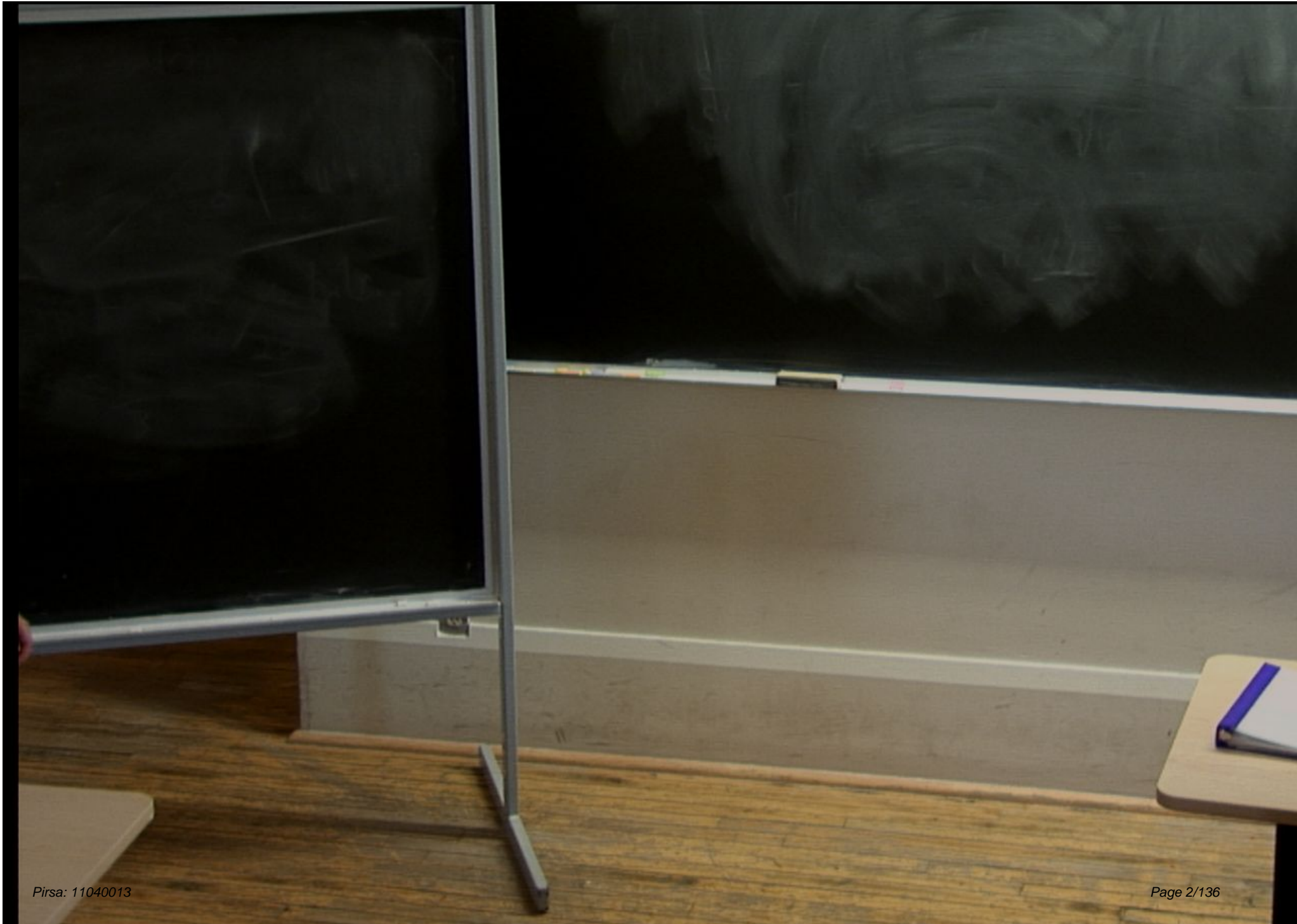


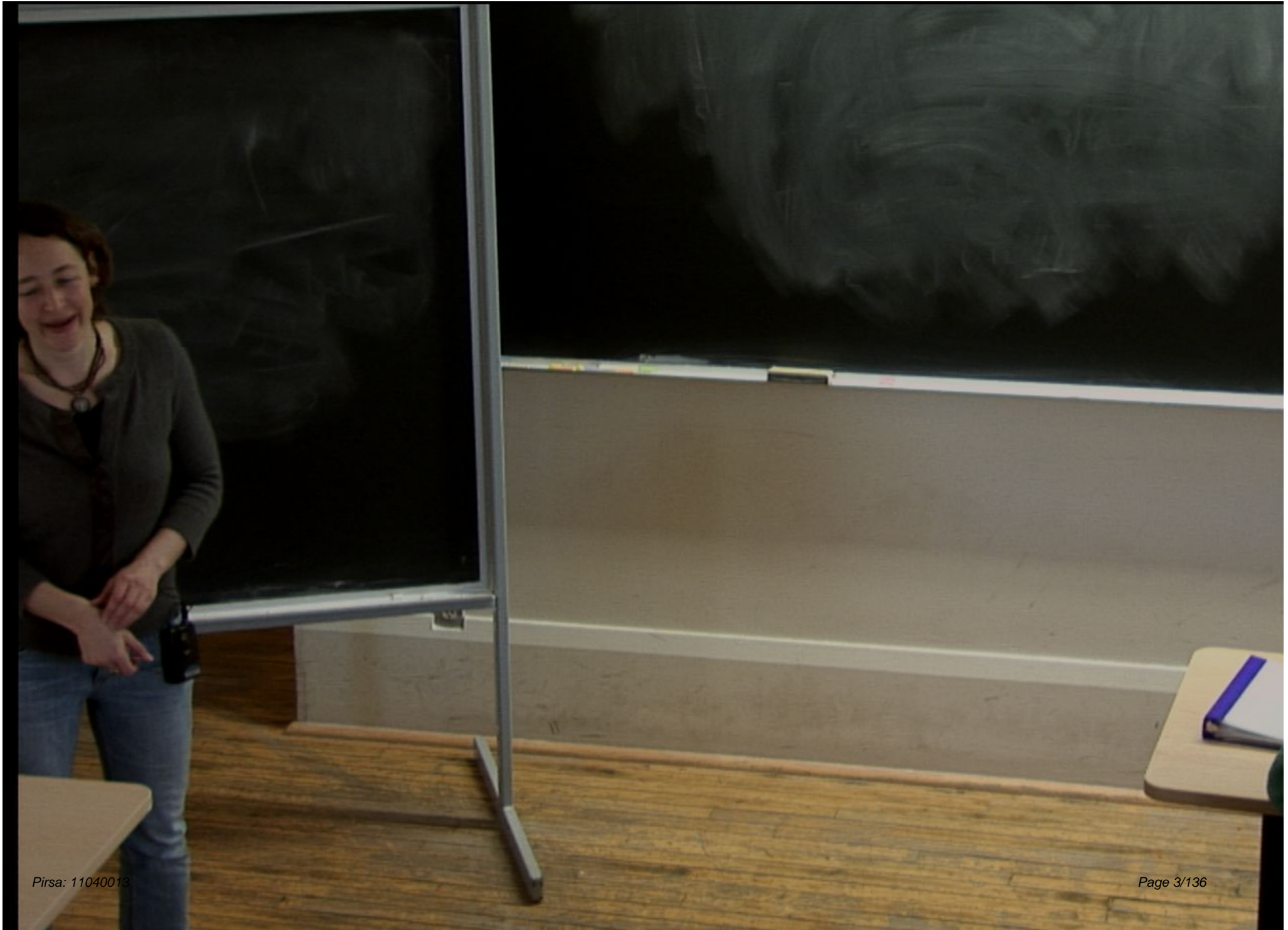
Title: Explorations in Cosmology - Lecture 9

Date: Apr 14, 2011 10:15 AM

URL: <http://pirsa.org/11040013>

Abstract:







Sarah Shandera

## Plan

- finish inflation discussion
- calculate power spectrum from inflation
- beyond  $P(k)$ : non-Gaussianity



Last time

"slow-roll"

Last time

"slow-roll" parameters



Last time

$$V' = \frac{dV}{d\phi}$$

"slow-roll" parameters

$$\textcircled{1} \quad \epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2$$



