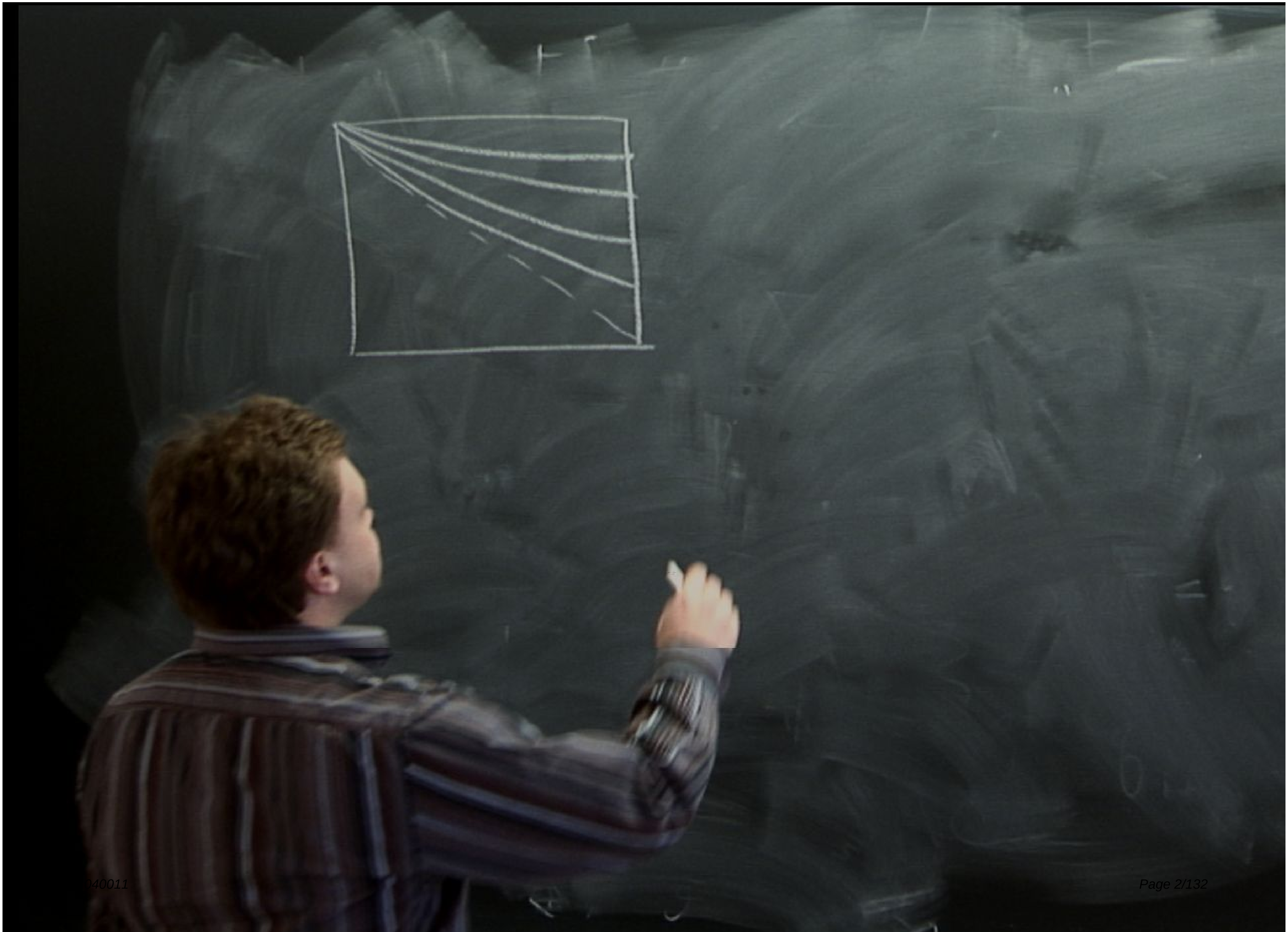
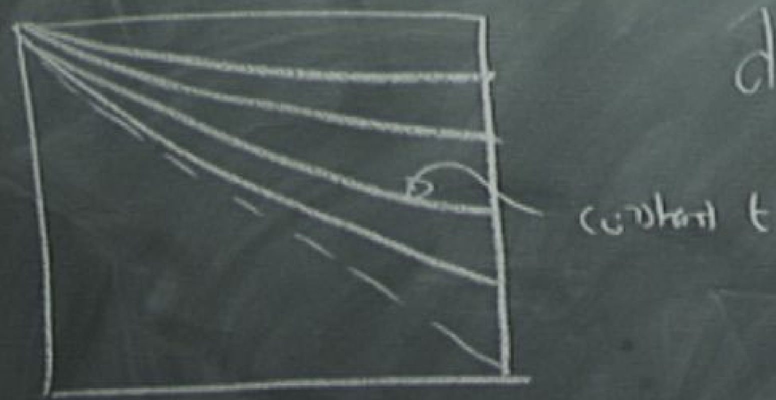
Title: Explorations in Cosmology - Lecture 7

Date: Apr 12, 2011 11:30 AM

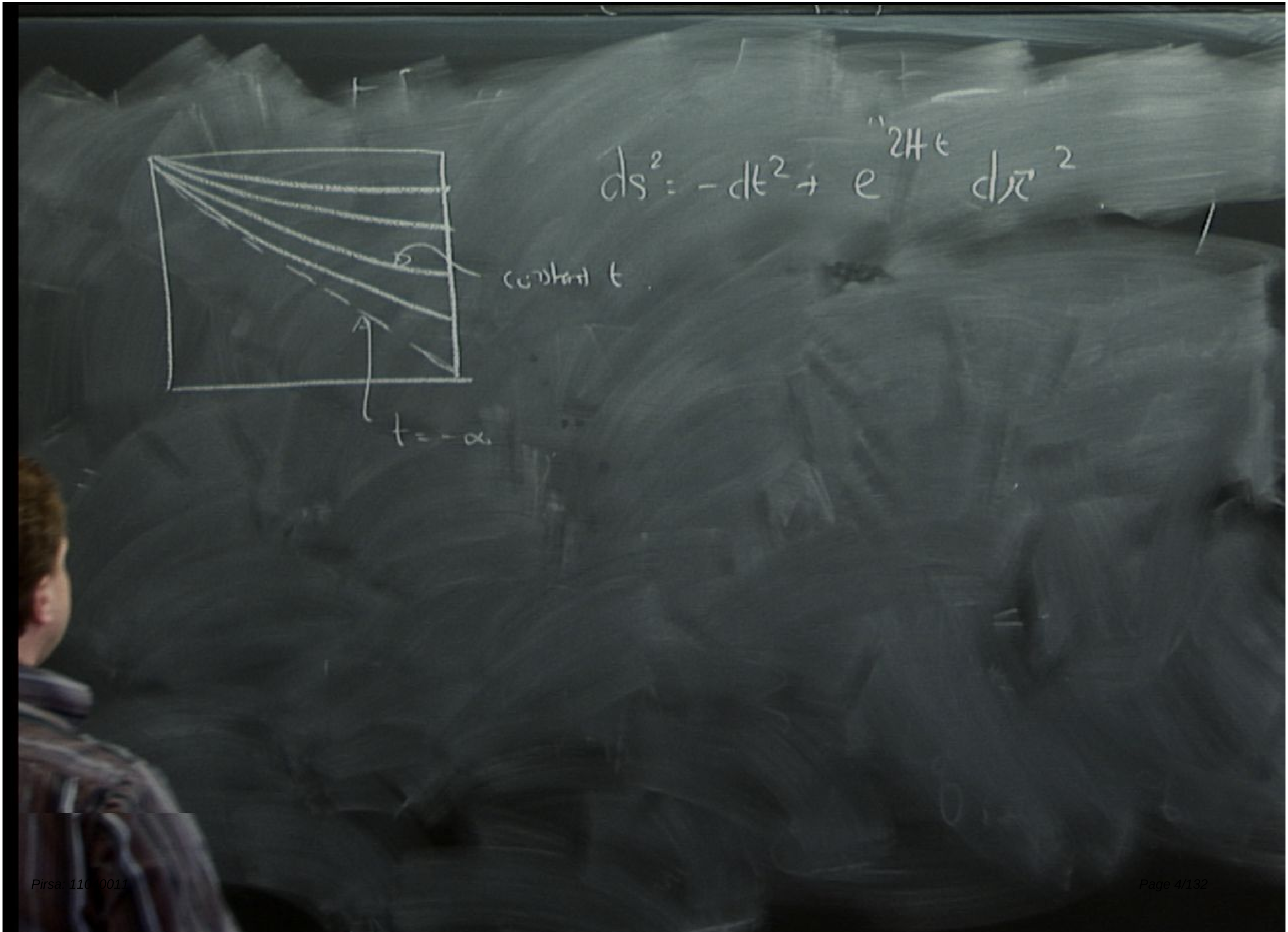
URL: http://pirsa.org/11040011

Abstract:

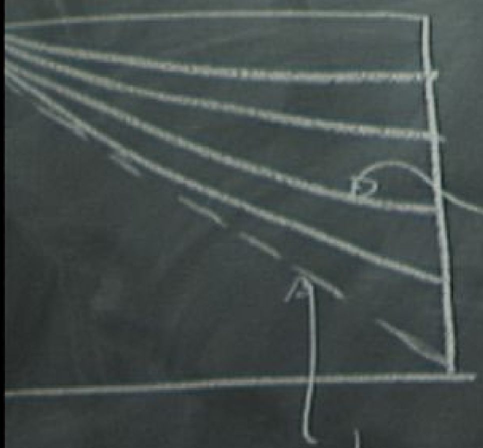




$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

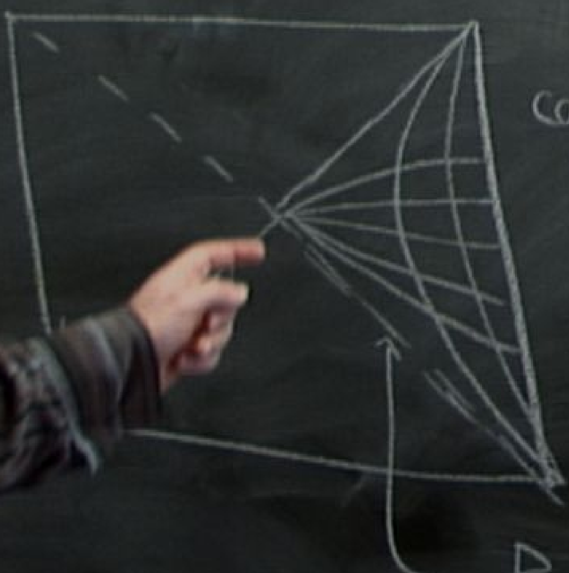

ds^2 = -dt^2 + e^2Ht dx^2
constant t
t = -∞

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$



constant t

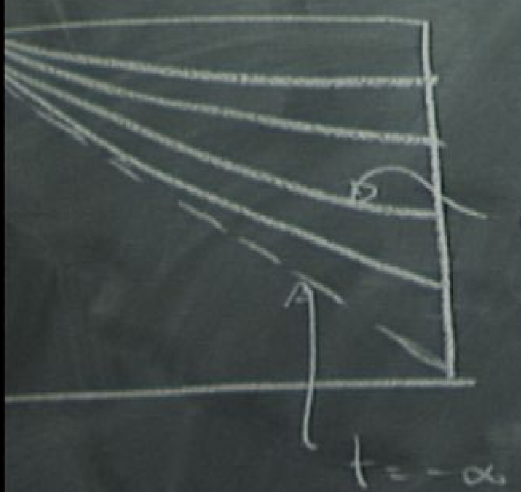
$t = -\infty$



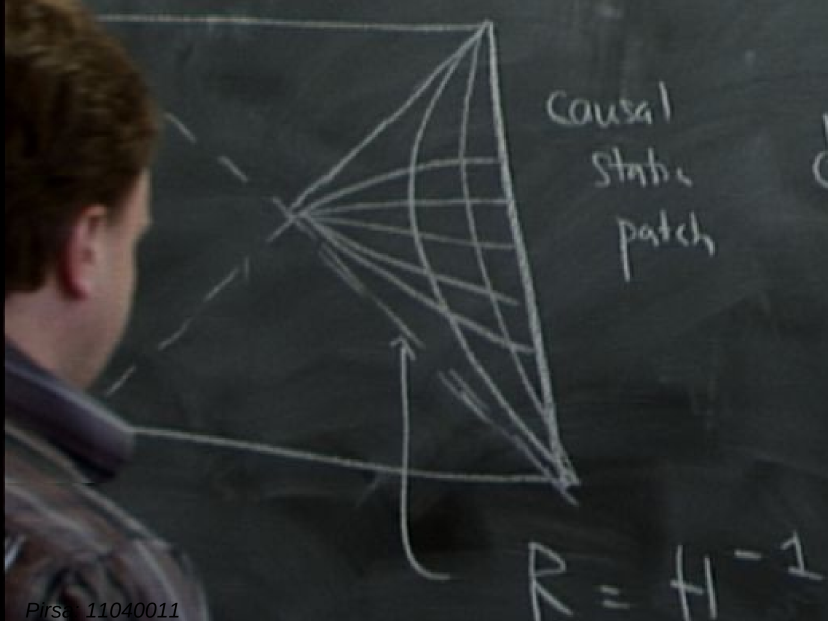
causal static patch

$$ds^2 = -f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2\, d^2\Omega$$

$R = H^{-1}$

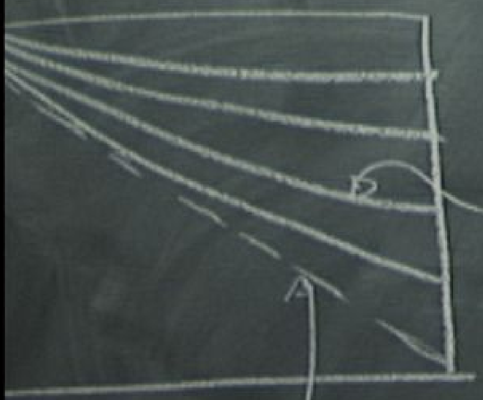


$$ds^2 = -dt^2 + e^{2Ht}\, dx^2$$



$$ds^2 = -f(R)\, dT^2 + \frac{1}{f(R)}\, dR^2 + R^2\, d^2\Omega$$

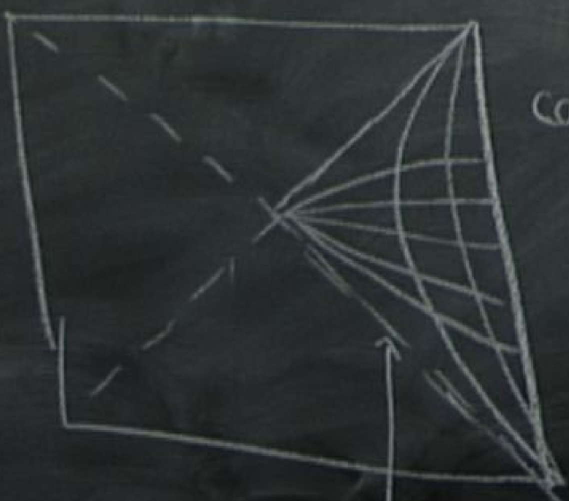
$$f(R) = 1 - R^2 H^2$$



constant t

$t = -\infty$

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

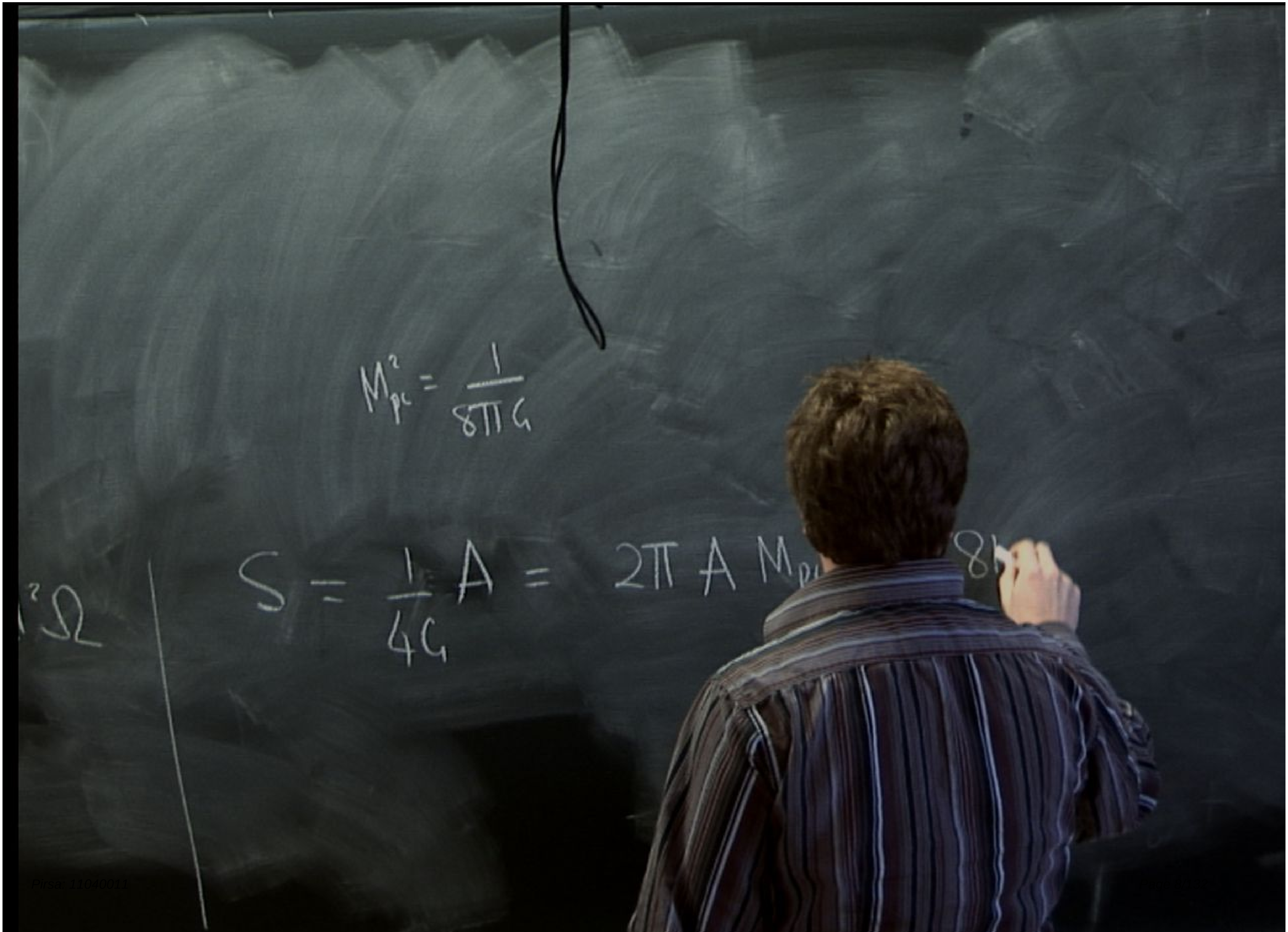


causal static patch

$$ds^2 = -f(R)\, dT^2 + \frac{1}{f(R)}\, dR^2 + R^2\, d^2\Omega$$

$$f(R) = 1 - R^2 H^2$$

$$R = H^{-1} \qquad A = 4\pi H^{-2}$$


$M_{pl}^2 = \frac{1}{8\pi G}$
$S = \frac{1}{4G} A = 2\pi A M_{pl}$
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$$M_{pl}^2 = \frac{1}{8\pi G}$$

Ω

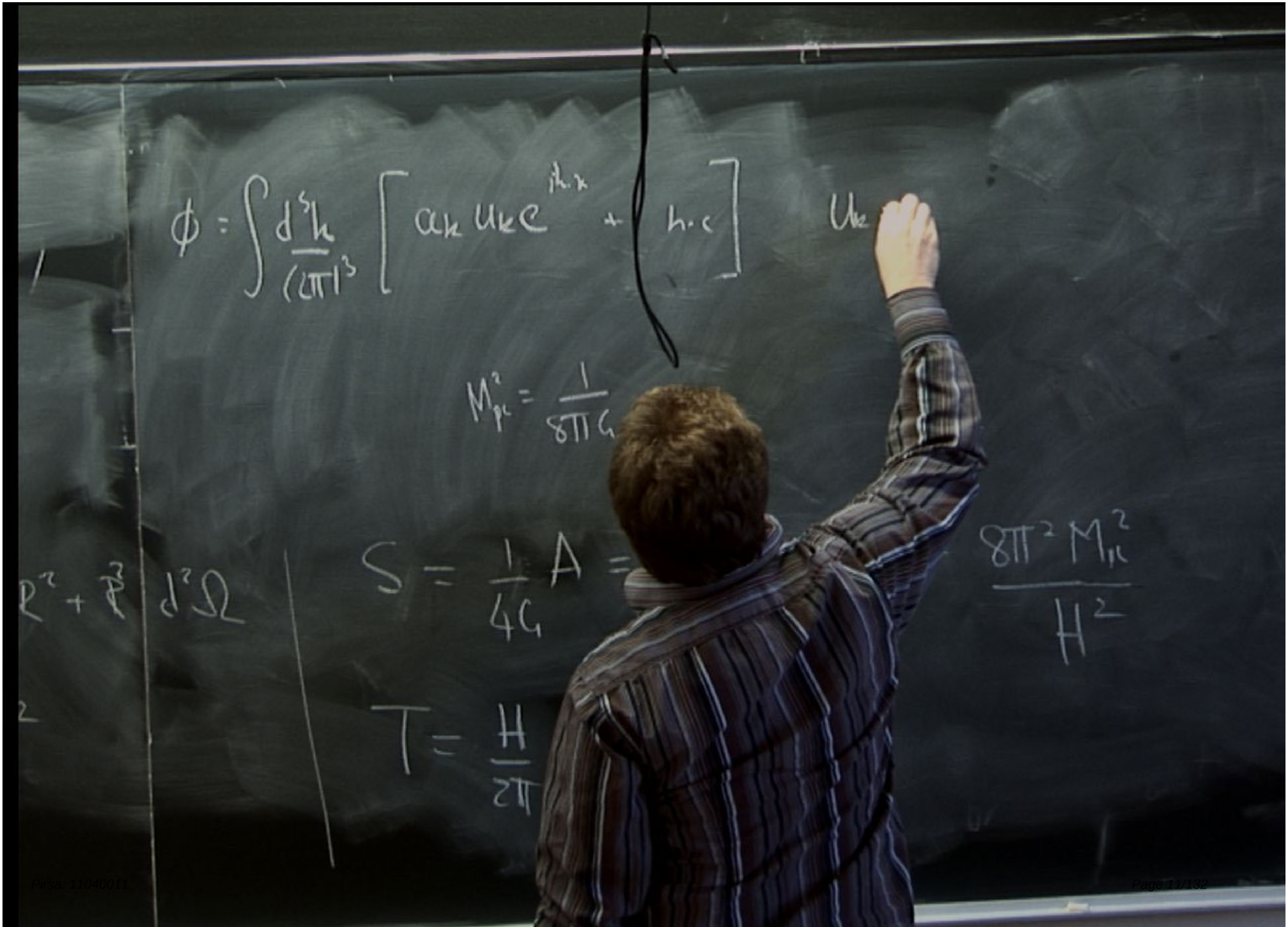
$$S = \frac{1}{4G} A = 2\pi A M_{pl}^2 = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$M_{pl}^2 = \frac{1}{8\pi G}$$

$${}^3\Omega$$

$$S = \frac{1}{4G} A = 2\pi A M_{pl}^2 = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$T = \frac{H}{2\pi}$$


$\phi = \int \frac{d^3k}{(2\pi)^3} \left[a_k u_k e^{ik.x} + h.c \right]$
u_k
$M_{pl}^2 = \frac{1}{8\pi G}$
$S = \frac{1}{4G} A =$
$\frac{8\pi^2 M_{pl}^2}{H^2}$
$T = \frac{H}{2\pi}$
$d^2\Omega$

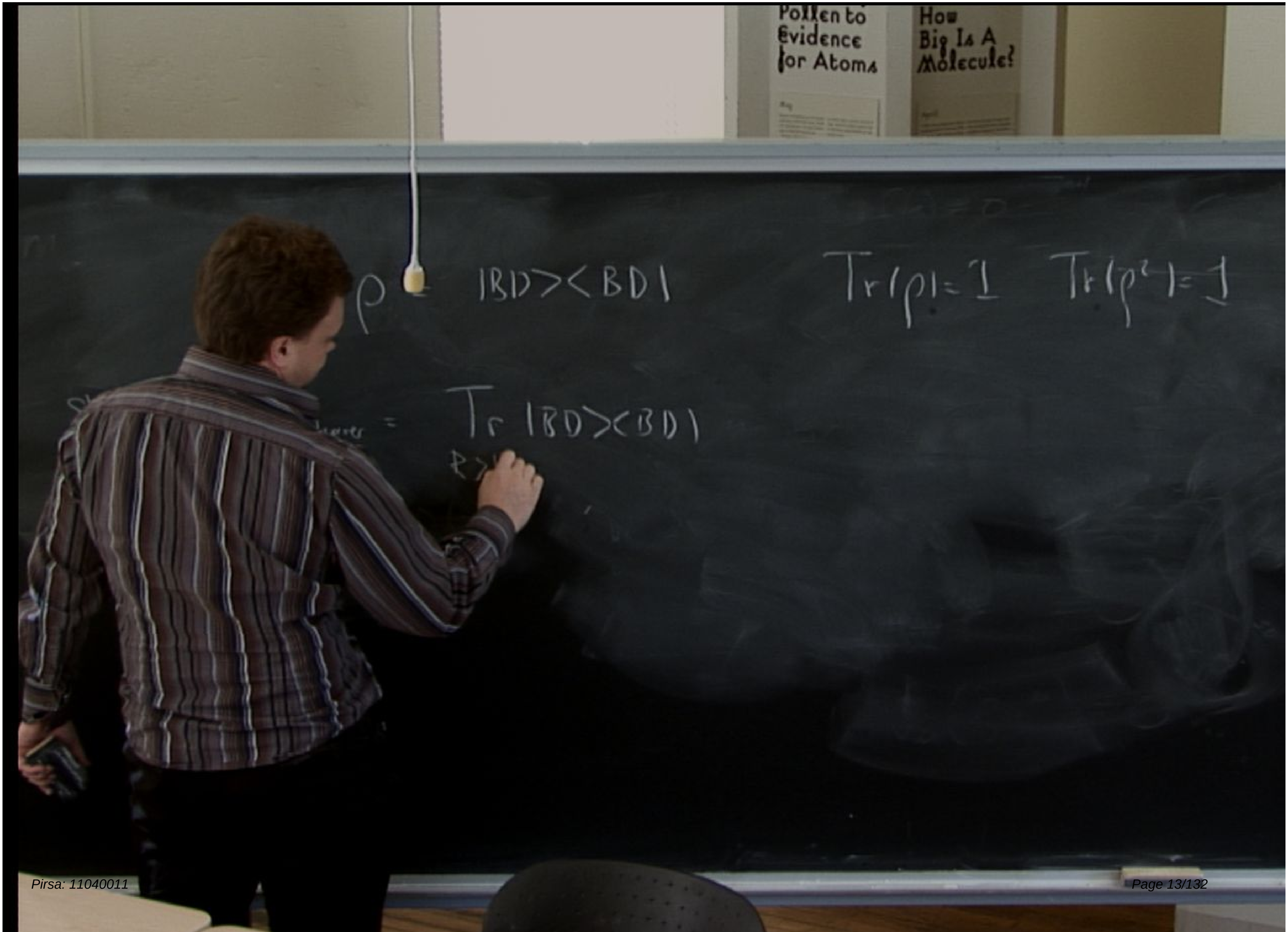
$$\phi = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[a_k u_k e^{ik\cdot x} + h.c \right] \qquad u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}$$

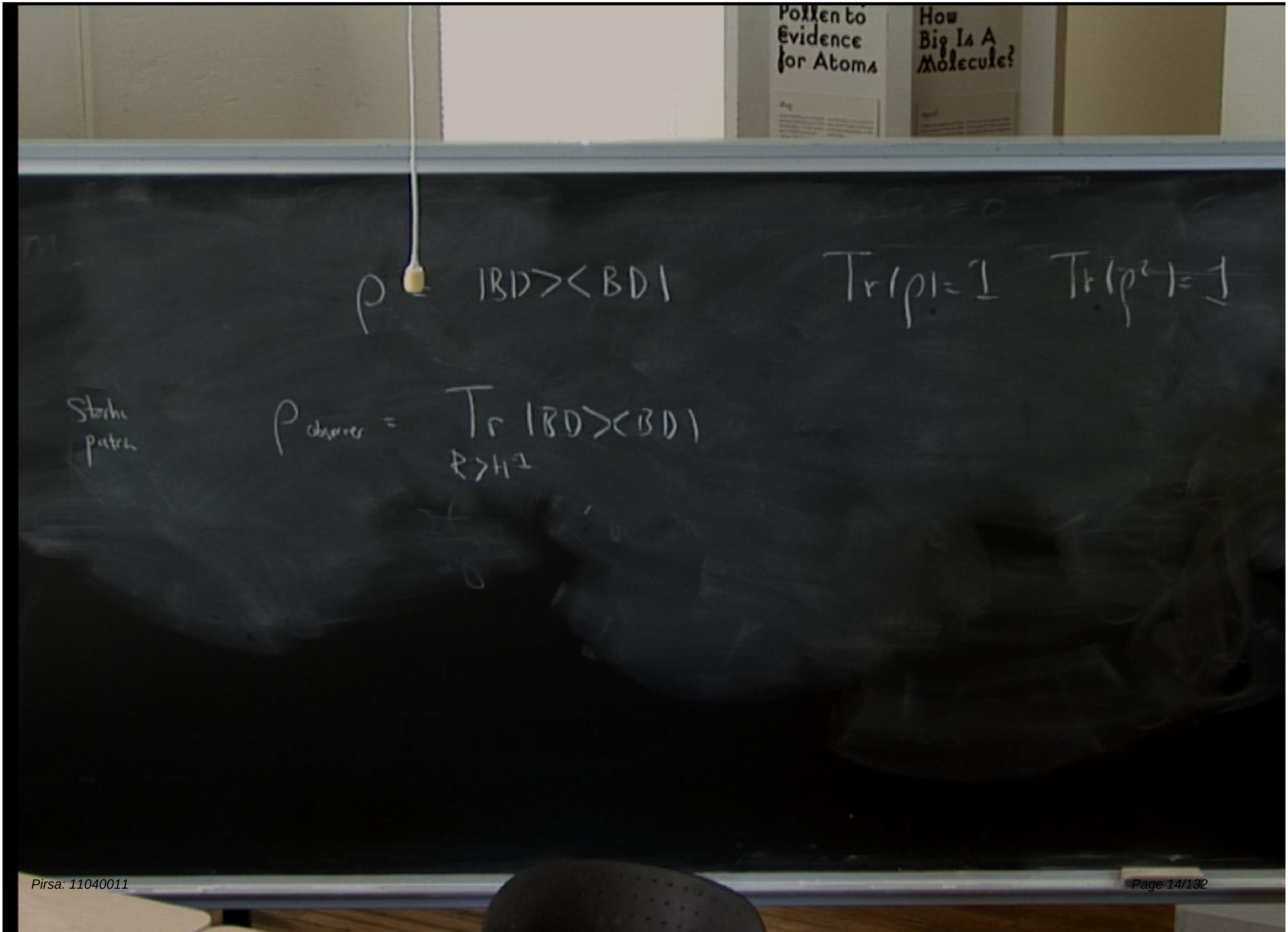
$$M_{pl}^2 = \frac{1}{8\pi G}$$

$$\ldots r^2 + r^2 \, d^2\Omega$$

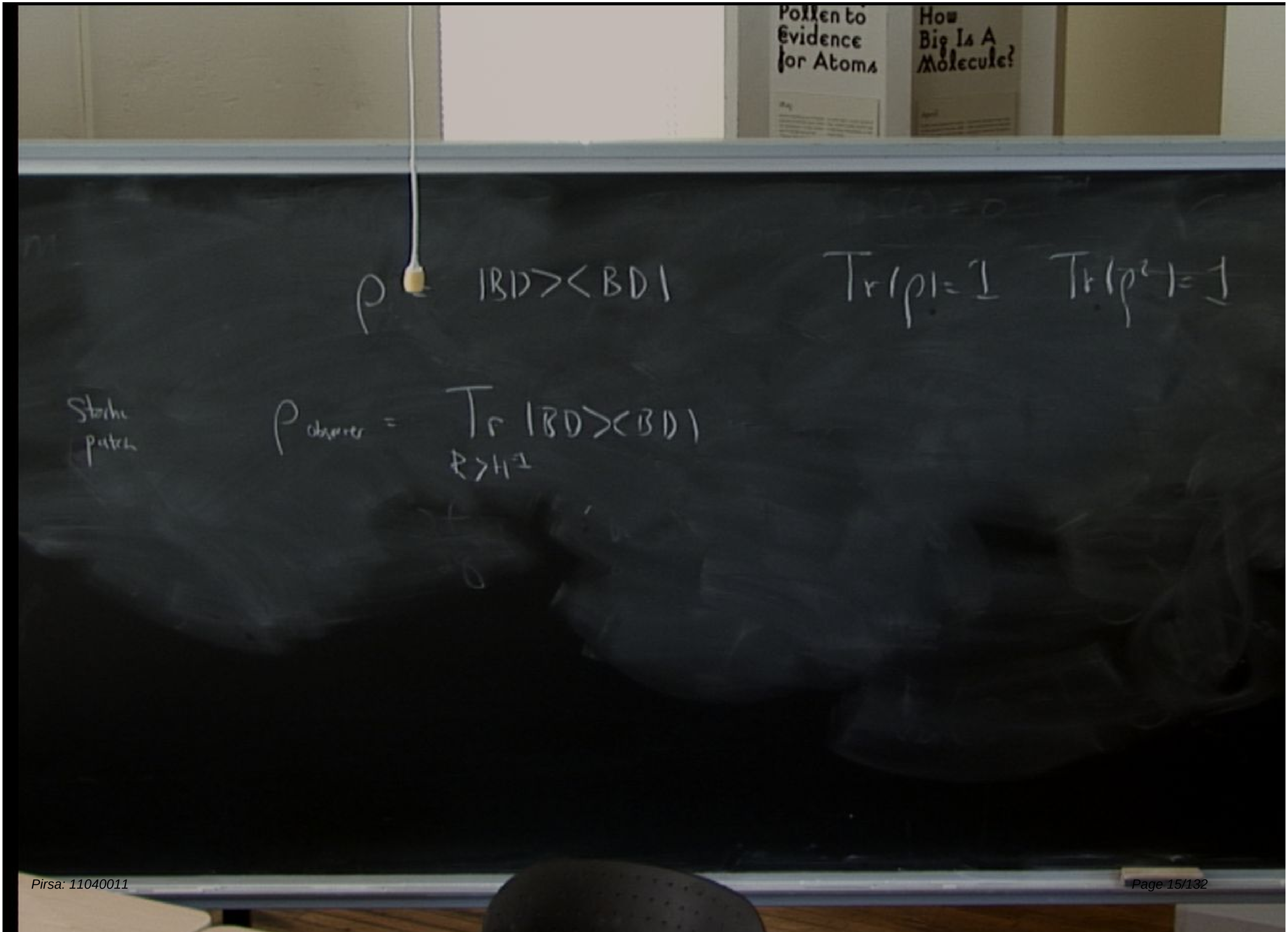
$$S = \frac{1}{4G} A = 2\pi A M_{pl}^2 = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$T = \frac{H}{2\pi}$$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
ρ = |BD⟩⟨BD|
Tr(ρ) = 1
Tr(ρ²) = 1
Tr |BD⟩⟨BD|



