

Title: Explorations in Cosmology - Lecture 11

Date: Apr 18, 2011 09:00 AM

URL: <http://pirsa.org/11040017>

Abstract:

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$\dot{H} \leq 0$$

$$\epsilon < 1 \Rightarrow \ddot{a}$$



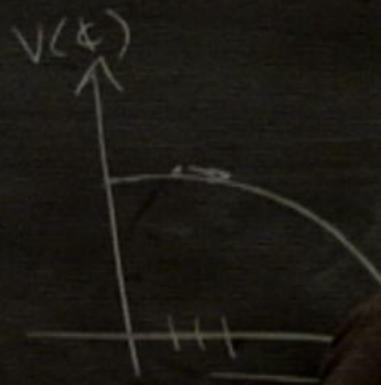
$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$\dot{H} \leq 0$$

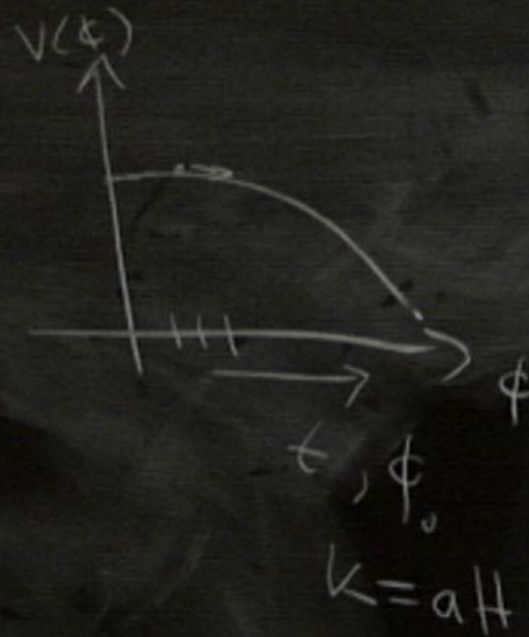
$$\epsilon < 1 \Rightarrow \ddot{a}$$

$$\Delta_s^2 = A_0 \left(\frac{k}{k_0} \right)^{n_s-1}$$

$$\propto \frac{H^2}{M_{Pl}^2}$$



$$\epsilon = -\frac{\dot{H}}{H^2}$$



$$\dot{H} \leq 0$$

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$$\Delta_s^2 = A_0 \left(\frac{k}{k_0} \right)^{n_s-1}$$

$$\propto \frac{H^2}{M_{\text{Pl}}^2 \epsilon}$$

$$n_s - 1 = \frac{d \ln \Delta^2}{d \ln k}$$

$$\eta =$$

$$\epsilon = -\frac{\dot{H}}{H^2}$$

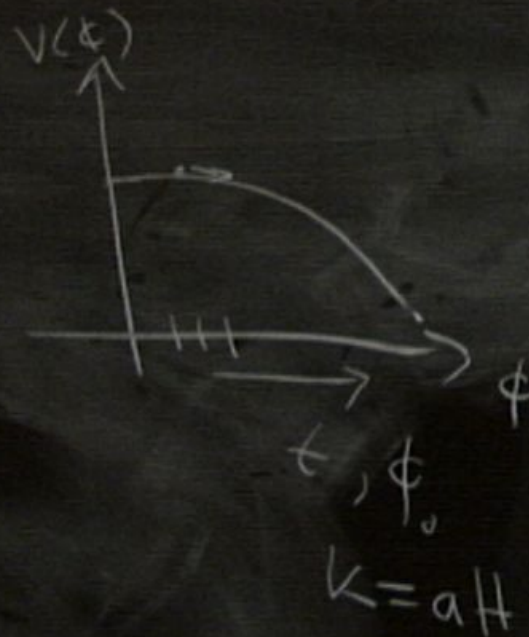
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$$\eta =$$

$$\frac{\epsilon}{H\epsilon}$$

$$\epsilon = -\frac{\dot{H}}{H^2}$$

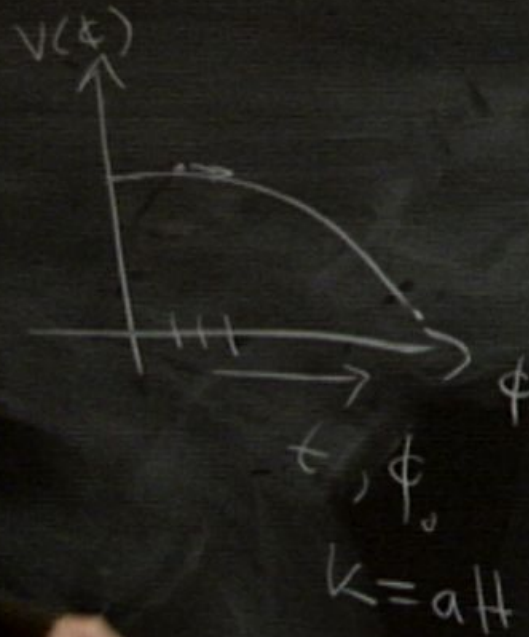
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Digression on $H \leq 0$

Last time was

Digression on $H \leq 0$

Last time was

$$p+p$$

Digression on $\dot{H} \leq 0$

Last time was

$$\rho + p \geq 0 \Rightarrow \dot{H} \leq 0$$

Digression on $\dot{H} \leq 0$

Last time was

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Null energy condition (NEC)

Digression on $\dot{H} \leq 0$

Last time was

$$\rho + p \geq 0 \Rightarrow \dot{H} \leq 0$$

Null energy condition (NEC)

More generally,

Digression on $\dot{H} \leq 0$

Last time was

$$\rho + p \geq 0 \Rightarrow \dot{H} \leq 0$$

Null energy condition (NEC)

generally,

we can find geometries that
are "weird" [acausal? Wormholes?]

Digression on $\dot{H} \leq 0$

Last time was

$$\rho + p \geq 0 \Rightarrow \dot{H} \leq 0$$

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Null energy condition (NEC)

More generally,

We can find geometries that

are "weird" [acausal? Wormholes?]

→ can we make those with "realistic" matter?

Look at perfect fluids

Look at perfect fluids

$$T_{\mu\nu} = (\rho + p)$$



Look at perfect fluids

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \quad ; \text{ in rest frame of fluid}$$

$\uparrow$
fluid 4-velocity

Look at perfect fluids

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \quad ; \text{ in rest frame of fluid}$$

↑
fluid 4-velocity

$$u^\mu = \frac{dx^\mu}{d\tau} = (1, 0, 0, 0)$$

?
matter?

Look at perfect fluids

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fluid 4-velocity

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$\uparrow$
proper time

normalise: $u_\mu u^\mu = -1$

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matter?

Look at perfect fluids

$$T_{\mu\nu} = (\rho + p) \underset{\substack{\uparrow \\ \text{fluid 4-velocity}}}{u_\mu} u_\nu + p g_{\mu\nu} \quad ; \text{ in rest frame of fluid}$$

$$u^\mu = \frac{dx^\mu}{d\tau} \underset{\substack{\uparrow \\ \text{proper time}}}{=} (1, 0, 0, 0)$$

$$\text{normalise: } u_\mu u^\mu = -1$$

More formally

$$\text{NEC} : T_{\mu\nu} l^\mu l^\nu \geq 0$$

↑
any null vector

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More formally

$$NEC : T_{\mu\nu} l^\mu l^\nu \geq 0$$

↑ any null vector

*relatively permissive condition

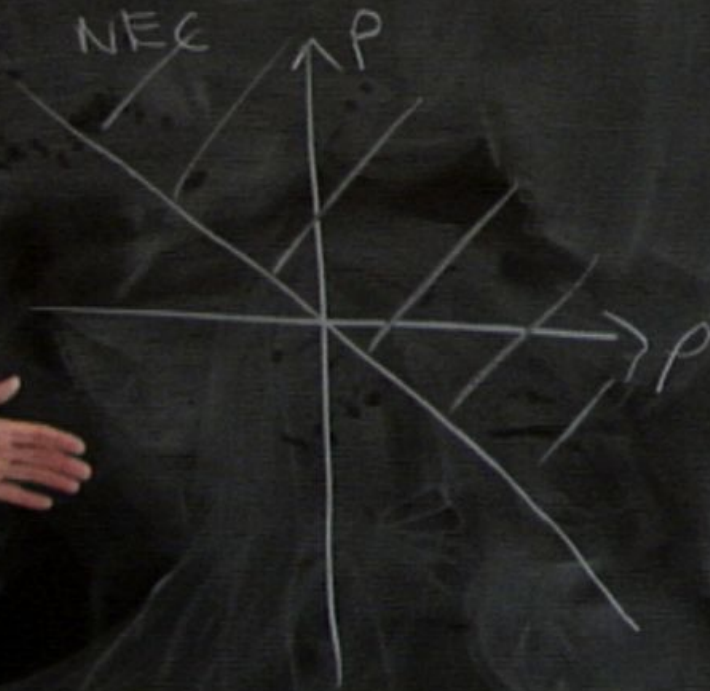


More formally

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↑
any null vector

*relatively permissive condition



Digression on $\dot{H} \leq 0$

Last time was

$$\rho + p \geq 0 \Rightarrow \dot{H} \leq 0$$

Null energy condition (NEC)

More generally,

We can find geometries that
are "weird" (acausal? Wormholes?)
→ can we make those with "realistic" matter?

