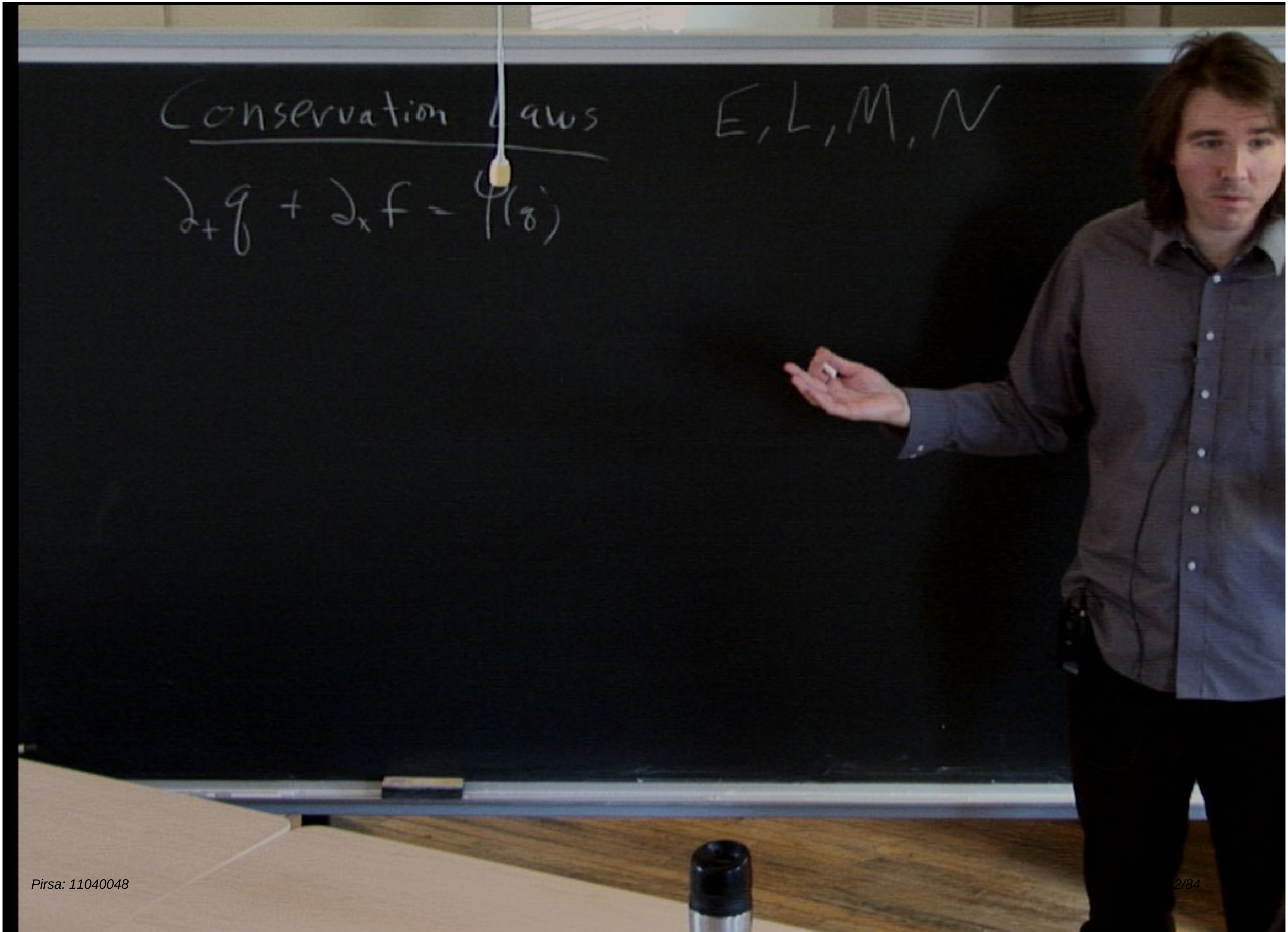
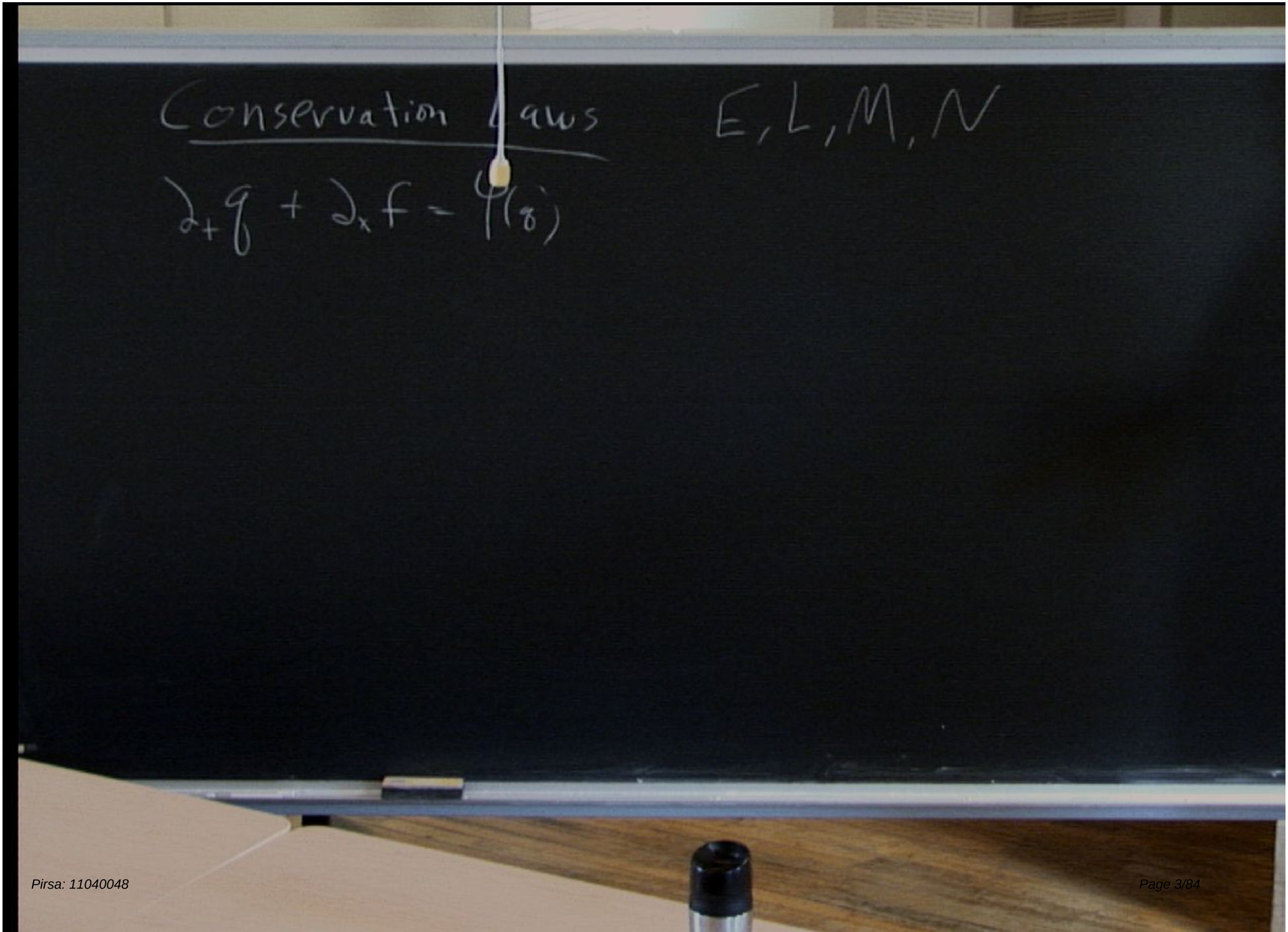
Title: Explorations in Numerical Relativity - Lecture 7

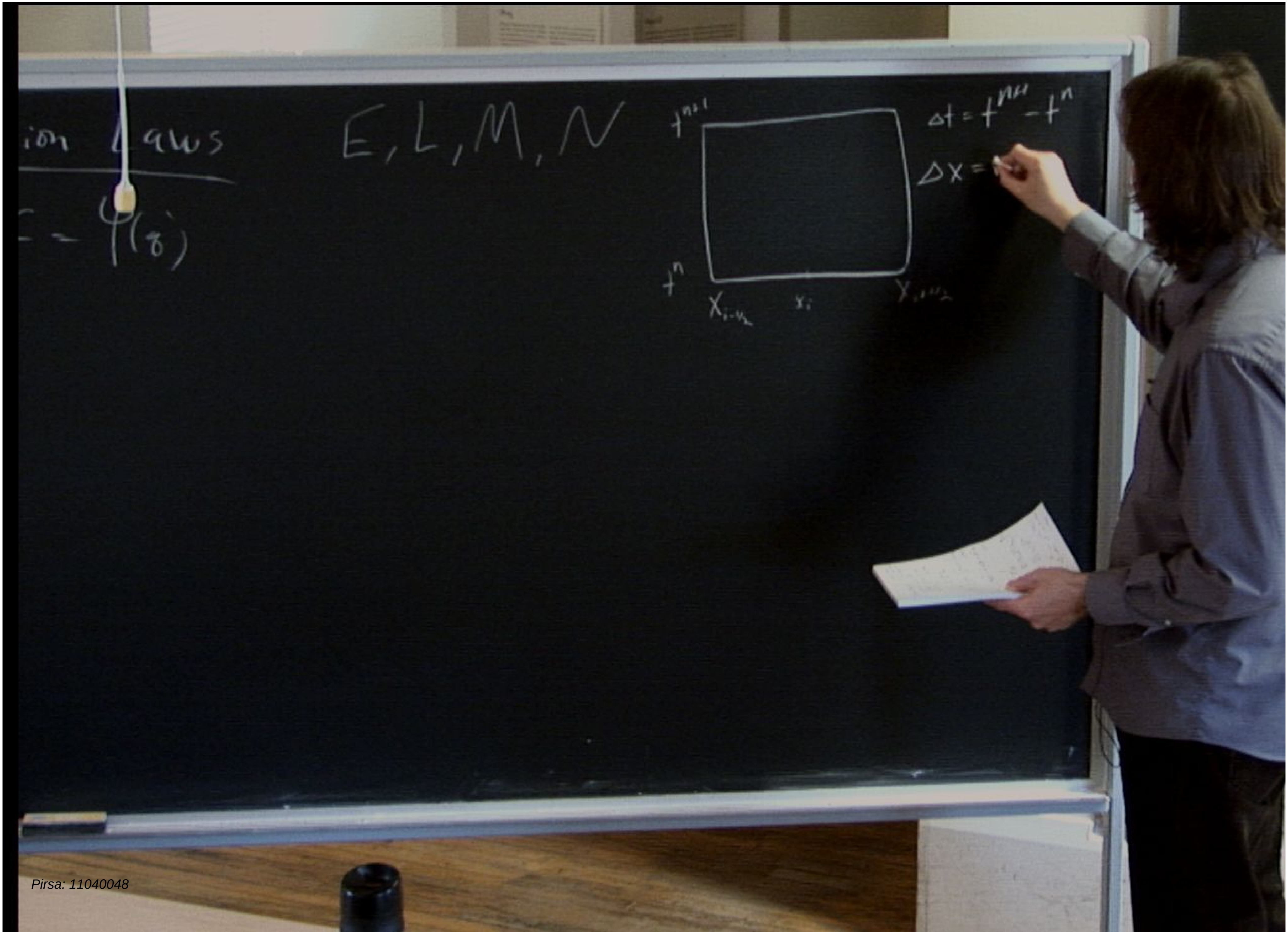
Date: Apr 12, 2011 09:00 AM

URL: http://pirsa.org/11040048

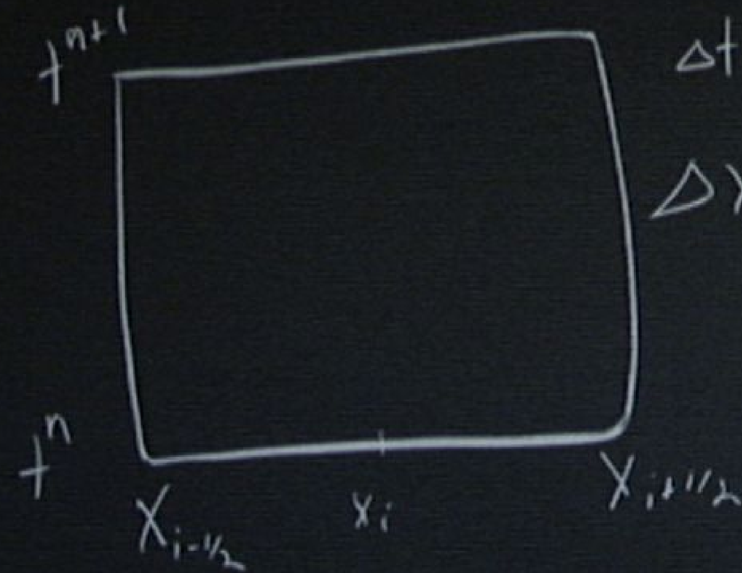
Abstract:


Conservation Laws
E, L, M, N
$\partial_t q + \partial_x f = \psi(q)$


Conservation Laws
E, L, M, N
$\partial_t q + \partial_x f = \psi(q)$


ion Laws
E, L, M, N
t^{n+1}
$\Delta t = t^{n+1} - t^n$
$\Delta x =$
t^n
$X_{i-1/2}$
x_i
$X_{i+1/2}$

, L, M, N



$\Delta t = t^{n+1} - t^n$

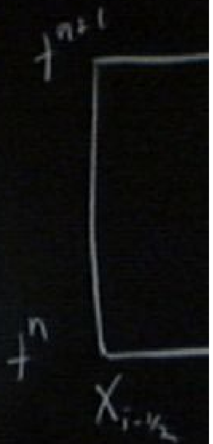
$\Delta x = x_{i+1/2} - x_{i-1/2}$

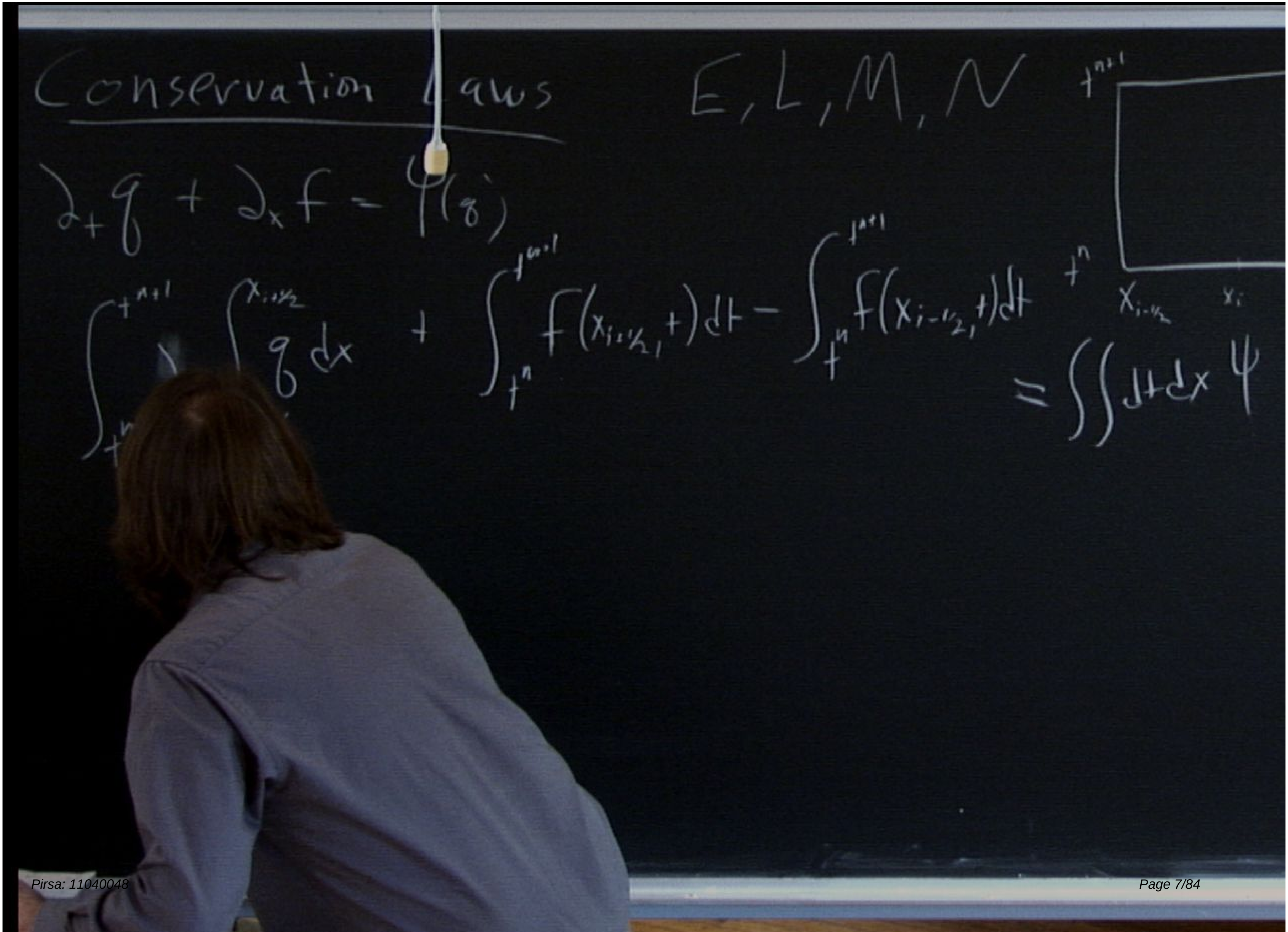
Conservation Laws

E, L, M, N

$$\partial_t q + \partial_x f = \psi(q)$$

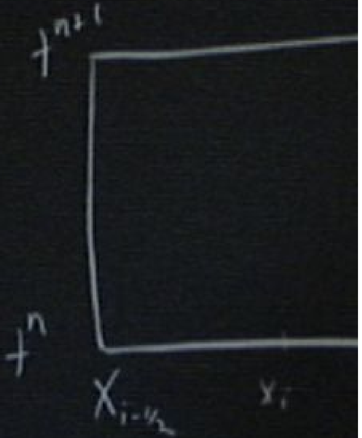
$$\int_{t^n}^{t^{n+1}} \partial_t \int_{x_{i-1/2}}^{x_{i+1/2}} q \, dx + \int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt - \int_{t^n}^{t^{n+1}} f(x_{i-1/2}, t)\,dt$$




Conservation Laws
E, L, M, N
$\partial_t q + \partial_x f = \psi(q)$
$\int_{t^n}^{t^{n+1}} \int^{x_{i+1/2}} q\,dx + \int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt - \int_{t^n}^{t^{n+1}} f(x_{i-1/2}, t)\,dt$
$= \iint dt\,dx\,\psi$
t^{n+1}
t^n
$x_{i-1/2}$
x_i

