Title: Explorations in Numerical Relativity - Lecture 11

Date: Apr 18, 2011 11:30 AM

URL: http://pirsa.org/11040054

Abstract:

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Explorations in Gravitational Physics: Numerical Relativity (week 3)

Perimeter Institute
Waterloo, Ontario
April 18-April 21

Outline and motivation for week's topics:

gravitational waves, NP formalism and Teukolsky equation, generalized harmonic & BSSN evolution, AMR, parallel computation, dynamical horizons, excision, ...

Matt Choptuik, UBC
Luis Lehner, Perimeter/Guelph
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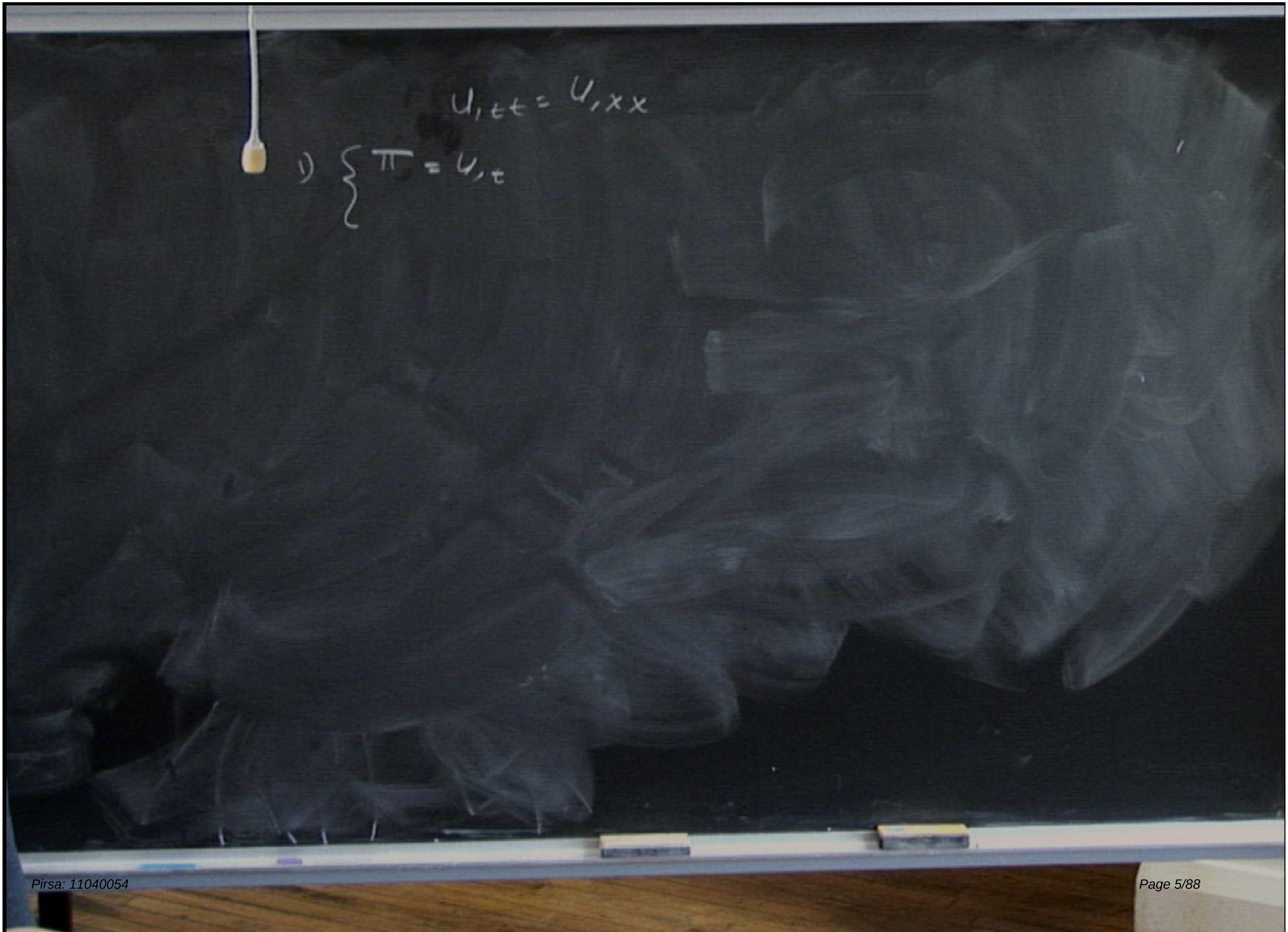
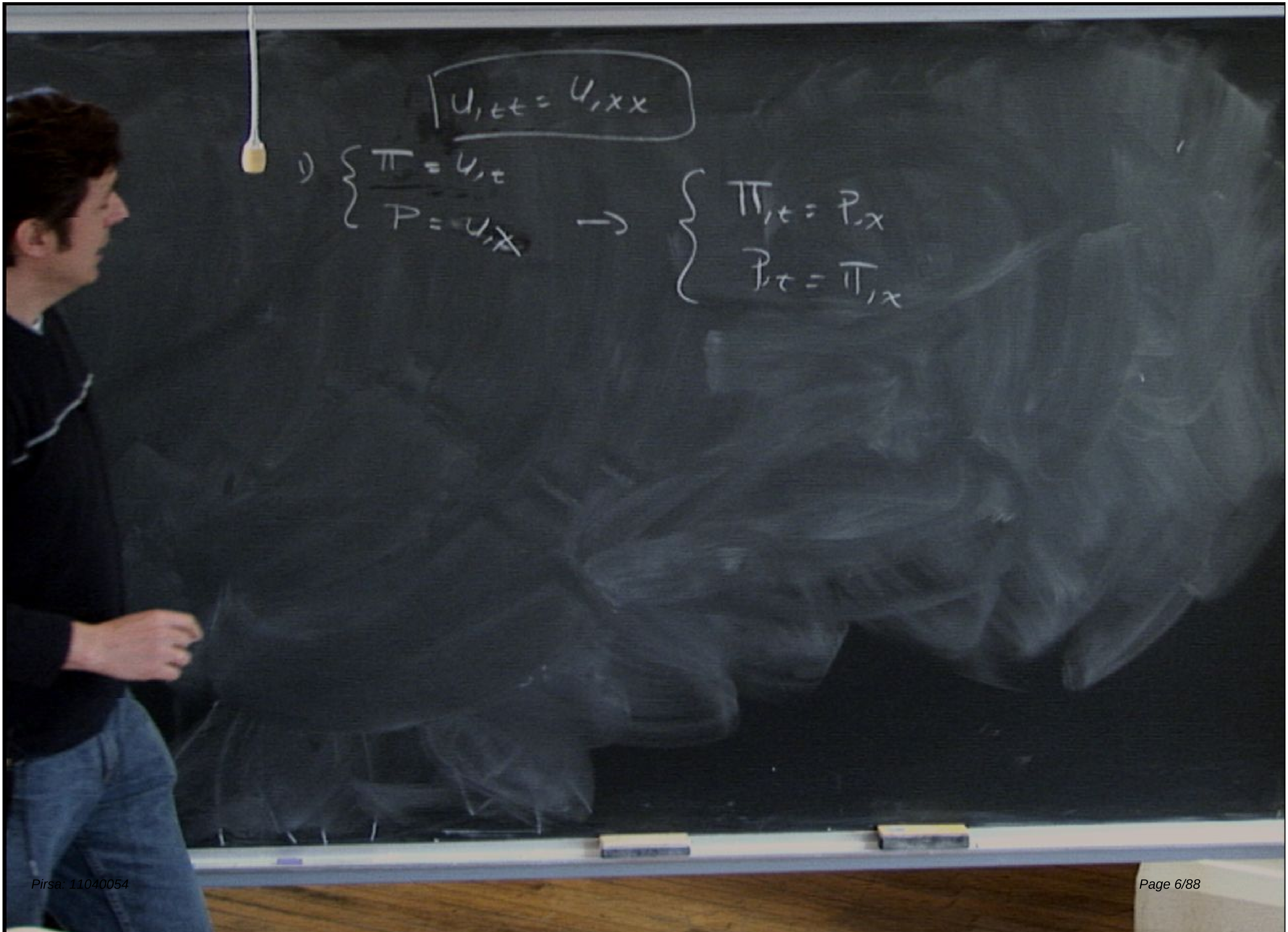
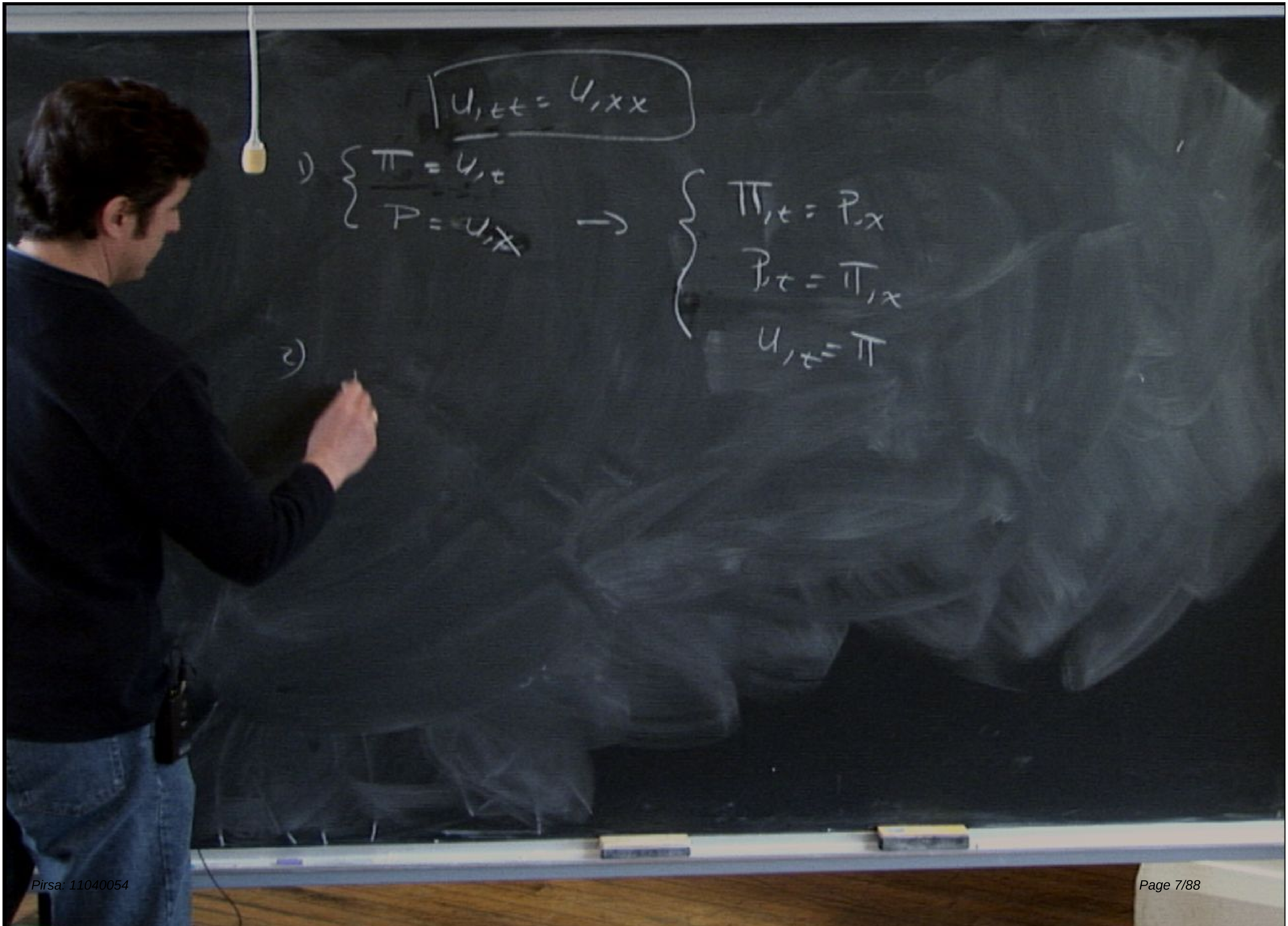
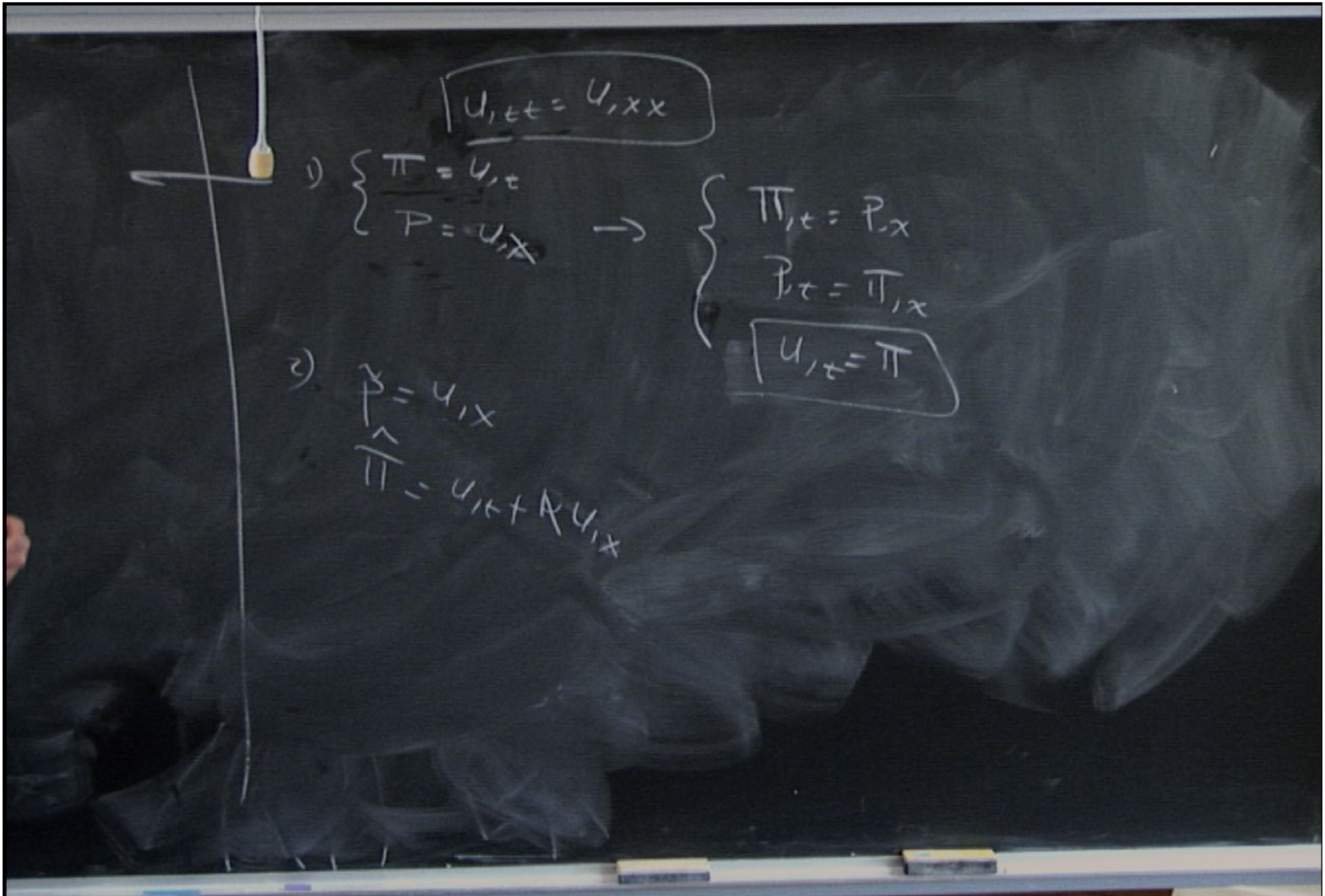
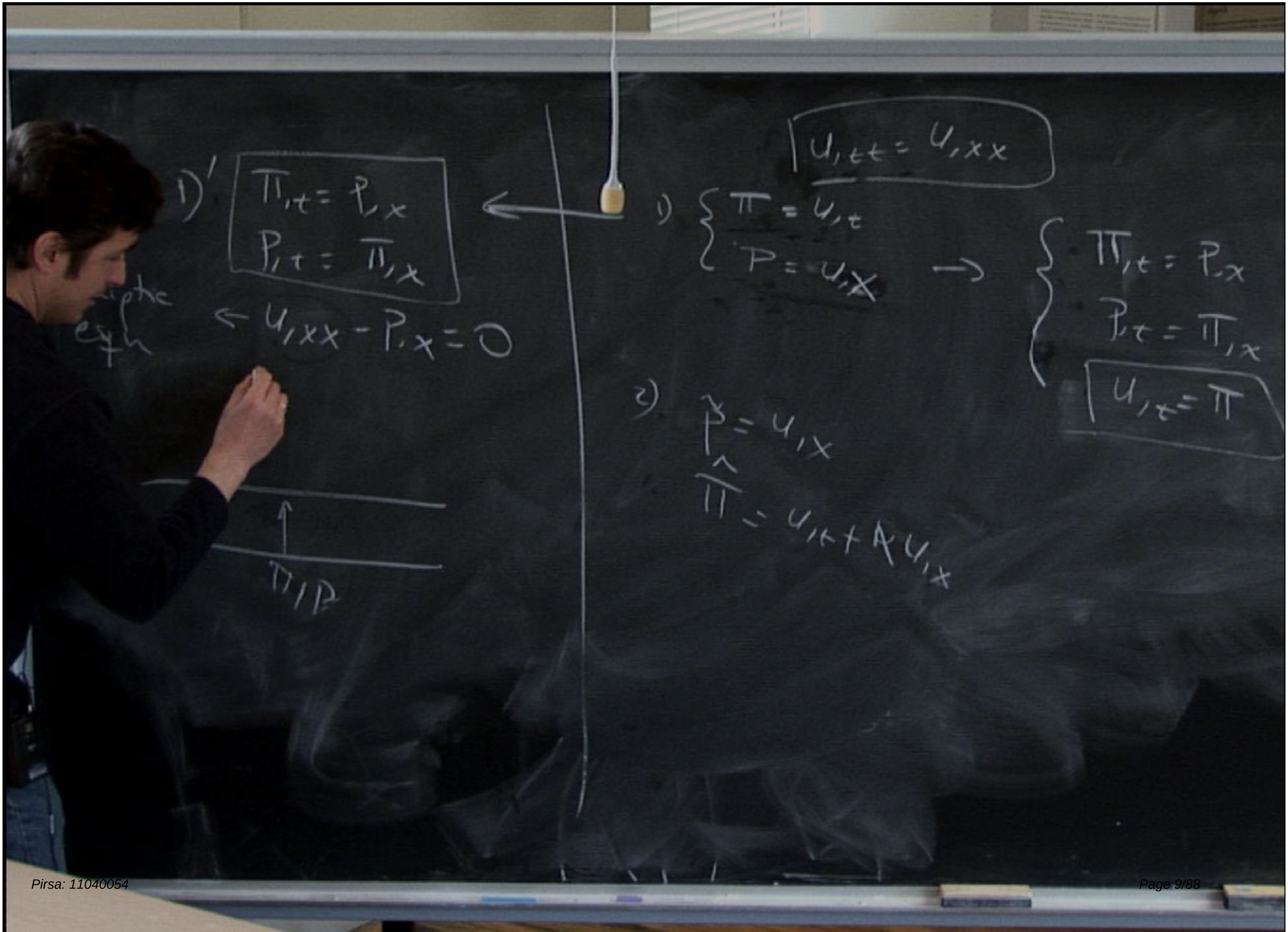
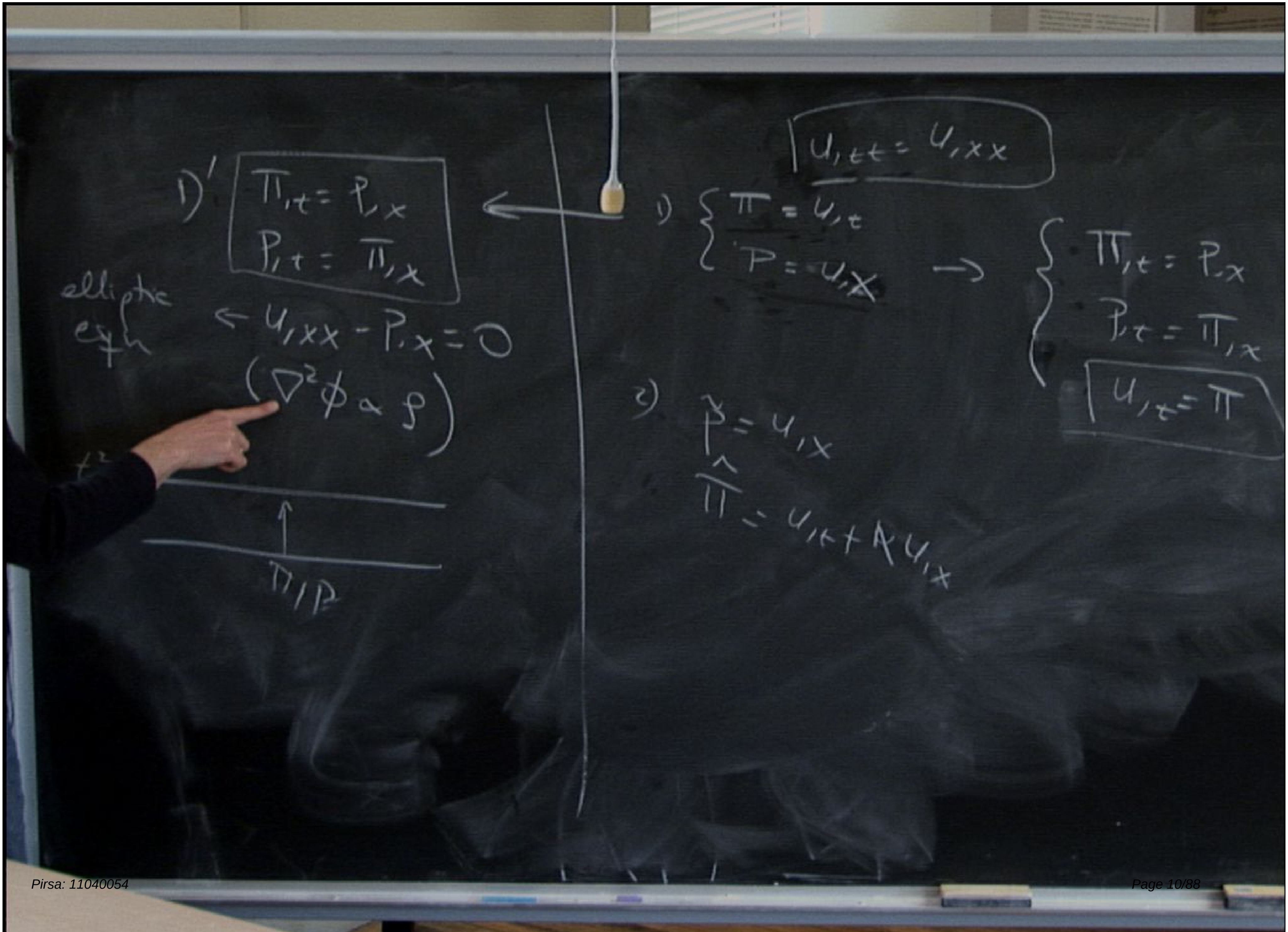

u,tt = u,xx
1) { π = u,t

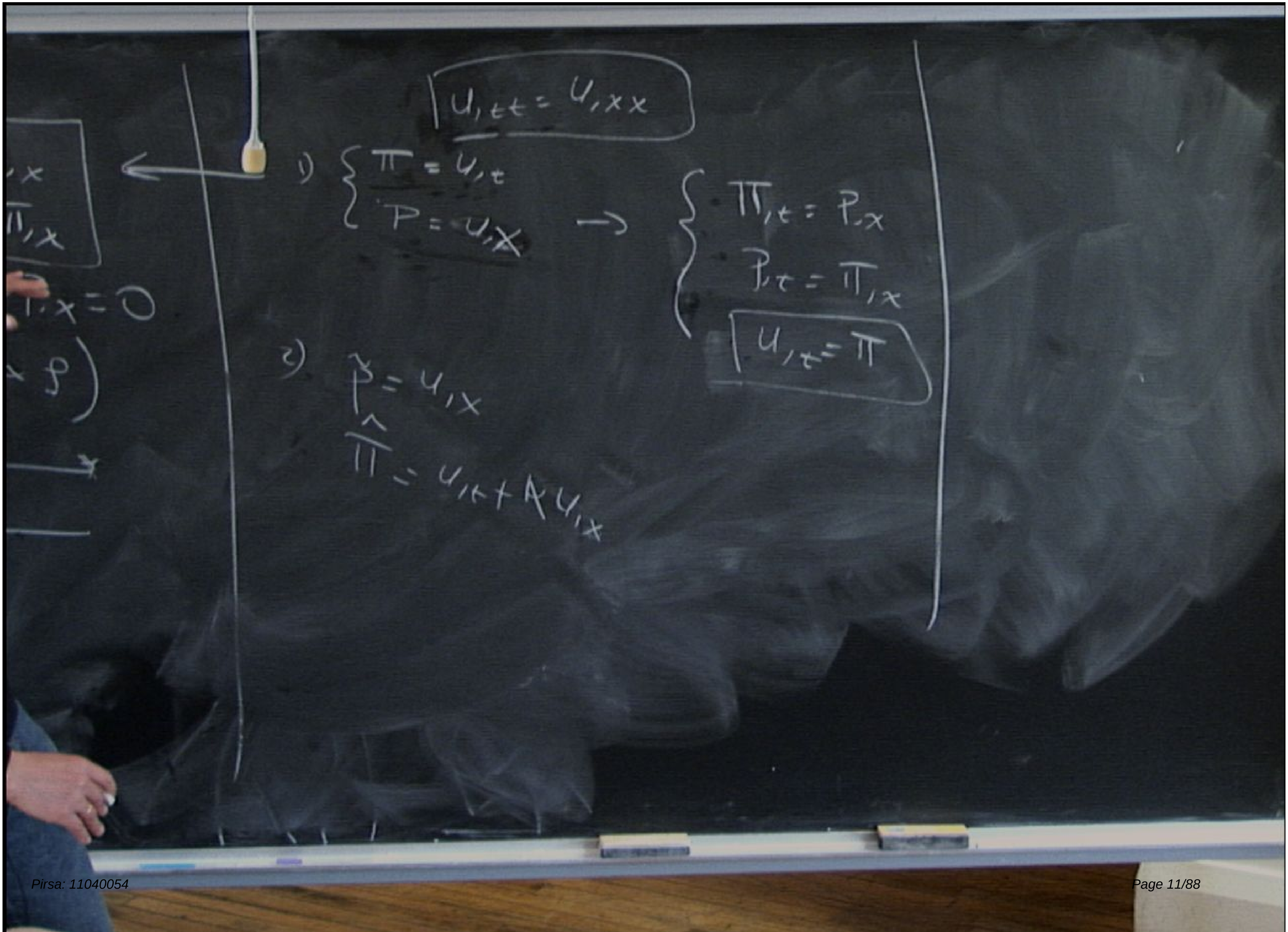

$u_{,tt} = u_{,xx}$
1) $\begin{cases} \pi = u_{,t} \\ P = u_{,x} \end{cases} \rightarrow \begin{cases} \pi_{,t} = P_{,x} \\ P_{,t} = \pi_{,x} \end{cases}$

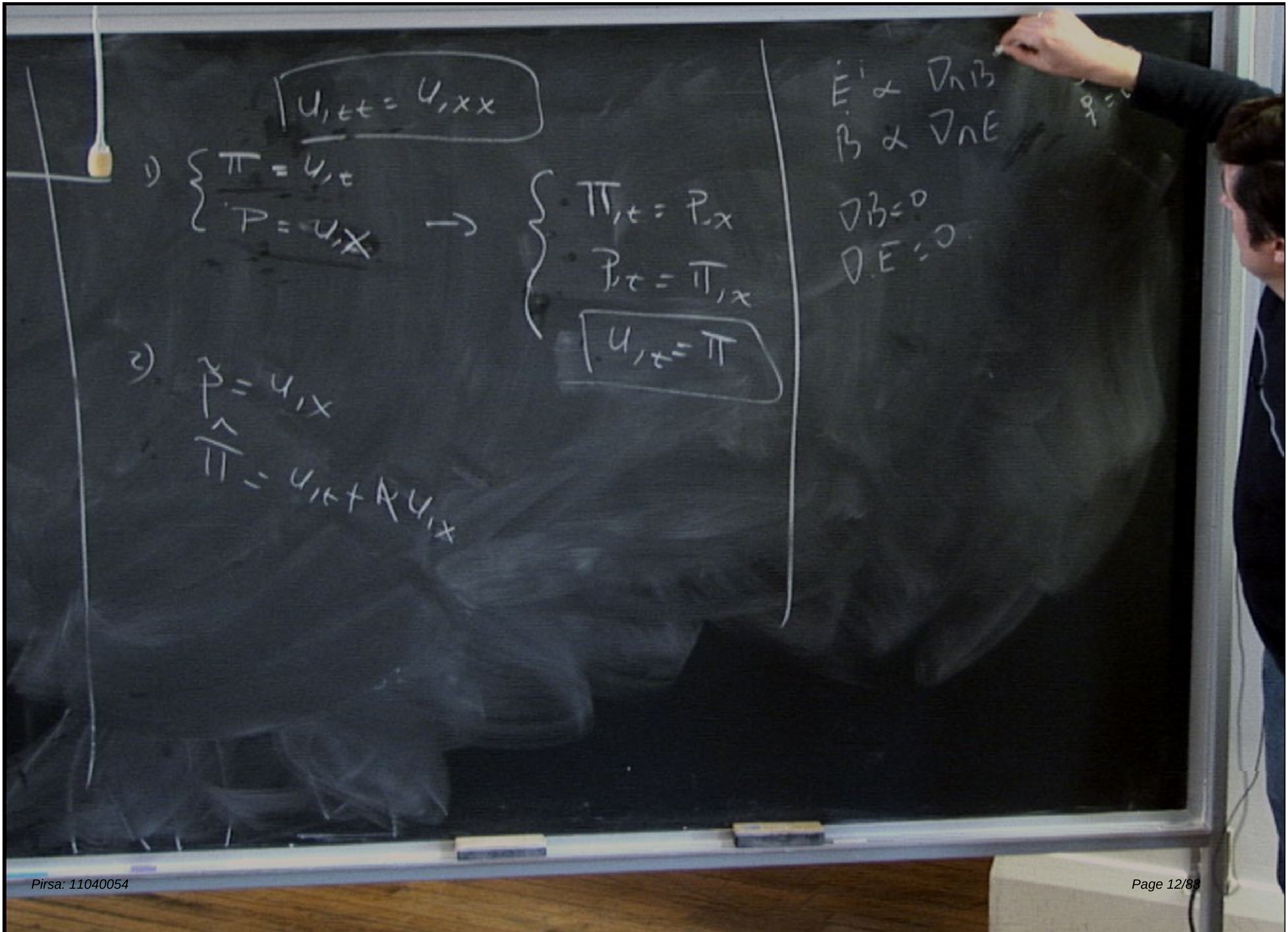

u,tt = u,xx
1) π = u,t
P = u,x
π,t = P,x
P,t = π,x
u,t = π
2)

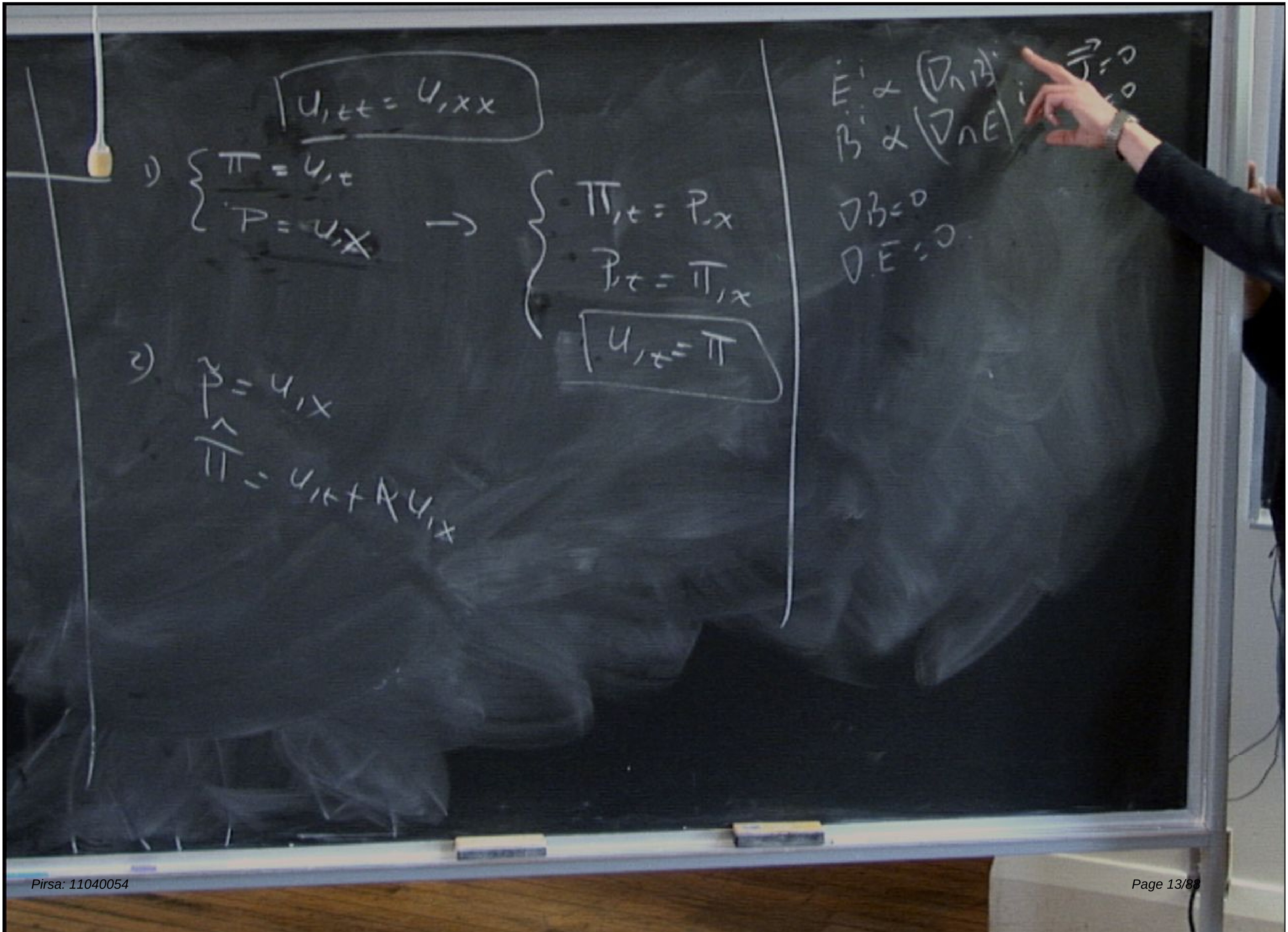

u,tt = u,xx
1) π = u,t
P = u,x
→
π,t = P,x
P,t = π,x
u,t = π
2) P̂ = u,x
π̂ = u,t + A u,x