More formally

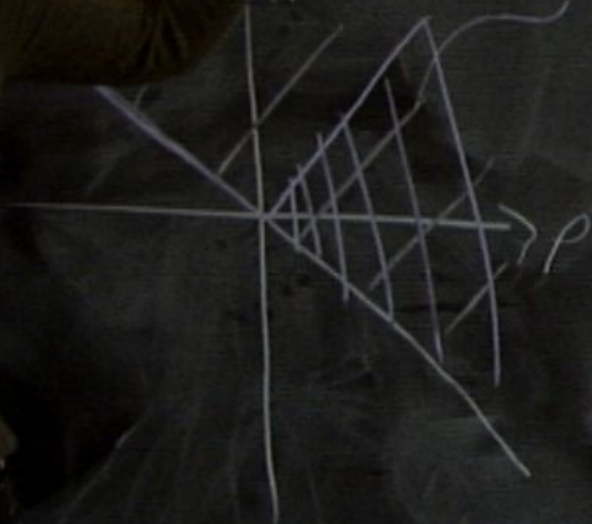
$$NEC : T_{\mu\nu} l^{\mu} l^{\nu} \geq 0$$

any null vector

* relatively permissive condition

NEC

Dominant energy condition



More formally

$$NEC : T_{\mu\nu} l^\mu l^\nu \geq 0$$

any null vector

*relatively permissive condition

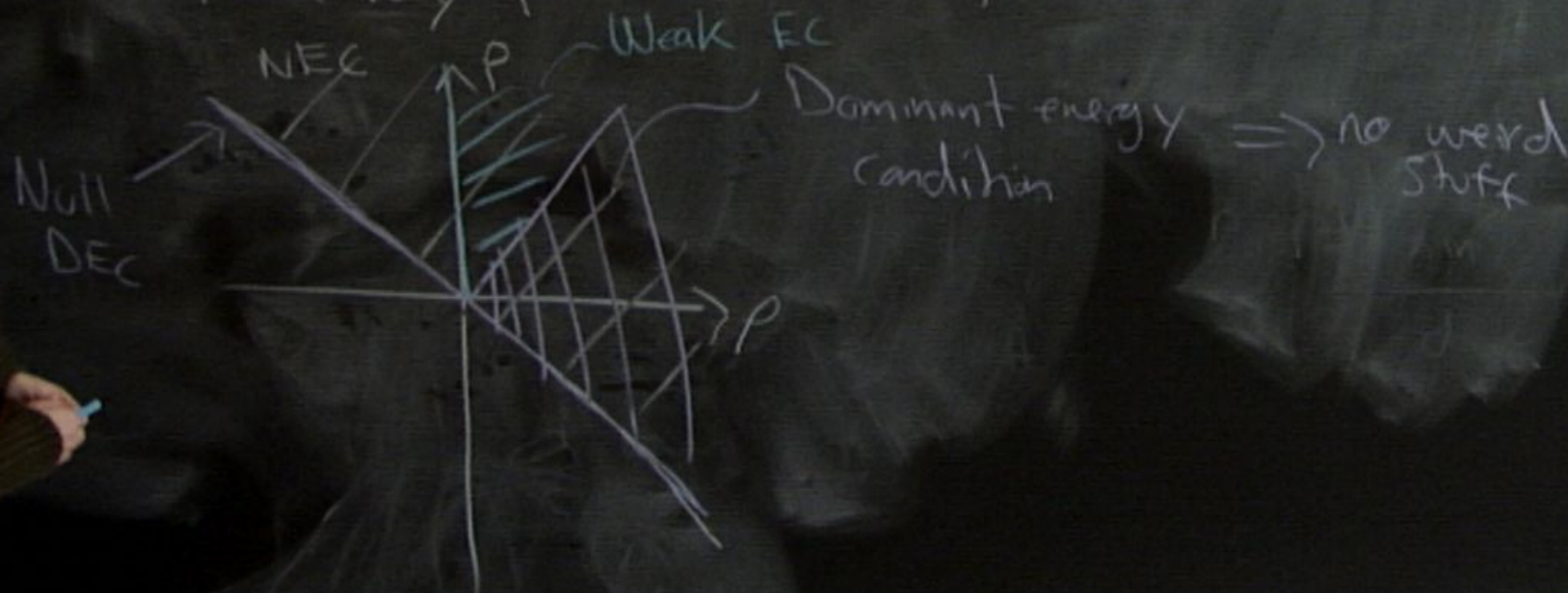


More formally

$$NEC : T_{\mu\nu} l^\mu l^\nu \geq 0$$

any null vector

*relatively permissive condition



Why do we care in cosmology?
↳ classical

Why do we care in cosmology?
→ classical matter
satisfies NEC

Why do we care in cosmology?

↳ classical matter
satisfies NEC

↳ Quantum matter fields
violate

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Why do we care in cosmology?

↳ classical matter
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violate these conditions
(*could figure out what
conditions you need to
avoid weirdness)

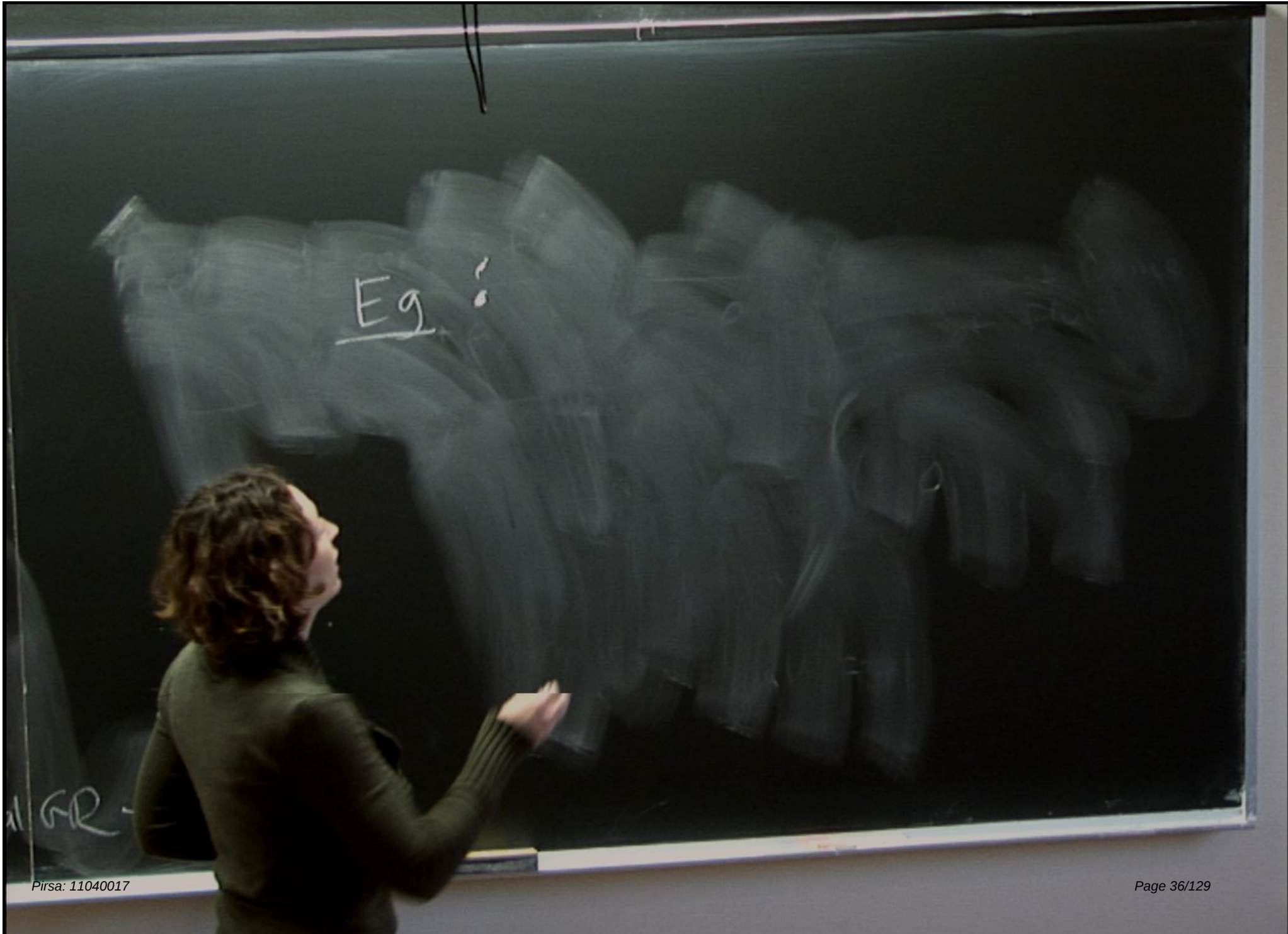
Why do we care in cosmology?

