

Title: Explorations in Numerical Relativity - Lecture 8

Date: Apr 13, 2011 09:00 AM

URL: <http://pirsa.org/11040049>

Abstract:

$$\partial_t g + \partial_r \underline{f} = \underline{4}$$

$$\partial_t g + \partial_r f = \underline{4}$$

$$\frac{-n+k}{g_1}$$

$$\partial_t \phi + \partial_x f = \psi$$

$$\bar{\phi}_i^{n+1/2} = \bar{\phi}_i^n + \frac{\Delta t}{2} L(\bar{\phi}_i^n)$$

$$\bar{\phi}_i^{n+1} = \bar{\phi}_i^n + \Delta t L(\bar{\phi}_i^{n+1/2})$$

$$\partial_t q + \partial_x f = \psi$$

$$L_i(\bar{q}^n) = - \frac{F_{i+1/2}^{Roe}(\bar{q}^n) - F_{i-1/2}(\bar{q}^n) + \bar{\psi}}{\Delta x}$$

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$$\bar{q}_i^{n+1} = \bar{q}_i^n + \Delta t L(\bar{q}^n)$$

$$\partial_t \phi + \partial_x f = \psi$$

$$L_i(\bar{q}^n) = - \frac{F_{i+1/2}^{Por}(\bar{q}^n) - F_{i-1/2}(\bar{q}^n)}{\Delta x} + \psi$$

$$\bar{q}_i^{n+1/2} = \bar{q}_i^n + \frac{\Delta t}{2} L(\bar{q}_i^n)$$

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$$\partial_t q + \partial_x f = \psi$$

$$L_i(\bar{q}^n) = - \frac{F_{i+1/2}^{roe}(\bar{q}^n) - F_{i-1/2}(\bar{q}^n)}{\Delta x} + \psi$$

$$F_{i+1/2}^{roe} = \frac{1}{2} [f(q_r) + f(q_l) -$$

$$\bar{q}_i^{n+1/2} = \bar{q}_i^n + \frac{\Delta t}{2} L(\bar{q}_i^n)$$

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$$\partial_t q + \partial_r f = \psi$$

$$L_i(\bar{q}^n) = - \frac{F_{i+1/2}^{roe}(\bar{q}^n) - F_{i-1/2}(\bar{q}^n)}{\Delta x} + \psi$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(q_r) + f(q_i) - \sum_a |\lambda_a| w_a \right]$$

$$q_r = q(p_r)$$

$$\bar{q}_i^{n+1/2} = \bar{q}_i^n + \frac{\Delta t}{2} L(\bar{q}_i^n)$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n + \Delta t L(\bar{q}_i^{n+1/2})$$

$$t^{n+1} - t^n = \Delta t$$

$$\partial_t \bar{g} + \partial_r \bar{f} = \Psi$$

$$\Psi(\bar{g}^n) = - \frac{F_{i+1/2}^{roe}(\bar{g}^n) - F_{i-1/2}(\bar{g}^n)}{\Delta x}$$

$$\bar{g}_i^{n+1/2} = \bar{g}_i^n + \frac{\Delta t}{2} L(\bar{g}_i^n)$$

$$\bar{g}_i^{n+1} = \bar{g}_i^n + \Delta t L(\bar{g}_i^{n+1/2})$$

$$t^{n+1} - t^n = \Delta t$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(\bar{g}_r) + f(\bar{g}_l) - \sum_a |\lambda_a| w_a r \right]$$

$$\bar{g}_r = \bar{g}(Pr)$$

$$\bar{g}_{i+1/2}^r - \bar{g}_{i+1/2}^l = \sum_a w_a r_a$$

$$g + \partial_r f = \psi$$

$$(\bar{g}^n) = - \frac{F_{i+1/2}^{roe}(\bar{g}^n) - F_{i-1/2}(\bar{g}^n)}{\Delta x} + \psi$$

$$\bar{g}_i^{n+1} = \bar{g}_i^n + \frac{\Delta t}{2} L(\bar{g}_i^n)$$

$$\bar{g}_i^{n+1} = \bar{g}_i^n + \Delta t L(\bar{g}_i^{n+1/2})$$

$$t^{n+1} - t^n = \Delta t$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(g_r) + f(g_o) - \sum_a |\lambda_a| w_a r_a \right]$$