Pollen to Evidence for Atoms
How Big Is A Molecule?
$\rho = |BD\rangle\langle BD|$
$Tr(\rho) = 1$
$Tr(\rho^2) = 1$
Static patch
$\rho_{observer} = Tr_{r > H^{-1}} |BD\rangle\langle BD|$


Pollen to
Evidence
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How
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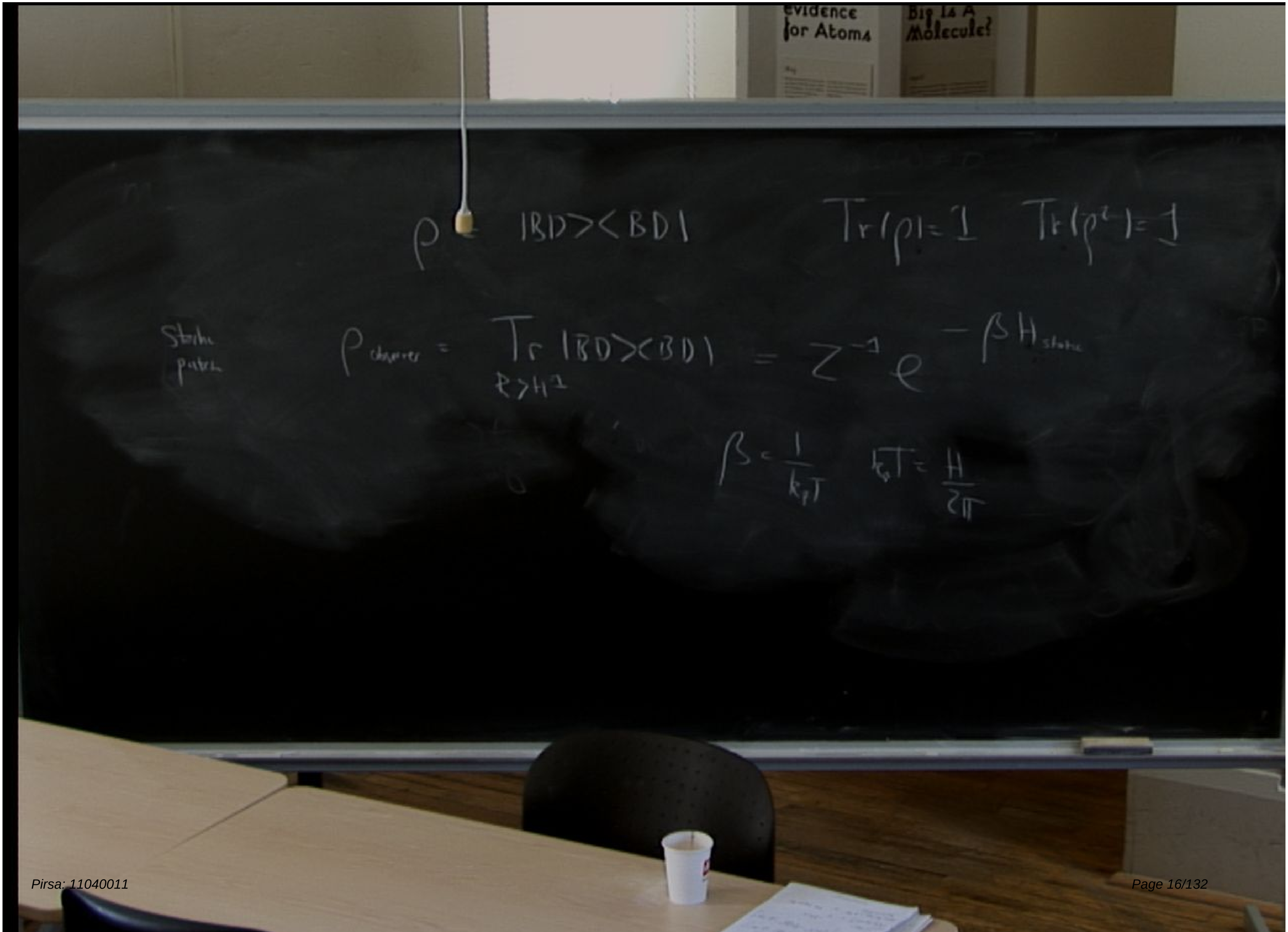
ρ |BD><BD|
Tr(ρ)=1 Tr(ρ²)=1
Static
patch
ρ observer = Tr |BD><BD|
R>H⁻¹


Evidence
for Atoms
$\rho = |BD\rangle\langle BD|$
$Tr(\rho) = 1$
$Tr(\rho^2) = 1$
Static patch
$\rho_{observer} = Tr_{r>H^{-1}} |BD\rangle\langle BD| = Z^{-1} e^{-\beta H_{static}}$
$\beta = \frac{1}{k_B T}$
$k_B T = \frac{H}{2\pi}$

$$\phi = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[a_k u_k e^{ik\cdot x} + h.c. \right] \qquad u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}$$

$$S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right]$$

$$\sqrt{-g} = R^2 \sin^2\theta$$

$$\phi = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[a_k u_k e^{ik\cdot x} + h.c. \right] \qquad u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}$$

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$$\sqrt{-g} = R^2 \sin^2\theta$$

$$d^2\Omega$$

$$d\theta^2 + \sin^2\theta\, d\phi^2$$

$$S = \int dR\, d T\, d\theta\, d\phi\, R^2 \sin^2\theta \left[\right.$$

$$\phi = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[a_k u_k e^{ik\cdot x} + h.c \right] \qquad u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}$$

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$$\sqrt{-g} = R^2 \sin^2\theta$$

$$d^2\Omega$$

$$d\theta^2 + \sin^2\theta d\phi^2$$

$$S = \int dR\, dT\, d\theta\, d\phi\, R^2 \sin^2\theta \left[\frac{1}{f(R)} \left(\frac{\partial\phi}{\partial T} \right)^2 - f(R) \left(\frac{\partial\phi}{\partial R} \right)^2 \right]$$

$$\phi = \frac{1}{a(\eta)}\int \frac{d^3k}{(2\pi)^3}\left[a_k u_k e^{ik\cdot x} + h.c.\right] \qquad u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta}$$

$$S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\, \partial_\nu\phi\right]$$

denote on S^2

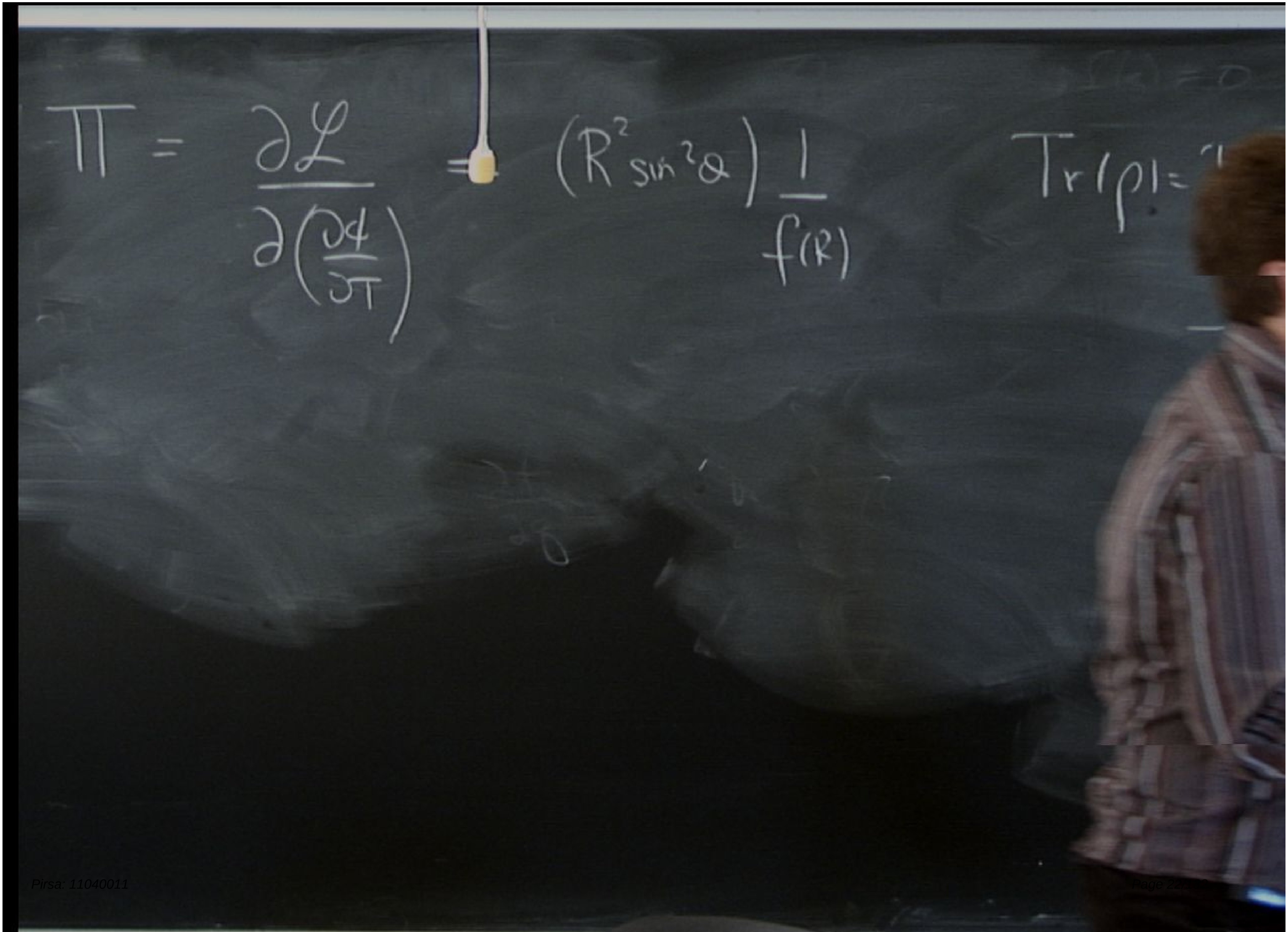
$$\sqrt{-g} = R^2 \sin^2\theta$$

$$\underbrace{d^2\Omega}_{d\theta^2 + \sin^2\theta\, d\phi^2}$$

$$S = \int dR\, dT\, d\theta\, d\phi\, R^2 \sin^2\theta \left[\frac{1}{f(R)}\left(\frac{\partial\phi}{\partial T}\right)^2 - f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 - \frac{1}{R^2}\left(\nabla_S\phi\right)^2\right]$$

$$\Pi = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial \tau}\right)} =$$

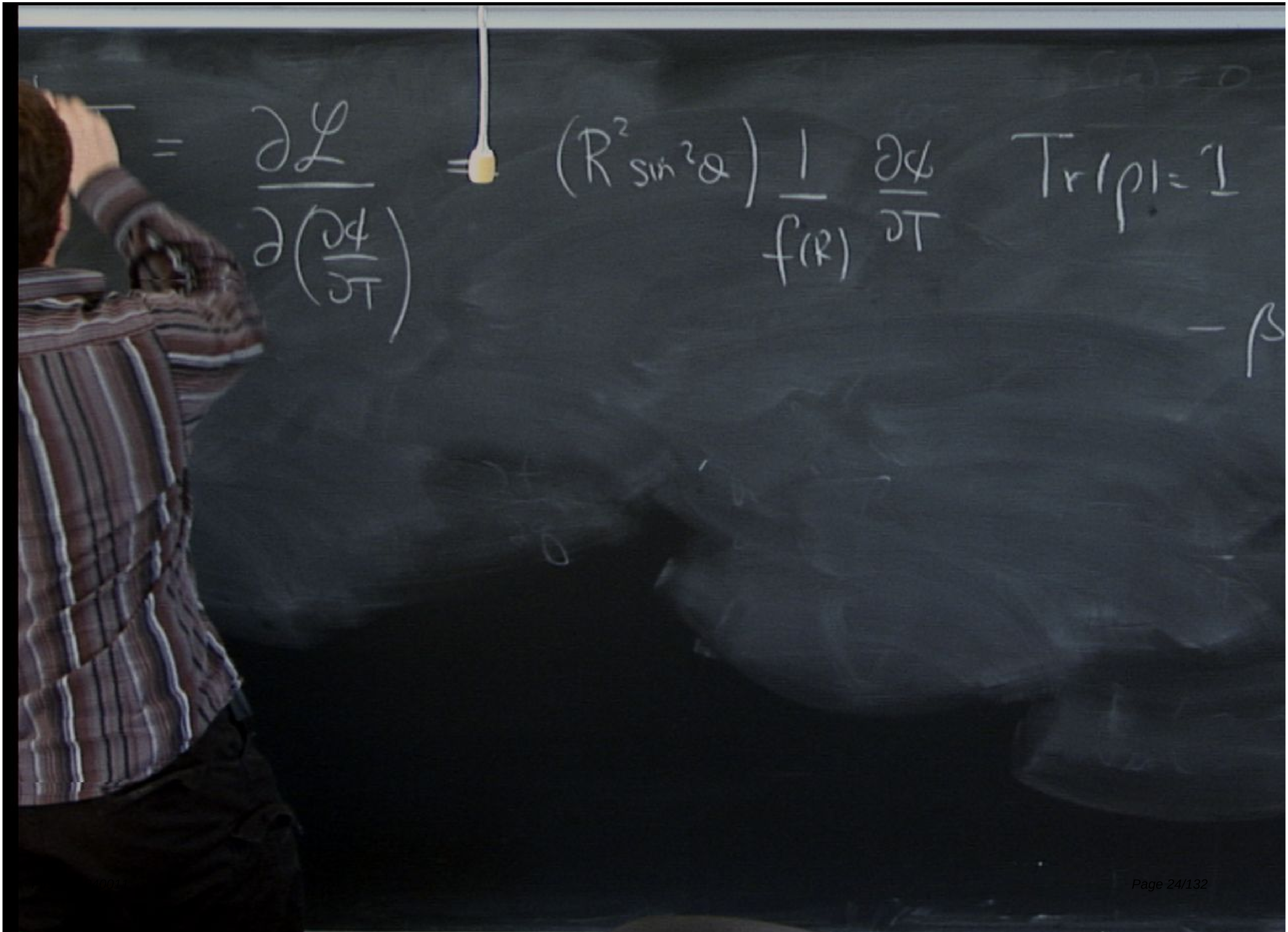
$\mathrm{Tr}(\rho) =$

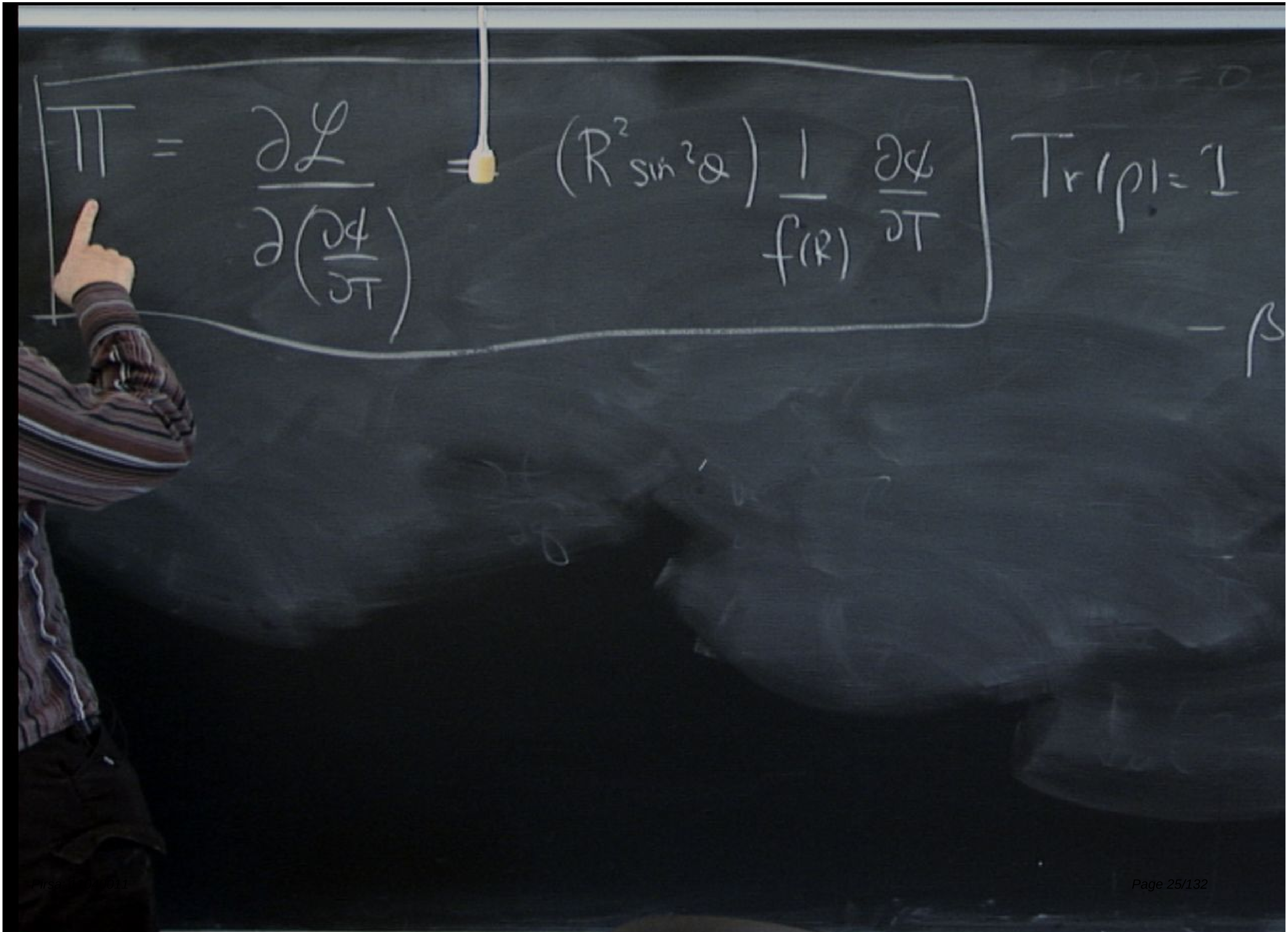

$\Pi = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial \tau}\right)} = (R^2 \sin^2\theta)\,\frac{1}{f(R)}$
$\mathrm{Tr}(\rho) =$

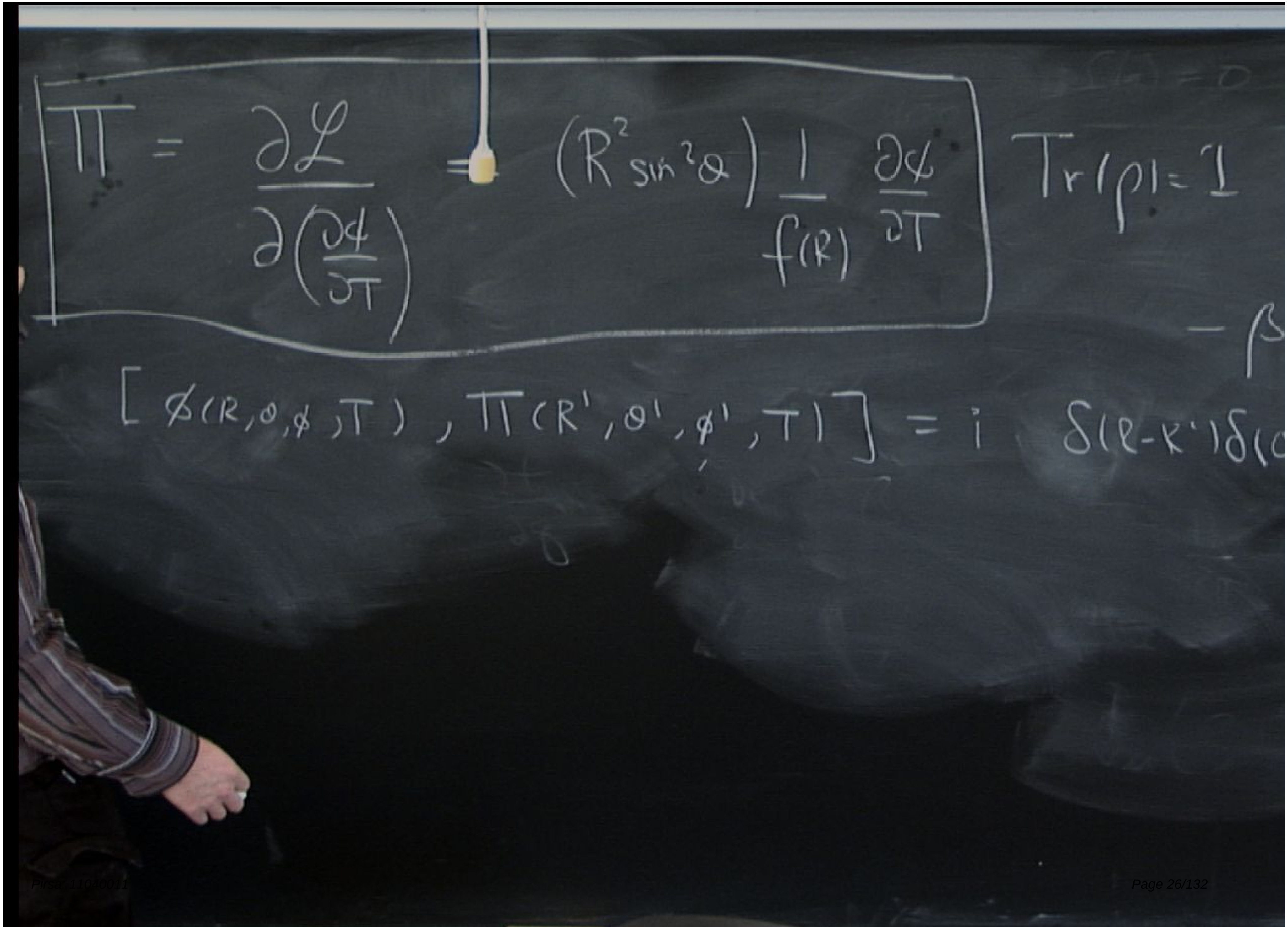
$$\Pi = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial\phi}{\partial\tau}\right)} = \left(R^2 \sin^2\theta\right)\frac{1}{f(R)}$$

$$\mathrm{Tr}(\rho) = 1$$

$$-\beta$$


$= \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial \tau}\right)} = (R^2 \sin^2\theta)\,\frac{1}{f(R)}\,\frac{\partial \phi}{\partial \tau}$
$Tr(\rho) = 1$
$-\beta$


$\Pi = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial \tau}\right)} = (R^2 \sin^2\theta) \frac{1}{f(R)} \frac{\partial \phi}{\partial \tau}$
$\mathrm{Tr}(\rho) = 1$


$\Pi = \dfrac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi}{\partial T}\right)} = (R^2 \sin^2\theta)\dfrac{1}{f(R)}\dfrac{\partial \phi}{\partial T}$
$Tr(\rho) = 1$
$[\phi(R,\theta,\phi,T), \Pi(R',\theta',\phi',T)] = i \quad \delta(R-R')\delta($

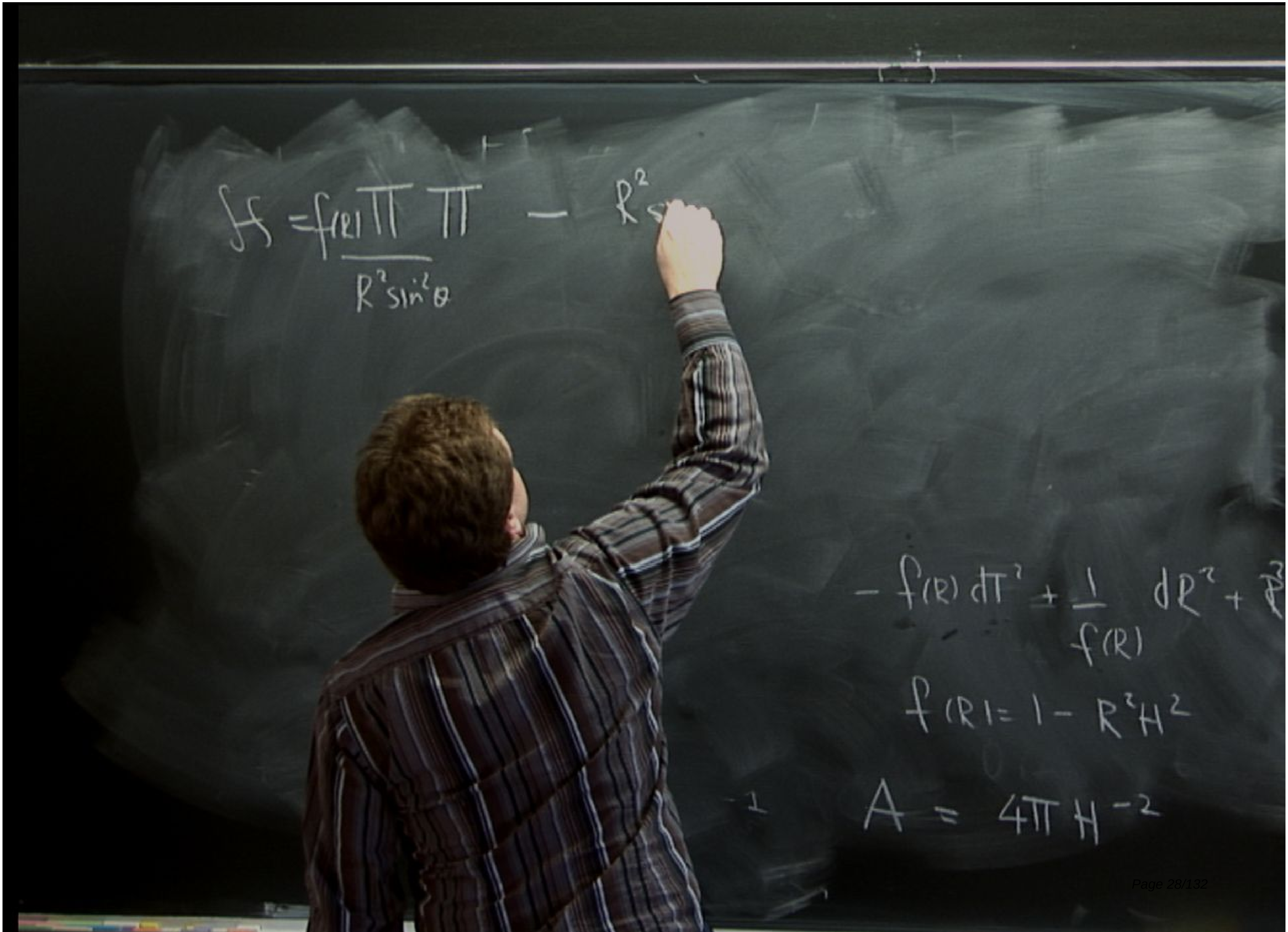
$$\boxed{\Pi = \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial\phi}{\partial T}\right)} = \left(R^2 \sin^2\theta\right)\frac{1}{f(R)}\frac{\partial\phi}{\partial T}}$$

$$\mathrm{Tr}(\rho) = 1 \qquad \mathrm{Tr}(\rho^2) = 1$$

$$-\beta H_{static}$$

$$[\phi(R,\theta,\phi,T), \Pi(R',\theta',\phi',T)] = i\,\delta(R-R')\,\delta(\theta-\theta')\,\delta(\phi-\phi')$$

$$\mathcal{H} = \Pi \frac{\partial\phi}{\partial T} - \mathcal{L}$$

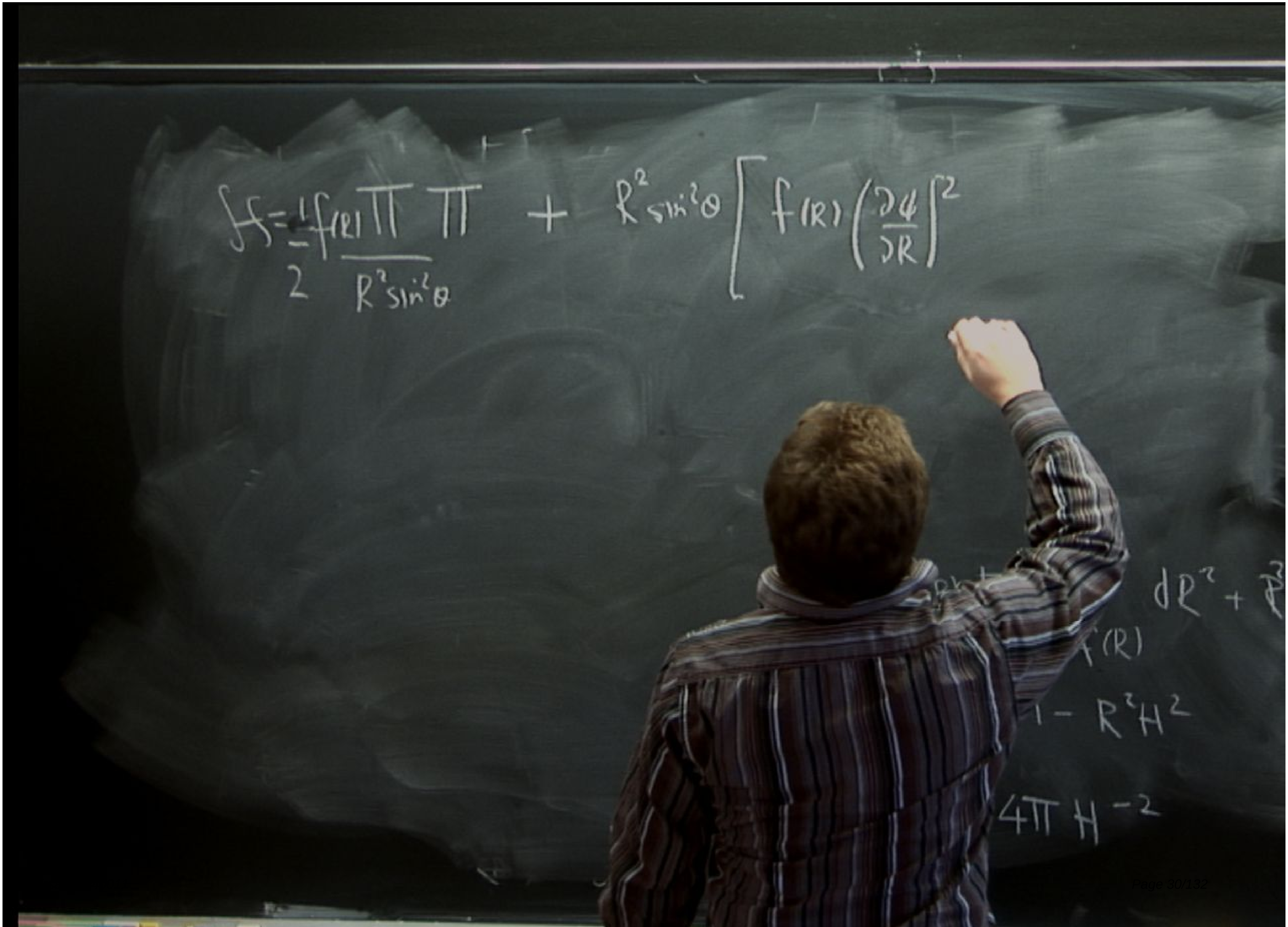

$-f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 +$
$f(R) = 1 - R^2H^2$
$A = 4\pi H^{-2}$

