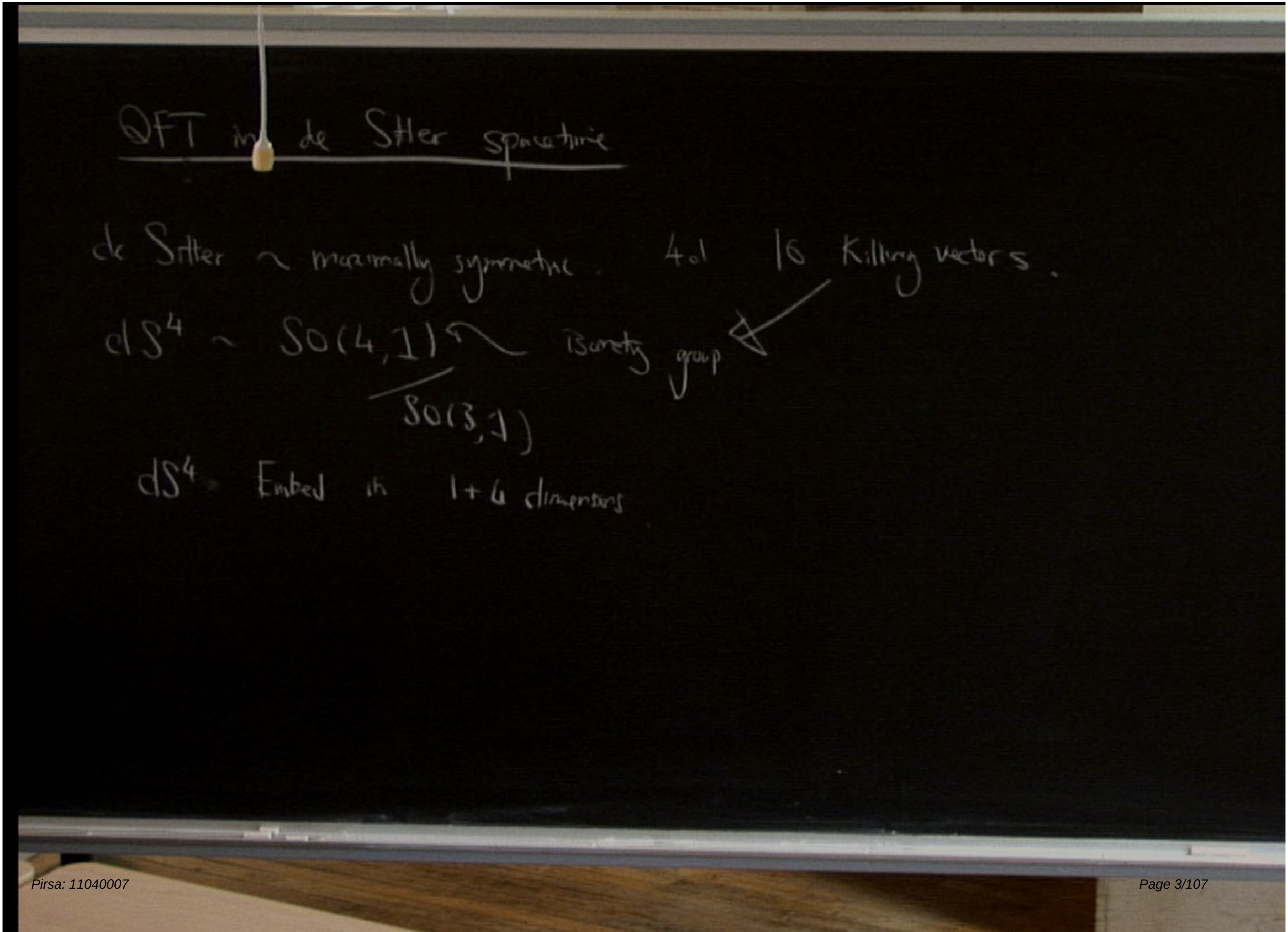
Title: Explorations in Cosmology - Lecture 5

Date: Apr 08, 2011 09:00 AM

URL: http://pirsa.org/11040007

Abstract:


QFT in de Sitter spacetime



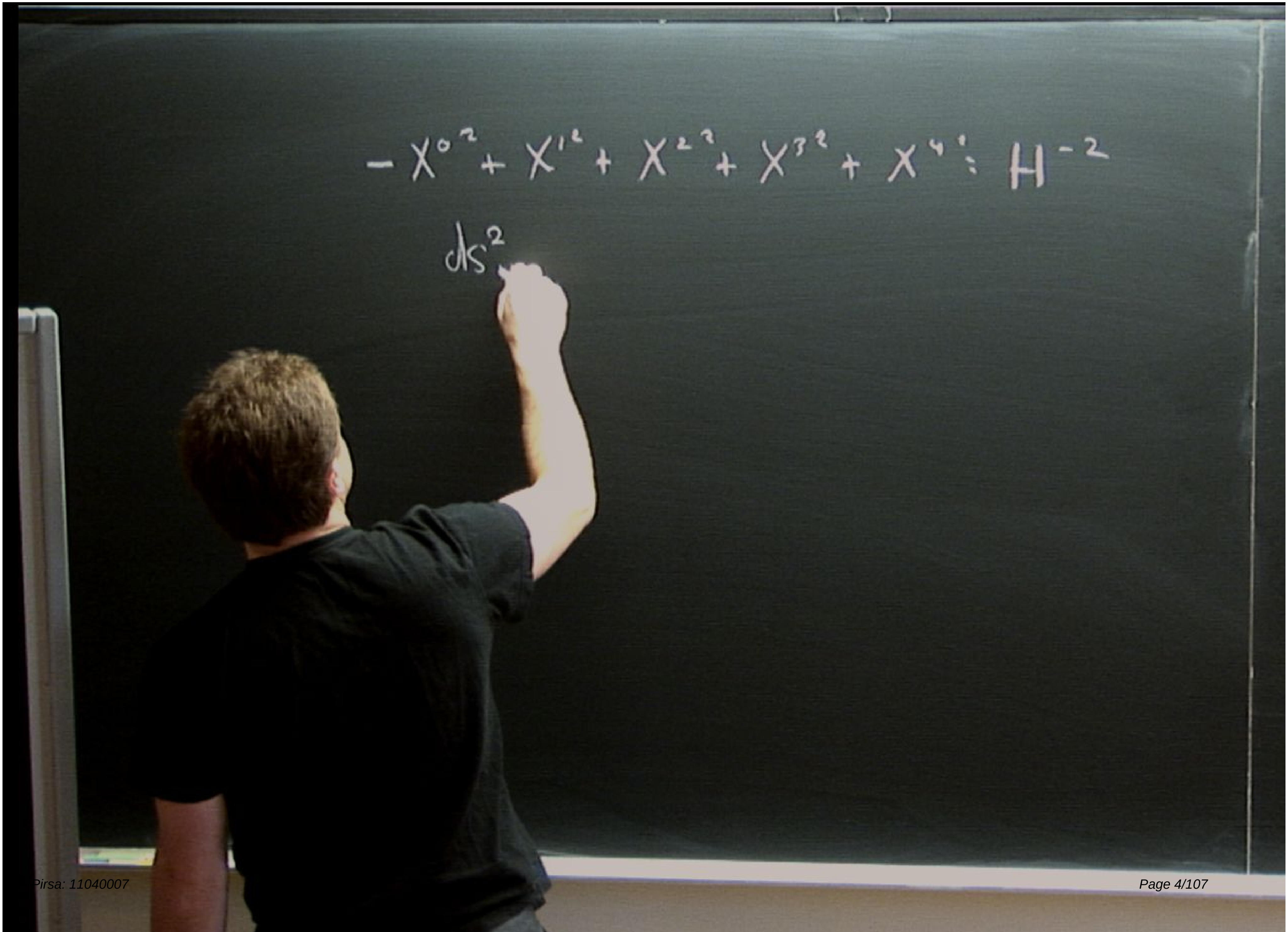
QFT in de Sitter spacetime

de Sitter ~ maximally symmetric. 4d 10 Killing vectors.

$dS^4 \sim SO(4,1)$ ~ Isometry group

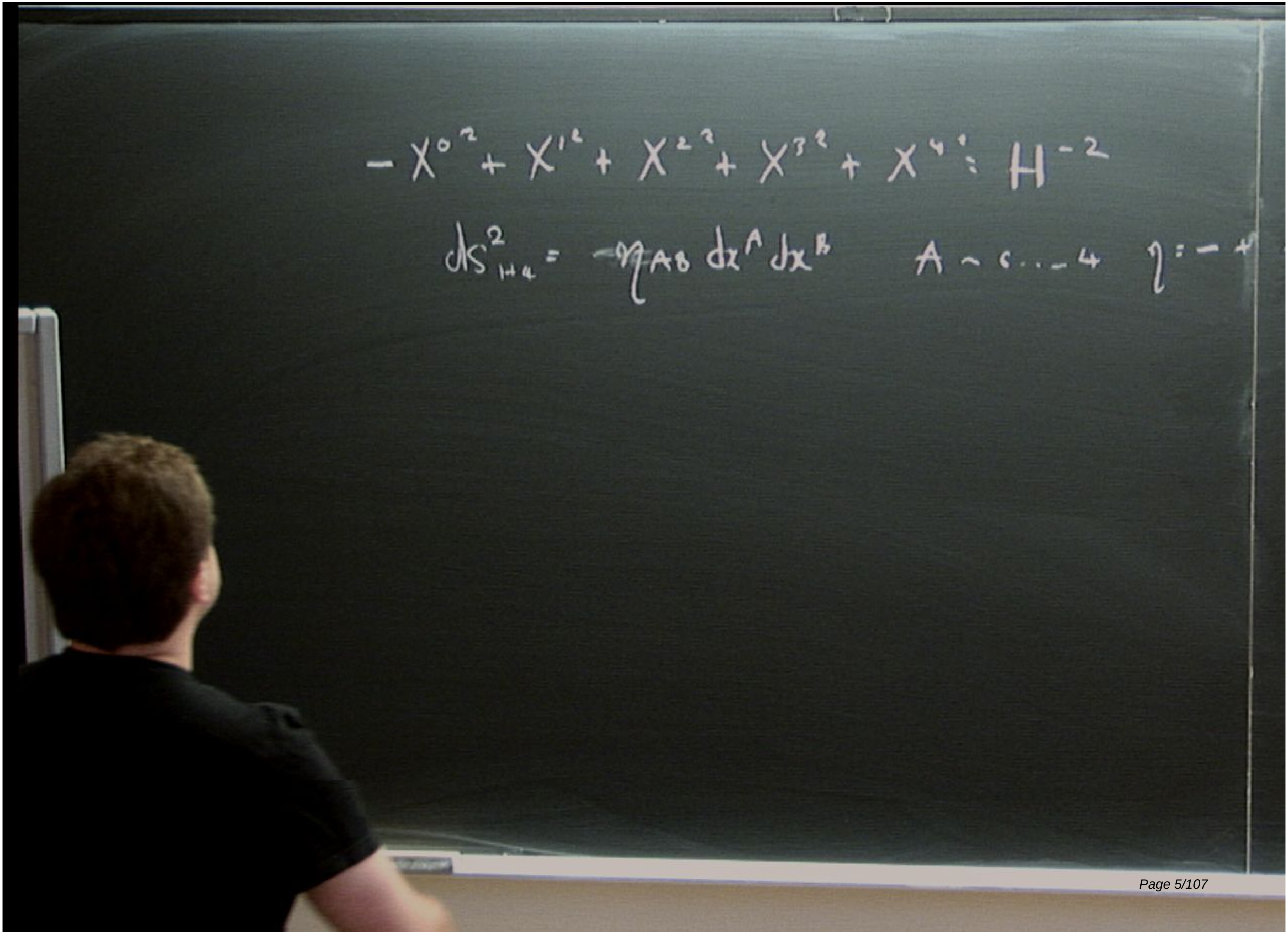
$SO(3,1))$

dS^4 Embed in 1+4 dimensions



$$-X^{0^2} + X^{1^2} + X^{2^2} + X^{3^2} + X^{4^2} = H^{-2}$$

$$ds^2$$



$$-X^{0\,2} + X^{1\,2} + X^{2\,2} + X^{3\,2} + X^{4\,2} = H^{-2}$$

$$ds^2_{1+4} = \eta_{AB}\, dx^A dx^B \qquad A \sim 0 \ldots 4 \qquad \eta = - +$$

$$-X^{0\,2} + X^{1\,2} + X^{2\,2} + X^{3\,2} + X^{4\,2} = H^{-2}$$

$$ds^2_{1+4} = \eta_{AB}\, dx^A dx^B \qquad A \sim 0 \ldots 4 \qquad \eta = - +$$

sign

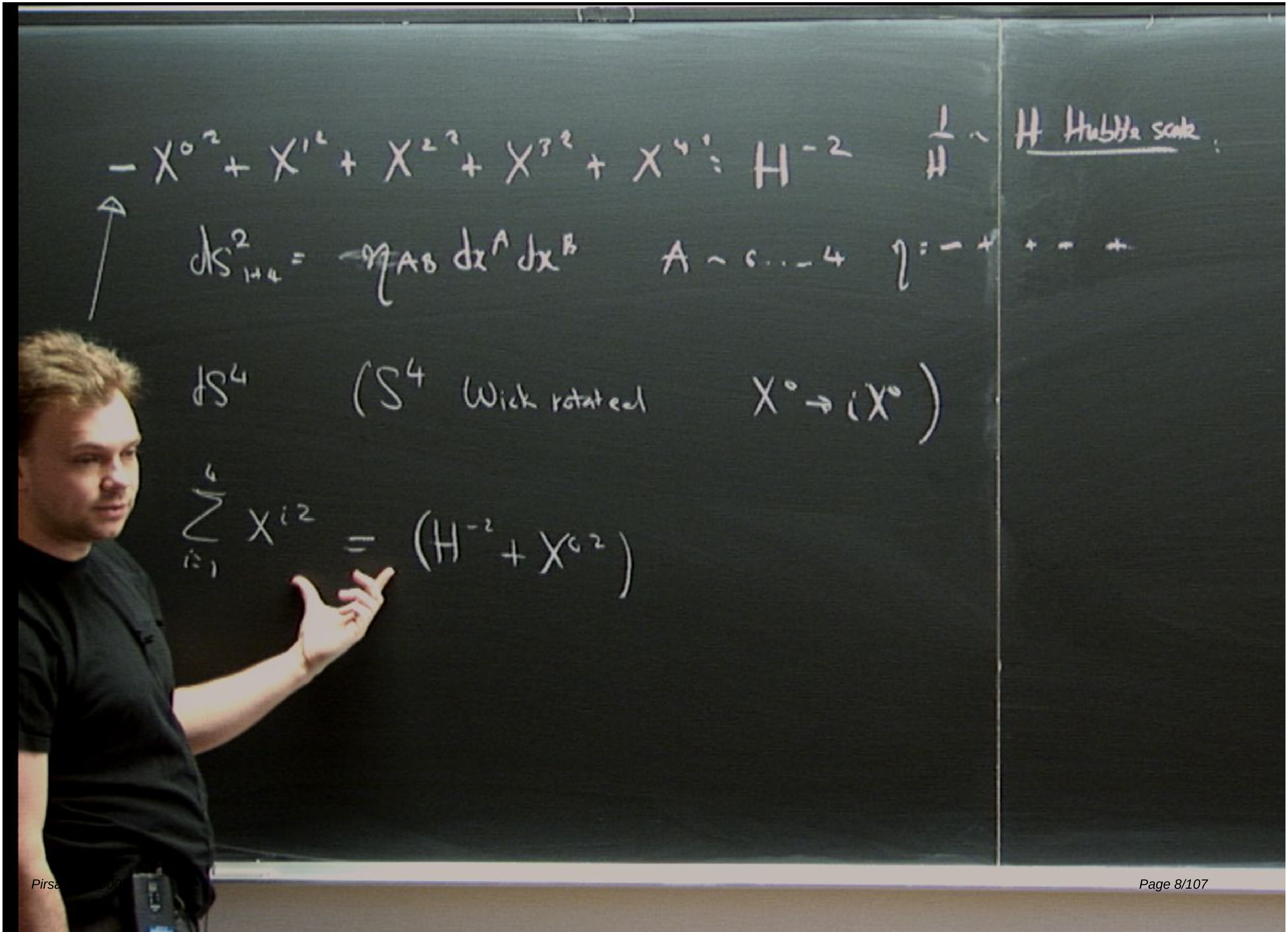
$$dS^4 \qquad (S^4 \text{ Wick rotated} \qquad X^0 \to iX^0)$$

$$-X^{0\,2} + X^{1\,2} + X^{2\,2} + X^{3\,2} + X^{4\,2} = H^{-2}$$

sign

$$ds^2_{1+4} = \eta_{AB}\, dx^A dx^B \qquad A \sim 0 \ldots 4 \qquad \eta : - +$$

$$dS^4 \qquad (S^4 \text{ Wick rotated} \qquad X^0 \to iX^0)$$

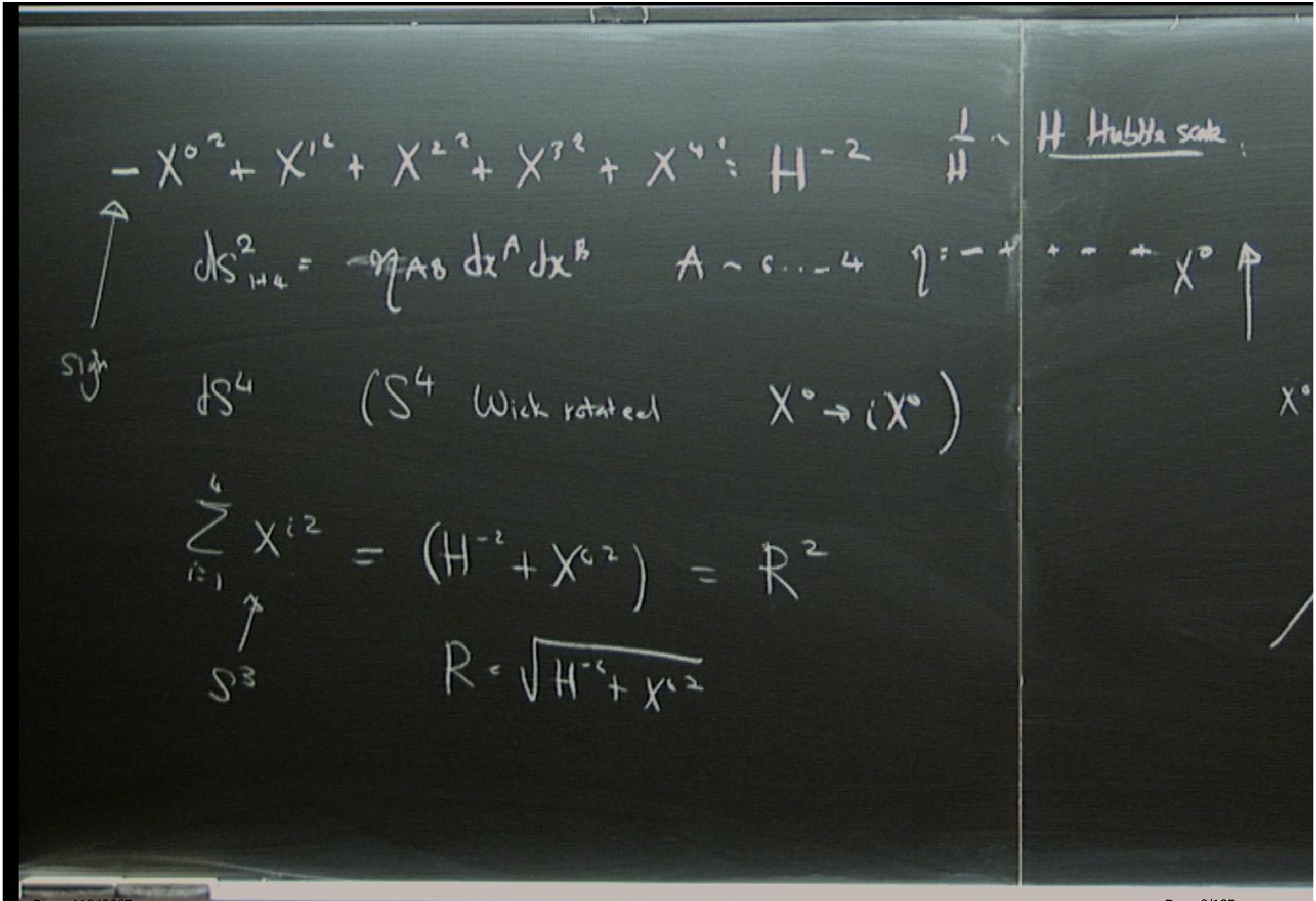


$-X^{0\,2} + X^{1\,2} + X^{2\,2} + X^{3\,2} + X^{4\,2} = H^{-2}$ $\frac{1}{H} \sim$ H Hubble scale

$ds^2_{1+4} = \eta_{AB}\, dx^A dx^B$ $A \sim 0 \ldots 4$ $\eta : - + + + +$

dS^4 $(S^4$ Wick rotated $X^0 \to iX^0)$

$\sum_{i=1}^{4} X^{i\,2} = (H^{-2} + X^{0\,2})$



$$-X^{0\,2} + X^{1\,2} + X^{2\,2} + X^{3\,2} + X^{4\,2} = H^{-2} \qquad \frac{1}{H} \sim$$

H Hubble scale

sign

$$ds^2_{1+4} = \eta_{AB}\, dx^A dx^B \qquad A \sim 0 \ldots 4 \qquad \eta: - + + + +$$

X^0

dS^4 (S^4 Wick rotated $X^0 \to i X^0$)

X^0

$$\sum_{i=1}^{4} X^{i\,2} = \left(H^{-2} + X^{0\,2}\right) = R^2$$

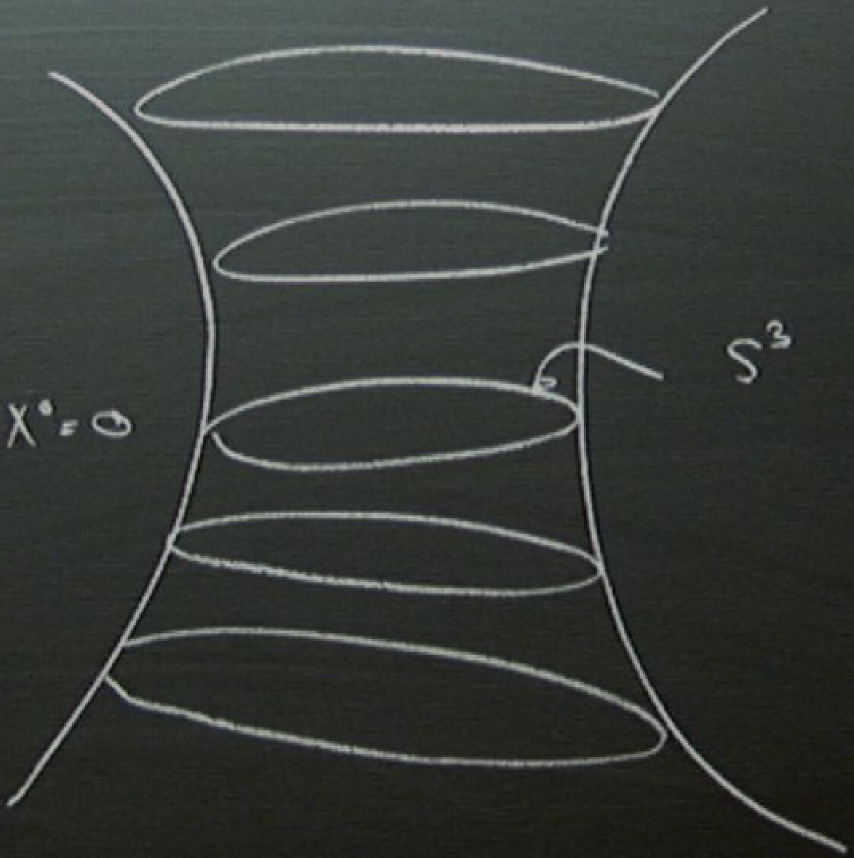
S^3

$$R = \sqrt{H^{-2} + X^{0\,2}}$$

$\frac{1}{H} \sim H$ Hubble scale :