$$g_r = g(p_r)$$

$$g_{i+1/2}^r - g_{i+1/2}^l = \sum_a w_a r_a$$

$$-q + 2r f = \psi \quad \left| \begin{array}{l} -^{n+1/2} \\ q_i \end{array} \right. = \bar{q}_i^n + \frac{\Delta t}{2} L(\bar{q}_i^n)$$

$$(\bar{q}^n) = - \frac{F_{i+1/2}^{roe}(\bar{q}^n) - F_{i-1/2}(\bar{q}^n)}{\Delta x} \quad \left| \begin{array}{l} - \\ q_i \end{array} \right. = \bar{q}_i^n + \Delta t L(\bar{q}_i^{n+1/2})$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(q_r) + f(q_o) - \sum_a |\lambda_a| w_a r_a \right] \quad \left. \begin{array}{l} - \\ q_i \end{array} \right\} \quad t^{n+1} - t^n = \Delta t$$

$$q_r = q(p_r)$$

$$q_{i+1/2}^r - q_{i+1/2}^l = \sum_a w_a r_a$$

A

$$A = \left. \frac{\partial f}{\partial g} \right|_{g = \frac{1}{2}(g_l + g_r)}$$

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$$g^r - g^l = \begin{pmatrix} \underline{r}_1 & \underline{r}_2 & \underline{r}_3 \end{pmatrix}$$

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BC's :

$$A = \frac{\partial f}{\partial g} \Big|_{g = \frac{1}{2}(g_l + g_r)}$$

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BC's :

$$\sigma \quad \begin{array}{l} P_1 = P_3 \\ P_2 = P_3 \end{array} \quad \begin{array}{l} P_N = P_{N-2} \\ P_{N-1} = P_{N-2} \end{array}$$

$$A = \left. \frac{\partial f}{\partial g} \right|_{g = \frac{1}{2}(g_l + g_r)}$$

$$g^r - g^l = \begin{pmatrix} r_1 & r_2 & r_3 & \dots \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

BC's :

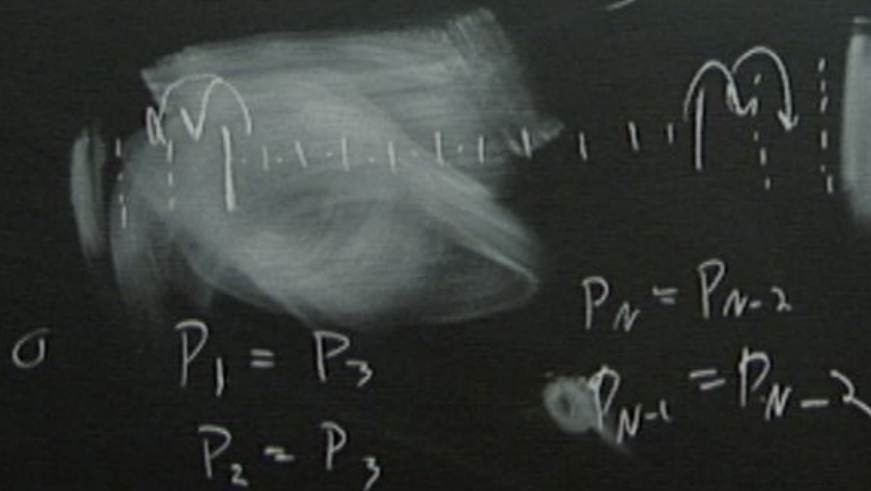


$P_1 = P_3$
 $P_2 = P_3$
 $P_N = P_{N-2}$
 $P_{N-1} = P_{N-2}$

$$A = \frac{\partial f}{\partial g} \Big|_{g = \frac{1}{2}(g_l + g_r)}$$

$$g^r - g^l = \begin{pmatrix} \underline{r}_1 & \underline{r}_2 & \underline{r}_3 \dots \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

BC's :



σ
 $P_1 = P_3$
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1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_0, q_r \rightarrow f(q_0), f(q_r)$

1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_0, q_r \rightarrow f(q_0), f(q_r)$

b)