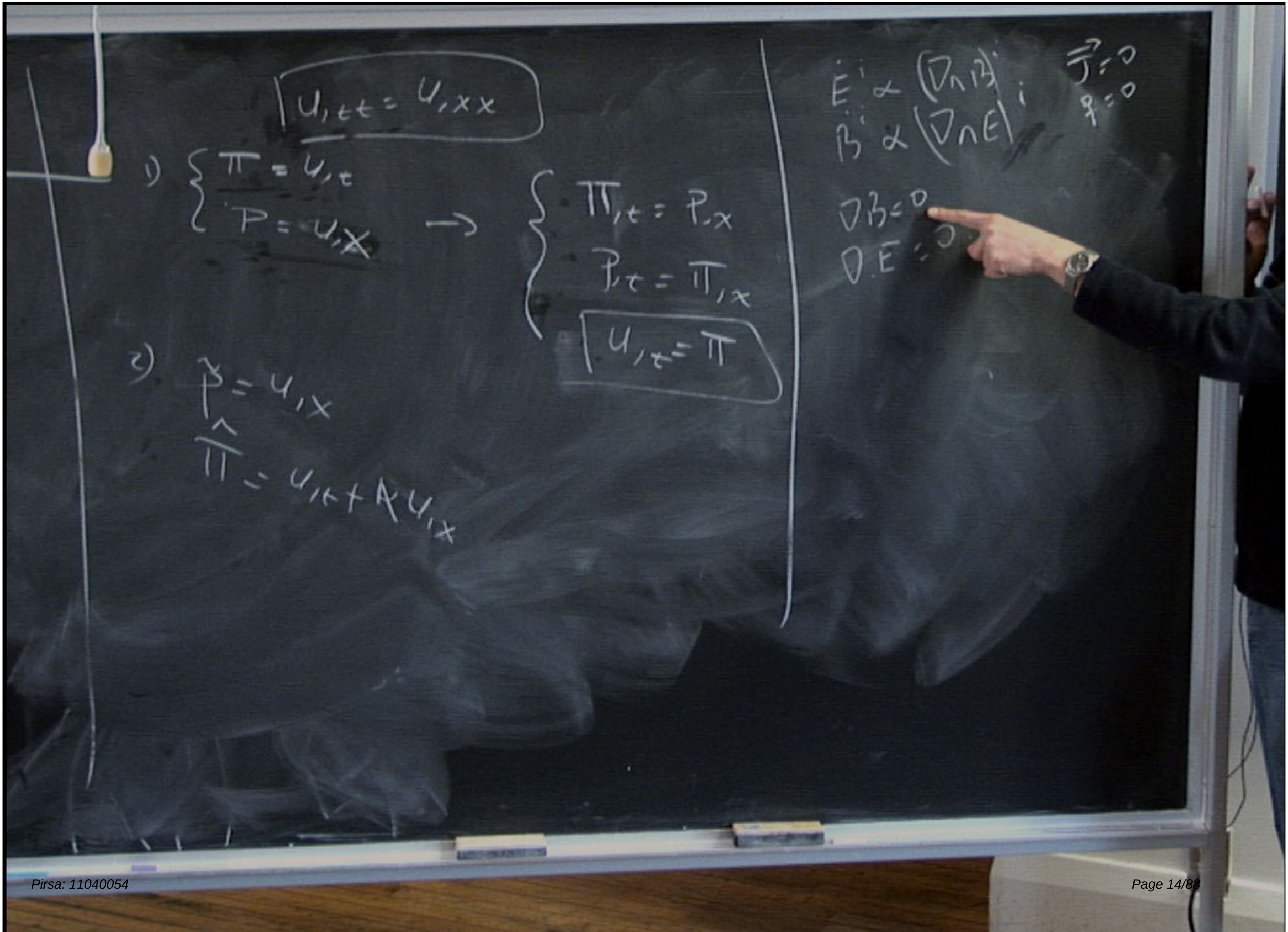

u,tt = u,xx
1)' Π,t = P,x
P,t = Π,x
← u,xx − P,x = 0
1) Π = u,t
P = u,x
→ Π,t = P,x
P,t = Π,x
u,t = Π
2) P̃ = u,x
Π̂ = u,t + k u,x
Π / P


1)'
Π,t = P,x
P,t = Π,x
elliptic eq'n
← u,xx − P,x = 0
(∇²φ ∝ ρ)
Π/P
u,tt = u,xx
1) { Π = u,t
P = u,x
→
{ Π,t = P,x
P,t = Π,x
u,t = Π
2)
P̃ = u,x
Π̂ = u,t + R u,x

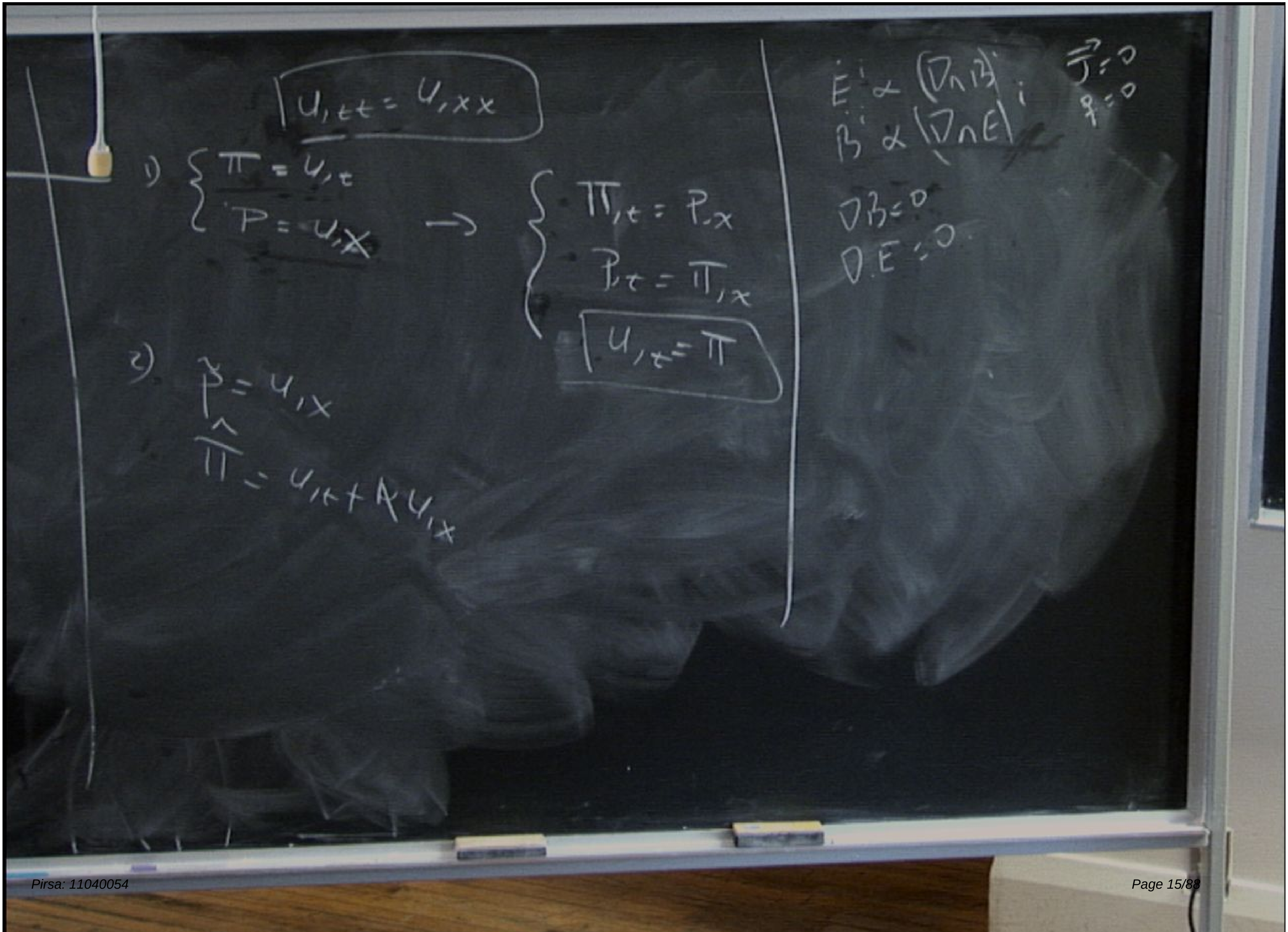

u,tt = u,xx
1) Π = u,t
P = u,x
Π,t = P,x
P,t = Π,x
u,t = Π
2) P̂ = u,x
Π̂ = u,t + k u,x
T,x = 0


u,tt = u,xx
1) π = u,t
P = u,x
→ π,t = P,x
P,t = π,x
u,t = π
2) P̂ = u,x
π̂ = u,t + k u,x
Ė ∝ ∇∧B
Ḃ ∝ ∇∧E
∇·B = 0
∇·E = 0


u,tt = u,xx
1) π = u,t
P = u,x
π,t = P,x
P,t = π,x
u,t = π
2) P = u,x
π = u,t + k u,x
E ∝ (∇∧B)
B ∝ (∇∧E)
J = 0
∇B = 0
∇E = 0



u,tt = u,xx
1) π = u,t
P = u,x
π,t = P,x
P,t = π,x
u,t = π
2) P = u,x
π̂ = u,t + Λ u,x
Ė' ∝ (∇∧B)'
Ḃ' ∝ (∇∧E)'
J = 0
ρ = 0
∇B = 0
∇.E = 0


u,tt = u,xx
1) { π = u,t ; P = u,x
→ { π,t = P,x ; P,t = π,x
u,t = π
2) P̂ = u,x
π̂ = u,t + R u,x
Ė^i ∝ (∇∧B)^i
Ḃ^i ∝ (∇∧E)^i
∇B = 0
∇.E = 0
J⃗ = 0
ρ = 0

NR…why?…*If we think hard enough we won't need a computer*

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With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with ***completely new and unexpected*** *things to think about*

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*With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with **completely new and unexpected** things to think about*

Motivation for the topics of this week's lectures

- Gravitational Wave Astrophysics – source simulations
  - compact object mergers
    - overview of nature of gravitational waves in GR
- Studies of dynamical, strong-field gravity
  - Examples:
  - high speed black hole collisions
  - higher dimensional gravity
  - critical collapse

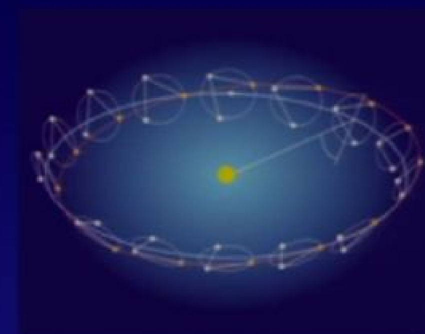
Gravitational Wave Astrophysics

LIGO/VIRGO/GEO/TAMA
ground based laser interferometers

LIGO Hanford

LIGO Livingston

LISA
space-based laser interferometer (hopefully with get funded for a 20?? Launch)

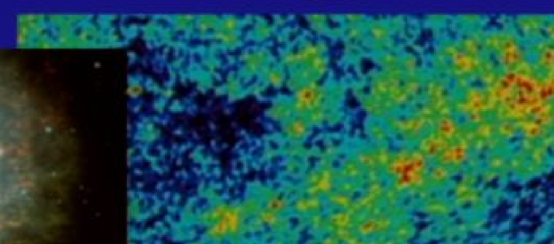


ALLEGRO/NAUTILUS/AURIGA/…
resonant bar detectors

AURIGA

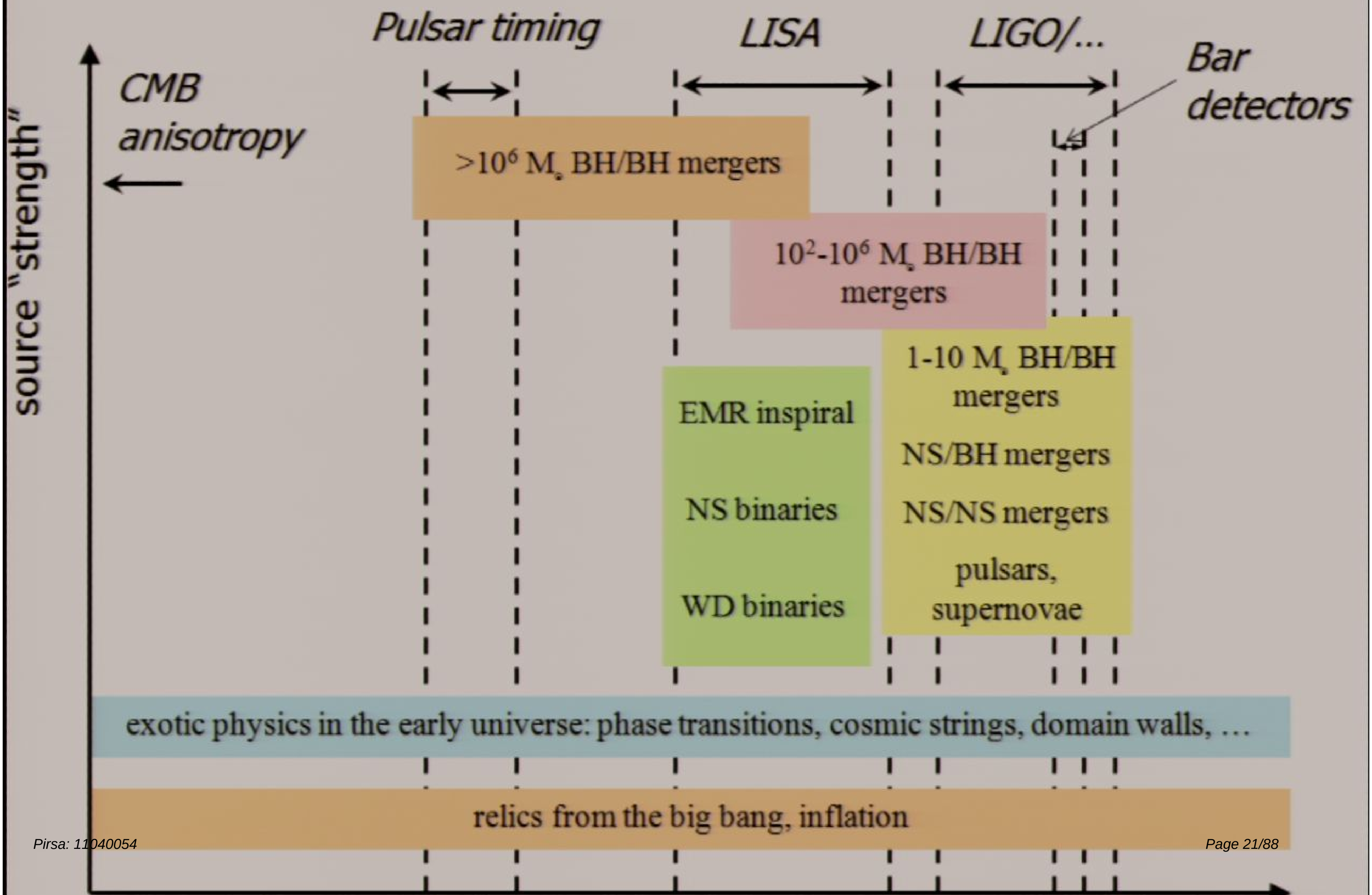
Pulsar timing network, CMB anisotropy

The Crab nebula … a supernovae



Segment of the CMB from WMAP

Overview of expected gravitational wave sources



Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - expected be the strongest source of gravitational waves in the universe
  - The binary BH is the "cleanest" simplest system, described exactly by vacuum GR
    - In the astrophysical system, will be a certain amount of matter (possibly circumbinary disk, ambient gas, CMB photons, ...), though for the most part is expected very low mass relative to the black holes and hence dynamically insignificant
  - With NS's, microphysics must be approximated, and the sky's the limit as to how complicated this can be
    - neutron star structure (crust, multi-component superfluid core, at nuclear densities with uncertain equations of state, magnetic fields, etc.)
    - post merger, radiation and neutrino physics, nuclear processes, MRI, etc. play a role

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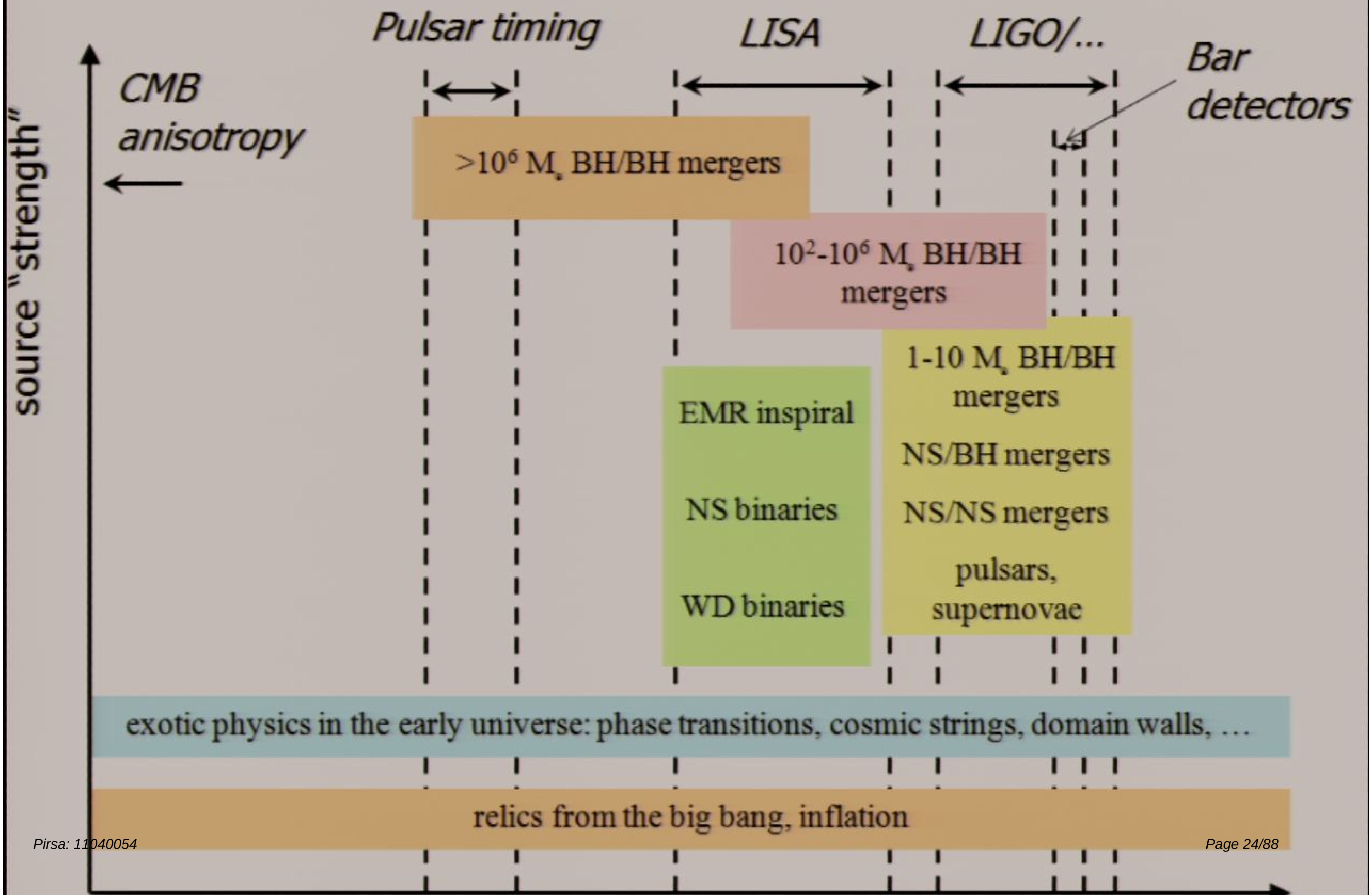
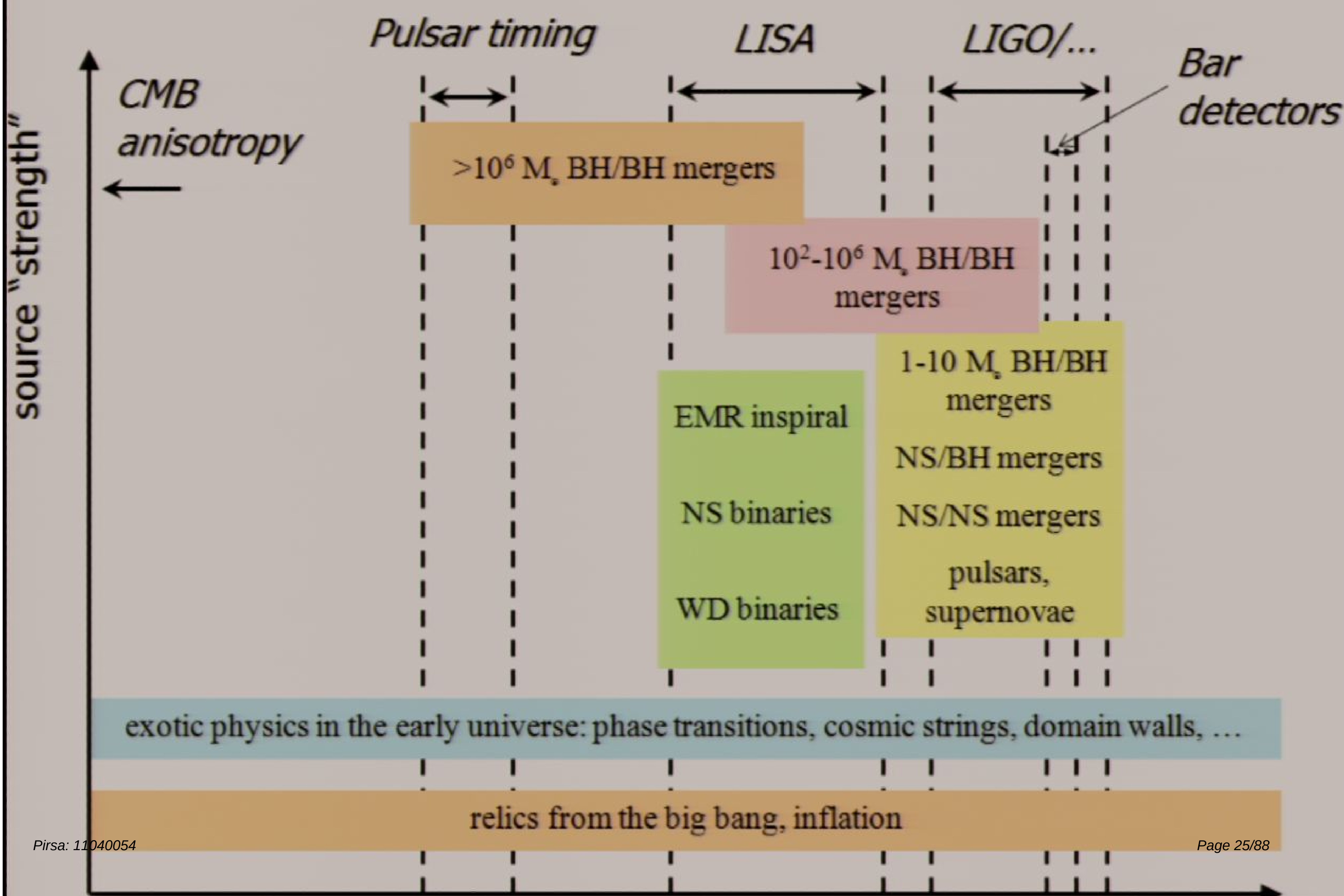
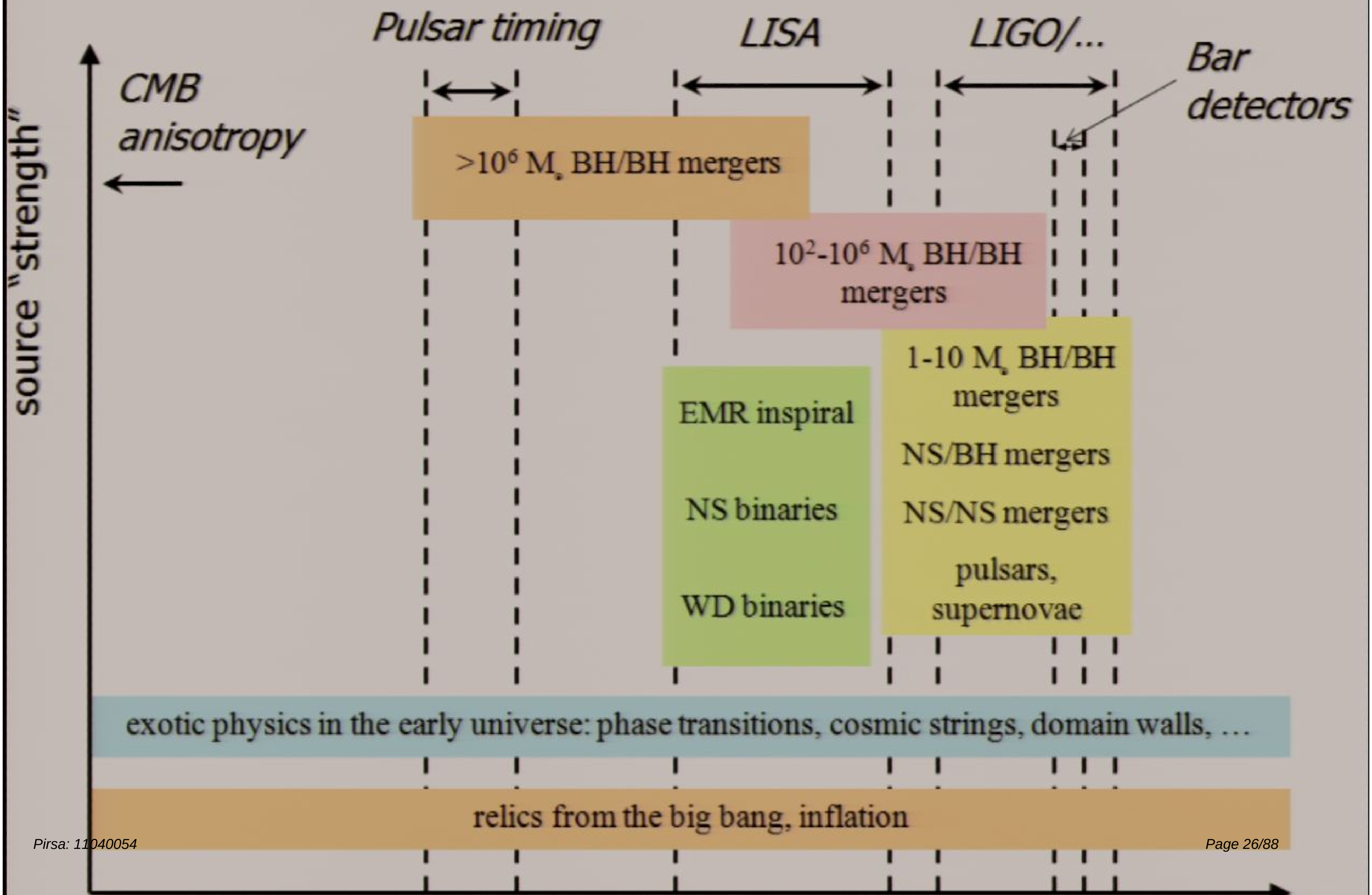
Overview of expected gravitational wave sources



Overview of expected gravitational wave sources



Overview of expected gravitational wave sources



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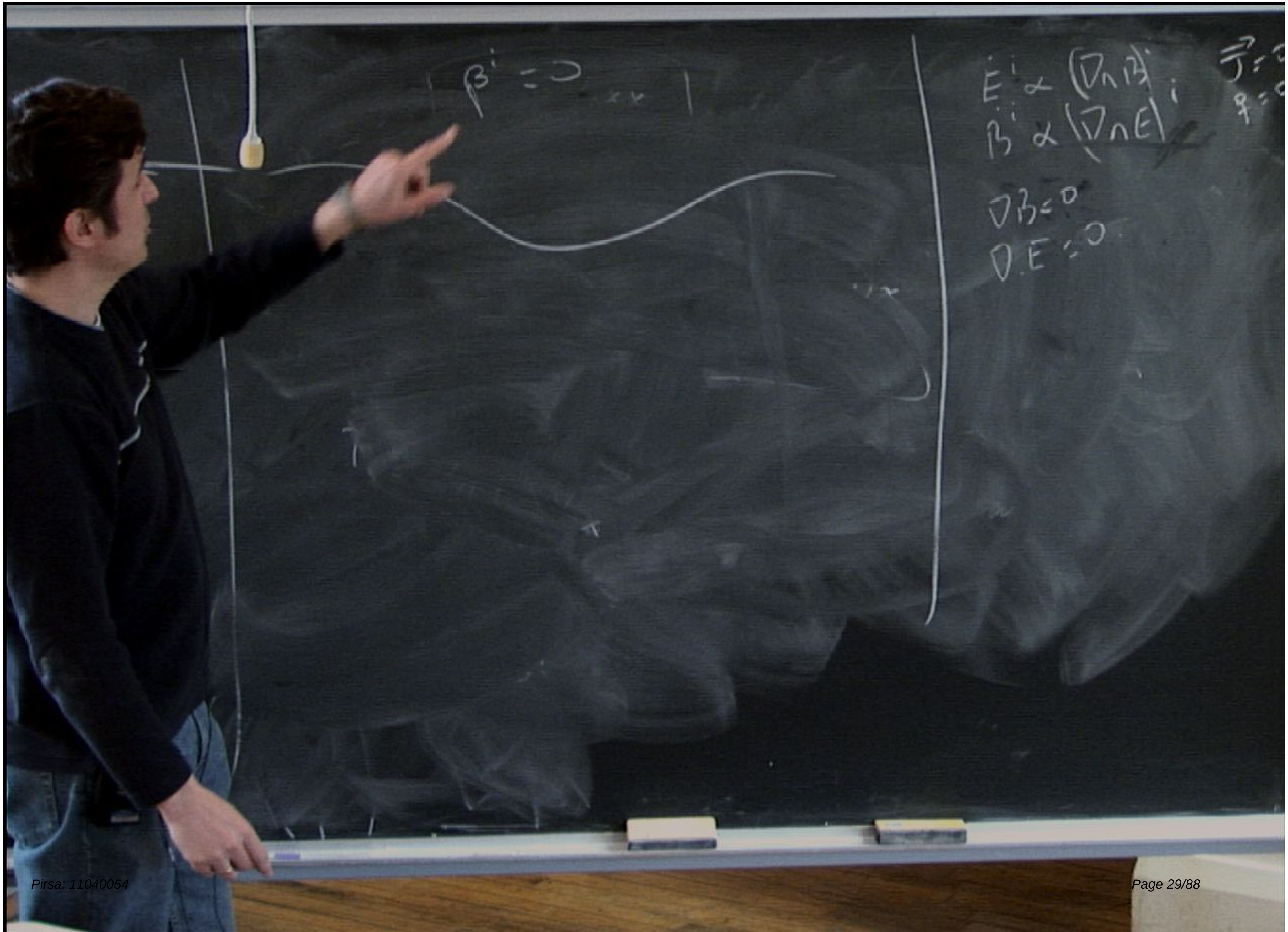
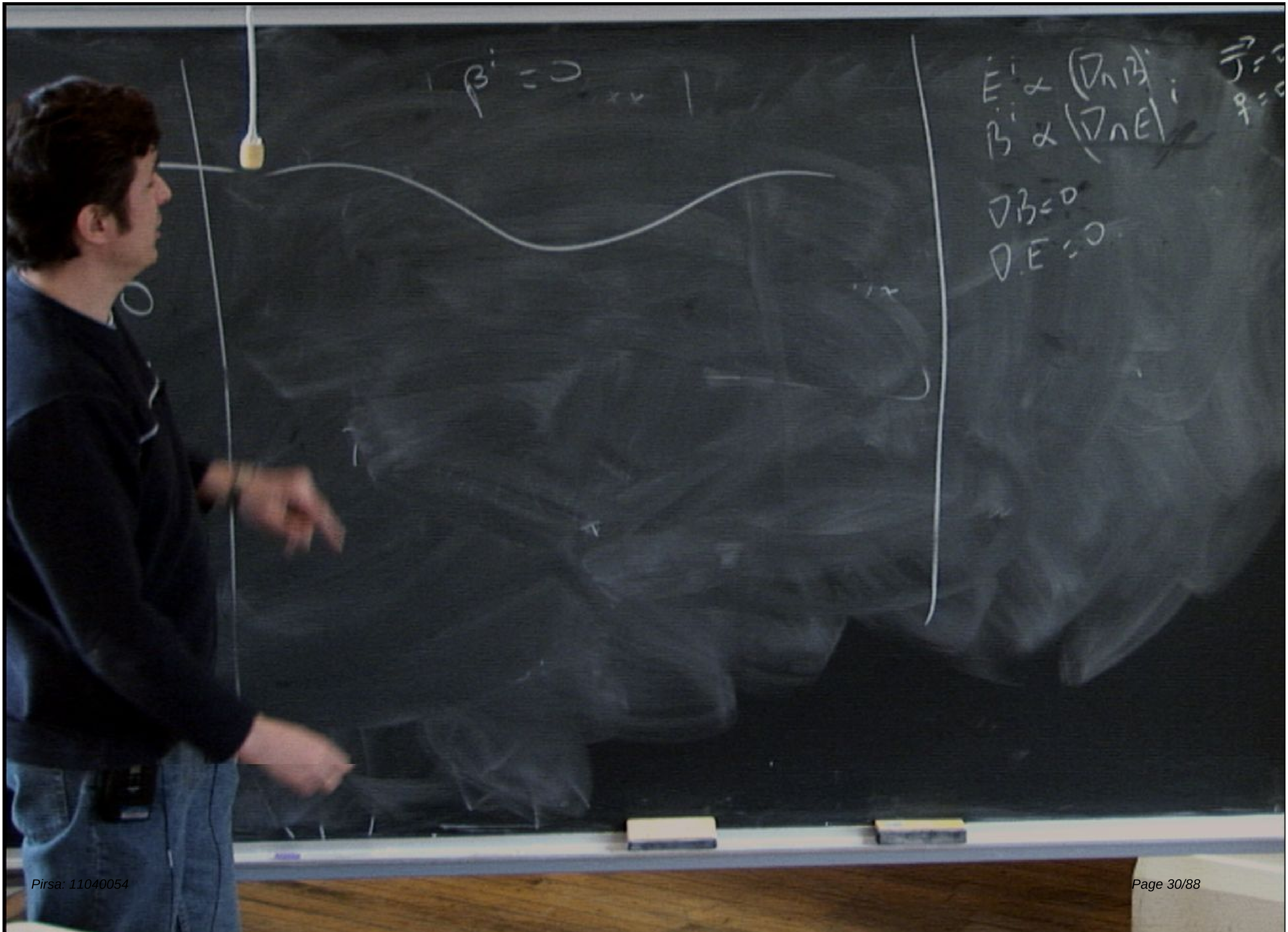
Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - final stages of merger within GR, plus "messy" astrophysics requires numerical solution
    - No symmetries : need a general 3+1 evolution code, which brings several new problems to the fore
      - ADM is only weakly hyperbolic in 3+1 : ***need reformulations***
      - *All* finite-difference/spectral numerical Cauchy solution methods can only satisfy the Einstein constraint equations to within truncation error, and the vast majority (100's – 2) of mathematically well-posed reformulations admit exponentially growing "constraint violating" solutions
        - » given truncation errors, no way the seeds of these modes can be eliminated in a numerical code
      - "nice" gauges, boundary conditions, methods of dealing with BH singularities that work well in spherical (or axi) symmetry often don't extend to the general 3+1 case


