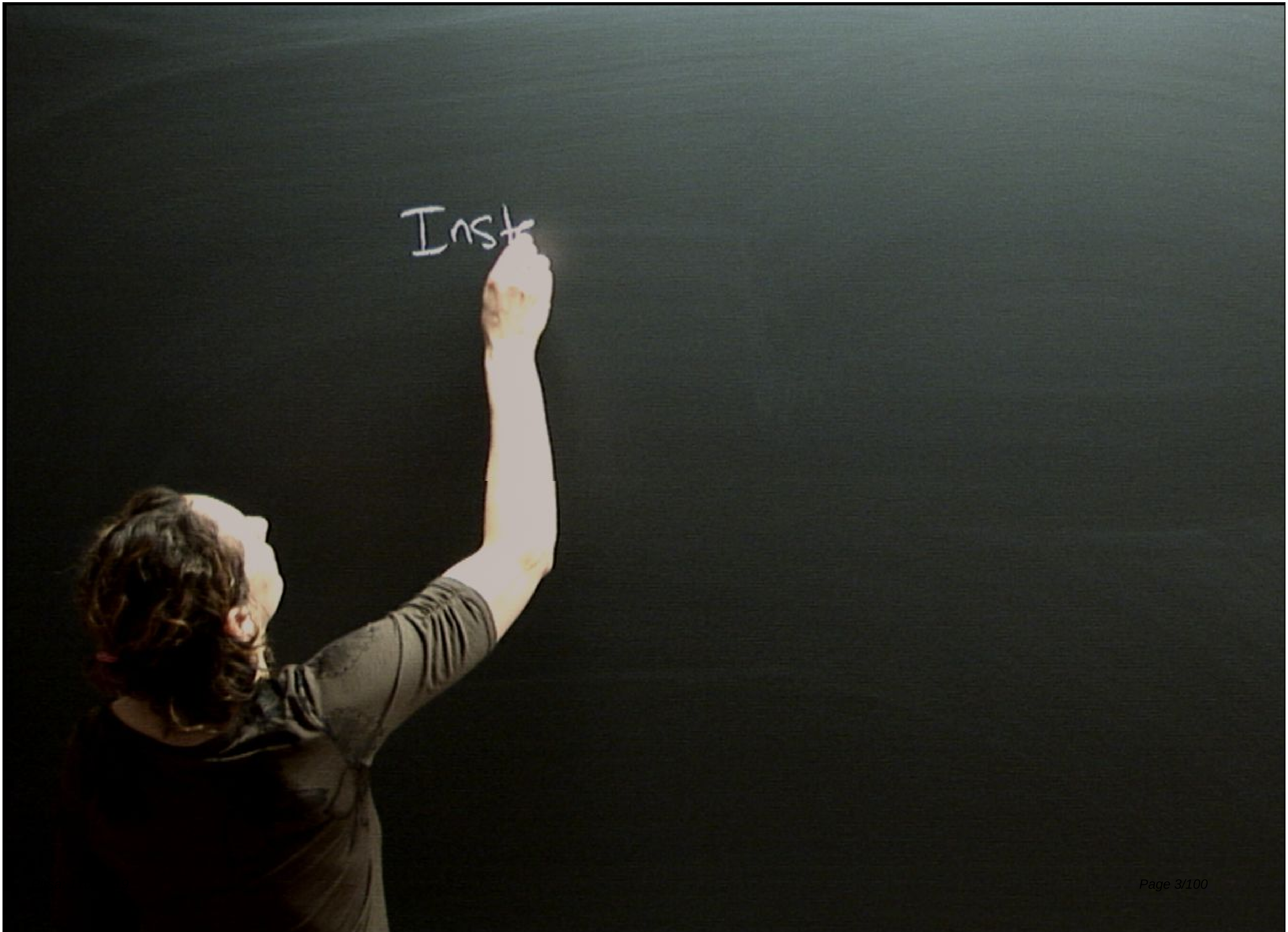
Title: Explorations in Cosmology - Lecture 13

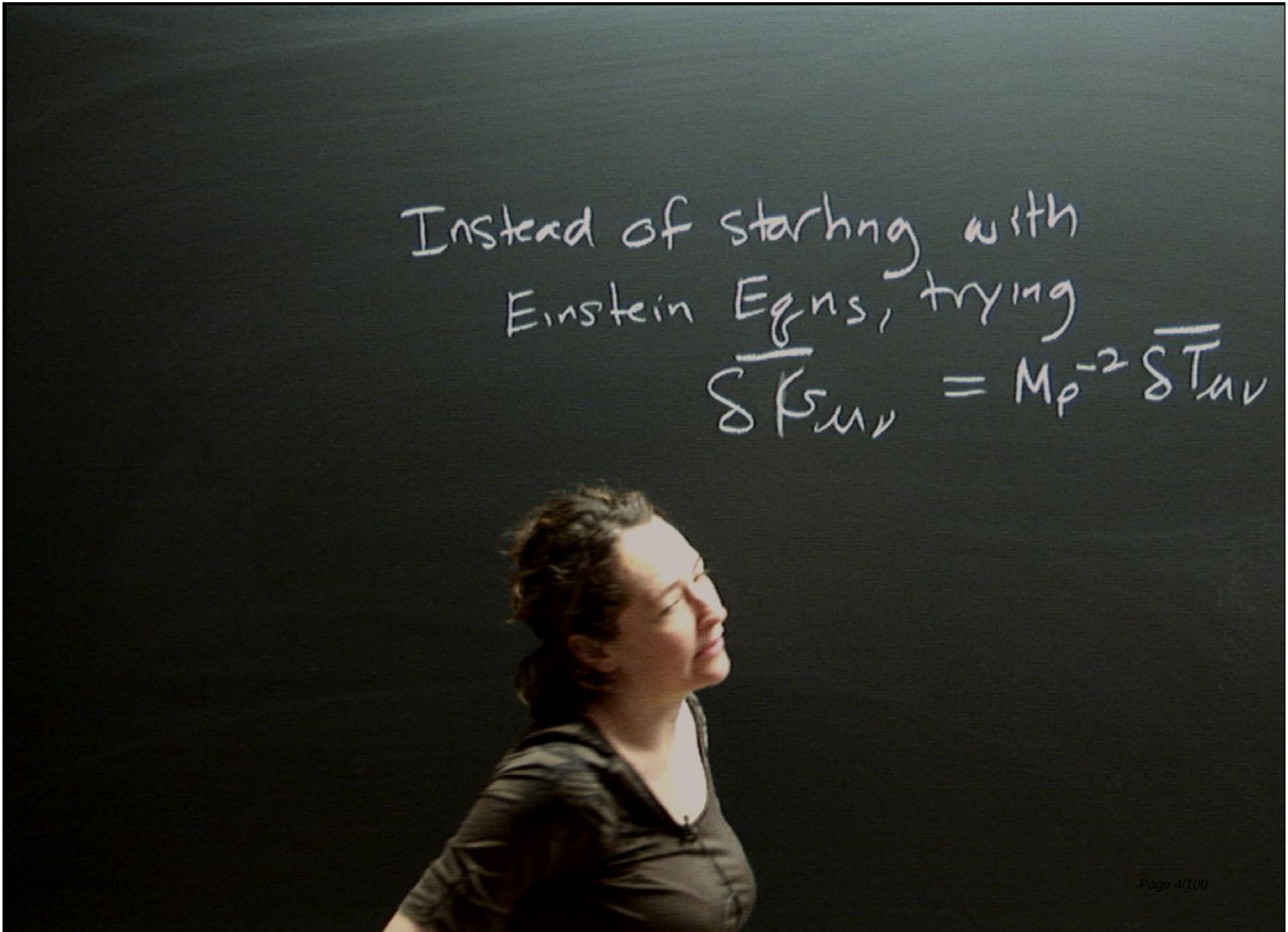
Date: Apr 20, 2011 09:00 AM

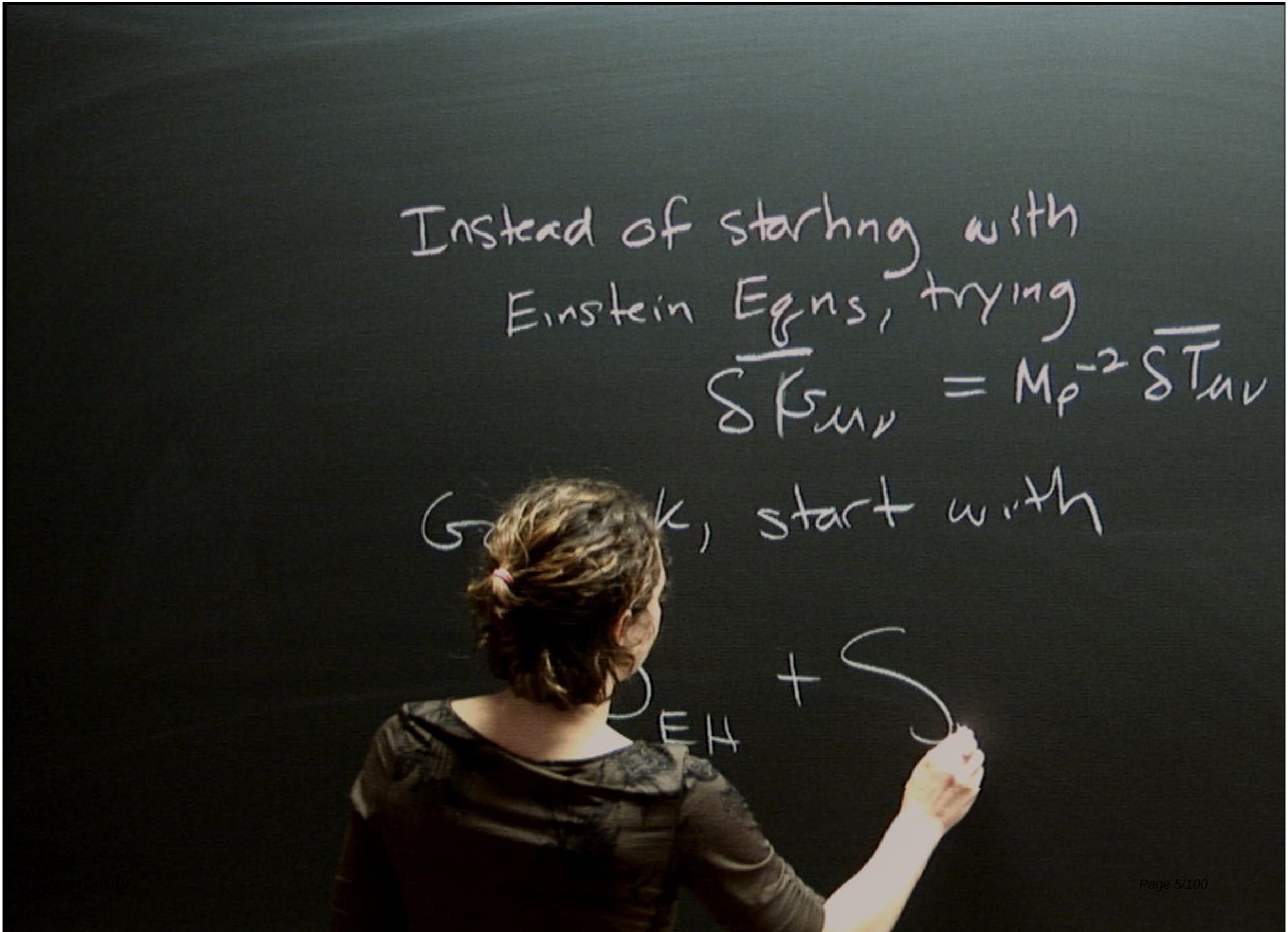
URL: http://pirsa.org/11040019

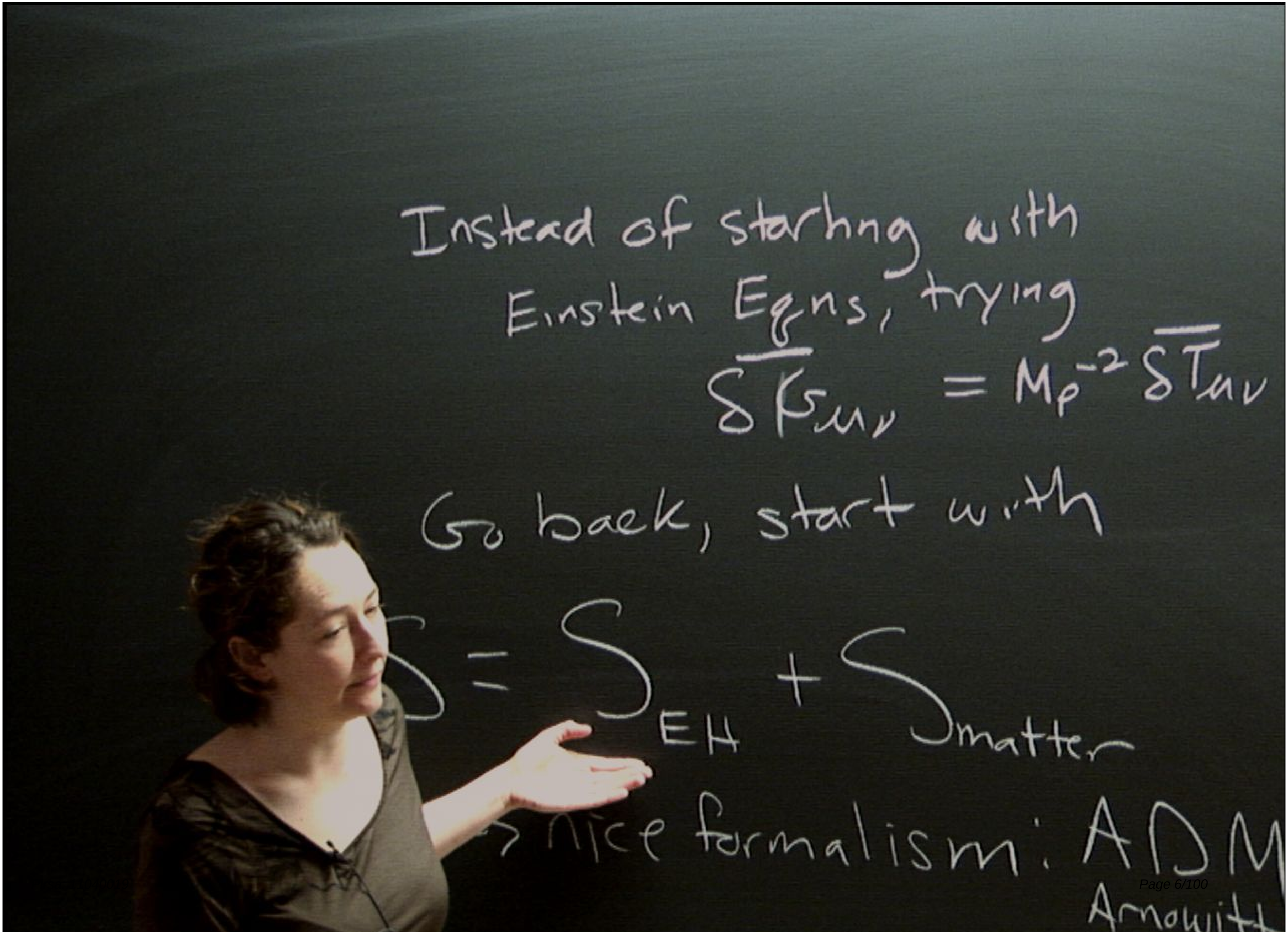
Abstract:

perimeter scholars
international


Inst


Instead of starting with
Einstein Eqns, trying
$\delta \bar{G}_{\mu\nu} = M_P^{-2} \delta \bar{T}_{\mu\nu}$


Instead of starting with
Einstein Eqns, trying
$\delta G_{\mu\nu} = M_p^{-2}\delta T_{\mu\nu}$
start with
EH + S


Instead of starting with
Einstein Eqns, trying
$\delta G_{\mu\nu} = M_p^{-2} \delta T_{\mu\nu}$
Go back, start with
$S = S_{EH} + S_{matter}$
→ nice formalism: ADM
Arnowitt

Instead of starting with
Einstein Eqns, trying

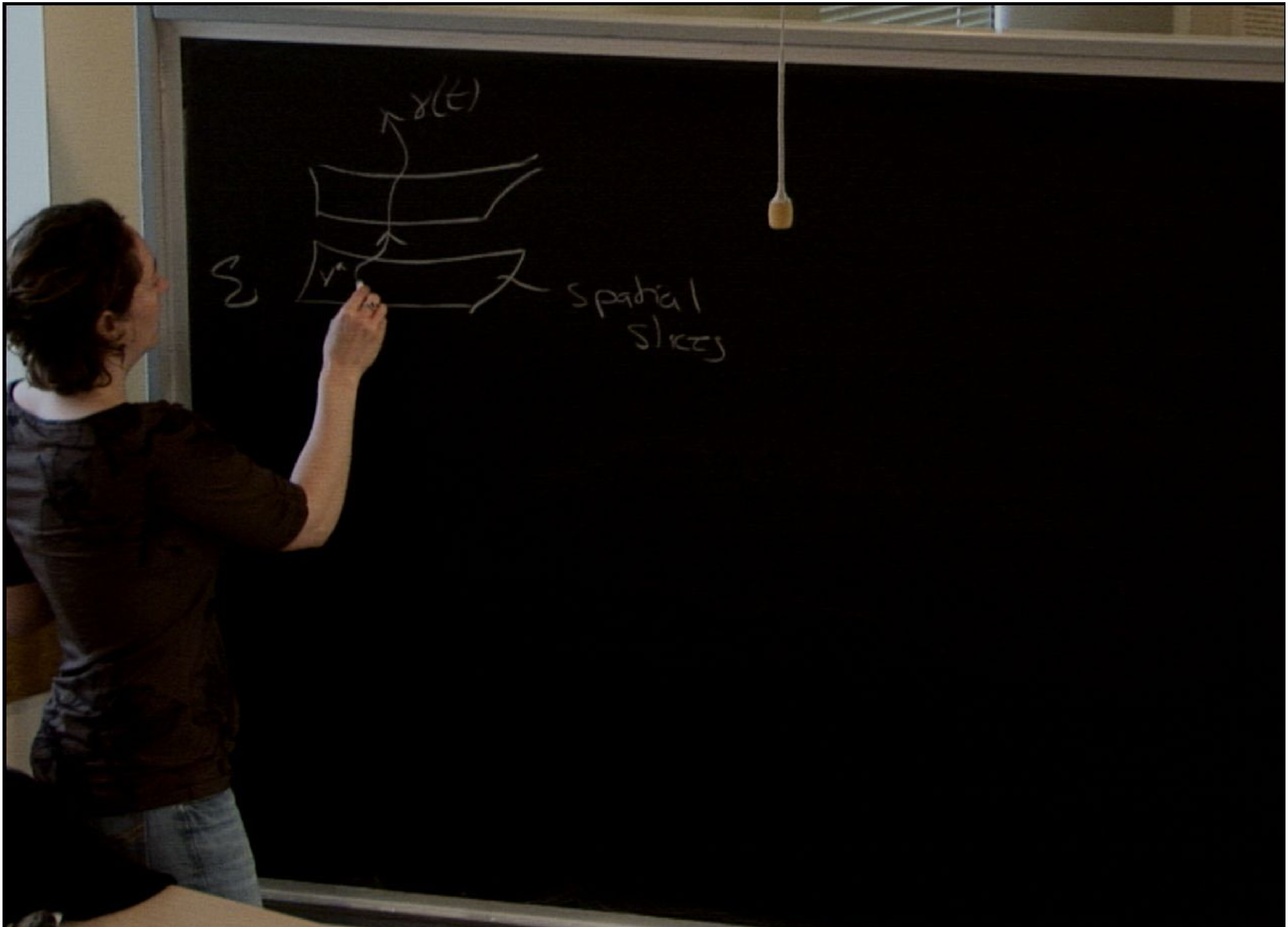
$$\delta \overline{G_{\mu\nu}} = M_p^{-2}\, \delta \overline{T_{\mu\nu}}$$