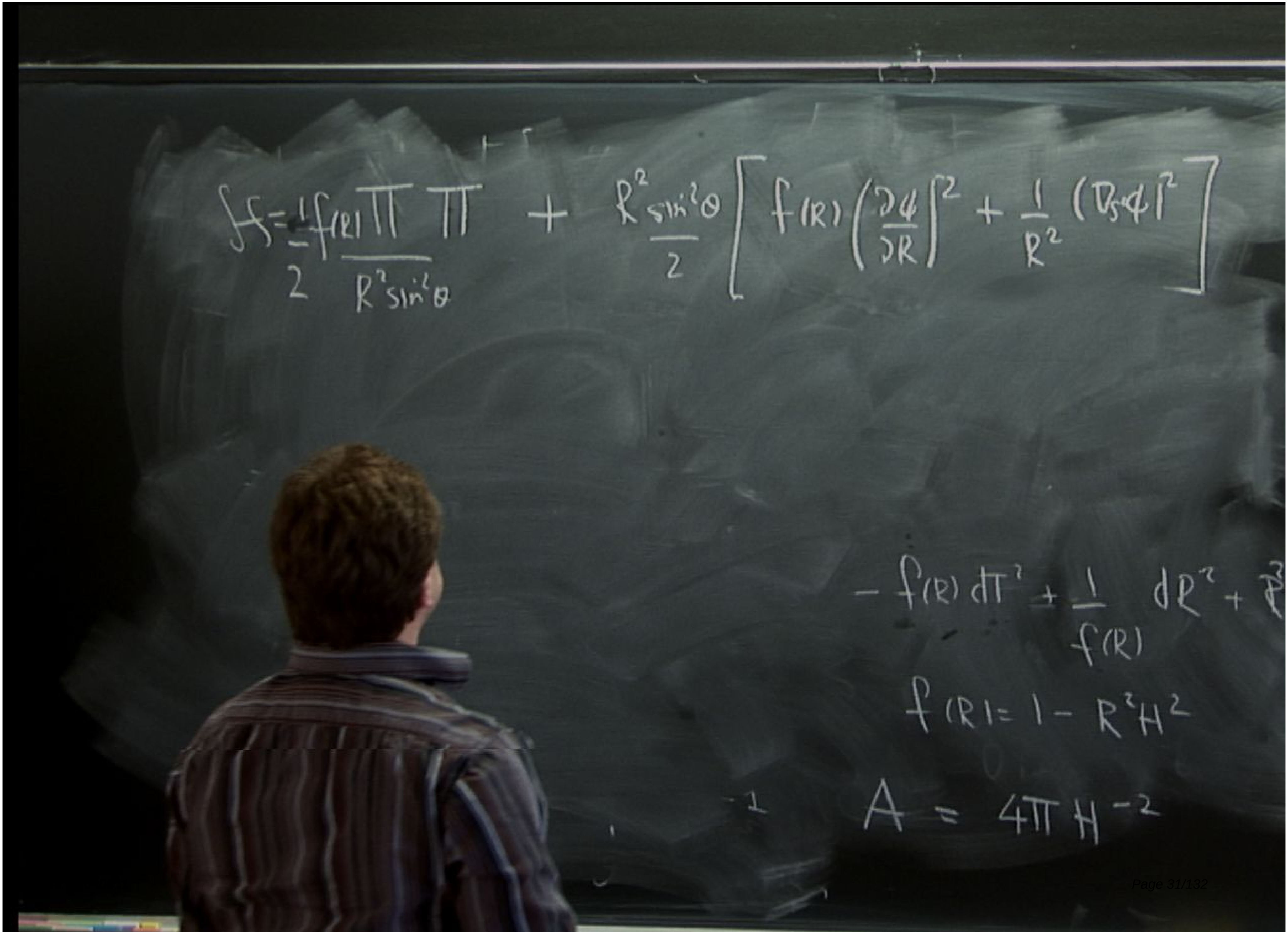
$$\delta S = f(R)\, \frac{\Pi\ \Pi}{R^2 \sin^2\theta} \quad - \quad R^2 \sin^2\theta \Big[$$

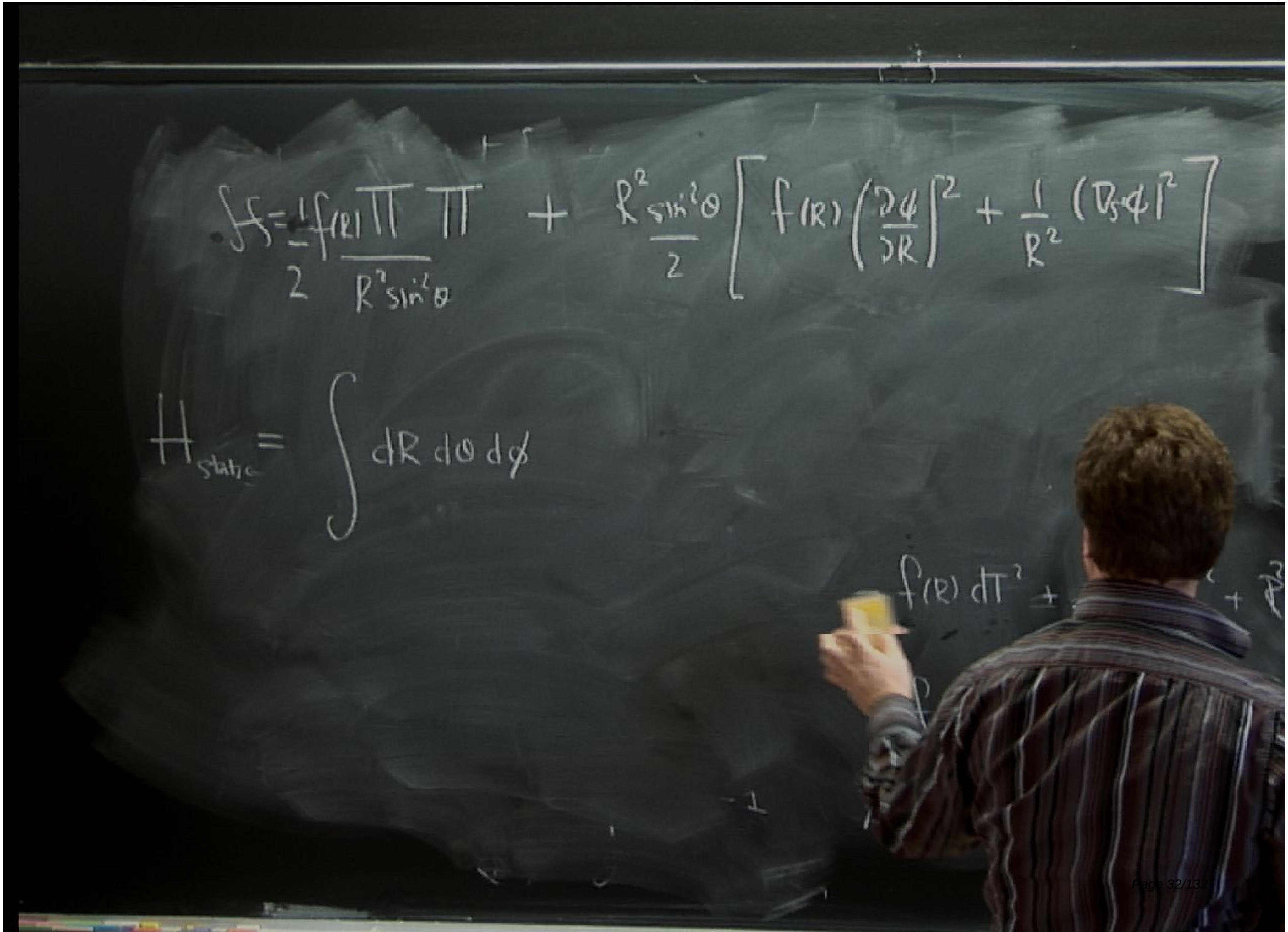
$$- f(R)\, dT^2 + \frac{1}{f(R)}\, dR^2 + R^2$$

$$f(R) = 1 - R^2 H^2$$

$$A = 4\pi H^{-2}$$

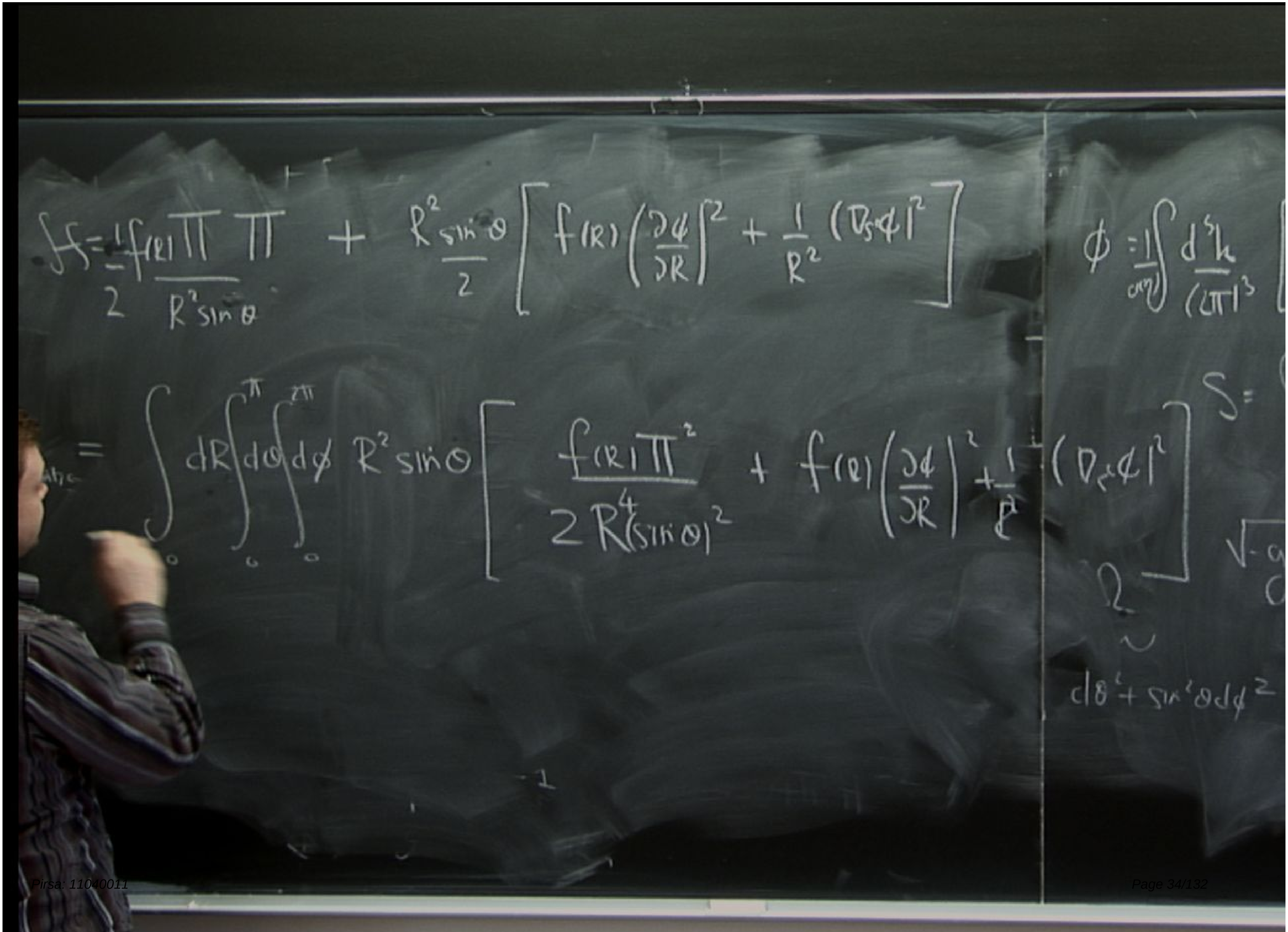

$\delta f = \frac{1}{2} \frac{f(R)\,\Pi\,\Pi}{R^2 \sin^2\theta} + R^2 \sin^2\theta \left[f(R) \left(\frac{\partial \psi}{\partial R} \right) \right]^2$
$dR^2 + R^2$
$f(R)$
$1 - R^2 H^2$
$4\pi H^{-2}$


$\mathcal{H} = \frac{1}{2} f(R) \frac{\Pi\,\Pi}{R^2 \sin^2\theta} + \frac{R^2 \sin^2\theta}{2}\left[f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S \phi)^2\right]$
$-f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2$
$f(R) = 1 - R^2H^2$
$A = 4\pi H^{-2}$


$S_f = \frac{1}{2} f(R) \frac{\Pi\,\Pi}{R^2 \sin^2\theta} + \frac{R^2 \sin^2\theta}{2}\left[f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S\phi)^2 \right]$
$H_{static} = \int dR\, d\theta\, d\phi$
$f(R)\, dT^2 +$

$$\mathcal{H} = \frac{1}{2} f(R) \frac{\Pi\,\Pi}{R^2 \sin\theta} + \frac{R^2 \sin^2\theta}{2}\left[f(R)\left(\frac{\partial \phi}{\partial R}\right)^2 + \frac{1}{R^2}\left(\nabla_S \phi\right)^2 \right]$$

$$H_{static} = \int dR\, d\theta\, d\phi$$



$\mathcal{H} = \frac{1}{2} f(R) \frac{\Pi\,\Pi}{R^2 \sin\theta} + \frac{R^2 \sin\theta}{2}\left[f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S\phi)^2\right]$
$\phi = \frac{1}{(2\pi)}\int \frac{d^3k}{(2\pi)^3}$
$= \int_0 dR \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\, R^2 \sin\theta \left[\frac{f(R)\Pi^2}{2R^4(\sin\theta)^2} + f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S\phi)^2 \right]$
$S =$
$\sqrt{-g}$
Ω
$d\theta^2 + \sin^2\theta\, d\phi^2$

$$\mathcal{L} = \frac{1}{2} f(R) \frac{\Pi\ \Pi}{R^2 \sin\theta} + \frac{R^2 \sin\theta}{2} \left[f(R) \left(\frac{\partial \phi}{\partial R} \right)^2 + \frac{1}{R^2} (\nabla_S \phi)^2 \right]$$

$$\phi = \frac{1}{(2\pi)} \int \frac{d^3 k}{(2\pi)}$$

$$H_{static} = \int_0^{H^{-1}} dR \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ R^2 \sin\theta \left[\frac{f(R)\,\Pi^2}{2 R^4 (\sin\theta)^2} + f(R) \left(\frac{\partial \phi}{\partial R} \right)^2 + \frac{1}{R^2} (\nabla_S \phi)^2 \right]$$

$$f(R) = 1 - R^2 H^2$$

$$R < H^{-1}$$

$$\Omega \sim d\theta^2 + \sin^2\theta\, d\phi$$

$$\mathcal{H} = \frac{1}{2}\frac{f(R)\,\Pi\,\Pi}{R^2\sin\theta} + \frac{R^2\sin\theta}{2}\left[f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S\phi)^2\right]$$

$$\phi = \frac{1}{(2\pi)}\int\frac{d^3k}{(2\pi)}$$

$$H_{static} = \int_0^{H^{-1}} dR\int_0^{\pi} d\theta\int_0^{2\pi} d\phi\; R^2\sin\theta\left[\frac{f(R)\,\Pi^2}{2R^4(\sin\theta)^2} + f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}(\nabla_S\phi)^2\right]$$

$$\Omega$$

$$d\theta^2 + \sin^2\theta\, d\phi$$

$$f(R) = 1 - R^2H^2$$

$$R < H^{-1}$$

$$\rho = Z^{-1}e^{-\beta H_{static}}$$

$$\mathcal{H} = \frac{1}{2} \frac{f(R)\,\Pi\,\Pi}{R^2 \sin\theta} + \frac{R^2 \sin\theta}{2}\left[f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}\left(\nabla_S \phi\right)^2 \right]$$

$$\phi = \frac{1}{(2\pi)}\int d^3k \; \frac{}{(2\pi)}$$

$$H_{static} = \int_0^{H^{-1}} dR \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \; R^2 \sin\theta \left[\frac{f(R)\,\Pi^2}{2R^4 (\sin\theta)^2} + f(R)\left(\frac{\partial\phi}{\partial R}\right)^2 + \frac{1}{R^2}\left(\nabla_S \phi\right)^2 \right]$$

S

Ω

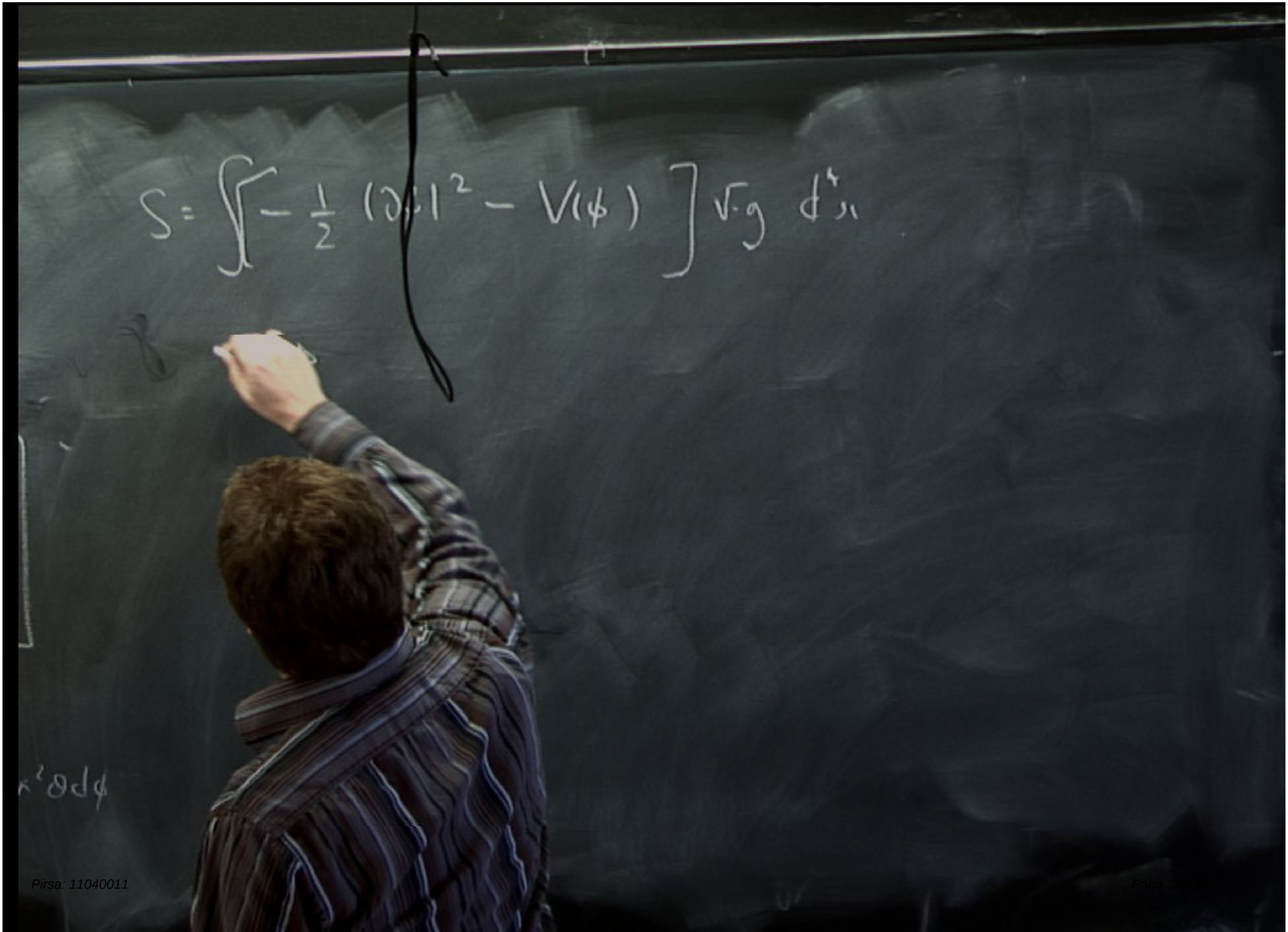
$d\theta^2 + \sin^2\theta \, d\phi$

$$f(R) = 1 - R^2 H^2$$

$$R < H^{-1}$$

$$\rho = Z^{-1} e^{-\beta H_{static}}$$

$$Z = \mathrm{Tr}\left(e^{-\beta H_{static}}\right)$$


$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$

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$r^2 \theta\, d\phi$

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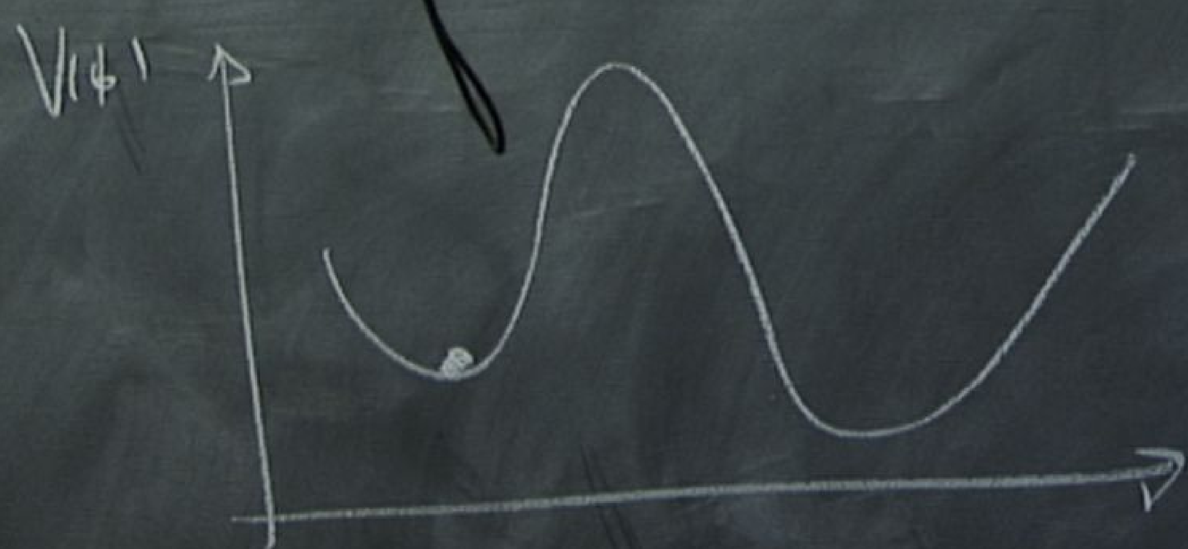

$$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$

φ

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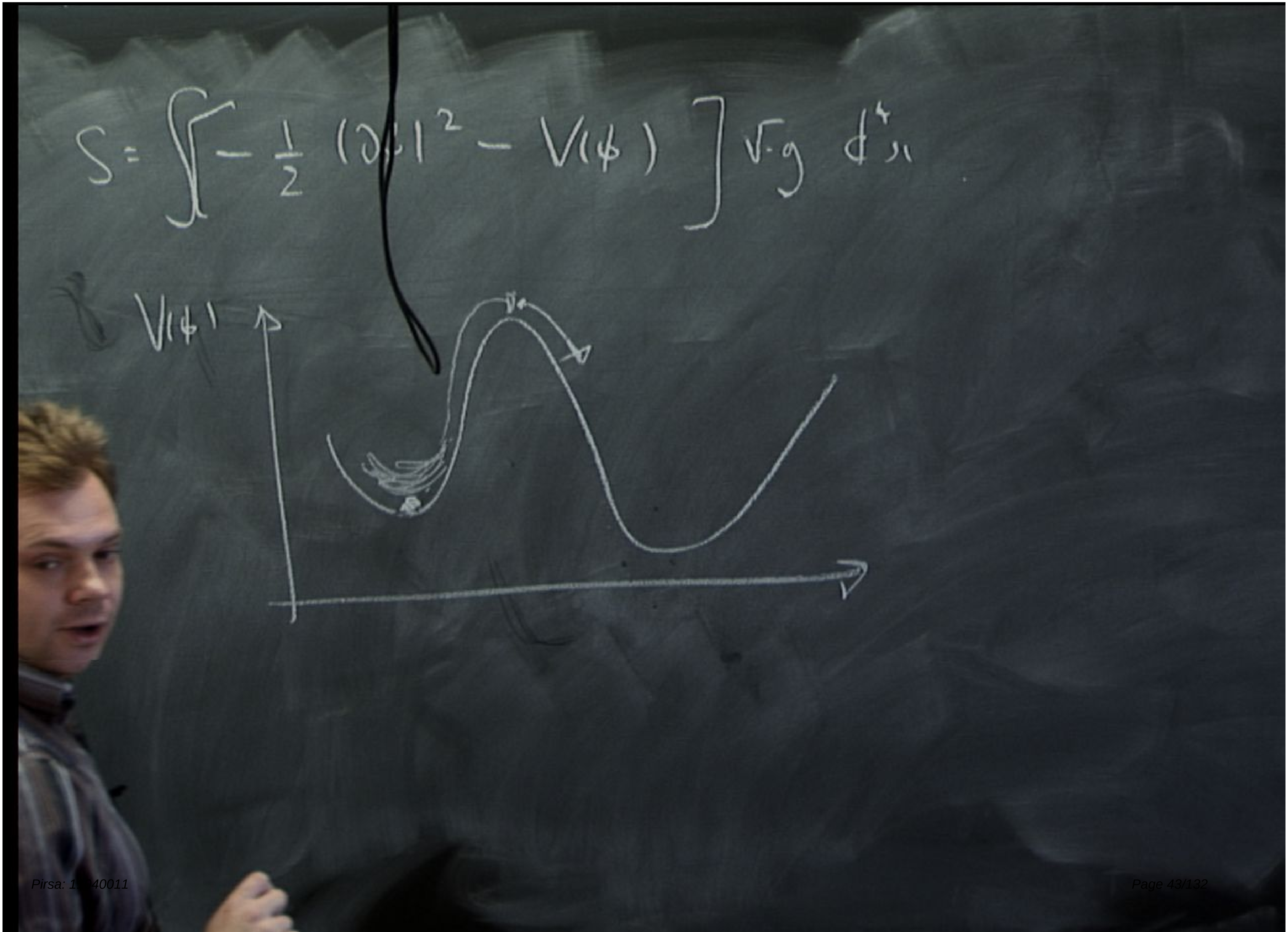
ϕ


$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$
$V(\phi)$

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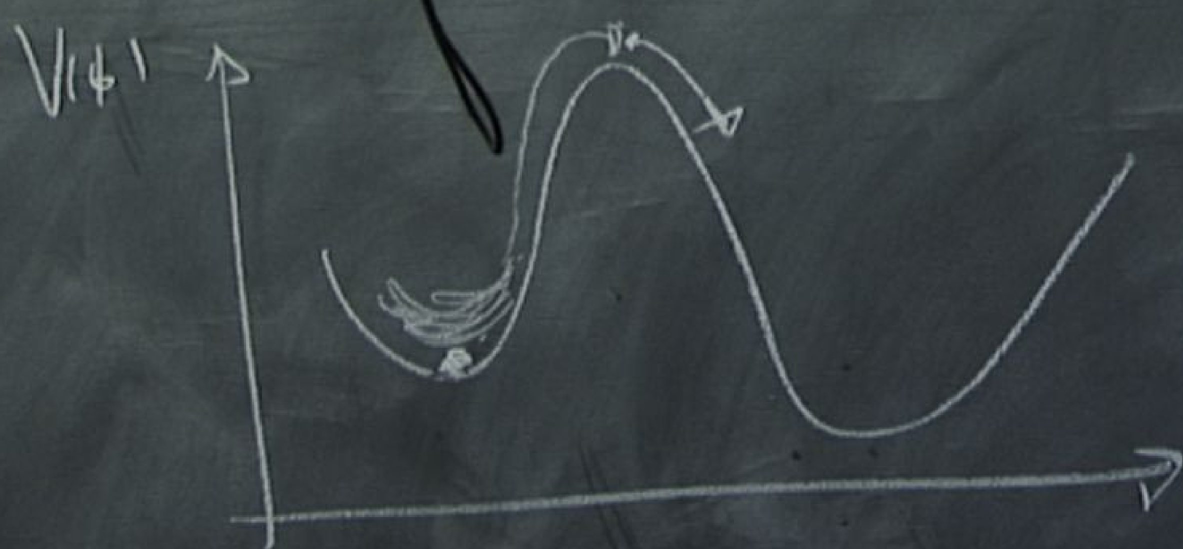
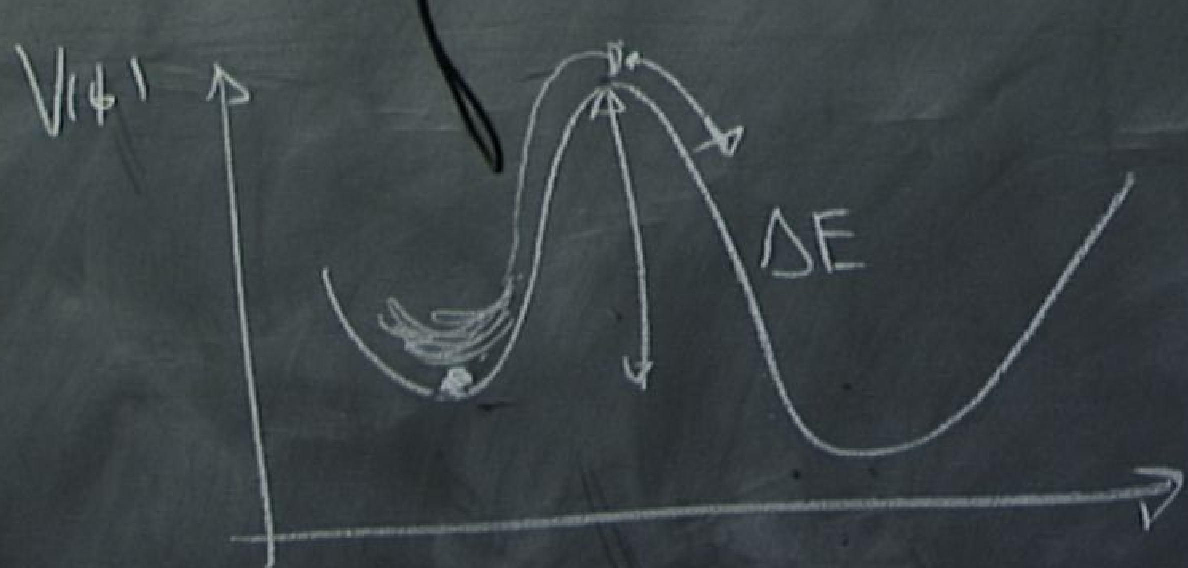