Conservation Laws

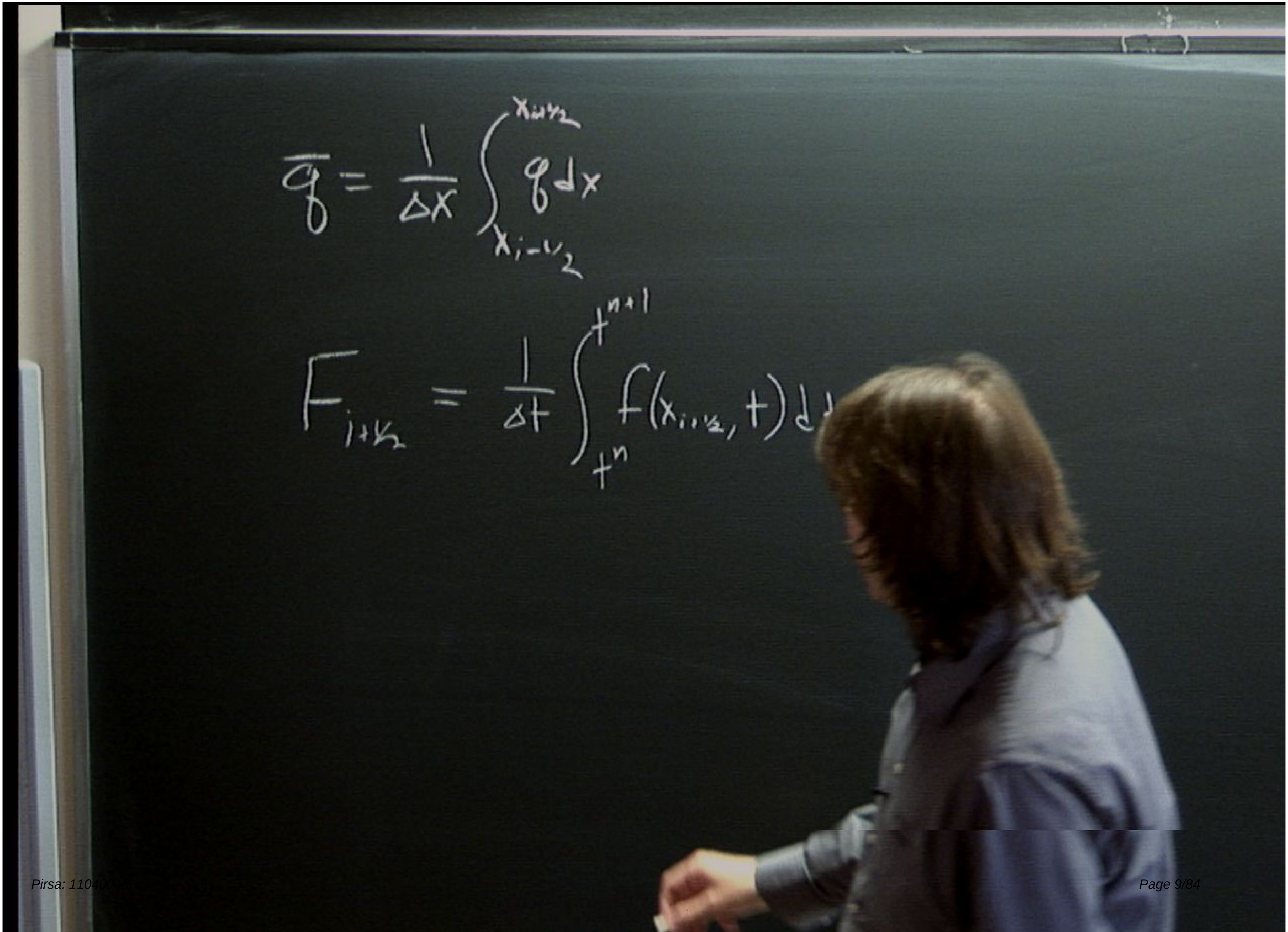
E, L, M, N



$$\partial_t q + \partial_x f = \psi(q)$$

$$\int_{t^n}^{t^{n+1}} \partial_t \int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx + \int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt - \int_{t^n}^{t^{n+1}} f(x_{i-1/2},t)\,dt = \iint dt\,dx\,\psi$$

$$\Delta x \int dt\, \partial_t \overline{q}$$


$\bar{q} = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} q \, dx$
$F_{i+1/2} = \frac{1}{\Delta t} \int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t) \, dt$

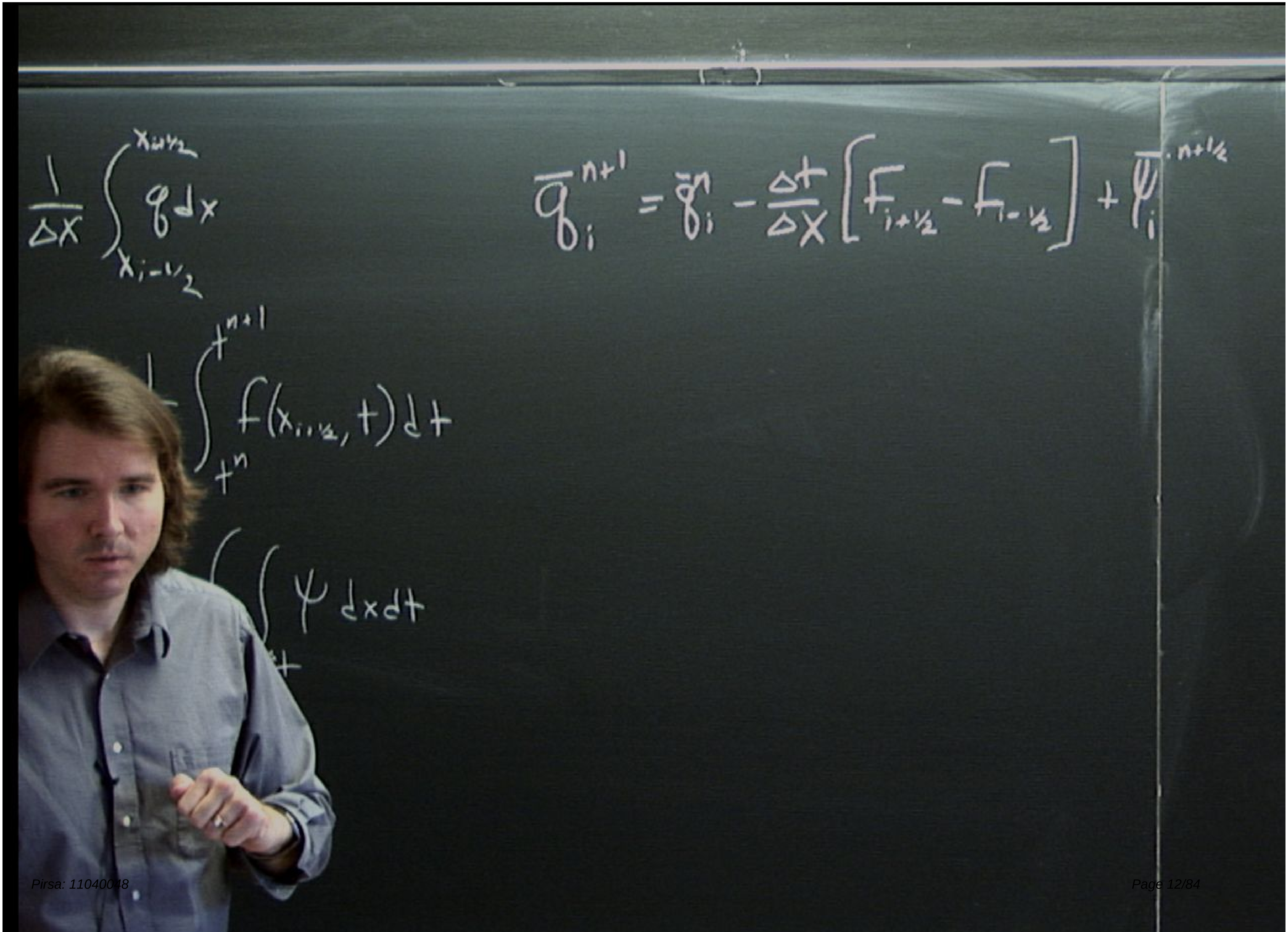
$$\bar{q} = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} q \, dx$$

$$F_{i+1/2} = \frac{1}{\Delta t} \int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t) \, dt$$

$$\bar{q} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx$$

$$F_{i+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$$

$$\bar{\psi} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi\,dx\,dt$$


$\frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx$
$\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$
$\iint \psi\,dx\,dt$
$\bar{q}_i^{n+1} = \bar{q}_i^{n} - \frac{\Delta t}{\Delta x}\left[F_{i+1/2} - F_{i-1/2}\right] + \bar{\psi}_i^{n+1/2}$

$$\bar{q} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx$$

$$F_{i+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$$

$$\bar{\psi} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi\,dx\,dt$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}$$

ervation Laws

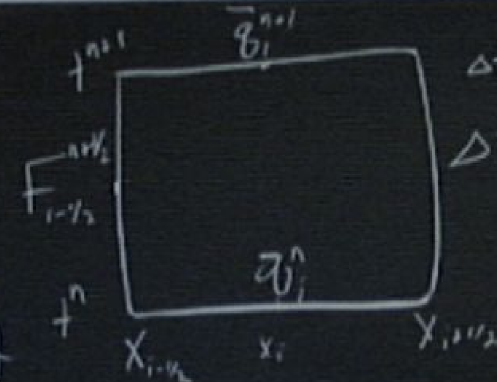
E, L, M, N

$\partial_x f = \psi(q)$

$$\int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx + \int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt - \int_{t^n}^{t^{n+1}} f(x_{i-1/2},t)\,dt$$

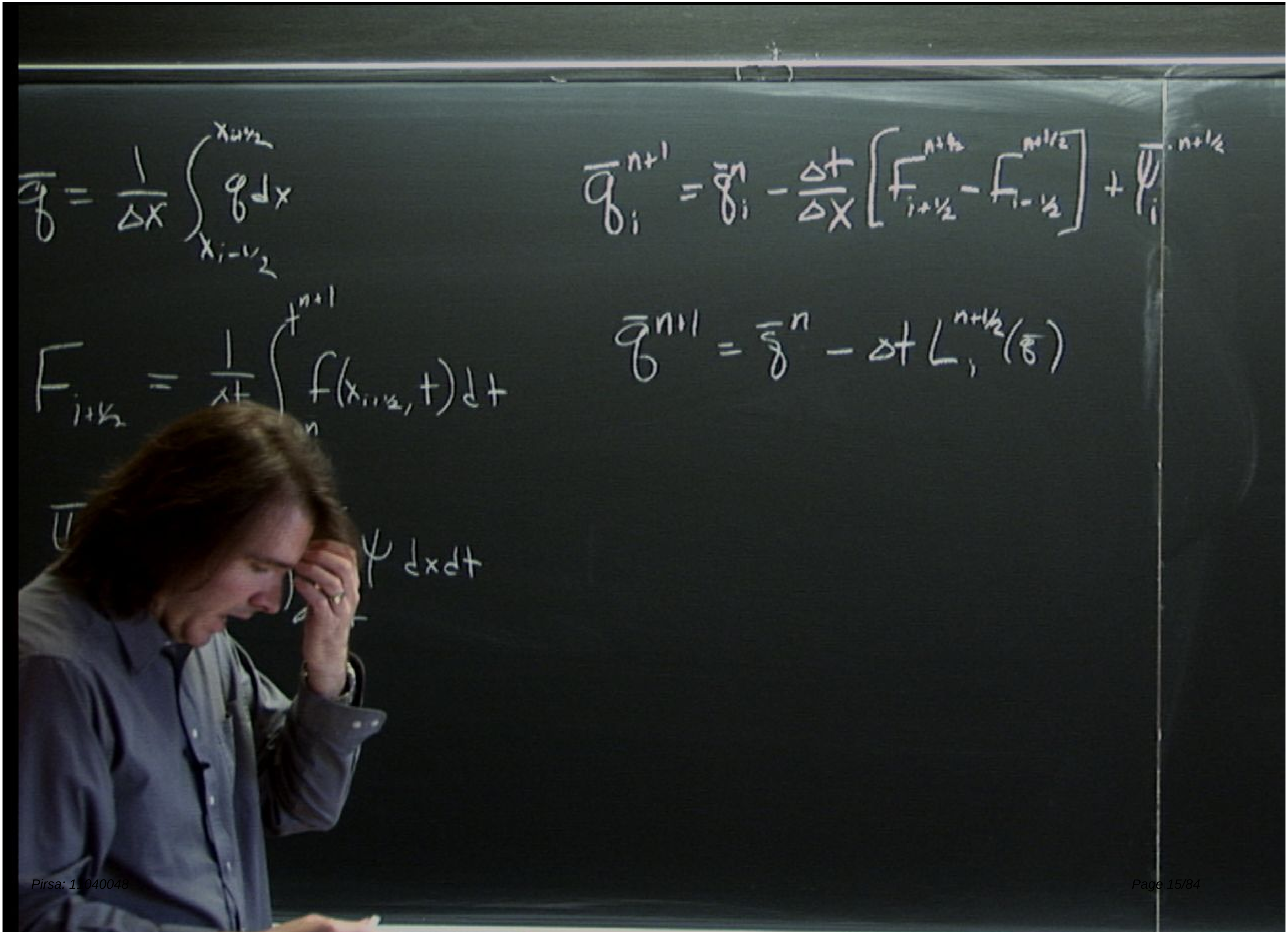
$$= \iint dt\,dx\,\psi$$

$dt\,\partial_t \bar{q}$



$\Delta t = t^{n+1} - t^n$

$\Delta x = x_{i+1/2} - x_{i-1/2}$


$\bar{q} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q\,dx$
$F_{i+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$
$\psi\,dx\,dt$
$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}$
$\bar{q}^{n+1} = \bar{q}^n - \Delta t\,L_i^{n+1/2}(\bar{q})$

$$\bar{q}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t^n)\,dx$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}$$

$$F_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt$$

$$\bar{q}^{n+1} = \bar{q}^n - \Delta t\, L_i^{n+1/2}(\bar{q})$$

$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi\,dx\,dt$$

$$\bar{q}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x, t^n)\,dx$$

$$F_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$$

$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi\,dx\,dt$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}\Delta t$$

$$\bar{q}^{n+1} = \bar{q}^n - \Delta t\, L_i^{n+1/2}(\bar{q})$$

$$\bar{q}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x, t^n)\,dx$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}\,\Delta t$$

$$F_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2}, t)\,dt$$

$$\bar{q}^{n+1} = \bar{q}^n - \Delta t\, L_i^{n+1/2}(\bar{q})$$

$$\partial_t q = L(q)$$

$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi\, dx\, dt$$

$$\bar{q}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t^n)\,dx$$

$$F_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt$$

$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi \, dx\,dt$$

$$\bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}\Delta t$$

$$\bar{q}^{n+1} = \bar{q}^n - \Delta t\, L_i^{n+1/2}(\bar{q})$$

$$\partial_t q = L(q)$$

$$\bar{q}_i^{n} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t^n)\,dx$$

$$\bar{q}_i^{n+1} = \bar{q}_i^{n} - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \bar{\psi}_i^{n+1/2}\,\Delta t$$

$$\bar{F}_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt$$

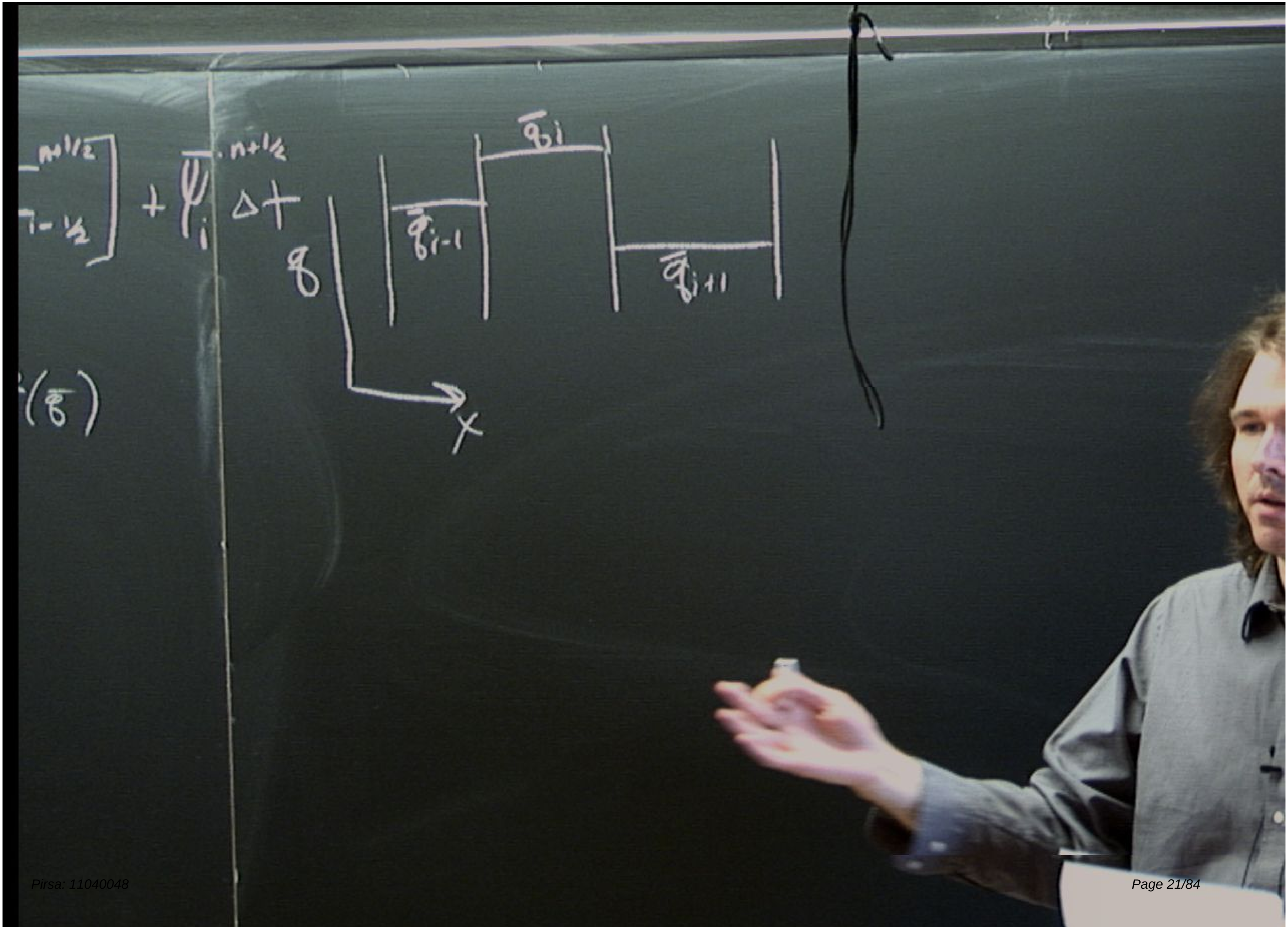
$$\bar{q}^{n+1} = \bar{q}^{n} - \Delta t\, L_i^{n+1/2}(\bar{q})$$

$$\partial_t q = L(q)$$

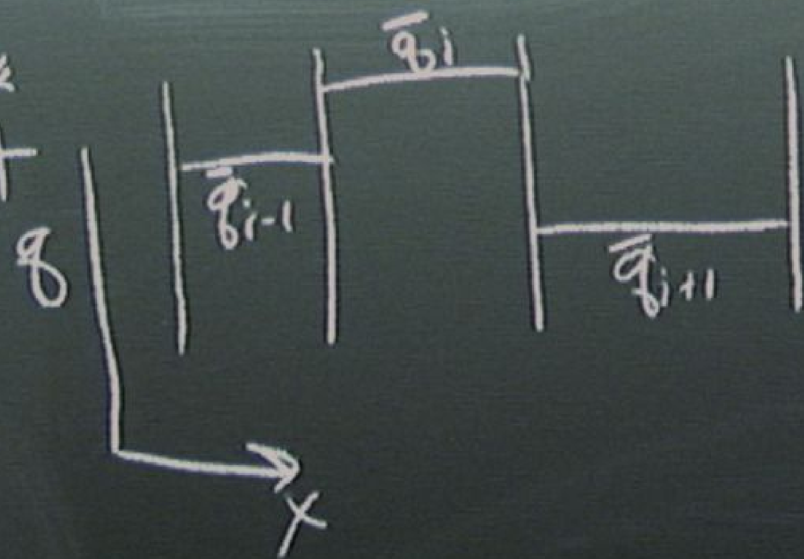
$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t}\psi\,dx\,dt$$

"Method of Lines"

MOL


$+ \bar{\psi}_i^{\,n+1/2} \Delta t$
q
$\bar{q}_{i-1}$
$\bar{q}_i$
$\bar{q}_{i+1}$
x

$$\left.\ldots^{n+1/2} - \ldots_{i-1/2}\right] + \bar{\psi}_i^{\,n+1/2}\,\Delta t$$



$\ldots(\bar{q})$

$$\bar{q}_i^{\,n} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t^n)\,dx$$

$$F_{i+1/2}^{\,n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt$$

$$\bar{\psi}_i^{\,n+1/2} = \frac{1}{\Delta x\,\Delta t}\int_{\Delta x}\int_{\Delta t}\psi\,dx\,dt$$