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"slow-roll" parameters

$$\textcircled{1} \quad \epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 < 1$$

condition for inflation

Last time

$$V' = \frac{\partial V}{\partial \phi}$$

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Last time

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sufficient inflation



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sufficient inflation

$$\frac{m^2}{H^2} \ll 1$$

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sufficient inflation

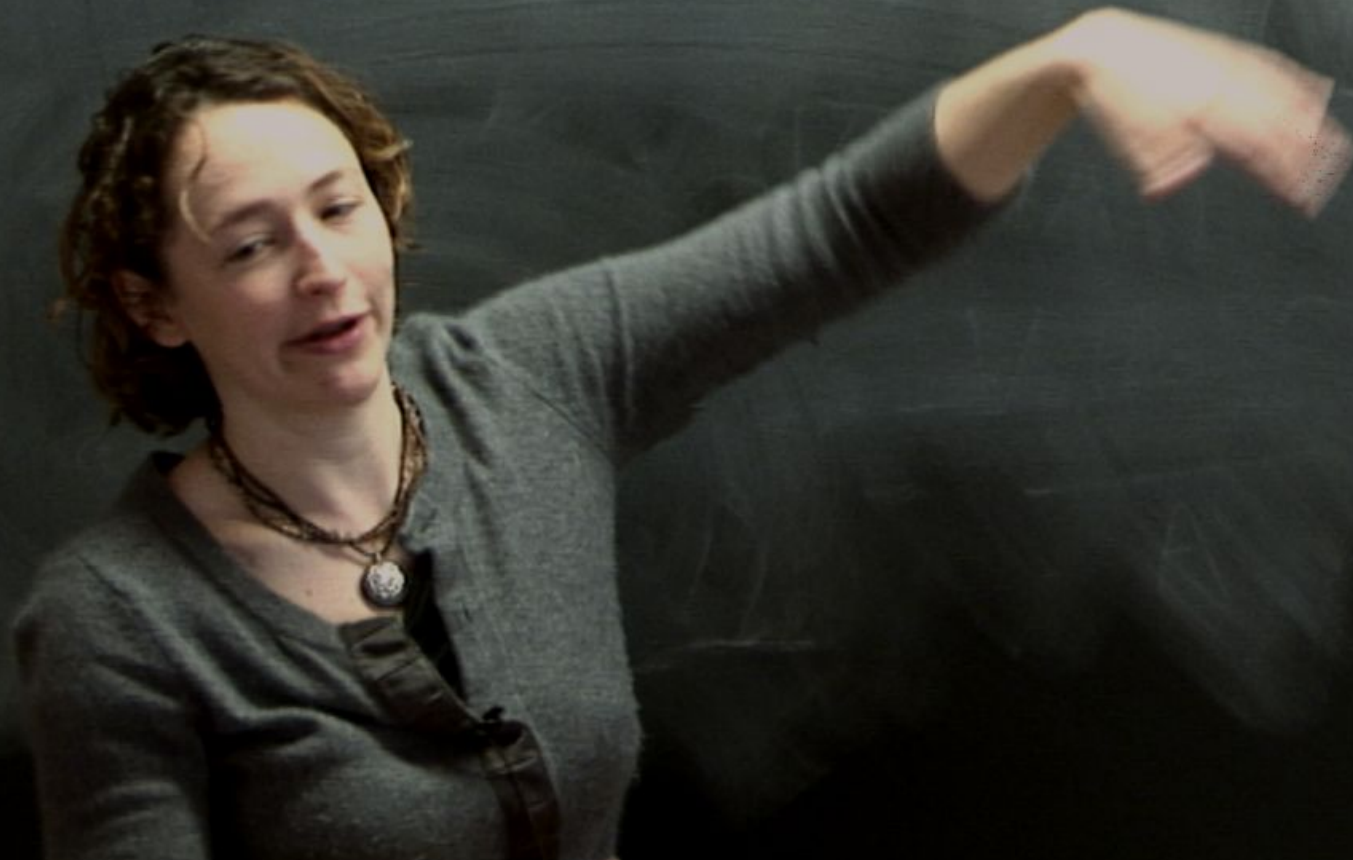
$$\frac{m^2}{H^2} \ll 1$$



Enough inflation  $\Rightarrow$  enough e-folds



Enough inflation  $\Rightarrow$  enough e-folds



Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

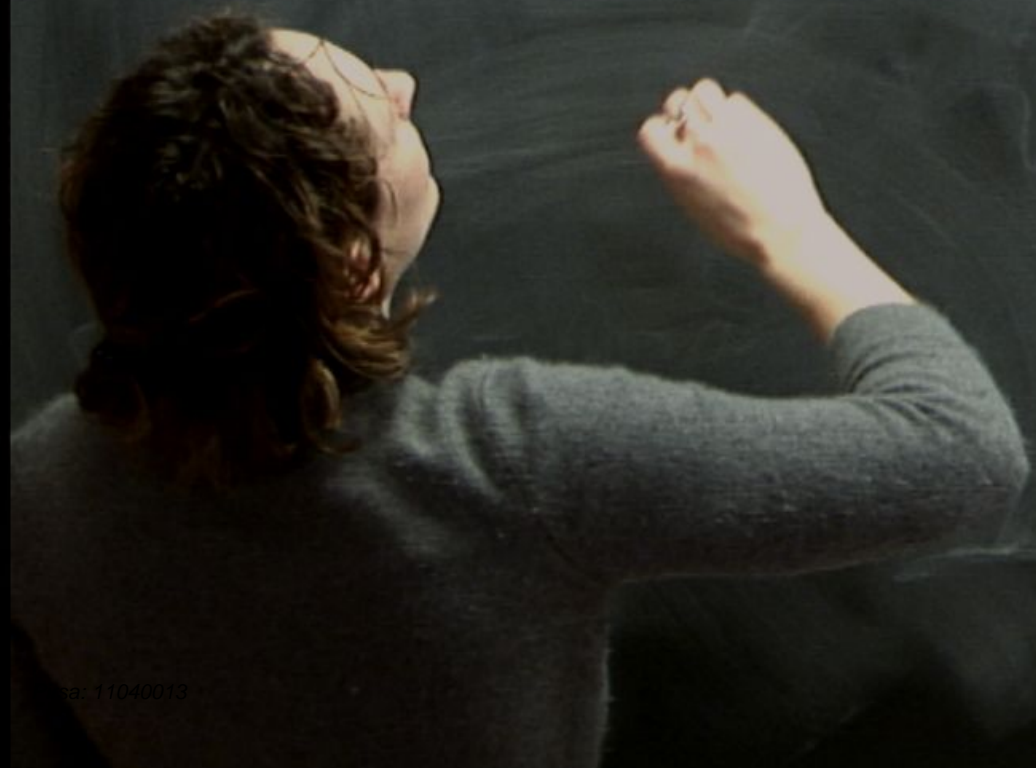
Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

$N_e =$



Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

$$N_e = \int d \ln a = \int \frac{da}{a}$$



Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

$$N_e = \int da \ln a = \int \frac{da}{a} = \int H dt$$





Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

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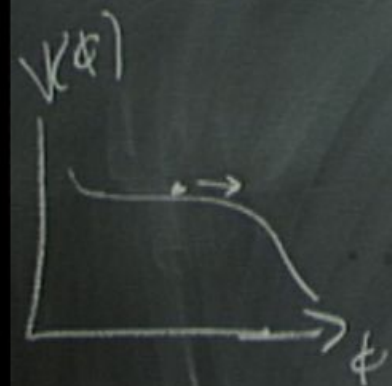
$$= \int H dt$$

Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

$$N_e = \int d \ln a = \int \frac{da}{a} = \int H dt$$

$$= \int \frac{H}{\dot{\phi}} d\phi$$

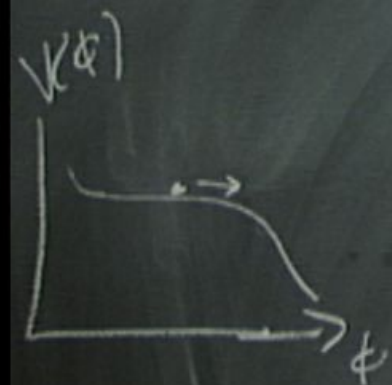




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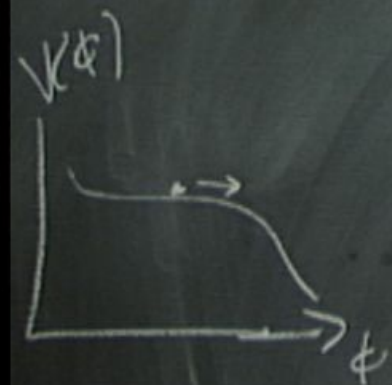


Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

$$N_e = \int da \ln a = \int \frac{da}{a} = \int H dt$$

$$= \int_{\phi_{\text{start}}}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi = \int H$$



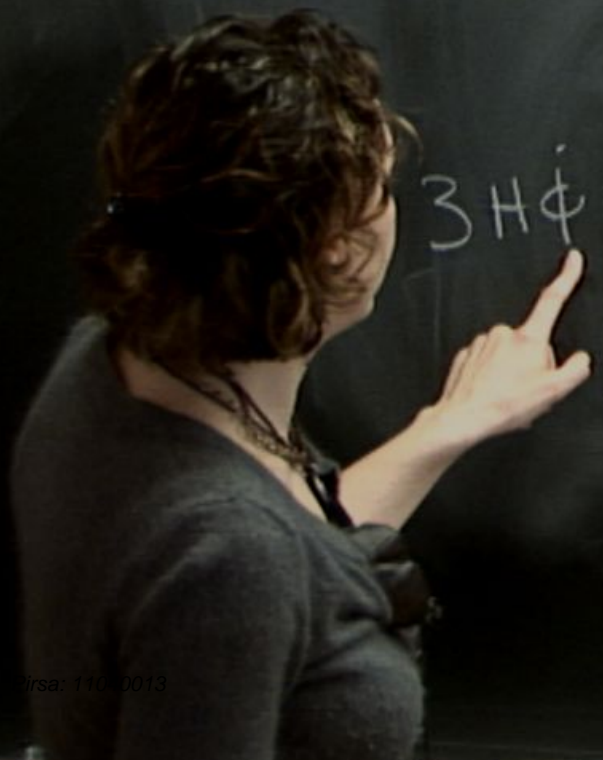


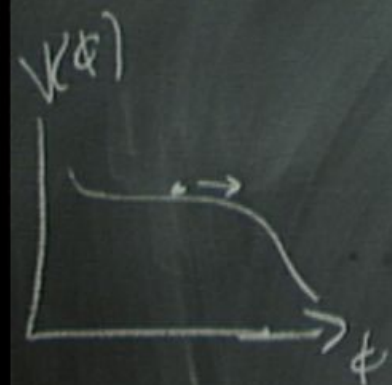
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$$3H\dot{\phi} \approx -V'(\phi)$$





Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

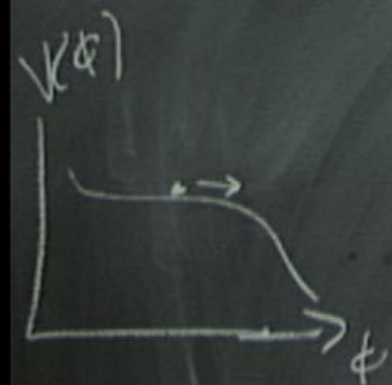
$$N_e = \int da \ln a = \int \frac{da}{a} = \int H dt$$

$$= \int_{\phi_{\text{start}}}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi = - \int \frac{3H^2}{V'} d\phi$$

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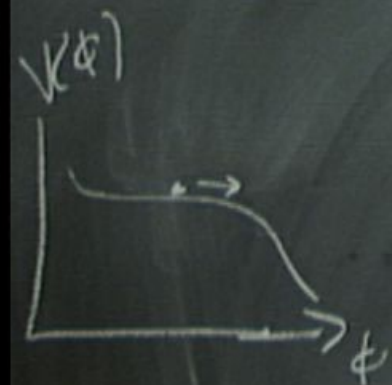
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$$= - \int \frac{1}{M_{\text{P}}^2} \frac{V}{V'} d\phi$$





Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

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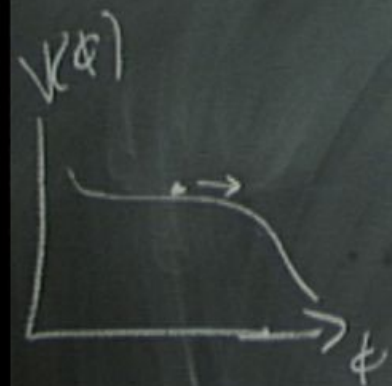
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$$= - \frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}}$$





Enough inflation  $\Rightarrow$  enough e-folds,  $N_e$

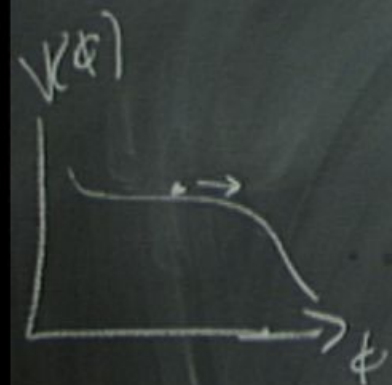
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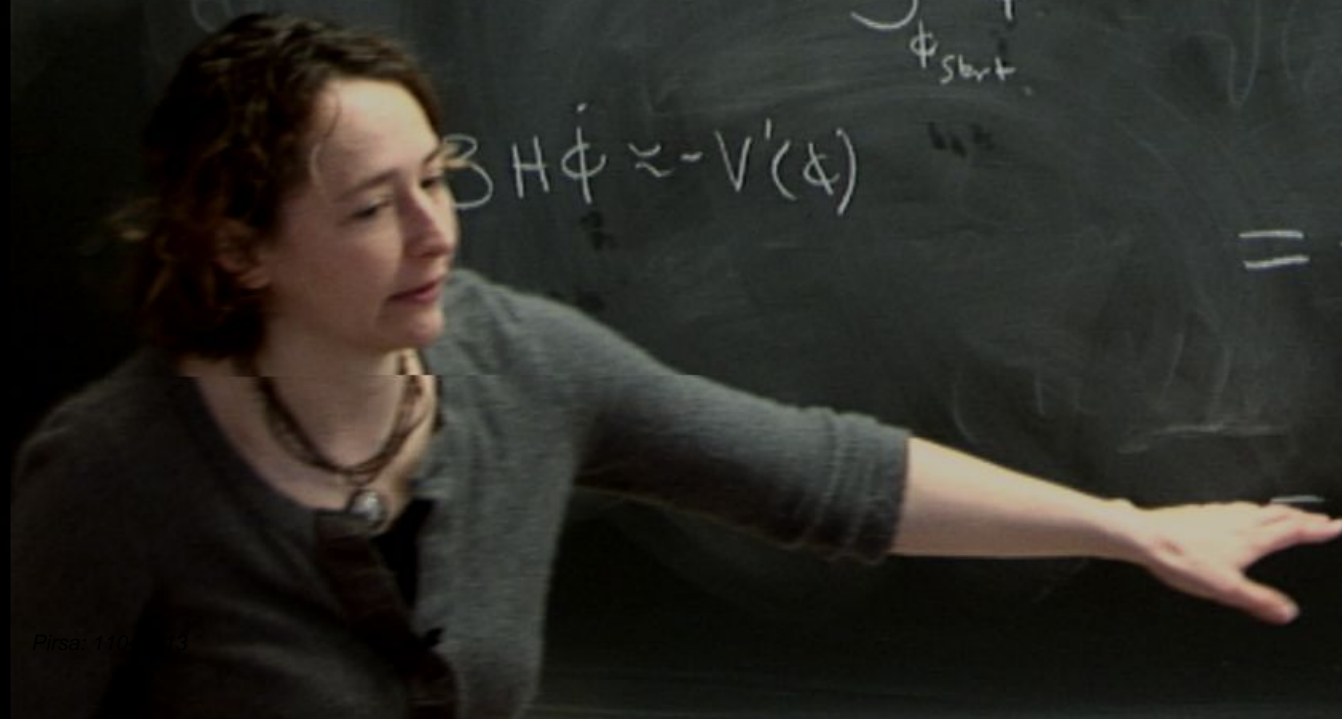
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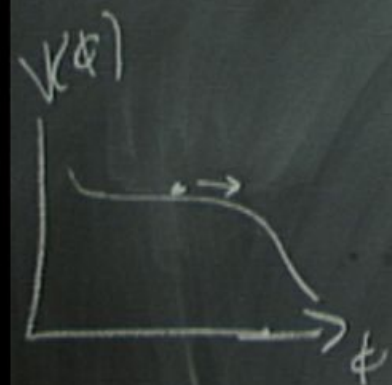
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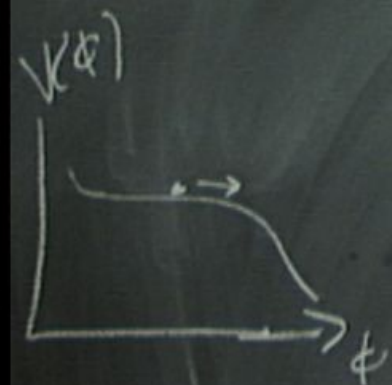
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3

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$$3M_p^2 H^2 = V(\phi)$$

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Example

$$V(\phi) = \frac{\lambda \phi^n}{n!}$$

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$$c = \frac{M_{pl}^2}{2}$$

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$$V(\phi) = \frac{\lambda}{n!} \phi^n$$

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Inflation ends when  $\epsilon = 1$



Example

$$V(\phi) = \frac{\lambda \phi^n}{n!}$$

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$$\phi^2 \approx$$

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Inflation ends when  $\epsilon = 1$

$$\phi^2 \approx \Theta(M_p^2)$$



## Example

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$$\Delta\phi \approx$$

$$\eta = M$$

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$$\epsilon =$$

Inflation ends when  $\epsilon = 1$

$$\phi^2 \approx \Theta(M_p^2)$$

$$\Delta\phi \approx M_p$$

"large field"



Example

$$V(\phi) = \frac{\lambda \phi^n}{n!}$$

$$V'(\phi) = \frac{n \lambda \phi^{n-1}}{n!}, \quad V''(\phi) = \frac{n(n-1) \lambda \phi^{n-2}}{n!}$$

$$\epsilon = \frac{M_p^2}{2} \left( \frac{n}{\phi} \right)^2$$

$$\eta = M_p^2 \left( \frac{n(n-1)}{\phi^2} \right)$$

$\Gamma$  ends when  $\epsilon = 1$

$$\approx \Theta(M_p^2)$$

field  $V = V_0 \left( 1 + \frac{\phi^2}{M_p^2} \right)$

## Example

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Inflation ends when  $\epsilon = 1$

$$\phi^2 \approx \mathcal{O}(M_p^2)$$

$$\phi - \phi_c = \Delta\phi \approx M_p$$

"large field"

$$V = V_0 \left( 1 + \frac{\phi^2}{M_p^2} \right)$$



## Example

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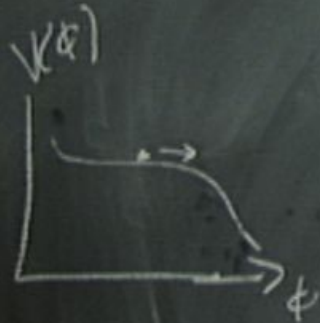
Inflation ends when  $\epsilon = 1$