1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_L, q_r \rightarrow f(q_L), f(q_r)$

b) $A, \bar{\tau}_\alpha, \tau_\alpha \rightarrow W_\alpha$

c) $F_{i+1/2}^{roe}$

3)

1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_0, q_r \rightarrow f(q_0), f(q_r)$

b) $A, \bar{r}_\alpha, \lambda_\alpha \rightarrow w_\alpha$

c) $F_{i+1/2}^{roe}$

3) $\Psi \approx \Psi(q_i)$

1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_0, q_r \rightarrow f(q_0), f(q_r)$

b) $A, \bar{r}_\alpha, \lambda_\alpha \rightarrow w_\alpha$

c) $F_{i+1/2}^{roe}$

3) $\psi \approx \psi(q_i)$

4) MOL $\rightarrow \bar{q}_i^{n+1/2}$

5) Repeat (1)-(4) $q^n \rightarrow q^{n+1/2}$

1) Set BC's, ghost zones

2) Calc. $F_{i+1/2}$, solve approx. Riem. Prob.

a) Recon. $p_L, p_r \rightarrow q_0, q_r \rightarrow f(q_0), f(q_r)$

b) $A, \bar{r}_\alpha, \lambda_\alpha \rightarrow w_\alpha$

c) $F_{i+1/2}^{roe}$

3) $\Psi \approx \Psi(q_i)$

4) MOL $\rightarrow \bar{q}_i^{n+1/2} \rightarrow \bar{p}^{n+1/2} = p(q^{n+1/2})$

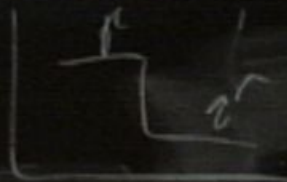
5) Repeat (1)-(4) $q^n \rightarrow q^{n+1/2}$

$$\nabla \cdot \mathbf{g} + \nabla \cdot (\mathbf{v} \mathbf{g}) = 0$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(\mathbf{g}_r) + f(\mathbf{g}_l) - \sum_a |\lambda_a| w_a \right]$$

$$2\phi + v\phi = 0$$

$$x - vt = k$$



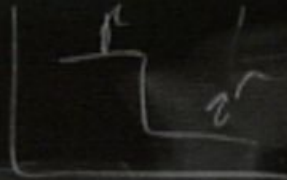
$$F_{roe} = \frac{1}{2} \left[f(\phi_n) + f(\phi_0) - \sum_a |\lambda_a| w_a \right]$$

$$2\psi + V\psi = 0$$

$$x - vt = k$$

$$q^{\pm} = \begin{cases} q_0 + \omega & \text{if } v < 0 \\ q_0 - \omega & \text{if } v > 0 \end{cases}$$

$$F_{\text{roe}} = \frac{1}{2} \left[f(q_0) + f(q_r) - \sum_a |\lambda_a| \omega_a \right]$$



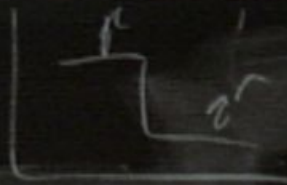
$$1 = g^v - g^l$$

$$2g + v_2 g = 0$$

$$x - vt = k$$

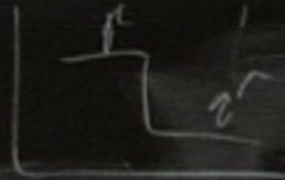
$$g^{\pm} = \begin{cases} g_0 + \omega & \text{if } v < 0 \\ g_0 - \omega & \text{if } v > 0 \end{cases}$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(g_r) + f(g_l) - \sum_a |\lambda_a| \omega_a \right]$$



$$1 = q^v - q^l$$

$$2 + q + v \bar{q} = 0$$



$$F(q^v) = v q^v$$

$$x - vt = k$$

$$q^* = \begin{cases} q_0 + w & \text{if } v < 0 \rightarrow q_r \\ q_r - w & \text{if } v > 0 \rightarrow q_l \end{cases}$$

$$= \begin{cases} -|v| q_r & v < 0 \end{cases}$$

$$F_{i+1/2}^{roe} = \frac{1}{2} \left[f(q_r) + f(q_l) - \sum_a |\lambda_a| w_a \bar{r} \right]$$