↳ classical matter
satisfies NEC

↳ Quantum matter fields
violate these conditions

(* could figure out what
conditions you need to
avoid weirdness)

* results proved in classical GR → violated



Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
BH evaporates

GR → violated

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② Inflation

ated

Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
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② Inflation : stochastic
eternal inflation

GR → violated

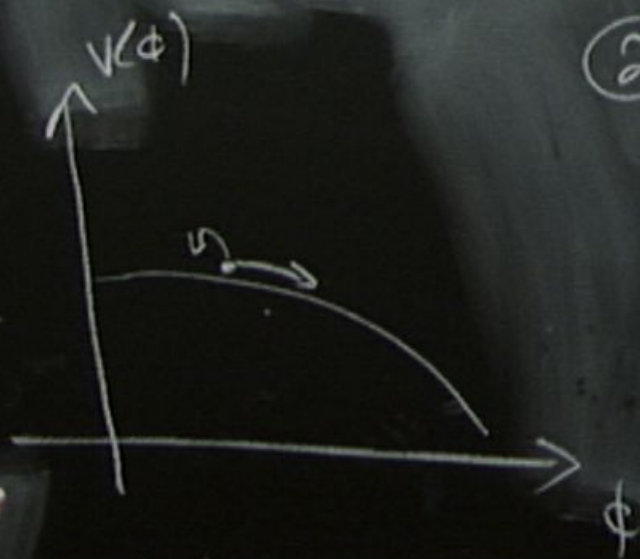
Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
BH evaporates

② Inflation : stochastic
eternal inflation
↳ quantum fluctuations
dominate

→ violated

Eg : ① Black holes : $\dot{A} \geq 0$
 $\hookrightarrow$ Quantum effect :
BH evaporates

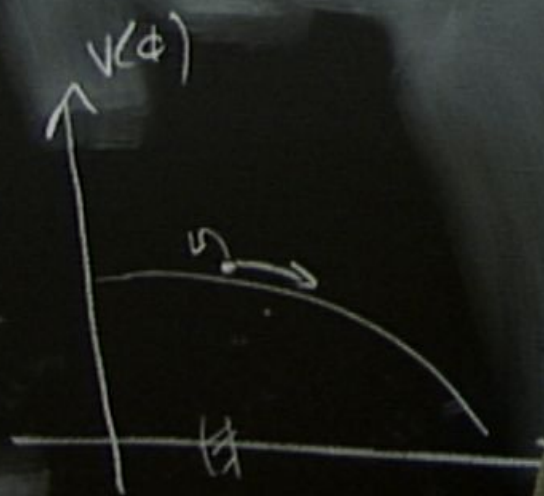
② Inflation : stochastic
eternal inflation
 $\hookrightarrow$ quantum fluctuations
dominate



$\rightarrow$ violated

Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
BH evaporates

② Inflation : stochastic
inflation
quantum fluctuations
dominate
fast



GR → violated

Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
BH evaporates

② Inflation : stochastic
eternal inflation
↳ quantum fluctuations
dominate

→ at least locally, in short times

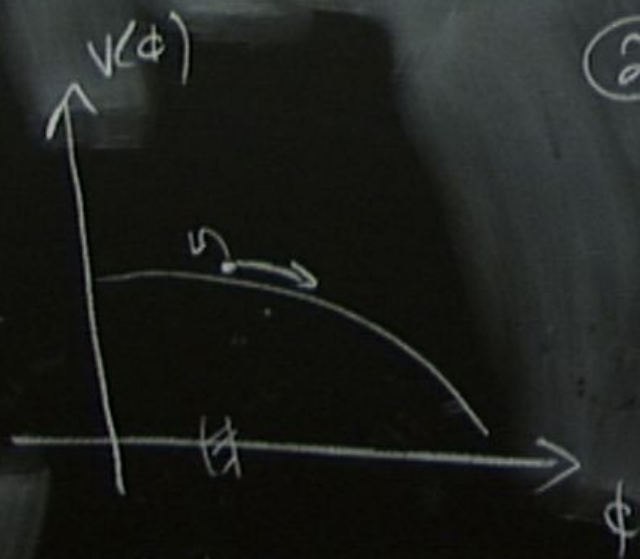


GR → violate

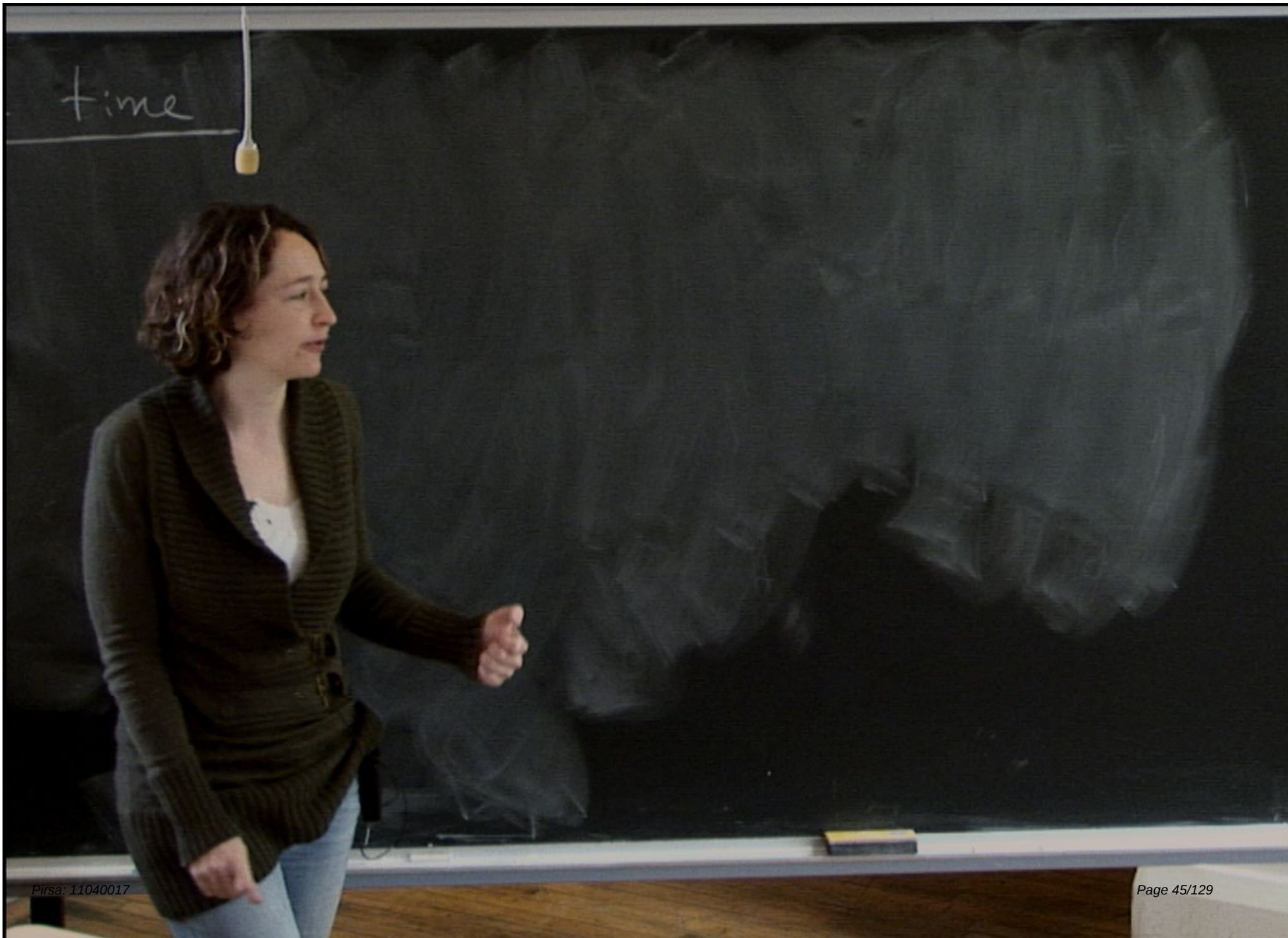
Eg : ① Black holes : $\dot{A} \geq 0$
↳ Quantum effect :
BH evaporates

