Title: Explorations in Cosmology - Lecture 8

Date: Apr 13, 2011 11:30 AM

URL: http://pirsa.org/11040012

Abstract:

Inflation

Inflation

Recap

Inflation

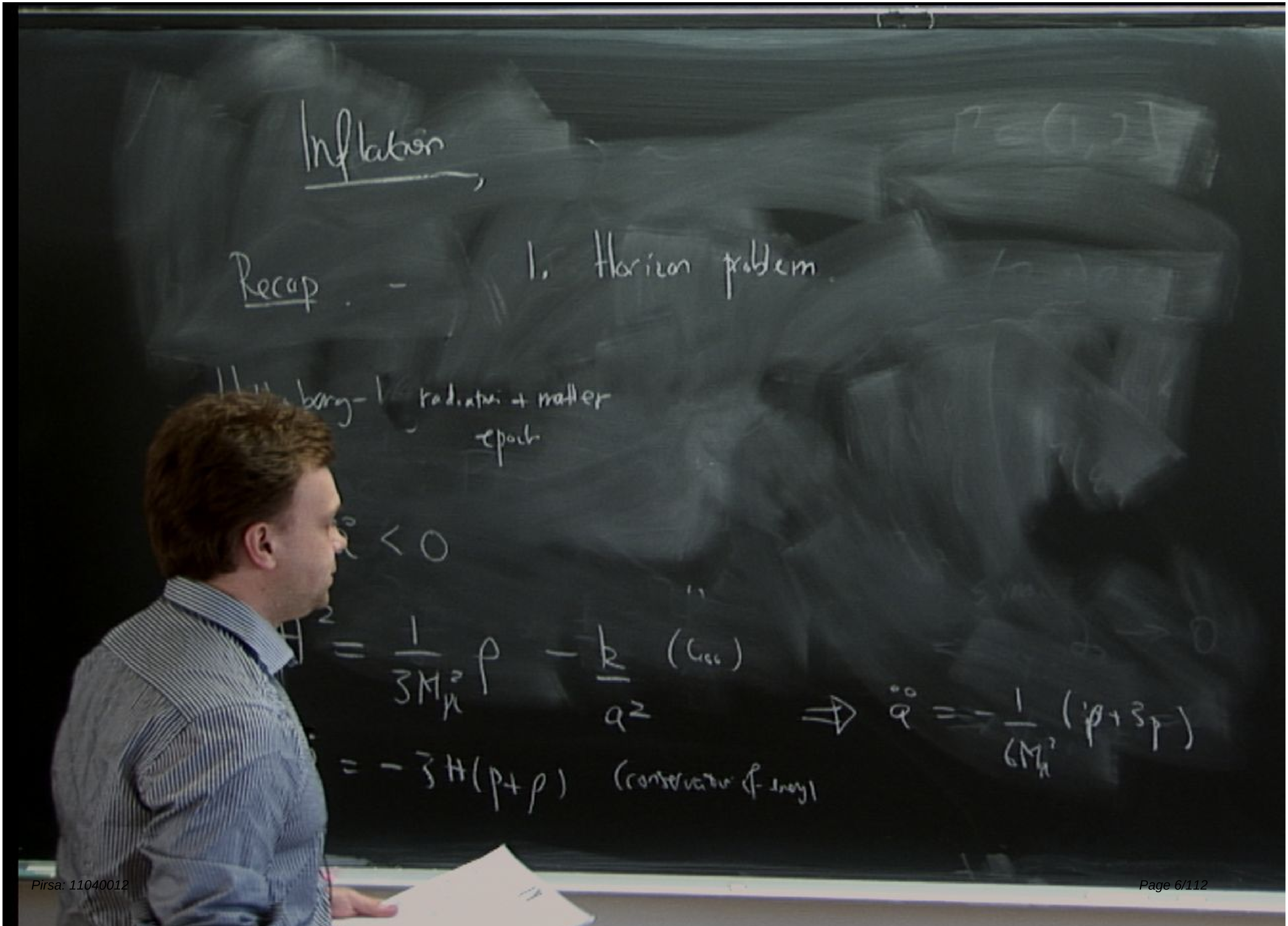
Recap. –   1. Horizon problem.

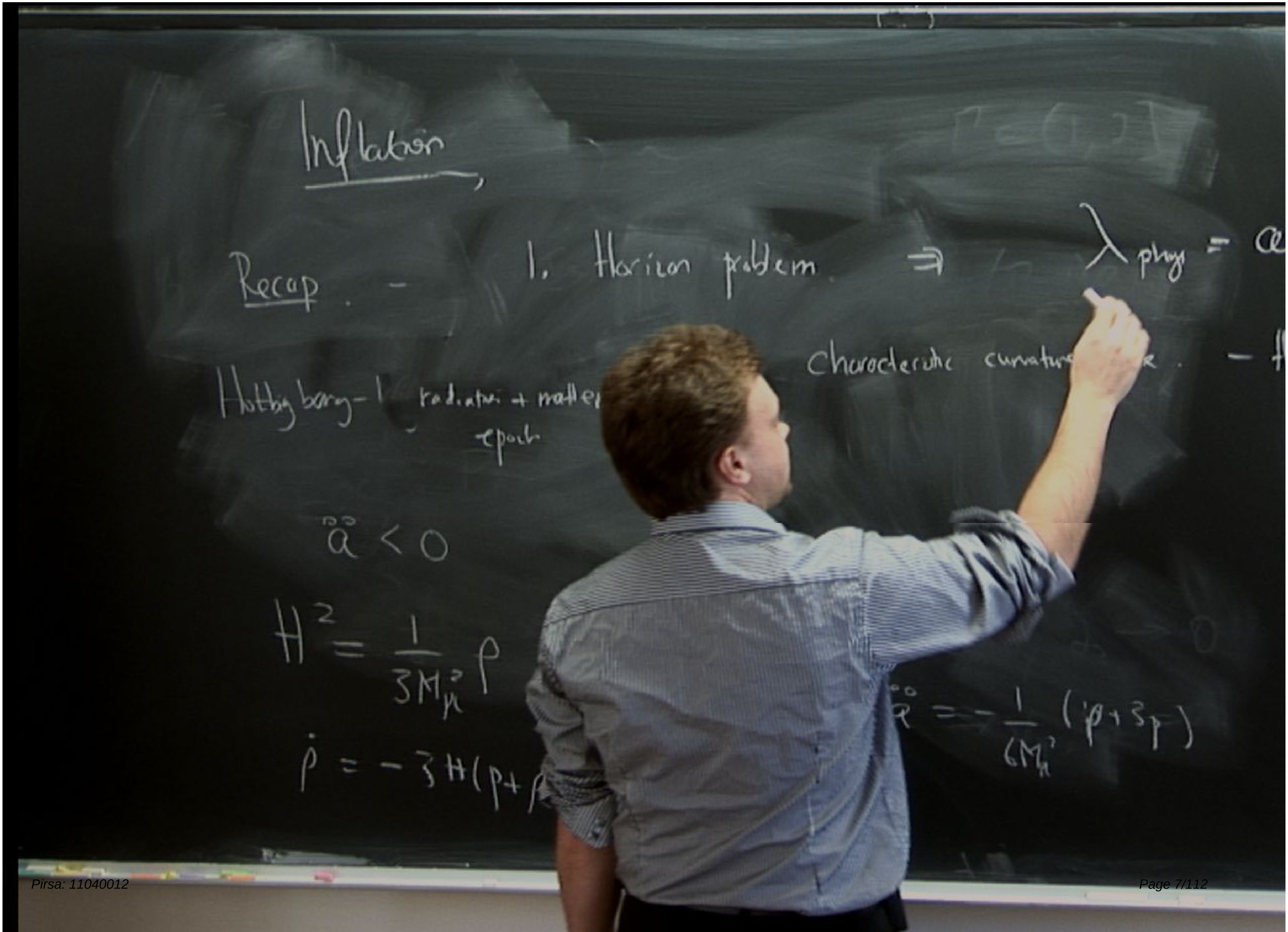
Inflation

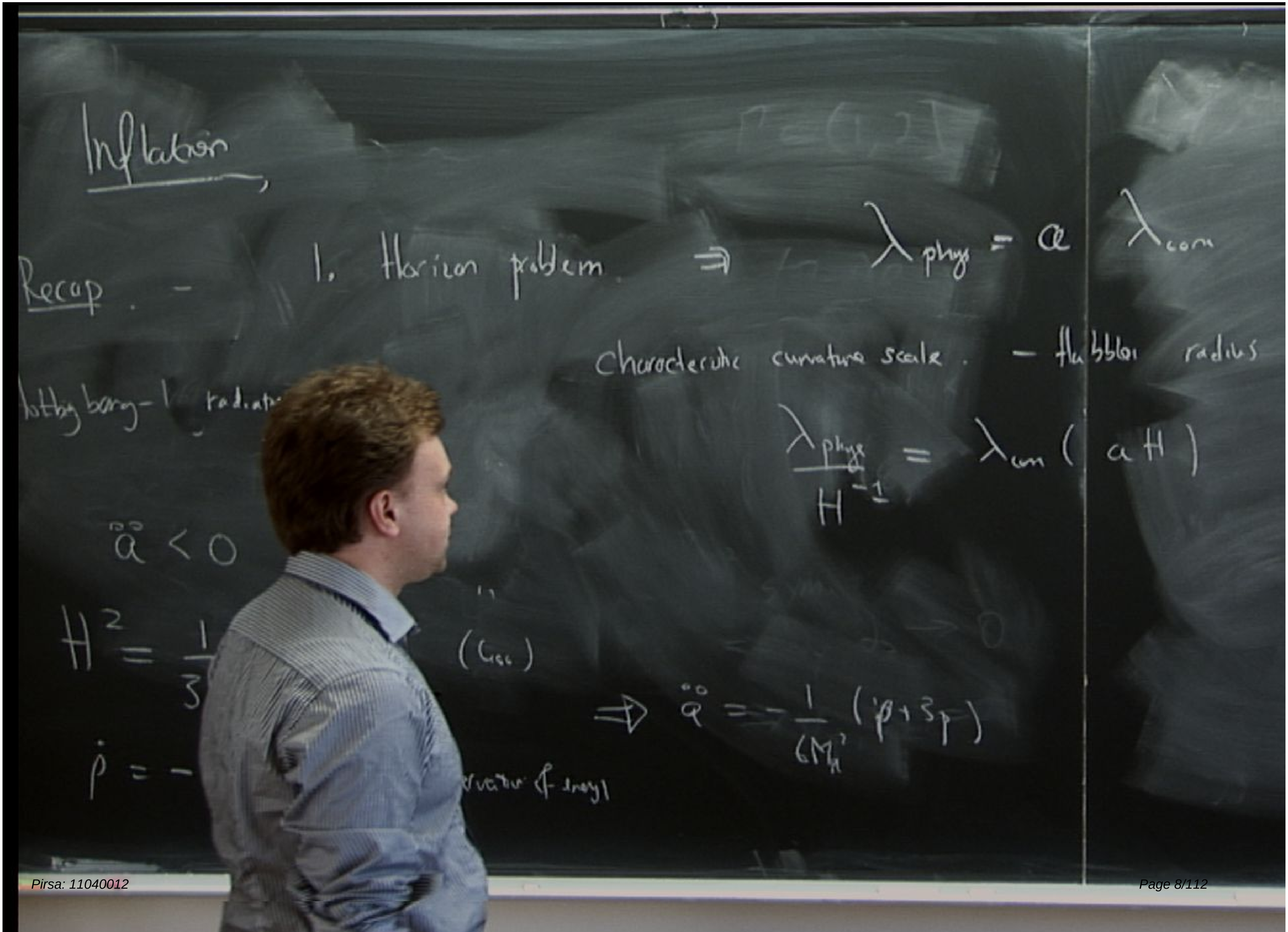
Recap. – 1. Horizon problem.

Hot big bang – radiation + matter epoch

$\ddot{a} < 0$


Inflation
Recap. - 1. Horizon problem.
radiation + matter
epoch
= 1/3M_pl² ρ − k/a²
⇒ ä = − 1/6M_pl² (ρ + 3p)
= −3H(p + ρ) (conservation of energy)


Inflation
Recap. - 1. Horizon problem ⇒ λ phys =
Characteristic curvature
Hot big bang - radiation + matter epoch
ä < 0
H² = 1/(3M_pl²) ρ
ρ̇ = -3H(ρ+
ä = -1/(6M_pl²)(ρ+3p)


Inflation
Recap. -
1. Horizon problem ⇒ λphys = a λcom
characteristic curvature scale. - Hubble radius
radiation
λphys / H^-1 = λcom (aH)
ä < 0
(Goo)
⇒ ä = - 1/(6Mpl^2) (ρ + 3p)

orizon problem $\Rightarrow$ $\lambda_{phy} = a \, \lambda_{com}$

Characteristic curvature scale. — Hubble radius H^{-1}

$$\frac{\lambda_{phys}}{H^{-1}} = \lambda_{com} (a H) = \lambda_{com} \dot{a}$$

$= \frac{\dot{a}}{a}$

$$\frac{d}{dt}\left(\frac{\lambda_{phys}}{H^{-1}} \right) = \lambda_{com} \ddot{a}$$

$\frac{k}{a^2}$ (G_{00})

$$\Rightarrow \ddot{a} = -\frac{1}{6M_p^2}(\rho + 3p)$$

(conservation of energy)

...flation

1. Horizon problem $\Rightarrow$ $\lambda_{phys} = a\ \lambda_{com}$

Characteristic curvature scale. — Hubble radius H^{-1}

...-1 radiation + matter epoch

$$\frac{\lambda_{phys}}{H^{-1}} = \lambda_{com}\,(aH) = \lambda_c\ldots$$

$$H = \frac{\dot a}{a}$$

$\ddot a < 0$

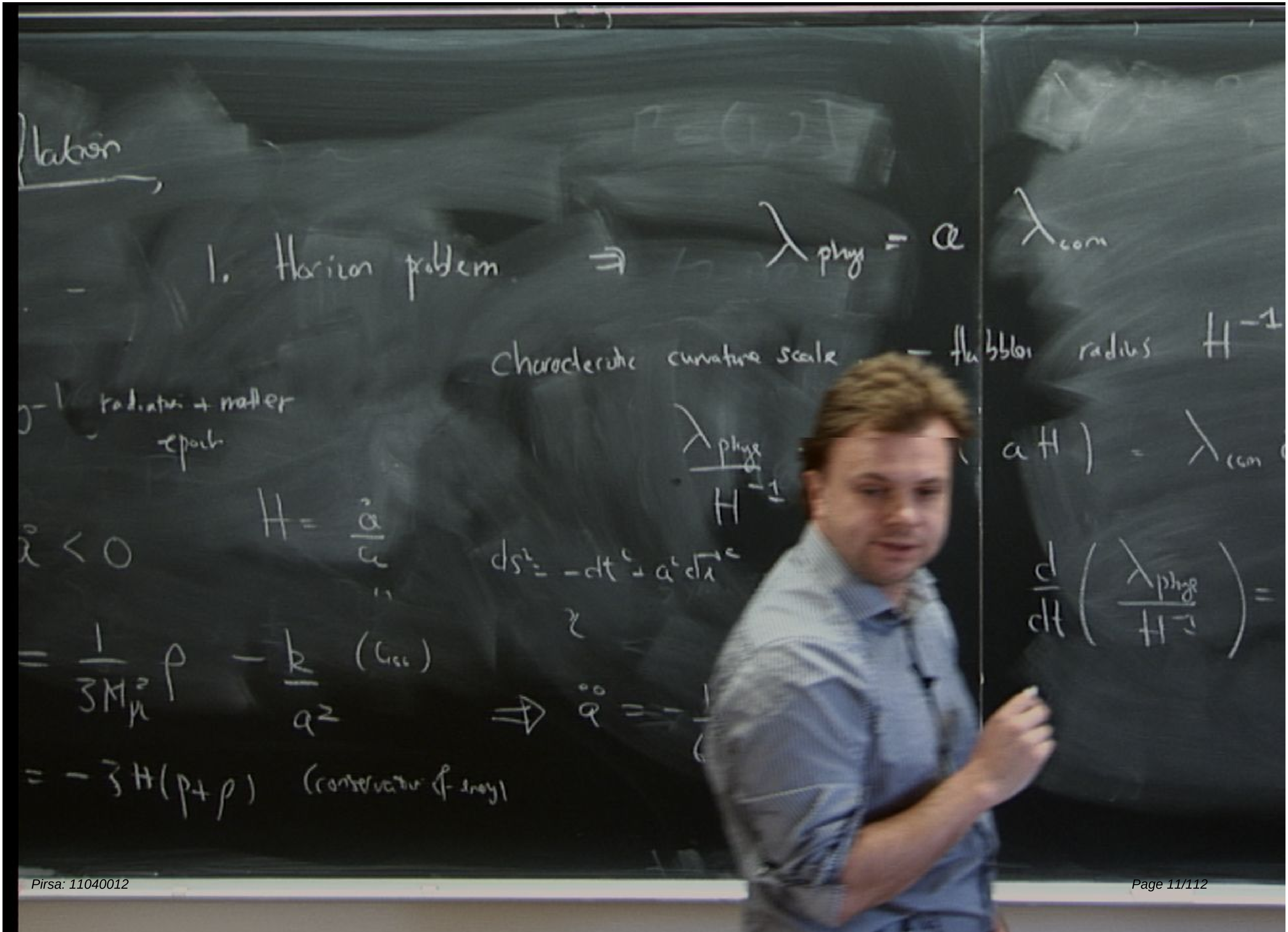
$$ds^2 = -dt^2 + a^2 dx^2$$

$$\frac{d}{dt}\left(\ldots\right.$$

$$= \frac{1}{3M_p^2}\rho - \frac{k}{a^2} \quad (G_{00})$$

$$\Rightarrow \ddot a = -\frac{1}{6M_p^2}(\rho + 3p)$$

$$= -3H(p+\rho) \quad \text{(conservation of energy)}$$


1. Horizon problem
λ phys = a λ com
Characteristic curvature scale = Hubble radius H^-1
radiation + matter epoch
H = ȧ/a
ds² = -dt² + a²dx²
(conservation of energy)

problem $\Rightarrow$ $\lambda_{phys} = a \, \lambda_{com}$

characteristic curvature scale. — Hubble radius H^{-1}

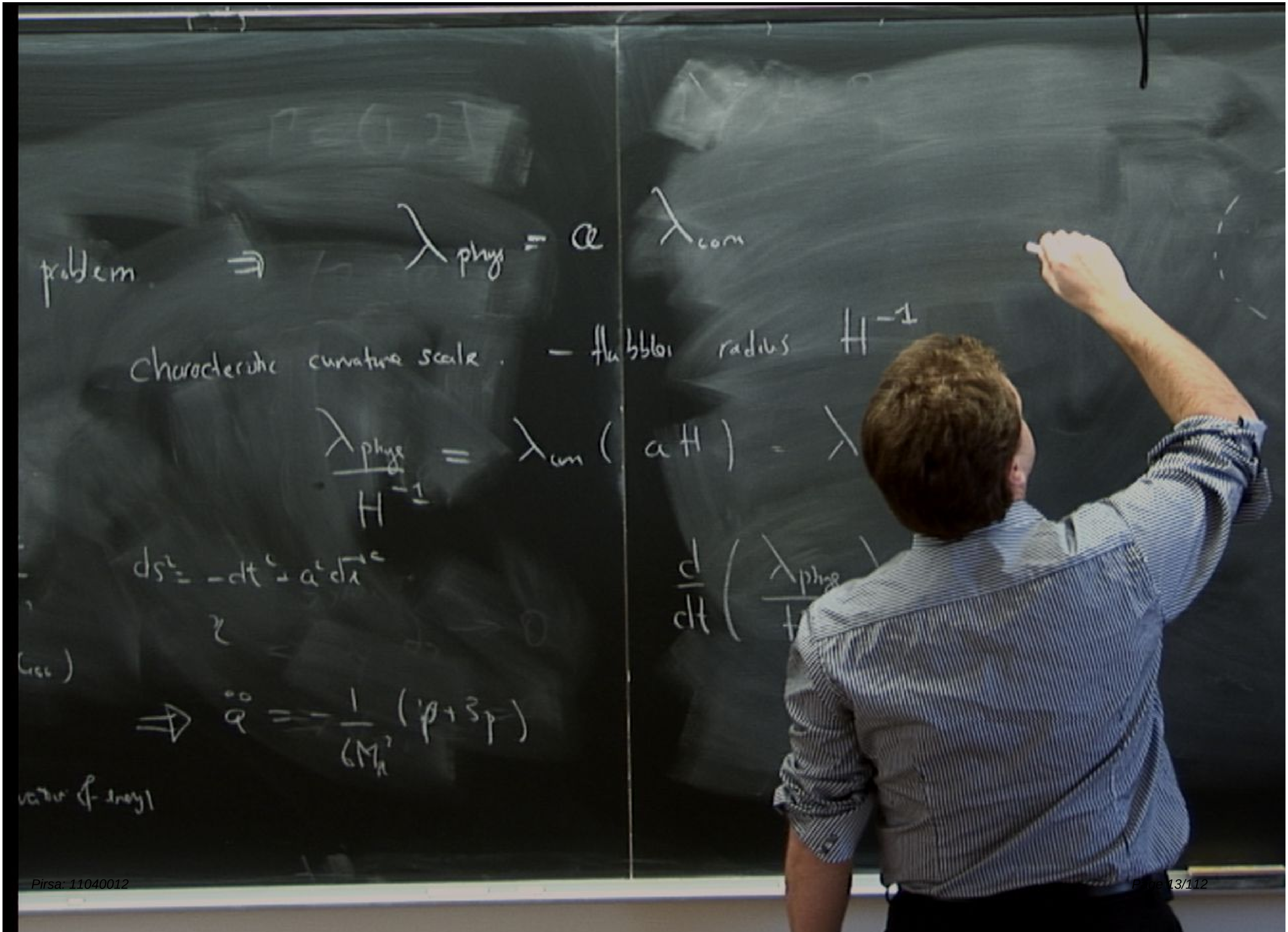
$$\frac{\lambda_{phys}}{H^{-1}} = \lambda_{com} (aH) = \lambda_{com} \dot{a}$$

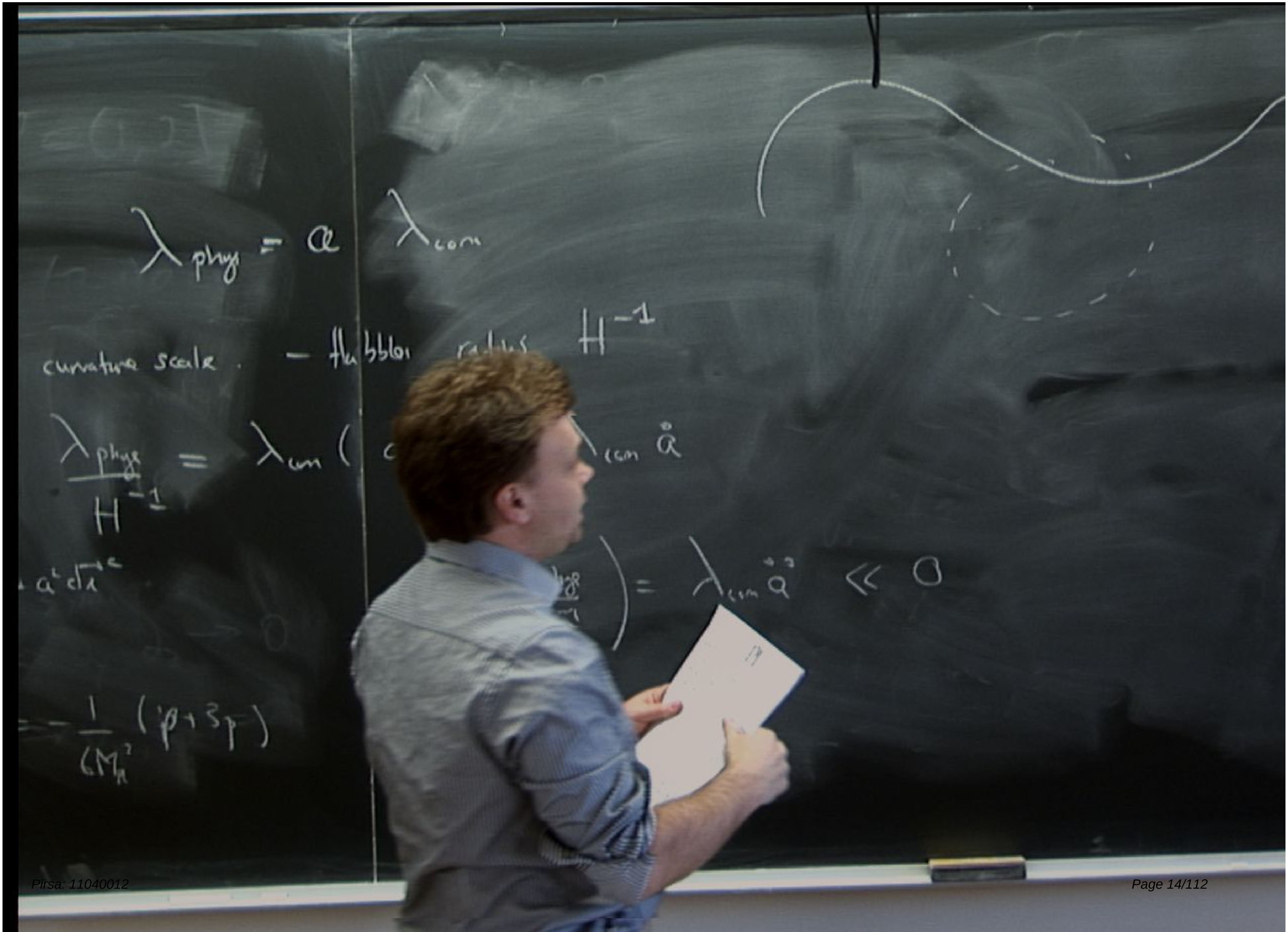
$ds^2 = -dt^2 + a^2 d\vec{x}^2$

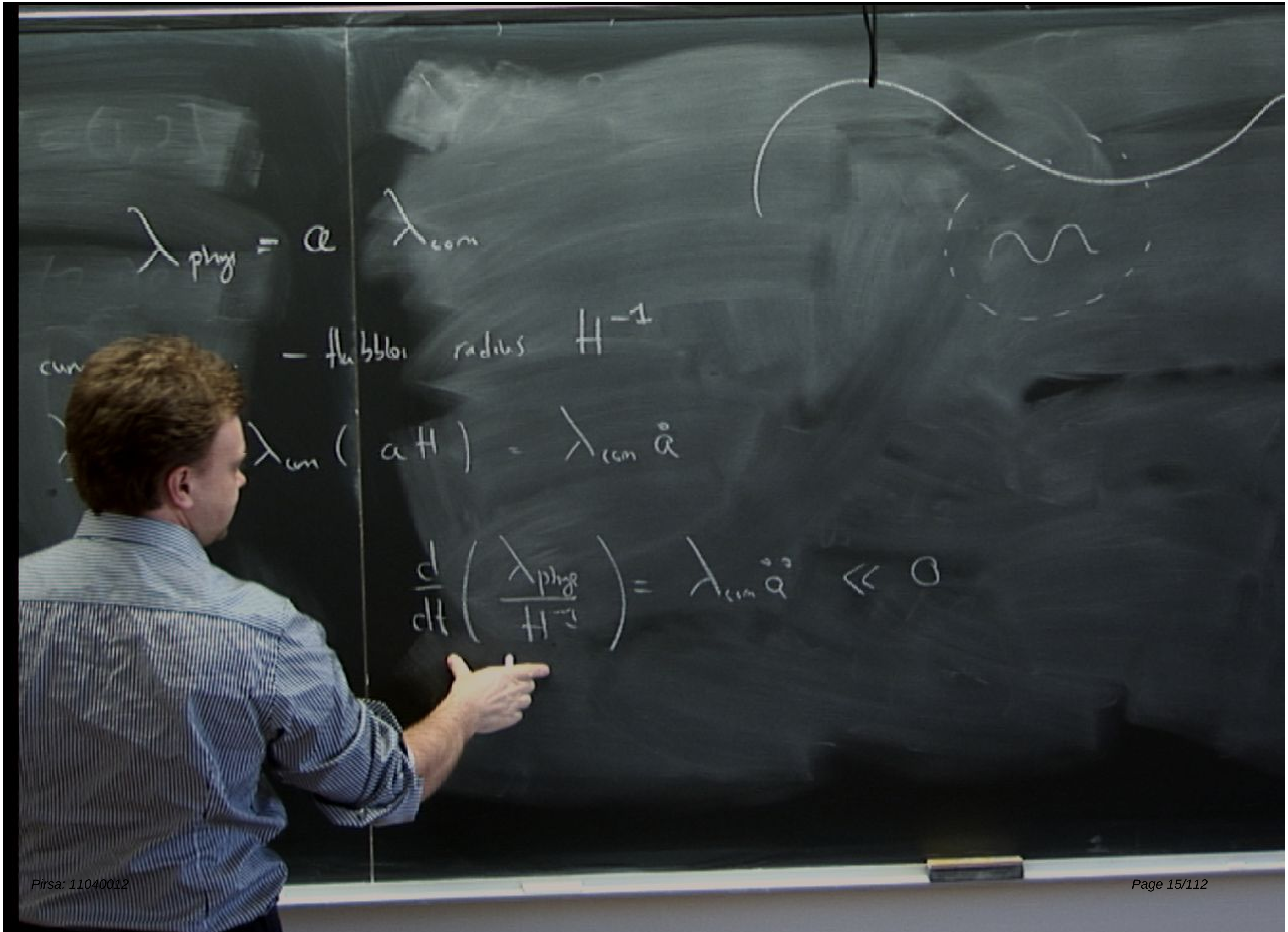
$$\frac{d}{dt}\left(\frac{\lambda_{phys}}{H^{-1}} \right) = \lambda_{com} \ddot{a} \ll 0$$