$$2\psi + v\psi = 0 \quad \rightarrow \quad \begin{array}{|c|} \hline \psi \\ \hline \end{array} \quad \psi$$

$$x - vt = k$$

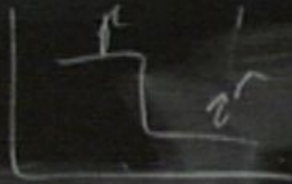
$$q^{\pm} = \begin{cases} q_0 + w & \text{if } v < 0 \rightarrow q_r \\ q_0 - w & \text{if } v > 0 \rightarrow q_l \end{cases}$$

$$f(q^{\pm}) = vq^{\pm}$$

$$= \begin{cases} -|v|q_r & v < 0 \rightarrow vq_r \\ |v|q_l = vq_l & v > 0 \end{cases}$$

$$F_{1+1/2}^{roe} = \frac{1}{2} \left[f(q_r) + f(q_l) - \sum_a |\lambda_a| w_a \right]$$

$$2\phi + v\phi = 0$$



$$f(\phi) = v\phi$$

$$x - vt = k$$

$$\phi = \begin{cases} \phi_0 + \omega & \text{if } v < 0 \rightarrow \phi_r \\ \phi_r - \omega & \text{if } v > 0 \rightarrow \phi_e \end{cases}$$

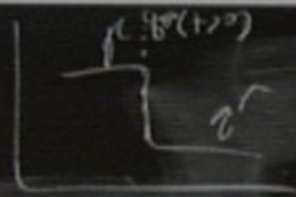
$$= \begin{cases} -|v|\phi_r & v < 0 \rightarrow v\phi_r \\ |v|\phi_r = v\phi_l & v > 0 \end{cases}$$

$$F_{\text{int}}^{\text{roe}} = \frac{1}{2} \left[f(\phi_r) + f(\phi_0) - \sum_a |\lambda_a| \omega_a \right] = \frac{1}{2} [v\phi_l + v\phi_r - |v|(\phi_r - \phi_e)]$$

$$2\psi + v\psi = 0$$

$$x - vt = k$$

$$q^{\pm} = \begin{cases} q_0 + \omega & \text{if } v < 0 \rightarrow q_r \\ q_r - \omega & \text{if } v > 0 \rightarrow q_e \end{cases}$$



$$f(q^{\pm}) = vq^{\pm}$$

$$= \begin{cases} -|v|q_r & v < 0 \rightarrow vq_r \\ |v|q_r = vq_l & v > 0 \end{cases}$$

$$F_{\text{roe}} = \frac{1}{2} \left[f(q_r) + f(q_0) - \sum_a |\lambda_a| \omega_a \right] = \frac{1}{2} [vq_l + vq_r - |v|(q_r - q_e)]$$

$$2\psi + v\psi = 0$$

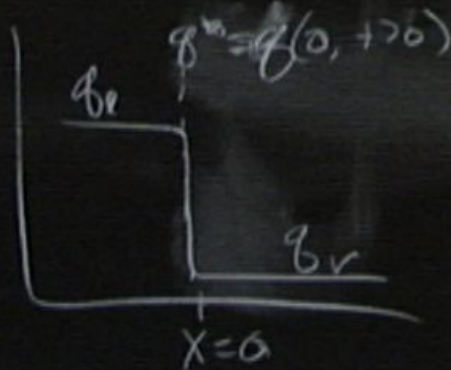
$$x - vt = k$$

$$q^{\pm} = \begin{cases} q_0 + \omega & \text{if } v < 0 \rightarrow q_r \\ q_r - \omega & \text{if } v > 0 \rightarrow q_e \end{cases}$$

$$f(q^{\pm}) = vq^{\pm}$$

$$= \begin{cases} -|v|q_r & v < 0 \rightarrow vq_r \\ |v|q_e = vq_e & v > 0 \end{cases}$$

$$F_{1+1/2}^{\text{roe}} = \frac{1}{2} \left[f(q_r) + f(q_0) - \sum_a |\lambda_a| \omega_a \right] = \frac{1}{2} [vq_e + vq_r - |v|(q_r - q_e)]$$



$$2\phi + v\phi = 0 \quad \rightarrow \quad \begin{array}{|c|} \hline \phi_r \\ \hline \end{array}$$