∇B = 0
∇.E = 0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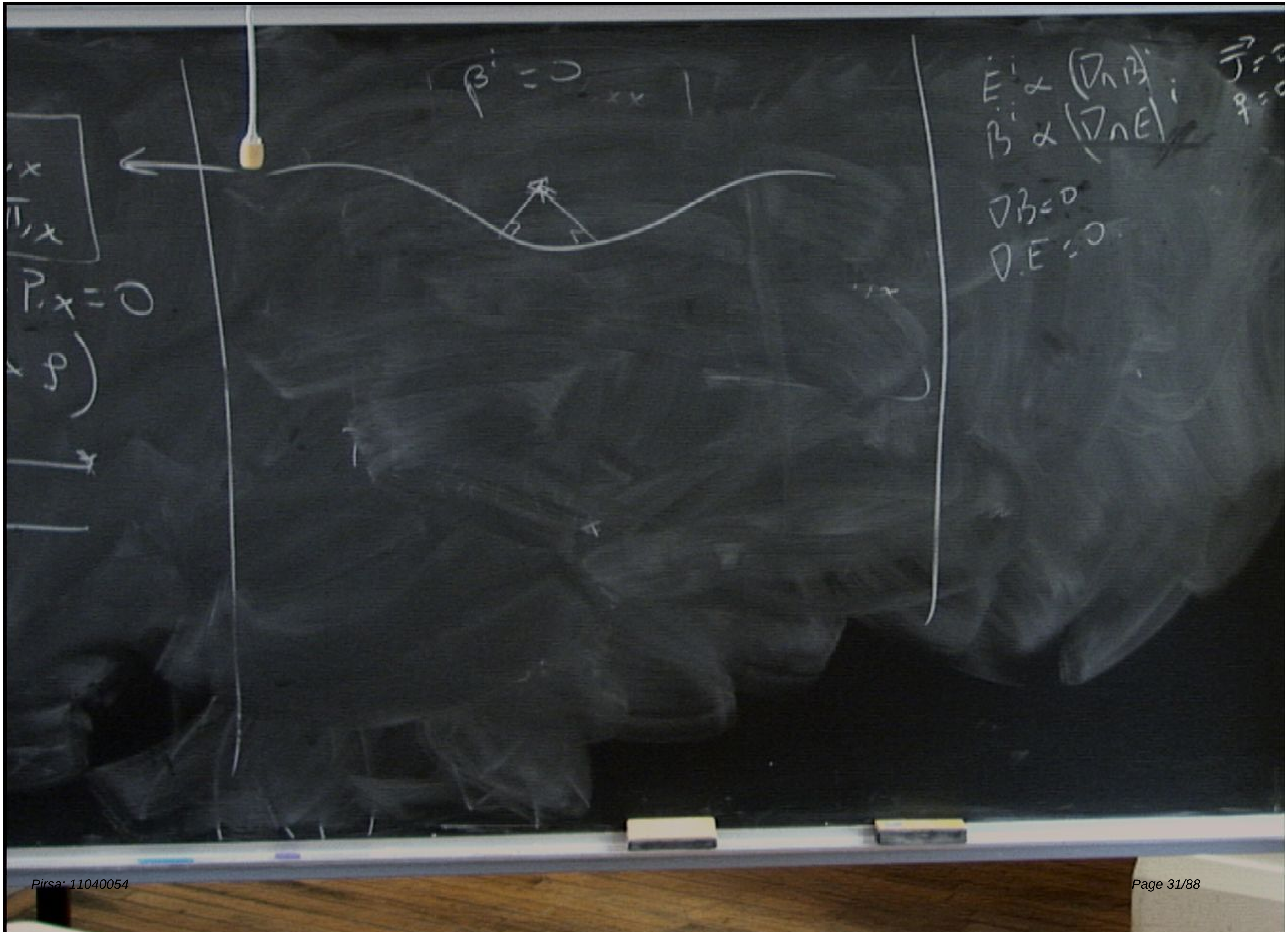
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P,x = 0

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Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - At minimum (vacuum), two orders of magnitude of relevant spatio-temporal length scales that need to be resolved:
    - spatial scales: smallest BH radius ~2M; intermediate orbital radius ~ 20M; largest "wave zone" ~ 200M
    - temporal scales : spatial scales/characteristic speed (1!) ~ spatial scales
  - Including matter (NS/NS, BH/NS), and depending on the kinds of questions one needs to answer (effects of microphysics, EM/neutrino signatures of events, delayed collapse of "hypermassiv" neutron stars following NS/NS mergers, formation of accretion disks/jets, etc), can increase the effective dimensionality of the problem, and add many orders of magnitude to the scales that need to be resolved

Need efficient algorithms (AMR), and terascale and upwards computing resources

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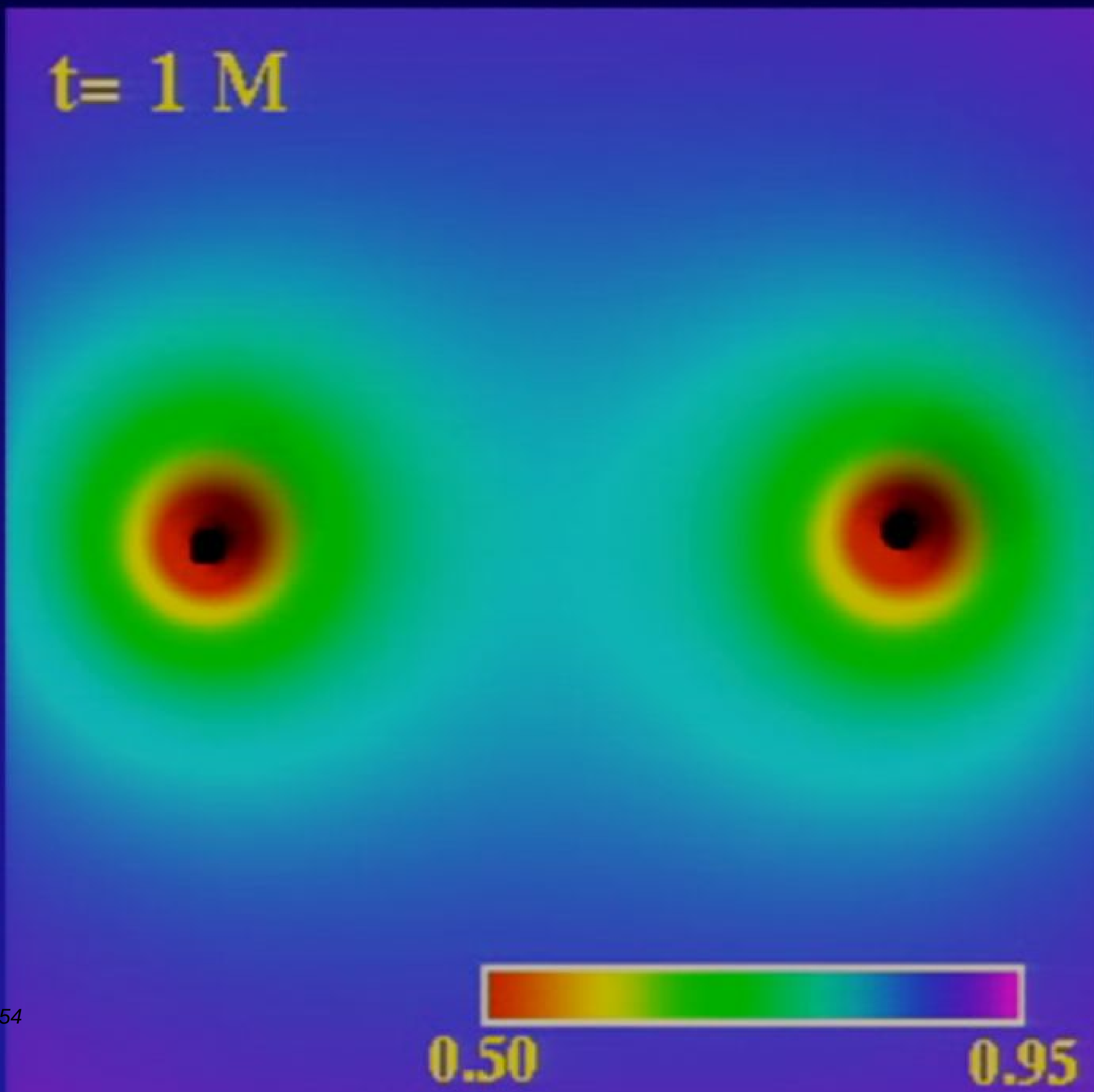
Sample evolution --- Cook-Pfeiffer Quasi-circular initial data

A. Buonanno, G.B. Cook and F.P.; Phys.Rev.D75:124018,2007

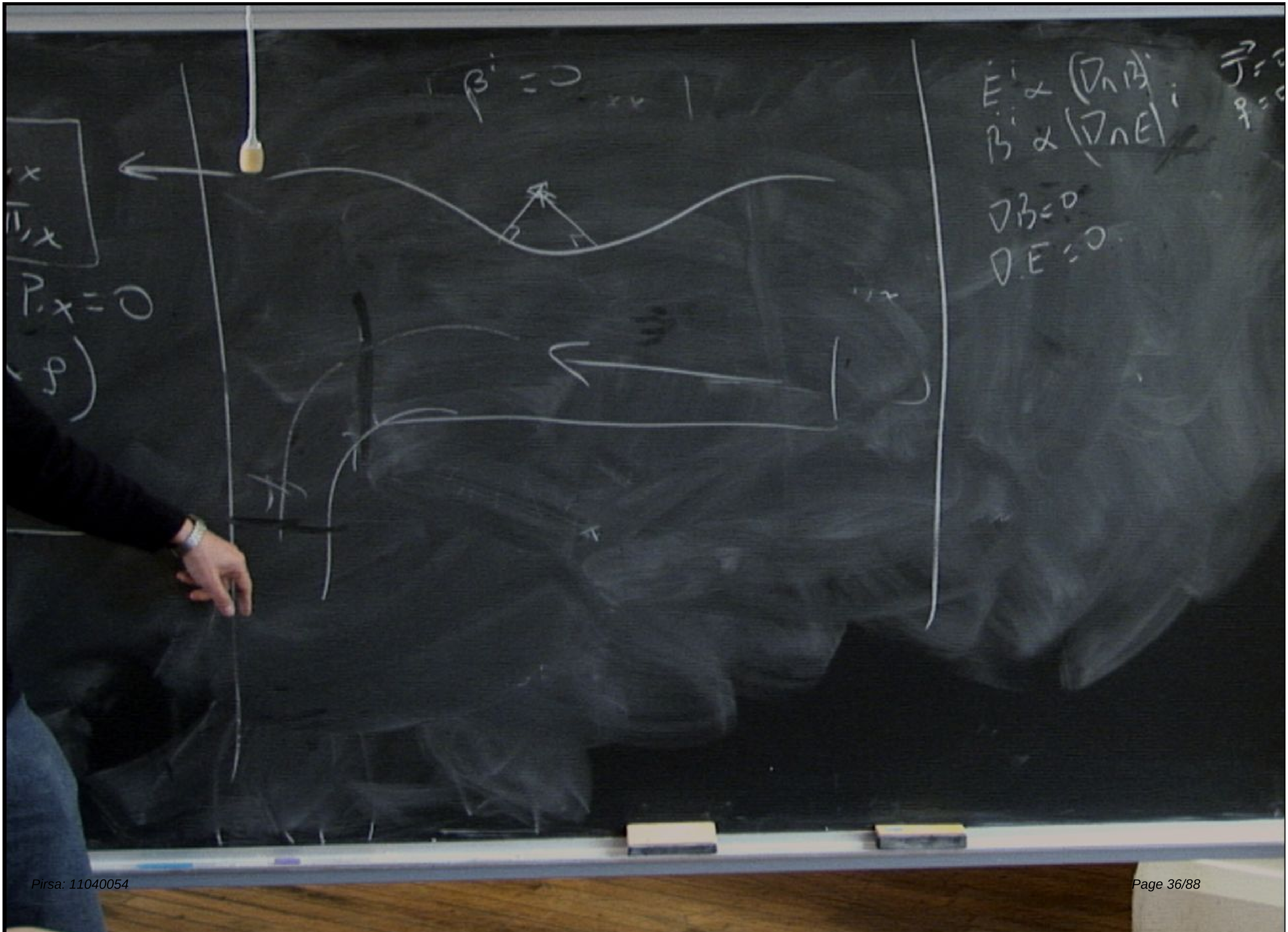


This animation shows the *lapse function* in the orbital plane.

The lapse function represents the relative time dilation between a hypothetical observer at the given location on the grid, and an observer situated very far from the system --- the redder the color, the slower local clocks are running relative to clocks at infinity

If this were in "real-time" it would correspond to the merger of two ~5000 solar mass black holes

Initial black holes are close to non-spinning Schwarzschild black holes; final black hole is a Kerr a black hole with spin parameter ~0.7, and ~4% of the total initial rest-mass of the system is emitted in gravitational waves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P,x = 0
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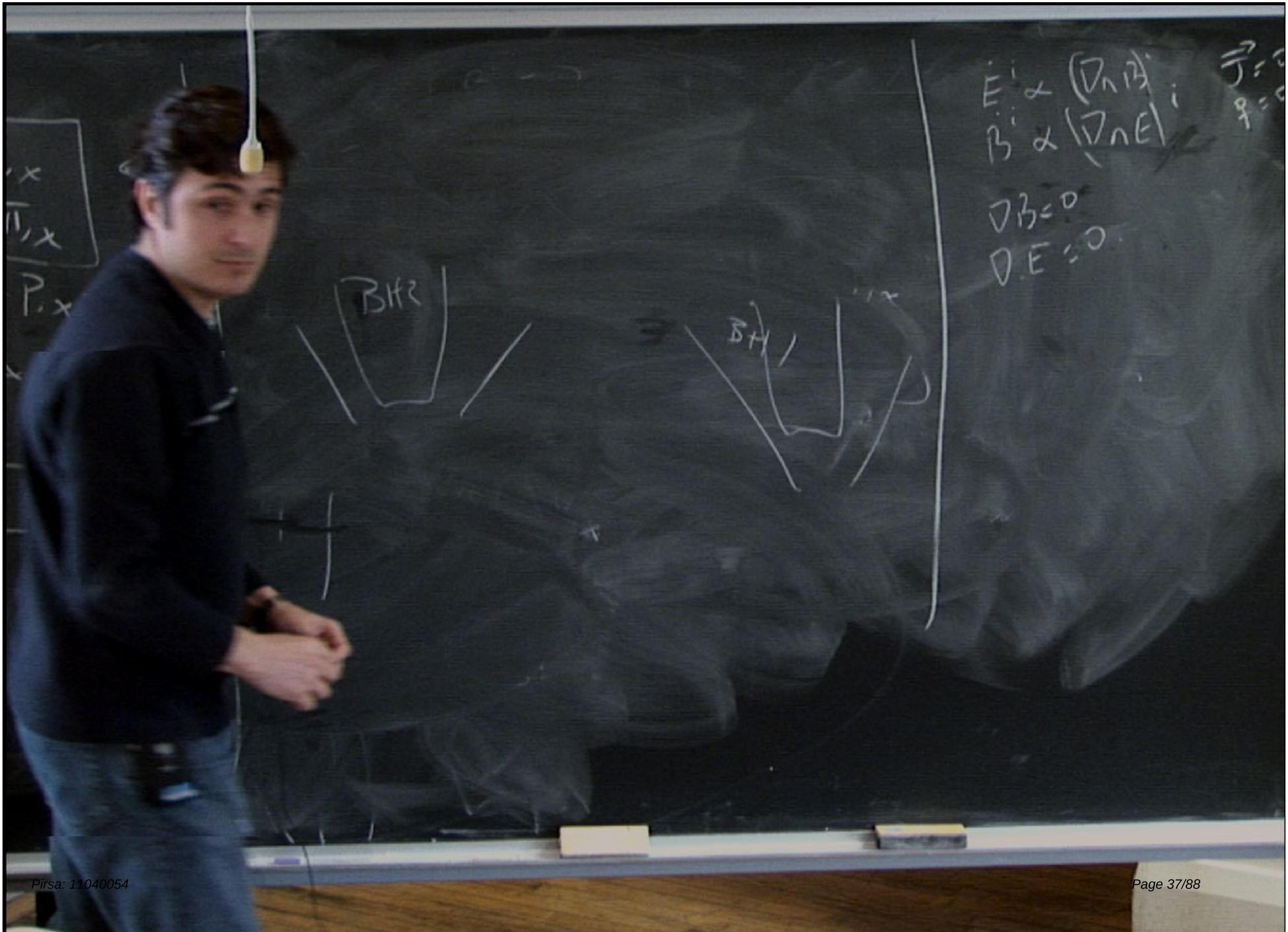
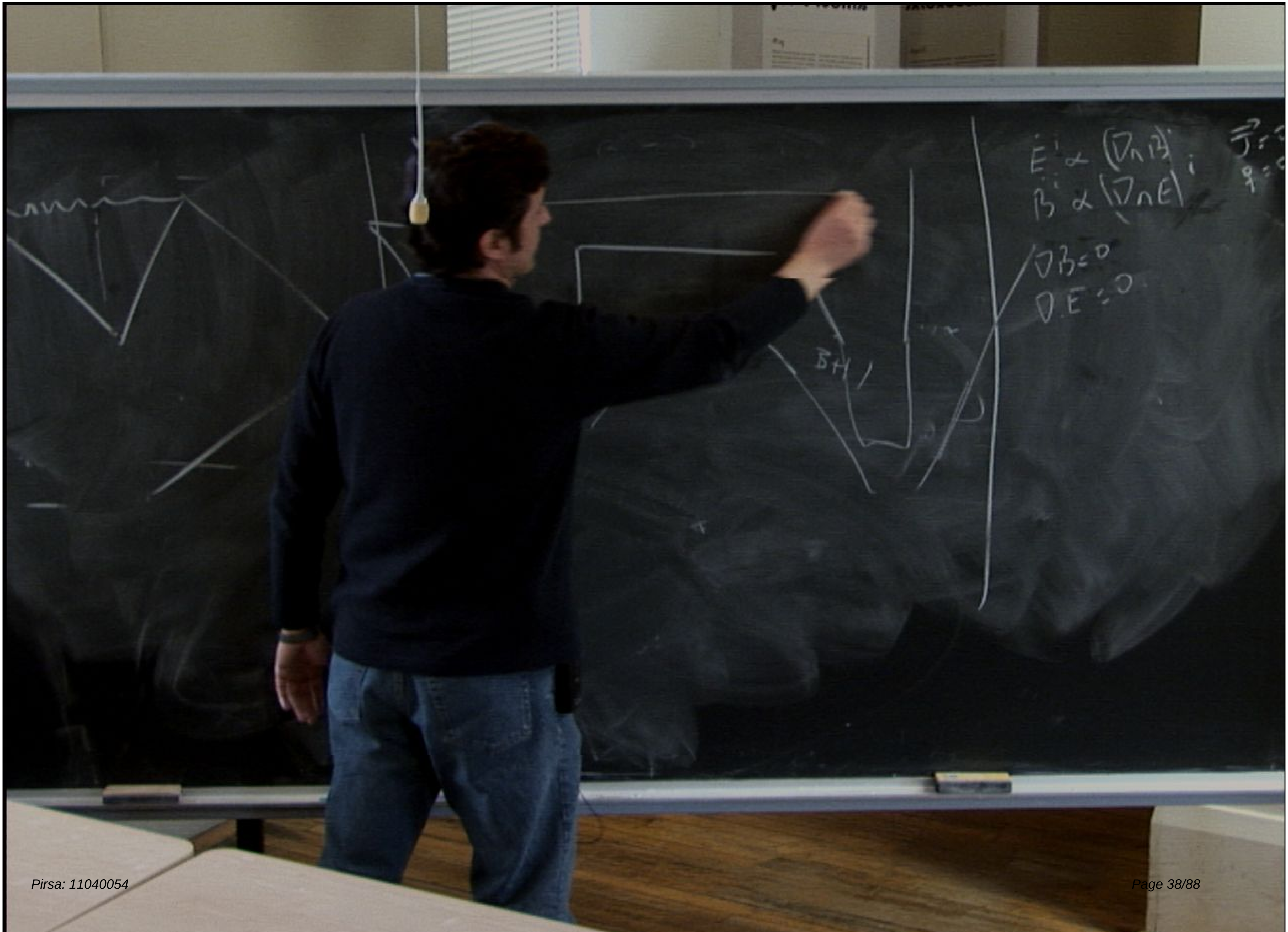
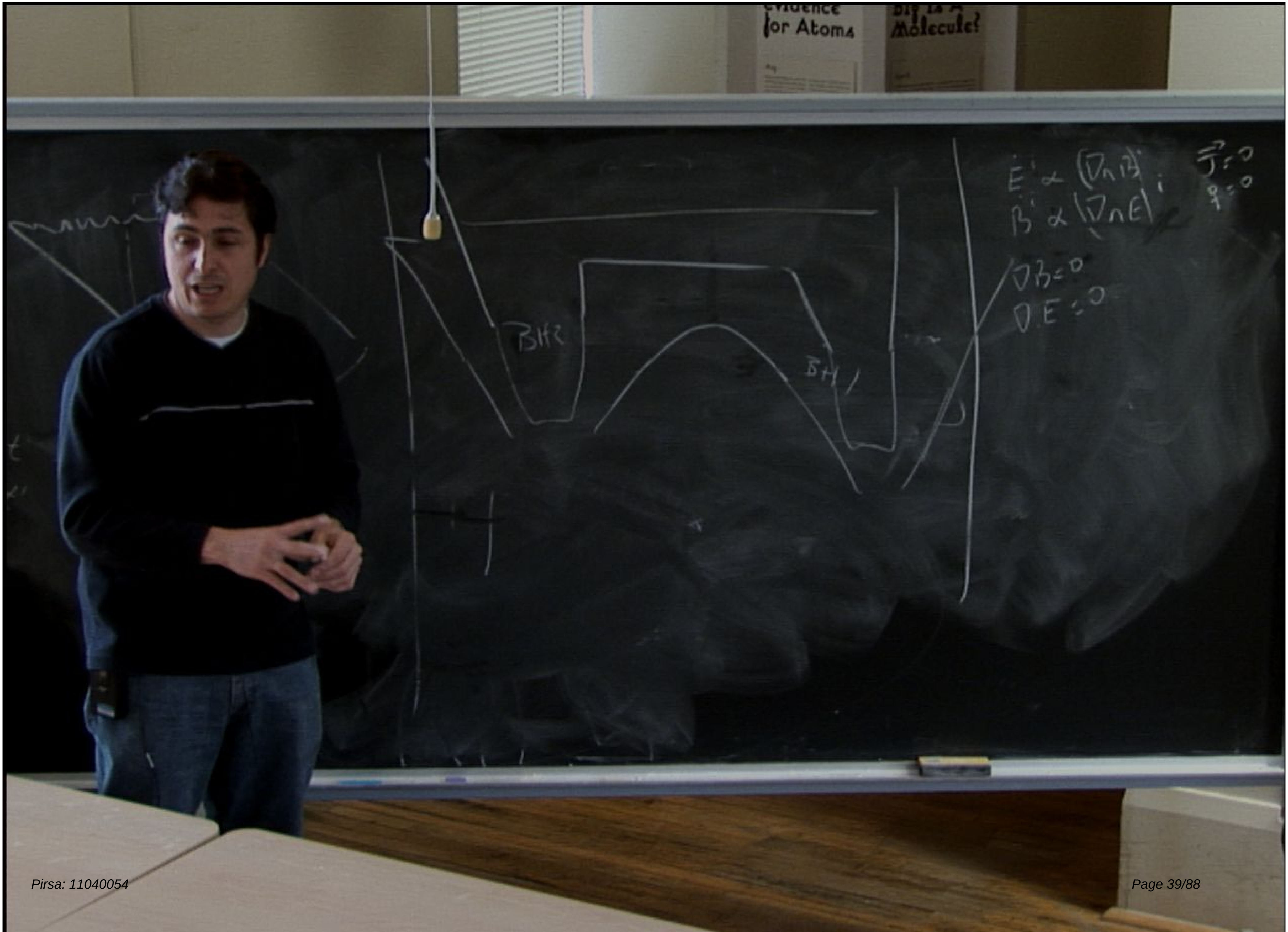

BH2
BH1
∇B = 0
∇·E = 0




for Atoms
Molecule?
BH2
BH1

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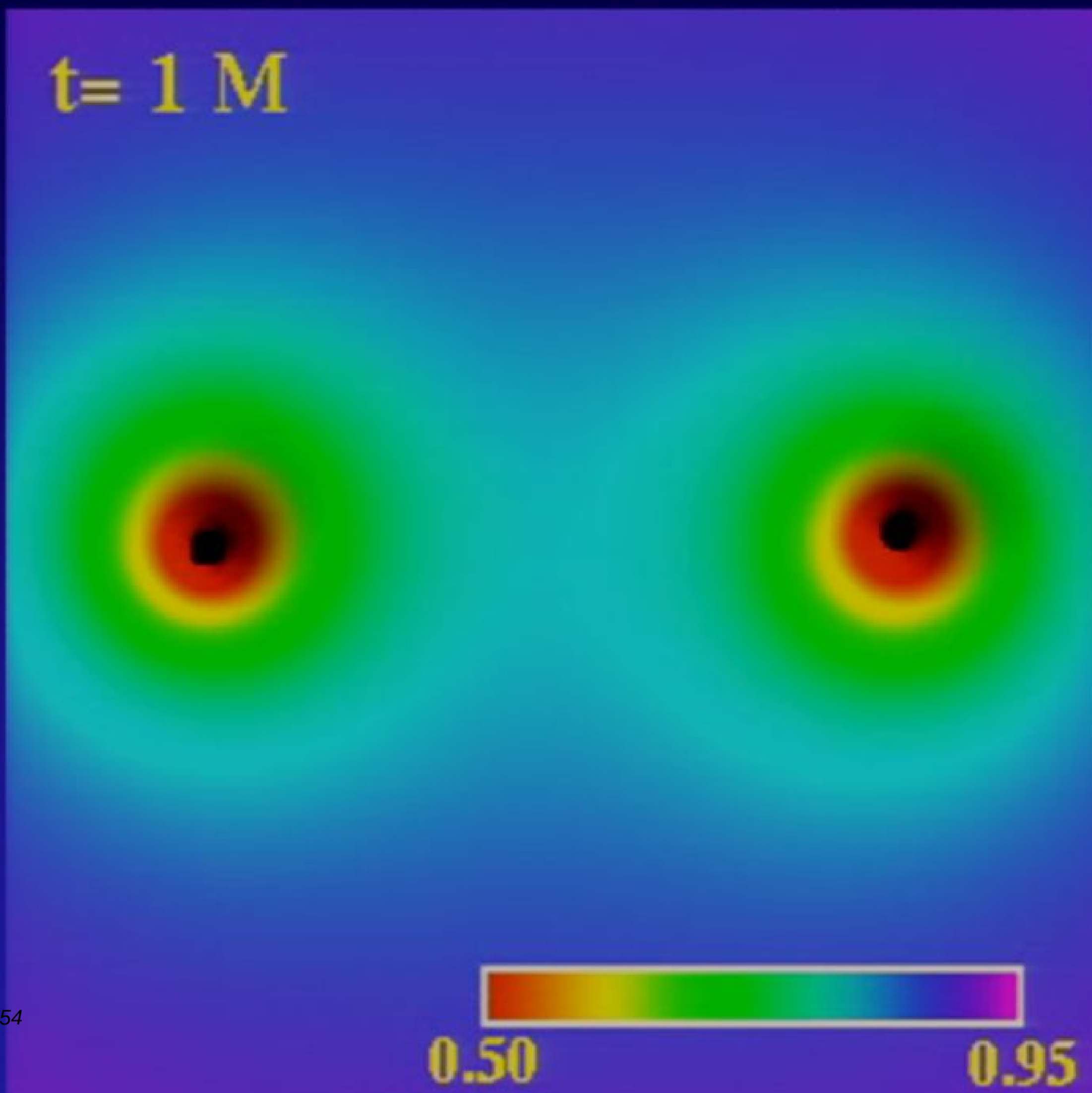
A. Buonanno, G.B. Cook and F.P.; Phys.Rev.D75:124018,2007