② Inflation : stochastic
eternal inflation
↳ quantum fluctuations
dominate

→ at least locally, in short times,
on Hubble scales $\dot{H} > 0$



GR → violated



Last time

scalars 4, Φ , Ψ , B , E

vectors 2, F

tensors

Last time

scalars

4, Φ, Ψ, B, E

tensors

2×2 ; F_i, S_i ($\partial_i F^i = 0$)

sensors

Last time

scalars

4, Φ , Ψ , B , E

vectors

2×2 , F_i , S_i ($\partial_i F^i = 0$)

tensors

Last time

scalars

4, ϕ , ψ , B , E

vectors

2×2 , F_i , S_i ($\partial_i F^i = 0$)

tensors

h_{ij}

Last time

scalars 4, ϕ, ψ, B, E

vectors

2×2 ; F_i, S_i ($\partial_i F^i = 0$)

tensors

$\frac{2}{10} h_{ij}$

Last time

scalars 4, ϕ , ψ , B , E

vectors

2×2 ; F_i , S_i ($\partial_i F^i = 0$)

tensors

$\frac{2}{10}$ h_{ij} (traceless, tran.)

Last time

Scalars 4, ϕ, ψ, B, E

vech 2x2, F_i, S_i ($\partial_i F^i = 0$)

$\frac{2}{10}$ h_{ij} ($\nabla_i h^i_j = 0$)

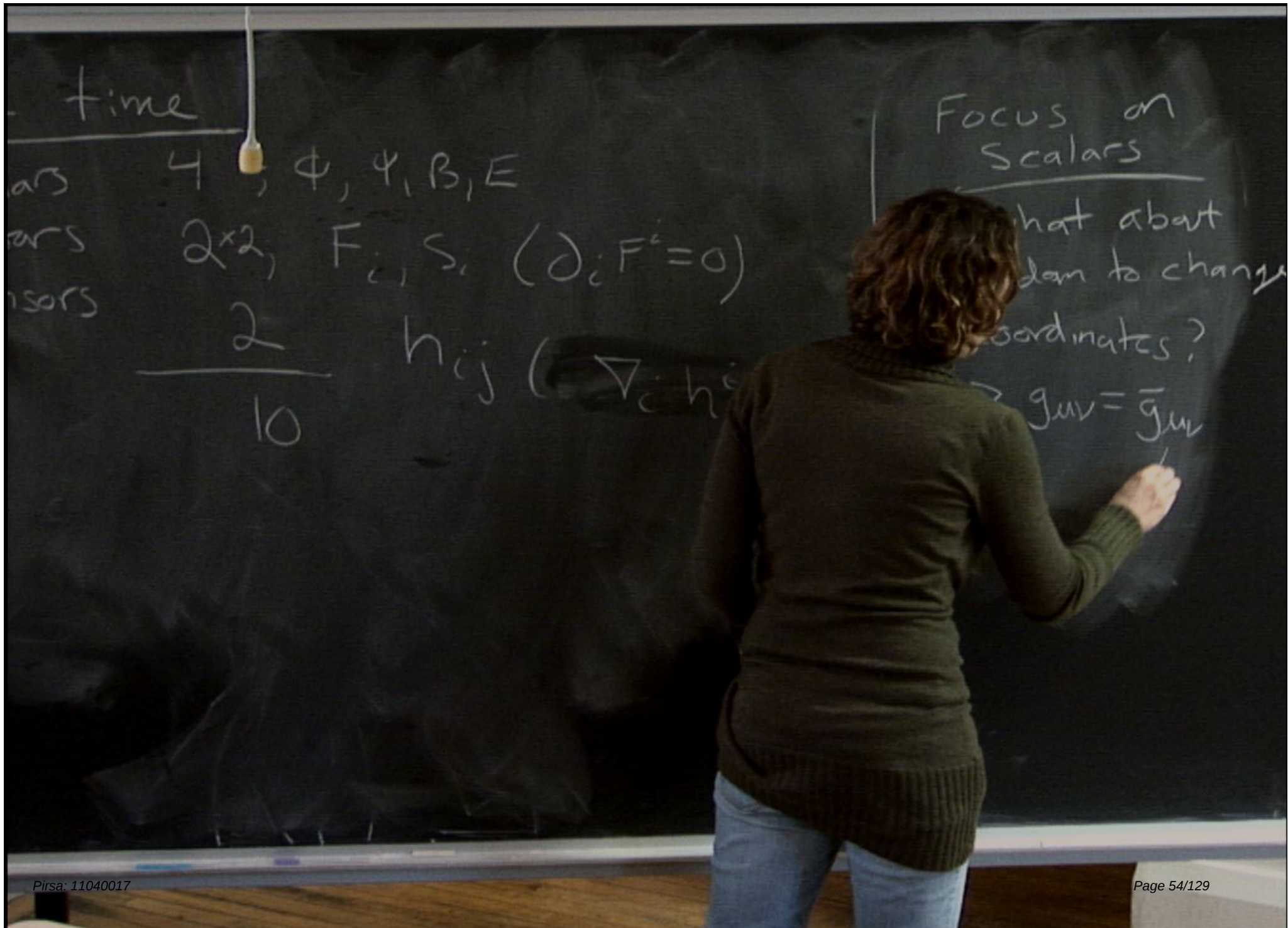
time

4, Φ , Ψ , B , E

2×2 , F_i , S_i ($\partial_i F^i = 0$)

2 h_{ij} ($\nabla_i h^i_j = 0$)