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$$V = V_0 \left( 1 + \frac{\phi^2}{M_p^2} \right)$$

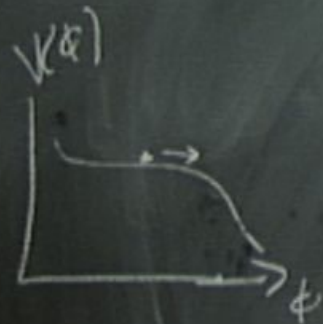


$$N_e =$$

$$= - \int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

$$= - \frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi$$

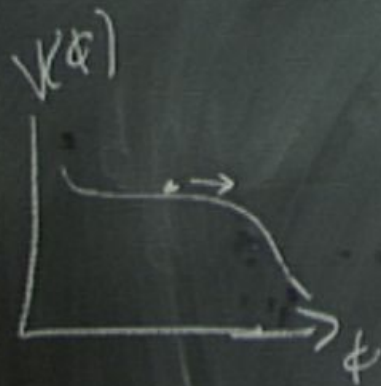




$$N_e = -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi =$$

$$= -\int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

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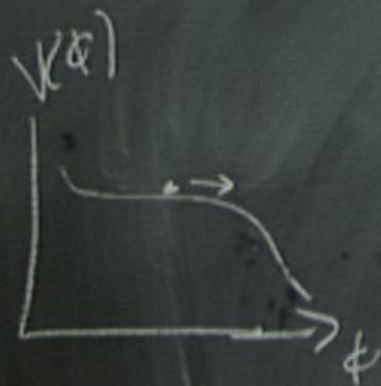


$$N_e = -\frac{1}{M_{\text{pl}}} \int \frac{1}{\sqrt{2\epsilon}} d\phi = -\frac{1}{M_{\text{pl}}^2} \int \phi d\phi$$

$d\phi$

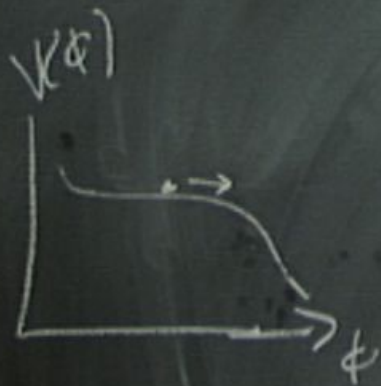
$d\phi$





$$N_e = -\frac{1}{m_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi = -\frac{1}{m_p^2} \int \phi d\phi$$

$$\int \frac{1}{\sqrt{2\epsilon}} d\phi$$



$$N_e = -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi \quad \rightarrow \quad -\frac{1}{M_p^2} \int \phi d\phi$$

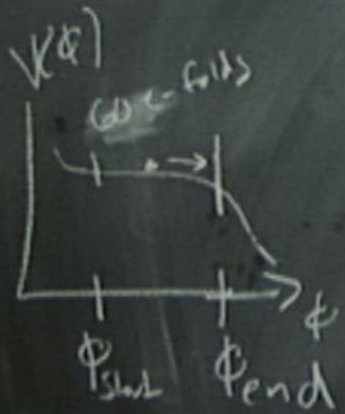
$$= - \int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

$$= -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi$$



$$N_e = -\frac{1}{M_P} \int \frac{1}{\sqrt{2\epsilon}} d\phi \quad \text{or} \quad -\frac{1}{M_P^2} \int \phi d\phi = \frac{\phi_s^2 - \phi_{in}^2}{M_P}$$

$$= -\int \frac{1}{M_P} \frac{V}{V'} d\phi$$

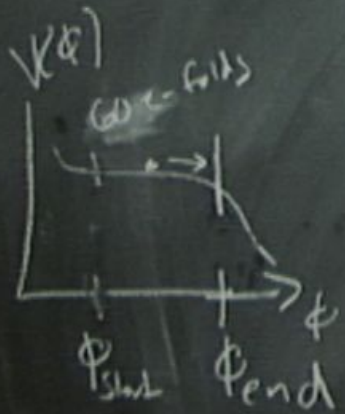


$$N_e = -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi = -\frac{1}{M_p^2} \int \phi d\phi = \frac{\phi_s^2 - \phi_{end}^2}{M_p}$$

$$= - \int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

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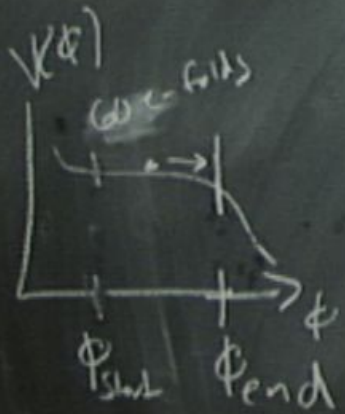


$$N_e = -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi \approx -\frac{1}{M_p^2} \int \phi d\phi = \frac{\phi_s^2 - \phi_{end}^2}{M_p}$$

$$N_e \approx 60$$

$$= - \int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

$$= - \frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi$$



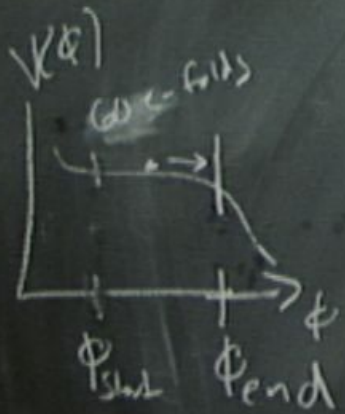
$$N_e = -\frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi \approx -\frac{1}{M_p^2} \int \phi d\phi = \frac{\phi_s^2 - \phi_{end}^2}{2M_p^2}$$

$N_e \approx 60$  ; sol

$$= - \int \frac{1}{M_p^2} \frac{V}{V'} d\phi$$

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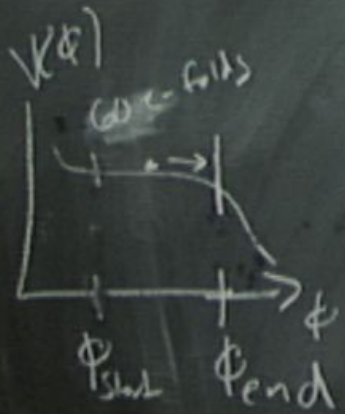


$$N_e = \frac{1}{M_p} \int \frac{1}{\sqrt{2\epsilon}} d\phi \approx -\frac{1}{M_p^2} \int \phi d\phi = \frac{\phi_{\text{start}}^2 - \phi_{\text{end}}^2}{2M_p^2}$$

$N_e \approx 60$  : solve horizon, flatness problems

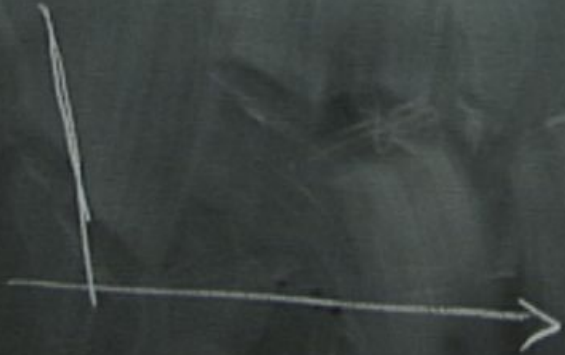
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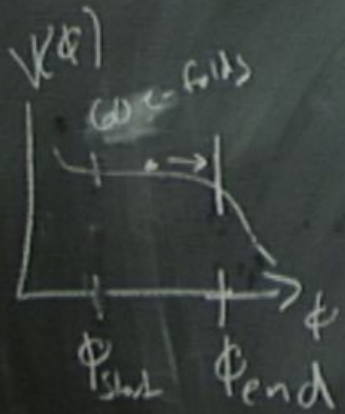


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$N_e \approx 60$  : solve horizon, flatness problems