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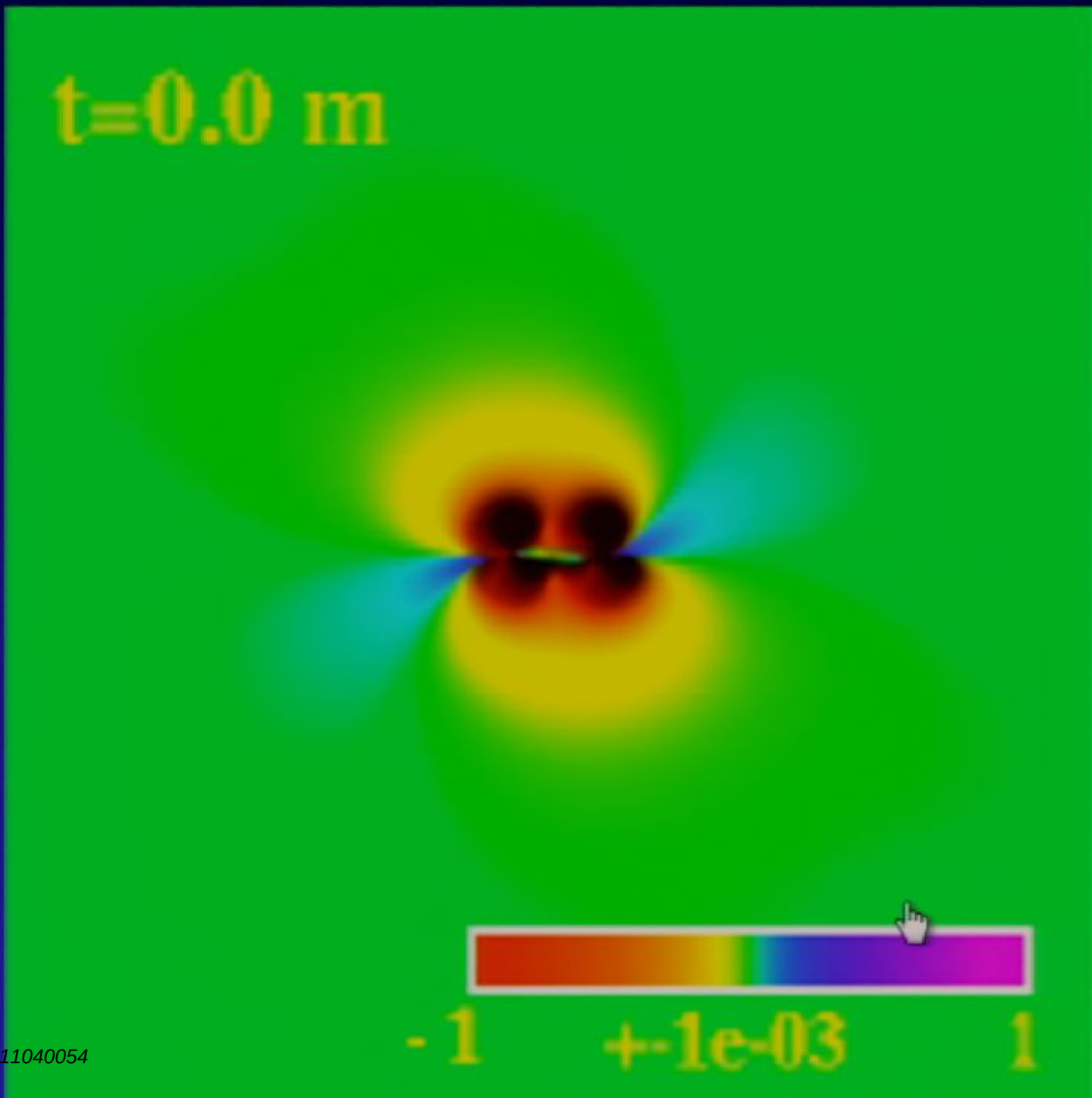
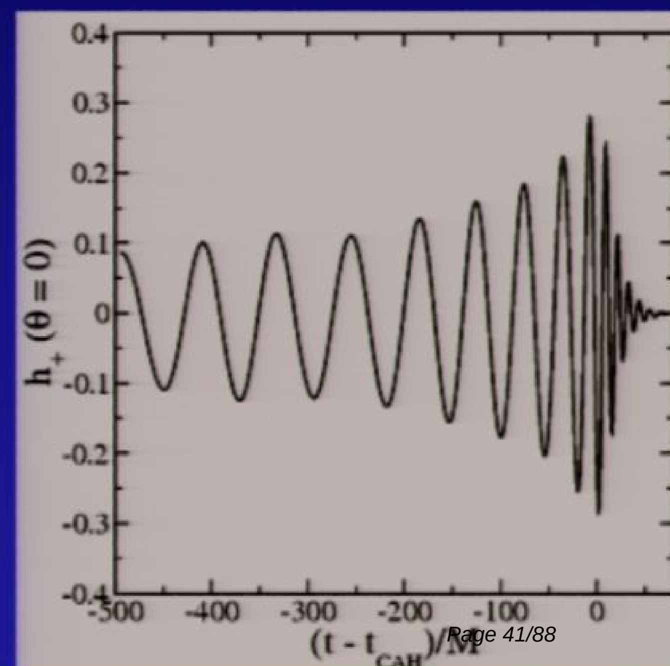
Gravitational waves from the simulation



A depiction of the gravitational waves emitted in the orbital plane of the binary. Shown is the real component of the Newman Penrose scalar ψ_4, which in the wave zone is proportional to the second time derivative of the usual plus-polarization



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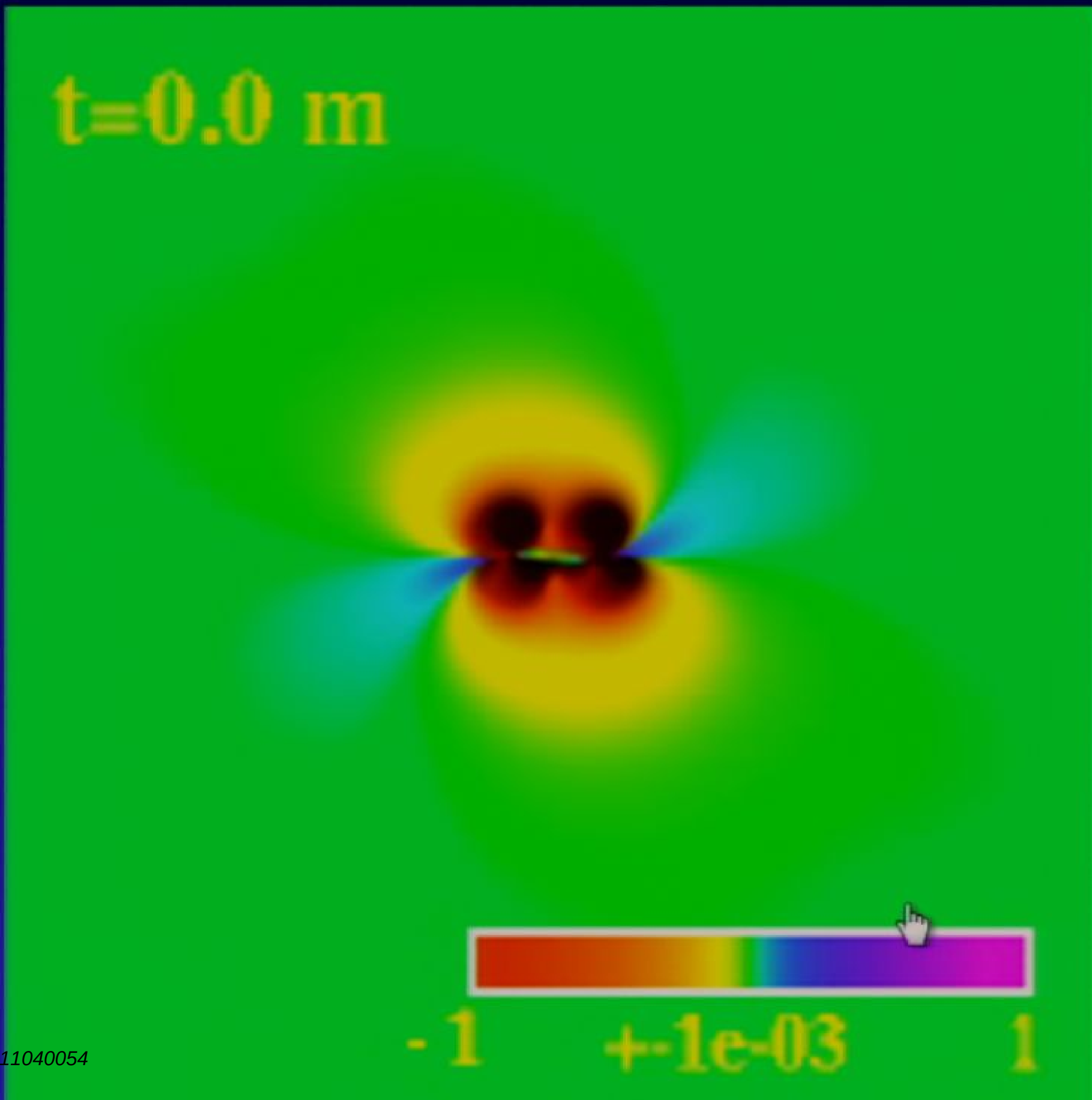

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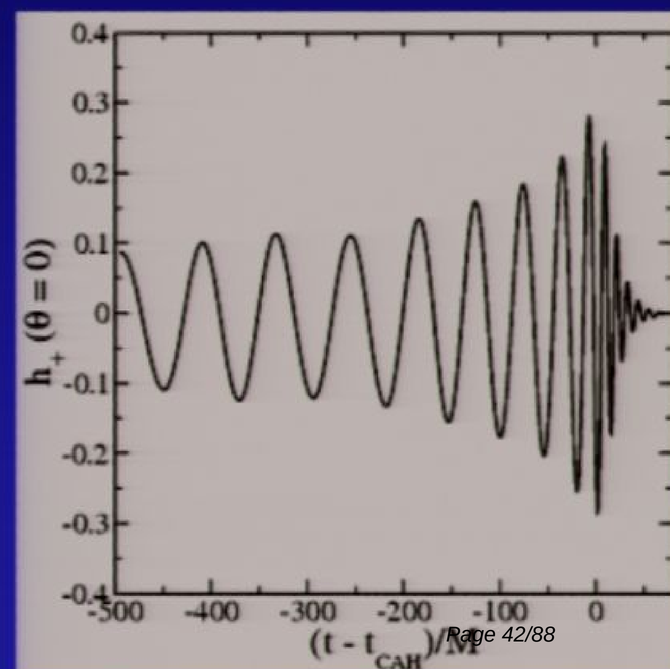

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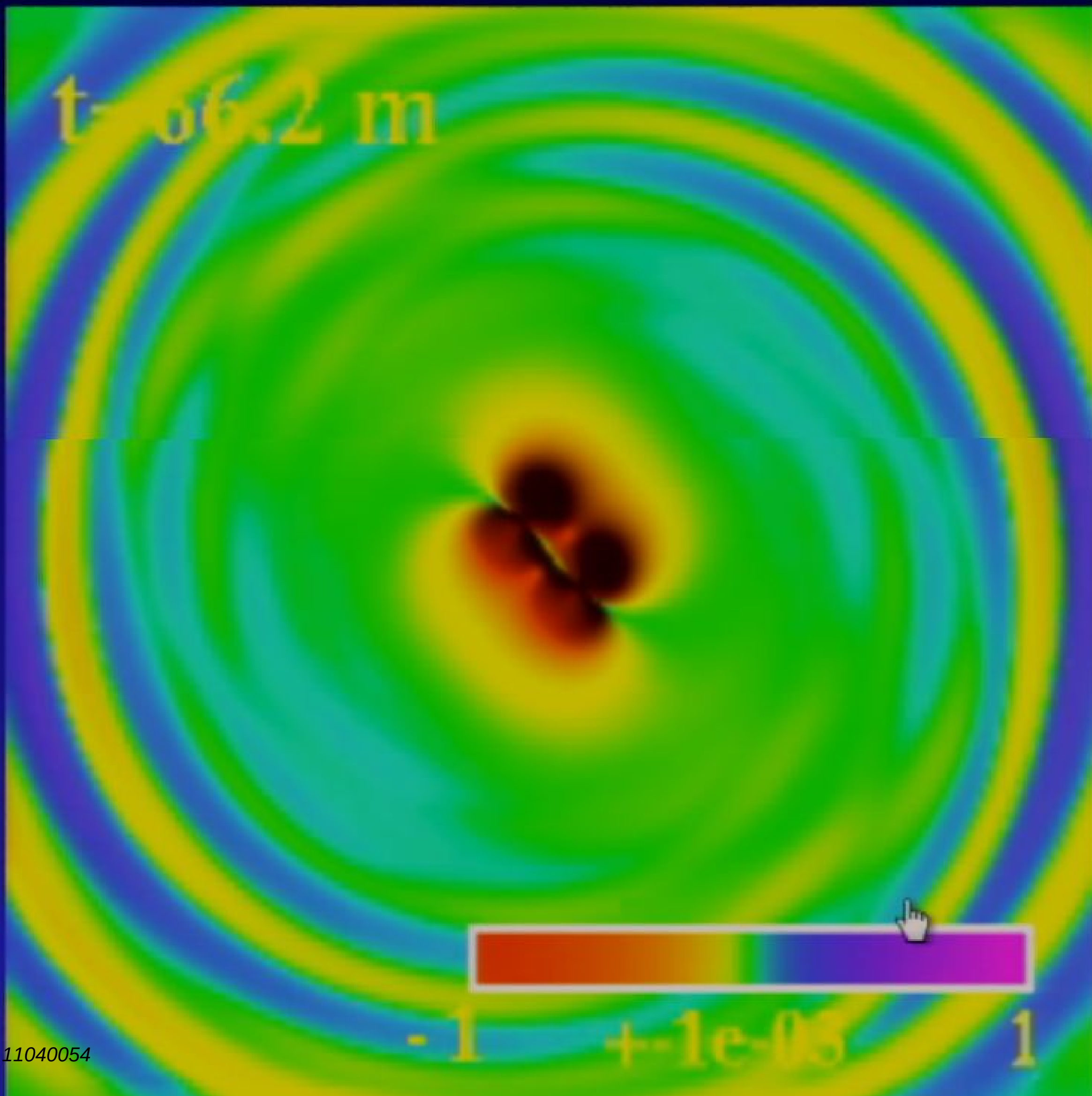

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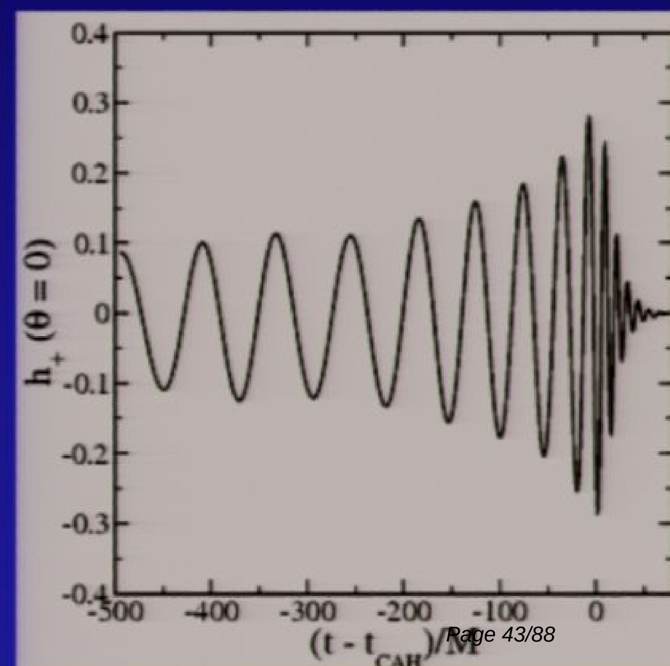

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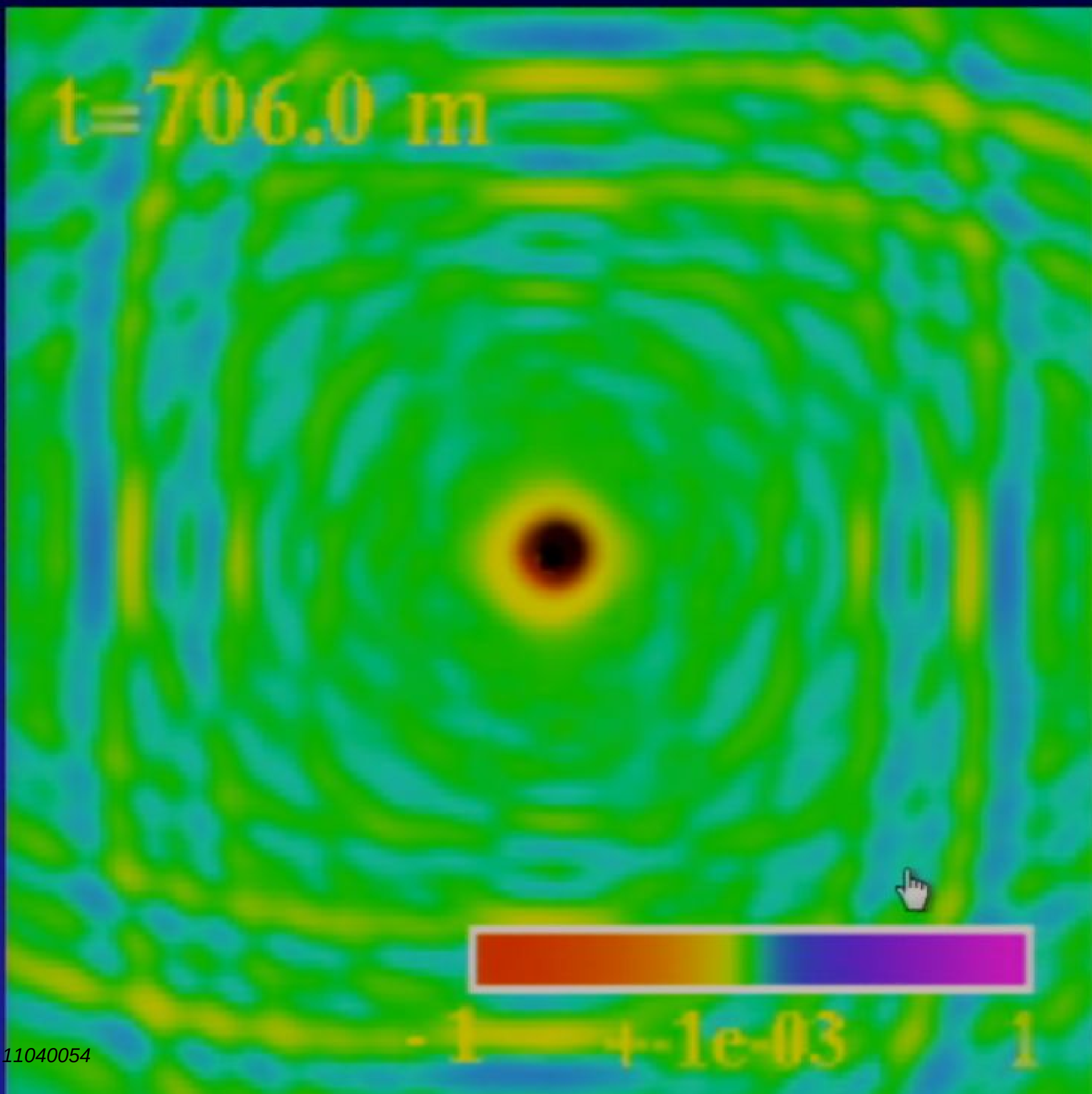

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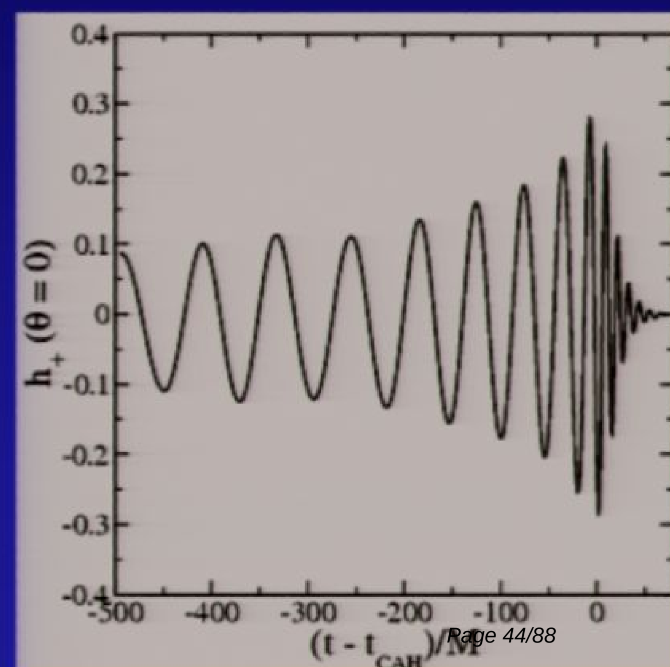
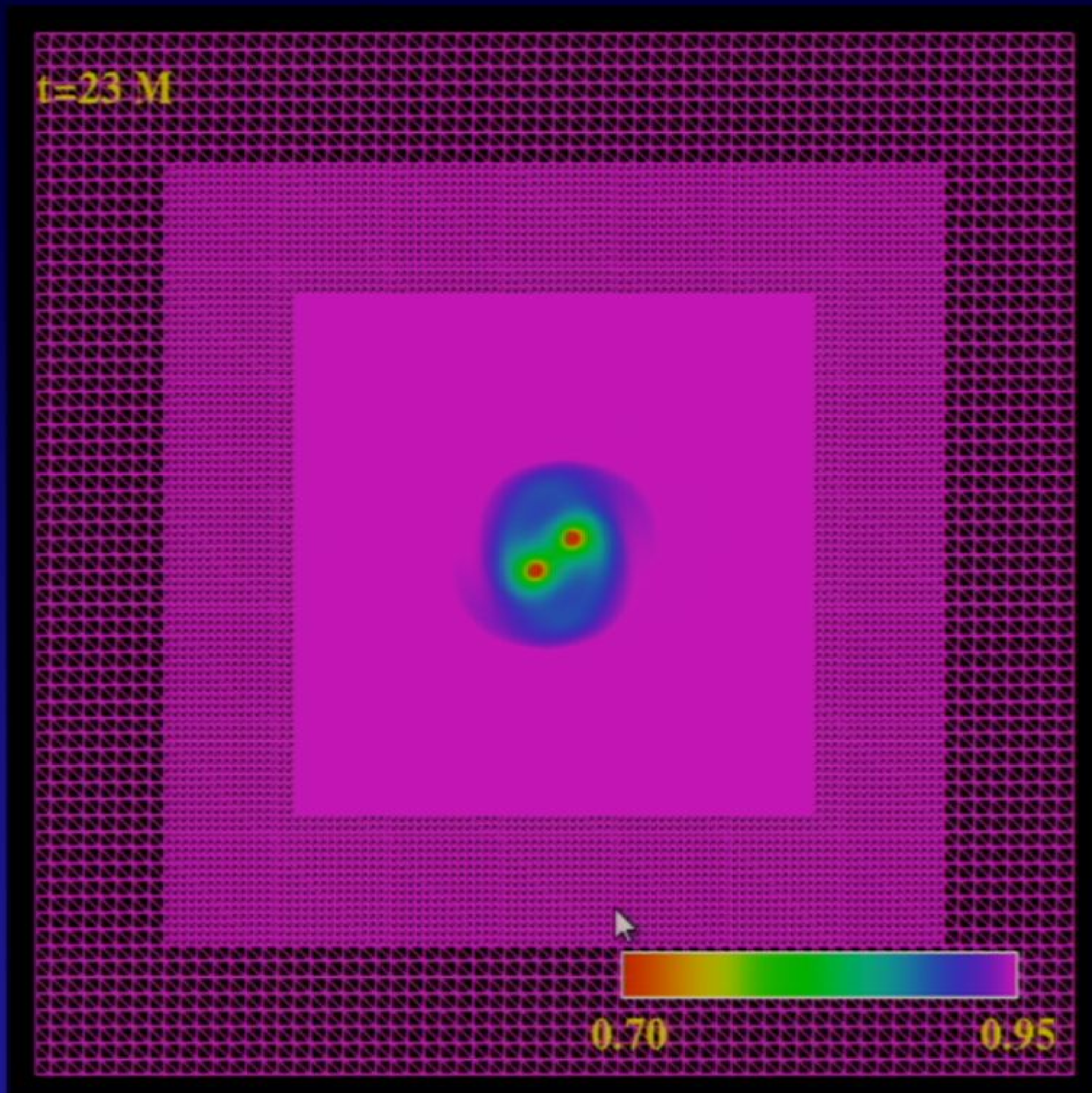


Sample mesh structure



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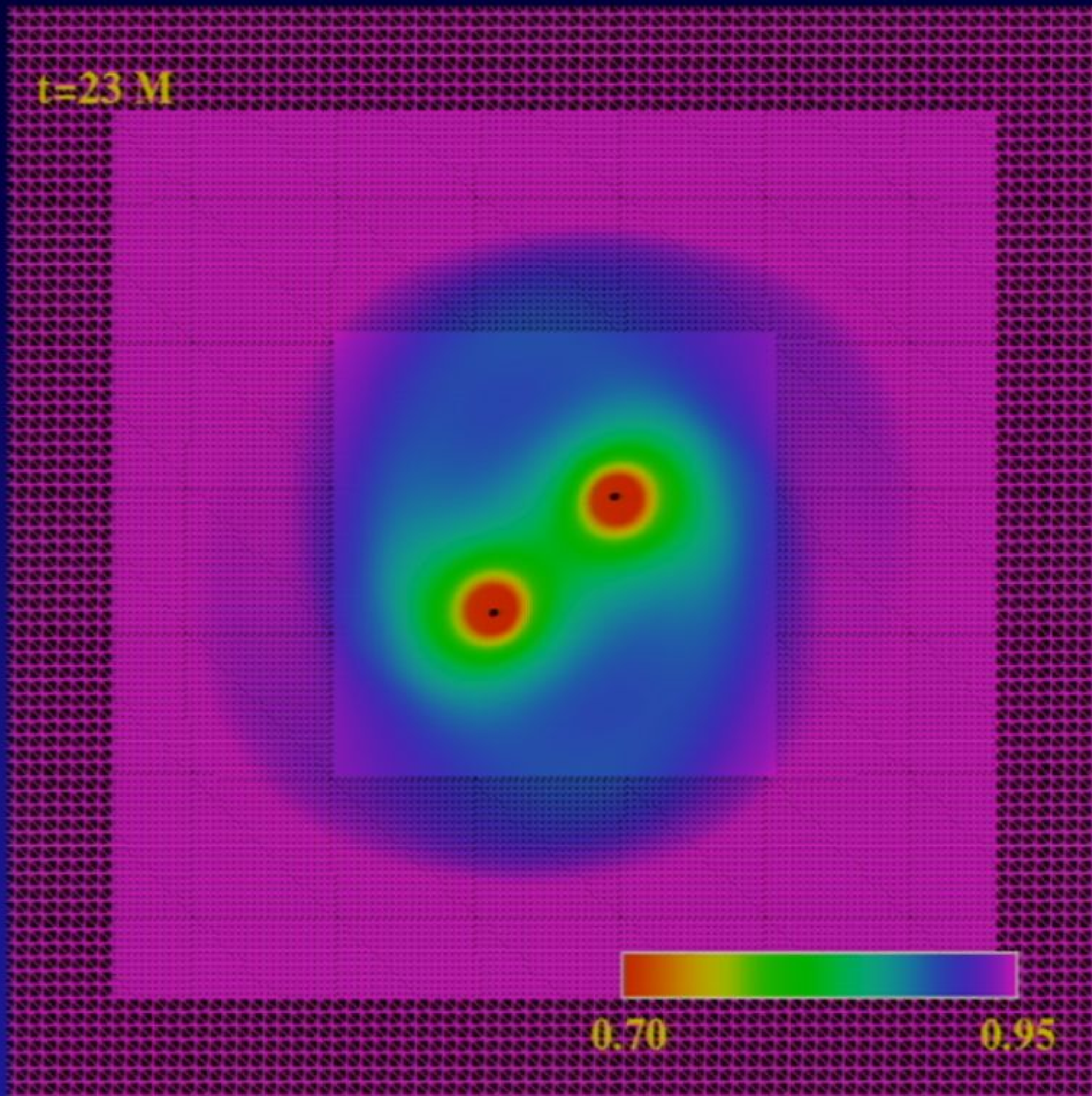
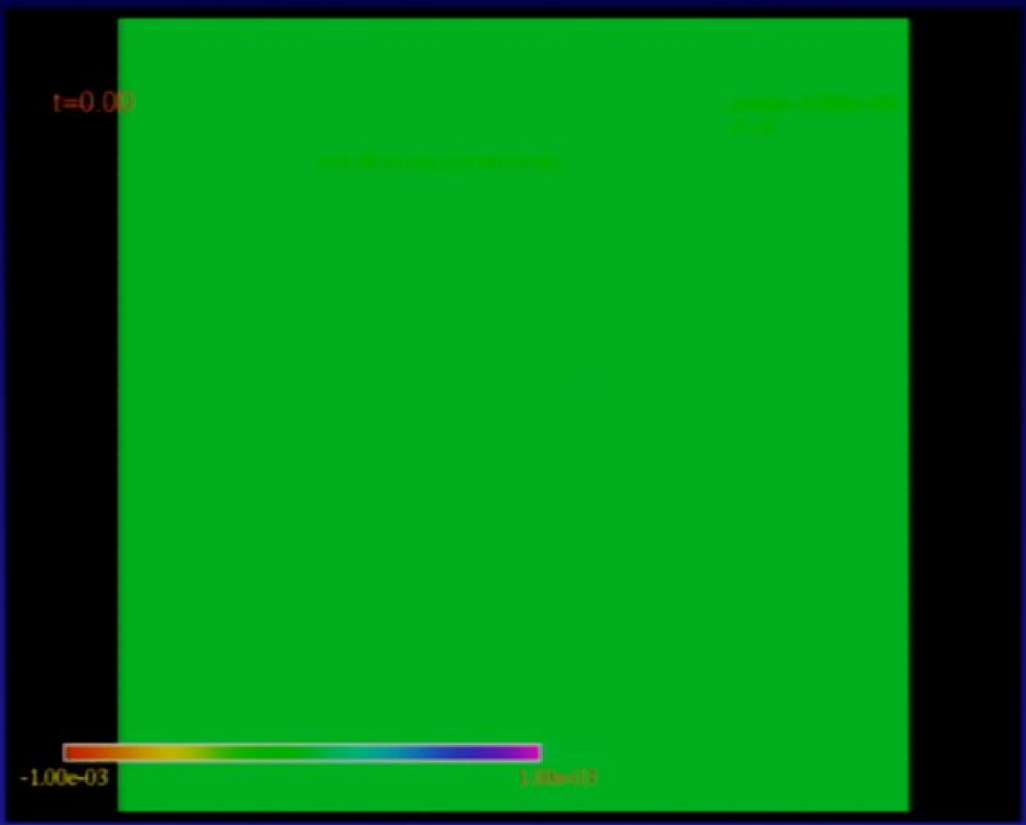
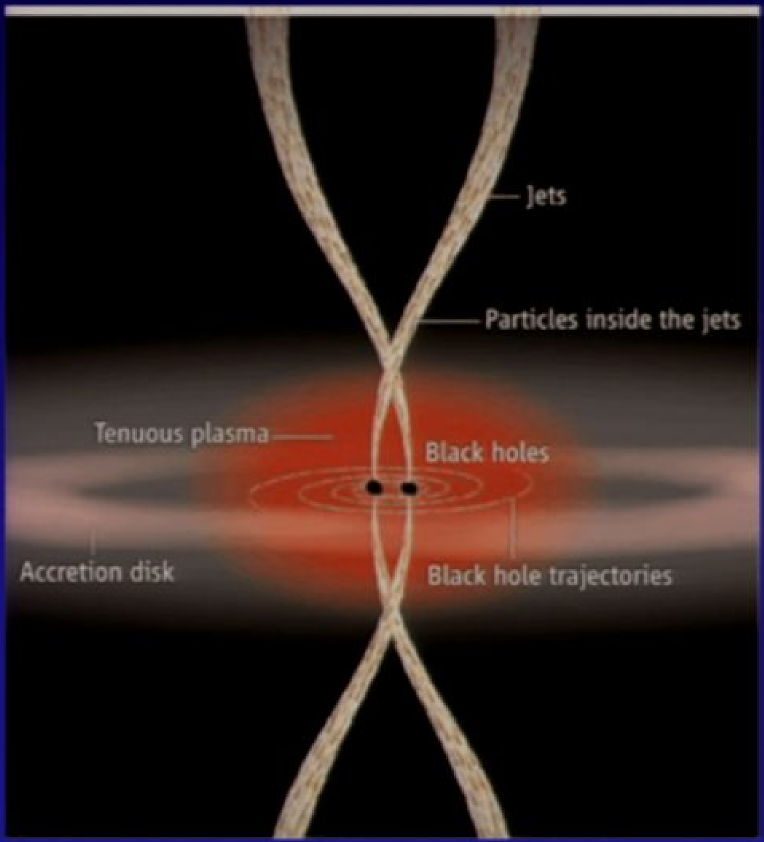


What else?

(think about what else!)