Focus on
Scalars



time 1
scalars 4, Φ, Ψ, B, E

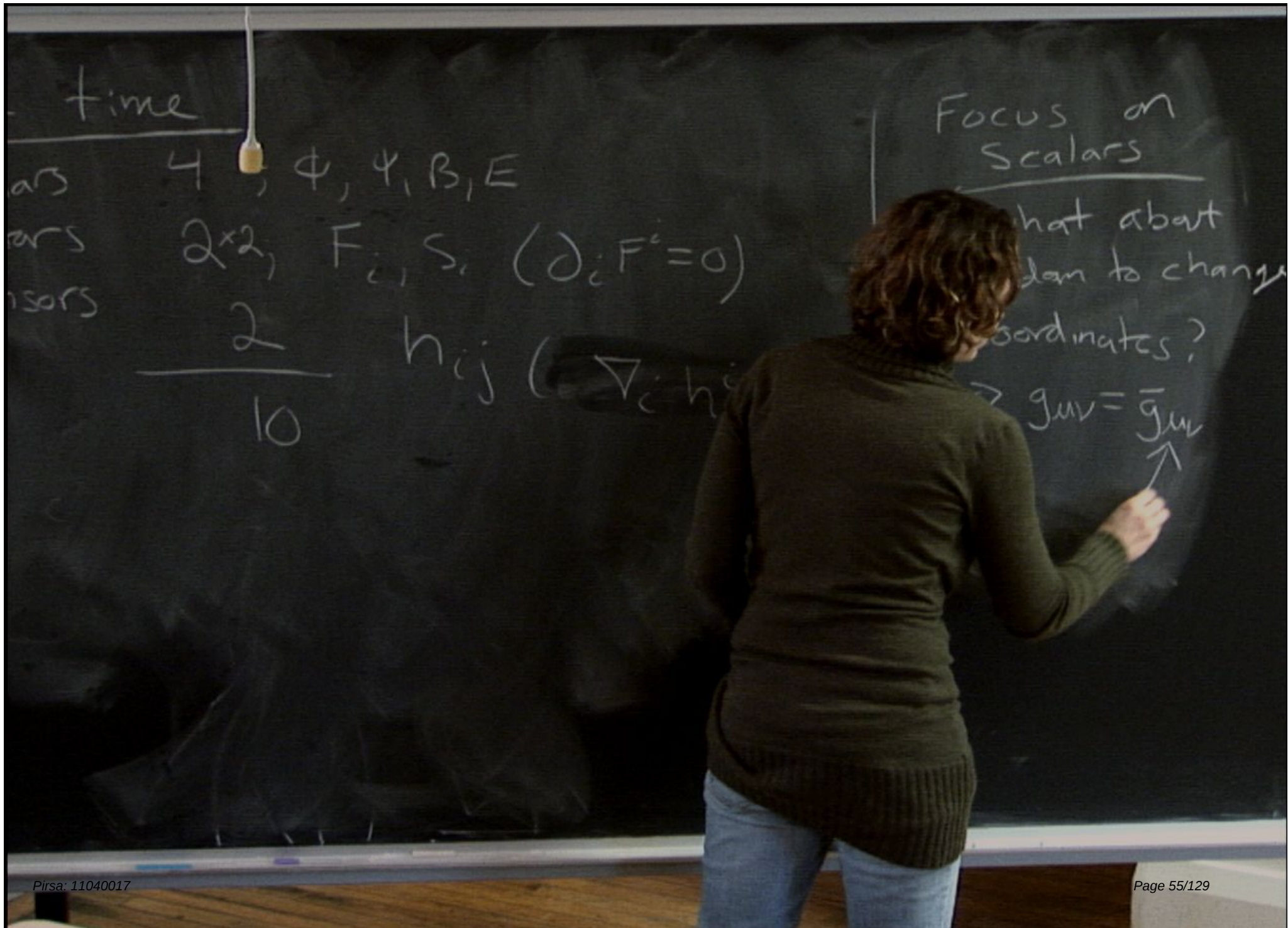
vectors 2×2 , F_i, S_i ($\partial_i F^i = 0$)

tensors $\frac{2}{10}$
 h_{ij} ($\nabla_i h^i_j = 0$)

Focus on
Scalars

what about
invariance to change
of coordinates?

$$g_{\mu\nu} = \bar{g}_{\mu\nu}$$



time
4, ϕ, ψ, B, E

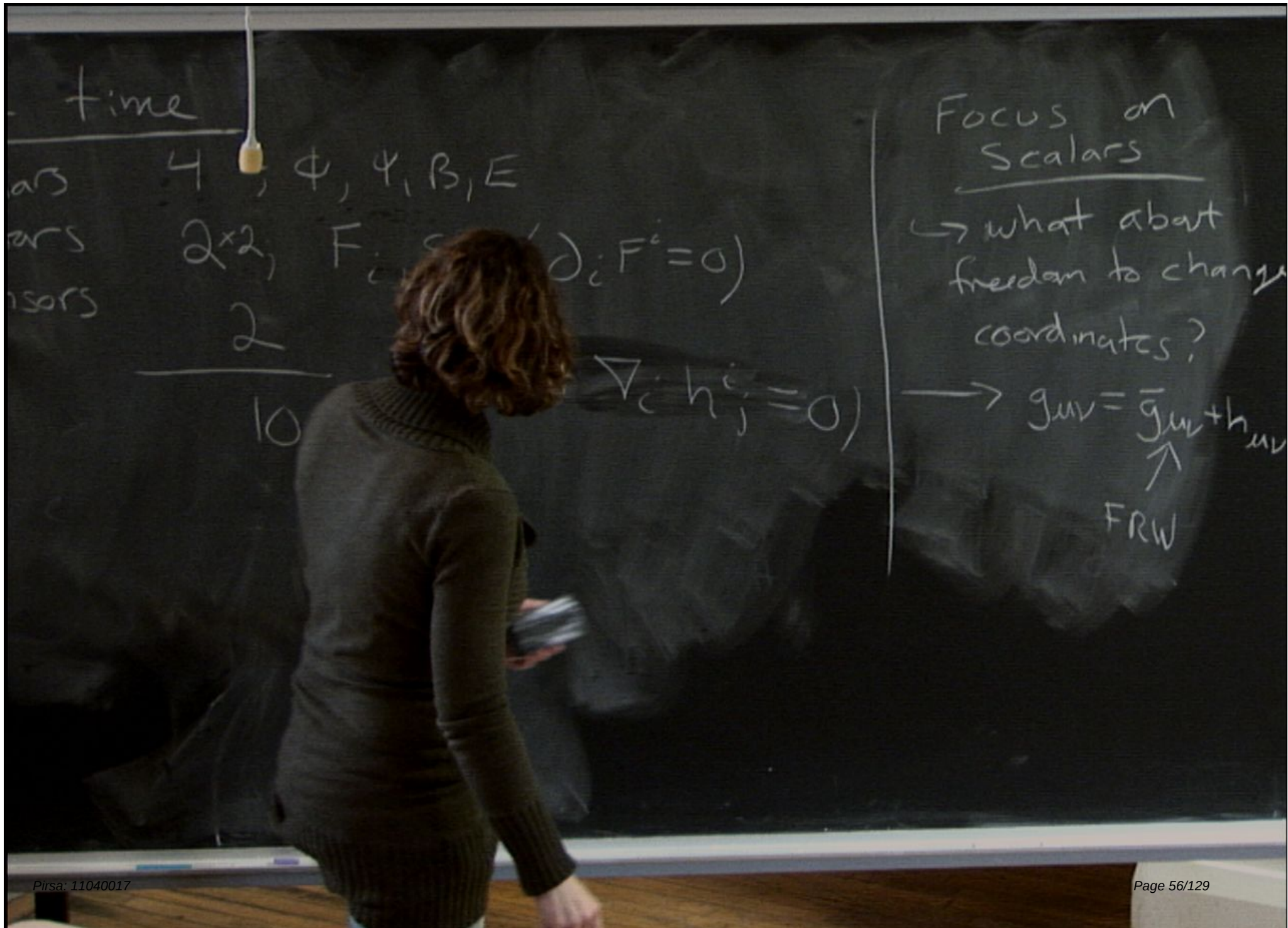
2x2, F_i, S_i ($\partial_i F^i = 0$)

$\frac{2}{10}$ h_{ij} ($\nabla_i h^i_j = 0$)

Focus on
Scalars

what about
can to change
coordinates?

$$g_{\mu\nu} = \bar{g}_{\mu\nu}$$



time
4, ϕ, ψ, B, E
2x2, F_{ij}
2

10

$$\partial_i F^i = 0$$
$$\nabla_i h^i_j = 0$$

Focus on Scalars

→ what about freedom to change coordinates?

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

FRW

Focus on Scalars

→ what about freedom to change coordinates?

