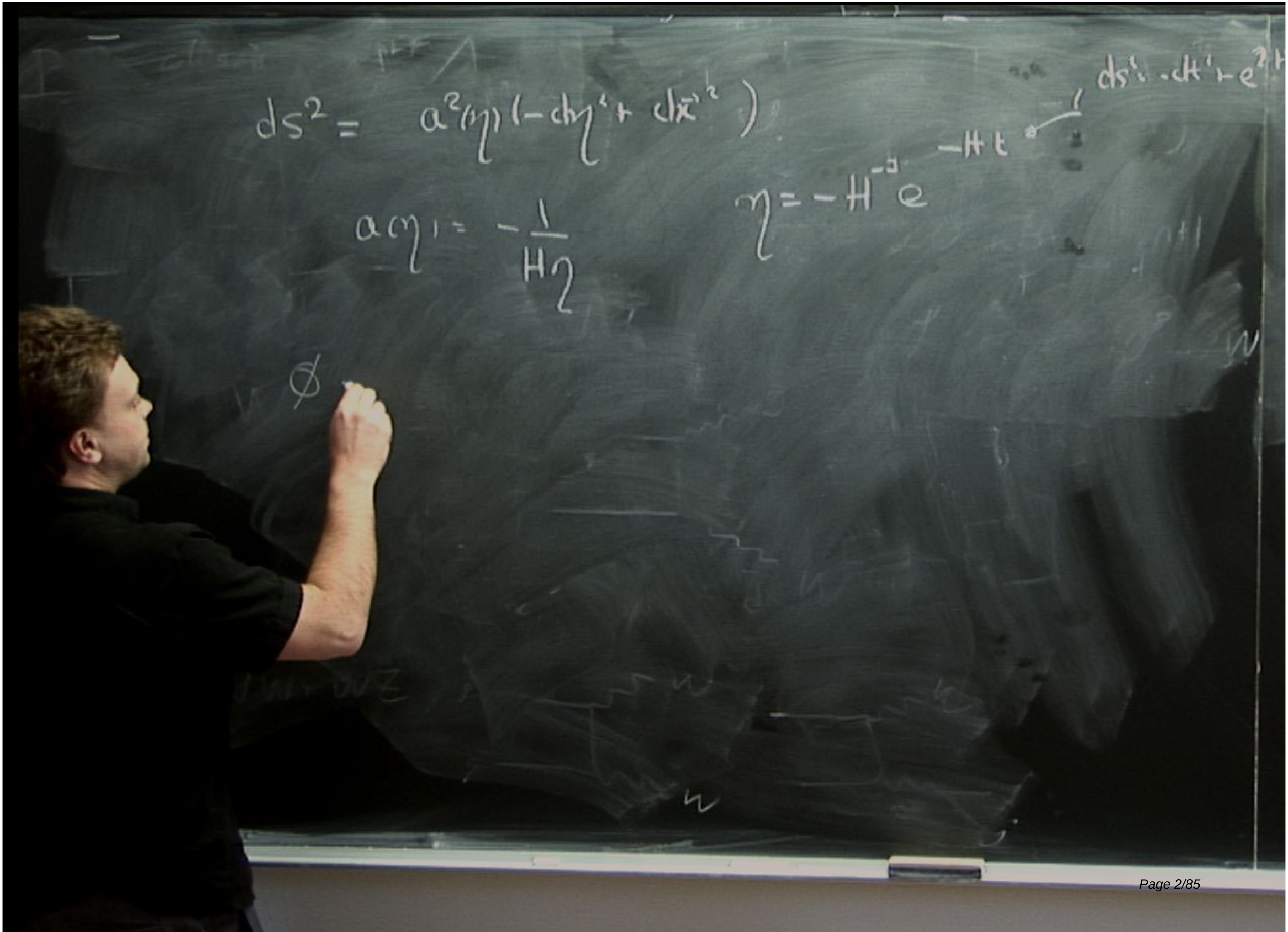
Title: Explorations in Cosmology - Lecture 6

Date: Apr 11, 2011 11:30 AM

URL: http://pirsa.org/11040010

Abstract:


$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$
$a(\eta) = -\frac{1}{H\eta}$
$\eta = -H^{-1} e^{-Ht}$
ϕ

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$$

$$ds^2 = -dt^2 + e^{2Ht} \ldots$$

$$a(\eta) = -\frac{1}{H\eta} \qquad \eta = -H^{-1} e^{-Ht}$$

$$\phi = \frac{u}{a} \qquad u_k'' = -\omega_k^2 u_k$$

$$\omega_k^2 = k^2 + m^2_{eff}$$

$$m^2_{eff}(\eta) = -\frac{a''}{a} = -\frac{2}{\eta^2}$$

$$u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

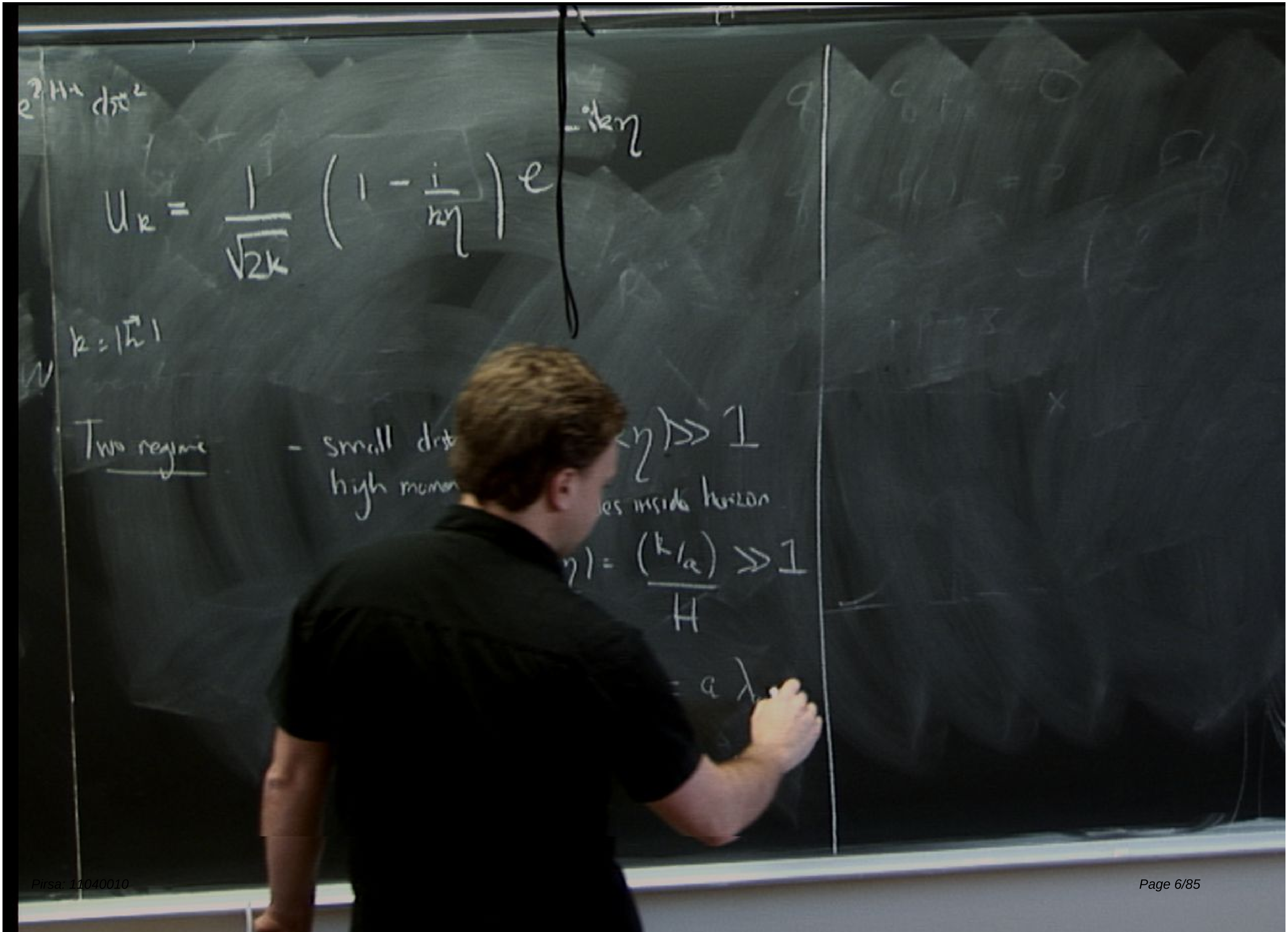
$|\vec{k}|$

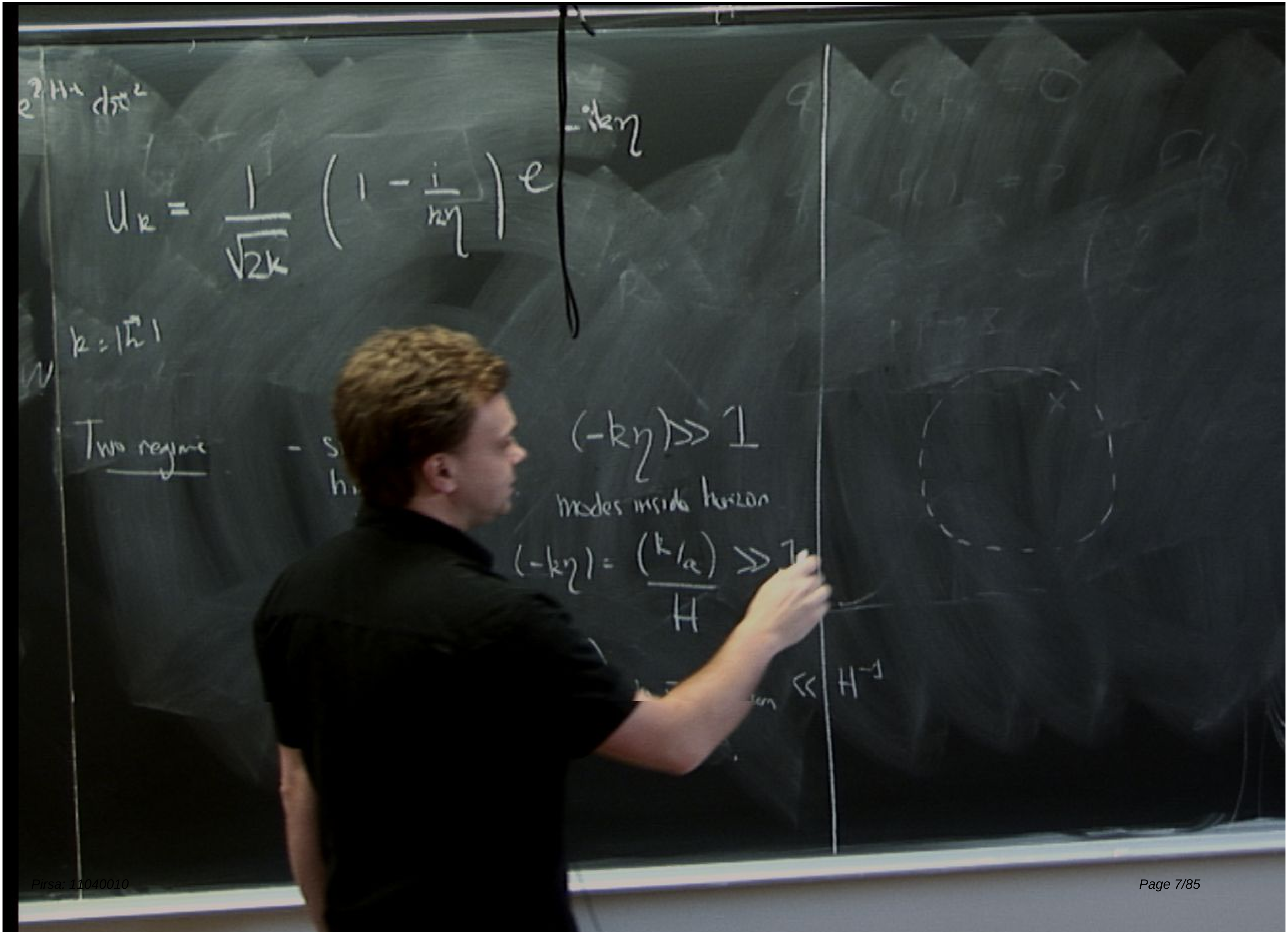
$e^{2Ht} dx^2$

$$U_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$k = |\vec{k}|$

Two regime — small distance, high momentum $\quad k\eta \gg 1$


$u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$
$k = |\vec{k}|$
Two regime
- small dist
high momen
es inside horizon


Two regime
(-kη) >> 1
modes inside horizon
H

$e^{2Ht} dx^2$

$$u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$k = |\vec{k}|$

Two regime — small distance, high momentum

$(-k\eta) \gg 1$

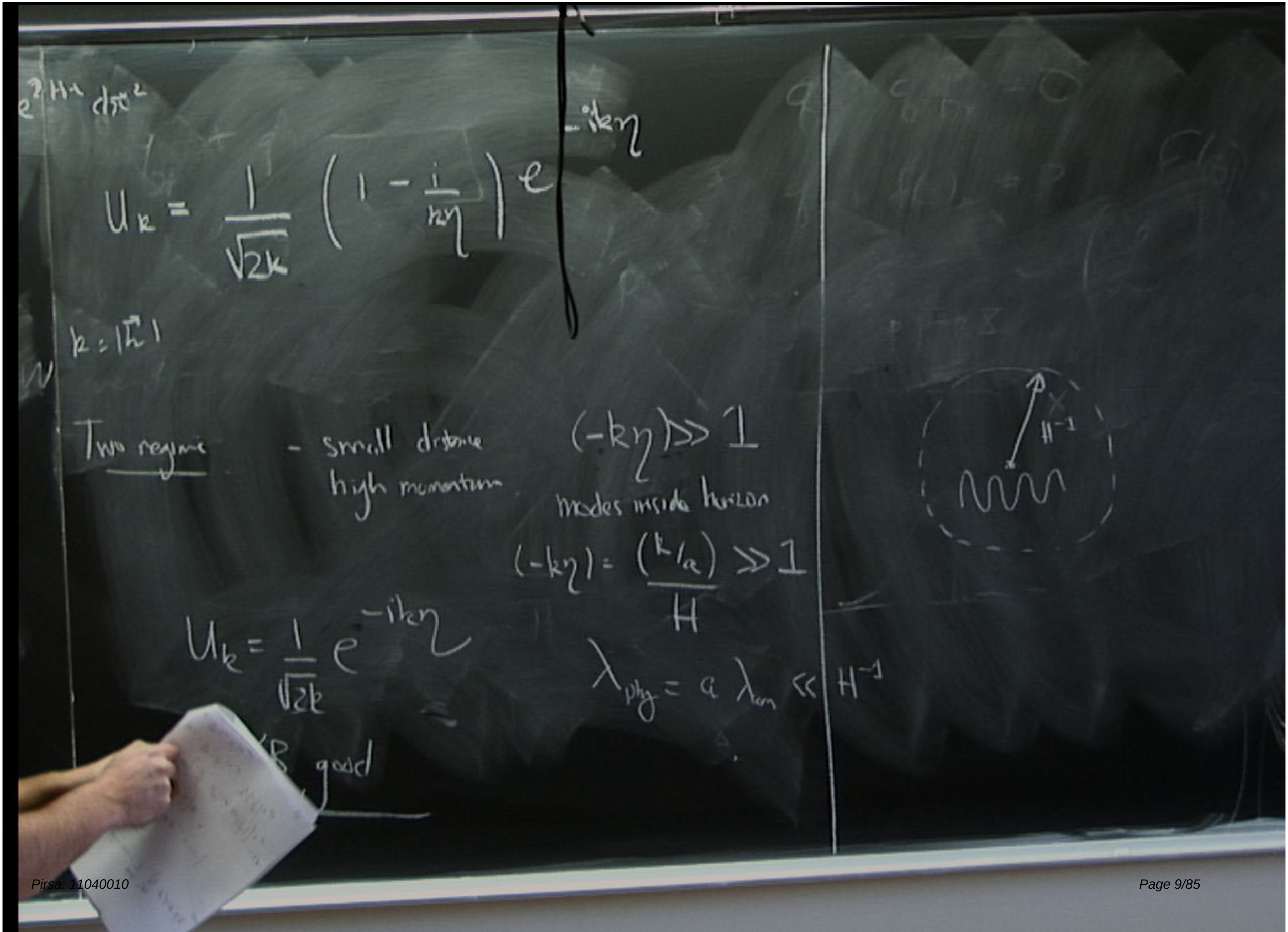
modes inside horizon

$$(-k\eta) = \frac{(k/a)}{H} \gg 1$$

$$u_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

$$\lambda_{phy} = a\,\lambda_{com} \ll H^{-1}$$

WKB good



Uk = 1/√2k (1 − i/kη) e^−ikη
k = |k⃗|
Two regime
− small distance
high momentum
(−kη) >> 1
modes inside horizon
(−kη) = (k/a)/H >> 1
Uk = 1/√2k e^−ikη
λphy = a λcom << H^−1
B good
H^−1

$$u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$k = |\vec{k}|$

Two regime

- small distance, high momentum

$(-k\eta) \gg 1$

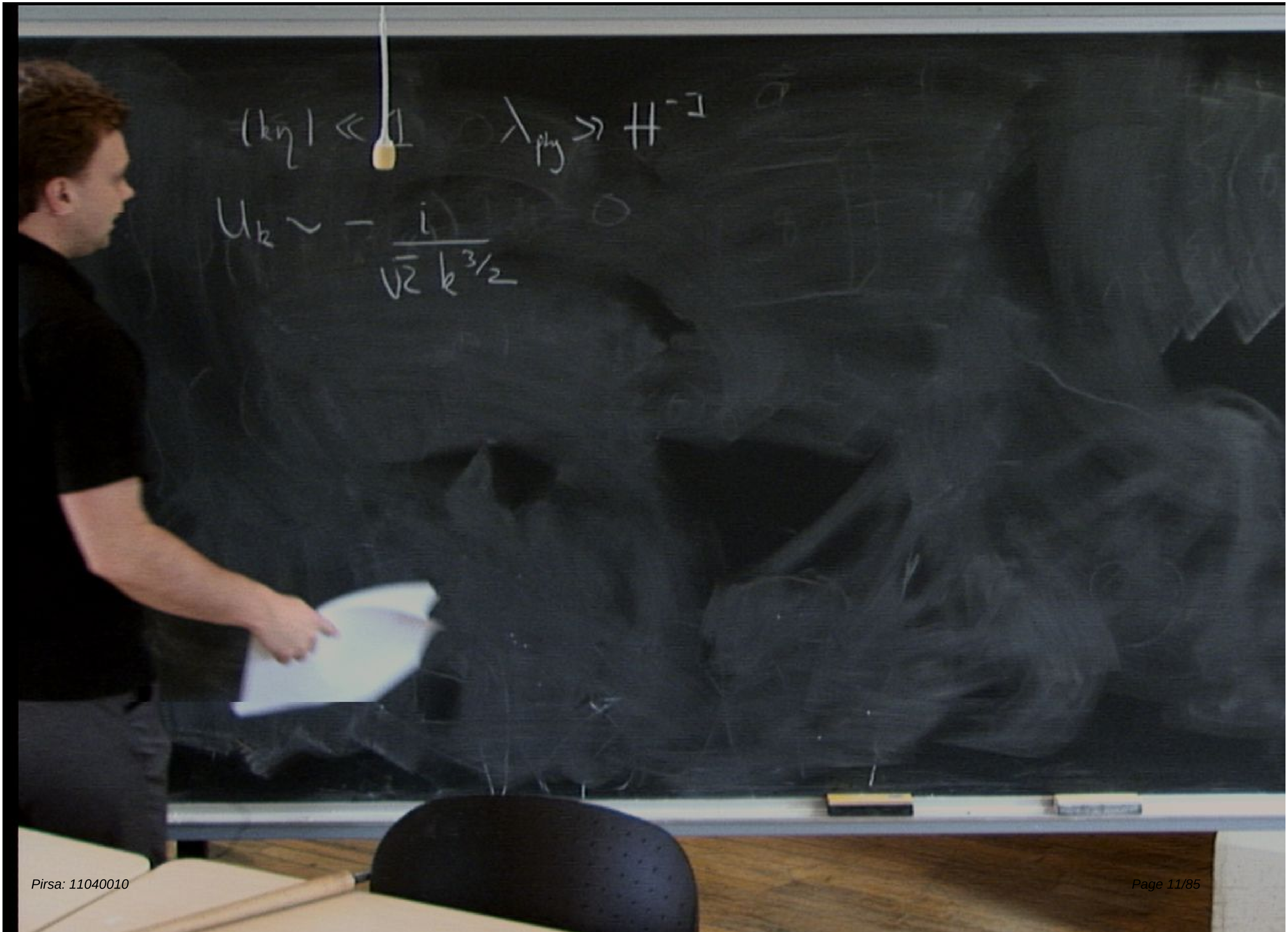
modes inside horizon

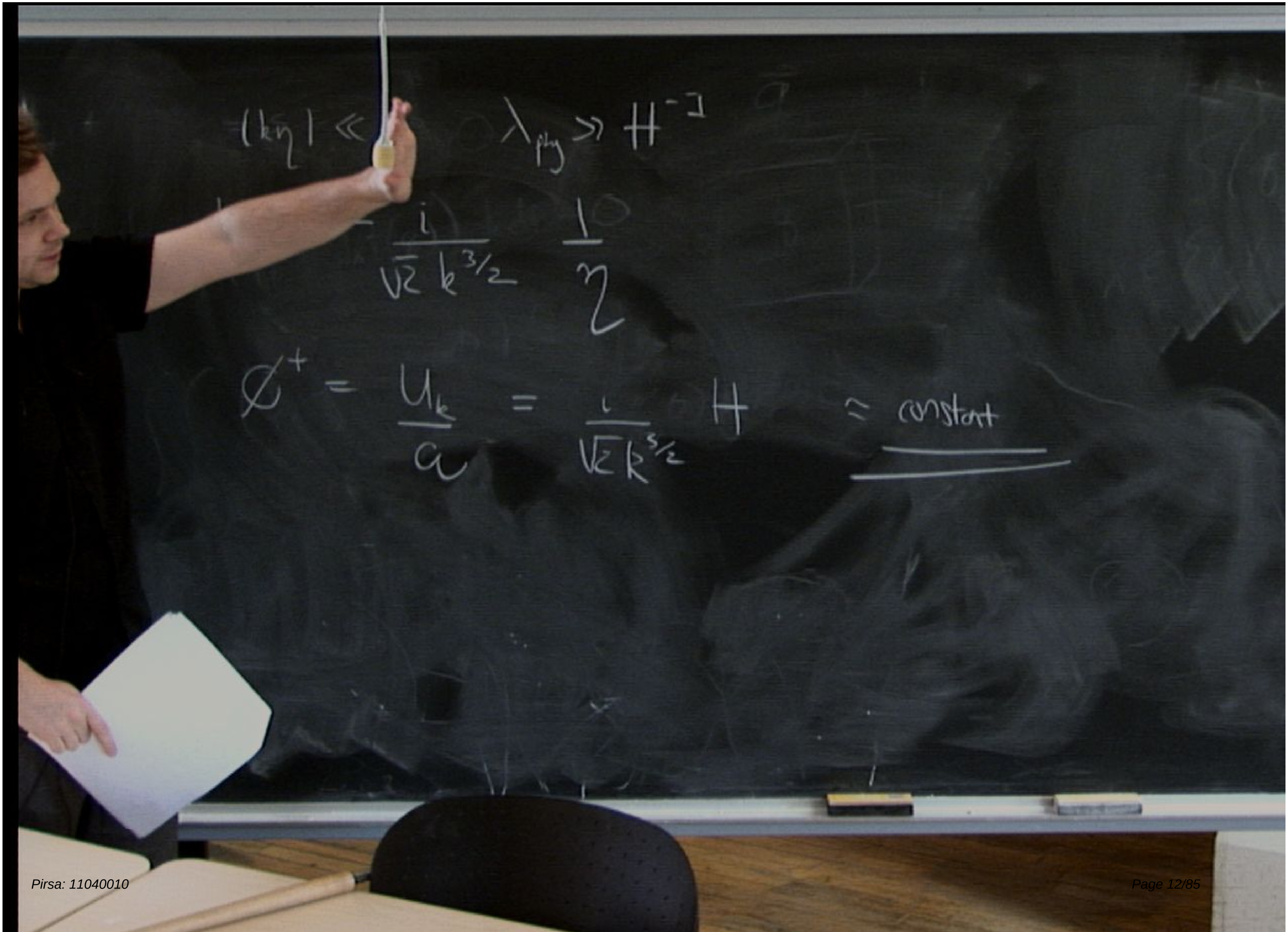
$$(-k\eta) = \frac{(k/a)}{H} \gg 1$$

$$u_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

WKB good

$$\lambda_{phy} = a\,\lambda_{com} \ll H^{-1}$$

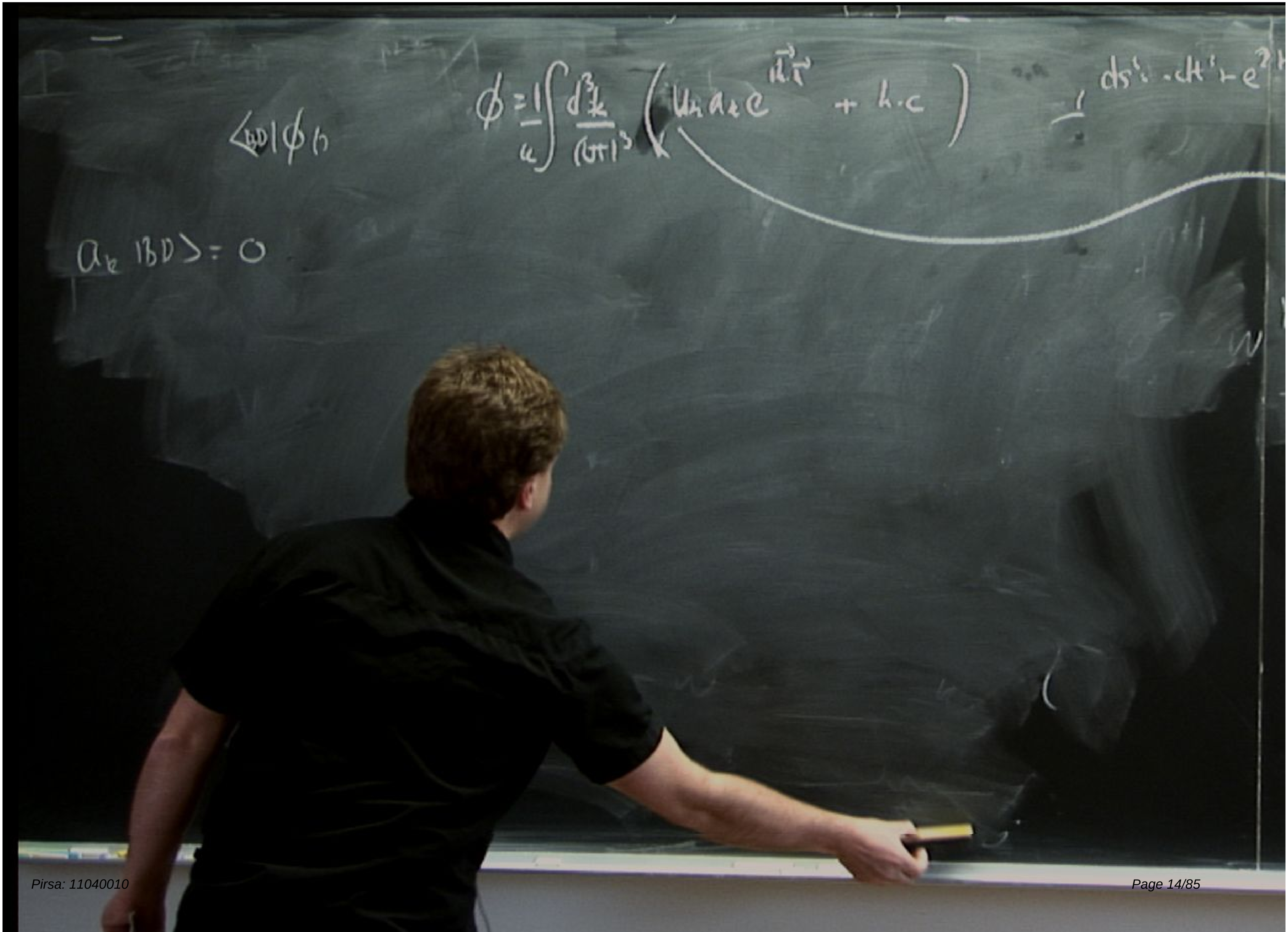

$|k\eta| \ll 1 \quad \lambda_{phy} \gg H^{-1}$
$u_k \sim -\frac{i}{\sqrt{2}\, k^{3/2}}$


|kη| ≪ 1   λ_phy ≫ H^-1
i/(√2 k^3/2) 1/η
φ+ = u_k/a = i/(√2 k^3/2) H ≃ constant

$$|k\eta| \ll 1 \qquad \lambda_{phy} \gg H^{-1}$$

$$u_k \sim -\frac{i}{\sqrt{2}\, k^{3/2}} \frac{1}{\eta}$$

$$\phi_k^{+} = \frac{u_k}{a} = \frac{i}{\sqrt{2}\, k^{3/2}} H \simeq \text{constant}$$