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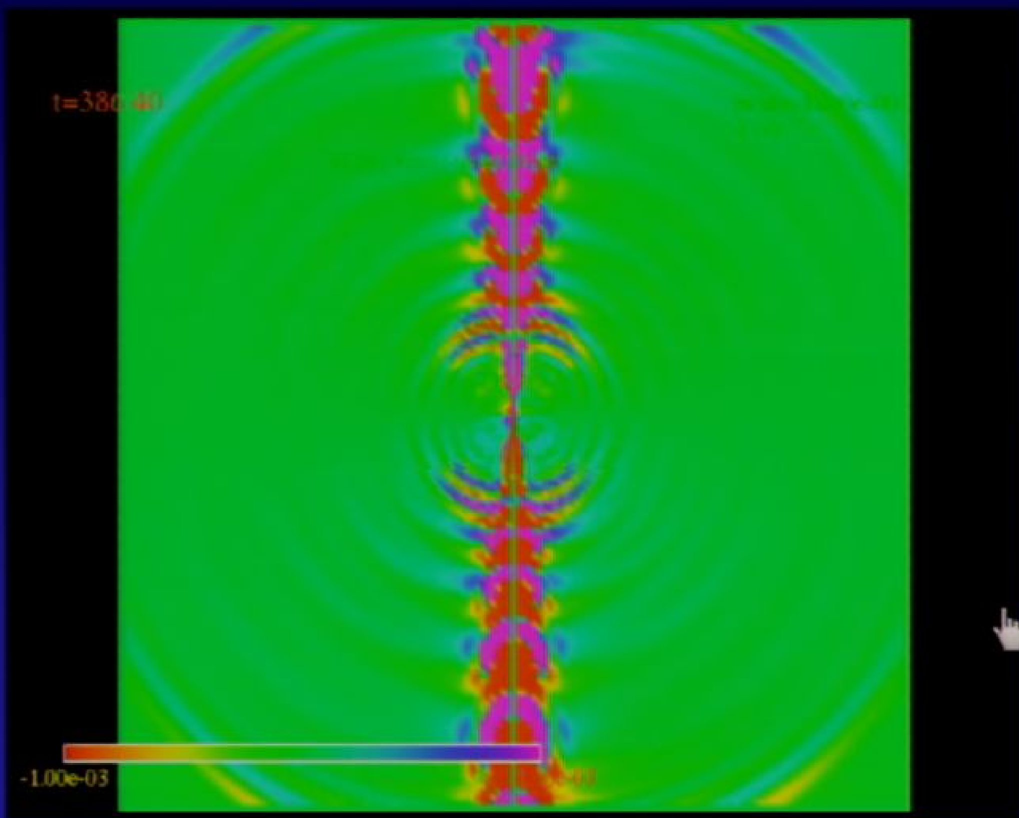

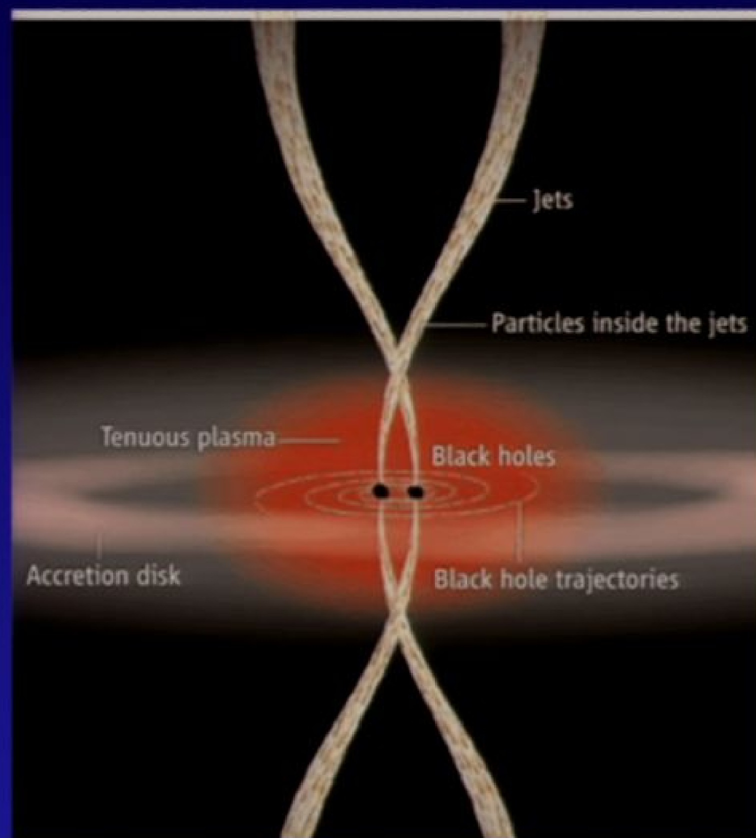

Gravitational Waves in General Relativity

- ***References (in addition to standard text-book accounts)***
  - *Flanagan & Hughes, qr-qc/0501041*
  - *Buonanno, arXiv:0709.4682[gr-qc]*

Black hole formation at the LHC and in the atmosphere?

large extra dimension scenarios [*N. Arkani-Hamed , S. Dimopoulos & G.R. Dvali, PLB429:263-272; L. Randall & R. Sundrum, PRL.83:3370-3373*] suggest the true Planck scale can be very different from what then would be an effective 4-dimensional Planck scale of *10^{19} GeV* calculated from the fundamental constants measured on our 4-D Brane

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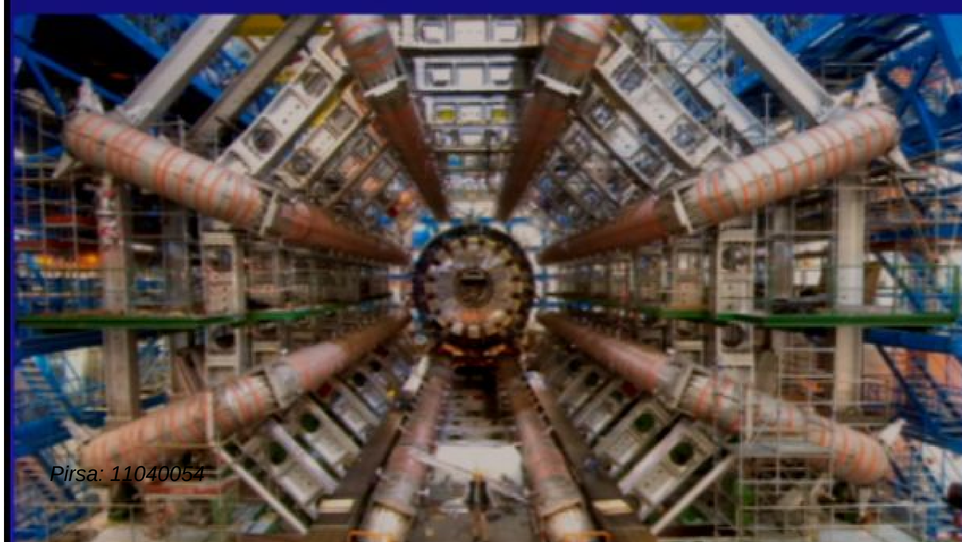
- collisions sufficiently above the Planck scale are expected to be dominated by the gravitational interaction, and arguments suggest that black hole formation will be the most likely result of the two-particle scattering event *[Banks & Fishler hep-th/9906038, Dimopoulos & Landsberg PRL 87 161602 (2001), Feng & Shapere, PRL 88 021303 (2002), ...]*

One of the water tanks at the Pierre Auger Observator



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- these black holes will be small and decay rapidly via

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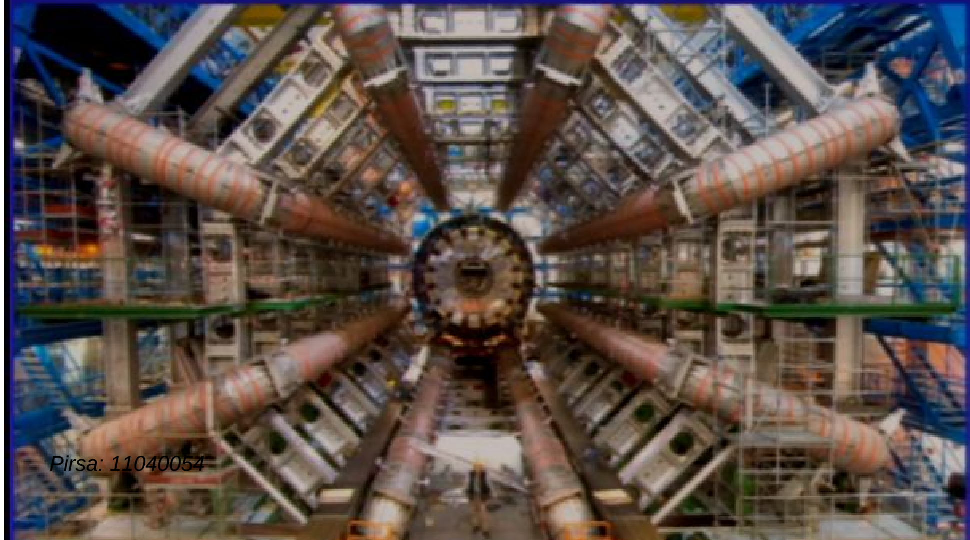

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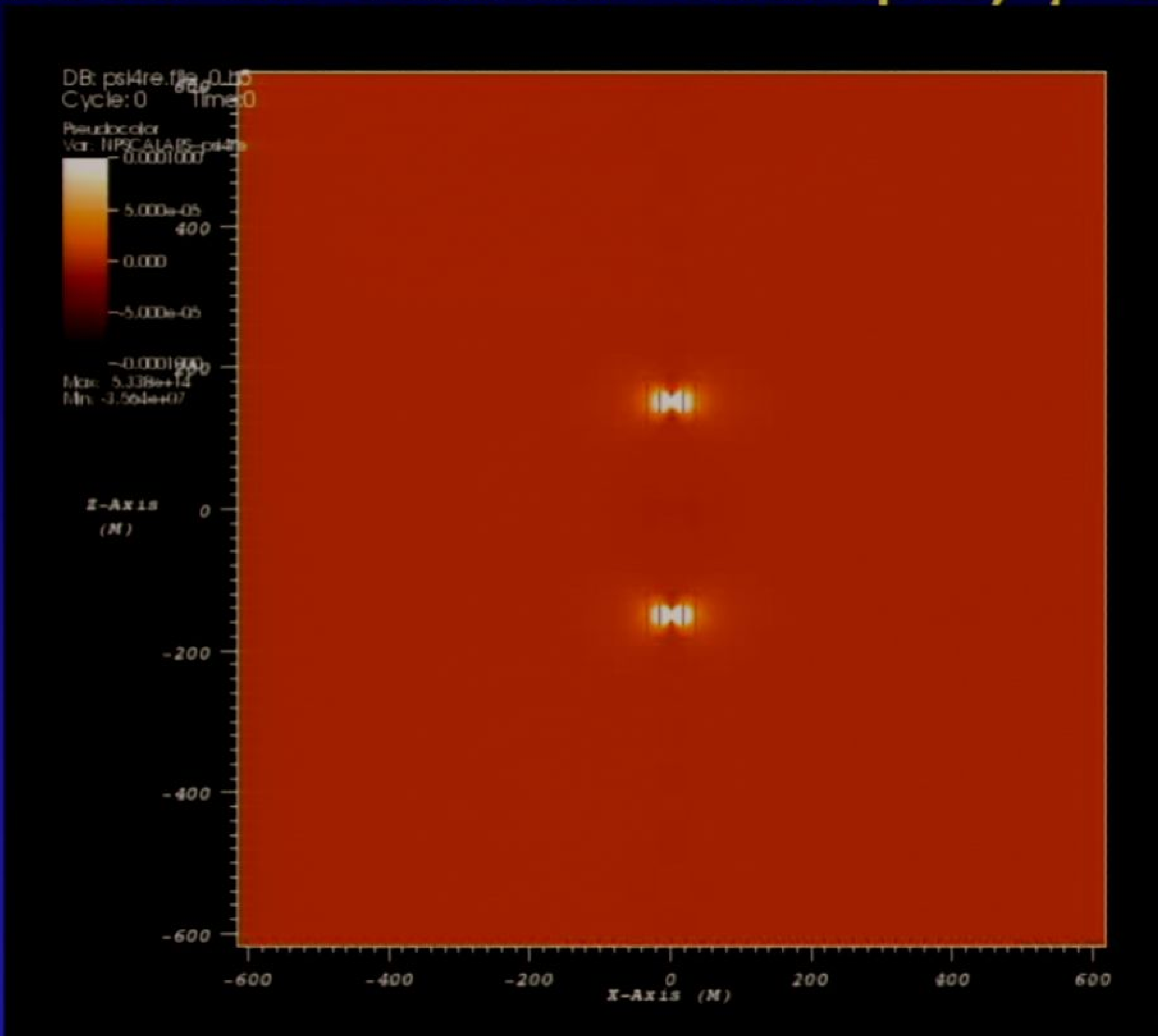
head-on collision example, γ=2.9



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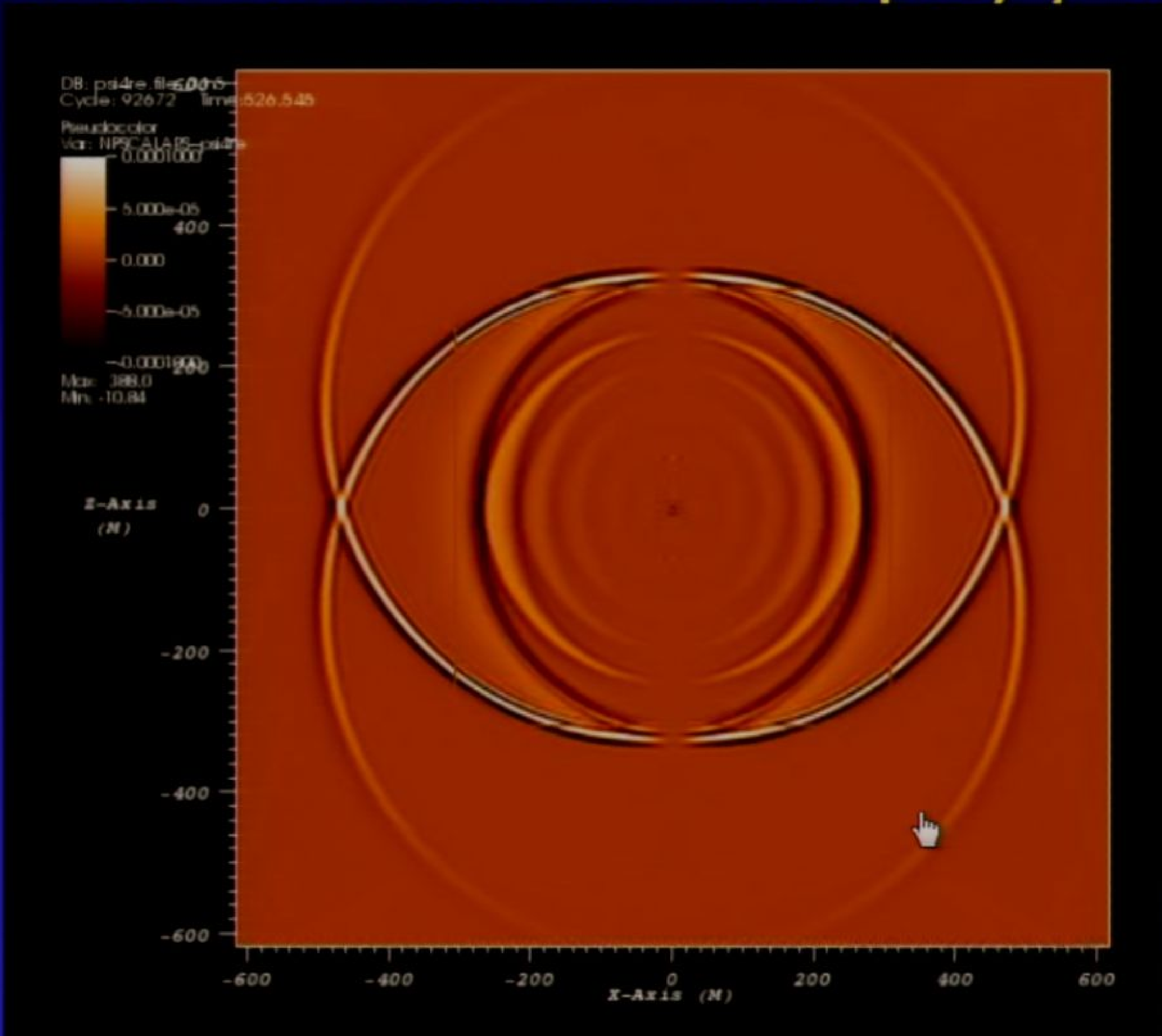

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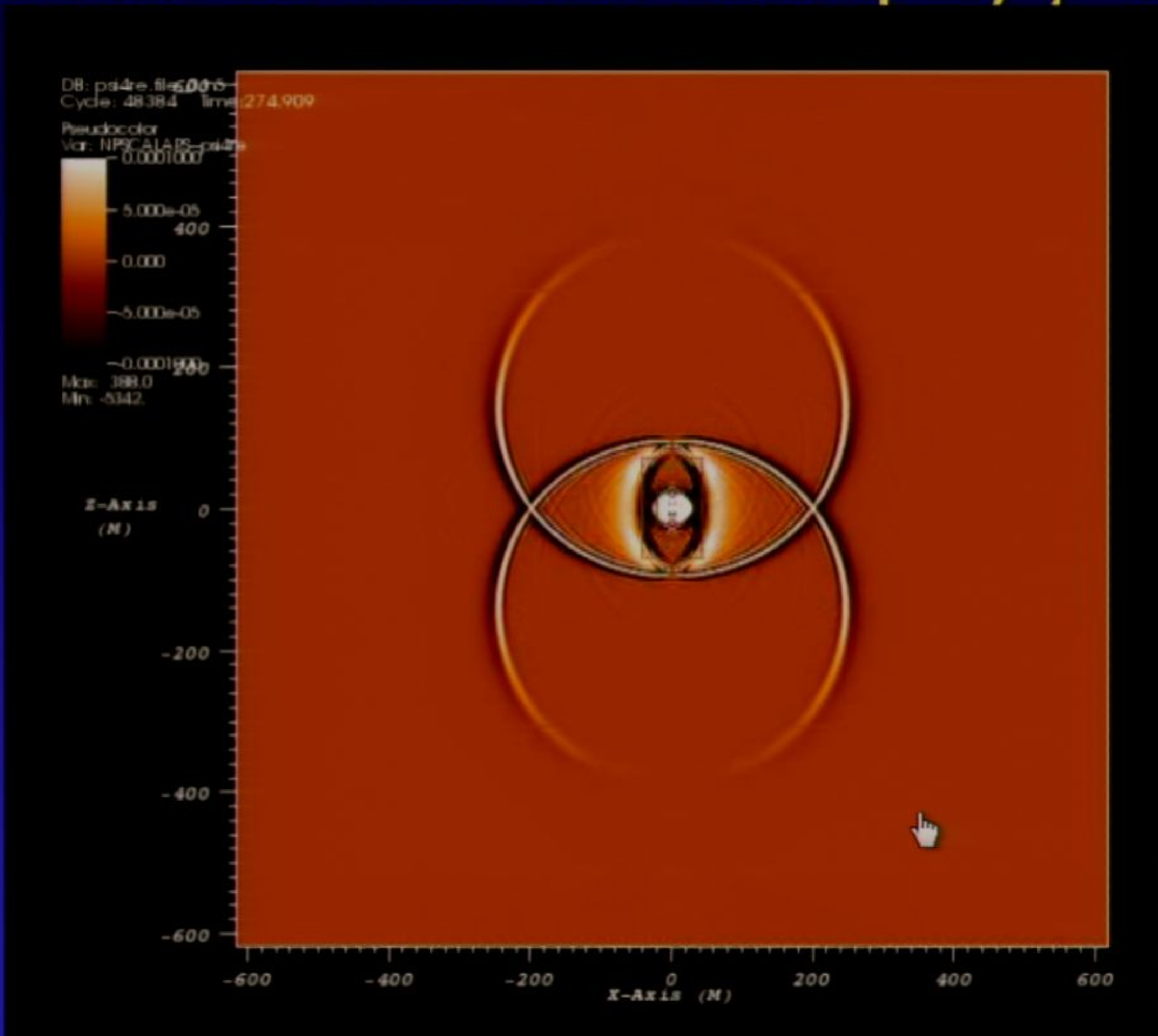
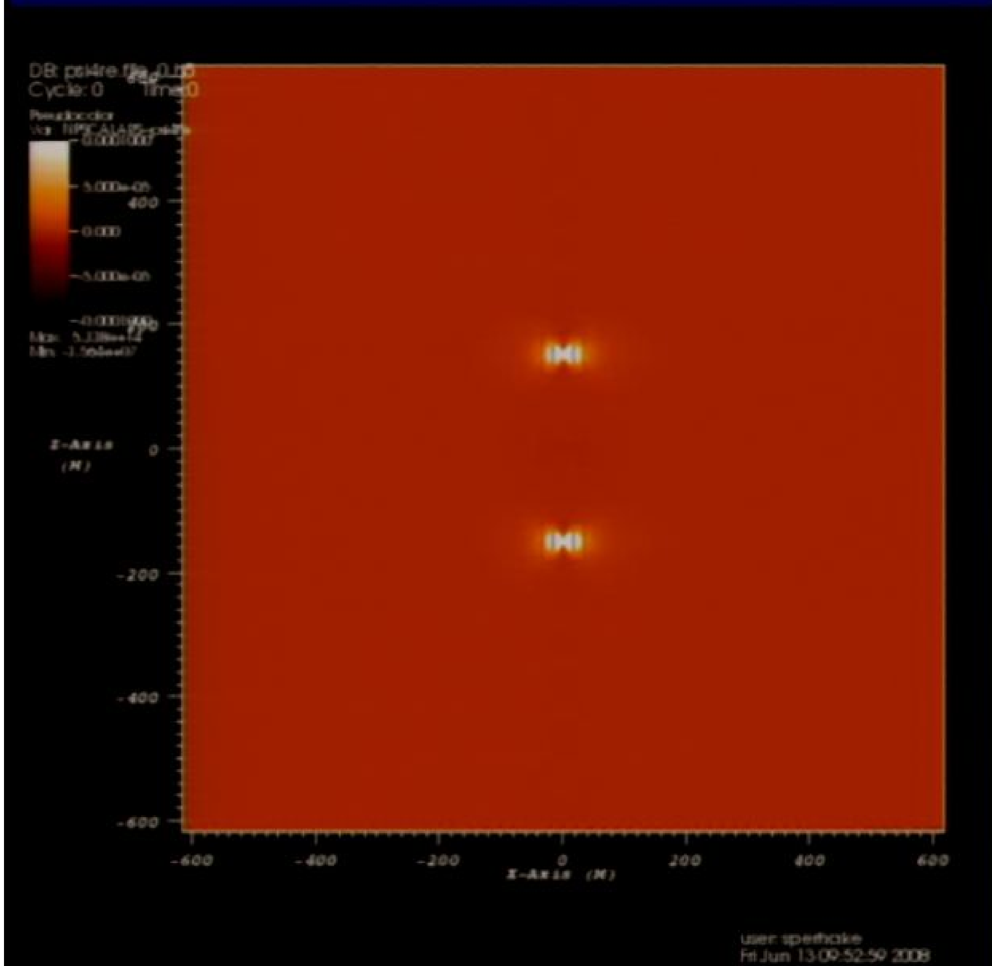


sample BH collisions

From U.Sperhake's BSSN based code



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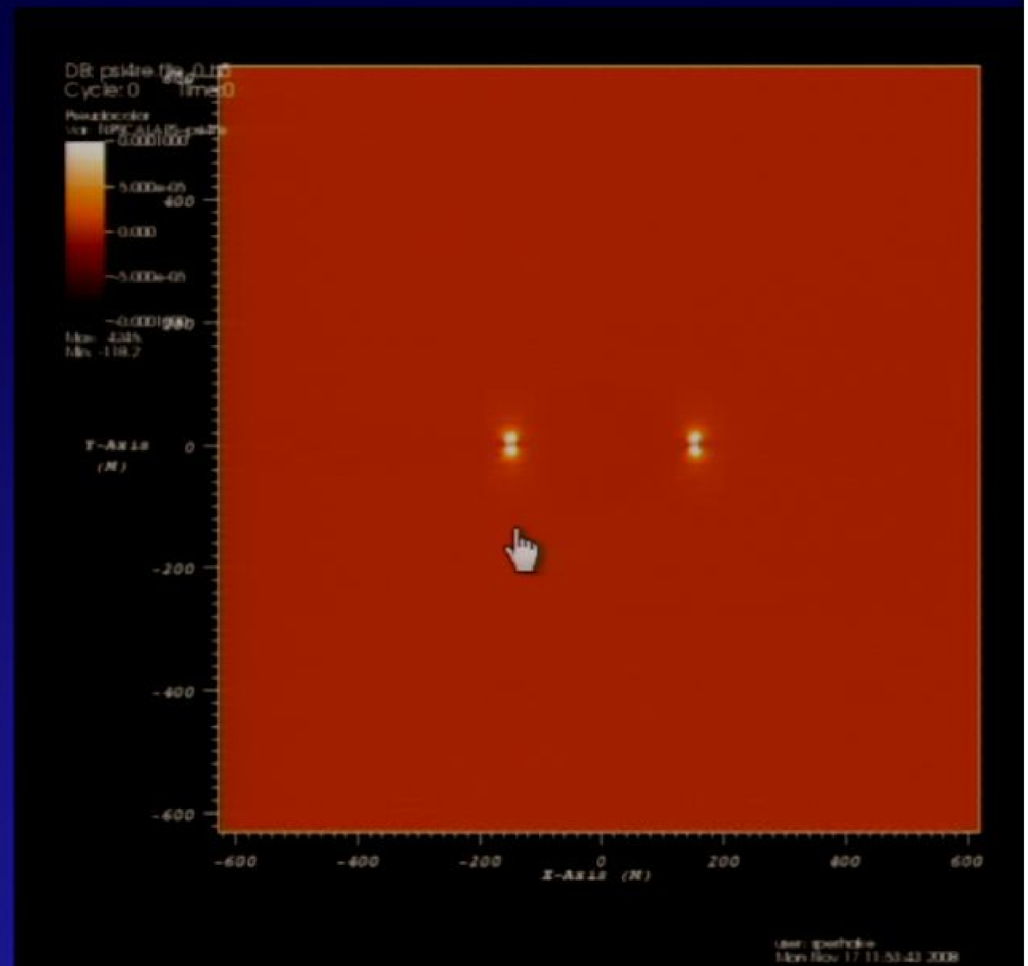

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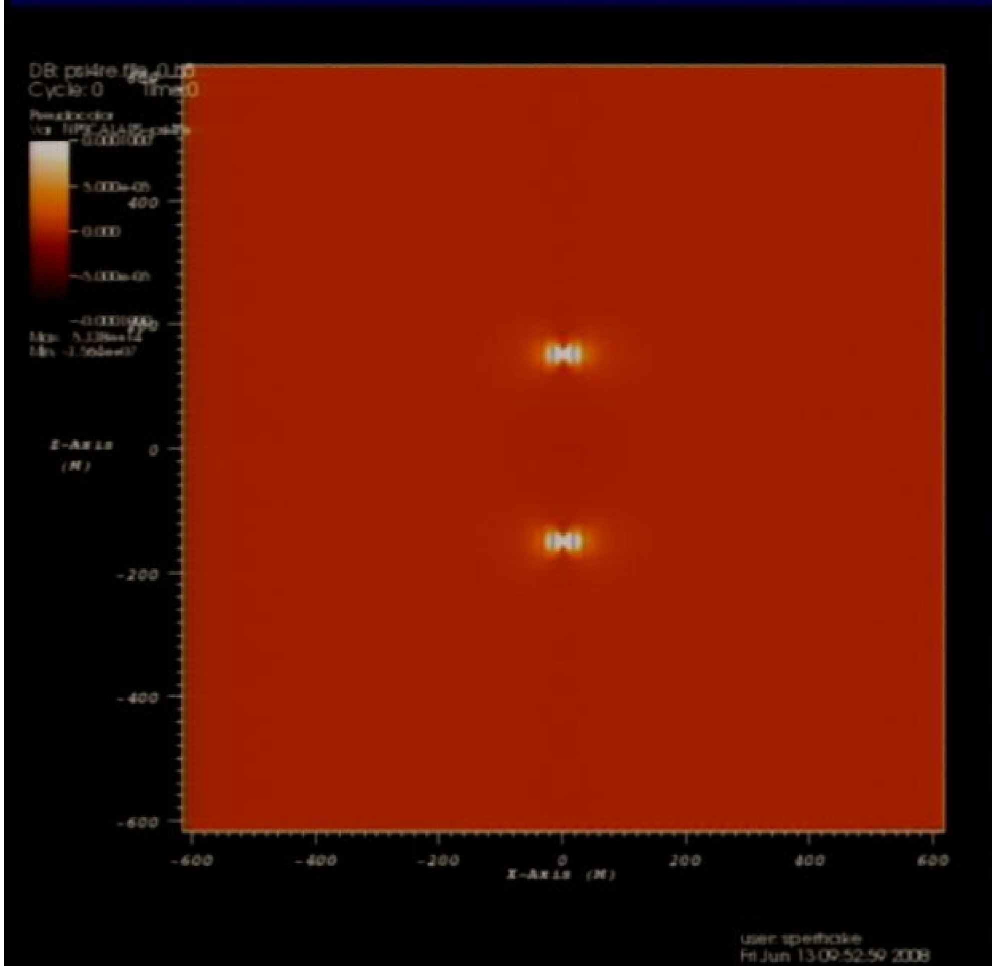

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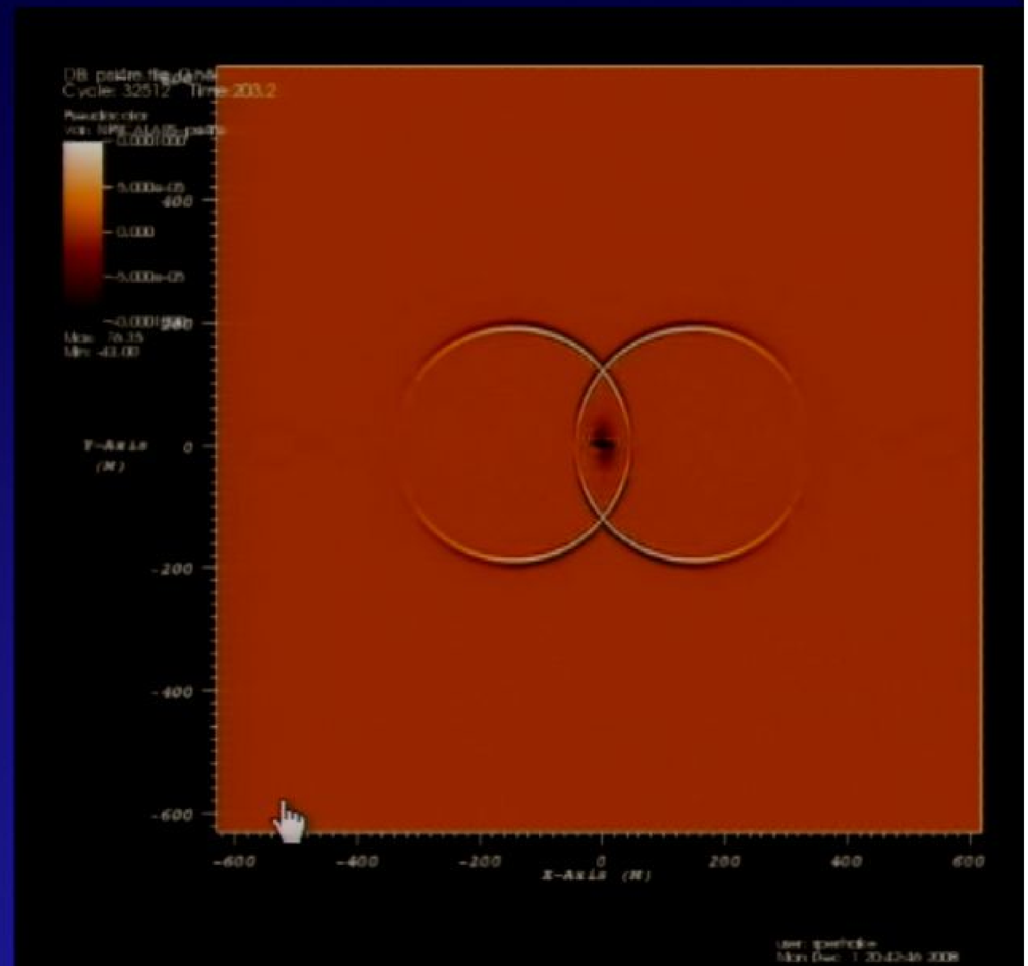

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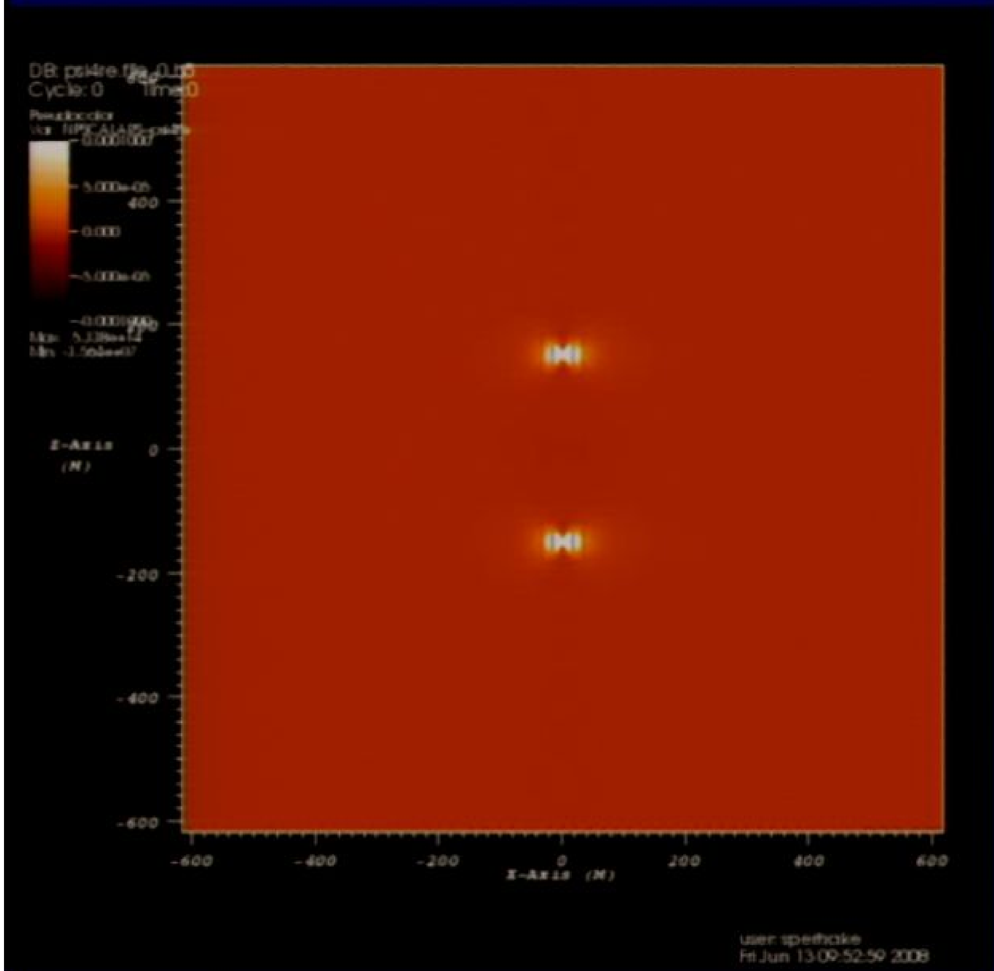