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

↑
FRW

just
scalars

$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta \right. \\ \left. + [(1-2\phi) \delta_{ij} + 2C_{ij}] dx^i dx^j \right]$$

Focus on
Scalars

→ what about
freedom to c
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu}$$

↑
FRW

just
scalars

$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta \right. \\ \left. + [(1-2\psi)S_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

Focus on
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Focus on
Scalars

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↑
FRW

just
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$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

Coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi$$

Focus on
Scalars

→ what about
freedom to c
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu}$$

↑
FRW

just
scalars

$$ds^2 = a^2 \left[- (1 - 2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1 - 2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

Focus on
Scalars

→ what about
freedom to choose
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} \quad \uparrow \text{FRW}$$

just
scalars

$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

Coordinate transformation

$$X^\mu \rightarrow \tilde{X}^\mu = x^\mu + \tilde{\xi}^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

$$\tilde{\xi}^i = \tilde{\xi}_\perp^i + \partial^i \xi \quad \leftarrow \text{scalar}$$

$$\partial_i(\tilde{\xi}_\perp^i) = 0$$

Focus on
Scalars

→ what about
freedom to c
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} \quad \uparrow \text{FRW}$$

just
scalars

$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

Coordinate transformation

$$\tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

$$\xi^i = \xi^i_{\perp} + \partial^i \xi \quad \leftarrow \text{scalar}$$

$$\partial_i(\xi^i_{\perp}) = 0$$

Focus on
Scalars

→ what about
freedom to choose
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} \quad \uparrow \text{FRW}$$

just
scalars

$$ds^2 = a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

Coordinate transformation

Scalar-Vector-
Tensor

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

(Spatial)

$$\xi^i = \xi^i_{\perp} + \partial^i \xi$$

$\uparrow$

$$\partial_i (\xi^i_{\perp}) = 0$$

$\nwarrow$ scalar

Focus on
Scalars

$\rightarrow$ what about
freedom to c
coordinates

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu}$$

$\uparrow$
FRW

just
Scalars

$$ds^2 = a^2 \left[-(1+2\phi) dt^2 - 2B_{,i} dx^i dt + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

At First
order

Coordinate transformation

Scalar-Vector-
Tensor

SUT decomposition (Spatial
theorem)

x^μ

$$x^\mu \rightarrow x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

$$\xi^\mu = \xi^\mu_{\perp} + \partial^\mu \xi$$

$$\partial_i(\xi^\mu_{\perp}) = 0$$

Scalar

Focus on
Scalars

→ what about
freedom to choose
coordinates

$$g_{\mu\nu} = \bar{g}_{\mu\nu} \quad \uparrow \quad \text{FRW}$$

$$a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x)$$

(Spatial)

$$\xi^i = \xi^i_\perp + \xi^i_\parallel$$

$\uparrow$

$$\partial_i(\xi^i_\parallel)$$

Focus on Scalars

→ what about freedom to change coordinates?

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$\uparrow$
FRW

$$1 + 2\phi) d\eta^2 - 2B_{,i} dx^i d\eta \\ [(1 - 2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j]$$

transformation

$$\rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

atial) $\xi^i = \xi^i_{\perp} + \partial^i \xi$

$\uparrow$ $\uparrow$ scalar

$$\partial_i(\xi^i_{\perp}) = 0$$

Focus on Scalars

$\rightarrow$ what about freedom to change coordinates?

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$\uparrow$
FRW

What happens to ϕ, ψ, B, E
under $x^\mu \rightarrow \tilde{x}^\mu$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

a, ψ

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = g_{\alpha\beta}(x)$$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

$$a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

gauge transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu \quad \leftarrow \text{infinitesimal}$$

initial) $\xi^i = \xi^i_{\perp} + \partial^i \xi$

$\uparrow$ $\uparrow$ scalar

$$\partial_i(\xi^i_{\perp}) = 0$$

Focus on Scalars

→ what about freedom to change coordinates?

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$\uparrow$
FRW

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$
$$= \left[\delta_{\mu}^{\nu} - \right]$$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\begin{aligned}\tilde{g}_{\mu\nu}(\tilde{x}^\mu) &= \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x) \\ &= \left[\delta_{\mu\nu} - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right]\end{aligned}$$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

$$= \left[\delta_\mu^\alpha - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right] \left[\delta_\nu^\beta - \frac{\partial \xi^\beta}{\partial \tilde{x}^\nu} \right] g_{\alpha\beta}(x)$$

What happens to ϕ, ψ, B, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

$$= \left[\delta_\mu^\alpha - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right] \left[\delta_\nu^\beta - \frac{\partial \xi^\beta}{\partial \tilde{x}^\nu} \right] g_{\alpha\beta}(x)$$

$$\approx g_{\mu\nu}(x) - g_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\mu} - g_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\nu}$$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

$$= \left[\delta_\mu^\alpha - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right] \left[\delta_\nu^\beta - \frac{\partial \xi^\beta}{\partial \tilde{x}^\nu} \right] g_{\alpha\beta}(x)$$

$$\tilde{g}_{\mu\nu}(\tilde{x}) + \delta \tilde{g}_{\mu\nu}(\tilde{x}) \approx \underbrace{g_{\mu\nu}(x)}_{\tilde{g}_{\mu\nu}(x) + \delta g_{\mu\nu}(x)} - g_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\mu} - g_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\nu}$$

Also need $\bar{g}_{\mu\nu}(x) \approx \bar{g}_{\mu\nu}$

$$g_{\alpha\beta}(x)$$

$$\frac{d\bar{g}}{dx^{i\mu}}$$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(\tilde{x})$

$g_{\alpha\beta}(x)$

$\frac{d\bar{g}_{\mu\nu}}{dx^i}$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(\tilde{x})$

→ And, claim, pert. around the same FRW
 background $\bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(\tilde{x})$

$$g_{\alpha\beta}(x)$$

$$\frac{d\xi^\mu}{dx^\mu}$$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(\tilde{x})$

→ And, claim, pert. around the same FRW
 background $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(\tilde{x})$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(x)$

→ And, claim, pert. around the same FRW
 background $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(x)$

Left over

$g_{\alpha\beta}(x)$

$\frac{d\xi^\mu}{dx^\mu}$

What happens to ϕ, ψ, B, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

$$= \left[\delta_\mu^\alpha - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right] \left[\delta_\nu^\beta - \frac{\partial \xi^\beta}{\partial \tilde{x}^\nu} \right]$$

$$\tilde{g}_{\mu\nu}(\tilde{x}^\mu) + \delta g_{\mu\nu}(\tilde{x}) \approx \underbrace{g_{\mu\nu}(x)}_{\tilde{g}_{\mu\nu}(x) + \delta g_{\mu\nu}(x)} - g_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\mu} - g_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\nu}$$

What happens to ϕ, ψ, β, E
under $x^\mu \rightarrow \tilde{x}^\mu$ (to 1st order)

$$\begin{aligned}\tilde{g}_{\mu\nu}(\tilde{x}^\mu) &= \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x) \\ &= \left[\delta_\mu^\alpha - \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} \right] \left[\delta_\nu^\beta - \frac{\partial \xi^\beta}{\partial \tilde{x}^\nu} \right] g_{\alpha\beta}(x)\end{aligned}$$

$$\begin{aligned}\tilde{g}_{\mu\nu}(\tilde{x}^\mu) + \delta g_{\mu\nu}(\tilde{x}) &\approx \underbrace{g_{\mu\nu}(x)}_{\tilde{g}_{\mu\nu}(x) + \delta g_{\mu\nu}(x)} - g_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\mu} - g_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\nu}\end{aligned}$$

$$\text{Also use } \bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\vec{x} - \vec{\xi}) \\ = \bar{g}_{\mu\nu}(\vec{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(x)$$

→ And, claim, pert. around the same FRW background $\Rightarrow \bar{g}_{\mu\nu}(\vec{x}) = \bar{g}_{\mu\nu}(\vec{x})$

Left over

$$\delta g_{\mu\nu} = \delta g_{\mu\nu}$$

$$g_{\mu\nu}(x)$$

$$\frac{\partial \bar{g}}{\partial x^\lambda}$$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(x)$

→ And, claim, pert. around the same FRW background $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(\tilde{x})$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}$$

$g_{\alpha\beta}(x)$

dx^μ

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(x)$

→ And, claim, pert. around the same FRW background $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(\tilde{x})$

Left over

$$\delta g_{\mu\nu} = \delta g_{\mu\nu} - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu} - \bar{g}_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\nu} - \bar{g}_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\mu}$$

$g_{\mu\beta}(x)$

$\frac{\partial \xi^\alpha}{\partial x^\mu}$

Let's work at $\delta \tilde{g}_{00}$

$$\delta \tilde{g}_{00} = -2a^2 \Phi$$

$$a^2 \left[-(1+2\phi) dm^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

(Spatial)

$$\xi^i = \xi^i_{\perp} + \partial^i \xi$$

$$\partial_i (\xi^i_{\perp}) = 0 \quad \leftarrow \text{scalar}$$

Focus on Scalars

→ what about freedom to change coordinates?