Thermal tunneling

Thermally activated tunneling.

$$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$

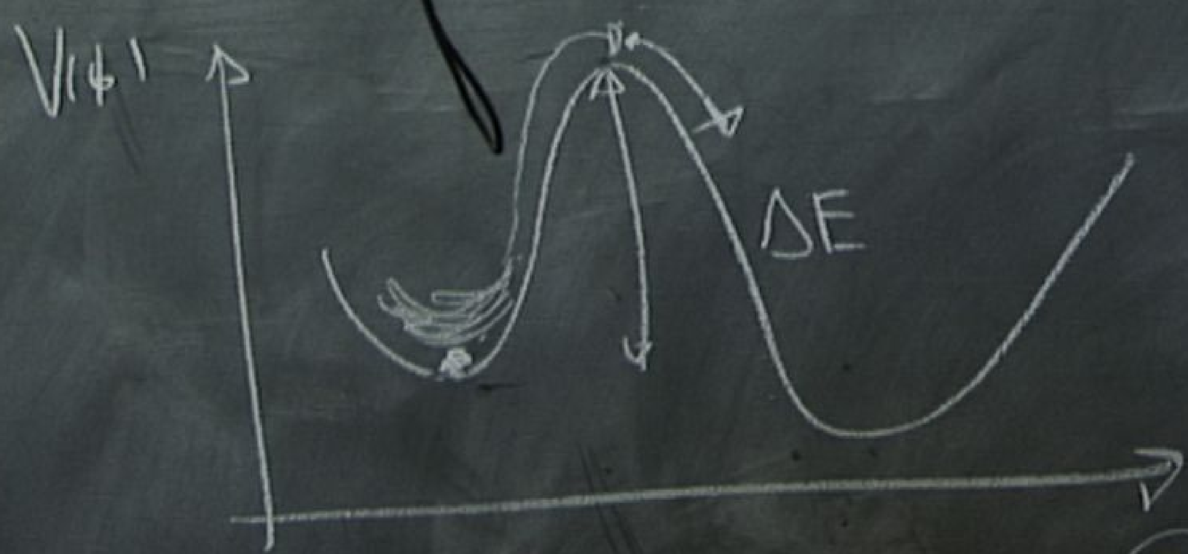


Thermal tunneling

Thermally activated tunneling.

$$\text{Tunneling rate} \sim e^{-\Delta E / kT}$$

$$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$



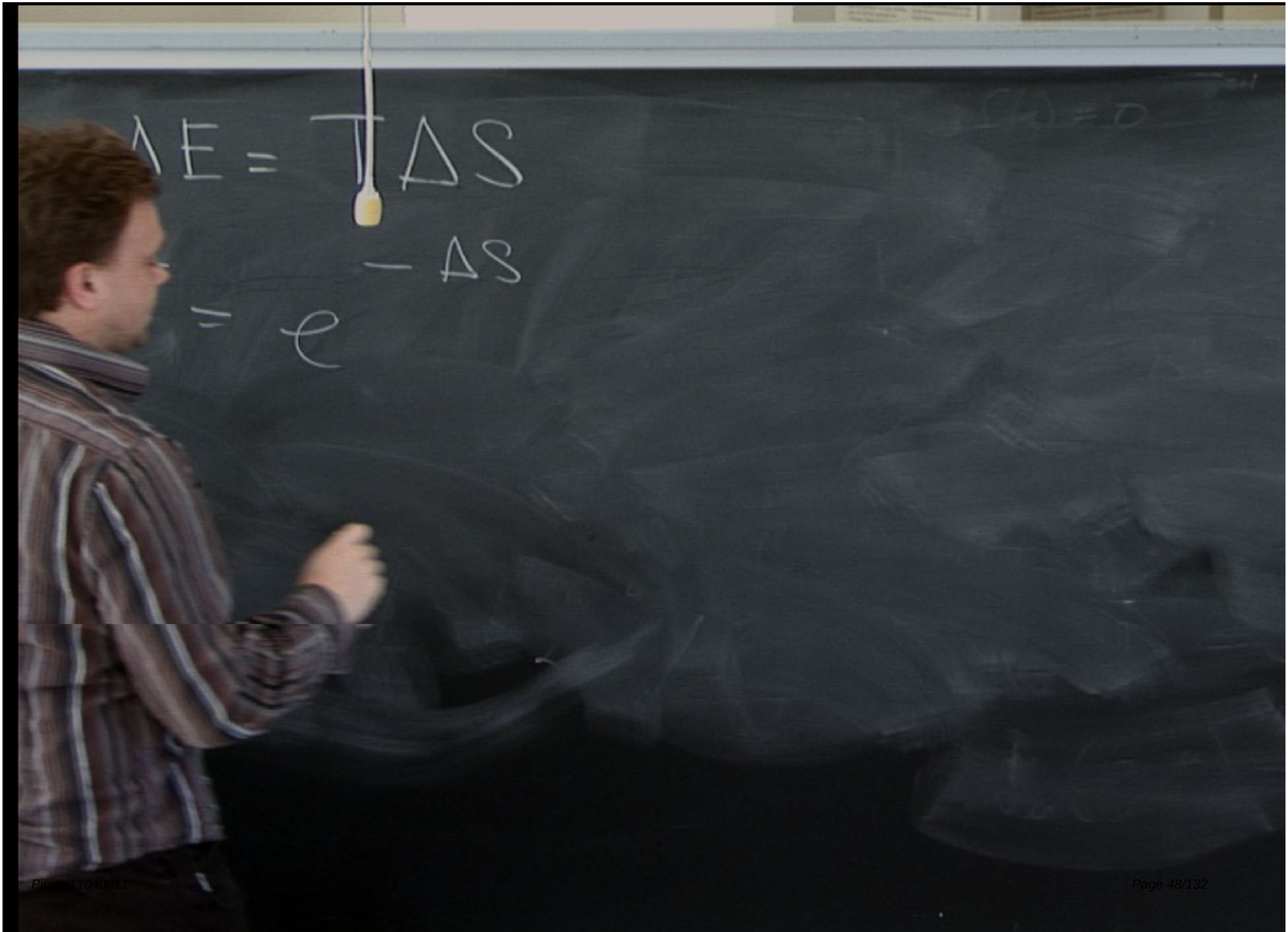
Thermal tunneling

Thermally activated tunneling.

Activation energy.

$$\text{Tunneling rate} \sim e^{-\Delta E / kT}$$

$$\Delta E = T \Delta S$$


ΔE = TΔS
− ΔS
= e

$$\Delta E = T \Delta S$$

$$S = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$\Gamma = e^{-\Delta S}$$

$$\Delta E = T \Delta S$$

$$S = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$\Gamma = e^{-\Delta S}$$

$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi\right.$$

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$$S = \frac{8\pi^2 M_{pl}^2}{H^2}$$

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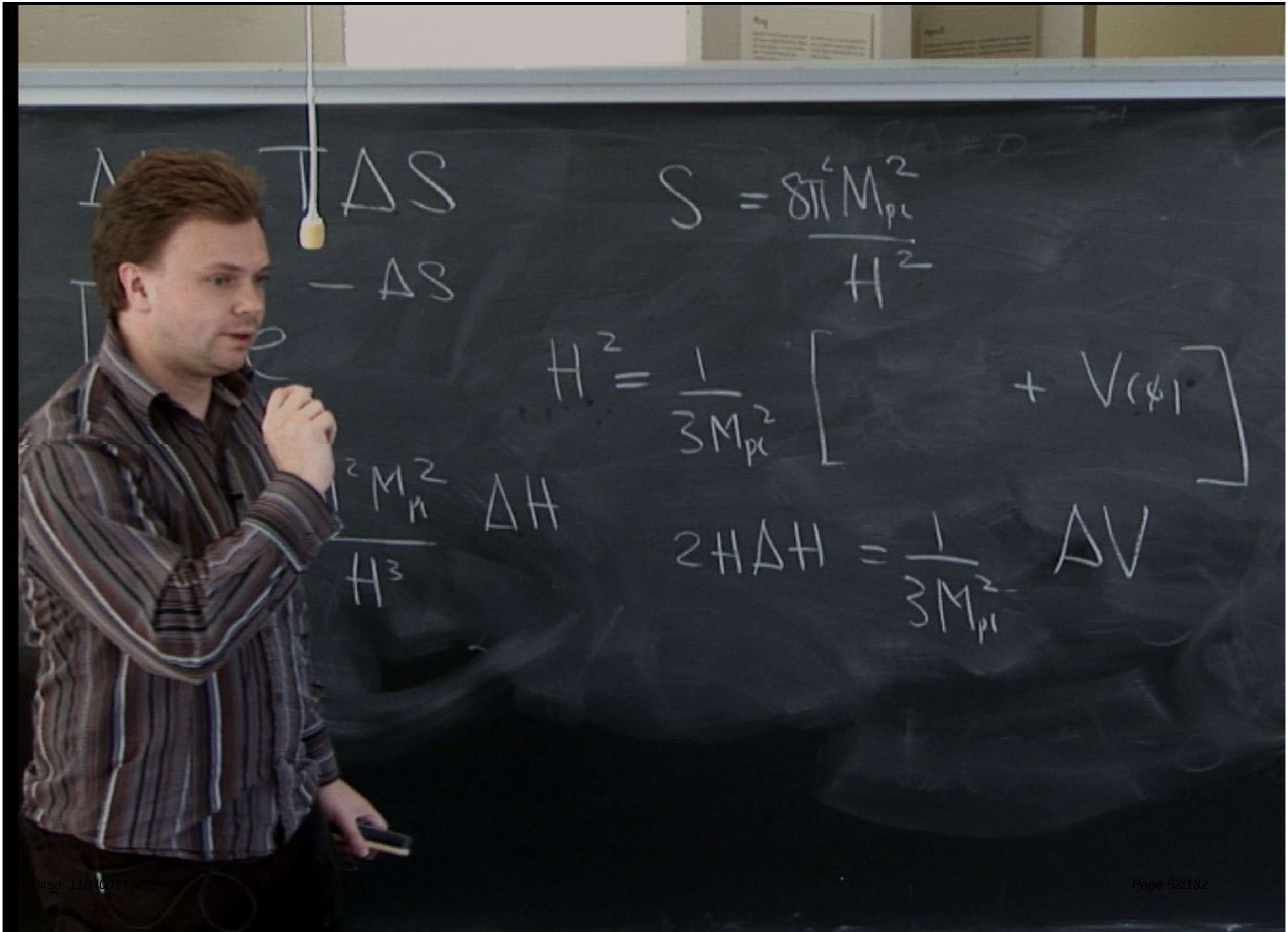
$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$


ΔS
− ΔS
S = 8π² M_pl² / H²
H² = 1 / 3M_pl² [+ V(φ)]
M_pl² / H³ ΔH
2HΔH = 1 / 3M_pl² ΔV

$$\Delta E = T \Delta S$$

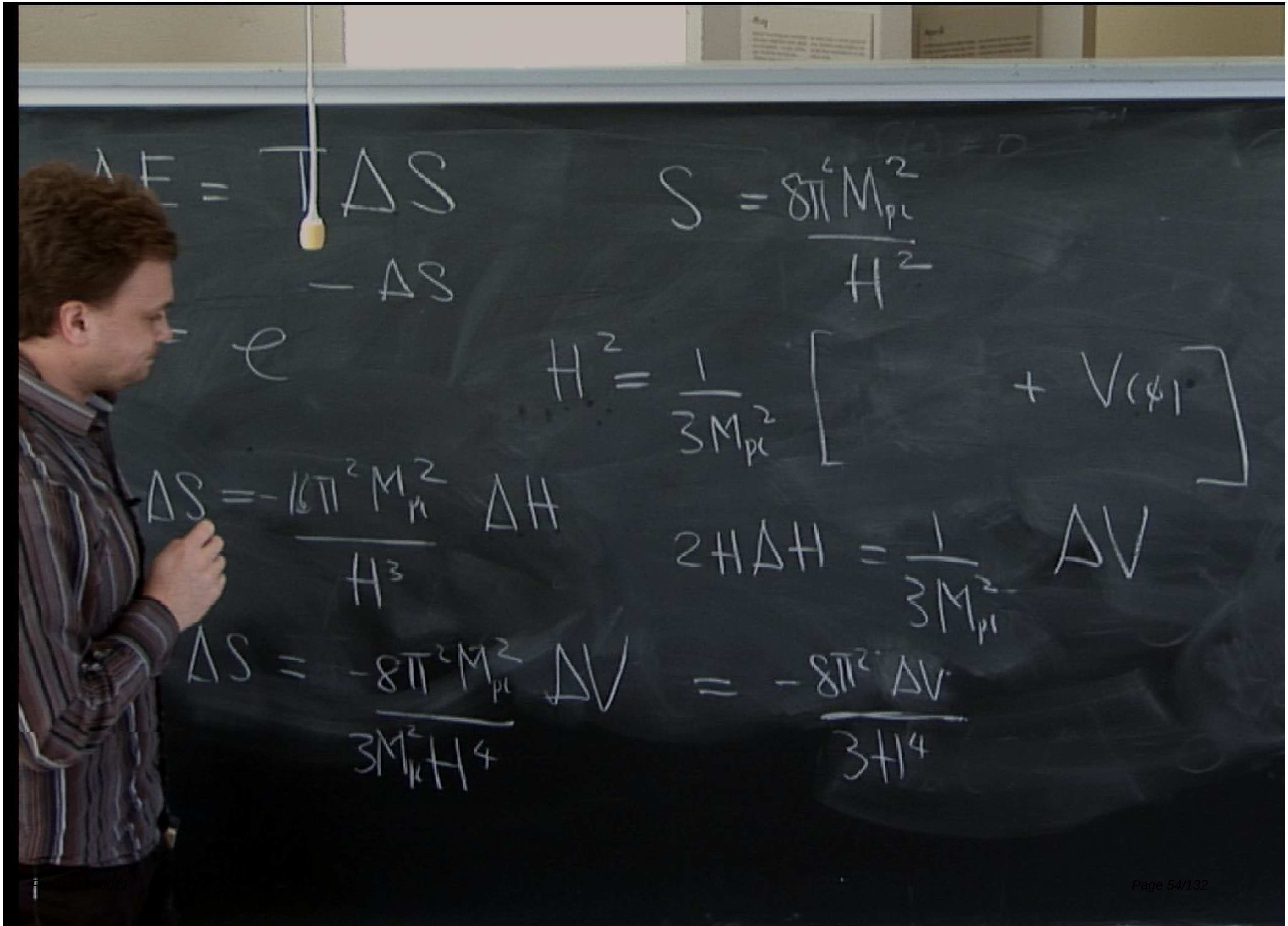
$$S = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$\Gamma = e^{-\Delta S}$$

$$H^2 = \frac{1}{3M_{pl}^2}\left[\qquad + V(\phi) \right]$$

$$\Delta S = -\frac{16\pi^2 M_{pl}^2}{H^3} \Delta H$$

$$2H\Delta H = \frac{1}{3M_{pl}^2} \Delta V$$


$\Delta E = T\Delta S$
$e^{-\Delta S}$
$S = \frac{8\pi^2 M_{pl}^2}{H^2}$
$H^2 = \frac{1}{3M_{pl}^2}\left[+ V(\phi)\right]$
$\Delta S = -\frac{16\pi^2 M_{pl}^2}{H^3}\Delta H$
$2H\Delta H = \frac{1}{3M_{pl}^2}\Delta V$
$\Delta S = -\frac{8\pi^2 M_{pl}^2}{3M_{pl}^2 H^4}\Delta V = -\frac{8\pi^2\Delta V}{3H^4}$

$$\Delta E = T \Delta S \qquad S = \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$\Gamma = e^{-\Delta S}$$

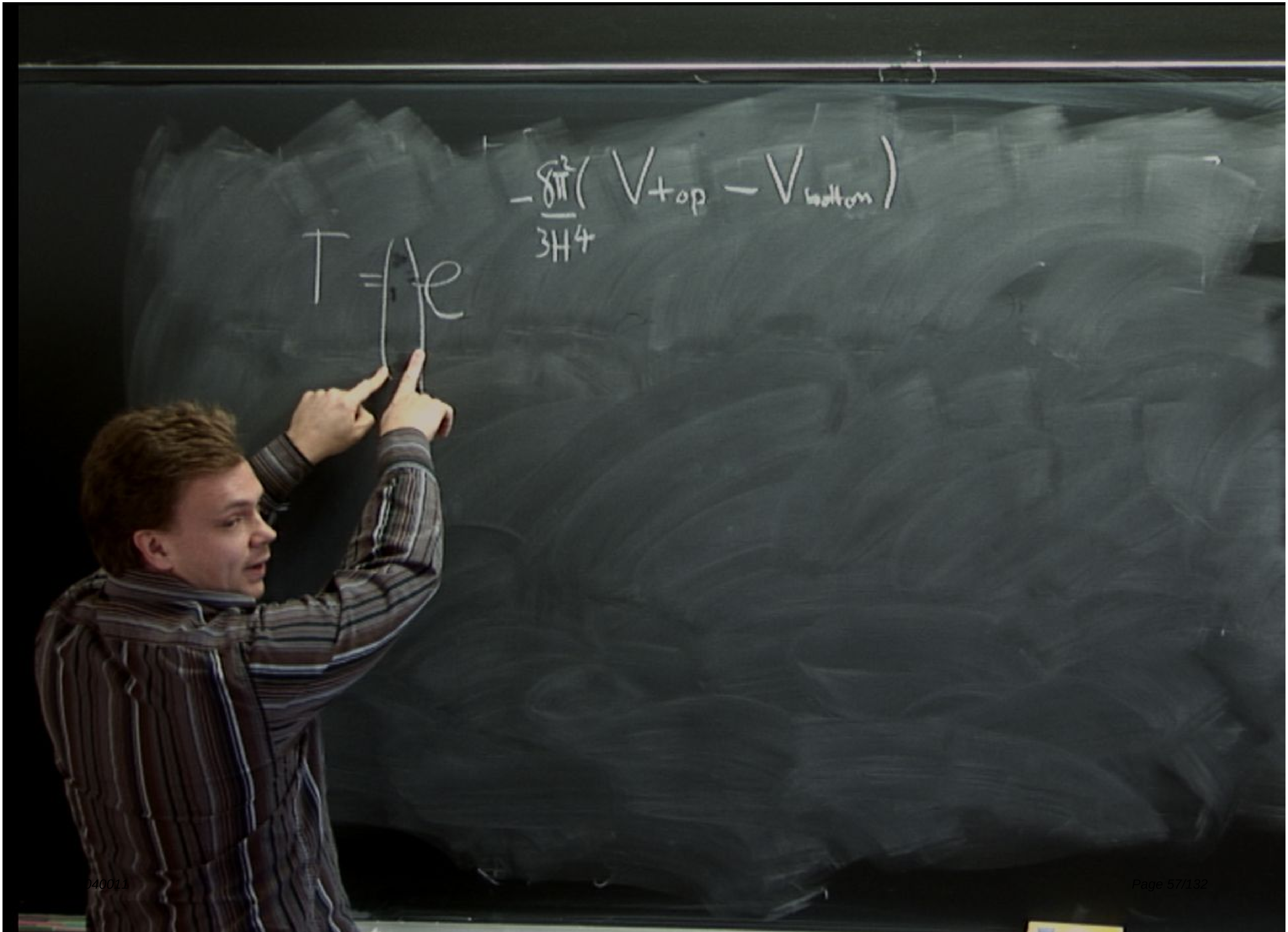
$$H^2 = \frac{1}{3M_{pl}^2}\left[\qquad + V(\phi) \right]$$

$$\Delta S = -\frac{16\pi^2 M_{pl}^2}{H^3}\Delta H$$

$$2H\Delta H = \frac{1}{3M_{pl}^2}\Delta V$$

$$\Delta S = -\frac{8\pi^2 M_{pl}^2}{3M_{pl}^2 H^4}\Delta V = -\frac{8\pi^2 \Delta V}{3H^4}$$

$$T = e^{-\frac{8\pi^2}{3H^4}\left(V_{top} - V_{bottom}\right)}$$


$T = (\ ) e^{-\frac{8\pi^2}{3H^4}(V_{top} - V_{bottom})}$

$$T = (\;)\, e^{-\frac{8\pi^2}{3H^4}\left(V_{top} - V_{bottom}\right)}$$

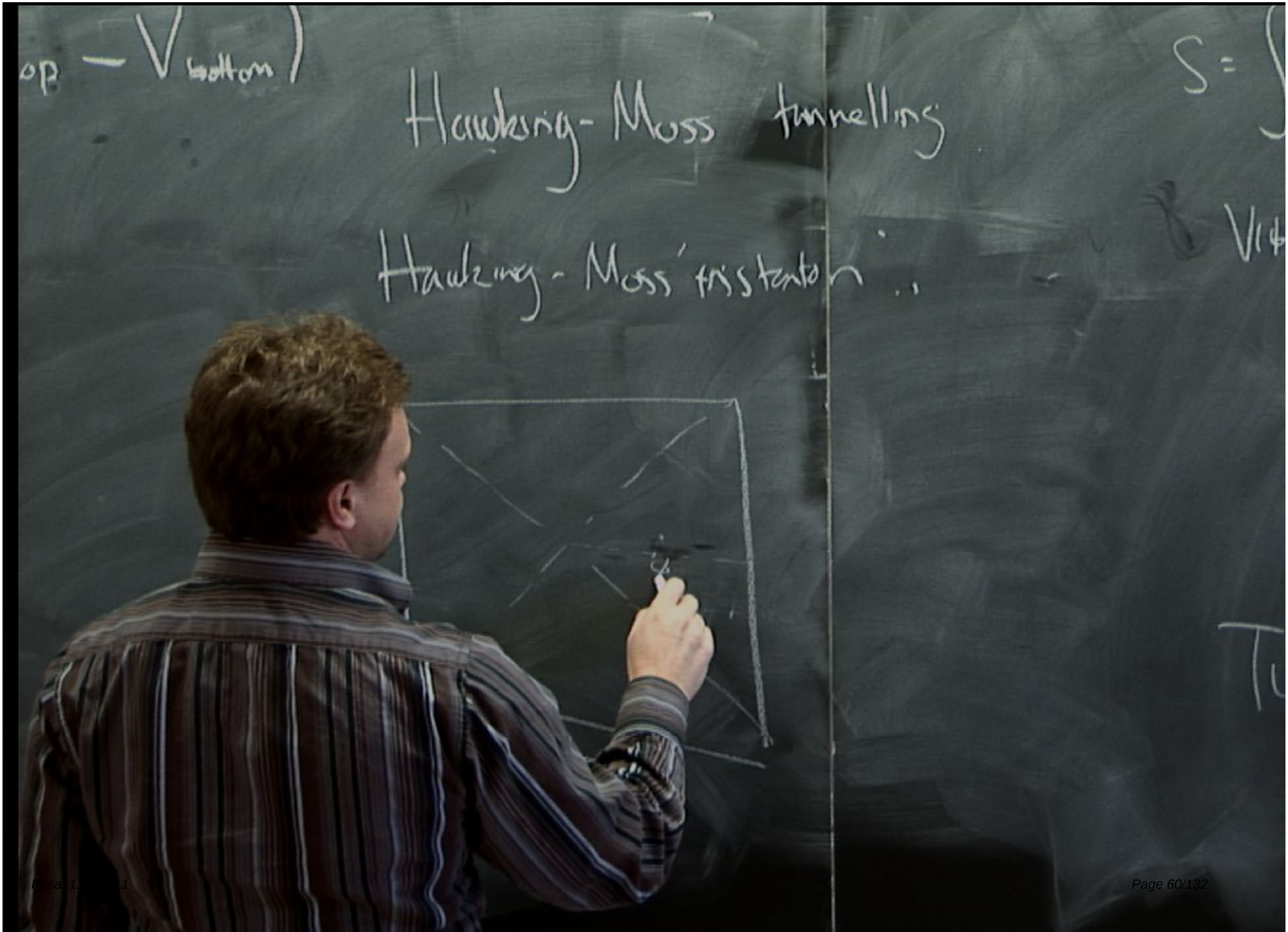
prefactor

$$\Gamma = (\;)\, e^{-\frac{8\pi^2}{3H^4}\left(V_{top} - V_{bottom}\right)}$$

prefactor

Hawking-Moss tunnelling

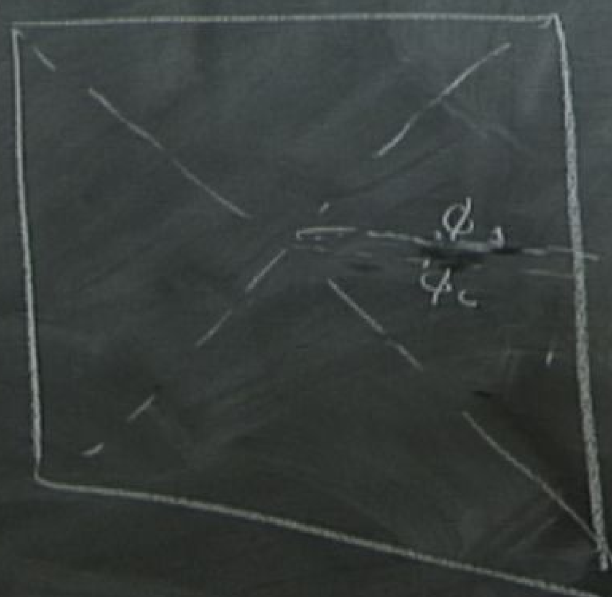
Hawking-Moss 'instanton' ..