$\eta = - + + - +$ $X^0 \uparrow$

$iX^0)$

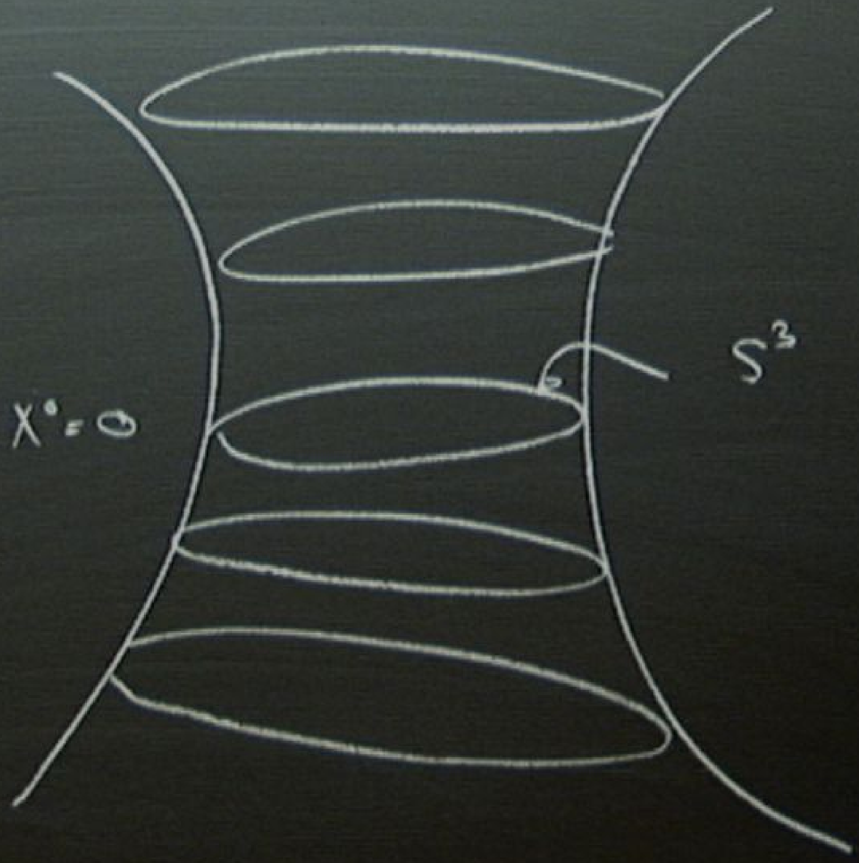


closed string & de Sit

$\frac{1}{H} \sim$ H Hubble scale :

$\eta = - + + + \cdots + \quad X^0 \uparrow$

$iX^0)$



$X^0 =$

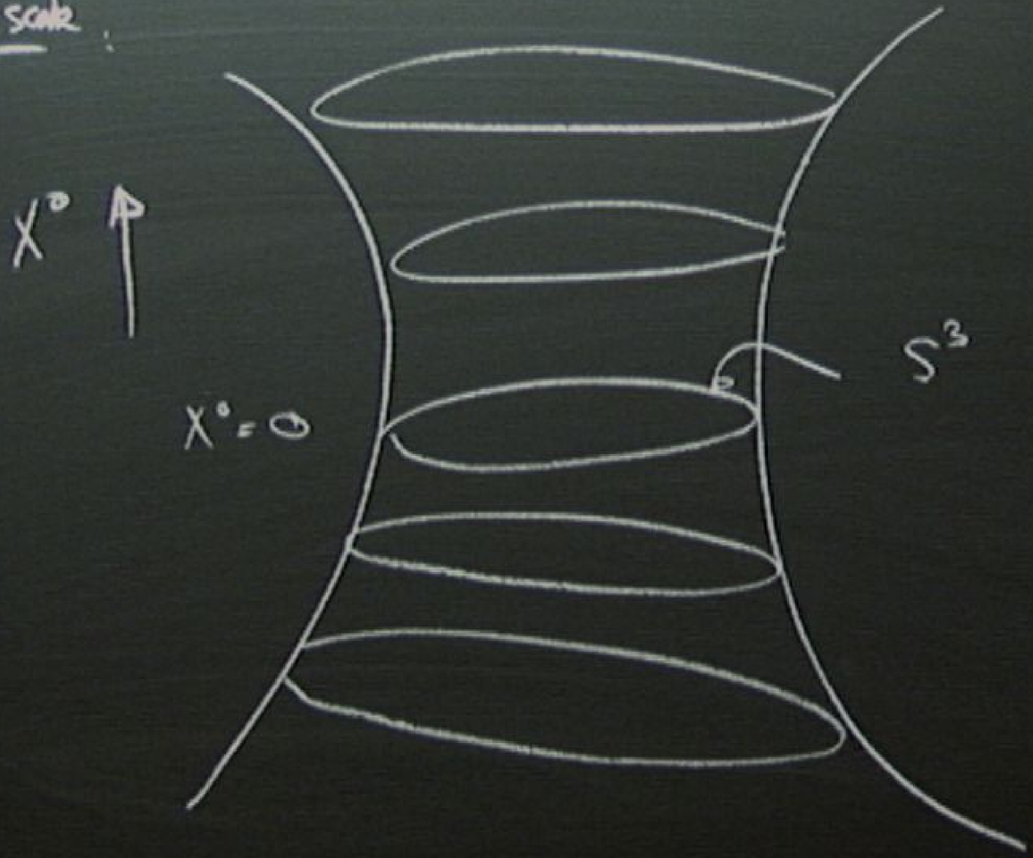
$X^i =$

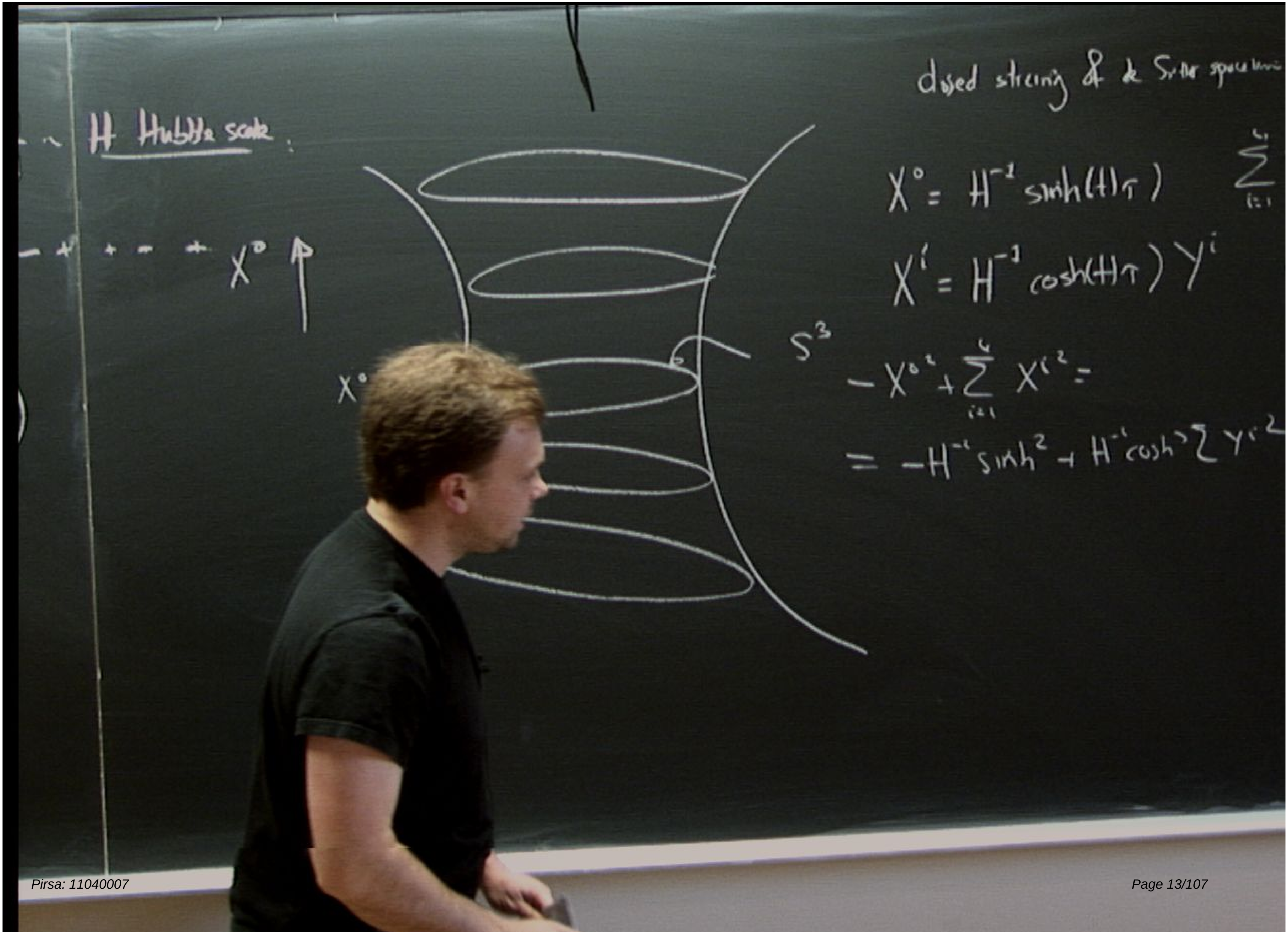
closed string & de Sitter spacetime

H Hubble scale :

$$X^0 = H^{-1} \sinh(H\tau)$$

$$X^i = H^{-1} \cosh(H\tau)$$





H Hubble scale :

closed string & de Sitter space

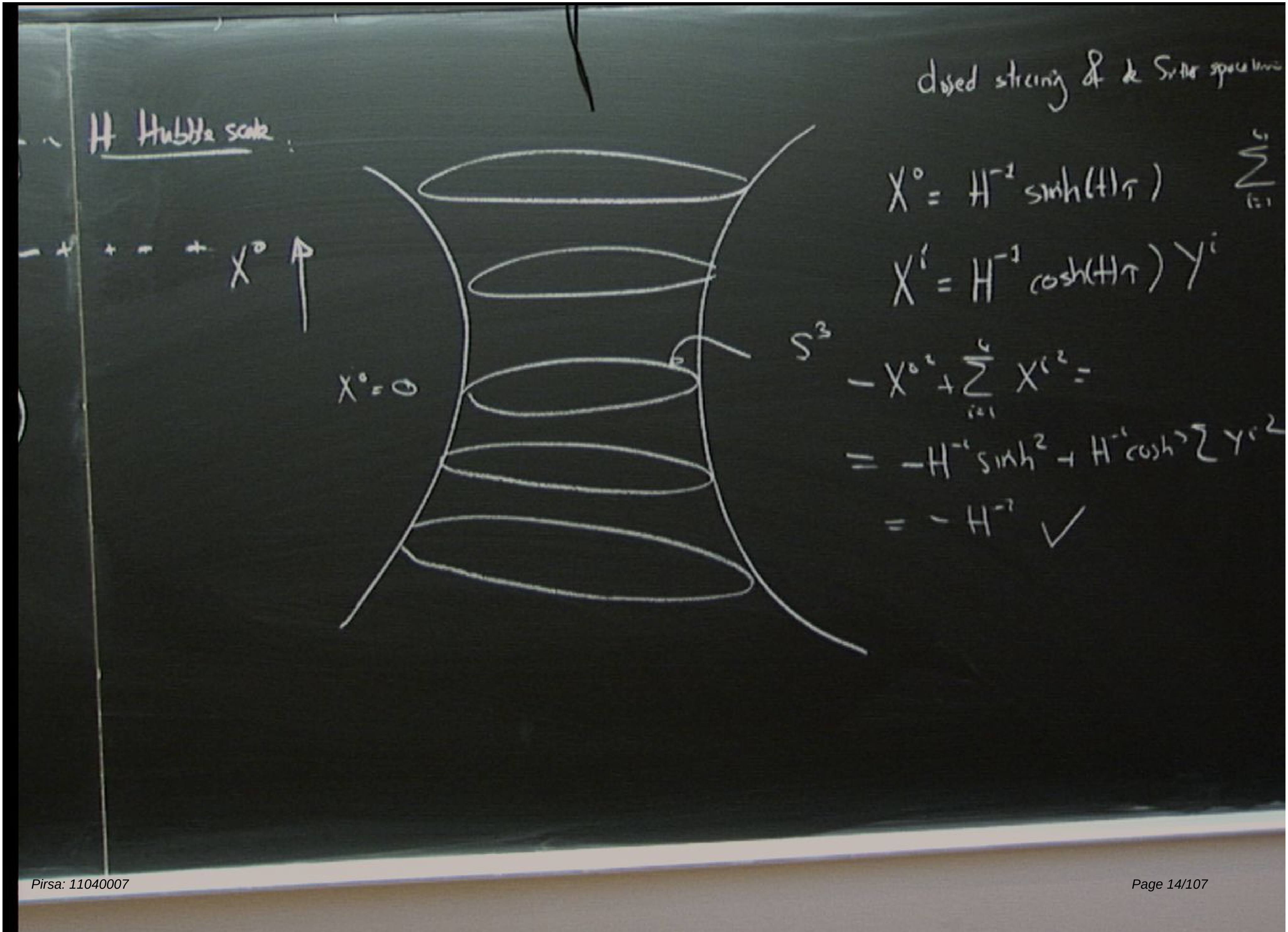
$$X^0 = H^{-1} \sinh(H\tau)$$

$$X^i = H^{-1} \cosh(H\tau)\, Y^i$$

$$\sum_{i=1}^{4}$$

$$-X^{0\,2} + \sum_{i=1}^{4} X^{i\,2} =$$

$$= -H^{-1} \sinh^2 + H^{-1} \cosh^2 \sum Y^{i\,2}$$



~ H Hubble scale :

− + + − − + X^0 ↑

closed string & de Sitter spacetime

$X^0 = H^{-1} \sinh(H\tau)$ $\sum_{i=1}^{4}$

$X^i = H^{-1} \cosh(H\tau)\, Y^i$

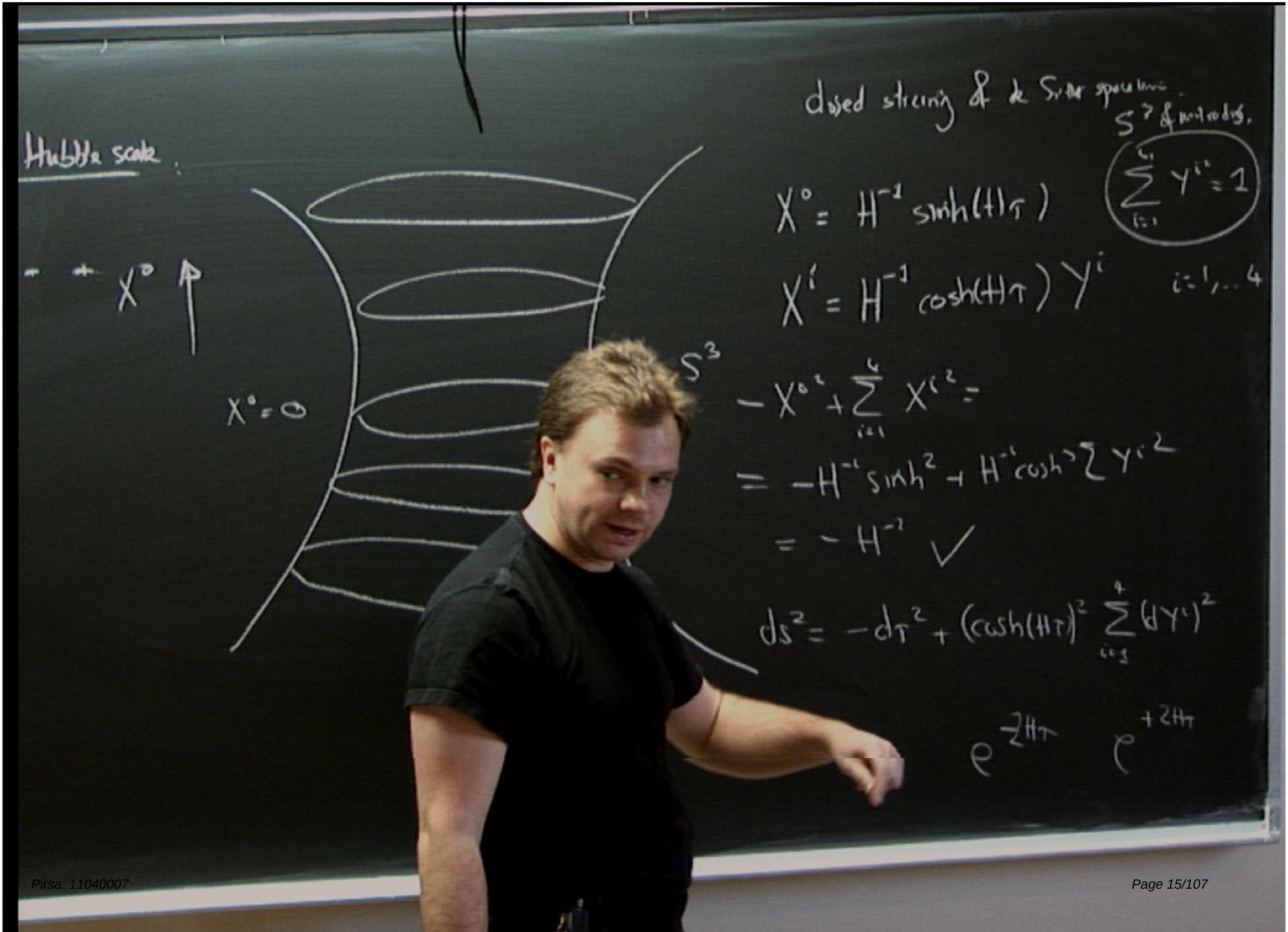
S^3

$X^0 = 0$

$-X^{0\,2} + \sum_{i=1}^{4} X^{i\,2} =$

$= -H^{-1}\sinh^2 + H^{-1}\cosh^2 \sum Y^{i\,2}$

$= -H^{-2}$ ✓



Hubble scale :

closed string & de Sitter spacetime.

S^3 & antipodes.

$$\boxed{\sum_{i=1}^{4} Y^{i^2} = 1}$$

$$X^0 = H^{-1} \sinh(H\tau)$$

$$X^i = H^{-1} \cosh(H\tau)\, Y^i \qquad i = 1, .. 4$$

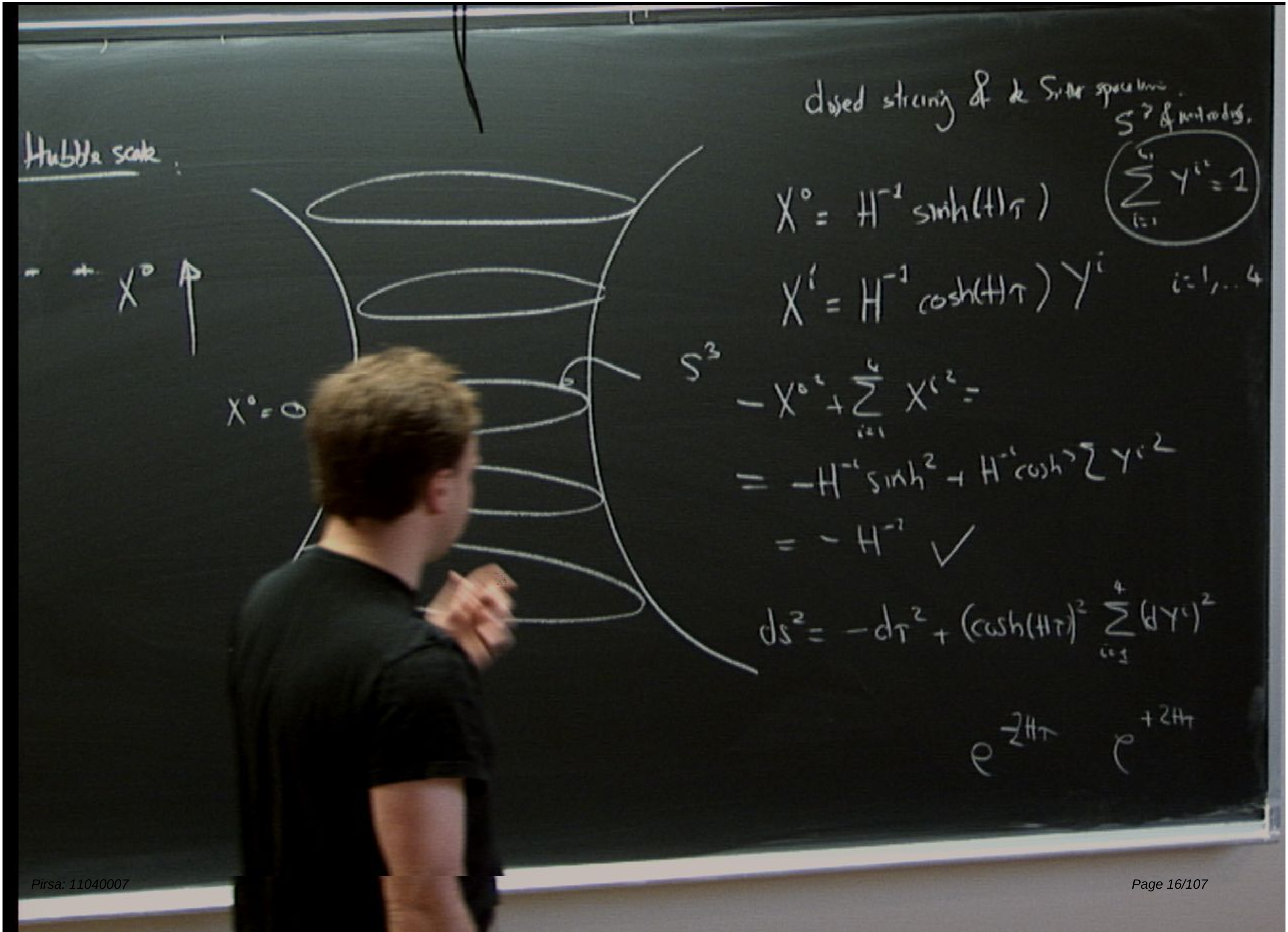
$$-X^{0^2} + \sum_{i=1}^{4} X^{i^2} =$$

$$= -H^{-1} \sinh^2 + H^{-1} \cosh^2 \sum Y^{i^2}$$

$$= -H^{-2} \quad \checkmark$$

$$ds^2 = -d\tau^2 + (\cosh(H\tau))^2 \sum_{i=1}^{4} (dY^i)^2$$

$$e^{-2H\tau} \qquad e^{+2H\tau}$$



closed string & de Sitter spacetime.

$S^?$ & instantons.

$$\boxed{\sum_{i=1}^{4} Y^{i^2} = 1}$$

$$X^0 = H^{-1} \sinh(H\tau)$$

$$X^i = H^{-1} \cosh(H\tau)\, Y^i \qquad i = 1, \dots 4$$

$$-X^{0^2} + \sum_{i=1}^{4} X^{i^2} =$$

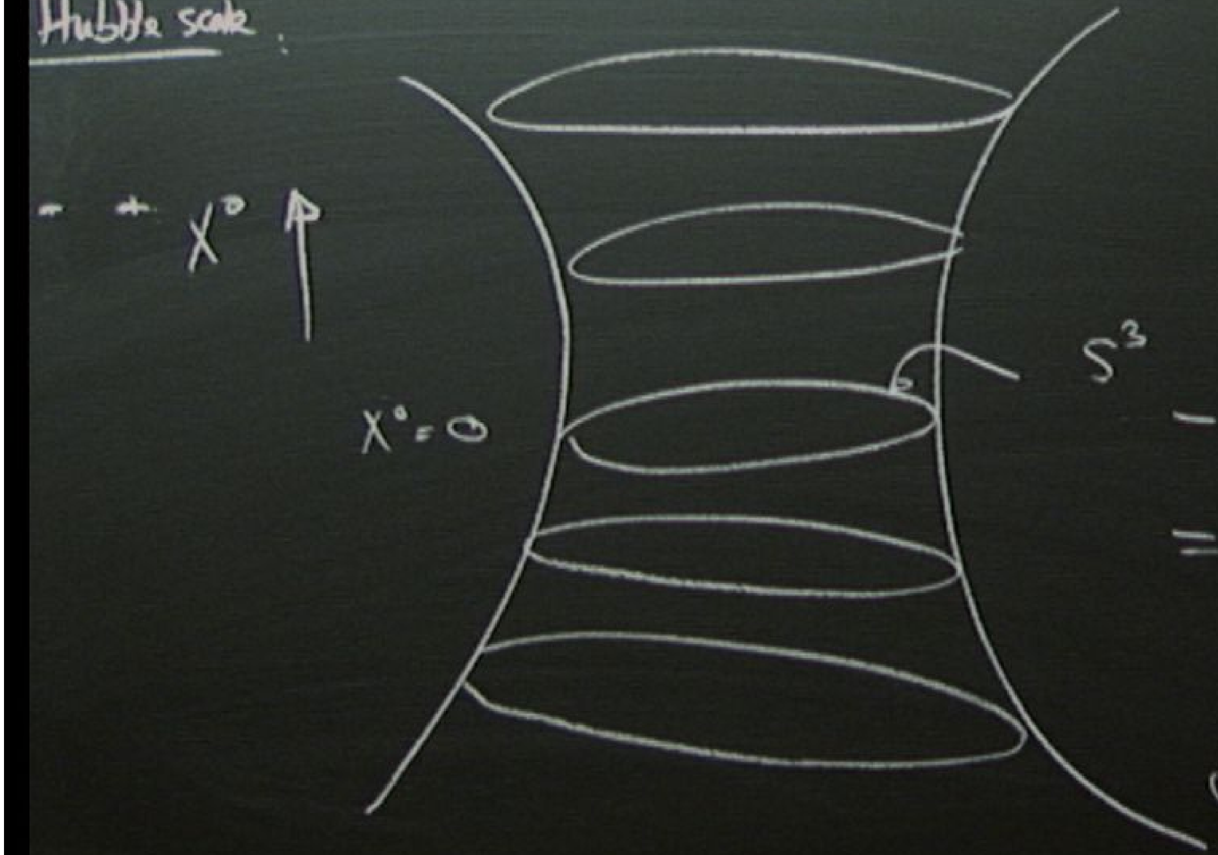
$$= -H^{-1} \sinh^2 + H^{-1} \cosh^2 \sum Y^{i^2}$$

$$= -H^{-1} \checkmark$$

$$ds^2 = -d\tau^2 + (\cosh(H\tau))^2 \sum_{i=1}^{4} (dY^i)^2$$

$$e^{-2H\tau} \qquad e^{+2H\tau}$$

Hubble scale :



closed string & de Sitter spacetime.