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

↑
FRW

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(n)]$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\Phi - \xi^\lambda \partial_\lambda [-a^2(x)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(n)]$$

$$- \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$= -2a^2\phi + \xi^0 \partial_0 [a^2(n)] + a^2 \frac{d\xi^0}{dx^0}$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(n)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2 = -2a^2\phi + \xi^0 \partial_0 [a^2(n)] + 2a^2 \frac{d\xi^0}{dx^0}$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(n)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2\tilde{\phi} = -2a^2\phi + \xi^0 \partial_0 [a^2(n)] + 2a^2 \frac{d\xi^0}{dx^0}$$

Let's work at $\delta \tilde{g}_{00}$

$$\delta \tilde{g}_{00} = -2a^2 \phi - \xi^\lambda \partial_\lambda [-a^2(n)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2 \tilde{\phi} = -2a^2 \phi + \xi^0 \partial_0 [a^2(n)] + 2a^2 \frac{d\xi^0}{dx^0} \\ \tilde{\phi} = \phi - \xi^0$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(n)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2\tilde{\phi} = -2a^2\phi + \xi^0 \partial_0 [a^2(n)] + 2a^2 \frac{d\xi^0}{dx^0} \\ \tilde{\phi} = \phi - \xi^0 \frac{a'}{a}$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(u)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2\tilde{\phi} = -2a^2\phi + \xi^0 \partial_0 [a^2(u)] + 2a^2 \frac{d\xi^0}{dx^0} \\ \tilde{\phi} = \phi - \xi^0 \frac{a'}{a} - (\xi^0)'$$

Let's work at $\tilde{\delta g}_{00}$

$$\tilde{\delta g}_{00} = -2a^2\phi - \xi^\lambda \partial_\lambda [-a^2(u)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2\tilde{\phi} = -2a^2\phi + \xi^0 \partial_0 [a^2(u)] + 2a^2 \frac{d\xi^0}{dx^0}$$

$$\tilde{\phi} = \phi - \xi^0 \frac{a'}{a} - (\xi^0)'$$

$$\boxed{\tilde{\phi} = \phi - \frac{1}{a} (a\xi^0)'}$$

$$(1+2\phi)dm^2 - 2B_{,i} dx^i d\eta$$

$$+ [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j$$

re transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu$$

(spatial)

$$\tilde{\xi}^i = \xi^i + \epsilon^i$$

↑

$\partial_i(\xi^i)$

Similarly

$\tilde{\psi}$

$$(1+2\phi)d\eta^2 - 2B_{,i}dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}]dx^i dx^j$$

re transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

spatial)

$$\xi^i = \xi^i_{\perp} + \partial^i \xi_{\parallel}$$

$\uparrow$
 $\partial_i(\xi^i_{\perp}) = 0$

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$(1+2\phi)dm^2 - 2B_{,i} dx^i d\eta \\ + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j]$$

transformation

$$\rightarrow \tilde{x}^\mu = x^\mu + \tilde{\xi}^\mu(x, t)$$

← infinitesimal

trial)

$$\tilde{\xi}^i = \tilde{\xi}_\perp^i + \partial^i \xi$$

↑ ↑
 $\partial_i(\tilde{\xi}_\perp^i) = 0$ scalar

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$\tilde{\phi} = \phi - \frac{1}{a} (a \xi^0)'$$

Let's work w/ $\delta \tilde{g}_{00}$

$$\delta \tilde{g}_{00} = -2a^2 \delta \phi - \xi^\lambda \partial_\lambda [-a^2(u)] \\ - \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$-2a^2 \tilde{\delta} \phi = -2a^2 \delta \phi + \xi^0 \partial_0 [a^2(u)] + 2a^2 \frac{d\xi^0}{dx^0} \\ \tilde{\delta} \phi = \delta \phi - \xi^0 \frac{a'}{a} - (\xi^0)'$$

$$\boxed{\hat{\delta} \phi = \delta \phi - \frac{1}{a} (a \xi^0)'}$$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(\tilde{x})$

→ And, claim, pert. around the same FRW
 background $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \eta_{\mu\nu} + \delta g_{\mu\nu}(\tilde{x})$ $\begin{bmatrix} -a^2 & & \\ & a^2 & \\ & & a^2 \end{bmatrix}$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} - \xi^\lambda \partial_\lambda \delta g_{\mu\nu} - \delta \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu} - \bar{g}_{\mu\nu} \partial_\lambda \xi^\lambda$$

Also need $\bar{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(\tilde{x} - \xi)$
 $= \bar{g}_{\mu\nu}(\tilde{x}) - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu}(x)$

→ And, claim, pert. around the same FRW
 back $\Rightarrow \bar{g}_{\mu\nu}(\tilde{x}) = \bar{g}_{\mu\nu}(\tilde{x})$ $\begin{bmatrix} -a^2 & & \\ & a^2 & \\ & & a^2 & \\ & & & a^2 \end{bmatrix}$

Left $\delta g_{\mu\nu} = \bar{g}_{\mu\nu} - \xi^\lambda \partial_\lambda \bar{g}_{\mu\nu} - \bar{g}_{\mu\beta} \frac{\partial \xi^\beta}{\partial x^\nu} - \bar{g}_{\alpha\nu} \frac{\partial \xi^\alpha}{\partial x^\mu}$

$$a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

(Spatial)

$$\xi^i = \xi^i_{\perp} + \partial^i \xi$$

$$\partial_i(\xi^i_{\perp}) = 0 \quad \leftarrow \text{scalar}$$

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$\tilde{\phi} = \phi - \frac{1}{a} (a \xi^0)'$$

$$+ \delta g_{00}$$

$$a^2 \phi - \xi^1 \partial_1 [a^2(u)]$$

$$- \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$a^2 \phi + \xi^0 \partial_0 [a^2(u)] +$$

$$- \xi^0 \frac{a'}{a} - (\xi^0)'$$

$$\phi - \frac{1}{a} (a \xi^0)'$$

Notice

① Transformations depend only on

ξ^0, ξ

$\Rightarrow$ choose coords

such

Also need $\bar{g}_{\mu\nu}(x)$

$\rightarrow$ And, claim, pert.
background $\Rightarrow \bar{g}_{\mu\nu}$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} -$$

$$+ \delta g_{00}$$

$$a^2 \phi - \xi^\lambda \partial_\lambda [-a^2(x)]$$

$$\bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$a^2 \phi + \xi^0 \partial_0 [2a^2 \frac{d\xi^0}{dx^0}]$$

$$= \xi^0 \frac{a'}{a}$$

$$\phi = \frac{1}{a}$$

Notice

① Transformations depend only on

$$\xi^0, \xi$$

$\Rightarrow$ choose coords

such that
two scalars
vanish

$\rightarrow$ to calculate,
people choose
a "gauge"

Also need $\bar{g}_{\mu\nu}(x)$

$\rightarrow$ And, claim, pert.
background $\Rightarrow \bar{g}_{\mu\nu}$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} -$$

$$+ \delta g_{00}$$

$$a^2 \phi - \xi^1 \partial_1 [-a^2(r)]$$

$$- \bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$a^2 \phi + \xi^0 \partial_0 [a^2(r)] + 2a^2 \frac{d\xi^0}{dx^0}$$

$$= \xi^0 \frac{a'}{a} - (\xi^0)'$$

$$\phi - \frac{1}{a} (a \xi^0)'$$

Notice

① Transformations depend only on

$$\xi^0, \xi$$

$\Rightarrow$ choose coords

such that
two scalars
vanish

$\rightarrow$ to calculate,
people choose

a "gauge"