Hawking-Moss tunnelling
Hawking-Moss instanton

$\text{op} - V_{\text{bottom}})$

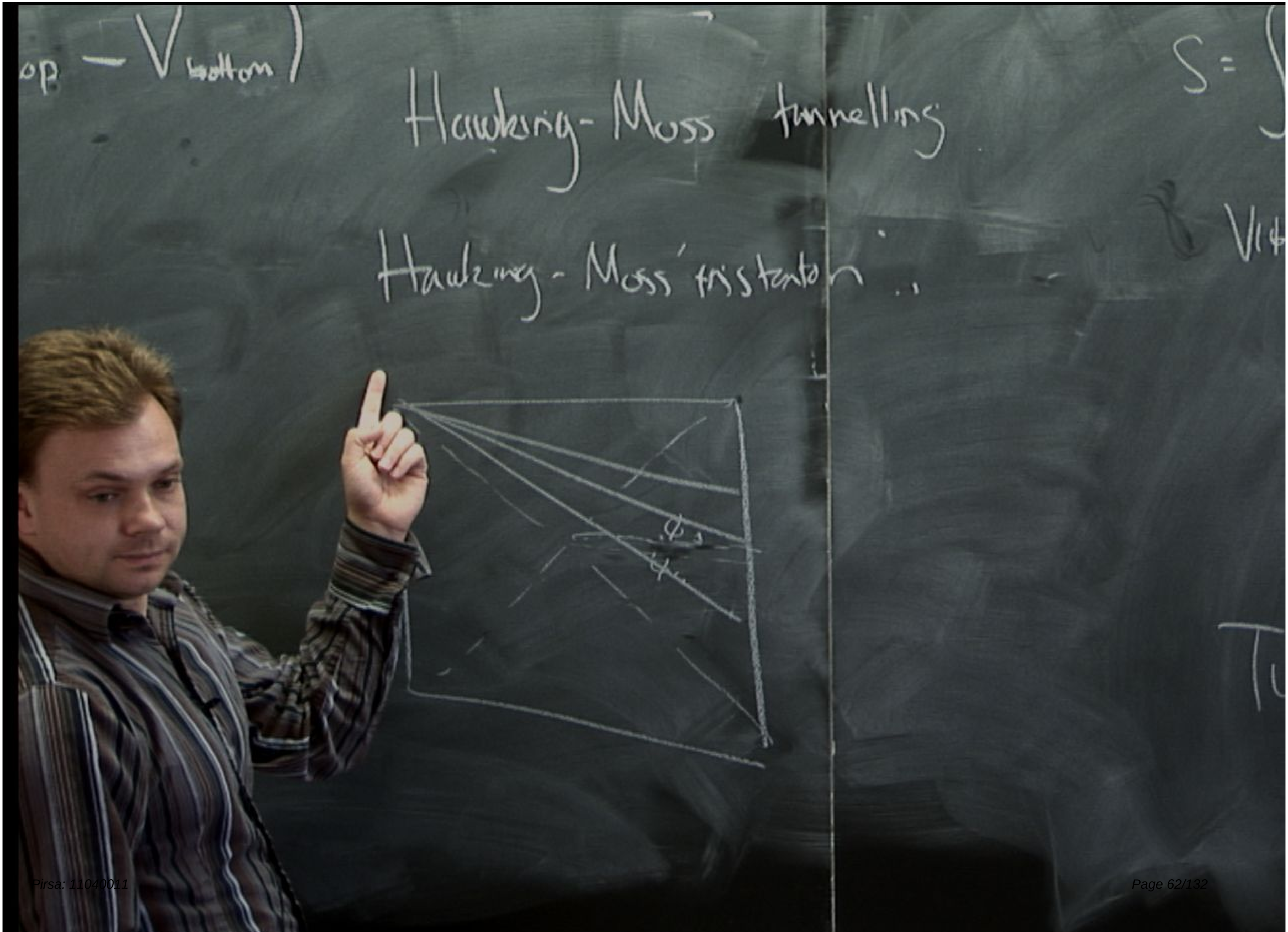
Hawking-Moss tunnelling

Hawking-Moss instanton



$S = \int$

$\& \; V(\phi$


op − V bottom)
Hawking-Moss tunnelling
Hawking-Moss instanton
S =

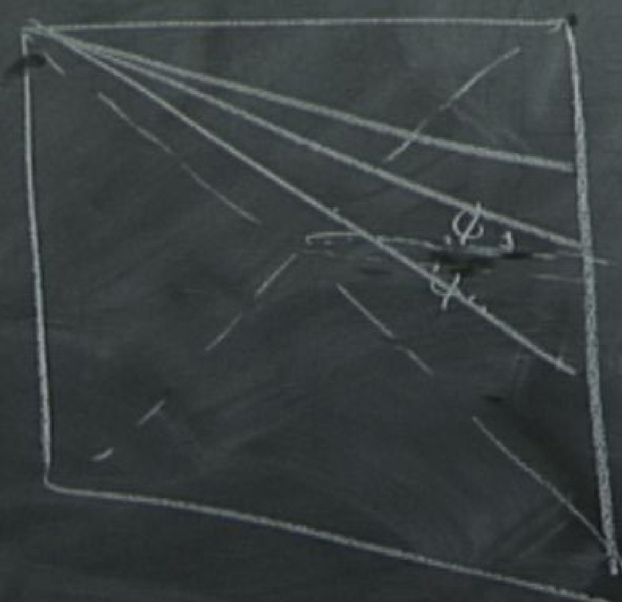
$\text{op} - V_{\text{bottom}})$

$S = \int$

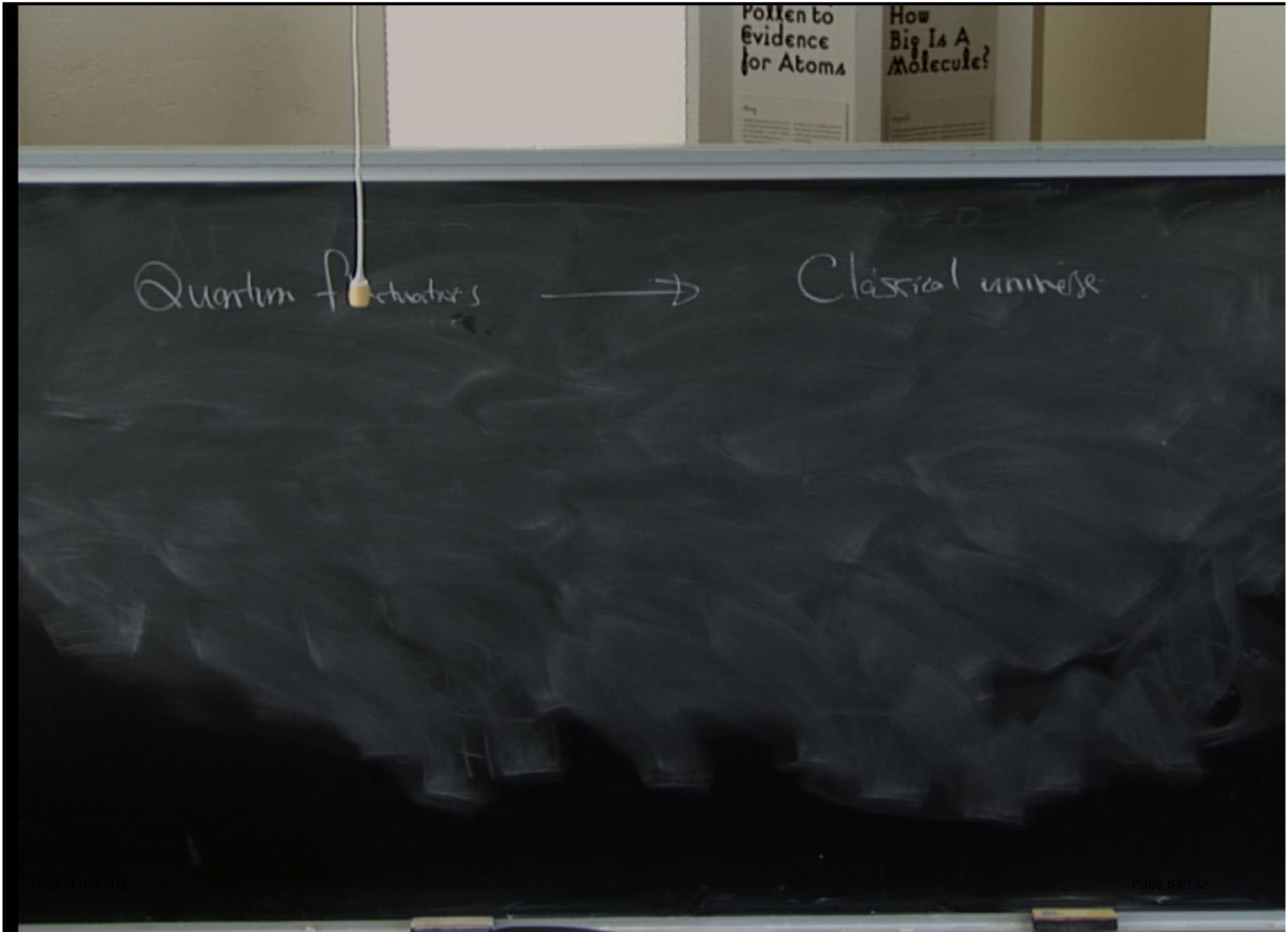
Hawking-Moss tunnelling

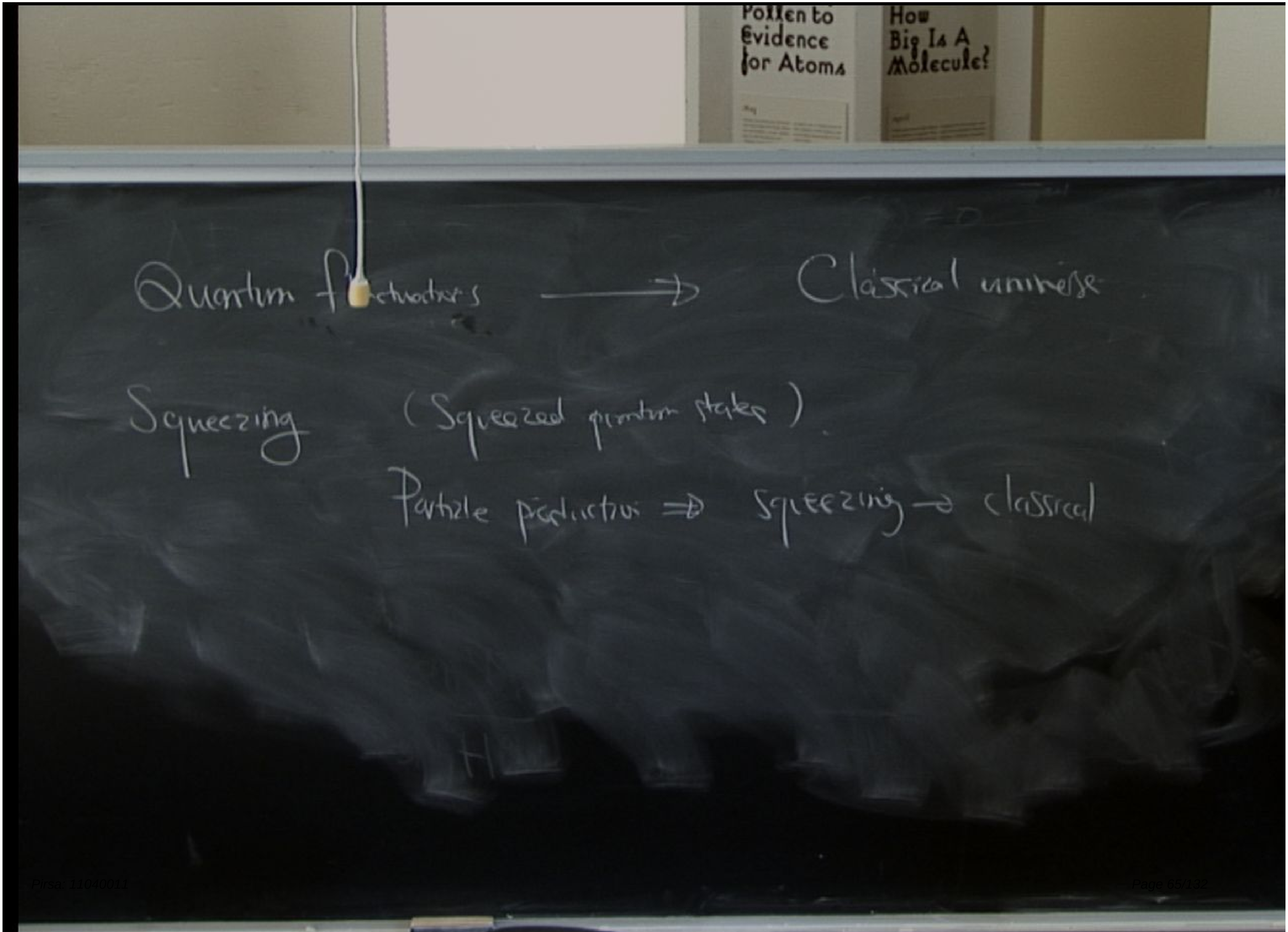
$V(\phi$

Hawking-Moss instanton ..



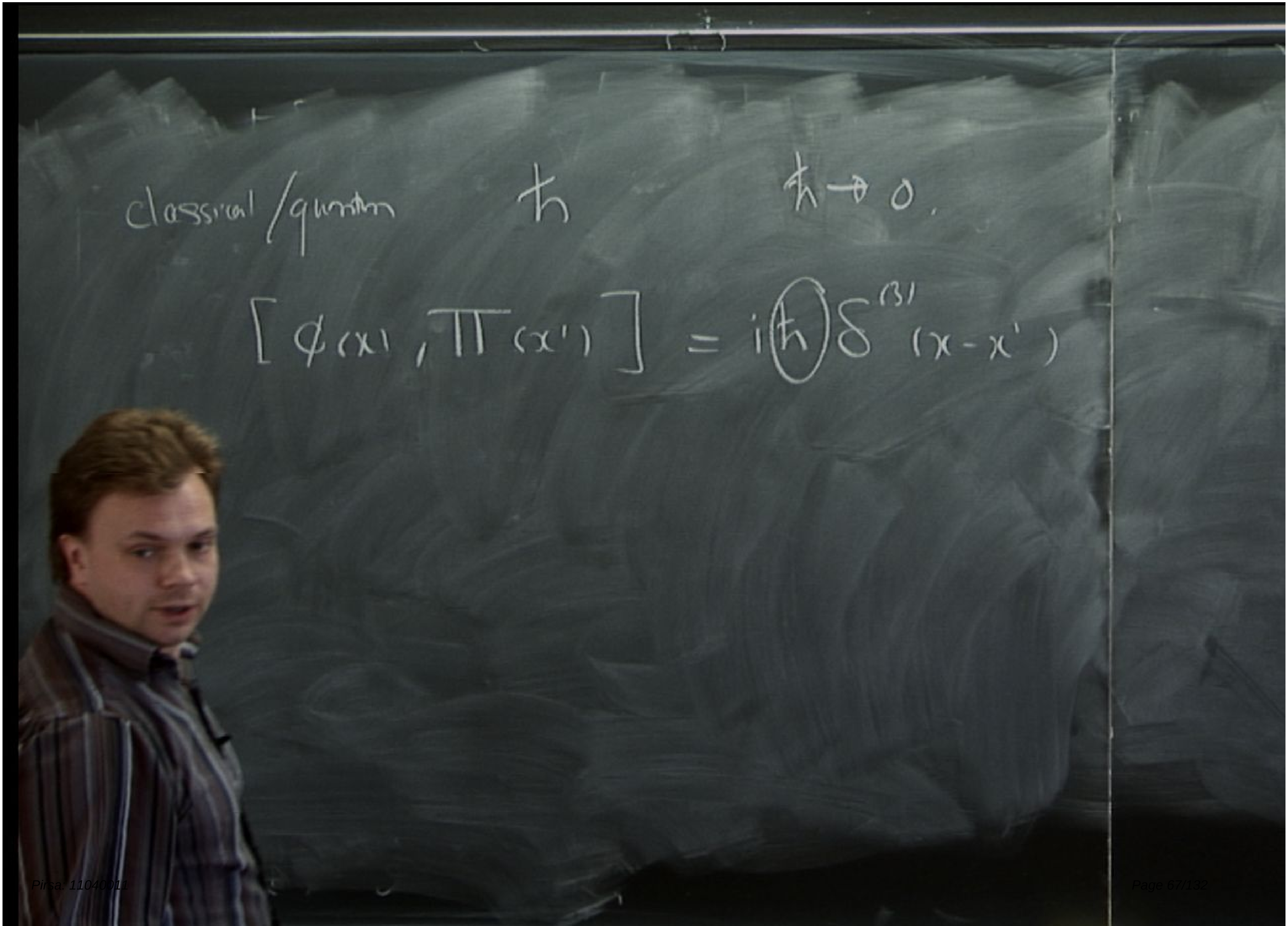
Tu


Pollen to Evidence for Atoms
How Big Is A Molecule?
Quantum fluctuations → Classical universe


Pollen to Evidence for Atoms
How Big Is A Molecule?
Quantum fluctuations → Classical universe
Squeezing (Squeezed quantum states)
Particle production ⇒ squeezing → classical

classical / quantum $\qquad \hbar \qquad \hbar \to 0.$

$[q$


classical / quantum
$\hbar$
$\hbar \to 0$.
$[\phi(x), \pi(x')] = i\hbar\,\delta^{(3)}(x - x')$

classical/quantum $\hbar$ $\hbar \to 0$.

$$[\phi(x), \Pi(x')] = i\hbar\,\delta^{(3)}(x-x')$$

classical / quantum $\hbar$ $\hbar \to 0.$

$$[\phi(x), \Pi(x')] = i\hbar\, \delta^{(3)}(x-x')$$

$$[x, p] = i\hbar$$

classical / quantum $\quad \hbar \quad \hbar \to 0.$

$$[\phi(x), \Pi(x')] = i\hbar\,\delta^{(3)}(x - x') \to 0$$

$$[x, p] = i\hbar \to 0$$

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$$[\phi(x), \Pi(x')] = i\hbar\,\delta^{(3)}(x - x') \to 0$$

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$$W^{+}(x,t;x',t') = \langle 0|\,\phi(x,t)\,\phi(x',t')\,|0\rangle$$

classical / quantum $\hbar$ $\hbar \to 0$.

$$[\phi(x), \Pi(x')] = i\hbar\, \delta^{(3)}(x-x') \to 0$$

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classical / quantum $\quad \hbar \quad \hbar \to 0.$

$$[\phi(x), \Pi(x')] = i\hbar\, \delta^{(3)}(x - x') \to 0$$

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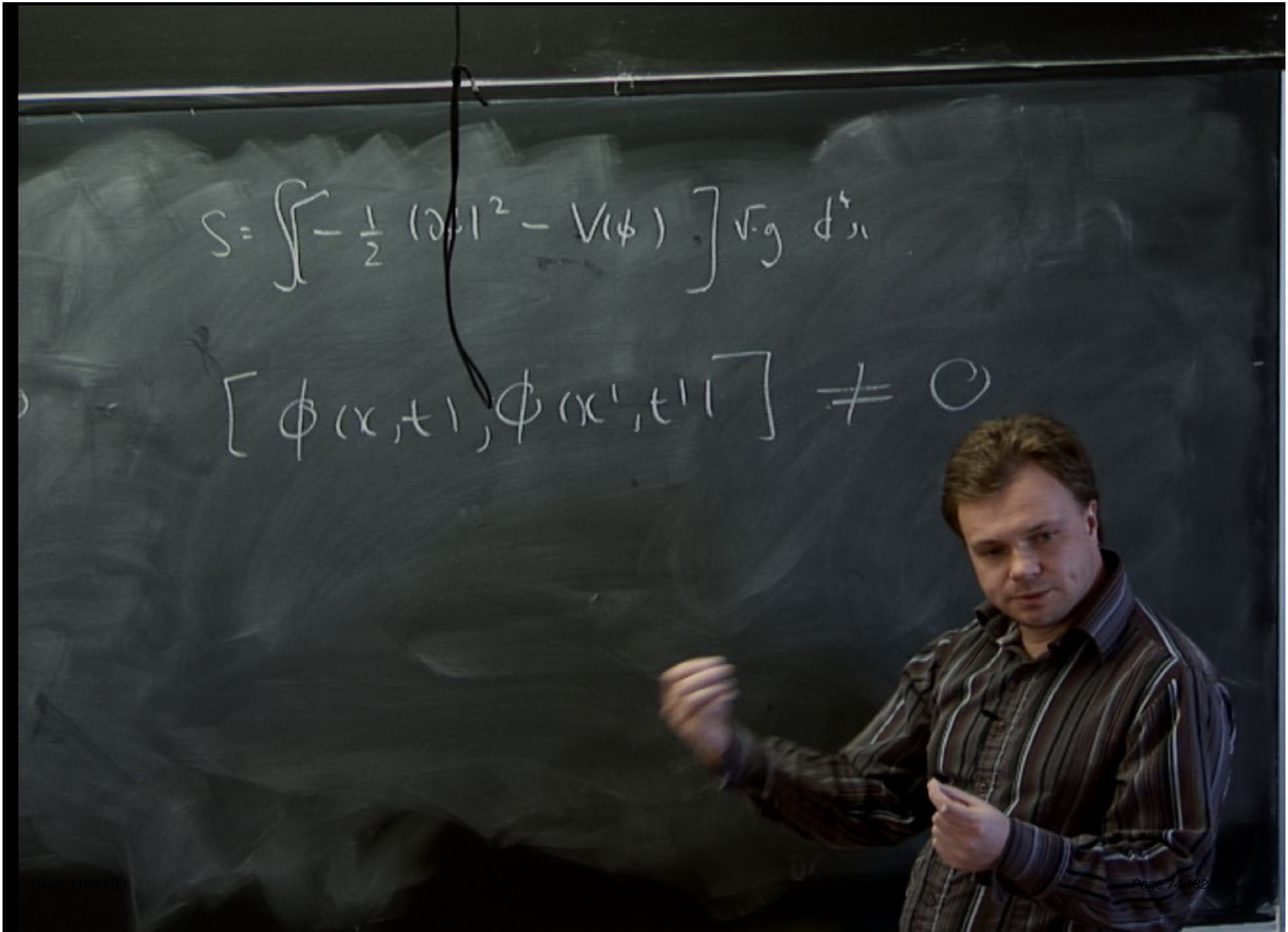
$$W^{+}(x,t; x',t') = \langle 0 | \phi(x,t)\, \phi(x',t') | 0 \rangle$$

$$W^{+}(x',t', x,t) \neq W^{+}(x,t; x',t')$$



$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$
$[\phi(x,t), \phi(x',t')] \neq 0$

$$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$

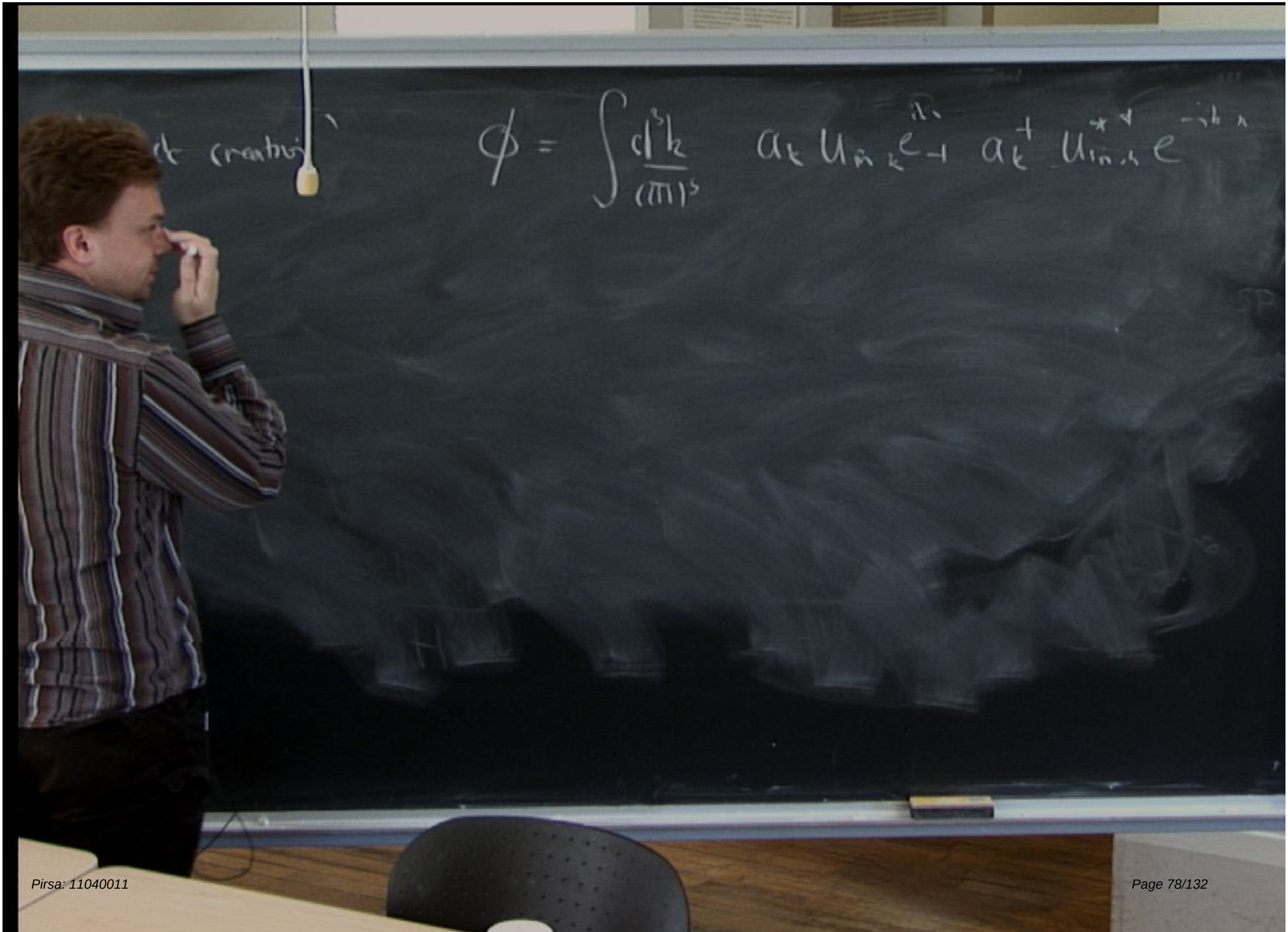
$$[\phi(x,t), \underbrace{\phi(x',t')}_{\text{depends on } \phi(x,t) \text{ and } \Pi(x,t)}] \neq 0$$

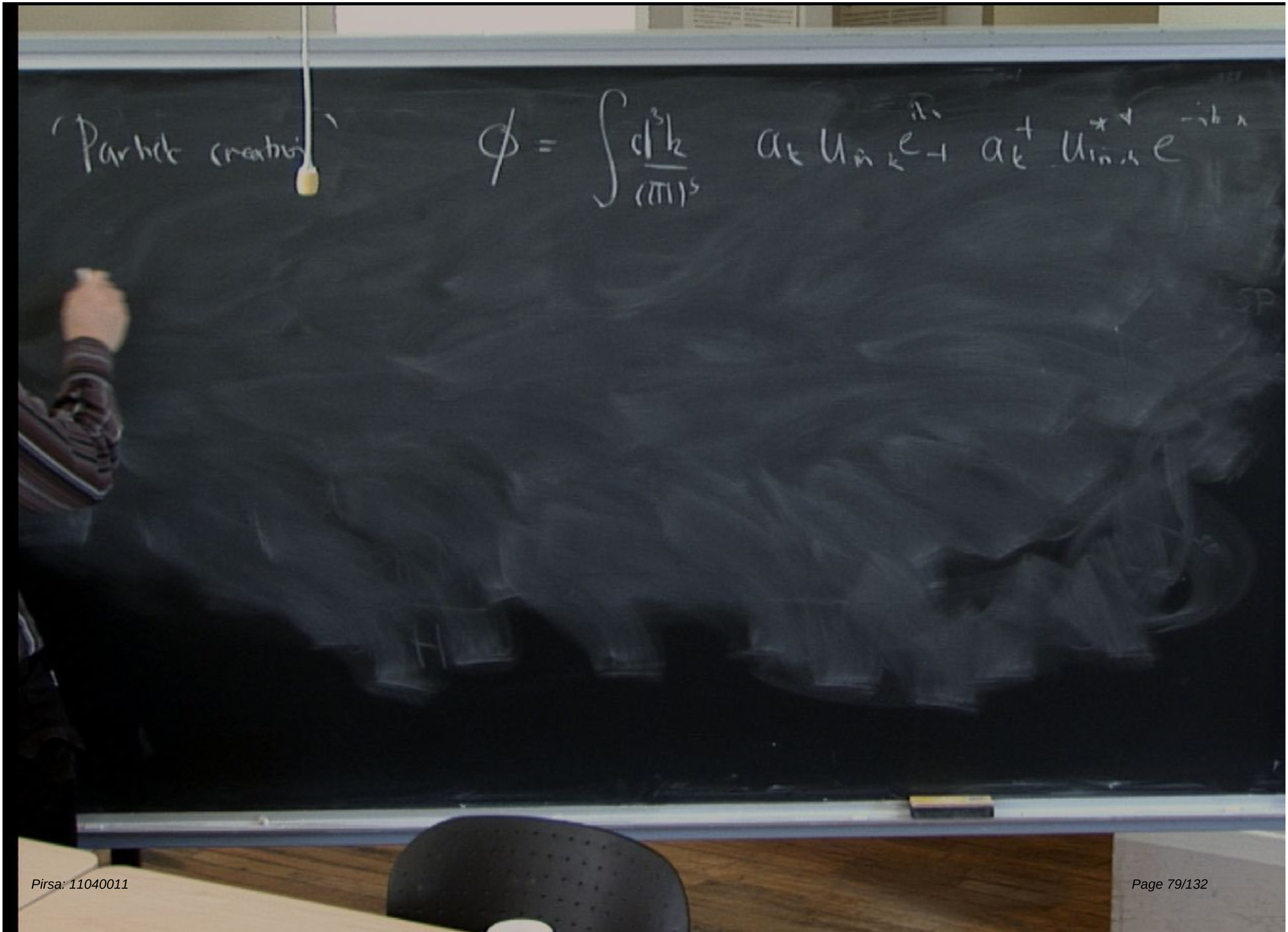
$$S = \int \left[-\frac{1}{2} (\partial \phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$

$$[\phi(x,t), \underbrace{\phi(x',t')}_{\text{depends on } \phi(x,t) \text{ and } \Pi(x,t)}] \neq 0 \; \hbar$$

$$S = \int \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \sqrt{-g}\, d^4x$$

$$\langle 0 | \left[\phi(x,t), \underbrace{\phi(x',t')}_{\text{depends on } \phi(x,t) \text{ and } \Pi(x,t)} \right] | 0 \rangle \neq 0 \quad \hbar$$

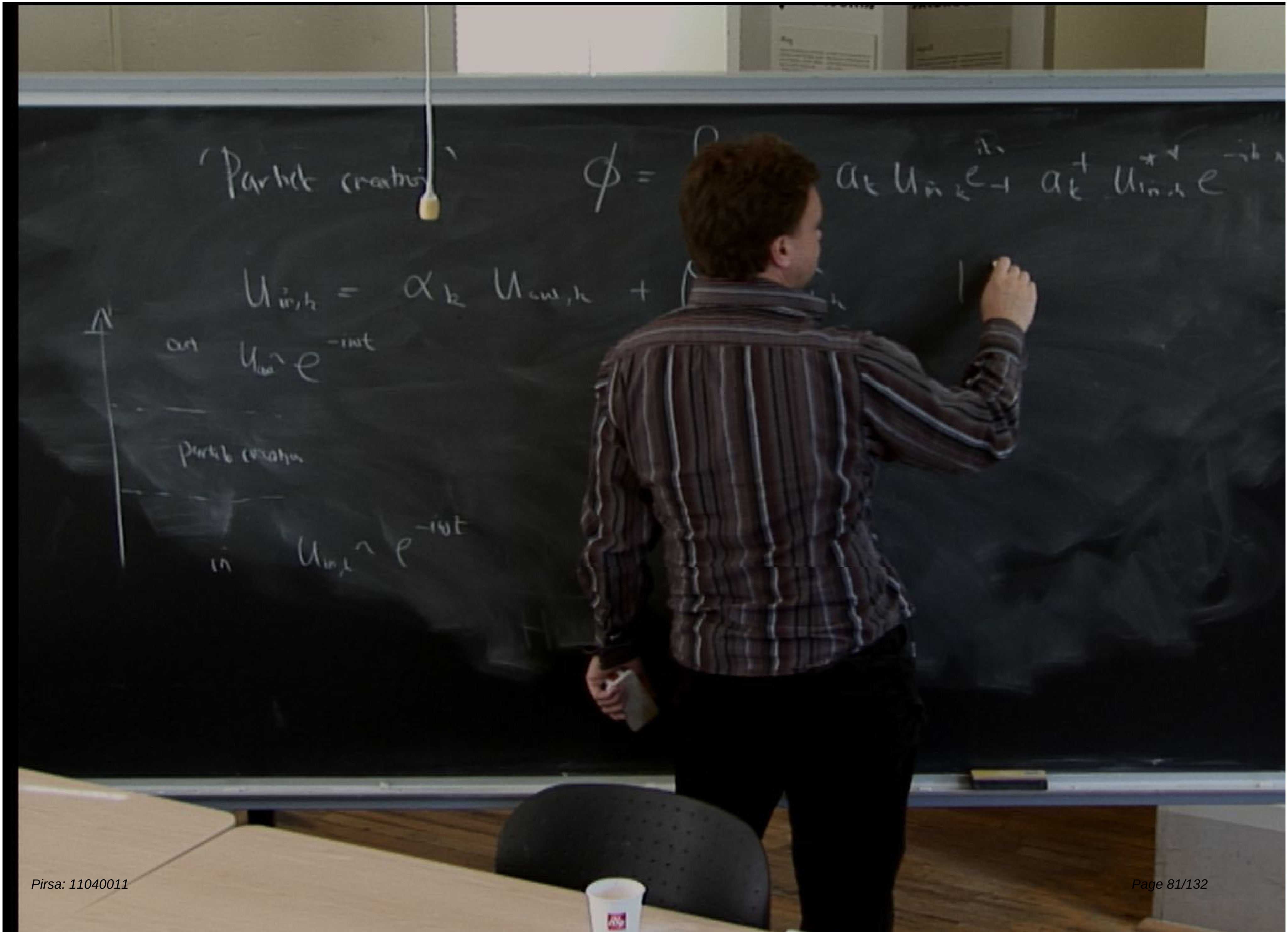

$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u_{in,k}^* e^{-ikx}$

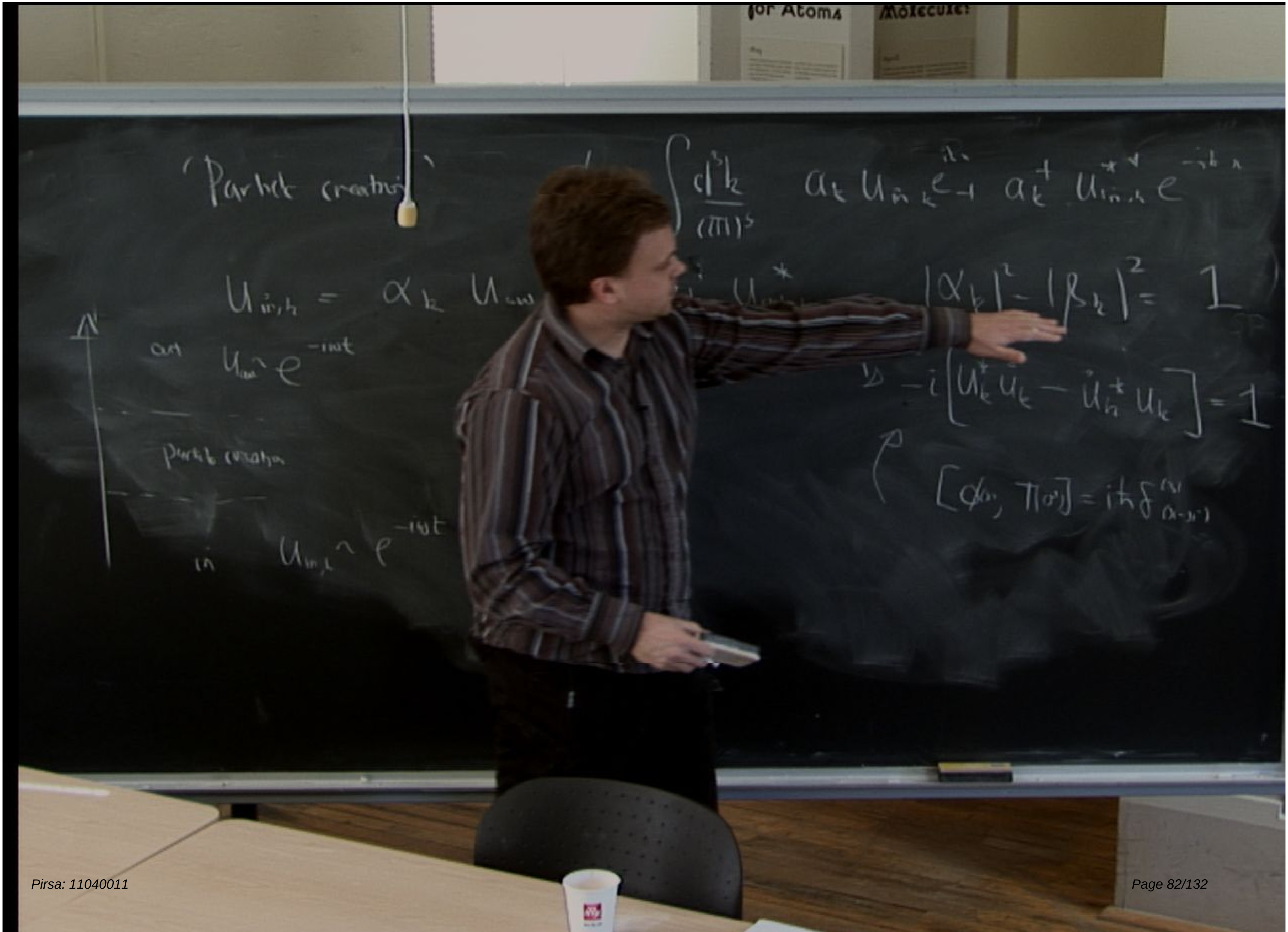

'Particle creation'
$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ikx} + a_k^\dagger u^{*}_{in,k} e^{-ikx}$