S^3 & [illegible]

$$X^0 = H^{-1} \sinh(H\tau) \qquad \boxed{\sum_{i=1}^{4} Y^{i^2} = 1}$$

$$X^i = H^{-1} \cosh(H\tau)\, Y^i \qquad i = 1, \ldots 4$$

$$-X^{0^2} + \sum_{i=1}^{4} X^{i^2} =$$

$$= -H^{-1} \sinh^2 + H^{-1} \cosh^2 \sum Y^{i^2}$$

$$= -H^{-2} \checkmark \qquad d^2\Omega_{S^3}$$

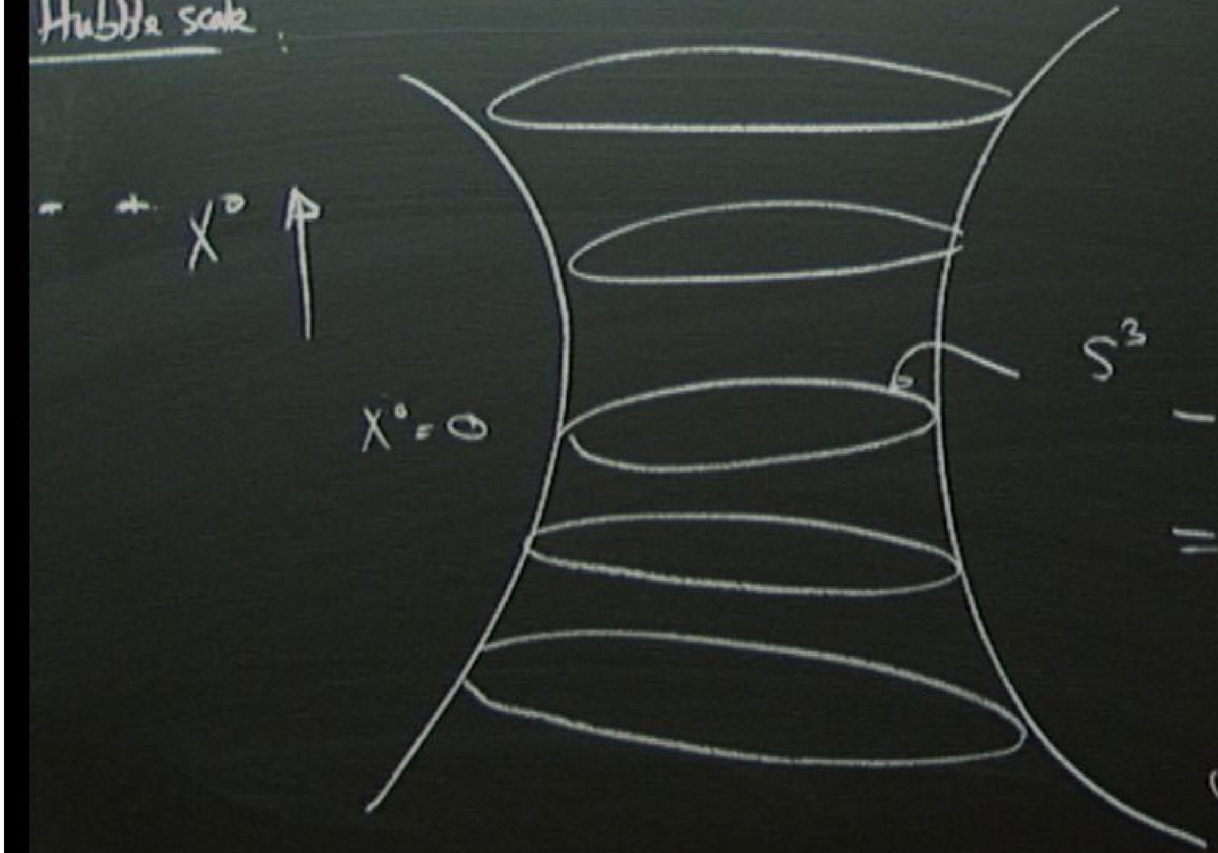
$$ds^2 = -d\tau^2 + (\cosh(H\tau))^2 \left(\sum_{i=1}^{4} (dY^i)^2 \right)$$

$$d^2\Omega_{S^3} = d\theta_1^2 + \sin^2(\theta_1) \left[d\theta_2^2 + \sin^2\theta_2\, d\phi \right] \quad e^{-2H\tau} \qquad e^{+2H\tau}$$

closed string & de Sitter spacetime.

S^7 & instantons.

Hubble scale :



$$X^0 = H^{-1} \sinh(H\tau)$$

$$\boxed{\sum_{i=1}^{4} Y^{i^2} = 1}$$

$$X^i = H^{-1} \cosh(H\tau)\, Y^i \qquad i = 1, \dots 4$$

$$-X^{0^2} + \sum_{i=1}^{4} X^{i^2} =$$

$$= -H^{-1} \sinh^2 + H^{-1} \cosh^2 \sum Y^{i^2}$$

$$= -H^{-1} \ ✓$$

$$d^2\Omega_{S^3}$$

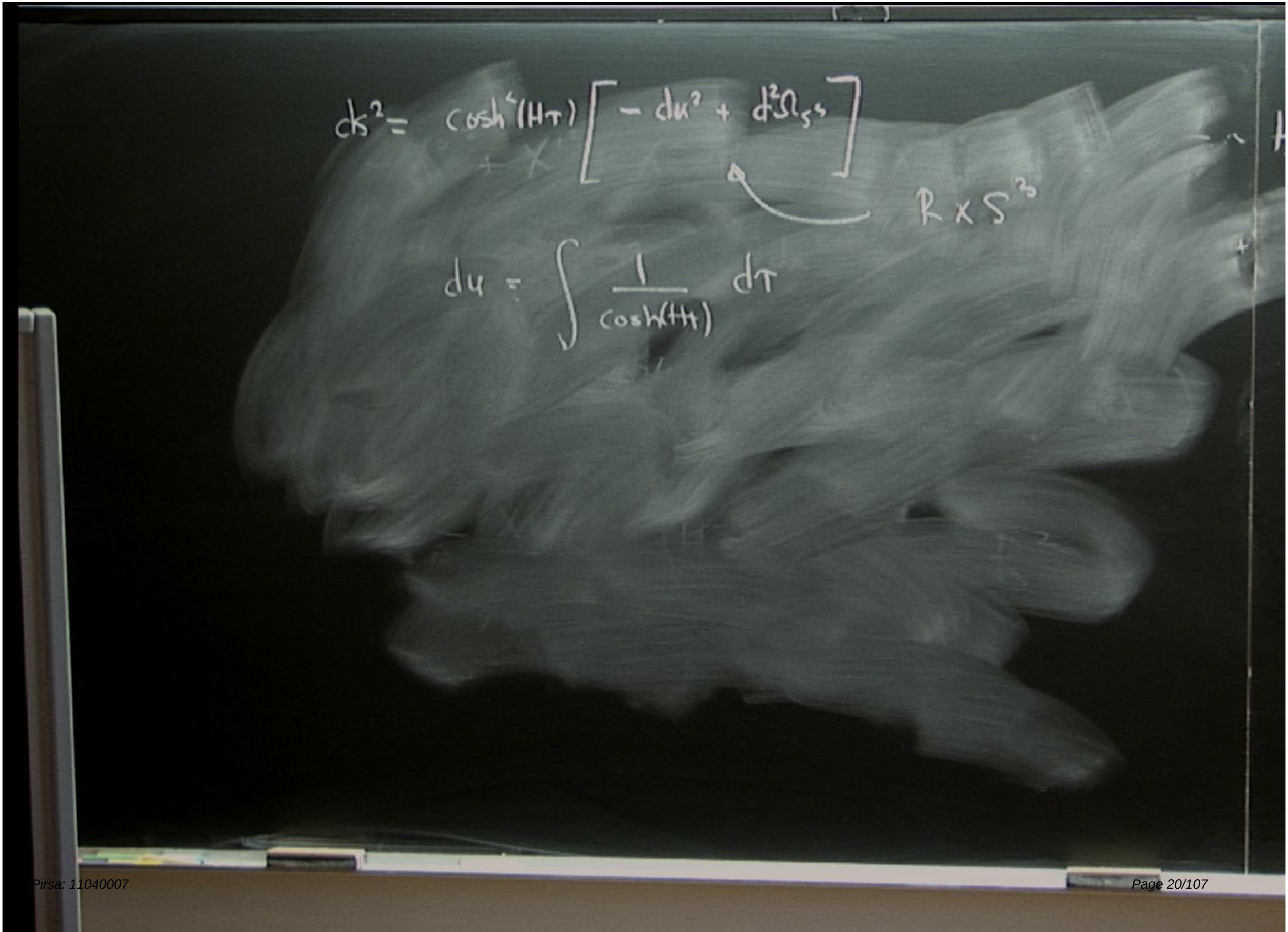
$$ds^2 = -d\tau^2 + (\cosh(H\tau))^2 \left(\sum_{i=1}^{4} (dY^i)^2 \right)$$

$$\sum (dY^i)^2 = d^2\Omega_{S^3} = d\theta_1^2 + \sin^2(\theta_1) \left[d\theta_2^2 + \sin^2\theta_2\, d\phi^2 \right] \quad e^{-2H\tau} \quad e^{+2H\tau}$$

$$ds^2 = \cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

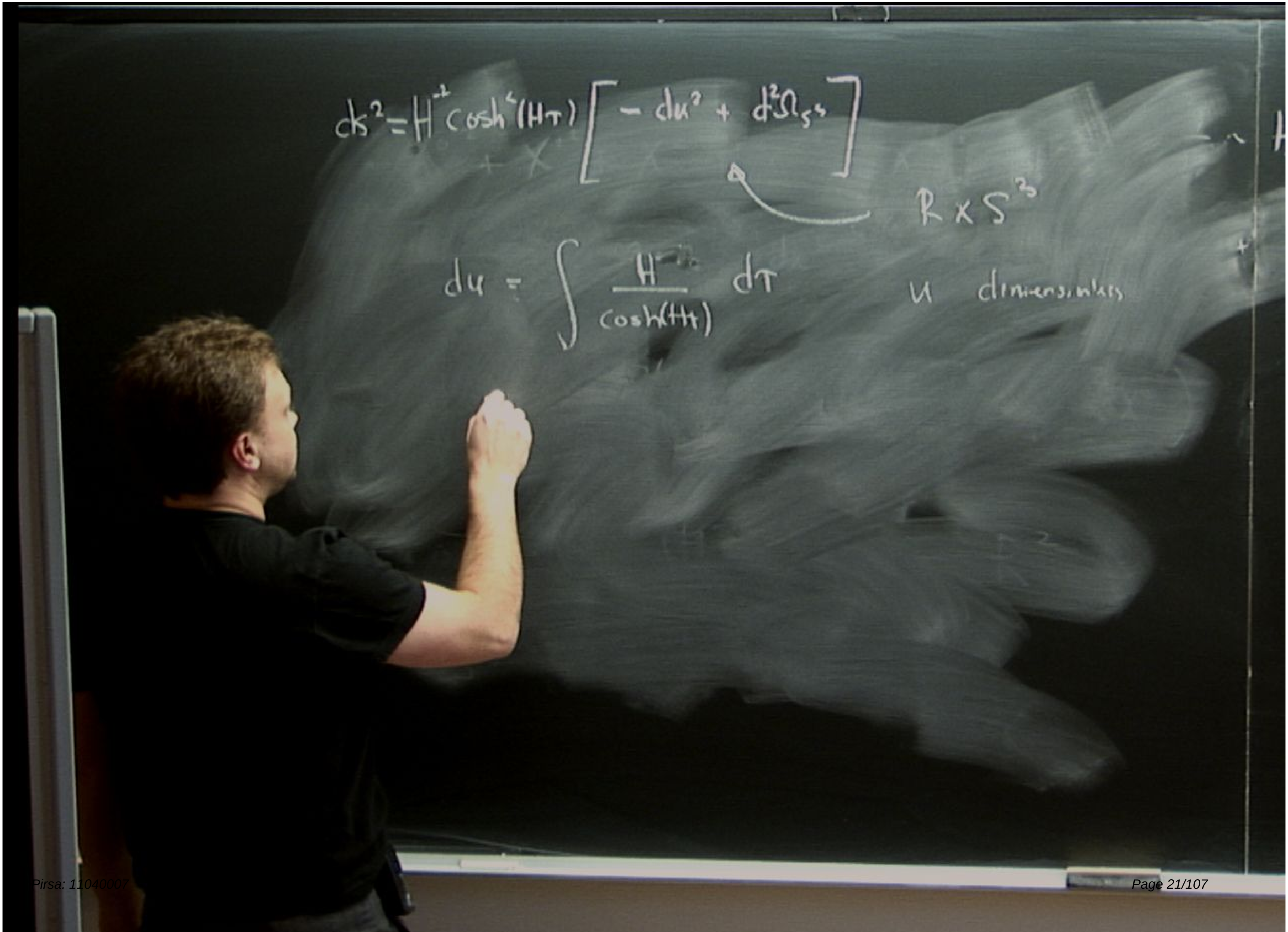
$$du = \int \frac{1}{\cosh(H\tau)}\, d\tau$$



$$ds^2 = \cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$du = \int \frac{1}{\cosh(H\tau)}\, d\tau$$

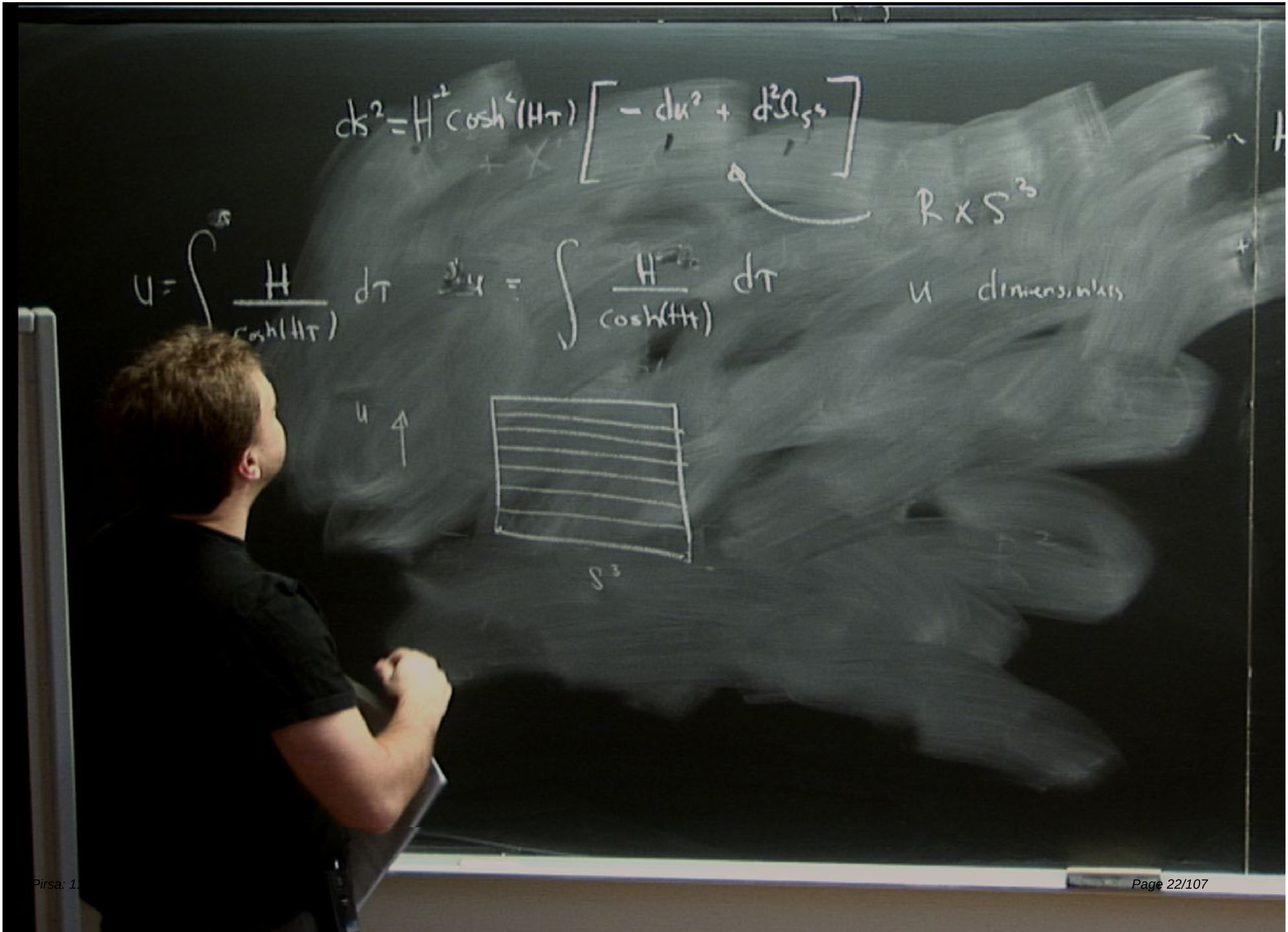


$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$du = \int \frac{H^{-}}{\cosh(H\tau)}\, d\tau$$

u dimensionless

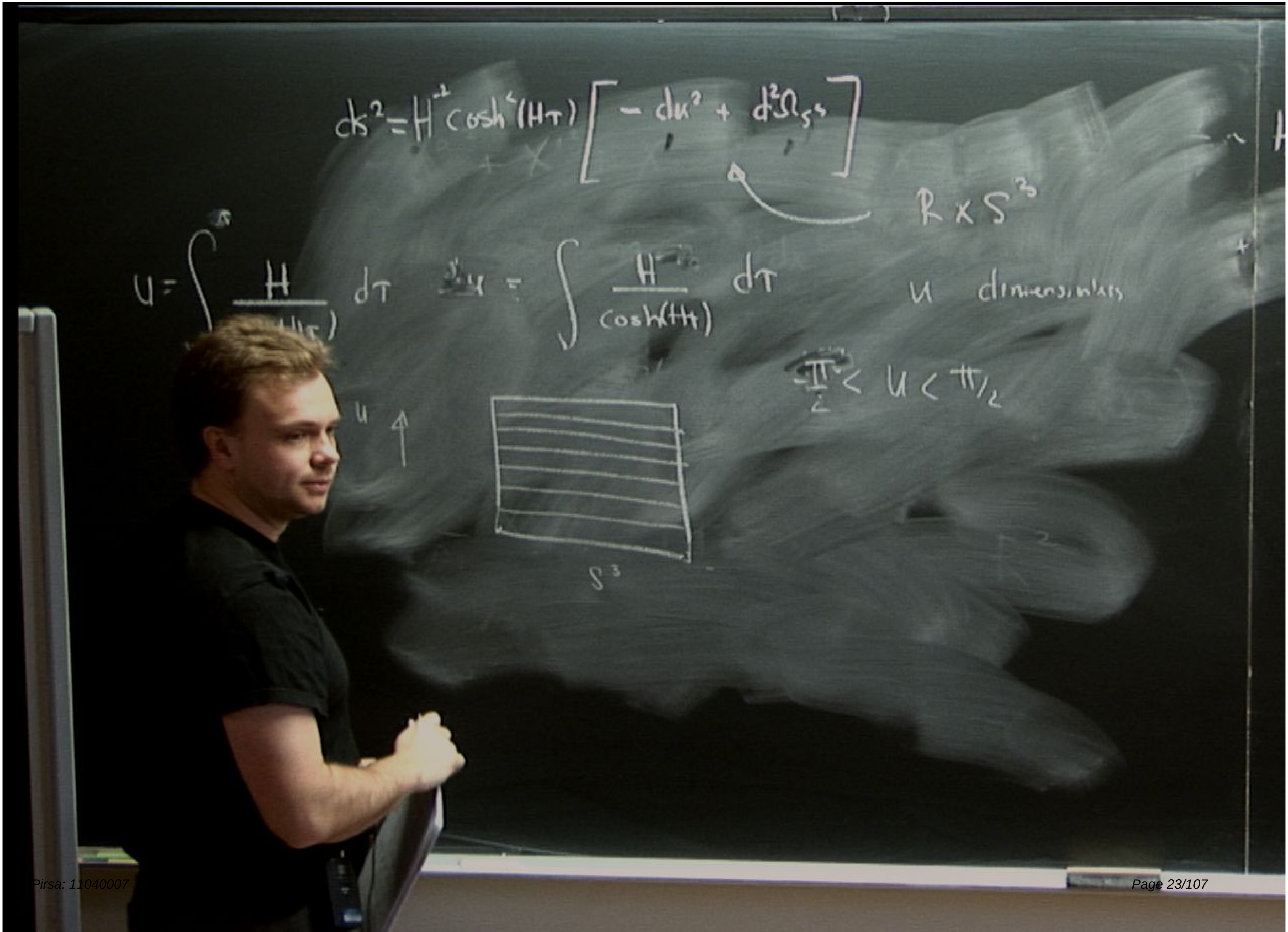


$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless



$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$

$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int_0^{\infty} \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless

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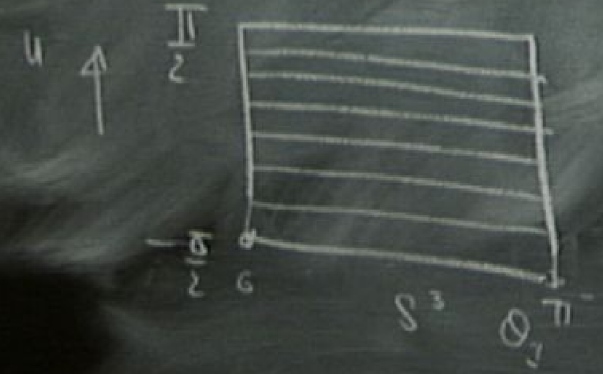
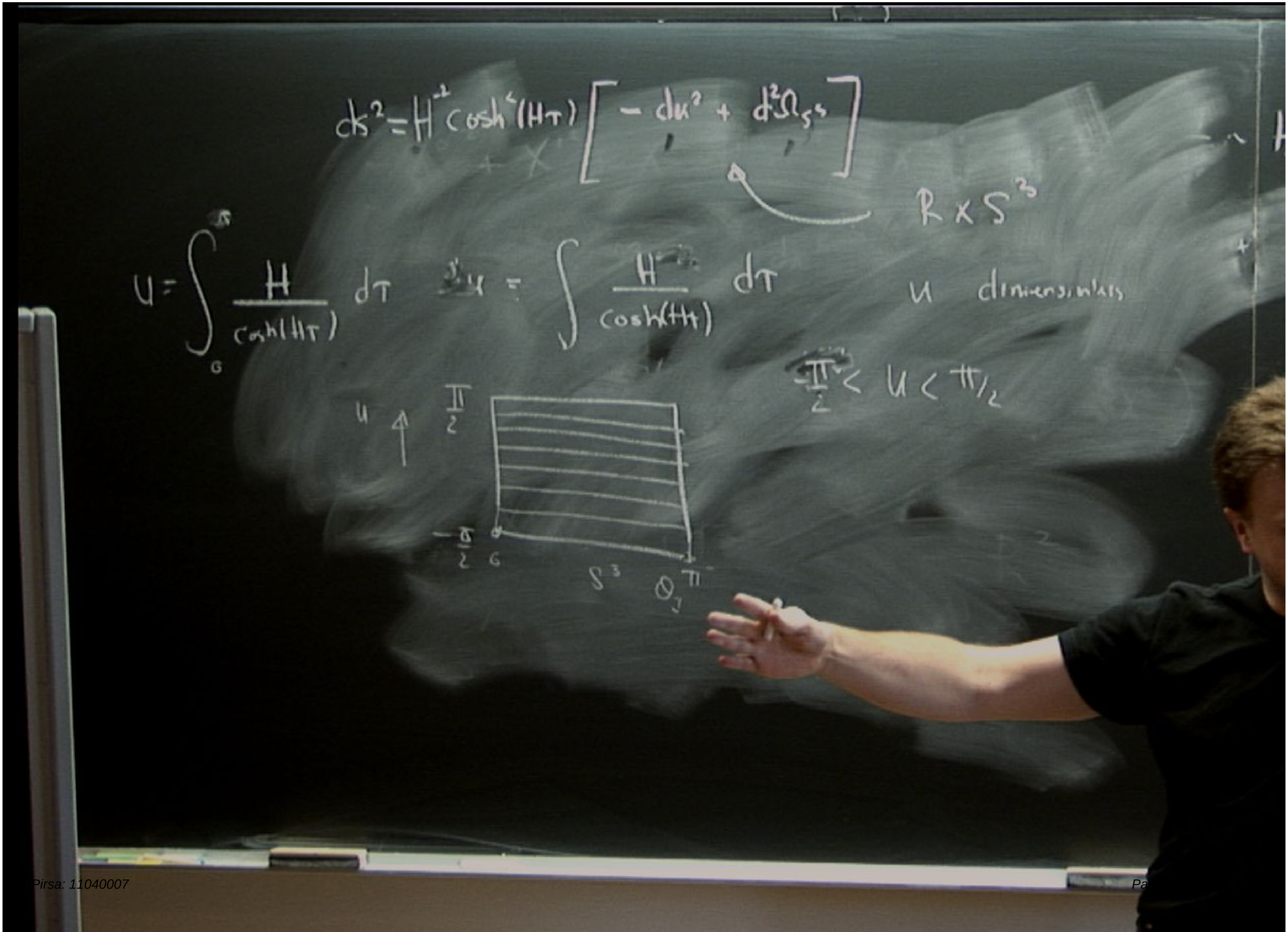