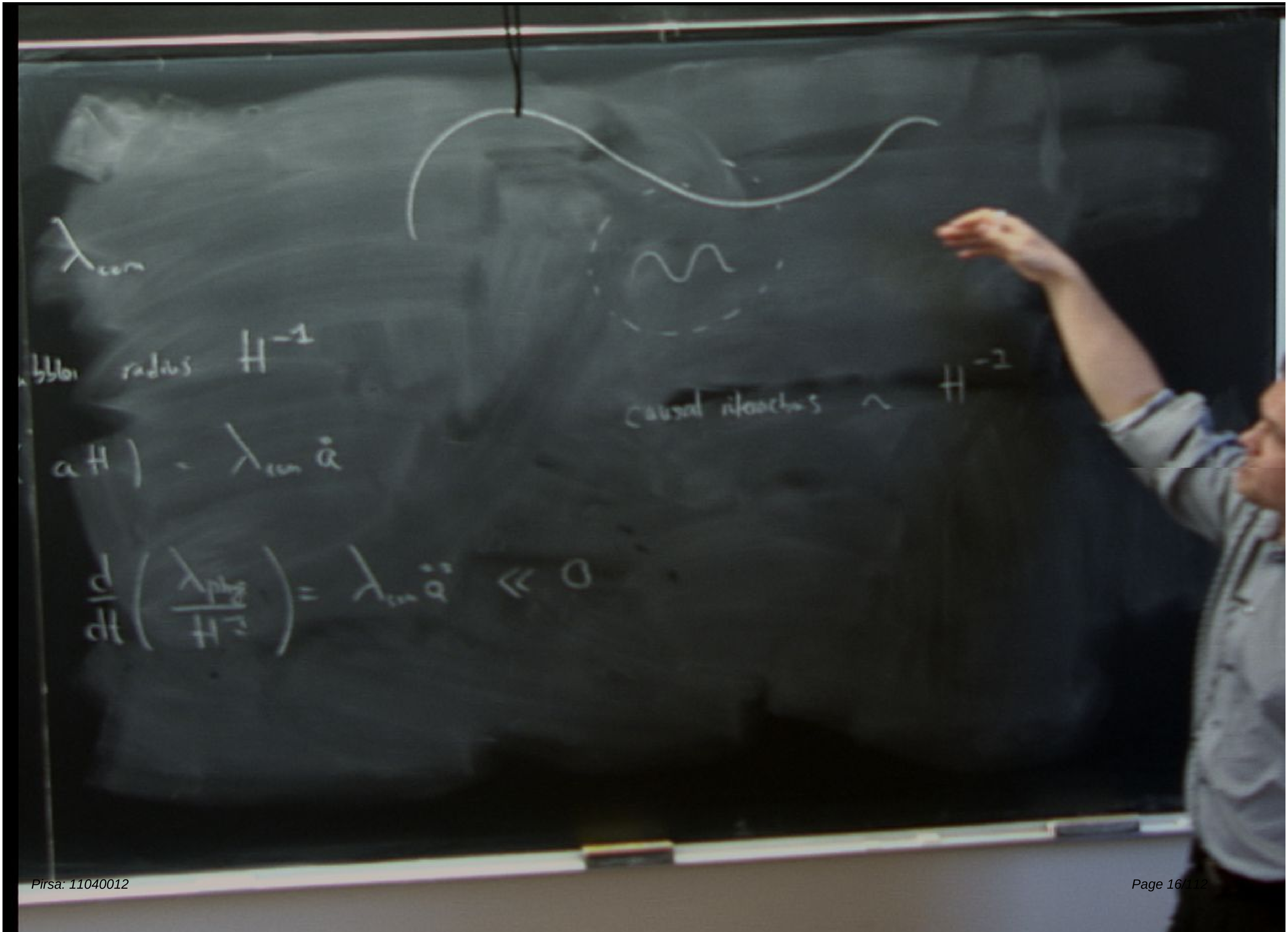
(GR)

$\Rightarrow$ $\ddot{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p)$


problem ⇒ λ_phys = a λ_com
characteristic curvature scale. – Hubble radius H^-1
λ_phys / H^-1 = λ_com (aH)
ds² = -dt² + a² dx²
d/dt (λ_phys / H^-1)
⇒ ä = -1/(6M_p²) (ρ + 3p)


λphys = a λcom
curvature scale
H⁻¹
≪ 0


λ phys = a λ com
Hubble radius H^-1
λ com (aH) = λ com ȧ
d/dt (λ phys / H^-1) = λ com ä ≪ 0


λ_{com}
radius H^{-1}
causal interactions $\sim H^{-1}$
$\frac{d}{dt}\left(\frac{\lambda_{phys}}{H^{-1}}\right) = \lambda_{com}\ddot{a} \ll 0$

λ_{com}

...bble radius H^{-1}

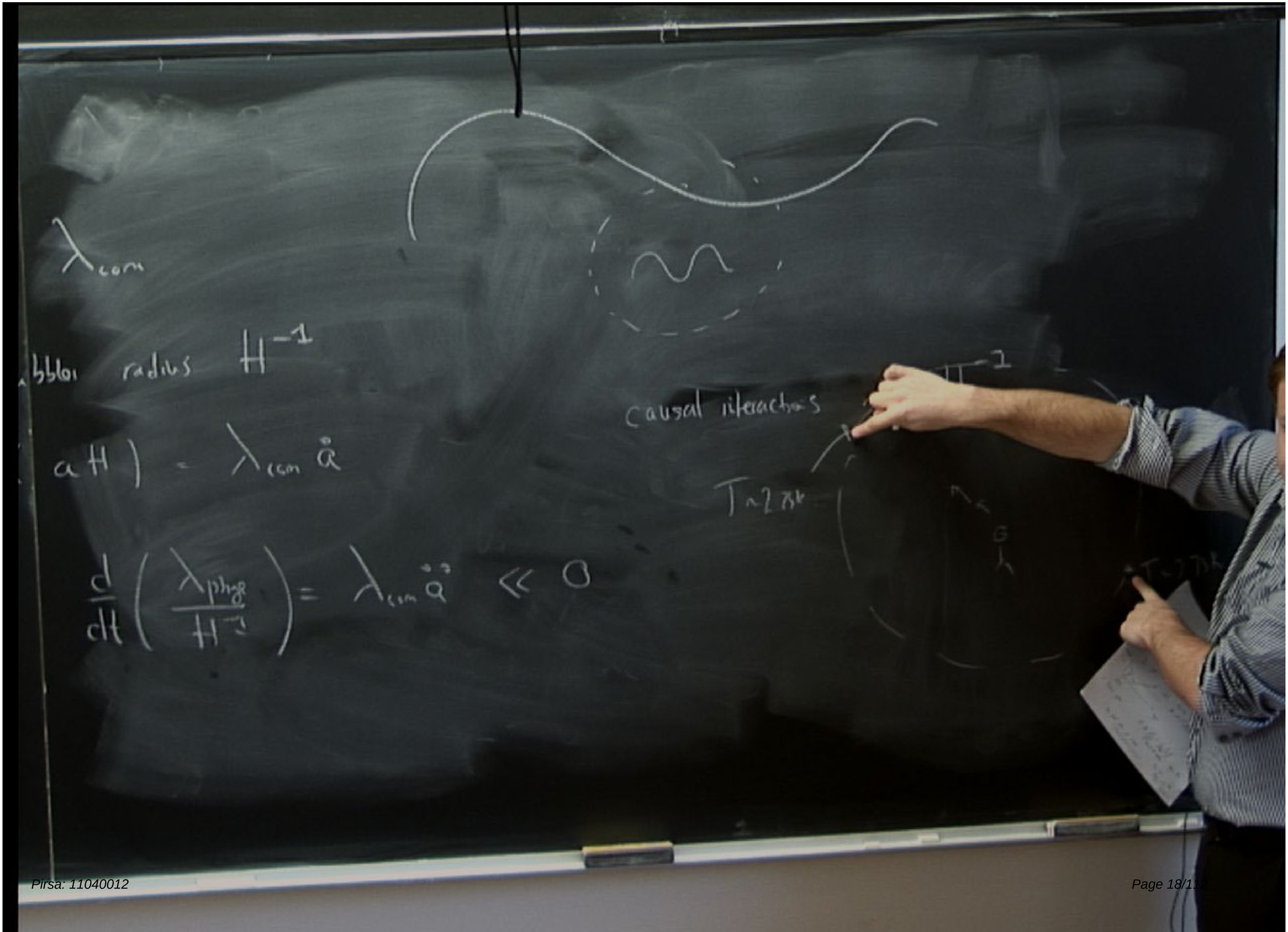
causal interactions $\sim H^{-1}$

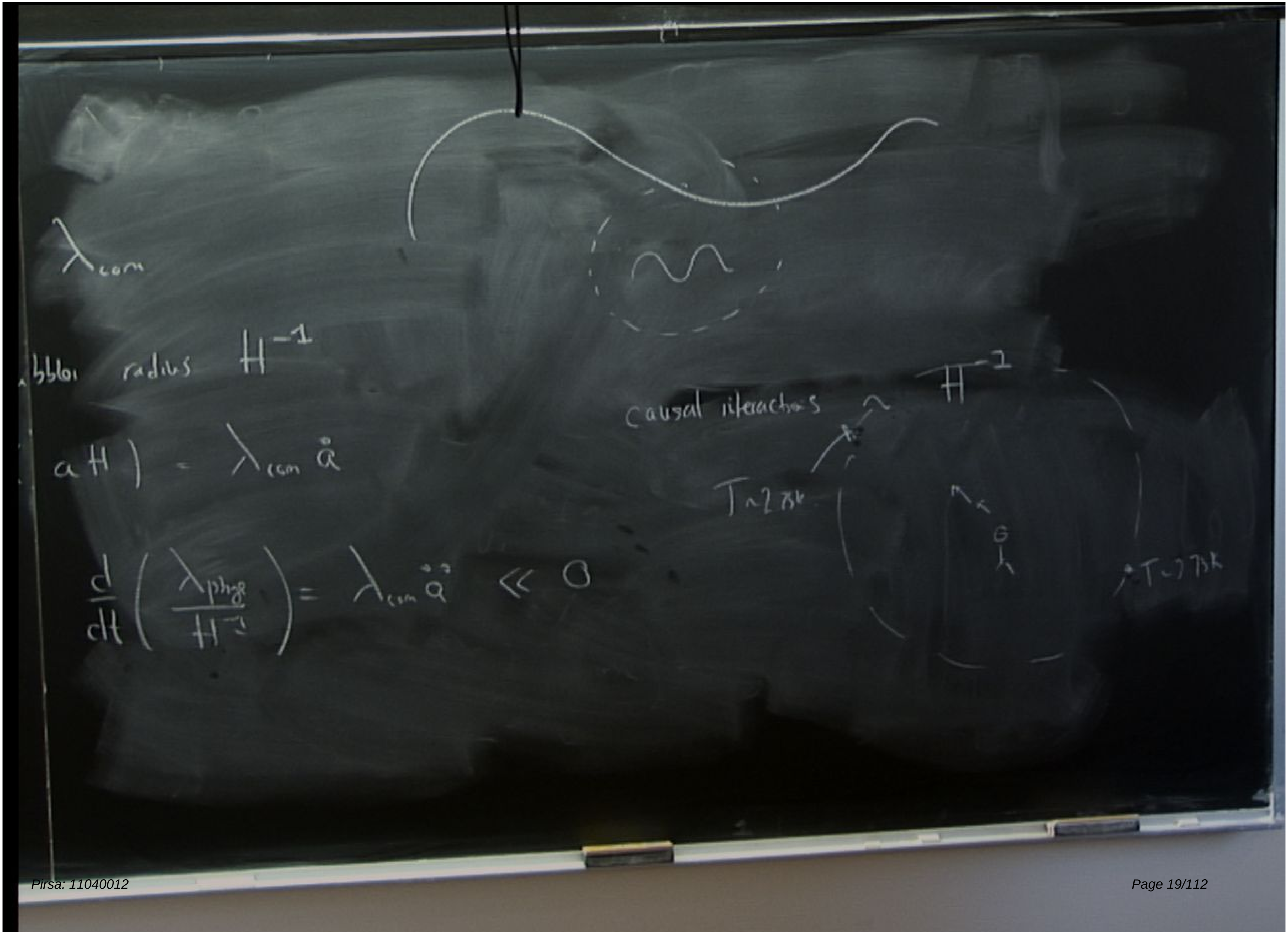
$T \sim 2.73k$

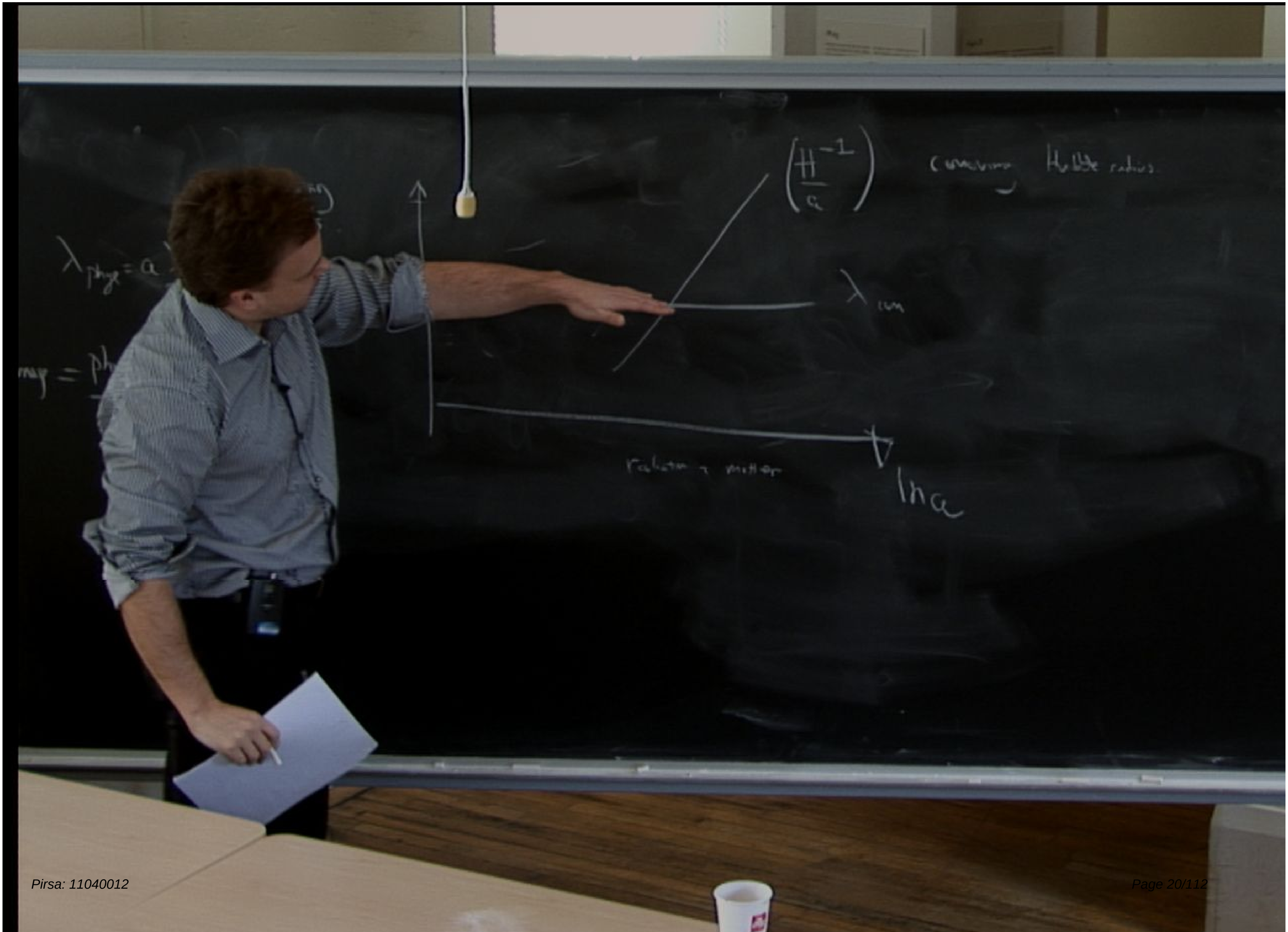
$T \sim 2.73k$

$\frac{d}{dt}(\lambda_{com}\, aH) = \lambda_{com}\,\ddot{a}$

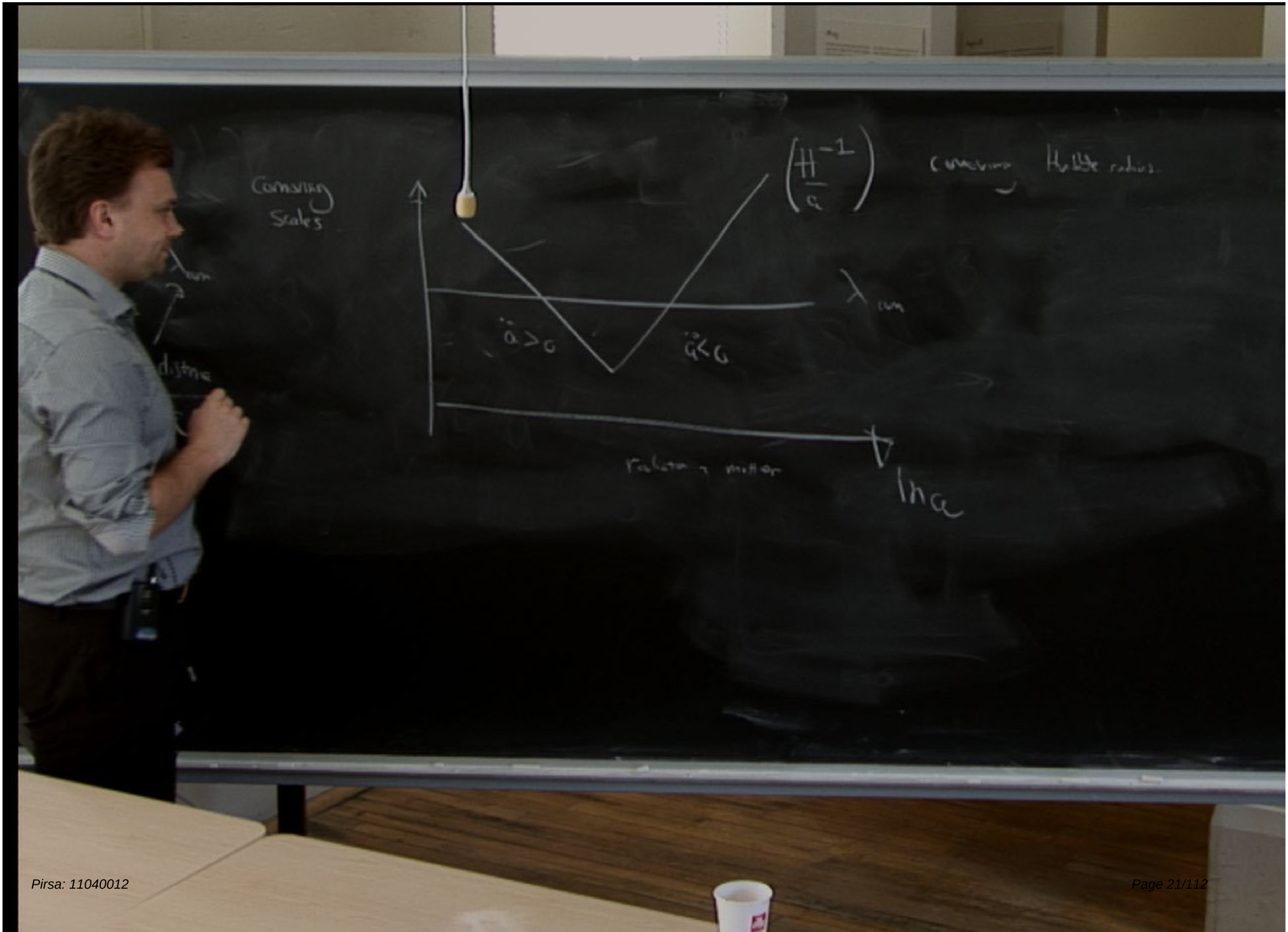
$\frac{d}{dt}\left(\frac{\lambda_{phys}}{H^{-1}}\right) = \lambda_{com}\,\ddot{a} \ll 0$


λcom
radius H⁻¹
causal interactions

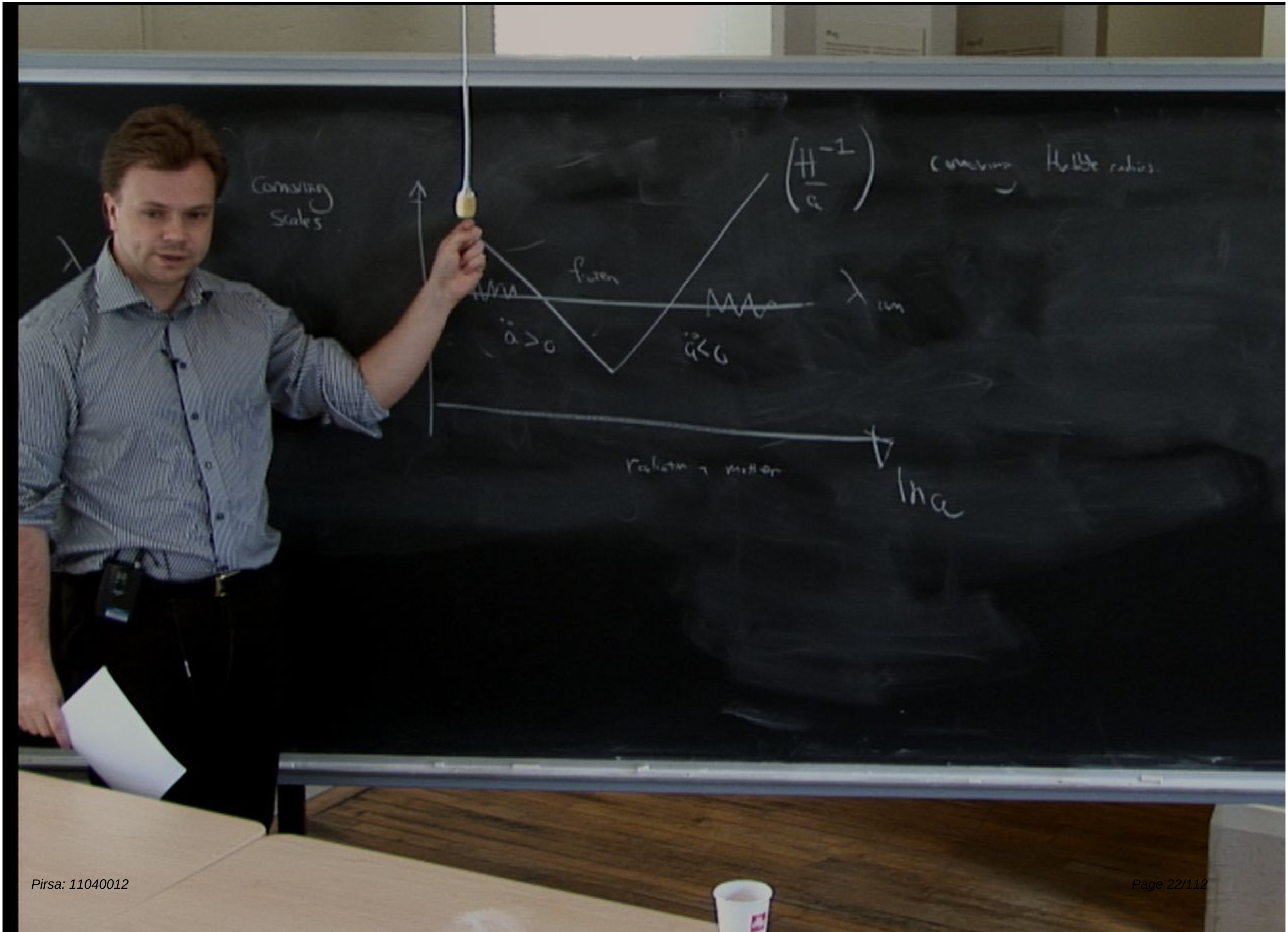

λ_{com}
radius H^{-1}
$aH) = \lambda_{com}\,\dot{a}$
$\frac{d}{dt}\left(\frac{\lambda_{phys}}{H^{-1}}\right) = \lambda_{com}\,\ddot{a} \ll 0$
causal interactions $\sim H^{-1}$

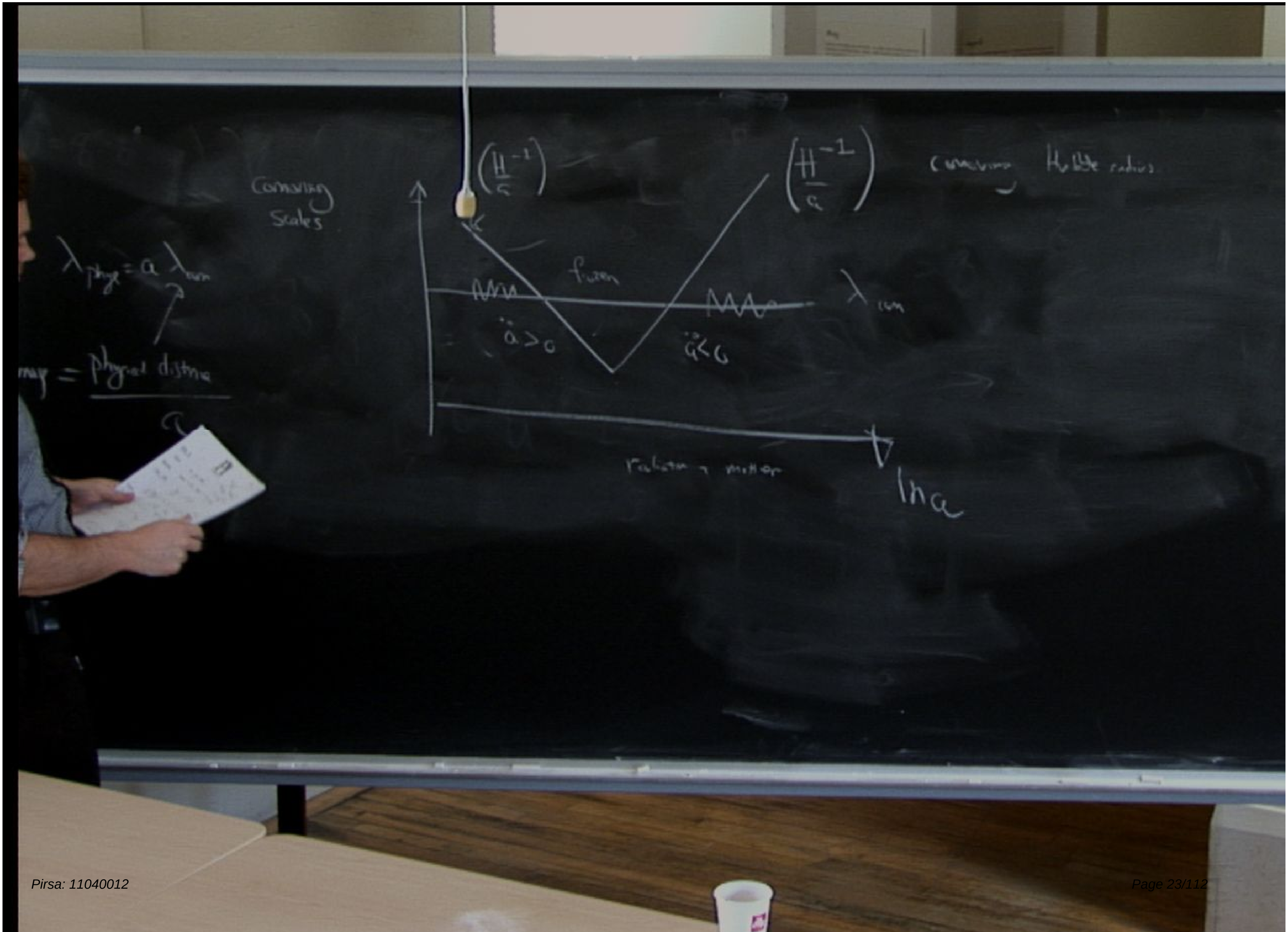


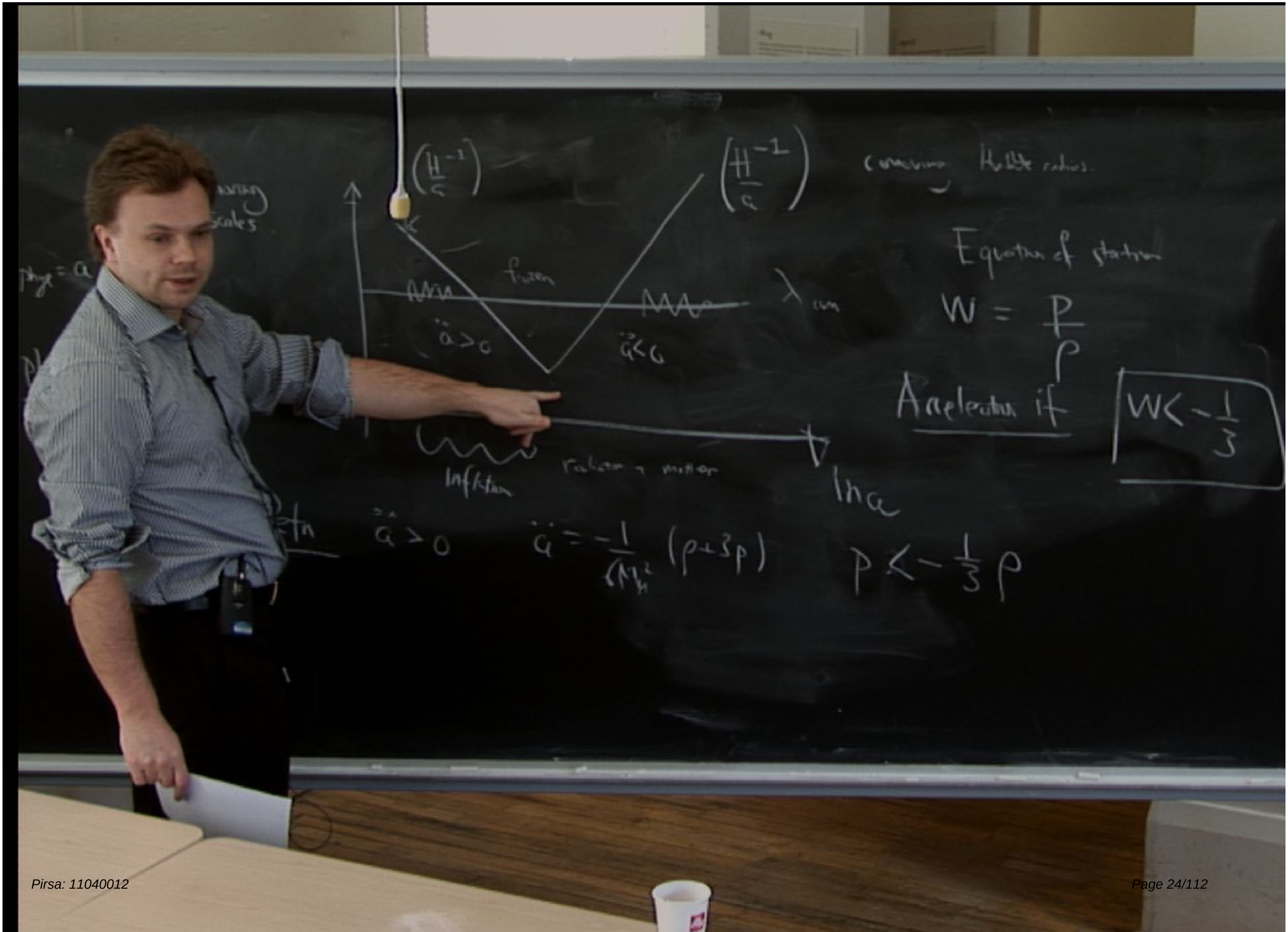
$\left(\frac{H^{-1}}{a}\right)$
comoving Hubble radius
λ_{com}
radiation + matter
$\ln a$