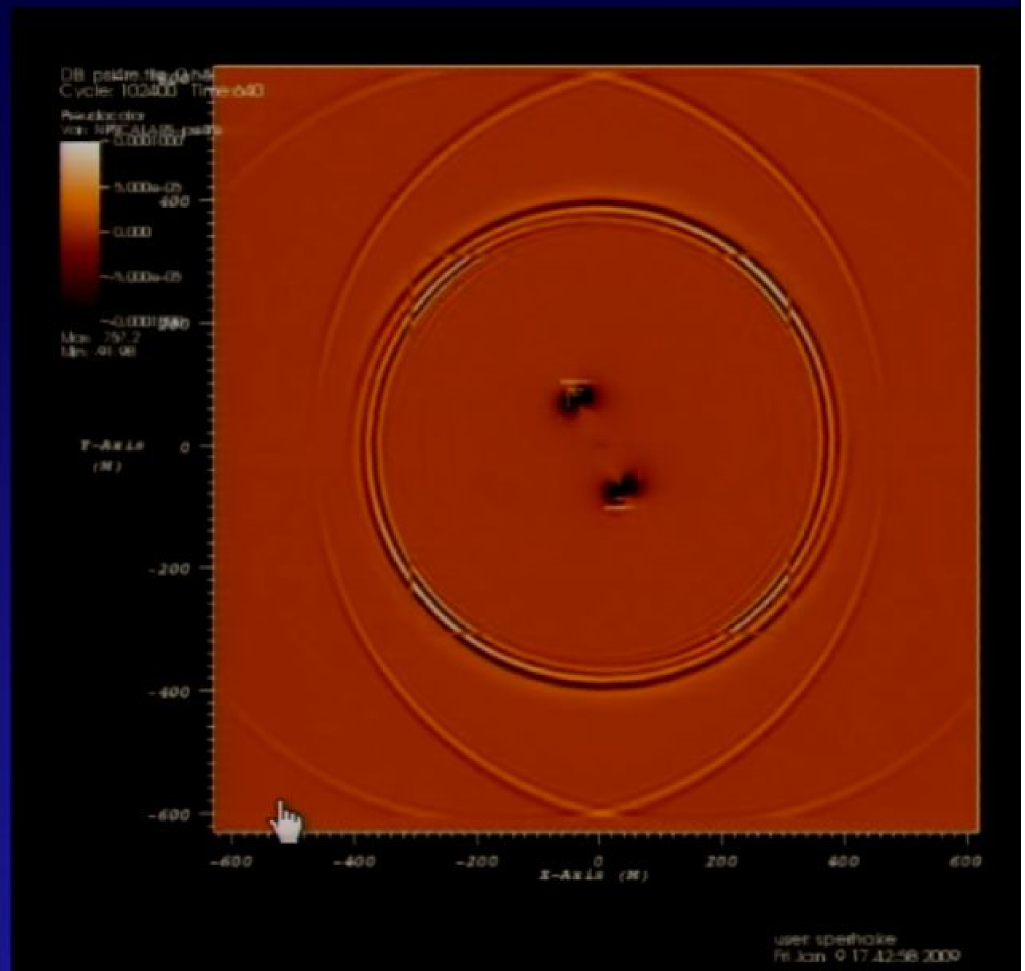

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Critical phenomena in gravitational collapse

- Discovered in 1993 by Choptuik, critical phenomena refers to interesting behavior observed at the *threshold of black hole formation* in gravitational collapse
- The question Choptuik was trying to answer was, "can one form black holes of arbitrarily small mass in scalar field collapse?" *(yes!)*
- In the process he discovered behavior that bears striking resemblance to critical phenomena observed at phase transitions in statistical mechanical systems:
  - power law scaling of order parameters (such as the black hole mass M) near threshold
  - universality of the threshold solution
  - scale invariance of the threshold solution
- Rare example in computational physics where a fundamentally new phenomena was discovered via purely numerical methods
  - Even though original example was in spherical symmetry, required AMR to resolve the exponentially rapid develop of features on small length scales

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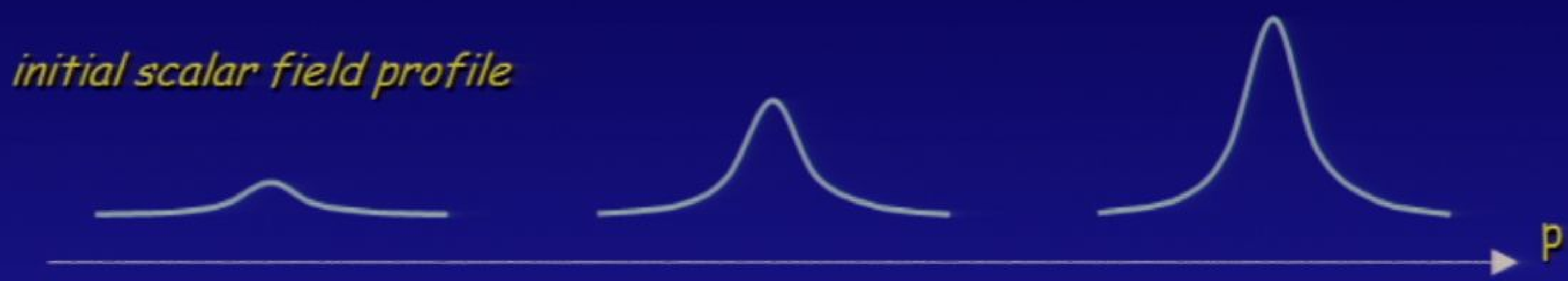
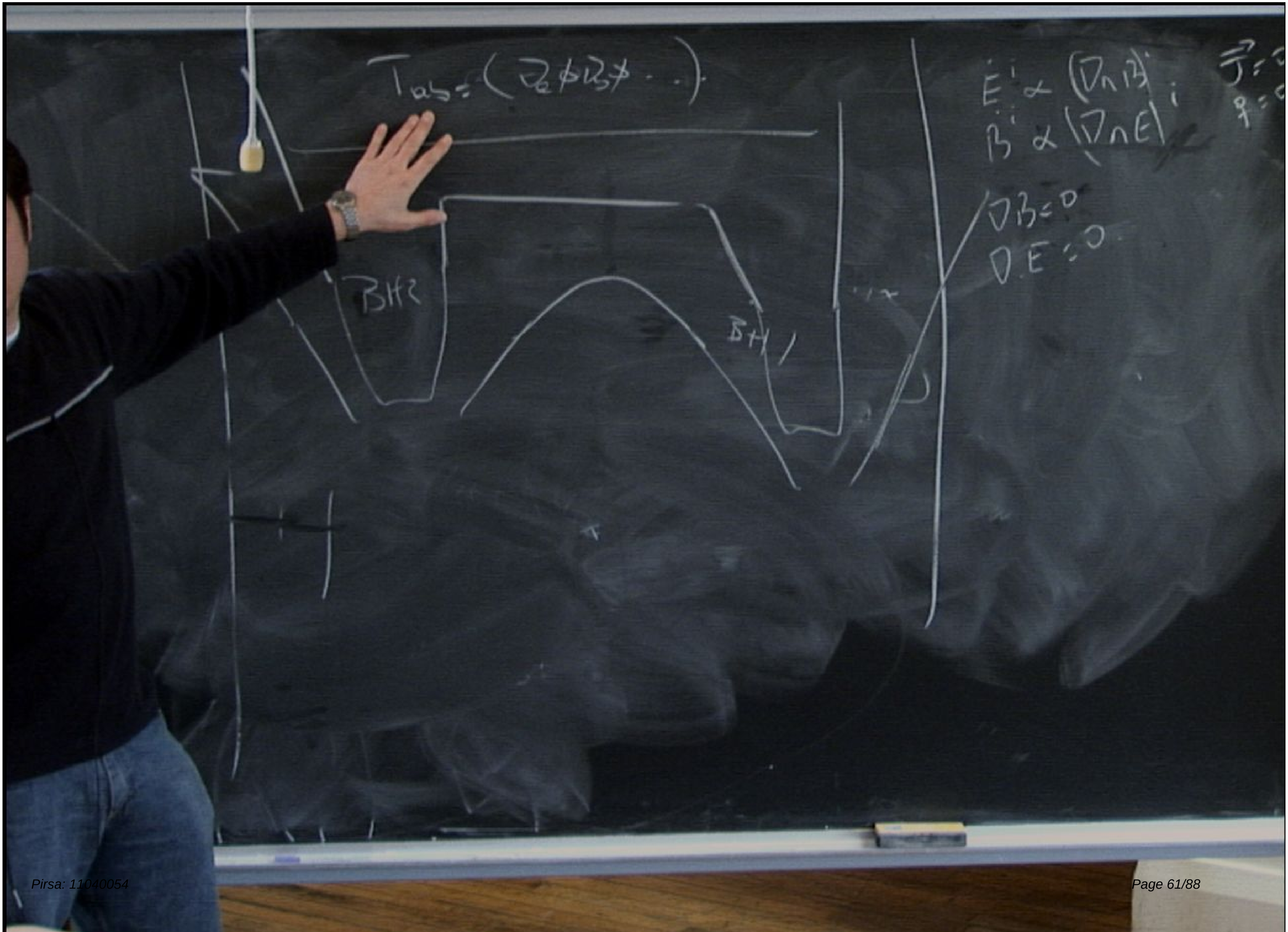
Finding the threshold of black hole formation

- Consider a smooth, one parameter (p) family of initial data, where p is, in some sense, related to the energy density of the initial configuration



- Then for $p<p^*$, evolution will lead to dispersal, while for $p>p^*$ a black hole will form — p^* labels the critical solution for this family

- In a numerical "experiment", p^* can be found via a bisection search


T_ab = (∂_a φ ∂_b φ . . .)
E^i ∝ (∇∧B)^i
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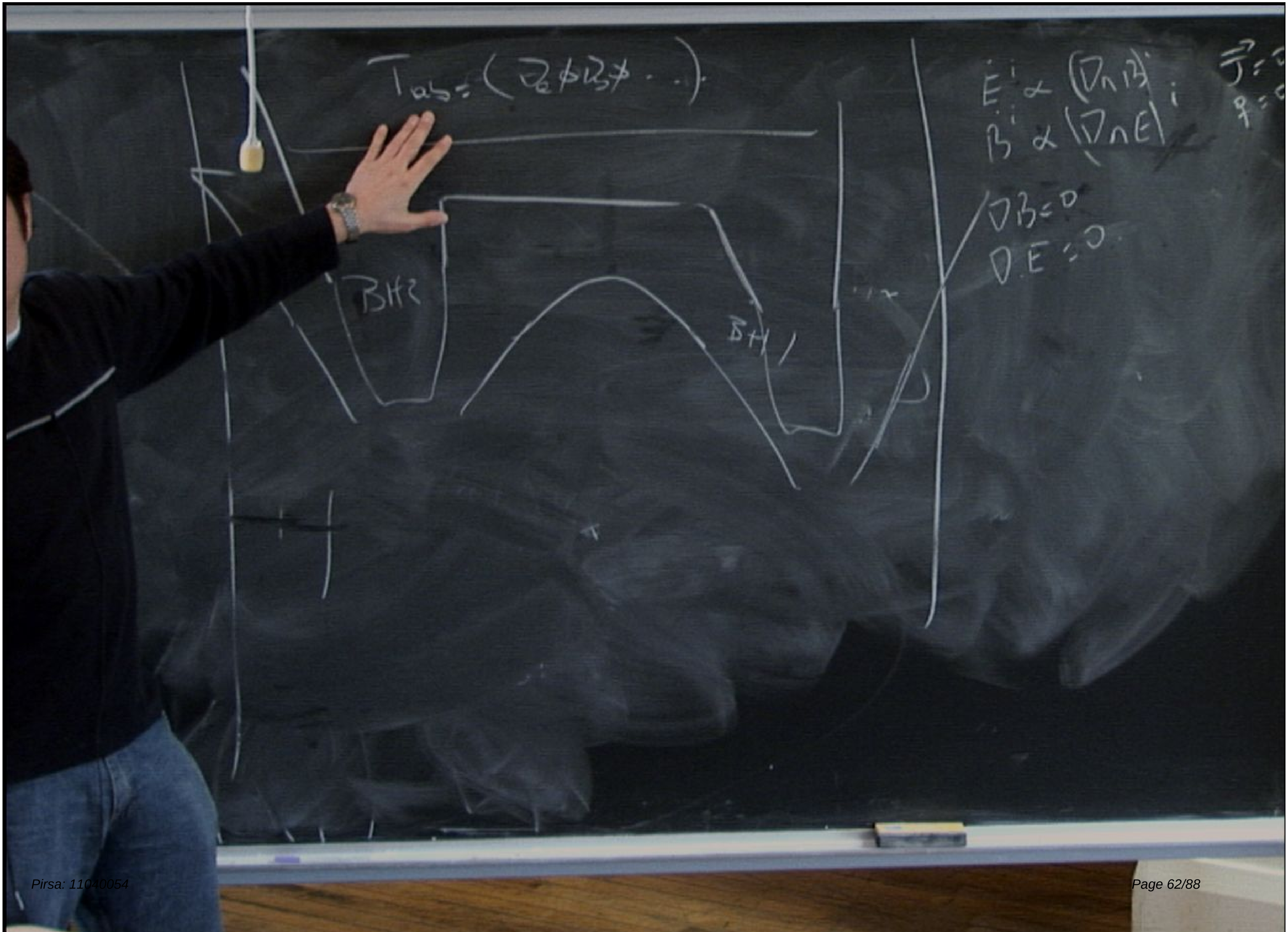
BH2
BH1



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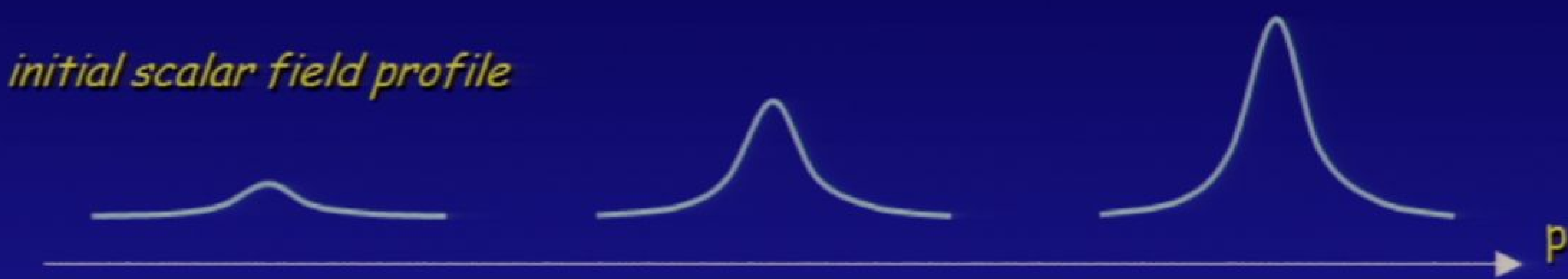

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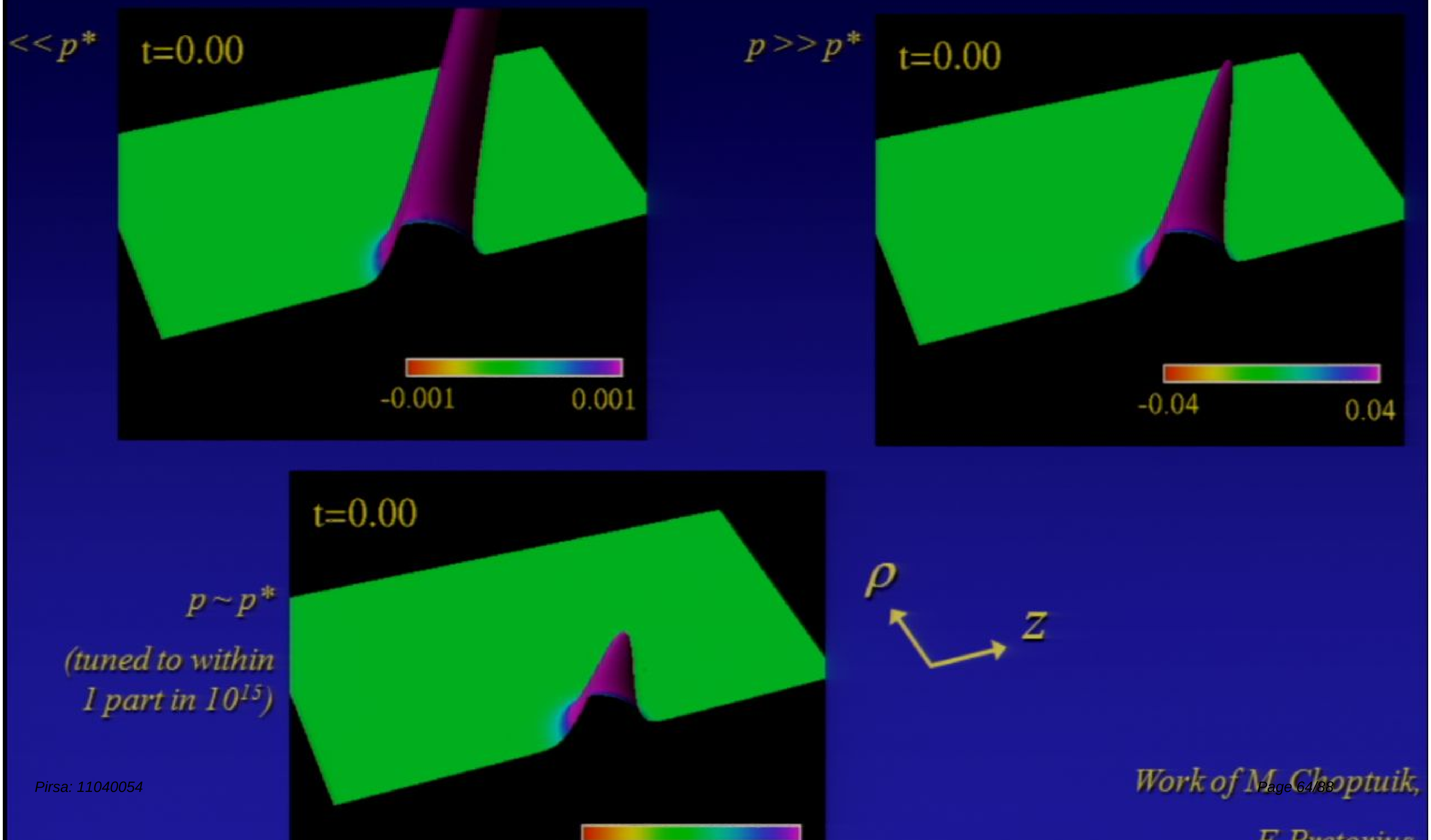
Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data



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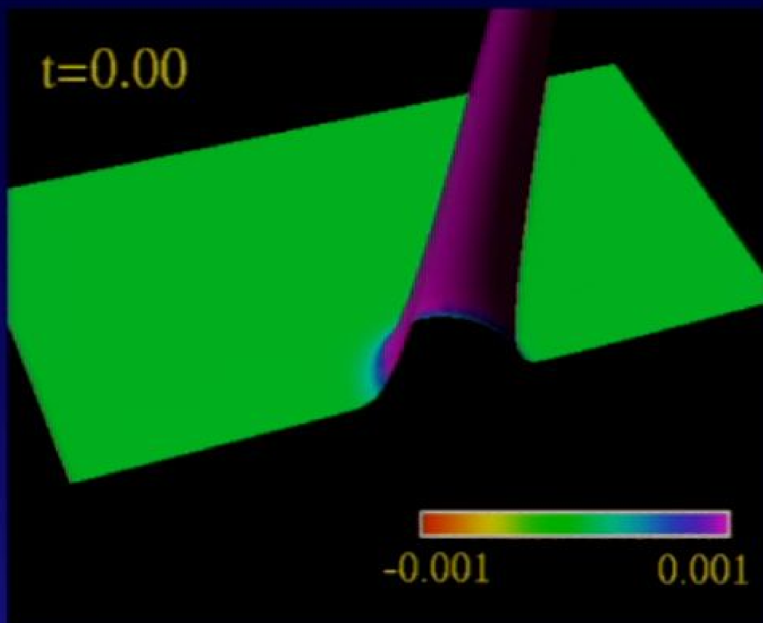

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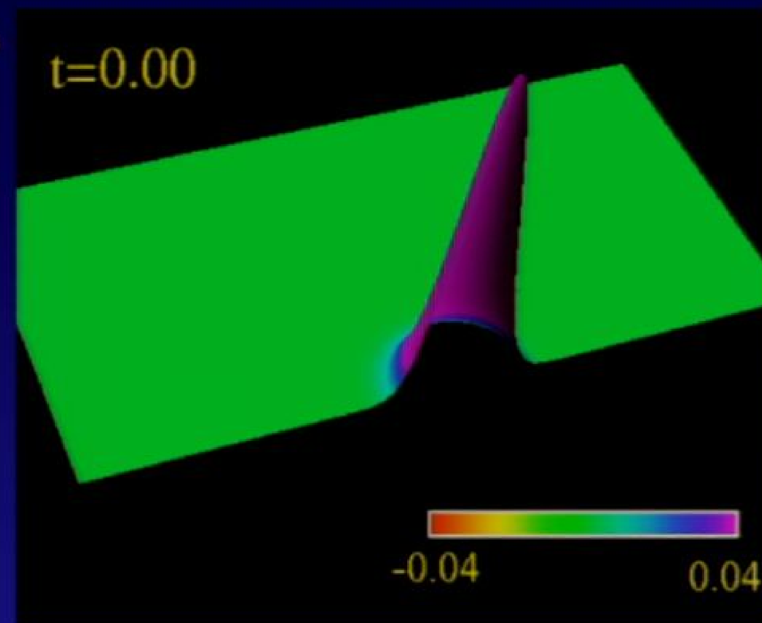

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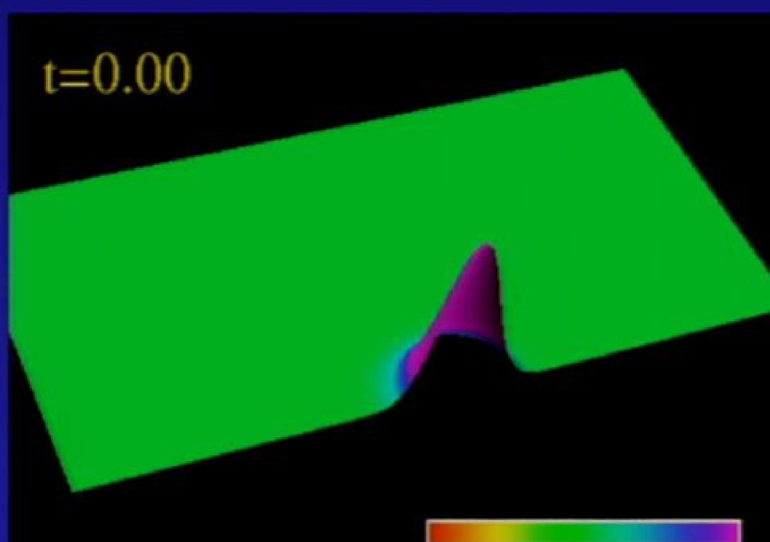
(tuned to within 1 part in 10^{15})



Work of M. Choptuik,

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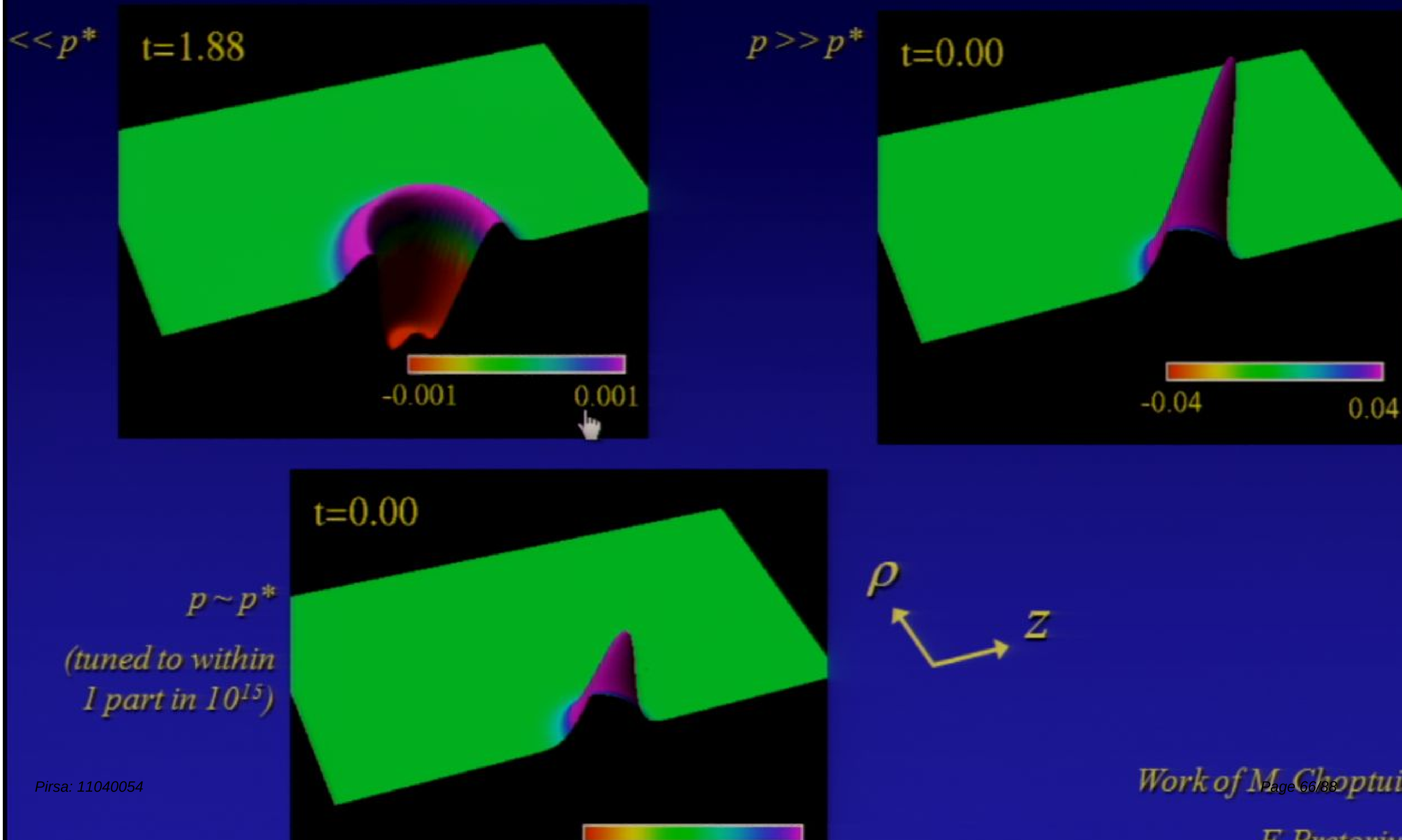

Work of M. Choptuik, F. Pretorius

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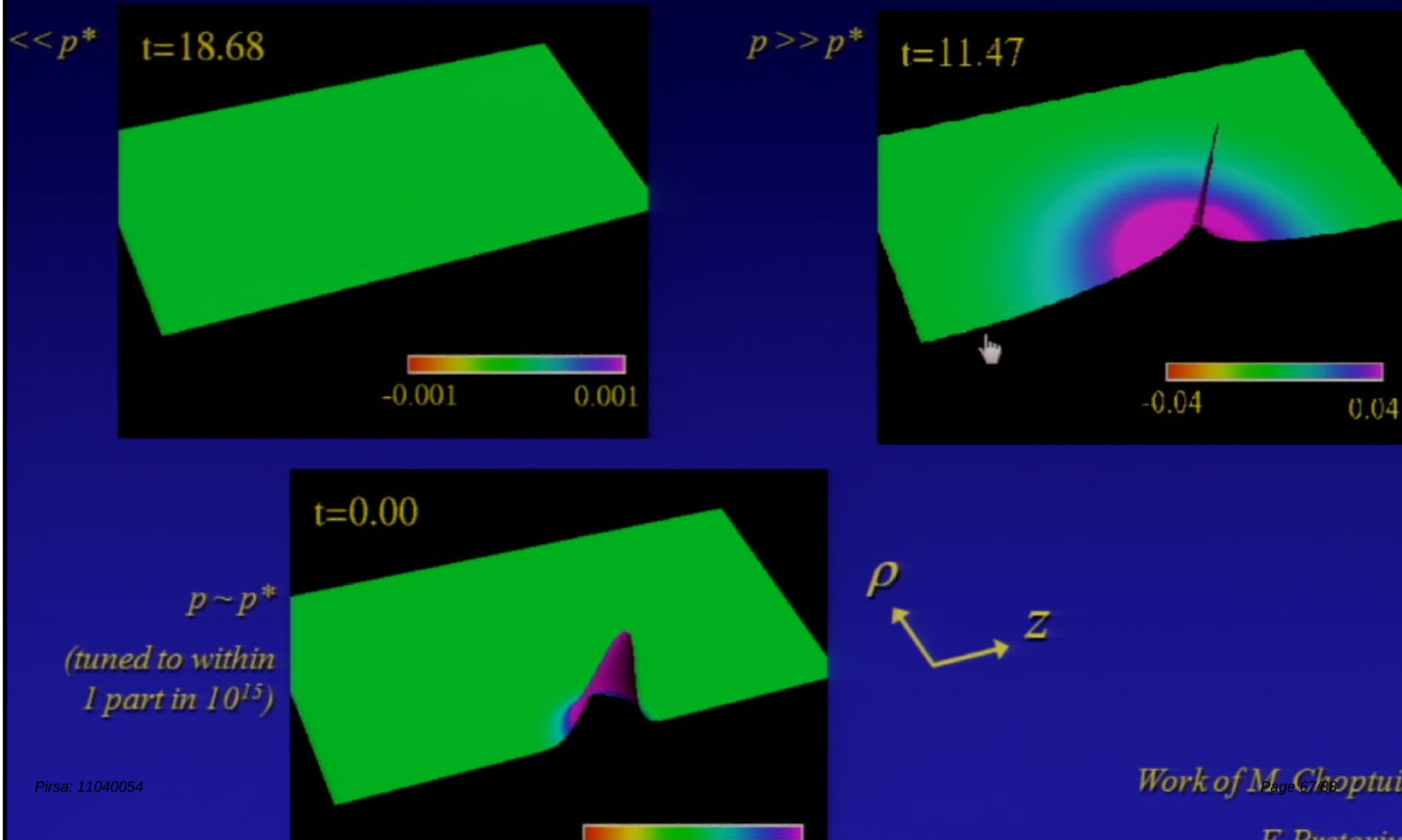
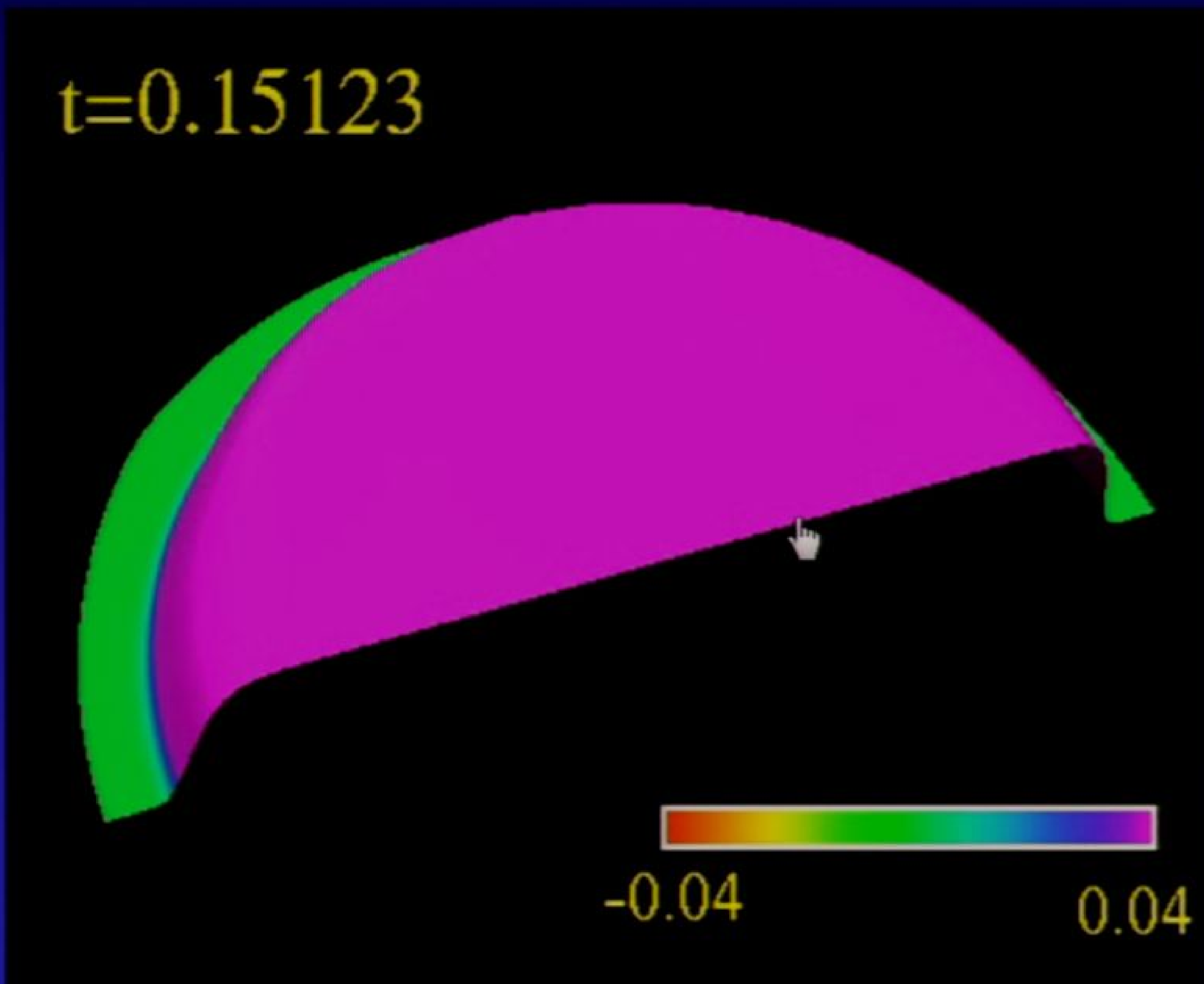