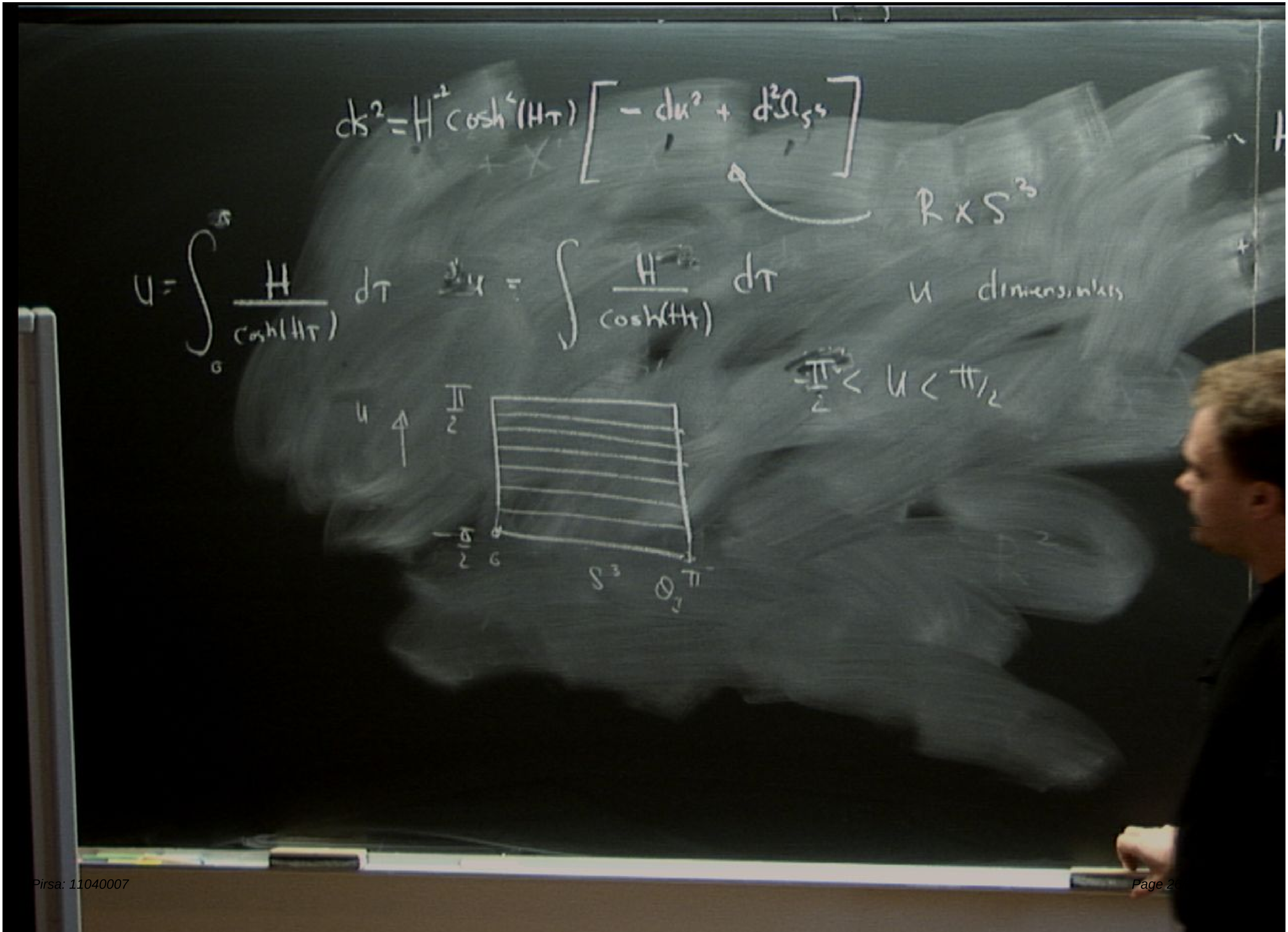
$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int_0^{\tau} \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H^{-2}}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$



$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

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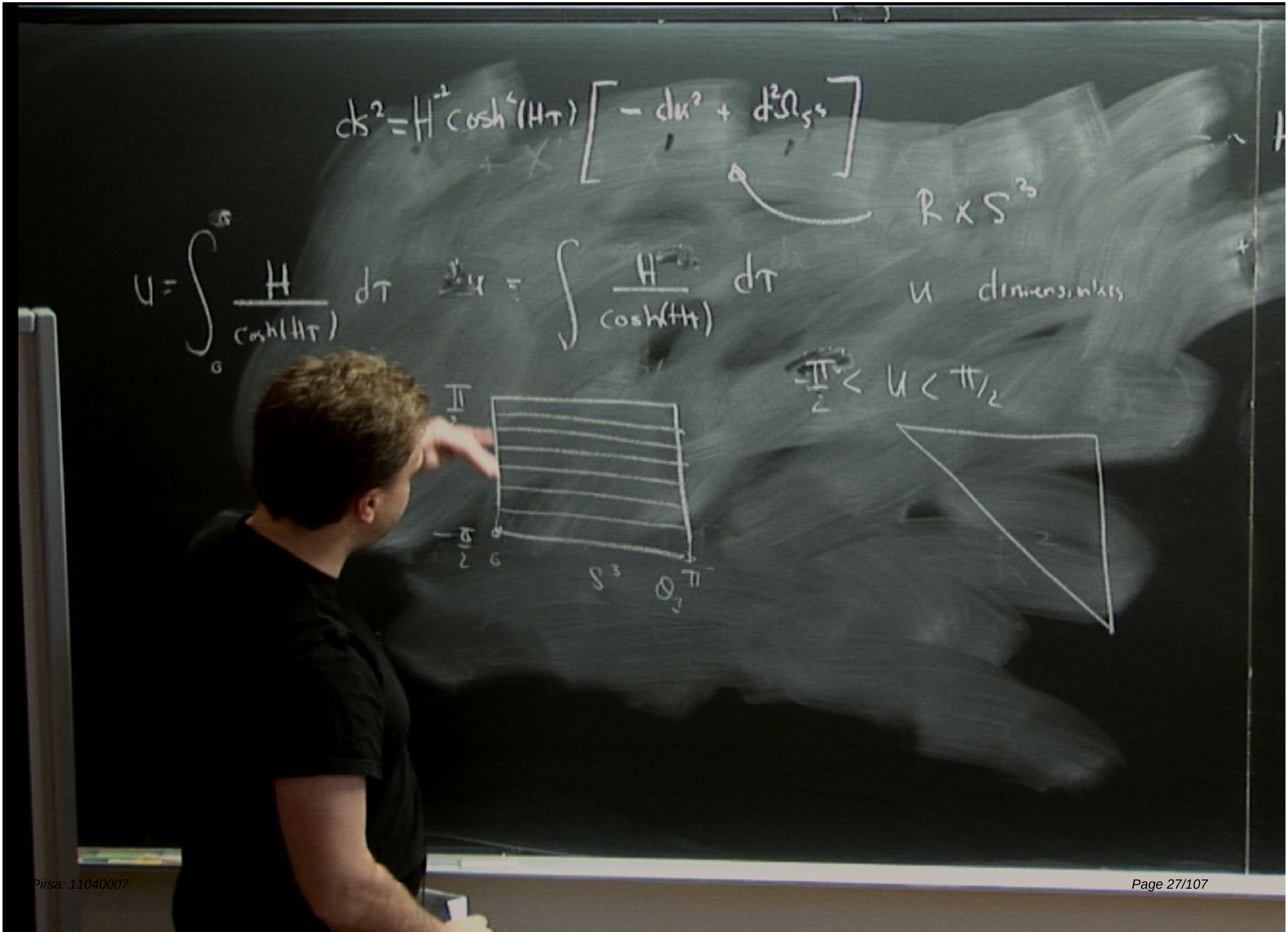
$$u = \int_0^{\infty} \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$



$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int_0^{\tau} \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$

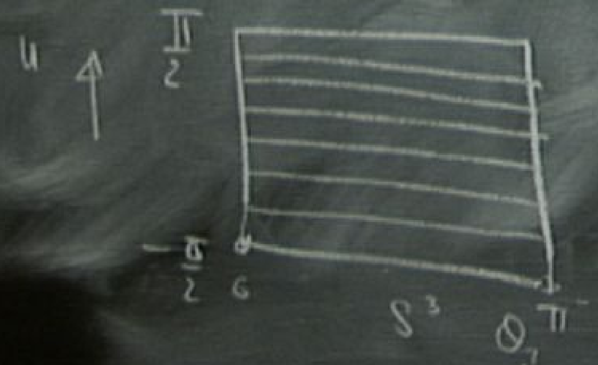
$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

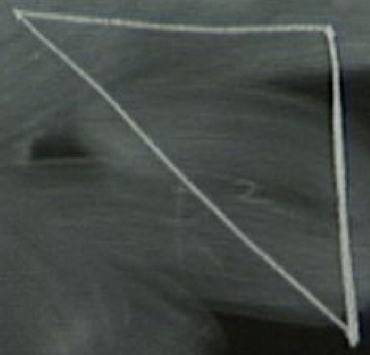
$R \times S^3$

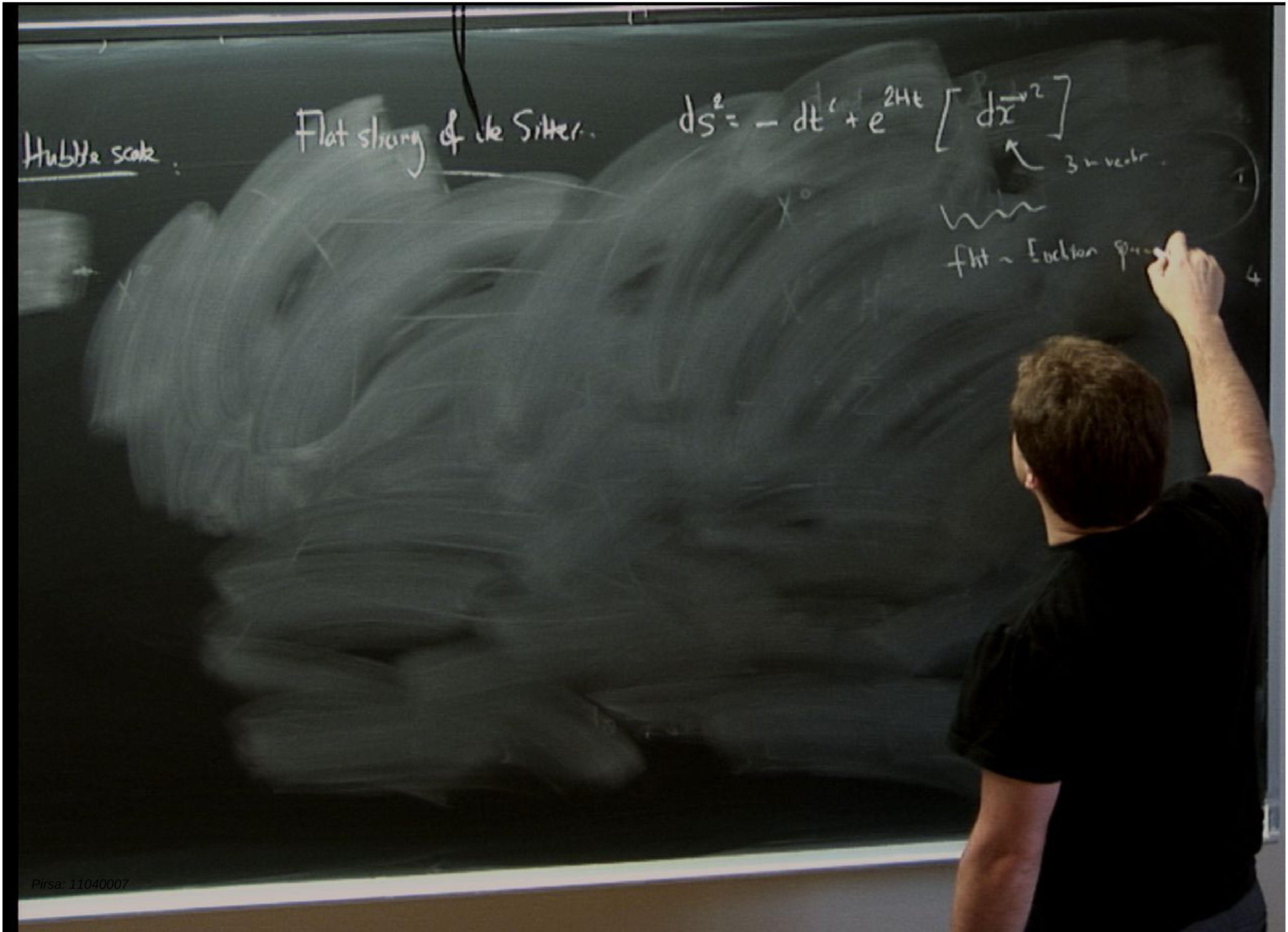
$$u = \int_0^\infty \frac{H}{\cosh(H\tau)}\, d\tau \quad = \int \frac{H^{-2}}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$







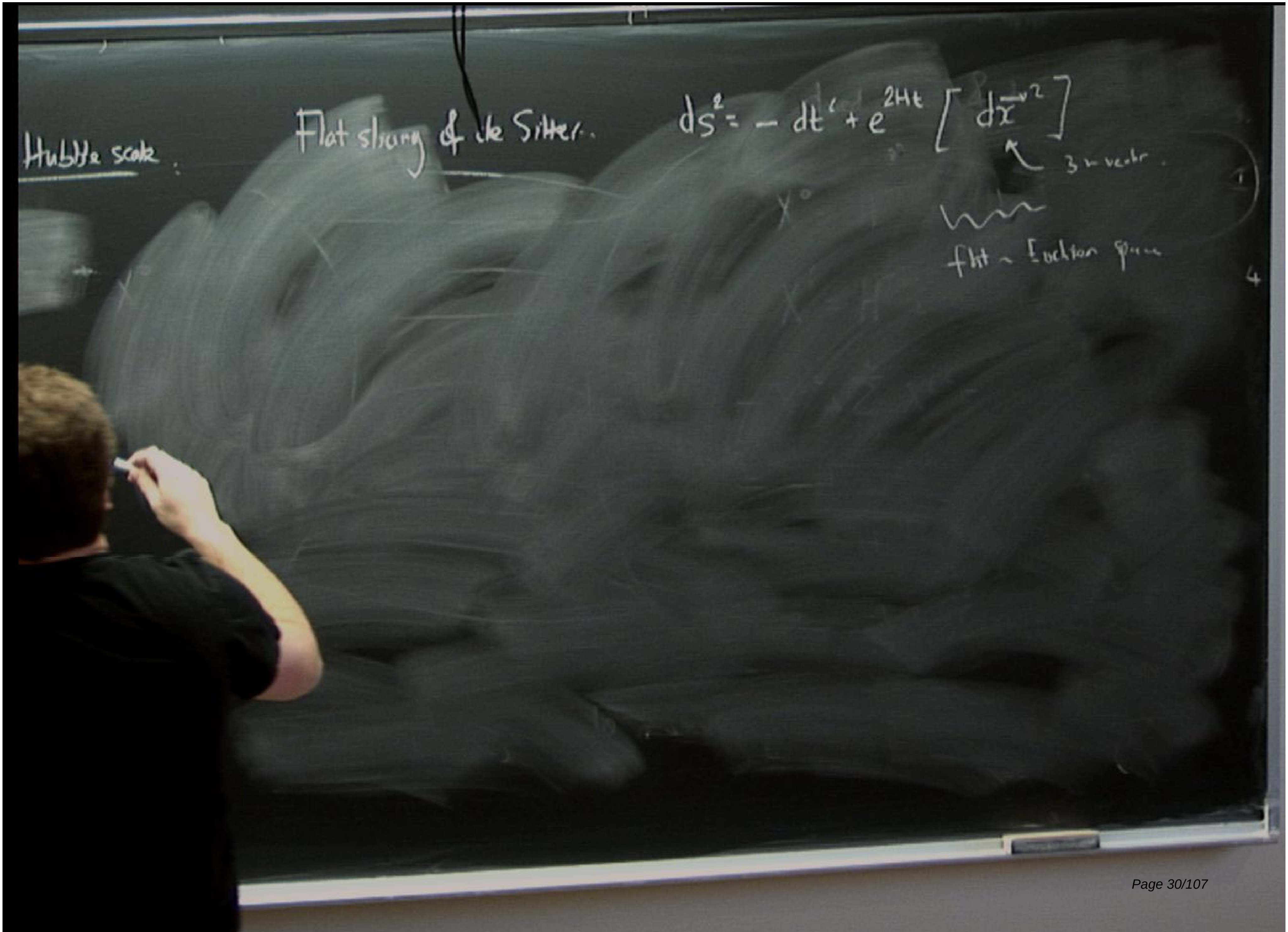
Hubble scale :

Flat slicing of de Sitter.

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector.

flat ~ Euclidean space



Hubble scale:

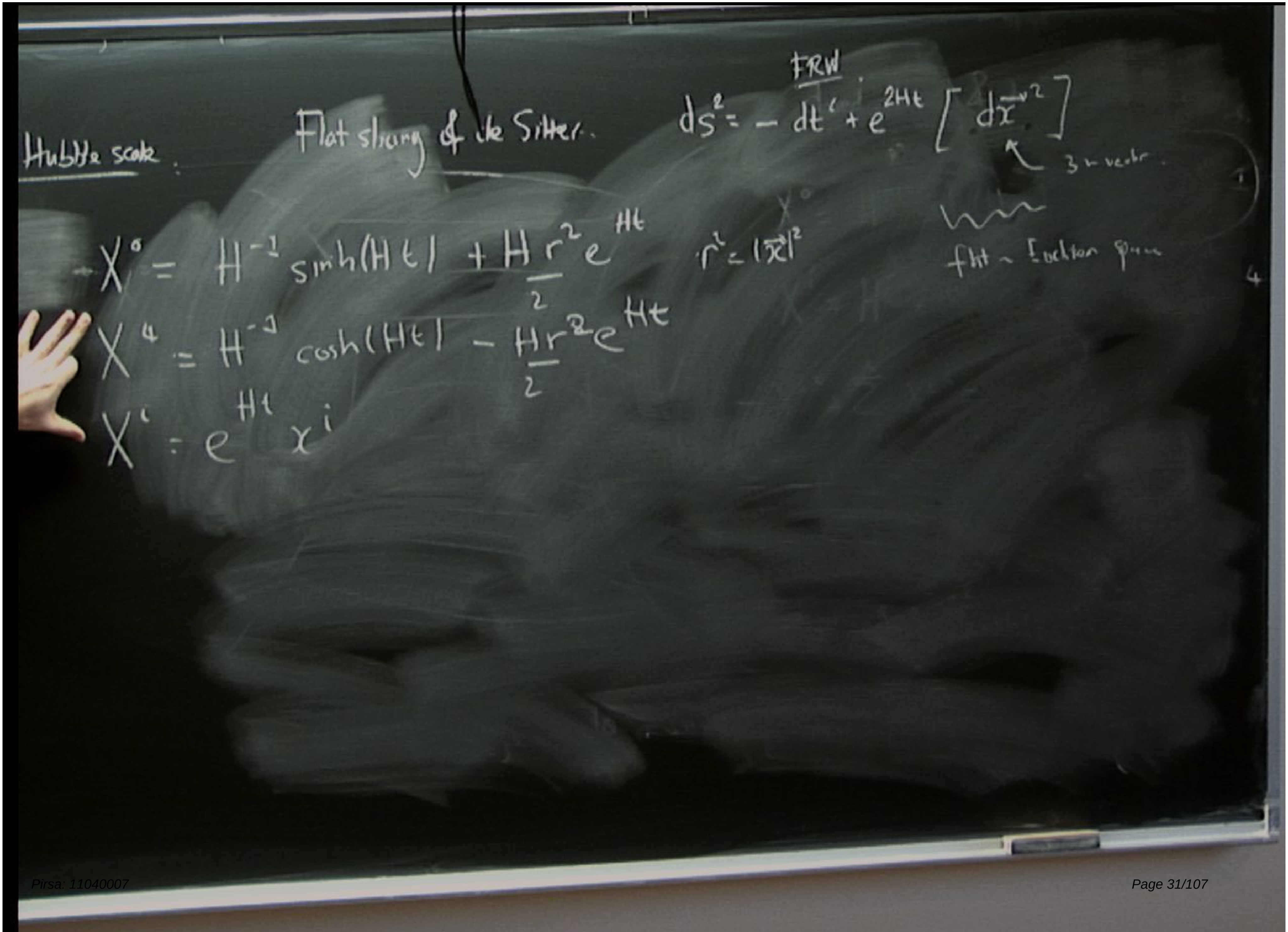
Flat slicing of de Sitter.

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector.

flat ~ Euclidean space

4



Hubble scale.

Flat slicing & de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3 - vector.