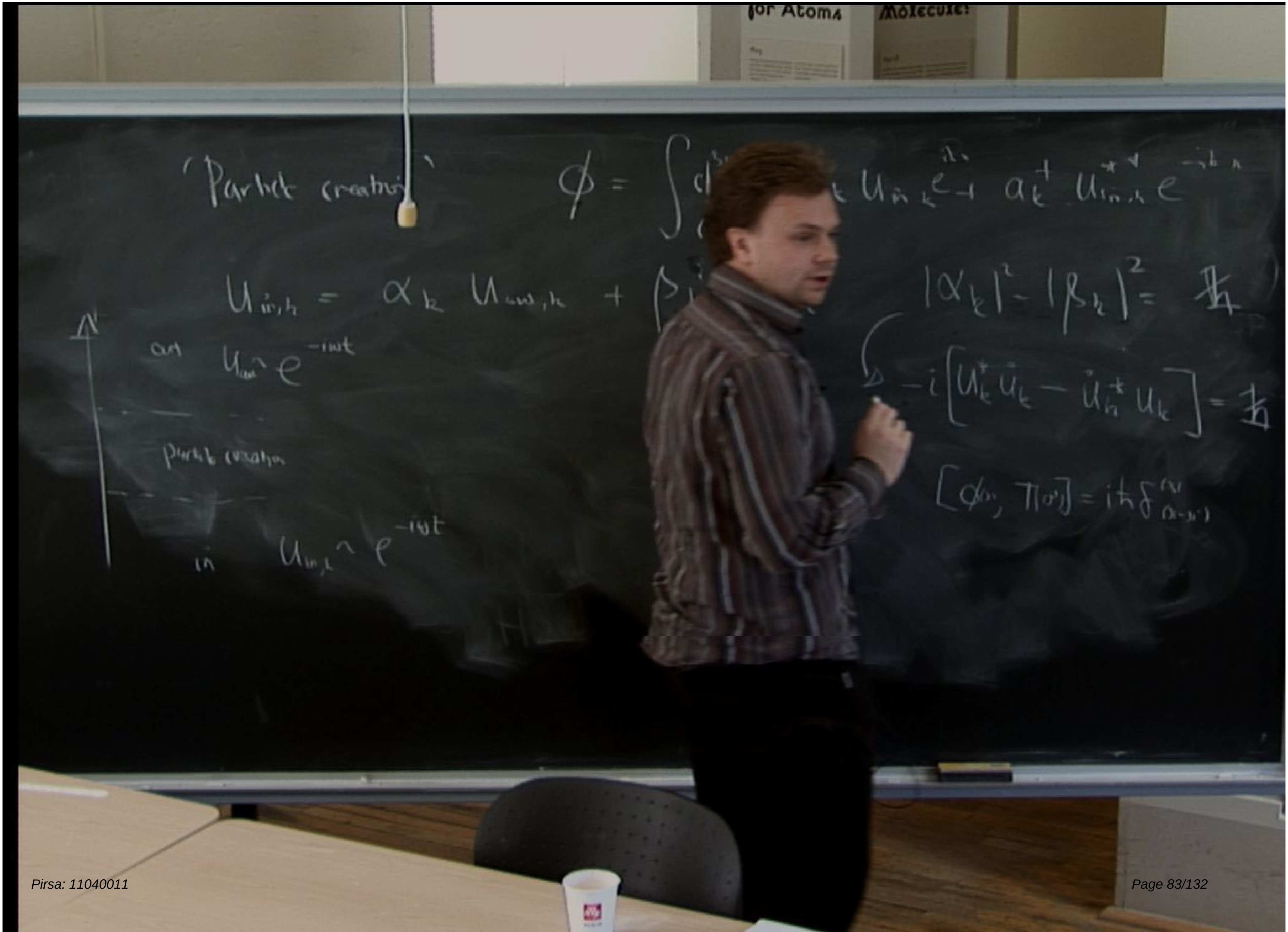
"Particle creation"

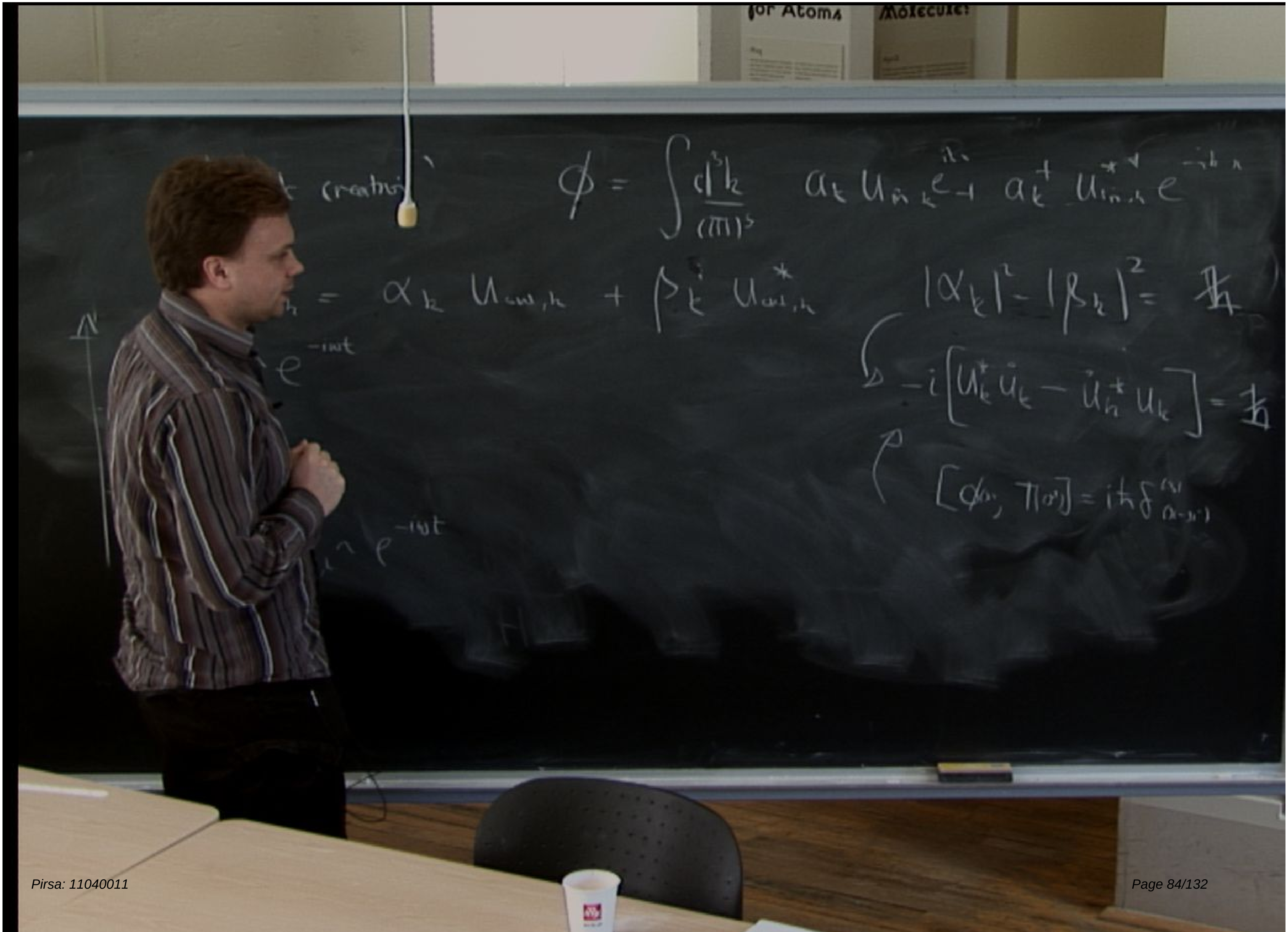
$$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$$

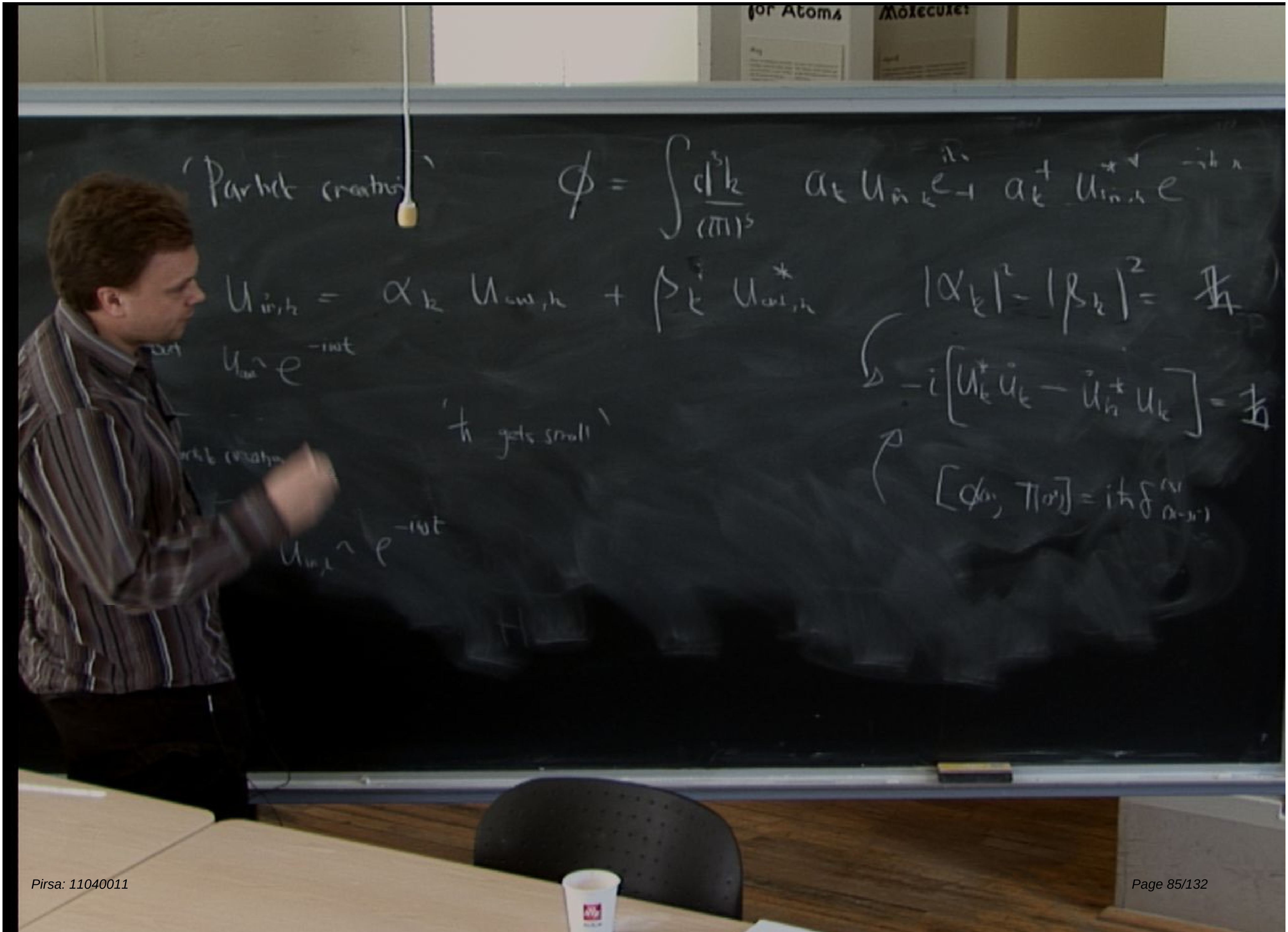
$$u_{in,k} = \alpha_k u_{out,k} + \beta_k u^*_{out,k}$$

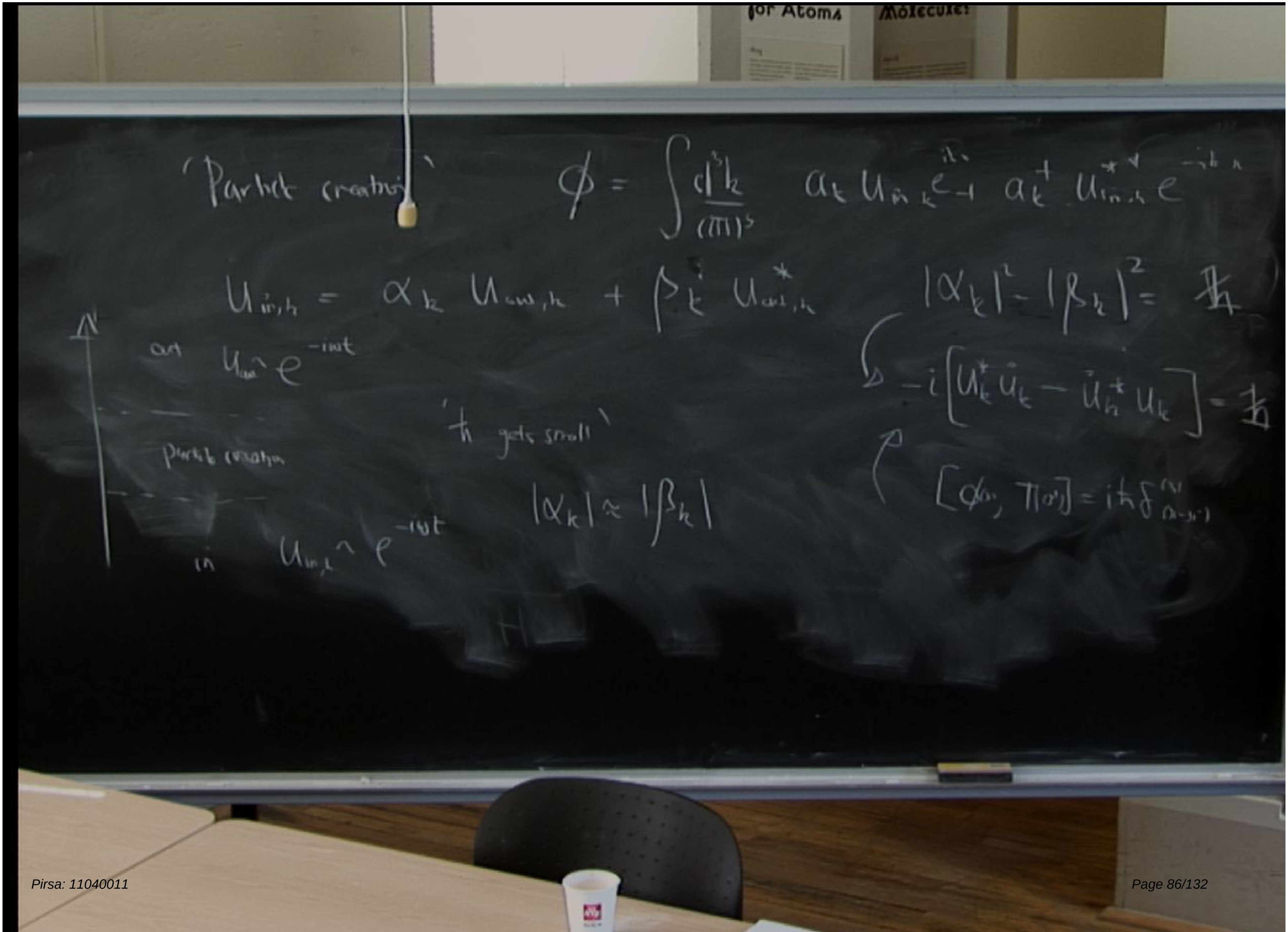



out
in


for Atoms
'Particle creation'
out
particle creation
in


for Atoms
Molecules
$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u_{in,k}^* e^{-ikx}$
$= \alpha_k u_{out,k} + \beta_k^* u_{out,k}^*$
$|\alpha_k|^2 - |\beta_k|^2 = \hbar$
$-i\left[u_k^* \dot{u}_k - \dot{u}_k^* u_k\right] = \hbar$
$[\phi(x), \pi(x')] = i\hbar\,\delta^{(3)}(x-x')$
$e^{-i\omega t}$


'Particle creation'
$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$
$u_{in,k} = \alpha_k u_{out,k} + \beta_k u^*_{out,k}$
$|\alpha_k|^2 - |\beta_k|^2 = \frac{1}{\hbar}$
$u_{out} \sim e^{-i\omega t}$
$-i\left[u^*_k \dot{u}_k - \dot{u}^*_k u_k\right] = \frac{1}{\hbar}$
'$\hbar$ gets small'
$[\phi(x), \pi(x')] = i\hbar\,\delta^{(3)}(x-x')$
$u_{in,k} \sim e^{-i\omega t}$


for Atoms
Molecules
'Particle creation'
$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$
$u_{in,k} = \alpha_k u_{out,k} + \beta_k u^*_{out,k}$
$|\alpha_k|^2 - |\beta_k|^2 = \hbar$
$-i[u_k^* \dot{u}_k - \dot{u}_k^* u_k] = \hbar$
$[\phi(x), \pi(x')] = i\hbar\delta^{(3)}(x-x')$
out $u_{out} \sim e^{-i\omega t}$
'$\hbar$ gets small'
particle creation
$|\alpha_k| \approx |\beta_k|$
in $u_{in,k} \sim e^{-i\omega t}$

...ct creation'

$$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$$

$$u_{in,k} = \alpha_k u_{out,k} + \beta_k u^*_{out,k}$$

$$|\alpha_k|^2 - |\beta_k|^2 = \hbar$$

$$\sim e^{-i\omega t}$$

$$-i\left[u_k^* \dot{u}_k - \dot{u}_k^* u_k\right] = \hbar$$

'$\hbar$ gets small'

...then

$$[\phi(x), \pi(y)] = i\hbar\,\delta^{(3)}(x-y)$$

$$|\alpha_k| \approx |\beta_k|$$

$$u_{in,k} \sim e^{-i\omega t}$$

'Particle creation'

$$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_{in,k} \ldots$$

$$u_{in,k} = \alpha_k \, u_{out,k} + \beta_k \, u^*_{out,k}$$

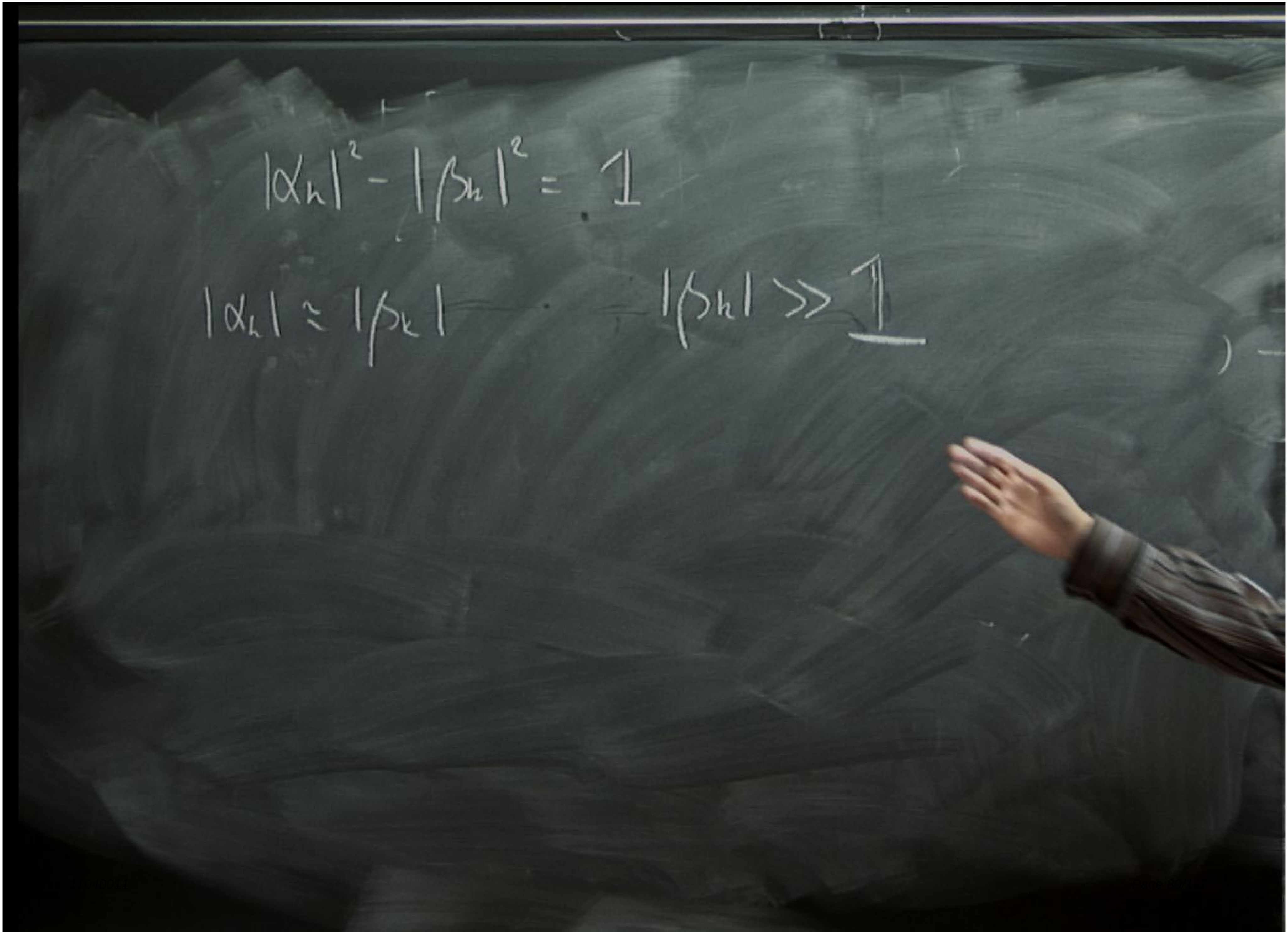
out $\quad u_{out} \sim e^{-i\omega t}$

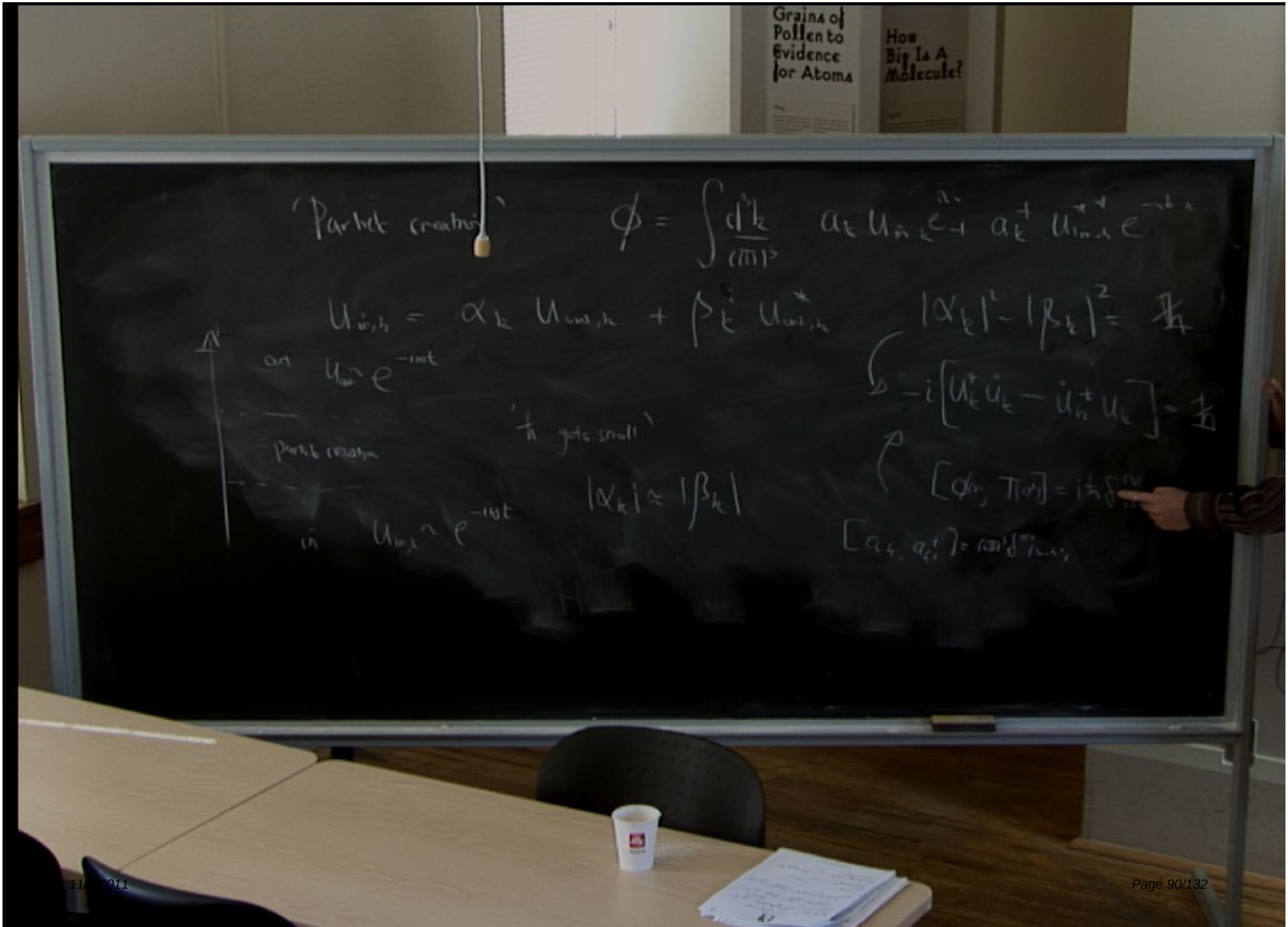
'$\hbar$ gets small'

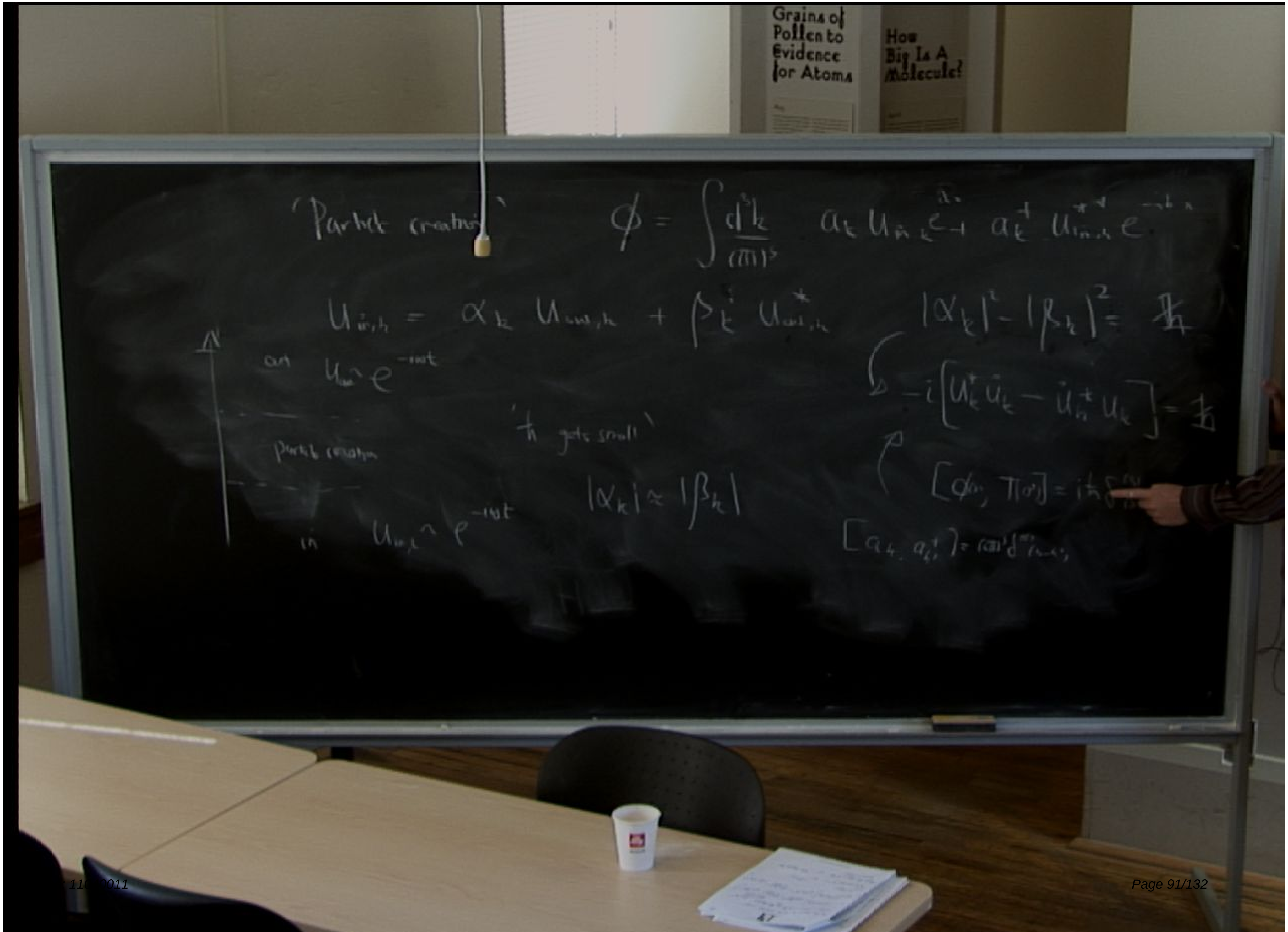
particle creation

$$|\alpha_k| \approx |\beta_k|$$

in $\quad u_{in,k} \sim e^{-i\omega t}$


$|\alpha_k|^2 - |\beta_k|^2 = 1$
$|\alpha_k| \simeq |\beta_k|$ — $|\beta_k| \gg 1$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
'Particle creation'
'ħ gets small'
particle creation
out
in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How Big Is A Molecule?
'Particle creation'
'ħ gets small'
particle creation
out
in