Work of M. Choptuik,

The scalar field threshold solution

Same near critical solution, transformed to spherical polar coordinates, and using logarithmic radial and time coordinates



θ $\ln(r+10^{-7})$

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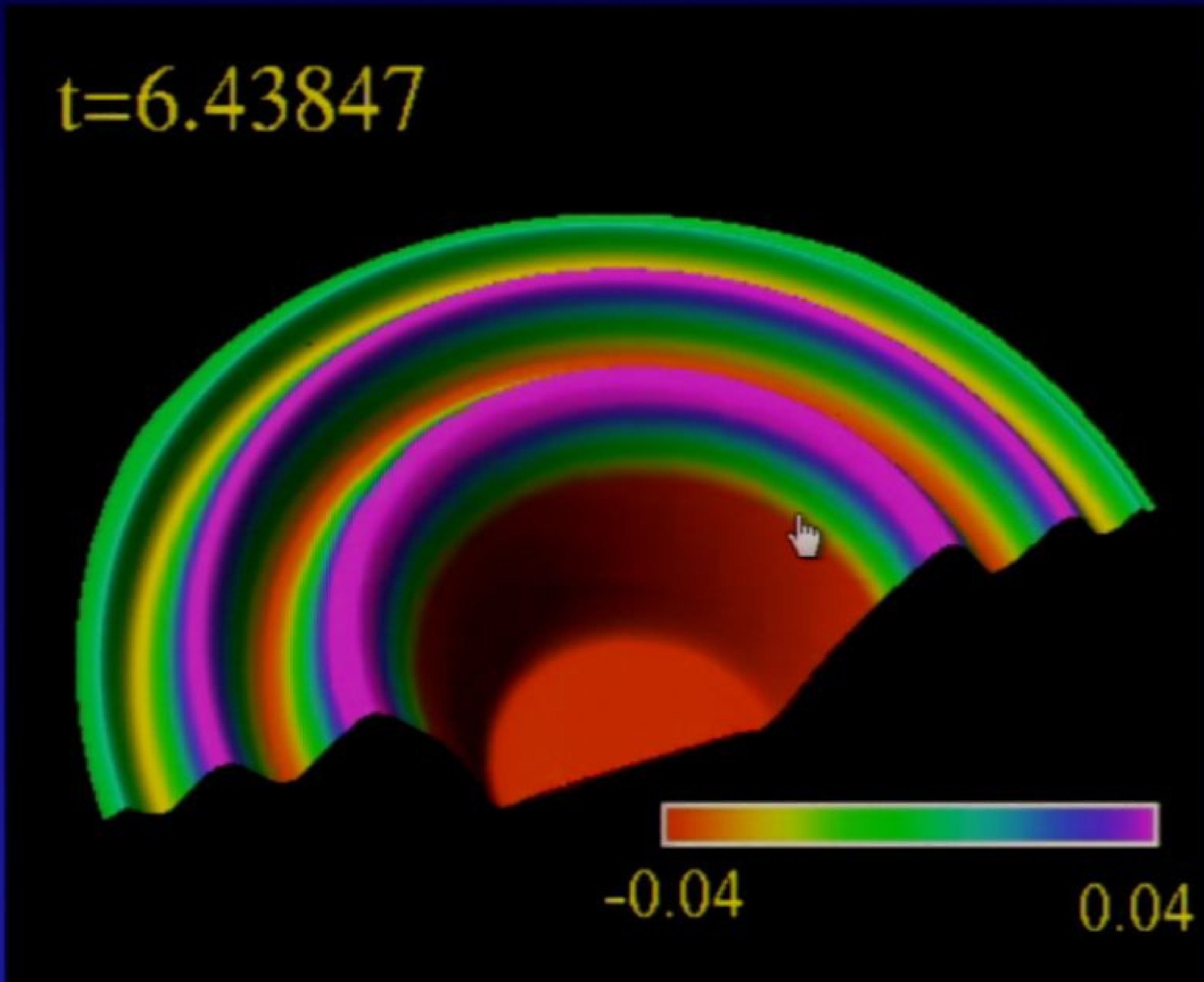

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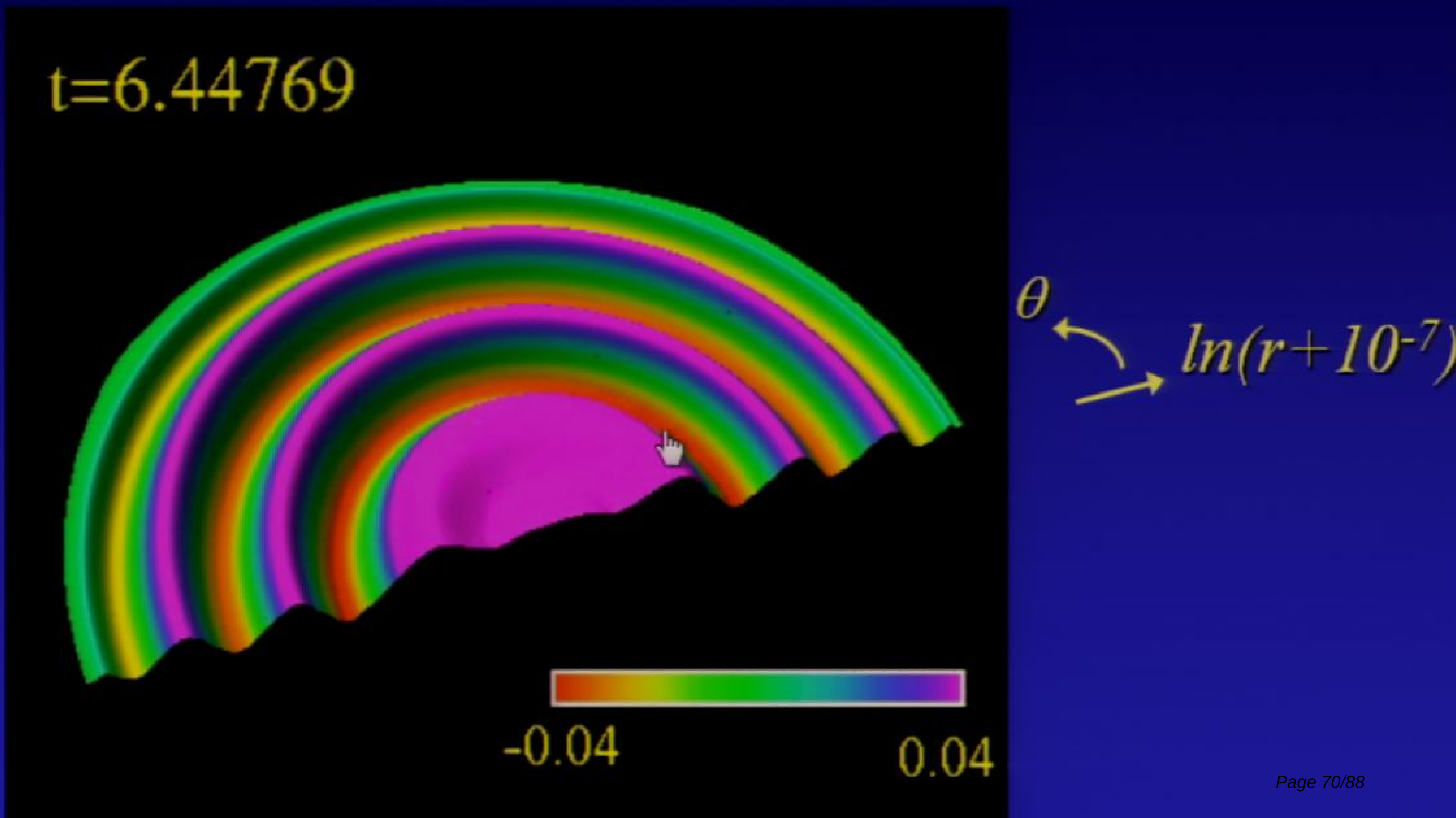
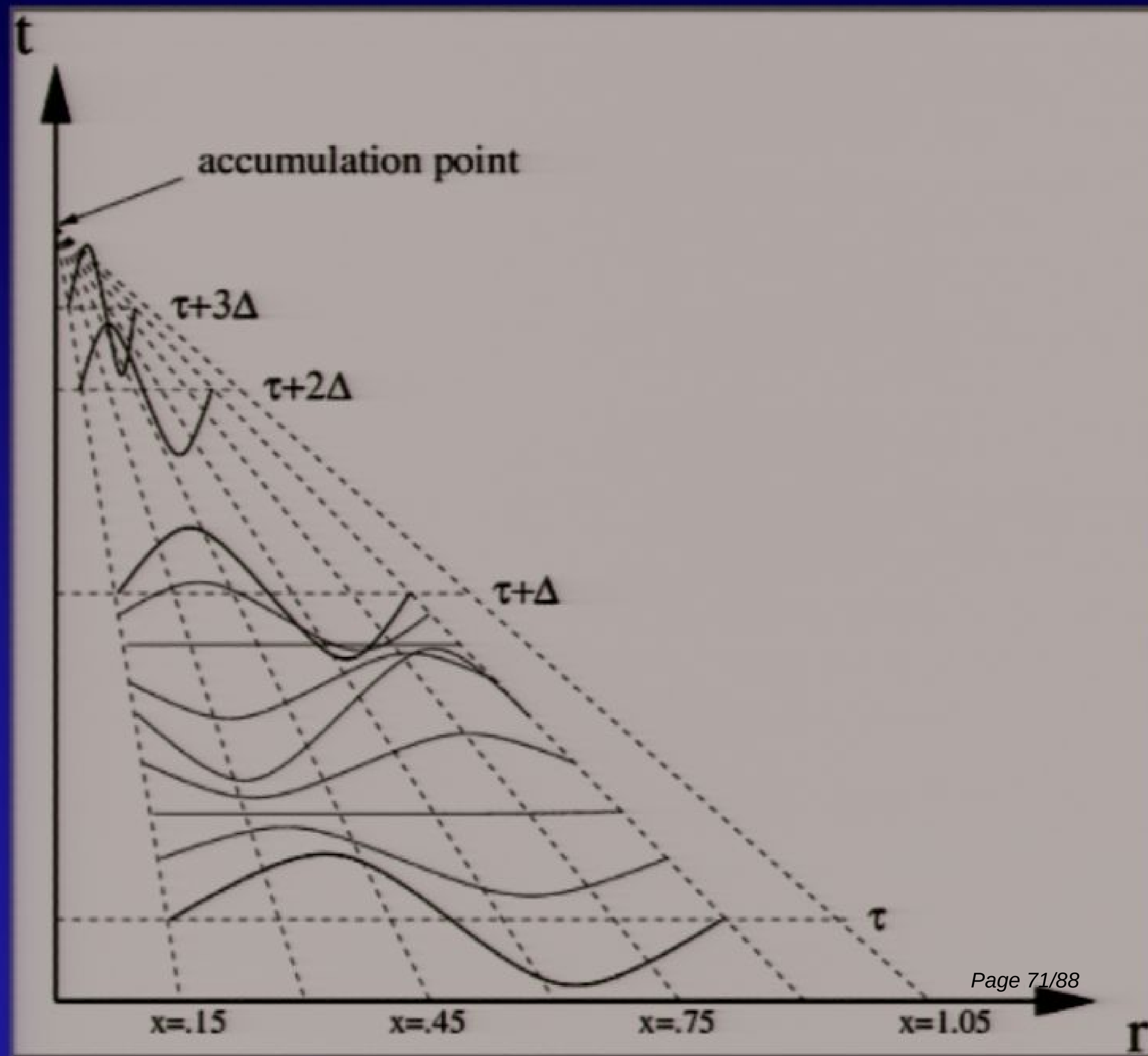


Properties of scalar field critical collapse

- the critical solution (scalar field and spacetime geometry) is spherically symmetric and scale invariant — specifically it is *discretely self-similar*
- example of a discretely self similar function $f(x, \tau)$
  - $f(x, t)$ is periodic in time τ with *echoing period* Δ
  - τ is related to the proper time t measured by a central observer (at radius $r=0$) via

    $$\tau = -\ln(-t)$$

  - x is a *dimensionless* variable, related to r and t via

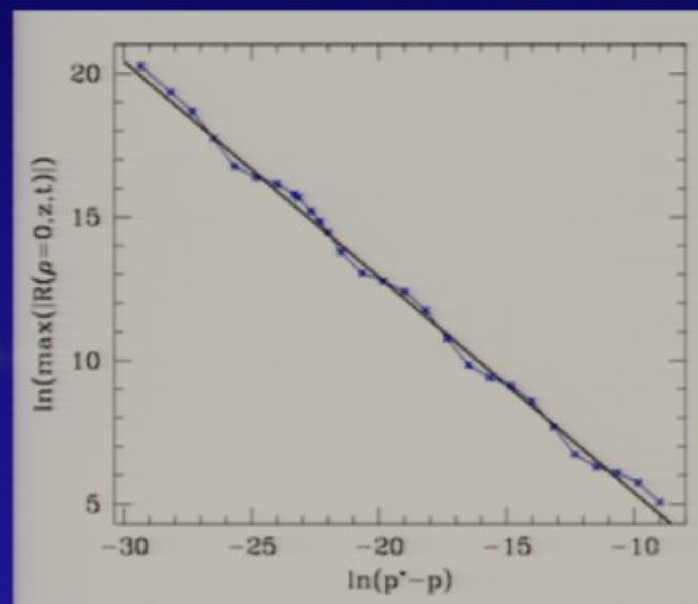


Properties of scalar field critical collapse

- Δ is equal to ~ 3.44
  - each "echo" of the field occurs on a scale 1/30th the previous, and in a 1/30th the time that of the previous echo
- The critical solution is (apparently) universal
  - the *same* solution is approached at threshold regardless of the initial conditions
- *Near* threshold, any length scale arising in the solution satisfies a universal power law relationship (to leading order)

$$M \propto (p - p^*)^{\gamma}, p > p^*$$

$$|R|_{\infty} \propto (p^* - p)^{-2\gamma}, p < p^*$$



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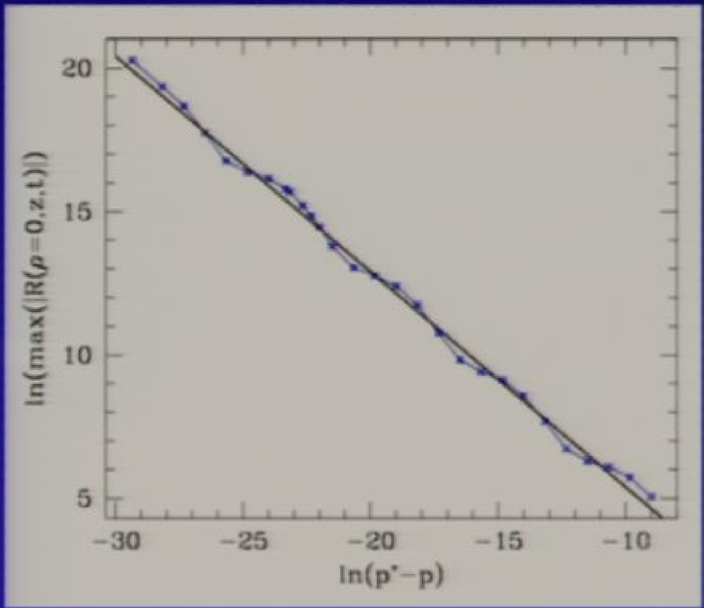

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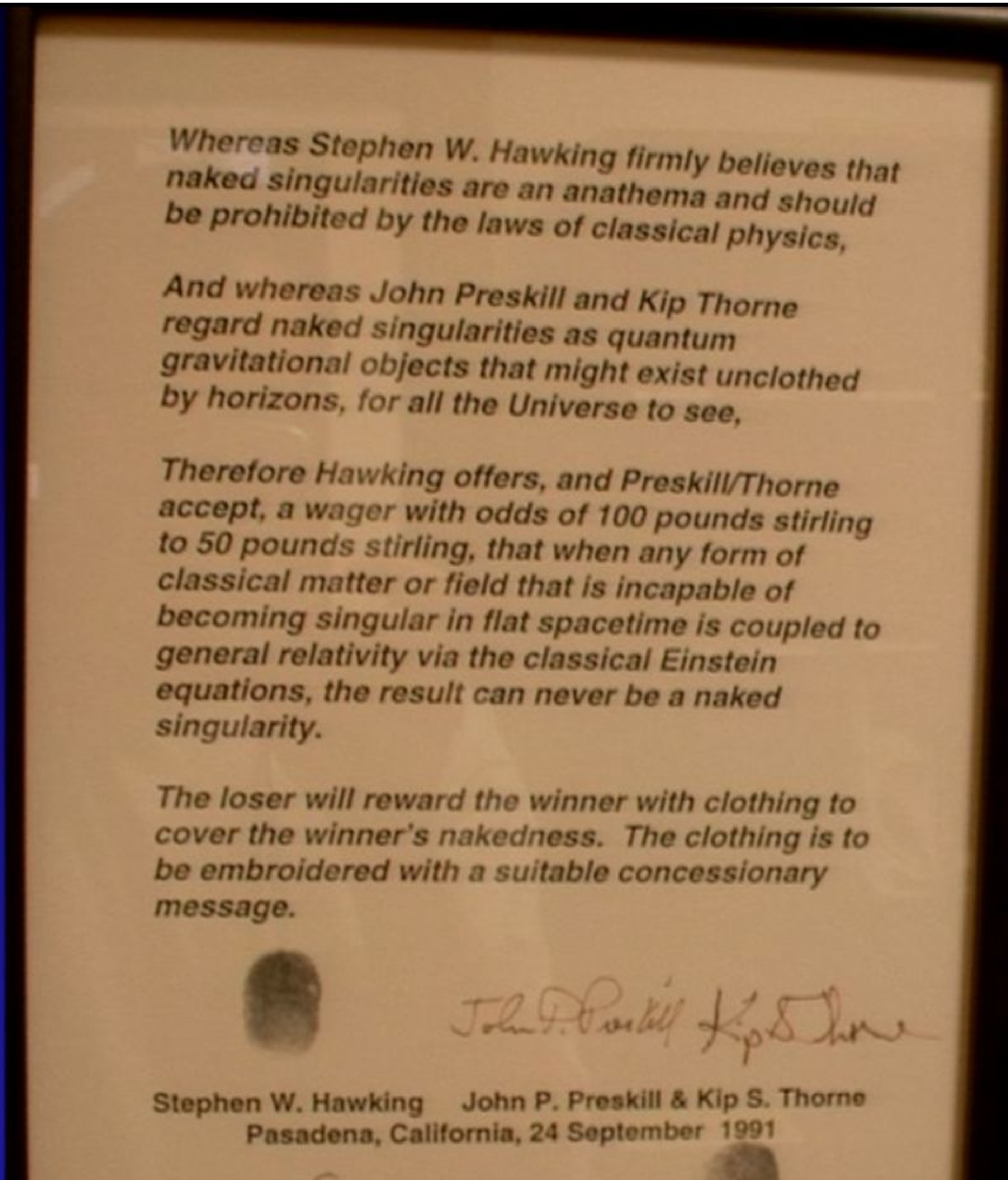


Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds stirling to 50 pounds stirling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

Stephen W. Hawking **John P. Preskill & Kip S. Thorne**

Pasadena, California, 24 September 1991

Beyond spherical symmetry

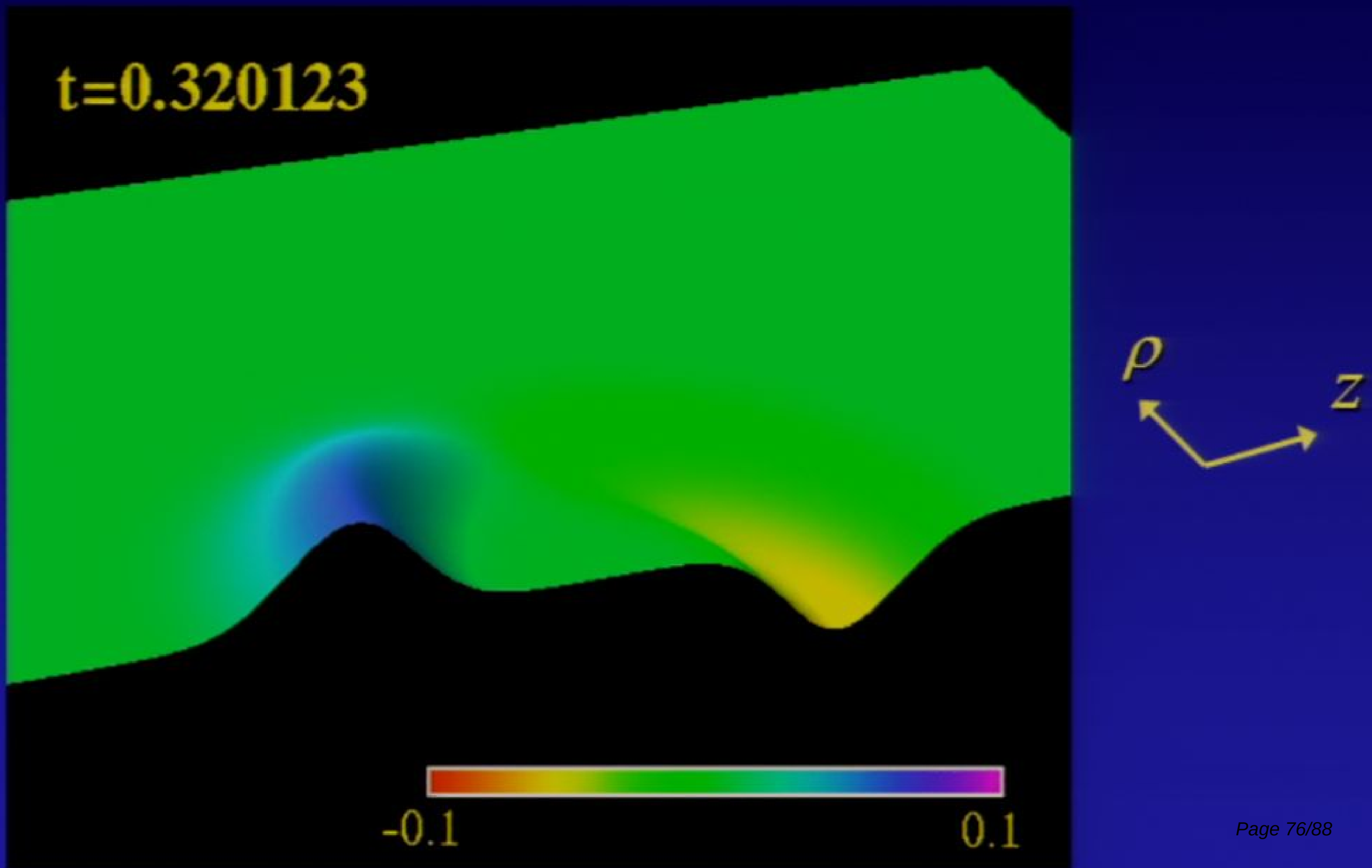
- Example in next slide was a (failed) attempt to "break" the universality conjecture
  - 2D, axisymmetric GR code, solved as a *constrained* evolution
    - 4 elliptic equations (3 constraints, plus an elliptic slicing condition), 3 hyperbolics
  - single CPU, takes from minutes (far from threshold) to days (close to threshold) for a solution

Evolution of plane anti-symmetric initial data

Initial data that is reflection anti-symmetric about z=0 (a conserved symmetry)

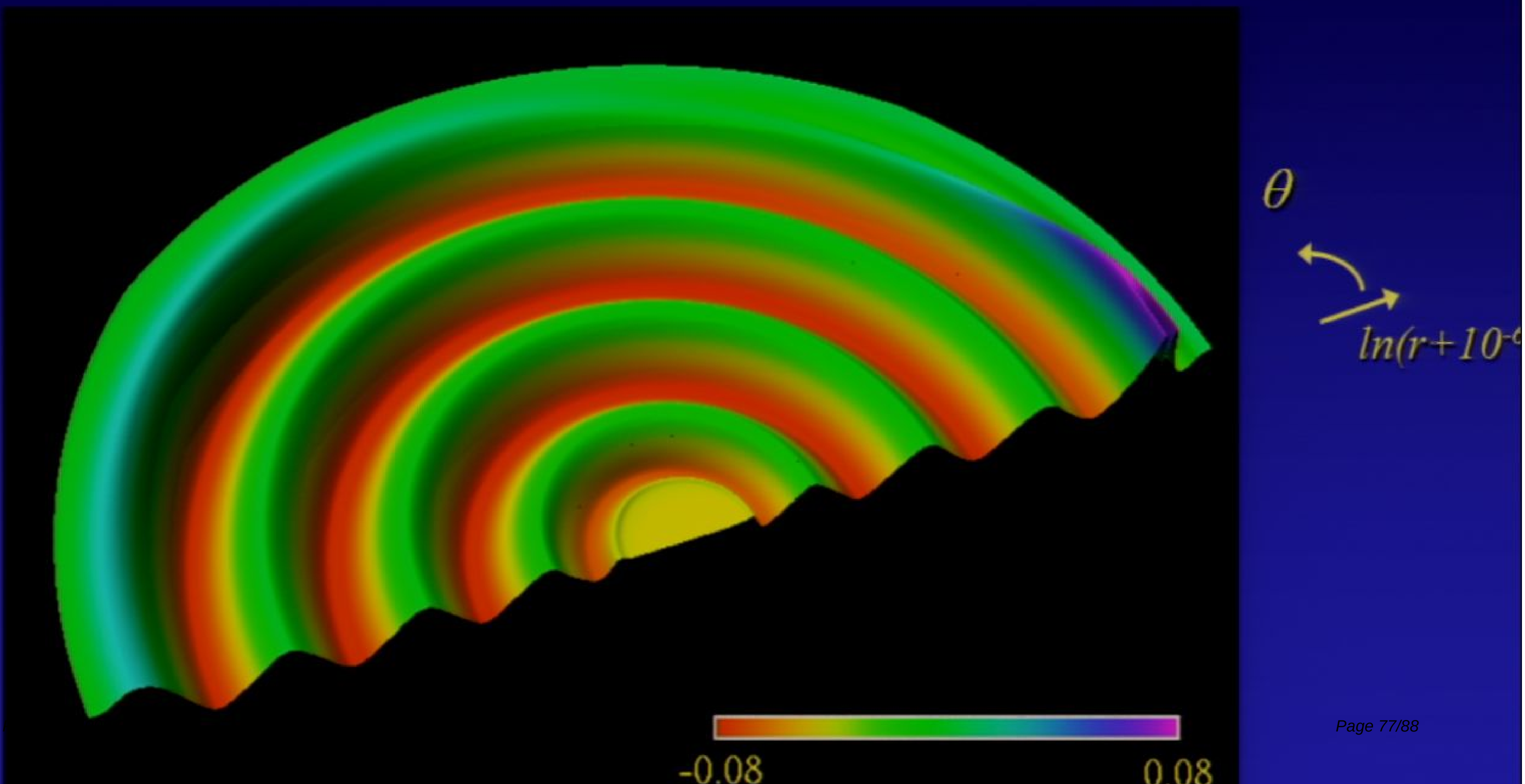
$p \sim p^*$

(tuned to within 1 part in 10^{15})



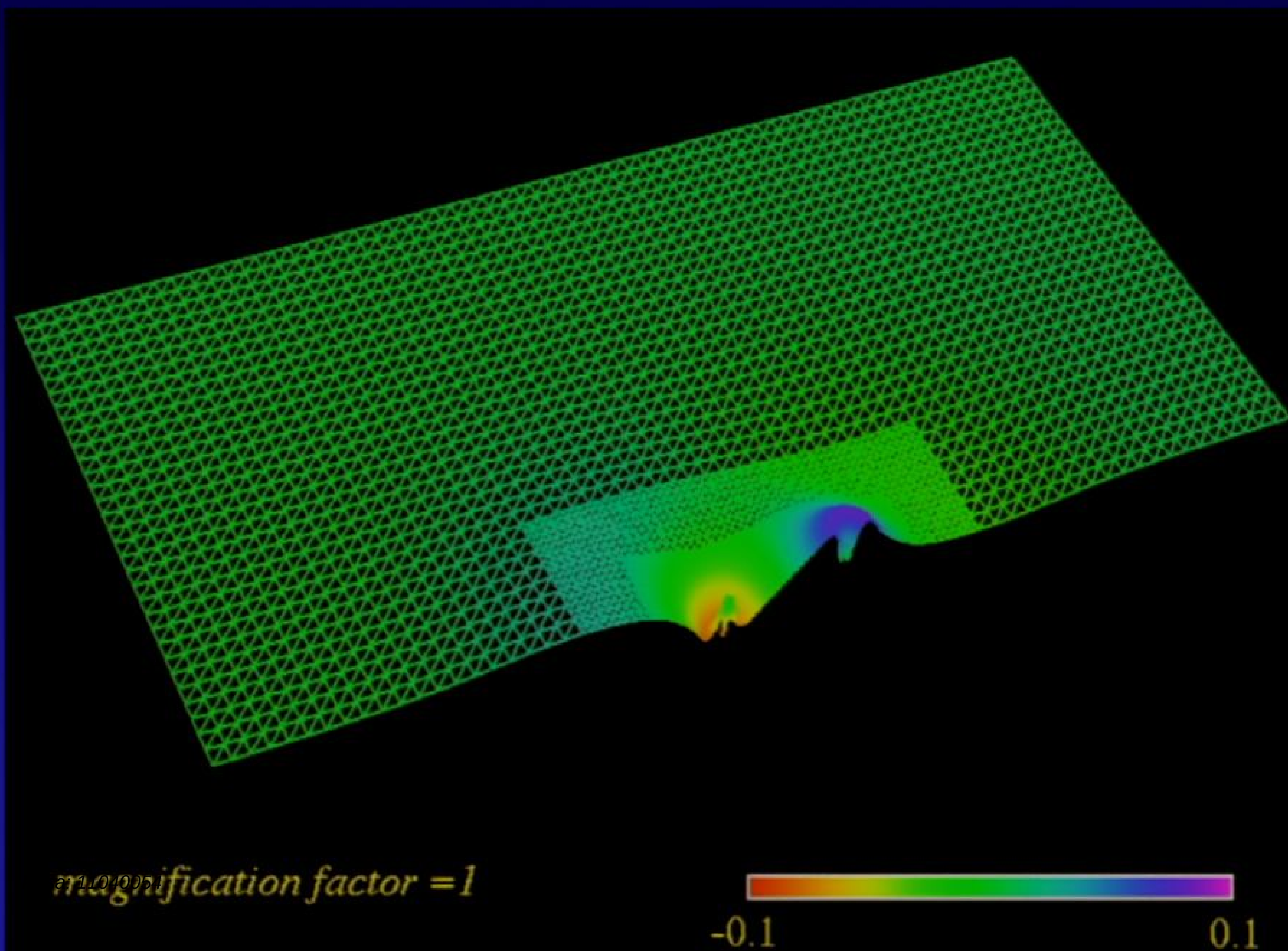
Evolution of plane anti-symmetric initial data

Last frame from the previous animation, transformed to a logarithmic radial coordinate centered about the left most echoer



Aside: AMR grid hierarchy sample

Last frame from the previous animation



25, 2:1 refined levels

(2:1 coarsened in figure)