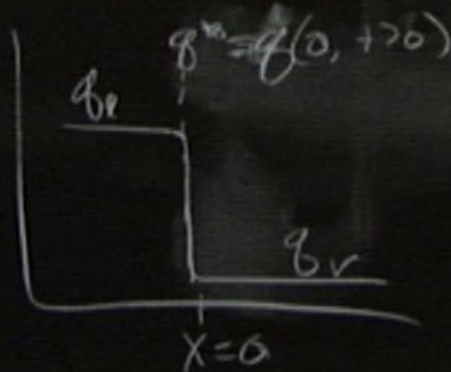
$$x - vt = k$$

$$q^{\pm} = \begin{cases} q_0 + w & \text{if } v < 0 \rightarrow q_r \\ q_0 - w & \text{if } v > 0 \rightarrow q_e \end{cases}$$

$$f(q^{\pm}) = vq^{\pm}$$

$$= \begin{cases} -|v|q_r & v < 0 \rightarrow vq_r \\ |v|q_e = vq_e & v > 0 \end{cases}$$

$$F_{\text{res}} = \frac{1}{2} \left[f(q_r) + f(q_e) - \sum_a |\lambda_a| w_a \right] = \frac{1}{2} [vq_e + vq_r - |v|(q_r - q_e)]$$



$$2\phi + v\phi = 0$$

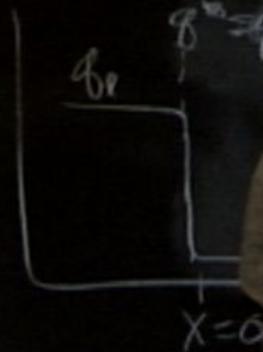
$$x - vt = k$$

$$q = \begin{cases} q_0 + \omega & \text{if } v < 0 \rightarrow q_r \\ q_r - \omega & \text{if } v > 0 \rightarrow q_e \end{cases}$$

$$F_{roe} = \frac{1}{2} \left[f(q_r) + f(q_0) - \frac{1}{a} (q_r - q_0) \right]$$

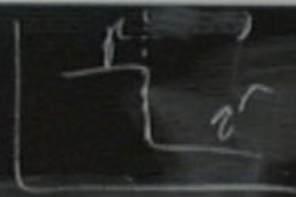
$$vq^0$$

$$-|v|q_r \quad v < 0 \Rightarrow vq_r \\ = vq_l \quad v > 0$$



$$2\psi + v\psi = 0$$

$$x - vt = k$$

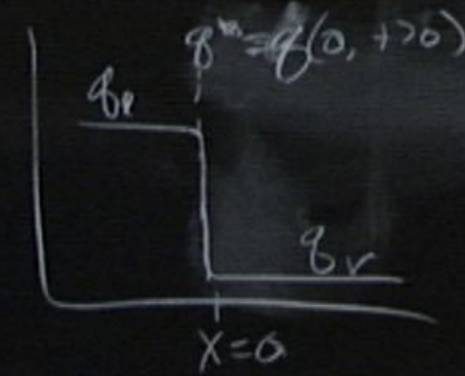


$$f(q^0) = vq^0$$

$$= \begin{cases} -|v|q_r & v < 0 \Rightarrow vq_r \\ |v|q_r = vq_l & v > 0 \end{cases}$$

$$q^0 = \begin{cases} q_l + w & \text{if } v < 0 \rightarrow q_r \\ q_r - w & \text{if } v > 0 \rightarrow q_l \end{cases}$$

$$F_{1/2}^{roe} = \frac{1}{2} \left[f(q_r) + f(q_l) - \sum_a |\lambda_a| w_a \right] = \frac{1}{2} [vq_l + vq_r - |v|(q_r - q_l)]$$



$$2 + 9 + \lambda_x f(9) = 0$$

$$f(9) = \frac{9^2}{2}$$

$$A = \frac{\partial f}{\partial 9} = 9$$

$$\lambda = 9$$

$$2 + \frac{1}{g} + \partial_x f(g) = 0$$

$$f(g) = \frac{g^2}{2}$$

$$A = \frac{\partial f}{\partial g} = g$$

$$\lambda = g \quad r = (1)$$

$$g^r - g^l = r\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{g_e^2}{2} + \frac{g_r^2}{2} - \right]$$