Godunov's Method

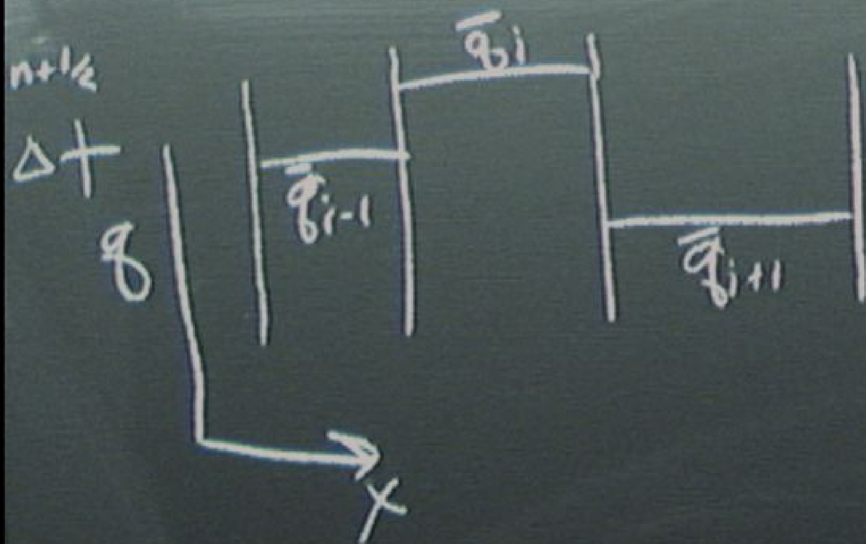
$$\bar{q}_i^{\,n+1} = \bar{q}_i^{\,n} - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{\,n+1/2} - F_{i-1/2}^{\,n+1/2}\right] + \bar{\psi}_i$$

$$\bar{q}^{\,n+1} = \bar{q}^{\,n} - \Delta t\, L_i^{\,n+1/2}(\bar{q})$$

$$\partial_t q = L(q)$$

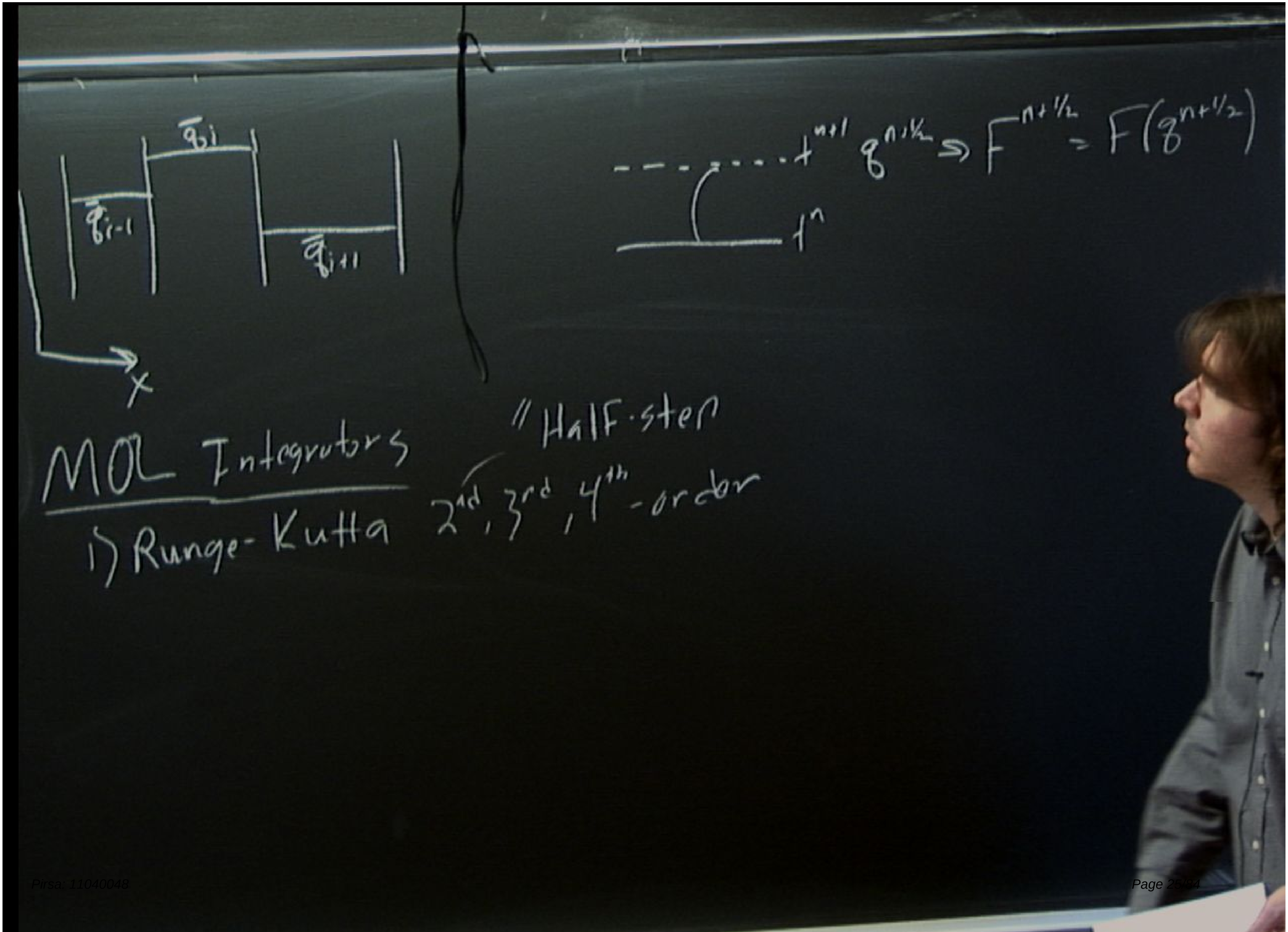
"Method of Lines"

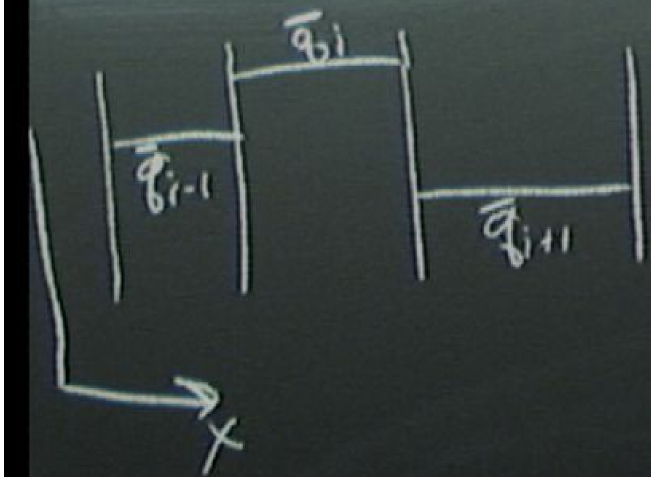
MOL

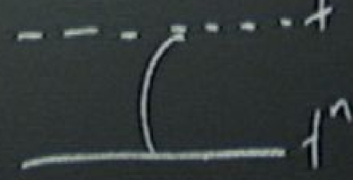


MOL Integrators

1) Runge-Kutta 2nd, 3rd, 4th-order


$\bar{q}_{i-1}$
$\bar{q}_i$
$\bar{q}_{i+1}$
x
t^{n+1} $q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$
t^n
"Half-step
MOL Integrators
1) Runge-Kutta 2nd, 3rd, 4th-order





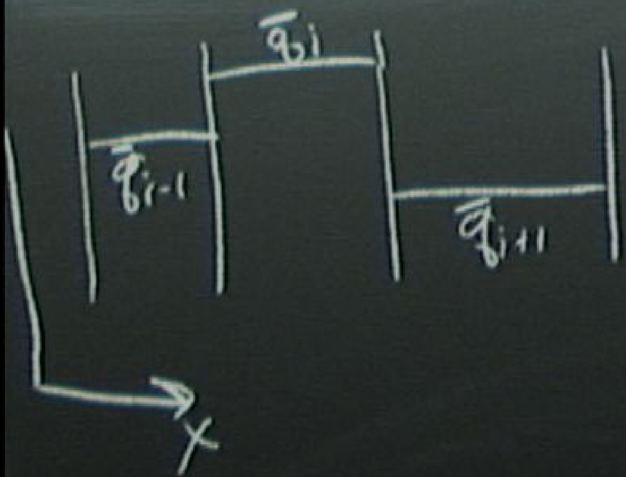
$q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

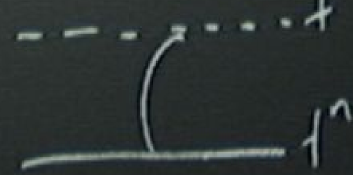
$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

MOL Integrators

"Half-step"

1) Runge-Kutta $2^{nd}, 3^{rd}, 4^{th}$-order





$q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

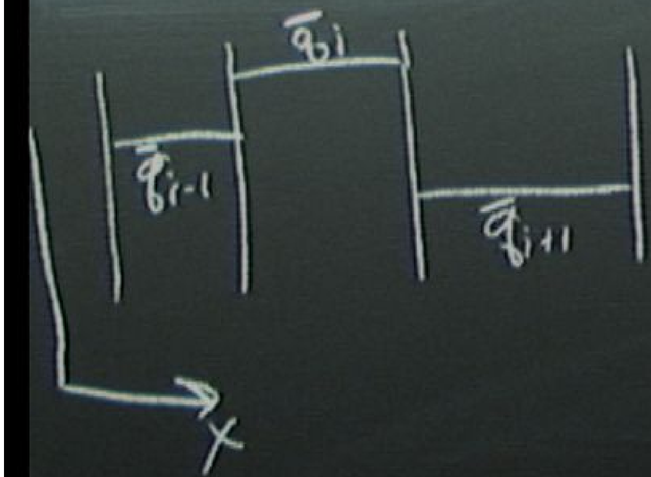
$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

MOL Integrators

// Half-step

1) Runge-Kutta 2nd, 3rd, 4th-order

2) Predictor-Corrector
   Euler's Method



t^{n+1} $q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

t^n

$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

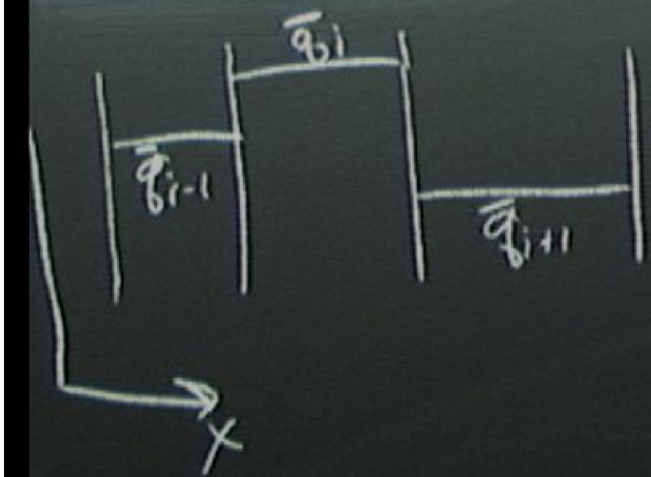
// Half-step

MOL Integrators

1) Runge-Kutta 2nd, 3rd, 4th-order

2) Predictor-Corrector
Euler's Method

$$\bar{\Psi}_i^{n+1/2} \simeq \Psi(\bar{q}_i^{n+1/2})$$



t^{n+1} $q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

t^n

$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$

"Half-step"

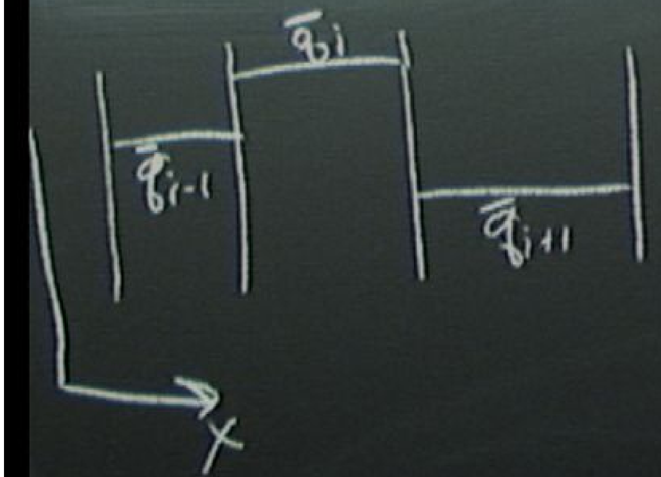
MOL Integrators

1) Runge-Kutta 2nd, 3rd, 4th-order

$\bar{\psi}_i^{n+1/2} \simeq \psi(\bar{q}_i^{n+1/2})$

2) Predictor-Corrector
Euler's Method

$q - \bar{q}$



$q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

"Half-step"

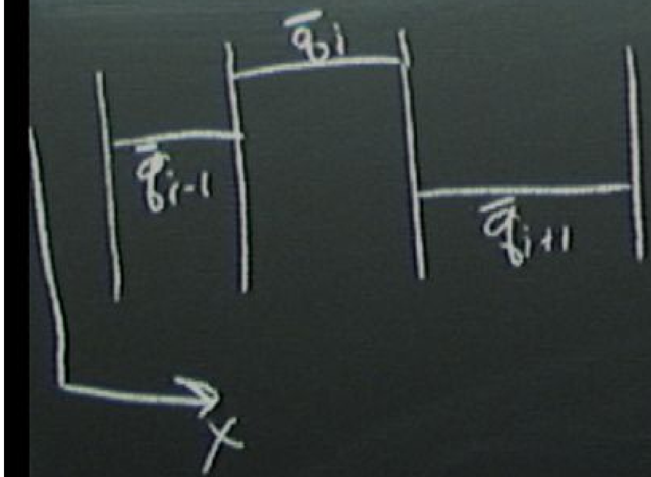
MOL Integrators

1) Runge-Kutta 2nd, 3rd, 4th-order

2) Predictor-Corrector
Euler's Method

$$\overline{\Psi}_i^{n+1/2} \simeq \Psi(\bar{q}_i^{n+1/2})$$

$$q - \bar{q} \sim \mathcal{O}($$



$$- - - - - - t^{n+1} \quad q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$$

$$t^n$$

$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

// Half-step

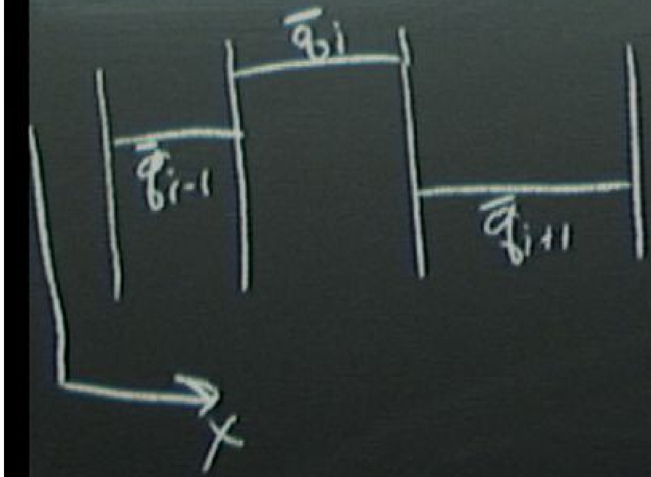
MOL Integrators

1) Runge-Kutta $2^{nd}, 3^{rd}, 4^{th}$-order

2) Predictor-Corrector
Euler's Method

$$\bar{\psi}_i^{n+1/2} \simeq \psi(\bar{q}_i^{n+1/2})$$

$$q - \bar{q} \sim \mathcal{O}($$



t^{n+1} $\quad q^{n+1/2} \Rightarrow F^{n+1/2} = F(q^{n+1/2})$

t^n

$$q^{n+1/2} = q^n - \frac{\Delta t}{2} L(q^n)$$

"Half-step"

MOL Integrators

1) Runge-Kutta 2nd, 3rd, 4th-order
2) Predictor-Corrector
   Euler's Method

$$\bar{\psi}_i^{n+1/2} \simeq \psi(\bar{q}_i^{n+1/2})$$

$$q - \bar{q} \simeq O(\Delta x^2) + O(\Delta t^2)$$

$$\bar{q}_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t^n)\,dx \qquad \bar{q}_i^{n+1} = \bar{q}_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right] + \ldots$$

$$F_{i+1/2}^{n+1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(x_{i+1/2},t)\,dt$$

"Numerical Flux"