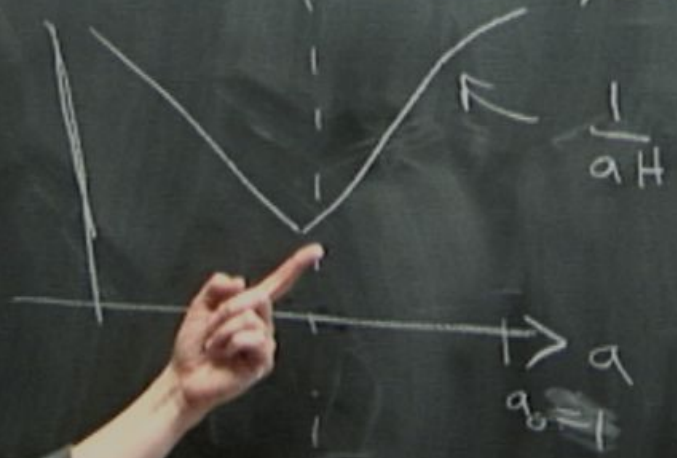


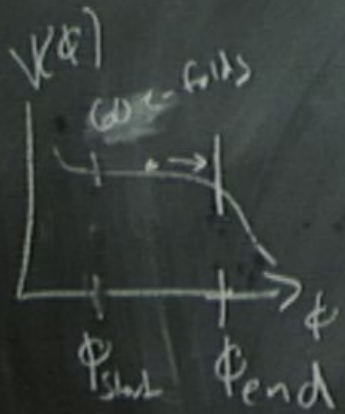


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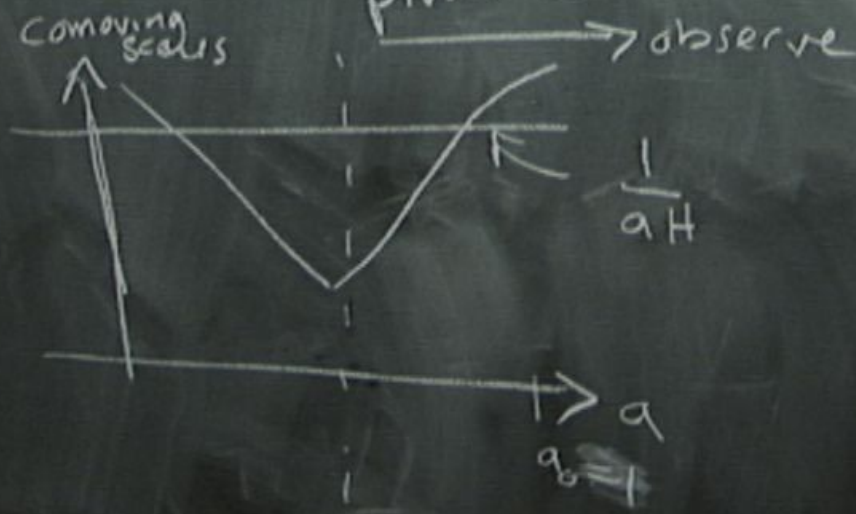
→ observe



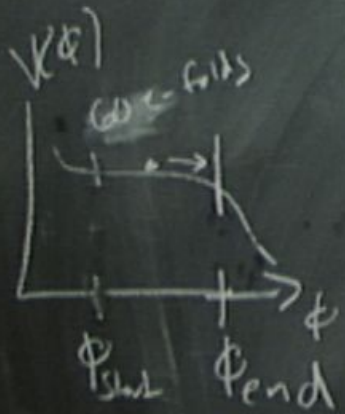


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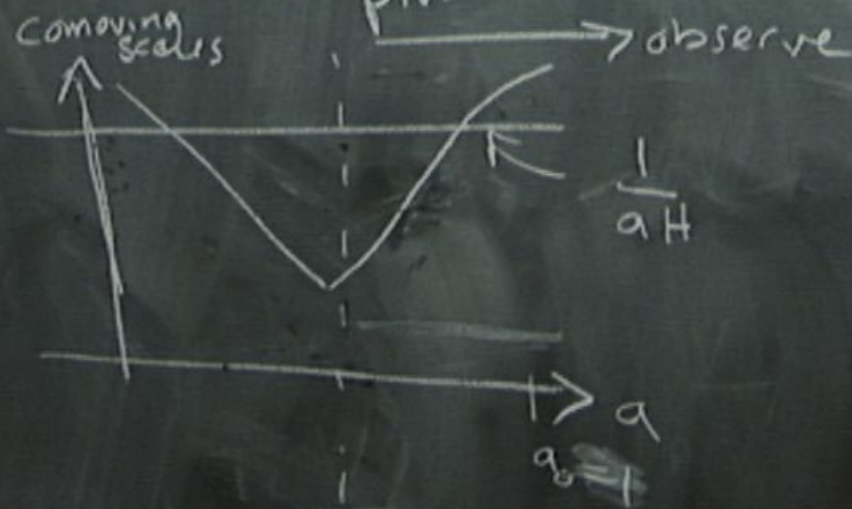


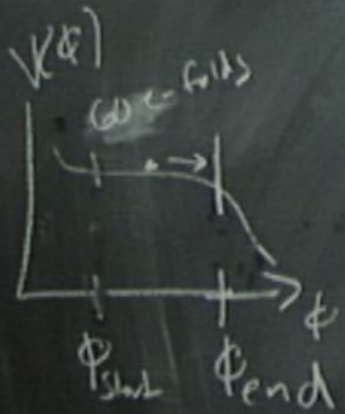




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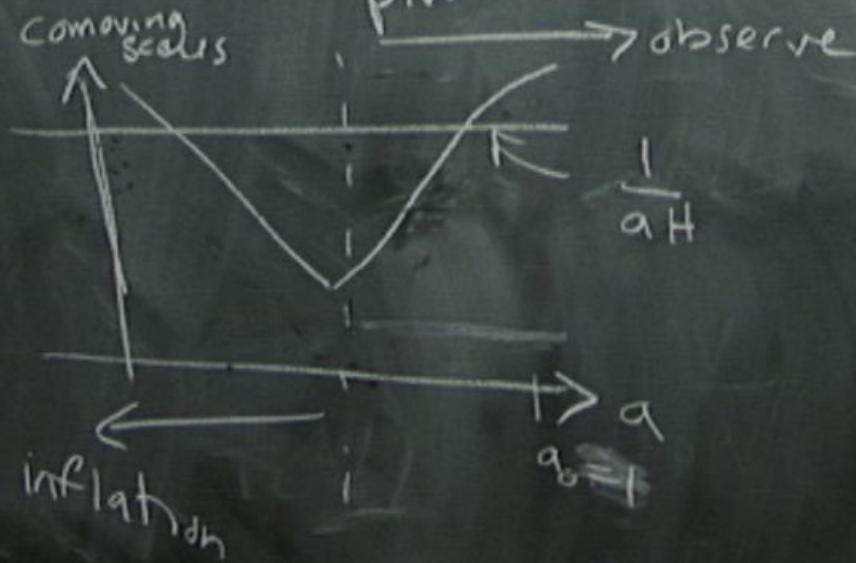
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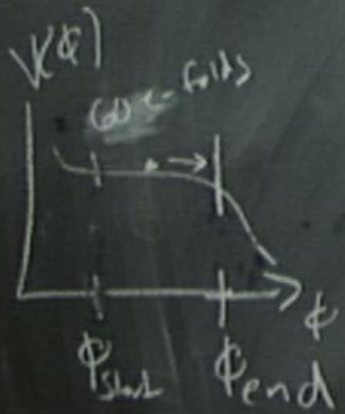


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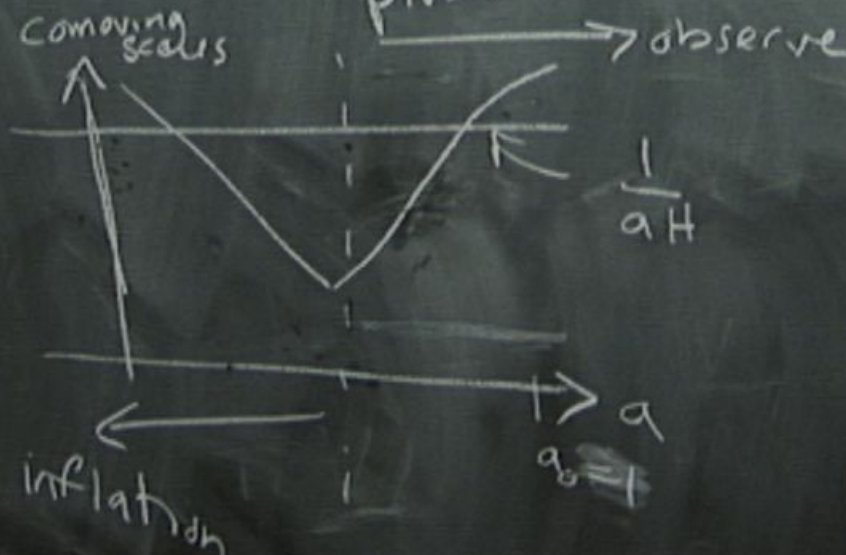






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$N_e \approx 60$  : solve horizon, flatness problems



approx. how many e-folds?  
assume  $t$  since in



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$$H \sim a^{-2}$$



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end of  
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$$\underline{a_0 H_0}$$



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$$T \sim \frac{1}{a}$$

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$$\frac{a_0 H_0}{a_e H_e} = \frac{a_e}{a_0} = \frac{T_0}{T_e} = 10^{-4} \text{ eV}$$



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$$\frac{a_0 H_0}{a_c H_c} = \frac{a_c}{a_0} = \frac{T_0}{T_c} = \frac{10^{-4} \text{ eV}}{10^{15} \text{ GeV}}$$