$$X^0 = H^{-1}\sinh(Ht) + \frac{Hr^2}{2}e^{Ht}$$

$r^2 = |\vec{x}|^2$

flat ~ Euclidean space

$$X^4 = H^{-1}\cosh(Ht) - \frac{Hr^2}{2}e^{Ht}$$

$$X^i = e^{Ht}x^i$$

1

4

Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3 - vector

$r^2 = |\vec{x}|^2$

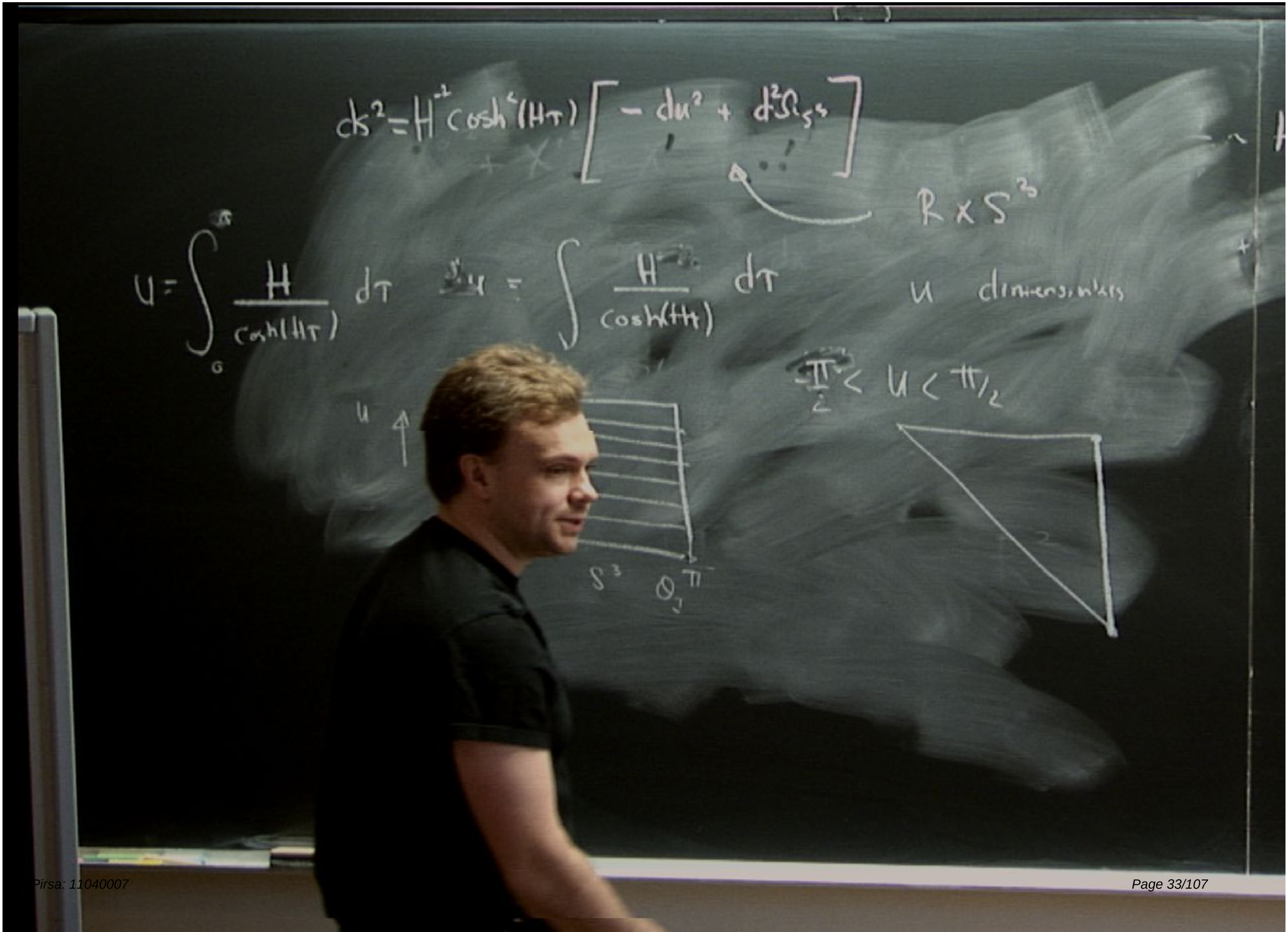
flat ~ Euclidean space

$$X^0 = H^{-1}\sinh(Ht) + \frac{Hr^2}{2}e^{Ht}$$

$$X^4 = H^{-1}\cosh(Ht) - \frac{Hr^2}{2}e^{Ht}$$

$$X^i = e^{Ht}x^i$$

$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2} \checkmark$$



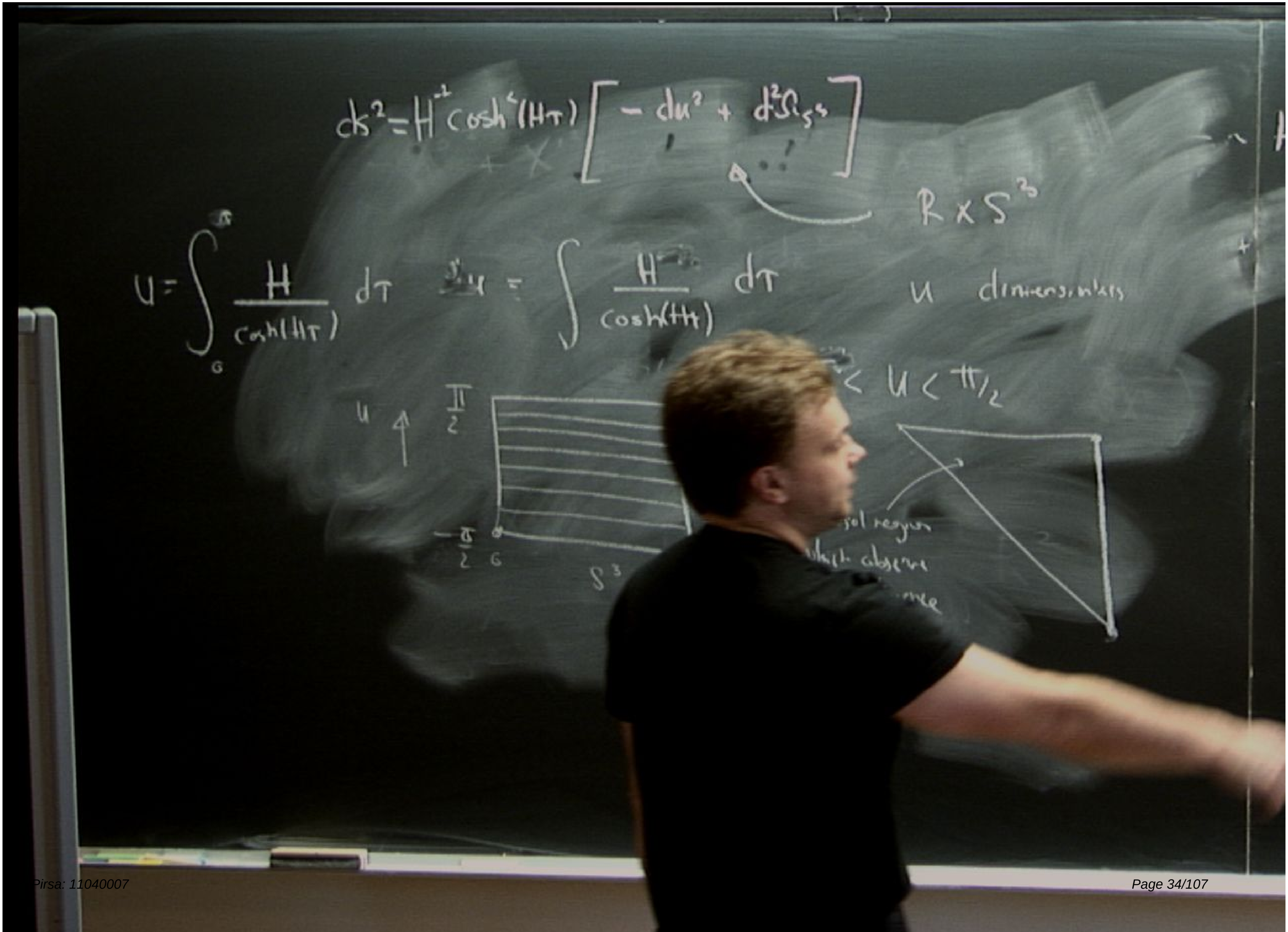
$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int_0^\infty \frac{H}{\cosh(H\tau)}\,d\tau \qquad u = \int \frac{H}{\cosh(H\tau)}\,d\tau$$

u dimensionless

$$-\frac{\pi}{2} < u < \pi/2$$



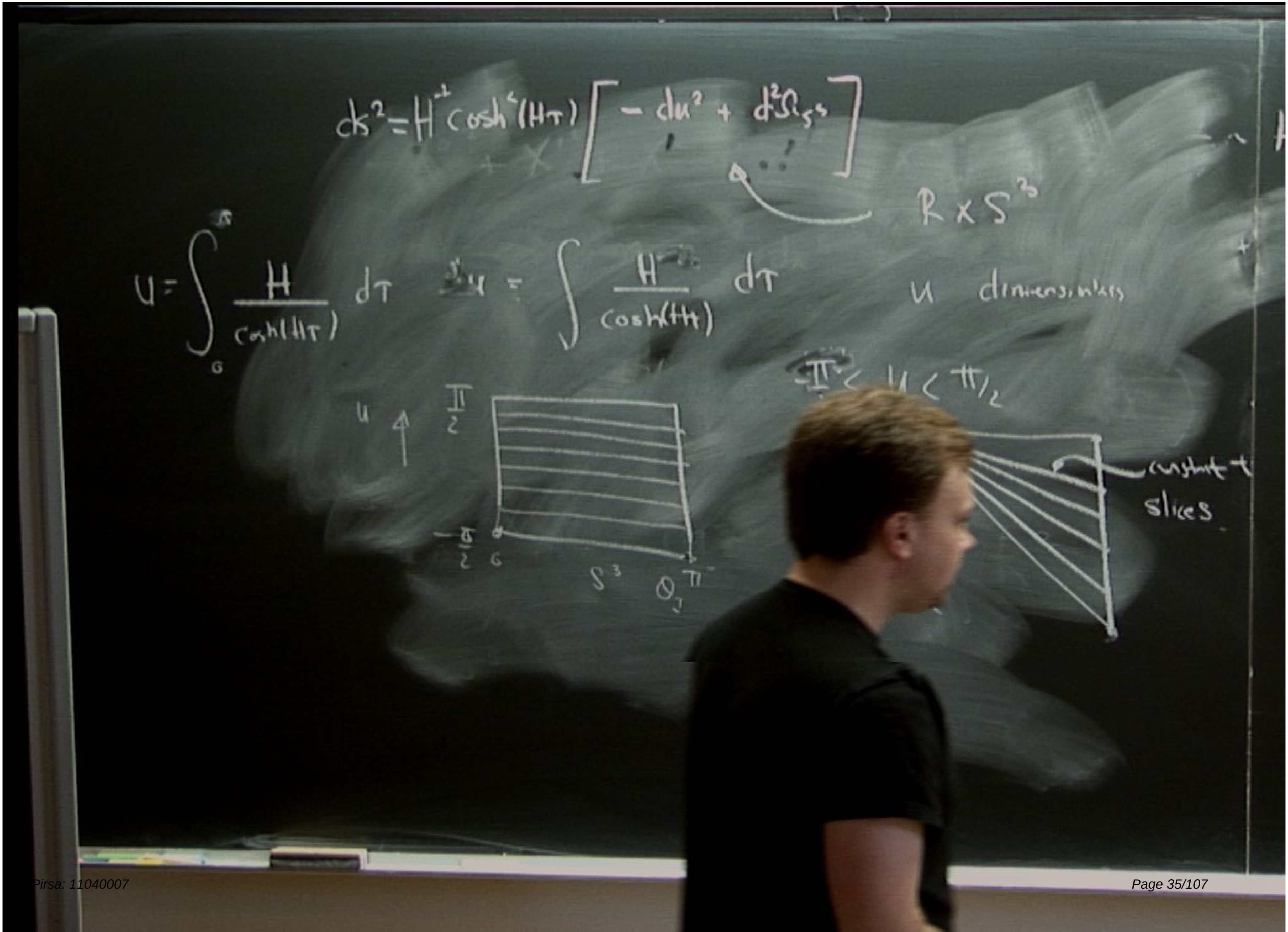
$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$R \times S^3$

$$u = \int_0^{\infty} \frac{H}{\cosh(H\tau)}\, d\tau \qquad = \int \frac{H}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$< u < \pi/2$



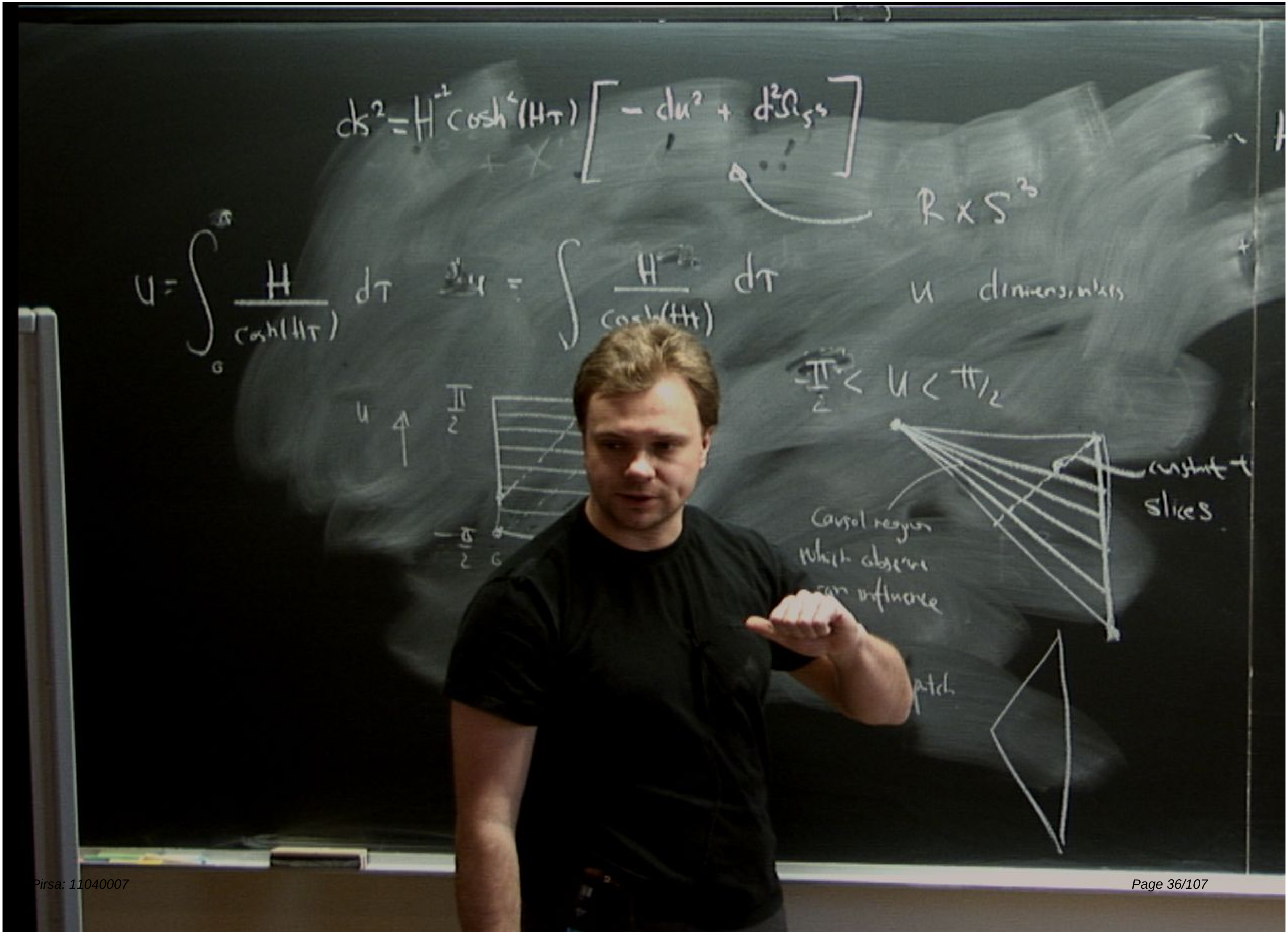
$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

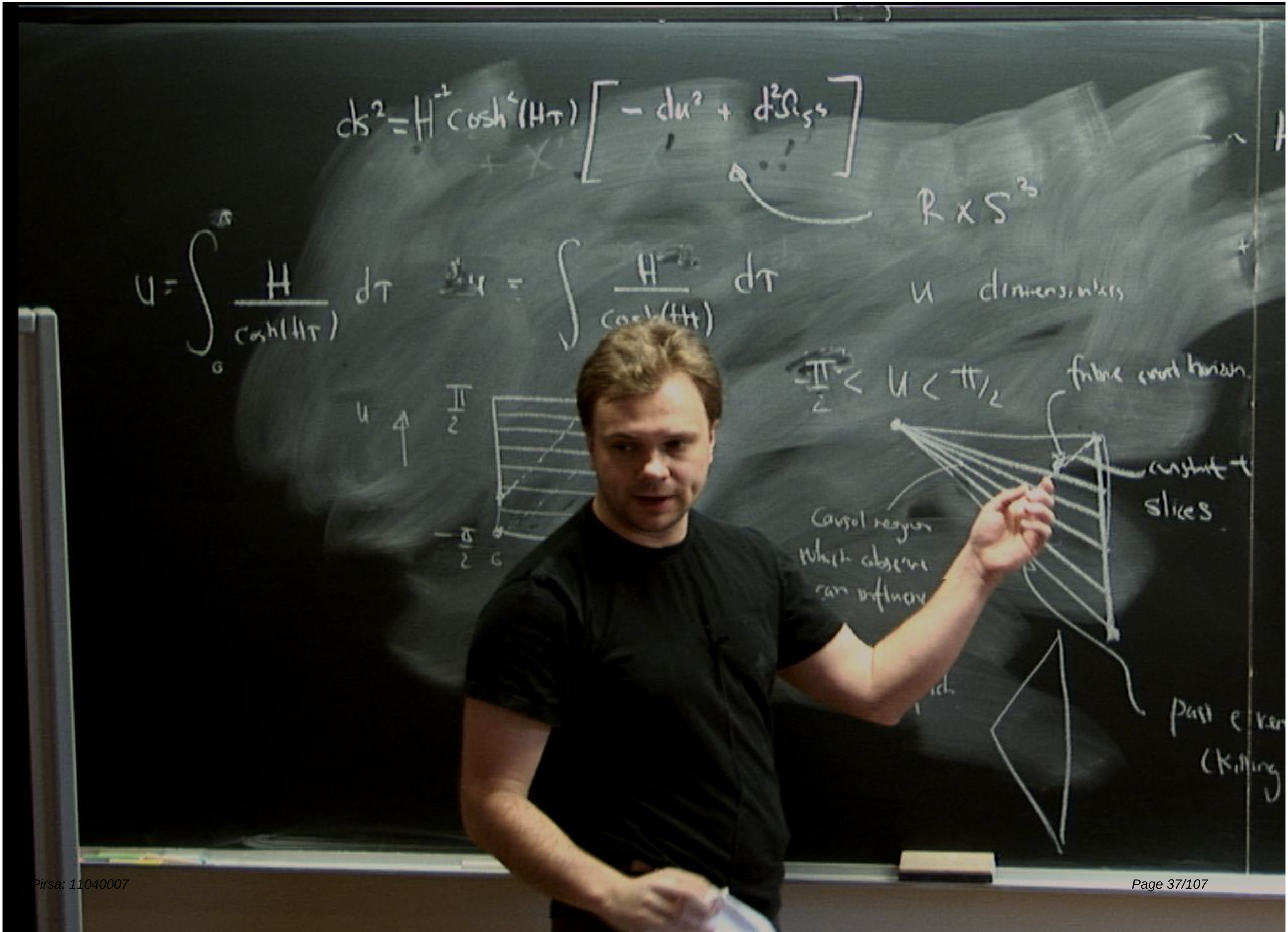
$R \times S^3$

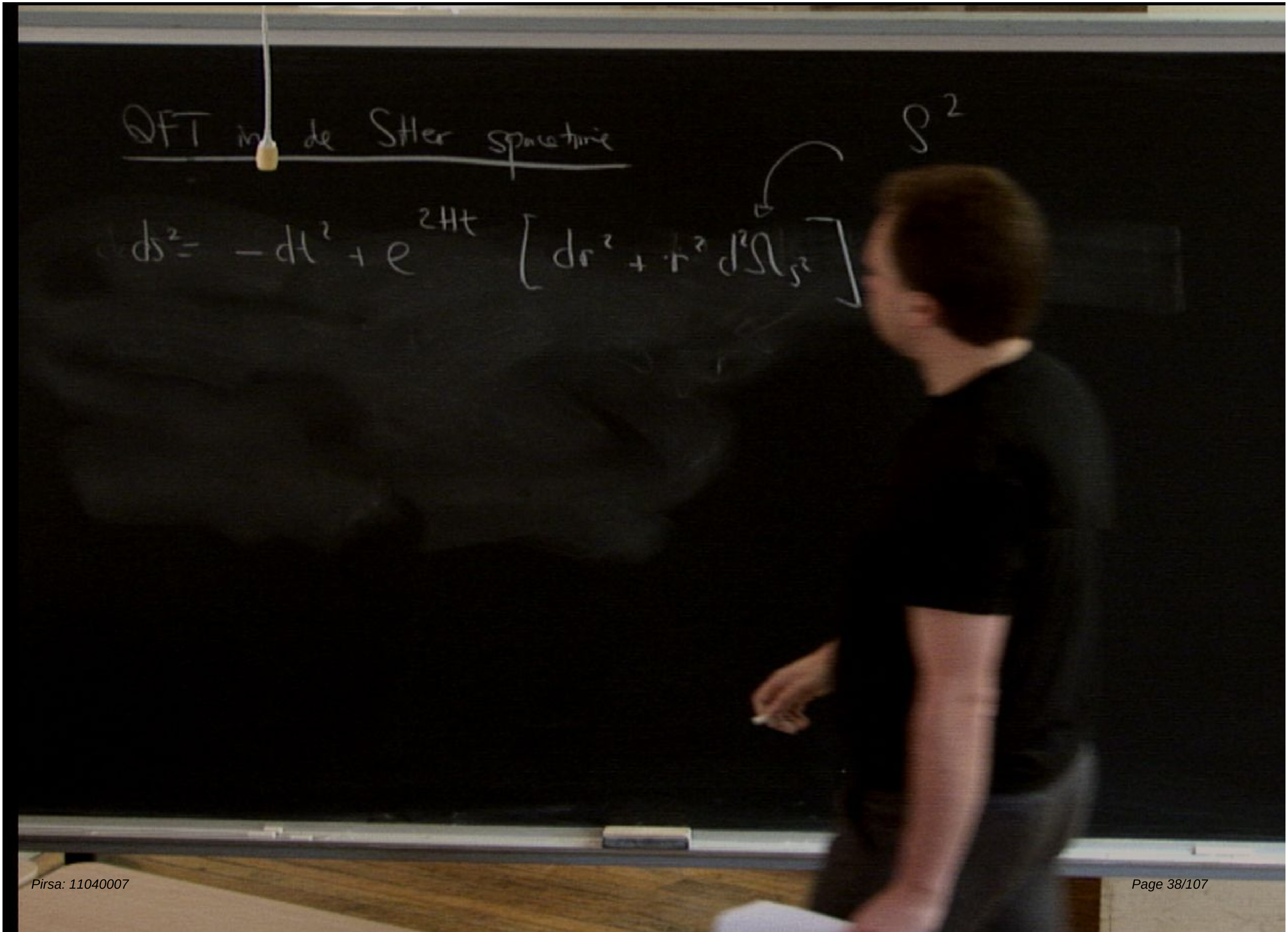
$$u = \int_0^{\tau} \frac{H}{\cosh(H\tau)}\, d\tau \qquad u = \int \frac{H^{-1}}{\cosh(H\tau)}\, d\tau$$

u dimensionless

$-\frac{\pi}{2} < u < \pi/2$


$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$
$R \times S^3$
$u = \int_0^{\infty} \frac{H}{\cosh(H\tau)}\, d\tau$
$= \int \frac{H^{-1}}{\cosh(H\tau)}\, d\tau$
u dimensionless
$-\frac{\pi}{2} < u < \pi/2$
u
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
Causal region
which observer
can influence
constant-τ
slices
patch

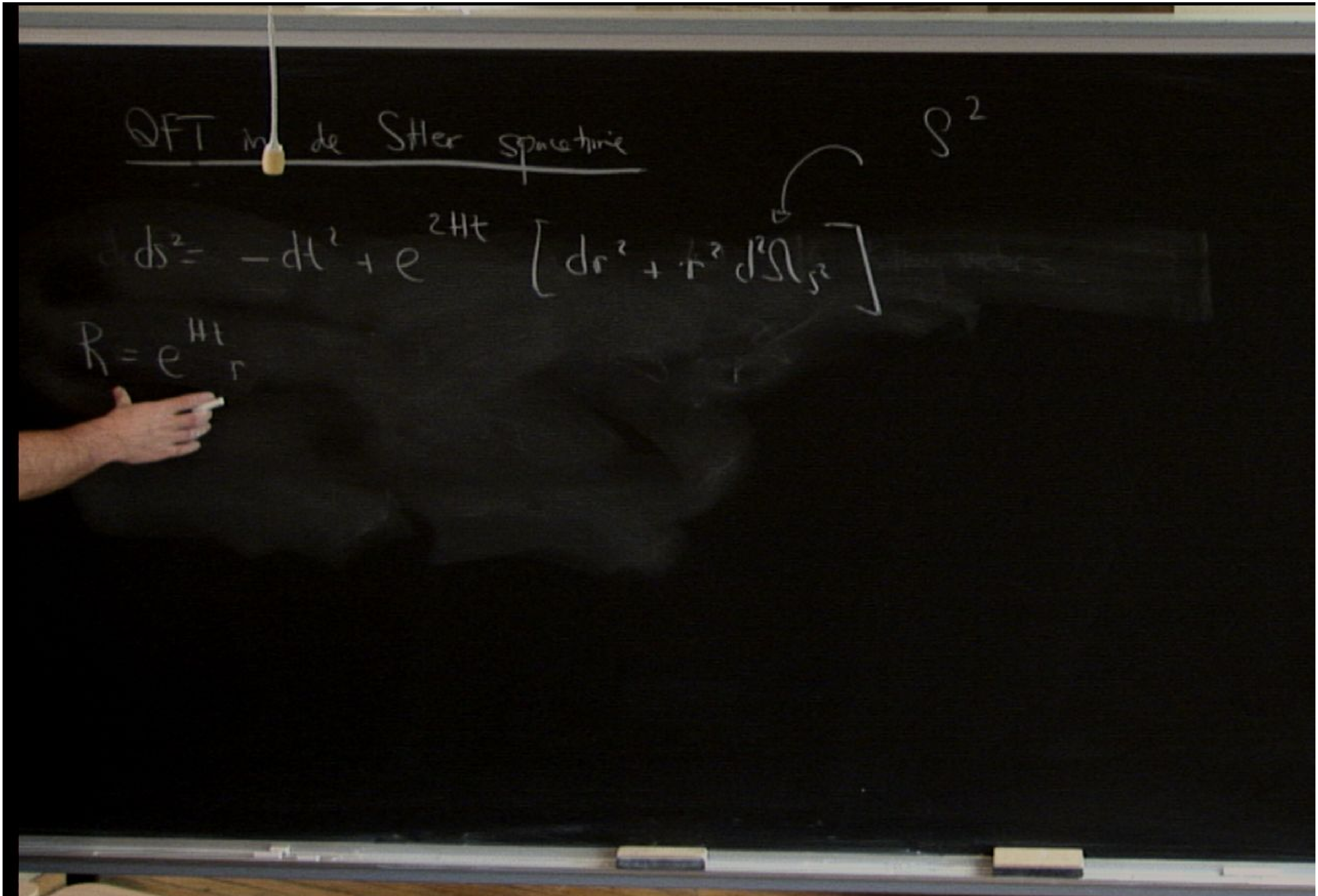

$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$
$R \times S^3$
$u = \int_0^\infty \frac{H}{\cosh(H\tau)}\, d\tau$
$u = \int \frac{H^{-1}}{\cosh(H\tau)}\, d\tau$
u dimensionless
$-\frac{\pi}{2} < u < \pi/2$
u
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
future event horizon
constant-τ
slices
Causal region
which observer
can influence
past event
(Killing