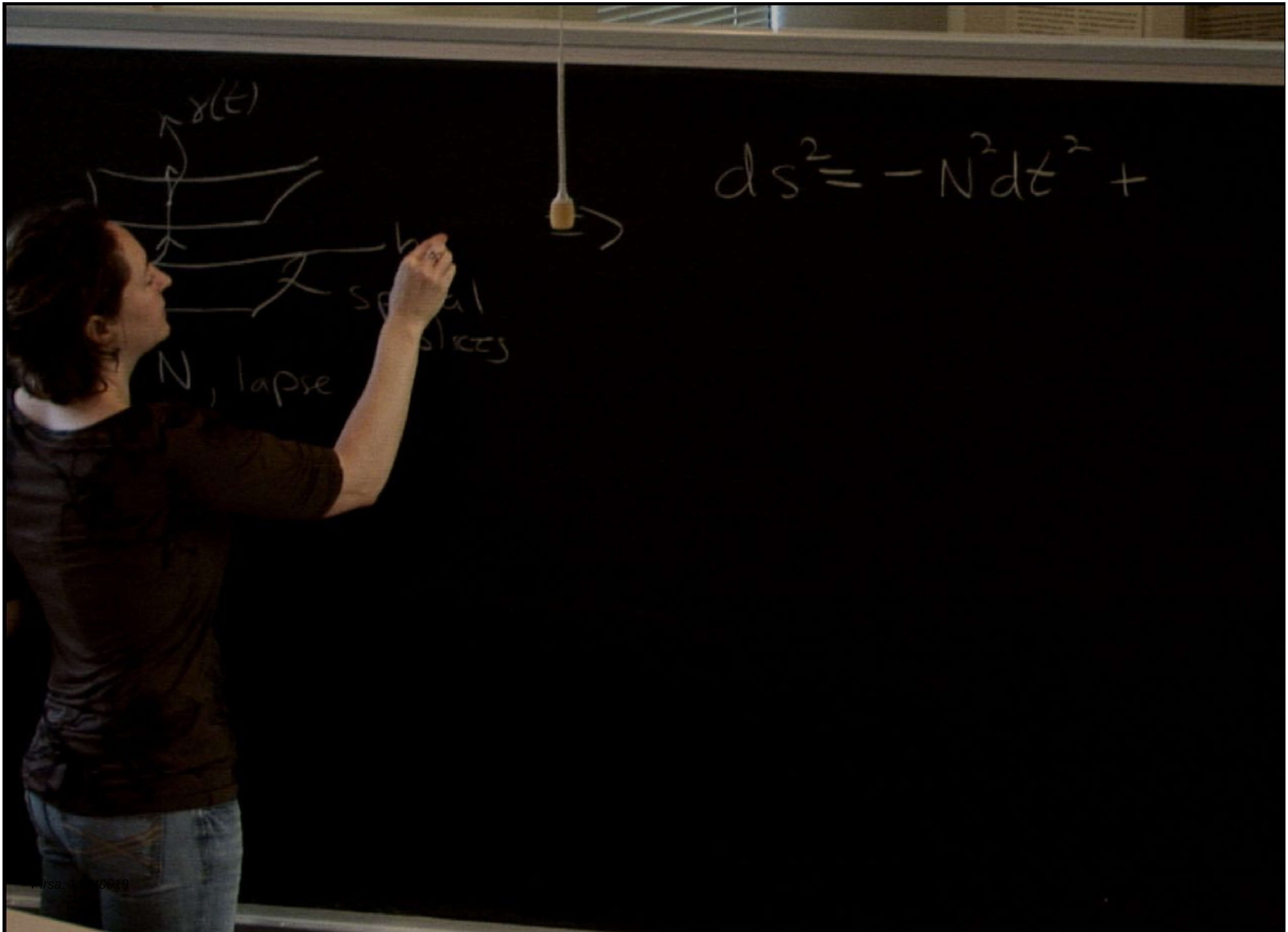
Go back, start with

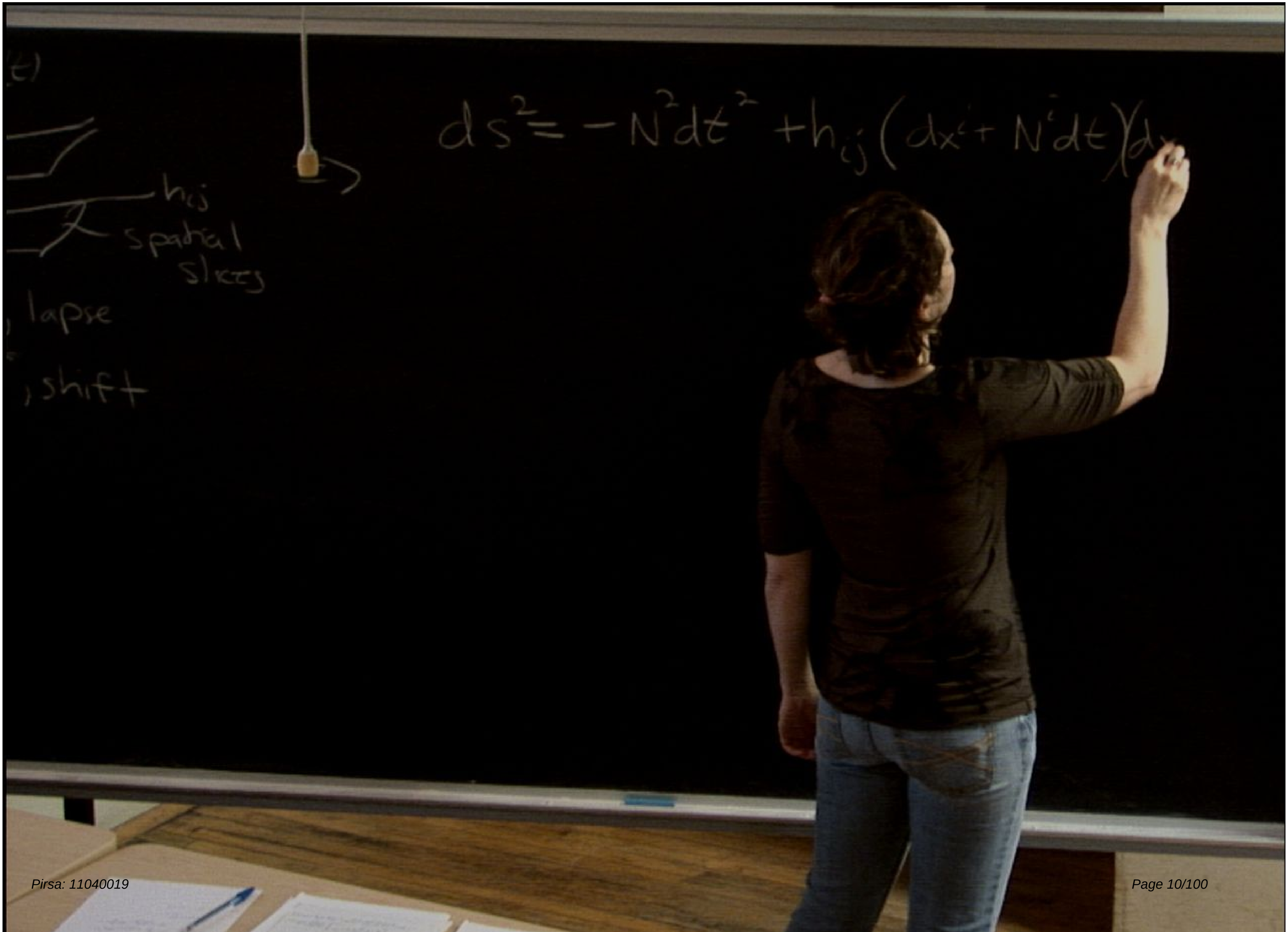
$$S = S_{EH} + S_{matter}$$

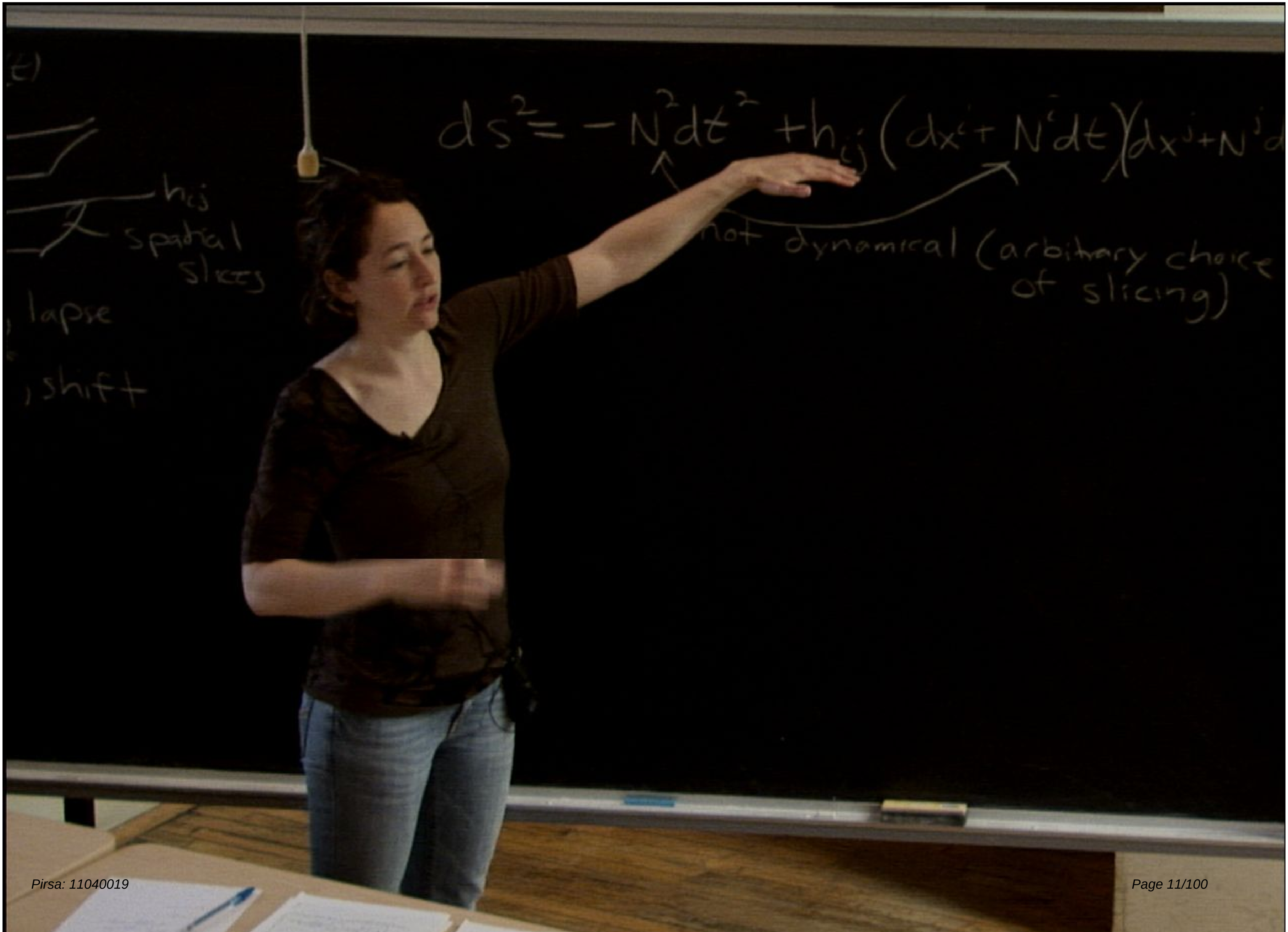
$\longrightarrow$ nice formalism: ADM

Arnowitt
Misner Deser

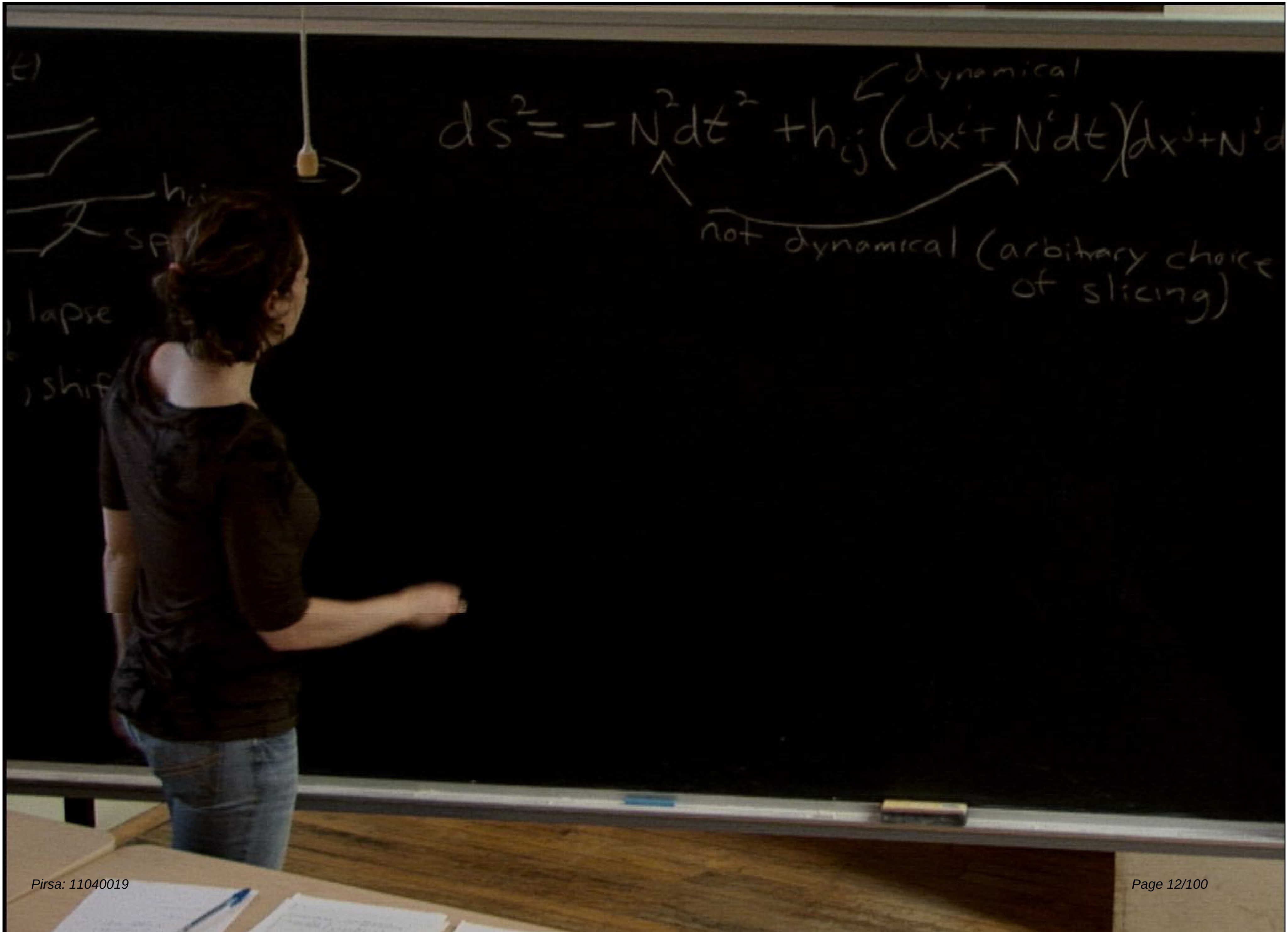

γ(t)
Σ
spatial
slices

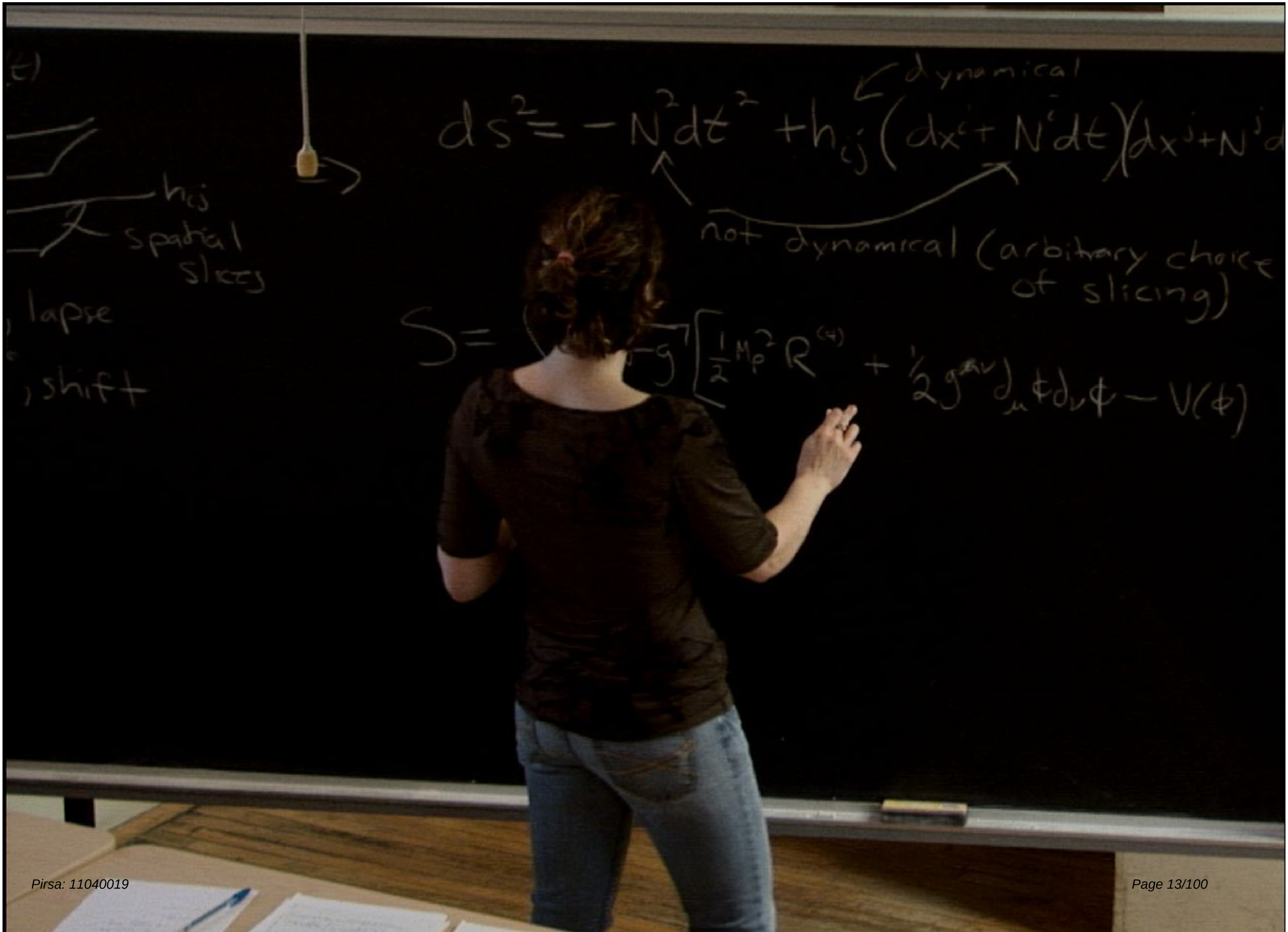

γ(t)
N, lapse
ds² = −N²dt² +

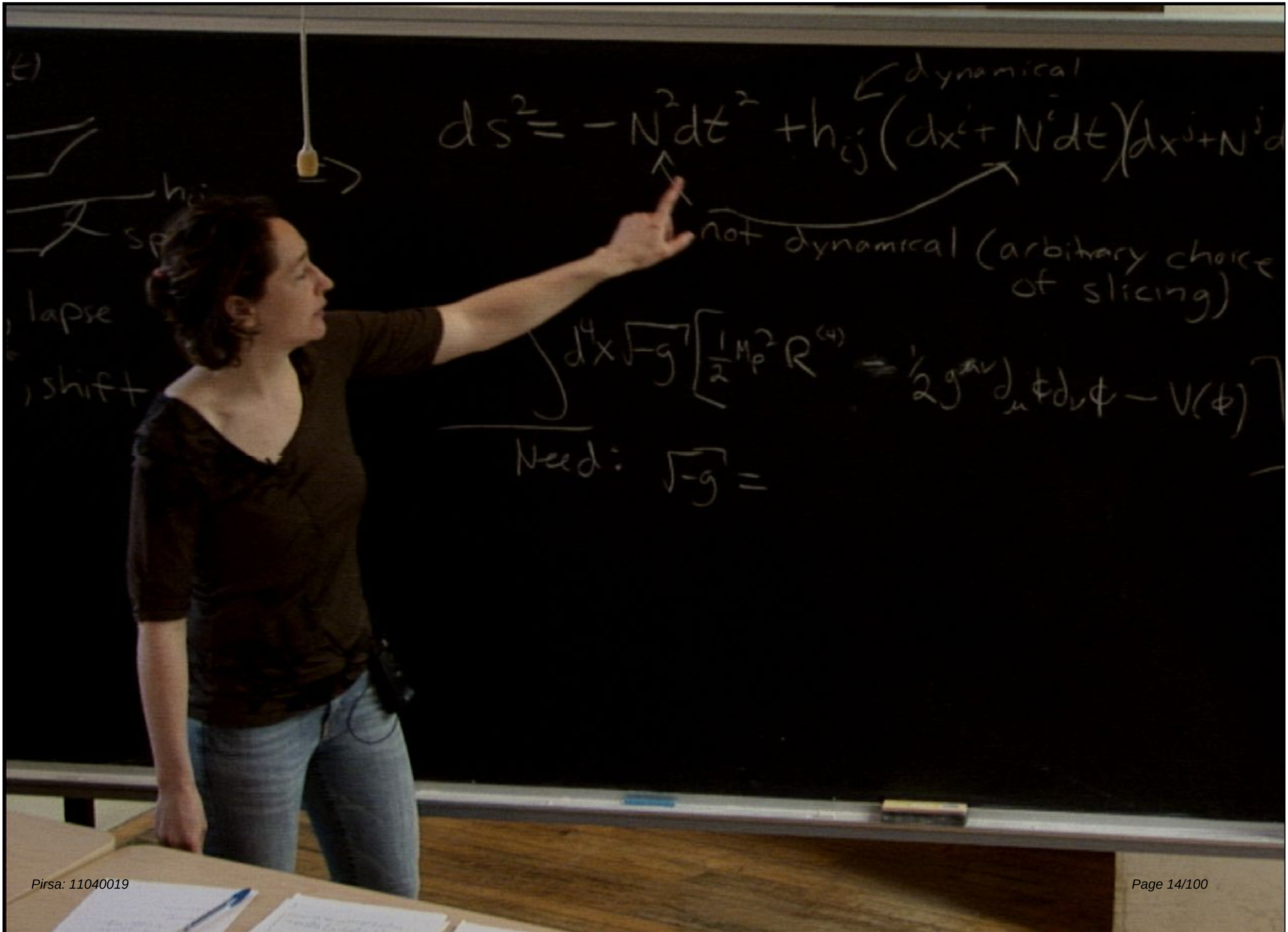

(t)
h_{ij}
spatial
slices
lapse
shift
$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx$

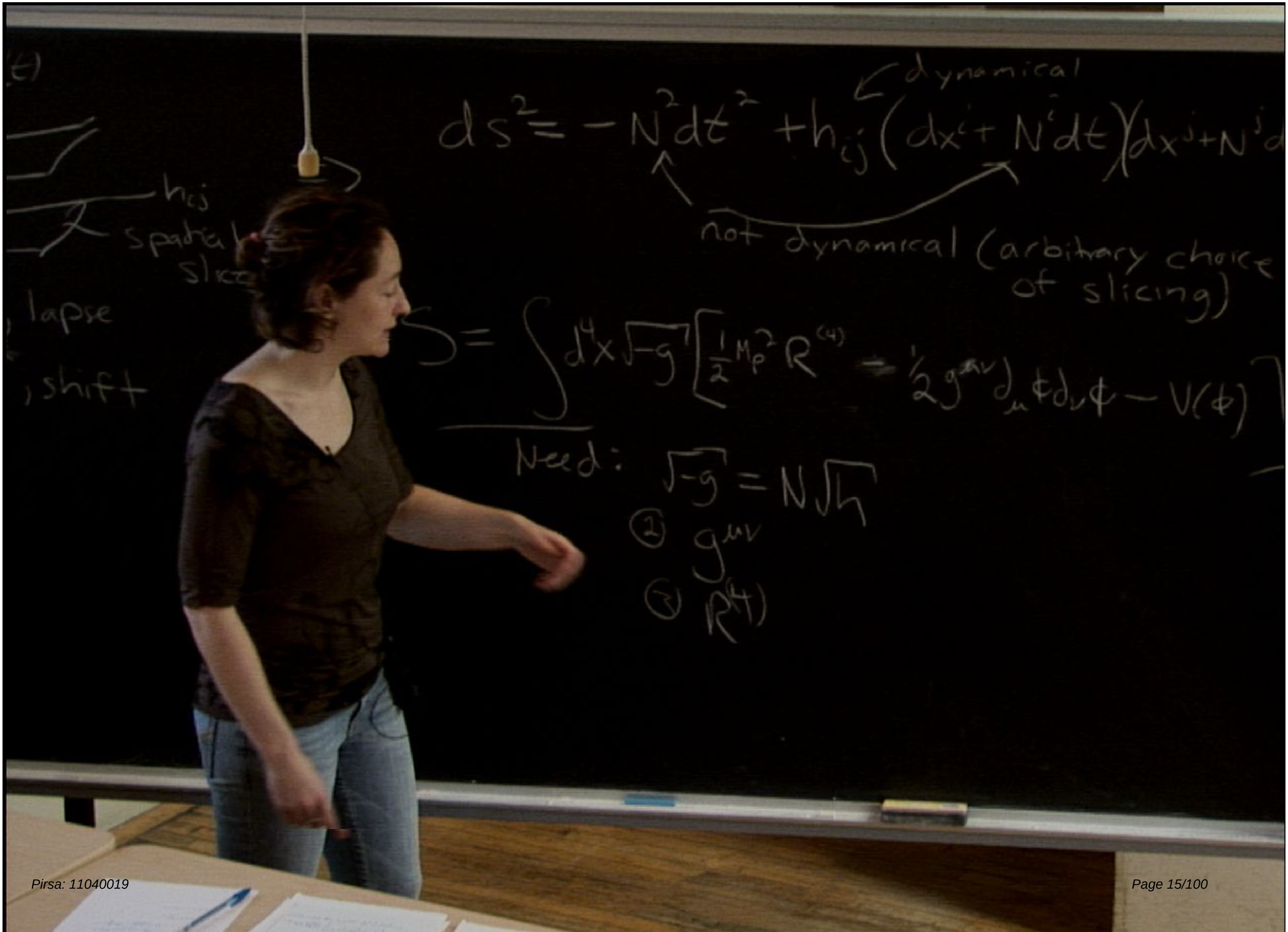


$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j d$
not dynamical (arbitrary choice of slicing)
h_{ij}
spatial slices
lapse
shift


dynamical
ds² = −N²dt² + h_ij (dx^i + N^i dt)(dx^j + N^j d
not dynamical (arbitrary choice of slicing)
lapse
shif


dynamical
$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j d$
h_{ij}
spatial slices
not dynamical (arbitrary choice of slicing)
lapse
shift
$S = \int \sqrt{-g}\left[\frac{1}{2} M_p^2 R^{(4)} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\right.$


dynamical
ds²= −N²dt² + hij(dxⁱ+ Nⁱdt)(dxʲ+Nʲd
hij
not dynamical (arbitrary choice of slicing)
lapse
shift
∫d⁴x √−g [½ Mp² R⁽⁴⁾ − ½ g^μν ∂μφ∂νφ − V(φ)]
Need: √−g =


dynamical
$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j d$
not dynamical (arbitrary choice of slicing)
h_{ij}
spatial slice
lapse
shift
$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_p^2 R^{(4)} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$
Need: $\sqrt{-g} = N\sqrt{h}$
② $g^{\mu\nu}$
③ $R^{(4)}$

(t)

h_{ij} spatial slices

lapse

shift

dynamical

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j d$$

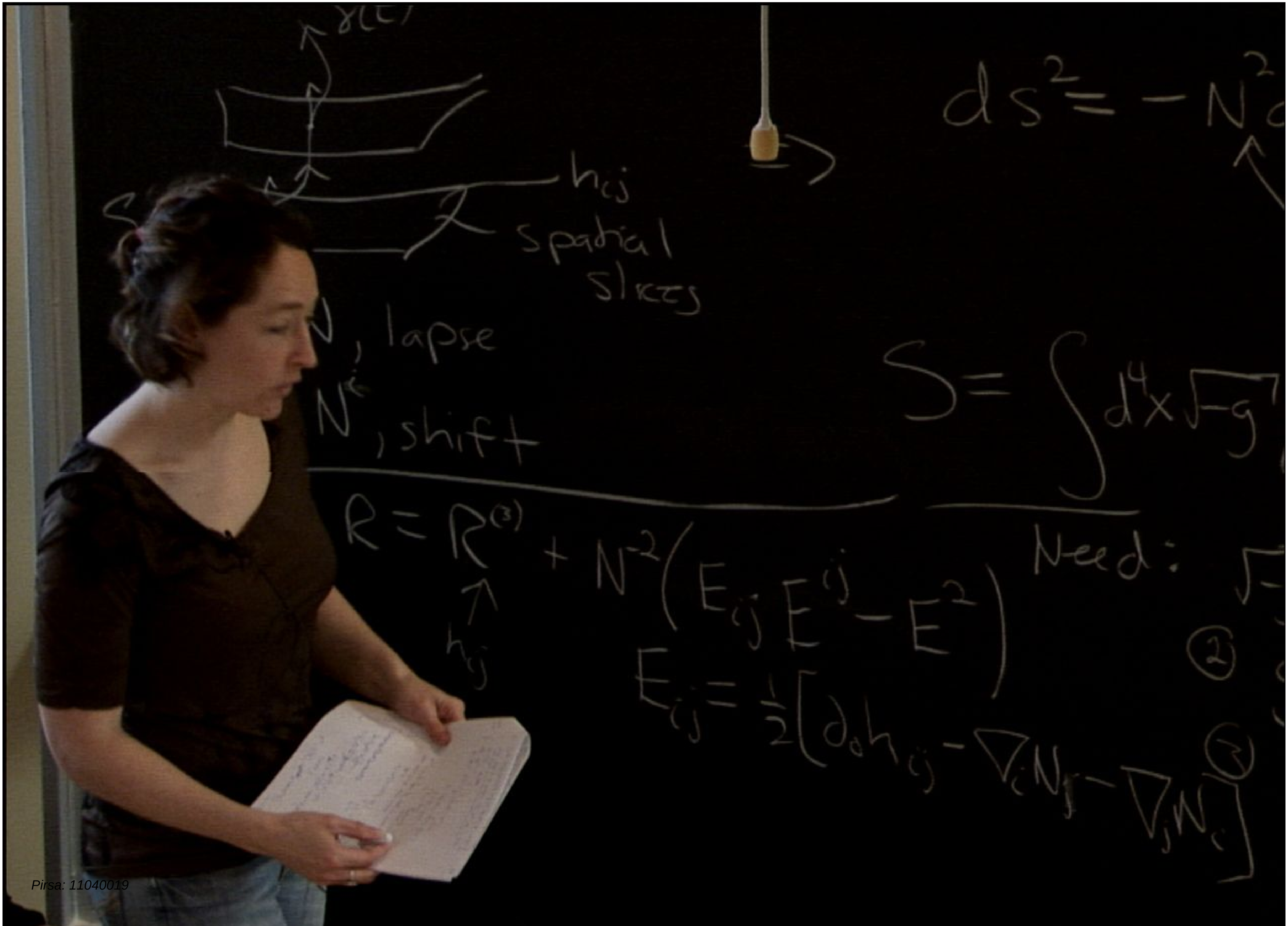
not dynamical (arbitrary choice of slicing)

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_p^2 R^{(4)} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

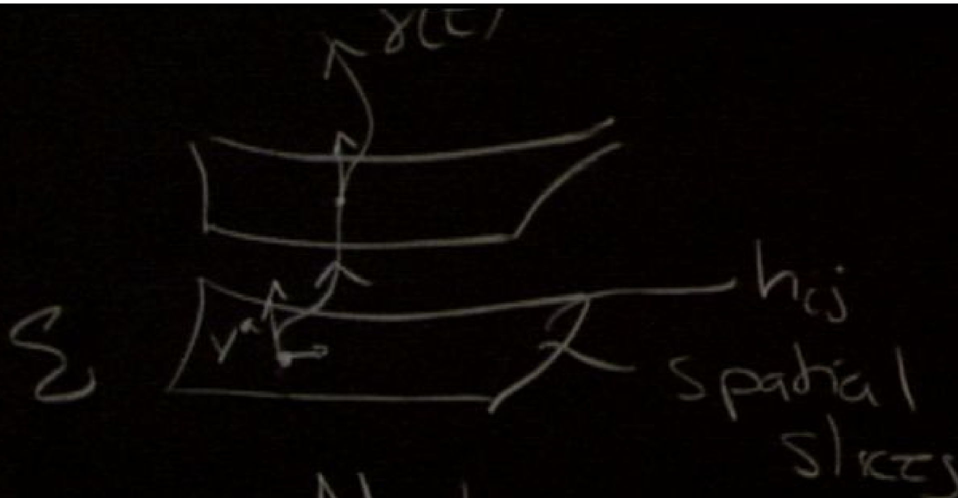
Need: $\sqrt{-g} = N\sqrt{h}$

① $g^{\mu\nu}$

② $R^{(4)} \longrightarrow N, R^{(3)}(h_{ij})$



$\gamma(t)$
h_{ij}
spatial slices
N, lapse
N^i, shift
$R = R^{(3)} + N^{-2}(E_{ij}E^{ij} - E^2)$
h_{ij}
$E_{ij} = \frac{1}{2}[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i]$
$ds^2 = -N^2 d$
$S = \int d^4x \sqrt{-g}$
Need:



$ds^2 = -N^2 d$

N, lapse

N^i, shift

$S = \int d^4x \sqrt{-g}$

$$R = \underset{h_{ij}}{R^{(3)}} + N^{-2}\left(E_{ij}E^{ij} - E^2\right)$$

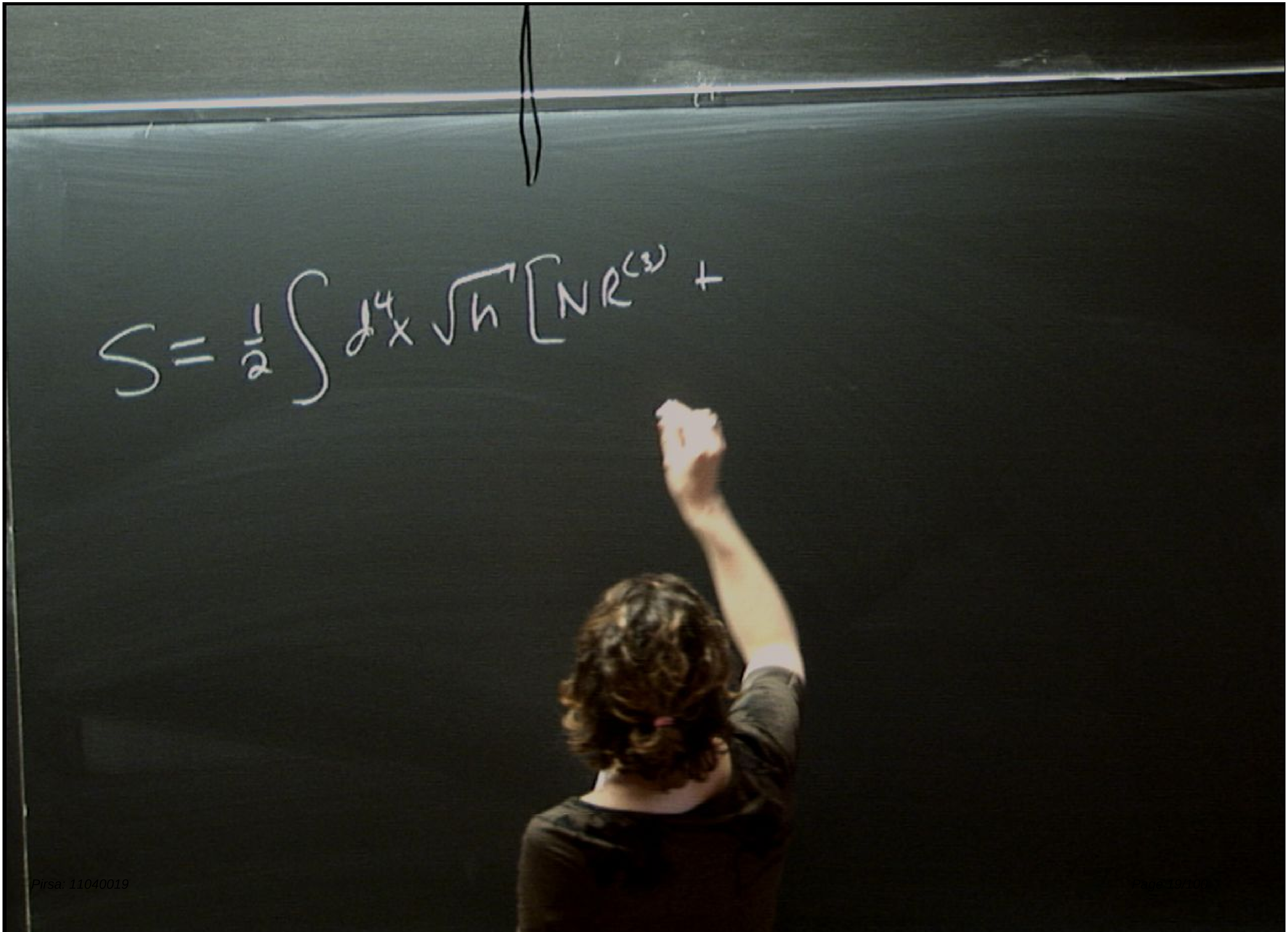
$$E_{ij} = \frac{1}{2}\left[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i\right]$$

h_{ij}

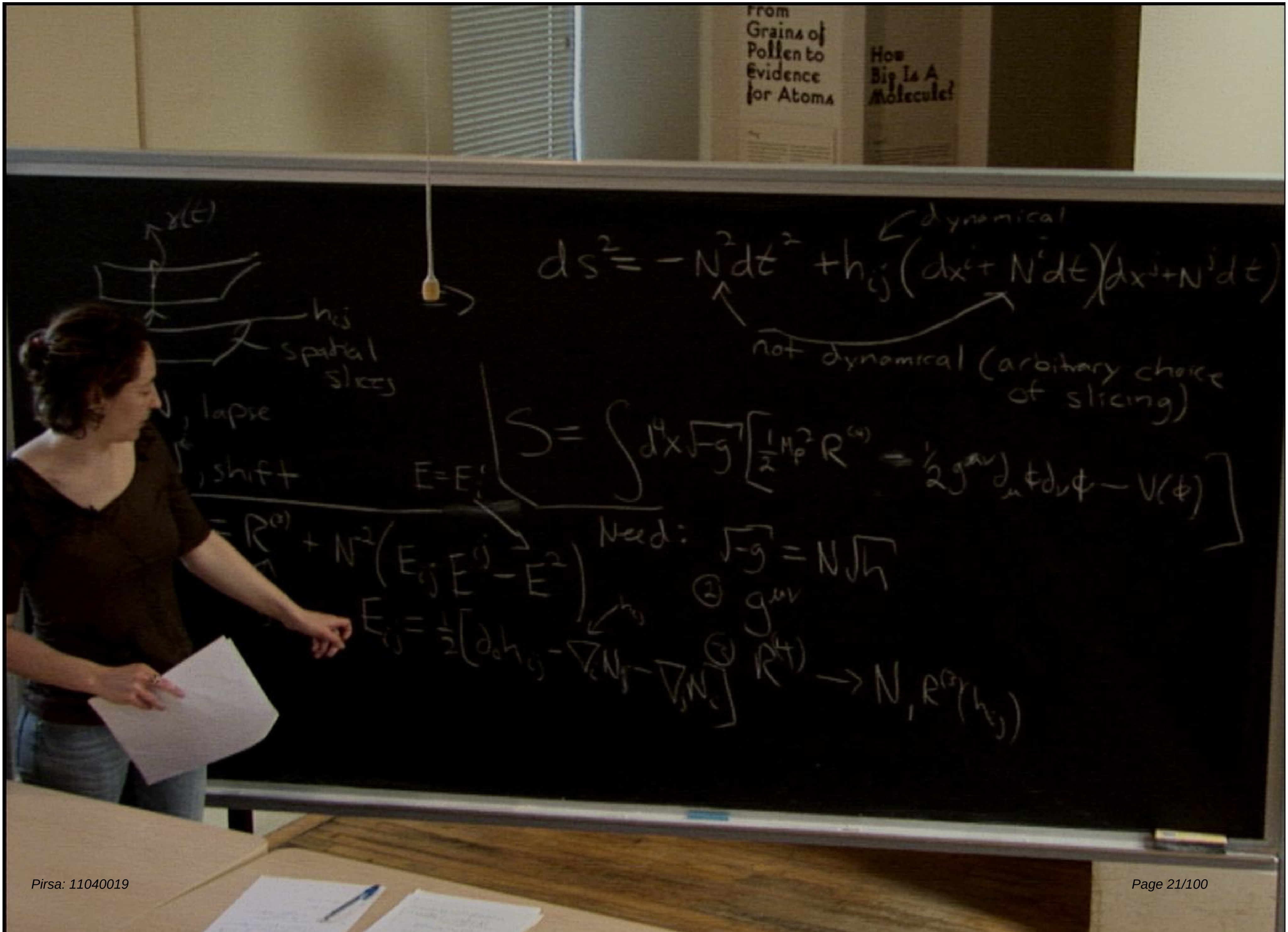
Need: $\sqrt{-}$

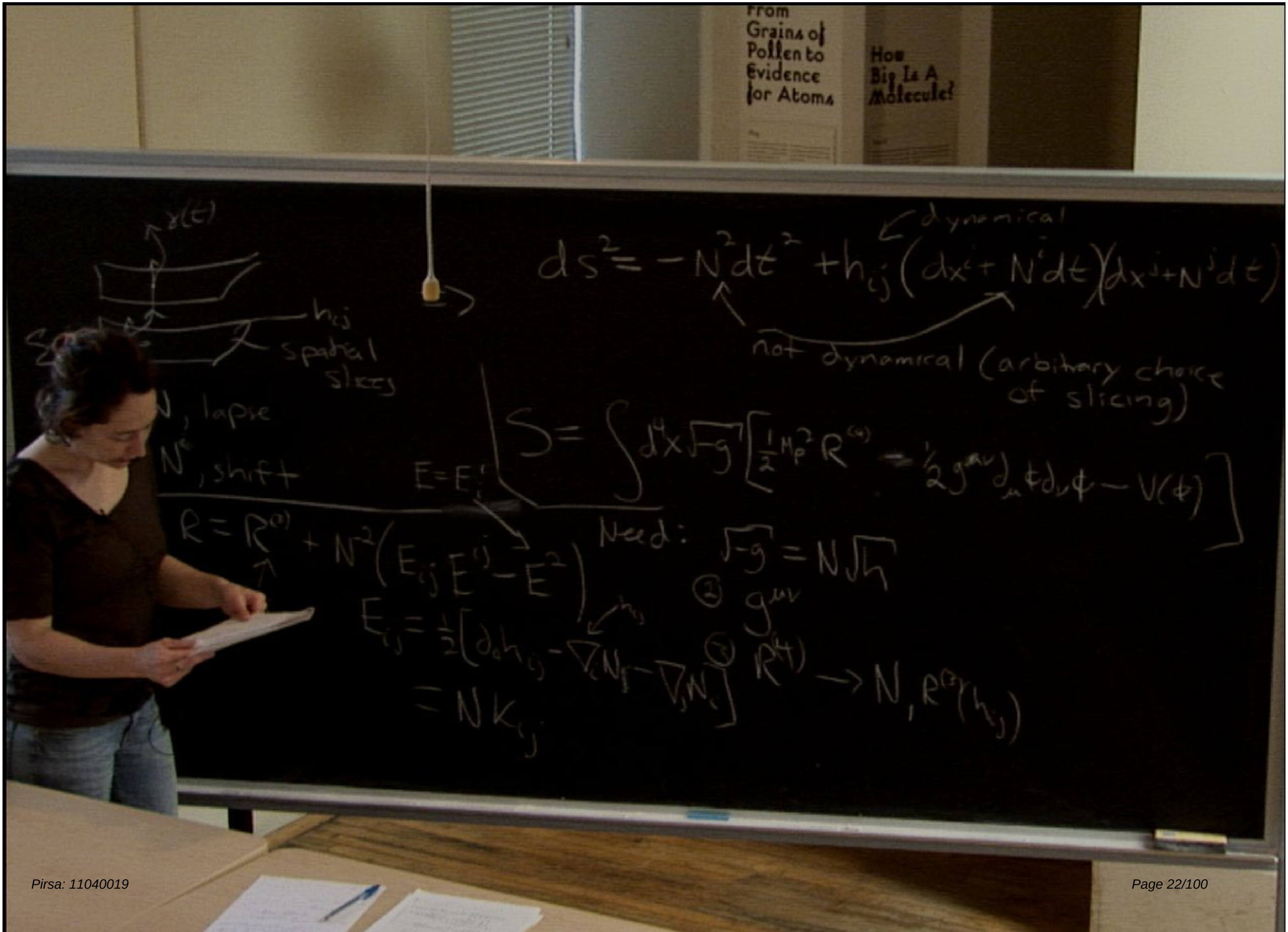
2

3


$S = \frac{1}{2}\int d^4x \sqrt{h}\left[N R^{(3)} + \right.$

$$S = \frac{1}{2}\int d^4x\,\sqrt{h}\,\left[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij} - E^2\right)\right.$$


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\gamma(t)$
h_{ij}
spatial slices
N, lapse
N^i, shift
dynamical
$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
not dynamical (arbitrary choice of slicing)
$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_p^2 R^{(4)} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$
$E = E_i^i$
$R^{(3)} + N^2(E_{ij}E^{ij} - E^2)$
$E_{ij} = \frac{1}{2}[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i]$
Need: $\sqrt{-g} = N\sqrt{h}$
② $g^{\mu\nu}$
③ $R^{(4)} \rightarrow N, R^{(3)}(h_{ij})$


From Grains of Pollen to Evidence for Atoms
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$E = E^i_i$
$R = R^{(3)} + N^2(E_{ij}E^{ij} - E^2)$
$E_{ij} = \frac{1}{2}[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i]$
$= N K_{ij}$
Need: $\sqrt{-g} = N\sqrt{h}$
② $g^{\mu\nu}$
③ $R^{(4)} \longrightarrow N, R^{(3)}(h_{ij})$

$t)$

h_{ij}

spatial slices

lapse

shift

$E_{ij}h^{ij} = E = E^i_i$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)$$

dynamical

not dynamical (arbitrar... of sl...