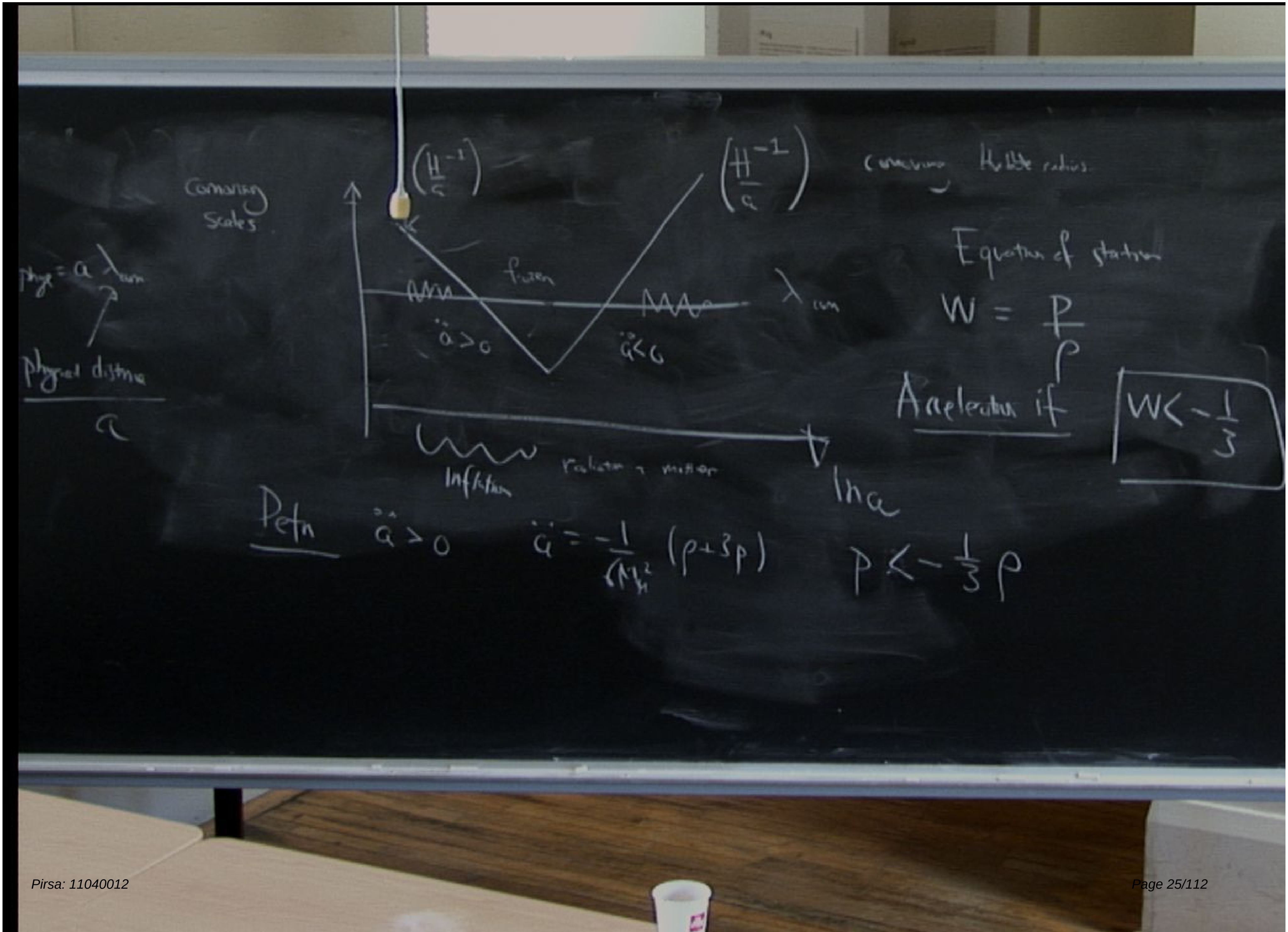


Comoving Scales
$\left(\frac{H^{-1}}{a}\right)$
comoving Hubble radius
λ_{com}
$\ddot{a}>0$
$\ddot{a}<0$
radiation + matter
$\ln a$


Comoving
Scales
λ
$\left(\frac{H^{-1}}{a}\right)$
comoving Hubble radius
frozen
λ_{cm}
$\ddot{a} > 0$
$\ddot{a} < 0$
radiation + matter
$\ln a$


Comoving scales
$\lambda_{phys} = a\, \lambda_{com}$
physical distance
a
$\left(\frac{H}{a}\right)^{-1}$
$\left(\frac{H}{a}\right)^{-1}$
comoving Hubble radius
frozen
λ_{com}
$\ddot{a} > 0$
$\ddot{a} < 0$
radiation + matter
ln a


$\left(\frac{H^{-1}}{a}\right)$
$\left(\frac{H^{-1}}{a}\right)$
comoving Hubble radius
Equation of state
$W = \frac{P}{\rho}$
Acceleration if
$\boxed{W < -\frac{1}{3}}$
frozen
λ_{com}
$\ddot{a} > 0$
$\ddot{a} < 0$
Inflation
radiation + matter
$\ln a$
$\ddot{a} > 0$
$\ddot{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p)$
$p < -\frac{1}{3}\rho$


Comoving Scales
(H^-1 / a)
comoving Hubble radius
frozen
λ com
ä > 0
ä < 0
physical distance / a
Inflation
radiation + matter
ln a
Equation of state
W = P / ρ
Acceleration if
W < -1/3
Defn ä > 0
ä = -1/(6M_pl^2) (ρ+3p)
p < -1/3 ρ

2 Flatness problem

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r\right.$$

Why is universe so flat.

$$\lambda_{phys} = a$$

$$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}$$ curvature of space

$- f$

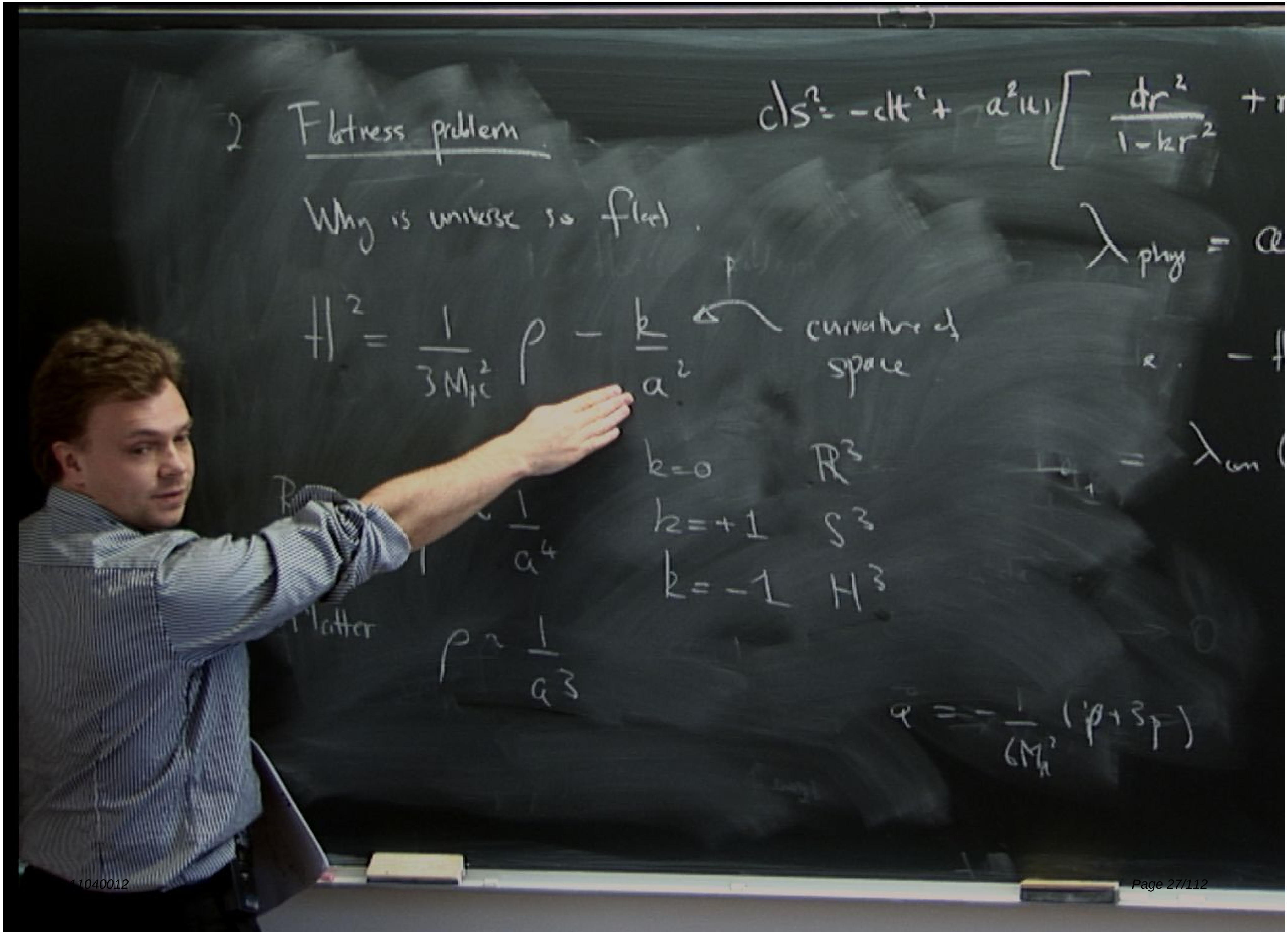
$k=0$ $\mathbb{R}^3$

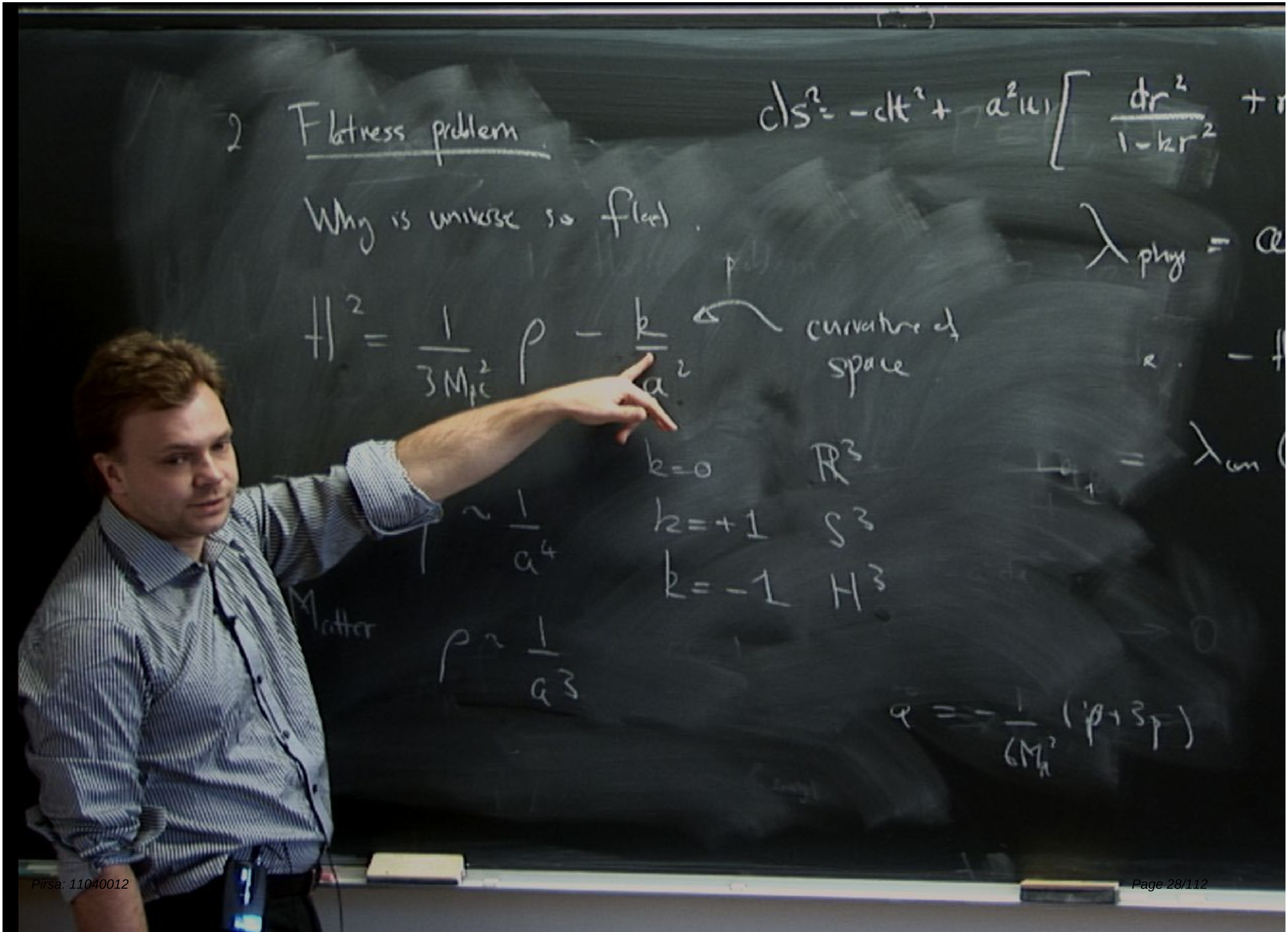
$= \lambda_{cm}($

$k=+1$ S^3

$k=-1$ H^3

$$\ddot{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p)$$


2 Flatness problem
Why is universe so flat.
$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}$
curvature of space
$k=0$ $\mathbb{R}^3$
$k=+1$ S^3
$k=-1$ H^3
Matter
$\rho \sim \frac{1}{a^3}$
$\frac{1}{a^4}$
$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + \right.$
$\lambda_{phys} =$
$\frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p)$


2 Flatness problem
Why is universe so flat.
ds² = −dt² + a²(t) [dr² / (1 − kr²) + r
H² = 1/(3M_pl²) ρ − k/a²
curvature of space
λ phys = a
k=0 R³
k=+1 S³
k=−1 H³
ρ ~ 1/a⁴
Matter
ρ ~ 1/a³
ä = − 1/(6M_pl²) (ρ + 3p)

2 Flatness problem

$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + \ldots\right.$

Why is universe so flat. $\boxed{k \approx 0}$

$$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}$$

curvature of space

$k=0$ $\mathbb{R}^3$

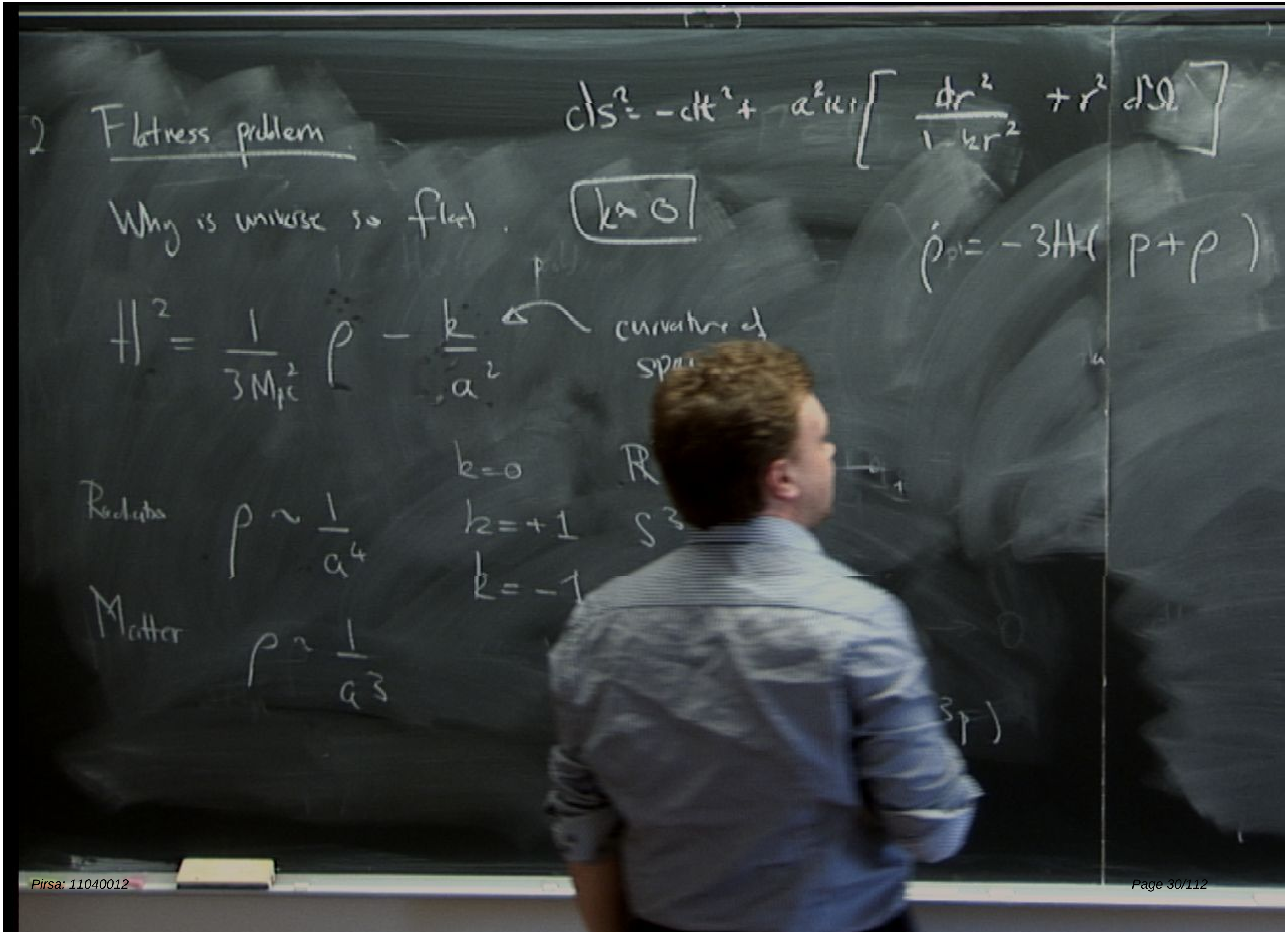
$k=+1$ S^3

$k=-1$ H^3

Radiation $\rho \sim \frac{1}{a^4}$

Matter $\rho \sim \frac{1}{a^3}$

$$\ddot{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p)$$


2 Flatness problem
$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d^2\Omega\right]$
Why is universe so flat
$\boxed{k \approx 0}$
$\dot{\rho} = -3H(p+\rho)$
$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}$
curvature of
$k=0$ $\mathbb{R}$
$k=+1$ S^3
$k=-1$
Matter
$\rho \sim \frac{1}{a^3}$

Flatness problem

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d^2\Omega\right]$$

Why is universe so flat.