QFT in de Sitter spacetime

S^2

$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$



QFT in de Sitter spacetime

S^2

$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

$$R = e^{Ht} r$$

QFT in de Sitter spacetime

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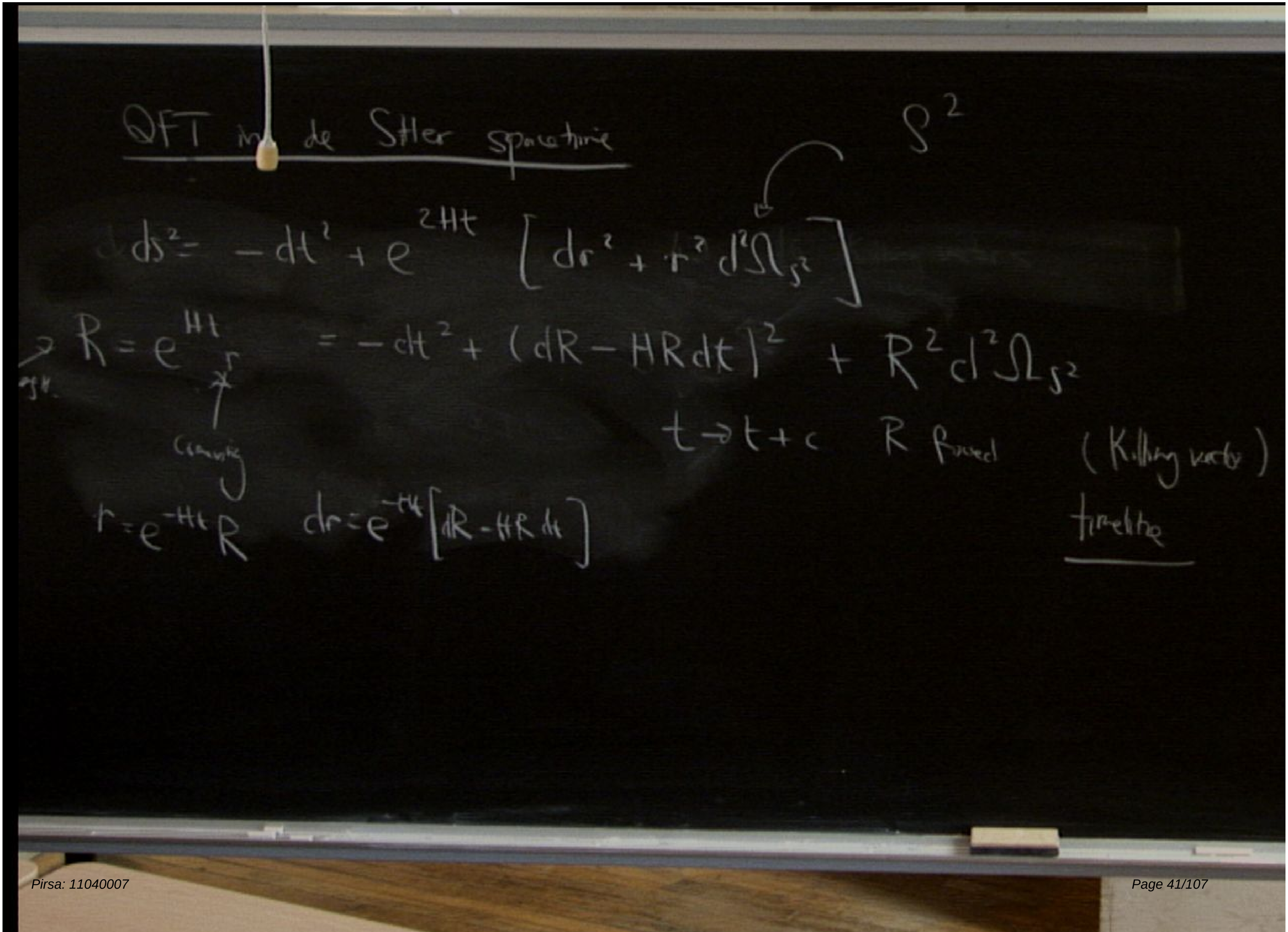
$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

$$\Rightarrow R = e^{Ht} r \qquad = -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S^2}$$

ving



QFT in de Sitter spacetime

S^2

$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

$R = e^{Ht} r$

$$= -dt^2 + (dR - HRdt)^2 + R^2 d^2\Omega_{S^2}$$

comoving

$r = e^{-Ht} R$ $\quad dr = e^{-Ht}\left[dR - HR\,dt\right]$

$t \to t + c$ $\quad R$ fixed

(Killing vector)

timelike

Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector

1

$$X^0 = H^{-1}\sinh(Ht) + \frac{Hr^2}{2}e^{Ht}$$

$r^2 = |\vec{x}|^2$

flat ~ Euclidean space

4

$$X^4 = H^{-1}\cosh(Ht) - \frac{Hr^2}{2}e^{Ht}$$

$$X^i = e^{Ht}x^i$$

$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2} \checkmark$$

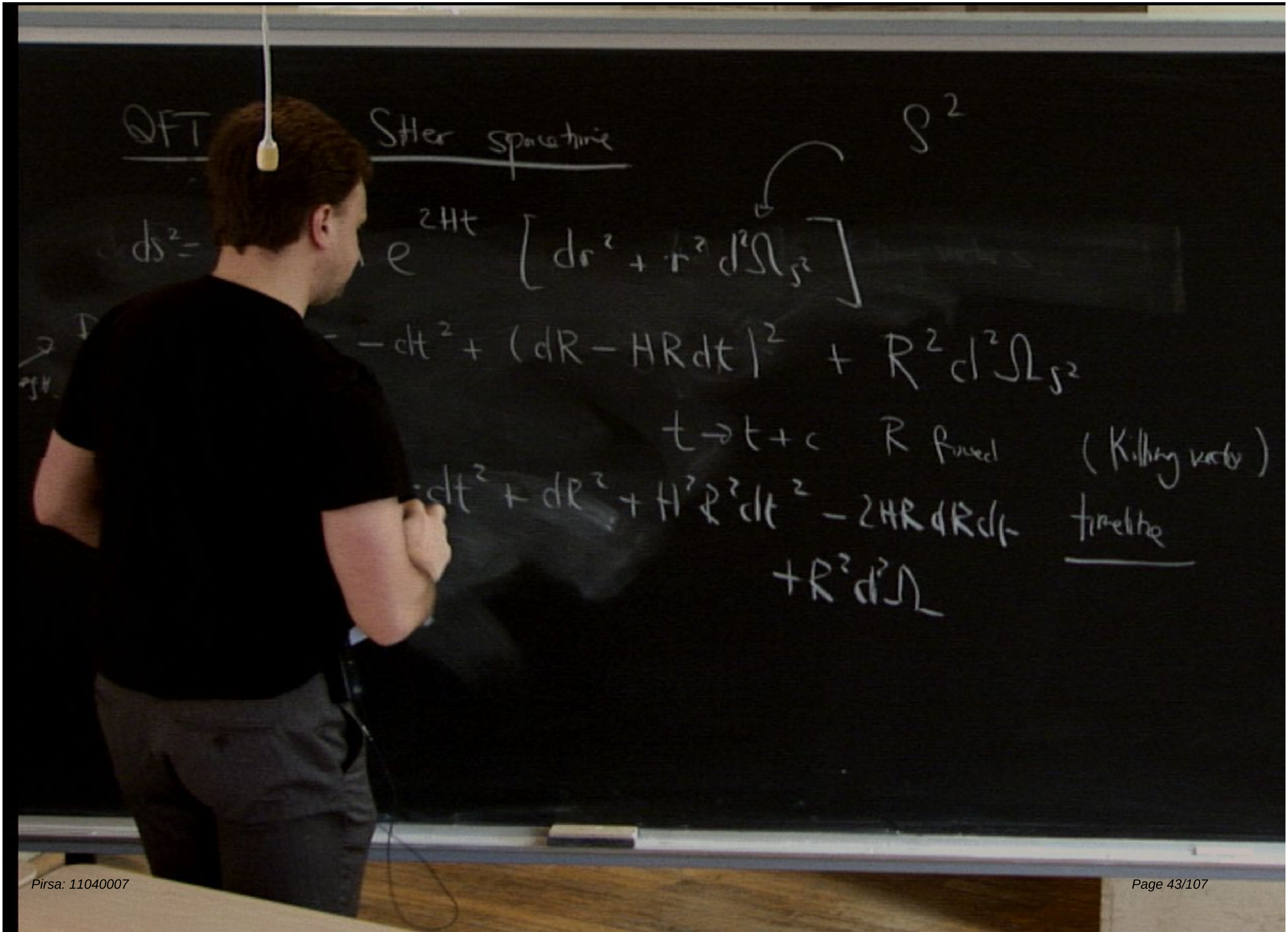
$$t \to t + c$$

$$\vec{x} \to \vec{x}\,e^{-Hc}$$

$$e^{2Ht}d\vec{x}^2 \to \cancel{e^{2Hc}}\,e^{2Ht}\,\cancel{e^{-2Hc}}\,d\vec{x}^2$$

nt horizon

) horizon)



QFT Sitter spacetime

S^2

$$ds^2 = \quad e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

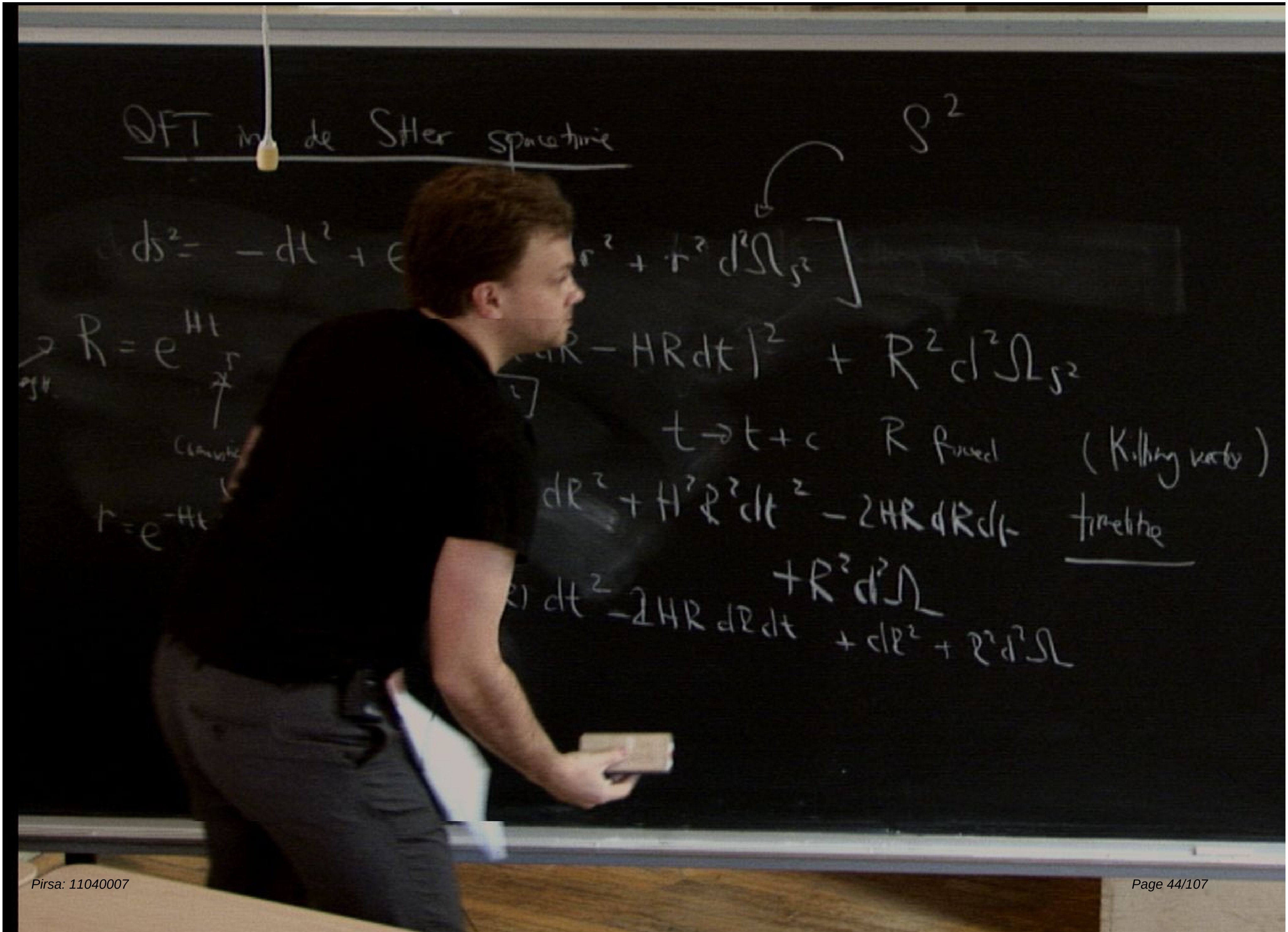
$$-dt^2 + (dR - HRdt)^2 + R^2 d^2\Omega_{S^2}$$

$t \to t + c$ R fixed (Killing vector)

$$dt^2 + dR^2 + H^2R^2dt^2 - 2HR\,dR\,dt$$

timelike

$$+R^2 d^2\Omega$$



QFT in de Sitter spacetime

S^2

$ds^2 = -dt^2 + e \ldots r^2 + r^2 d^2\Omega_{S^2}]$

$R = e^{Ht} r$

$\ldots R - HR\,dt)^2 + R^2 d^2\Omega_{S^2}$

$t \to t + c$ R fixed (Killing vector)

$\ldots dR^2 + H^2 R^2 dt^2 - 2HR\,dR\,dt$ timelike

$+R^2 d^2\Omega$

$\ldots) dt^2 - 2HR\,dR\,dt + dR^2 + R^2 d^2\Omega$

$r = e^{-Ht} \ldots$

QFT in de Sitter spacetime

S^2

$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

$R = e^{Ht} r$

comoving

$r = e^{-Ht} R$

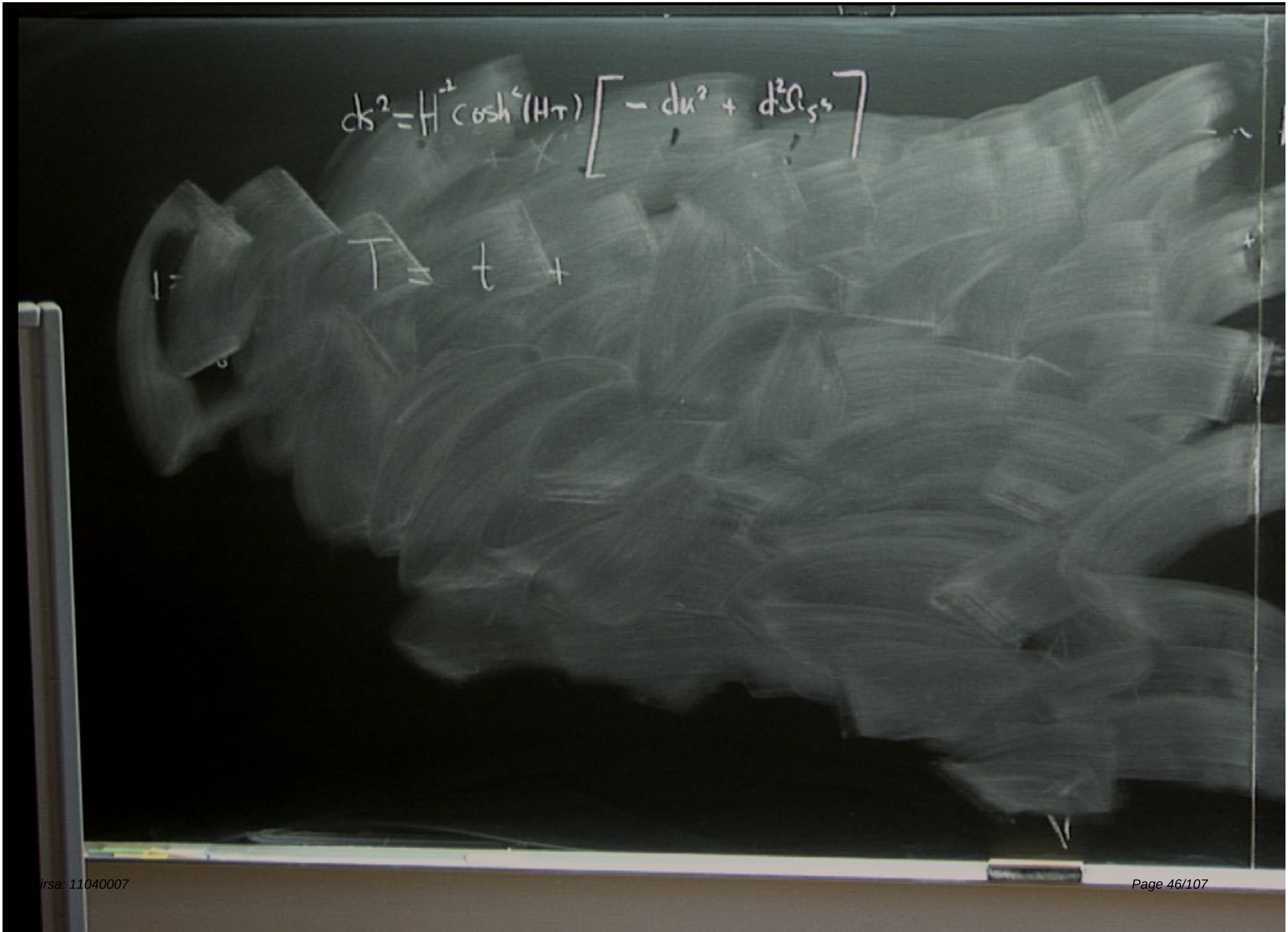
$$= -dt^2 + (dR - HR\,dt)^2 + R^2 d^2\Omega_{S^2}$$

$$\boxed{f(R) = 1 - H^2R^2}$$

$t \to t + c$ R fixed (Killing vector) timelike

$$= -dt^2 + dR^2 + H^2R^2dt^2 - 2HR\,dR\,dt + R^2d^2\Omega$$

$$= -f(R)\,dt^2 - 2HR\,dR\,dt + dR^2 + R^2d^2\Omega$$



$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^4}\right]$$

$$T = t +$$

QFT in de Sitter spacetime

S^2

$$ds^2 = -dt^2 + e^{2Ht}\left[dr^2 + r^2 d^2\Omega_{S^2}\right]$$

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ngH.

comoving

$$\boxed{f(R) = 1 - H^2R^2}$$

$t \to t + c$ R fixed (Killing vector)

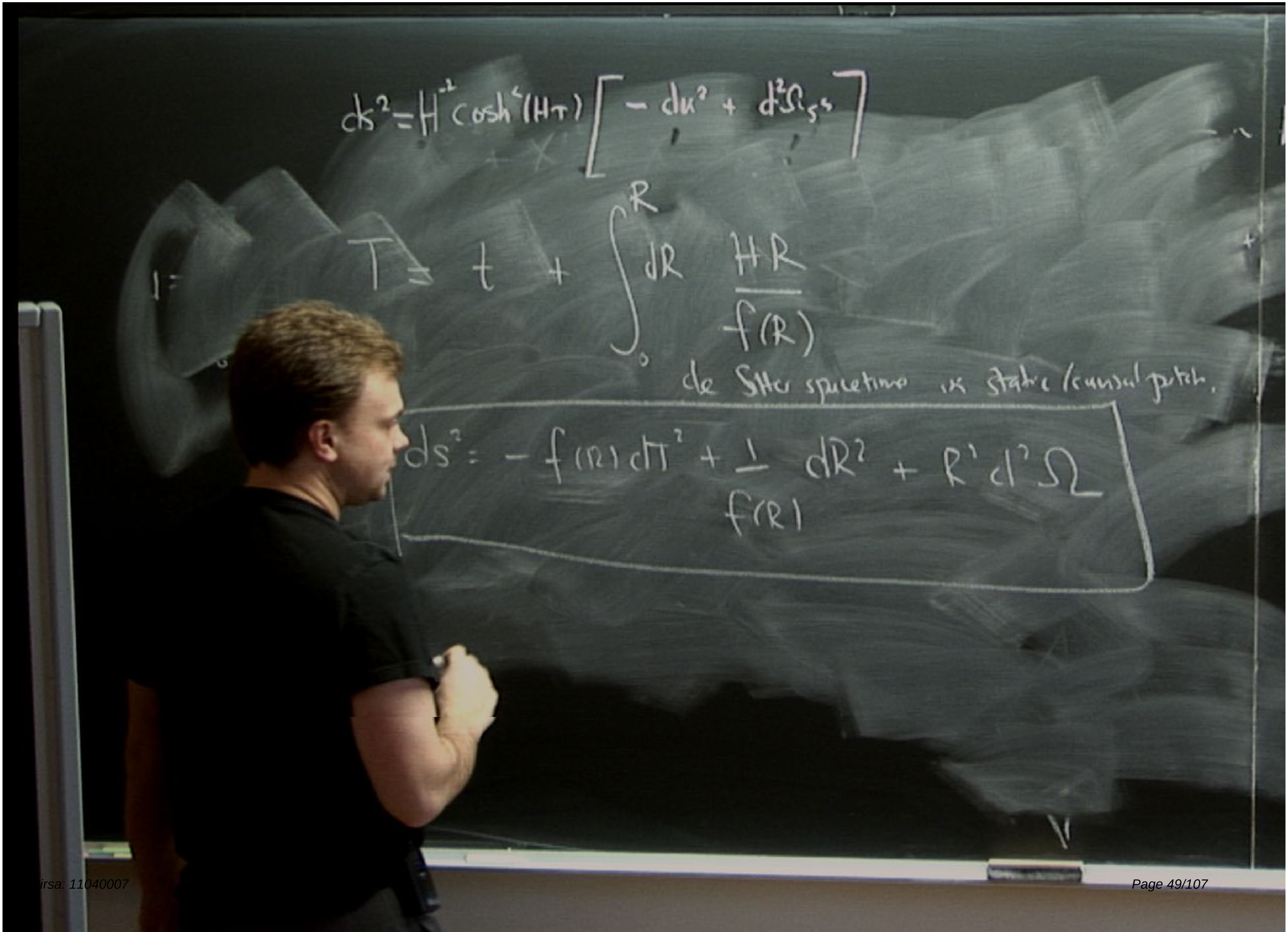
$$r = e^{-Ht} R \qquad = -dt^2 + dR^2 + H^2R^2dt^2 - 2HR\,dR\,dt + R^2 d^2\Omega$$

timelike

$$= -f(R)\,dt^2 - 2HR\,dR\,dt + dR^2 + R^2 d^2\Omega$$

$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^5}\right]$$

$$T = t + \int_0^R dR\,\frac{HR}{f(R)}$$

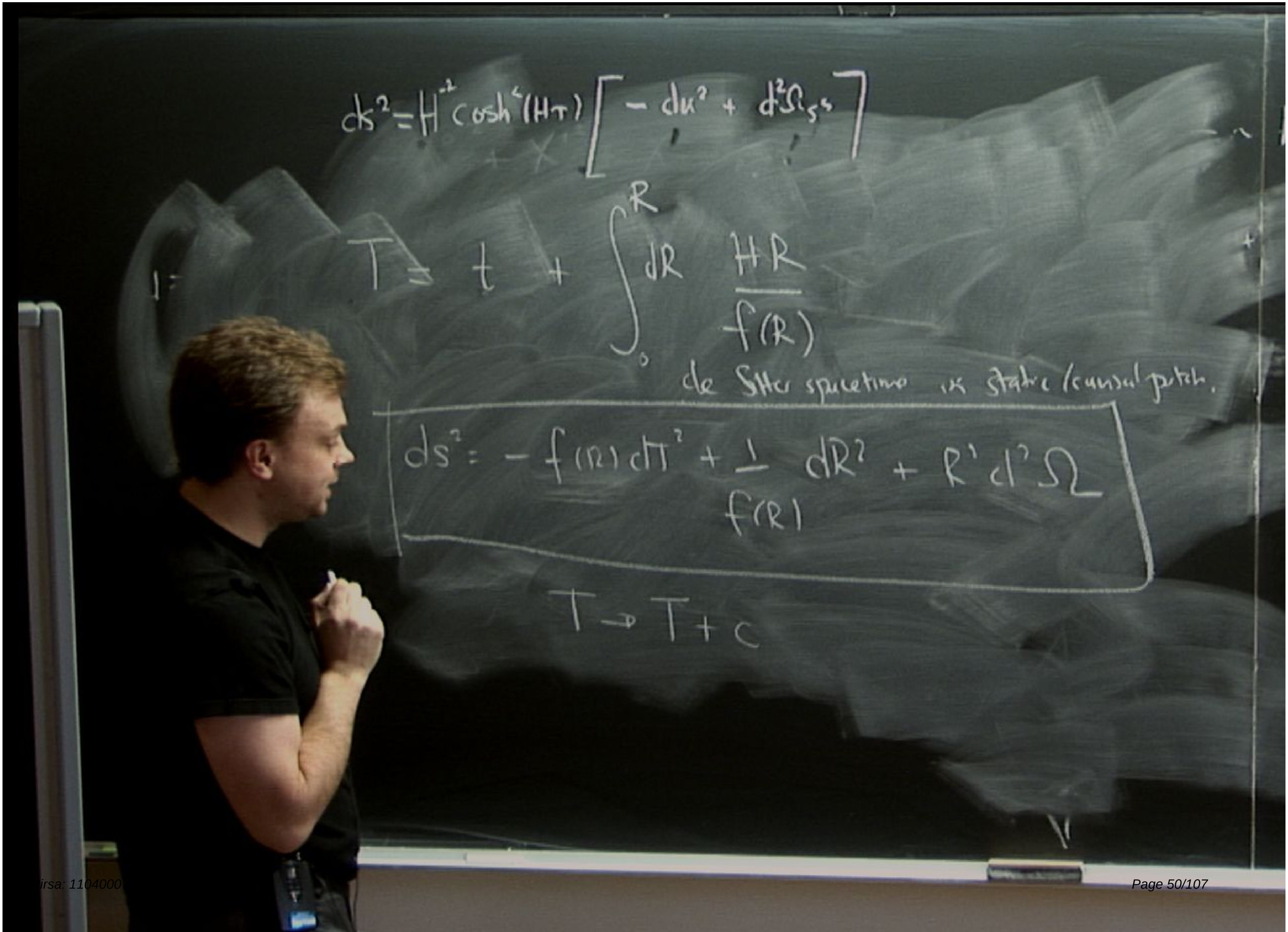


$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^3}\right]$$

$$T = t + \int_0^R dR\,\frac{HR}{f(R)}$$

de Sitter spacetime in static (causal) patch.

$$\boxed{ds^2 = -f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2 d^2\Omega}$$



$$ds^2 = H^{-2}\cosh^2(H\tau)\left[-du^2 + d^2\Omega_{S^4}\right]$$

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de Sitter spacetime in static (causal) patch.

