b \nabla_b C_a + C^b \nabla_{(a} C_{b)} = 0$


$\partial_t^2 A_i = -\partial^j \partial_j A_i + \partial_i \partial^j A_j$
$(\partial_i \partial_e \gamma_{jm} - \partial_j \gamma_{ie}) + l.o. = 0$
$\gamma_{je} - \partial_j \gamma_{ie}) + l.o. = 0$
$-\partial_i \partial_e \gamma_{mj} - \partial_j \partial_e \gamma_{mi}) + \partial_i \partial_j \alpha +$ lower order terms
$\{\partial_e \gamma, \gamma, \partial_i \alpha\}$
where we use that $H_{ADM} + M_{gauge} \Rightarrow \partial_i H - \delta^{jm} \partial_e h_{im} = 0$
$D^b D_b C_a + C^b D_{(a} C_{b)} = 0$
$\Box C_a - C^2$