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_p^2 R^{(4)} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right.$$

$$R = R^{(3)} + N^{-2}\left(E_{ij}E^{ij} - E^2\right)$$

h_{ij}

$$E_{ij} = \frac{1}{2}\left[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i\right]$$

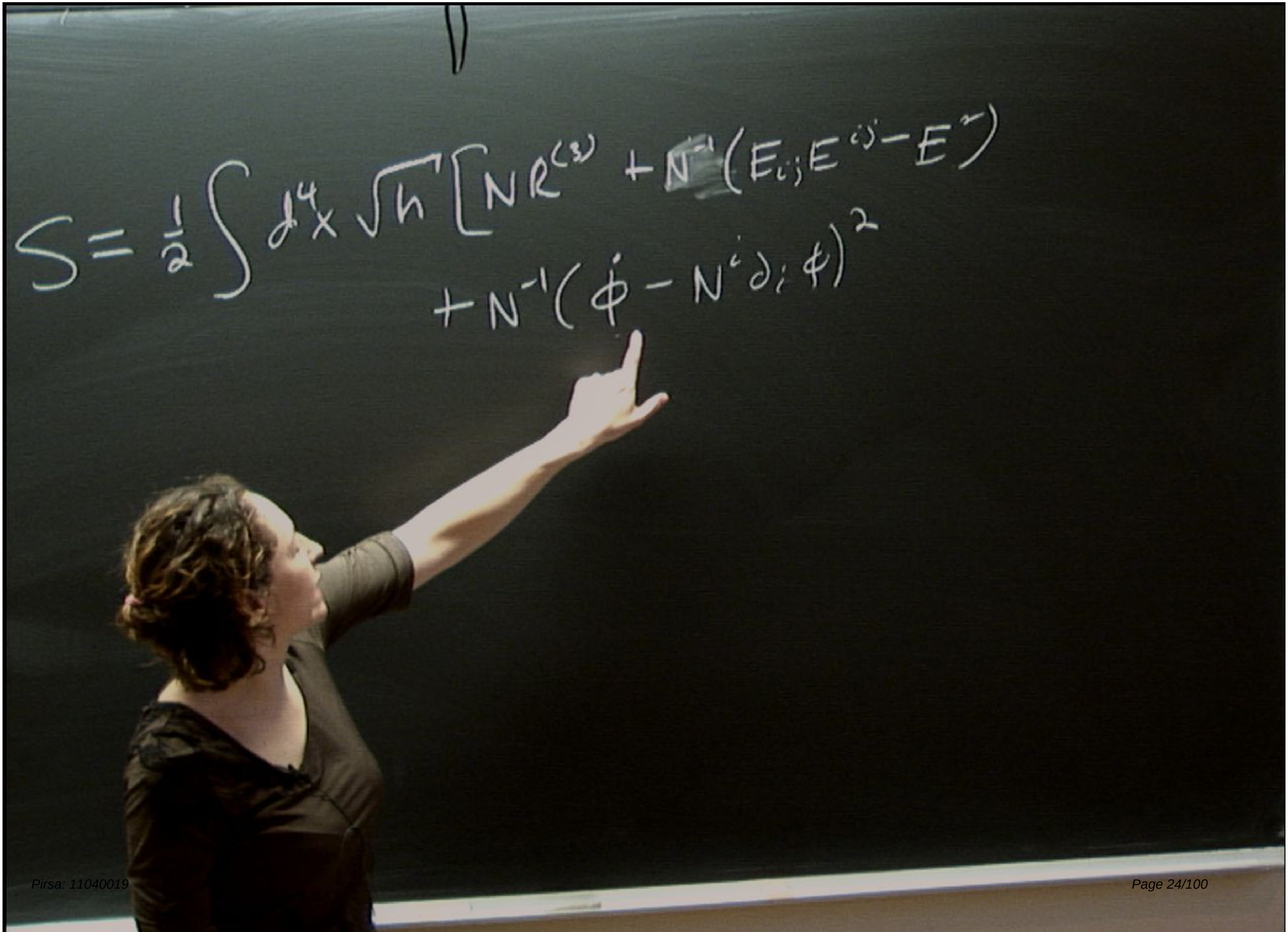
$$= N K_{ij}$$

h_{ij}

Need: $\sqrt{-g} = N\sqrt{h}$

(2) $g^{\mu\nu}$

(3) $R^{(4)} \longrightarrow N, R^{(3)}(h_{ij})$


$S = \frac{1}{2}\int d^4x \sqrt{h}\,[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2)$
$+ N^{-1}(\dot{\phi} - N^i \partial_i \phi)^2$

$$S = \frac{1}{2}\int d^4x \sqrt{h}\,\Big[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij} - E^2\right) - N h^{ij}\partial_i\phi\,\partial_j\phi + N^{-1}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - 2NV(\phi) \Big]$$

$$S = \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i \partial_i \phi)^2 - N h^{ij}\partial_i\phi\,\partial_j\phi - 2N V(\phi)\Big]$$

$N, N^i \longrightarrow 4$ doF

$$S = \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij} - E^2\right) - N h^{ij}\partial_i\phi\,\partial_j\phi + N^{-1}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - 2NV(\phi)\Big]$$

N, N^i [illegible] 4 doF (constraints for those, do the same as our choice to define Φ, Ψ)

$$S = \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) - N h^{ij}\partial_i\phi\,\partial_j\phi$$
$$+ N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2$$
$$-2NV(\phi)\Big]$$

$\underbrace{N, N^i}$ $\longrightarrow$ 4 doF (constraints for those, do the same as our choice to define Φ, Ψ)

$\downarrow$ 2 scalars
1 vector $(\partial^i w_i = 0)$

$$S = \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i \partial_i \phi)^2 - N h^{ij}\partial_i\phi\partial_j\phi - 2NV(\phi) \Big]$$

$\underbrace{N, N^i}$ $\longrightarrow$ 4 doF (constraints for those, do the same as our choice to define Φ, Ψ)

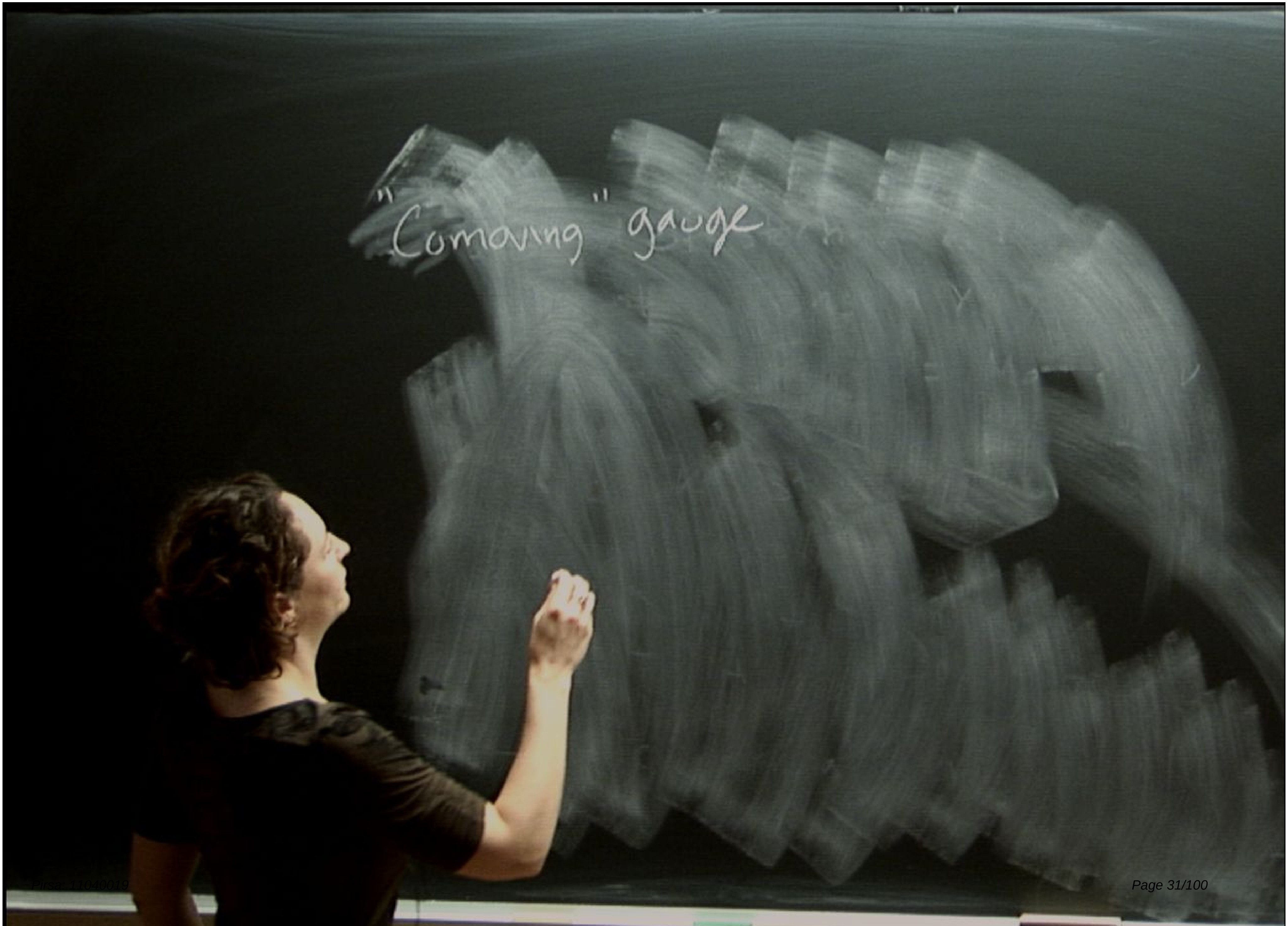
$\downarrow$ 2 scalars, 1 vector ($\partial^i \omega_i = 0$)

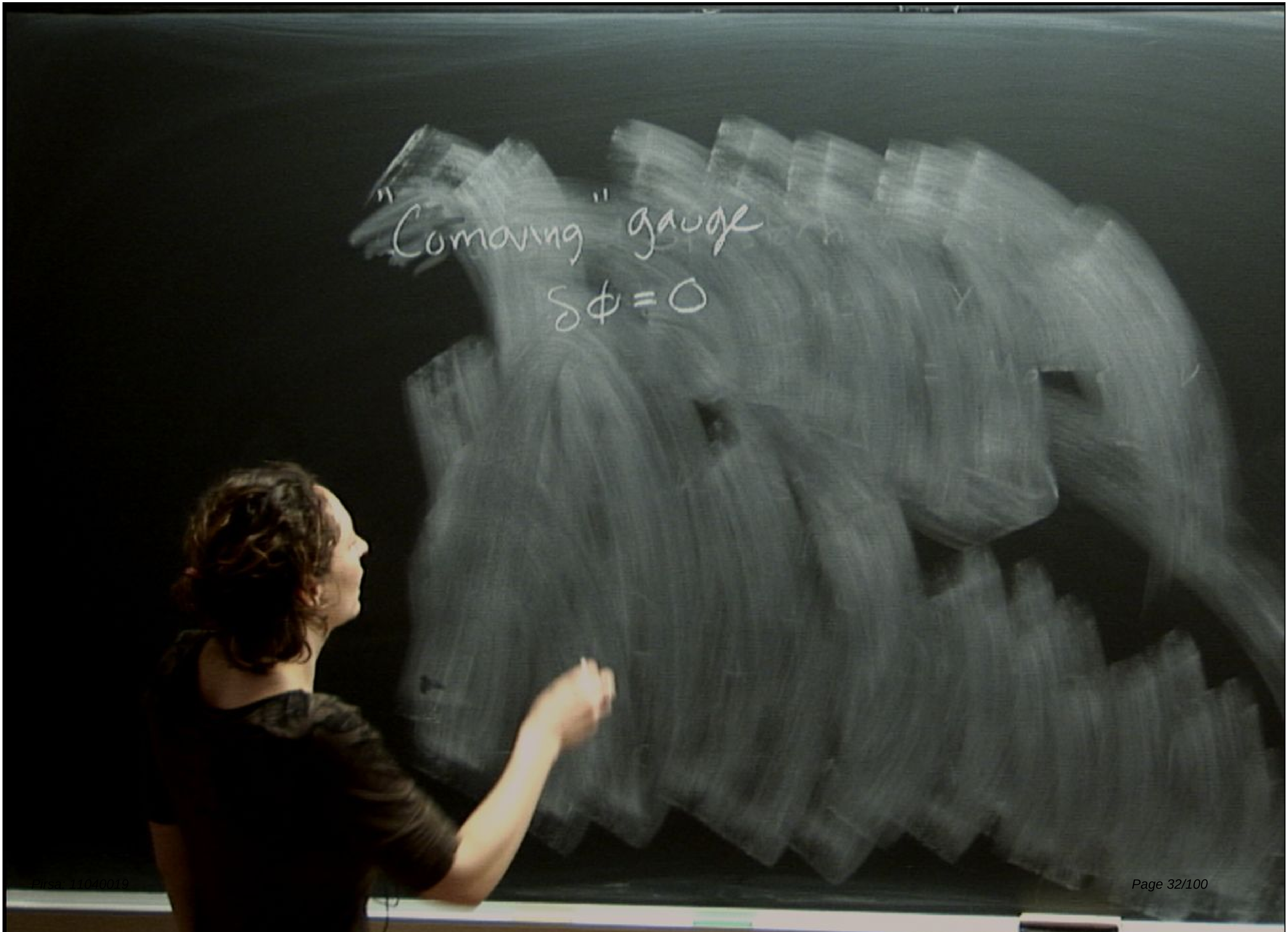
$\longrightarrow$ still need to choose a gauge

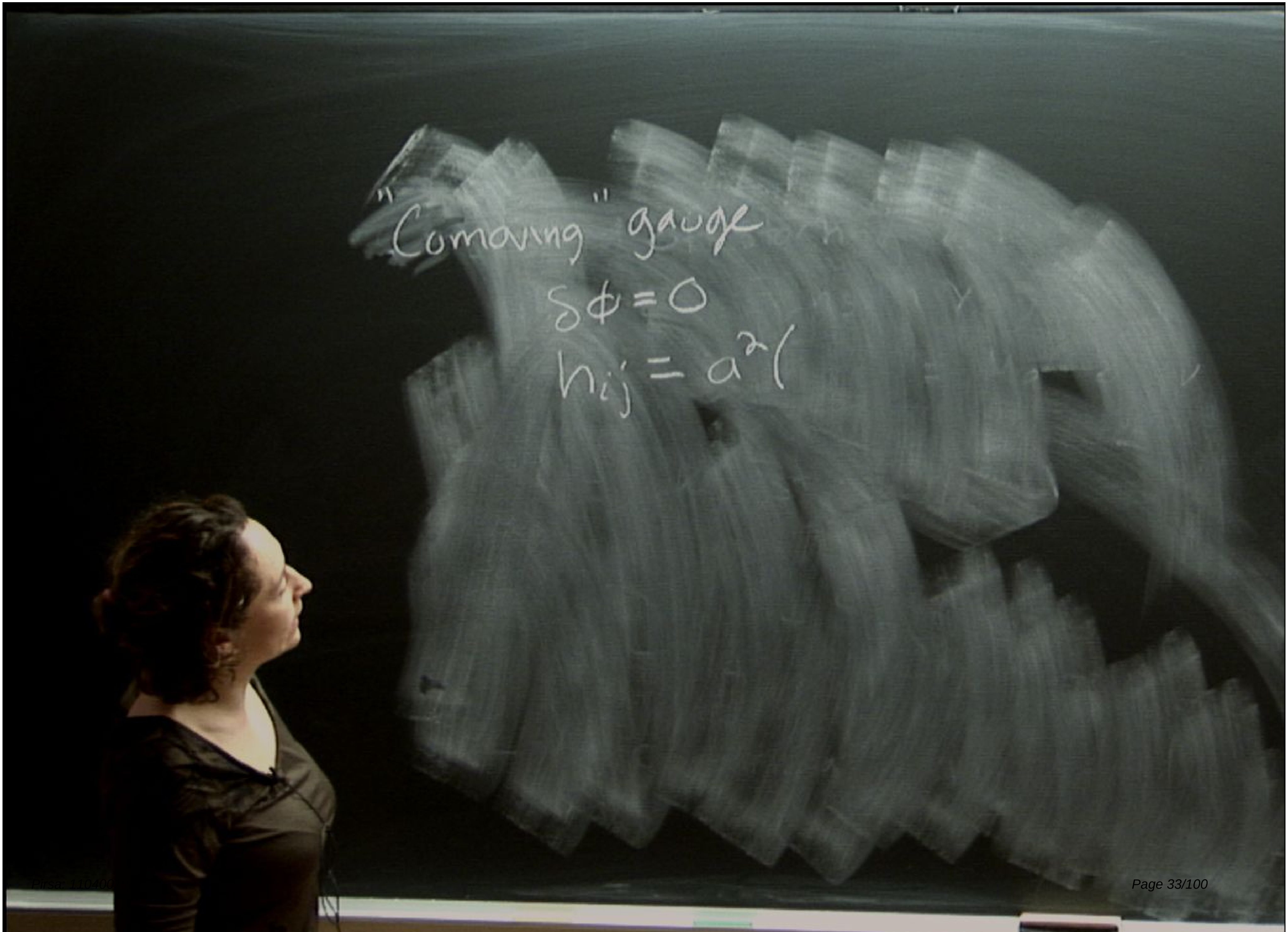
$$S = \frac{1}{2}\int d^4x \sqrt{h}\,\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) - N h^{ij}\partial_i\phi\,\partial_j\phi$$
$$+ N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2$$
$$-2NV(\phi)\Big]$$

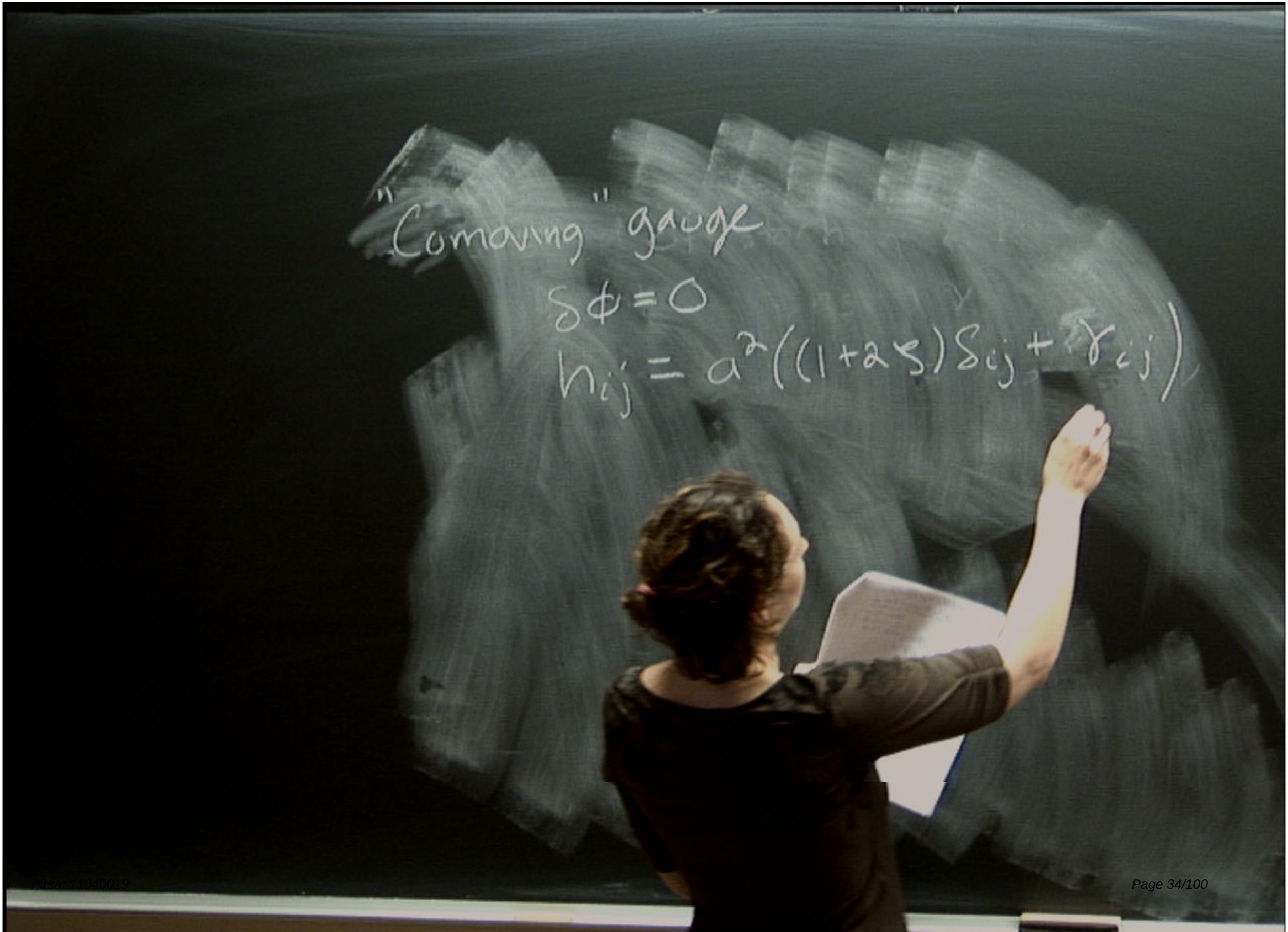
$\underbrace{N, N^i}$ $\longrightarrow$ 4 doF (constraints for those, do the same as our choice to define Φ, Ψ)
$\downarrow$
2 scalars
1 vector ($\partial^i w_i = 0$)

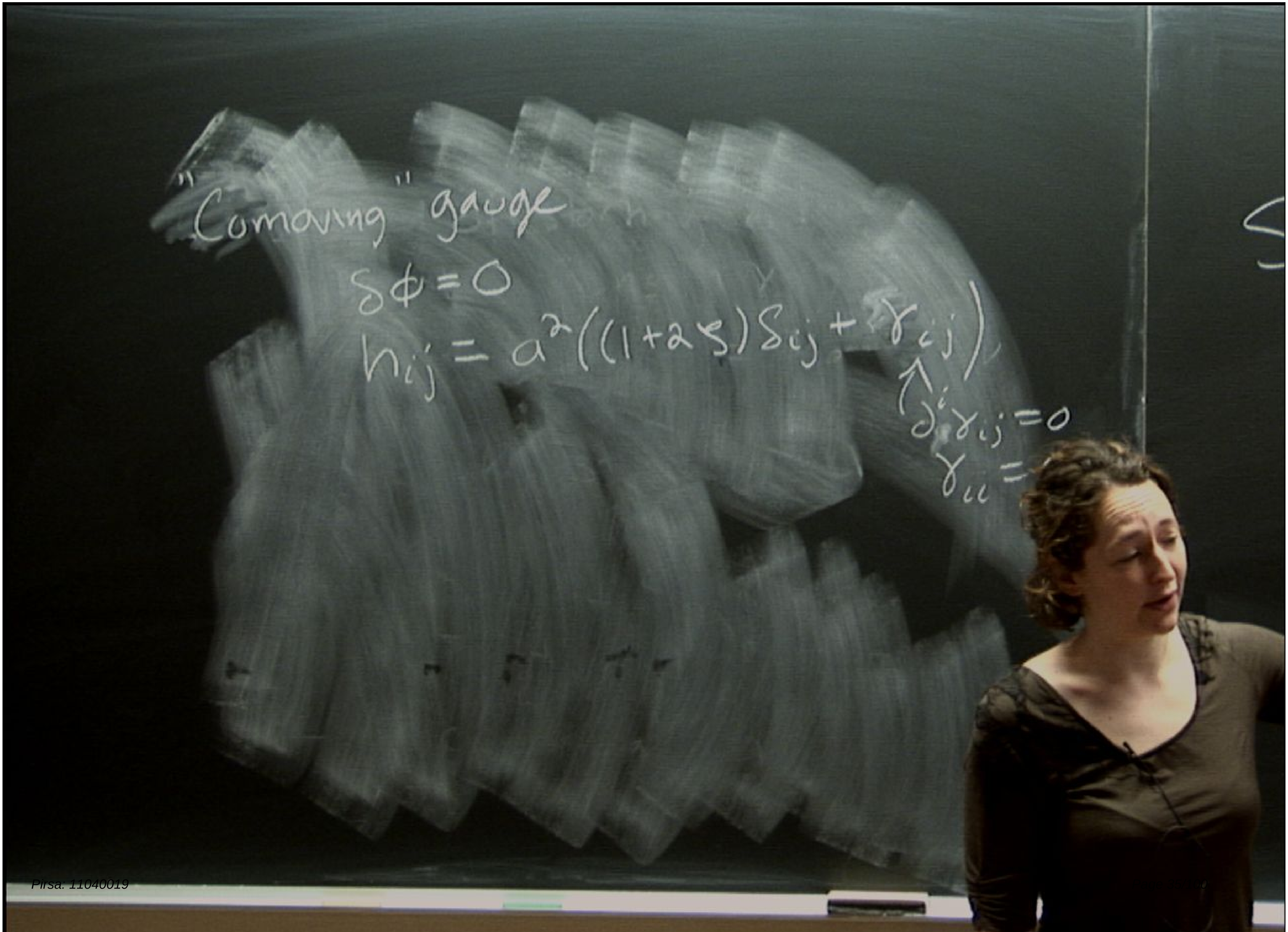
$\longrightarrow$ still need to choose a gauge

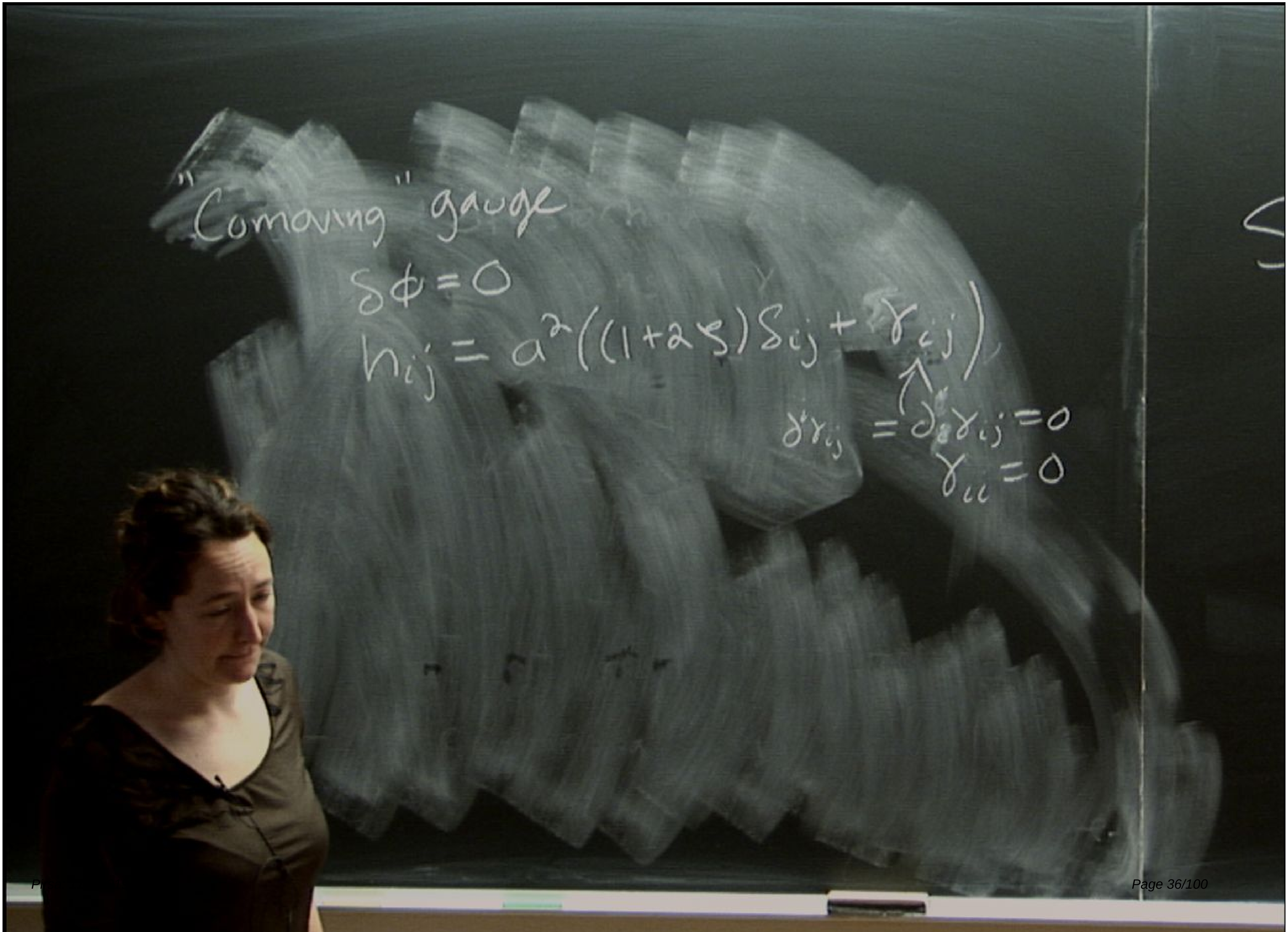

"Comoving" gauge

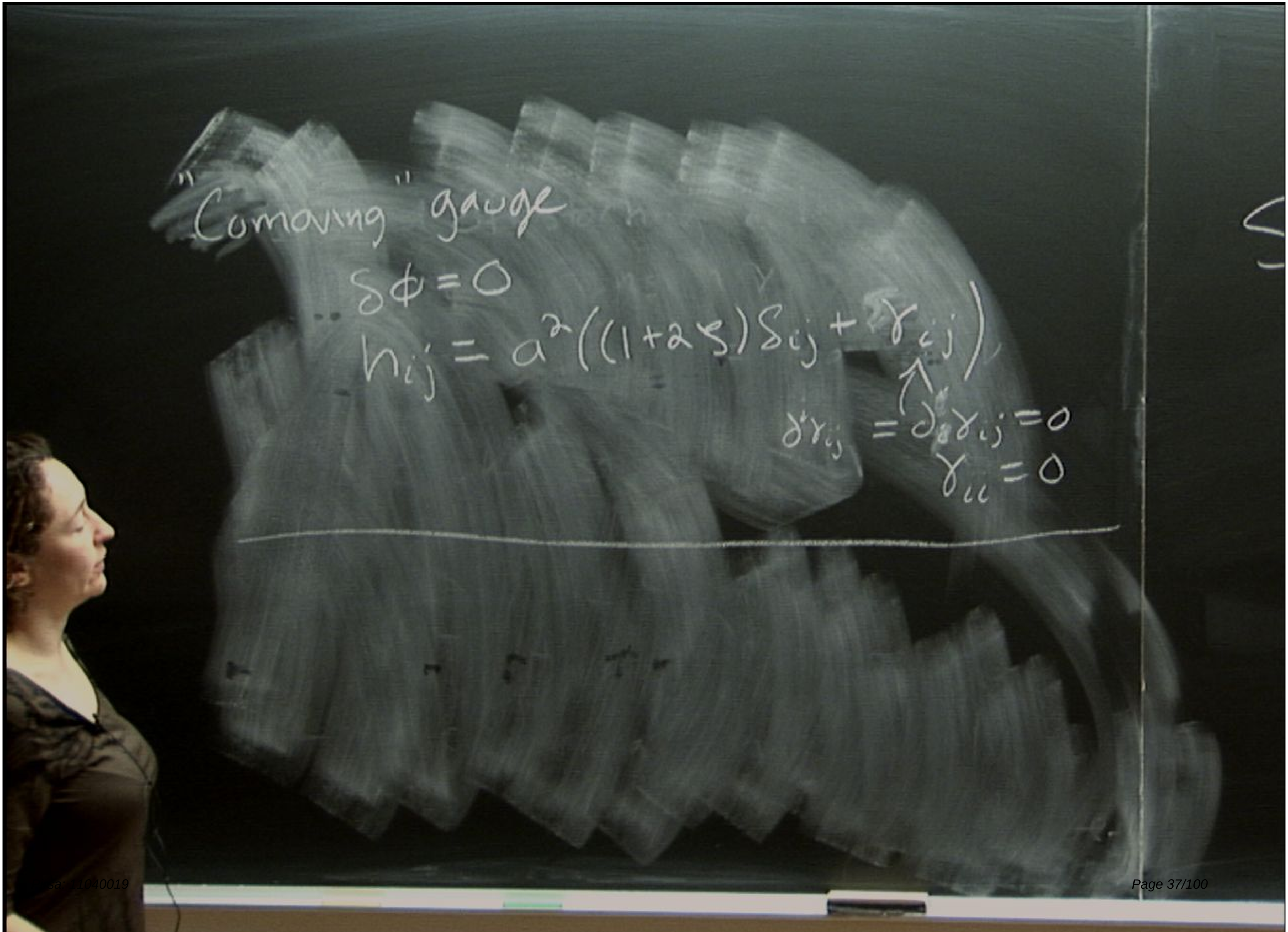

"Comoving" gauge
$\delta\phi = 0$


"Comoving" gauge
δφ = 0
h_ij = a^2(


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$


"Comoving" gauge
δφ = 0
h_ij = a²((1+2ζ)δ_ij + γ_ij)
∂^i γ_ij = 0
γ_ii =


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$
$\gamma_{ii} = 0$


"Comoving" gauge
$\delta\phi = 0$
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$\partial^i\gamma_{ij} = \partial_i\gamma_{ij} = 0$
$\gamma_{ii} = 0$