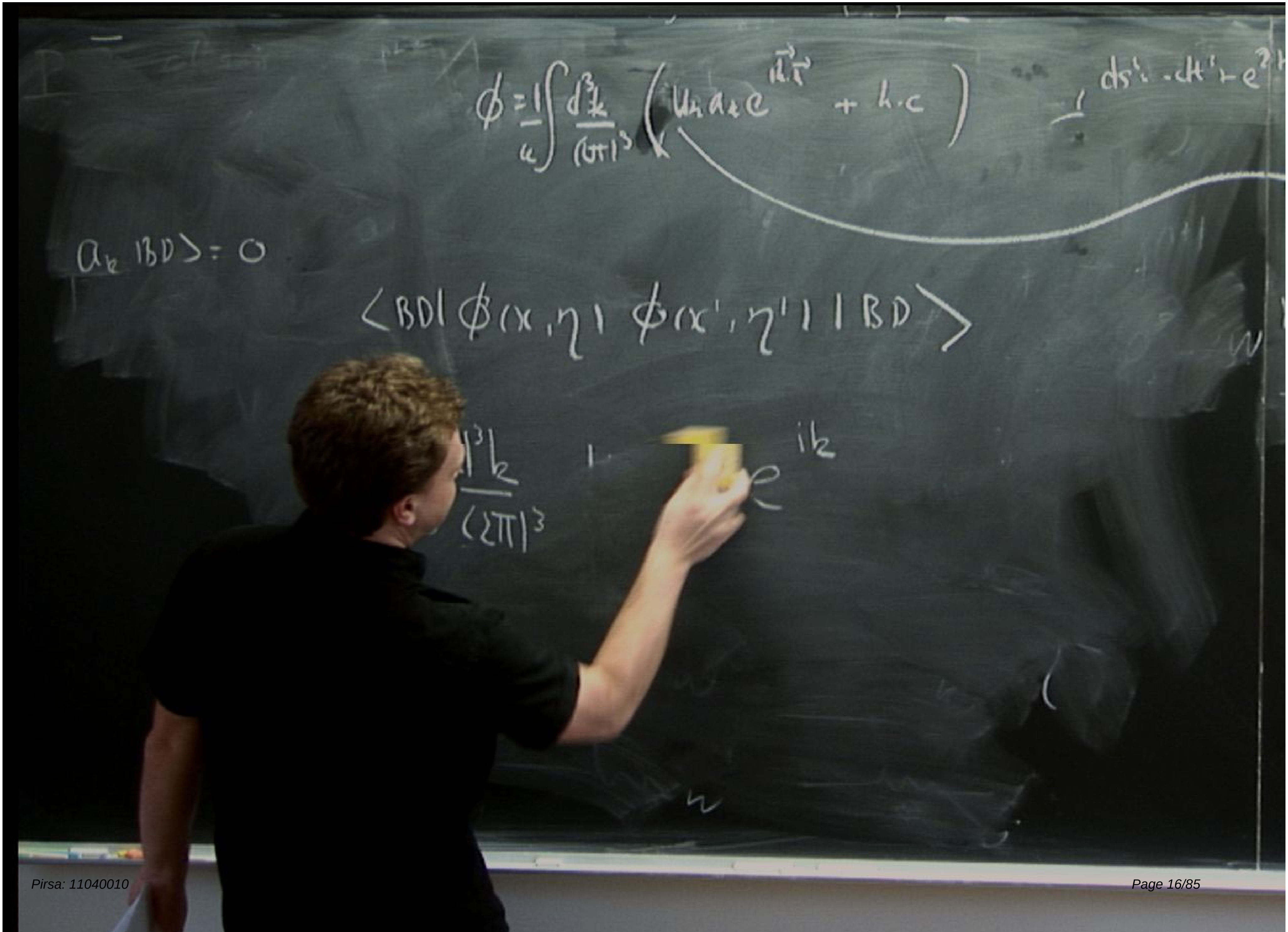

a_k |BD> = 0

$$\phi = \frac{1}{a}\int \frac{d^3k}{(2\pi)^3}\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c\right)$$

$ds^2 = -dt^2 + e^{2Ht}$

$$a_k |BD\rangle = 0$$

$$\langle BD|\,\phi(x,\eta)\,\phi(x',\eta')\,|BD\rangle$$


a_k |BD> = 0
<BD| φ(x, η) φ(x', η') |BD>

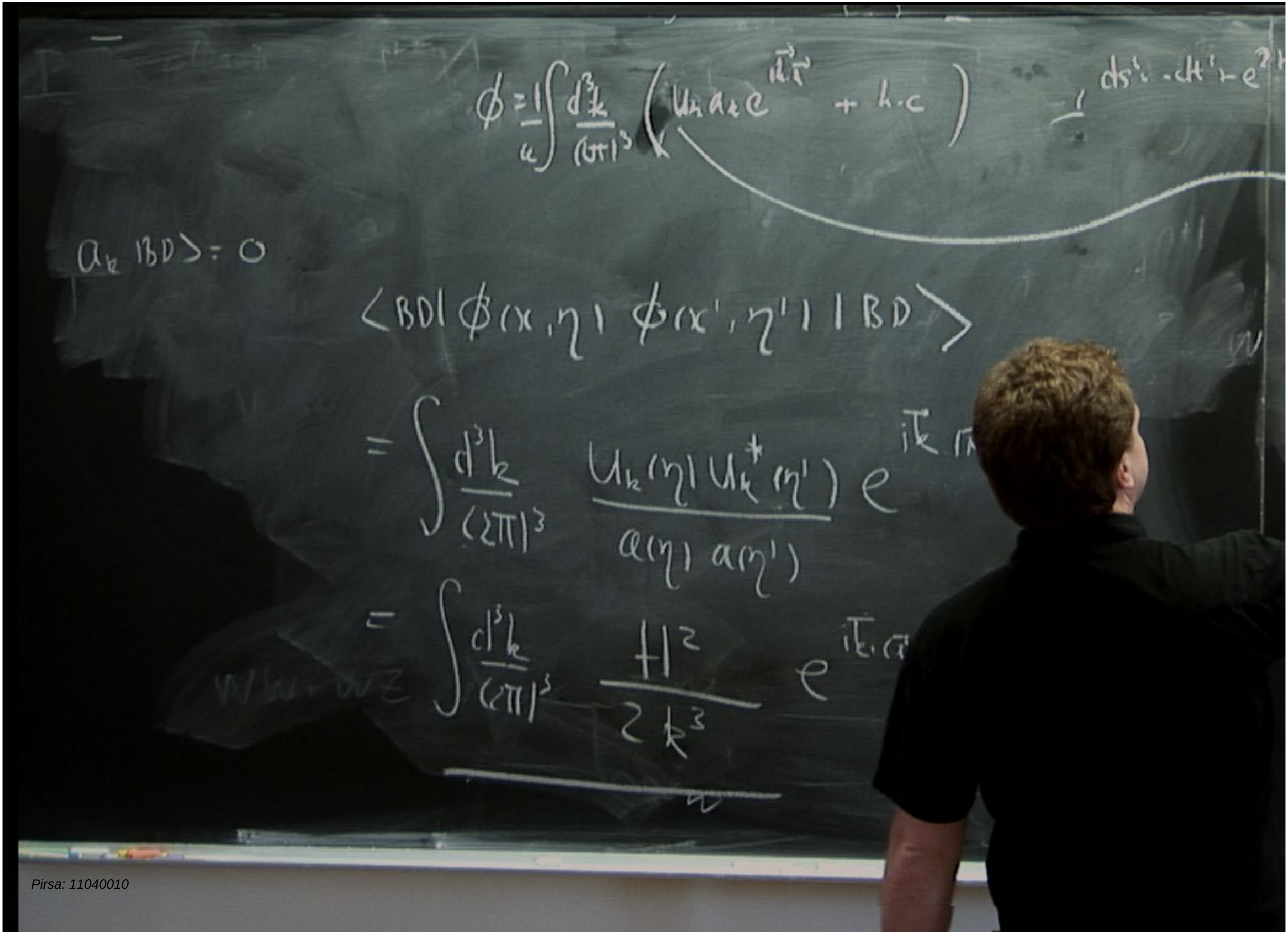
$ds^2 = -dt^2 + e^{2H}$

$$\phi = \frac{1}{a}\int \frac{d^3k}{(2\pi)^3}\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c. \right)$$

$a_k |BD\rangle = 0$

$$\langle BD | \phi(x,\eta)\, \phi(x',\eta') | BD \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{u_k(\eta)\, u_k^*(\eta')}{a(\eta)\, a(\eta')} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$


$\phi = \frac{1}{a}\int \frac{d^3k}{(2\pi)^3}\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c\right)$
$ds^2 = dt^2 - e^{2H}$
$a_k |BD\rangle = 0$
$\langle BD| \phi(x,\eta)\, \phi(x',\eta') |BD\rangle$
$= \int \frac{d^3k}{(2\pi)^3} \frac{u_k(\eta) u_k^*(\eta')}{a(\eta)\, a(\eta')} e^{i\vec{k}\cdot(\vec{x}}$
$= \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} e^{i\vec{k}\cdot(\vec{x}}$

$$\int \frac{d^3k}{(2\pi)^3} \left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c. \right)$$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$\to u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e$$

$$\eta)\, \phi(x', \eta')\, | BD \rangle$$

$$k \to \lambda k$$

$$\frac{u_k(\eta)\, u_k^*(\eta')}{a(\eta)\, a(\eta')}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$$

$$\frac{H^2}{2k^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$e^{2Ht} d\vec{x}^2$

$\to U_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$

$k \to \lambda k$

$\int d^3k \, \frac{1}{k^3} \to \int d^3k \, \frac{1}{k^3}$

$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$

$t \to t + c \qquad \vec{x} \to e^{-2Hc} \vec{x}$

$|5\pi|^2$

$$U_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta}$$

$$k \to \lambda k$$

$$\int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$t \to t + c \quad \vec{x} \to e^{-2Hc} \vec{x}'$$

$$\vec{k} \to e^{+2Hc} \vec{k}$$

$$e^{i\vec{k}\cdot\vec{x}}$$

$$U_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$$k \to \lambda k$$

$$\int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$t \to t + c \quad \vec{x} \to e^{-Hc} \vec{x}'$$

$$\vec{k} \to e^{+Hc} \vec{k}$$

$$e^{i\vec{k}\cdot\vec{x}}$$

$$\phi = \frac{1}{a}\int \frac{d^3k}{(2\pi)^3}\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c\right)$$

$ds^2 = -dt^2 + e^{2H}$

$a_k |BD\rangle = 0$

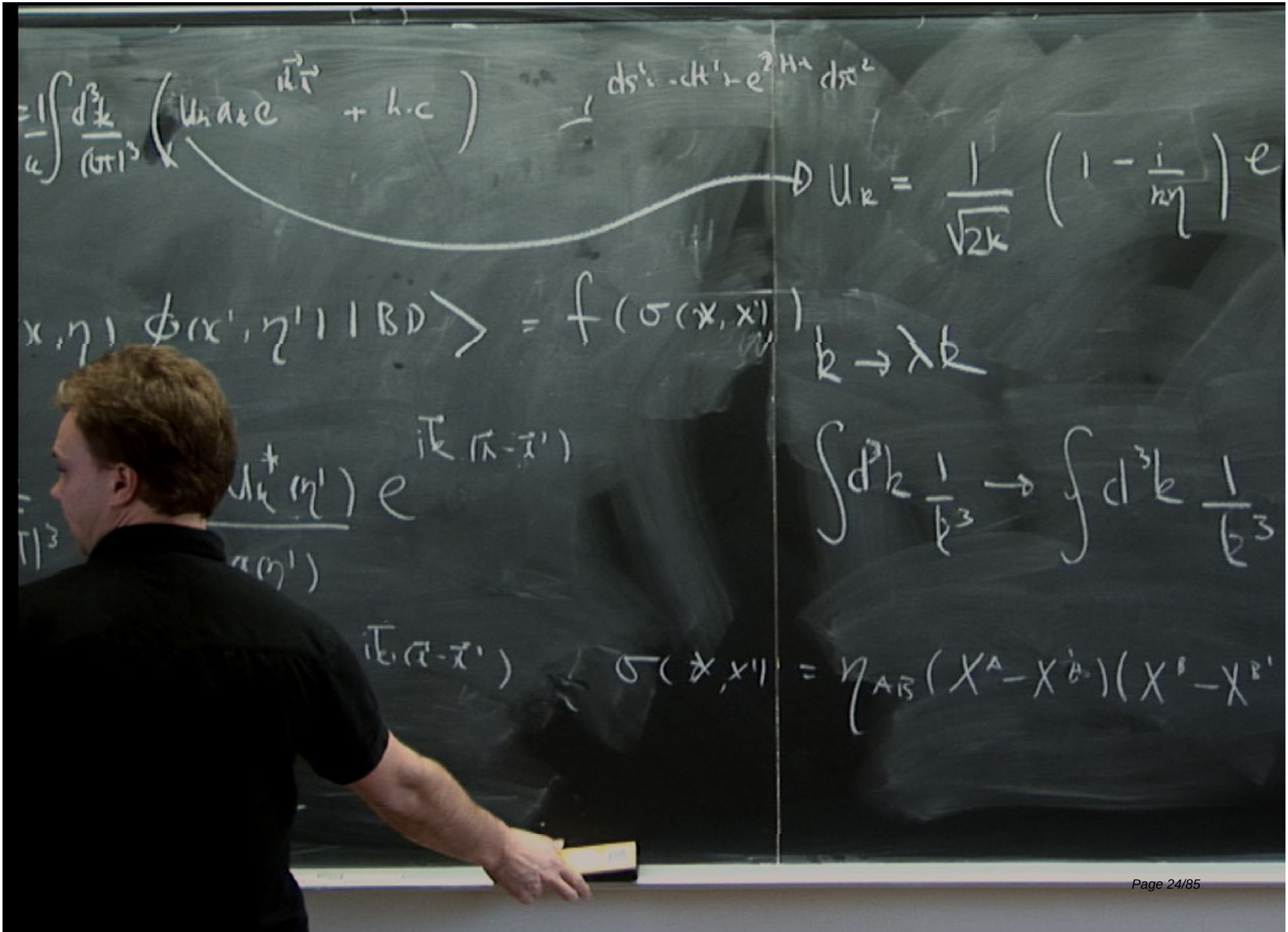
$$\langle BD|\phi(x,\eta)\,\phi(x',\eta')|BD\rangle = f(\sigma(x,x'))$$

$|BD\rangle$ is de Sitter invariant

annihilated by generators of de Sitter group.

$$= \int \frac{d^3k}{(2\pi)^3}\,\frac{u_k(\eta)\,u_k^*(\eta')}{a(\eta)\,a(\eta')}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$= \int \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$


+ h.c
k → λk
σ(x, x') = η_AB (X^A − X^A')(X^B − X^B')

$$\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c \right) \qquad ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$\Rightarrow u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}$$

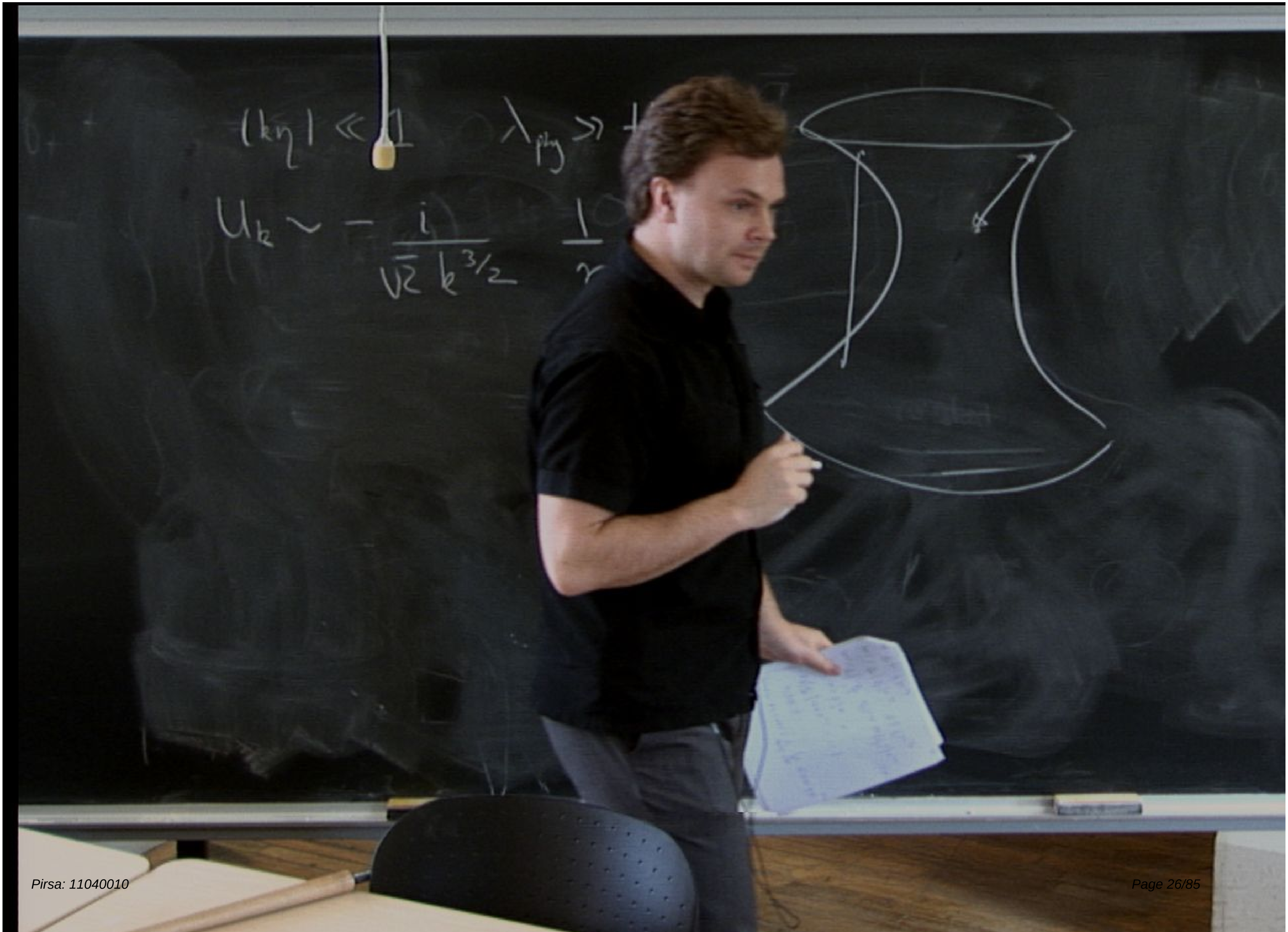
$$\phi(x', \eta') | BD \rangle = f(\sigma(x, x'))$$

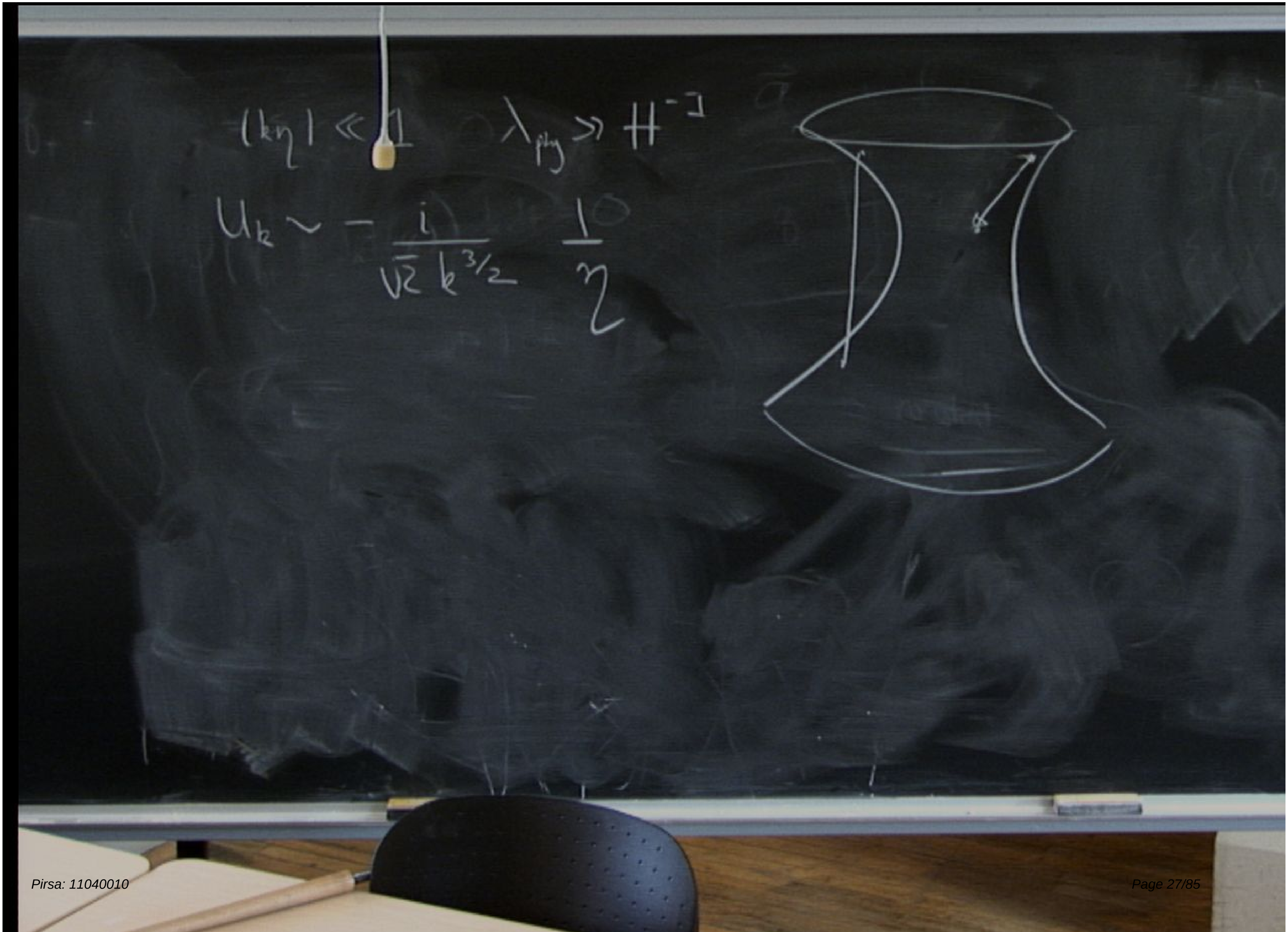
$$k \to \lambda k$$

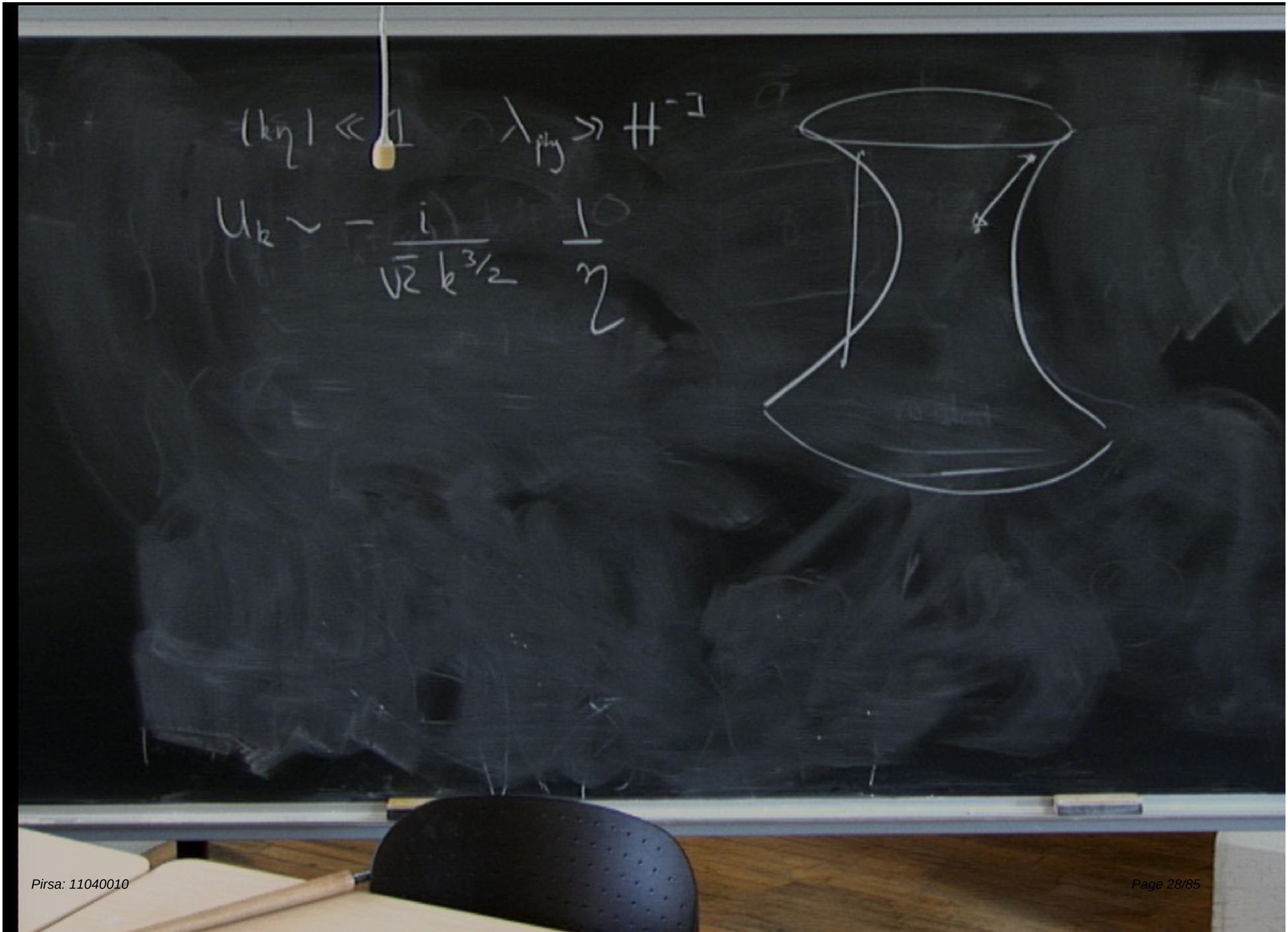
$$(\eta) u_k^*(\eta') e^{i\vec{k}(\vec{x}-\vec{x}')} \qquad \int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$$

$$\frac{}{(\eta)\, a(\eta')}$$

$$\frac{H^2}{k^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \qquad \sigma(x, x') = \eta_{AB}(X^A - X^{A'})(X^B - X^{B'})$$