Particle creation

$$\phi = \int \frac{d^3k}{(2\pi)^3}\; a_k u_{in,k} e^{ikx} + a_k^\dagger u_{in,k}^{*} e^{-ikx}$$

$$u_{in,k} = \alpha_k u_{out,k} + \beta_k u_{out,k}^{*}$$

$$|\alpha_k|^2 - |\beta_k|^2 = \hbar$$

$$u_{out} \sim e^{-i\omega t}$$

$$-i\left[u_k^{*}\dot{u}_k - \dot{u}_k^{*} u_k\right] = \hbar$$

'$\hbar$ gets small'

b creation

$$[\phi(x), \pi(y)] = i\hbar\,\delta^{(3)}(x-y)$$

$$|\alpha_k| \approx |\beta_k|$$

$$u_{in,k} \sim e^{-i\omega t}$$

$$[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(k-k')$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

$$|\alpha_k| \simeq |\beta_k| \qquad |\beta_k| \gg 1$$

$$) \to 0$$

$$n_k = |\beta_k|^2$$

number of particle created per unit volume per range of k

$$\int \frac{d^3k}{(2\pi)^3} |\beta_k|^2$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

$$|\alpha_k| \simeq |\beta_k| \qquad |\beta_k| \gg 1$$

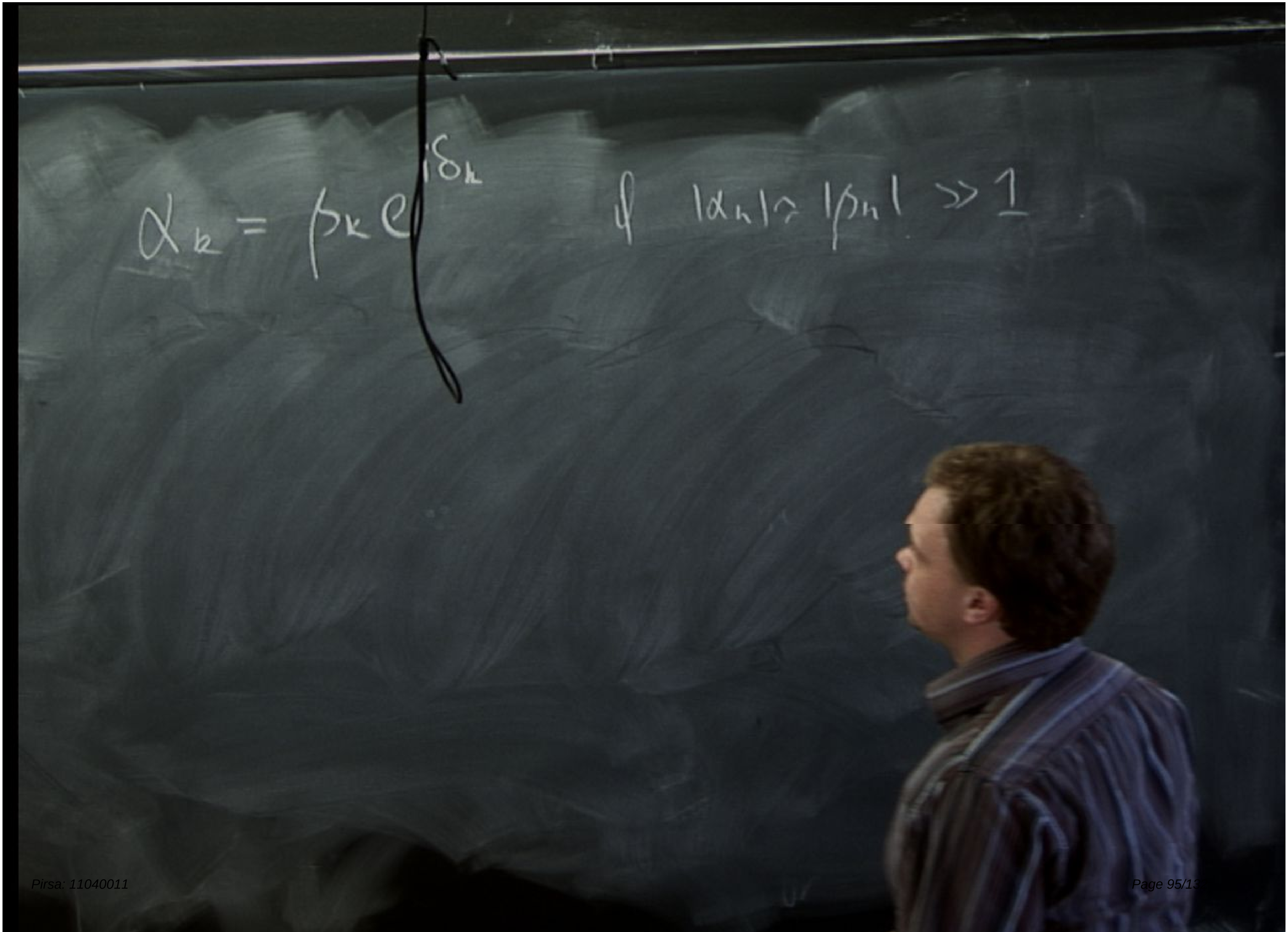
$) \longrightarrow 0$

$$n_k = |\beta_k|^2$$

number of particle created per unit volume per range of k

occupation numbers

$$\int \frac{d^3k}{(2\pi)^3} \, |\beta_k|^2$$


$\alpha_k = \beta_k e^{i\delta_k}$ if $|\alpha_k| \simeq |\beta_k| \gg 1$

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_k| \simeq |\beta_k| \gg 1$$

$$u_{in,k} = |\beta_k| \left(u_k + e^{i\delta_k} u_k^* \right)$$

u

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_k| \simeq |\beta_k| \gg 1$$

$$u_{in,k} = \beta_k \left(u_k + e^{i\delta_k} u_k^* \right)$$

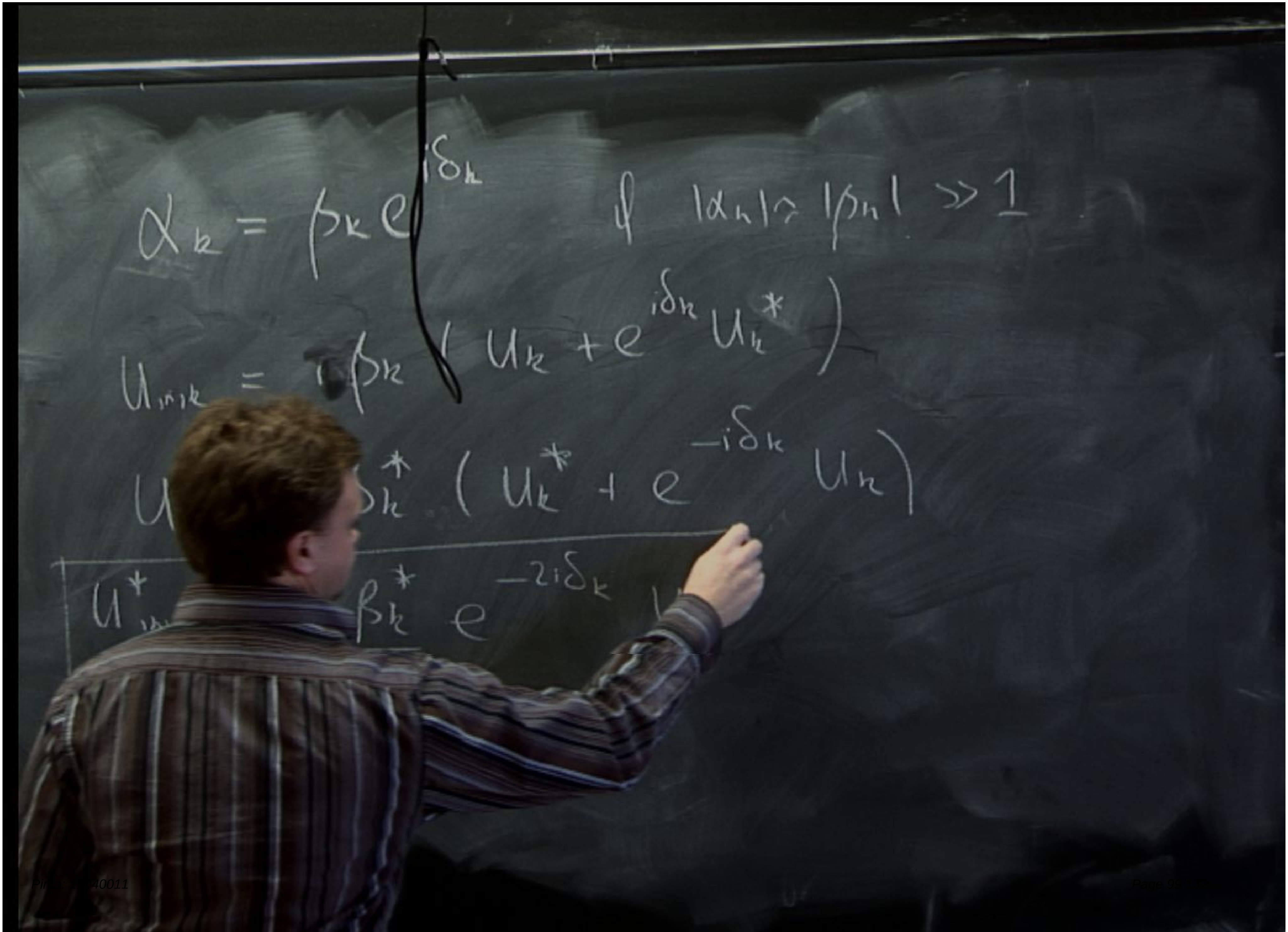
$$u_{in,k} = \beta_k^* \left(u_k^* + e^{-i\delta_k} u_k \right)$$

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_k| \simeq |\beta_k| \gg 1$$

$$u_{in,k} = \beta_k \left(u_k + e^{i\delta_k} u_k^* \right)$$

$$u^*_{in,k} = \beta_k^* \left(u_k^* + e^{-i\delta_k} u_k \right)$$

$$u^*_{in,k} = \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} u_{in,k}$$

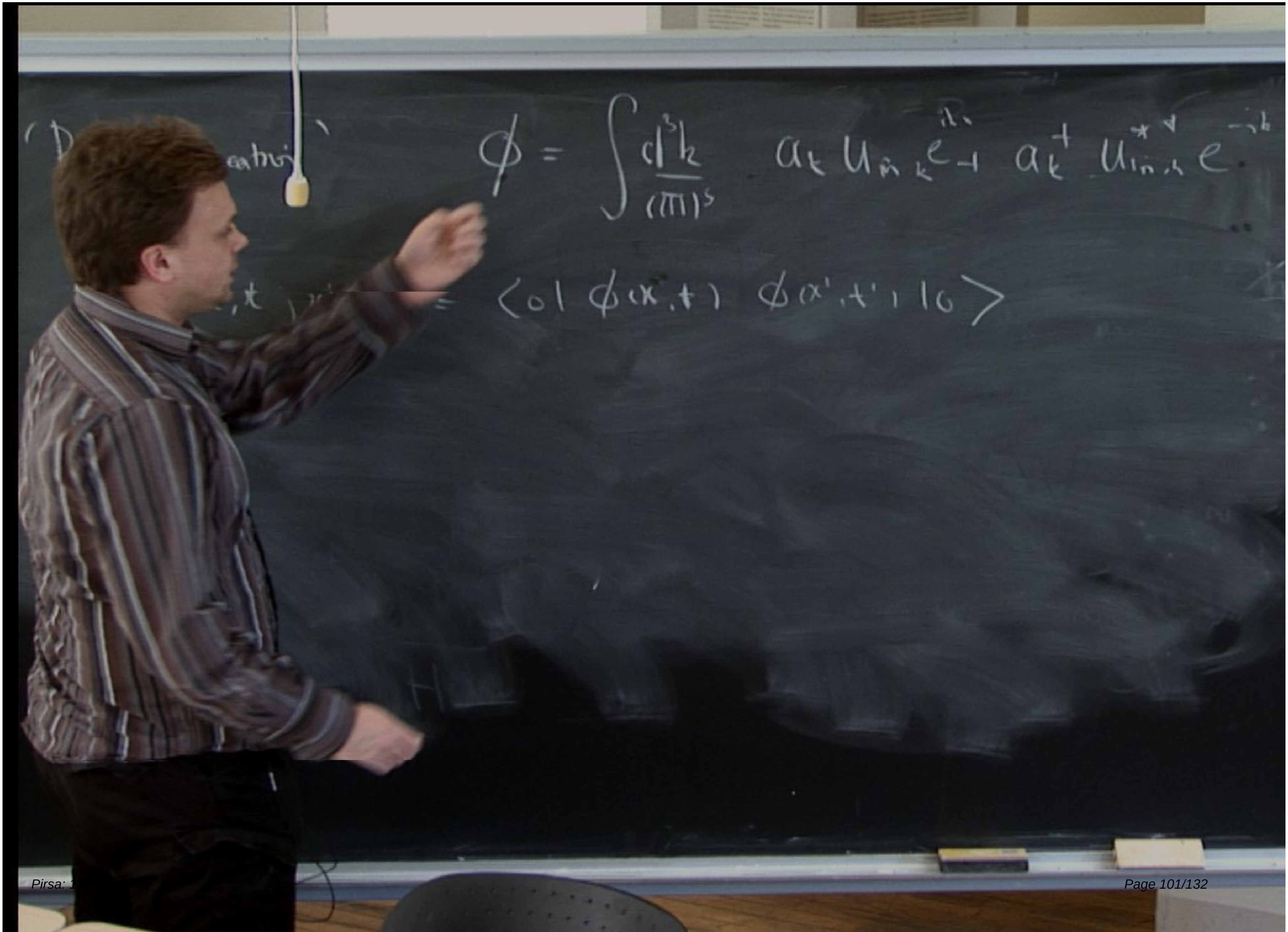

$\alpha_k = \beta_k e^{i\delta_k}$ if $|\alpha_k| \approx |\beta_k| \gg 1$
$u_{in,k} = \beta_k (u_k + e^{i\delta_k} u_k^*)$
$u^*_{in,k} = \beta_k^* (u_k^* + e^{-i\delta_k} u_k)$
$u^*_{in,k} = \beta_k^* e^{-2i\delta_k}$

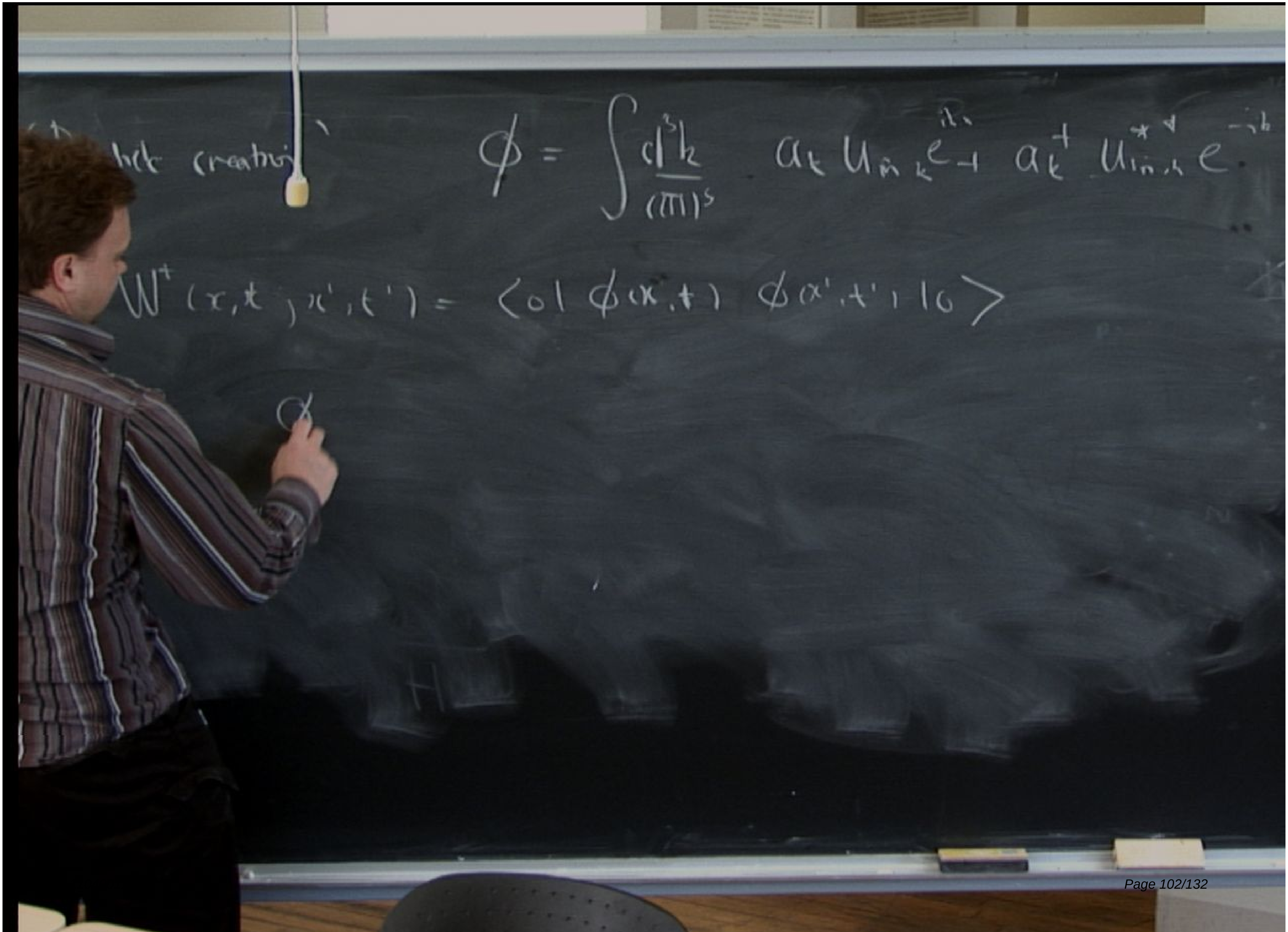
$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if } |\alpha_k| \simeq |\beta_k| \gg 1$$

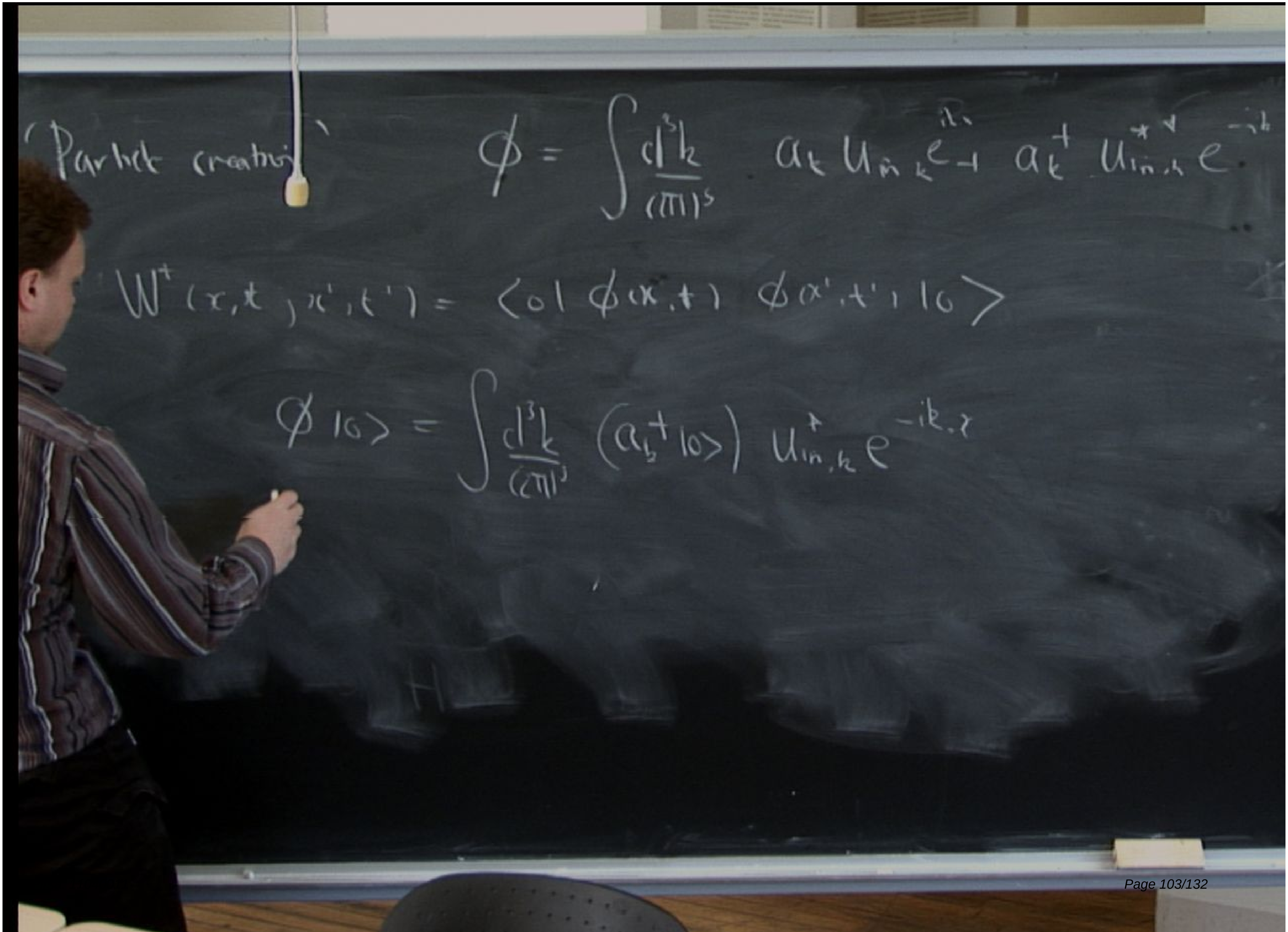
$$u_{in,k} = \beta_k \left(u_k + e^{i\delta_k} u_k^* \right)$$

$$u^*_{in,k} = \beta_k^* \left(u_k^* + e^{-i\delta_k} u_k \right)$$

$$\boxed{u^*_{in,k} = \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} u_{in,k}}$$


$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ik} + a_k^\dagger u^*_{in,k} e^{-ik}$
$\langle 0 | \phi(x,t) \; \phi(x',t') | 0 \rangle$


$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ik} + a_k^\dagger u^*_{in,k} e^{-ik}$
$W^+(x,t;x',t') = \langle 0 | \phi(x,t) \, \phi(x',t') | 0 \rangle$


(Particle creation)
$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ik\cdot x} + a_k^\dagger u_{in,k}^* e^{-ik\cdot x}$
$W^+(x,t;x',t') = \langle 0| \phi(x,t) \, \phi(x',t') |0\rangle$
$\phi|0\rangle = \int \frac{d^3k}{(2\pi)^3} \left(a_k^\dagger |0\rangle\right) u_{in,k}^* e^{-ik\cdot x}$

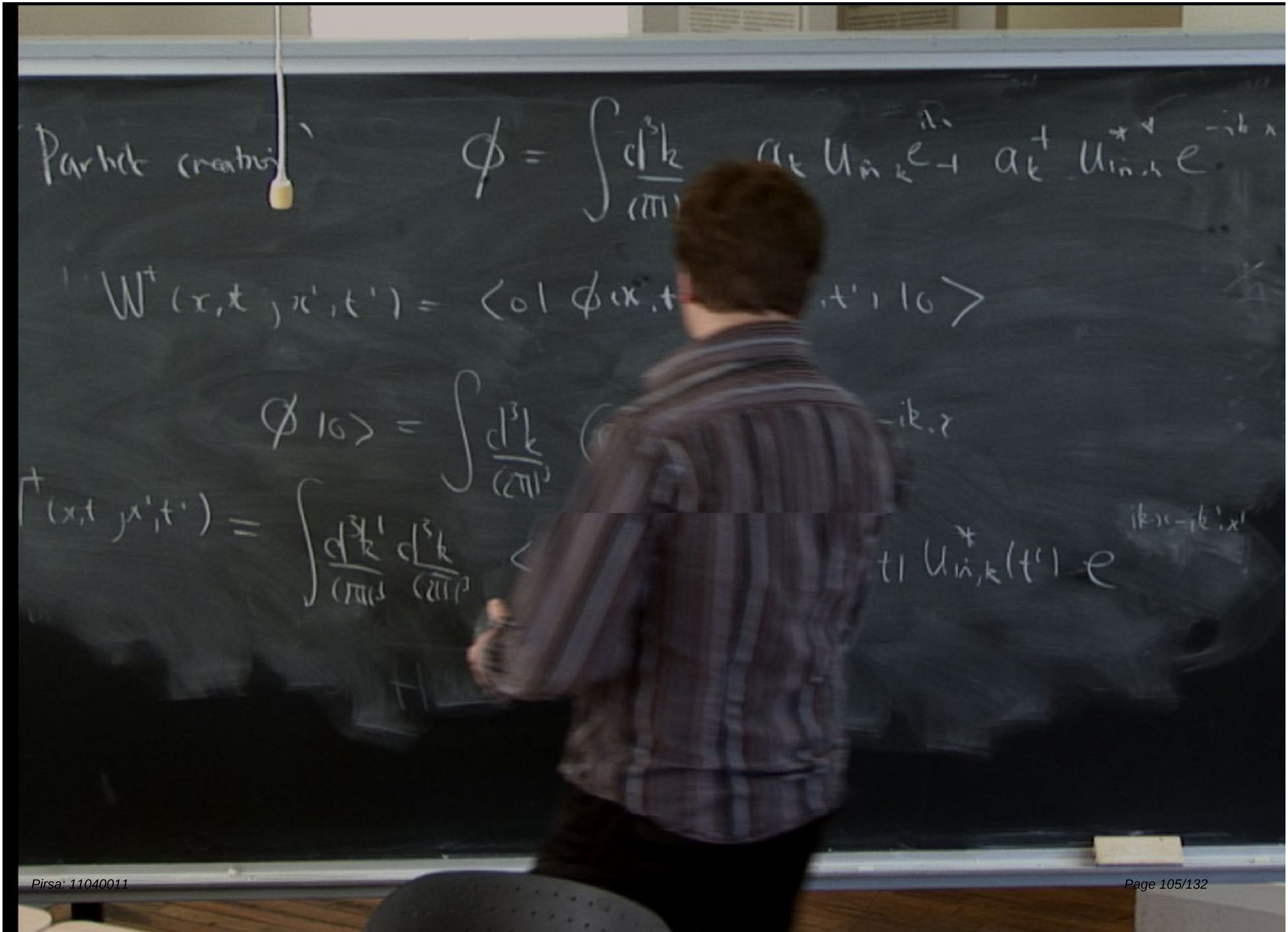
'Particle creation'

$$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ik\cdot x} + a_k^\dagger u^*_{in,k}$$

$$W^+(x,t;x',t') = \langle 0|\phi(x,t)\,\phi(x',t')|0\rangle$$

$$\phi|0\rangle = \int \frac{d^3k}{(2\pi)^3} \left(a_k^\dagger|0\rangle\right) u^*_{in,k} e^{-ik\cdot x}$$

$$W^+(x,t;x',t') = \int \frac{d^3k'}{(2\pi)^3}\frac{d^3k}{(2\pi)^3} \langle 0|a_k a_{k'}^\dagger|0\rangle\, u_{in,k}(t)\, u^*_{in,k}(t')\, e^{ik\cdot(}$$



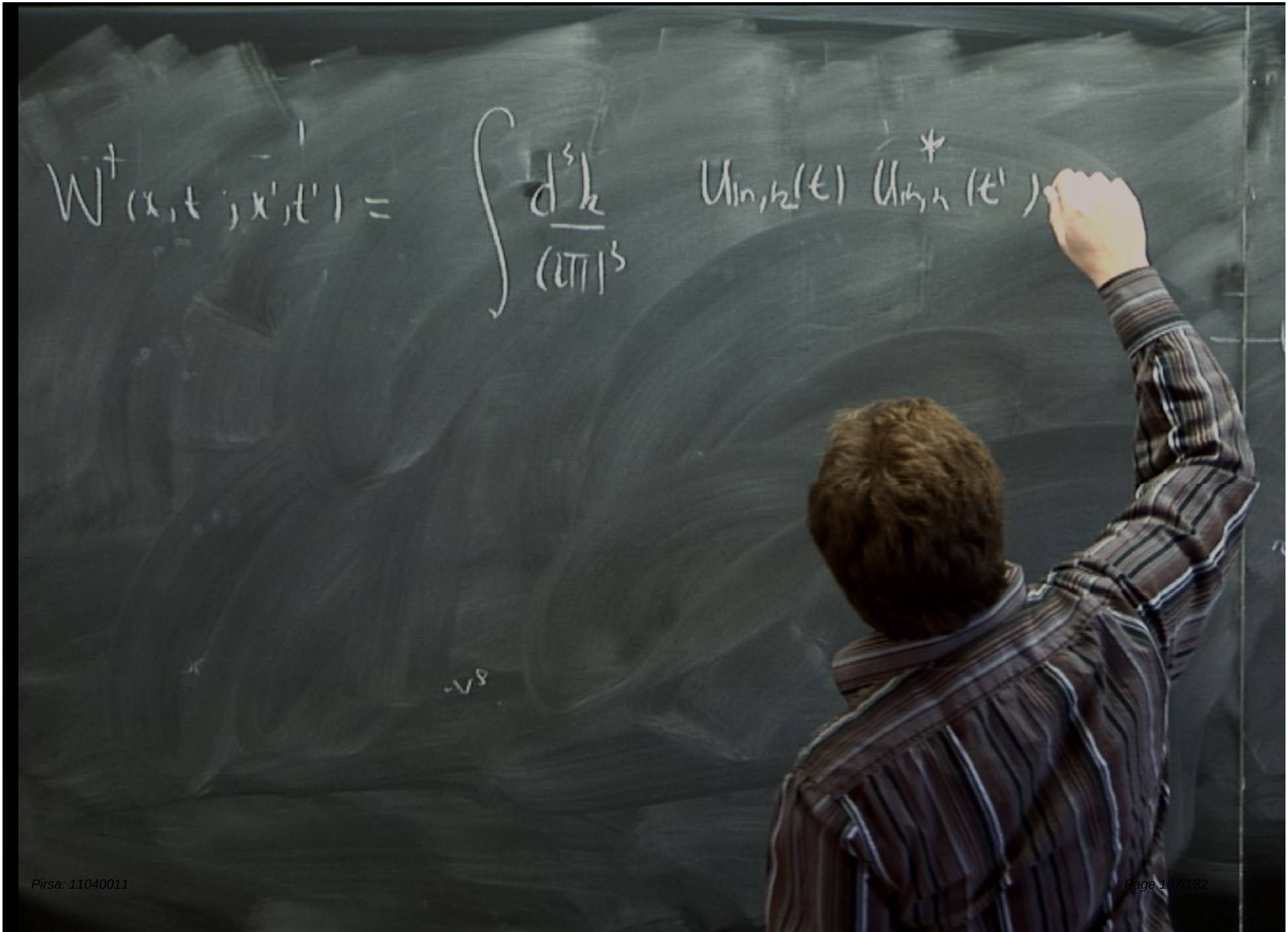
'Particle creation'

$$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ik\cdot x} + a_k^\dagger u_{in,k}^{*} e^{-ik\cdot x}$$

$$W^+(x,t;x',t') = \langle 0| \phi(x,t)\, \phi(x',t') |0\rangle$$

$$\phi |0\rangle = \int \frac{d^3k}{(2\pi)^3} \left(a_k^\dagger |0\rangle\right) u_{in,k}^{*} e^{-ik\cdot x}$$

$$W^+(x,t;x',t') = \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \underbrace{\langle 0| a_k a_{k'}^\dagger |0\rangle}_{(2\pi)^3 \delta^{(3)}(k-k')} u_{in,k}(t)\, u_{in,k}^{*}(t')\, e^{ik\cdot x - ik'\cdot x'}$$


$W^{+}(x,t;x',t') = \int \frac{d^3k}{(2\pi)^3} \; u_{in,k}(t)\; u^{*}_{in,k}(t')$

$$W^{+}(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik\cdot(x-x')}$$

$$\boxed{W^{+}(x,t;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik(x-x')}}$$

$i\langle[\phi(x,t),\phi(x'$

$$\boxed{W^{+}(x,t;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik\cdot(x-x')}}$$

$$\langle 0 | [\phi(x,t), \phi(x',t)] | 0 \rangle$$

$$\boxed{W^{+}(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik\cdot(x-x')}}$$

$$\langle 0|[\phi(x,t),\phi(x',t')]|0\rangle$$

$$= W^{+}(x,t\,;x',t') - W^{+}(x',t'\,;x,t)$$

$$\boxed{W^{+}(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik(x-x')}}$$

$$\langle 0|[\phi(x,t),\phi(x',t')]\,|0\rangle$$

$$= W^{+}(x,t\,;x',t') - W^{+}(x',t'\,;x,t)$$

$$=$$

$$\boxed{W^+(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^*_{in,k}(t')\, e^{ik(x-x')}}$$

$$\ldots\,[\phi(x,t), \phi(x',t')]\,|0\rangle$$

$$W^+(x,t\,;x',t') - W^+(x',t'\,;x,t)$$

$$= \int \frac{d^3k}{(2\pi)^3}\left[u_{in,k}(t)\, u^*_{in,k}(t') - u_{in,k}(t')\, u^*_{in,k}(t)\right] e^{ik(x-\ldots}$$

$$W^{+}(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\; U_{in,k}(t)\, U^{*}_{in,k}(t')\, e^{ik\cdot(x-x')}$$

$$\langle 0|[\phi(x,t),\phi(x',t')]|0\rangle$$

$$= W^{+}(x,t\,;x',t') - W^{+}(x',t'\,;x,t)$$

$$= \int \frac{d^3k}{(2\pi)^3}\left[U_{in,k}(t)\,U^{*}_{in,k}(t') - U_{in,k}(t')\,U^{*}_{in,k}(t)\right] e^{ik\cdot(x-x')}$$

$$u_{in,k}(t)\, u^*_{in,k}(t')\, e^{ik(x-x')}\Big] \to 0$$

$$, x, t)$$

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_k|$$

$$u_{in,k} = \beta_k \left(u_k + e^{i\delta_k} u_k^* \right.$$

$$u^*_{in,k} = \beta_k^* \left(u_k^* + e^{-i\delta_k} \right.$$

$$\boxed{u^*_{in,k} = \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} u_{in,k}}$$

$$u_{in,k}(t')\, u^*_{in,k}(t)\Big] e^{ik(x-x')} = \int \frac{d^3k}{(2\pi)^3} \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} \Big[$$