$$\boxed{k \approx 0}$$

$$\dot\rho = -3H(p+\rho)$$

$$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}$$

curvature of space

... of state $w = \frac{p}{\rho}$

$$\ldots -3H\rho(1+w)$$

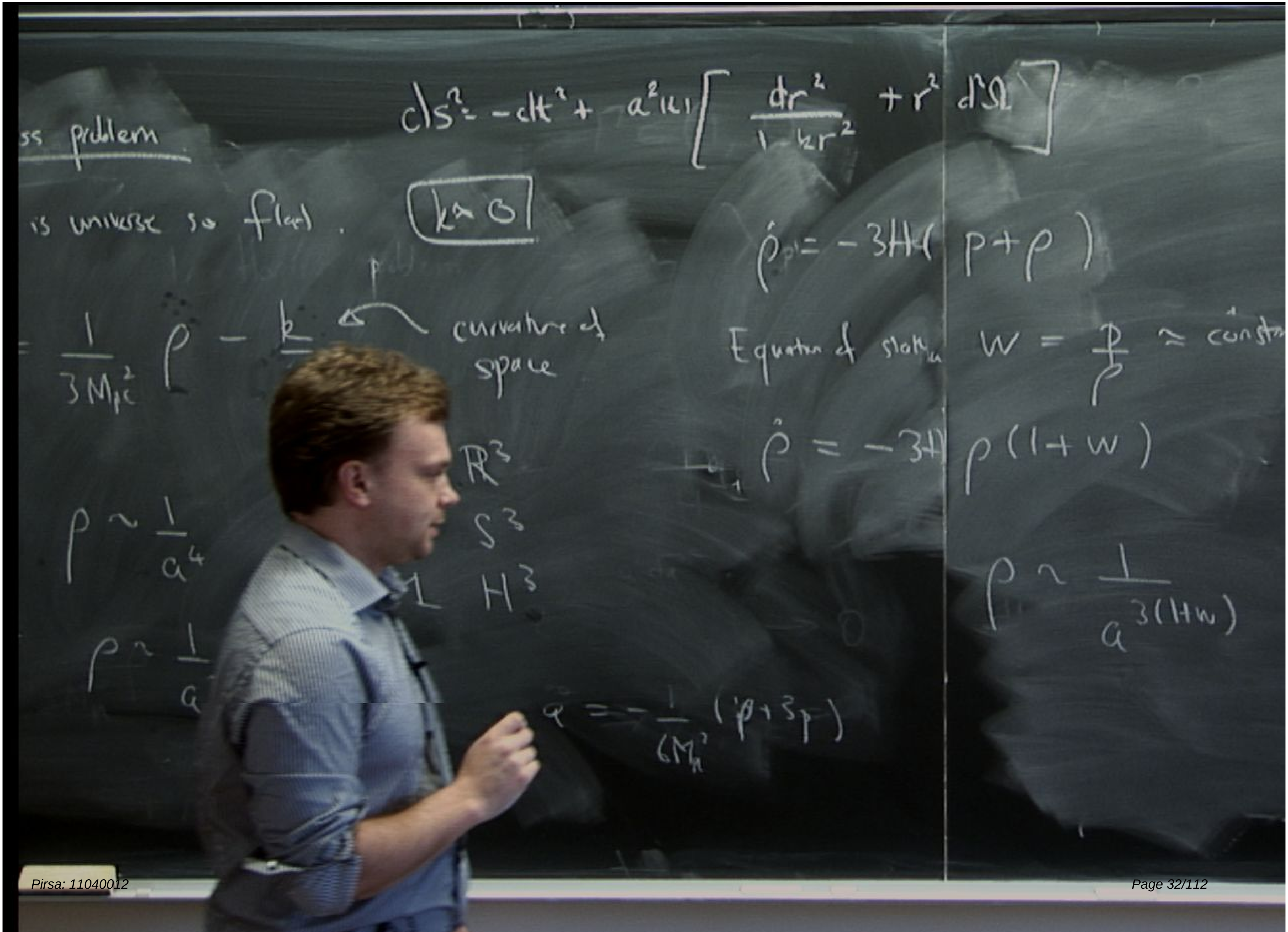
$k=0$ $\mathbb{R}^3$

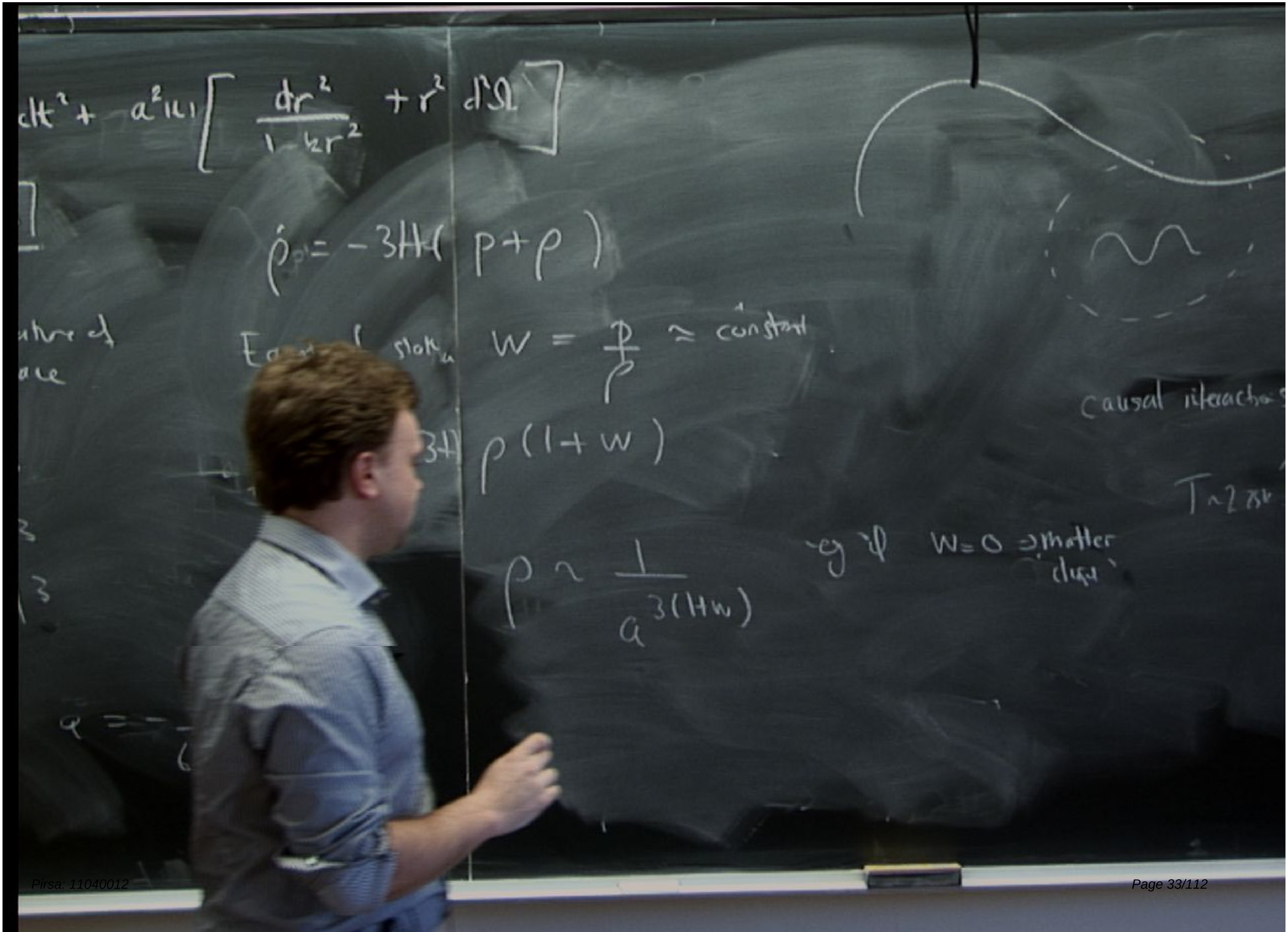
$k=+1$ S^3

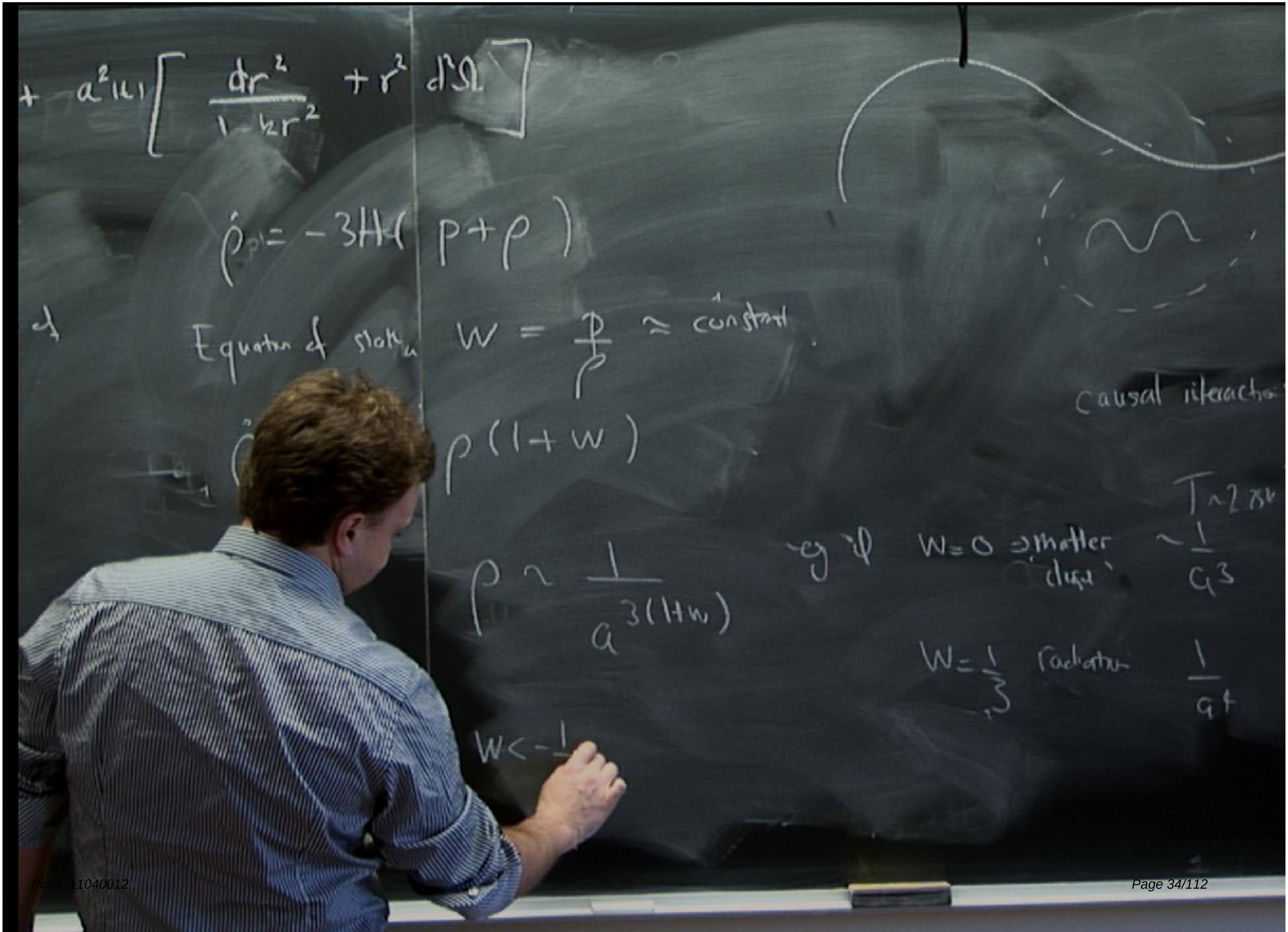
$k=-1$ H^3

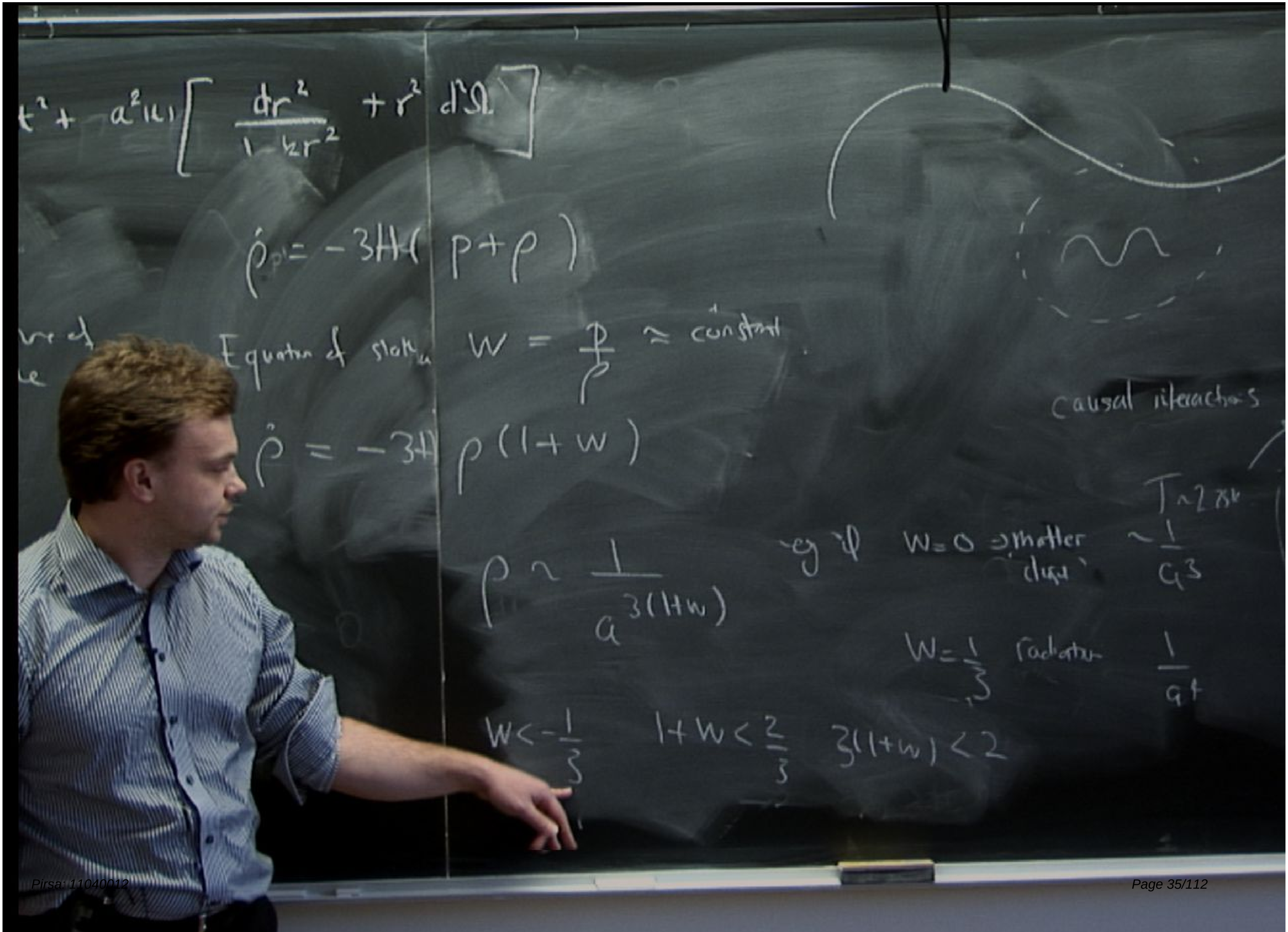
Radiation $\rho \sim \frac{1}{a^4}$

Matter $\rho \sim \frac{1}{a^3}$


ss problem
$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d^2\Omega\right]$
is universe so flat.
$\boxed{k=0}$
curvature of space
$= \frac{1}{3M_{pl}^2}\rho - k$
$\mathbb{R}^3$
S^3
H^3
$\rho \sim \frac{1}{a^4}$
$\rho \sim \frac{1}{a}$
$\dot\rho = -3H(p+\rho)$
Equation of state $W = \frac{p}{\rho} \simeq$ const
$\dot\rho = -3H\rho(1+w)$
$\rho \sim \frac{1}{a^{3(1+w)}}$
$\ddot a = -\frac{1}{6M_{pl}^2}(\rho+3p)$


$dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d^2\Omega\right]$
$\dot\rho = -3H(p+\rho)$
$w = \frac{p}{\rho} \simeq$ constant
$\rho(1+w)$
$\rho \sim \frac{1}{a^{3(1+w)}}$
$w=0 \Rightarrow$ matter
causal interaction


Equation of state
W = p/ρ ≃ constant
causal interaction
W = 0 ⇒ matter ~ 1/a³
W = 1/3 radiation 1/a⁴
W < -1


t² + a²(t) [dr² / (1 − kr²) + r² d²Ω]
ρ̇ = −3H (p + ρ)
Equation of state w = p/ρ ≃ constant
ρ̇ = −3H ρ (1 + w)
ρ ∼ 1 / a^3(1+w)
eg if w = 0 ⇒ matter 'dust' ∼ 1/a³
w = 1/3 radiation 1/a⁴
w < −1/3   1 + w < 2/3   3(1+w) < 2
causal interactions
T ∼ 2.7K

$+ r^2 \, d^2\Omega \,]$

$3H (p + \rho)$

Most inflation

$w \simeq -1$

slow... $w = \frac{p}{\rho} \simeq$ constant

$-3H \, \rho (1 + w)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w = 0 \Rightarrow$ matter 'dust'

$w = \frac{1}{3}$ radiation

$w < -\frac{1}{3}$ $\quad 1 + w < \frac{2}{3}$ $\quad 3(1+w) < 2$

$+ r^2 \, d^2\Omega \,]$

$3H(p+\rho)$

Most inflation

slow roll $\quad W = \frac{p}{\rho} \simeq$ constant

$W \simeq -1$

$-3H\rho(1+w)$

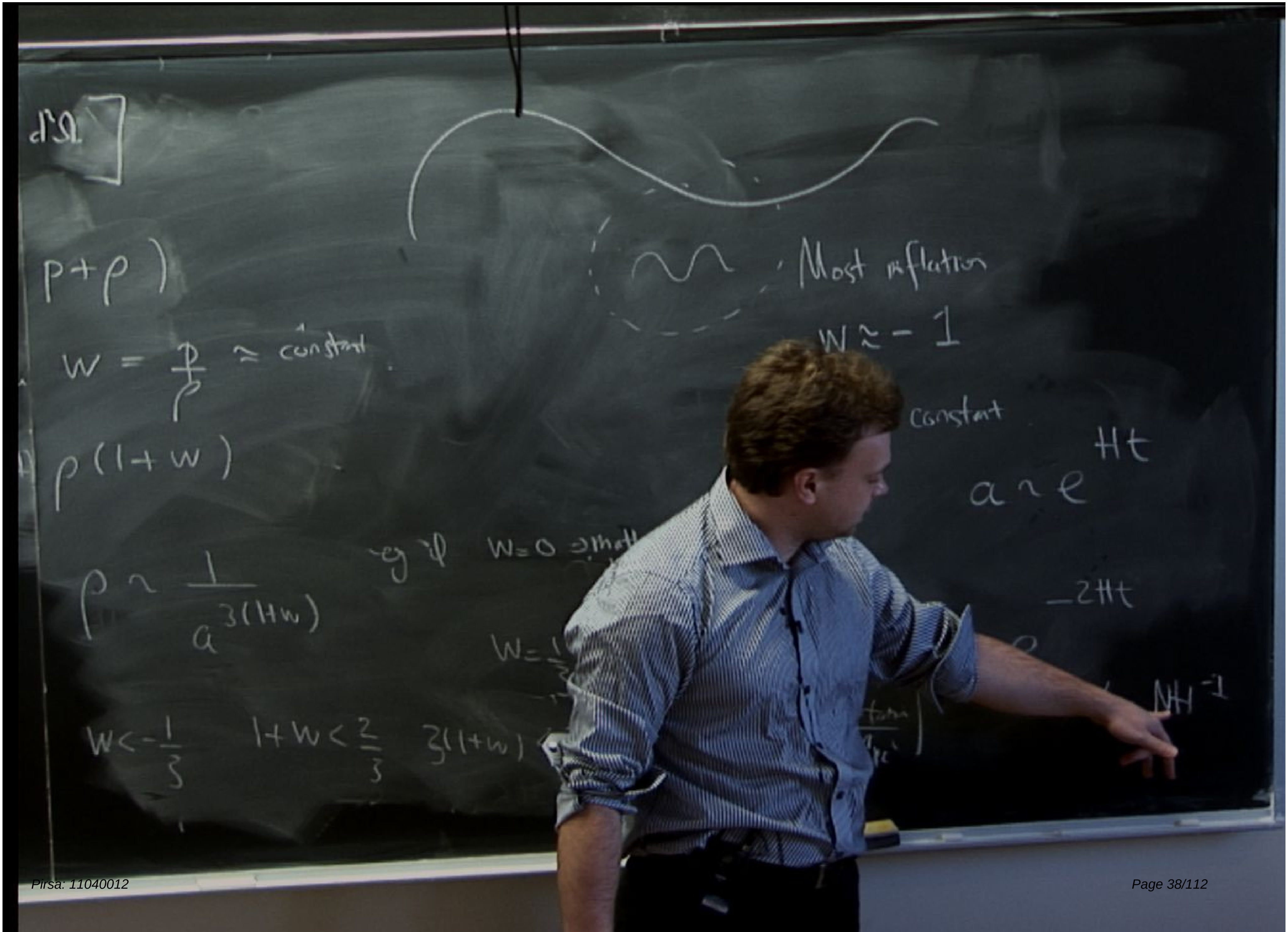
$\rho \simeq$ constant

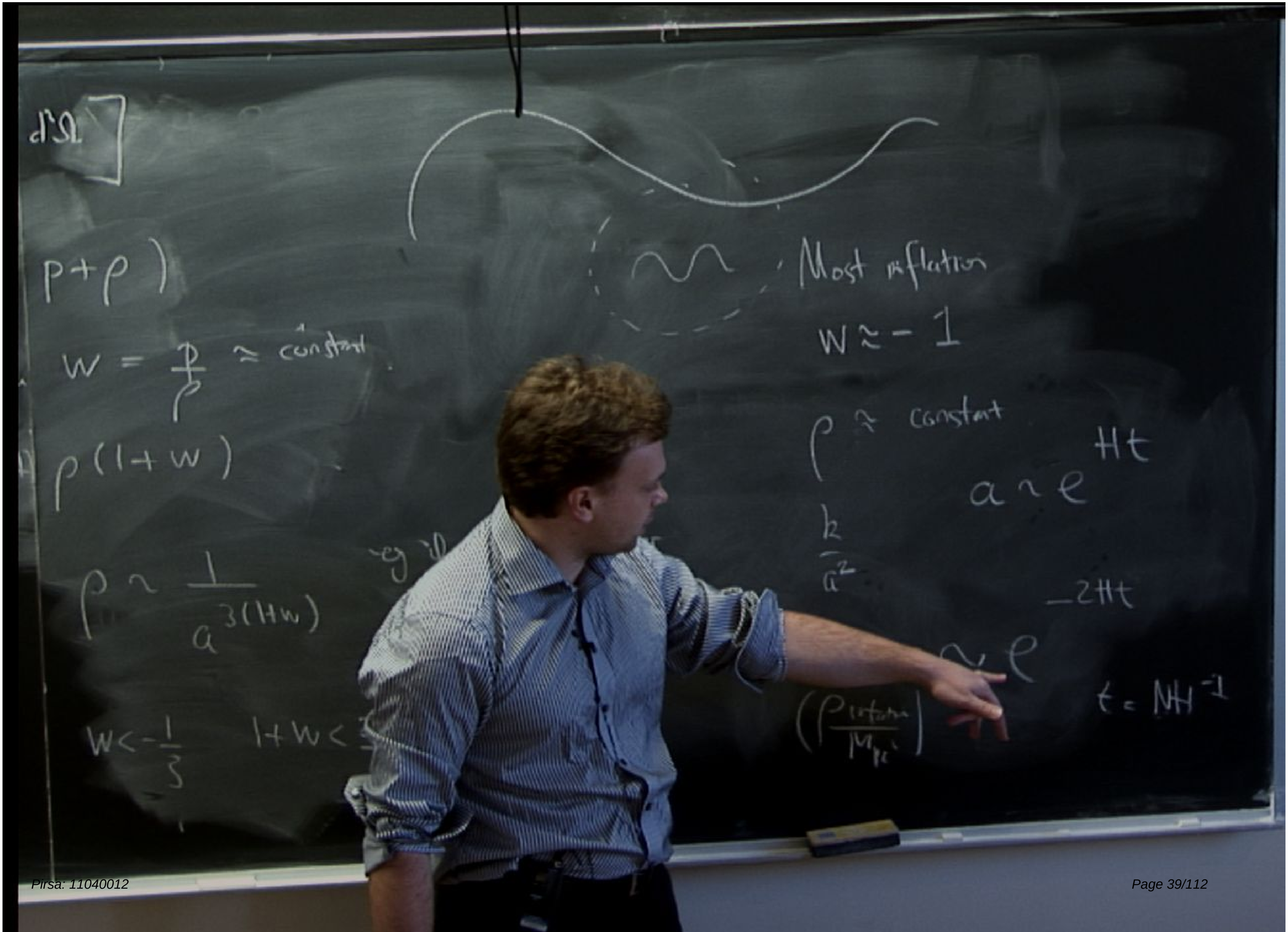
$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $W=0 \Rightarrow$ matter 'dust'

$W = \frac{1}{3}$ radiation

$W < -\frac{1}{3} \quad 1+W < \frac{2}{3} \quad 3(1+w) < 2$


p + ρ)
w = p/ρ ≃ constant
ρ (1 + w)
ρ ~ 1/a^{3(1+w)}
eg if w = 0 small
w < -1/3   1 + w < 2/3   3(1+w)
Most inflation
w ≃ -1
constant
a ~ e^{Ht}
-2Ht
NH^{-1}


p+ρ)
w = p/ρ ≃ constant
ρ(1+w)
ρ ~ 1/a^{3(1+w)}
w < -1/3
Most inflation
w ≃ -1
ρ ≃ constant
a ~ e^{Ht}
e^{-2Ht}
t = NH^{-1}

$d^3\Omega$

$p+\rho)$

$w = \frac{p}{\rho} \simeq$ constant

$w)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w=0 \Rightarrow$ matter "dust"

$w=\frac{1}{3}$ radiation

$w<-\frac{1}{3}$ $\quad 1+w<\frac{2}{3}$ $\quad 3(1+w)<2$

Most inflation

$w \simeq -1$

$\rho \simeq$ constant

$a \sim e^{Ht}$

$\frac{k}{a^2}$

$\frac{k/a^2}{\left(\rho_{\text{inflation}}/M_{pl}^2\right)} \sim e^{-2Ht}$

$t = NH^{-1}$

$\boxed{d^3\Omega}$

$p+\rho)$

$w = \frac{p}{\rho} \simeq$ constant

$\rho(1+w)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w=0 \Rightarrow$ matter "dust"

$w=\frac{1}{3}$ radiation

$w<-\frac{1}{3}$ $\quad 1+w<\frac{2}{3}$ $\quad 3(1+w)<2$

Most inflation

$w \simeq -1$

$\rho \simeq$ constant

$a \sim e^{Ht}$

$\frac{k}{a^2}$

$\frac{k/a^2}{\left(\frac{\rho_{\text{inflation}}}{M_{pl}^2}\right)} \sim e^{-2Ht}$

$t = NH^{-1}$

$d^3\Omega$

$p+\rho\,)$

$w = \frac{p}{\rho} \simeq$ constant

$\rho\,(1+w\,)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w=0 \Rightarrow$ matter dust

$w=\frac{1}{3}$ radiation

$w<-\frac{1}{3}$ $\quad 1+w<\frac{2}{3}$ $\quad 3(1+w)<2$

Most inflation

$w \simeq -1$

$\rho \simeq$ constant

$a \sim e^{Ht}$

$\frac{k}{a^2}$

$\frac{k/a^2}{\left(\frac{\rho_{\text{inflation}}}{M_{pl}^2}\right)} \sim e^{-2Ht}$

$t = NH^{-1}$

N

$d^2\Omega$]

$p+\rho$)

$w = \frac{p}{\rho} \simeq$ constant

$\rho(1+w)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w=0 \Rightarrow$ matter dust