$|k\eta| \ll 1$ $\lambda_{phy} \gg$
$U_k \sim -\frac{i}{\sqrt{2}\,k^{3/2}}$ $\frac{1}{\eta}$


$|k\eta| \ll 1 \quad \lambda_{phy} \gg H^{-1}$
$u_k \sim -\frac{i}{\sqrt{2}\, k^{3/2}} \frac{1}{\eta}$


$|k\eta| \ll 1 \quad \lambda_{phy} \gg H^{-1}$
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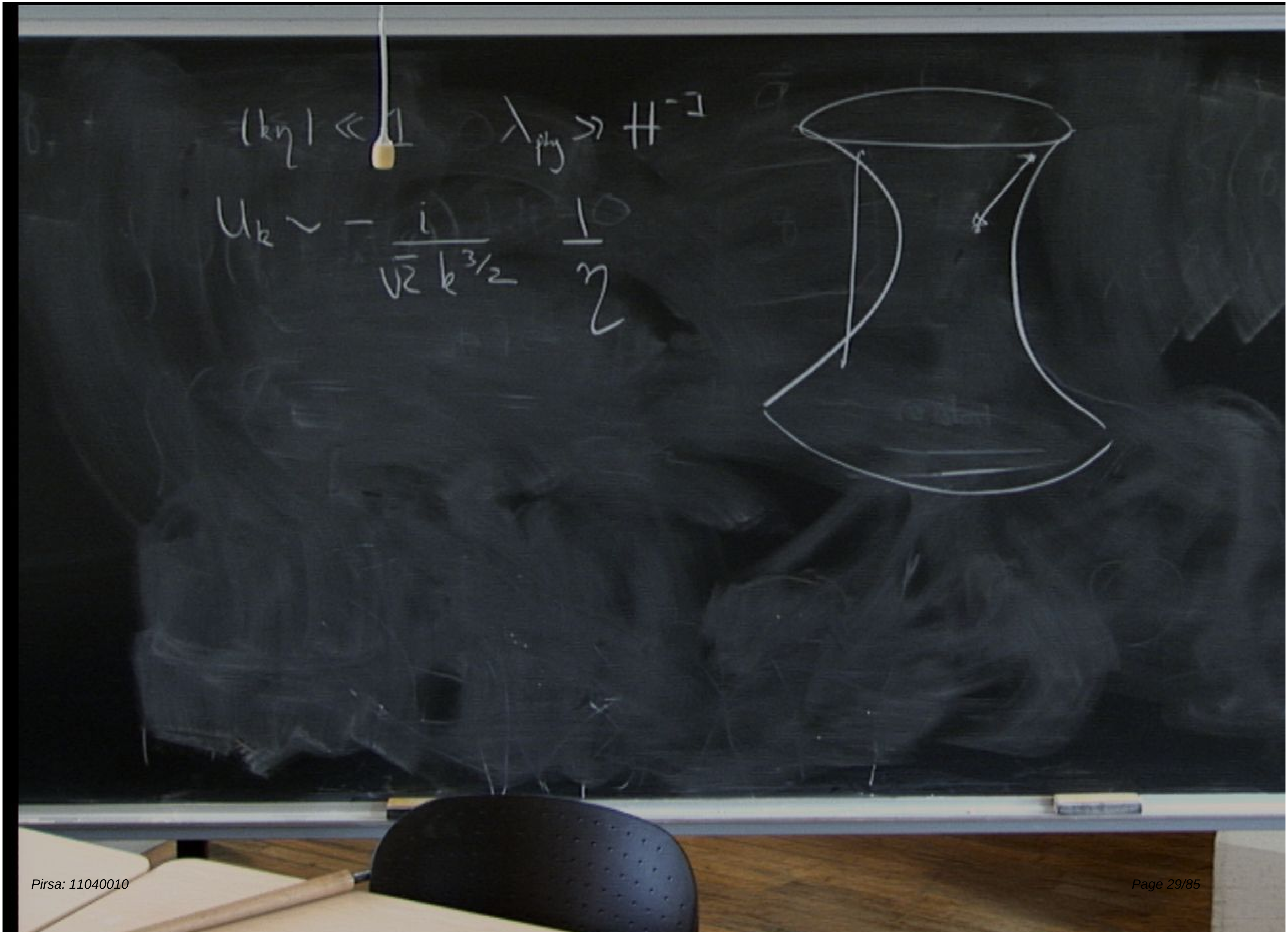
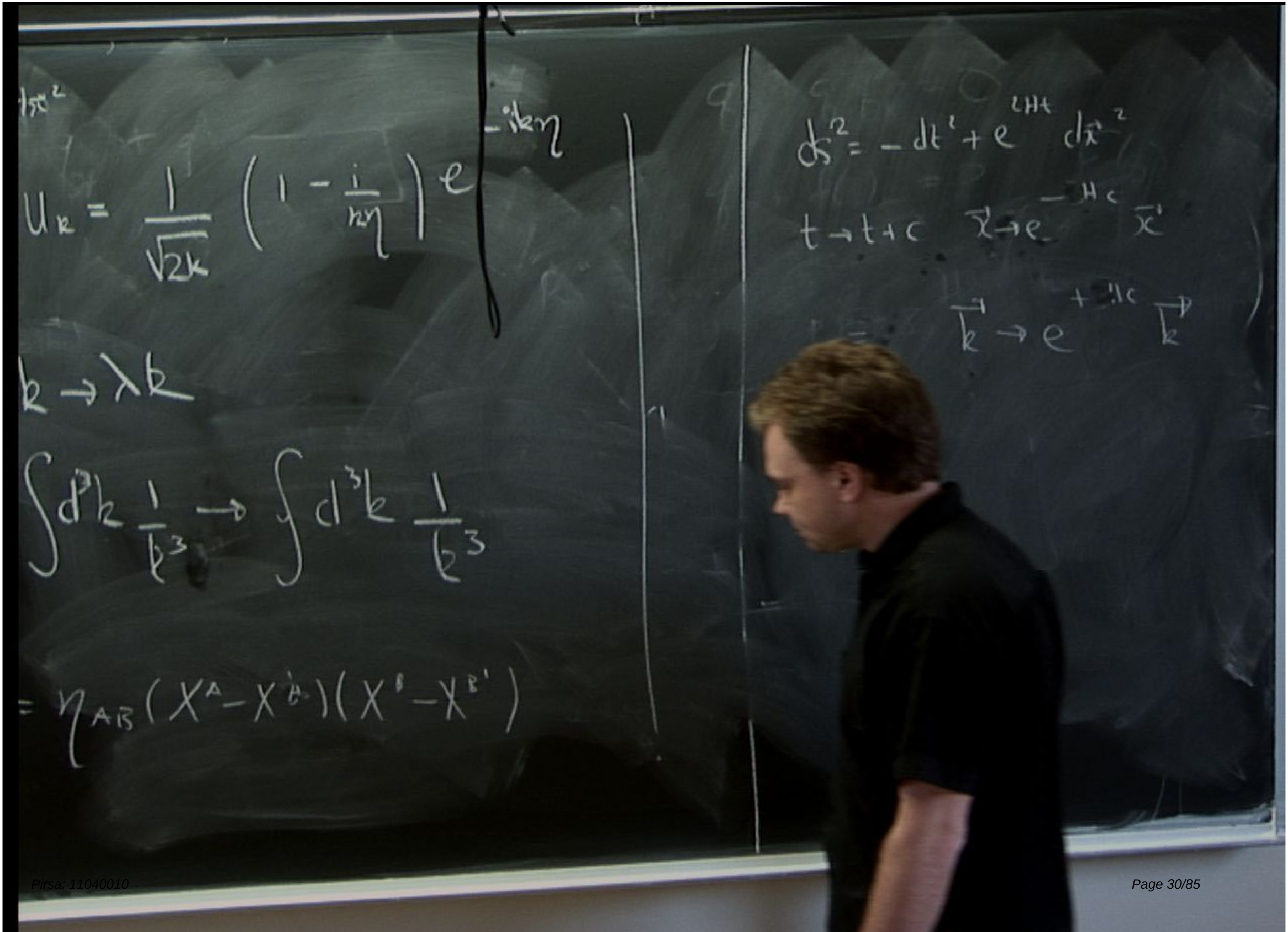

|kη| ≪ 1   λphy ≫ H^-1
Uk ~ − i / (√2 k^3/2) · 1/η



$u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$
$k \to \lambda k$
$\int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$
$= \eta_{AB}(X^A - X^{A'})(X^B - X^{B'})$
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
$t \to t + c \quad \vec{x} \to e^{-Hc} \vec{x}'$
$\vec{k} \to e^{+Hc} \vec{k}$

$$u_k = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$$k \to \lambda k$$

$$\int d^3k \frac{1}{k^3} \to \int d^3k \frac{1}{k^3}$$

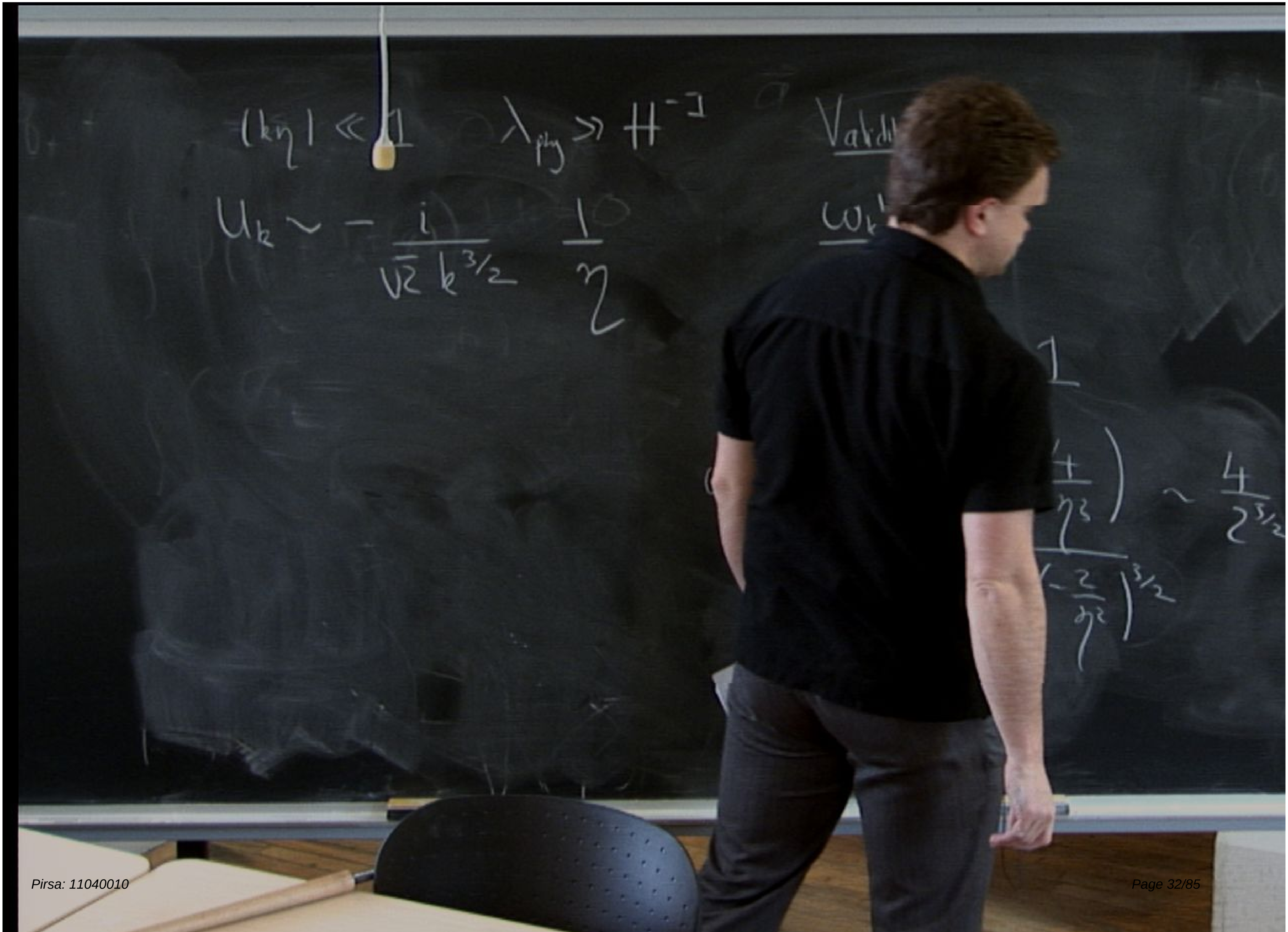
$$= \eta_{AB}(X^A - X^{A'})(X^B - X^{B'})$$

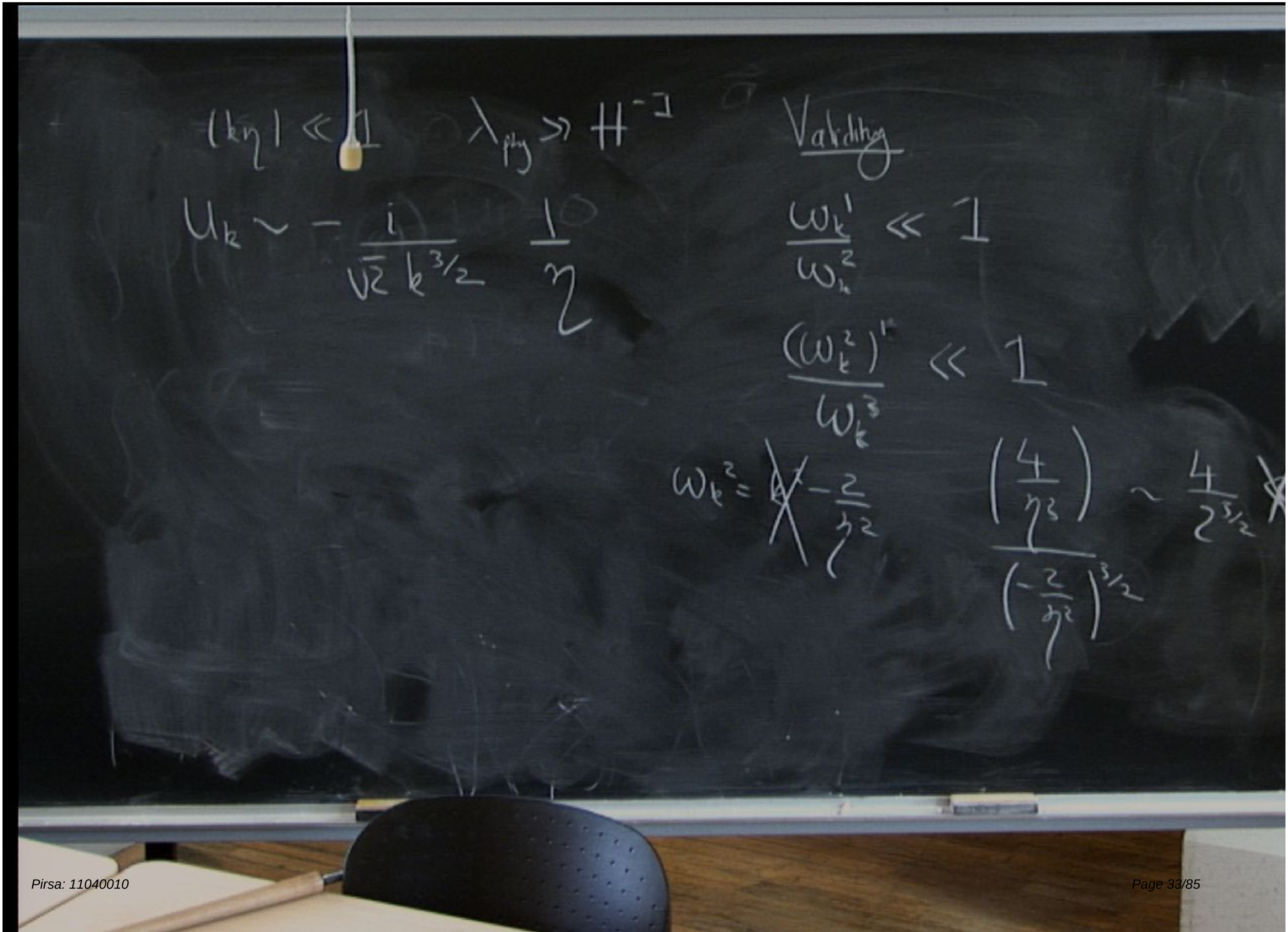
$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

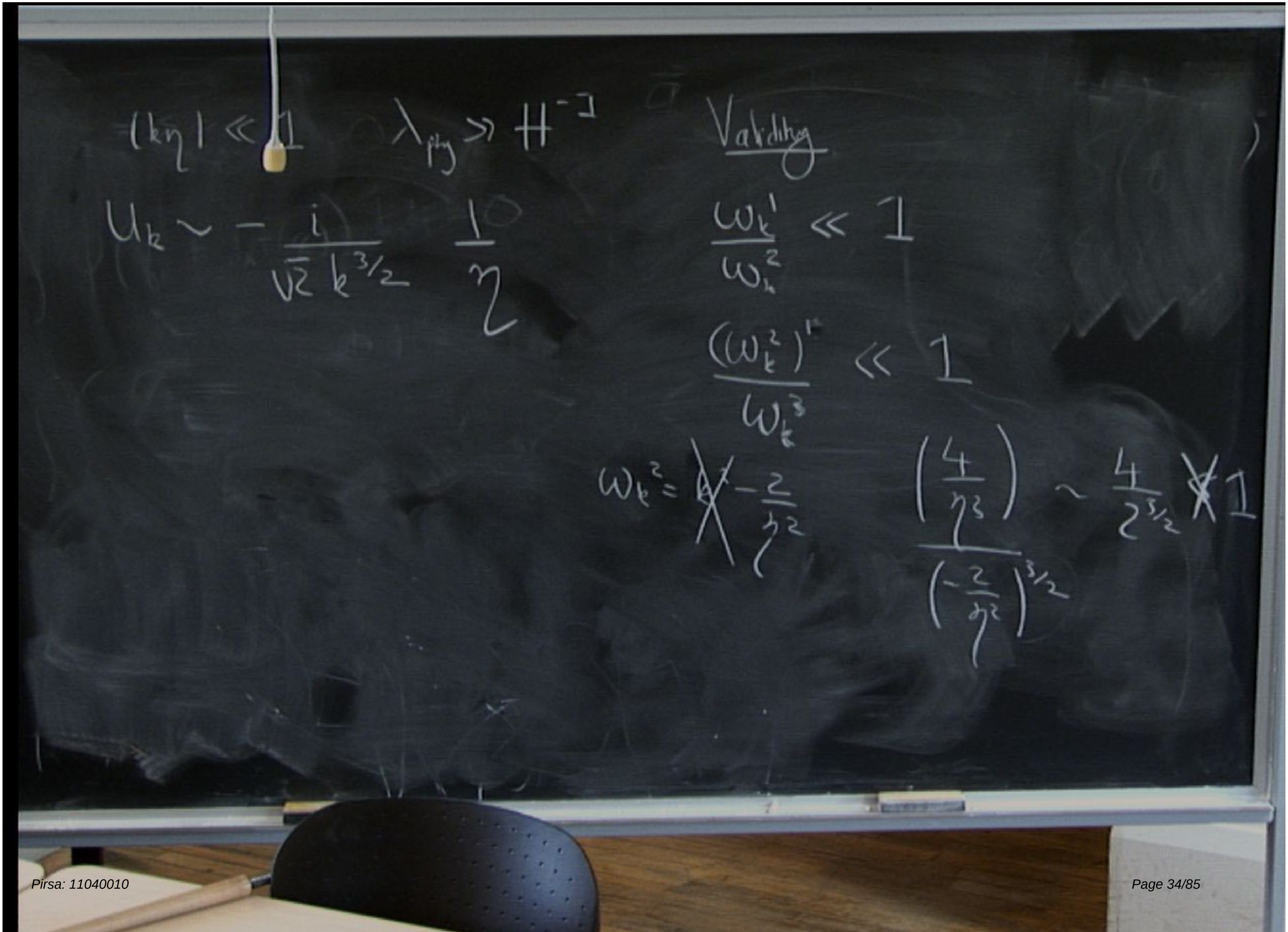
$$t \to t + c \quad \vec{x} \to e^{-Hc} \vec{x}'$$