$$\bar{q}^{n+1} = \bar{q}^n - \Delta t\, L_i^{n+1/2}(\bar{q})$$

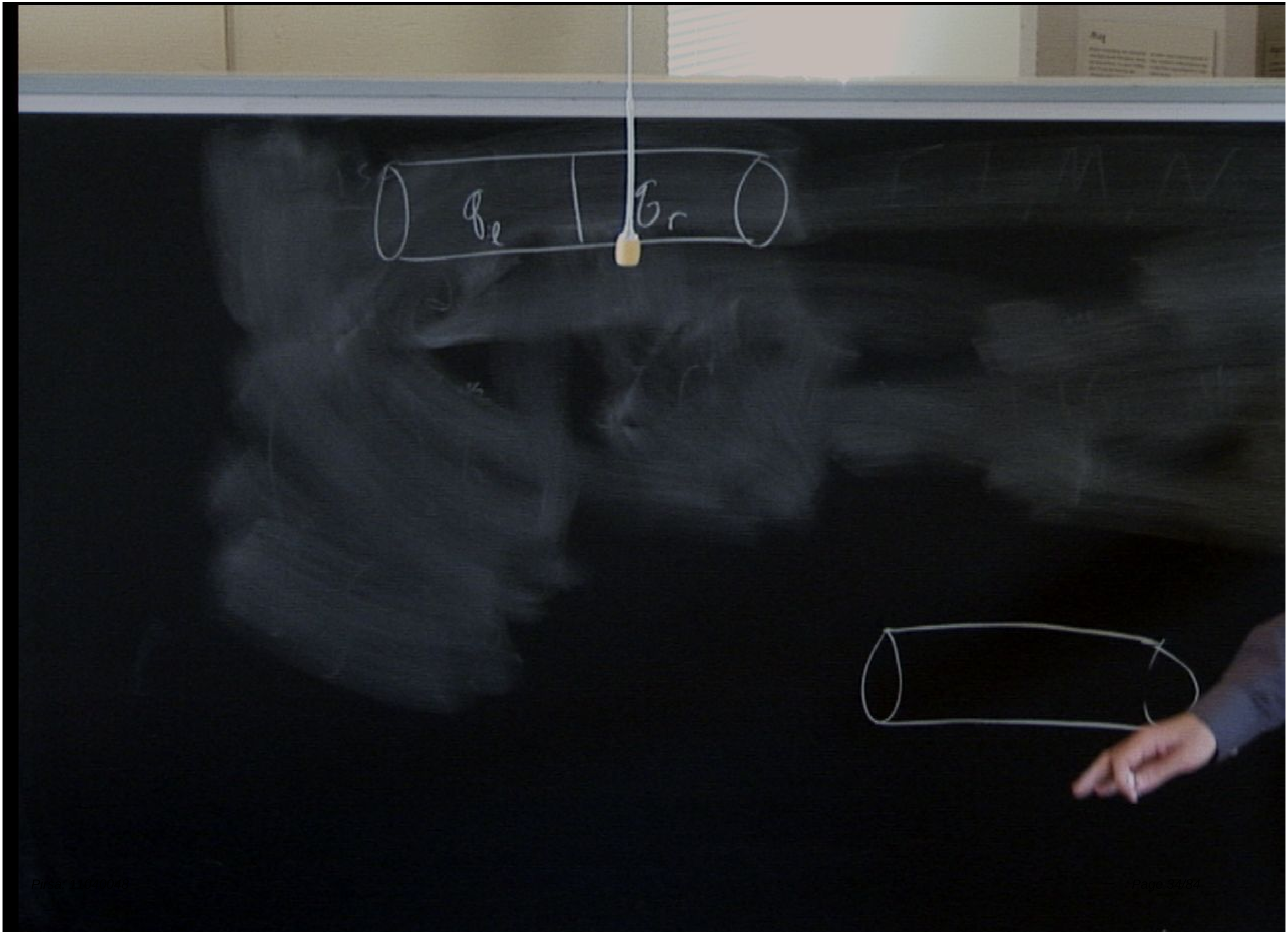
$$\partial_t q = L(q)$$

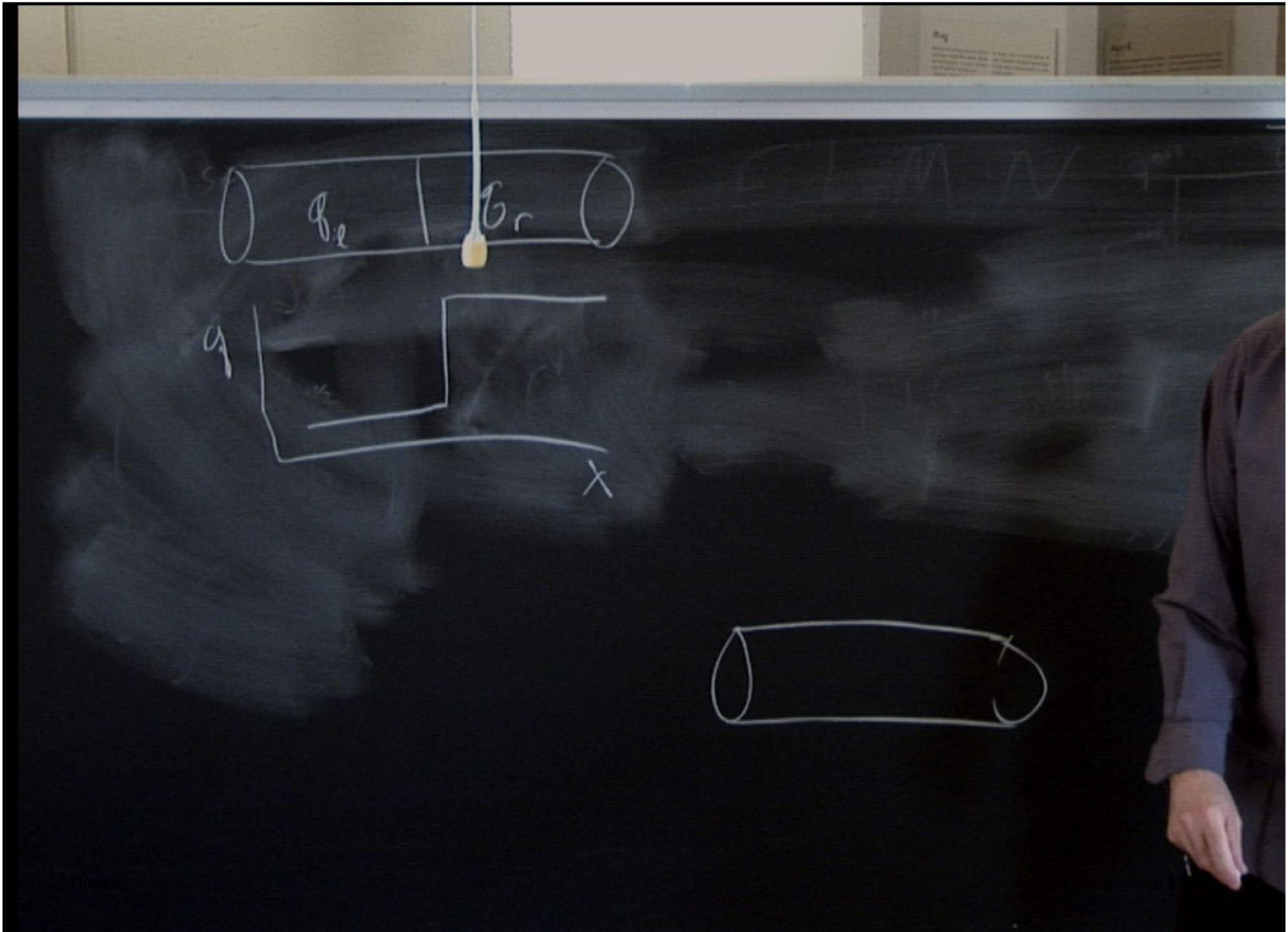
$$\bar{\psi}_i^{n+1/2} = \frac{1}{\Delta x \Delta t}\int_{\Delta x}\int_{\Delta t} \psi \, dx\, dt$$

Godunov's Method

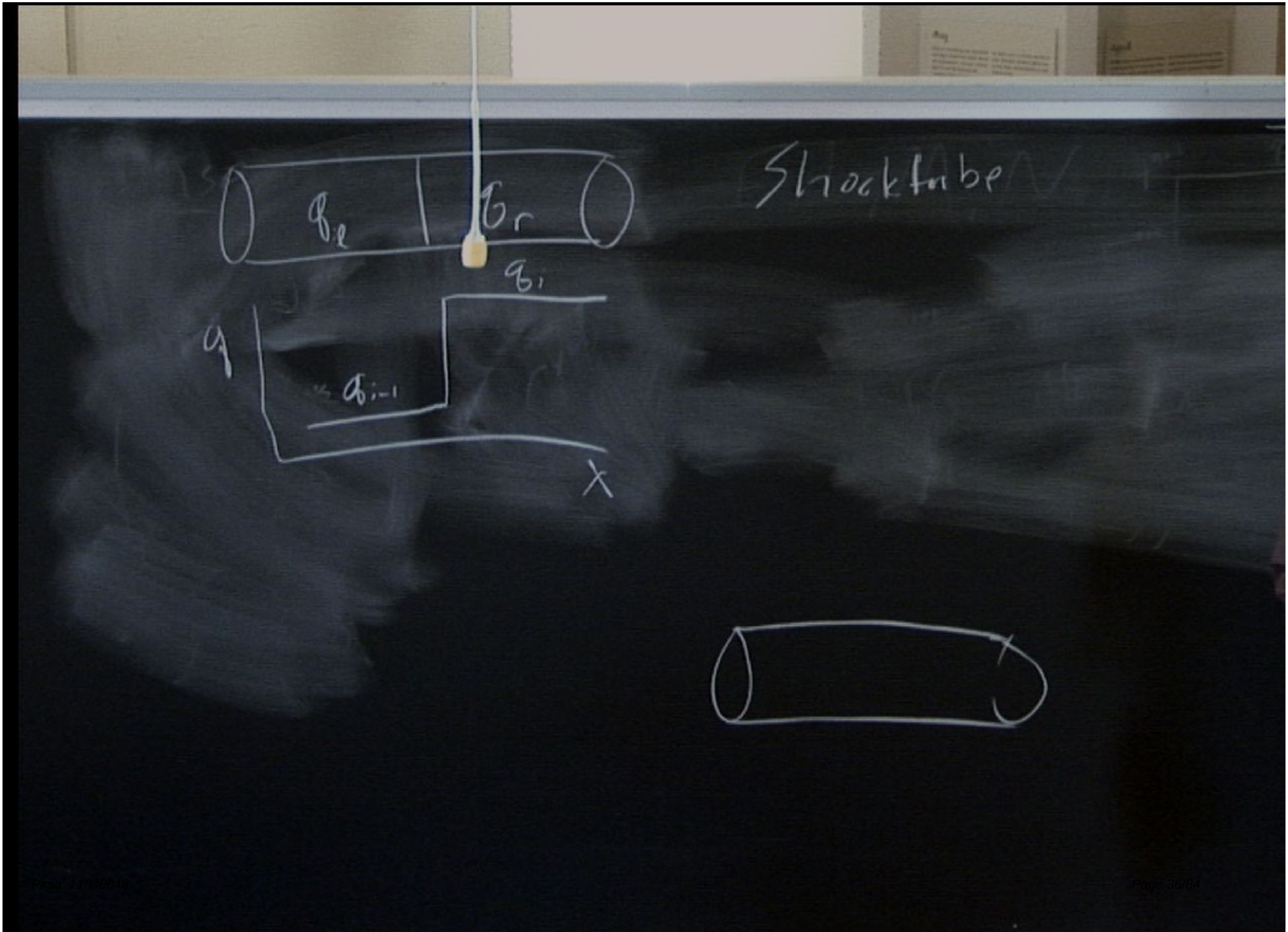
"Method of Lines"

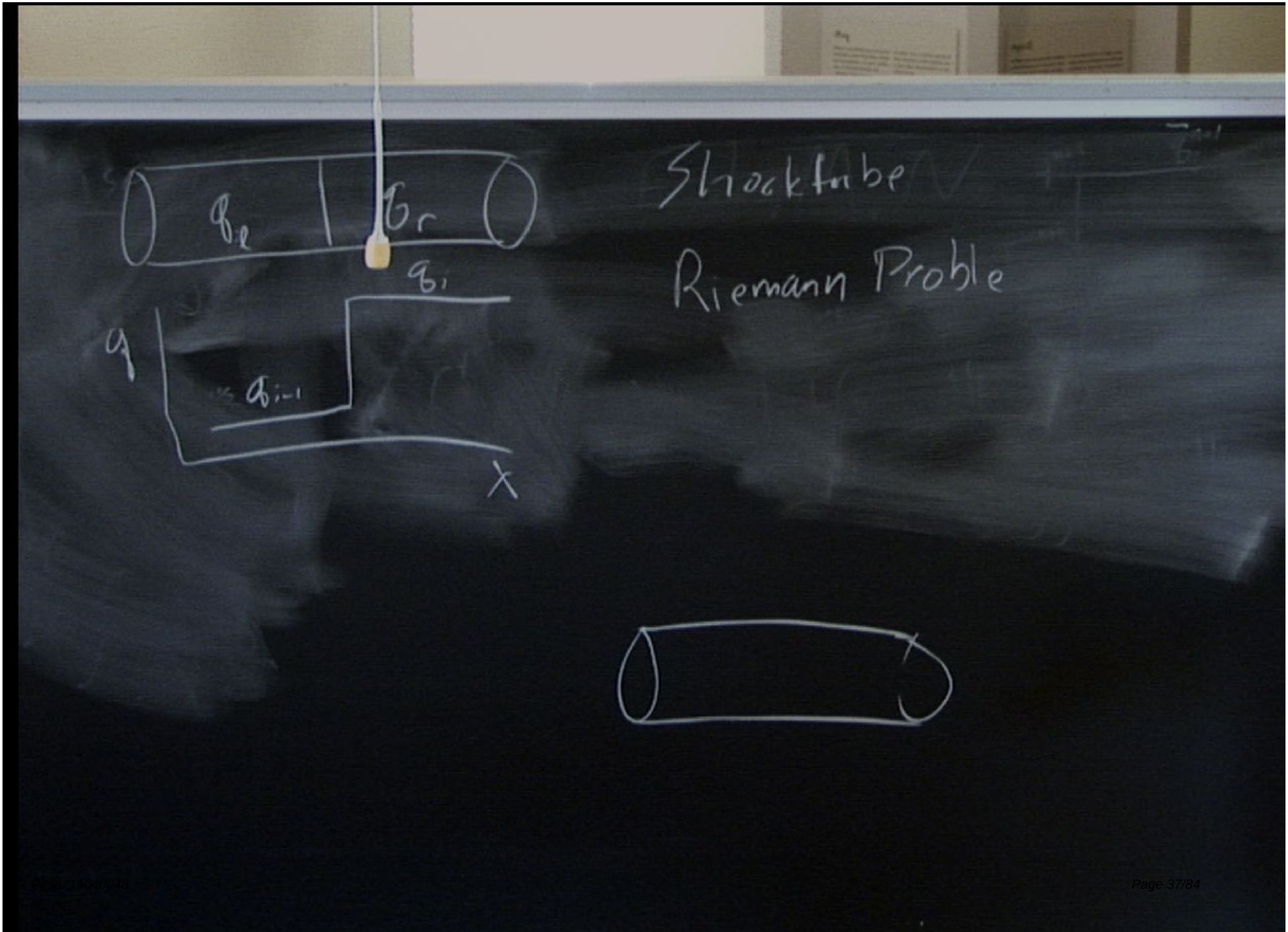
MOL

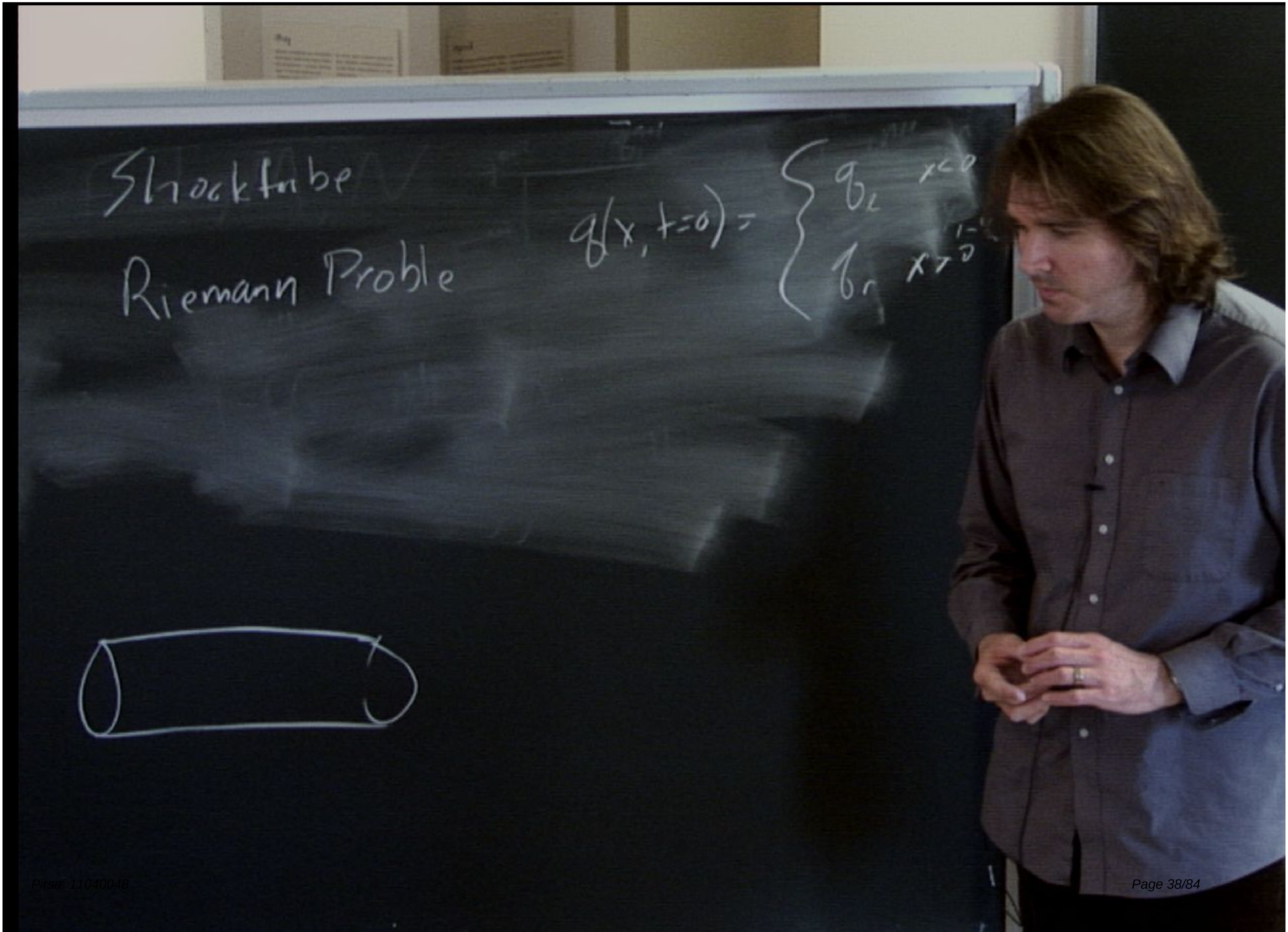

q_ℓ
q_r

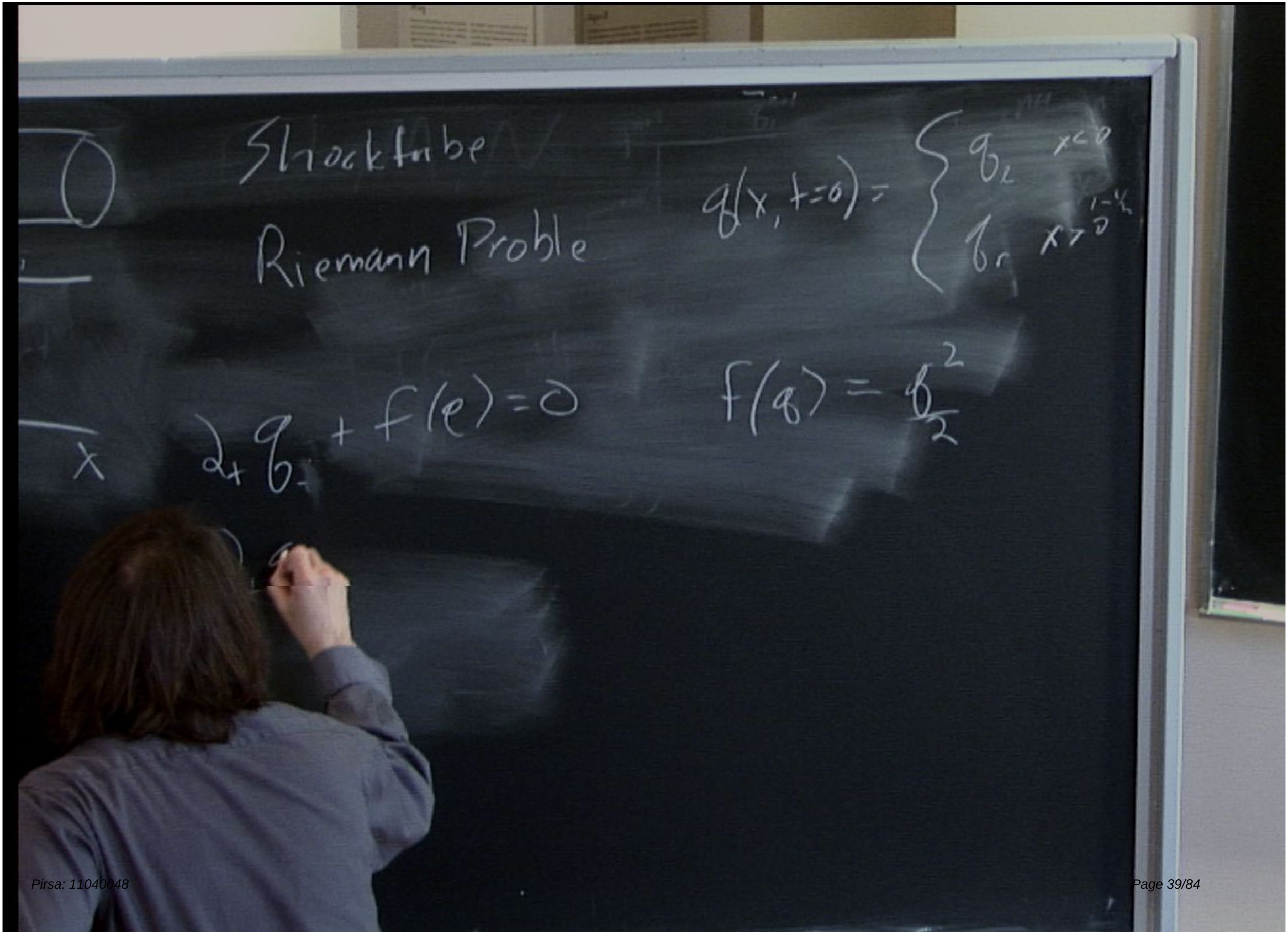


q_l
q_r
q
x


Shocktube
q_l
q_r
q_i
q
q_{i-1}
x


Shocktube
Riemann Proble
q_l
q_r
q_i
q_{i-1}
q
x


Shocktube
Riemann Proble
$q(x, t=0) = \begin{cases} q_L & x<0 \\ q_r & x>0 \end{cases}$