$$\lambda + g^2 + \lambda_x f(g) = 0$$

$$f(g) = \frac{g^2}{2}$$

$$A = \frac{\partial f}{\partial g} = g$$

$$\lambda = g \quad r = (1)$$

$$g^r - g^l = r\omega = \omega$$

$$F^{roe} = \frac{1}{2} \left[\frac{g_e^2}{2} + \frac{g_r^2}{2} - |g|(g_r - g_e) \right]$$

$$2 + \lambda + \partial_x f(\lambda) = 0$$

$$f(\lambda) = \frac{\lambda^2}{2}$$

$$A = \frac{\partial f}{\partial \lambda} = \lambda$$

$$\lambda = 8 \quad r = (1)$$

$$q^r - q^l = r\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$2 + q^2 + \partial_x f(q) = 0$$

$$f(q) = \frac{q^2}{2}$$

$$A = \frac{\partial f}{\partial q} = q$$

$$\lambda = q \quad r = (1)$$

$$q^n - q^l = n\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^{n+1/2} (q_i^{n+1/2} - q_{i-1}^{n+1/2}) \quad - \text{1st-order}$$

$$2 + q^2 + \partial_x f(q) = 0$$

$$f(q) = \frac{q^2}{2}$$

$$A = \frac{\partial f}{\partial q} = q$$

$$\lambda = q \quad r = (1)$$

$$q^n - q^k = n\omega = \omega$$

$$F^{roe} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^{n+1/2} (q_i^{n+1/2} - q_{i-1}^{n+1/2}) \quad - \text{1st-order}$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x}$$

$$2 + q^2 + \partial_x f(q) = 0$$

$$f(q) = \frac{q^2}{2}$$

$$A = \frac{\partial f}{\partial q} = q$$

$$\lambda = q \quad r = (1)$$

$$q^n - q^l = n\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^{n+1/2} (q_i^{n+1/2} - q_{i-1}^{n+1/2})$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} \left[\frac{1}{2} (q_i^{n+1/2})^2 - \frac{1}{2} (q_{i-1}^{n+1/2})^2 \right]$$

$$2 + q^2 + \partial_x f(q) = 0$$

$$f(q) = \frac{q^2}{2}$$

$$A = \frac{\partial f}{\partial q} = q$$

$$\lambda = q \quad r = (1)$$

$$q^n - q^L = n\omega = \omega$$

$$F^{roe} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^{n+1/2} (q_i^{n+1/2} - q_{i-1}^{n+1/2}) \quad - \text{1st-order}$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} \left[\frac{1}{2} (q_i^{n+1/2})^2 - \frac{1}{2} (q_{i-1}^{n+1/2})^2 \right]$$

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$$\lambda + g^2 + \lambda_x f(g) = 0$$

$$f(g) = \frac{g^2}{2}$$

$$A = \frac{\partial f}{\partial g} = g$$

$$\lambda = g \quad r = (1)$$

$$g^n - g^k = n\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{g_e^2}{2} + \frac{g_r^2}{2} - |g|(g_r - g_e) \right]$$

$$g_i^{n+1} = g_i^n - \frac{\Delta t}{\Delta x} g_i^{n+1/2} (g_i^{n+1/2} - g_{i-1}^{n+1/2}) \quad \text{--- 1st-order}$$

$$g_i^{n+1} = g_i^n - \frac{\Delta t}{\Delta x} \left[\frac{1}{2} (g_i^{n+1/2})^2 - \frac{1}{2} (g_{i-1}^{n+1/2})^2 \right]$$

$$\lambda + q + \partial_x f(q) = 0$$

$$f(q) = \frac{q^2}{2}$$

$$A = \frac{\partial f}{\partial q} = q$$

$$\lambda = q \quad r = (1)$$

$$q^n - q^k = n\omega = \omega$$

$$F_{\text{roe}} = \frac{1}{2} \left[\frac{q_e^2}{2} + \frac{q_r^2}{2} - |q|(q_r - q_e) \right]$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} q_i^{n+1/2} (q_i^{n+1/2} - q_{i-1}^{n+1/2}) \quad \text{--- 1st-order}$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x} \left[\frac{1}{2} (q_i^{n+1/2})^2 - \frac{1}{2} (q_{i-1}^{n+1/2})^2 \right]$$