$w=\frac{1}{3}$ radiation

$w<-\frac{1}{3}$ $\quad 1+w<\frac{2}{3}$ $\quad 3(1+w)<2$

Most inflation

$w \simeq -1$

$\rho \simeq$ constant

$a \sim e^{Ht}$

$\frac{k}{a^2}$

$\frac{k/a^2}{\left(\frac{\rho_{\text{inflaton}}}{M_{pl}}\right)} \sim e^{-2Ht}$

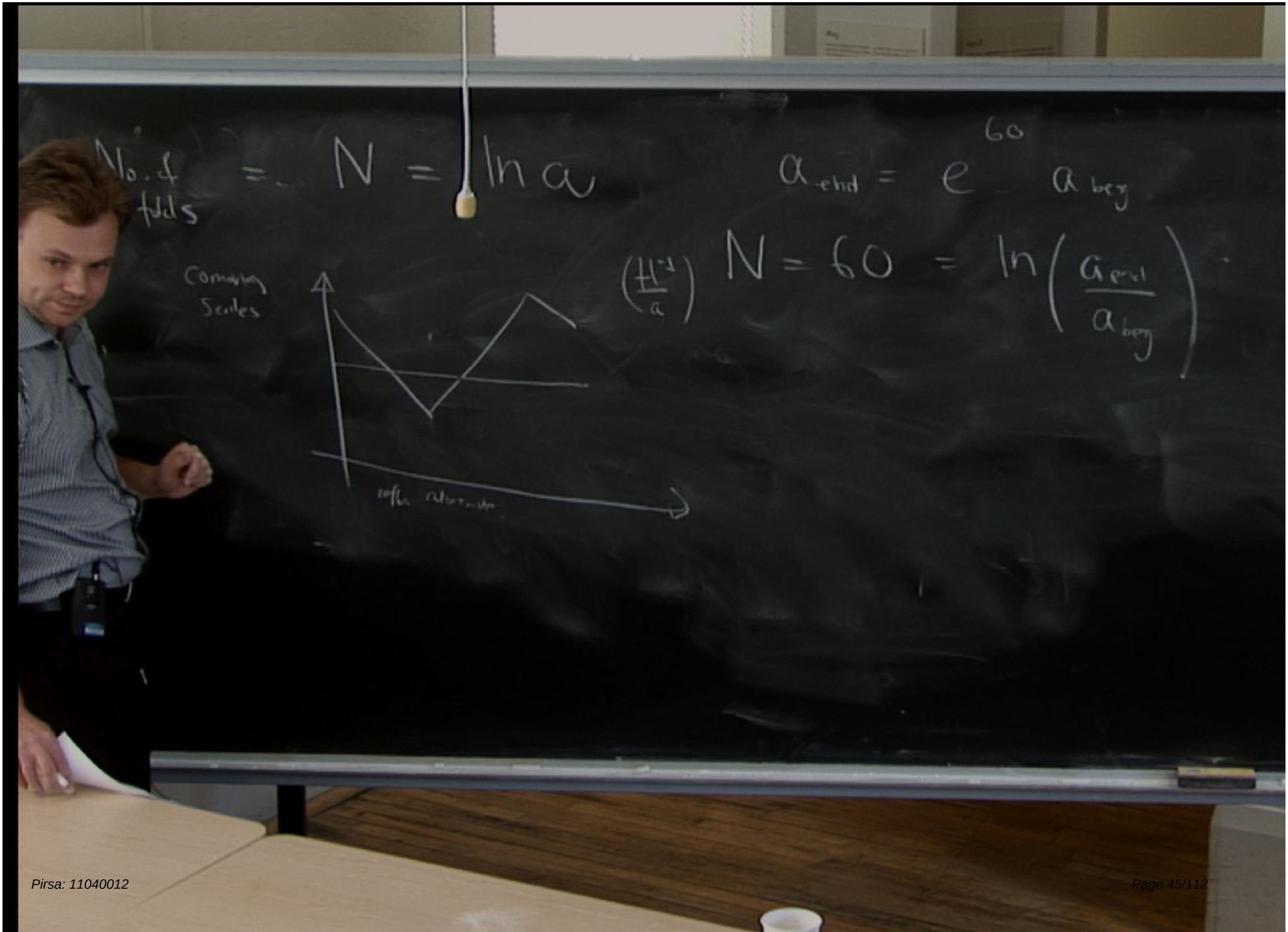
$t = NH^{-1}$

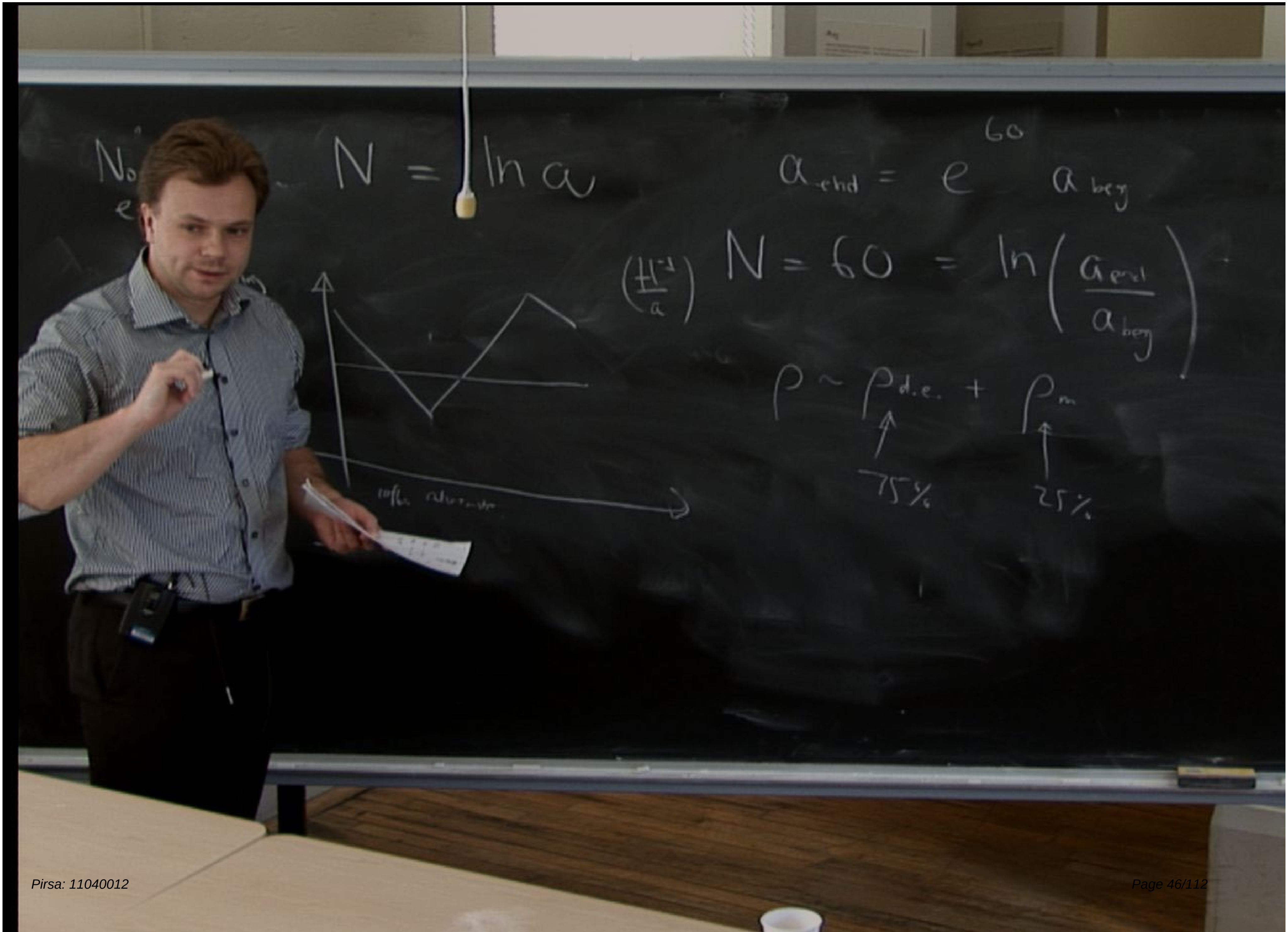
N = No. of e-folds ~ 60

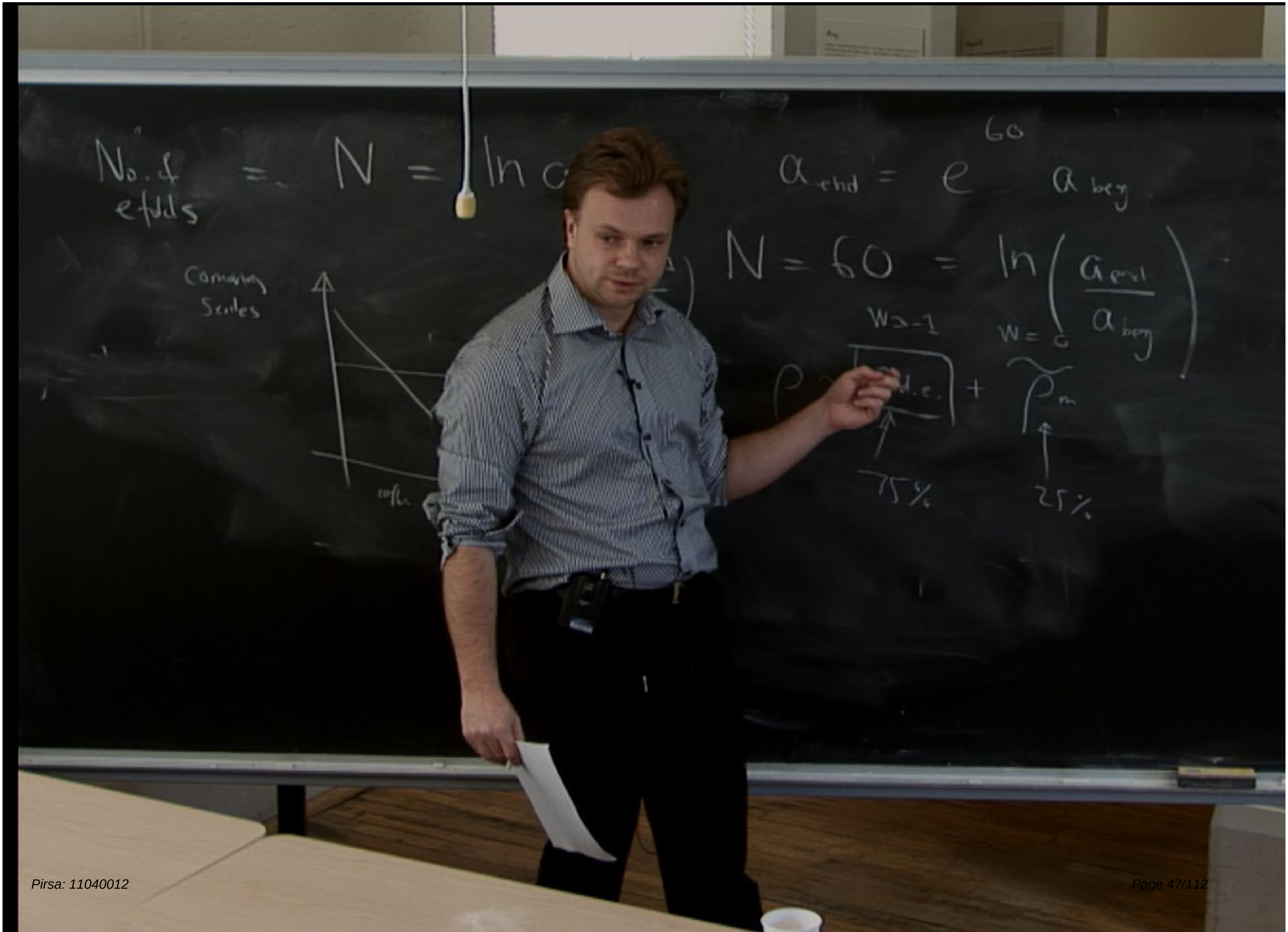
No. of efolds $= N = \ln a$

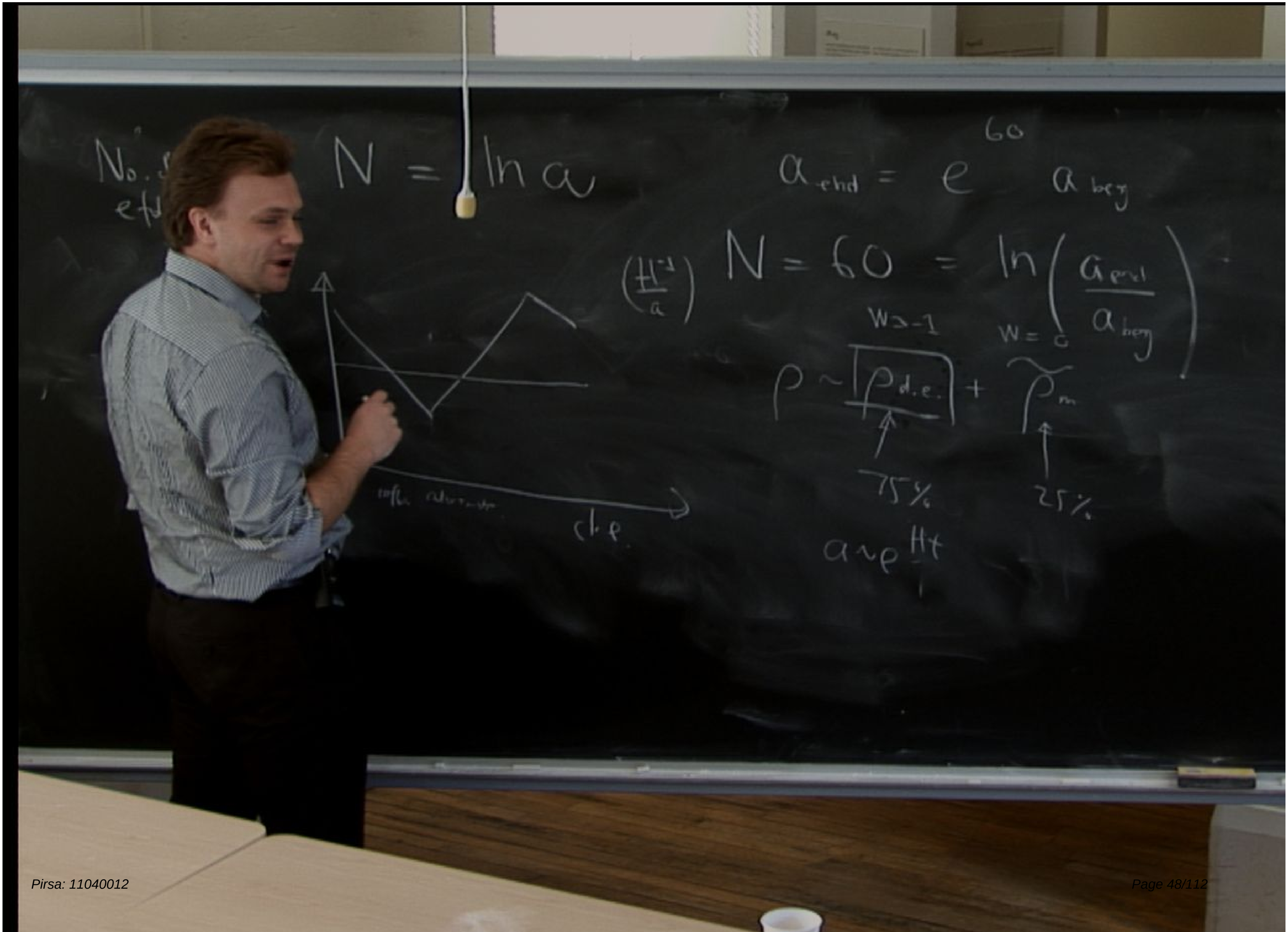
$$a_{end} = e^{60} a_{beg}$$

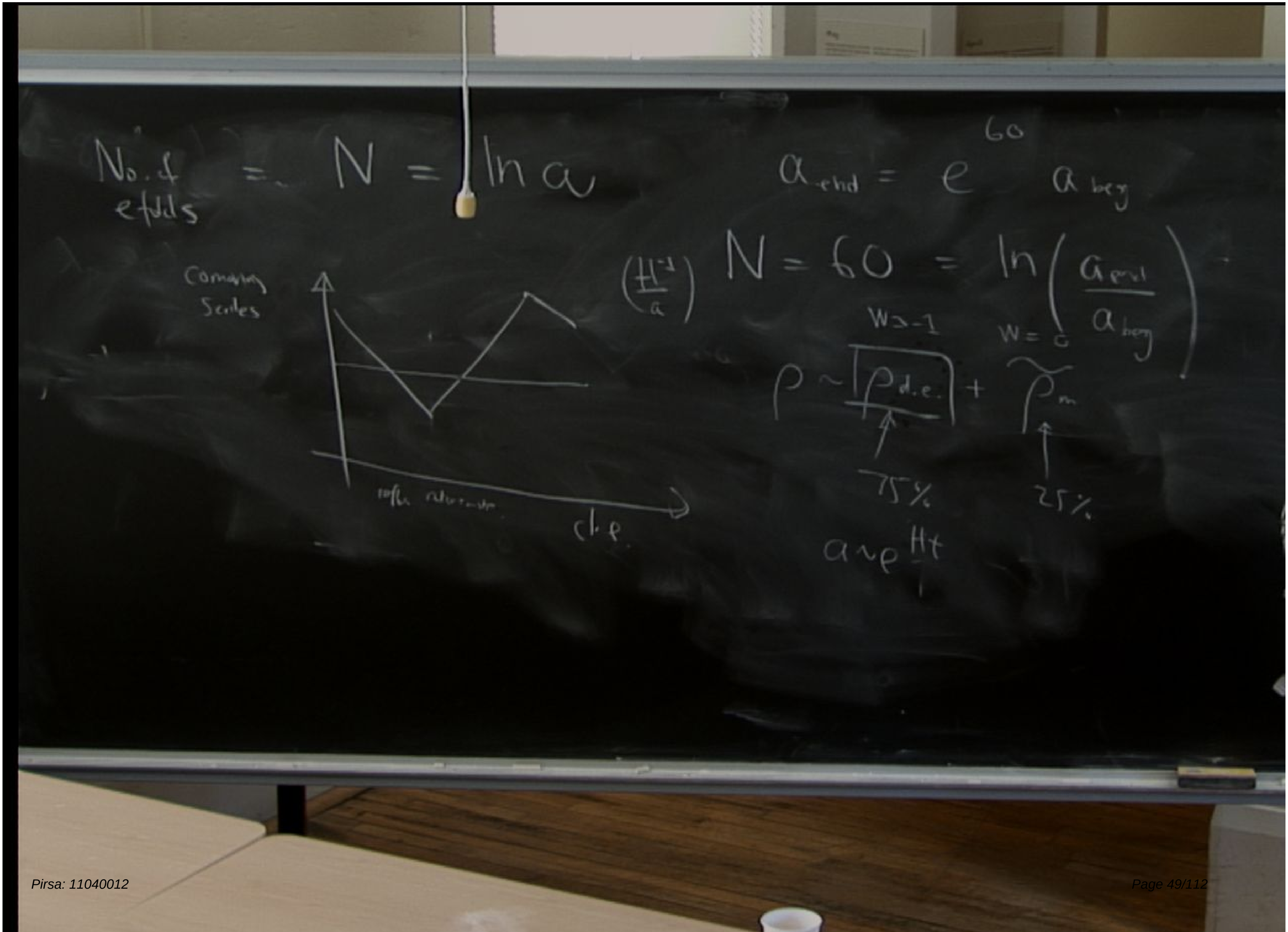
$$N = 60 = \ln\left(\frac{a_{end}}{a_{beg}}\right)$$


No. of folds = N = ln a
a end = e 60 a beg
N = 60 = ln (a end / a beg)
(H^-1 / a)
Comoving Scales


N = ln a
a_end = e^60 a_beg
N = 60 = ln(a_end / a_beg)
ρ ~ ρ_d.e. + ρ_m
75%
25%


No. of efolds = N = ln
$a_{end} = e^{60} a_{beg}$
$N = 60 = \ln\left(\frac{a_{end}}{a_{beg}}\right)$
Comoving Scales
w≈-1
w=0
d.e.
75%
25%


N = ln a
a end = e 60 a beg
N = 60 = ln (a end / a beg)
w = -1
w = 0
ρ ~ ρ d.e. + ρ m
75%
25%


No. of efolds = N = ln a
a_end = e^60 a_beg
N = 60 = ln(a_end / a_beg)
(H^-1 / a)
Coming Series
w = -1
w = 0
ρ ~ ρ_d.e. + ρ_m
75%
25%
a ~ e^Ht

$d^2\Omega$]

$p+\rho$)

$w = \frac{p}{\rho} \simeq$ constant

$\rho(1+w)$

$\rho \sim \frac{1}{a^{3(1+w)}}$

eg if $w=0 \Rightarrow$ matter "dust"

$w=\frac{1}{3}$ radiation

$w<-\frac{1}{3}$ $\quad 1+w<\frac{2}{3}$ $\quad 3(1+w)<2$

Most inflation

$w \simeq -1$

$\rho \simeq$ constant

$a \sim e^{Ht}$

$\frac{k}{a^2}$

$\frac{k/a^2}{\left(\rho_{inflation}/M_{pl}^2\right)} \sim e^{-2Ht}$

$t = NH^{-1}$

N = No. of e-folds ~ 60

No. of efolds $= N = \ln a$

$$a_{end} = e^{60}\, a_{beg}$$

$$= \ln\left(\frac{a_{end}}{a_{beg}}\right)$$

$$\rho \sim \boxed{\underset{75\%}{\overset{w \simeq -1}{\rho_{d.e.}}}} + \underset{25\%}{\overset{w = 0}{\rho_m}}$$

$$a \sim e^{Ht}$$

3 Monopole problem

Matter / dust $\quad w = 0 \qquad \rho_{mon} \sim \frac{1}{a^3}$

$$\frac{\rho_{monopole}}{\rho_{inflaton}} \sim$$

3. Monopole problem

Matter / dust $\quad w = 0 \quad \rho_{mon} \sim \frac{1}{a^3}$

$$\frac{\rho_{monopole}}{\rho_{inflaton}} \sim e^{-3N}$$

3 Monopole problem

Matter / dust $w = 0$ $\quad \rho_{mon} \sim \frac{1}{a^3}$

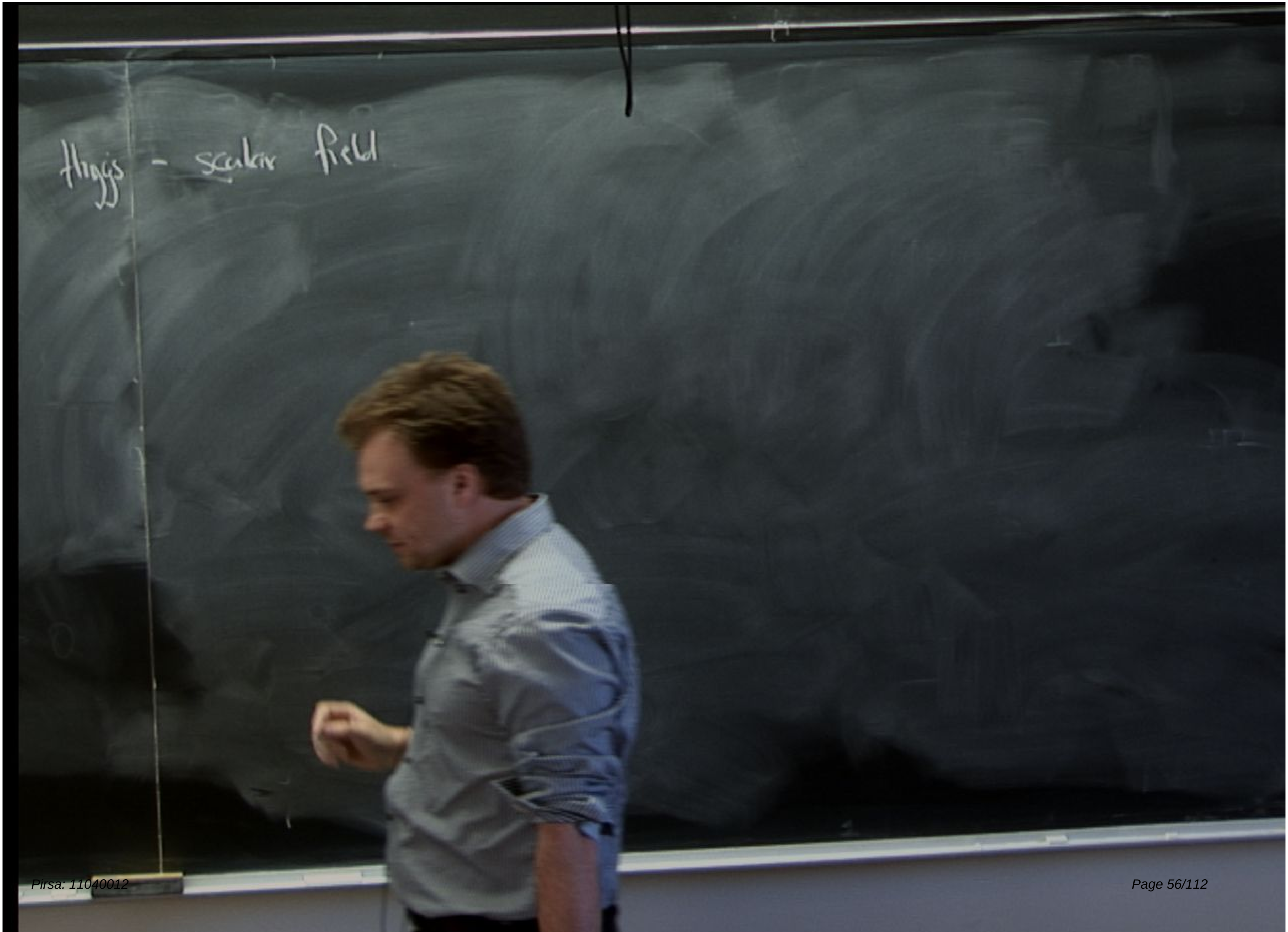
$$\frac{\rho_{monopole}}{\rho_{inflaton}} \sim e^{-3N}$$

3. Monopole problem

Matter / dust $w \simeq 0$ $\rho_{mono} \sim \frac{1}{a^3}$

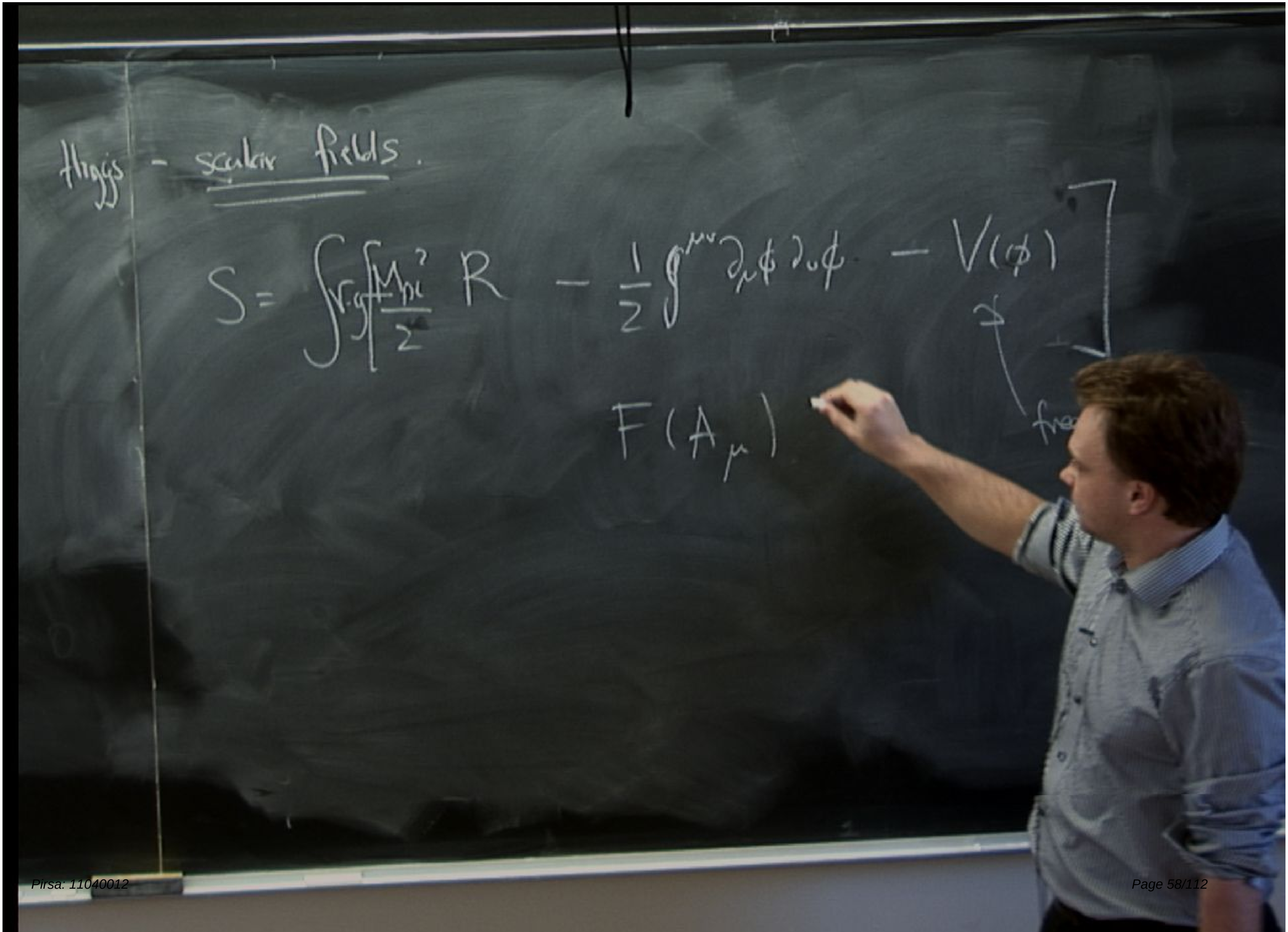
$$\frac{\rho_{\text{monopole}}}{\rho_{\text{inflaton}}} \sim e^{-3N}$$

$w < -\frac{1}{3}$


Higgs - scalar field

Higgs - scalar fields.

$$S = \int \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$


Higgs - scalar fields.
$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$
$F(A_\mu)$

Higgs - scalar fields.

$$S = \int \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu \phi\, \partial_\nu \phi - V(\phi)\right]$$

ϕ

freedom

$\cancel{F(A_\mu)}$

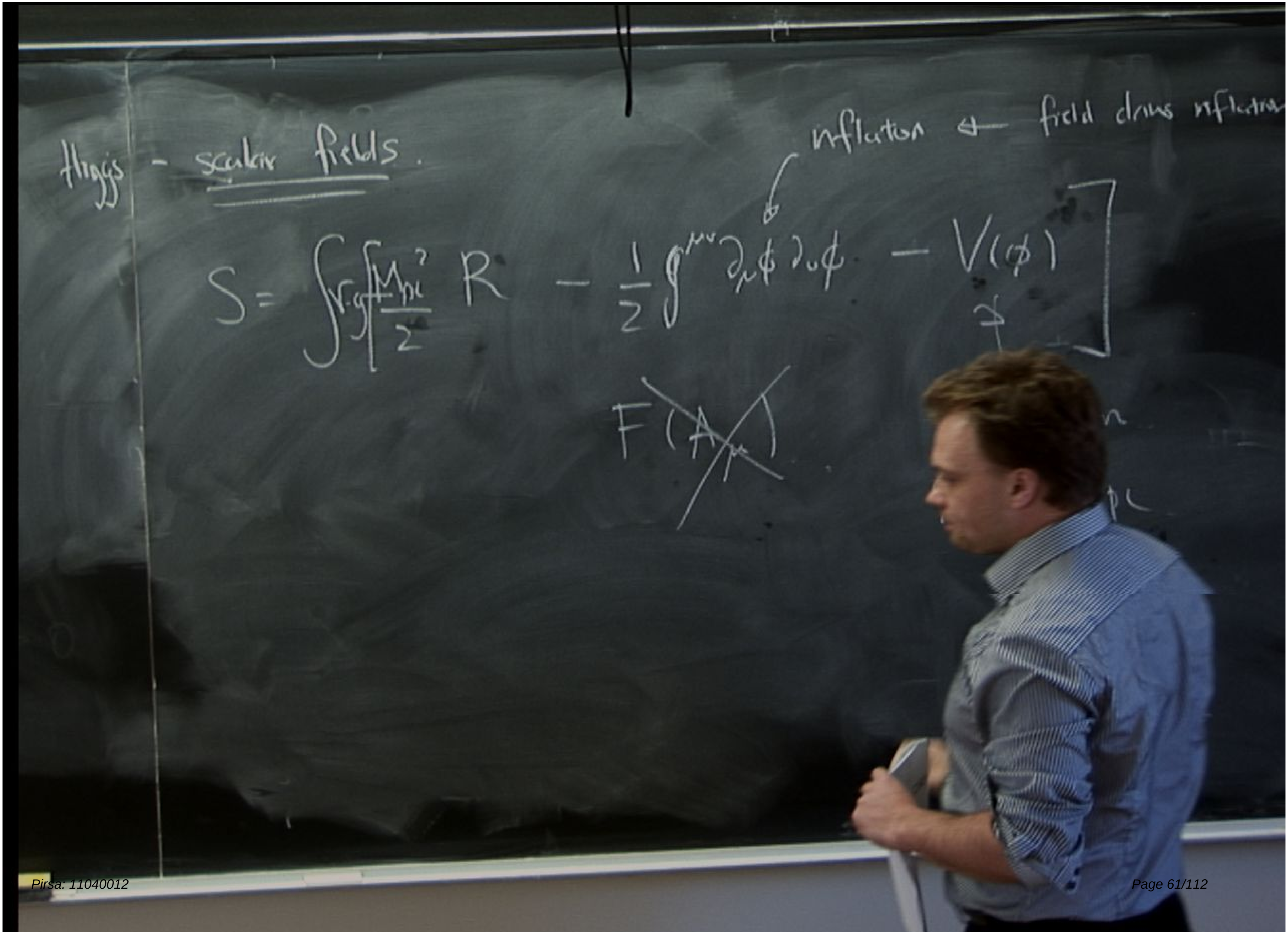
Higgs - scalar fields.

$$S = \int \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

$\cancel{F(A_\mu)}$

freedom

$\phi \gg M_{pl}$



Higgs - scalar fields.
inflaton ← field drives
$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$
$\cancel{F(A_\mu)}$

Higgs - scalar fields.

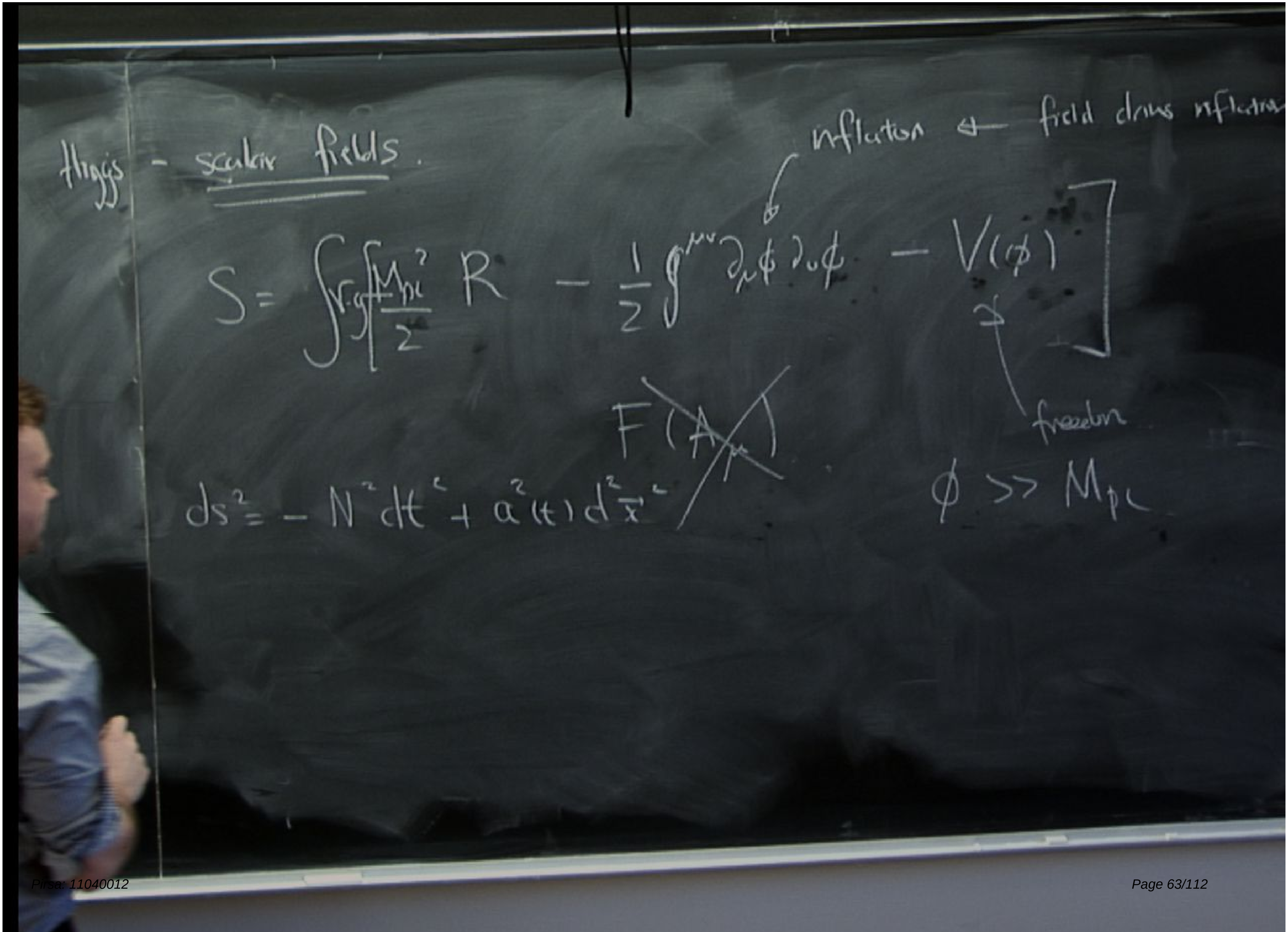
inflaton $\leftarrow$ field drives inflation

$$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

$\cancel{F(A_\mu)}$

freedom

$\phi \gg M_{pl}$


Higgs - scalar fields.
inflaton ← field drives inflation
$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$
$\cancel{F(A_\mu)}$
freedom
$ds^2 = -N^2 dt^2 + a^2(t) d\vec{x}^2$
$\phi >> M_{pl}$

Higgs - scalar fields.

inflaton ← field drives inflation

$$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

$\cancel{F(A_\mu)}$

freedom

$$\phi \gg M_{pl}$$

$$ds^2 = -N^2 dt^2 + a^2(t) \, d\vec{x}^2$$

spatially flat

$k = 0$

Higgs - scalar fields.

inflaton ← field drives inflation

$$S = \int \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

$\cancel{F(A_\mu)}$

freedom

$\phi \gg M_{pl}$

$ds^2 = -N^2(t)dt^2 + a^2(t)\,d\vec{x}^2$

lapse

spatially flat

$k=0$

Higgs – scalar fields.

inflaton ← field drives inflation

$$S = \int \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

freedom

$$\cancel{F(A_\mu)}$$

$$\phi \gg M_{pl}$$

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,d^2\vec{x}^2$$

lapse

spatially flat

$k=0$

cosmic time $N=1$

conformal time $N=a$

Higgs - scalar fields.