Shocktube
Riemann Proble
q(x, t=0) = q_l x<0
q_r x>0
f(q) = q^2/2

Shocktube

Riemann Proble

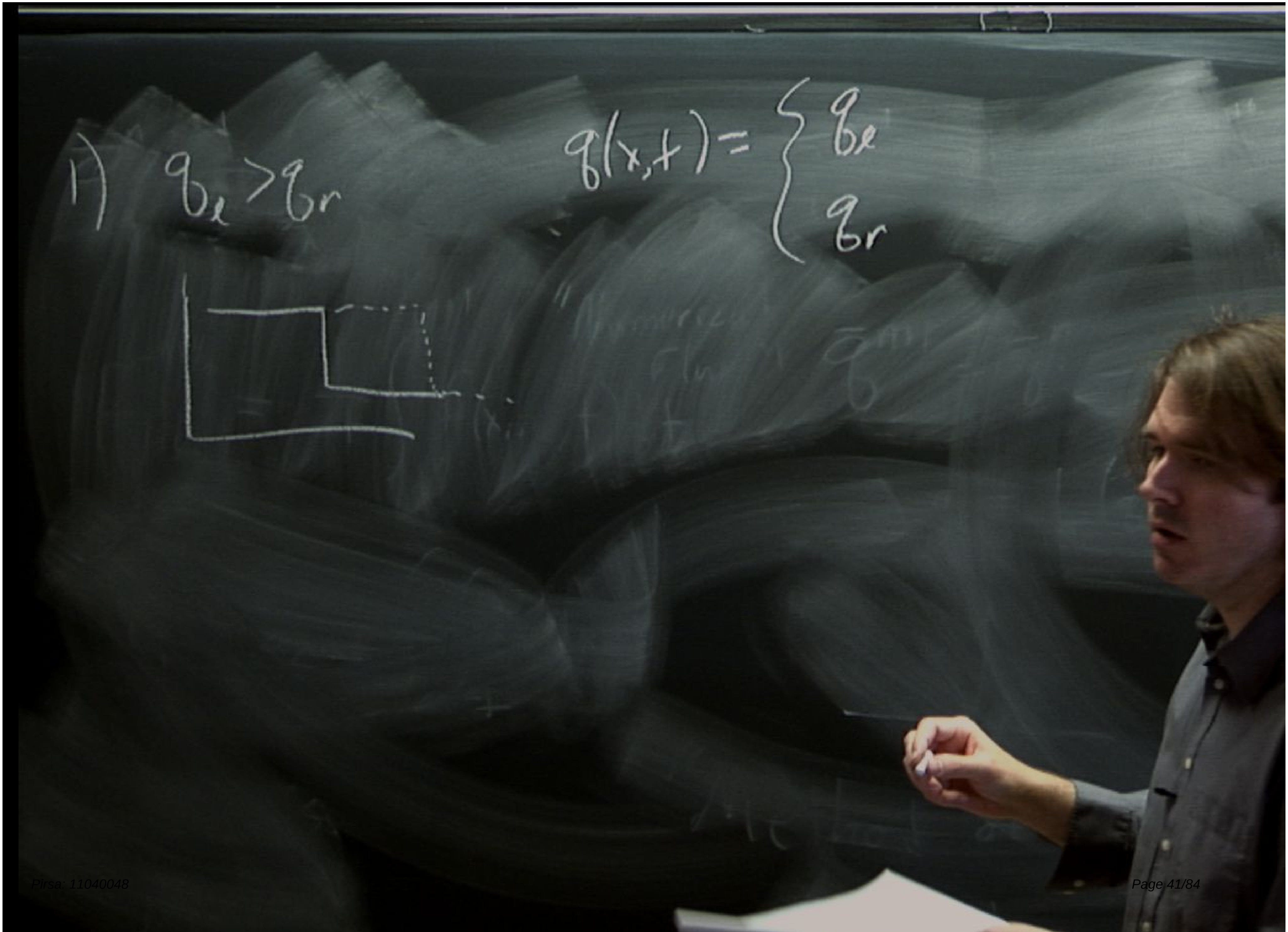
$$q(x, t=0) = \begin{cases} q_l & x<0 \\ q_r & x>0 \end{cases}$$

x

$$\partial_t q + f(q) = 0$$

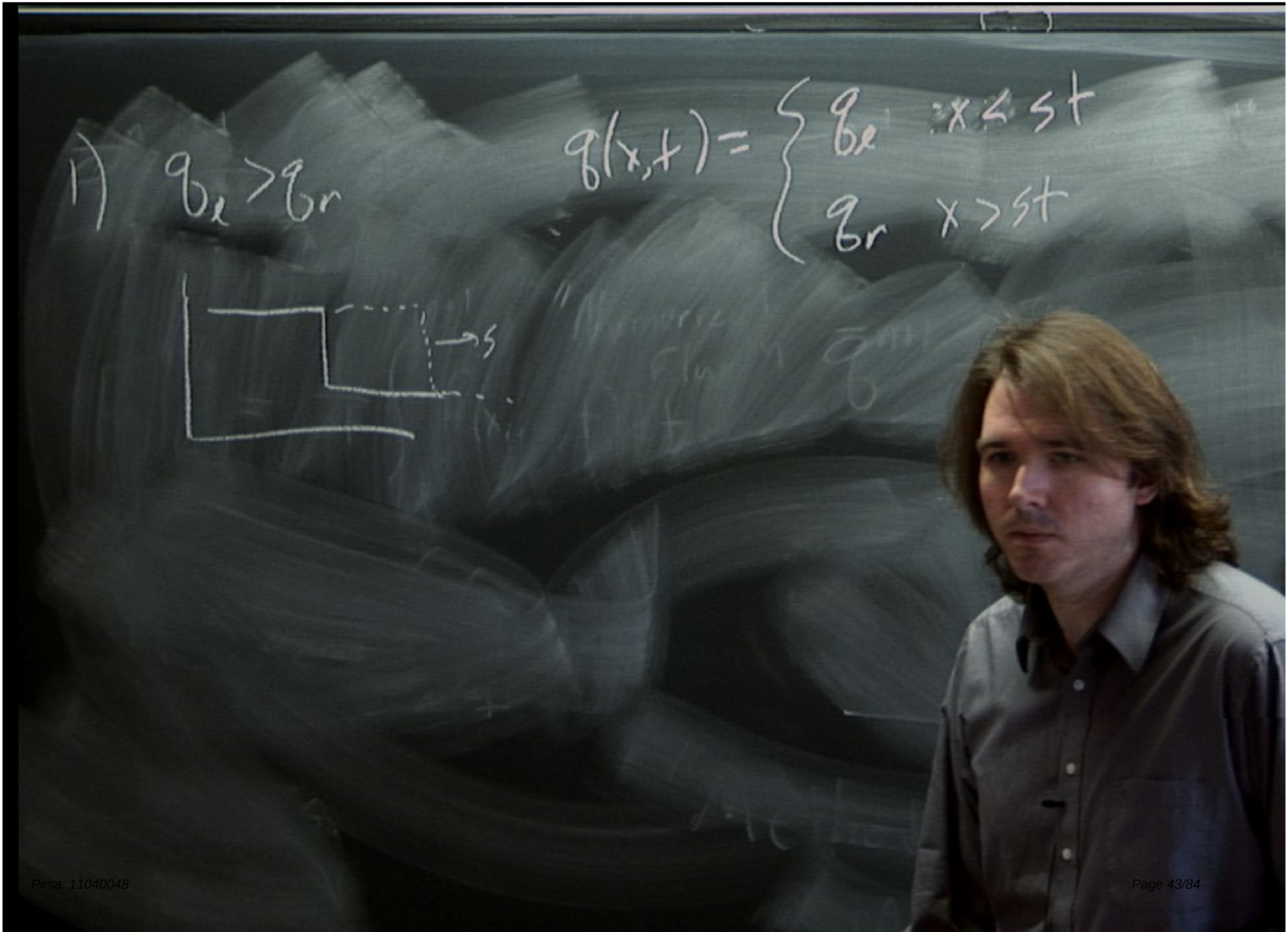
$$f(q) = \frac{q^2}{2}$$

$$\partial_t q + q\,\partial_x q = 0$$


1) q_l > q_r
q(x,t) = { q_l
q_r

1) $q_l > q_r$

$$q(x,t) = \begin{cases} q_l & x < st \\ q_r & x > st \end{cases}$$

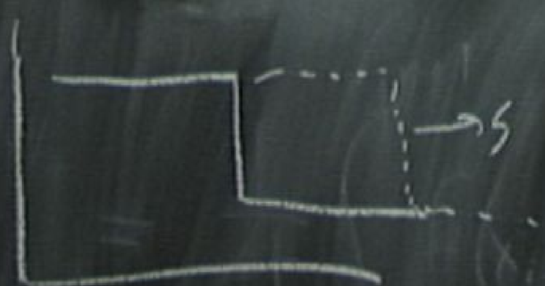

1) $q_l > q_r$
$q(x,t) = \begin{cases} q_l & x < st \\ q_r & x > st \end{cases}$
s

1) $q_\ell > q_r$

$$q(x,t) = \begin{cases} q_\ell & x < st \\ q_r & x > st \end{cases}$$

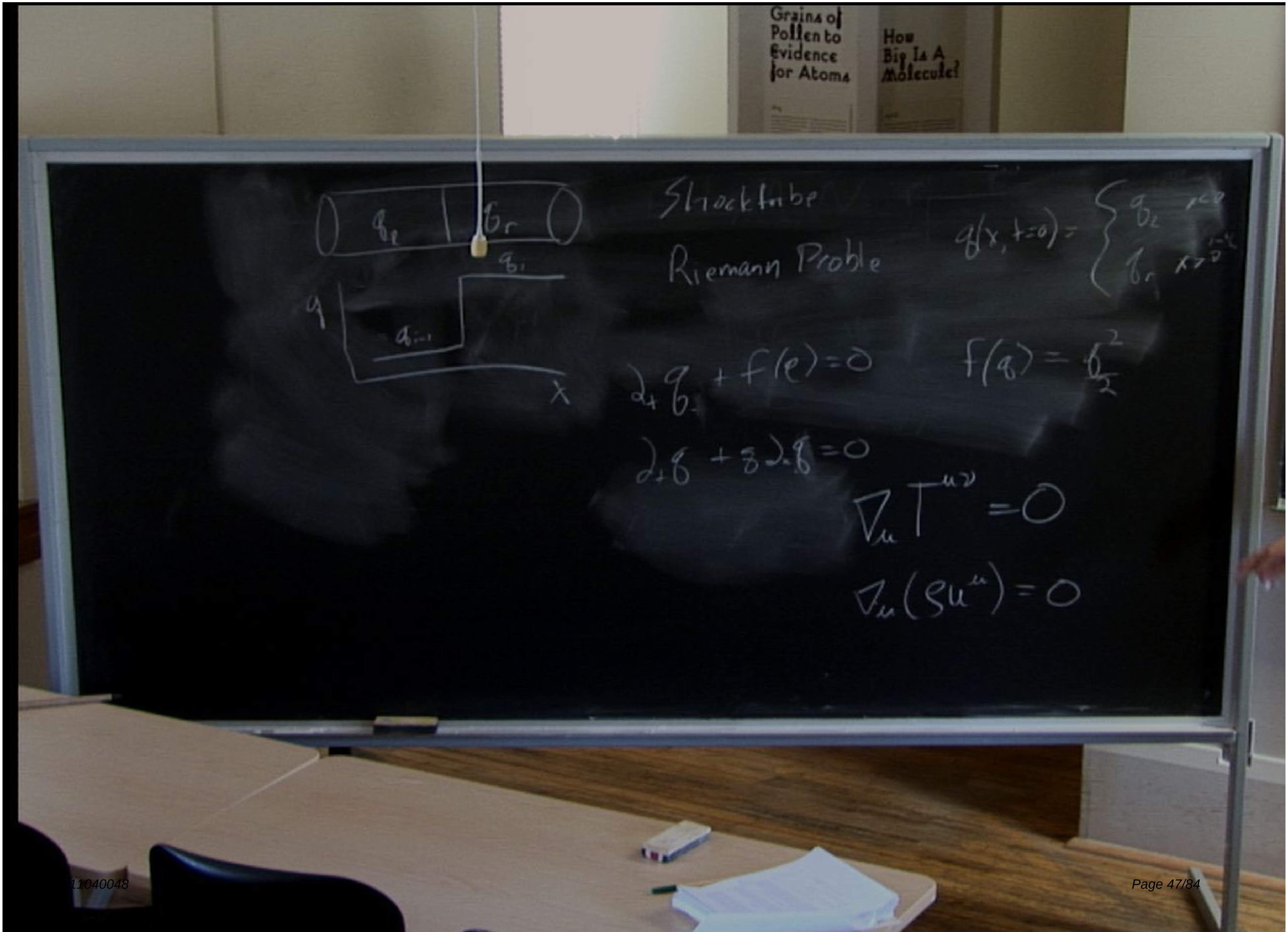
1) $q_\ell > q_r$

$$q(x,t) = \begin{cases} q_\ell & x < st \\ q_r & x > st \end{cases}$$



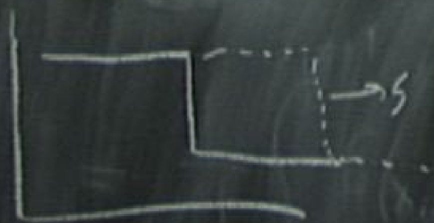
1) $q_l > q_r$

$$q(x,t) = \begin{cases} q_l & x < st \\ q_r & x > st \end{cases}$$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Shocktube
Riemann Proble
∂t q + f(q) = 0
∂t q + q ∂x q = 0
f(q) = q²/2
∇μ T^μν = 0
∇μ (ρ u^μ) = 0

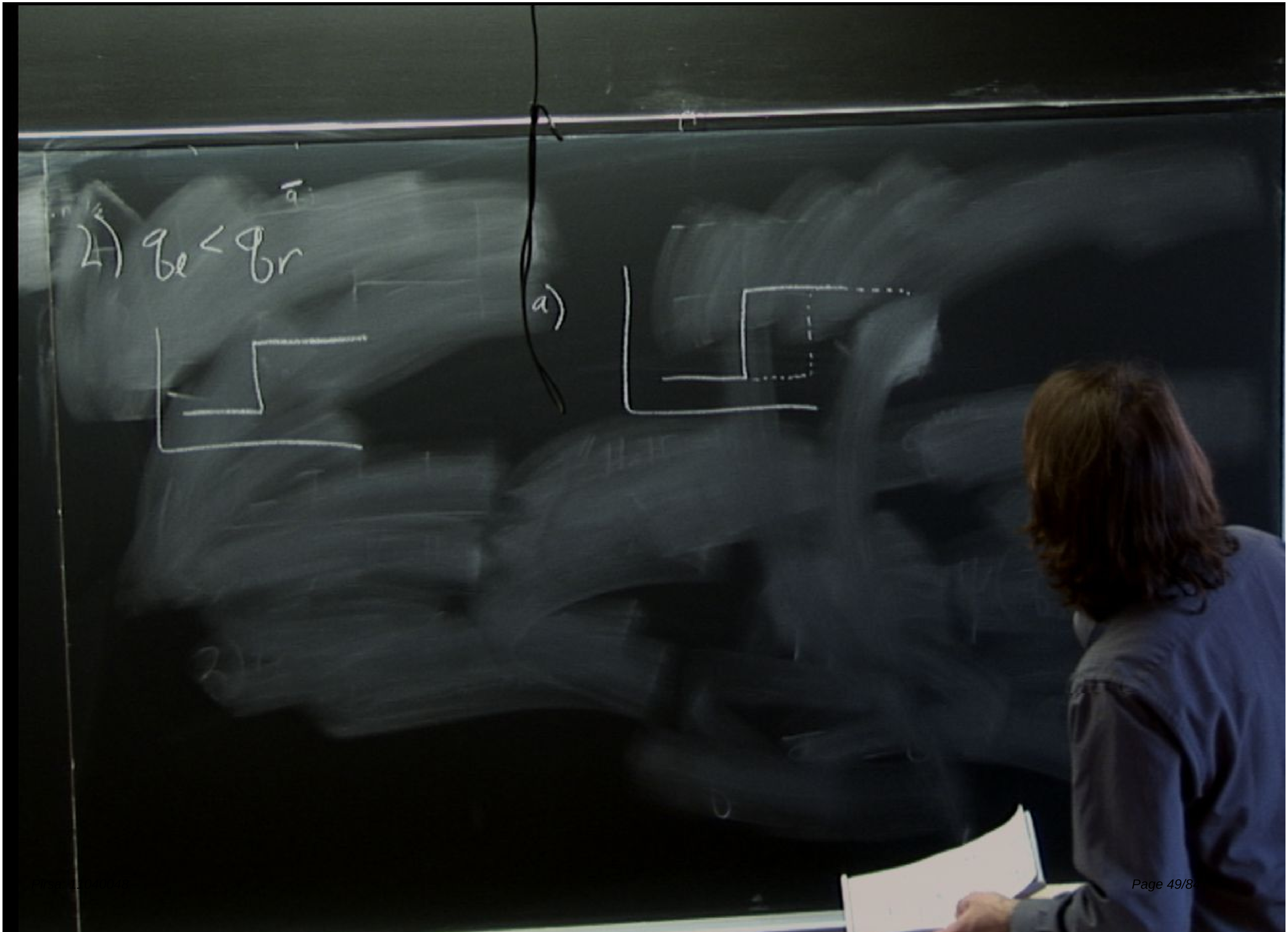
1) $q_l > q_r$

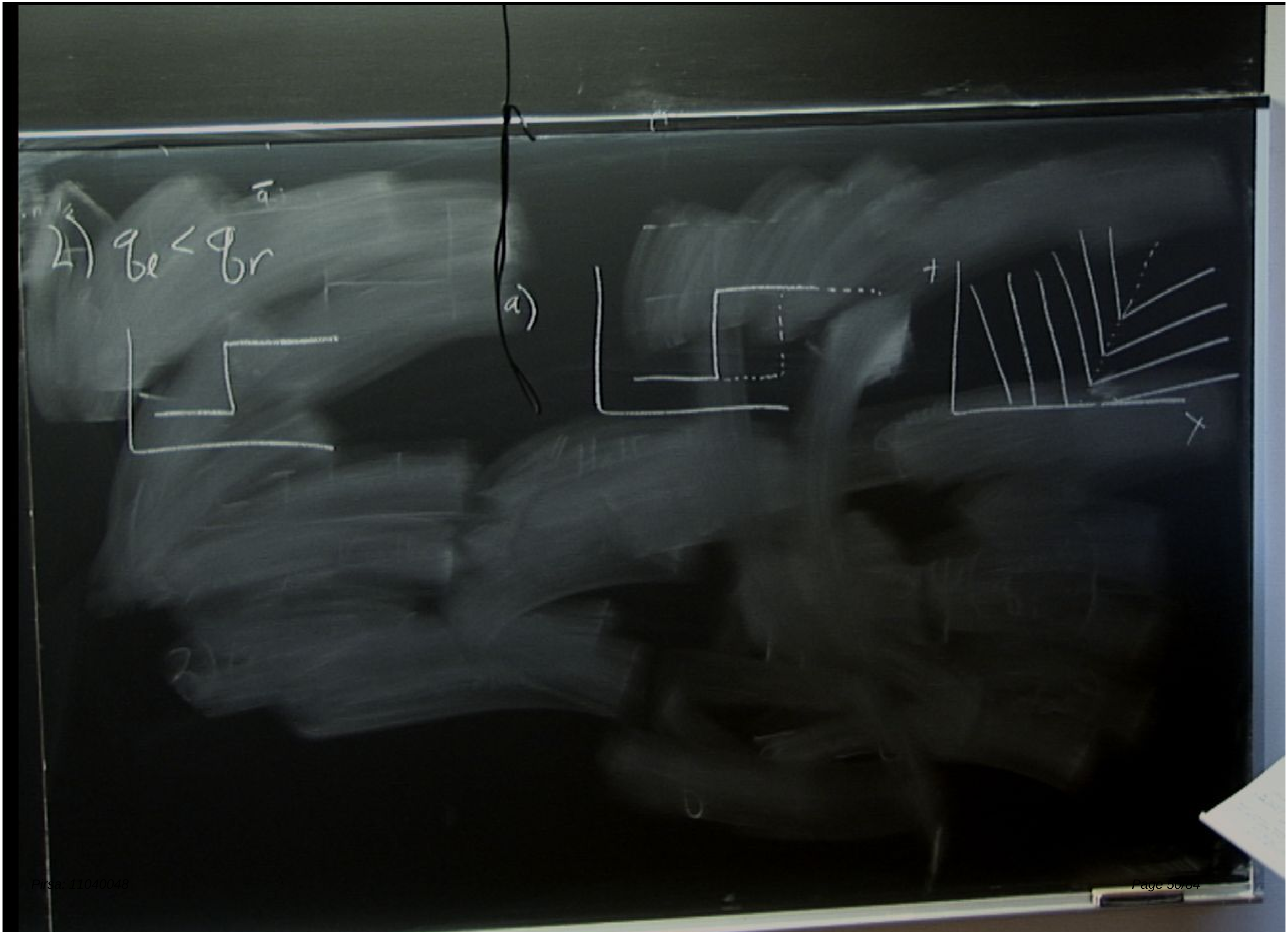
$$q(x,t) = \begin{cases} q_l & x < st \\ q_r & x > st \end{cases}$$