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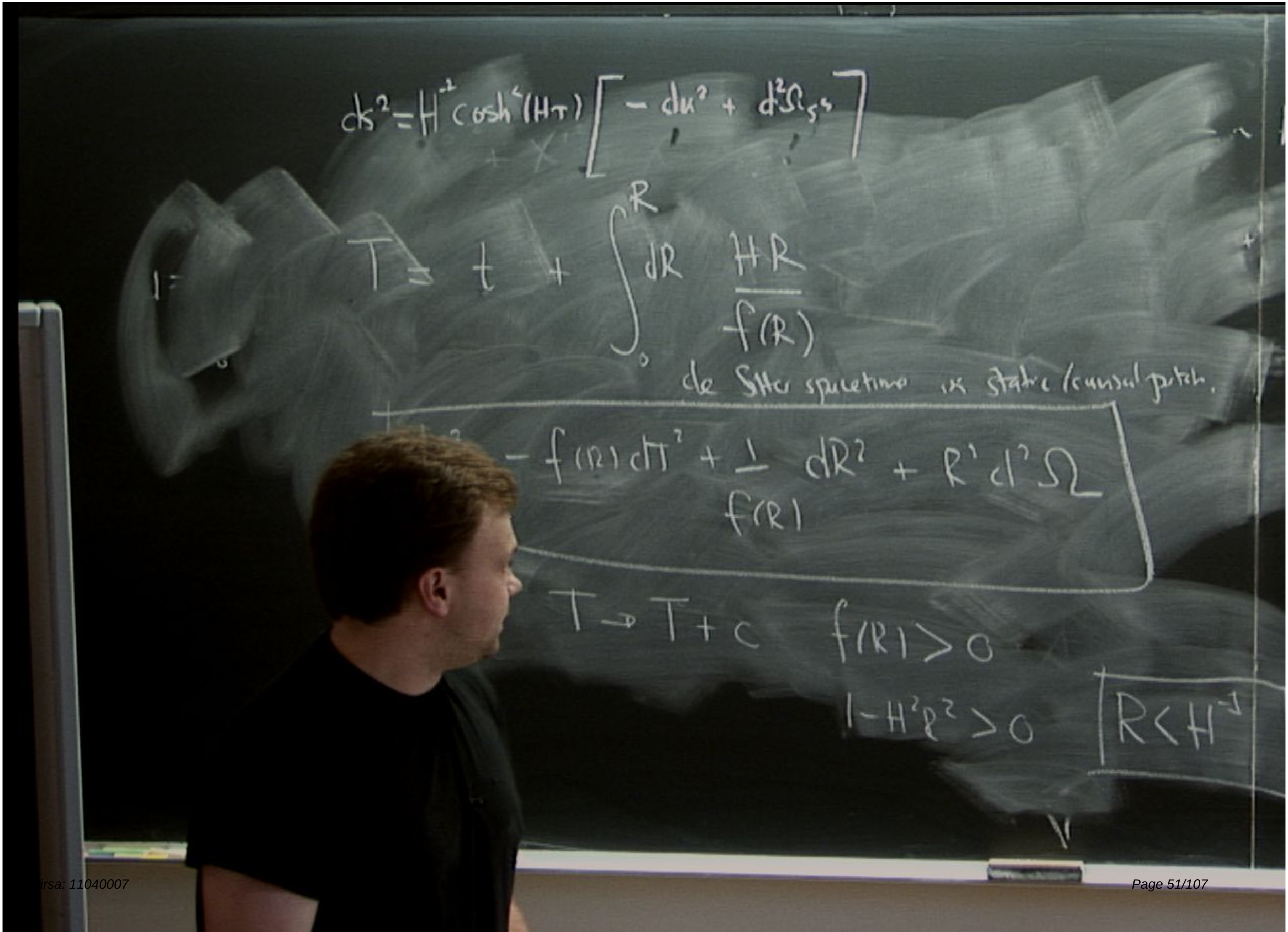
$$T \to T + c$$



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de Sitter spacetime in static (causal) patch.

$$\boxed{-f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2 d^2\Omega}$$

$$T \to T + c \qquad f(R) > 0$$

$$1 - H^2R^2 > 0 \qquad \boxed{R < H^{-1}}$$

$$\tau) \left[- du^2 + d^2\Omega_{S^4} \right]$$

$$+ \int_0^R dR \, \frac{HR}{f(R)}$$

de Sitter spacetime in static (causal) patch.

$$\boxed{f(R)\, dT^2 + \frac{1}{f(R)} dR^2 + R^2 d^2\Omega}$$

$$T \to T + c \qquad f(R) > 0$$

$$1 - H^2 R^2 > 0 \qquad \boxed{R < H^{-1}}$$

H Hubble scale:

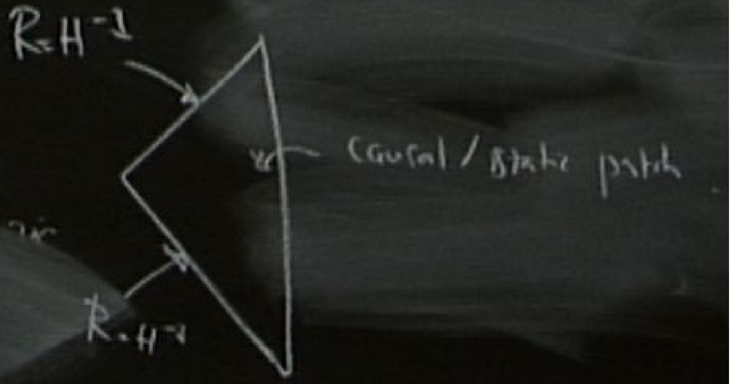
Flat slicing & de

$$X^0 = H^{-1} \sinh(Ht) + H$$

$$X^4 = H^{-1} \cosh(Ht) -$$

$$X^i = e^{Ht} x^i$$

$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2} \checkmark$$



$$T)\left[-du^2+d^2\Omega_{S^3}\right]$$

$$+\int_0^R dR\,\frac{HR}{f(R)}$$

de Sitter spacetime in static (causal) patch.

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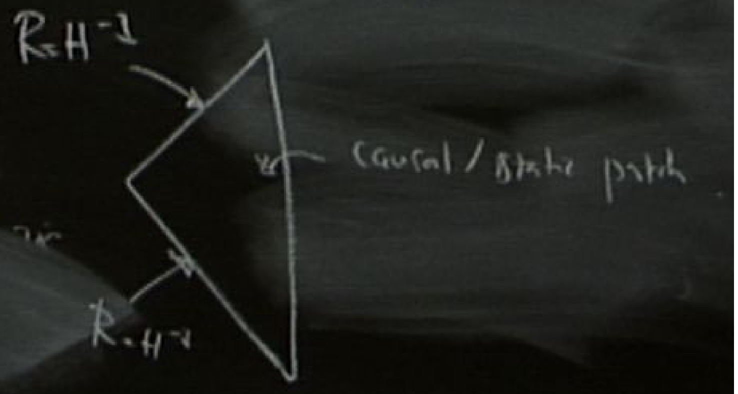

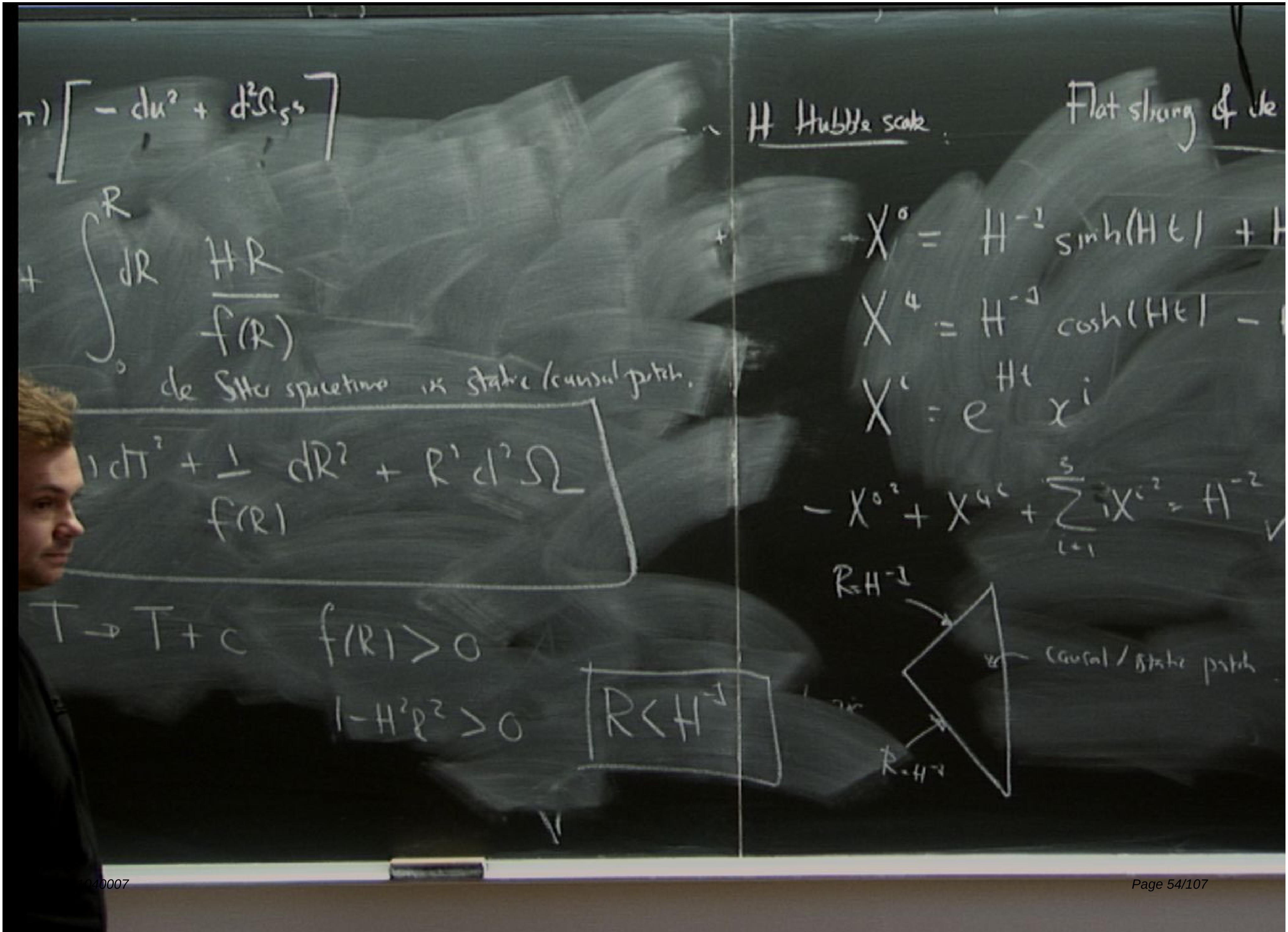

$\tau)\left[-du^2 + d^2\Omega_{S^4} \right]$

$+ \int_0^R dR \, \frac{HR}{f(R)}$

de Sitter spacetime is static (causal patch.

$$\boxed{)\,dT^2 + \frac{1}{f(R)} dR^2 + R^2 d^2\Omega}$$

$T \to T + c \qquad f(R) > 0$

$1 - H^2 R^2 > 0 \qquad \boxed{R < H^{-1}}$

H Hubble scale:

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$X^0 = H^{-1} \sinh(Ht) + H$

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$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2}$

$$\tau)\left[-du^2 + d^2\Omega_{S^4}\right]$$

$$+\int_0^R dR\,\frac{HR}{f(R)}$$

de Sitter spacetime in static (causal) patch.

$$\boxed{f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2\,d^2\Omega}$$

$$T \to T + c \qquad f(R) > 0$$

$$1 - H^2R^2 > 0 \qquad \boxed{R < H^{-1}}$$

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$$X^0 = H^{-1}\sinh(Ht) + H$$

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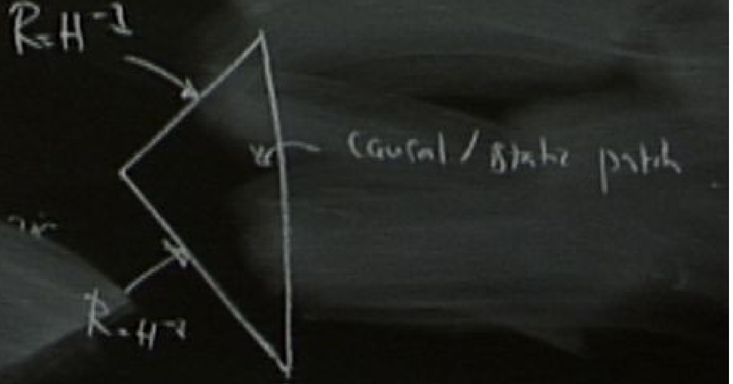
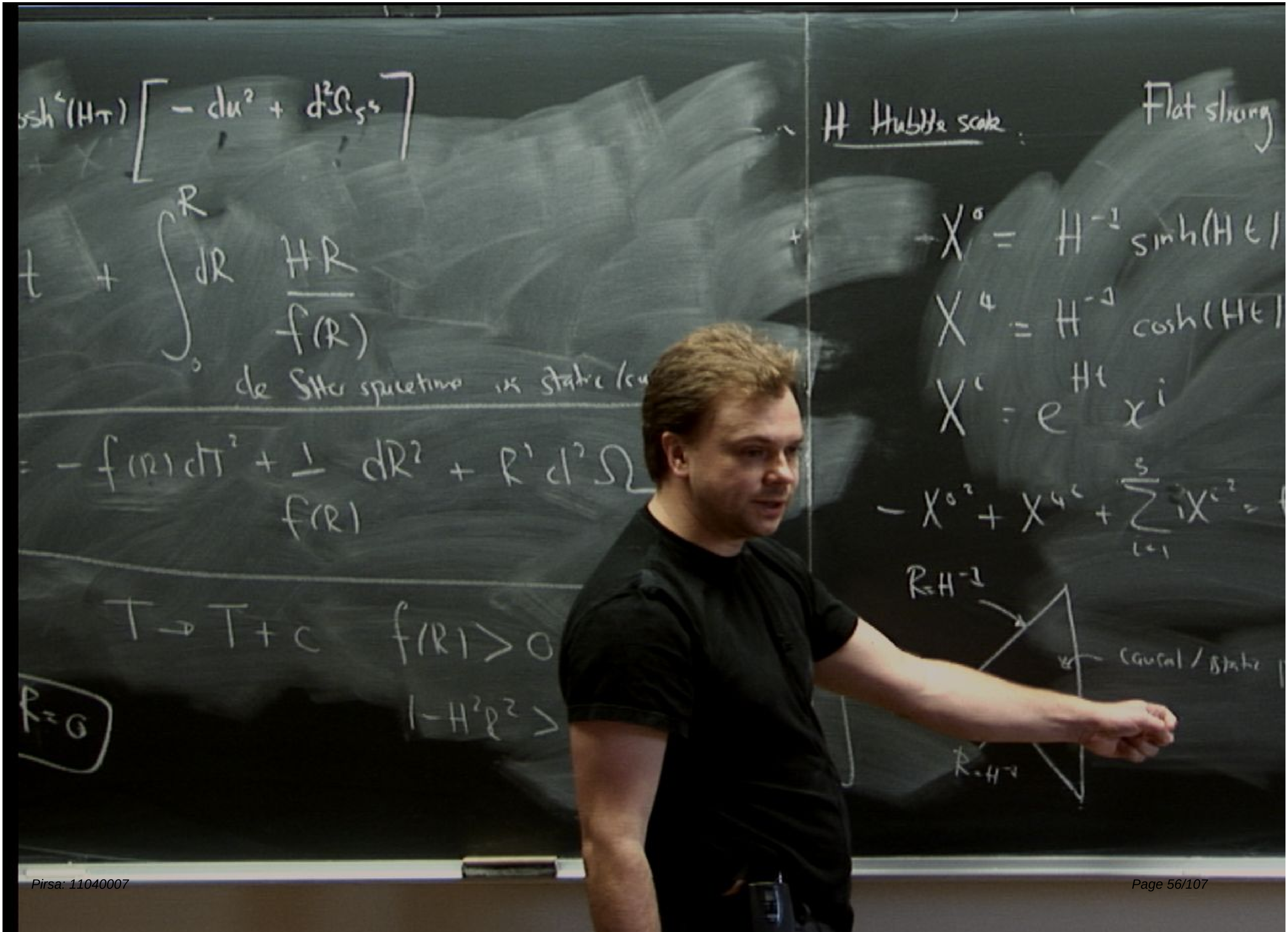
$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2} \quad \checkmark$$





$\text{osh}^2(H\tau)\left[-du^2 + d^2\Omega_{S^4}\right]$

$t + \int_0^R dR \frac{HR}{f(R)}$

de Sitter spacetime is static (cu

$= -f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2 d^2\Omega$

$T \to T + c \qquad f(R) > 0$

$\boxed{R=0}$

$1 - H^2R^2 >$

H Hubble scale :

Flat slicing

$X^0 = H^{-1}\sinh(Ht)$

$X^4 = H^{-1}\cosh(Ht)$

$X^i = e^{Ht}x^i$

$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} =$

$R = H^{-1}$

causal / static p

$R = H^{-1}$

$$sh^4(H\tau)\left[-du^2 + d^2\Omega_{S^4}\right]$$

$$t + \int_0^R dR\,\frac{HR}{f(R)}$$

de Sitter spacetime in static (causal) patch.

$$\boxed{= -f(R)\,dT^2 + \frac{1}{f(R)}\,dR^2 + R^2 d^2\Omega}$$

$$T \to T + c \qquad f(R) > 0$$

$$\boxed{R=0} \qquad 1 - H^2R^2 > 0 \qquad \boxed{R<H^{-1}}$$

H Hubble scale:

Flat slicing

$$X^0 = H^{-1}\sinh(Ht)$$

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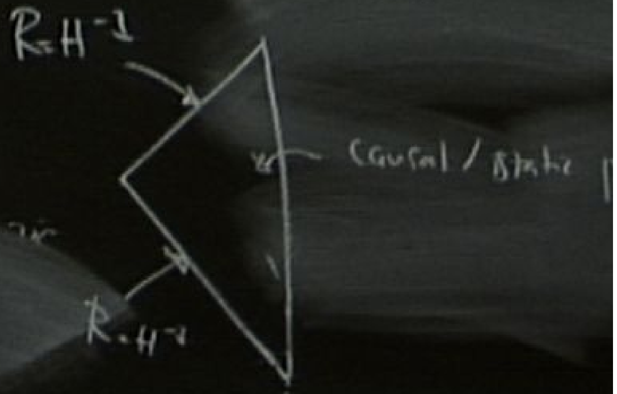

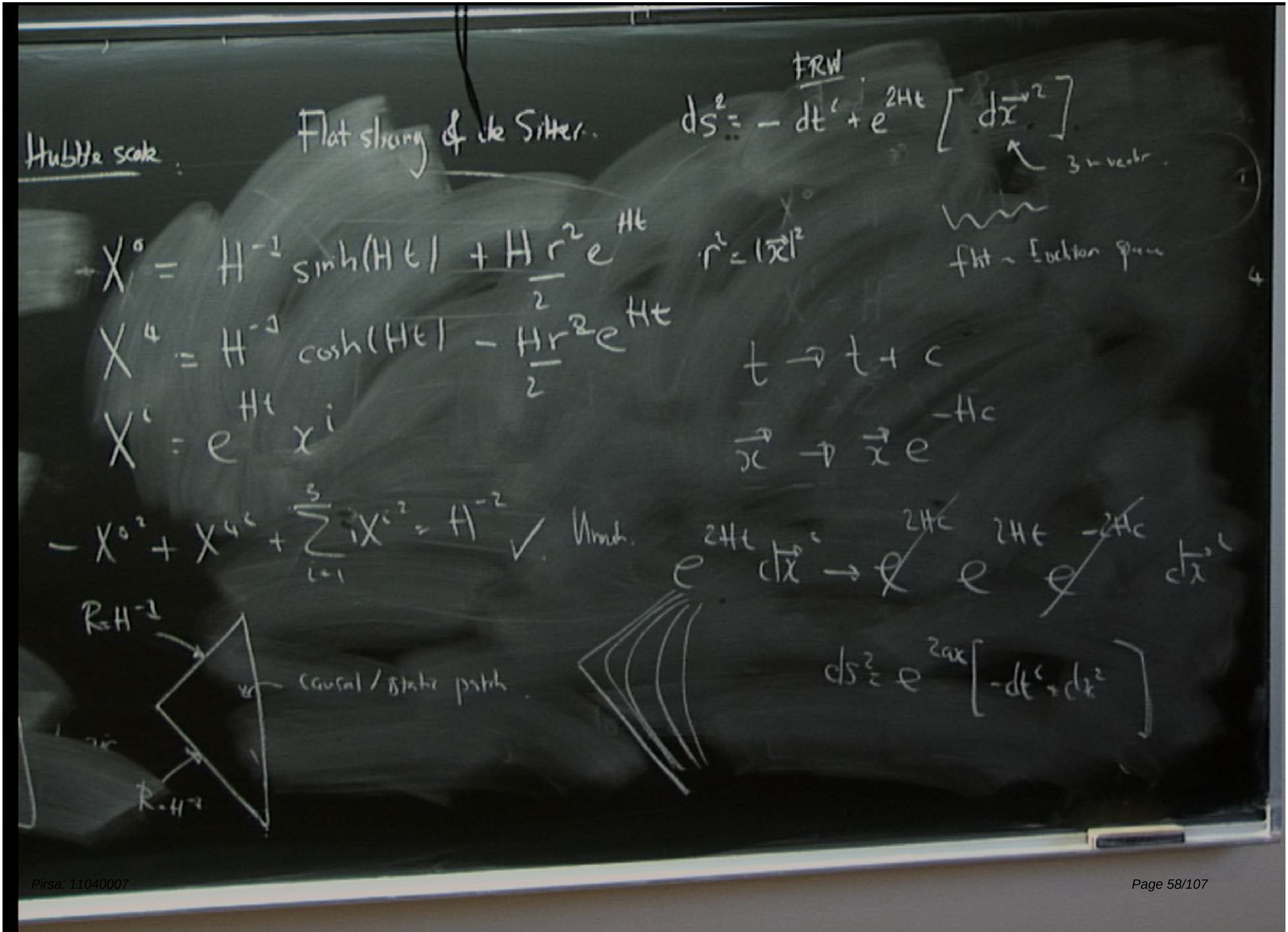

Hubble scale:

Flat slicing of de Sitter.