inflaton ← field drives inflation

$$S = \int \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

freedom

$$\cancel{F(A_\mu)}$$

$$\phi \gg M_{pl}$$

spatially flat

$k = 0$

$$ds^2 = -N^2(t) dt^2 + a^2(t) d^2\vec{x}^2$$

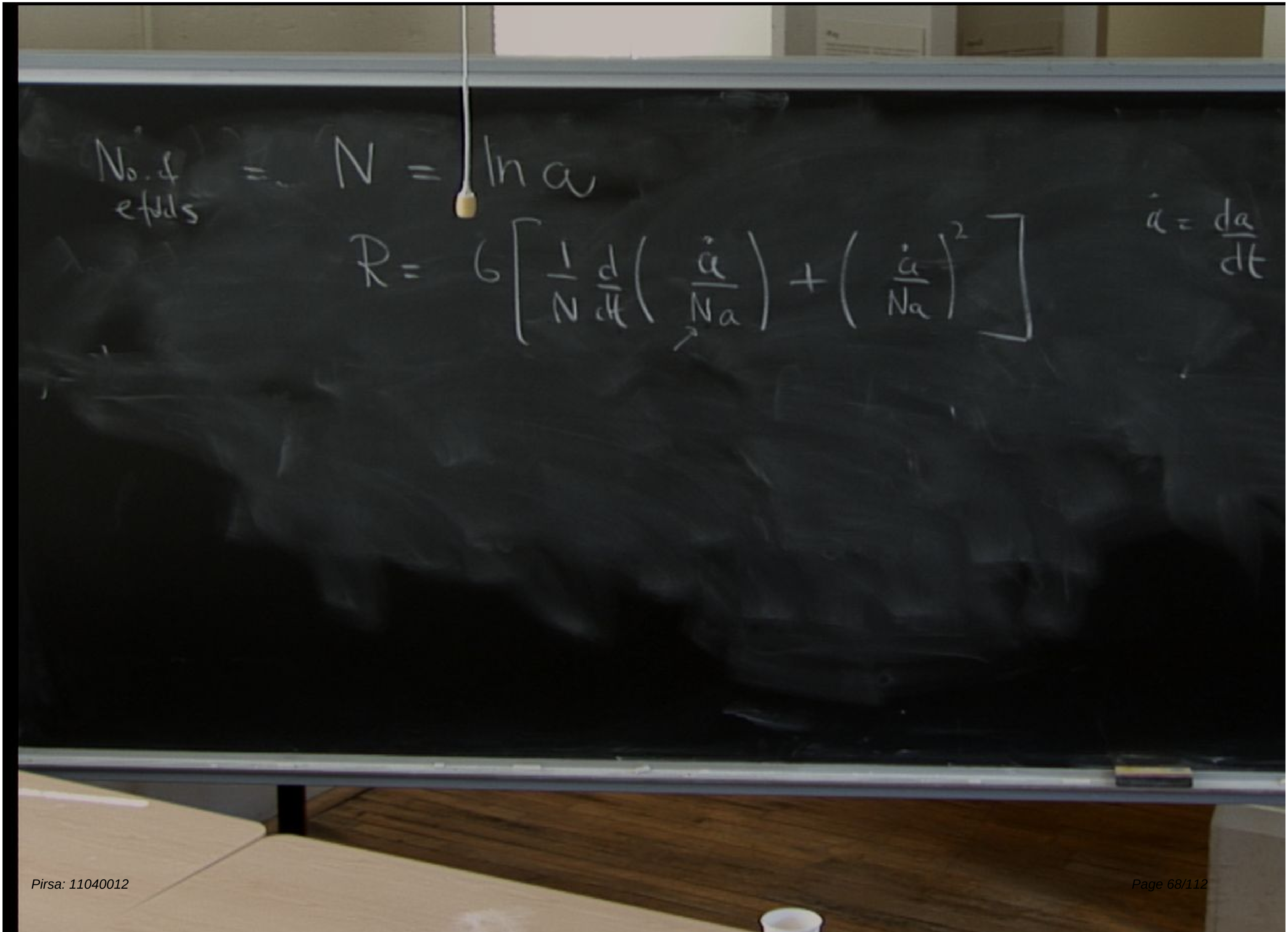
lapse

cosmic time $N = 1$

conformal time $N = a$

$$\phi = \phi(t)$$

'background evolution'

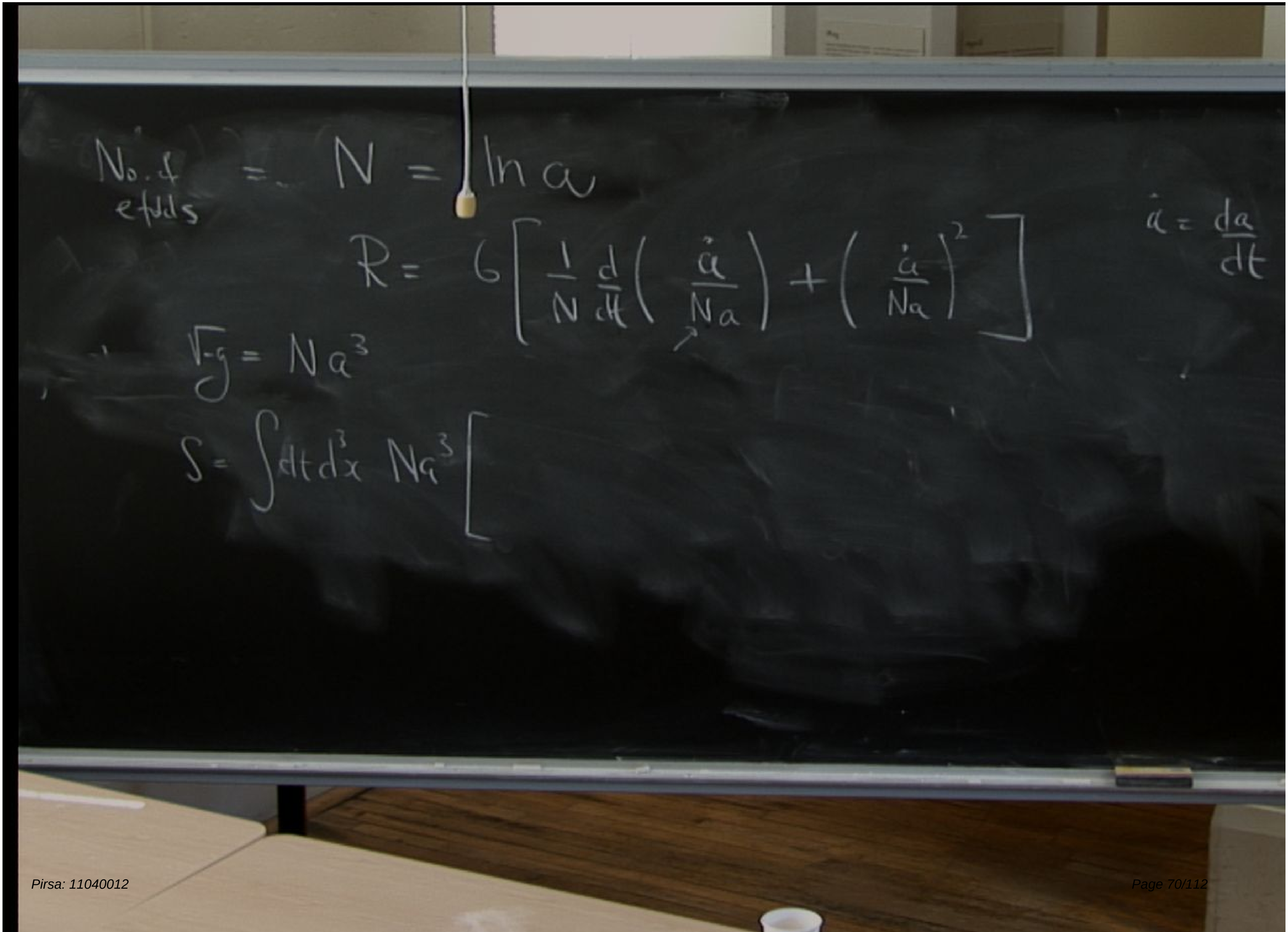

No. of efolds $= N = \ln a$
$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right]$
$\dot{a} = \frac{da}{dt}$

No. of efolds $= N = \ln a$

$$\dot{a} = \frac{da}{dt}$$

$$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right]$$

$$\sqrt{-g} = N a^3$$


No. of efolds $= N = \ln a$
$\dot{a} = \frac{da}{dt}$
$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right]$
$\sqrt{-g} = N a^3$
$S = \int dt\, d^3x\, N a^3 \Big[$

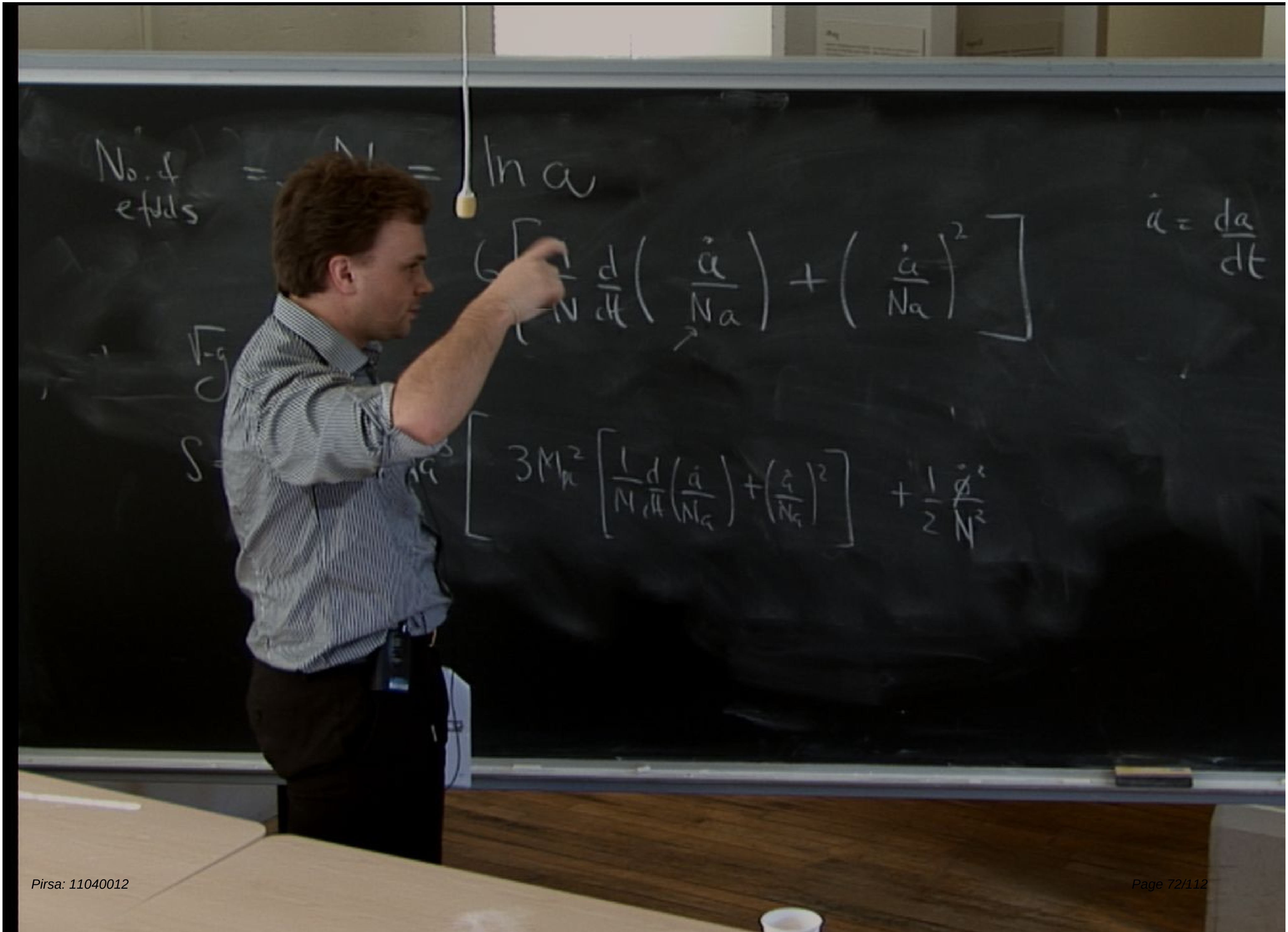
No. of efolds $= \; N = \ln a$

$\dot{a} = \frac{da}{dt}$

$$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right]$$

$$\sqrt{-g} = N a^3$$

$$S = \int dt\, d^3x\, N a^3 \left[3 M_p^2 \left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right] + \frac{1}{2}\frac{\dot{\phi}^2}{a^2}\right.$$


No. of
efolds
ln a
ȧ = da/dt

No. of efolds $= N = \ln a$

$$\dot a = \frac{da}{dt}$$

$$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot a}{Na}\right) + \left(\frac{\dot a}{Na}\right)^2\right]$$

$$\sqrt{-g} = N a^3$$

$$S = \int dt\, d^3x\, N a^3 \left[3M_p^2 \left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot a}{Na}\right) + \left(\frac{\dot a}{Na}\right)^2\right] + \frac{1}{2}\frac{\dot\phi^2}{N^2} - V(\phi) \right]$$

No. of efolds $= N = \ln a$

$$\dot a = \frac{da}{dt}$$

$$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot a}{Na}\right) + \left(\frac{\dot a}{Na}\right)^2\right]$$

$$\sqrt{-g} = N a^3$$

$$S = \int dt\, d^3x\, N a^3 \left[3M_p^2\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot a}{Na}\right) + \left(\frac{\dot a}{Na}\right)^2\right] + \frac{1}{2}\frac{\dot\phi^2}{N^2} - V(\phi)\right]$$

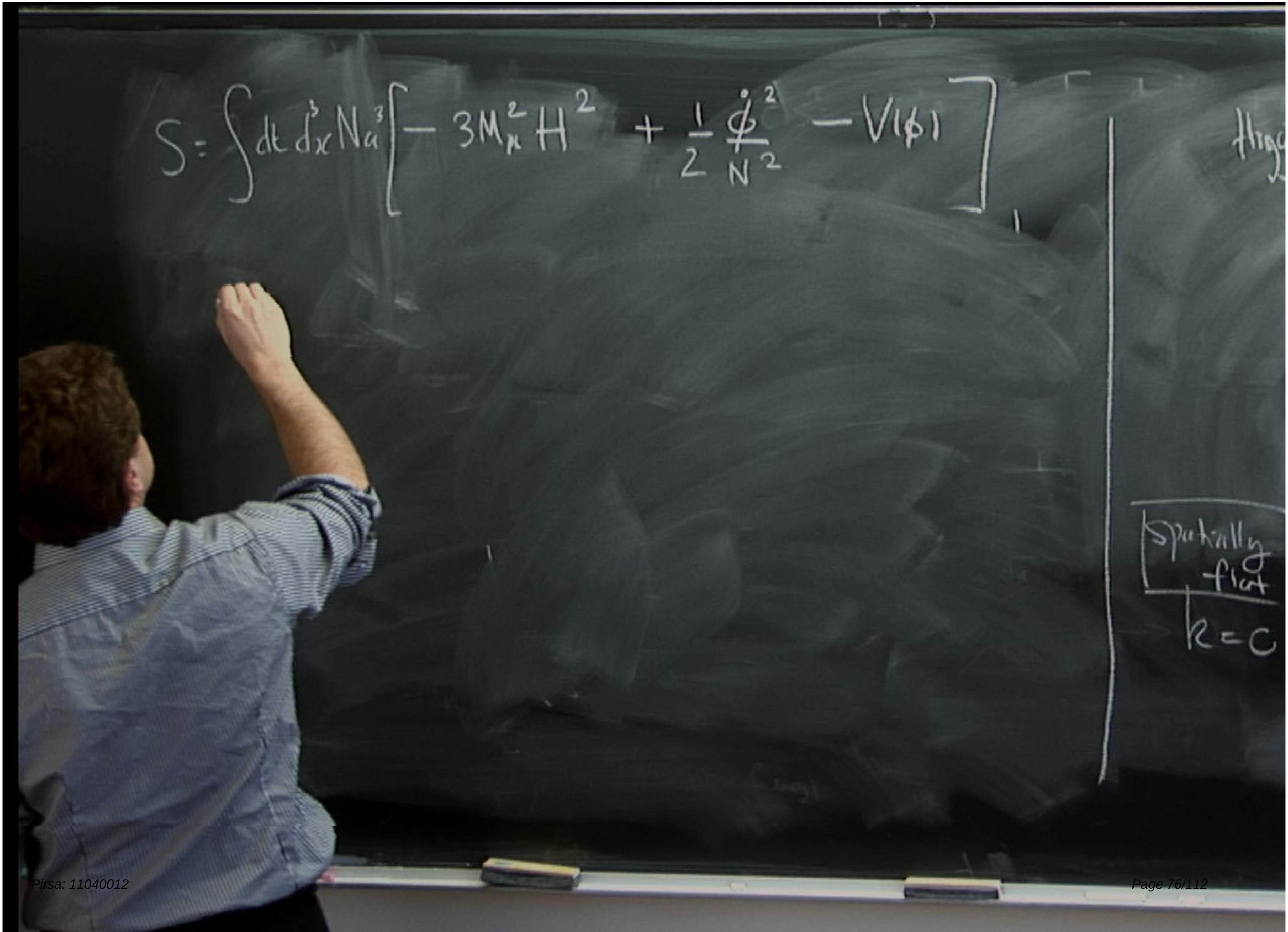
No. of efolds $= N = \ln a$

$\dot{a} = \frac{da}{dt}$

$$R = 6\left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right]$$

$$\sqrt{-g} = N a^3$$

$$S = \int dt\, d^3x\, N a^3 \left[3M_p^2 \left[\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{a}}{Na}\right) + \left(\frac{\dot{a}}{Na}\right)^2\right] + \frac{1}{2}\frac{\dot{\phi}^2}{N^2} - V(\phi) \right]$$


$S = \int dt\, d^3x\, N a^3 \left[-3M_p^2 H^2 + \frac{1}{2}\frac{\dot{\phi}^2}{N^2} - V(\phi) \right]$
Spatially flat

$$S = \int dt\, d^3x\, N a^3 \left[-3 M_{pl}^2 H^2 + \frac{1}{2}\frac{\dot{\phi}^2}{N^2} - V(\phi) \right]$$

$$H = \frac{\dot{a}}{Na}$$ 'Hubble constant'

Spatially flat

$k = 0$

$$S = \int dt\, d^3x\, N a^3 \left[-3M_{pl}^2 H^2 + \frac{1}{2}\frac{\dot{\phi}^2}{N^2} - V(\phi) \right]$$

Hubble

Spatially flat

$k=0$

$$H = \frac{\dot{a}}{Na}$$ 'Hubble constant'

$N dt \to N' dt'$ $\quad t \to t'(t)$

$$S = \int dt\, d^3x\, N a^3 \left[-3M_p^2 H^2 + \frac{1}{2}\frac{\dot{\phi}^2}{N^2} - V(\phi) \right]$$

Minisuperspace action

$$H = \frac{\dot{a}}{Na}$$ 'Hubble constant'

$$N dt \rightarrow N' dt' \qquad t \rightarrow t'(t)$$

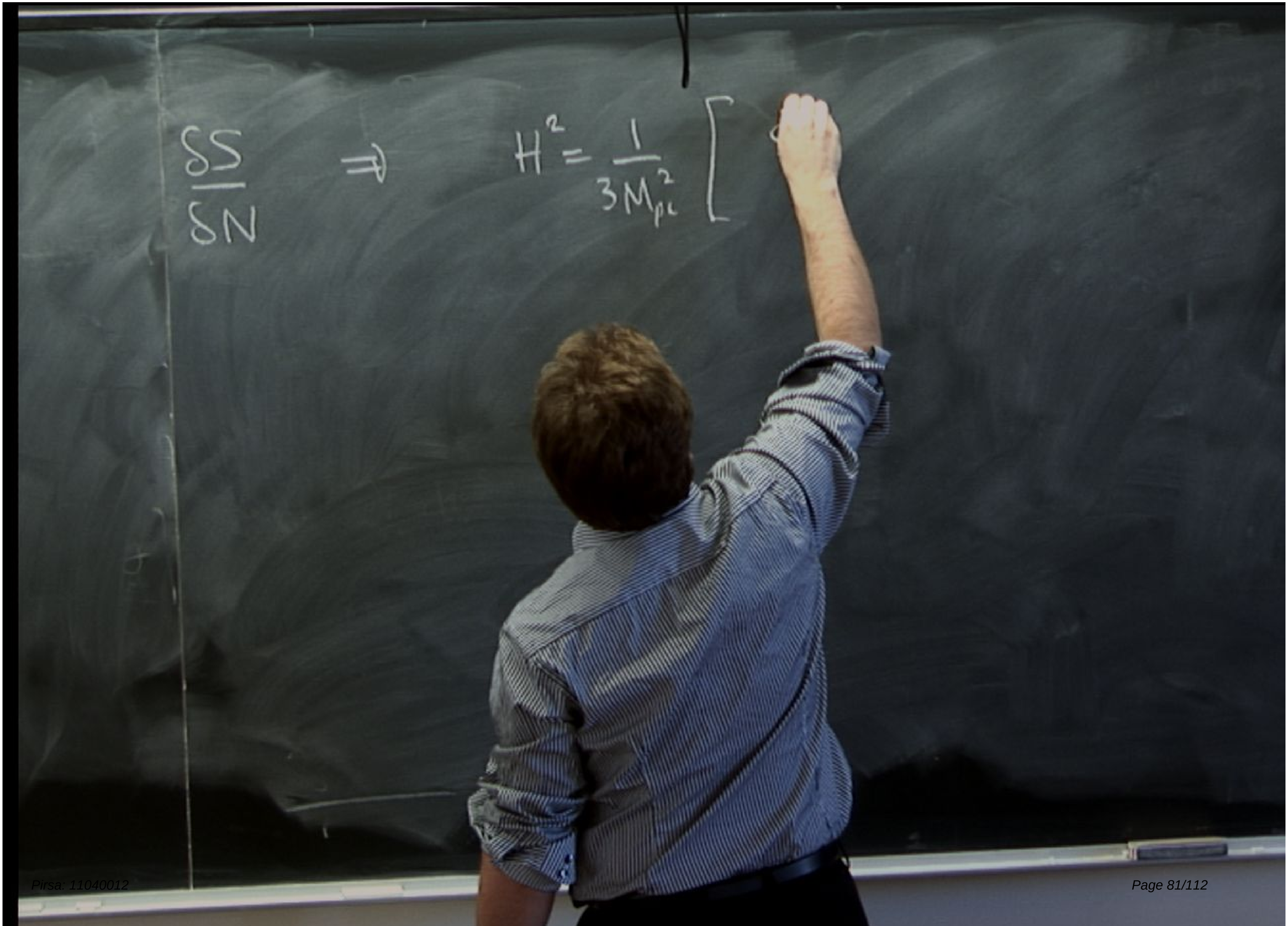
$$H^2 + \frac{1}{2}\frac{\dot\phi^2}{N^2} - V(\phi)\Big]$$

Minisuperspace action

bble constant

dt' $\quad t \to t'(t)$

$$\frac{\delta S}{\delta N}$$


$\frac{\delta S}{\delta N} \Rightarrow H^2 = \frac{1}{3M_{pl}^2}\Big[$

$$\frac{\delta S}{\delta N} \Rightarrow \quad H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2N^2}\dot{\phi}^2 + V(\phi)\right]$$

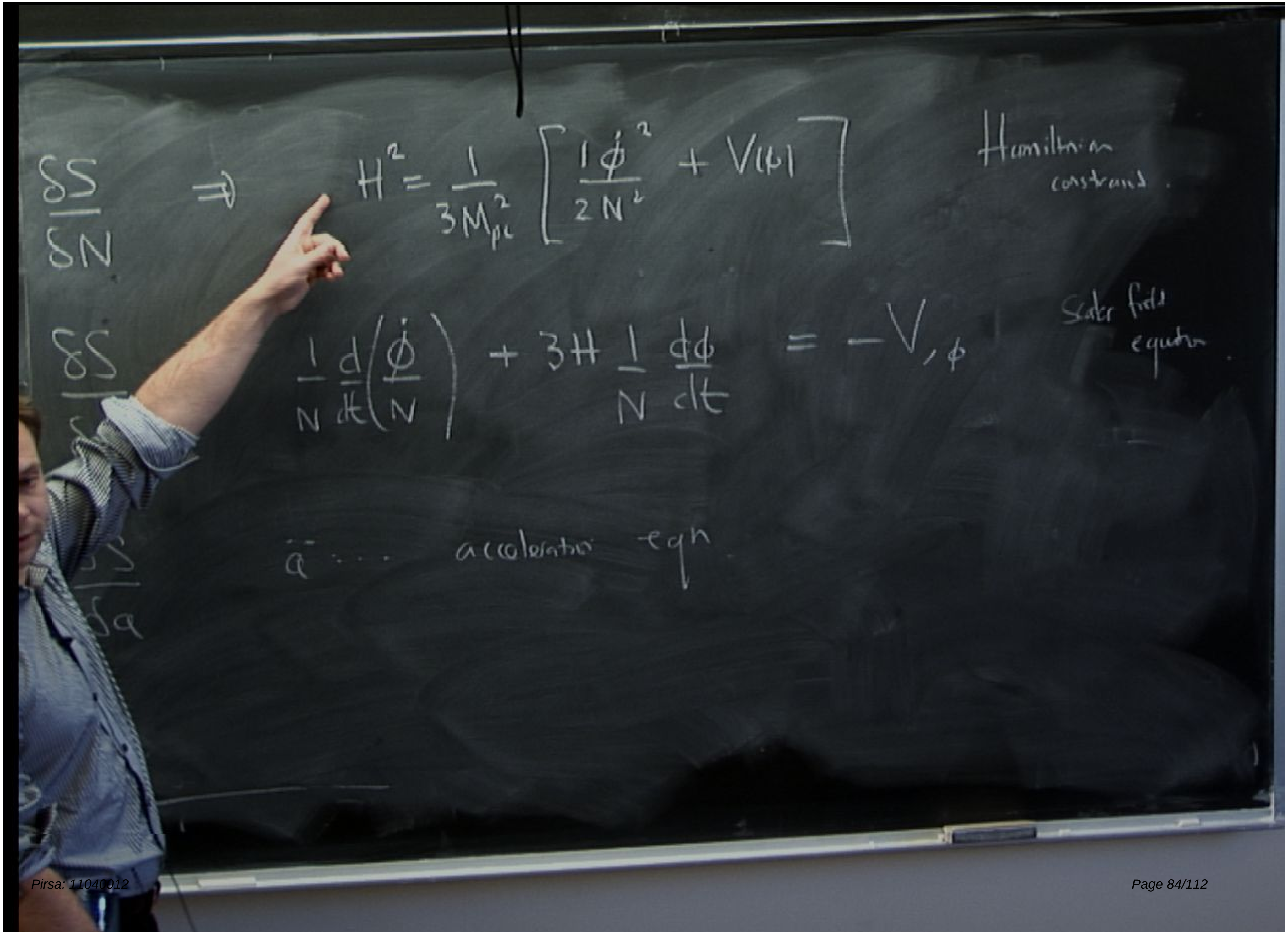
Hamilton
cons

$$\frac{\delta S}{\delta \phi} \qquad \frac{1}{N}\frac{d}{dt}\left(\frac{\dot{\phi}}{N}\right)$$

$$\frac{\delta S}{\delta N} \Rightarrow \qquad H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\frac{\dot{\phi}^2}{N^2} + V(\phi)\right]$$

Hamilton
cons

$$\frac{\delta S}{\delta \phi} \qquad \frac{1}{N}\frac{d}{dt}\left(\frac{\dot{\phi}}{N}\right) + 3H\frac{1}{N}\frac{d\phi}{dt} = -V_{,\phi}$$


$\frac{\delta S}{\delta N} \Rightarrow H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\frac{\dot{\phi}^2}{N^2} + V(\phi)\right]$
Hamiltonian constraint.
$\frac{\delta S}{\delta \phi}$
$\frac{1}{N}\frac{d}{dt}\left(\frac{\dot{\phi}}{N}\right) + 3H\frac{1}{N}\frac{d\phi}{dt} = -V_{,\phi}$
Scalar field equation.
$\frac{\delta S}{\delta a}$
$\ddot{a}$ ... acceleration eqn.

$$\frac{\delta S}{\delta N} \Rightarrow \quad H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2N^2}\dot{\phi}^2 + V(\phi)\right]$$

Hamiltonian constraint.

$$\frac{\delta S}{\delta \phi} \quad \frac{1}{N}\frac{d}{dt}\left(\frac{\dot{\phi}}{N}\right) + 3H\frac{1}{N}\frac{d\phi}{dt} = -V_{,\phi}$$

Scalar field equation.

$$\frac{\delta S}{\delta a} \quad \ddot{a} \ldots$$

acceleration eqn.

$$\frac{\delta S}{\delta N} \Rightarrow \quad H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\frac{\dot{\phi}^2}{N^2} + V(\phi)\right]$$

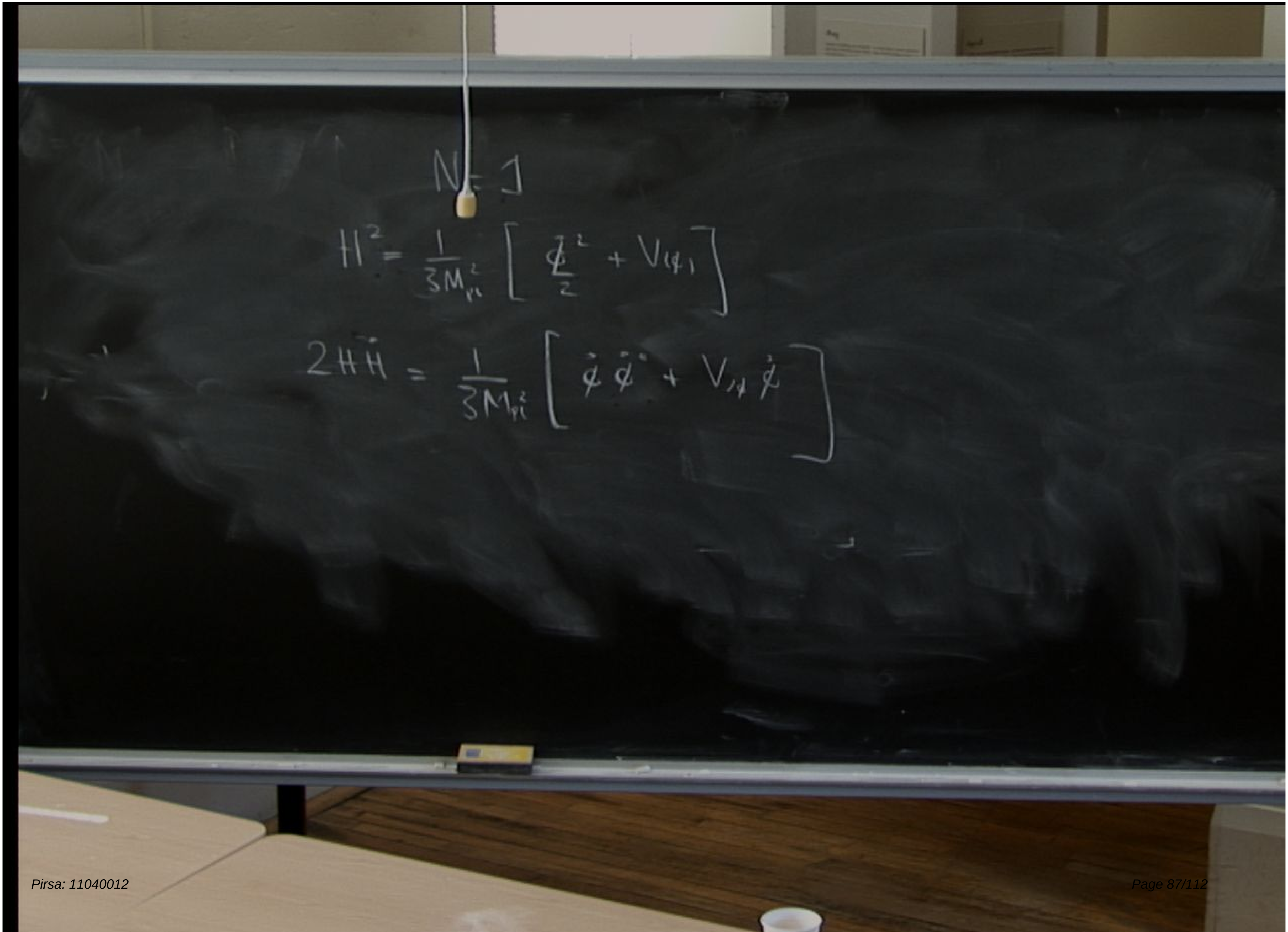
Hamiltonian constraint.