"Comoving" gauge

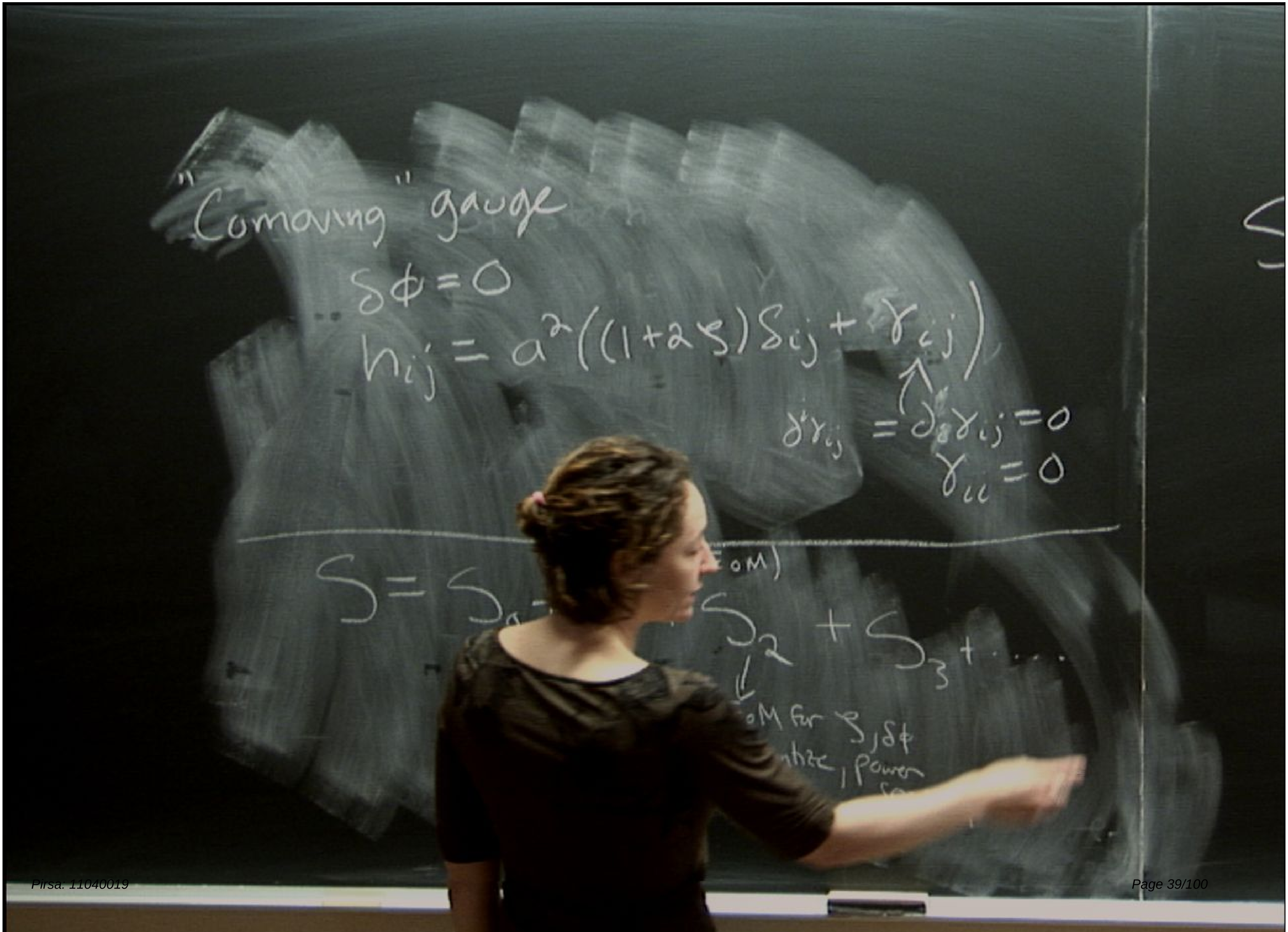
$$\delta\phi = 0$$

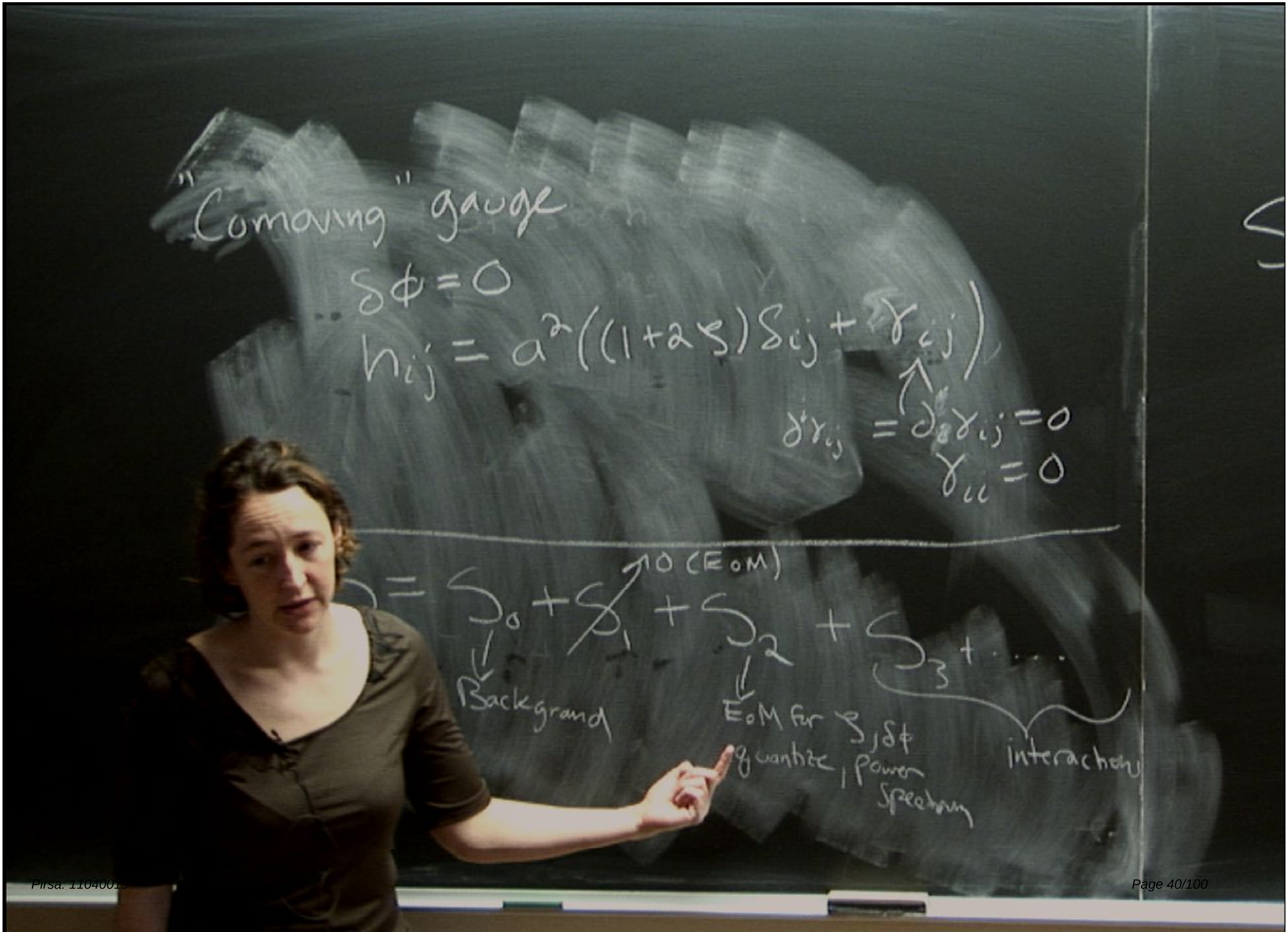
$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

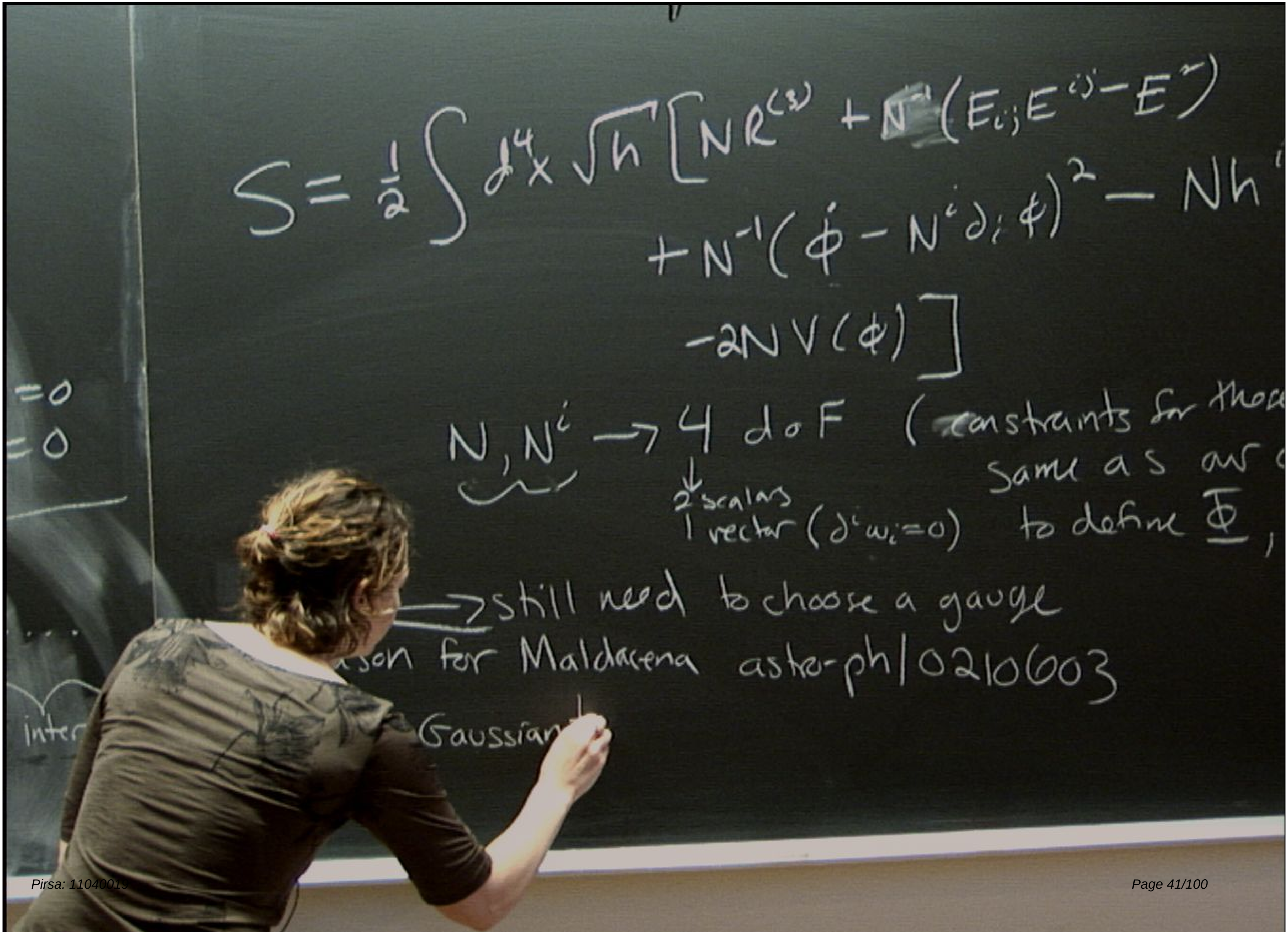
$$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$$

$$\gamma_{ii} = 0$$

$$S = \underbrace{S_0}_{\text{Background}} + \overbrace{\cancel{S_1}}^{0\ (\text{EoM})}$$


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i \gamma_{ij} = \partial_j \gamma_{ij} = 0$
$\gamma_{ii} = 0$
$S = S_0 + \ldots S_2 + S_3 + \ldots$
for $\zeta, \delta\phi$
power


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i\gamma_{ij} = \partial_i\gamma_{ij} = 0$
$\gamma_{ii} = 0$
0 (EoM)
$S = S_0 + \cancel{S_1} + S_2 + S_3 + \ldots$
Background
EoM for $\zeta, \delta\phi$
quantize, power spectrum
interactions


S = ½ ∫ d⁴x √h [N R⁽³⁾ + N⁻¹ (E_ij E^ij − E²)
− N h
+ N⁻¹ (φ̇ − Nⁱ ∂ᵢ φ)²
− 2N V(φ)]
N, Nⁱ → 4 doF (constraints for those
same as our
2 scalars
1 vector (∂ⁱwᵢ = 0)
to define Φ,
→ still need to choose a gauge
son for Maldacena astro-ph/0210603
Gaussian
=0
=0
inter

$$S = \frac{1}{2}\int d^4x \sqrt{h}\,\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) - N h^{i}$$
$$+ N^{-1}(\dot\phi - N^i \partial_i \phi)^2$$
$$- 2NV(\phi)\Big]$$

=0

=0

$\underbrace{N, N^i}$ $\longrightarrow$ 4 doF (constraints for those

same as our

$\downarrow$ 2 scalars

1 vector ($\partial^i w_i = 0$) to define Φ,

$\longrightarrow$ still need to choose a gauge

$\rightarrow$ reason for Maldacena astro-ph/0210603

interactions $\rightarrow$ Non-Gaussianity!

$$= \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot\phi - N^i\partial_i\phi)^2 - N h^{ij}\partial_i\phi\partial_j\phi - 2NV(\phi)\Big]$$

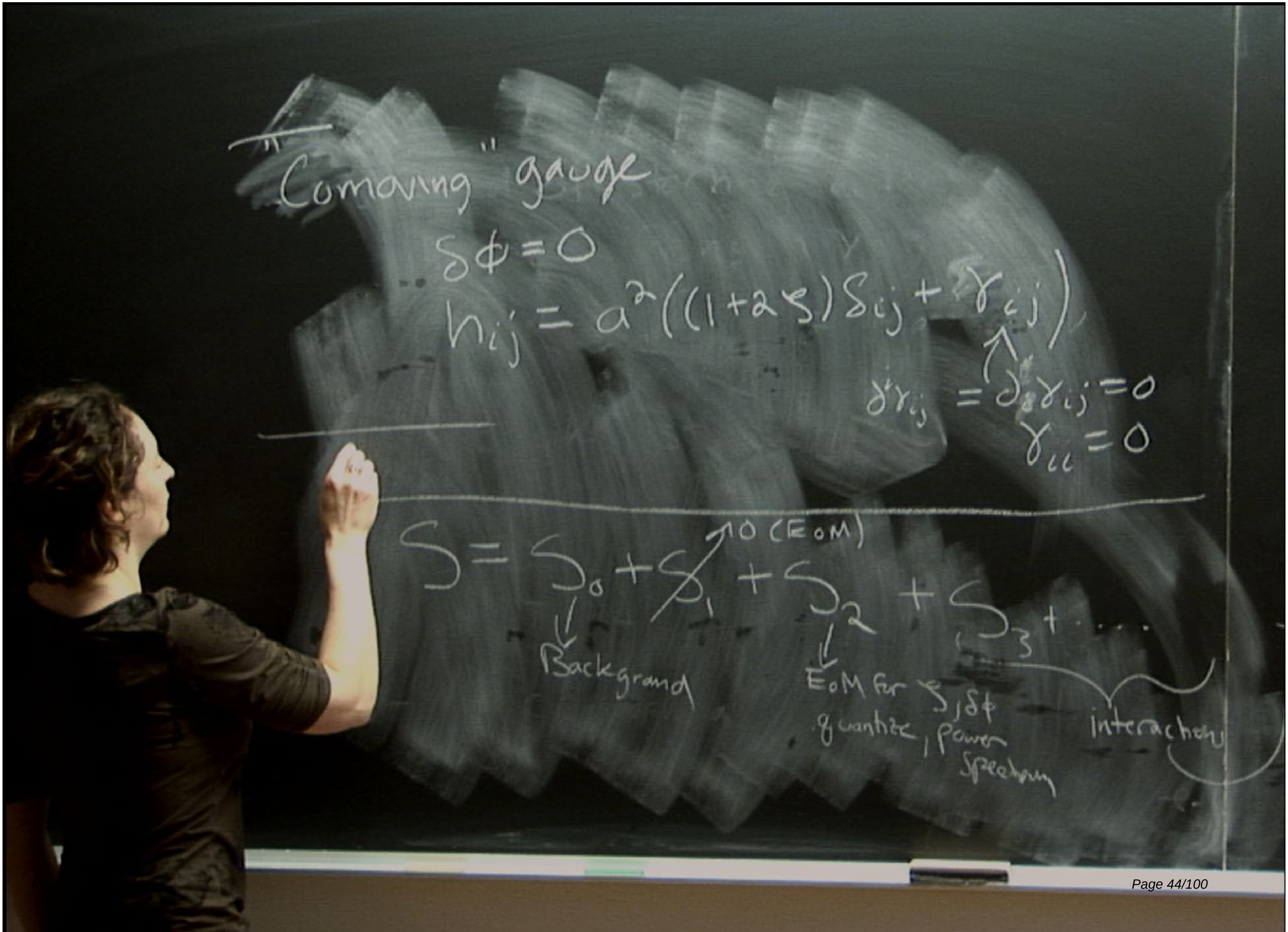
$\underbrace{N, N^i}$ $\longrightarrow$ 4 doF (constraints for those, do the same as our choice to define Φ, Ψ)

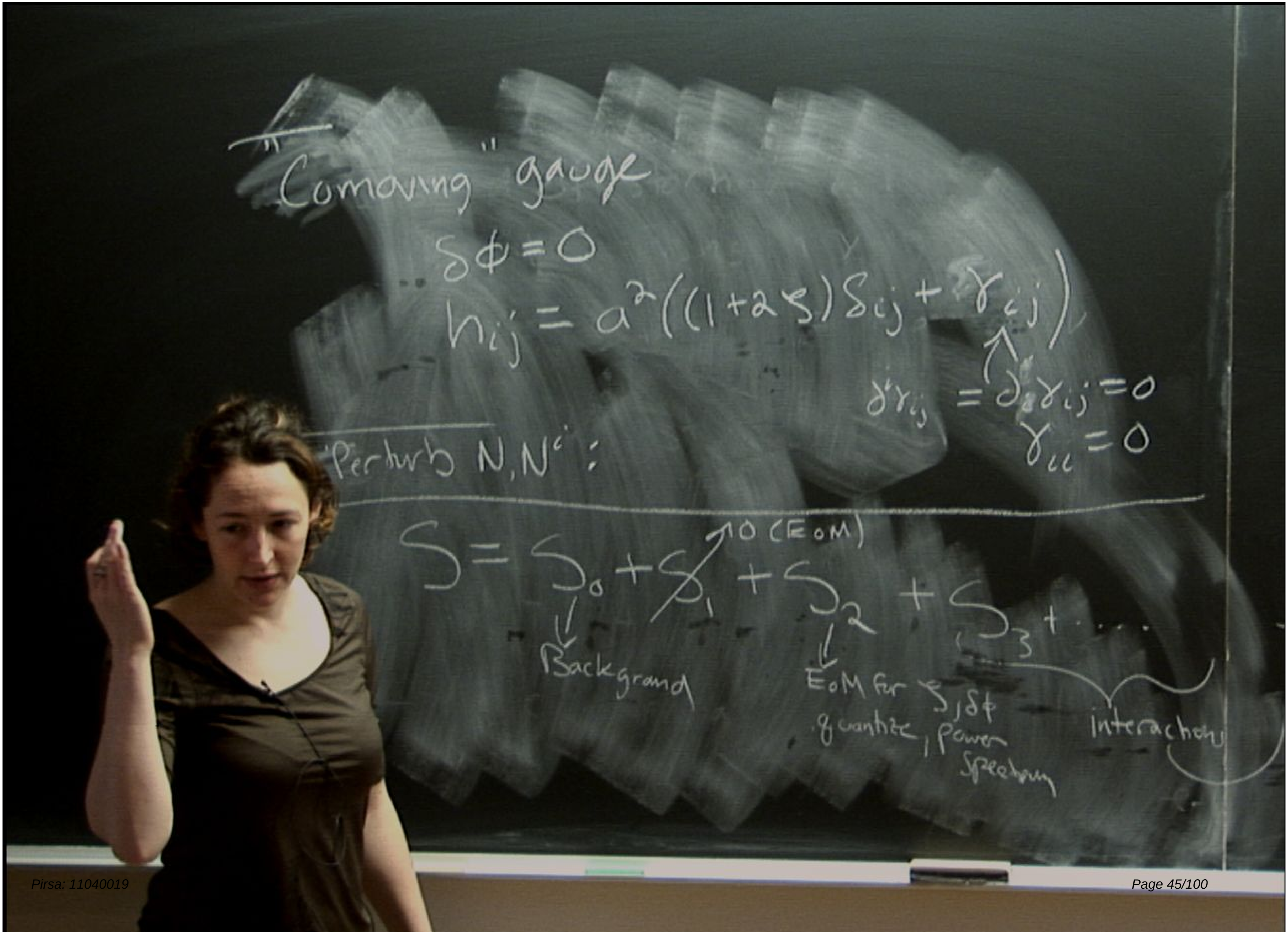
$\downarrow$ 2 scalars, 1 vector ($\partial^i w_i = 0$)

$\longrightarrow$ still need to choose a gauge

Reason for Maldacena astro-ph/0210603

Non-Gaussianity! Only need first order pert. in N, N^i, ϕ to get S_3


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$
$\gamma_{ii} = 0$
$S = S_0 + \cancel{S_1} + S_2 + S_3 + \dots$
0 (EoM)
Background
EoM for $\zeta, \delta\phi$
quantize, power spectrum
interactions


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i\gamma_{ij} = \partial_i\gamma_{ij} = 0$
$\gamma_{ii} = 0$
Perturb N, N^i :
0 (EoM)
$S = S_0 + \cancel{S_1} + S_2 + S_3 + \dots$
Background
EoM for $\zeta, \delta\phi$
quantize, power spectrum
interactions

"Comoving" gauge

$\delta\phi = 0$

$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$

$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$

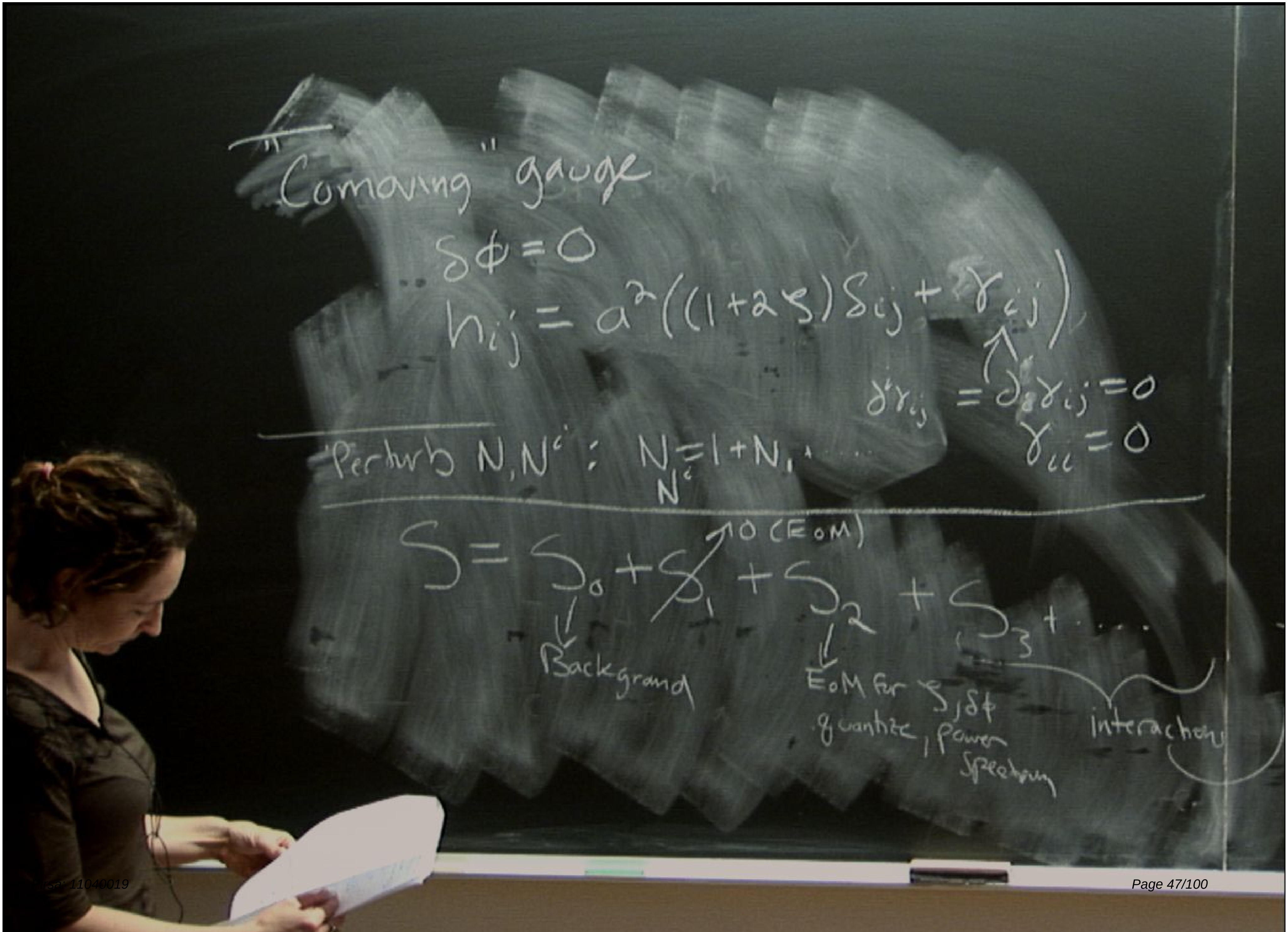
$\gamma_{ii} = 0$

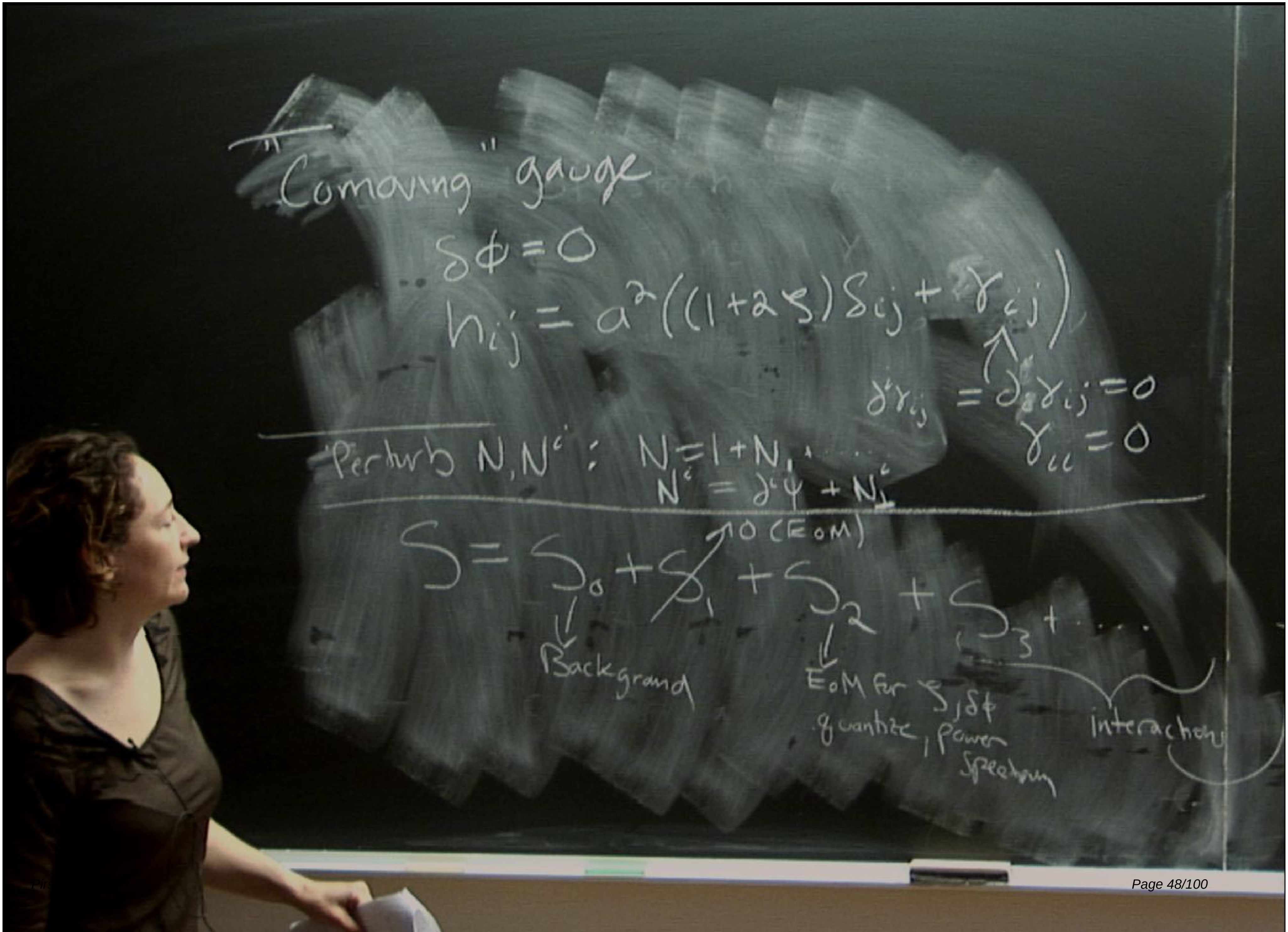
Perturb N, N^i : $N=1$

$S = S_0 + \cancel{S_1}^{\,0\ (\mathrm{EoM})} + S_2 + \underbrace{S_3 + \ldots}_{\text{interactions}}$

S_0: Background

S_2: EoM for $\zeta, \delta\phi$ quantize, power spectrum


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
$\partial^i\gamma_{ij} = \partial_i\gamma_{ij} = 0$
$\gamma_{ii} = 0$
Perturb N, N^i : $N = 1 + N_1 \ldots$
N^i
0 (EoM)
$S = S_0 + \cancel{S_1} + S_2 + S_3 + \ldots$
Background
EoM for $\zeta, \delta\phi$
quantize, power spectrum
interactions


"Comoving" gauge
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N^i = ∂^i ψ + N_1^i
S = S_0 + S_1 + S_2 + S_3 + ...
0 (EoM)
Background
EoM for ζ, δφ
quantize, power spectrum
interactions

"Comoving" gauge

$$\delta\phi = 0$$

$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

$$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$$

$$\gamma_{ii} = 0$$

Perturb N, N^i : $N = 1 + N_1 + \cdots$

$$N^i = \partial^i \psi + N_1^i$$

$$S = \underbrace{S_0}_{\text{Background}} + \overset{\to 0\ (\text{EoM})}{\cancel{S_1}} + \underbrace{S_2}_{\text{EoM for } \zeta, \delta\phi} + \underbrace{S_3 + \cdots}_{\text{interactions}}$$

quantize, power spectrum

...ning" gauge

$\delta\phi = 0$

$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$

$\partial^i \gamma_{ij} = \partial_i \gamma_{ij} = 0$

$\gamma_{ii} = 0$

...urb N, N^i : $N = 1 + N_1 + \ldots$

$N^i = \partial^i \psi + N_\perp^i + \ldots$

$\partial_i (N_\perp^i) = 0$

$S = S_0 + \cancel{S_1} + S_2 + S_3 + \ldots$

$S_1 \to 0$ (EoM)

S_0: Background

S_2: EoM for $\zeta, \delta\phi$, quantize, power spectrum

$S_3 + \ldots$: interaction

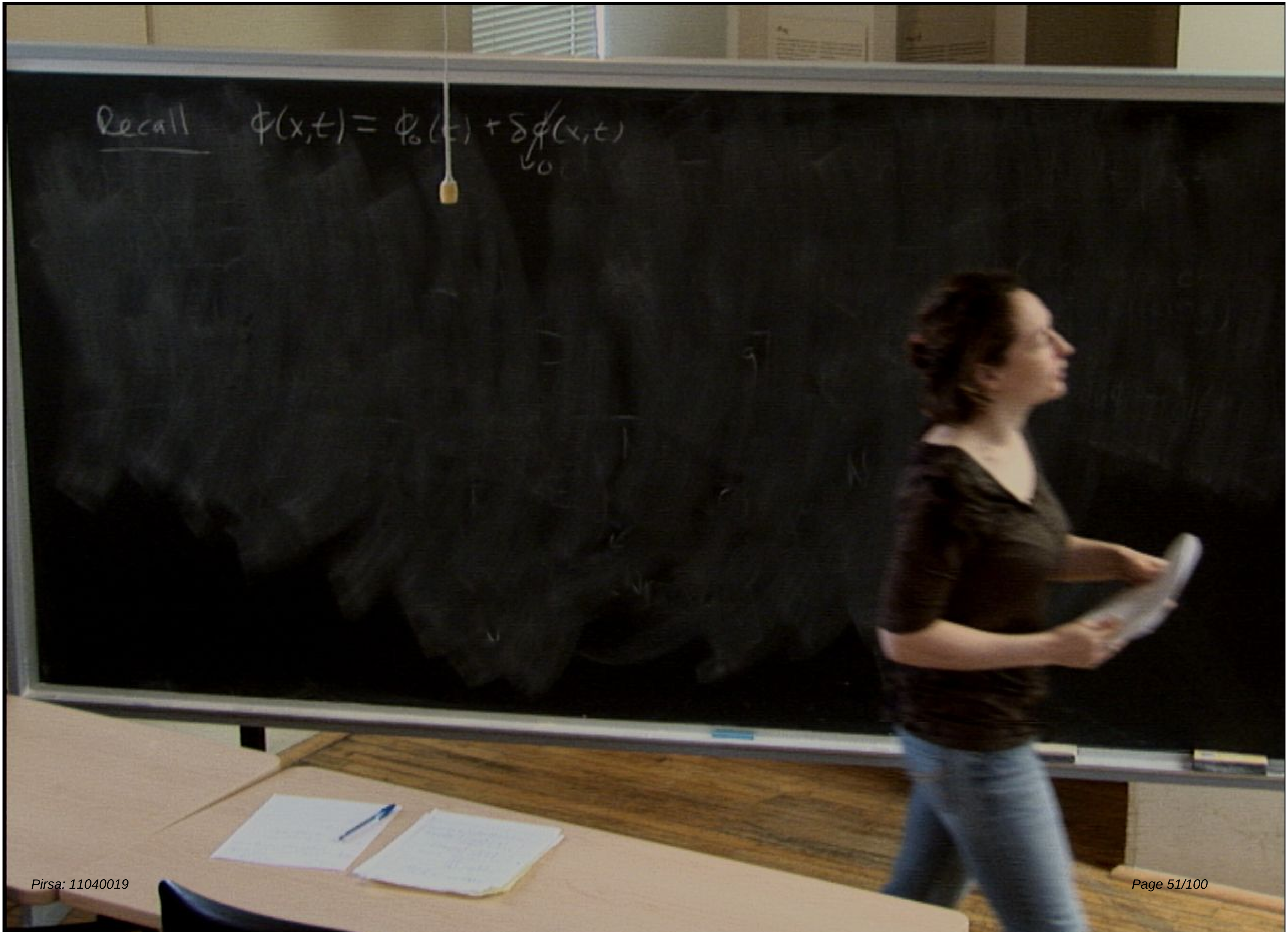
$S = \frac{1}{2}\int d\ldots$

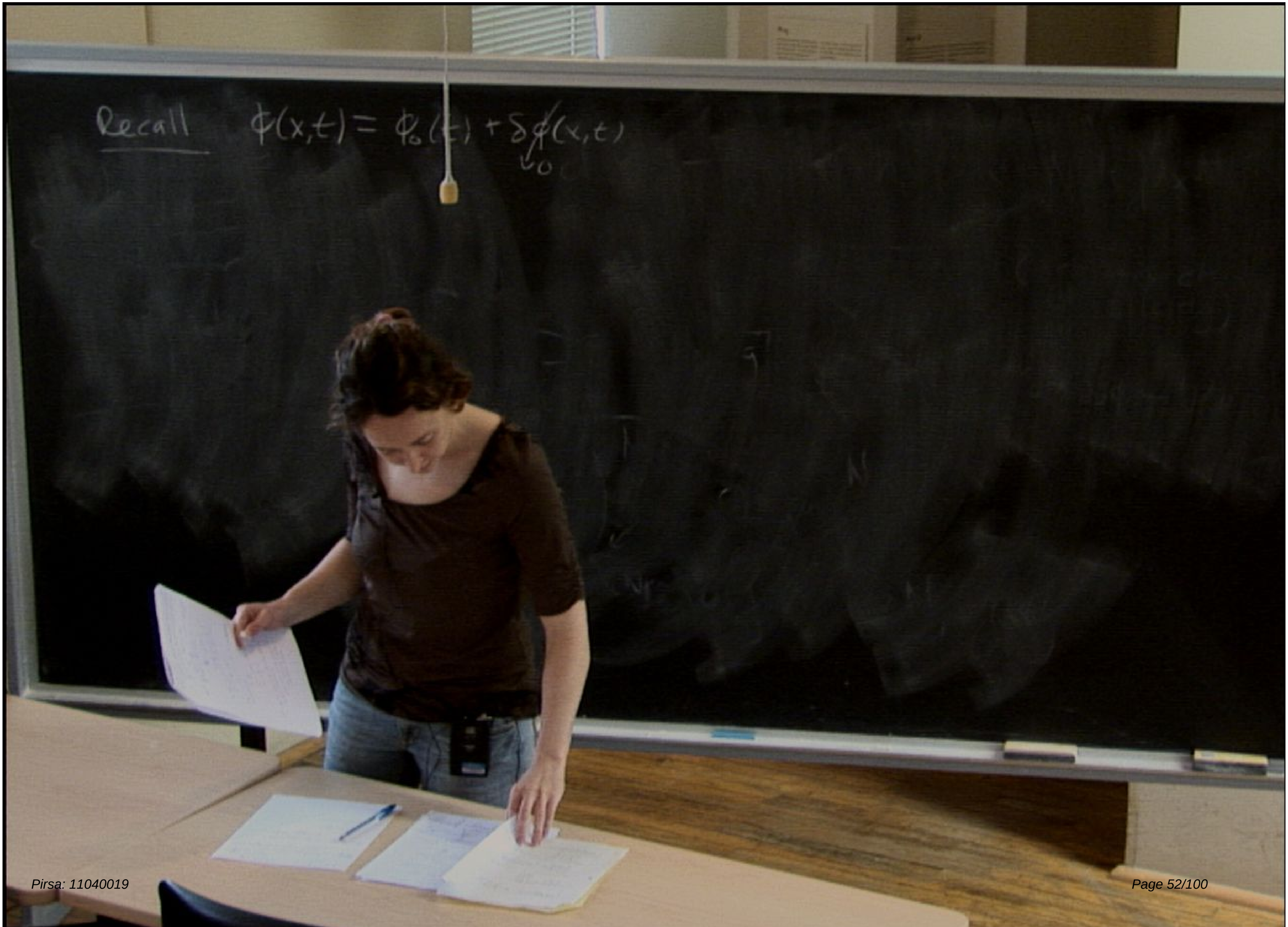
$N, \ldots$

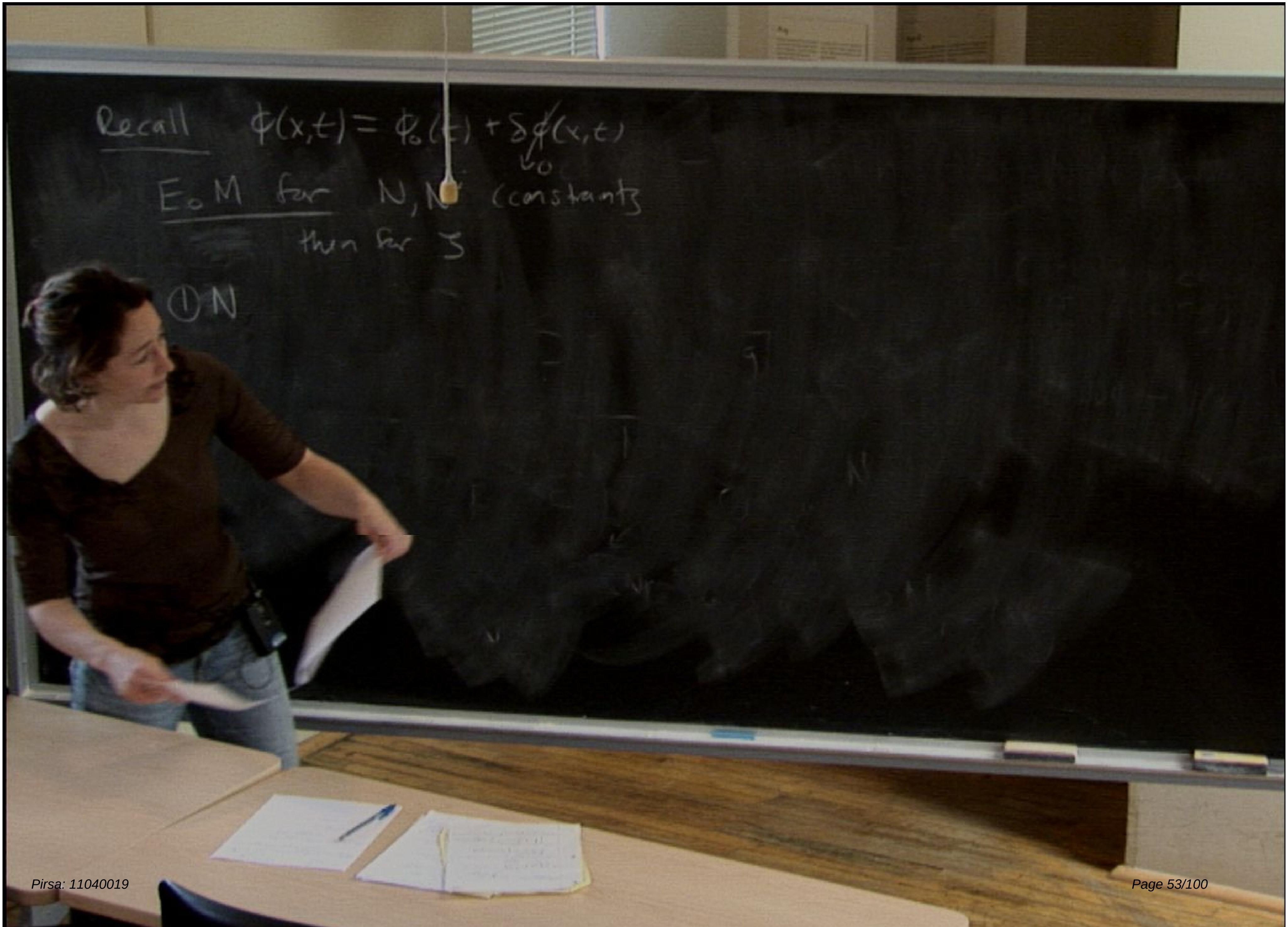
$\to$ st...