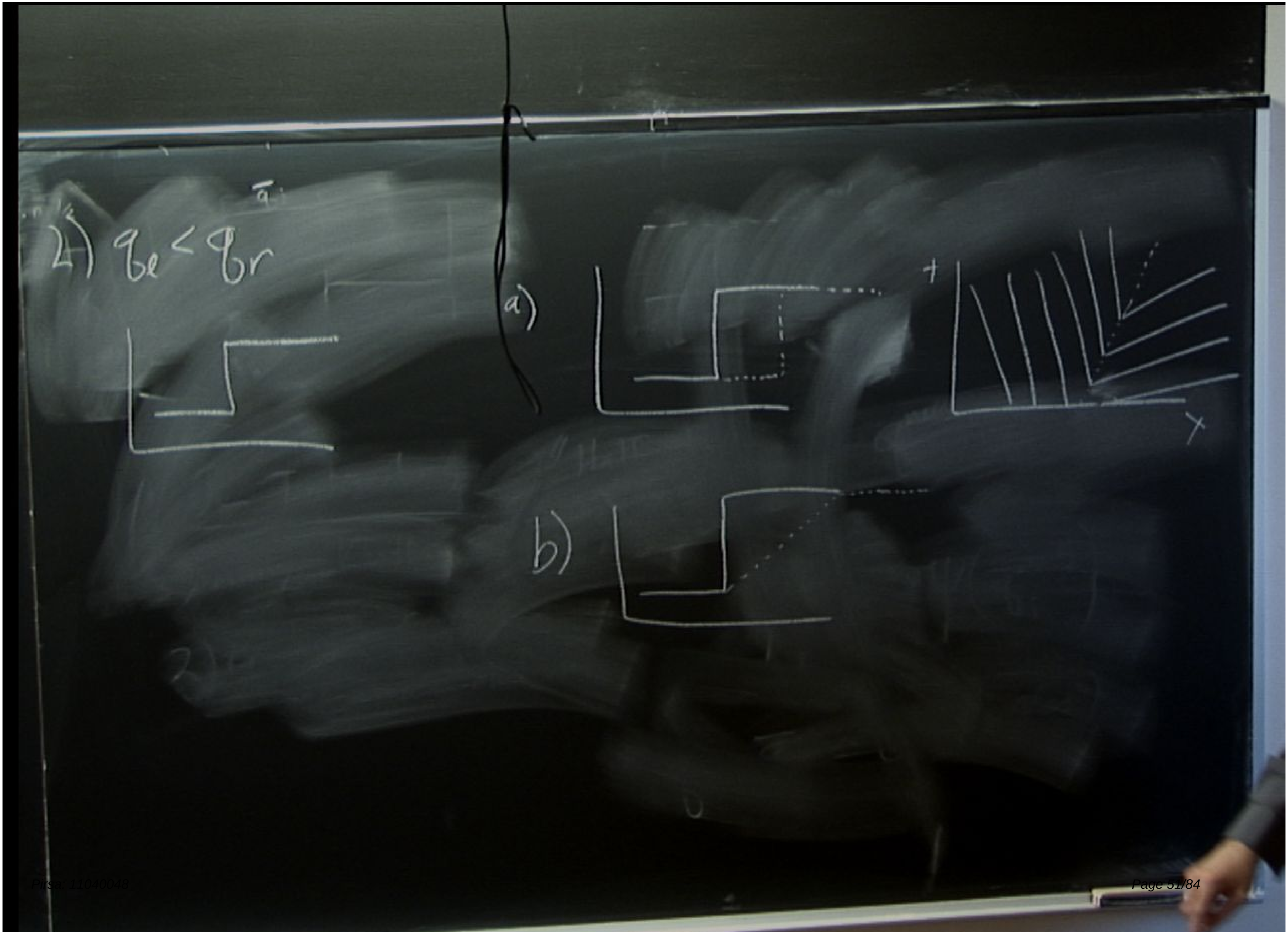


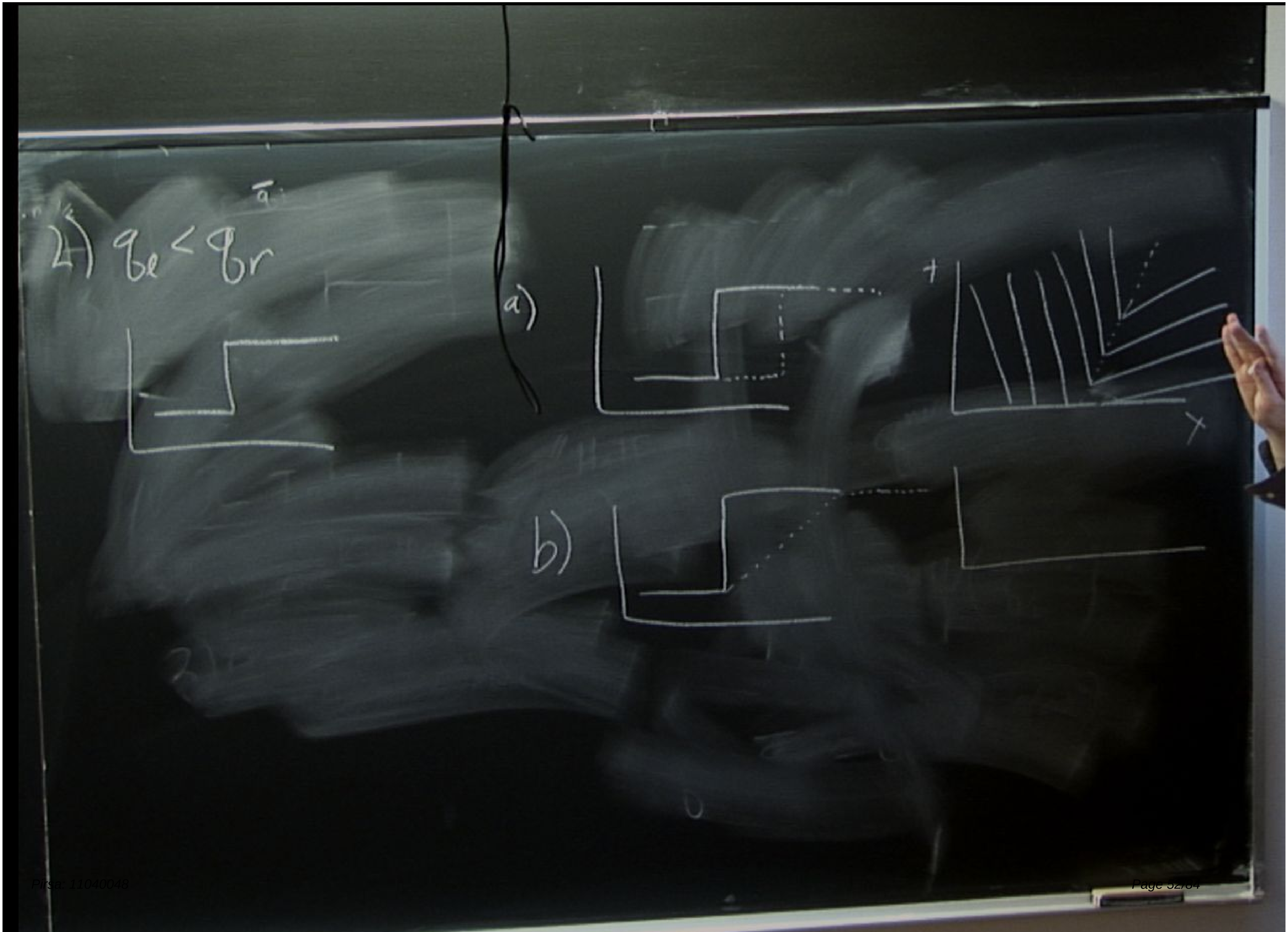
Rankine-Hugoniot :

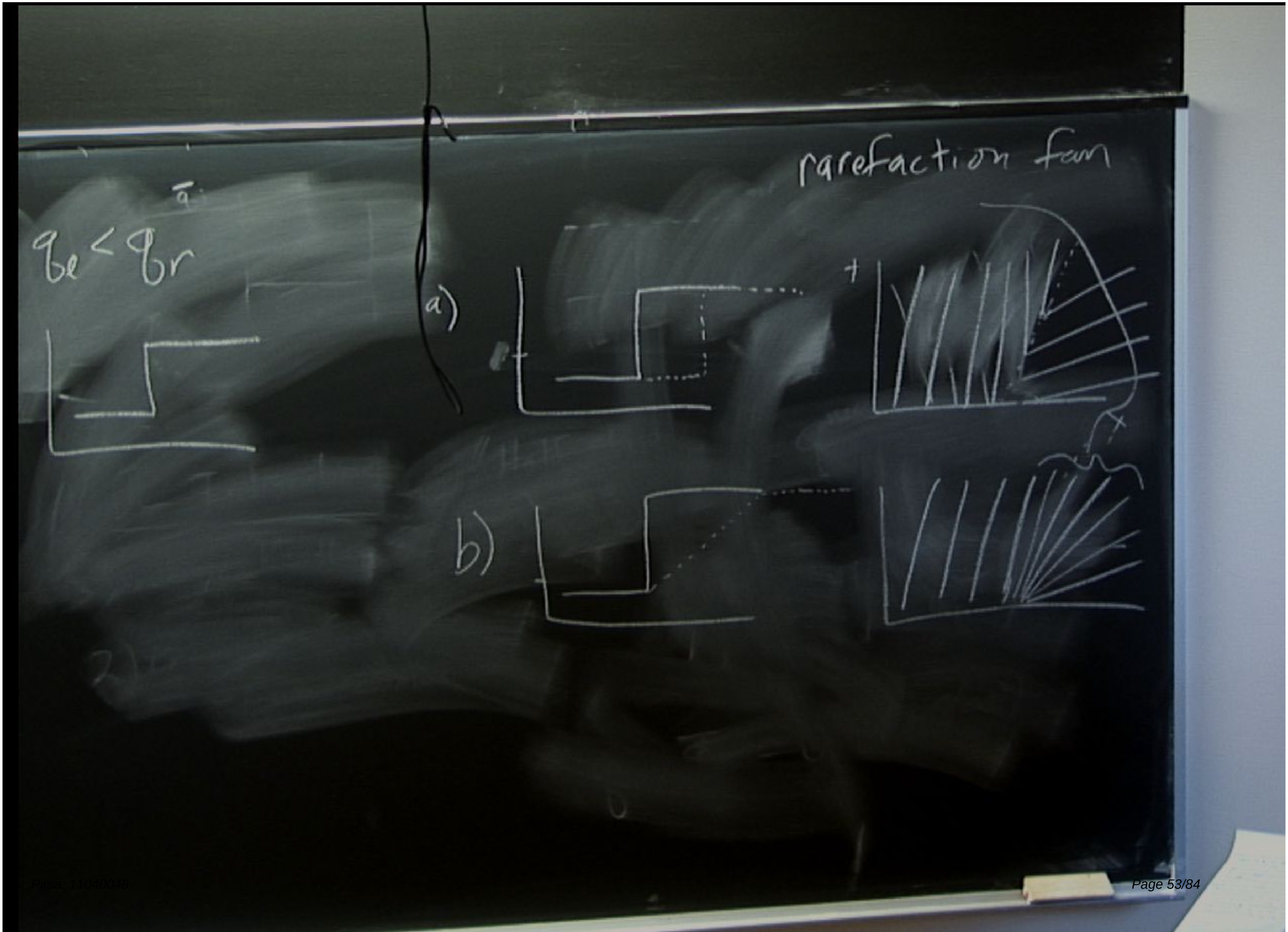
$$f(q_r) - f(q_l) = s(q_r - q_l)$$


4) $q_e < q_r$
a)


2) q_e < q_r
a)
t
x


2) q_l < q_r
a)
b)
t
x

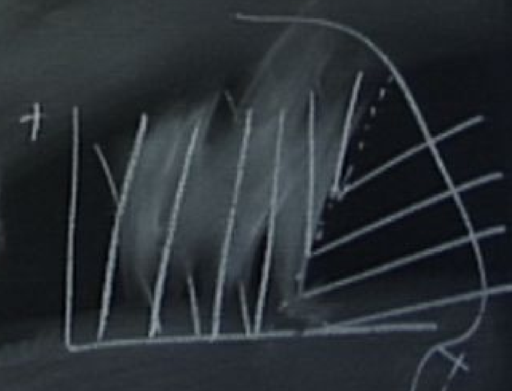

2) $q_e < q_r$
a)
b)
t
x


rarefaction fan
ql < qr
a)
b)

rarefaction fan

$q_\ell < q_r$

a)



b)

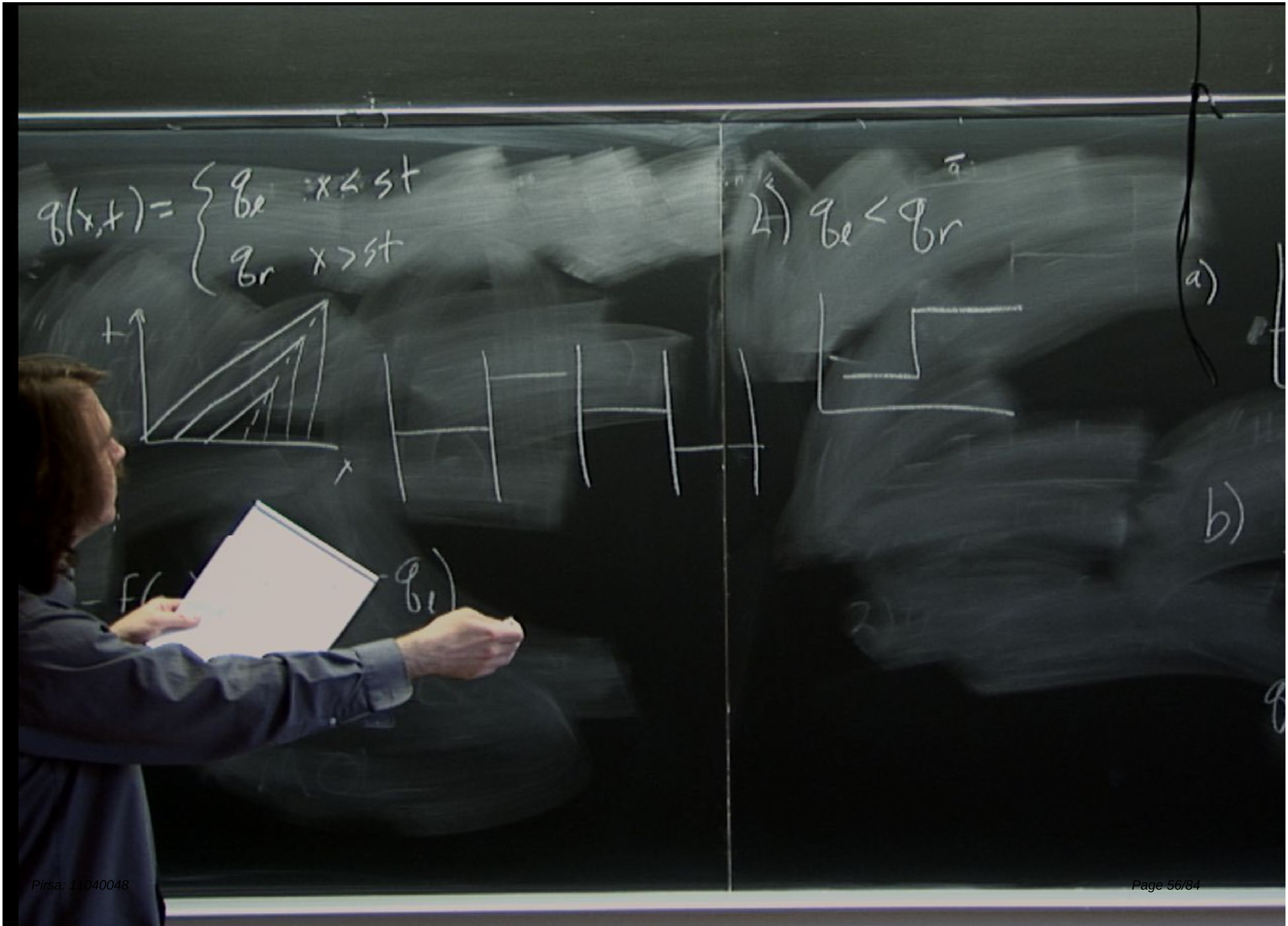
$$q(x,t) = \begin{cases} q_\ell & x < q_\ell t \\ x/t & q_\ell t < x < q_r t \\ q_r & x > q_r t \end{cases}$$

rarefaction fan

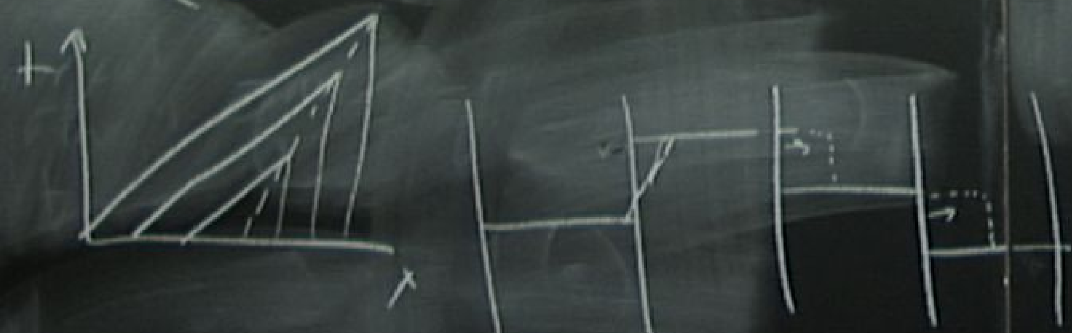
a)

b)

$$q(x,t) = \begin{cases} q_l & x < q_l t \\ x/t & q_l t < x < q_r t \\ q_r & x > q_r t \end{cases}$$


q(x,t) = { q_l x ≤ st
q_r x > st
2) q_l < q_r
a)
b)

$$q(x,t) = \begin{cases} q_l & x < st \\ q_r & x > st \end{cases}$$



$$f(q_l) = s(q_r - q_l)$$

2) $q_l < q_r$

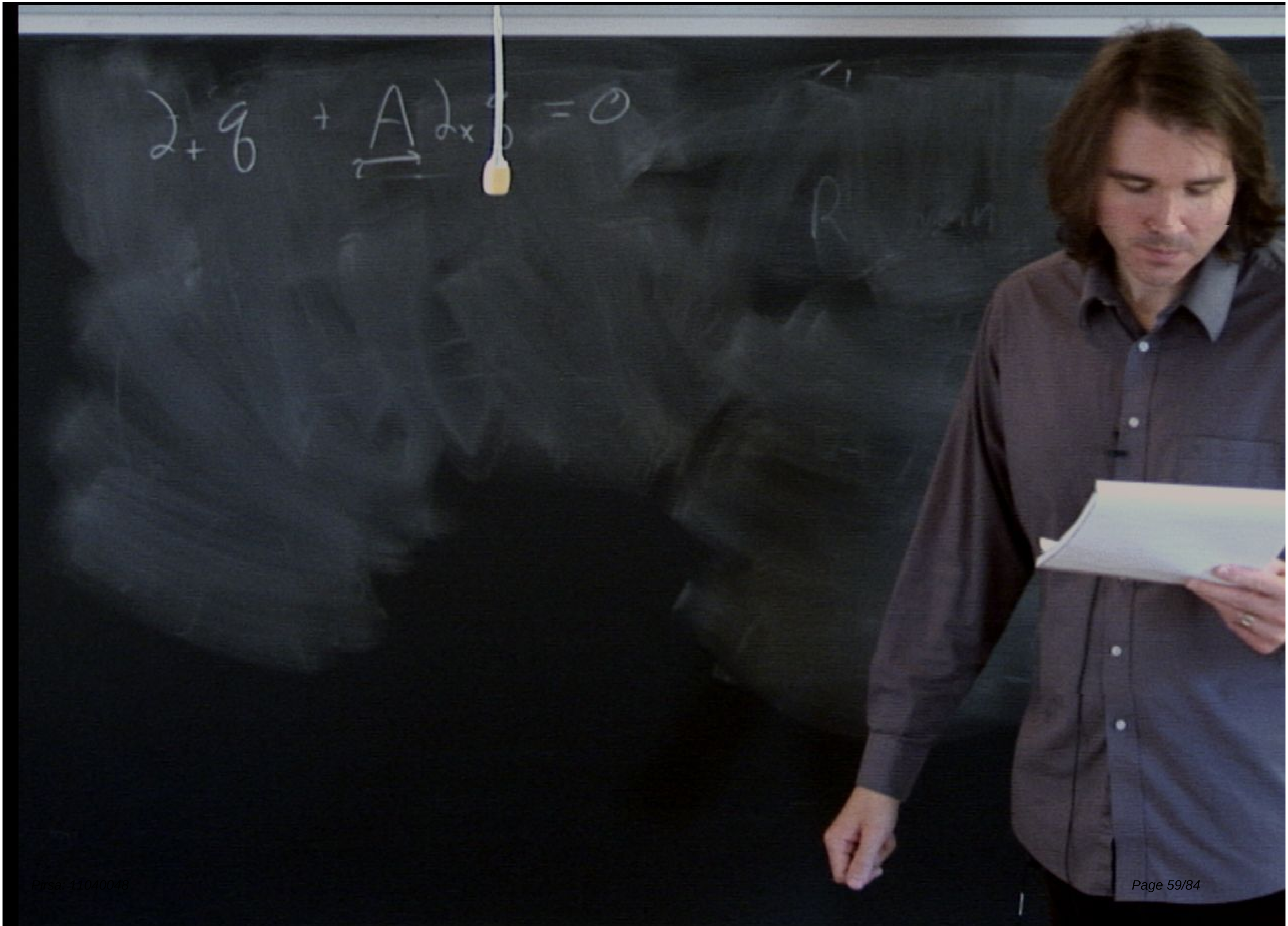
a)