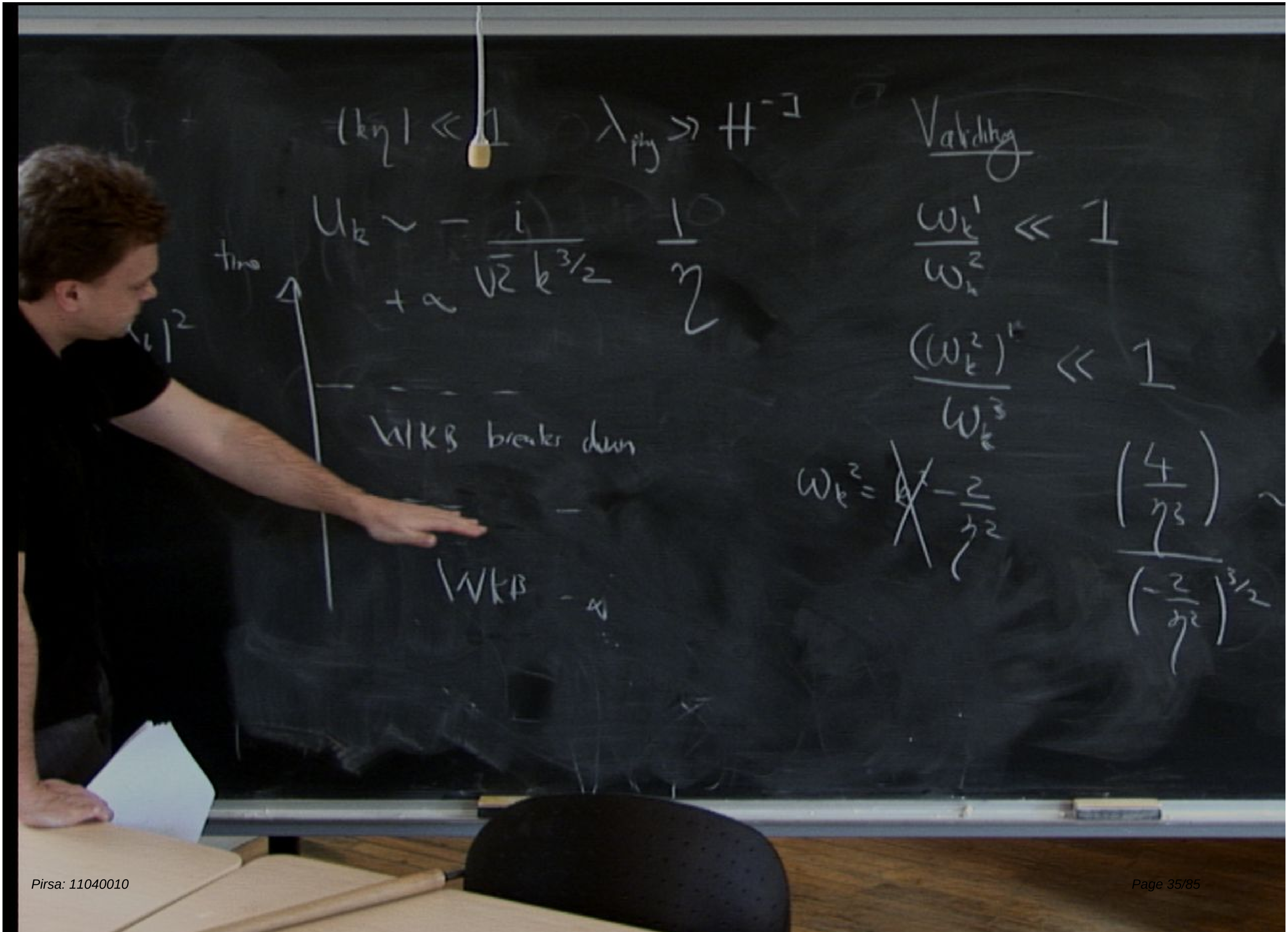
$$\vec{k} \to e^{+Hc} \vec{k}$$

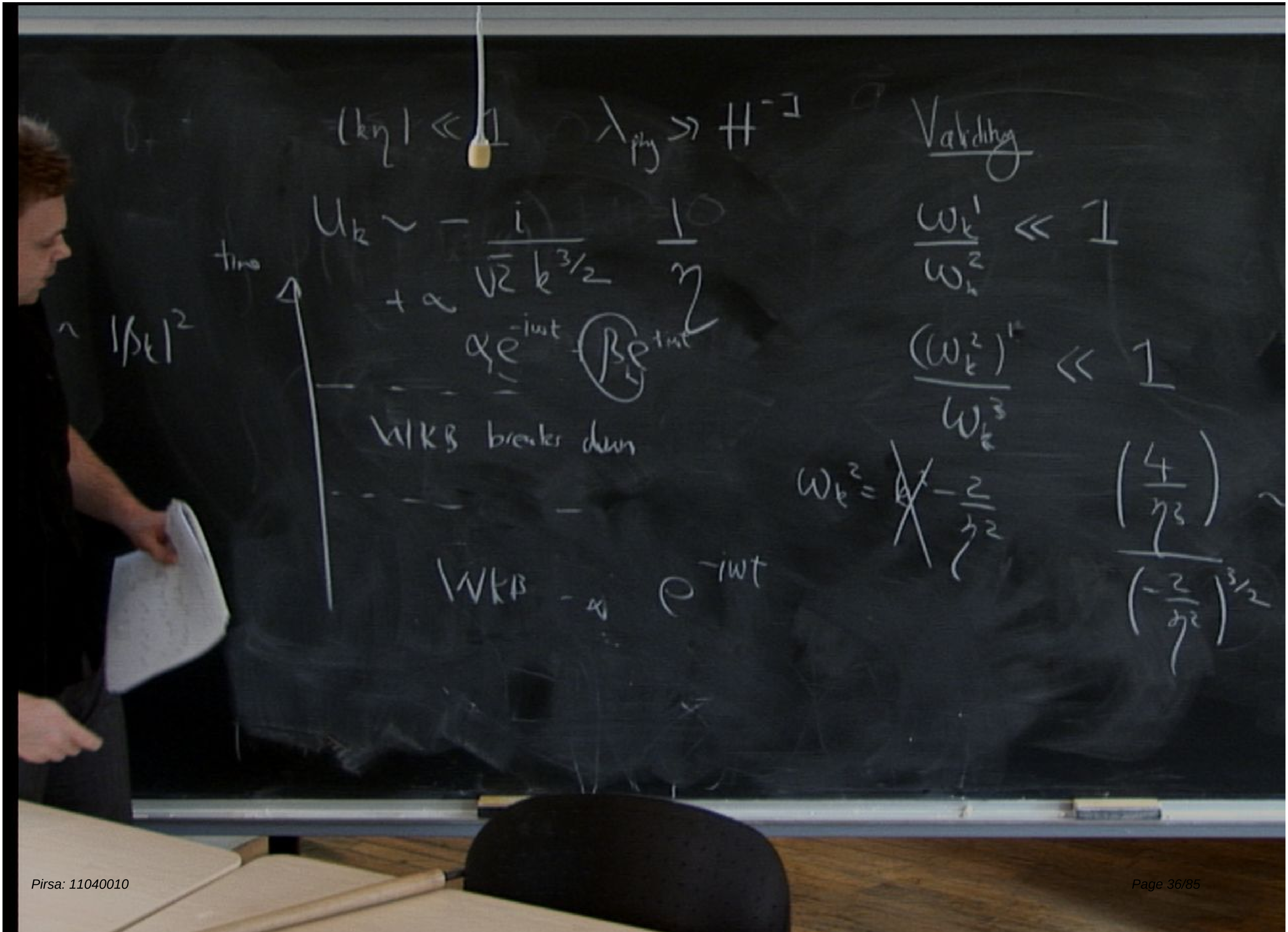
$$e^{i\vec{k}\cdot\vec{x}}$$

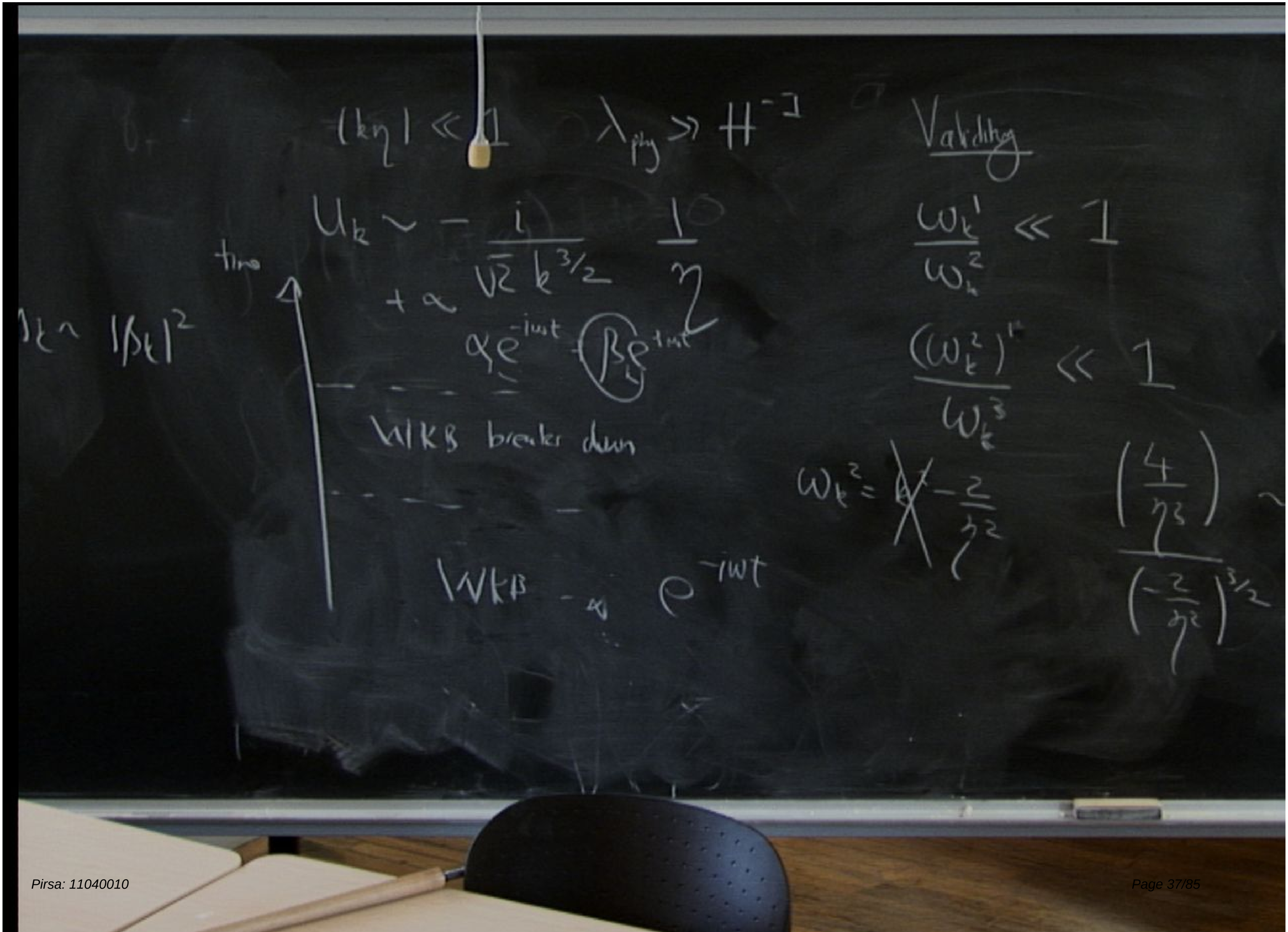

|kη| ≪ 1   λ_phy ≫ H^-1
U_k ~ − i / (√2 k^3/2) · 1/η

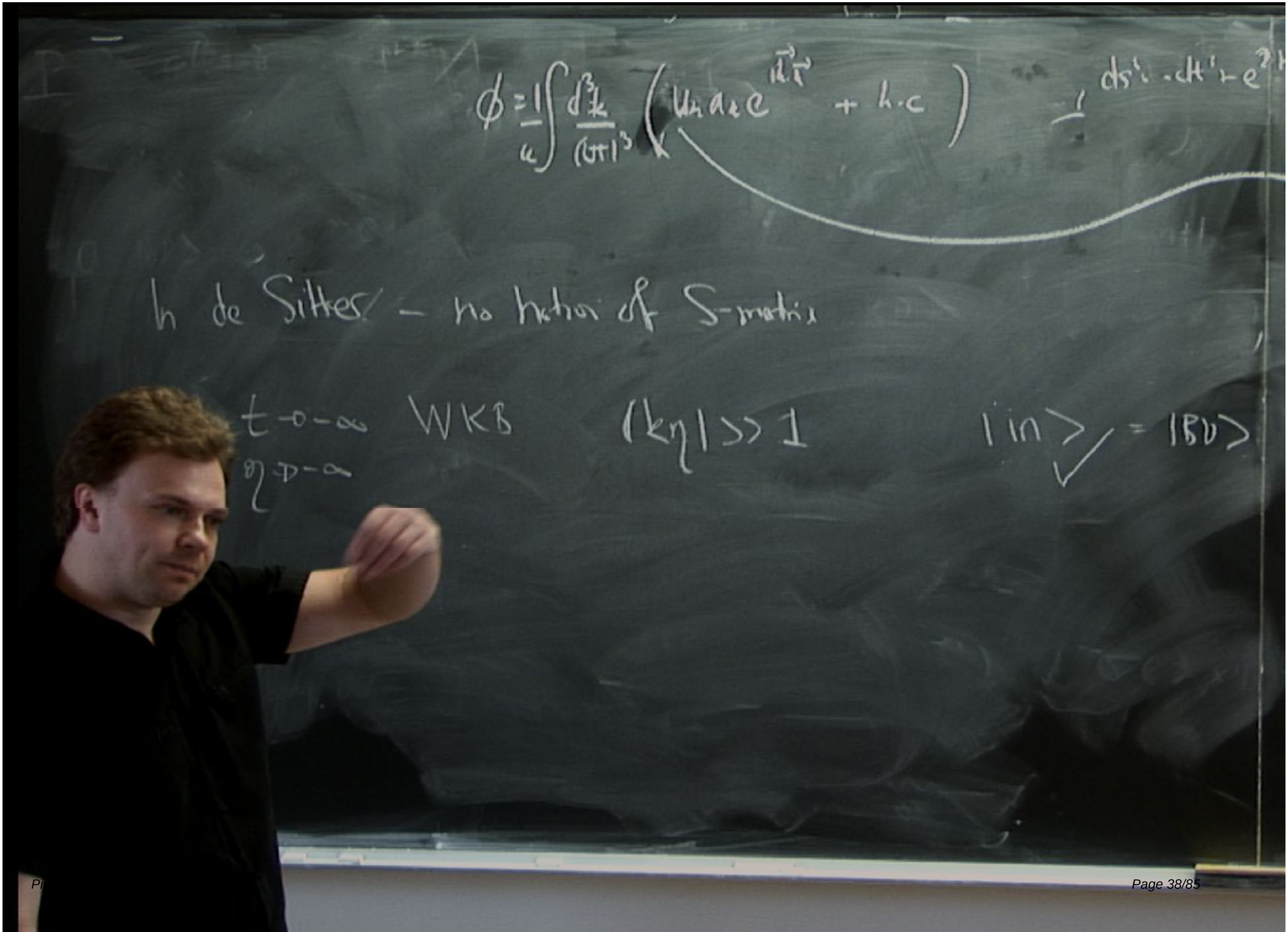

|kη| ≪ 1   λ_phy ≫ H^-1
Validity
u_k ~ − i/(√2 k^3/2) 1/η
ω_k'/ω_k^2 ≪ 1
((ω_k^2)')/ω_k^3 ≪ 1
ω_k^2 = k^2 − 2/η^2
(4/η^3)/(−2/η^2)^3/2 ~ 4/2^3/2


$|k\eta| \ll 1$
$\lambda_{phy} \gg H^{-1}$
Validity
$u_k \sim -\frac{i}{\sqrt{2}\,k^{3/2}} \frac{1}{\eta}$
$\frac{\omega_k'}{\omega_k^2} \ll 1$
$\frac{(\omega_k^2)'}{\omega_k^3} \ll 1$
$\omega_k^2 = k^2 - \frac{2}{\eta^2}$
$\frac{\left(\frac{4}{\eta^3}\right)}{\left(-\frac{2}{\eta^2}\right)^{3/2}} \sim \frac{4}{2^{3/2}} \not\ll 1$


|kη| ≪ 1
λ_phy ≫ H^-1
Validity
U_k ~ − i / (√2 k^3/2) · 1/η
+ α
time
WKB breaks down
WKB
ω_k' / ω_k^2 ≪ 1
(ω_k^2)'' / ω_k^3 ≪ 1
ω_k^2 = k^2 − 2/η^2
(4/η^3) / (−2/η^2)^3/2


|kη| ≪ 1
λ_phy ≫ H^-1
Validity
U_k ~ − i/(√2 k^3/2) · 1/η
time
~ |β_k|^2
αe^-iωt + β_k e^+iωt
WKB breaks down
WKB
e^-iωt
ω_k' / ω_k^2 ≪ 1
((ω_k^2)' / ω_k^3) ≪ 1
ω_k^2 = k^2 − 2/η^2
(4/η^3) / (−2/η^2)^3/2


|kη| ≪ 1
λ_phy ≫ H^-1
Validity
time
WKB breaks down
WKB
ω_k' / ω_k^2 ≪ 1
(ω_k^2)'' / ω_k^3 ≪ 1


In de Sitter - no notion of S-matrix
WKB
|in> = |BD>

$$\phi = \frac{1}{a}\int \frac{d^3k}{(2\pi)^3}\left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c.\right)$$

$ds^2 = -dt^2 + e^{2H\ldots}$

In de Sitter — no notion of S-matrix

$t \to -\infty$ WKB $\quad |k\eta| \gg 1 \qquad |in\rangle = |BD\rangle$ ✓

$\eta \to -\infty$

$$|\ldots n_{k_1 k_2}; n_{k_1 k_1}\rangle = \ldots \frac{(a^\dagger_{k_2})^{n_{k_2}}}{\sqrt{n_{k_2}!}}\frac{(a^\dagger_{k_1})^{n_{k_1}}}{\sqrt{n_{k_1}!}}|BD\rangle$$

$$\phi = \frac{1}{a} \int \frac{d^3k}{(2\pi)^3} \left(u_k a_k e^{i\vec{k}\cdot\vec{x}} + h.c \right)$$

$ds^2 = -dt^2 + e^{2H} \ldots$

In de Sitter - no notion of S-matrix

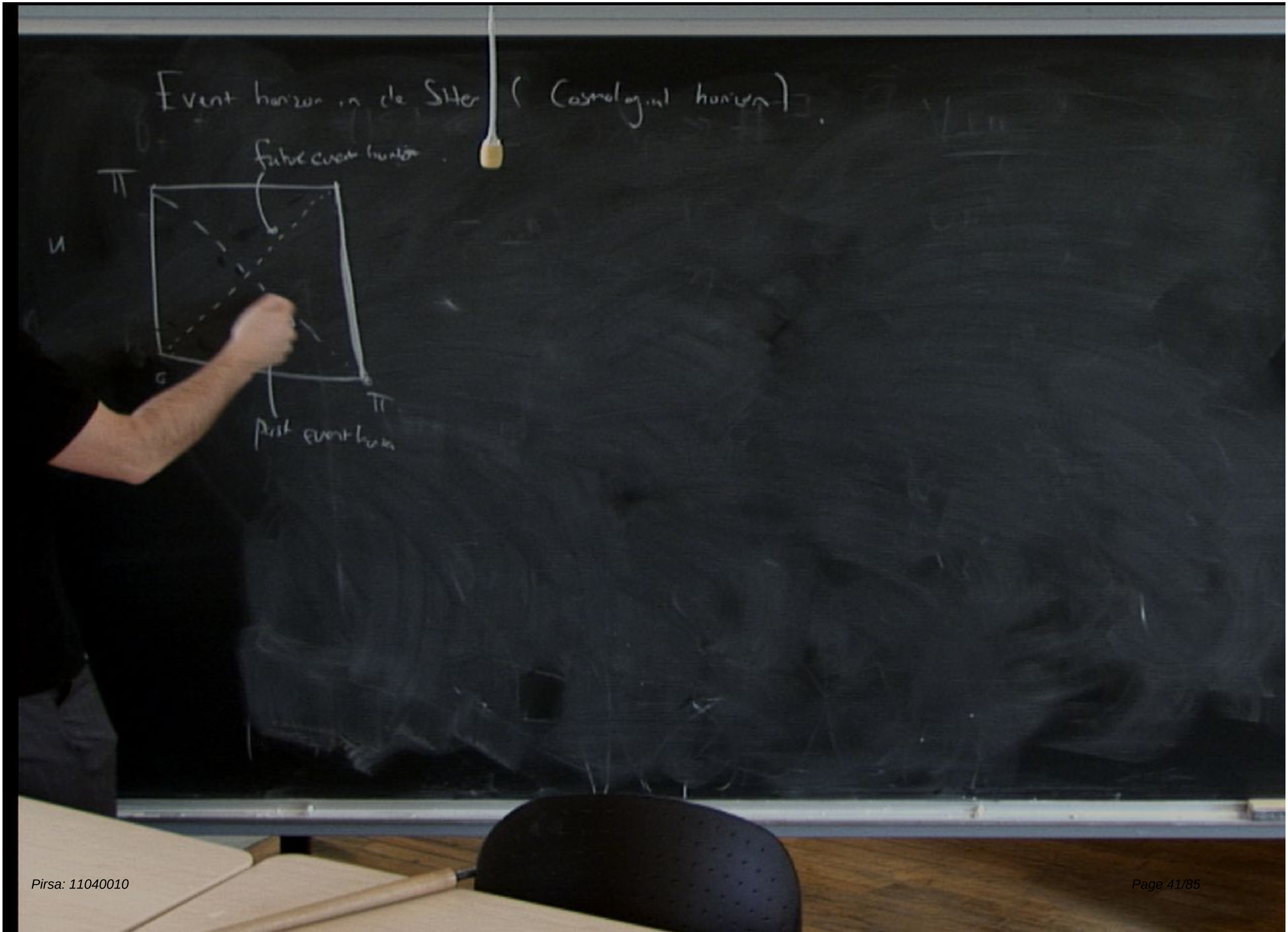
$t \to -\infty$ WKB $|k\eta| \gg 1$ $|in\rangle = |BD\rangle$ ✓

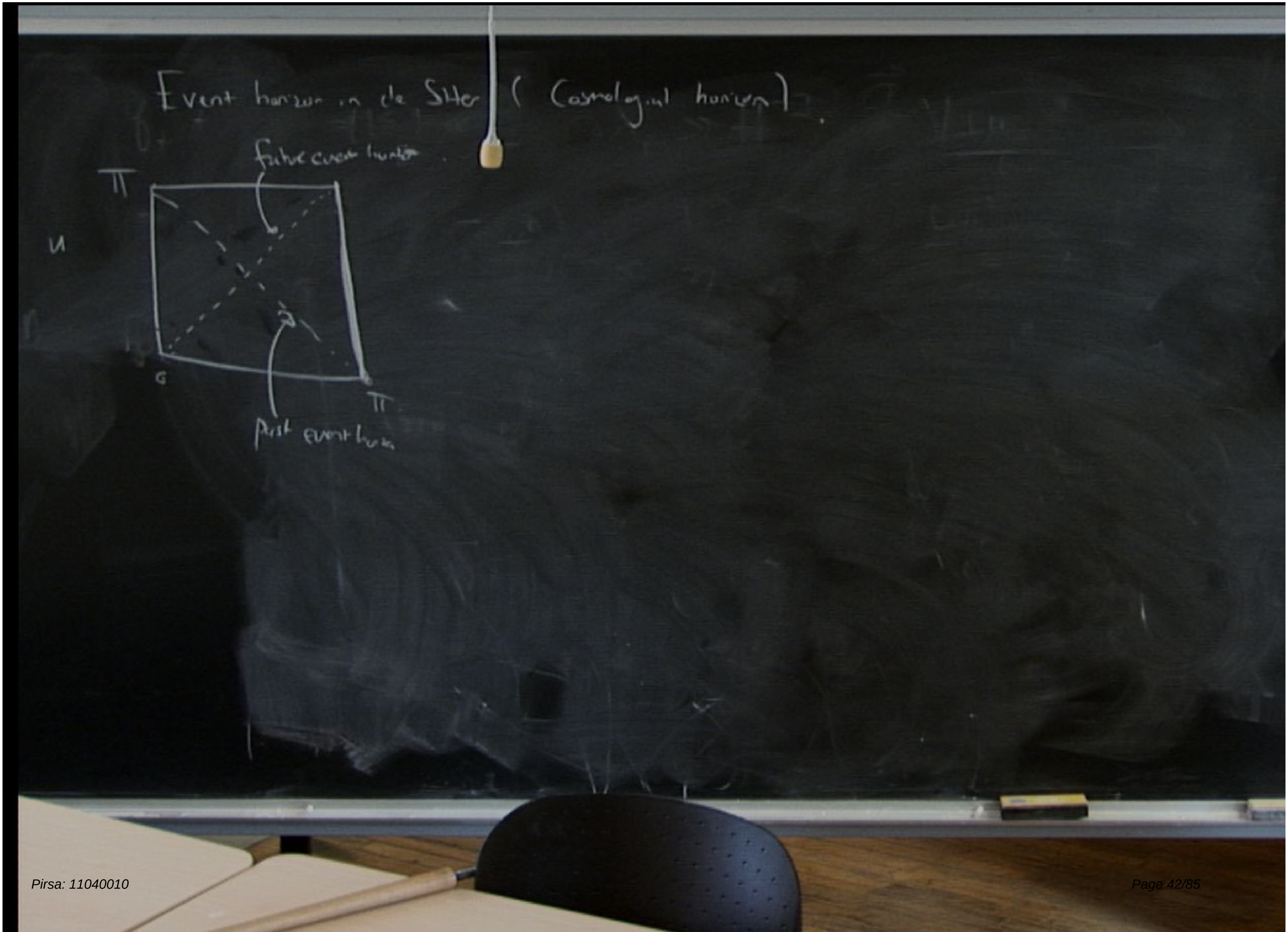
$\eta \to -\infty$

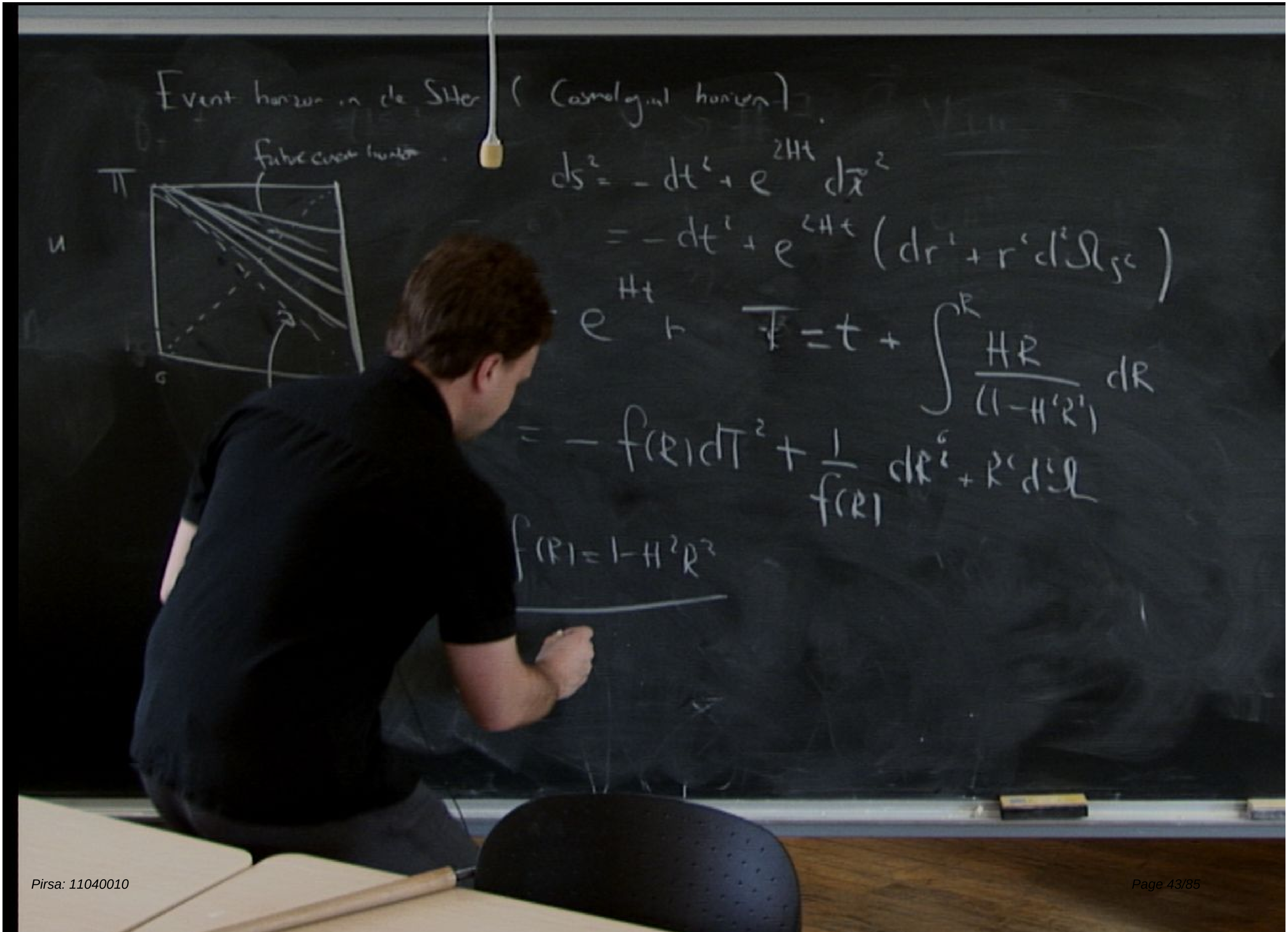
$$| \ldots n_{k_2, k_2} ; n_{k_1 k_1} \rangle = \ldots \frac{(a^+_{k_2})^{n_{k_2}}}{\sqrt{n_{k_2}!}} \frac{(a^+_{k_1})^{n_{k_1}}}{\sqrt{n_{k_1}!}} |BD\rangle$$

$t \to +\infty$ No notion of particles

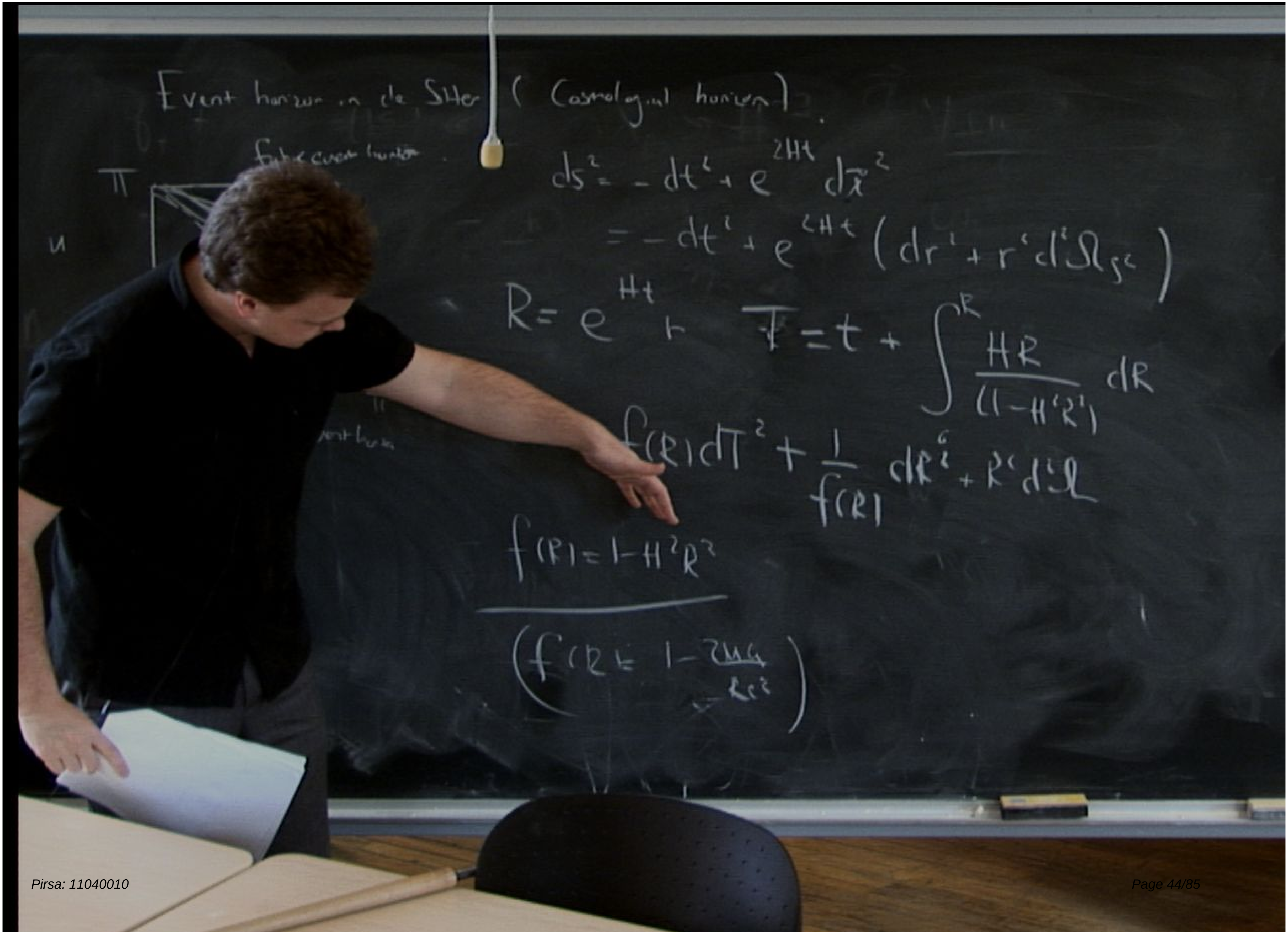
$\eta \to 0$ No notion $|out\rangle$

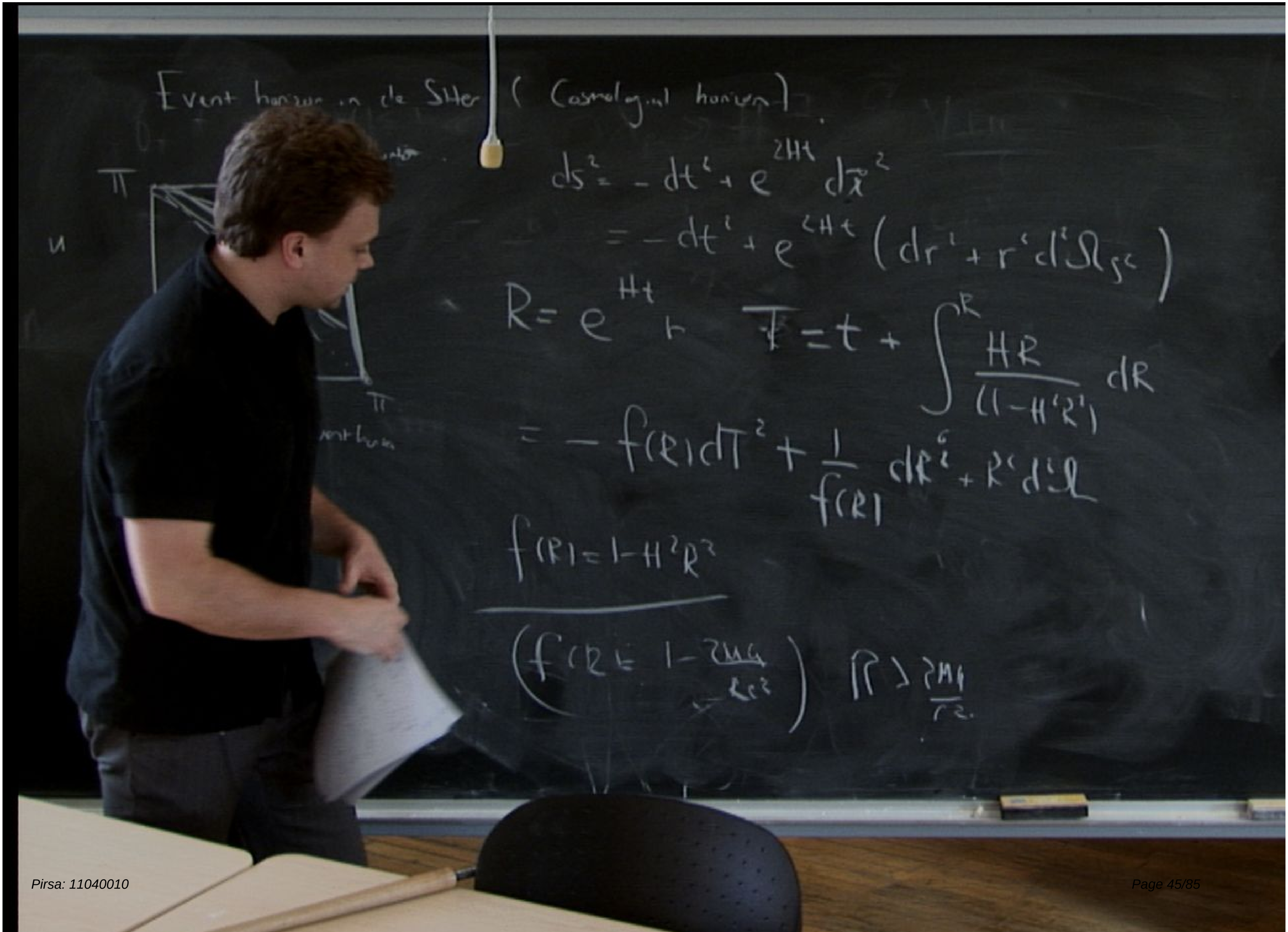

Event horizon in de Sitter (Cosmological horizon)
future event horizon
π
π
past event horizon