$$\frac{\delta S}{\delta \phi} \quad \frac{1}{N}\frac{d}{dt}\left(\frac{\dot{\phi}}{N}\right) + 3H\frac{1}{N}\frac{d\phi}{dt} = -V_{,\phi}$$

Scalar field equation.

$$\frac{\delta S}{\delta a} \quad \ddot{a} \ldots$$

acceleration eqn


$N = 1$
$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$
$2H\dot{H} = \frac{1}{3M_{pl}^2}\left[\dot{\phi}\ddot{\phi} + V_{,\phi}\dot{\phi}\right]$

$$N = 1$$

$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{\dot\phi^2}{2} + V(\phi)\right]$$

$$2H\dot H = \frac{1}{3M_{pl}^2}\left[\dot\phi\ddot\phi + V_{,\phi}\dot\phi\right] \qquad \ddot\phi + 3H\dot\phi = -V_{,\phi}$$

$$= \frac{1}{3M_{pl}^2}\left[-3H\dot\phi^2\right] = -\frac{H\dot\phi^2}{M_{pl}^2} \qquad \boxed{\dot H = -\frac{\dot\phi^2}{2M_{pl}^2}}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi).$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w < -\frac{1}{3} \qquad \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}$$

$$\boxed{\dot{\phi}^2 \ll V(\phi)}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi).$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w < -\frac{1}{3} \qquad \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}$$

$$\boxed{\dot{\phi}^2 \ll V(\phi)}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w < -\frac{1}{3} \qquad \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3} \qquad w \approx -1$$

$$\boxed{\dot{\phi}^2 \ll V(\phi)}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi).$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

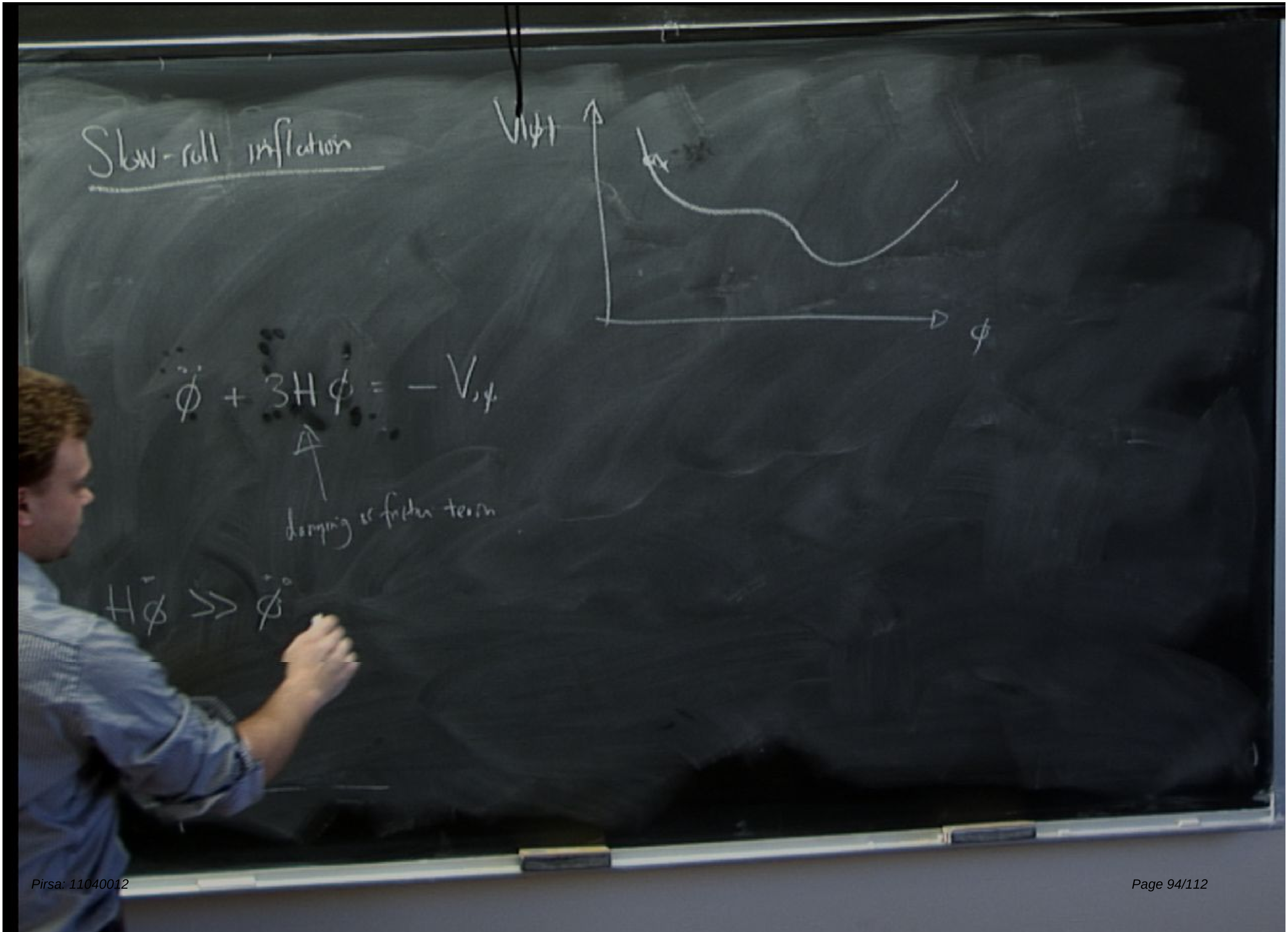
$$w < -\frac{1}{3} \qquad \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3} \qquad w \approx -1.$$

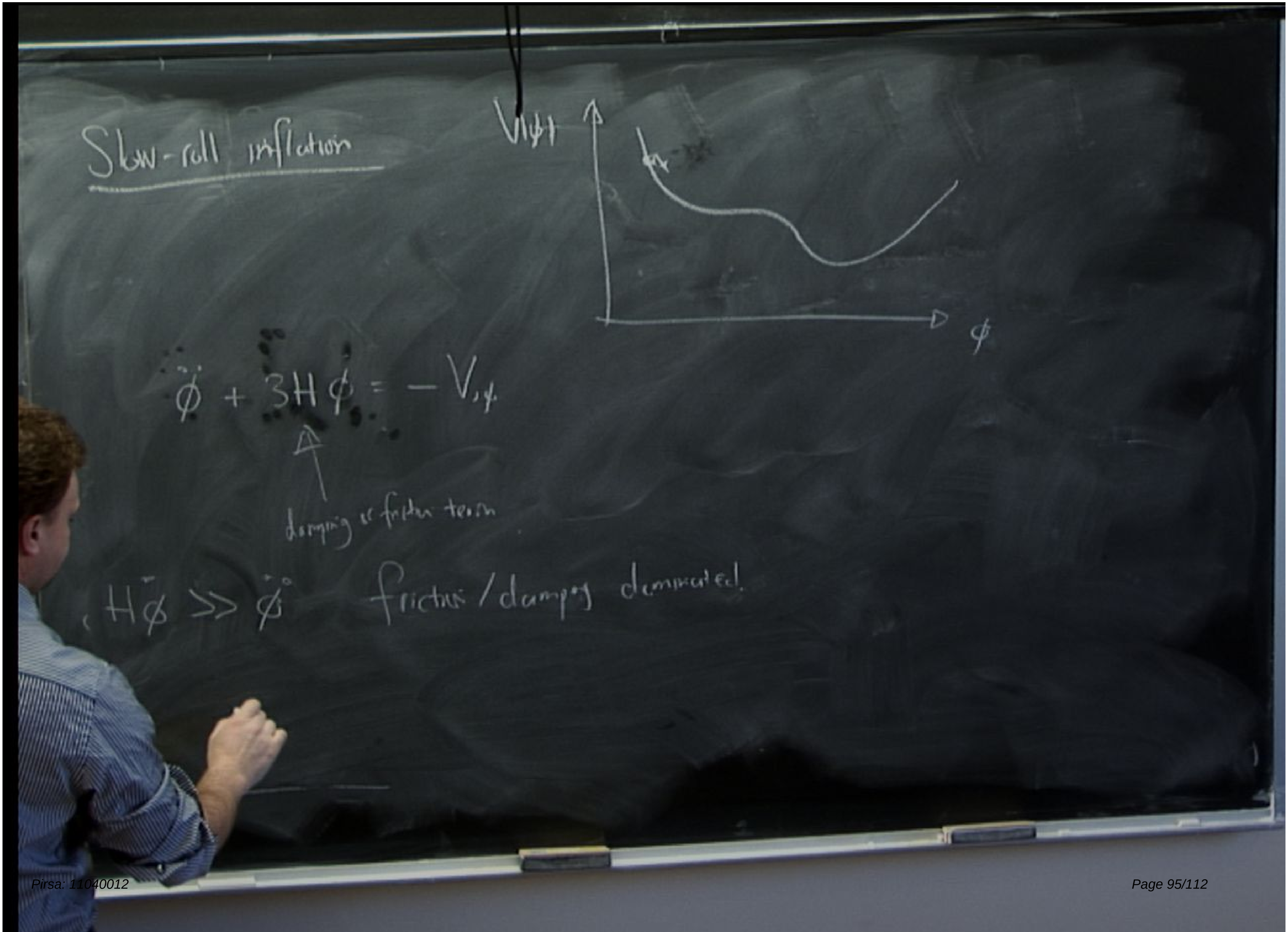
$$\boxed{\dot{\phi}^2 \ll V(\phi)}$$

Slow-roll inflation

$$\ddot{\phi} + \cdots = -V_{,\phi}$$

damping or friction term


Slow-roll inflation
V(φ)
φ
$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}$
damping or friction term
$H\dot{\phi} \gg \ddot{\phi}$



Slow-roll inflation
V(φ)
φ
φ̈ + 3Hφ̇ = −V,φ
damping or friction term
Hφ̇ >> φ̈
friction / damping dominated!

Slow-roll inflation

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}$$

damping or friction term

$$H\dot{\phi} \gg \ddot{\phi}$$ friction/damping dominated

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H}$$

Slow-roll inflation

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}$$

damping or friction term

$$H\dot{\phi} \gg \ddot{\phi} \quad \text{friction / damping dominated}$$

$$\dot{\phi} \cong -\frac{V_{,\phi}}{3H}$$

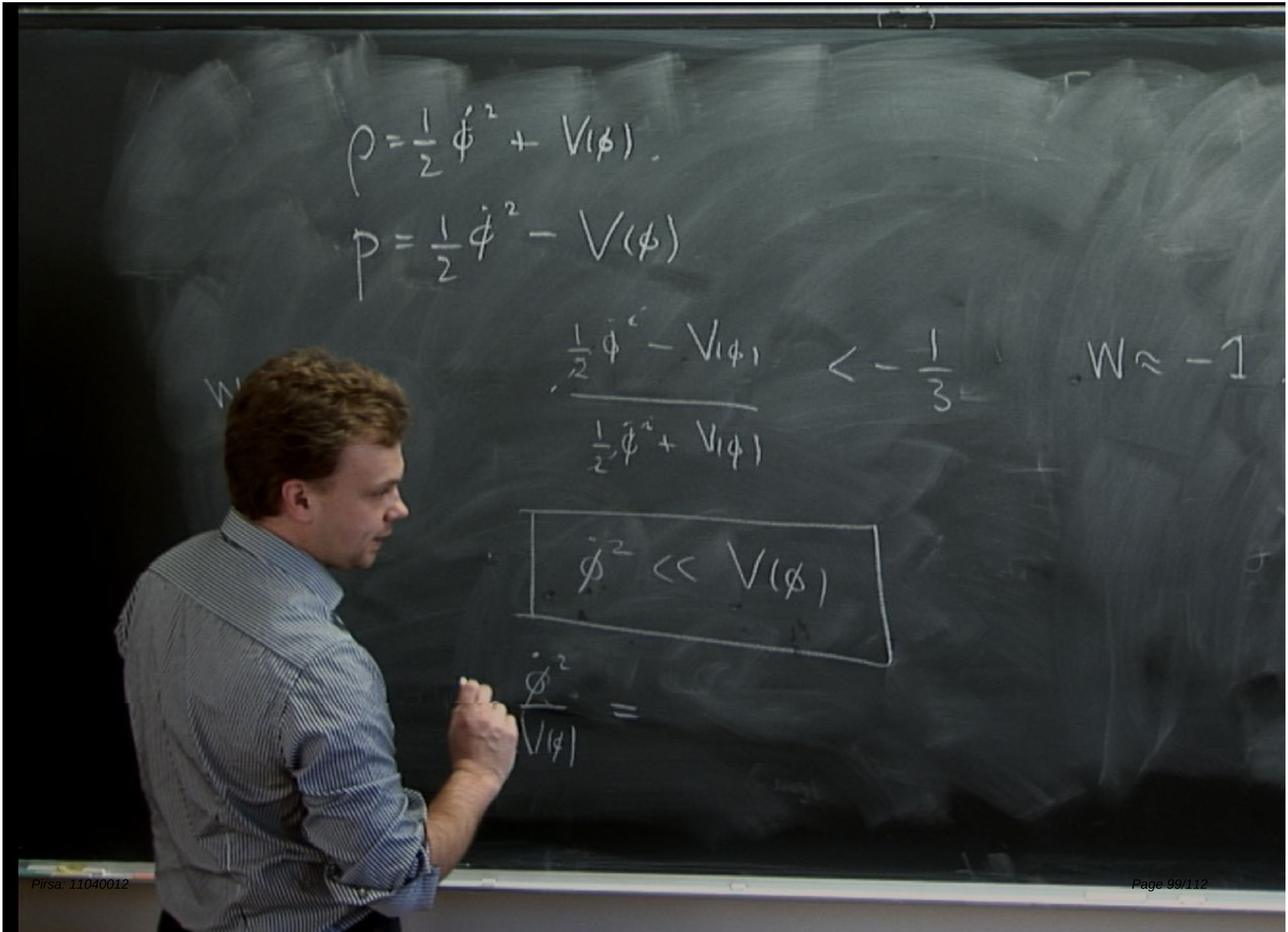
Slow-roll inflation

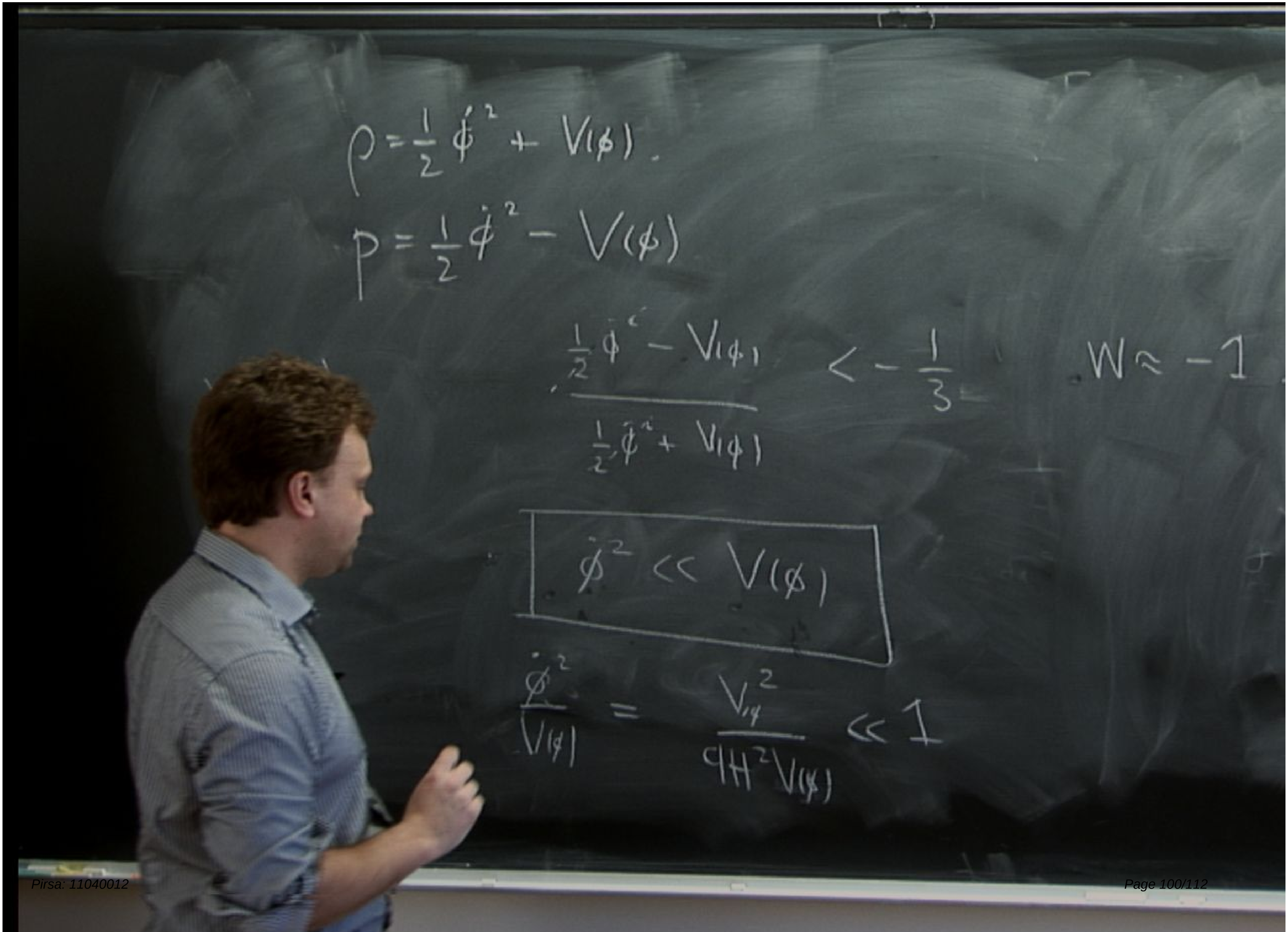
$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}$$

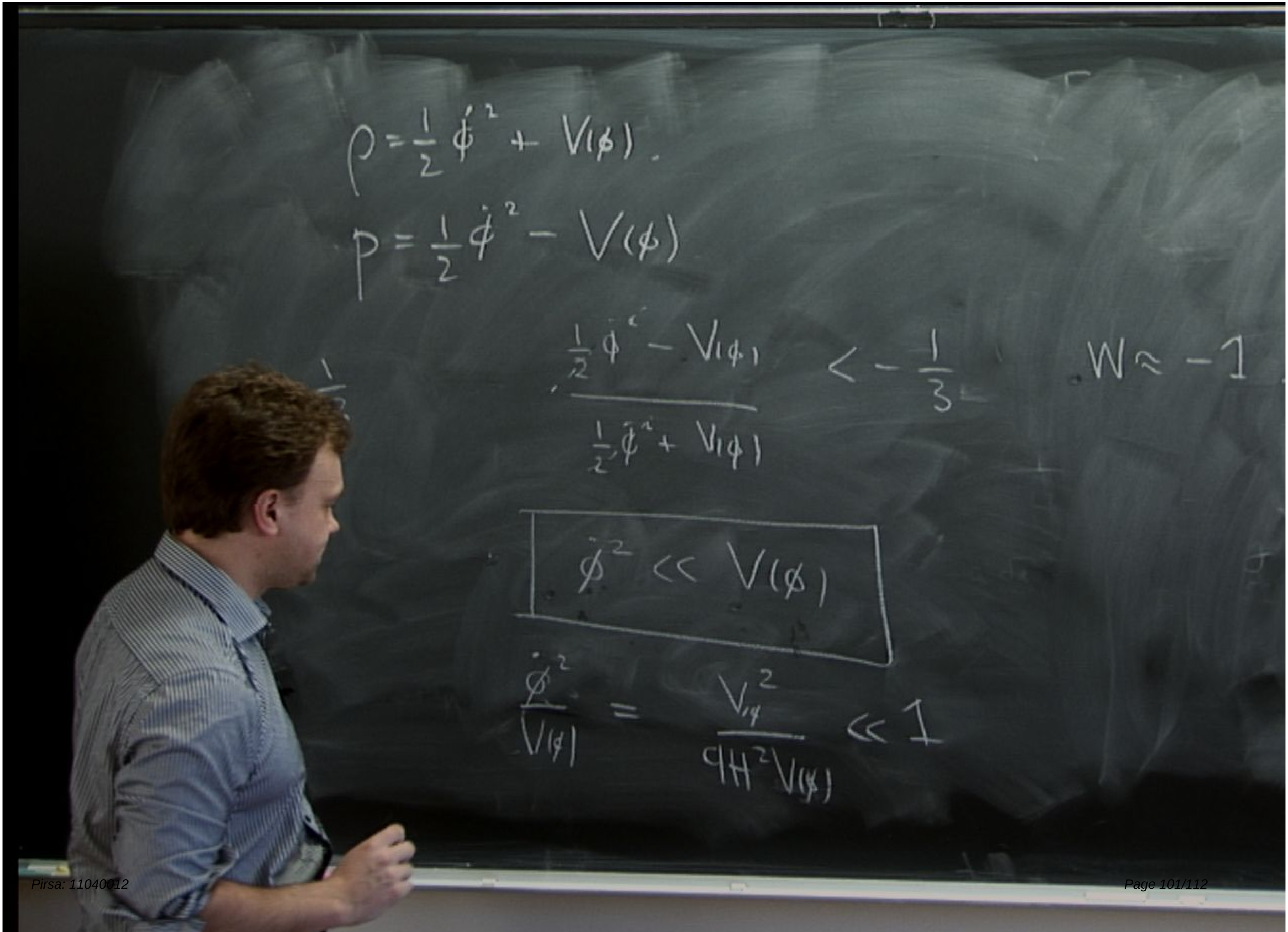
damping or friction term

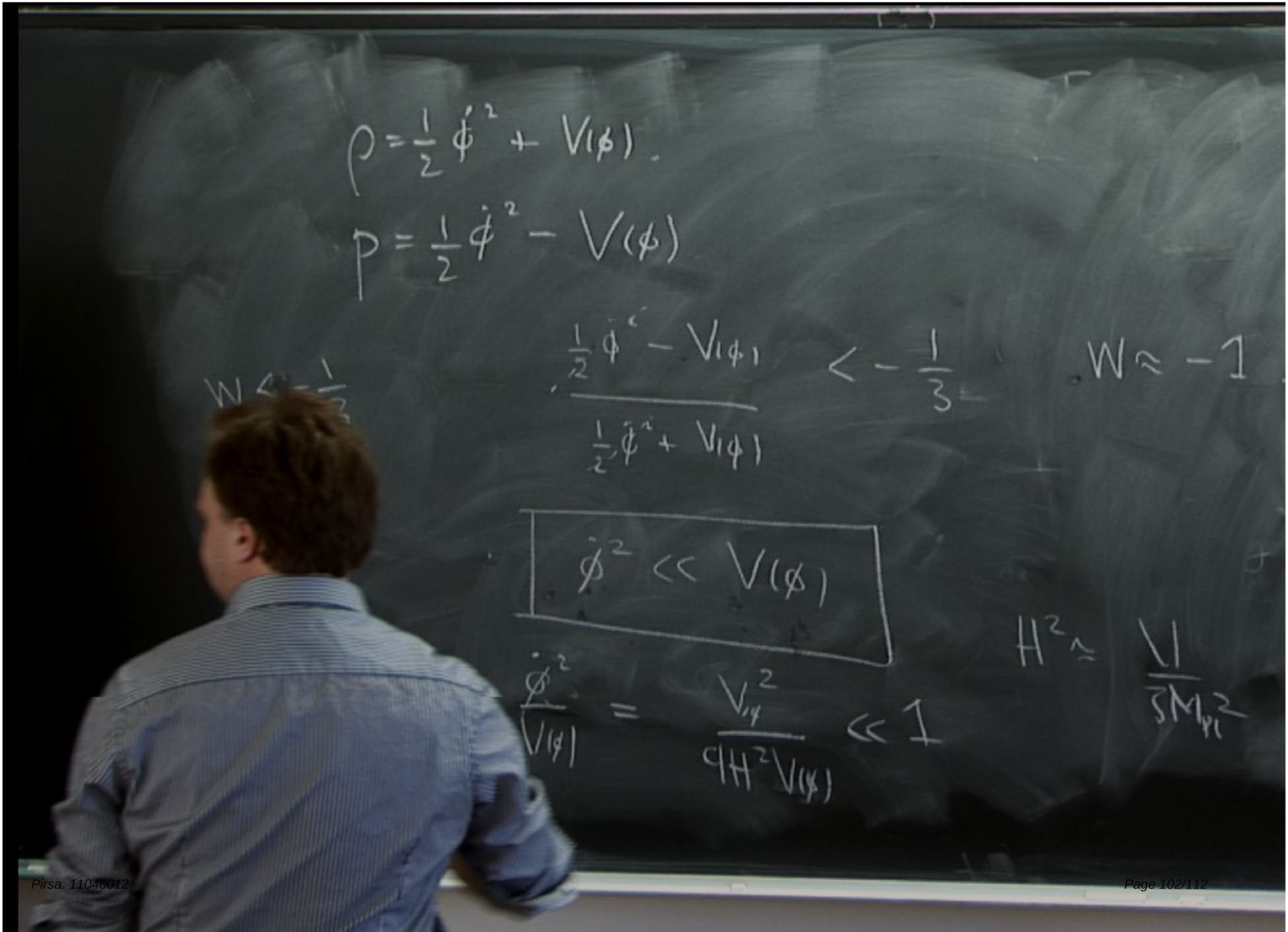
$H\dot{\phi} \gg \ddot{\phi}$ friction / damping dominated

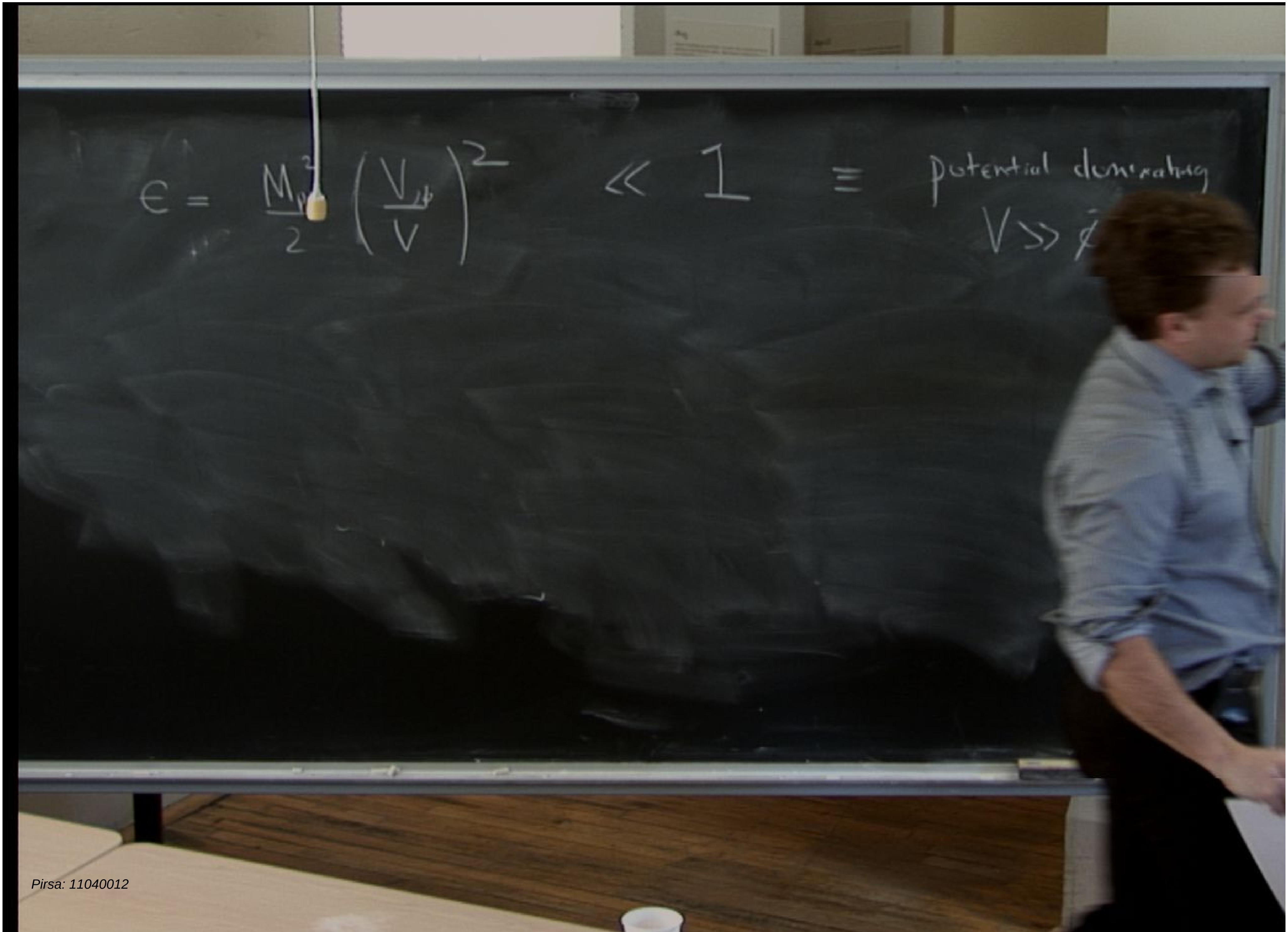
$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H}$$