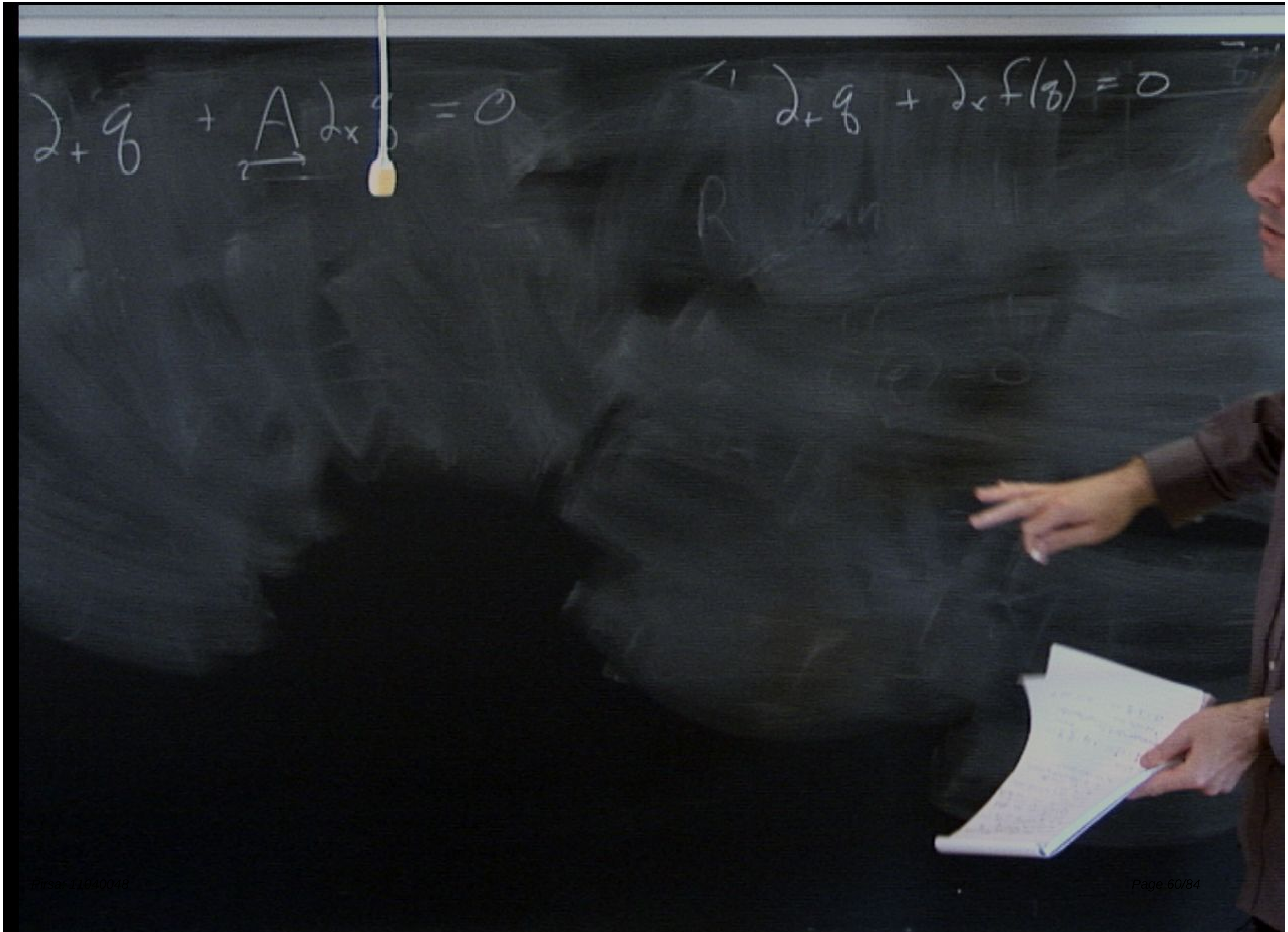
b)

$x > st$

$4)\ q_l < q_r$

$(q_r - q_l)$


$\partial_t q + \underline{A}\, \partial_x q = 0$
R



$\partial_t q + \underline{A}\,\partial_x q = 0$
$\partial_t q + \partial_x f(q) = 0$

$$\partial_t q + \underline{A}\, \partial_x q = 0$$

$$\partial_t q + \partial_x f(q) = 0$$

1) $A(q_l, q_r)(q_r - q_l) = f(q_r) - f(q_l)$

$$\partial_t q + \underline{A}\, \partial_x q = 0 \qquad \partial_t q + \partial_x f(q) = 0$$

1) $A(q_l, q_r)(q_r - q_l) = f(q_r) - f(q_l)$

2) A is diagonalizable $\lambda_\alpha \in \mathbb{R}$

$$\partial_t q + \underline{A}\, \partial_x q = 0$$

$$\partial_t q + \partial_x f(q) = 0$$

1) $A(q_l, q_r)(q_r - q_l) = f(q_r) - f(q_l)$

2) A is diagonalizable $\lambda_\alpha \in \mathbb{R}$

3) $A \to \dfrac{\partial f}{\partial q}$

$$\partial_t q + \underline{A}\, \partial_x q = 0 \qquad \partial_t q + \partial_x f(q) = 0$$

1) $A(q_l, q_r)(q_r - q_l) = f(q_r) - f(q_l)$

2) A is diagonalizable $\lambda_\alpha \in \mathbb{R}$

$$q_l, q_r \to \bar{q}$$

3) $A \to \dfrac{\partial f}{\partial q}$

$$\partial_t q + \partial_x f(q) = 0$$

$$= f(q_r) - f(q_l)$$

$\ldots$lizable $\lambda_\alpha \in \mathbb{R}$

$$q_l, q_r \to \bar{q}$$

$$A(q_l, q_r) = \left.\frac{\partial f}{\partial q}\right|_{q = \frac{1}{2}(q}$$

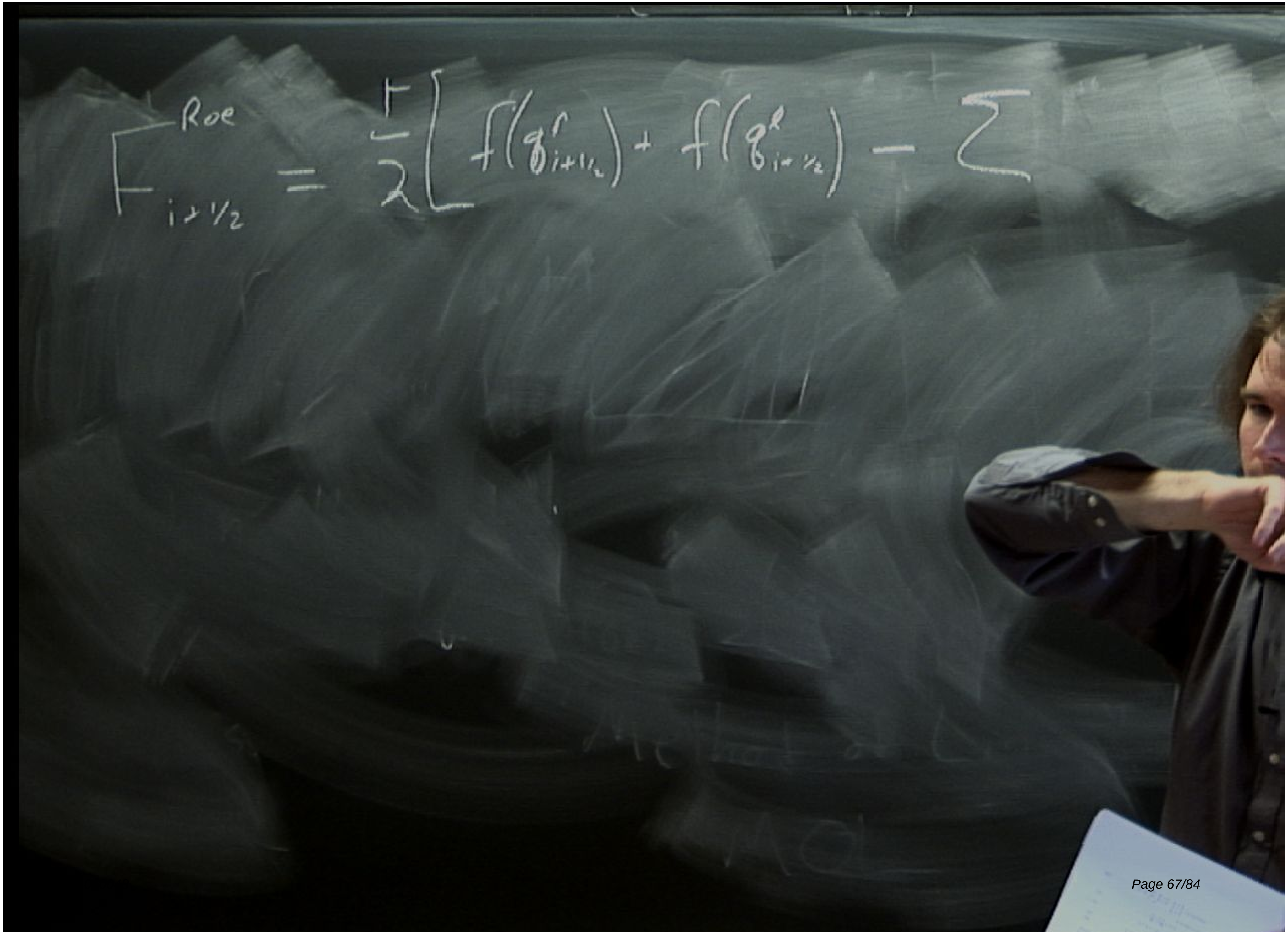
$$\partial_t q + \partial_x f(q) = 0$$

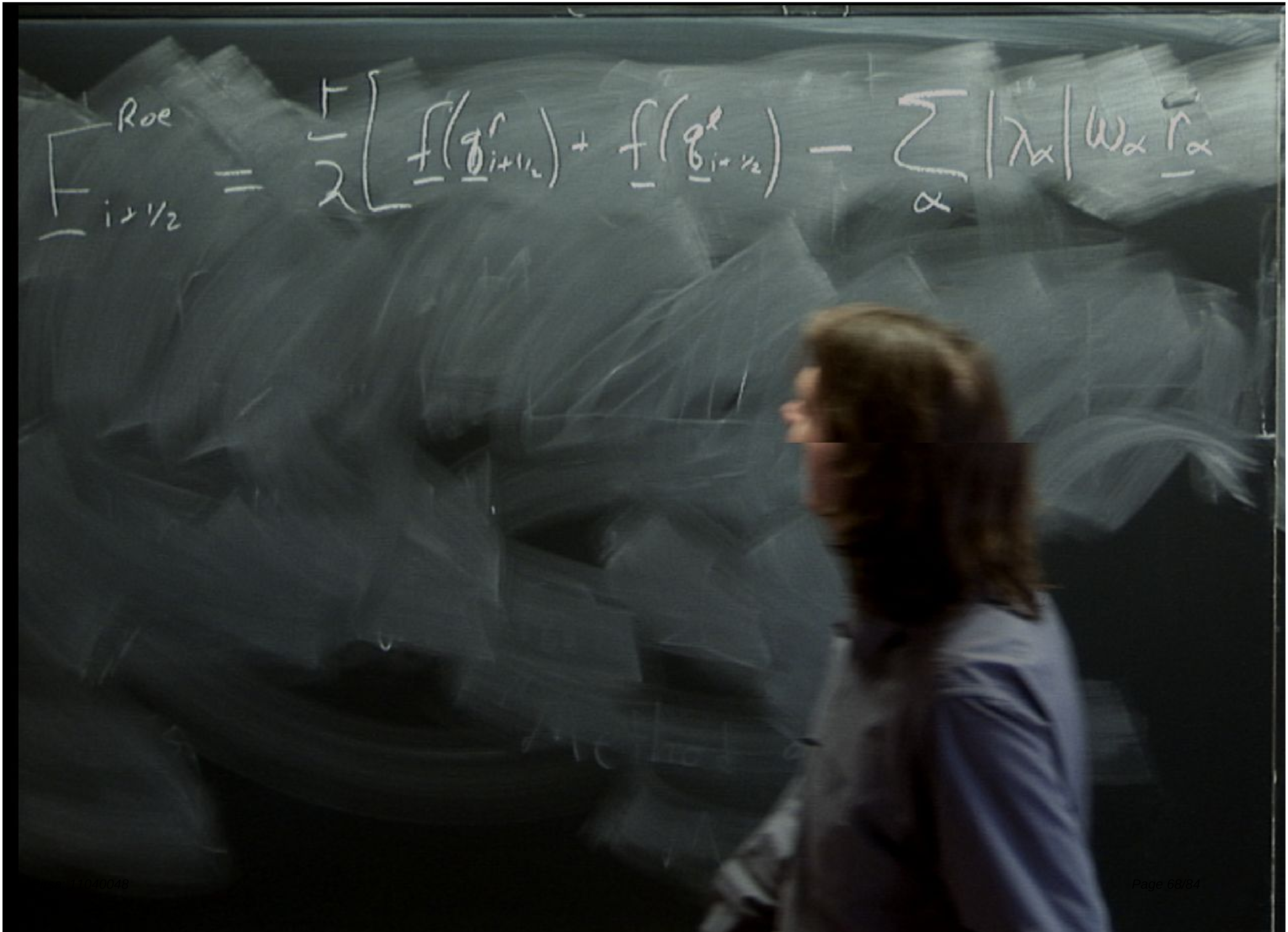
$$A(q_l, q_r) = \left.\frac{\partial f}{\partial q}\right|_{q=\frac{1}{2}(q_l+q_r)}$$

$= f(q_r) - f(q_l)$

...lizable $\lambda_\alpha \in \mathbb{R}$

$$q_l, q_r \rightarrow \bar{q}$$


$F^{Roe}_{i+1/2} = \frac{1}{2}\Big[f(q^r_{i+1/2}) + f(q^l_{i+1/2}) - \sum$


$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{\ell}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \omega_\alpha \bar{\underline{r}}_\alpha\right.$

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\Big[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{l}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \, \omega_\alpha \, \bar{\underline{r}}_\alpha$$

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{l}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \, \omega_\alpha \, \underline{\bar{r}}_\alpha \right]$$

right eigenvector of A

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{\ell}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| w_\alpha \underline{\bar{r}}_\alpha \right.$$

right eigenvector of A

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\Big[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{\ell}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \, \omega_\alpha \, \underline{\bar{r}}_\alpha$$

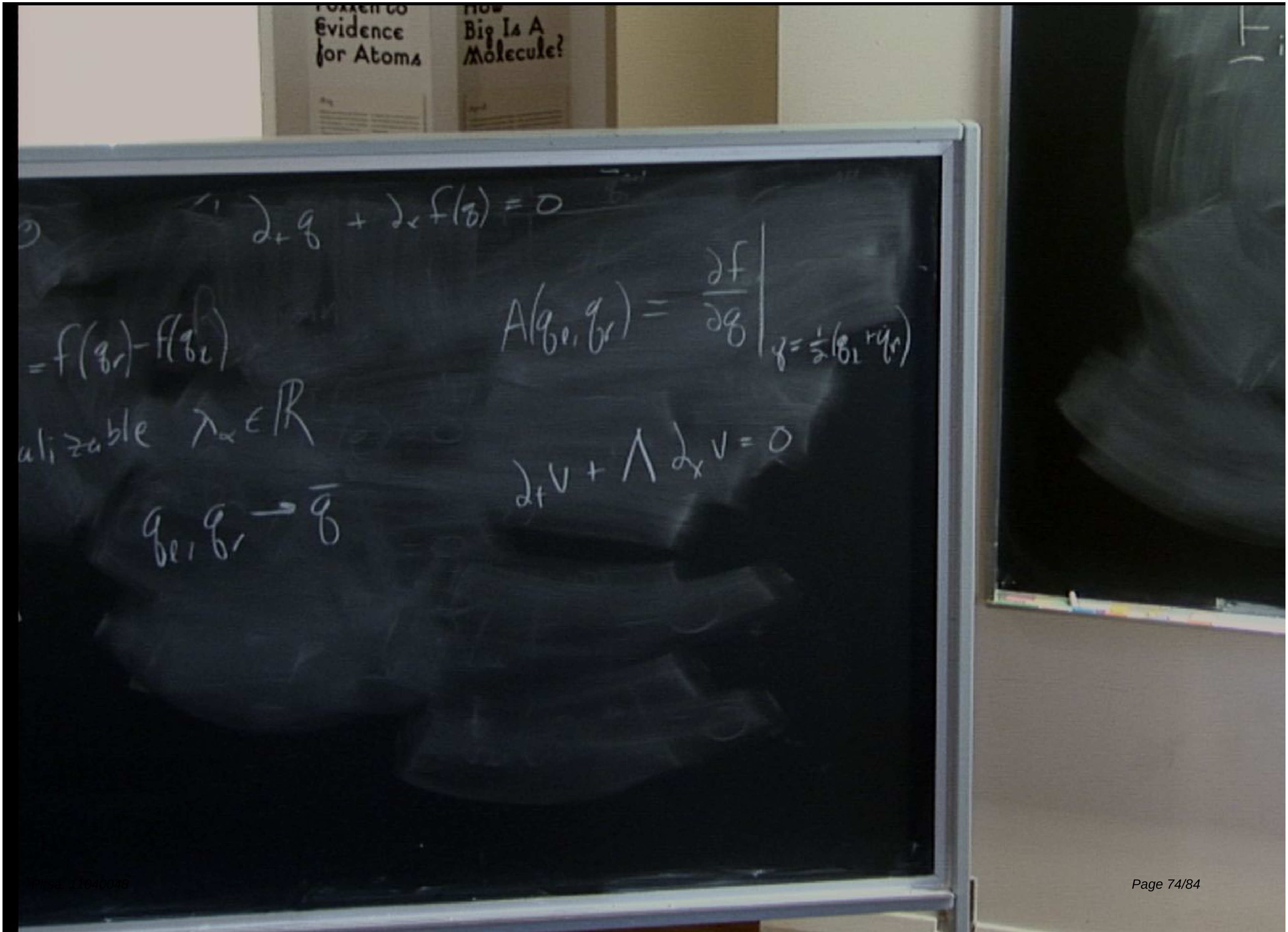
right eigenvector of A

$$q^{r}_{i+1/2} - q^{L}_{i+1/2} = \sum_{\alpha} \omega_\alpha \vec{r}_\alpha$$

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{l}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \, \omega_\alpha \, \underline{\hat{r}}_\alpha \right]$$

right eigenvector of A

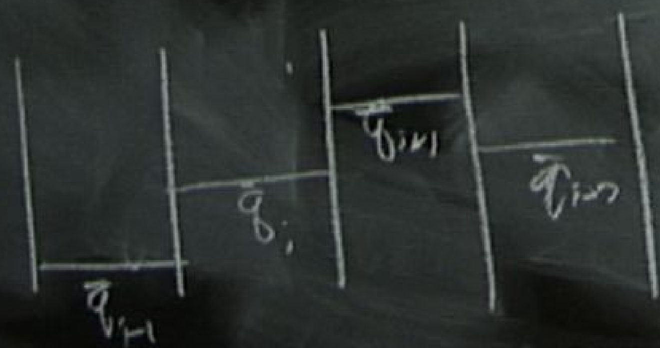
$$q^{r}_{i+1/2} - q^{L}_{i+1/2} = \sum_{\alpha} \omega_\alpha \vec{r}_\alpha$$