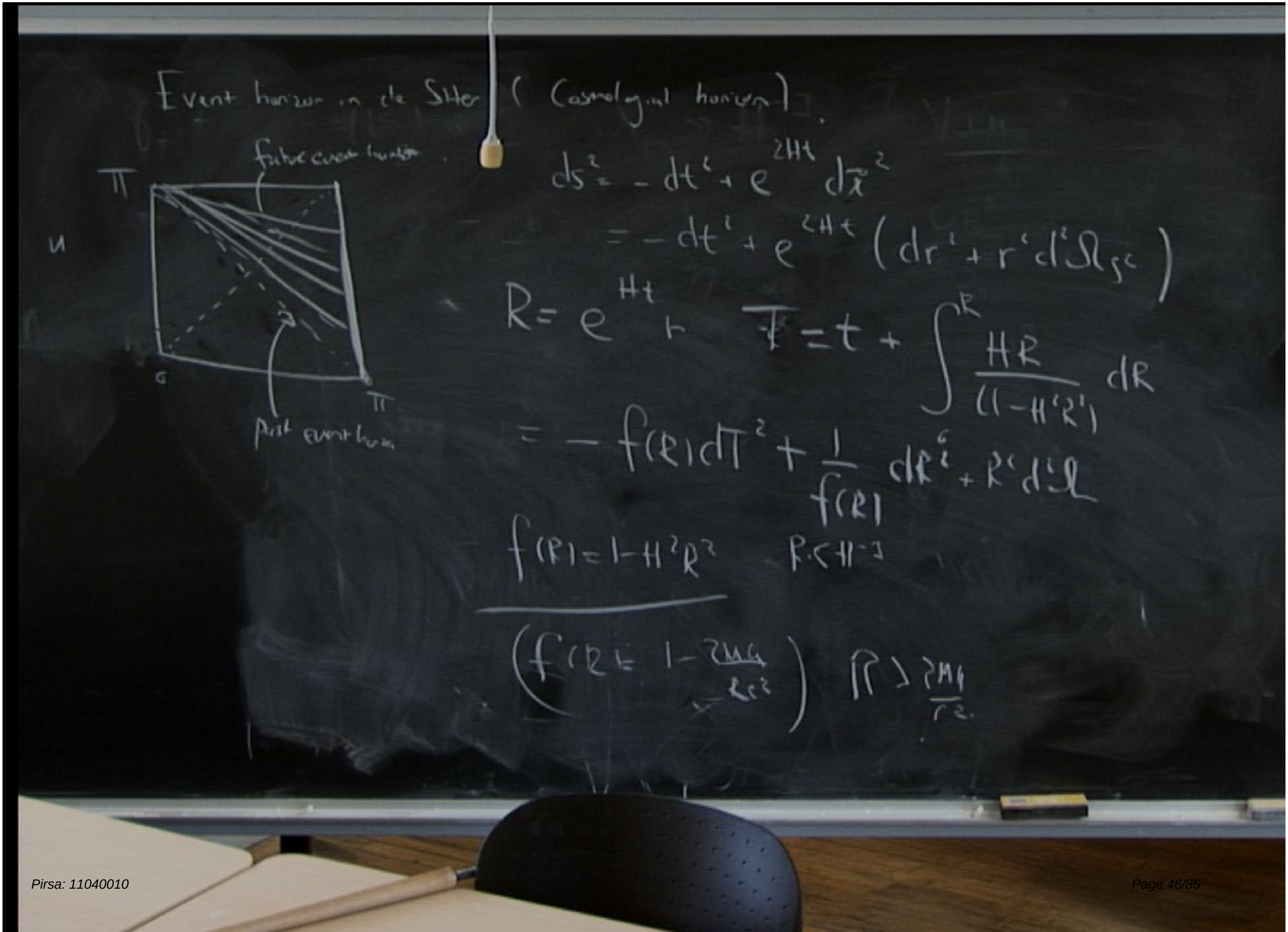

Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
π
π

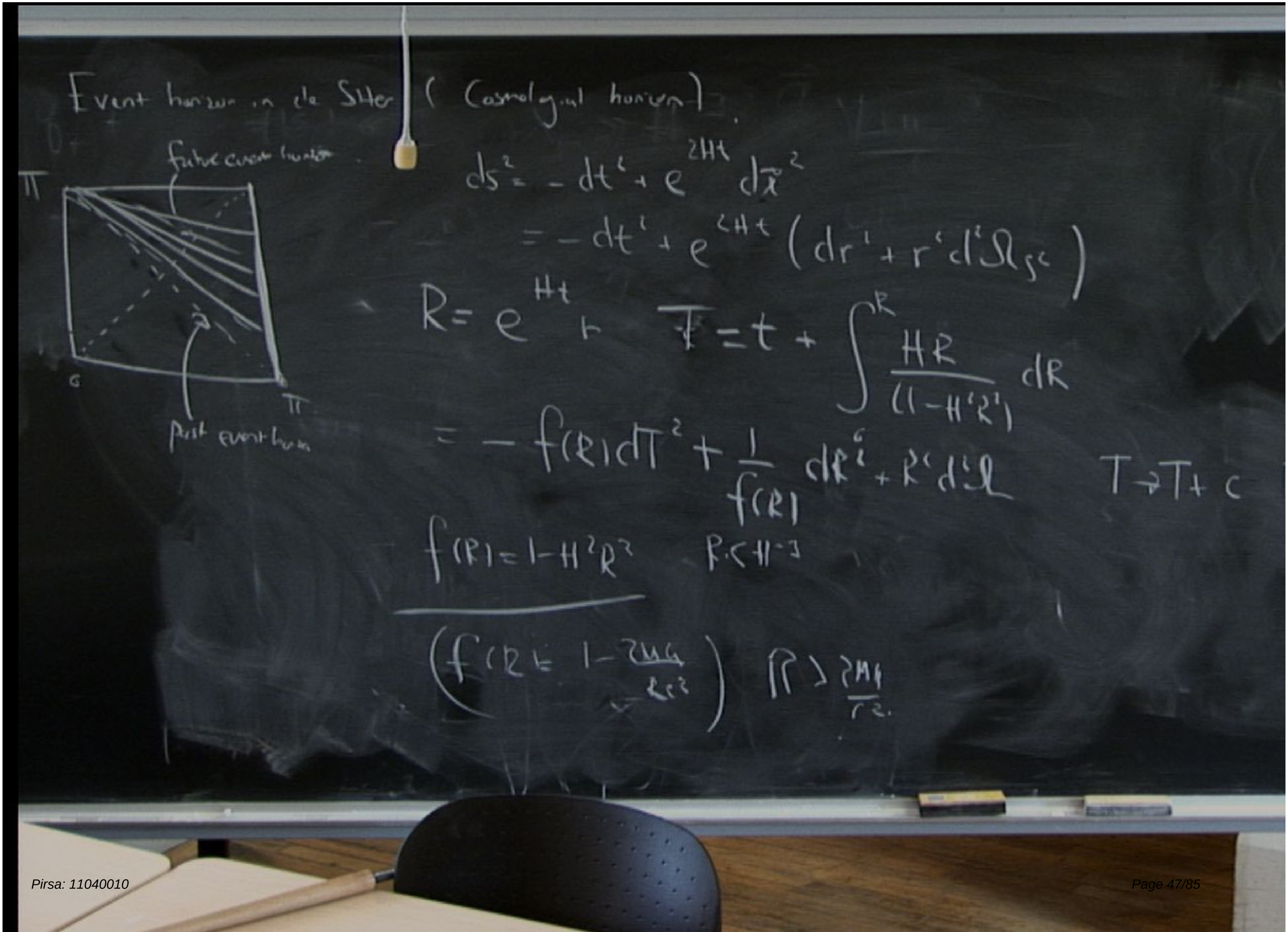


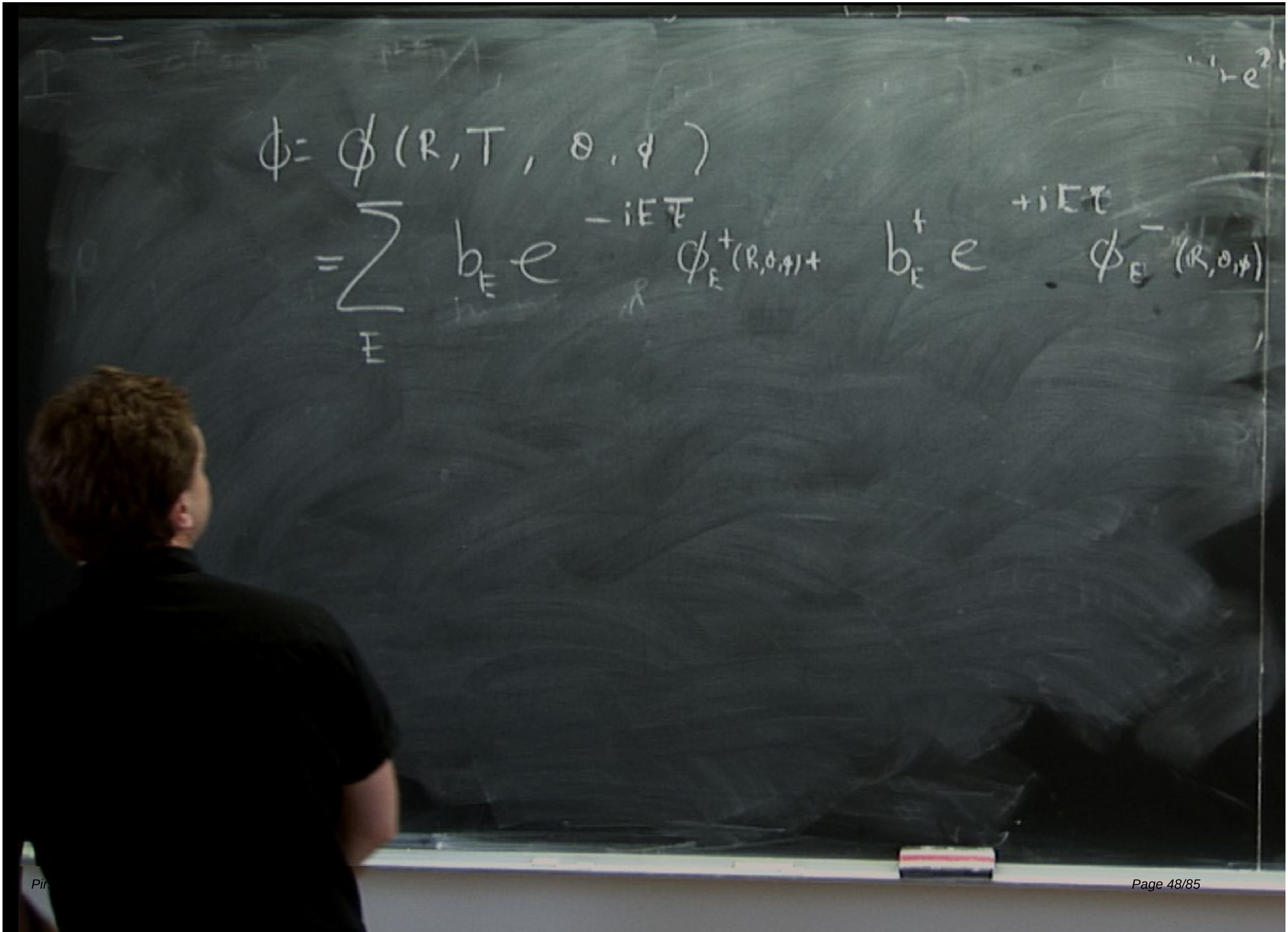
Event horizon in de Sitter (Cosmological horizon)
future event horizon
ds² = − dt² + e^2Ht dx²
= − dt² + e^2Ht (dr² + r² d²Ω)
= − f(R) dT² + 1/f(R) dR² + R² d²Ω
f(R) = 1 − H²R²


Event horizon in de Sitter (Cosmological horizon)
f(R) = 1 − H²R²


Event horizon in de Sitter (Cosmological horizon)
ds² = − dt² + e^{2Ht} dx²
= − dt² + e^{2Ht} (dr² + r² d²Ω)
R = e^{Ht} r
= − f(R) dT² + 1/f(R) dR² + R² d²Ω
f(R) = 1 − H²R²

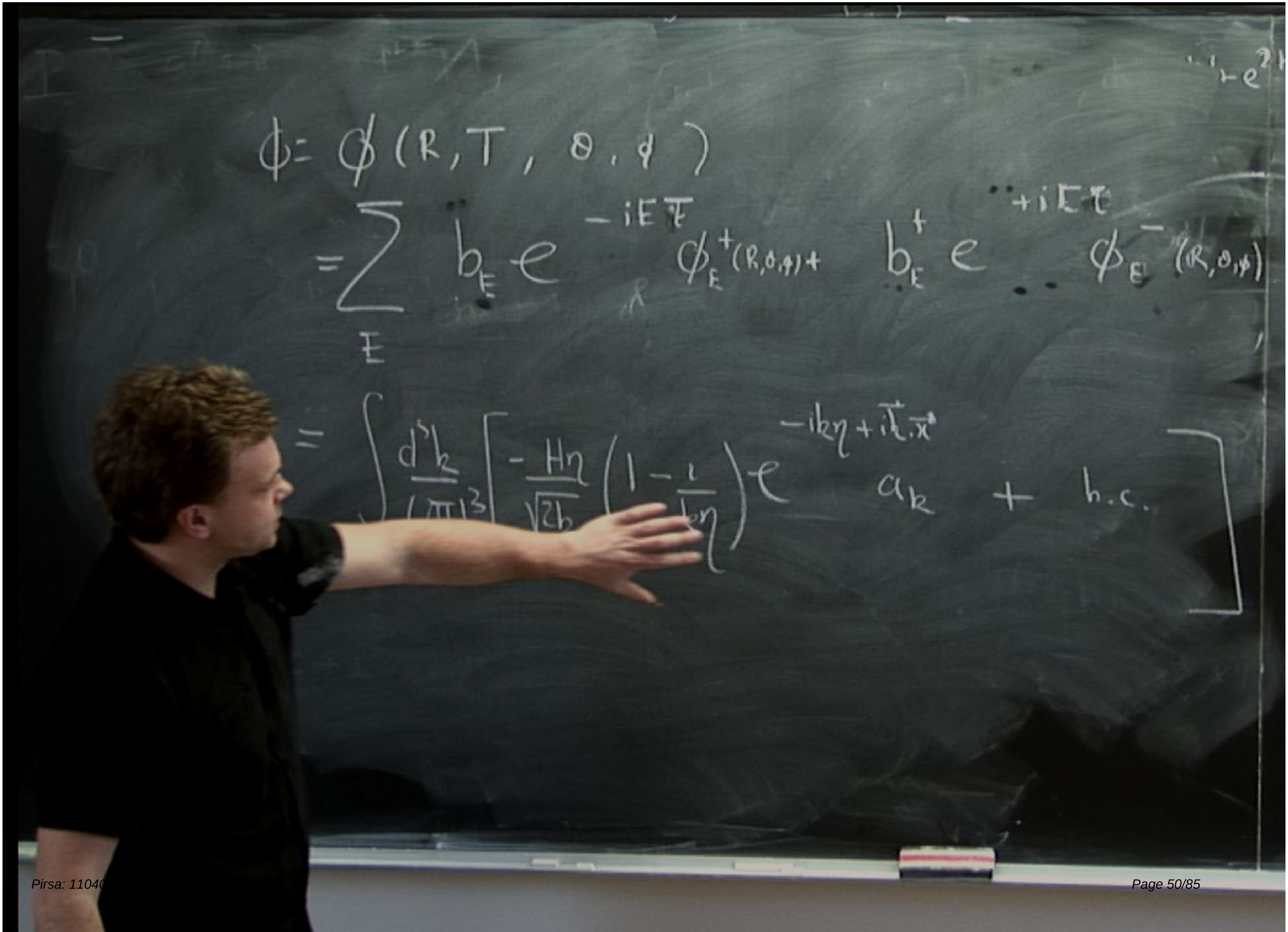

Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
$= -dt^2 + e^{2Ht}(dr^2 + r^2 d^2\Omega_{S^2})$
$R = e^{Ht} r$
$T = t + \int^R \frac{HR}{(1 - H^2R^2)} dR$
$= -f(R) dT^2 + \frac{1}{f(R)} dR^2 + R^2 d^2\Omega$
$f(R) = 1 - H^2R^2$


Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
T → T + c


ϕ = ϕ (R, T, θ, φ)
= Σ_E b_E e^{-iEτ} ϕ_E^+ (R, θ, φ) + b_E^† e^{+iEτ} ϕ_E^- (R, θ, φ)

$$\phi = \phi(R, T, \theta, \phi)$$

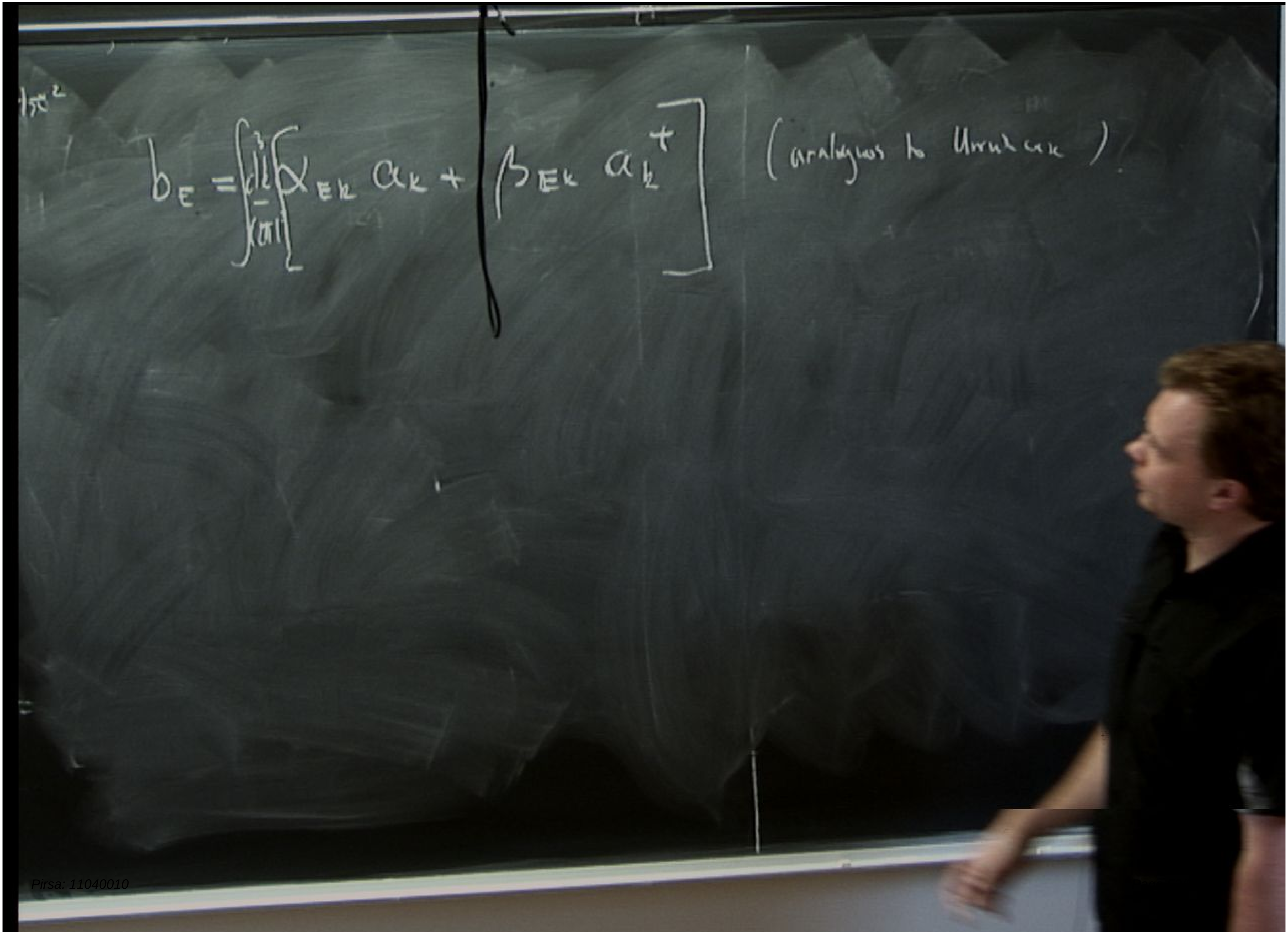
$$= \sum_E b_E e^{-iET} \phi_E^{+}(R,\theta,\phi) + b_E^{\dagger} e^{+iET} \phi_E^{-}(R,\theta,\phi)$$


$\phi = \phi(R, T, \theta, \varphi)$
$= \sum_E b_E e^{-iET} \phi_E^+(R,\theta,\varphi) + b_E^\dagger e^{+iET} \phi_E^-(R,\theta,\varphi)$
$= \int \frac{d^3k}{(2\pi)^3} \left[-\frac{H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta + i\vec{k}\cdot\vec{x}} a_k + h.c. \right]$

$$\phi = \phi(R, T, \theta, \phi)$$

$$= \sum_E b_E \, e^{-iET} \phi_E^{+}(R,\theta,\phi) + b_E^{\dagger} \, e^{+iET} \phi_E^{-}(R,\theta,\phi)$$

$$= \int \frac{d^3k}{(2\pi)^3} \left[\frac{-H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta + i\vec{k}\cdot\vec{x}} a_k + h.c. \right]$$


$b_E = \int \frac{d^3k}{(2\pi)^3} \left[\alpha_{Ek} a_k + \beta_{Ek} a_k^\dagger \right]$
(analogous to Unruh case)

$$b_E = \int \frac{d^3k}{(2\pi)^3}\left[\alpha_{Ek}\, a_k + \beta_{Ek}\, a_k^\dagger\right]$$

(analogous to Unruh case)

$\int |\beta_{Ek}|^2 \frac{d^3k}{(2\pi)^3}$ no. of particles seen by a 'static' observer in de Sitter spacetime.

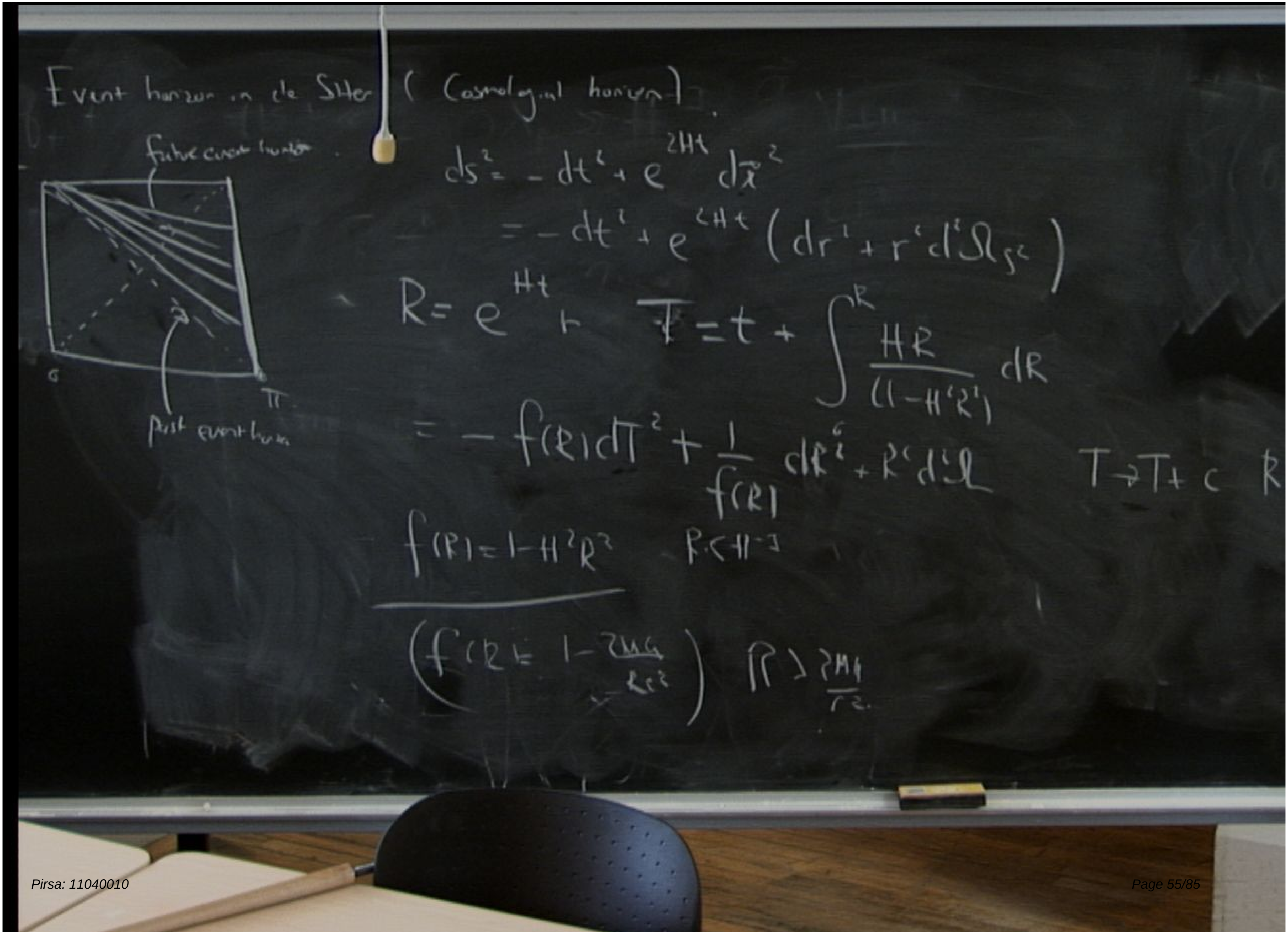
'true defn.'