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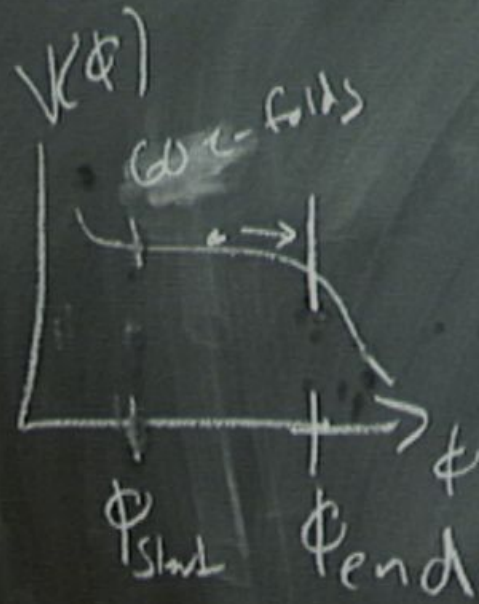
$$T \sim \frac{1}{a}$$

end of  
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$$\frac{H_0}{H_e} = \frac{a_e^2}{a_0^2}$$

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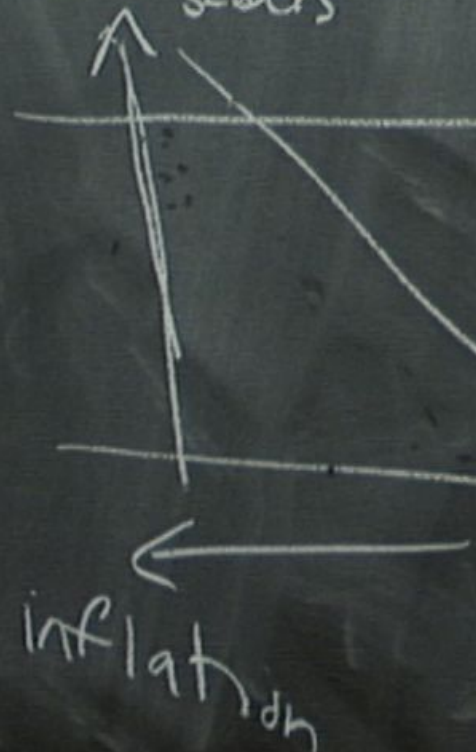
( $10^{15} \cdot 10^9 \text{ eV}$ )



$$N_e = \frac{1}{M_p}$$

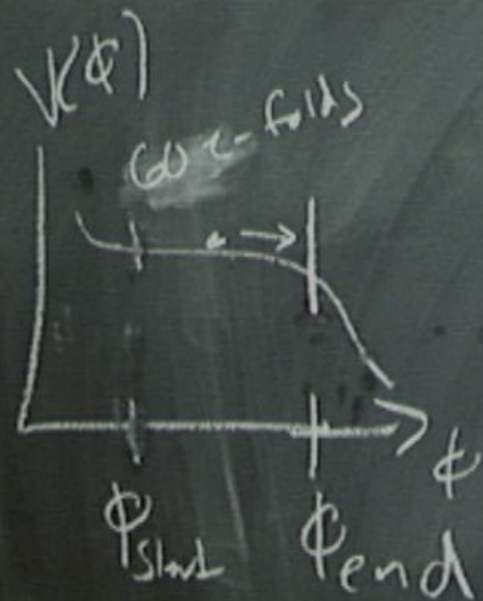
$$N_e \approx 60 :$$

comoving scales



$$3M_p^2 H^2 = V(\phi) = m^2 \phi^2$$

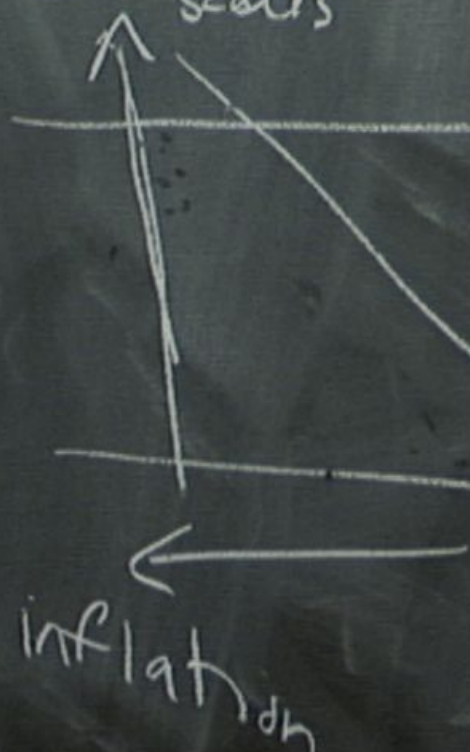




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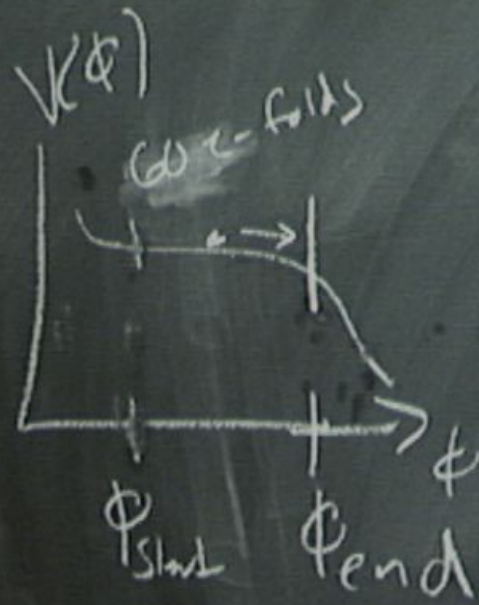
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↑ fixed by measuring fluctuations



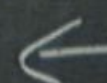
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$$\frac{H_0}{H_{\text{pl}}^{\text{today}}} = \frac{a_c}{a_0} = \frac{T_0}{T_e} = \frac{10^{-4} \text{ eV}}{10^{15} \text{ GeV}} = 10^{-28}$$

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$(10^{15} \cdot 10^9 \text{ eV})$

st time

low-roll" parameters

$$\equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 < 1$$

$$\equiv M_p^2 \frac{V''}{V}$$

inflation

$H \sim \text{constant}$



slow-roll time

slow-roll parameters

$$\epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 < 1$$

$$\eta \equiv M_p^2 \frac{V''}{V} \ll 1$$

inflation

$H \sim \text{constant}$   
 $a \sim e^{Ht}$

$M_p$

st time

low-roll" parameters

$$\equiv \frac{M_p^2}{2} \left( \frac{\delta \phi}{M_p} \right)^2$$

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$$N_e = \ln a$$



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$$\frac{a_{\text{start}}}{a_{\text{end}}} =$$



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$$a \sim e^{Ht}$$

$$N_e = \ln a_s/a_e$$

$$\frac{a_{\text{start}}}{a_{\text{end}}} = 10^{28} = \left( \frac{a_e H t_e}{a_o H t_o} \right)$$

$$N_e = \ln(10^{28})$$

$$\approx 64$$

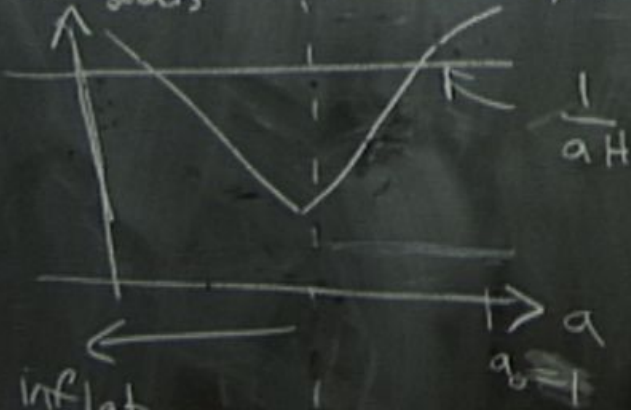


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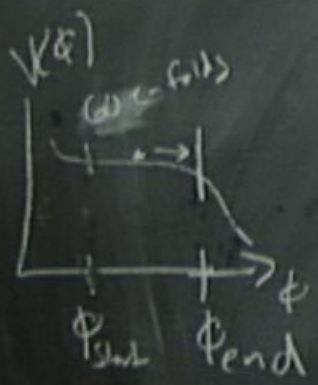
comoving scales

observe



$\phi$   
 $\approx \phi^2$

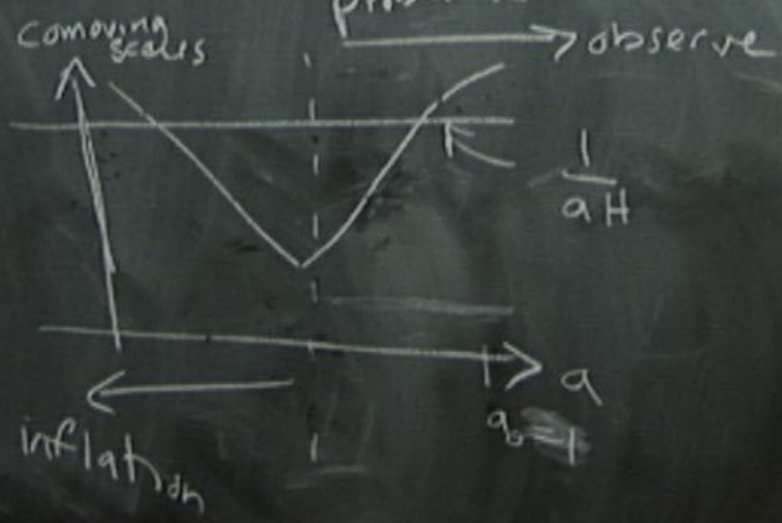
fixed  
by measuring  
fluctuations



$$V(\phi) = \frac{1}{n!} \phi^n$$

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$$3M_p^2 H^2 = V(\phi) = m^2 \phi^2$$

↑ fixed by measuring fluctuations



$$\phi_{\text{end}} \approx n M_p$$

$$\frac{\Delta(\phi^2)}{M_p^2} \sim 64$$

---

upshot

---

$$\Delta\phi \sim M_p$$

$$\phi_s > \phi_{\text{end}} > M_p$$

large field models

---

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$$\Delta\phi \sim M_p$$

$$\phi_{\text{start}} > \phi_{\text{end}}$$

large field

$$V(\phi) = m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 + \frac{\lambda_6}{6!} \phi^6$$

+ ...

$$\approx V_0(\phi)$$



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+ ...

$$\approx V_0 \left( 1 + \frac{\phi^2}{M_p^2} + \dots \right)$$

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for large field models  
Symmetries important!

$$\phi \rightarrow \phi + c$$



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So far

① What do we want for  $H(\mathbb{Q})$ ?