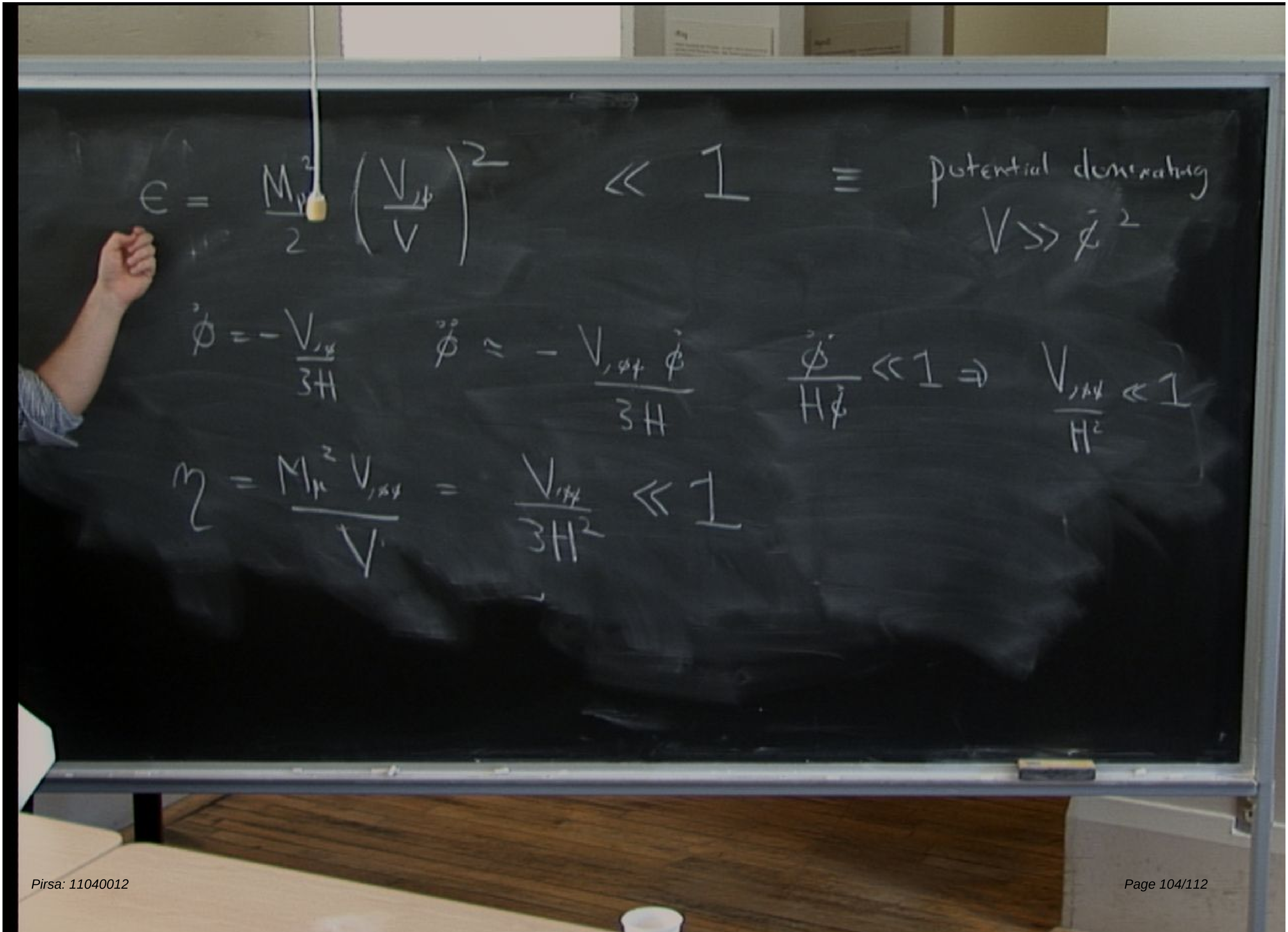

$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
$\frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}$
$w \approx -1$
$\boxed{\dot{\phi}^2 \ll V(\phi)}$
$\frac{\dot{\phi}^2}{V(\phi)} =$


$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
$\frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}$
$w \approx -1$
$\boxed{\dot{\phi}^2 \ll V(\phi)}$
$\frac{\dot{\phi}^2}{V(\phi)} = \frac{V_\phi^2}{9H^2 V(\phi)} \ll 1$


$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
$\frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}$
$w \approx -1$
$\boxed{\dot{\phi}^2 \ll V(\phi)}$
$\frac{\dot{\phi}^2}{V(\phi)} = \frac{V_{,\phi}^2}{9H^2 V(\phi)} \ll 1$


$\rho = \frac{1}{2}\dot\phi^2 + V(\phi)$
$p = \frac{1}{2}\dot\phi^2 - V(\phi)$
$w < -\frac{1}{3}$
$\frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} < -\frac{1}{3}$
$w \approx -1$
$\boxed{\dot\phi^2 \ll V(\phi)}$
$H^2 \simeq \frac{V}{3M_{pl}^2}$
$\frac{\dot\phi^2}{V(\phi)} = \frac{V_\phi^2}{9H^2V(\phi)} \ll 1$


$\epsilon = \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2 \ll 1 \equiv$ potential dominating
$V \gg \dot{\phi}$

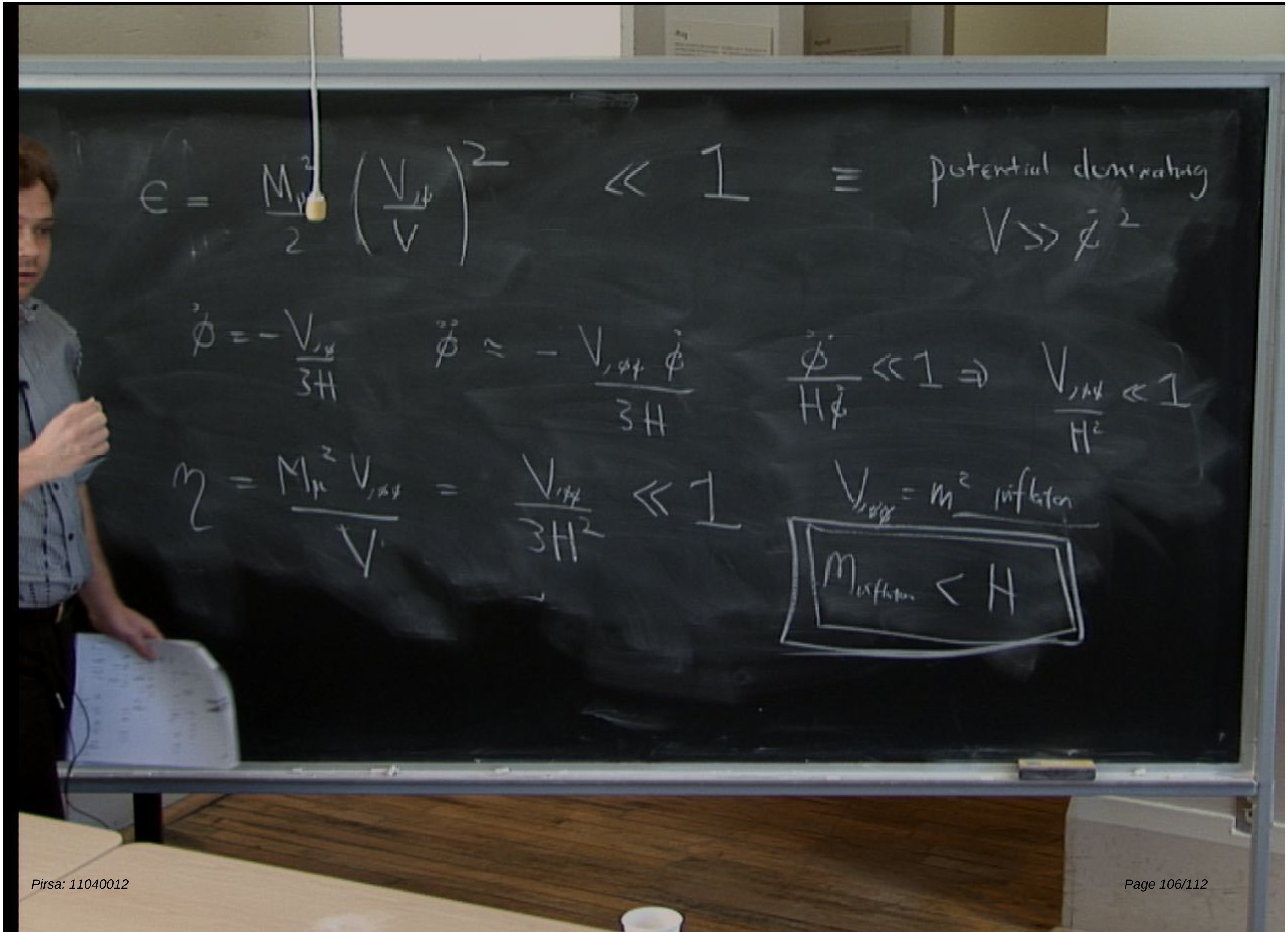

$\epsilon = \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \equiv$ potential dominating
$V \gg \dot{\phi}^2$
$\dot{\phi} = -\frac{V_{,\phi}}{3H}$
$\ddot{\phi} \simeq -\frac{V_{,\phi\phi}\dot{\phi}}{3H}$
$\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \Rightarrow \frac{V_{,\phi\phi}}{H^2} \ll 1$
$\eta = \frac{M_p^2 V_{,\phi\phi}}{V} = \frac{V_{,\phi\phi}}{3H^2} \ll 1$

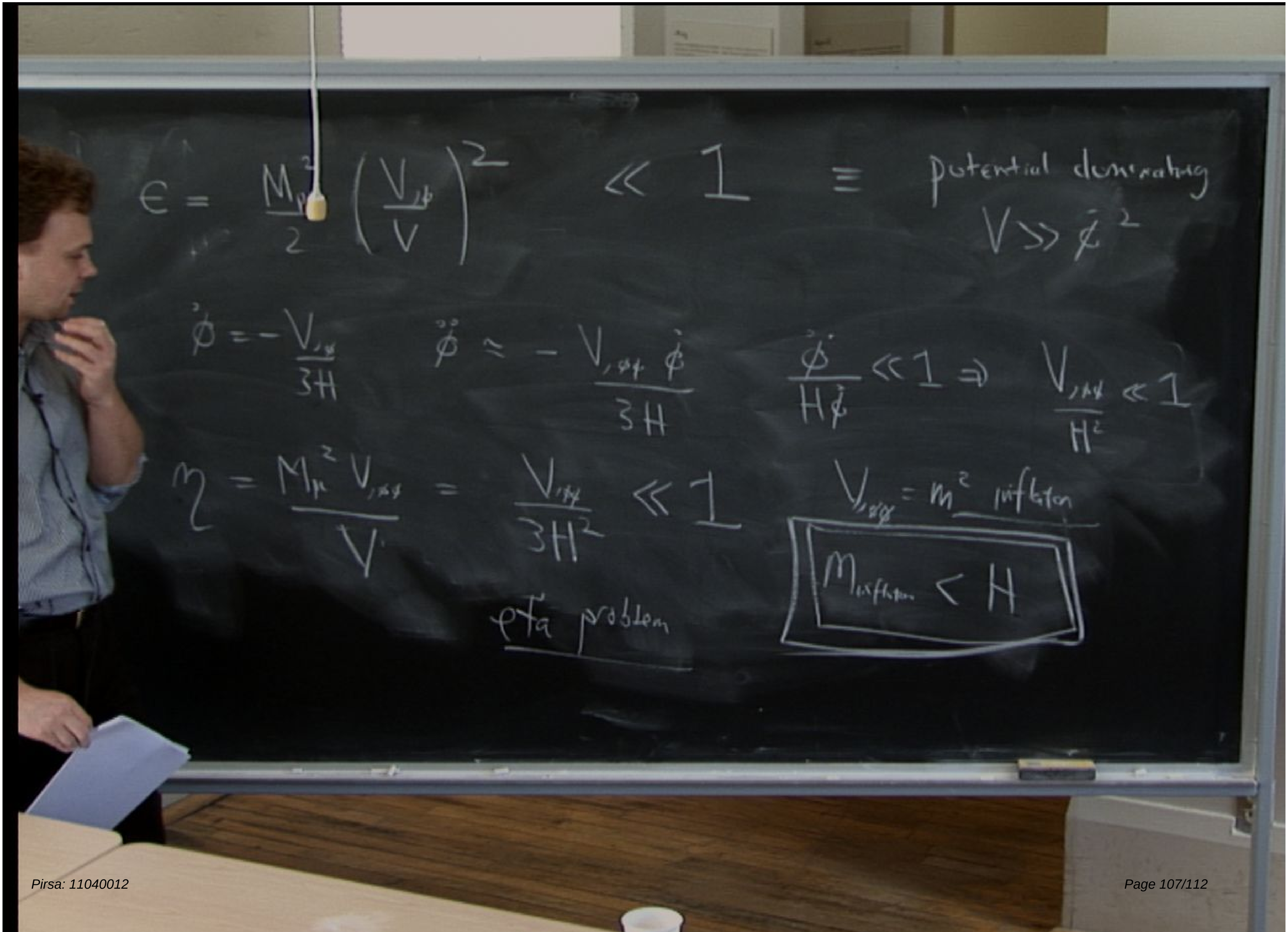
$$\epsilon = \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \equiv \text{potential dominating}$$

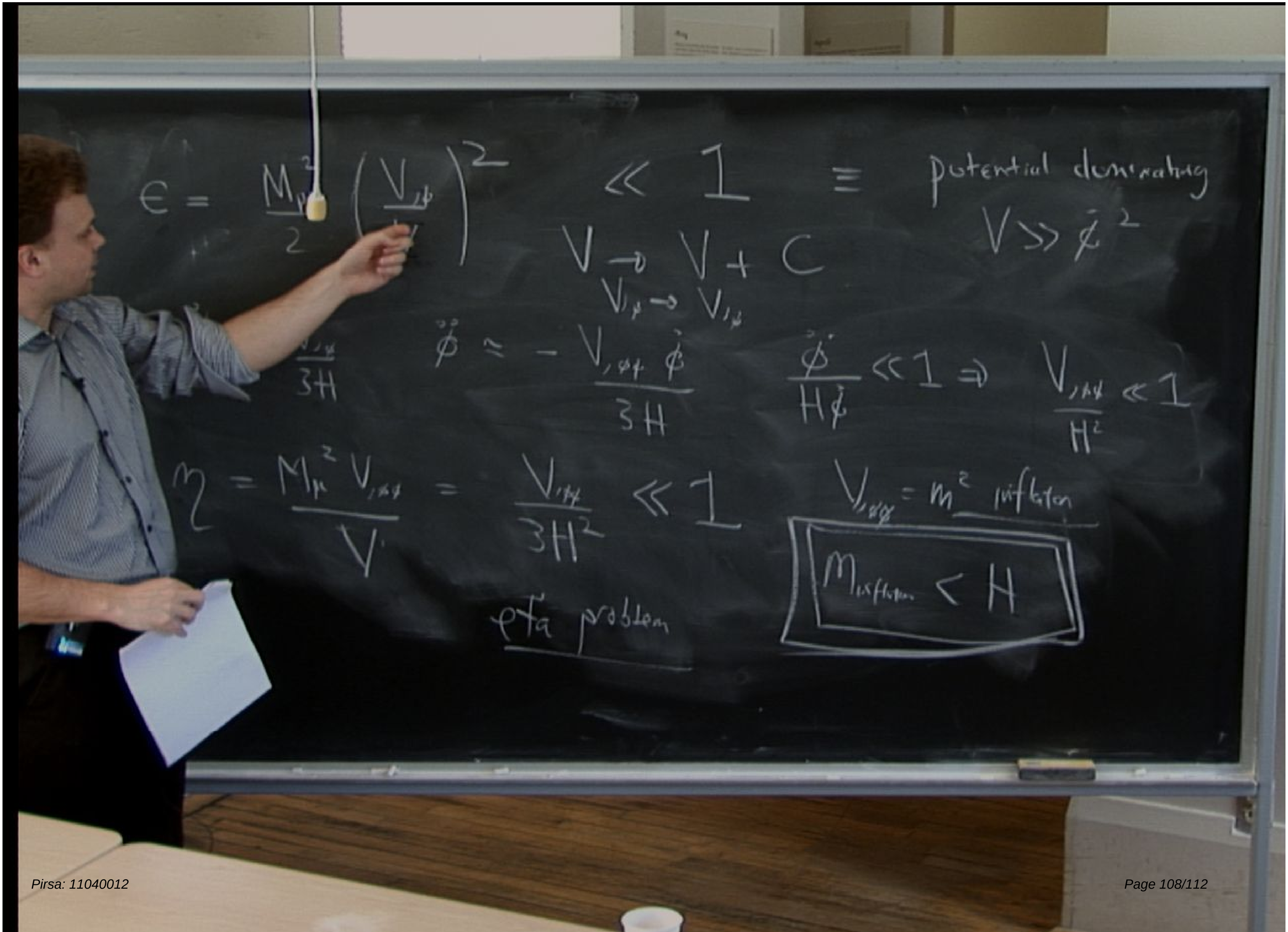
$$V \gg \dot{\phi}^2$$

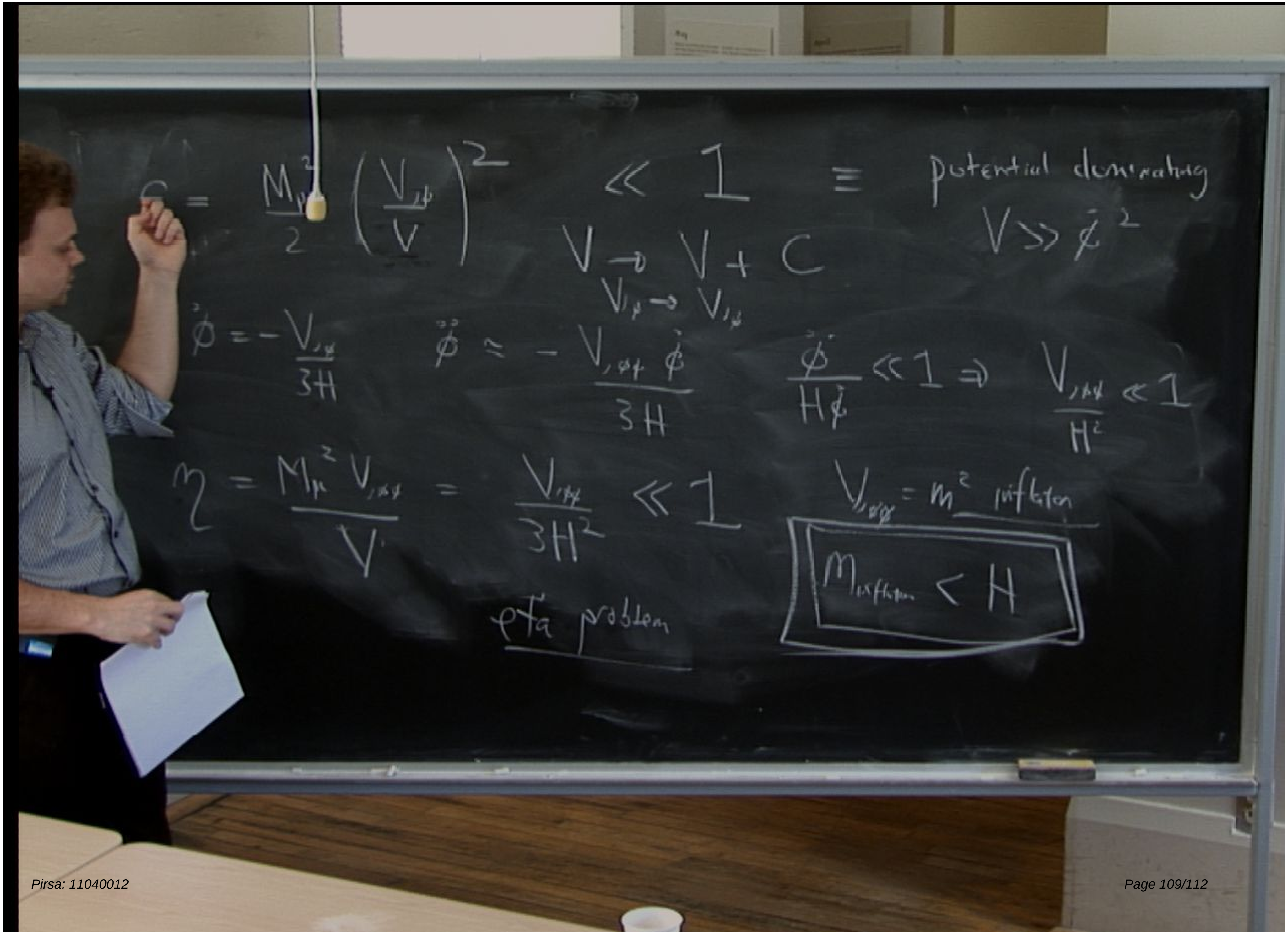
$$\dot{\phi} = -\frac{V_{,\phi}}{3H} \qquad \ddot{\phi} \simeq -\frac{V_{,\phi\phi}\,\dot{\phi}}{3H} \qquad \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \Rightarrow \frac{V_{,\phi\phi}}{H^2} \ll 1$$

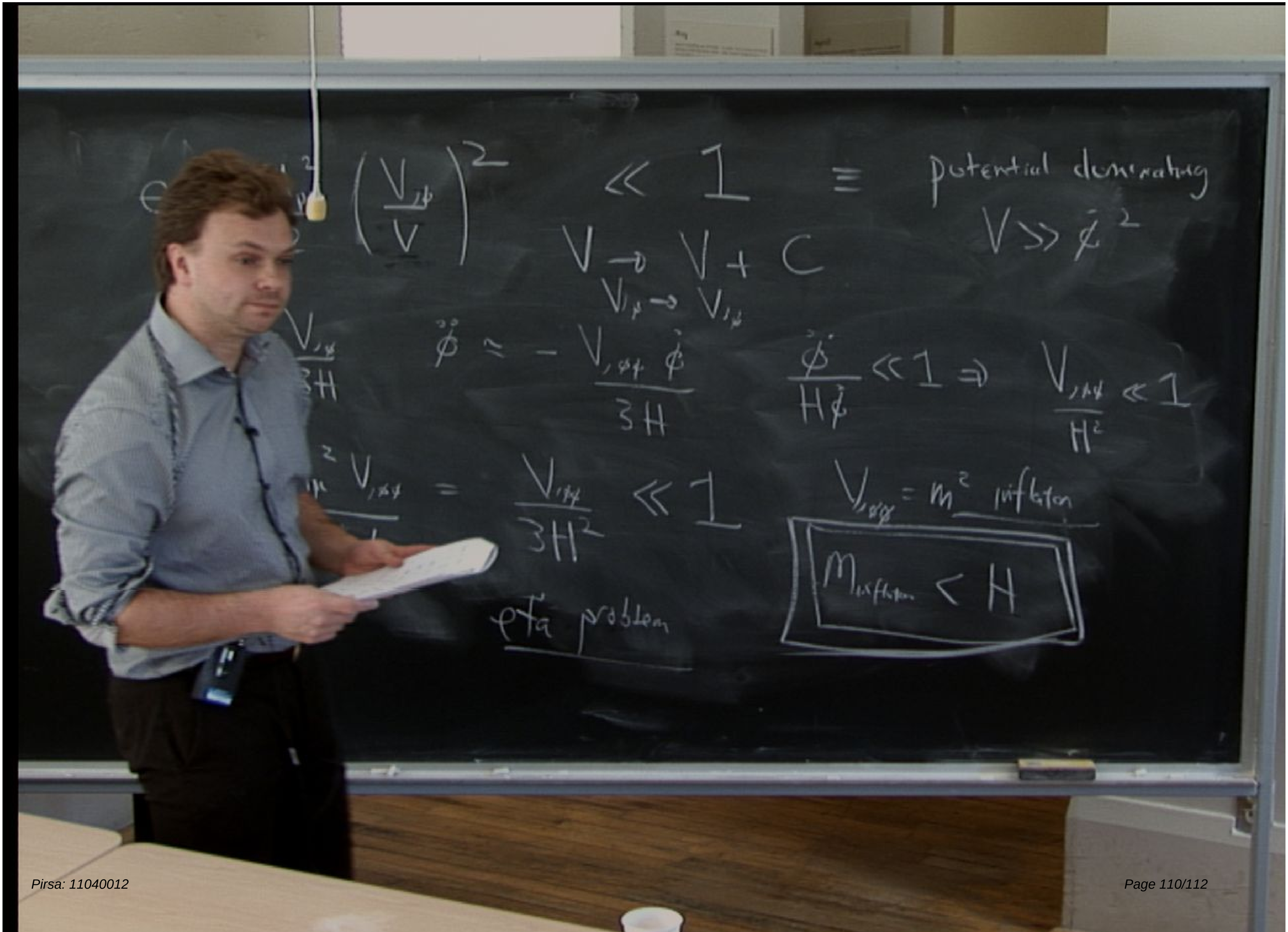
$$\eta = \frac{M_p^2 V_{,\phi\phi}}{V} = \frac{V_{,\phi\phi}}{3H^2} \ll 1$$


$\epsilon = \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \equiv$ potential dominating
$V \gg \dot{\phi}^2$
$\dot{\phi} = -\frac{V_{,\phi}}{3H}$
$\ddot{\phi} \simeq -\frac{V_{,\phi\phi}\,\dot{\phi}}{3H}$
$\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \Rightarrow \frac{V_{,\phi\phi}}{H^2} \ll 1$
$\eta = \frac{M_p^2 V_{,\phi\phi}}{V} = \frac{V_{,\phi\phi}}{3H^2} \ll 1$
$V_{,\phi\phi} = m^2$ inflaton
$m_{inflaton} < H$


ε = Mp²/2 (V,φ/V)² ≪ 1 ≡ potential dominating
V ≫ φ̇²
φ̇ = −V,φ/3H
φ̈ ≃ −V,φφ φ̇/3H
φ̈/Hφ̇ ≪ 1 ⇒ V,φφ/H² ≪ 1
η = Mp² V,φφ/V = V,φφ/3H² ≪ 1
V,φφ = m² inflaton
M inflaton < H
eta problem


ε = M_p^2/2 (V_{,φ}/V)^2 ≪ 1 ≡ potential dominating
V ≫ φ̇^2
V → V + C
V_{,φ} → V_{,φ}
φ̈ ≃ − V_{,φφ} φ̇ / 3H
φ̈ / Hφ̇ ≪ 1 ⇒ V_{,φφ}/H^2 ≪ 1
η = M_p^2 V_{,φφ} / V = V_{,φφ}/3H^2 ≪ 1
V_{,φφ} = m^2 inflaton
M_inflaton < H
eta problem


$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \equiv$ potential dominating
$V \gg \dot{\phi}^2$
$V \to V + C$
$V_{,\phi} \to V_{,\phi}$
$\dot{\phi} = -\frac{V_{,\phi}}{3H}$
$\ddot{\phi} \simeq -\frac{V_{,\phi\phi}\dot{\phi}}{3H}$
$\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \Rightarrow \frac{V_{,\phi\phi}}{H^2} \ll 1$
$\eta = \frac{M_{pl}^2 V_{,\phi\phi}}{V} = \frac{V_{,\phi\phi}}{3H^2} \ll 1$
$V_{,\phi\phi} = m^2$ inflaton
$m_{inflaton} < H$
eta problem


≪ 1 ≡ potential dominating
V ≫ φ̇²
V → V + C
V,φ → V,φ
φ̈ ≃ − V,φφ φ̇ / 3H
φ̈ / Hφ̇ ≪ 1 ⇒ V,φφ / H² ≪ 1
V,φφ / 3H² ≪ 1
V,φφ = m² inflaton
M inflaton < H
eta problem

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \equiv \text{potential dominating}$$

$$V \gg \dot{\phi}^2$$

$$V \to V + C$$

$$V_{,\phi} \to V_{,\phi}$$

$$\dot{\phi} = -\frac{V_{,\phi}}{3H} \qquad \ddot{\phi} \simeq -\frac{V_{,\phi\phi}\,\dot{\phi}}{3H} \qquad \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \Rightarrow \frac{V_{,\phi\phi}}{H^2} \ll 1$$

$$\eta = \frac{M_{pl}^2 V_{,\phi\phi}}{V} = \frac{V_{,\phi\phi}}{3H^2} \ll 1 \qquad V_{,\phi\phi} = m^2_{\text{inflaton}}$$

$$\boxed{m_{\text{inflaton}} < H}$$

eta problem

$$H \sim 10^{-6} M_{pl}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w < -\frac{1}{3} \qquad \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3} \qquad w \approx -1$$

Axion models

$$V \sim -V_0 \cos\left(\frac{\phi}{f}\right) \qquad \boxed{\dot{\phi}^2 \ll V(\phi)}$$

$$\phi \to \phi + 2\pi f$$

$$\frac{\dot{\phi}^2}{V(\phi)} = \frac{V_{,\phi}^2}{9H^2 V(\phi)} \ll 1 \qquad H^2 \simeq \frac{V}{3M_{pl}^2}$$