$\to$ reason for ...

$\to$ Non-Gaussi...


Recall
$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$


Recall
$\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
0


Recall $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
$\to 0$
EoM for N, N^i (constraints)
then for ζ
① N

$$S=\frac{1}{2}\int d^4x\,\sqrt{h}\Big[N R^{(3)} + N^{-1}\left(E_{ij}E^{ij}-E^2\right) + N^{-1}\left(\dot{\phi}-N^i\partial_i\phi\right)^2 - N h^{ij}\partial_i\phi\,\partial_j\phi - 2NV(\phi)\Big]\,, \quad E_{ij}=\frac{1}{2}\Big[$$

$N, N^i \longrightarrow 4$ doF (… nts for … hose, do the … as … choice

$\downarrow$ 2 scalars

1 vector ($\partial^i w_i = 0$ … Ψ]

$\longrightarrow$ still need to choose …

$\rightarrow$ reason for Maldacena astro-ph …

$\rightarrow$ Non-Gaussianity! Only need … N^i, ϕ to get S_3

$$S = \frac{1}{2}\int d^4x \sqrt{h}\Big[N R^{(3)} + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i \partial_i \phi)^2 - N h^{ij}\partial_i\phi\partial_j\phi - 2NV(\phi)\Big] \;, \quad E_{ij} = \frac{1}{2}\left[\partial_0 h_{ij} - \nabla_i N_j - \nabla_j N_i\right]$$

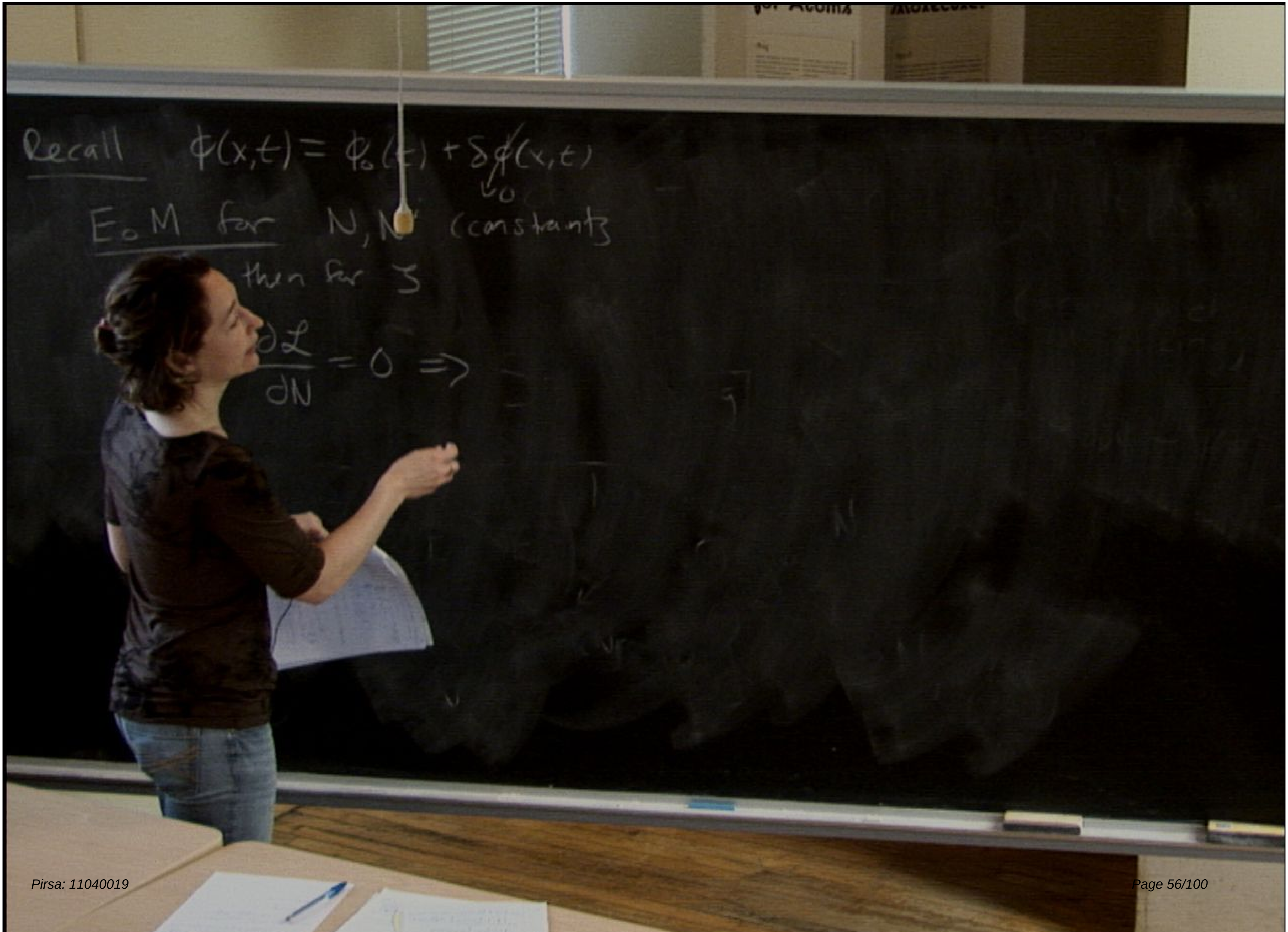
$N, N^i \rightarrow 4$ doF (constraints for these, do the same as our choice to define Φ, Ψ)

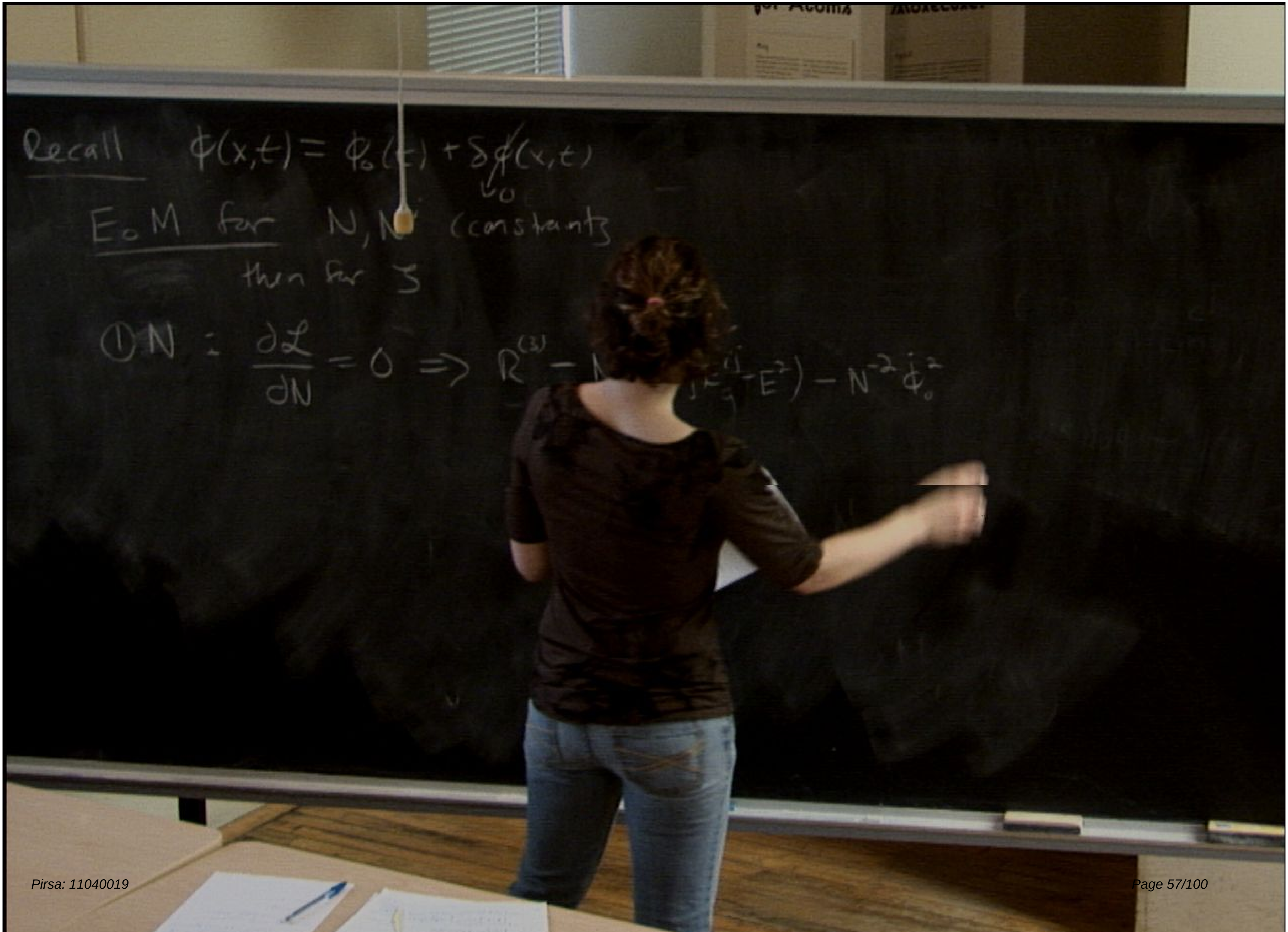
$\downarrow$ 2 scalars, 1 vector ($\partial^i w_i = 0$)

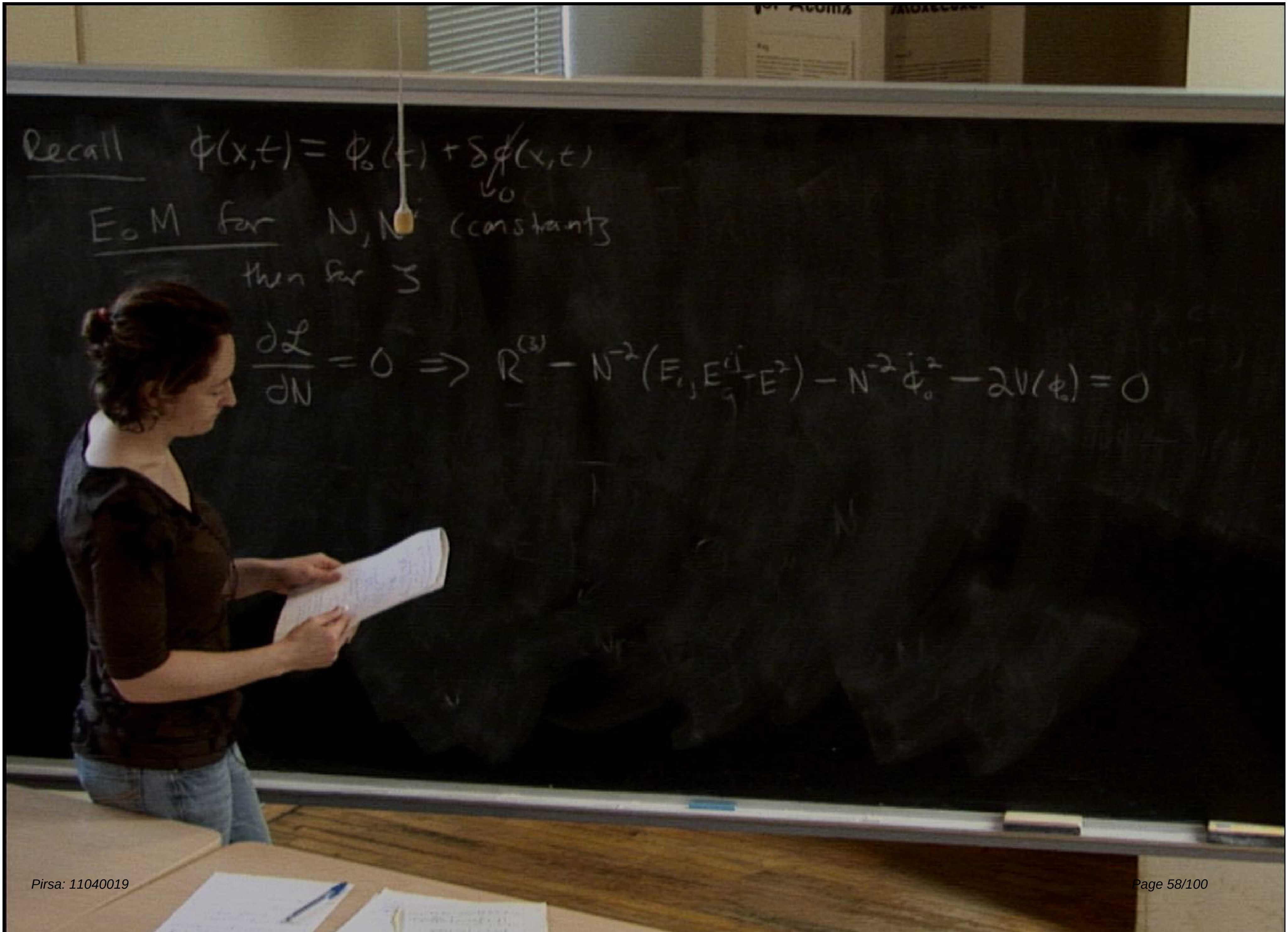
$\rightarrow$ still need to choose a gauge

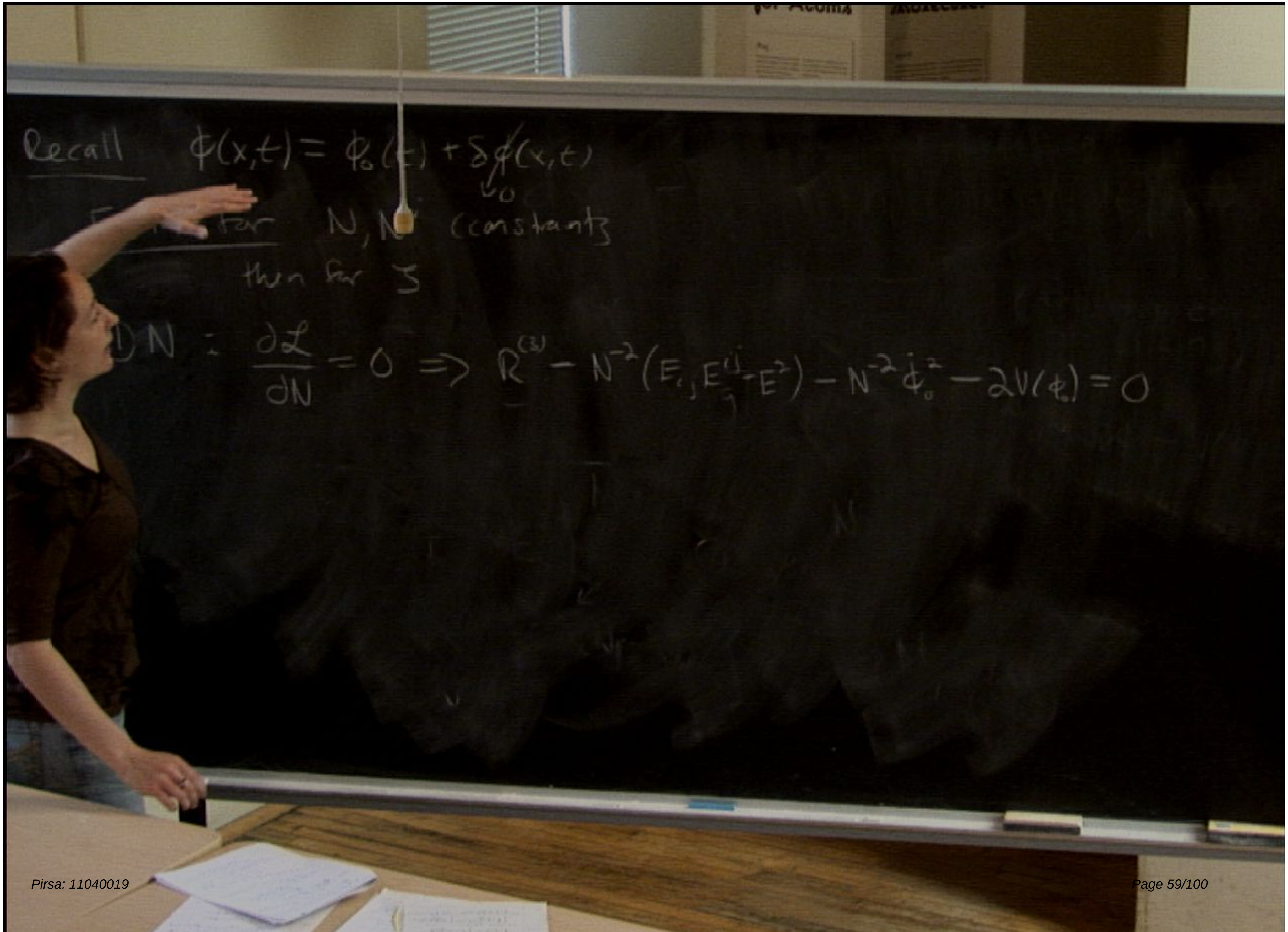
$\rightarrow$ reason for Maldacena astro-ph/0210603

$\rightarrow$ Non-Gaussianity! Only need first order pert. in N, N^i, ϕ to get S_3


Recall $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
$\to 0$
EoM for N, N^i (constraints)
then for ζ
$\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow$


Recall φ(x,t) = φ₀(t) + δφ(x,t)
EoM for N, N (constants
then for ζ
① N : ∂L/∂N = 0 ⇒ R(3)


Recall $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
$\to 0$
EoM for N, N^i (constraints)
then for ζ
$\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow R^{(3)} - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}_0^2 - 2V(\phi_0) = 0$


Recall $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
$\to 0$
for N, N^i (constraints)
then for ζ
N : $\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow R^{(3)} - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}_0^2 - 2V(\phi_0) = 0$

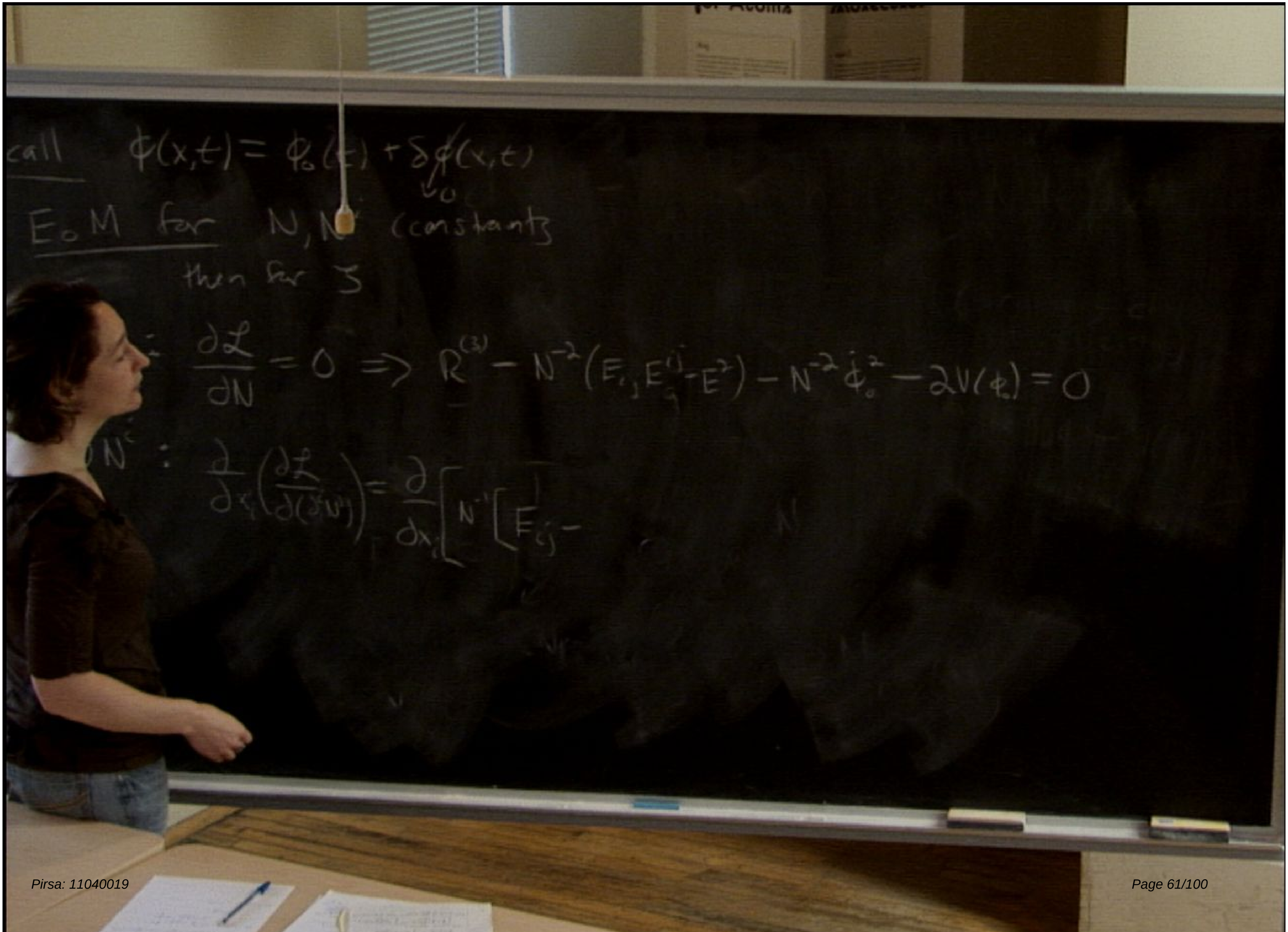
Recall $\phi(x,t) = \phi_0(t) + \underset{\to 0}{\cancel{\delta\phi}}(x,t)$

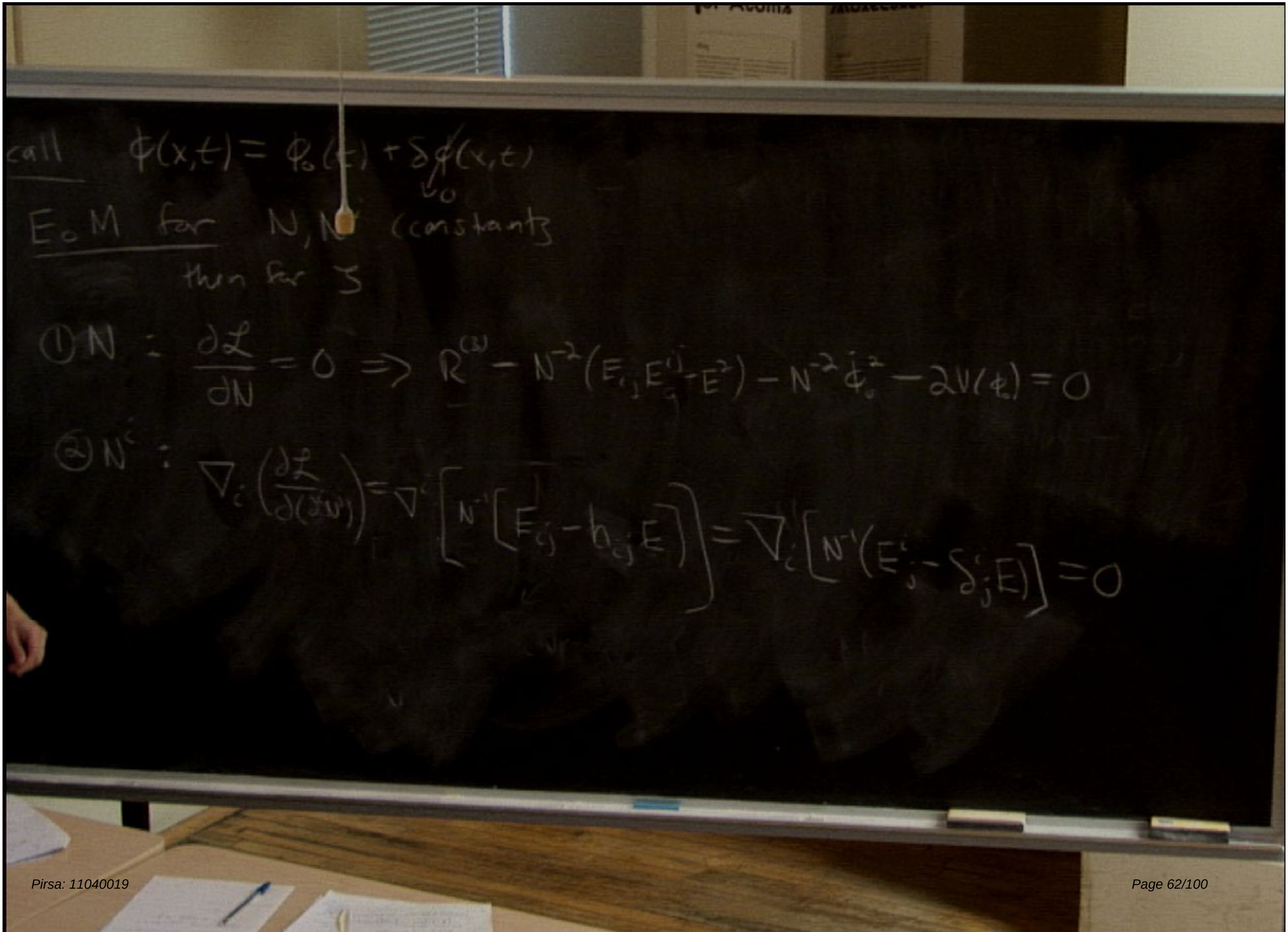
EoM for N, N^i (constraints)

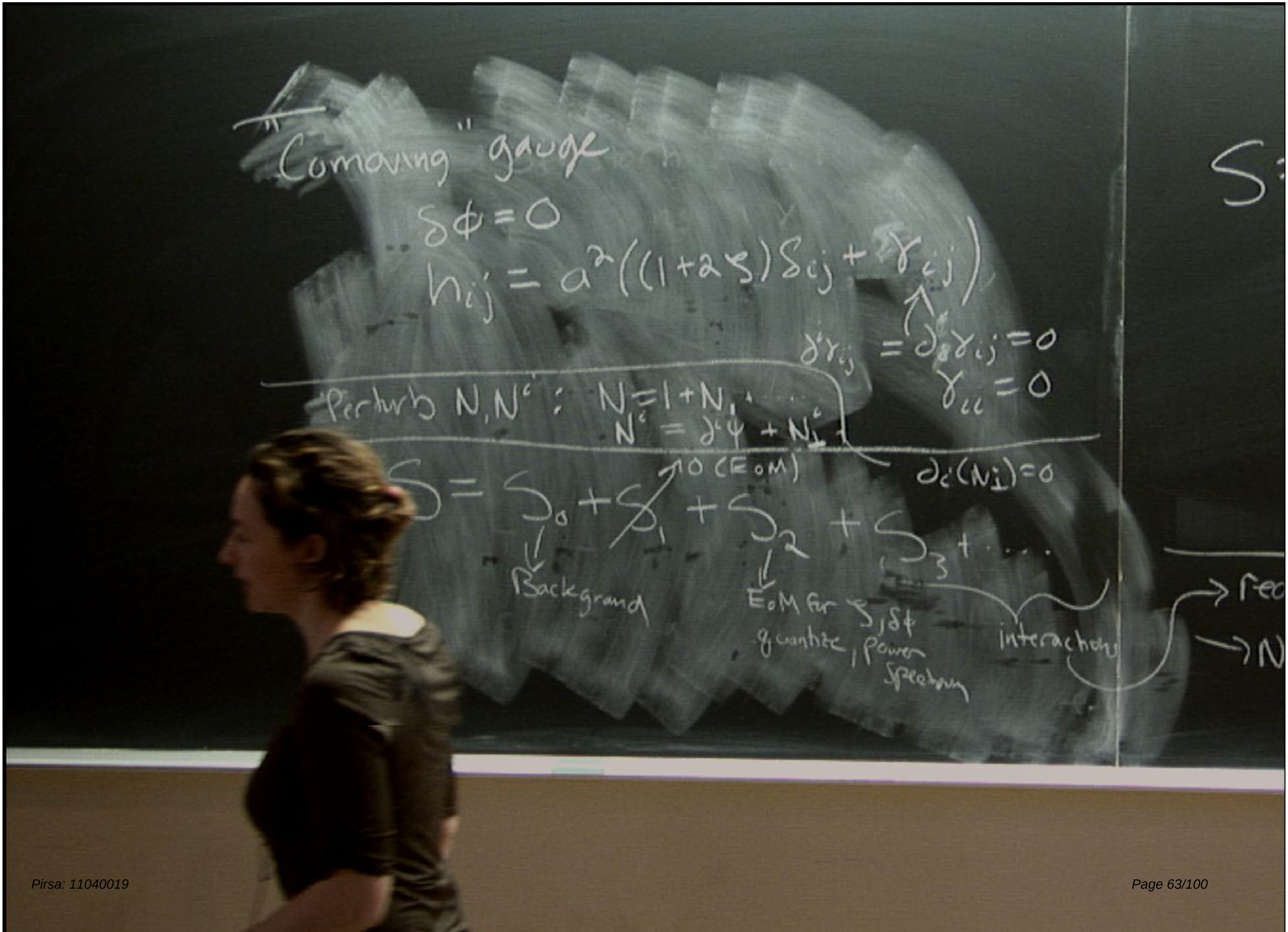
then for ζ

① N : $\dfrac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow R^{(3)} - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}_0^2 - 2V(\phi_0) = 0$

② N^i :


call φ(x,t) = φ0(t) + δφ(x,t)
EoM for N, N^i (constraints)
then for ζ
∂L/∂N = 0 => R^(3) − N^-2 (E_ij E^ij − E^2) − N^-2 φ̇0^2 − 2V(φ0) = 0
N^i : ∂/∂x_i (∂L/∂(∂_j N^i)) = ∂/∂x_i [N^-1 [E_ij −


call $\phi(x,t) = \phi_0(t) + \delta\phi(x,t)$
$\to 0$
EoM for N, N^i (constraints)
then for ζ
① N : $\frac{\partial \mathcal{L}}{\partial N} = 0 \Rightarrow R^{(3)} - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}_0^2 - 2V(\phi_0) = 0$
② N^i : $\nabla_i\left(\frac{\partial \mathcal{L}}{\partial(\partial_i N^j)}\right) = \nabla^i\left[N^{-1}\left[E_{ij} - h_{ij}E\right]\right] = \nabla_i\left[N^{-1}(E^i_j - \delta^i_j E)\right] = 0$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N^i = ∂^i ψ + N_⊥^i
∂_i(N_⊥^i) = 0
S = S_0 + S_1 + S_2 + S_3 + ...
0 (EoM)
Background
EoM for ζ, δφ
quantize, power spectrum
interactions
S