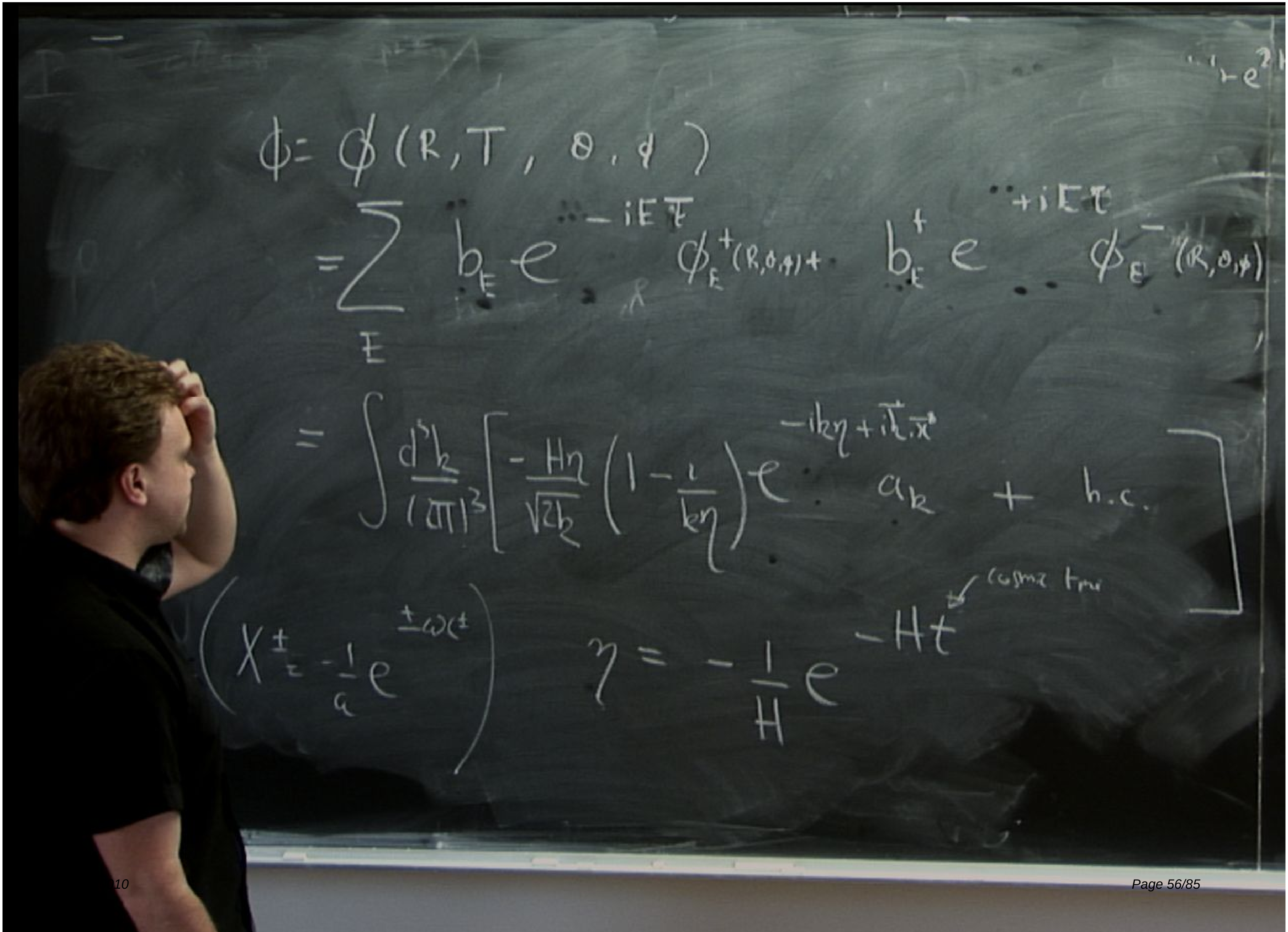
$$b_E = \int \frac{d^3k}{(2\pi)^3} \left[\alpha_{Ek}\, a_k + \beta_{Ek}\, a_k^\dagger \right]$$

(analogous to Unruh case)

observer defn of particle annihilation operator

$\int |\beta_{Ek}|^2 \frac{d^3k}{(2\pi)^3}$ no. of particles seen by a 'static' observer in de Sitter spacetime.


Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$
$= -dt^2 + e^{2Ht} (dr^2 + r^2 d^2\Omega_{S^2})$
$R = e^{Ht} r$
$T = t + \int^R \frac{HR}{(1-H^2R^2)} dR$
$= -f(R) dT^2 + \frac{1}{f(R)} dR^2 + R^2 d^2\Omega$
$T \to T + c$
$f(R) = 1 - H^2R^2$



$\phi = \phi(R, T, \theta, \phi)$
$= \sum_E b_E e^{-iET} \phi_E^+(R,\theta,\phi) + b_E^\dagger e^{+iET} \phi_E^-(R,\theta,\phi)$
$= \int \frac{d^3k}{(2\pi)^3} \left[-\frac{H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta + i\vec{k}\cdot\vec{x}} a_k + h.c. \right]$
$\eta = -\frac{1}{H} e^{-Ht}$
cosmic time

$d\vec{x}^2$

'true defn.'

$$b_E = \int \frac{d^3k}{(2\pi)^3} \left[\alpha_{Ek}\, a_k + \beta_{Ek}\, a_k^\dagger \right]$$

(analogous to Unruh case)

observer defn of particle annihilation opers.

$\int |\beta_{Ek}|^2 \frac{d^3k}{(2\pi)^3}$ no. of particles seen by a 'static' observer in de Sitter spacetime.

$$\vec{x}=0 \equiv R=0$$

dx^2

'true defn.'

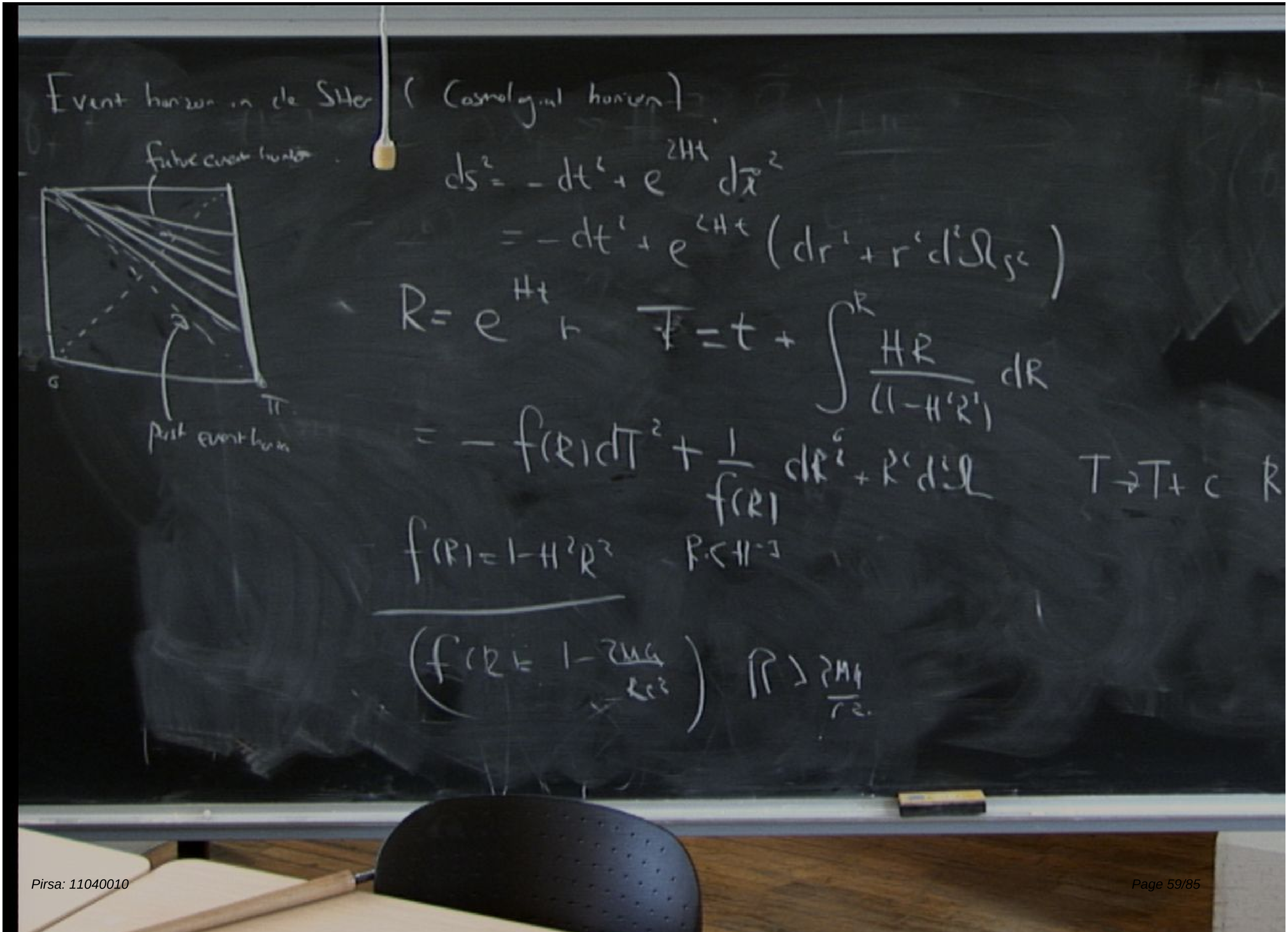
$$b_E = \int \frac{d^3k}{(2\pi)^3} \left[\alpha_{Ek}\, a_k + \beta_{Ek}\, a_k^\dagger \right]$$

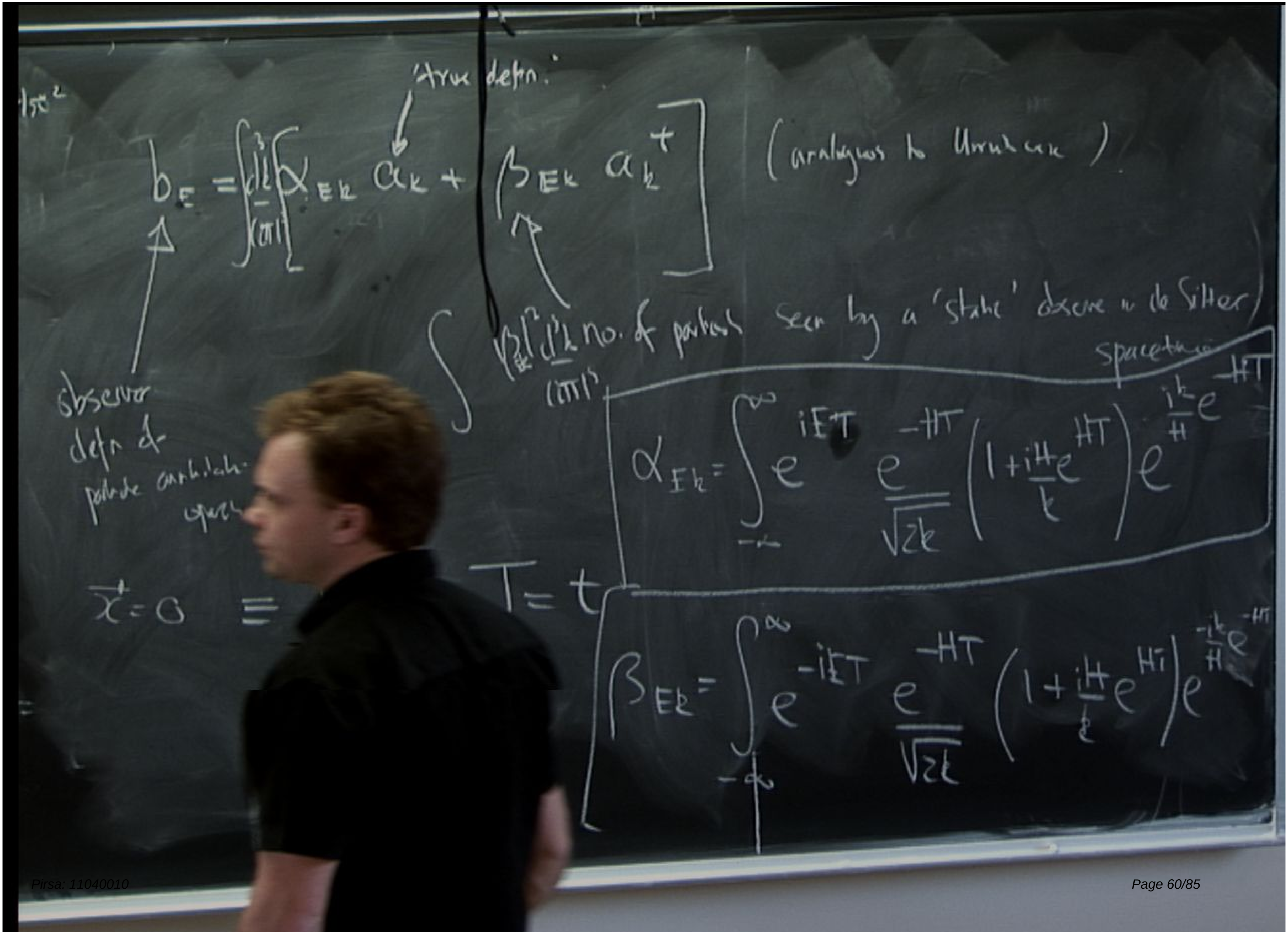
(analogous to Unruh case)

observer defn of particle annihilation operator.

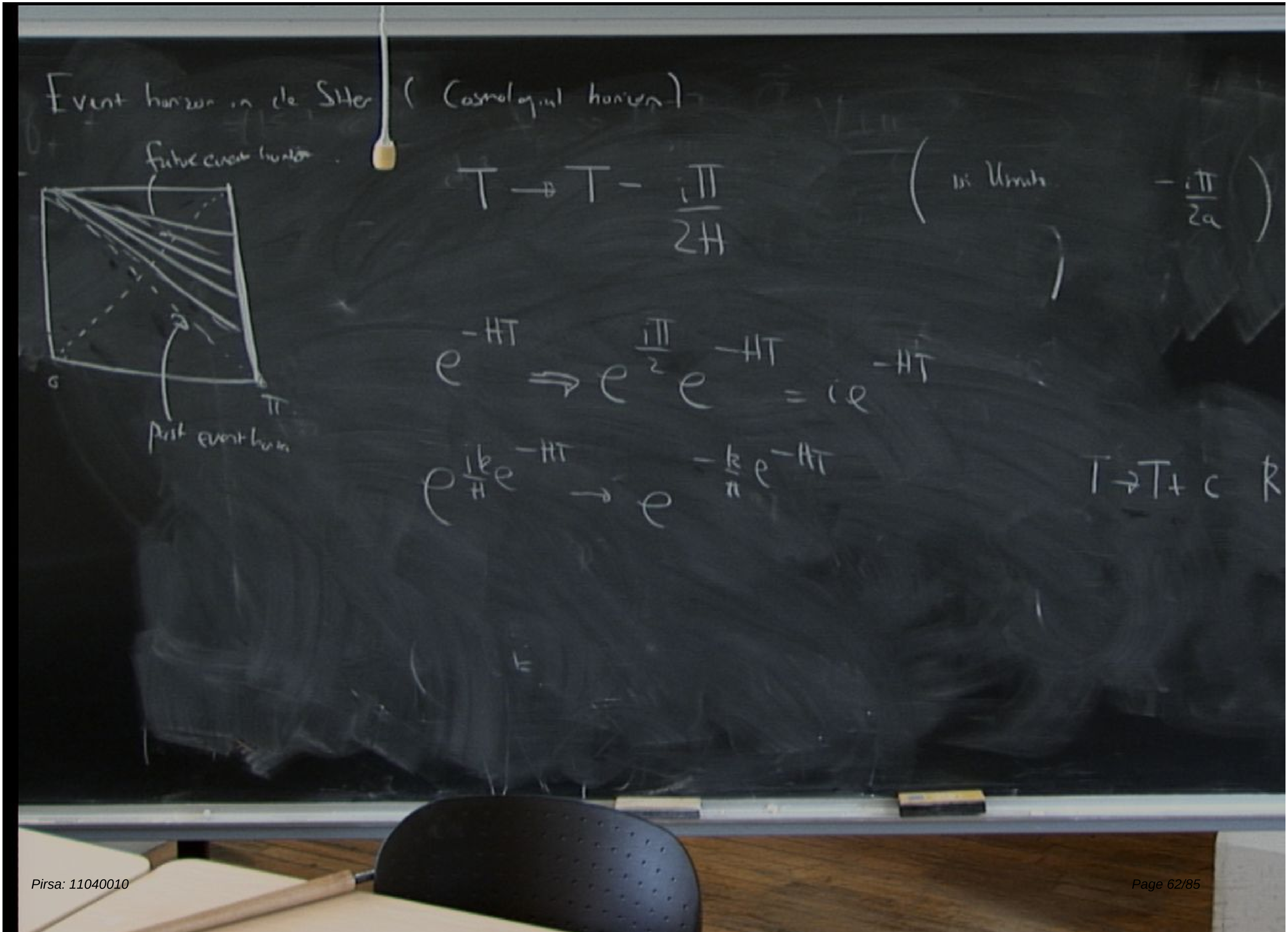
$$\int |\beta_{Ek}|^2 \frac{d^3k}{(2\pi)^3}$$ no. of particles seen by a 'static' observer in de Sitter spacetime.

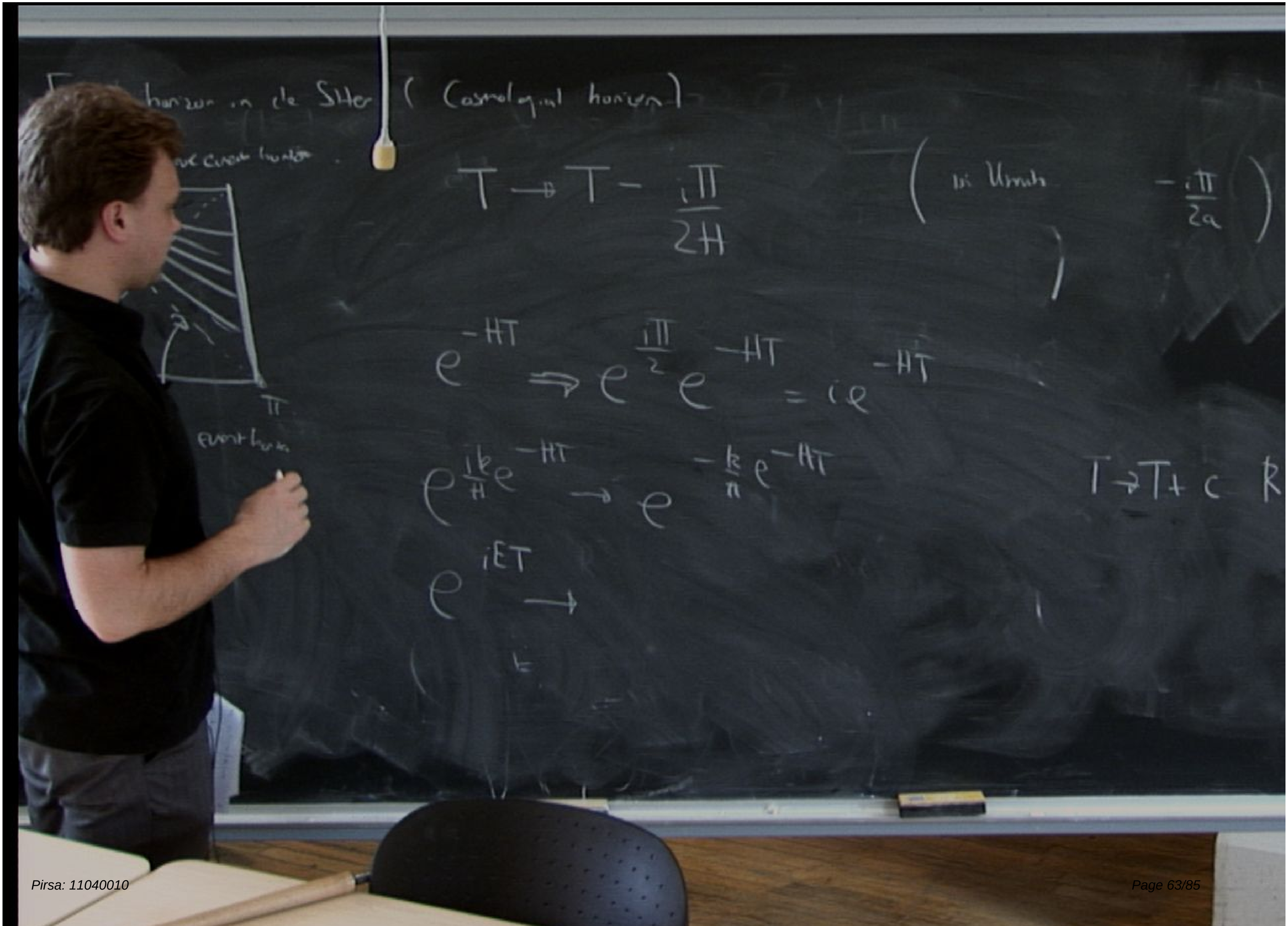
$$\vec{x}=0 \equiv R=0 \quad T=t$$

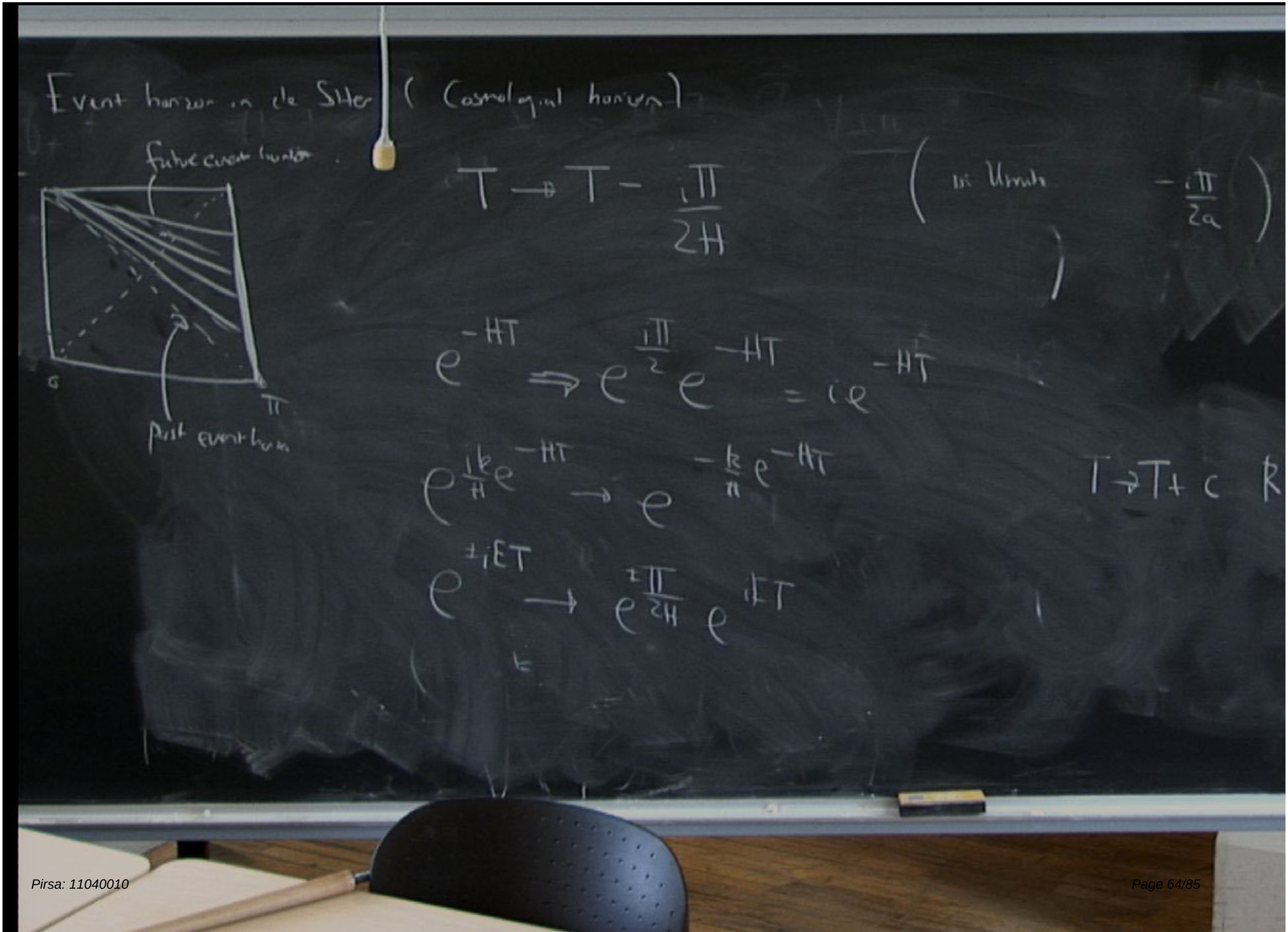

Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
ds² = -dt² + e^2Ht dx²
= -dt² + e^2Ht (dr² + r² d²Ω_S²)
R = e^Ht r   T = t + ∫^R HR/(1-H²R²) dR
= -f(R) dT² + 1/f(R) dR² + R² d²Ω   T → T + c
f(R) = 1 - H²R²   R < H⁻¹