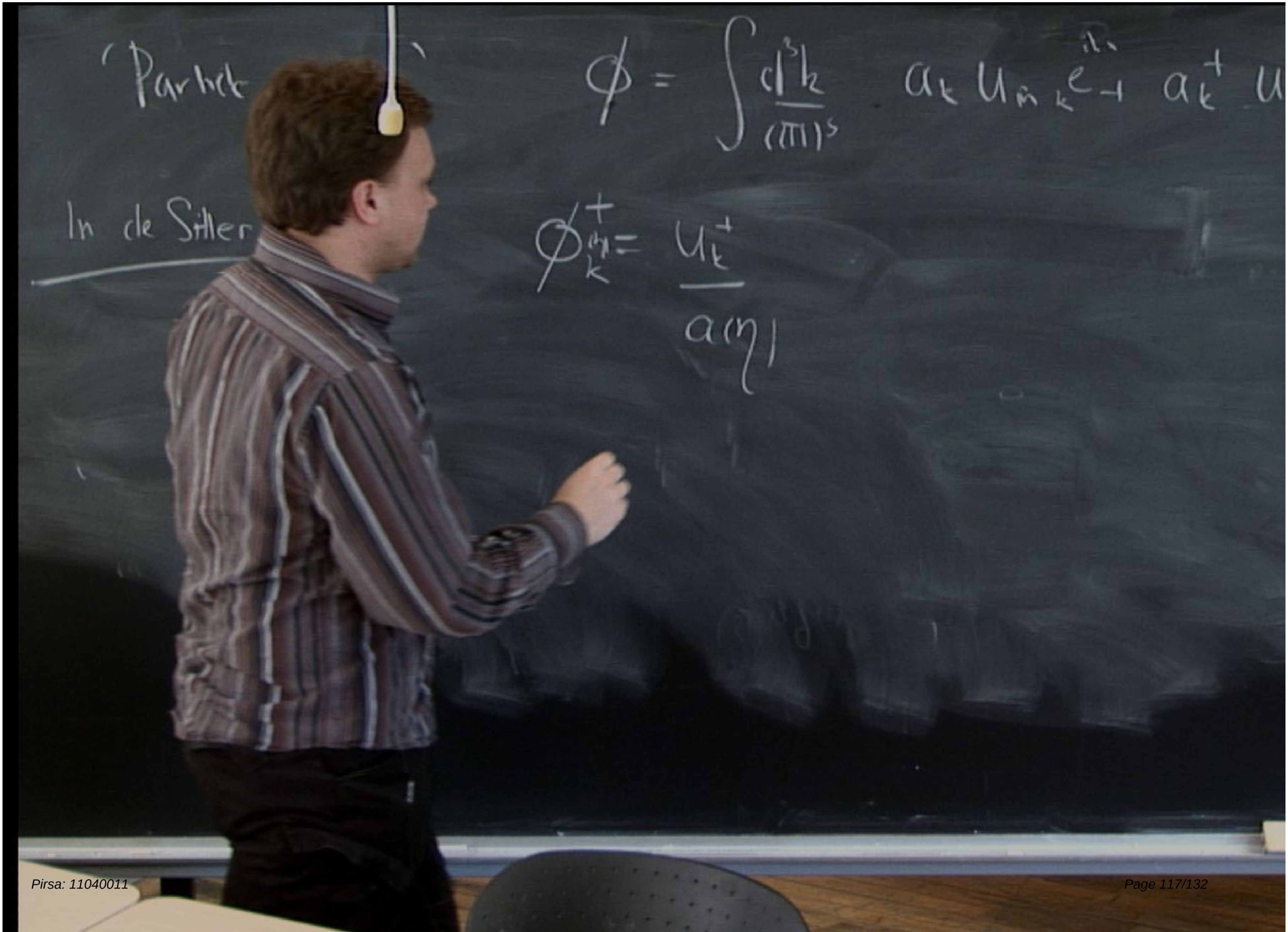
$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if } |\alpha_k| \simeq |\beta_k| \gg 1$$

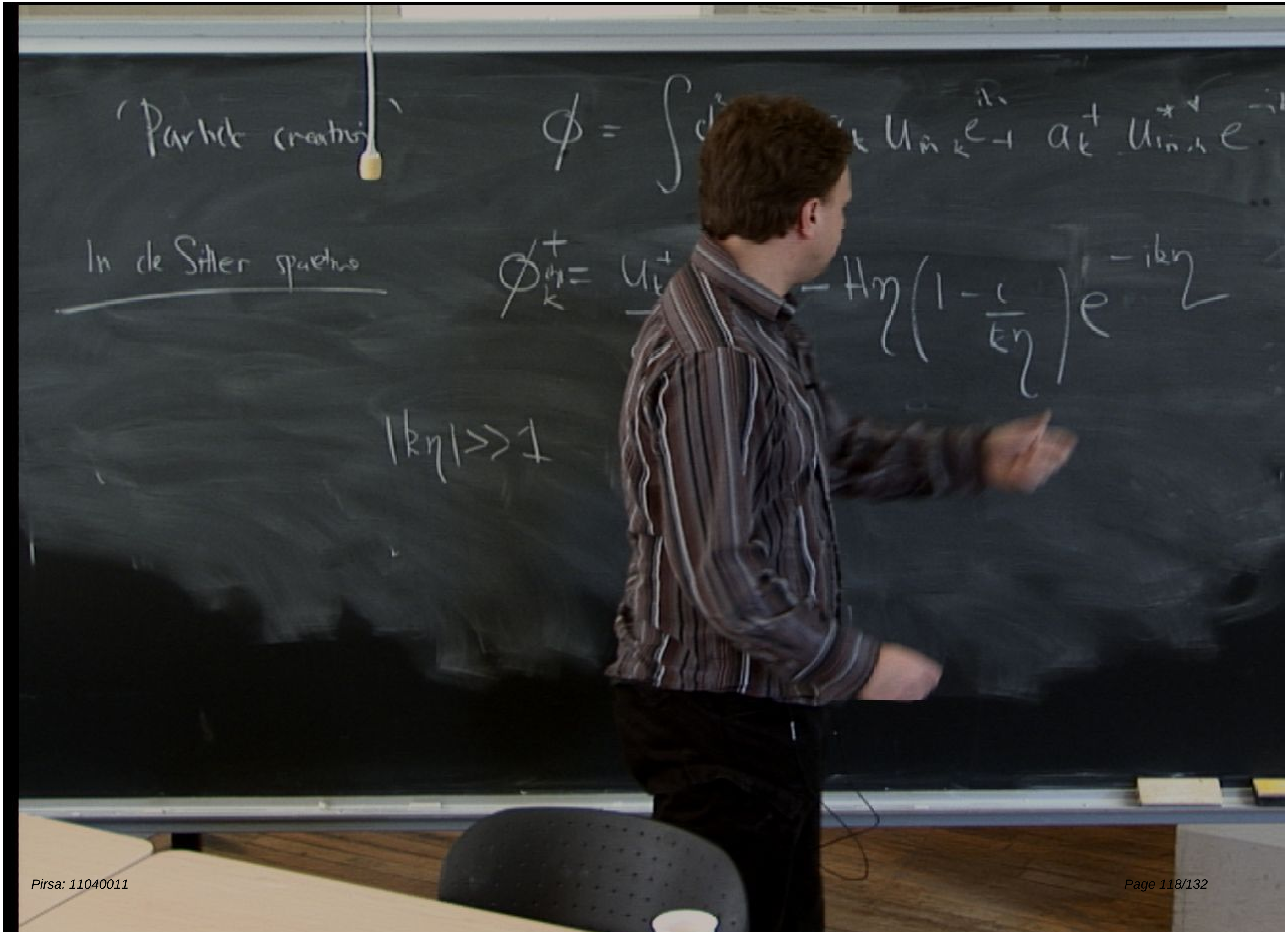
$$u_{in,k} = \beta_k \left(u_k + e^{i\delta_k} u_k^* \right)$$

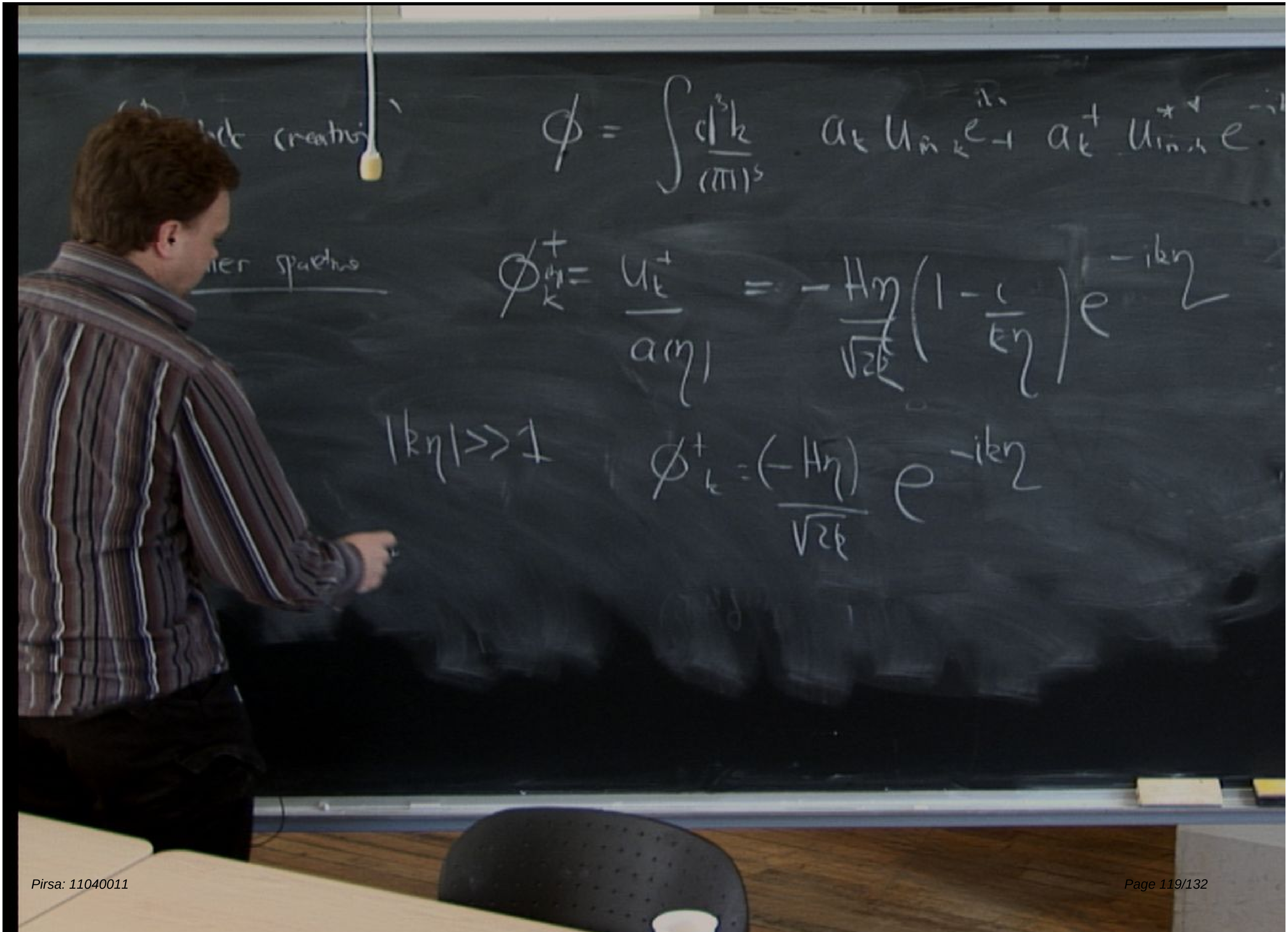
$$u^*_{in,k} = \beta_k^* \left(u_k^* + e^{-i\delta_k} u_k \right)$$

$$\boxed{u^*_{in,k} = \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} u_{k_{in}}}$$

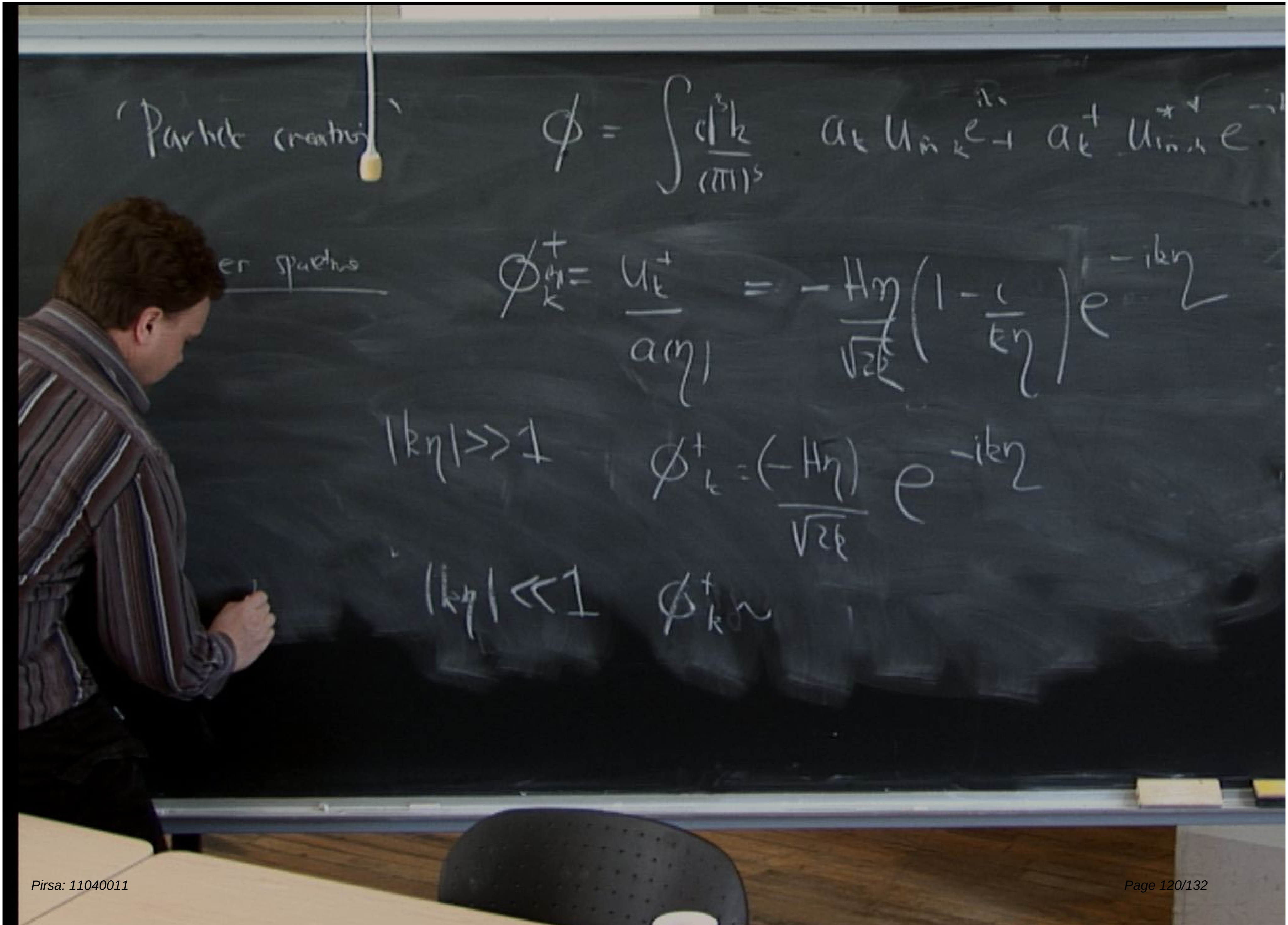
$$= \int \frac{d^3k}{(2\pi)^3} \frac{\beta_k^*}{\beta_k} e^{-2i\delta_k} \left[u_{k,in}(t)\, u_{k,in}(t') - u_{k,in}(t')\, u_{k,in}(t) \right] \to 0$$

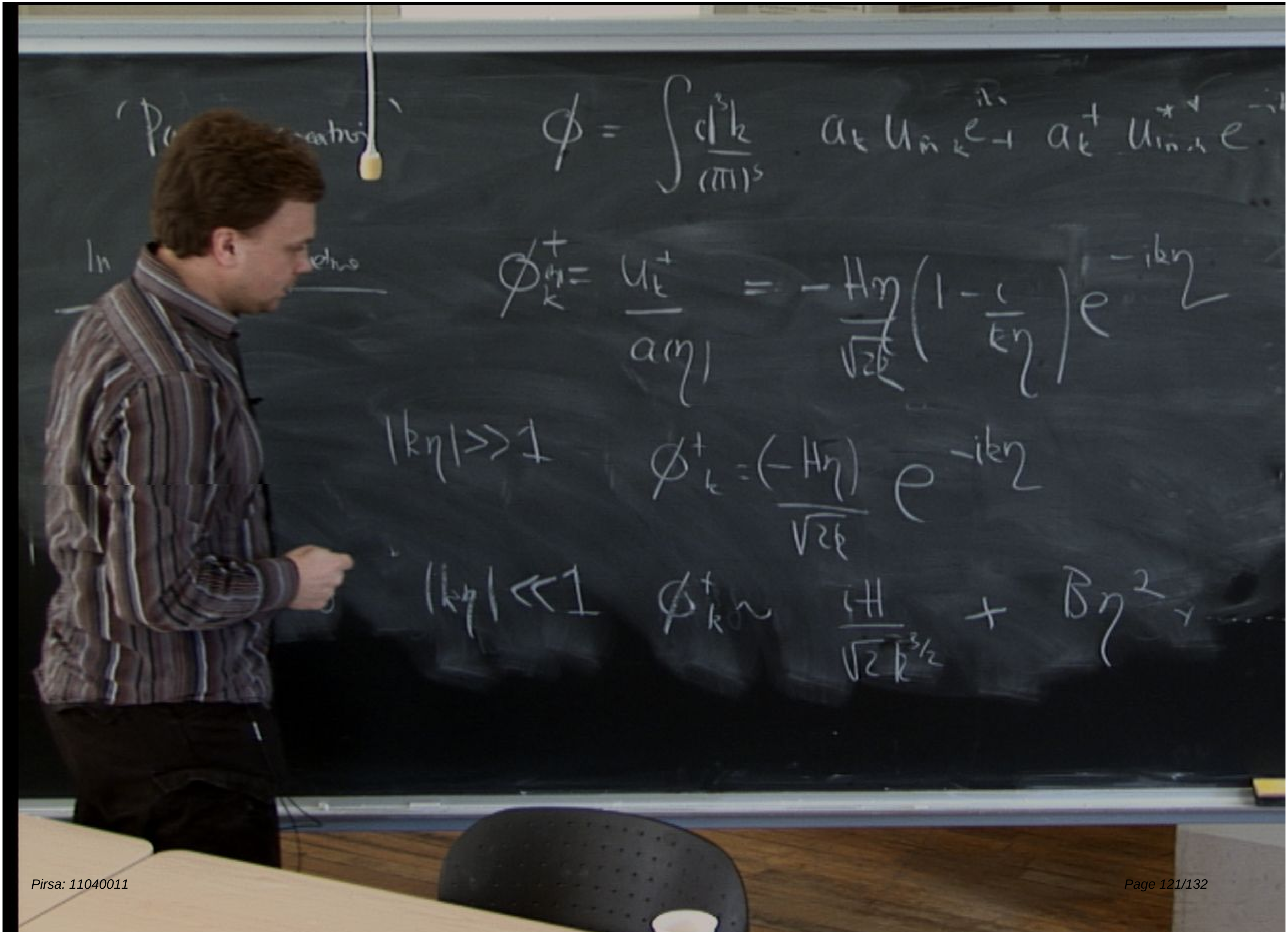

'Partic
In de Sitter
$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{m\,k} e^{ik\cdot x} + a_k^\dagger u$
$\phi^+_{m\,k} = \frac{u_k^+}{a(\eta)}$


'Particle creation'
In de Sitter spacetime
|kη|>>1



$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{ikx} + a_k^+ u^*_{in,k} e^{-ikx}$
$\phi^+_{in,k} = \frac{u_k^+}{a(\eta)} = -\frac{H\eta}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$
$|k\eta| >> 1$
$\phi^+_k = \frac{(-H\eta)}{\sqrt{2k}} e^{-ik\eta}$


'Particle creation'
$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{i k \cdot x} + a_k^\dagger u^*_{in,k} e^{-ik\cdot x}$
er spectrum
$\phi^+_k(\eta) = \frac{u_k^+}{a(\eta)} = -\frac{H\eta}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$
$|k\eta| \gg 1 \quad \phi^+_k = \left(\frac{-H\eta}{\sqrt{2k}}\right) e^{-ik\eta}$
$|k\eta| \ll 1 \quad \phi^+_k \sim$



φ = ∫ d³k/(2π)³ a_k u_in,k e^{ik·x} + a_k† u*_in,k e^{-ik·x}
φ_k^+ = u_k^+ / a(η) = −Hη/√2k (1 − i/kη) e^{−ikη}
|kη| >> 1   φ_k^+ = (−Hη)/√2k e^{−ikη}
|kη| << 1   φ_k^+ ~ iH/(√2 k^{3/2}) + Bη² + …

'Particle creation'

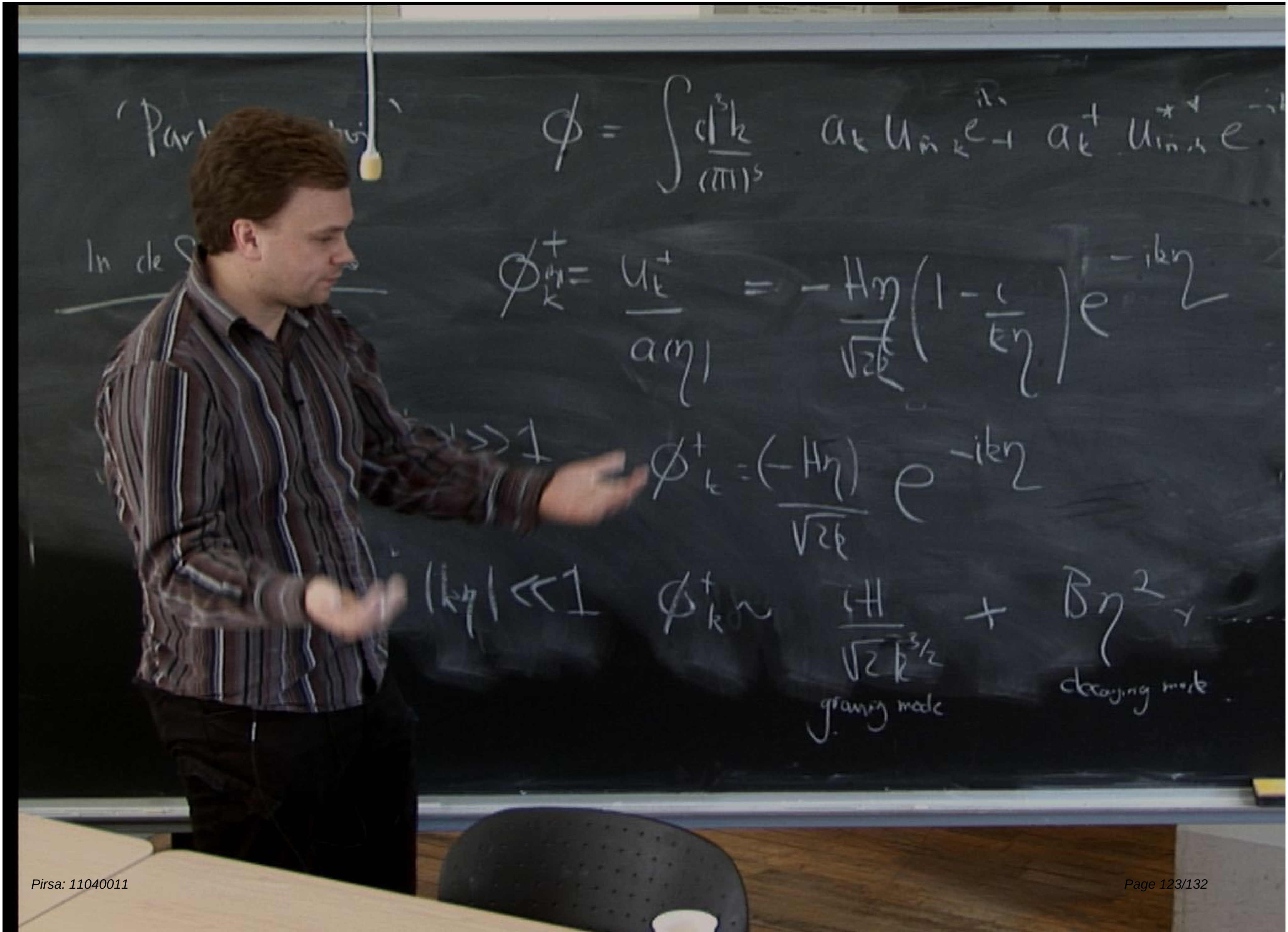
$$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$$

In de Sitter spacetime

$$\phi^{+}_{in,k} = \frac{u_k^+}{a(\eta)} = -\frac{H\eta}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$|k\eta| \gg 1$ $\quad \phi_k^+ = \frac{(-H\eta)}{\sqrt{2k}} e^{-ik\eta}$

$\eta \to 0$ $\quad |k\eta| \ll 1$ $\quad \phi_k^+ \sim \frac{iH}{\sqrt{2}\,k^{3/2}} + B\eta^2 + \dots$


$\phi = \int \frac{d^3k}{(2\pi)^3} \; a_k u_{in,k} e^{i k x} + a_k^\dagger u^*_{in,k} e^{-ikx}$
$\phi^+_{k} = \frac{u_k^+}{a(\eta)} = -\frac{H\eta}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$
$\phi^+_k = \frac{(-H\eta)}{\sqrt{2k}} e^{-ik\eta}$
$|k\eta| \ll 1$
$\phi^+_k \sim \frac{iH}{\sqrt{2}\,k^{3/2}} + B\eta^2$
growing mode
decaying mode

'Particle creation'

$$\phi = \int \frac{d^3k}{(2\pi)^3} \, a_k u_{in,k} e^{ikx} + a_k^\dagger u^*_{in,k} e^{-ikx}$$

In de Sitter spacetime

$$\phi^+_{in,k} = \frac{u^+_k}{a(\eta)} = -\frac{H\eta}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$$|k\eta| \gg 1 \qquad \phi^+_k = \frac{(-H\eta)}{\sqrt{2k}} \, e^{-ik\eta}$$

$$\eta \to 0 \qquad |k\eta| \ll 1 \qquad \phi^+_k \sim \underbrace{\frac{iH}{\sqrt{2}\,k^{3/2}}}_{\text{growing mode}} + \underbrace{B\eta^2}_{\text{decaying mode}} + \dots$$

$$\boxed{W^{+}(x,t\,;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik\cdot(x-x')}}$$

original scalar in de Sitter

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\nabla^2\phi = 0$$

$$\boxed{W^{+}(x,t;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik(\ldots)}}$$

original scalar on de Sitter

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^2$$

$$\ddot{\phi} + 3H\dot{\phi} \cancel{- e^{-2Ht}\nabla^2\phi} = 0$$

$$k\eta \ll 1$$

$x')$] in limit $\to 0$.

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if } |\alpha_k| \simeq |\beta_k| \gg 1$$

$e^{i\delta_k} u_k^*)$

$(\ln \dot\phi) = -3H =$

$e^{-i\delta_k} u_k)$

$\ln \dot\phi = -3Ht + \ln(\text{constant})$

$$\boxed{e^{-i\delta_k} \, u_{k,\text{in}}}$$

$u_{k,\text{in}}(t) \, u_{k,\text{in}}(t') - u_{k,\text{in}}(t') \, u$

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_k| \simeq |\beta_k| \gg 1$$

$$e^{i\delta_k} u_k^*)$$

$$\to 0 .$$

$$e^{-i\delta_k} u_k)$$

$$(\ln\dot\phi) = -3H =$$

$$\ln\dot\phi = -3Ht + \ln(\text{constant}) \quad -e^{-i\delta_k} u_{k,\text{in}}$$

$$\dot\phi = C e^{-3Ht}$$

$$u_{k,\text{in}}(t)\, u_{k,\text{in}}^*(t') - u_{k,\text{in}}(t')\, u \ldots$$

$x')$] $\overrightarrow{\text{in limit}}$ 0 .

$\alpha_k = \beta_k e^{i\delta_k}$ if $|\alpha_k| \simeq |\beta_k| \gg 1$

$e^{i\delta_k} u_k^{*})$

$(\ln\dot\phi) = -3H =$ $e^{-i\delta_k} u_k)$

$\ln\dot\phi = -3Ht + \ln(\text{constant})$ $e^{-i\delta_k}$ $u_{k,in}$

$\dot\phi = Ce^{-3Ht}$

$\phi = A + C'e^{-3Ht}$

$=$

$u_{k,in}(t)\, u_{k,in}^{*}(t') - u_{k,in}(t')\, u$

$$\boxed{W^{+}(x,t;x',t') = \int \frac{d^3k}{(2\pi)^3}\, u_{in,k}(t)\, u^{*}_{in,k}(t')\, e^{ik(x-x')}}$$

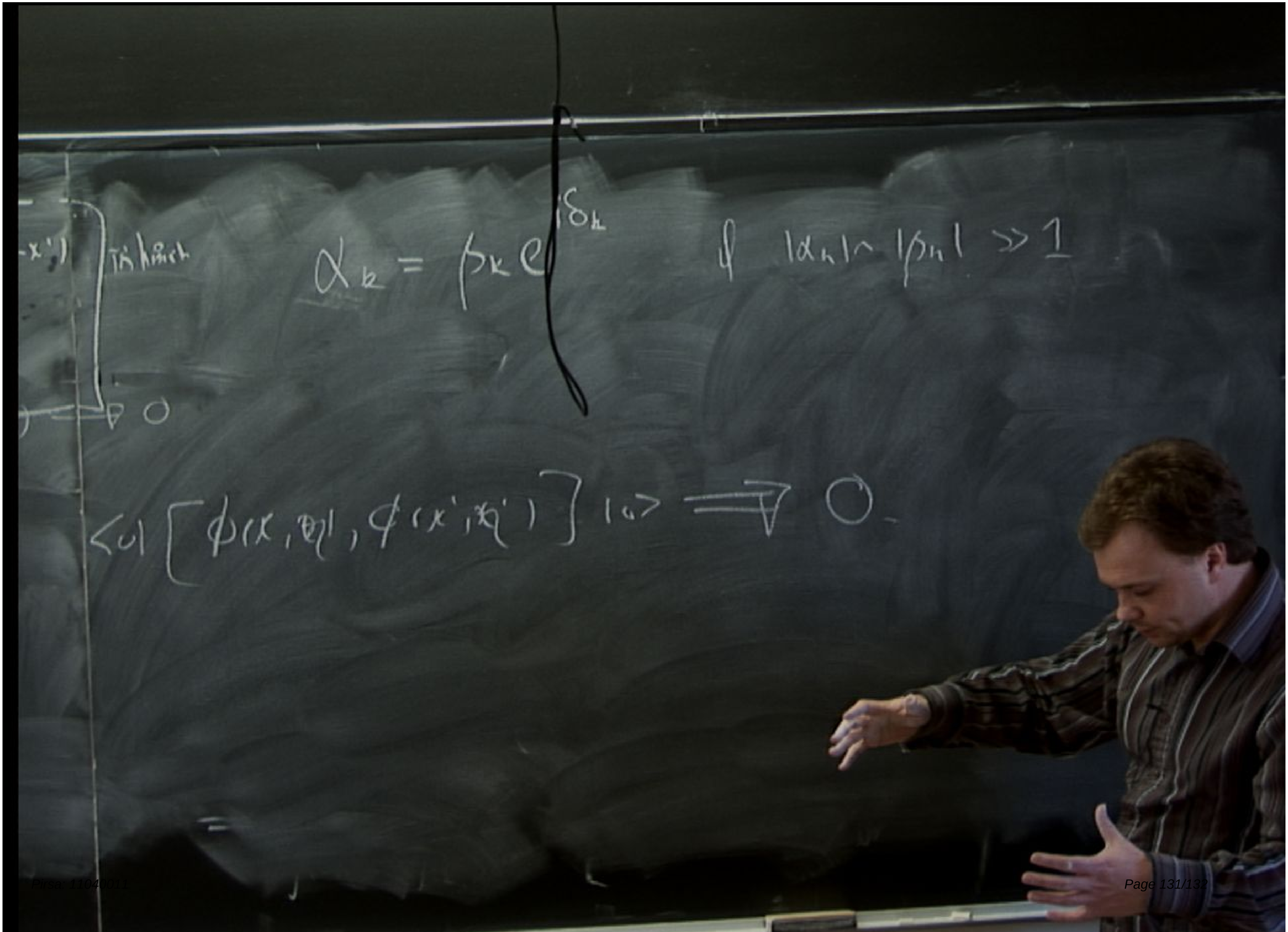
minimal scalar on de Sitter

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^2$$

$$\ddot{\phi} + 3H\dot{\phi} - \cancel{e^{-2Ht}\nabla^2\phi} = 0$$

$k\eta \ll 1$

$$(\phi_k^{+})^{*} = -(\phi_k^{+})$$


which
$\alpha_k = \beta_k e^{i\delta_k}$
if $|\alpha_k| \sim |\beta_k| \gg 1$
$\langle 0| [\phi(x,\eta), \phi(x',\eta')] |0\rangle \Rightarrow 0$

inhomish

$$\alpha_k = \beta_k e^{i\delta_k} \quad \text{if} \quad |\alpha_n| \sim |\beta_n|$$

$\to 0$

$$\langle u | [\phi(x,t), \phi(x',t')] | u\rangle \Rightarrow 0.$$