$$\nabla_m (g_{\alpha\beta} u^\alpha) = 0$$
$$\nabla_m T^{\mu\nu} = 0$$

$$\nabla_\mu (g_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$g_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$\nabla_\mu (g_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (g + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$g_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$g = g_0(1 + \epsilon)$$

$$\nabla_\mu (\rho_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\rho_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$\rho = \rho_0 (1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$\nabla_\mu (\rho_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\rho_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$\rho = \rho_0 (1 + \epsilon)$$

$$\epsilon = \text{specific energy in}$$

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\begin{pmatrix} -\rho & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_\mu (s_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$s_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$s = s_0(1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$T^{\mu\nu} = (s + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\begin{pmatrix} -s & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_\mu (g_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$g_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$g = g_0(1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$T^{\mu\nu} = (g + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\begin{pmatrix} -g & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$\delta = \delta_0(1+\varepsilon) \simeq \delta_0 \varepsilon$$

$$g = g_0(1+\varepsilon) \approx g_0\varepsilon$$

$$P = P(g_0, \varepsilon)$$

ideal-gas :
"gamma-law"

$$P = (\gamma - 1)g_0\varepsilon$$

$$s = s_0(1+\varepsilon) \approx s_0\varepsilon$$

$$P = P(s_0, \varepsilon)$$

ideal-gas :
"gamma-law"

$$P = (\gamma - 1) s_0 \varepsilon$$

adiabatic index γ

$$\gamma = \gamma(\varepsilon)$$

$$s = s_0(1+\varepsilon) \approx s_0\varepsilon$$

$$P = P(s_0, \varepsilon)$$

ideal-gas :
"gamma-law"

$$P = (\gamma - 1) s_0 \varepsilon$$

adiabatic index γ

$$\gamma = \gamma(\varepsilon)$$

$$s = s_0(1+\varepsilon) \simeq s_0\varepsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(s_0, \varepsilon)$$

ideal-gas :
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$$P = (\Gamma - 1) s_0 \varepsilon$$

adiabatic index γ

$$\Gamma = \Gamma(\varepsilon)$$

$$\rho = \rho_0(1+\epsilon) \approx \rho_0 \epsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(\rho_0, \epsilon)$$

ideal-gas :
"gamma-law"

$$P = (\Gamma - 1) \rho_0 \epsilon$$

adiabatic index γ

$$\text{ultra-rel EOS : } P = (\Gamma - 1) \rho$$

$$\Gamma = \Gamma(\epsilon)$$

$$s = s_0(1+\epsilon) \approx s_0\epsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(s_0, \epsilon)$$

ideal-gas :
"gamma-law"

$$P = (\Gamma - 1) s_0 \epsilon$$

adiabatic index γ

ultra-rel
EOS : $P = (\Gamma - 1) s$

$$\Gamma = \Gamma(\epsilon)$$

$$\nabla_\mu (s_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$s_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$s = s_0(1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$T^{\mu\nu} = (s + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\begin{pmatrix} -s & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$\nabla_\mu (s_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$s_0 = m_b n$$

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$$s = s_0(1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$T^{\mu\nu} = (s + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\begin{pmatrix} -s & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

$$\nabla_\mu (g_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (g + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g u^\mu) = 0$$

$$g_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$g = g_0 (1 + \epsilon)$$

$$\epsilon = g^{\mu\nu} \partial_\mu \partial_\nu \phi$$

$$\begin{pmatrix} -g & P & 0 \\ 0 & P & P \end{pmatrix}$$

$$\nabla_\mu (s_0 u^\mu) = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (s + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$u^\mu = [-1, 0, 0, 0]$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} s u^\mu) = 0$$

$$s_0 = m_b n$$

$$u^\mu = 4\text{-vel}$$

$$s = s_0 (1 + \varepsilon)$$

$$\varepsilon = \text{specific energy}$$

$$\begin{pmatrix} -s & 0 \\ 0 & P \end{pmatrix}$$

$$\rho = \rho_0(1+\varepsilon) \approx \rho_0 \varepsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(\rho_0, \varepsilon)$$

ideal-gas :
"Gamma-law"

$$P = (\Gamma - 1) \rho_0 \varepsilon$$

adiabatic index Γ

ultra-rel
EOS : $P = (\Gamma - 1) \rho$

$$\Gamma = \Gamma(\varepsilon)$$

$$\frac{\partial \rho}{\partial t} + \partial_x f = \psi$$

- slab sym.