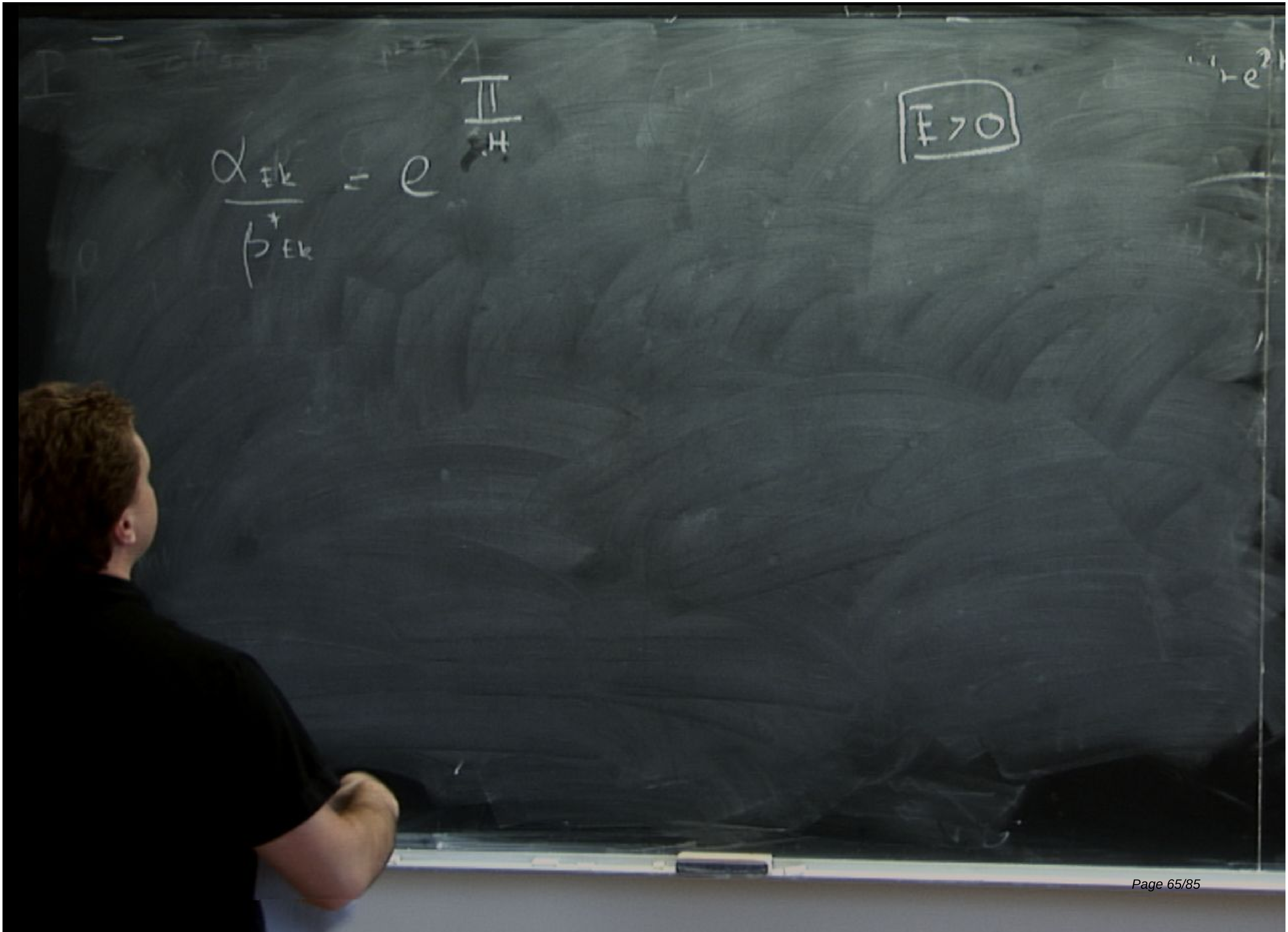

'true' defn:
(analogous to Unruh case)
observer
T = t

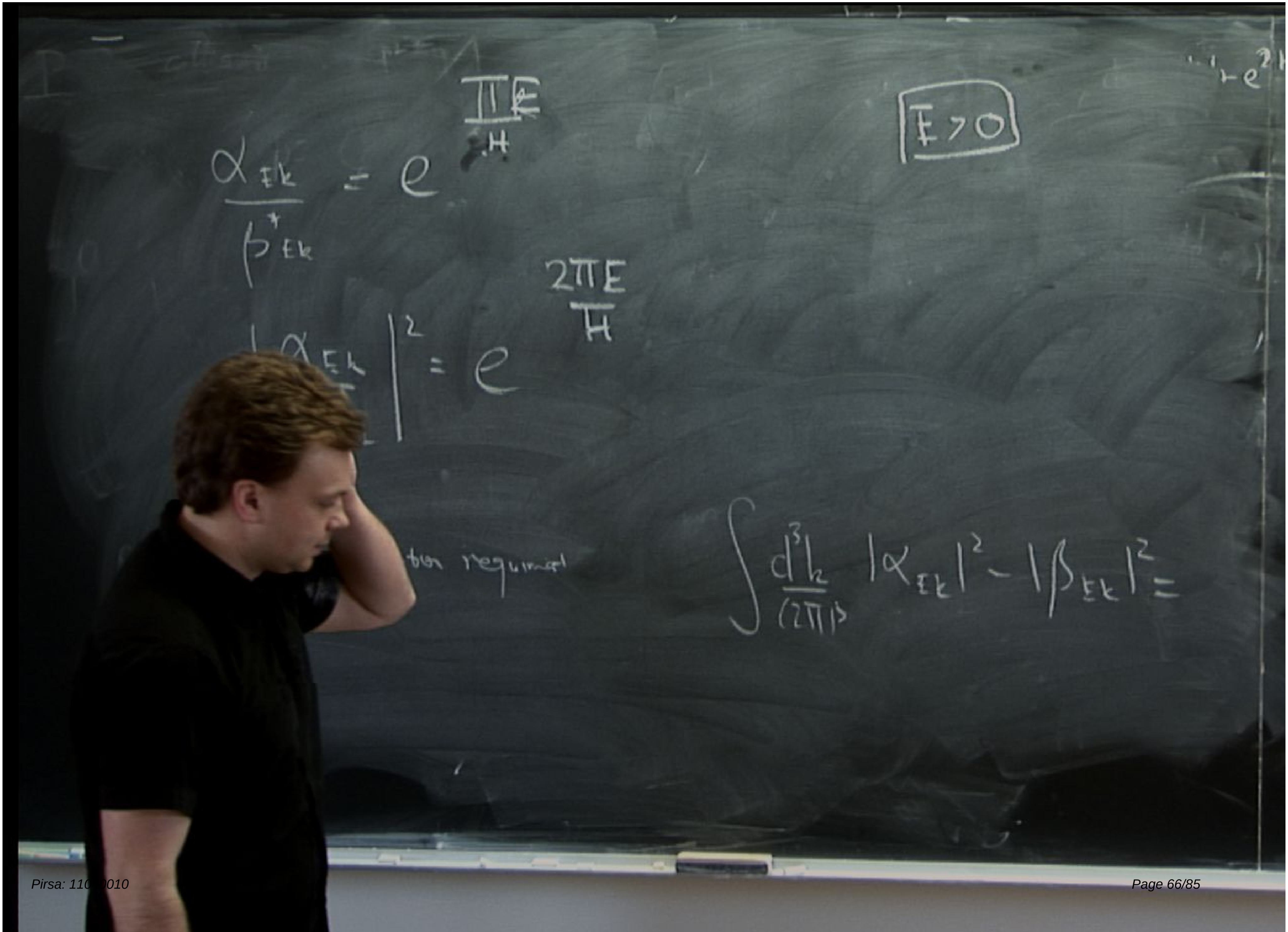
Event horizon
(Cosmological horizon)
$T \to T - \frac{i\pi}{2H}$
$T \to T + c$


Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
0
π
T → T − iπ/2H
(in Unruh − iπ/2a)
e^{−HT} ⇒ e^{iπ/2} e^{−HT} = i e^{−HT}
T → T + c


horizon in de Sitter (Cosmological horizon)
T → T − iπ/2H
in Unruh
− iπ/2a
e^{−HT} ⇒ e^{iπ/2} e^{−HT} = i e^{−HT}
event horizon
e^{(ik/H) e^{−HT}} → e^{−(k/H) e^{−HT}}
T → T + c
e^{iET} →


Event horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
π
T → T − iπ/2H
− iπ/2a
e^{−HT} ⇒ e^{iπ/2} e^{−HT} = i e^{−HT}
e^{ik/H e^{−HT}} → e^{−k/H e^{−HT}}
T → T + c
e^{±iET} → e^{±π/2H} e^{iET}


$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi}{H}}$
E > 0


$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$
$\boxed{E > 0}$
$|\alpha_{Ek}|^2 = e^{\frac{2\pi E}{H}}$
$\int \frac{d^3k}{(2\pi)^3} |\alpha_{Ek}|^2 - |\beta_{Ek}|^2 =$
tion required

$E > 0$

$$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$$

$$\left|\frac{\alpha_{Ek}}{\beta_{Ek}}\right|^2 = e^{\frac{2\pi E}{H}}$$

Normalization requirement

$$\int \frac{d^3k}{(2\pi)^3} |\alpha_{Ek}|^2 - |\beta_{Ek}|^2 =$$

$$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$$

$$\left|\frac{\alpha_{Ek}}{\beta_{Ek}}\right|^2 = e^{\frac{2\pi E}{H}}$$

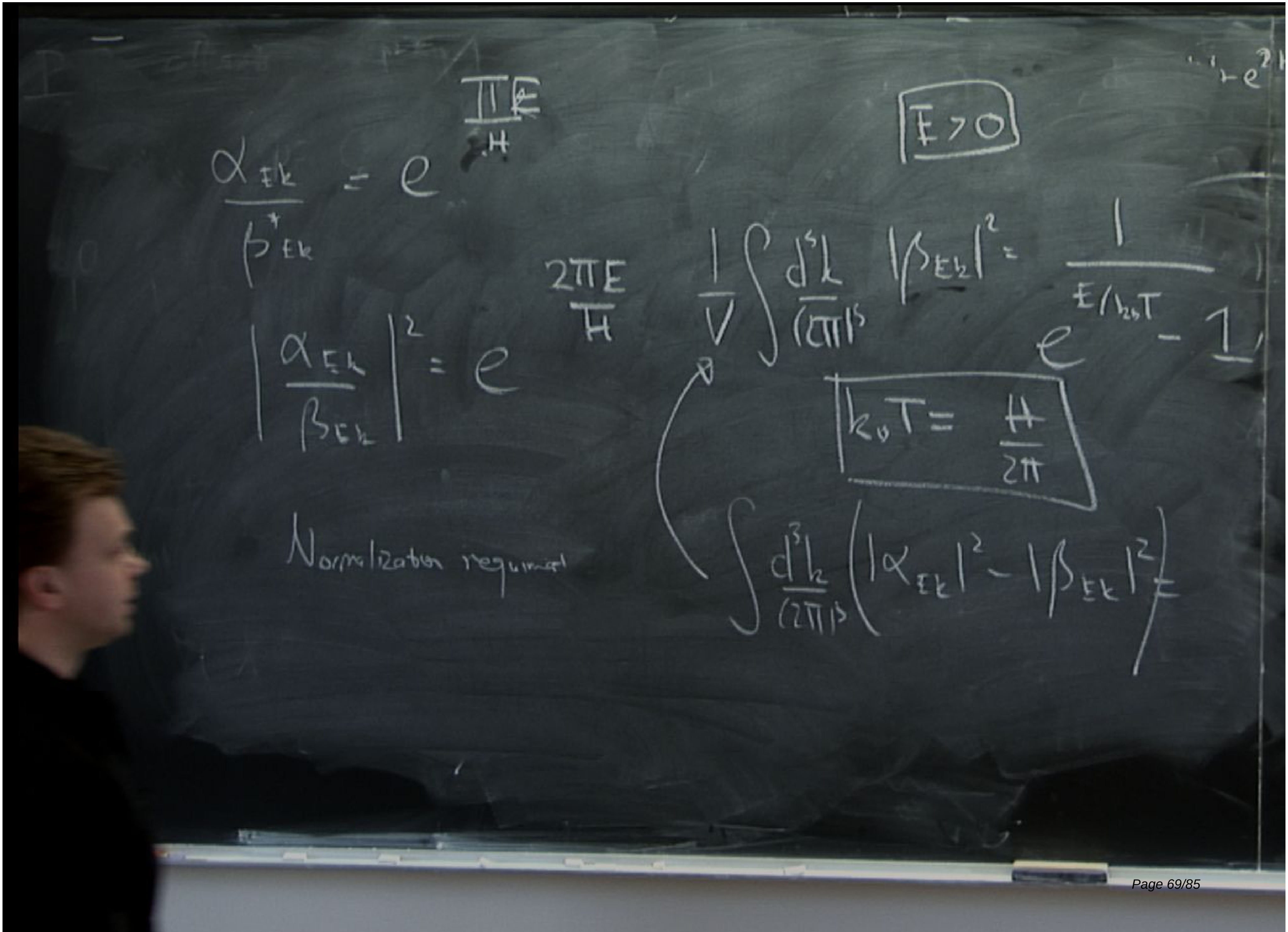
Normalization required

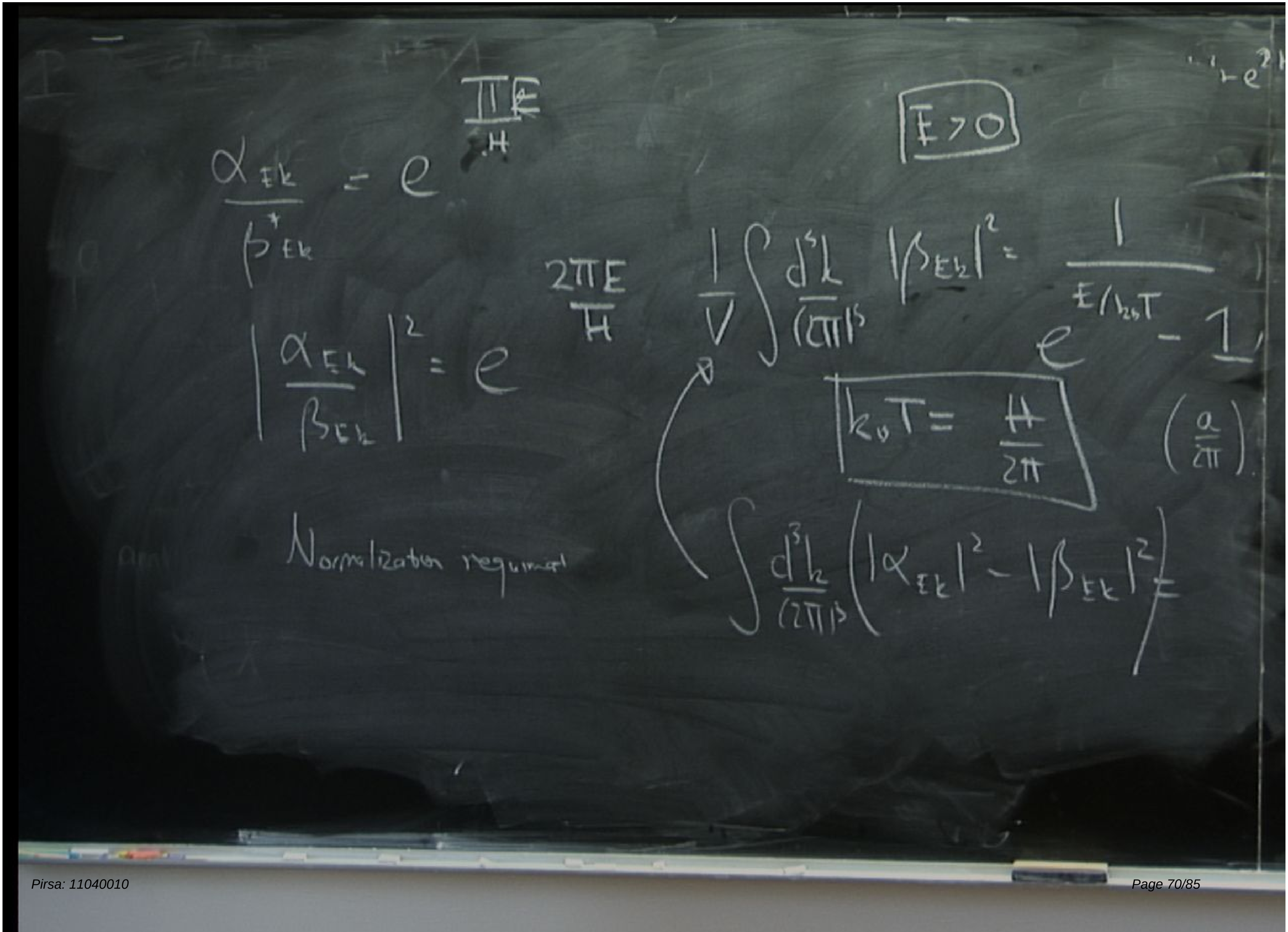
$$\boxed{E > 0}$$

$$\int \frac{d^3k}{(2\pi)^3} |\beta_{Ek}|^2 = \frac{1}{e^{E/k_B T} - 1}$$

$$\boxed{k_B T = \frac{H}{2\pi}}$$

$$\int \frac{d^3k}{(2\pi)^3} \left(|\alpha_{Ek}|^2 - |\beta_{Ek}|^2 \right) =$$


$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$
$\left|\frac{\alpha_{Ek}}{\beta_{Ek}}\right|^2 = e^{\frac{2\pi E}{H}}$
Normalization required
$E > 0$
$\frac{1}{V}\int \frac{d^3k}{(2\pi)^3} |\beta_{Ek}|^2 = \frac{1}{e^{E/k_bT} - 1}$
$k_bT = \frac{H}{2\pi}$
$\int \frac{d^3k}{(2\pi)^3}\left(|\alpha_{Ek}|^2 - |\beta_{Ek}|^2\right) =$


α_Ek / β*_Ek = e^(πE/H)
E > 0
|α_Ek / β_Ek|^2 = e^(2πE/H)
1/V ∫ d^3k/(2π)^3 |β_Ek|^2 = 1/(e^(E/k_B T) − 1)
k_B T = H/2π
(a/2π)
Normalization requirement
∫ d^3k/(2π)^3 (|α_Ek|^2 − |β_Ek|^2) =

$$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$$

$E > 0$

$$\left|\frac{\alpha_{Ek}}{\beta_{Ek}}\right|^2 = e^{\frac{2\pi E}{H}}$$

$$\frac{1}{V}\int \frac{d^3k}{(2\pi)^3} |\beta_{Ek}|^2 = \frac{1}{e^{E/k_B T} - 1}$$

$$k_B T = \frac{H}{2\pi} \quad \left(\frac{a}{2\pi}\right)$$

Normalization required

$$\int \frac{d^3k}{(2\pi)^3}\left(|\alpha_{Ek}|^2 - |\beta_{Ek}|^2\right) =$$

$$\mathrm{Tr}\left(|BD\rangle\langle BD|\right) = Z^{-1} e^{-\frac{H_{\text{static patch}}}{k_B T}}$$

outside horizon

$R > H^{-1}$

$$\frac{\alpha_{Ek}}{\beta^*_{Ek}} = e^{\frac{\pi E}{H}}$$

$$\boxed{E > 0}$$

$$\left|\frac{\alpha_{Ek}}{\beta_{Ek}}\right|^2 = e^{\frac{2\pi E}{H}}$$

$$\frac{1}{V}\int \frac{d^3k}{(2\pi)^3} |\beta_{Ek}|^2 = \frac{1}{e^{E/k_B T} - 1}$$

$$\boxed{k_B T = \frac{H}{2\pi}} \qquad \left(\frac{a}{2\pi}\right)$$

Normalization required

$$\int \frac{d^3k}{(2\pi)^3}\left(|\alpha_{Ek}|^2 - |\beta_{Ek}|^2\right) =$$

$$\mathrm{Tr}_{\substack{\text{outside}\\ \text{horizon}\\ R > H^{-1}}}\left(|BD\rangle\langle BD|\right) = Z^{-1} e^{-\frac{H_{\text{static patch}}}{k_B T}}$$

'true defn.'

$$b_E = \int \frac{d^3k}{(2\pi)^3} \left[\alpha_{Ek}\, a_k + \beta_{Ek}\, a_k^{\dagger} \right]$$

(analogous to Unruh case)

$\int |\beta_{Ek}|^2 \frac{d^3k}{(2\pi)^3}$ no. of particles seen by a 'static' observer in de Sitter spacetime

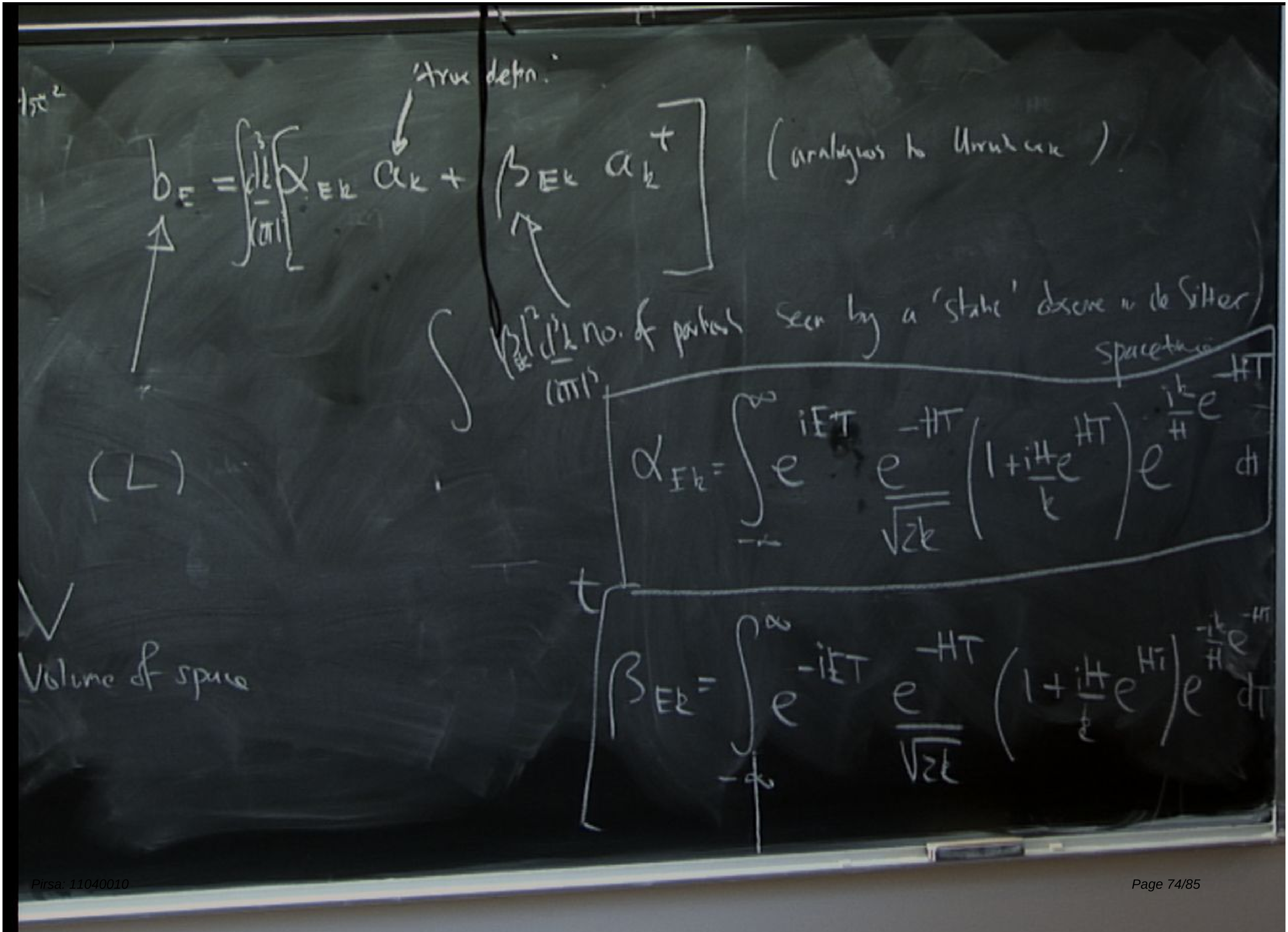
(L)