①, ② in terms of N, ϕ

①, ② in terms of N_1, ψ

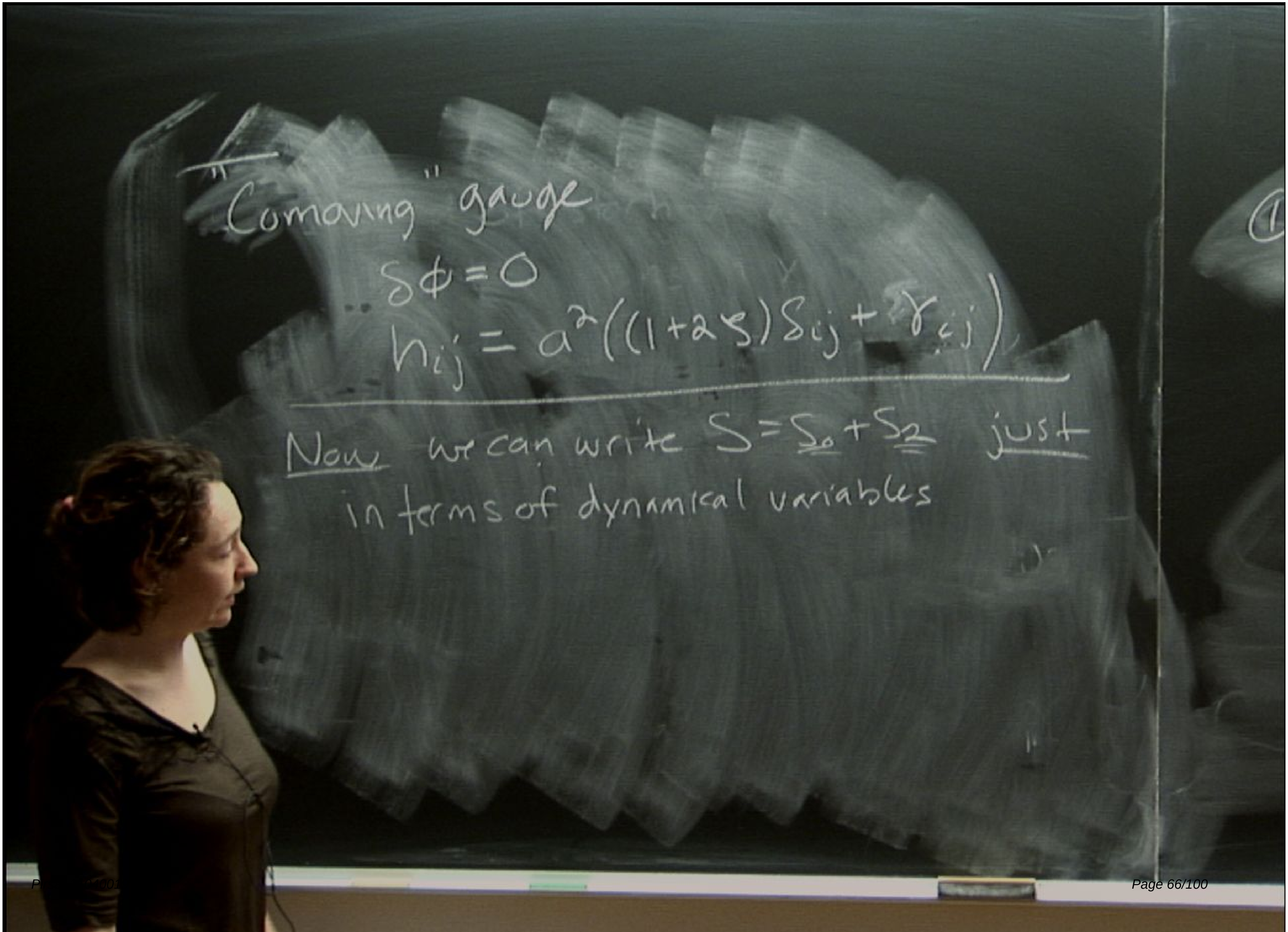
give

$$N_1 = \frac{\dot{\zeta}}{H}$$

$$N_\perp^i = 0$$

$$\psi = a^{-2}\frac{\zeta}{H} + \chi$$

$$\hookrightarrow \partial^2 \chi = \frac{\dot{\phi}_0^2}{2H}\frac{\dot{\zeta}}{M_p^2}$$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$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
Now we can write $S = S_0 + S_2$ just
in terms of dynamical variables

"Comoving" gauge

$$\delta\phi = 0$$

$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

Now we can write $S = S_0 + S_2$ just in terms of dynamical variables

$\rightarrow$ Put solutions for N, N^i back in action

find $S_2 =$

"Comoving" gauge

$$\delta\phi = 0$$

$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

Now we can write $S = \underline{S_0} + \underline{\underline{S_2}}$ just in terms of dynamical variables

→ Put solutions for N, N^i back in action

find "$S_2 = M_p^2 \int d^4x \sqrt{-\bar{g}}$"

BG FRW

"Comoving" gauge

$$\delta\phi = 0$$

$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

Now we can write $S = S_0 + S_2$ just in terms of dynamical variables

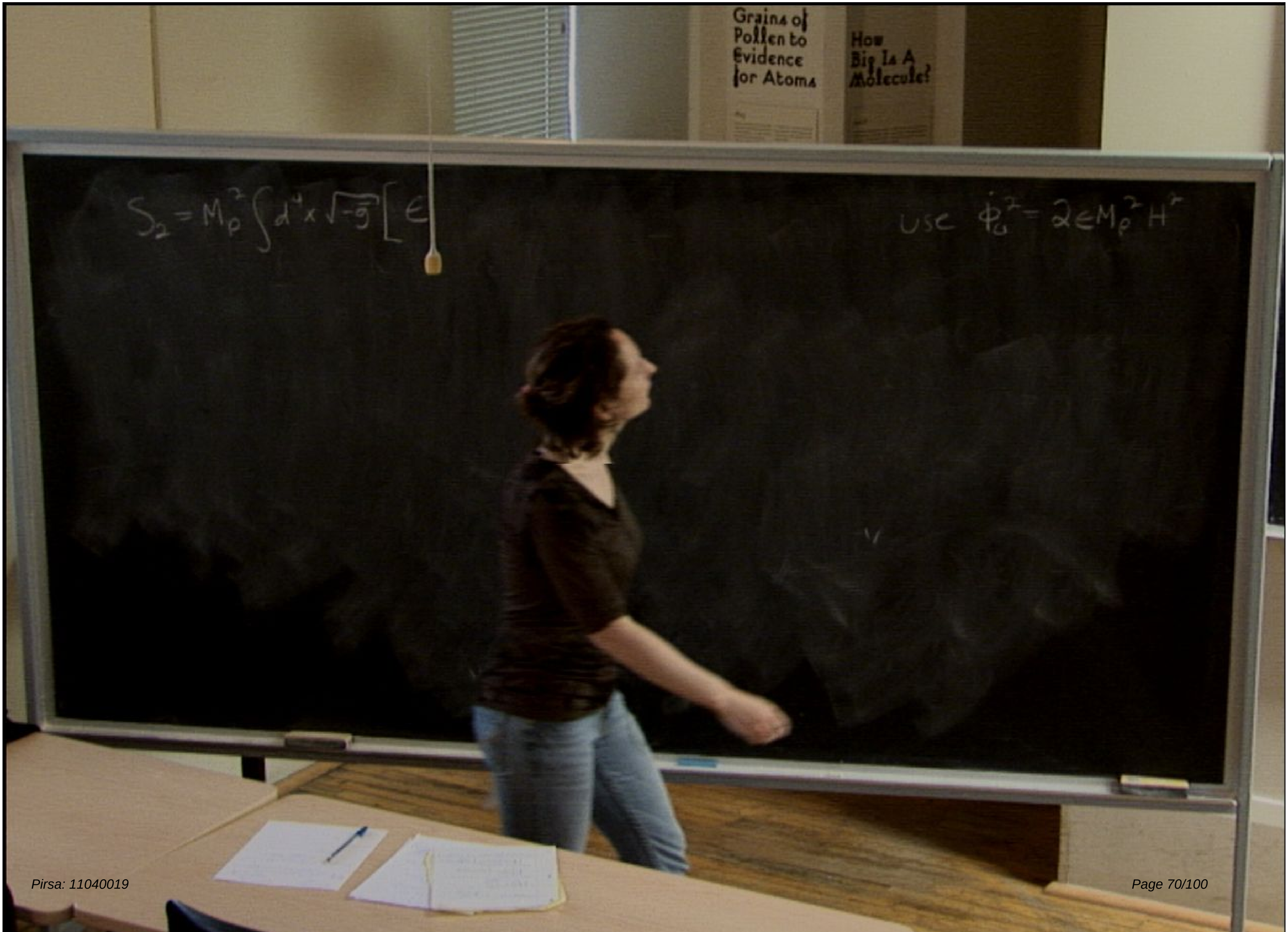
$\rightarrow$ Put solutions for N, N^i back in action

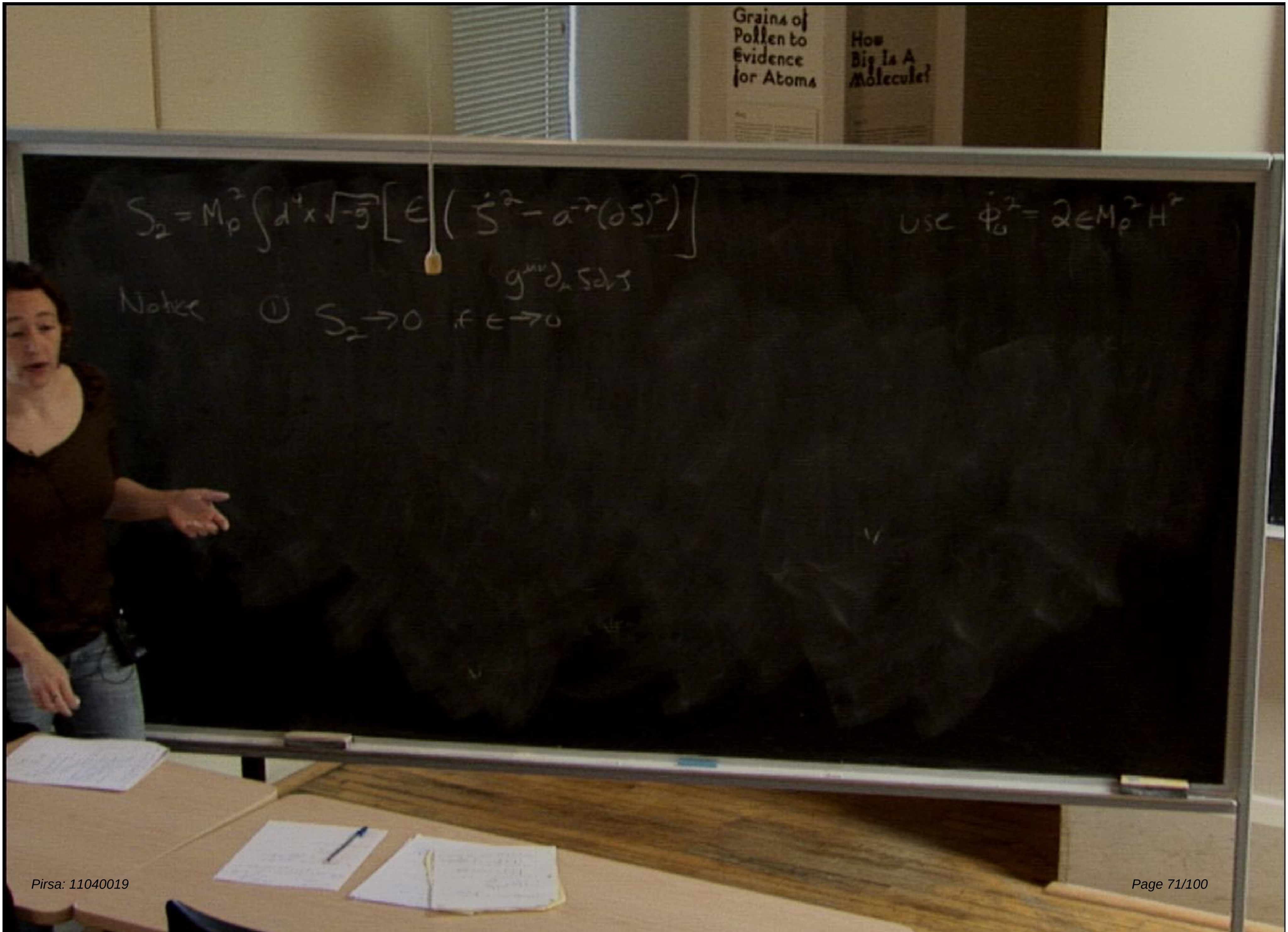
find $S_2 = M_p^2 \int d^4x \sqrt{-g}\left(f(\zeta, \zeta')\right)$

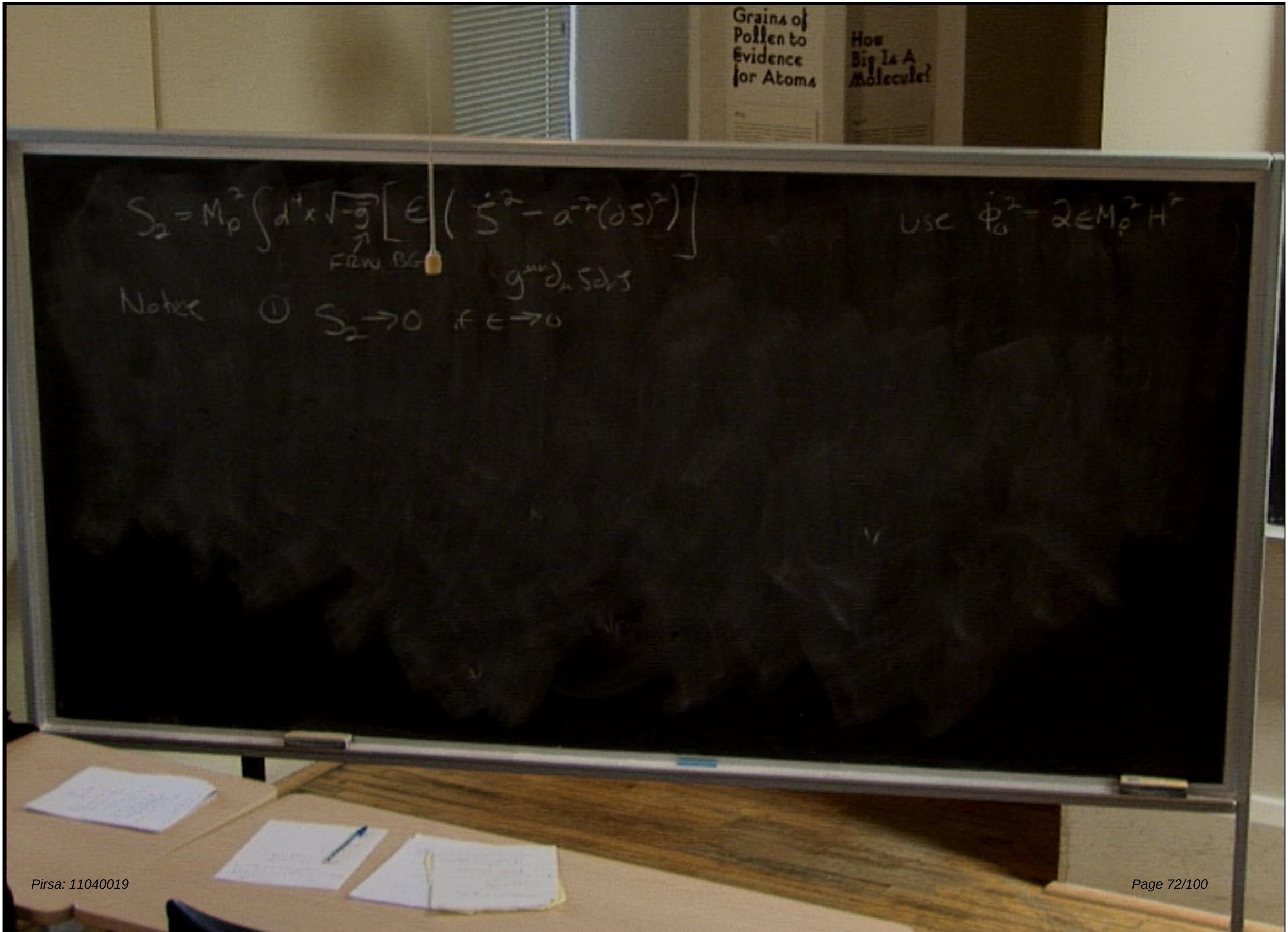
BG FRW

①,② in term

give


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$S_2 = M_p^2 \int d^4x \sqrt{-g} [\epsilon$
use $\dot{\phi}_0^2 = 2\epsilon M_p^2 H^2$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$S_2 = M_p^2 \int d^4x \sqrt{-g} \left[\epsilon \left(\dot{S}^2 - a^{-2} (\partial S)^2 \right) \right]$
$g^{\mu\nu} \partial_\mu S \partial_\nu S$
use $\dot{\phi}_0^2 = 2 \epsilon M_p^2 H^2$
Notice ① $S_2 \to 0$ if $\epsilon \to 0$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$S_2 = M_p^2 \int d^4x \sqrt{-g} \left[\epsilon \left(\dot{S}^2 - a^{-2}(\partial S)^2 \right) \right]$
FRW BG
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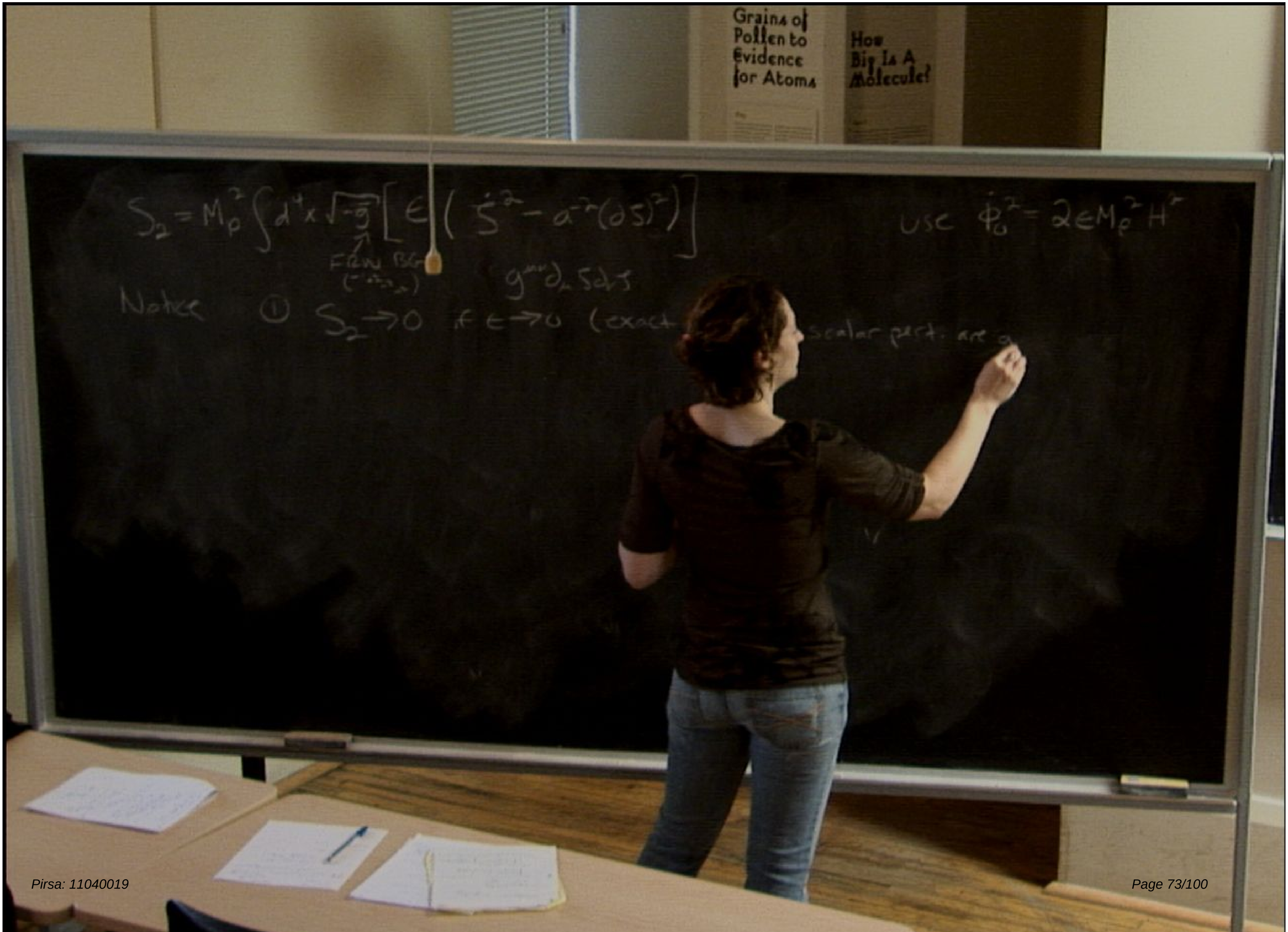
Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
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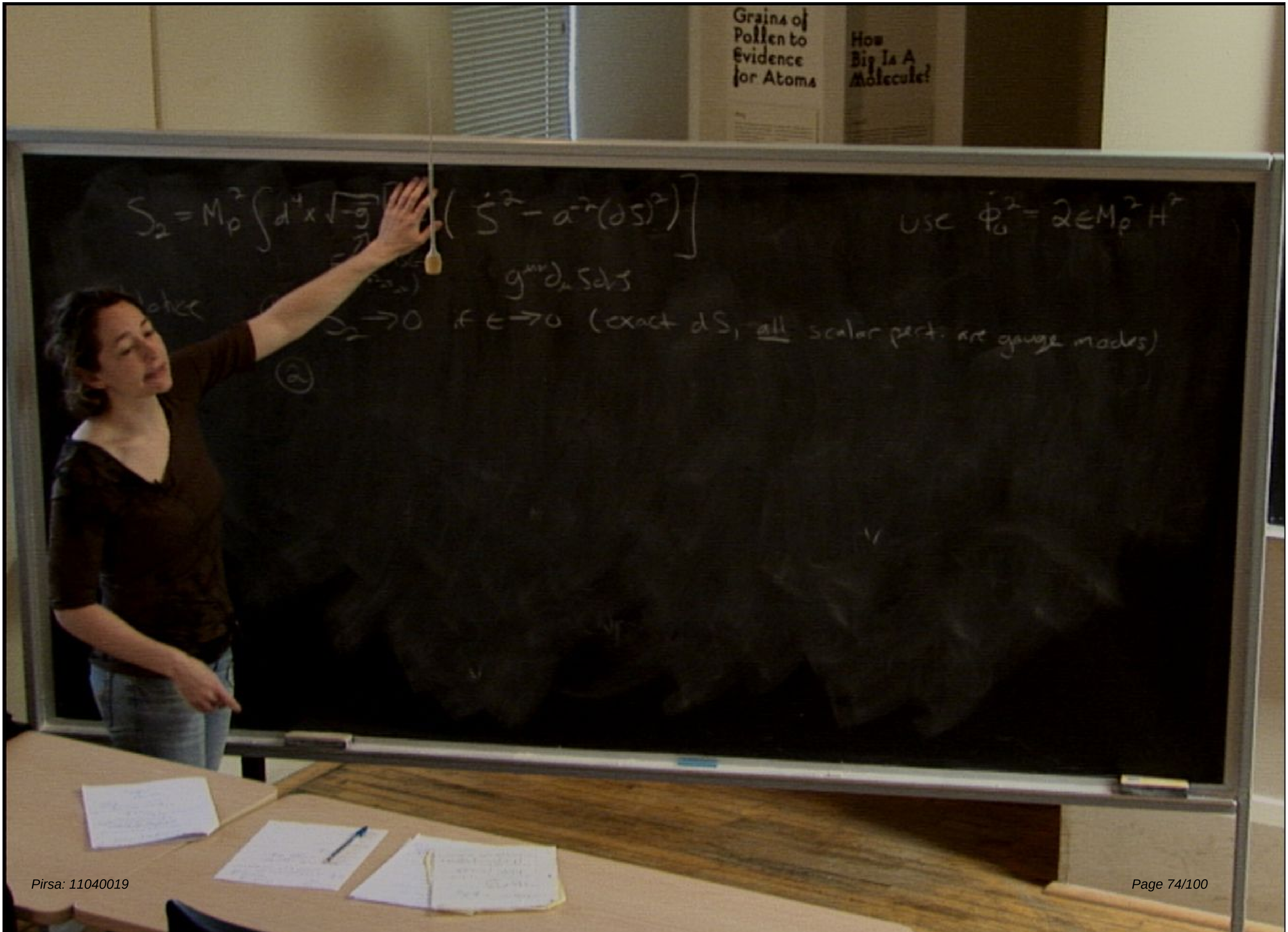
FRW BG
use $\dot{\phi}_0^2 = 2 \epsilon M_p^2 H^2$
Notice ① $S_2 \to 0$ if $\epsilon \to 0$ (exact
scalar pert. are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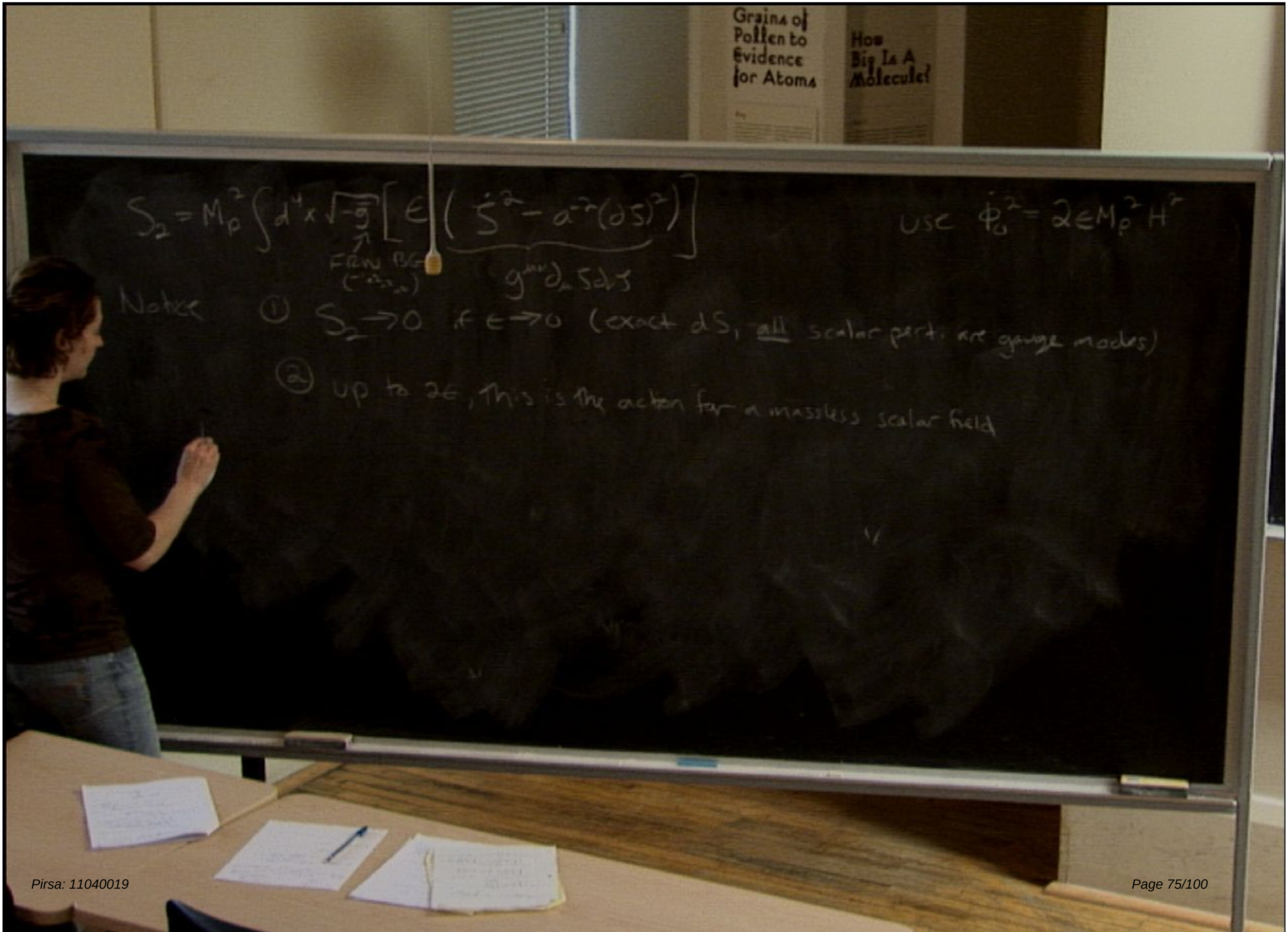
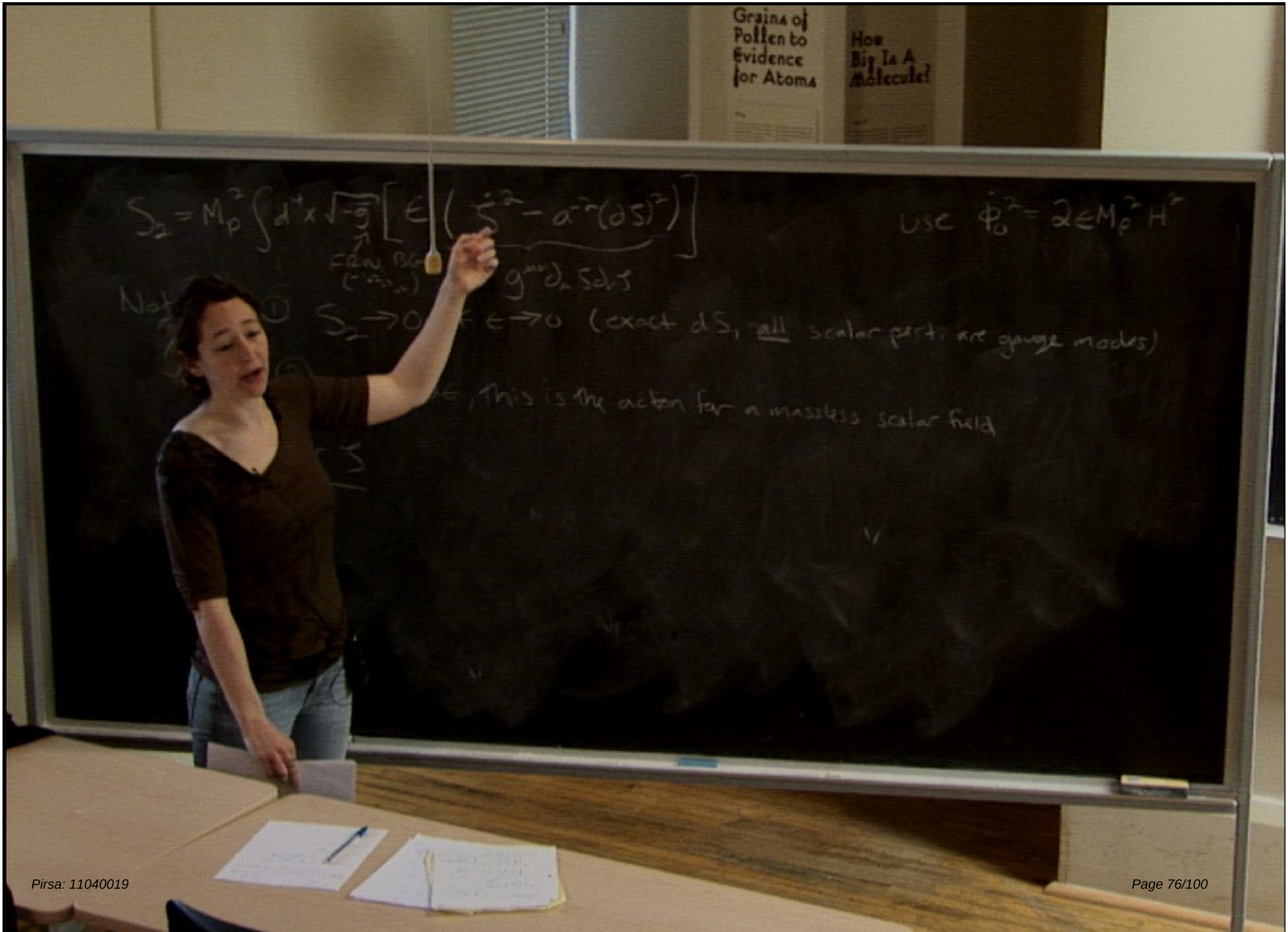
How Big Is A Molecule?
$S_2 = M_p^2 \int d^4x \sqrt{-g} \, [\; (\dot{S}^2 - a^{-2}(\partial S)^2)]$
use $\dot{\phi}_0^2 = 2\epsilon M_p^2 H^2$
$g^{\mu\nu}\partial_\mu S \partial_\nu S$
$S_2 \to 0$ if $\epsilon \to 0$ (exact dS, all scalar pert. are gauge modes)
(a)


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$S_2 = M_p^2 \int d^4x \sqrt{-g} \left[\epsilon \left(\dot{\zeta}^2 - a^{-2}(\partial\zeta)^2 \right) \right]$
FRW BG
$g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta$
use $\dot{\phi}_0^2 = 2\epsilon M_p^2 H^2$
Notice
① $S_2 \to 0$ if $\epsilon \to 0$ (exact dS, all scalar pert. are gauge modes)
② up to 2ϵ, this is the action for a massless scalar field



Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$S_2 = M_p^2 \int d^4x \sqrt{-g} \left[\epsilon \left(\dot{\zeta}^2 - a^{-2}(\partial\zeta)^2 \right) \right]$
FRW BG
$g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta$
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$S_2 \to 0$ if $\epsilon \to 0$ (exact dS, all scalar pert. are gauge modes)
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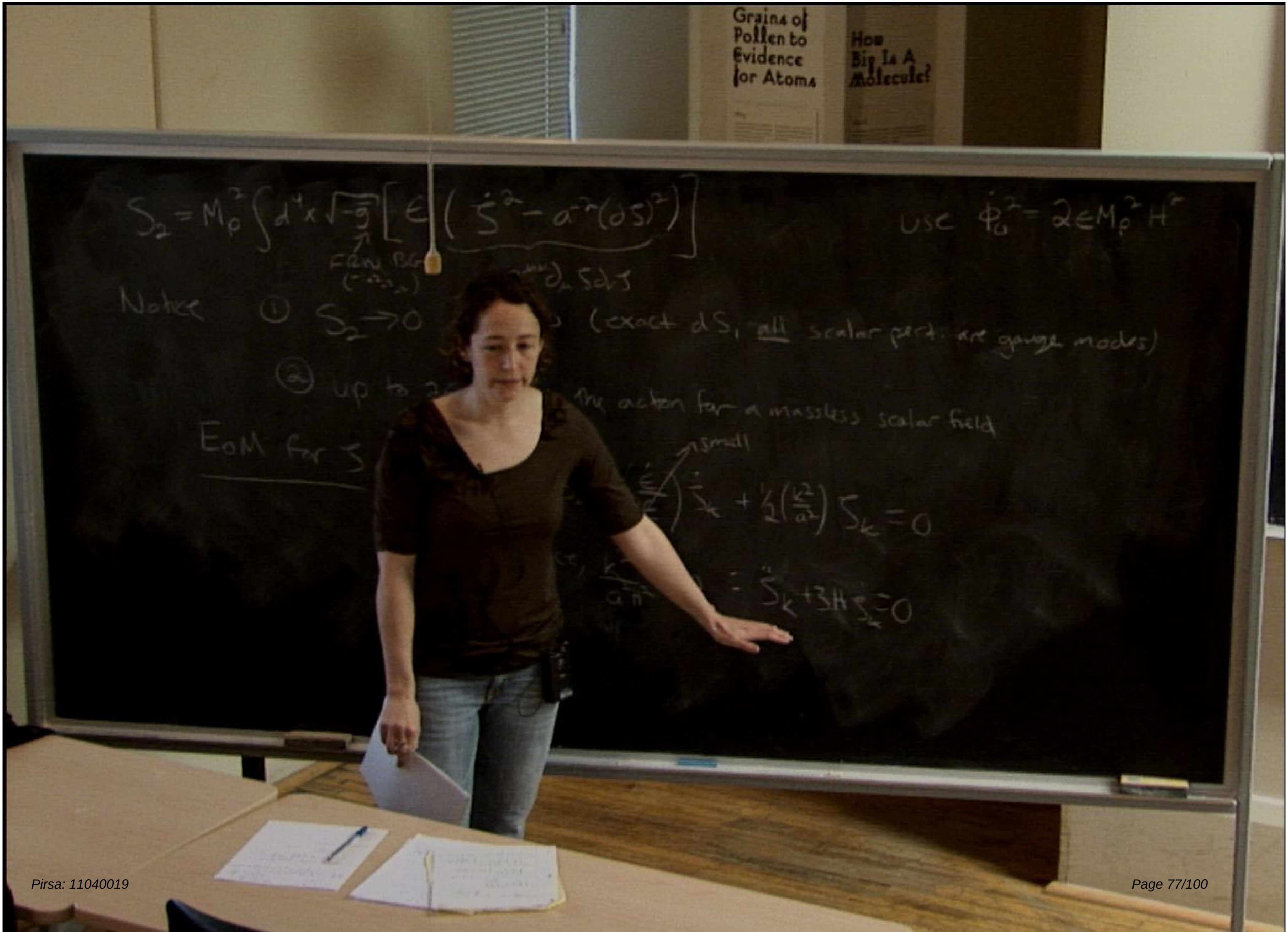

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
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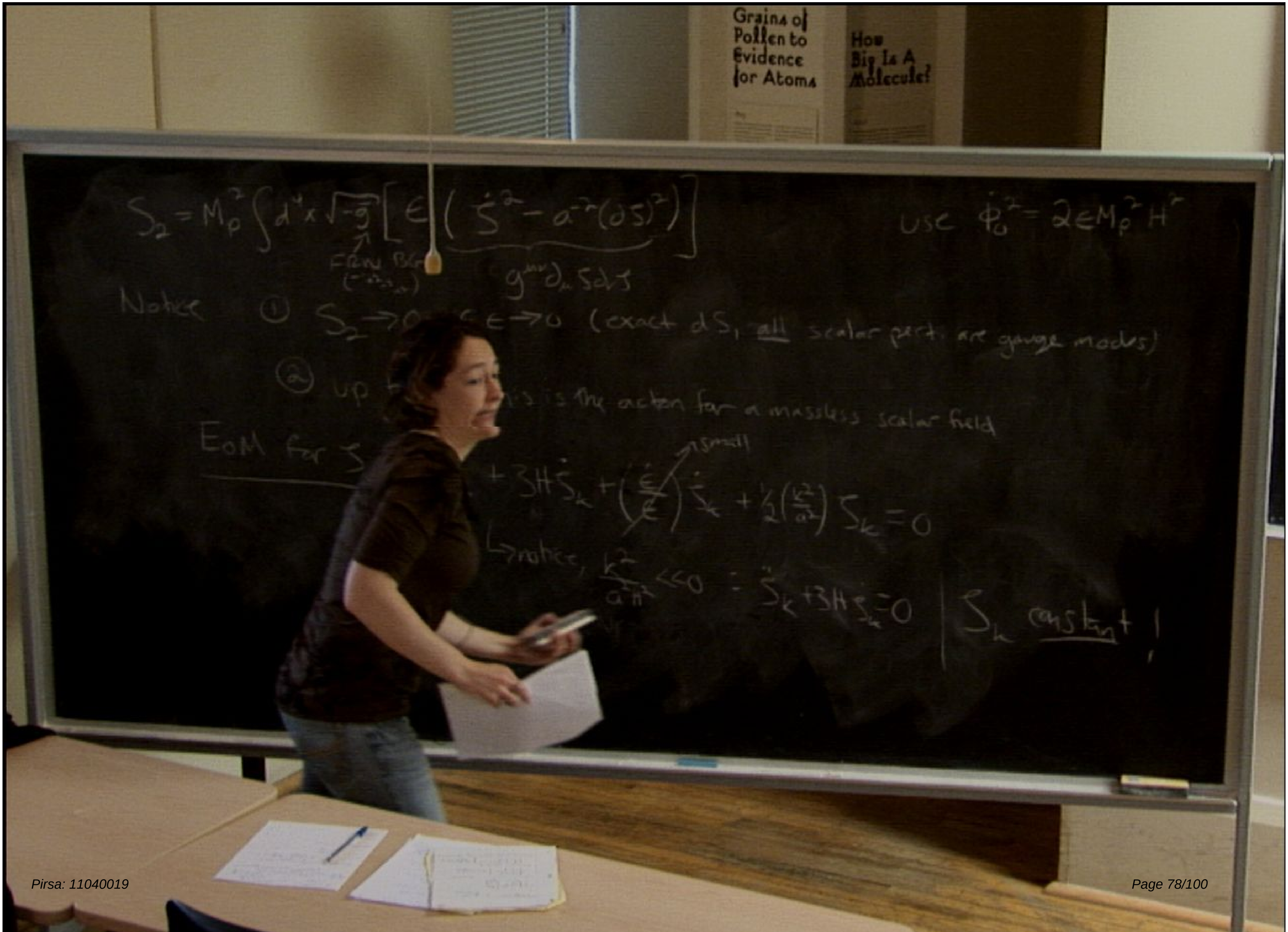
use $\dot{\phi}_0^2 = 2\epsilon M_p^2 H^2$
Notice ① $S_2 \to 0$
(exact dS, all scalar pert. are gauge modes)
② up to
the action for a massless scalar field
EoM for ζ
small
$\dot{\zeta}_k + \frac{1}{2}\left(\frac{k^2}{a^2}\right) \zeta_k = 0$
$\ddot{\zeta}_k + 3H\dot{\zeta}_k = 0$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
S2 = Mp^2 ∫ d^4x √-g [ε (ζ̇^2 − a^-2 (∂ζ)^2)]
FRW BG
g^μν ∂μ ζ ∂ν ζ
Use φ̇0^2 = 2εMp^2 H^2
Notice ① S2 → 0 as ε → 0 (exact dS, all scalar pert. are gauge modes)
② up to ... this is the action for a massless scalar field
EoM for ζ
+ 3H ζ̇k + (ε̇/ε) ζ̇k + ½ (k^2/a^2) ζk = 0
small
notice, k^2/(a^2 H^2) << 0 : ζ̈k + 3H ζ̇k = 0
ζk constant!

$$S_2 = M_p^2 \int d^4x \underset{\text{from BG}}{\sqrt{-g}} \left[\epsilon \underbrace{\left(\dot{\zeta}^2 - a^{-2}(\partial\zeta)^2 \right)}_{g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta} \right]$$

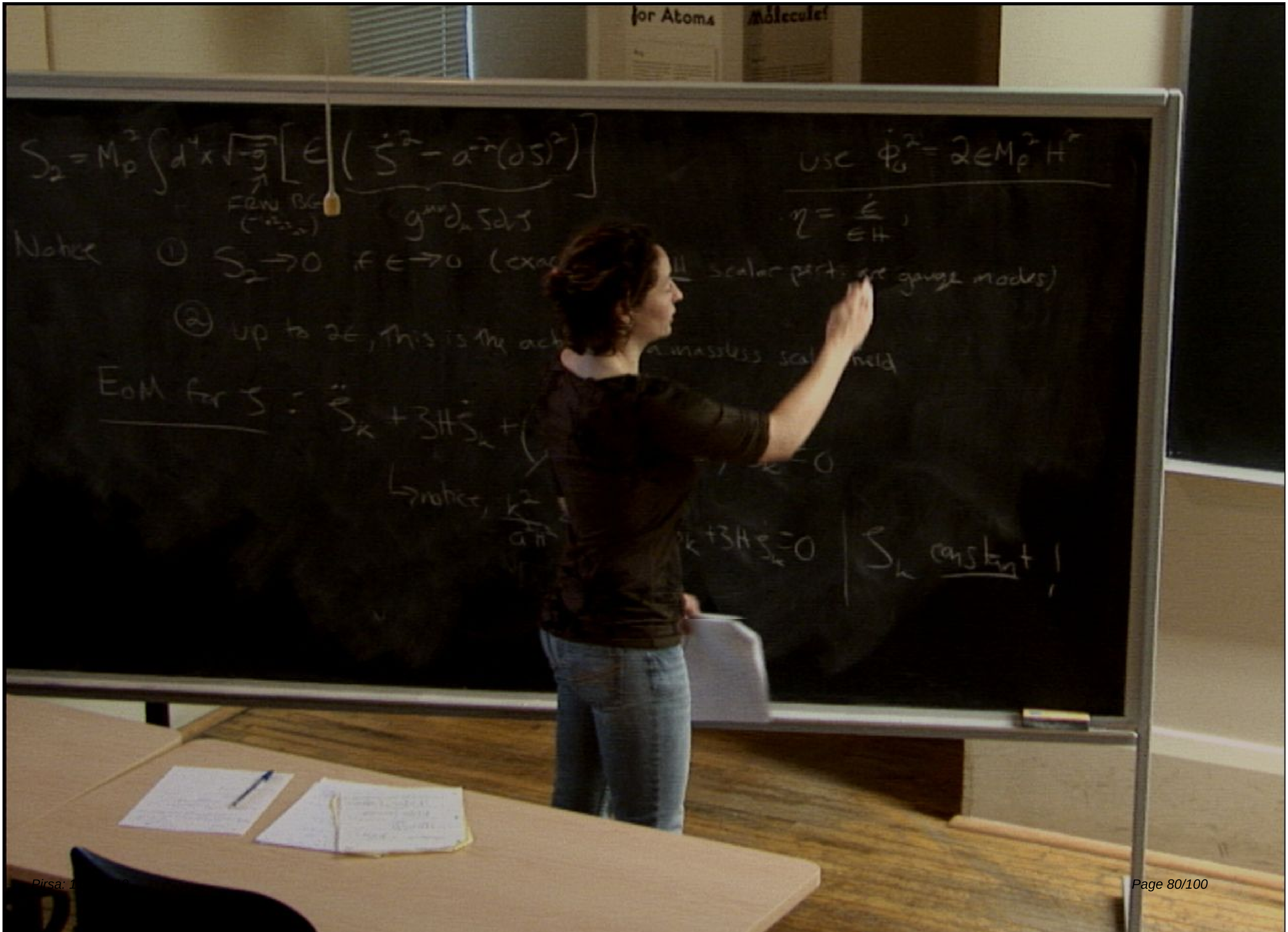
Use $\dot{\phi}_0^2 = 2\epsilon M_p^2 H^2$

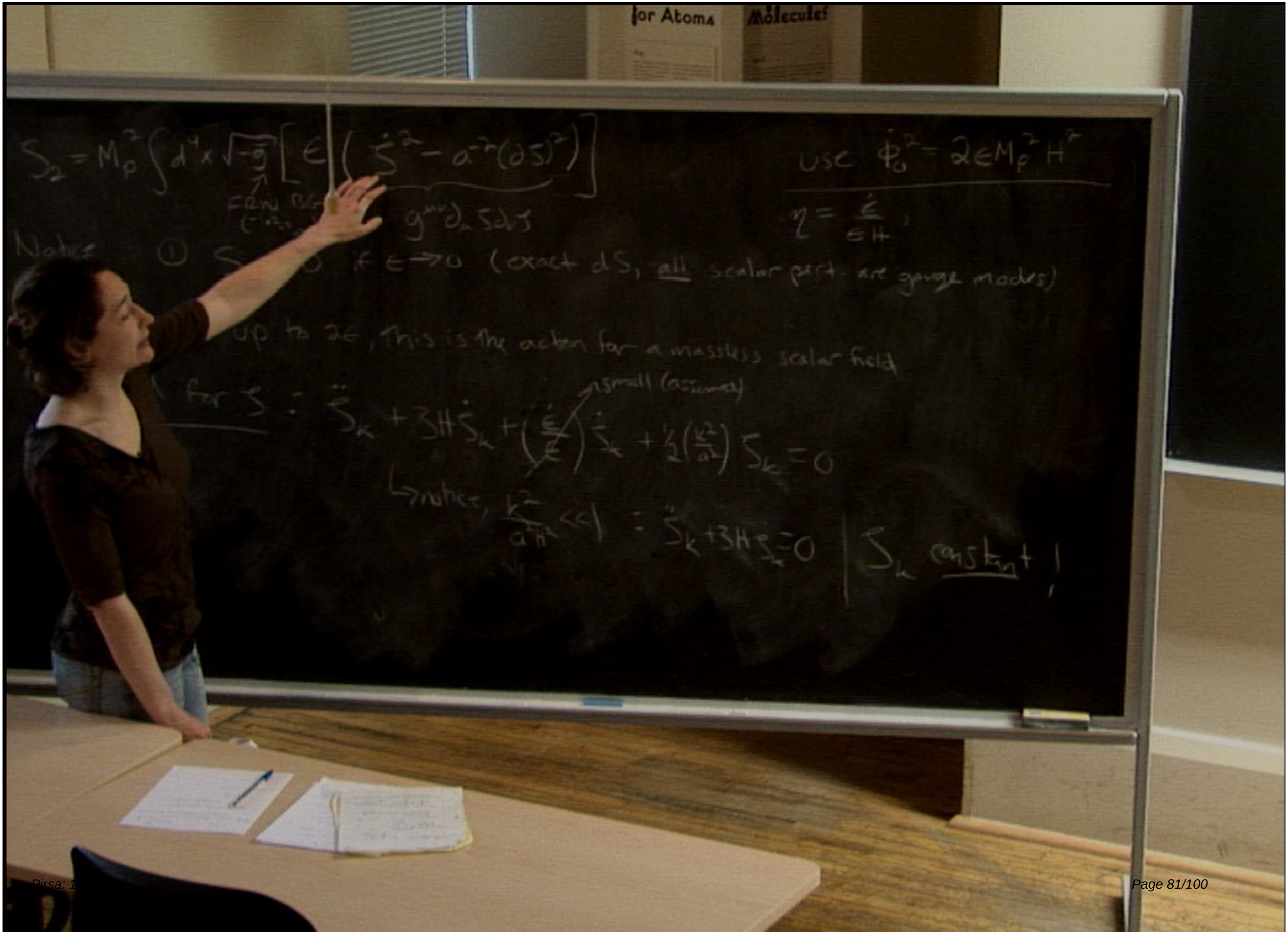
Notice ① $S_2 \to 0$ if $\epsilon \to 0$ (exact dS, all scalar pert. are gauge modes)

② up to 2ϵ, this is the action for a massless scalar field

EoM for ζ : $\ddot{\zeta}_k + 3H\dot{\zeta}_k + \cancel{\left(\frac{\dot{\epsilon}}{\epsilon}\right)}^{\text{small}}\dot{\zeta}_k + \frac{1}{2}\left(\frac{k^2}{a^2}\right)\zeta_k = 0$

↳ notice, $\frac{k^2}{a^2H^2} \ll 0$: $\ddot{\zeta}_k + 3H\dot{\zeta}_k = 0$ | ζ_k constant!


for Atoms
Molecules
Notice
scalar parts are gauge modes)
up to 2ε, This is the action
massless scal field
EoM for ζ :
notice,
constant !


for Atoms
Molecules
use $\dot\phi_0^2 = 2\epsilon M_p^2 H^2$
Notice
up to 2ϵ, This is the action for a massless scalar field
small (assumed)
notice,
constant!

$$S_2 = M_P^2 \int d^4x \sqrt{-g} \left[\epsilon \underbrace{\left(\dot{\zeta}^2 - a^{-2} (\partial \zeta)^2 \right)}_{g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta} \right]$$

FRW BG

use $\dot{\phi}_0^2 = 2\epsilon M_P^2 H^2$

$\eta = \frac{\dot{\epsilon}}{\epsilon H}$

Notice ① $S_2 \to 0$ if $\epsilon \to 0$ (exact dS, all scalar pert. are gauge modes)

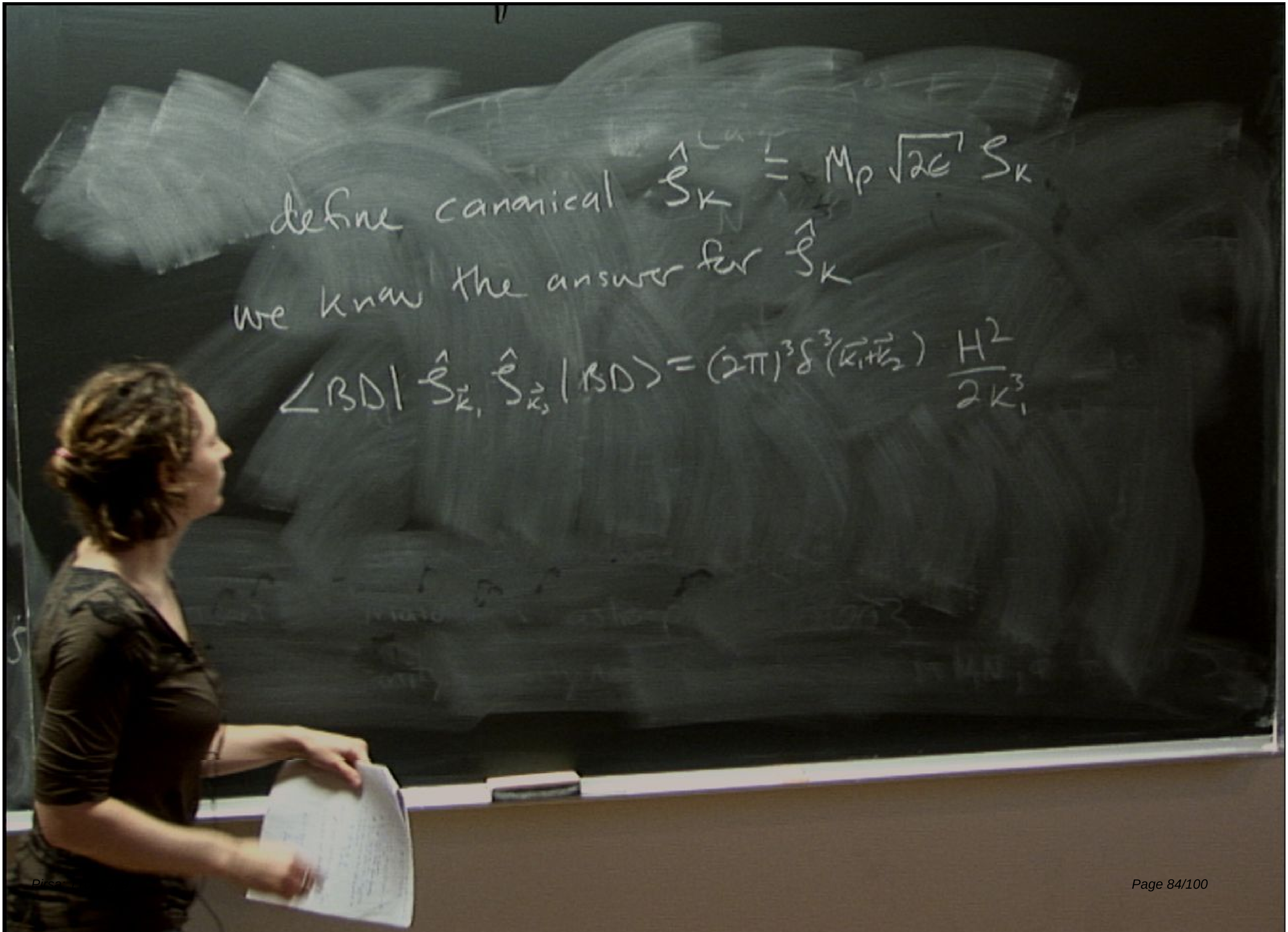
② up to 2ϵ, this is the action for a massless scalar field

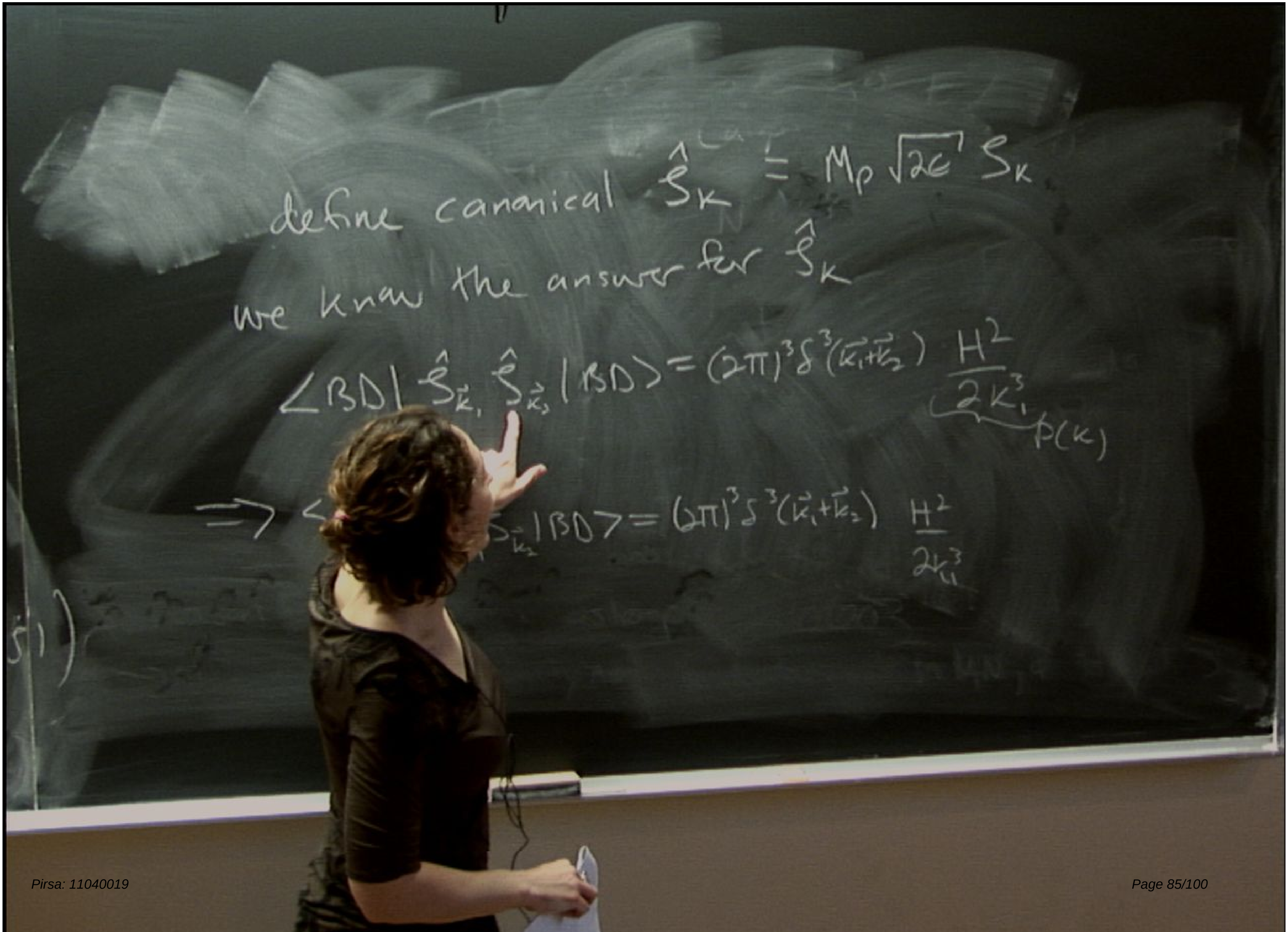
EoM for ζ : $\ddot{\zeta}_k + 3H\dot{\zeta}_k + \left(\cancel{\frac{\dot{\epsilon}}{\epsilon}}\right)\dot{\zeta}_k + \frac{1}{2}\left(\frac{k^2}{a^2}\right)\zeta_k = 0$

small (assumed)

↳ notice, $\frac{k^2}{a^2H^2} \ll 1$: $\ddot{\zeta}_k + 3H\dot{\zeta}_k = 0$ | ζ_k constant!

define canonical $\hat{\zeta}_k \equiv M_p \sqrt{2\epsilon}\, \zeta_k$


define canonical $\hat{\zeta}_k \equiv M_p \sqrt{2\epsilon}\, \zeta_k$
we know the answer for $\hat{\zeta}_k$
$\langle BD | \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \frac{H^2}{2k_1^3}$



define canonical $\hat{\zeta}_k = M_p \sqrt{2\epsilon}\, \zeta_k$
we know the answer for $\hat{\zeta}_k$
$\langle BD | \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{H^2}{2k_1^3}}_{P(k)}$
$\Rightarrow \langle \quad \zeta_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \frac{H^2}{2k_1^3}$

define canonical $\hat{\zeta}_k \doteq M_p \sqrt{2\epsilon}\, \zeta_k$

we know the answer for $\hat{\zeta}_k$

$$\langle BD | \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{H^2}{2k_1^3}}_{P(k)}$$

$$\Rightarrow \langle BD | \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \frac{H^2}{2k_1^3} \frac{1}{2\epsilon M_p^2}$$

$$\Delta_k^2 = \frac{k^3 P(k)}{2\pi^2}$$

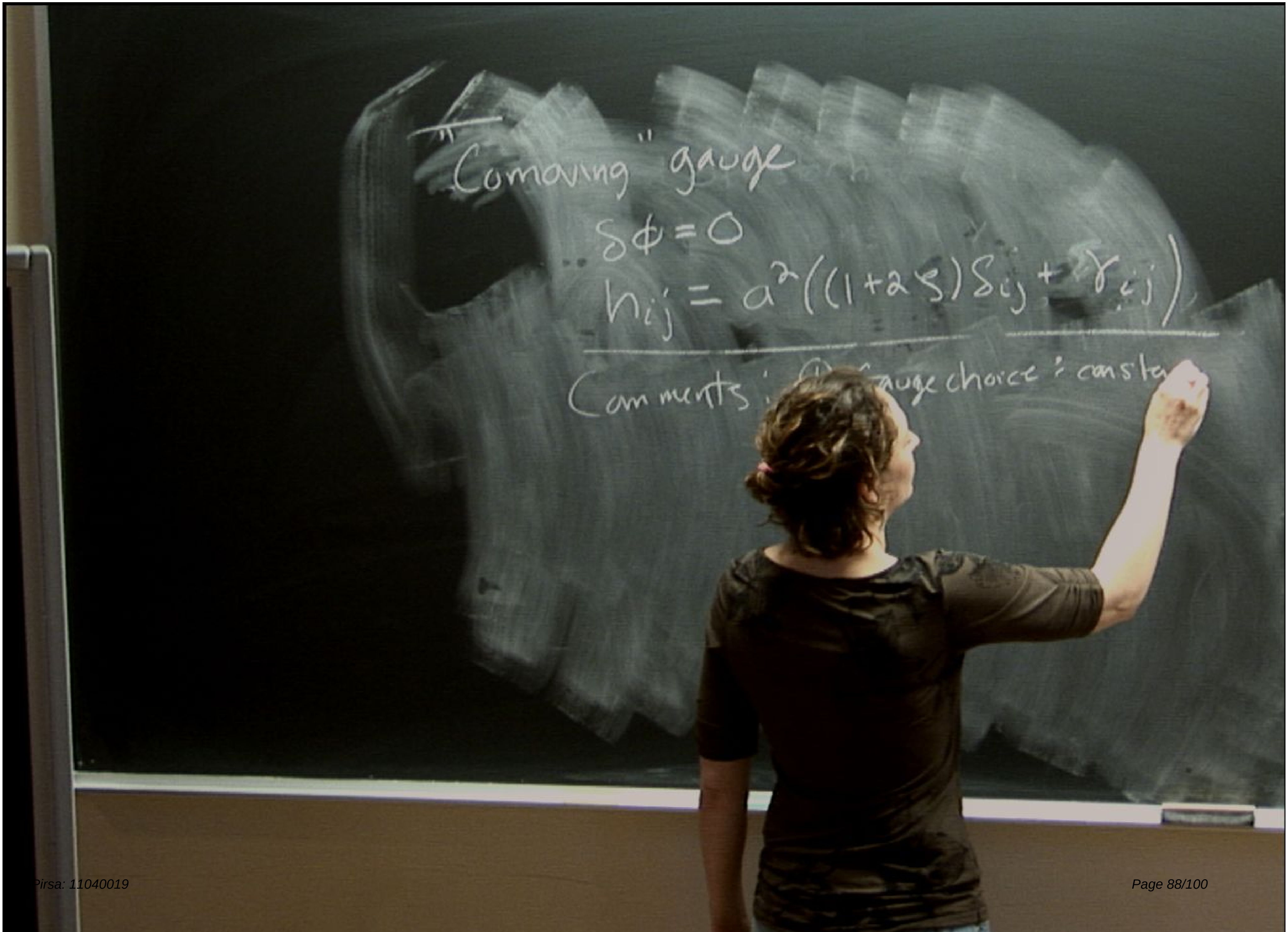
define canonical $\hat{\zeta}_k = M_p \sqrt{2\epsilon}\, \zeta_k$