$$\Rightarrow \partial_y, \partial_z \rightarrow 0$$

$$\rho = \rho_0(1+\epsilon) \approx \rho_0 \epsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(\rho_0, \epsilon)$$

ideal-gas :
"gamma-law"

$$P = (\Gamma - 1) \rho_0 \epsilon$$

adiabatic index $\times$

ultra-rel
EOS : $P = (\Gamma - 1) \rho$

$$\Gamma = \Gamma(\epsilon)$$

$$\frac{\partial \rho}{\partial t} + \partial_x f = \psi$$

$P_x = P_{x-2}$
- slab sym.

$$\Rightarrow \partial_y, \partial_z \rightarrow 0$$

$$\Rightarrow v_y, v_z = 0$$

$$\rho = \rho_0(1+\epsilon) \approx \rho_0 \epsilon$$

$$\Gamma \in (1, 2]$$

$$P = P(\rho_0, \epsilon)$$

ideal-gas :
"Gamma-law"

$$P = (\Gamma - 1) \rho_0 \epsilon$$

adiabatic index γ

ultra-rel
EOS : $P = (\Gamma - 1) \rho$

$$\Gamma = \Gamma(\epsilon)$$

$$\frac{\partial \rho}{\partial t} + \partial_x f = \psi$$

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ideal-gas :
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$$\Gamma = \Gamma(\epsilon)$$

$$\frac{\partial \rho}{\partial t} + \partial_x f = \psi$$

- slab sym.

$$\Rightarrow \partial_y, \partial_z \rightarrow 0$$

$$\Rightarrow v_y, v_z = 0$$

$$E = (\rho + P)W^2 - P$$

$$S = vW^2(\epsilon + P)$$

$$W^2 = \frac{1}{1 - v^2}$$

$$v = \frac{u^x}{u^t}$$

$$T = (\rho + P)W^2 - P$$

$$S = vW^2(\epsilon + P)$$

$$W^2 = \frac{1}{1-v^2} = (u')^2 \quad v = \frac{u^x}{u^t}$$

$$\mathcal{E} = (\rho + P)W^2 - P$$

$$S = vW^2(\epsilon + P)$$

$$T^{++} = \mathcal{E}$$

$$T^{+x} = S$$

$$T^{xx} = S v + P$$

$$W^2 = \frac{1}{1-v^2} = (\gamma)^2 \quad v = \frac{u^x}{u^t}$$

$$\mathcal{E} = (\rho + P)W^2 - P$$

$$S = vW^2(\epsilon + P)$$

$$T^{++} = \mathcal{E}$$

$$T^{+x} = S$$

$$T^{xx} = S_v + P$$

$$W^2 = \frac{1}{1-v^2} = (u')^2 \quad v = \frac{u^x}{u^t}$$

$$\partial_t \mathcal{E} + \partial_x S = 0$$

$$\partial_t S + \partial_x (S_v + P) = 0$$

$$g = \begin{bmatrix} \mathcal{E} \\ S \end{bmatrix}, \quad f = \begin{bmatrix} S \\ S_v + P \end{bmatrix}, \quad \psi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathcal{L} = (\rho + P)W^2 - P$$

$$S = vW^2(\epsilon + P)$$

$$T^{++} = \mathcal{L}$$

$$T^{+x} = S$$

$$T^{xx} = S_v + P$$

$$W^2 = \frac{1}{1-v^2} = (\gamma)^2 \quad v = \frac{u^x}{u^+}$$

$$\partial_+ \mathcal{L} + \partial_x S = 0$$

$$\partial_+ S + \partial_x (S_v + P) = 0$$

$$g = \begin{bmatrix} \tau \\ s \end{bmatrix}$$

$$f = \begin{bmatrix} s \\ s_v + P \end{bmatrix}$$

$$p = \begin{bmatrix} s \\ v \end{bmatrix}$$

$$\psi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$