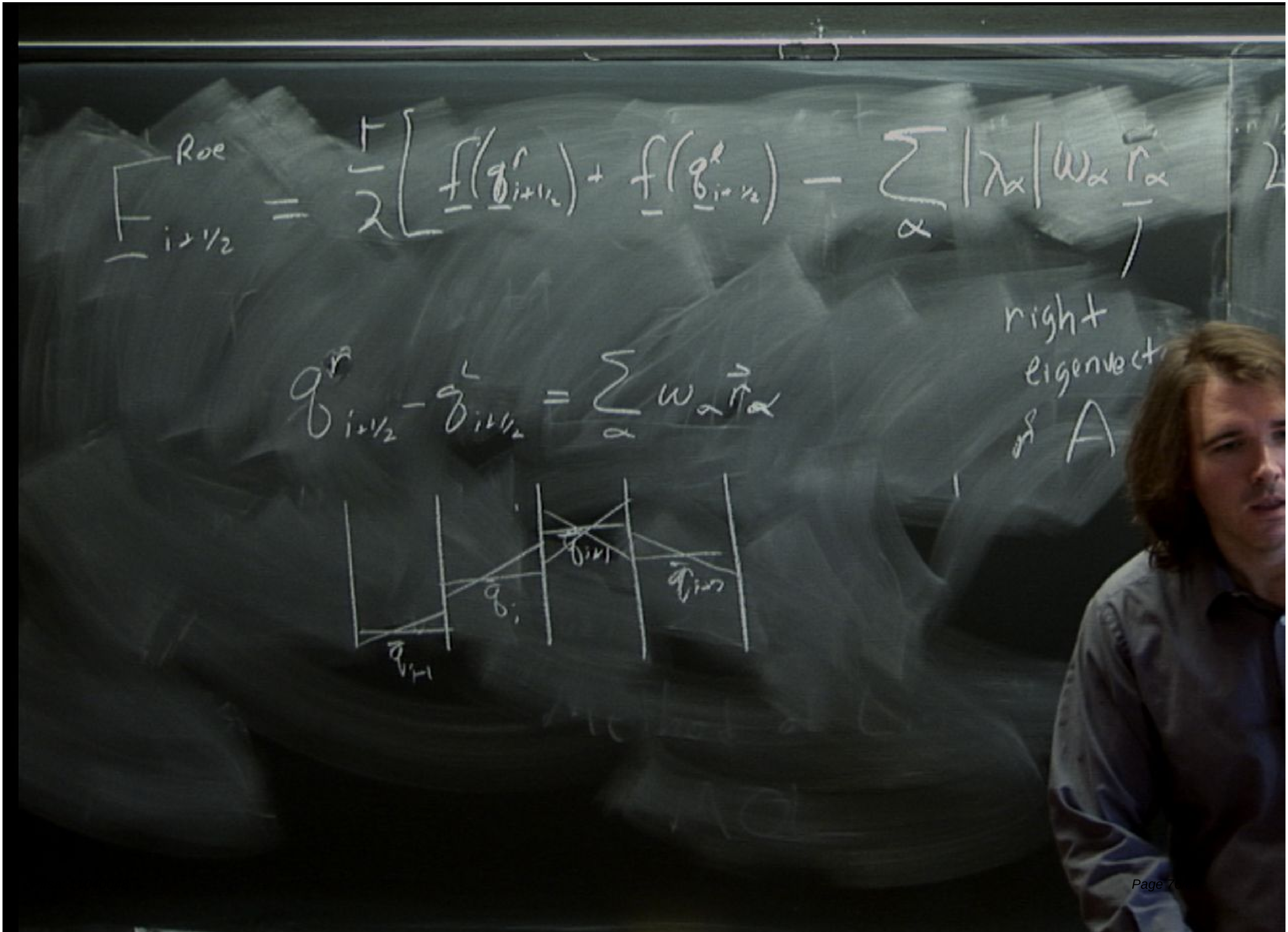

Evidence
for Atoms
Big Is A
Molecule?
∂t q + ∂x f(q) = 0
A(ql, qr) = ∂f/∂q | q = ½(ql + qr)
= f(qr) − f(ql)
alizable λα ∈ ℝ
∂t v + Λ ∂x v = 0
ql, qr → q̄

$$\underline{F}^{Roe}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^r_{i+1/2}) + \underline{f}(\underline{q}^l_{i+1/2}) - \sum_\alpha |\lambda_\alpha| w_\alpha \underline{\vec{r}}_\alpha \right]$$

right eigenvector of A

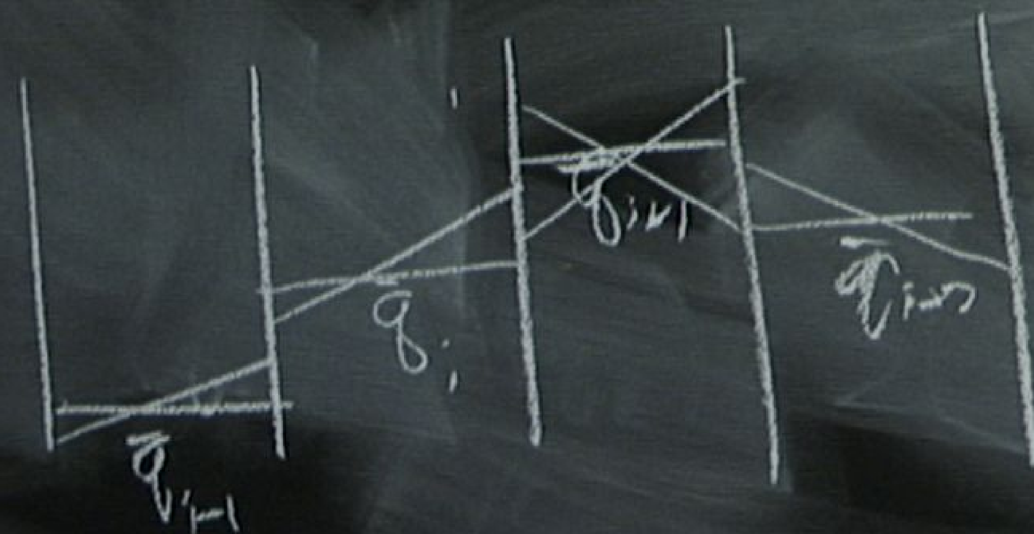
$$q^r_{i+1/2} - q^l_{i+1/2} = \sum_\alpha w_\alpha \vec{r}_\alpha$$

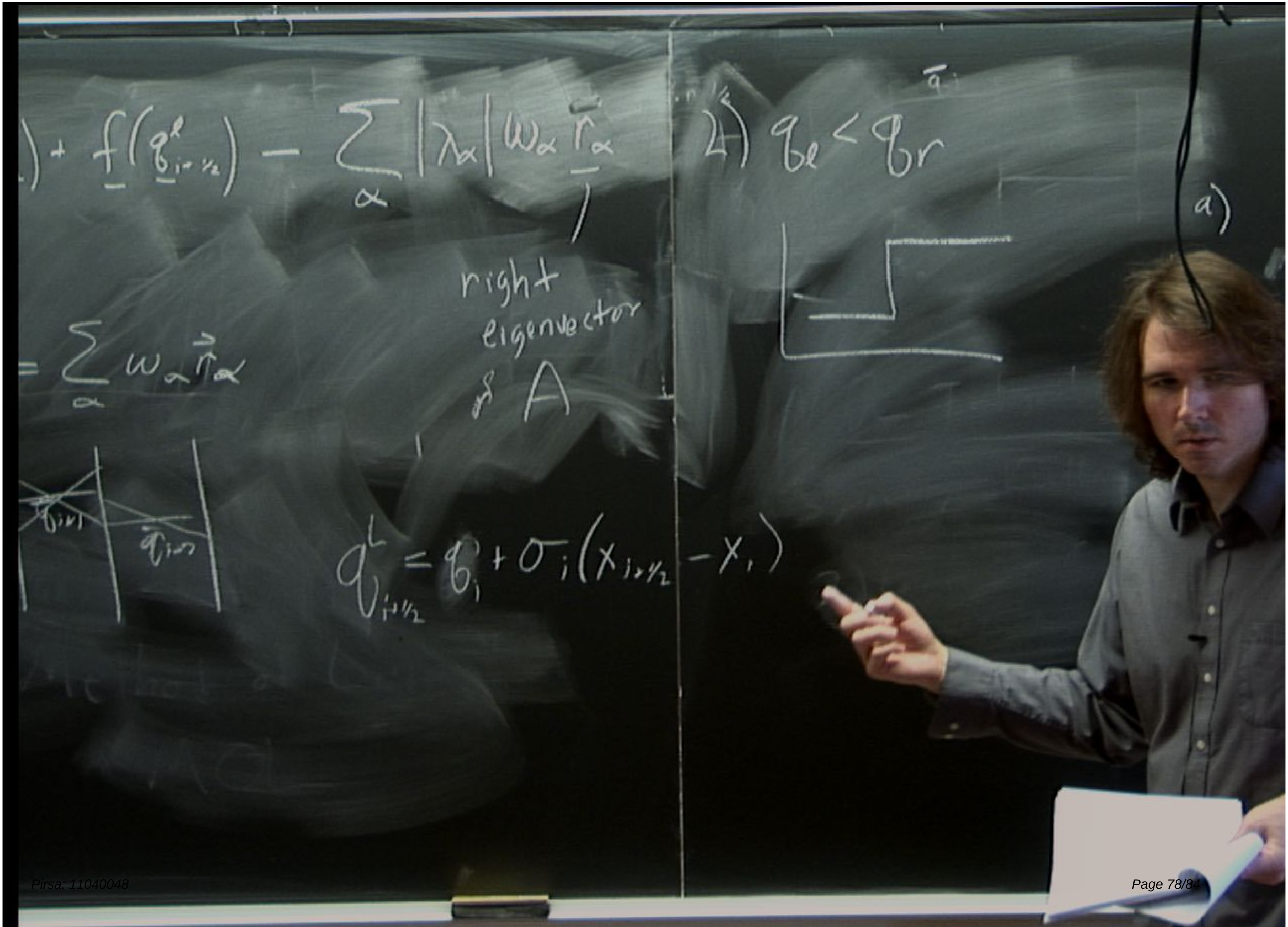



Roe
$\underline{F}_{i+1/2} = \frac{1}{2}\left[\underline{f}(\underline{q}^{r}_{i+1/2}) + \underline{f}(\underline{q}^{l}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| \omega_\alpha \underline{\vec{r}}_\alpha\right]$
right
eigenvect
of A
$q^{r}_{i+1/2} - q^{l}_{i+1/2} = \sum_{\alpha} \omega_\alpha \vec{r}_\alpha$
$\bar{q}_{i-1}$
$\bar{q}_{i}$
$\bar{q}_{i+1}$
$\bar{q}_{i+2}$

$i+1/2$

$$q^{r}_{i+1/2} - q^{L}_{i+1/2} = \sum_{\alpha} w_{\alpha} \vec{r}_{\alpha}$$

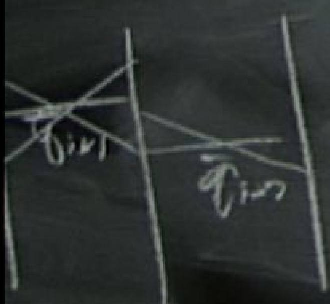



right
eigenvector
of A
2) q_l < q_r
a)

$$) + \underline{f}(\underline{q}^{l}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| w_\alpha \underline{\bar{r}}_\alpha$$

right eigenvector of A

$$= \sum_{\alpha} w_\alpha \vec{r}_\alpha$$



$$q^{L}_{i+1/2} = q_i + \sigma_i (x_{i+1/2} - x_i)$$

$$q^{R}_{i+1/2} = q_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_{i+1})$$

2) $q_l < q_r$

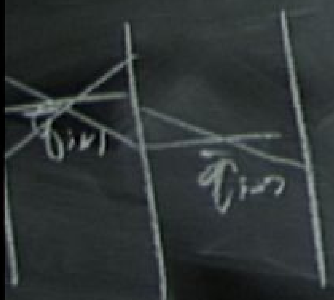
a)

b)

$$) + \underline{f}(\underline{q}^{\ell}_{i+1/2}) - \sum_{\alpha} |\lambda_\alpha| w_\alpha \underline{r}_\alpha$$

right eigenvector of A

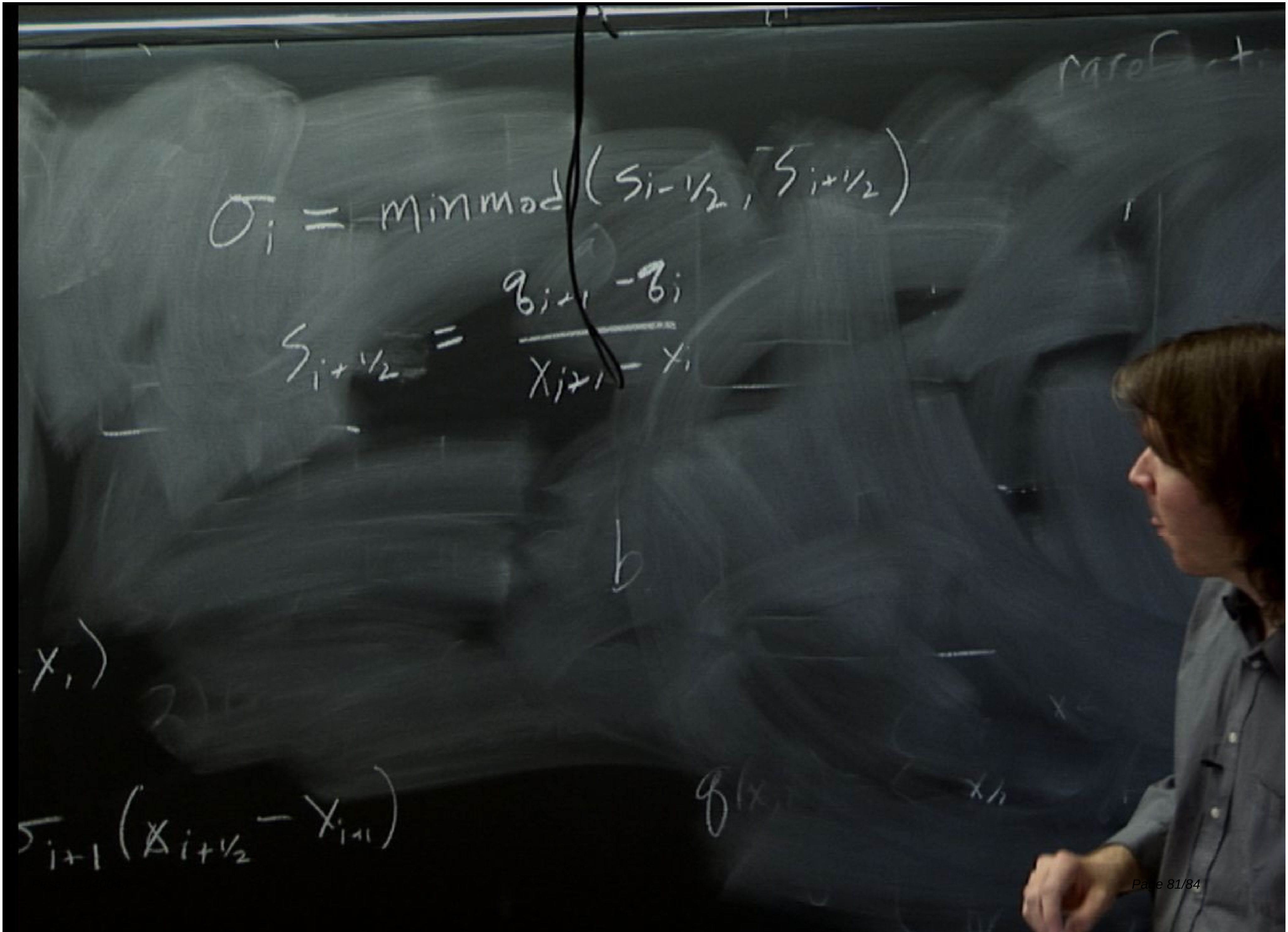
$$= \sum_{\alpha} w_\alpha \vec{r}_\alpha$$



b)

$$q^{L}_{i+1/2} = q_i + \sigma_i (x_{i+1/2} - x_i)$$

$$q^{R}_{i+1/2} = q_{i+1} + \sigma_{i+1} (x_{i+1/2} - x_{i+1})$$


rarefact
$\sigma_i = \text{minmod}(s_{i-1/2}, s_{i+1/2})$
$s_{i+1/2} = \frac{q_{i+1} - q_i}{x_{i+1} - x_i}$
b
$x_1)$
$\sigma_{i+1}(x_{i+1/2} - x_{i+1})$
$q(x_i$
x_i

rarefaction

$$= \text{minmod}(s_{i-1/2}, s_{i+1/2})$$

$$s_{i+1/2} = \frac{q_{i+1} - q_i}{x_{i+1} - x_i}$$

$$\text{minmod}(a, b) = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \ \& \ ab > 0 \\ b & \text{if } |b| < |a| \ \& \ ab > 0 \end{cases}$$

$x_{i+1})$ $q(x_i$

rarefaction

$= \text{minmod}(s_{i-1/2}, s_{i+1/2})$ $\vec{p} = \{p, \rho, v^i\}$

$\vec{q} = \{T^{\mu\nu}, \rho u^\mu)$

$s_{i+1/2} = \dfrac{q_{i+1} - q_i}{x_{i+1} - x_i}$

$$\text{minmod}(a, b) = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \ \& \ ab > 0 \\ b & \text{if } |b| < |a| \ \& \ ab > 0 \end{cases}$$

$q(x_i$

x_i

$x_{i+1})$

rare of $\vec{q} \to \vec{q}(\vec{p})$

$$= \mathrm{minmod}(s_{i-1/2}, s_{i+1/2}) \qquad \vec{p} = \{p, \rho, v^i\}$$

$$s_{i+1/2} = \frac{q_{i+1} - q_i}{x_{i+1} - x_i} \qquad \vec{q} = \{T^{\mu\nu}, \rho u^\mu\}$$

$$\mathrm{minmod}(a,b) = \begin{cases} 0 & \text{if } ab < 0 \\ a & \text{if } |a| < |b| \ \& \ ab > 0 \\ b & \text{if } |b| < |a| \ \& \ ab > 0 \end{cases}$$

$x_{i+1})$ $\qquad q(x$ $\qquad x_i$