②

Also need $\bar{g}_{\mu\nu}(x)$

$\rightarrow$ And, claim, pert.
background $\Rightarrow \bar{g}_{\mu\nu}$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} -$$

$$+ \delta g_{00}$$

$$a^2 \phi - \xi^1 \partial_1 [-a^2(x)]$$

$$\bar{g}_{0\beta} \frac{\partial \xi^\beta}{\partial x^0} - \bar{g}_{\alpha 0} \frac{\partial \xi^\alpha}{\partial x^0}$$

$$a^2 \phi + \xi^0 \partial_0 [1 + 2a^2 \frac{d\xi^0}{dx^0}]$$

$$- \xi^0 \frac{a'}{a}$$

$$\phi - \frac{1}{a} (a \xi^0)$$

Notice

① Transformations depend only on ξ^0, ξ

$\Rightarrow$ choose coords such that two scalars vanish

$\rightarrow$ to calculate, people choose

② a "gauge"
Can choose combinations

Also need $\bar{g}_{\mu\nu}(x)$

$\rightarrow$ And, claim, pert. background $\Rightarrow \bar{g}_{\mu\nu}$

Left over

$$\tilde{\delta} g_{\mu\nu} = \delta g_{\mu\nu} -$$

that are invariant

Noti
① To

②

that are invariant

$$\Phi \equiv$$

$$\Psi \equiv$$

Noti
① To

②

that are invariant

$$\overline{\Phi} \equiv \phi - \frac{1}{a}$$

$$\overline{\Psi} \equiv \psi$$

Notice
① To

②

that are invariant

$$\bar{\Phi} \equiv \phi - \frac{1}{a} [a(\beta - E')]']$$

$$\bar{\Psi} \equiv \psi + \frac{a'}{a} [\beta - E']$$

Notic
① To

②

that are invariant

$$\bar{\Phi} \equiv \phi - \frac{1}{a} [a(B-E')]'$$

$$\bar{\Psi} \equiv \psi + \frac{a'}{a} [B-E']$$

Notes
① To

②

that are invariant

$$\underline{\Phi} \equiv \phi - \frac{1}{a} [a(B-E')]'$$

$$\underline{\Psi} \equiv \psi + \frac{a'}{a} [B-E']$$

Common choice "Newtonian gauge"

Notation
① To

②

that are invariant

$$\bar{\Phi} \equiv \phi - \frac{1}{a} [a(B-E')]'$$

$$\bar{\Psi} \equiv \psi + \frac{a'}{a} [B-E']$$

* useful for easy coordinate changes

Common choice "Newtonian gauge"

$$E=B=0$$

Notes
① To

②

$$a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

(Spatial)

$$\xi^i = \xi^i_{\perp} + \partial^i \xi \quad \leftarrow \text{scalar}$$

$$\partial_i (\xi^i_{\perp}) = 0$$

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$\tilde{\phi} = \phi - \frac{1}{a} (a \xi^0)'$$

that are invariant

$$\underline{\Phi} \equiv \phi - \frac{1}{a} [a(B-E')]'$$

$$\underline{\Psi} \equiv \psi + \frac{a'}{a} [B-E']$$

* useful for easy coordinate changes

Common choice "Newtonian gauge"

$$E=B=0$$

→ Careful! Eg "Synchronous gauge"

Notes
① To

②

$$a^2 \left[-(1+2\phi) d\eta^2 - 2B_{,i} dx^i d\eta + [(1-2\psi)\delta_{ij} - 2E_{,ij}] dx^i dx^j \right]$$

coordinate transformation

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \tilde{\xi}^\mu(x, t) \quad \leftarrow \text{infinitesimal}$$

(Spatial)

$$\tilde{\xi}^i = \tilde{\xi}_\perp^i + \partial^i \xi$$

$\uparrow$ $\perp$ $\uparrow$
 $\partial_i(\tilde{\xi}_\perp^i) = 0$ Scalar

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$\tilde{\phi} = \phi - \frac{1}{a} (a \xi^0)'$$

$$B_{,i} dx^i d\eta$$

$$2E_{,ij} dx^i dx^j]$$

$\eta \leftarrow \text{infinitesimal}$

$$\xi^\mu(x, t)$$

$$\xi^i + \partial^i \xi$$

$\uparrow$ scalar

$$\partial_i(\xi^i_{\perp}) = 0$$

Similarly

$$\tilde{\psi} = \psi + \frac{a'}{a} \xi^0$$

$$\tilde{B} = B + \xi' - \xi^0$$

$$\tilde{E} = E + \xi$$

$$\hat{\phi} = \phi - \frac{1}{a} (a \xi^0)'$$

that are invariant

$$\bar{\Phi} = \Phi - \frac{1}{a} [a(B-E')]'$$

$$\bar{\Psi} = \Psi + \frac{a'}{a} [B-E']$$

* useful for easy coordinate changes

Choice "Newtonian gauge"

$$E=B=0$$

"Synchronous gauge"

$\Phi_s = B_s = 0$ allows coord. changes

Notice
① To

②

ce
informations
depend only on
 ξ^0, ξ

⇒ choose coords
such that
two scalars
vanish

→ to calculate,
people choose

a "gauge"
Can choose
combinations

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

ce
informations
depend only on
 ξ^0, ξ

$\Rightarrow$ choose coords

such that

$\rightarrow$ $\dots$
 p

$$\overline{G}_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

ce
informations
depend only on
 ξ^0, ξ

$\Rightarrow$ choose coords
such that
two scalars
vanish
 $\rightarrow$ to calculate,
people choose
a "gauge"
Can choose
combinations

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$
$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = M_P^{-2} [\bar{T}_{\mu\nu} + \delta T_{\mu\nu}]$$

ce
informations
depend only on
 ξ^0, ξ

$\Rightarrow$ choose coords
such that
two scalars
vanish

$\rightarrow$ to calculate,
people choose

a "gauge"
can choose
combinations

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = M_P^{-2} [\bar{T}_{\mu\nu} + \delta T_{\mu\nu}]$$

$$\bar{\rho} = \frac{1}{2} \dot{\phi}_0^2 + V(\phi_0)$$

$$\rho = \frac{1}{2} \dot{\phi}_0^2 - V(\phi_0)$$

ce
 informations
 depend only on
 ξ^0, ξ

$\Rightarrow$ choose coords
 such that
 two scalars
 vanish
 $\rightarrow$ to calculate
 people choose
 a "gauge"
 can choose
 combination

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = M_P^{-2} [\bar{T}_{\mu\nu} + \delta T_{\mu\nu}]$$

$$\begin{aligned} \bar{\rho} &= \frac{1}{2} \dot{\phi}_0^2 + V(\phi_0) \\ \rho &= \frac{1}{2} \dot{\phi}_0^2 - V(\phi_0) \end{aligned}$$

Still need $\delta T_{\mu\nu}$

give $\delta G_{\mu\nu} = \delta T_{\mu\nu}$

ce
informations
depend only on
 ξ^0, ξ

→ choose coords
such that
two scalars
vanish

→ to calculate,
people choose

a "gauge"
can choose
combinations

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = M_P^{-2} [\bar{T}_{\mu\nu} + \delta T_{\mu\nu}]$$

$$\bar{\rho} = \frac{1}{2} \dot{\phi}_0^2 + V(\phi_0)$$

$$\rho = \frac{1}{2} \dot{\phi}_0^2 - V(\phi_0)$$

Still need $\delta T_{\mu\nu}$

and solve $\delta G_{\mu\nu} = \delta T_{\mu\nu}$

ce
informations
depend only on
 ξ^0, ξ

→ choose coords
such that
two scalars
vanish

→ to calculate,
people choose

a "gauge"
can choose
combinations

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = M_P^{-2} [\bar{T}_{\mu\nu} + \delta T_{\mu\nu}]$$

$$\bar{\rho} = \frac{1}{2} \dot{\phi}_0^2 + V(\phi_0)$$

$$\rho = \frac{1}{2} \dot{\phi}_0^2 - V(\phi_0)$$

Still need $\delta T_{\mu\nu}$

and solve $\delta G_{\mu\nu} = \delta T_{\mu\nu}$