So far:

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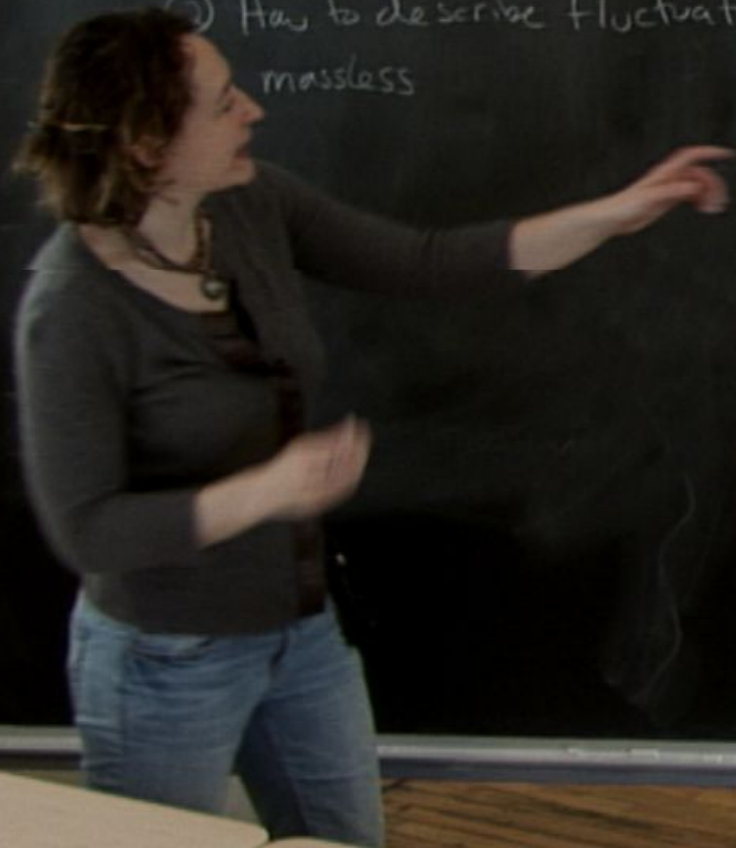
②



So far:

① What do we want for  $H(\phi)$ ? ← Hubble

② How to describe fluctuations of massless



So far:

← Hubble ( $t$ )

- ① What do we want for  $H(\phi)$ ?
- ② How to describe fluctuations of massless scalar field in exact dS?





So far:

← Hubble ( $t$ )

① What do we want for  $H(\phi)$ ?

② How to describe fluctuations of massless scalar field in exact dS?  $H = \text{const.}$

For Let the scalar field source inflation, + fluctuate

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Fun! Let the scalar field source inflation, + fluctuate  
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So far:

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Why?  
 $\frac{\delta\phi}{\delta\phi} \rightarrow \delta \approx -\frac{H}{\dot{\phi}} \delta\phi$

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So far:

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Fun! Let the scalar field source of  
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← Hubble (t)

Why?  $\delta\phi \rightarrow \delta \dot{\phi} \approx -\frac{H}{\phi} \delta\phi$

← log of perturbation



So far:

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② How to describe fluctuations of massless scalar field in exact de Sitter.

Fun! Let the scalar field source inflation  
→ Source of support for

← Hubble (t)

Why?  $\delta\phi \rightarrow \delta \approx -\frac{H}{\dot{\phi}} \delta\phi$  ← grad pot / curvature

$\frac{\delta T}{T}$



Why?

$$\frac{\delta\phi}{\dot{\phi}} \rightarrow \delta \approx -\frac{H}{\dot{\phi}} \delta\phi$$

grad pot / curvature

$$\frac{\delta T}{T}$$

CMB

WMAP satellite  
Planck satellite

= const.

at

ple (t)

Why?

grav pot / curvature

$$\frac{\delta\phi}{\dot{\phi}} \rightarrow \delta \approx -\frac{H}{\dot{\phi}} \delta\phi$$

$$\frac{\delta T}{T}$$

CMB

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Satellite

Planck satellite (2013)

H = const.

fluctuate



ple (t)

Why?

gal pot/cumtore

$$\frac{\delta\phi}{\dot{\phi}} \rightarrow \delta \approx -\frac{H}{\dot{\phi}} \delta\phi$$

ds const.

$$\frac{\delta T}{T}$$

$$\left(\frac{\delta\rho}{\rho}\right)$$

CDM

CMB

WMAP

Satellite

Planck

Satellite

Structure Surveys

(2013)



ple (t)

$dS$  ?  $H = \text{const.}$

ion, + fluctuate  
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CDM

CMB

WMAP

Satellite

Planck

Satellite

Structure  
Surveys

(2013)



What to compute? Statistics!

let mean  $\bar{0}$

(1) amplitude of fluctuations & power spectrum

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let mean 0

(1) amplitude of fluctuations : power spectrum

$$\text{say } S \equiv \left( \frac{\delta p}{\rho} \right)_{\text{com}}$$

$$\langle S(\omega) \rangle$$



What to compute? Statistics!

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(1) amplitude of fluctuations : power spectrum

$$\text{say } \delta \equiv \left( \frac{\delta \rho}{\rho} \right)_{\text{com}}$$

$$\langle \delta(\vec{x}) \delta(\vec{y}) \rangle$$

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(1) amplitude of fluctuations & power spectrum

say  $S \equiv \left( \frac{\delta \rho}{\rho} \right)_{\text{com}}$   
real space correlation fcn

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What to compute? Statistics!

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$$\delta(x) = \int \frac{d^3k}{(2\pi)^3} \delta_{\vec{k}} e^{i\vec{k} \cdot \vec{x}}$$

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What to compute? Statistics!

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space correlation fcn

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What to compute? Statistics!

let mean 0

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$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle$$



What to compute? Statistics!

let mean 0

(1) amplitude of fluctuations: power spectrum

say  $S \equiv \left( \frac{\delta \rho}{\rho} \right)_{\text{com}}$   
real space correlation fcn

$$\delta(x) = \int \frac{d^3 k}{(2\pi)^3} \delta_{\vec{k}} e^{i\vec{k} \cdot \vec{x}}$$

$$\langle \delta(\vec{x}) \delta(\vec{y}) \rangle = \left\langle \int \frac{d^3 k_1}{(2\pi)^3} \delta_{\vec{k}_1} e^{i\vec{k}_1 \cdot \vec{x}} \int \frac{d^3 k_2}{(2\pi)^3} \delta_{\vec{k}_2} e^{i\vec{k}_2 \cdot \vec{y}} \right\rangle$$

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle = (2\pi)^3 \delta^3$$

What to compute? Statistics!

let mean 0

(1) amplitude of fluctuations: power spectrum

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$$\langle \delta(\vec{x}) \delta(\vec{y}) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{k^3} P(k)$$



What to compute? Statistics!

let mean 0

(1) amplitude of fluctuations: power spectrum

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real space correlation fcn

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$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{P(k)}{k^3}}_{P(k)}$$

What to compute? Statistics!