V

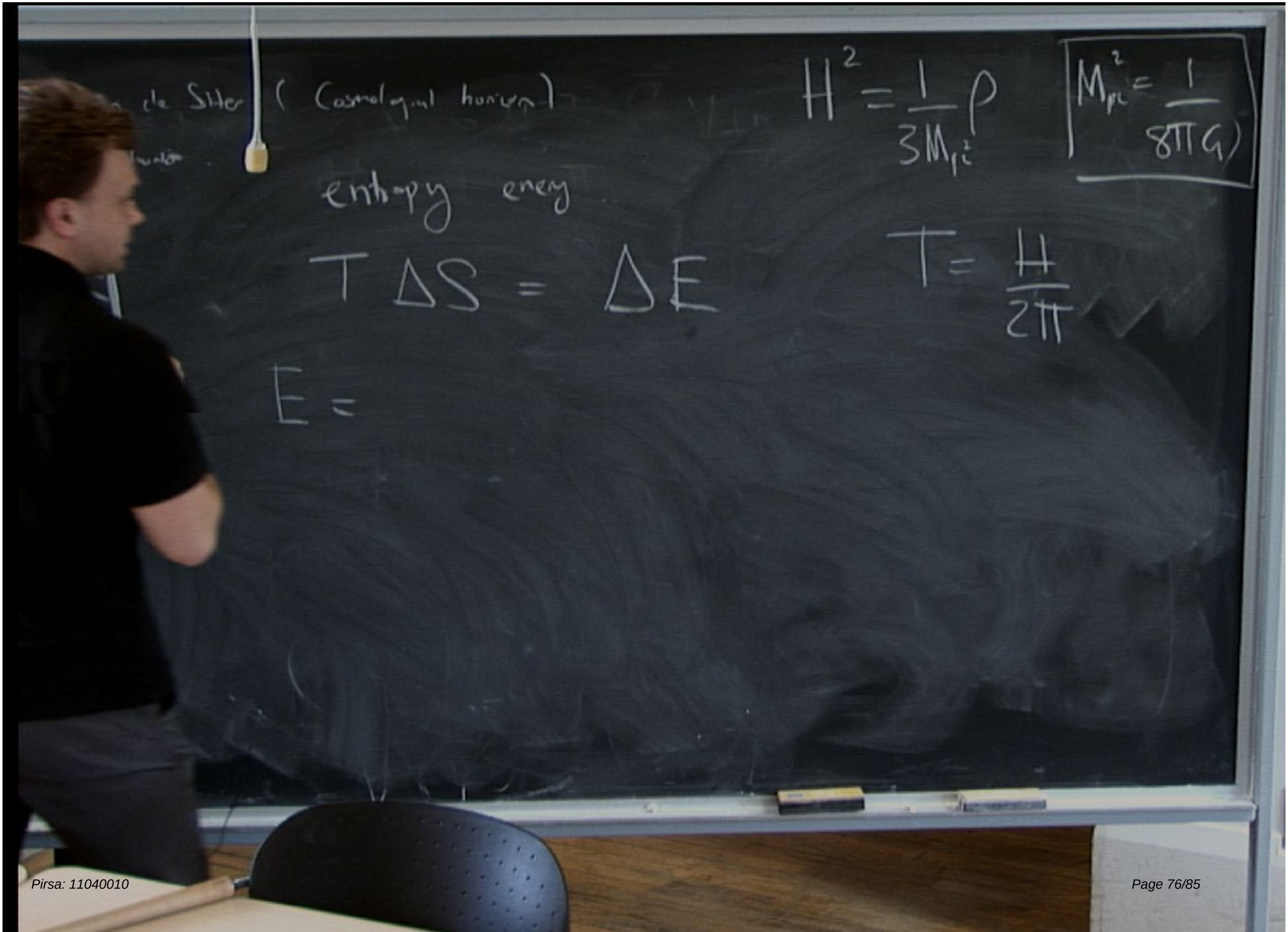
Volume of space

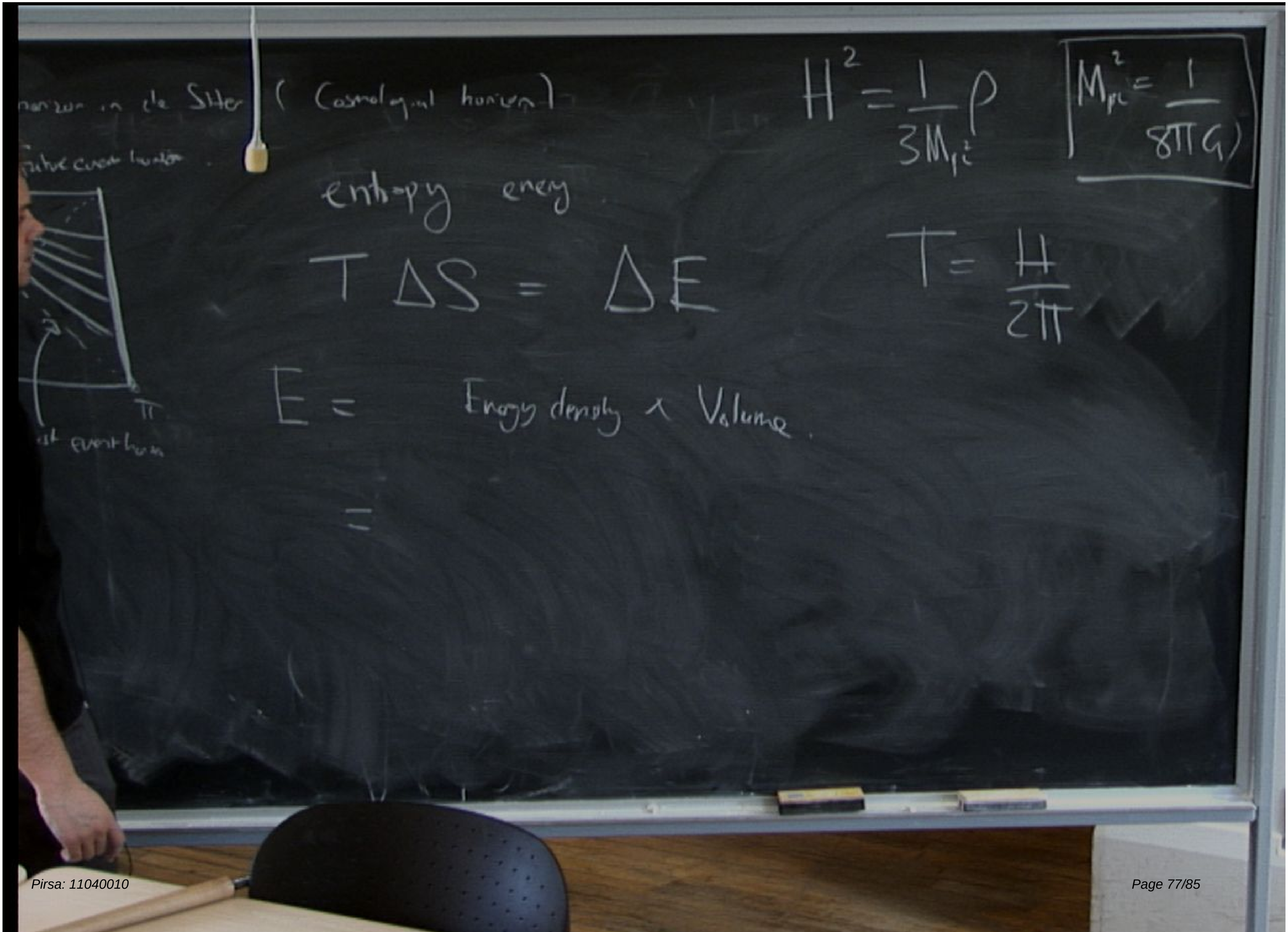
$$\alpha_{Ek} = \int_{-\infty}^{\infty} e^{iET} \frac{e^{-HT}}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{HT}\right) e^{\frac{ik}{H} e^{-HT}}\, dt$$

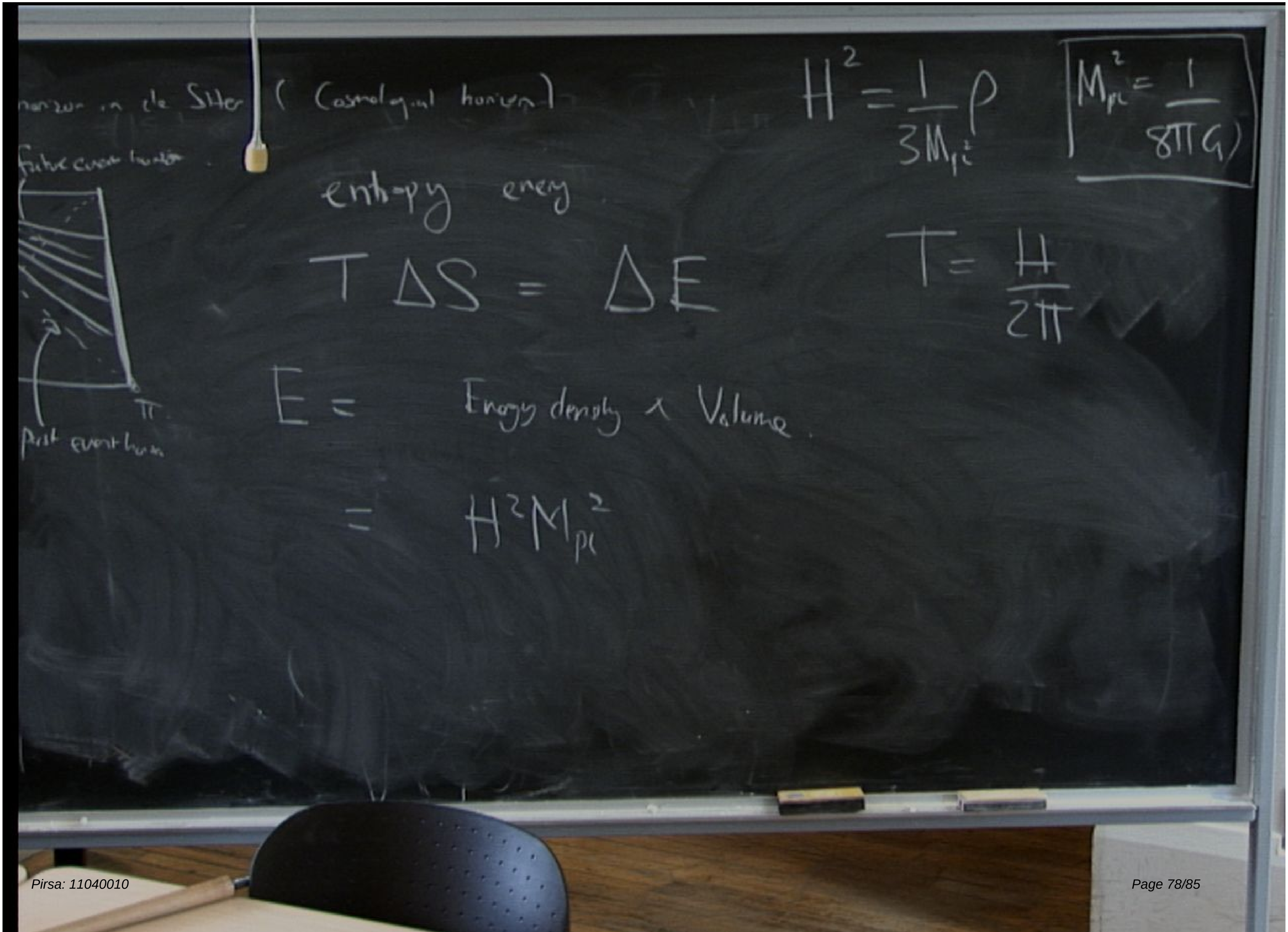
$$\beta_{Ek} = \int_{-\infty}^{\infty} e^{-iET} \frac{e^{-HT}}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{HT}\right) e^{-\frac{ik}{H} e^{-HT}}\, dt$$

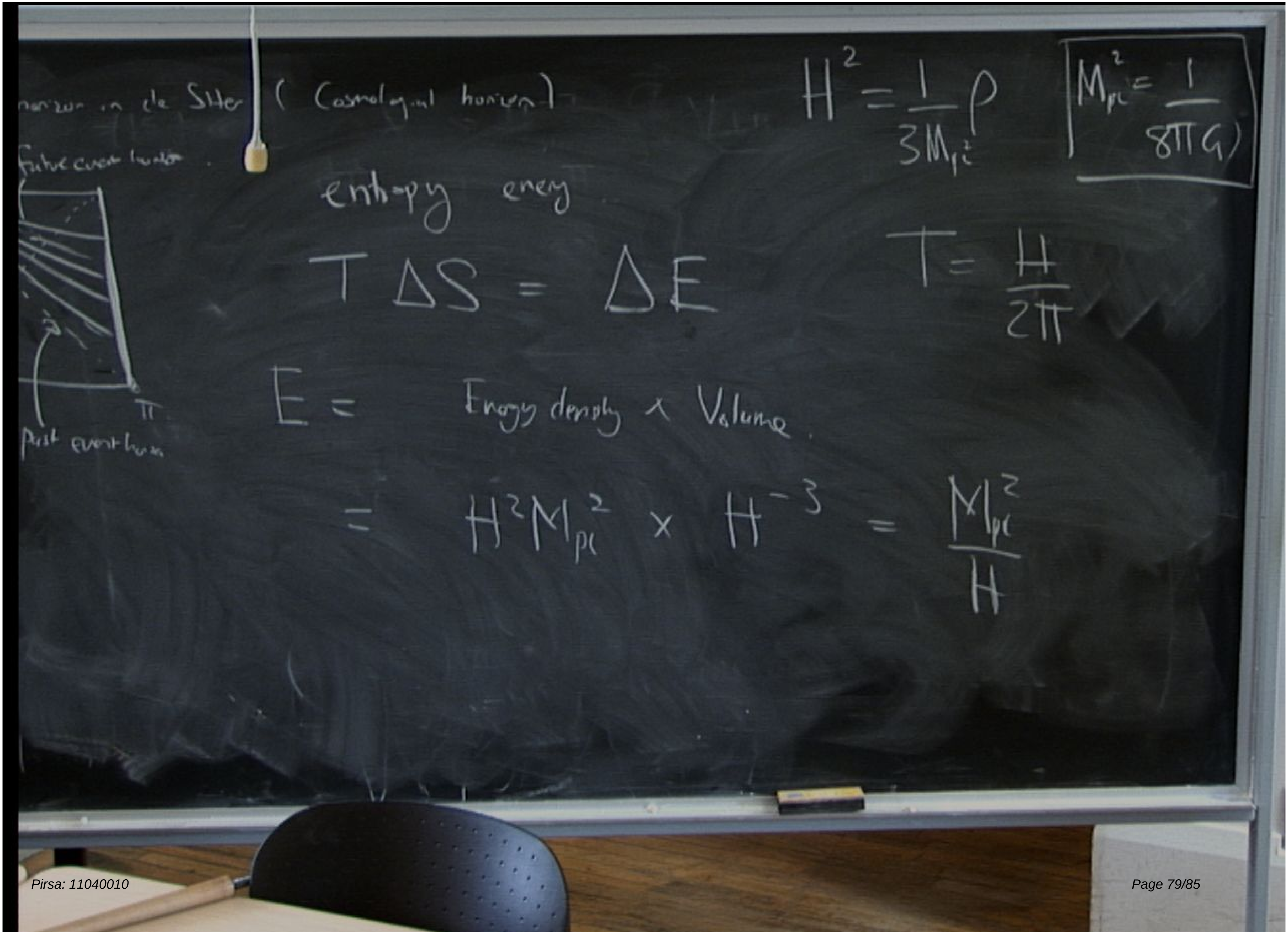

'true defn.'
b_E = ∫ d³k/(2π)³ [α_Ek a_k + β_Ek a_k†]
(analogous to Unruh case)
∫ |β_Ek|² d³k/(2π)³ no. of particles seen by a 'static' observer in de Sitter spacetime
(L)
α_Ek = ∫_{-∞}^{∞} e^{iET} e^{-HT}/√2k (1 + iH/k e^{HT}) e^{ik/H e^{-HT}} dT
β_Ek = ∫_{-∞}^{∞} e^{-iET} e^{-HT}/√2k (1 + iH/k e^{HT}) e^{-ik/H e^{-HT}} dT
Volume of space

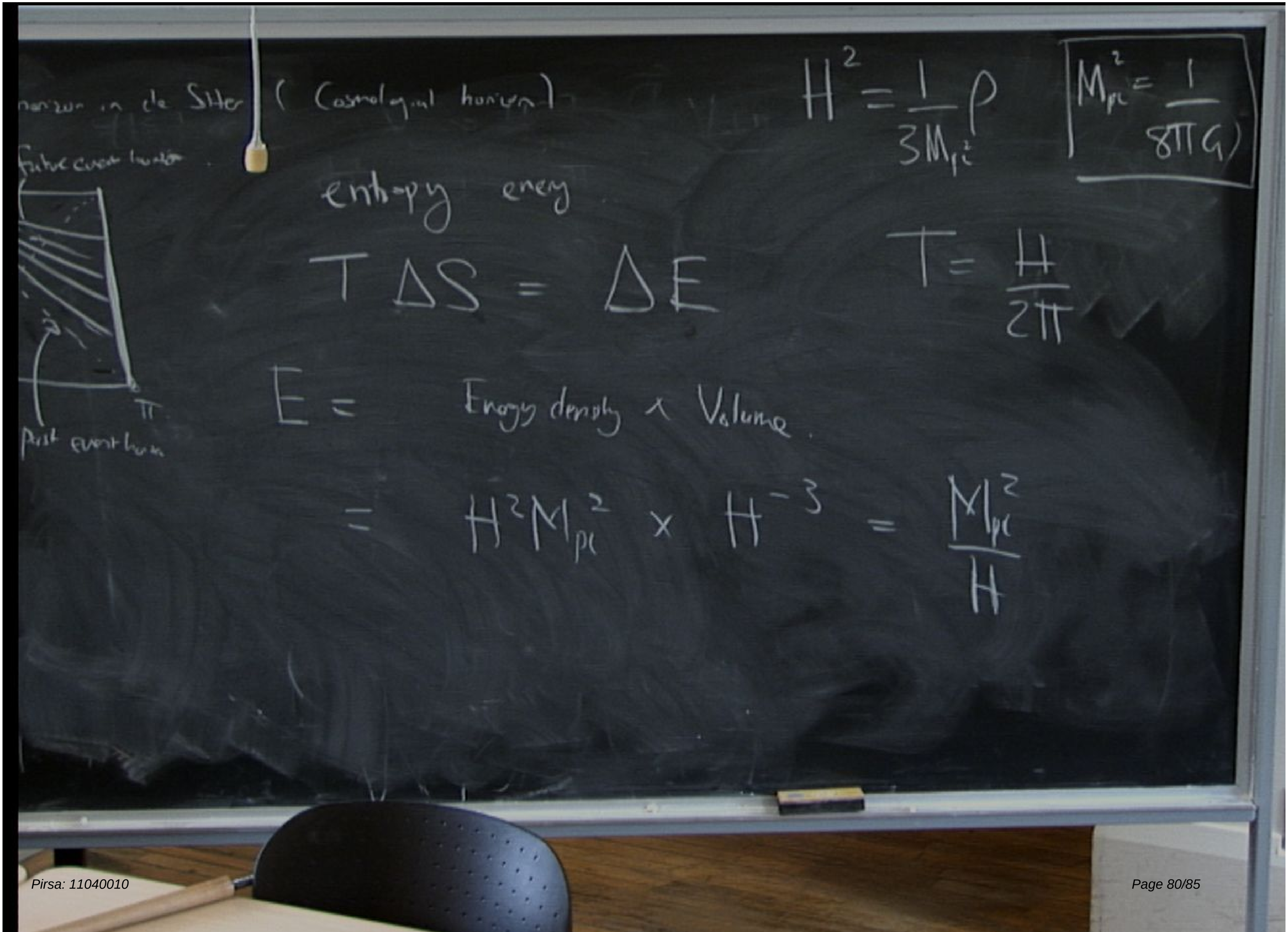

Event horizon in de Sitter (Cosmological horizon)
entropy energy
T ΔS = ΔE
T = H/2π

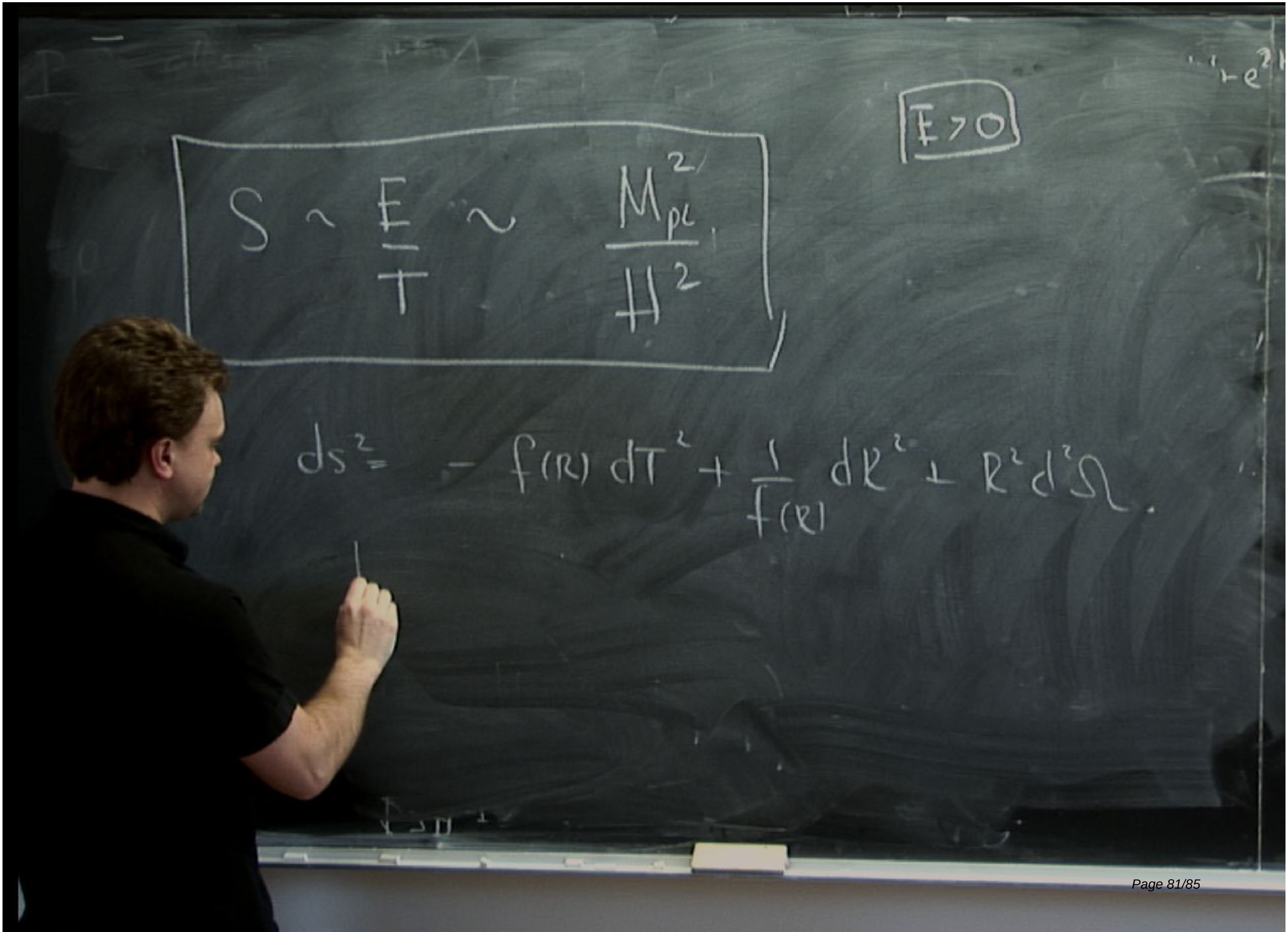

de Sitter (Cosmological horizon)
entropy energy
T ΔS = ΔE
E =
H² = 1/3M_pl² ρ
M_pl² = 1/8πG)
T = H/2π

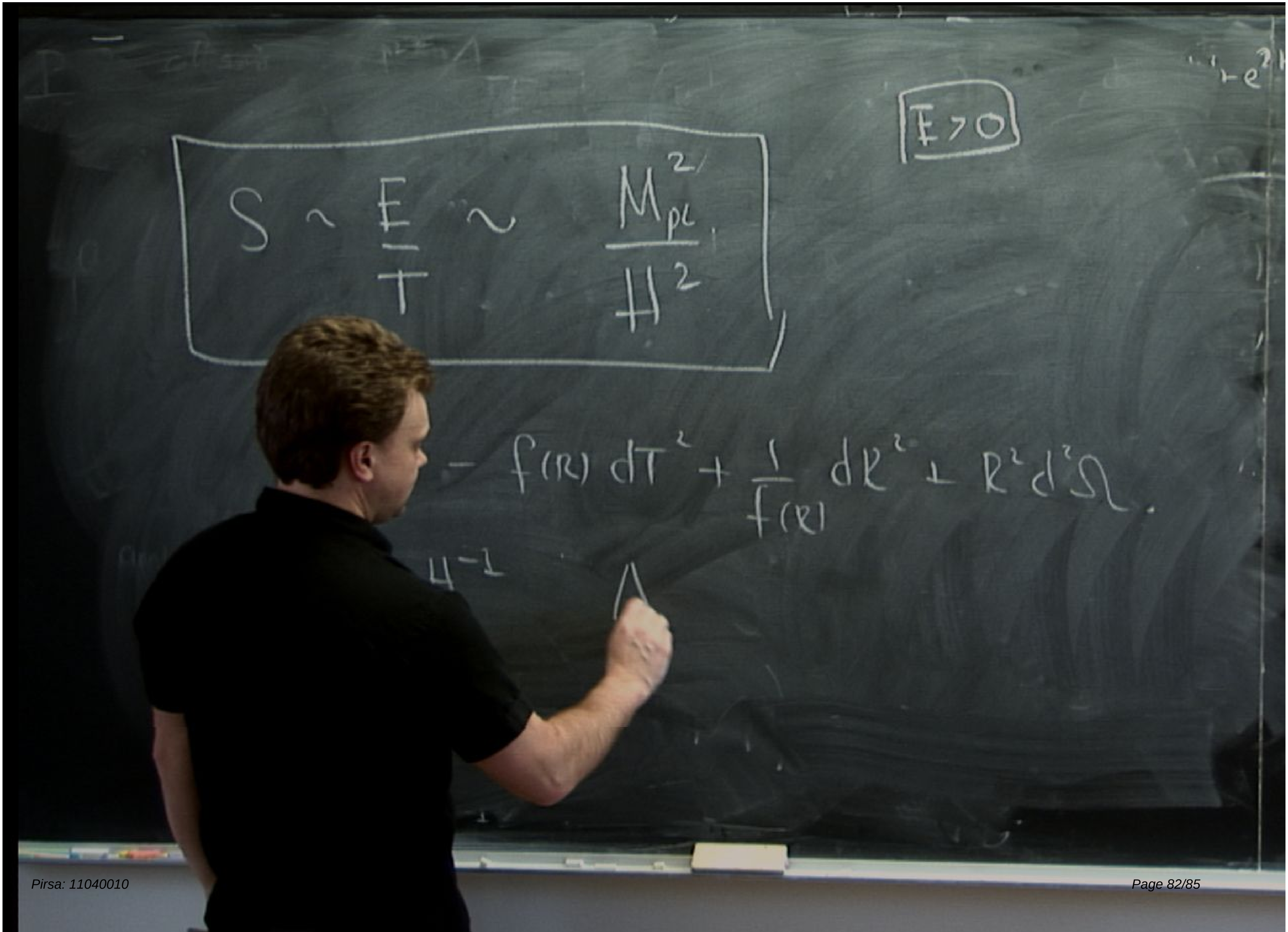

(Cosmological horizon)
$H^2 = \frac{1}{3M_{pl}^2}\rho$
$M_{pl}^2 = \frac{1}{8\pi G}$
entropy energy
$T\Delta S = \Delta E$
$T = \frac{H}{2\pi}$
E = Energy density x Volume


(Cosmological horizon)
$H^2 = \frac{1}{3M_{pl}^2}\rho$
$M_{pl}^2 = \frac{1}{8\pi G}$
future event horizon
entropy energy
$T\Delta S = \Delta E$
$T = \frac{H}{2\pi}$
$E =$ Energy density x Volume
$= H^2 M_{pl}^2$
past event horizon


horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
entropy energy
$T \Delta S = \Delta E$
$H^2 = \frac{1}{3M_{pl}^2}\rho$
$M_{pl}^2 = \frac{1}{8\pi G}$
$T = \frac{H}{2\pi}$
$E =$ Energy density × Volume
$= H^2 M_{pl}^2 \times H^{-3} = \frac{M_{pl}^2}{H}$


horizon in de Sitter (Cosmological horizon)
future event horizon
past event horizon
π
$H^2 = \frac{1}{3M_{pl}^2}\rho$
$M_{pl}^2 = \frac{1}{8\pi G}$
entropy energy
$T\Delta S = \Delta E$
$T = \frac{H}{2\pi}$
E = Energy density x Volume
$= H^2 M_{pl}^2 \times H^{-3} = \frac{M_{pl}^2}{H}$


E > 0
S ~ E/T ~ M_pl^2/H^2
ds^2 = - f(R) dT^2 + 1/f(R) dR^2 + R^2 d^2Ω


S ~ E/T ~ M_pl^2/H^2
E > 0
− f(R) dT^2 + 1/f(R) dR^2 + R^2 d^2Ω
H^-1
Λ

$$\boxed{\bar{E} > 0}$$

$$\boxed{S \sim \frac{E}{T} \sim \frac{M_{pl}^2}{H^2}}$$

$$ds^2 = -f(R)\, dT^2 + \frac{1}{f(R)}\, dR^2 + R^2 d^2\Omega$$

$$R = H^{-1}$$

$$\boxed{A_H = 4\pi H^{-2}}$$

'true data'

$$S = \frac{1}{4G} A_{\text{horizon}} = \frac{\pi}{G H^{-2}}$$

$$M_{pl}^2 = \frac{1}{8\pi G}$$

$$= \frac{8\pi^2 M_{pl}^2}{H^2}$$

... in de Sitter spacetime

$$\frac{e}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{H\tau}\right) e^{\frac{ik}{H} e^{-H\tau}} d\tau$$

$$S_{Ek} = \int_{-\infty}^{\infty} e^{-iE\tau} \frac{e^{-H\tau}}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{H\tau}\right) e^{\frac{-ik}{H} e^{-H\tau}} d\tau$$

'true data:'

$$S = \frac{1}{4G} A_{\text{horizon}} = \frac{\pi}{G H^{-2}}$$

$$= \frac{8\pi^2 M_{pl}^2}{H^2}$$

$$M_{pl}^2 = \frac{1}{8\pi G}$$

[...] in de Sitter spacetime

$$\frac{[\ldots]}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{H\tau}\right) e^{\frac{ik}{H} e^{-H\tau}} d\tau$$

$$S_{Ek} = \int_{-\infty}^{\infty} e^{-iE\tau} \frac{e^{-H\tau}}{\sqrt{2k}} \left(1 + \frac{iH}{k} e^{H\tau}\right) e^{-\frac{ik}{H} e^{-H\tau}} d\tau$$