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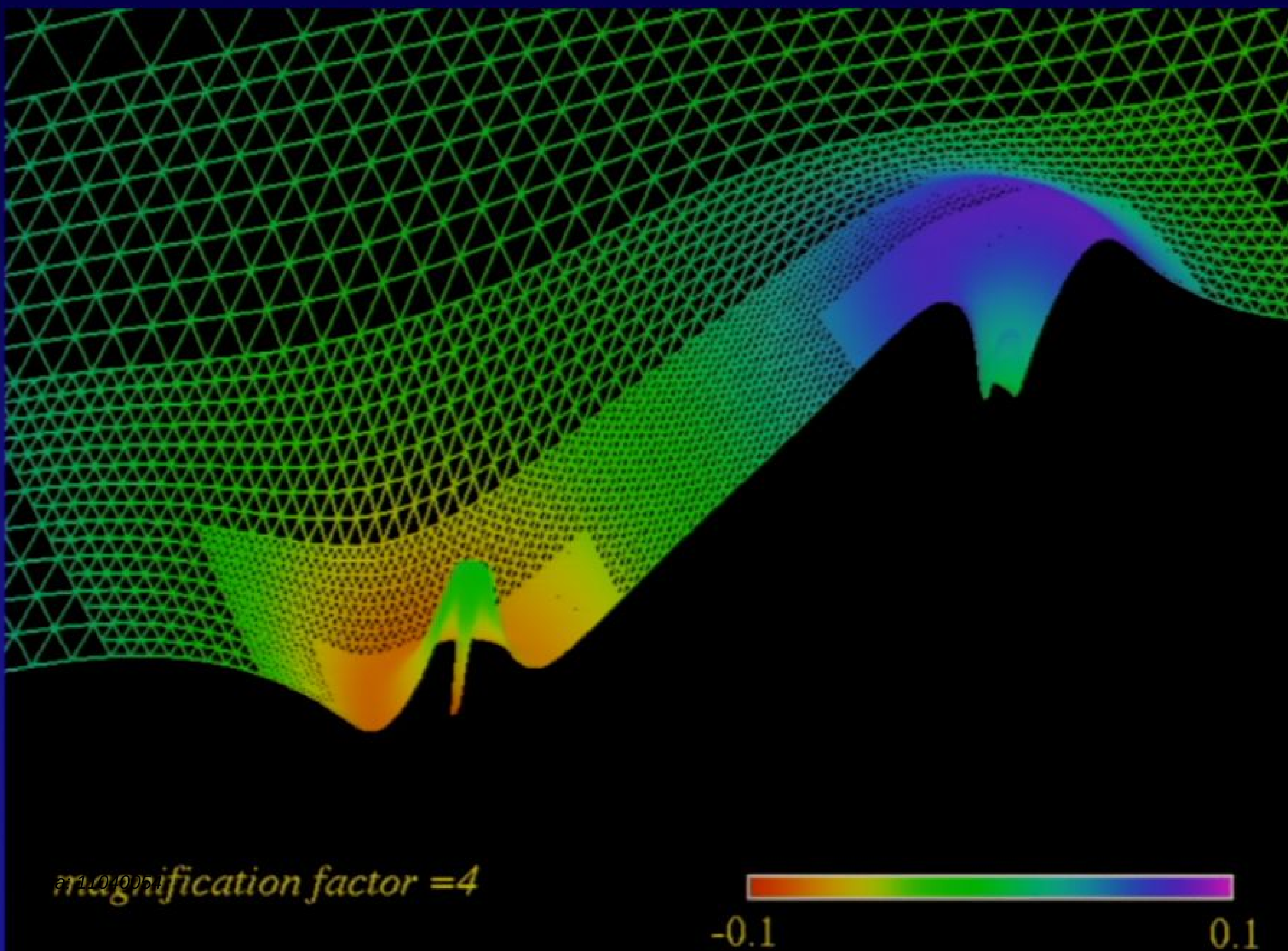

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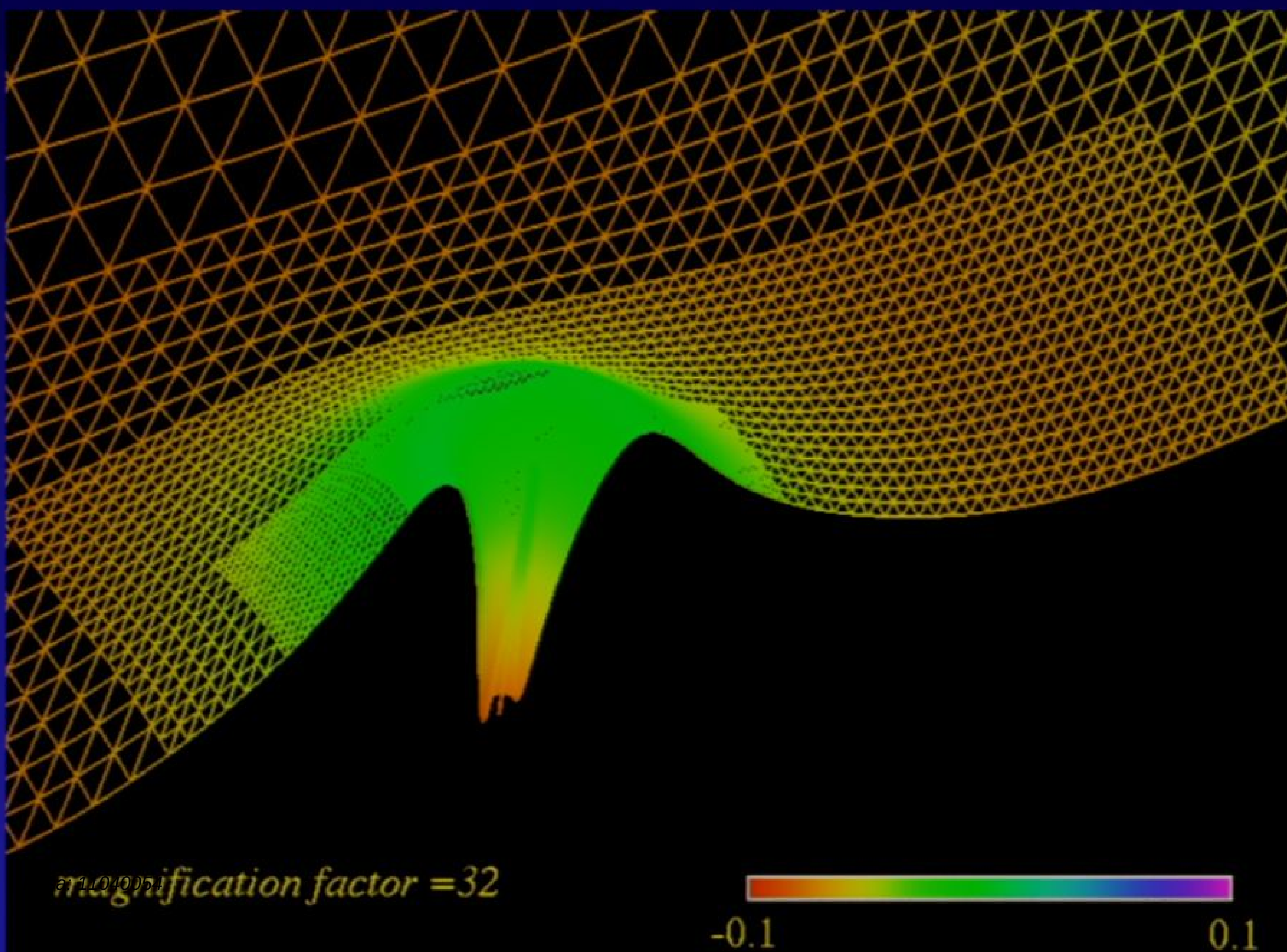

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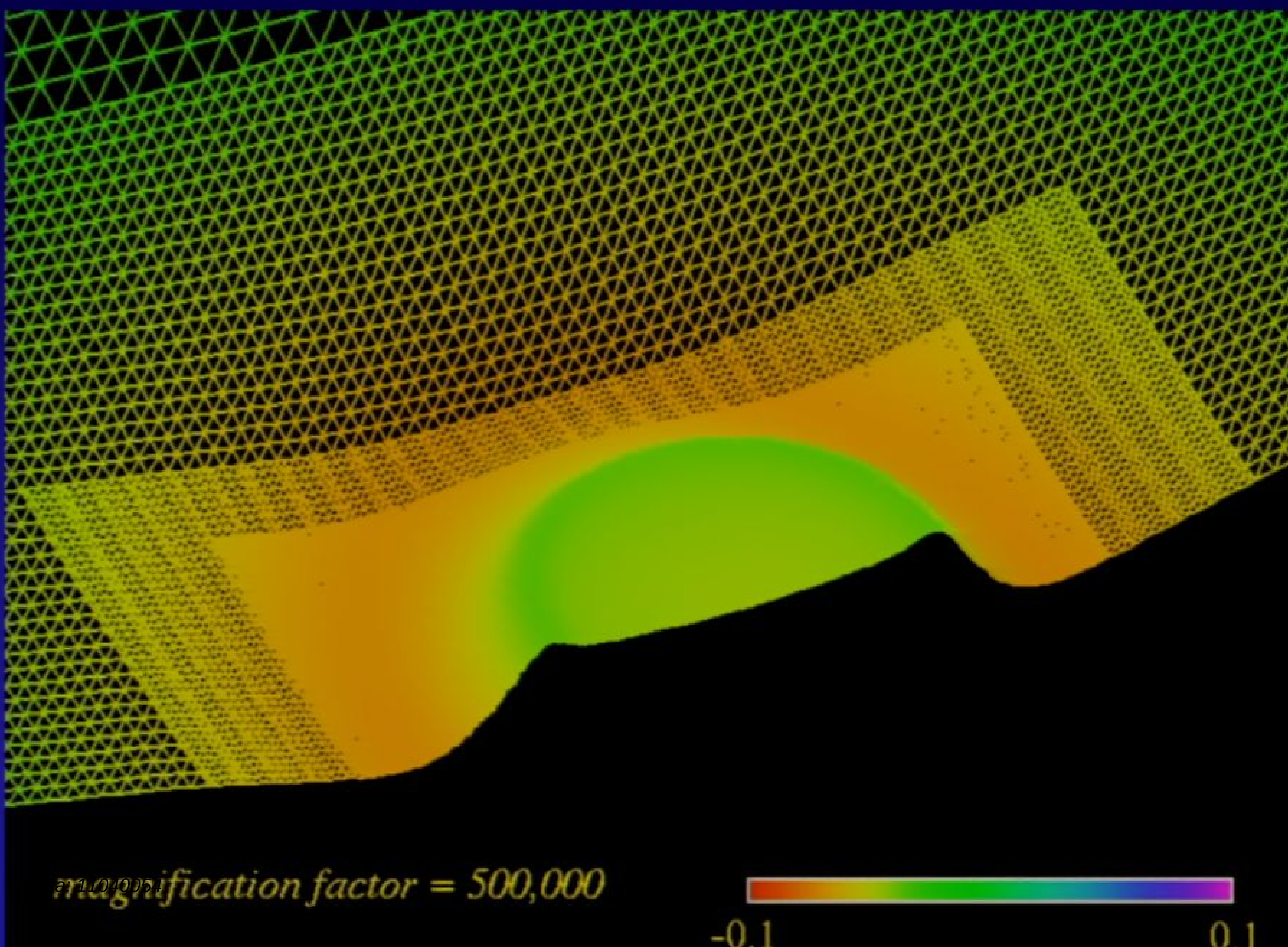

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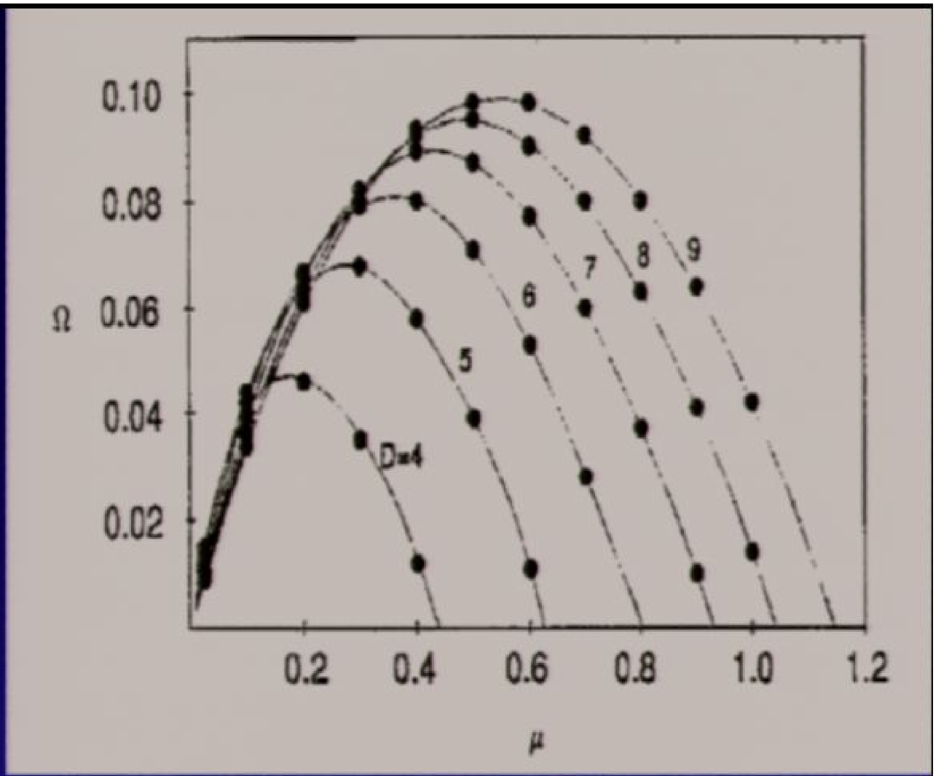
Contain singularities
Ruled by null-rays
Non-unique even in spherical symm



ability?

- Black string perturbations admit exponential growth for L > L_c (Gregory-Laflamme)
- Entropy $S_{BS} < S_{BH}$ (for a given M) [bs ~ M^2/L ; bh ~ $M^{3/2}$]

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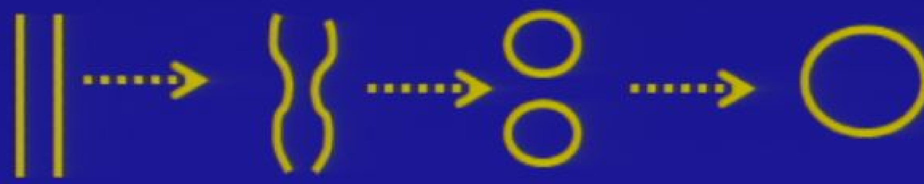

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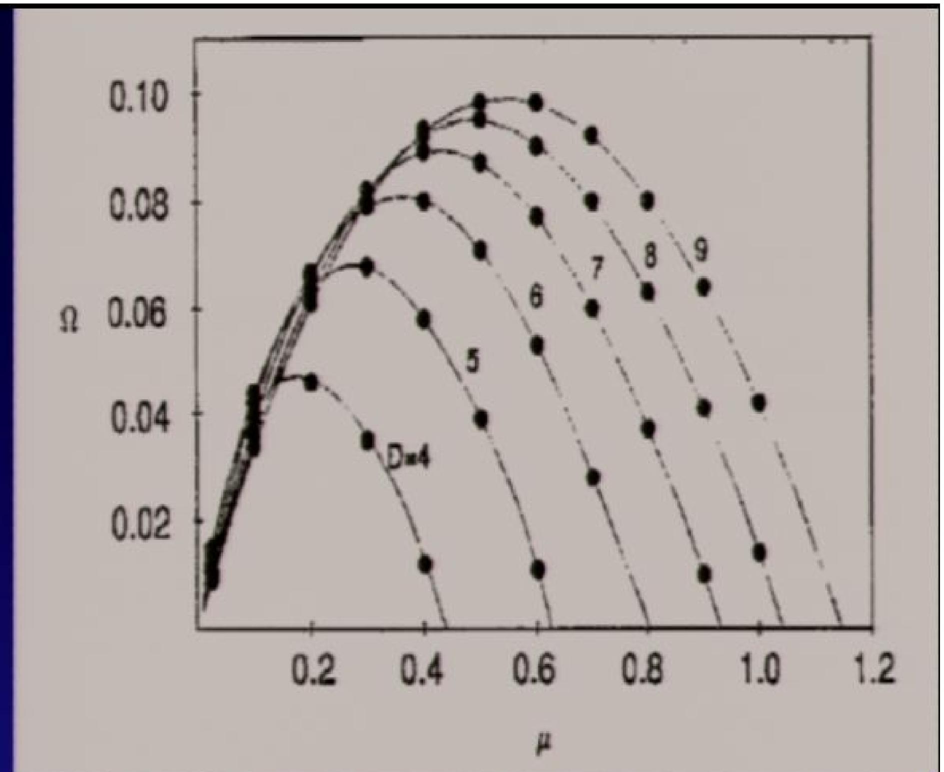

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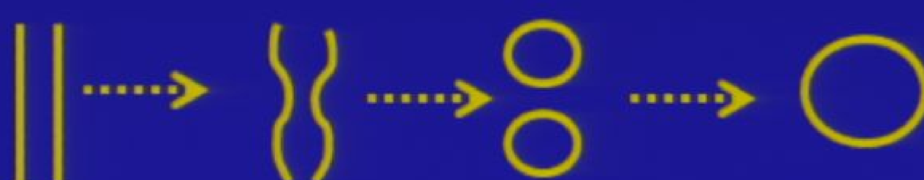
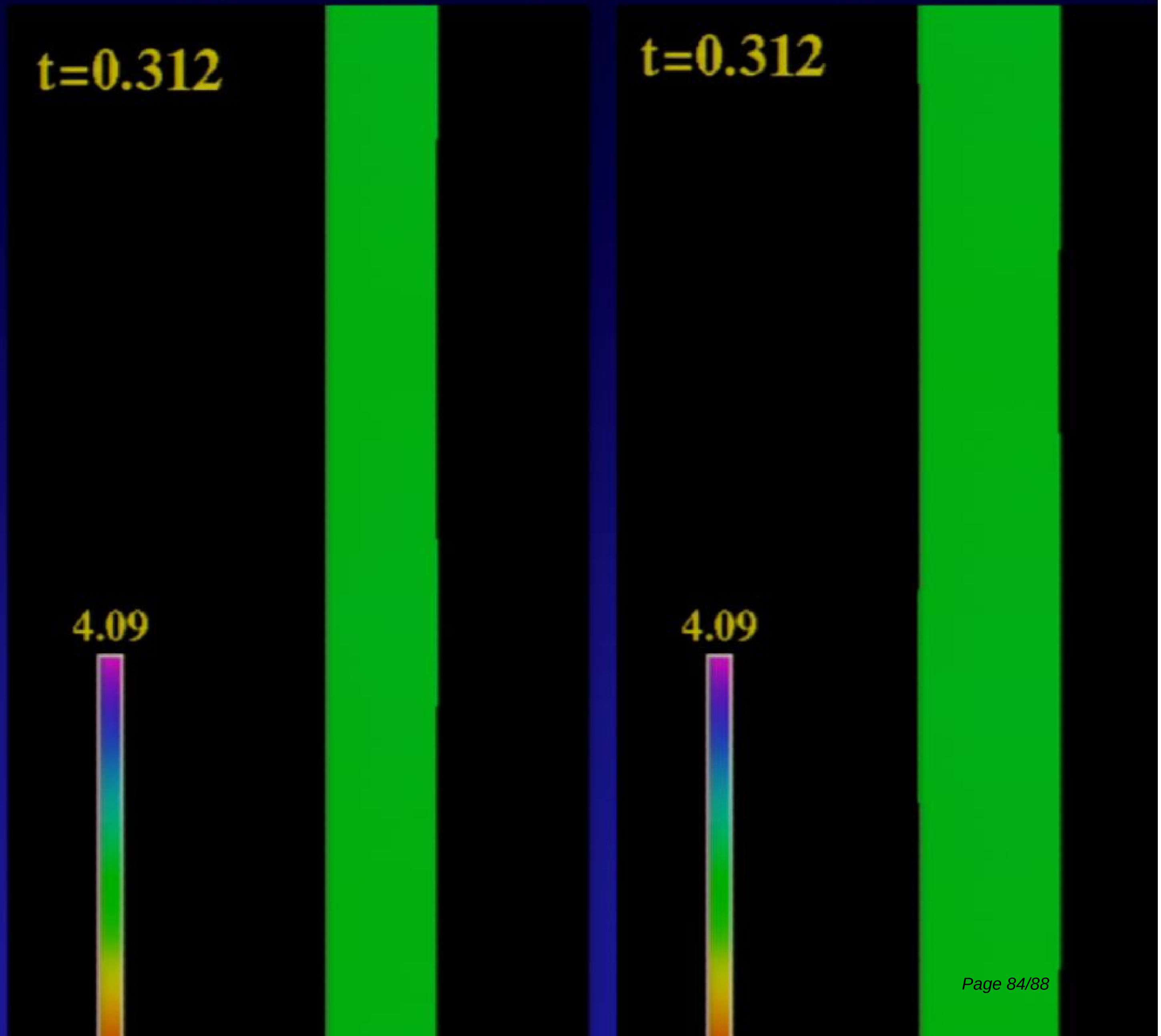


Finishing it up…[LL,Pretorius]

m = 20; $L_c \sim 14$ m

= $4 \pi (2m)^2 L$

w… $S_{BH}/S_{BS} = 1.374\ldots$



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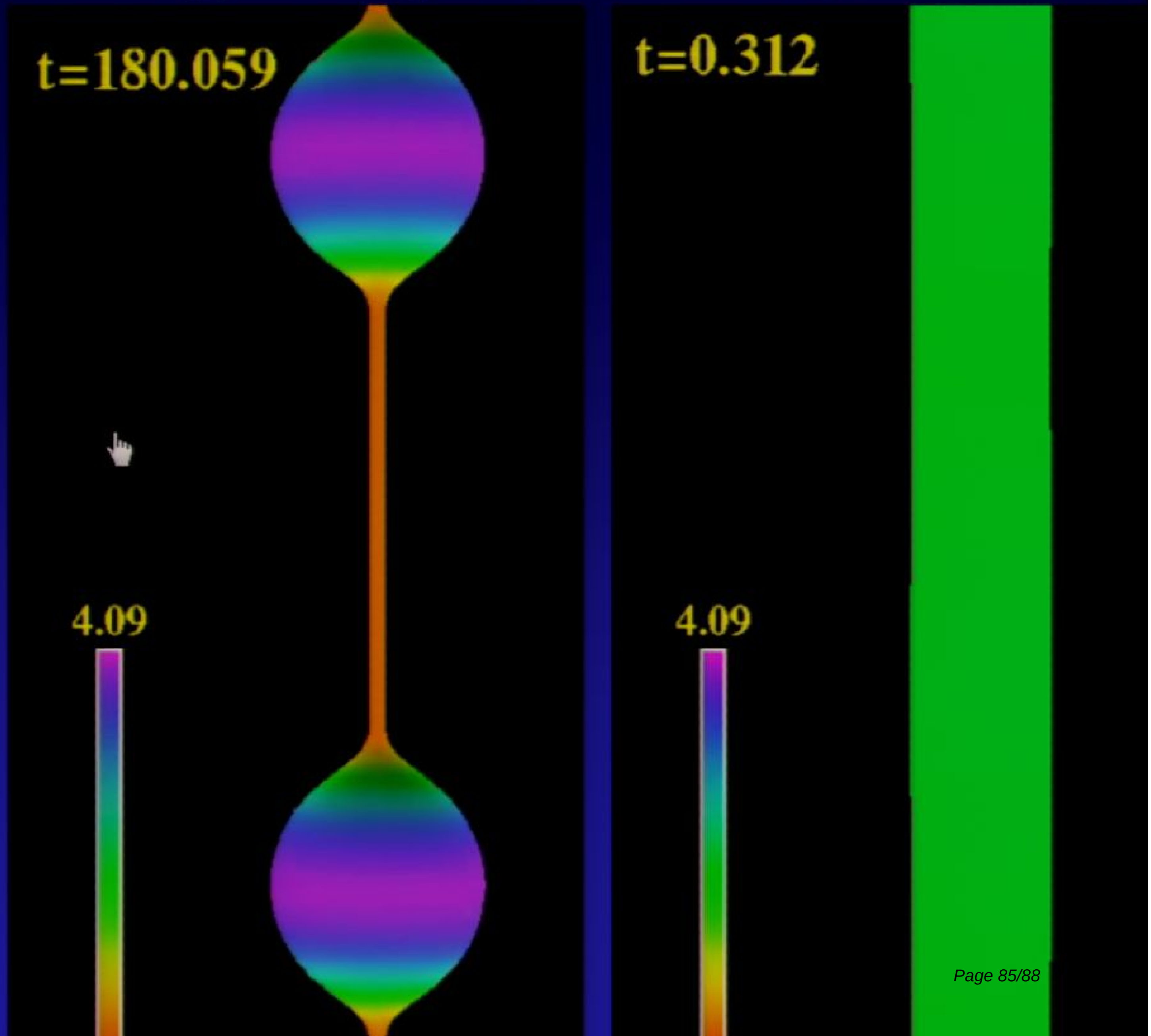

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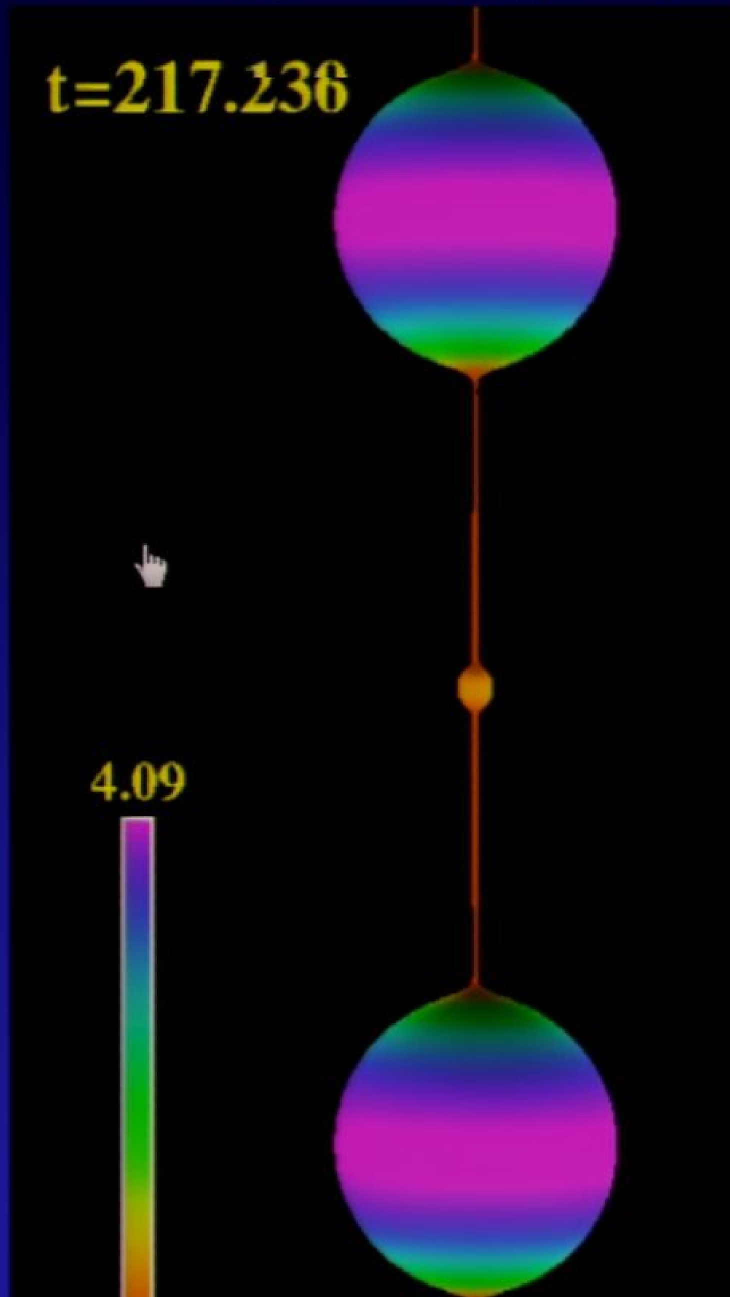

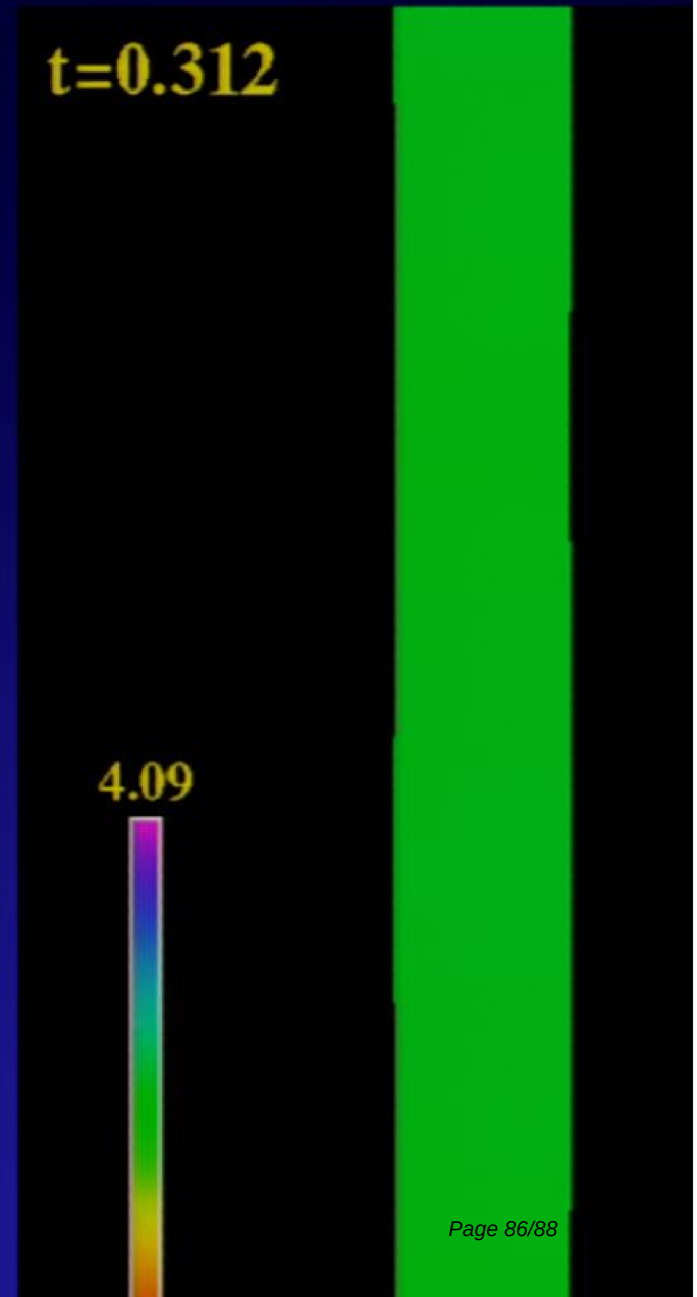

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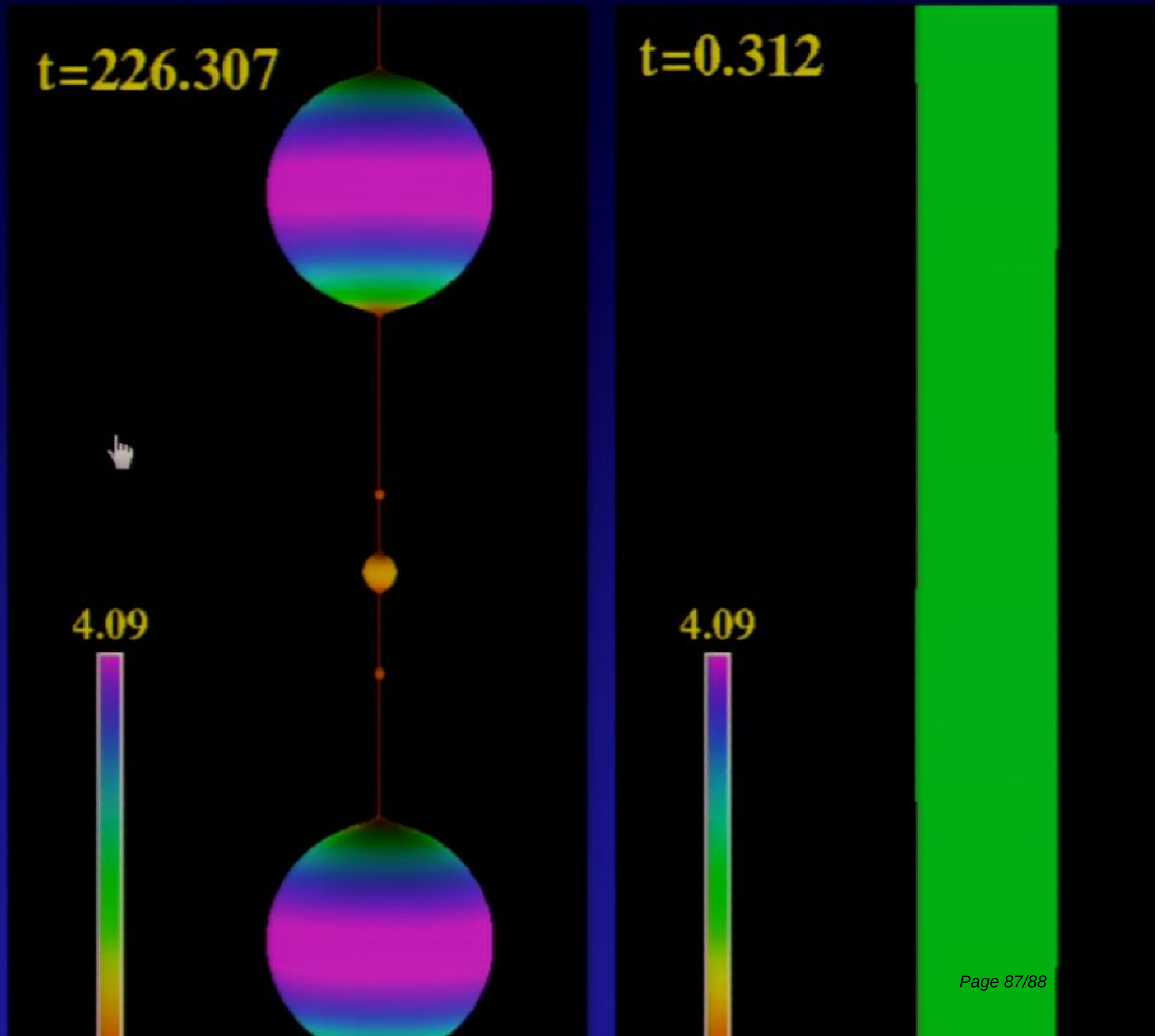

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