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(1) amplitude of fluctuations: power spectrum

say  $\delta \equiv \left(\frac{\delta \rho}{\rho}\right)_{\text{com}}$   
real space correlation fcn

$$\delta(x) = \int \frac{d^3k}{(2\pi)^3} \delta_{\vec{k}}$$

$$\underbrace{\xi(|\vec{x} - \vec{y}|)}_r = \langle \delta(\vec{x}) \delta(\vec{y}) \rangle = \left\langle \int \frac{d^3k_1}{(2\pi)^3} \delta_{\vec{k}_1} e^{i\vec{k}_1 \cdot \vec{x}} \int \frac{d^3k_2}{(2\pi)^3} \delta_{\vec{k}_2} e^{i\vec{k}_2 \cdot \vec{y}} \right\rangle$$

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) \underbrace{\frac{P(k)}{k^3}}_{P(k)}$$



$$\xi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{ik \cdot r}$$

simplify using spherical coord. ( $\hat{k} = \hat{e}$ )

$$= \int \underline{dk}$$



$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

simplify using spherical coord. ( $\hat{k} = \hat{r}$ )

$$= \int \frac{dk}{k}$$

$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\vec{k}\cdot\vec{r}}$$

simplify using spherical coord. ( $\hat{k} = \hat{z}$ )

$$\frac{dk}{k} \quad \frac{4\pi}{(2\pi)^3} \quad \varphi(k) \quad \frac{\sin(kr)}{kr}$$



$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

simplify using spherical coord. ( $\hat{k} = \hat{r}$ )

$$= \int \frac{dk}{k} \frac{4\pi}{(2\pi)^3} \varphi(k) \frac{\sin(kr)}{kr}$$

$$\rightarrow \int \frac{dk}{k} \underbrace{\frac{1}{2\pi^2}} \varphi(k)$$

$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\vec{k}\cdot\vec{r}}$$

simplify using spherical coord. ( $\hat{k} = \hat{e}$ )

$$= \int \frac{dk}{k} \frac{4\pi}{(2\pi)^3} \varphi(k) \frac{\sin(kr)}{kr}$$

$$\xrightarrow{r \rightarrow 0} \int \frac{dk}{k} \underbrace{\frac{1}{2\pi^2}}_{\Delta^2(k)} \varphi(k)$$



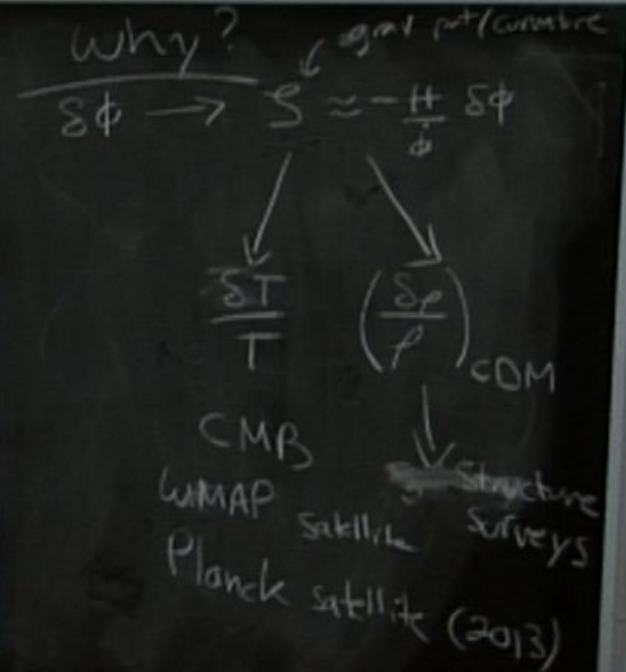
$$\rightarrow \xi(r) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^3} \varphi(k) e^{i\vec{k}\cdot\vec{r}}$$

simplify using spherical coord. ( $\hat{k} = \hat{z}$ )

$$= \int \frac{dk}{k} \frac{4\pi}{(2\pi)^3} \varphi(k) \frac{\sin(kr)}{kr}$$

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Dimensionless power spectrum  
parametrized





Dimensionless power spectrum  
parametrized

$$\Delta_{\delta}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s-1}$$

↑ amplitude

Why?

$$\delta\phi \rightarrow S \approx -\frac{H}{\dot{\phi}} \delta\phi$$

$$\frac{\delta T}{T}$$

$$\left( \frac{\delta \rho}{\rho} \right)_{\text{COM}}$$

CMB  
WMAP satellite  
Planck satellite (2013)

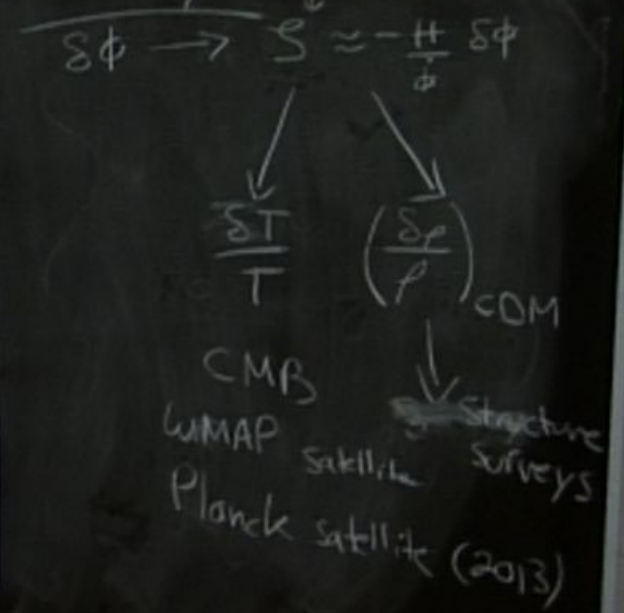
Structure  
Surveys

Dimensionless power spectrum  
parametrized

$$\Delta_{\mathcal{S}}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude      ← spectral index  
↑ pivot point

Why? ← gal pot / curvature





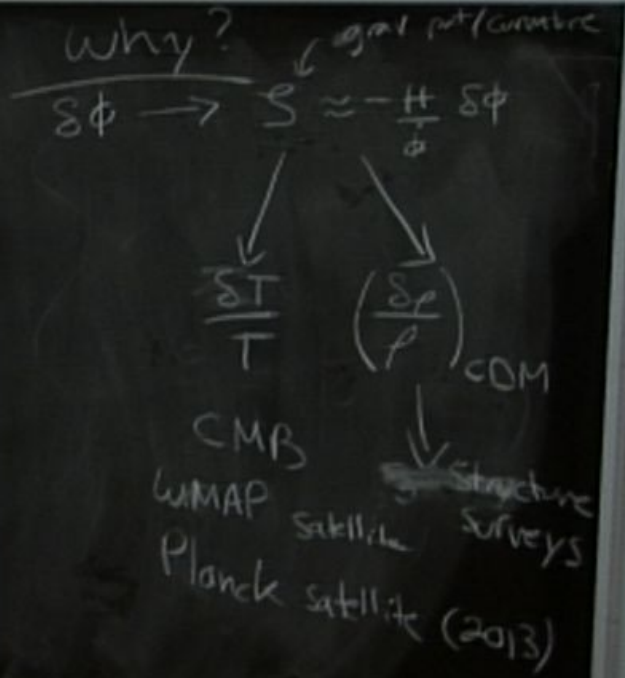
Dimensionless power spectrum  
parametrized

$$\Delta_{\delta}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude
← spectral index  
↑ pivot point

WMAP;  $\downarrow 10^{-9}$

Why?



Dimensionless power spectrum  
parametrized

$$\Delta_{\delta}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude      ← spectral index  
↑ pivot point

WMAP;  $\downarrow 10^{-9}$

$\downarrow 0.04$

Why?

$$\delta\phi \rightarrow S \approx -\frac{H}{\dot{\phi}} \delta\phi$$

$$\frac{\delta T}{T}$$

$$\left( \frac{\delta \rho}{\rho} \right)_{\text{CDM}}$$

CMB  
WMAP satellite  
Planck satellite (2013)

Structure  
Surveys



Dimensionless power spectrum  
parametrized

$$\Delta_{\mathcal{P}}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude      ↑ pivot point      ← spectral index

WMAP;  $\downarrow 10^{-5}$

$\downarrow 0.04$

Goal match theory  
to observation  
↳ compute  $A_0, n_s - 1$

Dimensionless power spectrum  
parametrized

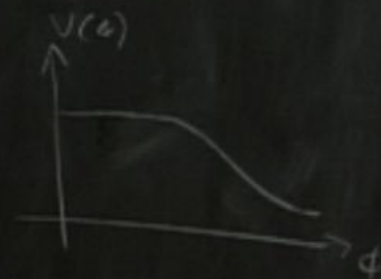
$$\Delta_{\omega}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude      ← pivot point

← spectral index

WMAP;  $\downarrow 10^{-9}$

Goal match theory  
to observation  
↳ compute  $A_0, n_s - 1$





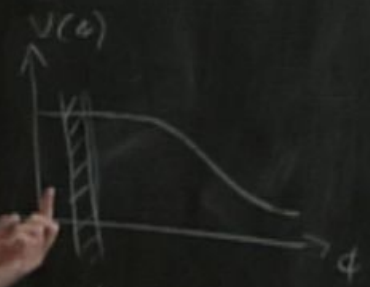
Dimensionless power spectrum  
parametrized

$$\Delta_{\ell}^2 = A_0 \left( \frac{\ell}{\ell_0} \right)^{n_s - 1}$$

↑ amplitude      ↑ pivot point      ← spectral index

WMAP;  $\downarrow 10^{-5}$

Goal match theory  
to observation  
↳ compute  $A_0, n_s - 1$



Dimensionless power spectrum  
parametrized

$$\Delta_{\mathcal{L}}^2 = A_0 \left( \frac{k}{k_0} \right)^{n_s - 1}$$

↑ amplitude      ↑ pivot point      ← spectral index

WMAP:  $\downarrow 10^{-5}$        $\downarrow 0.00$

Goal match theory  
to observation  
↳ compute  $A_0, n_s - 1$