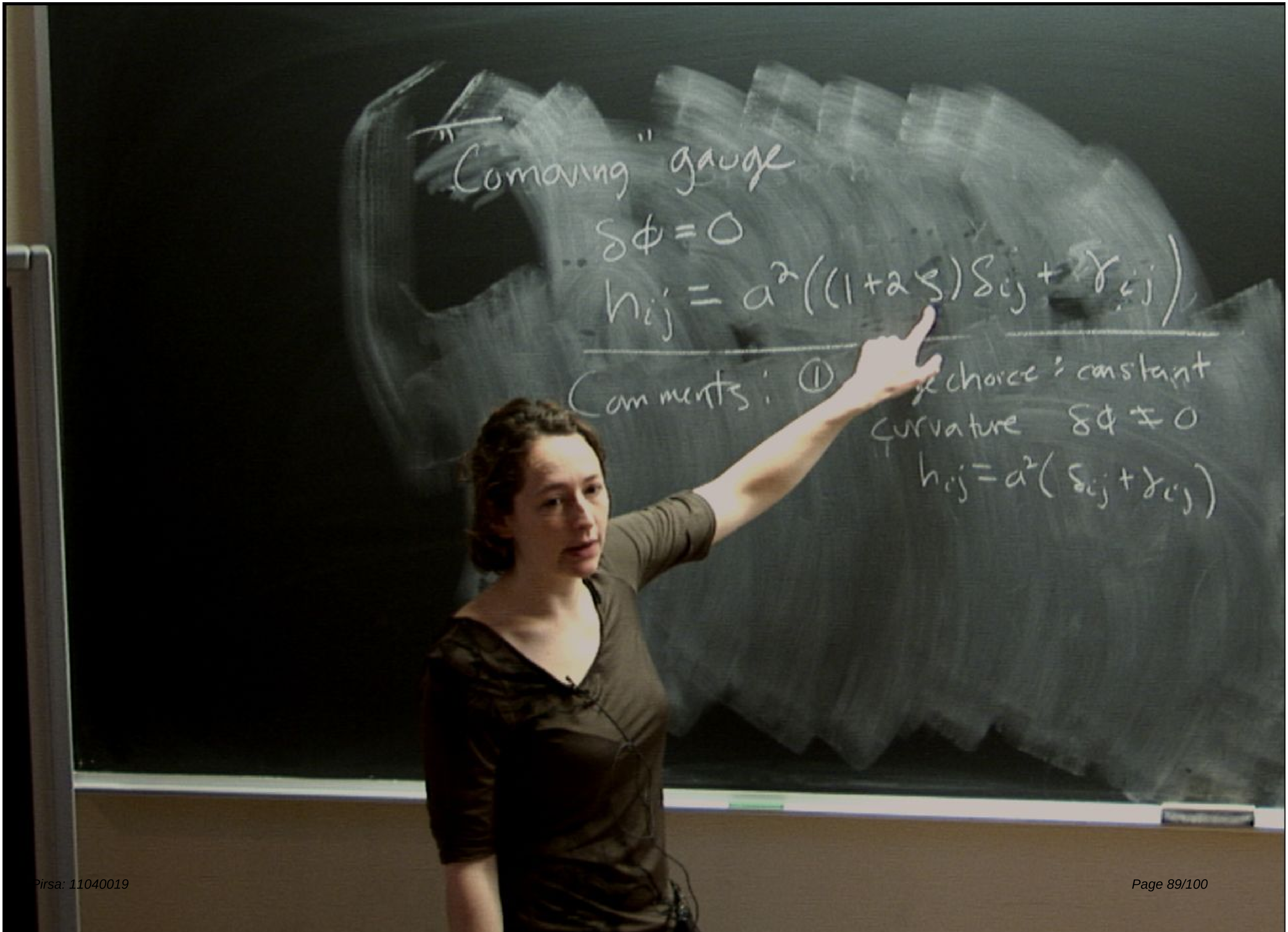
we know the answer for $\hat{\zeta}_k$

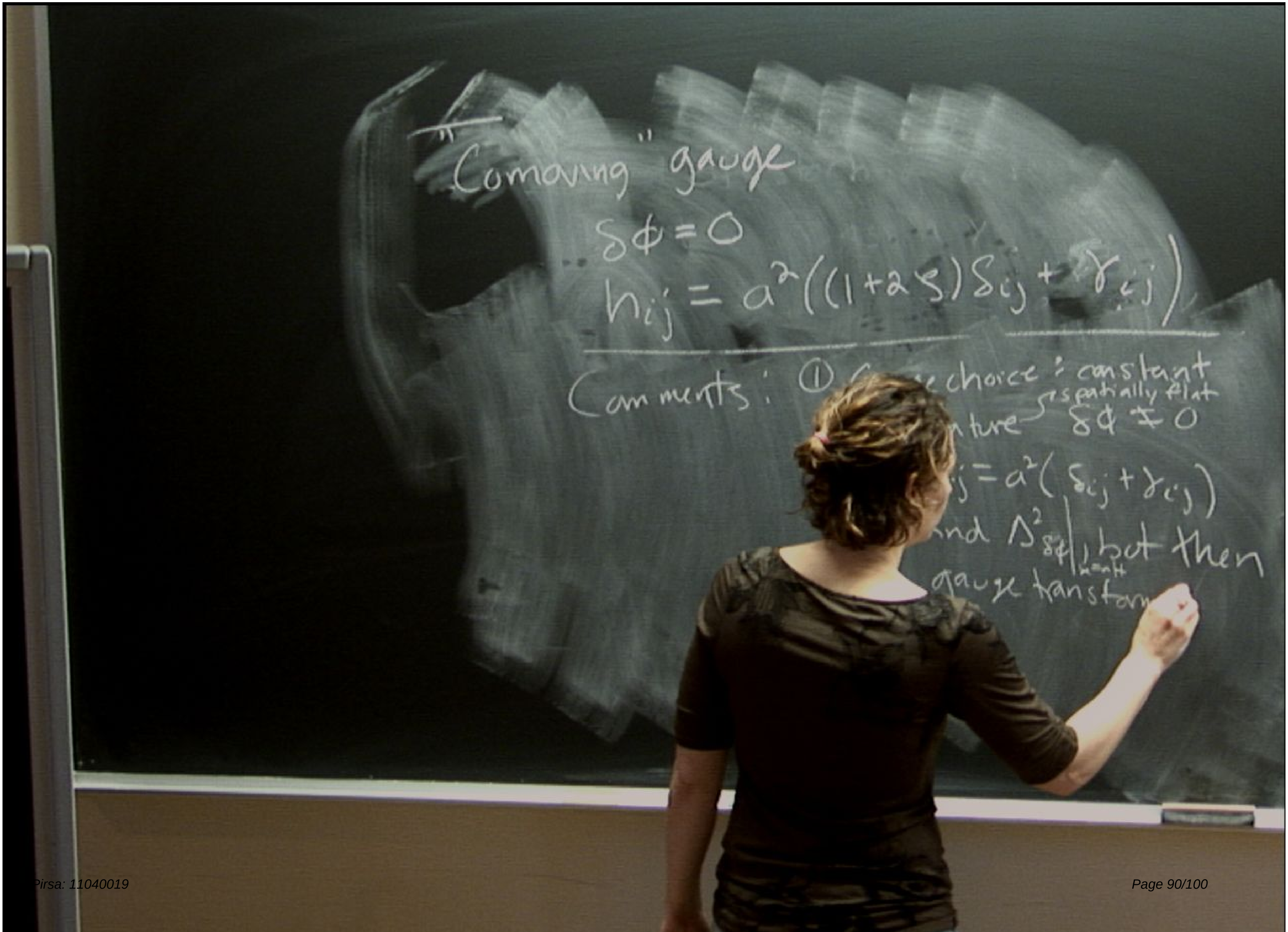
$$\langle BD | \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{H^2}{2k_1^3}}_{P(k)}$$

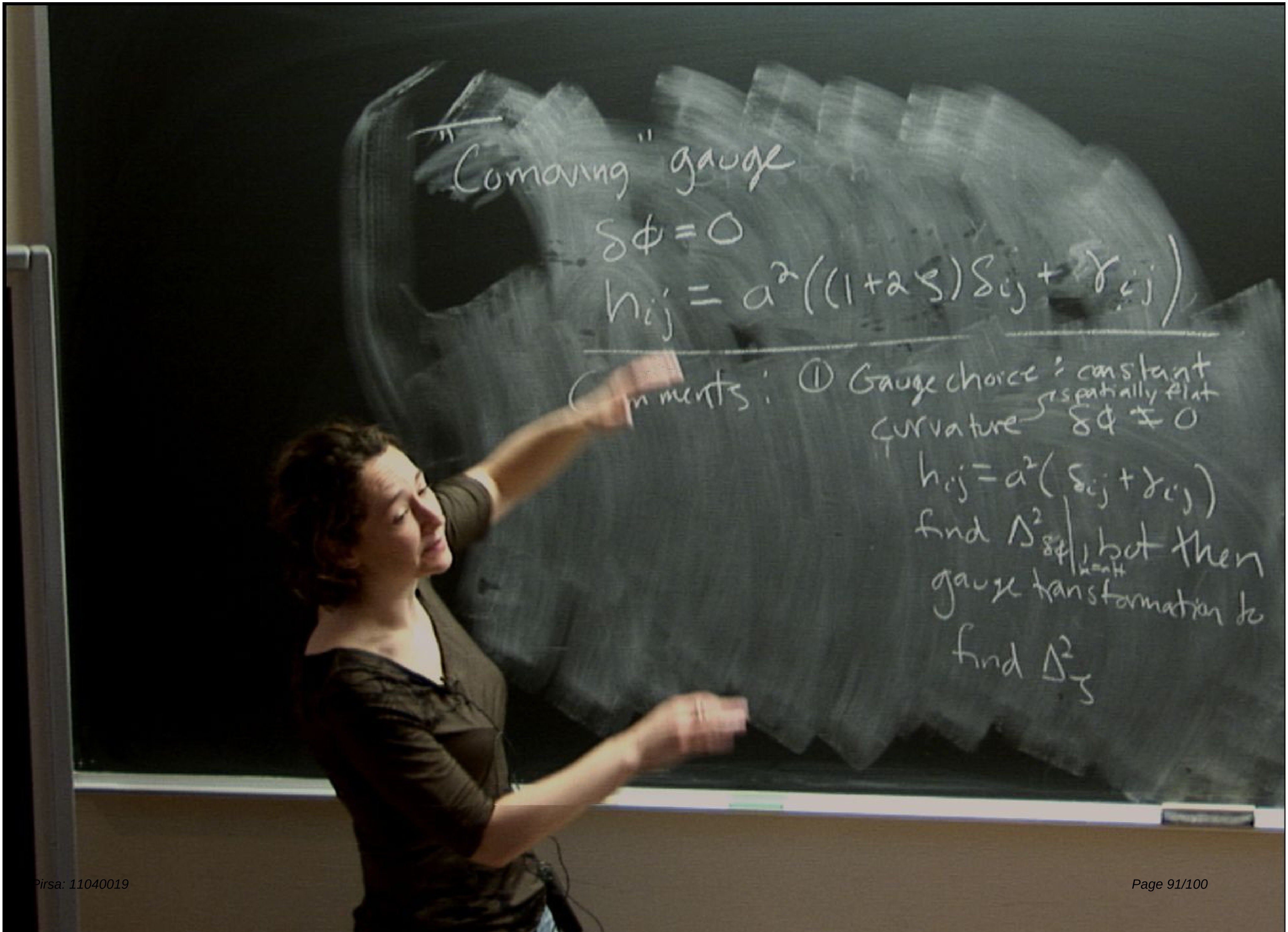
$$\Rightarrow \langle BD | \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \frac{H^2}{2k_1^3} \frac{1}{2\epsilon M_p^2}$$

$$\boxed{\Delta^2_{\zeta}(k) = \frac{k^3 P(k)}{2\pi^2} = \frac{H^2}{8\pi^2 \epsilon M_p^2}}$$


"Comoving" gauge
δφ = 0
h_ij = a^2((1+2ζ)δ_ij + γ_ij)
Comments: ① Gauge choice : const


"Comoving" gauge
δφ = 0
h_ij = a²((1+2ζ)δ_ij + γ_ij)
Comments: ①
choice : constant curvature δφ ≠ 0
h_ij = a²(δ_ij + γ_ij)


"Comoving" gauge
δφ = 0
h_ij = a²((1+2ζ)δ_ij + γ_ij)
Comments: ①
choice: constant
spatially flat
δφ ≠ 0
= a²(δ_ij + γ_ij)
Δ²_δφ|_k=aH but then
gauge transform


"Comoving" gauge
$\delta\phi = 0$
$h_{ij} = a^2((1+2\zeta)\delta_{ij} + \gamma_{ij})$
ments: ① Gauge choice: constant curvature ≡ spatially flat
$\delta\phi \neq 0$
$h_{ij} = a^2(\delta_{ij} + \gamma_{ij})$
find $\Delta^2_{\delta\phi}\big|_{k=aH}$ but then
gauge transformation to
find Δ^2_{ζ}

"Comoving" gauge

$$\delta\phi = 0$$

$$h_{ij} = a^2\left((1+2\zeta)\delta_{ij} + \gamma_{ij}\right)$$

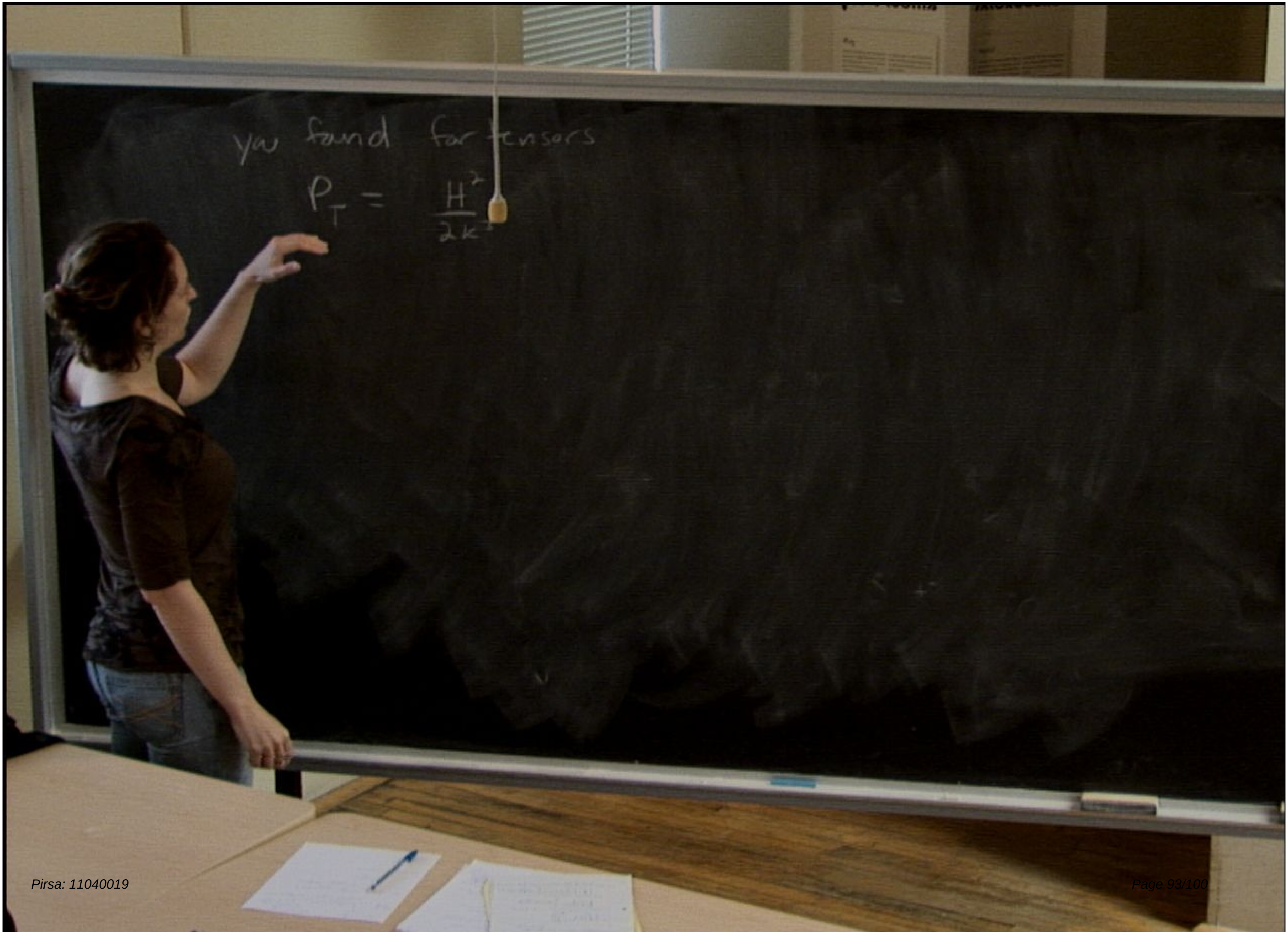
Comments: ① Gauge choice: constant curvature is spatially flat

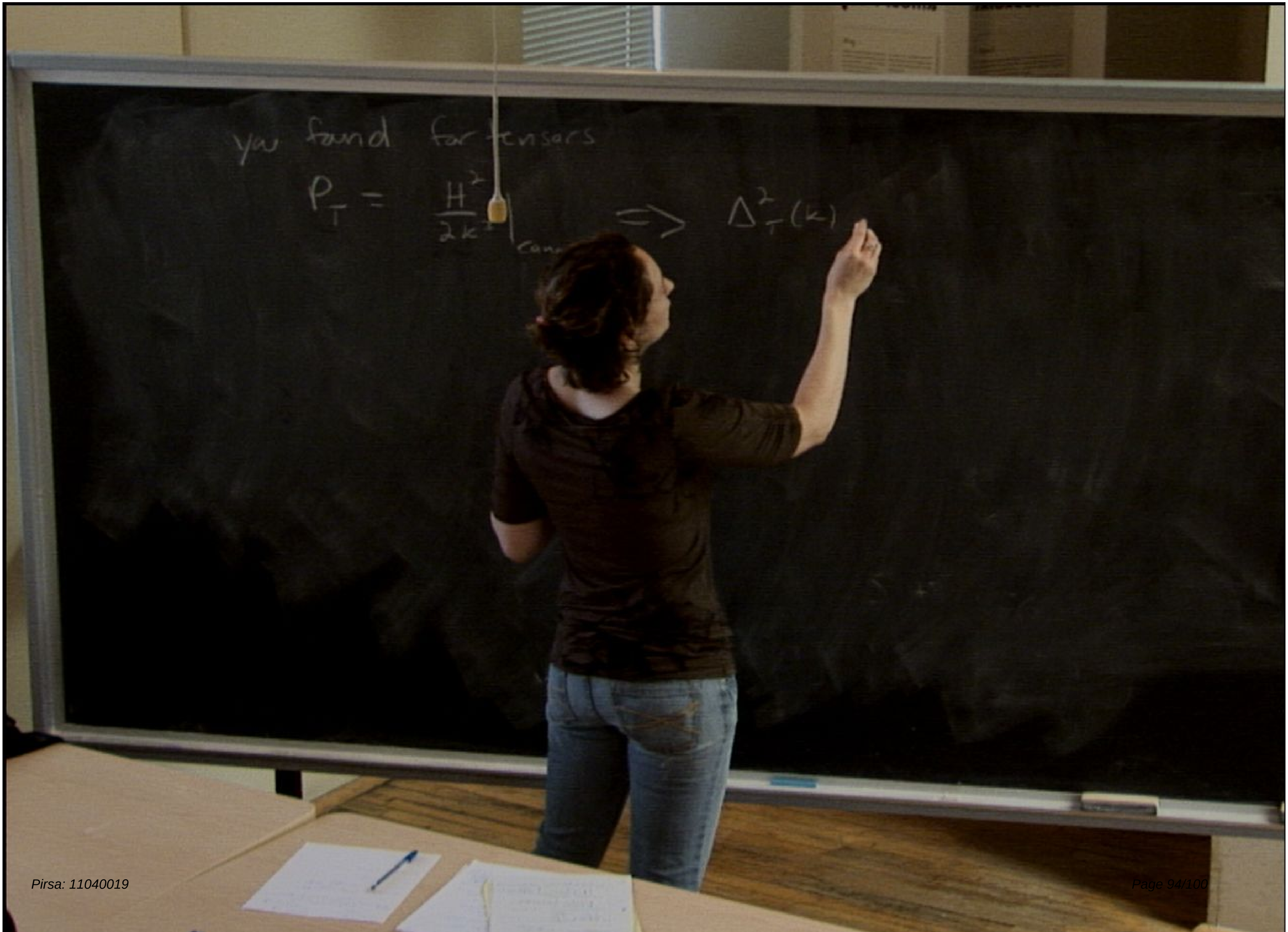
$$\delta\phi \neq 0$$

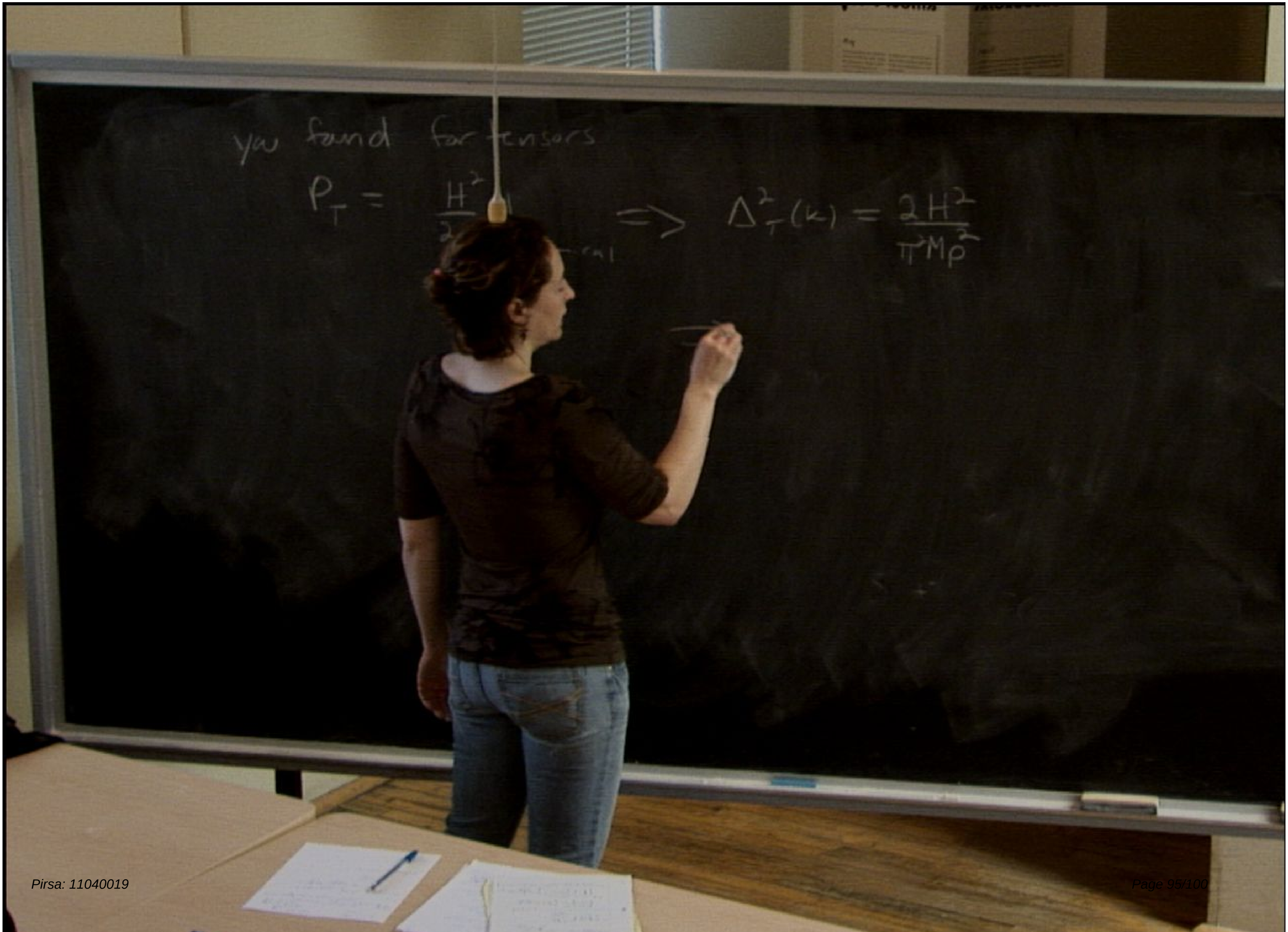
$$h_{ij} = a^2(\delta_{ij} + \gamma_{ij})$$

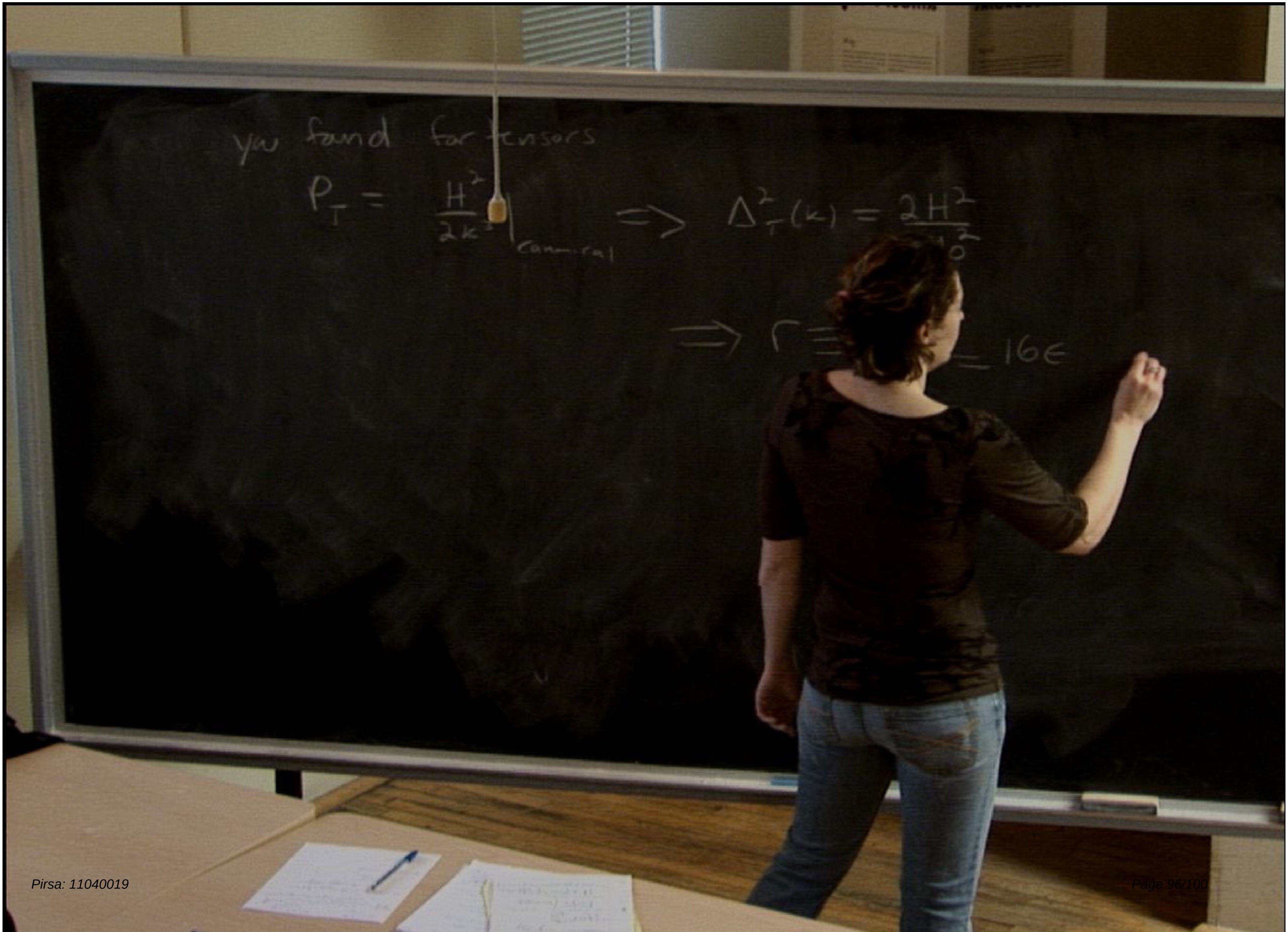
find $\Delta^2_{\delta\phi}\big|_{k=aH}$ but then gauge transformation to find Δ^2_{ζ}

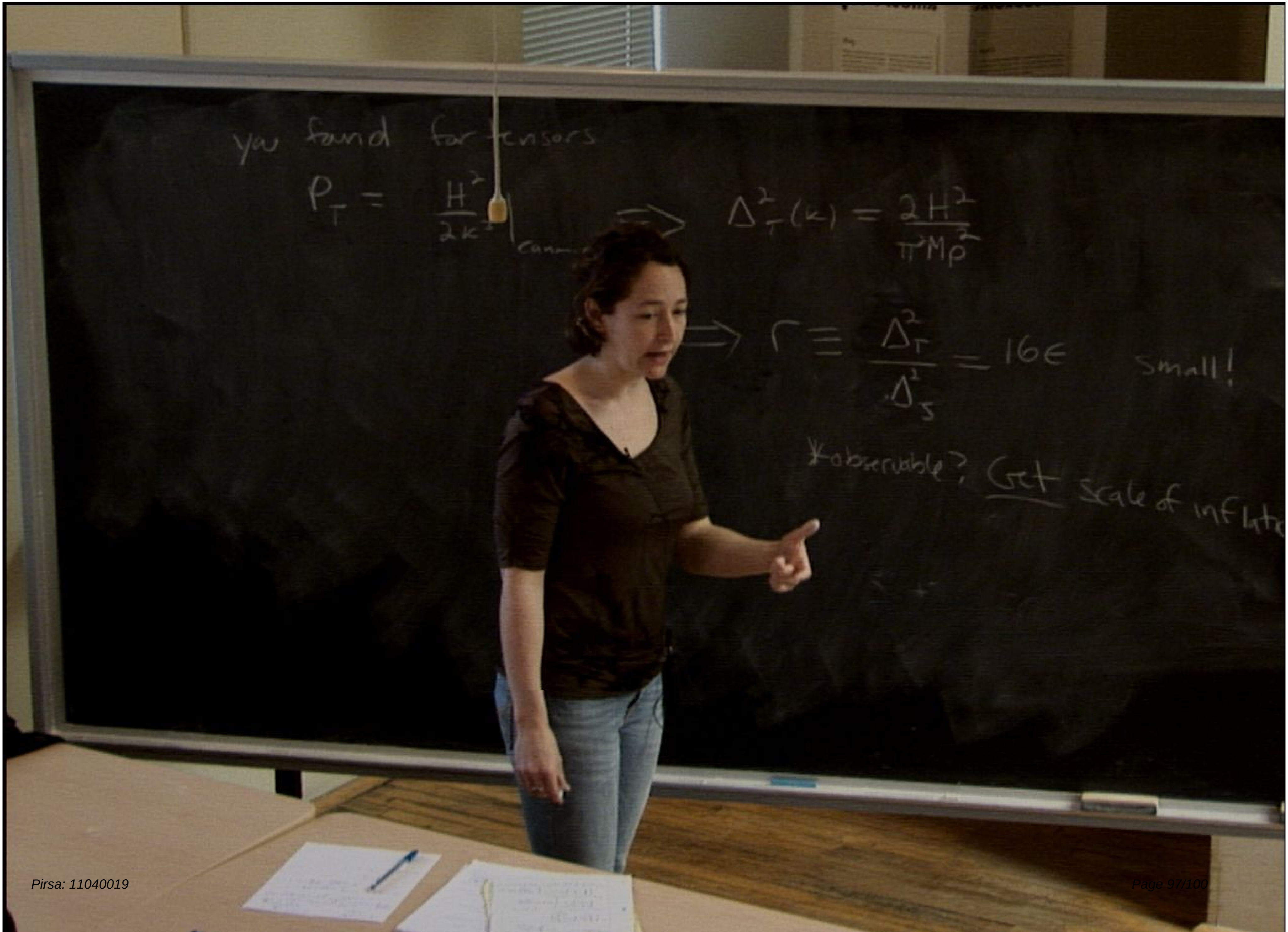
$$\zeta_k = -\frac{H}{\dot{\bar{\phi}}}\,\delta\phi_k$$

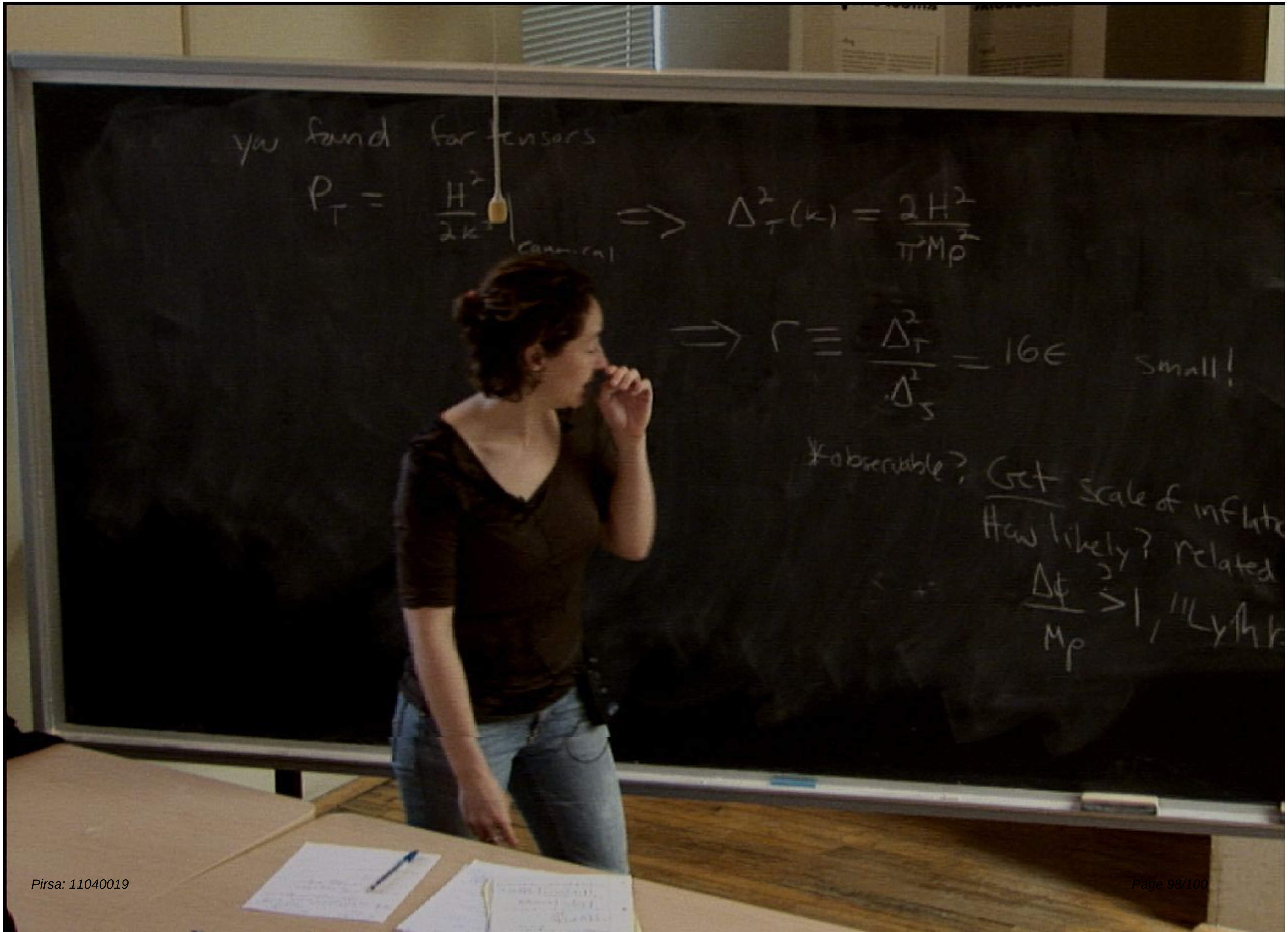

you found for tensors
$P_T = \frac{H^2}{2k}$


you found for tensors
$P_T = \frac{H^2}{2k^3}\Big|$
$\Rightarrow \Delta_T^2(k)$


you found for tensors
$P_T =$
$\Rightarrow \Delta_T^2(k) = \frac{2H^2}{\pi^2 M_p^2}$


you found for tensors
canonical
16ε


you found for tensors
$P_T = \frac{H^2}{2k^3}\Big|_{\text{canon}}$
$\Rightarrow \Delta_T^2(k) = \frac{2H^2}{\pi^2 M_p^2}$
$\Rightarrow r \equiv \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon$
small!
*observable? Get scale of inflat


you found for tensors
P_T = H^2/2k^3 | canonical
⟹ Δ_T^2(k) = 2H^2/π^2 Mp^2
⟹ r ≡ Δ_T^2/Δ_S^2 = 16ϵ
small!
observable? Get scale of inf
How likely? related
Δφ/Mp >? 1,

you found for tensors

$$P_T = \frac{H^2}{2k^3}\Big|_{\text{canonical}} \Rightarrow \Delta_T^2(k) = \frac{2H^2}{\pi^2 M_p^2}$$

$$\Rightarrow r \equiv \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon$$ small!

*observable? Get scale of inflation (H)!

How likely? related to $\frac{\Delta\phi}{M_p} \overset{?}{>} 1$, "Lyth bound" define canonical $\hat{\zeta}_k \equiv M_p \sqrt{2\epsilon}\, \zeta_k$

we know the answer for $\hat{\zeta}_k$

$$\langle BD | \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{H^2}{2k_1^3}}_{P(k)}$$

$$\Rightarrow \langle BD | \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} | BD \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \frac{H^2}{2k_1^3} \frac{1}{2\epsilon M_p^2}$$

$$\boxed{\Delta_\zeta^2(k) = \frac{k^3 P(k)}{2\pi^2} = \frac{H^2}{8\pi^2 \epsilon M_p^2}} = A_0 \left(\frac{k}{k_0}\right)^{n_s - 1}$$