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$$X^i = e^{Ht} x^i$$

$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2}$$ ✓ Umch.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^{\,2}\right]$$

3-vector.

fixed t ~ Euclidean space

$$r^2 = |\vec{x}|^2$$

$$t \to t + c$$

$$\vec{x} \to \vec{x}\, e^{-Hc}$$

$$e^{2Ht}\, d\vec{x}^{\,2} \to \cancel{e^{2Hc}}\, e^{2Ht}\, \cancel{e^{-2Hc}}\, d\vec{x}^{\,2}$$

$$ds^2 = e^{2ax}\left[-dt^2 + dx^2\right]$$

Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector.

flat ~ Euclidean space

$$X^0 = H^{-1}\sinh(Ht) + \frac{H r^2}{2} e^{Ht} \qquad r^2 = |\vec{x}|^2$$

$$X^4 = H^{-1}\cosh(Ht) - \frac{H r^2}{2} e^{Ht}$$

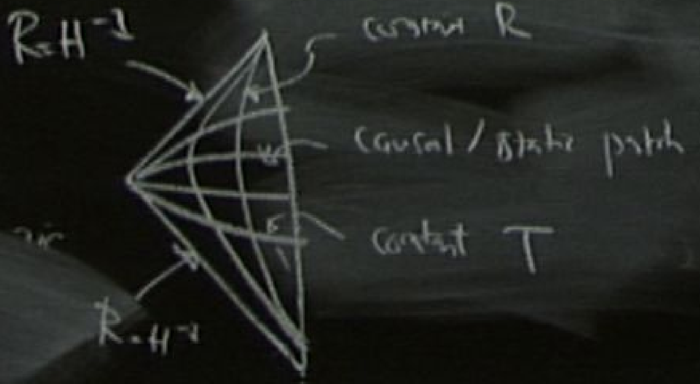
$$X^i = e^{Ht} x^i$$

$$-X^{0^2} + X^{4^2} + \sum_{i=1}^{3} X^{i^2} = H^{-2} \checkmark$$ Unruh.

$$t \to t + c$$

$$\vec{x} \to \vec{x}\, e^{-Hc}$$

$$e^{2Ht} d\vec{x}^2 \to \cancel{e^{2Hc}}\, e^{2Ht}\, \cancel{e^{-2Hc}}\, d\vec{x}^2$$



$$ds^2 = e^{2ax}\left[-dt^2 + dx^2\right]$$

4

Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

↖ 3-vector.

flat ~ Euclidean space

$$X^0 = H^{-1}\sinh(Ht) + \frac{Hr^2}{2}e^{Ht}$$

$$r^2 = |\vec{x}|^2$$

$$X^4 = H^{-1}\cosh(Ht) - \frac{Hr^2}{2}e^{Ht}$$

$$X^i = e^{Ht}x^i$$

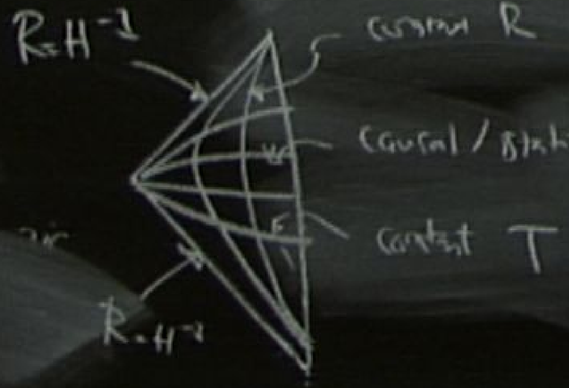
$$-X^{0\,2} + X^{4\,2} + \sum_{i=1}^{3} X^{i\,2} = H^{-2}$$ ✓ Umnh.

$$t \to t + c$$

$$\vec{x} \to \vec{x}e^{-Hc}$$

$$e^{2Ht}d\vec{x}^2 \to \cancel{e^{2Hc}}\,e^{2Ht}\,\cancel{e^{-2Hc}}\,d\vec{x}^2$$

4



$$ds^2 = e^{2ax}\left[-dt^2 + dx^2\right]$$

Hubble scale :

Flat slicing & de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector.

①

$$X^0 = H^{-1}\sinh(Ht) + \frac{Hr^2}{2}e^{Ht}$$

$$r^2 = |\vec{x}|^2$$

flat ~ Euclidean space

4

$$X^4 = H^{-1}\cosh(Ht) - \frac{Hr^2}{2}e^{Ht}$$

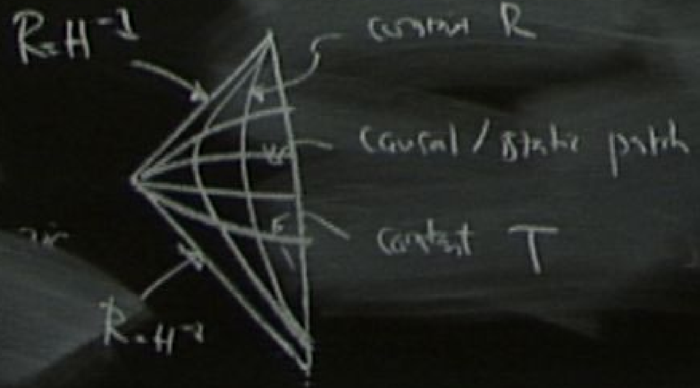
$$X^i = e^{Ht}x^i$$

$$t \rightarrow t + c$$

$$\vec{x} \rightarrow \vec{x}e^{-Hc}$$

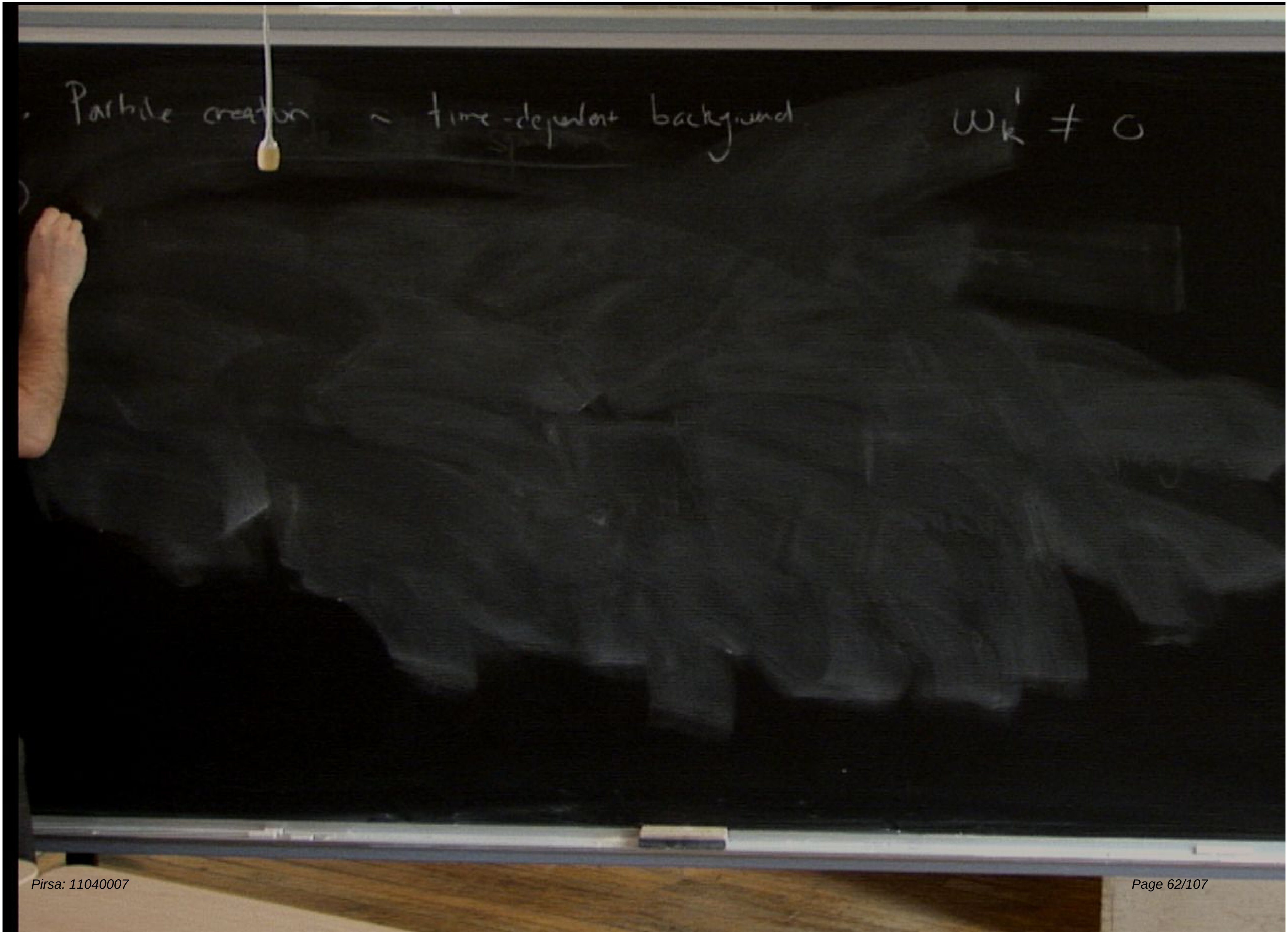
$$-X^{0^2} + X^{4^2} + \sum_{i=1}^{3} X^{i^2} = H^{-2}$$ ✓ Hmm.

$$e^{2Ht}d\vec{x}^2 \rightarrow \cancel{e^{2Hc}}\,e^{2Ht}\,\cancel{e^{-2Hc}}\,d\vec{x}^2$$

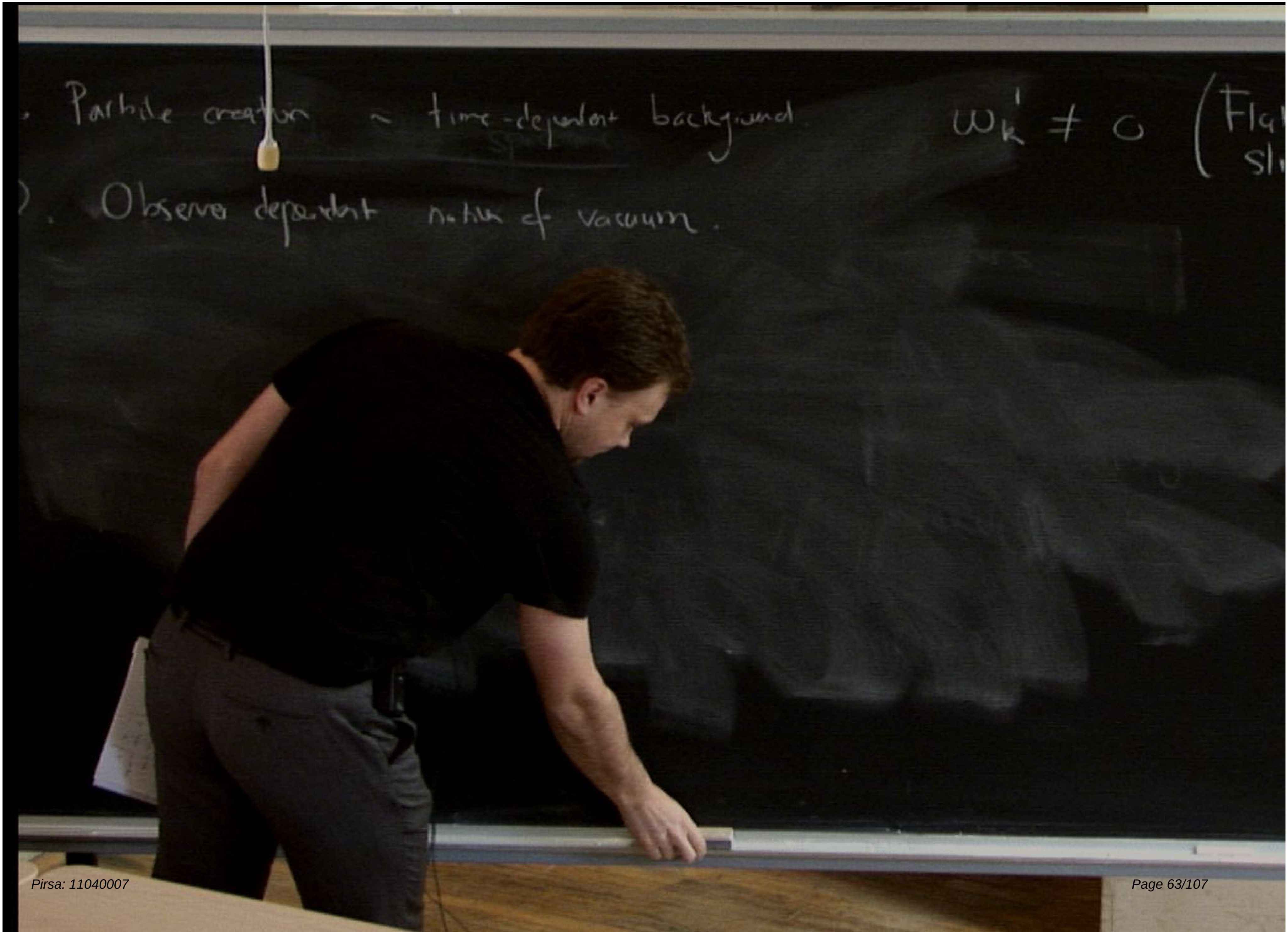


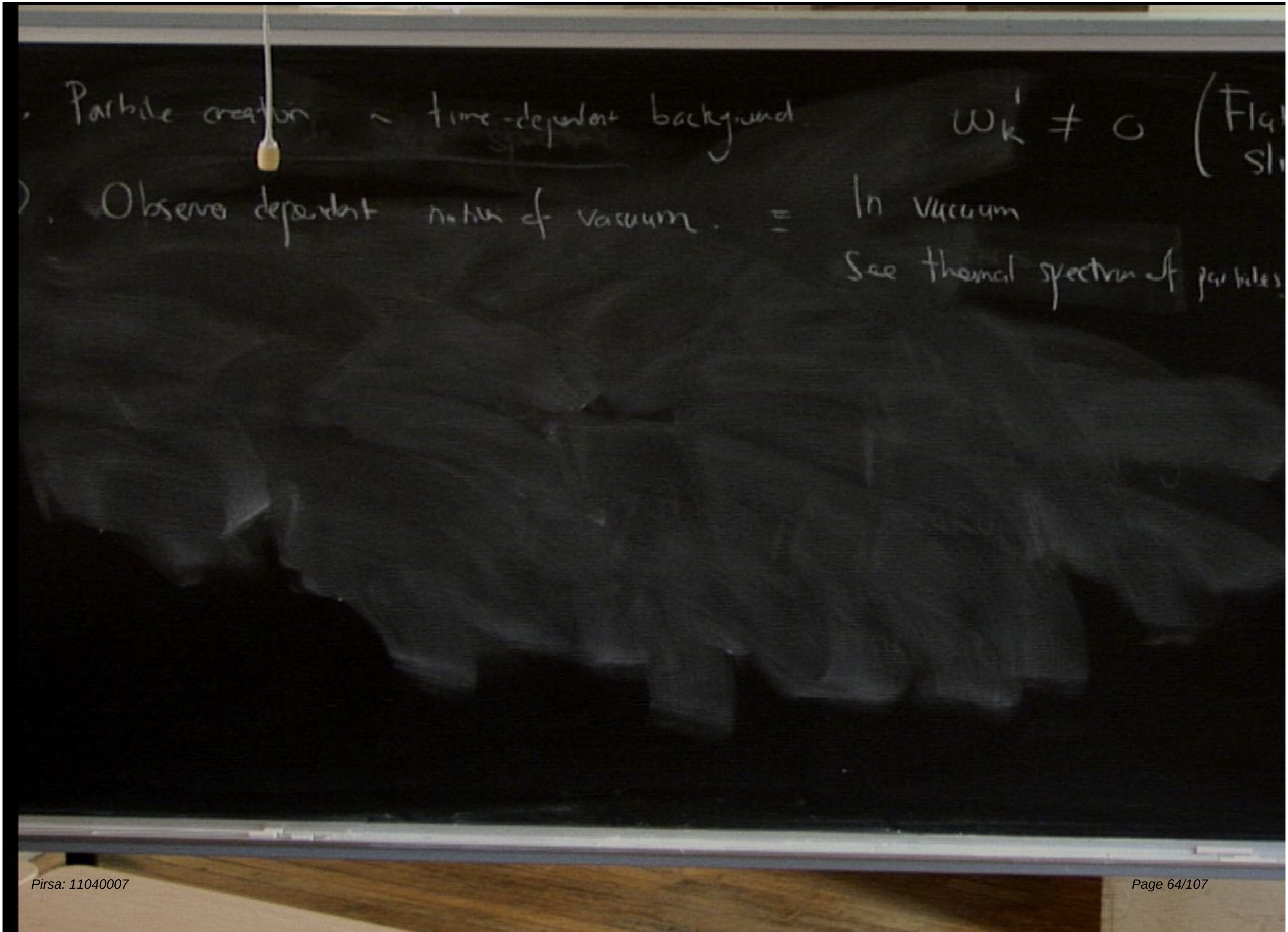
$$ds^2 = e^{2ax}\left[-dt^2 + dx^2\right]$$

$$= \left[-\rho^2 dt^2 + d\rho^2\right]$$



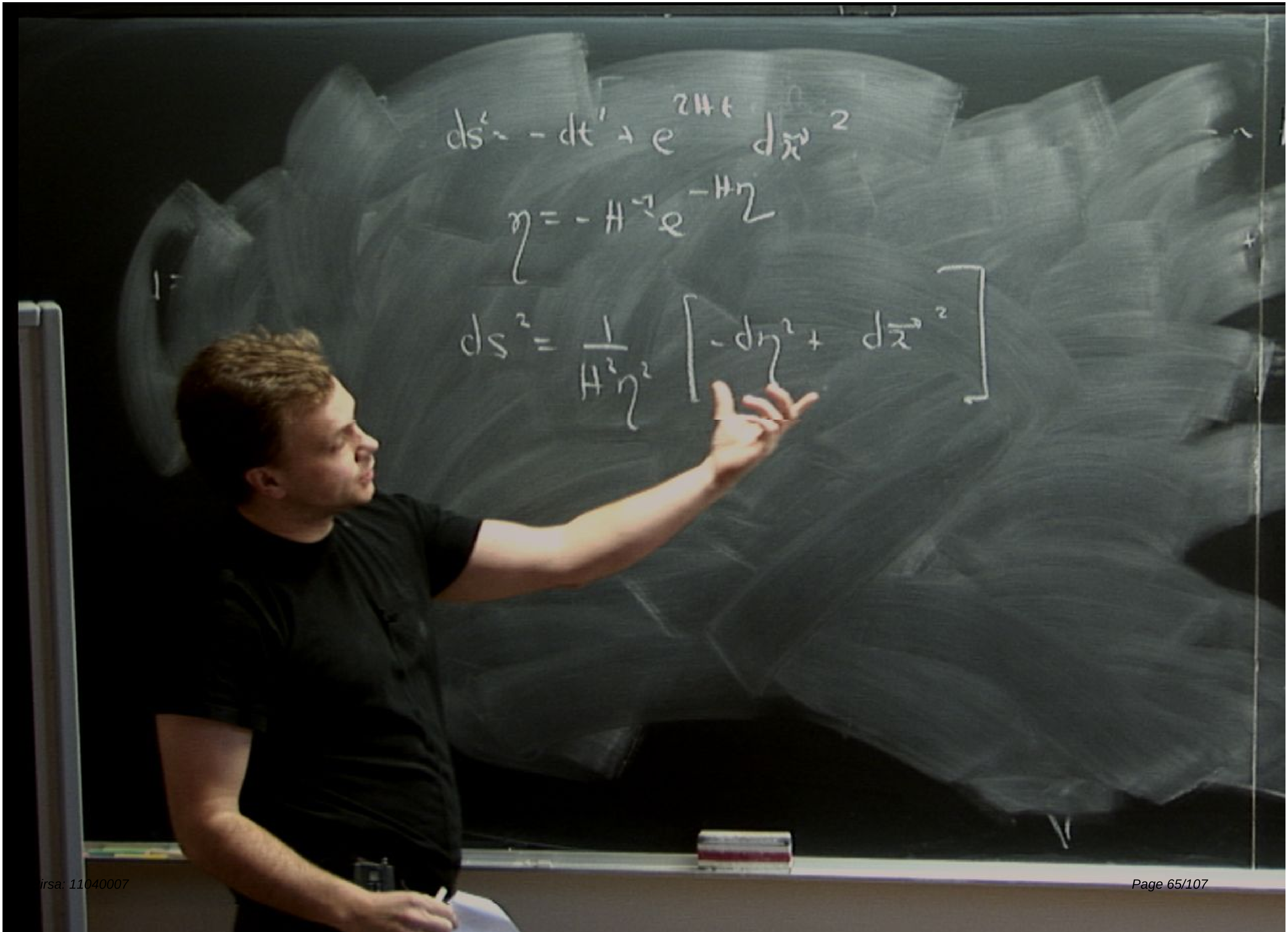
- Particle creation ~ time-dependent background $\omega_k' \neq 0$


. Particle creation ~ time-dependent background.
$\omega_k' \neq 0$ (Fla
sl
. Observer dependent notion of vacuum.



. Particle creation ~ time-dependent background $\omega_k' \neq 0$ (Fla
sl

). Observer dependent notion of vacuum. ≡ In vacuum
See thermal spectrum of particles.



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$\eta = -H^{-1} e^{-Ht}$$

$$ds^2 = \frac{1}{H^2\eta^2}\left[-d\eta^2 + d\vec{x}^2\right]$$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{\pm}}$$

$$\eta = -H^{-1} e^{-Ht}$$

$$ds^2 = \frac{1}{H^2 \eta^2} \Big[\underbrace{-d\eta^2 + d\vec{x}^2}_{\text{Minkowsk.}} \Big]$$

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2} \qquad X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{a}}$$

$$\eta = -H^{-1} e^{-Ht}$$

$$ds^2 = \frac{1}{H^2\eta^2}\Big[\underbrace{-d\eta^2 + d\vec{x}^{\,2}}_{\text{Minkowsk.}}\Big] = a^2(\eta)\Big[-d\eta^2$$

<u>Massless scalar in 4d</u>

$$\phi = \frac{u}{a(\eta)}$$

$$u_k'' = -\omega_k^2 u_k \qquad \omega_k^2 = k^2 - \frac{a''}{a}$$

$$ds^2 = -dt^2 + e^{2Ht} \, d\vec{x}^{\,2}$$

$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm i k x^a}$$

$$\eta = -H^{-1} e^{-Ht}$$

$$ds^2 = \frac{1}{H^2 \eta^2} \Big[\underbrace{-d\eta^2 + d\vec{x}^{\,2}}_{\text{Minkowsk.}} \Big] = a^2(\eta) \Big[-d\eta^2 +$$

<u>Massless scalar in 4d</u>

$$\phi = \frac{u}{a(\eta)}$$

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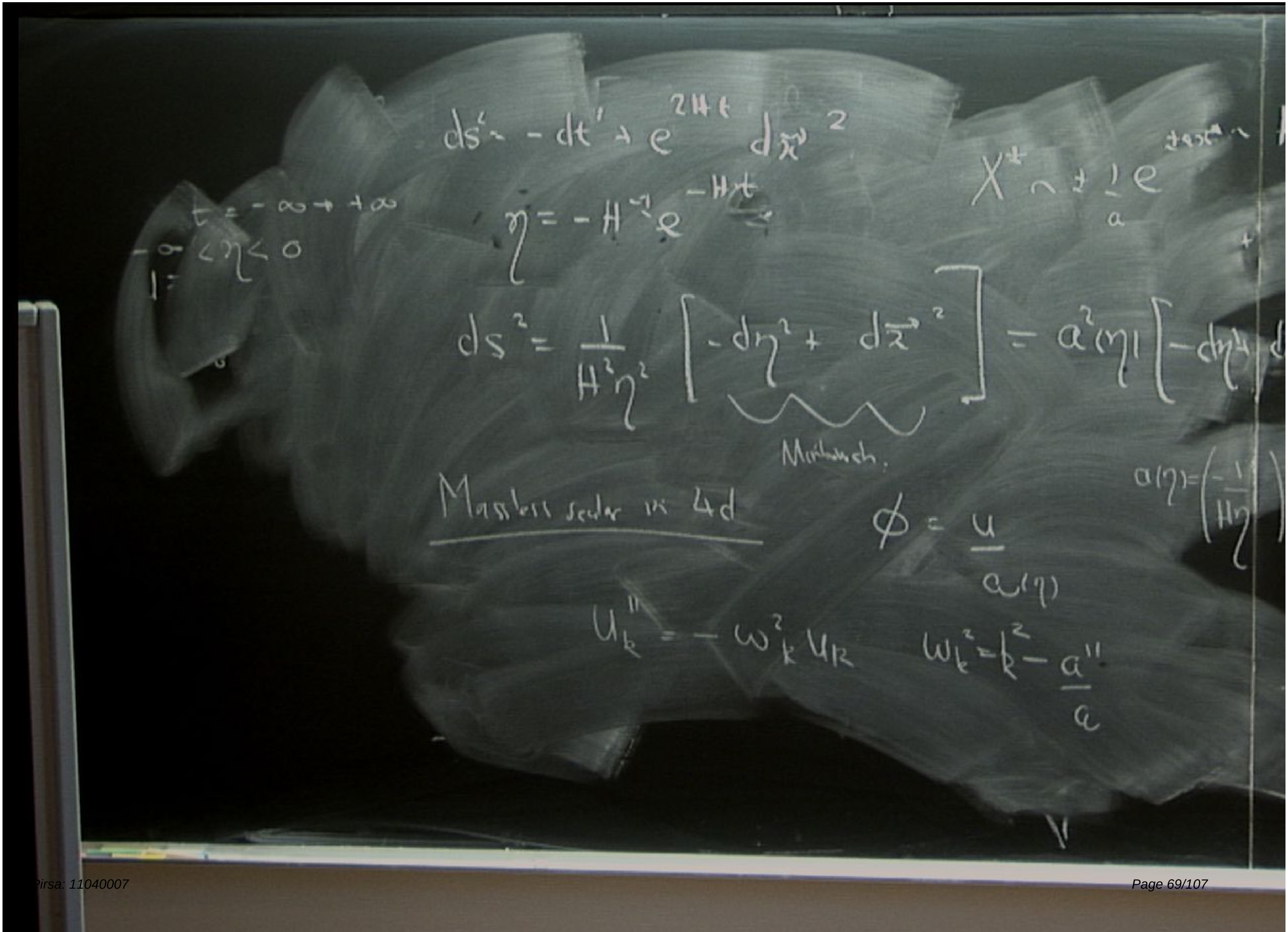


$t = -\infty \to +\infty$

$-\infty < \eta < 0$

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^{\,2}$$

$$\eta = -H^{-1} e^{-Ht}$$

$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{\mp}}$$

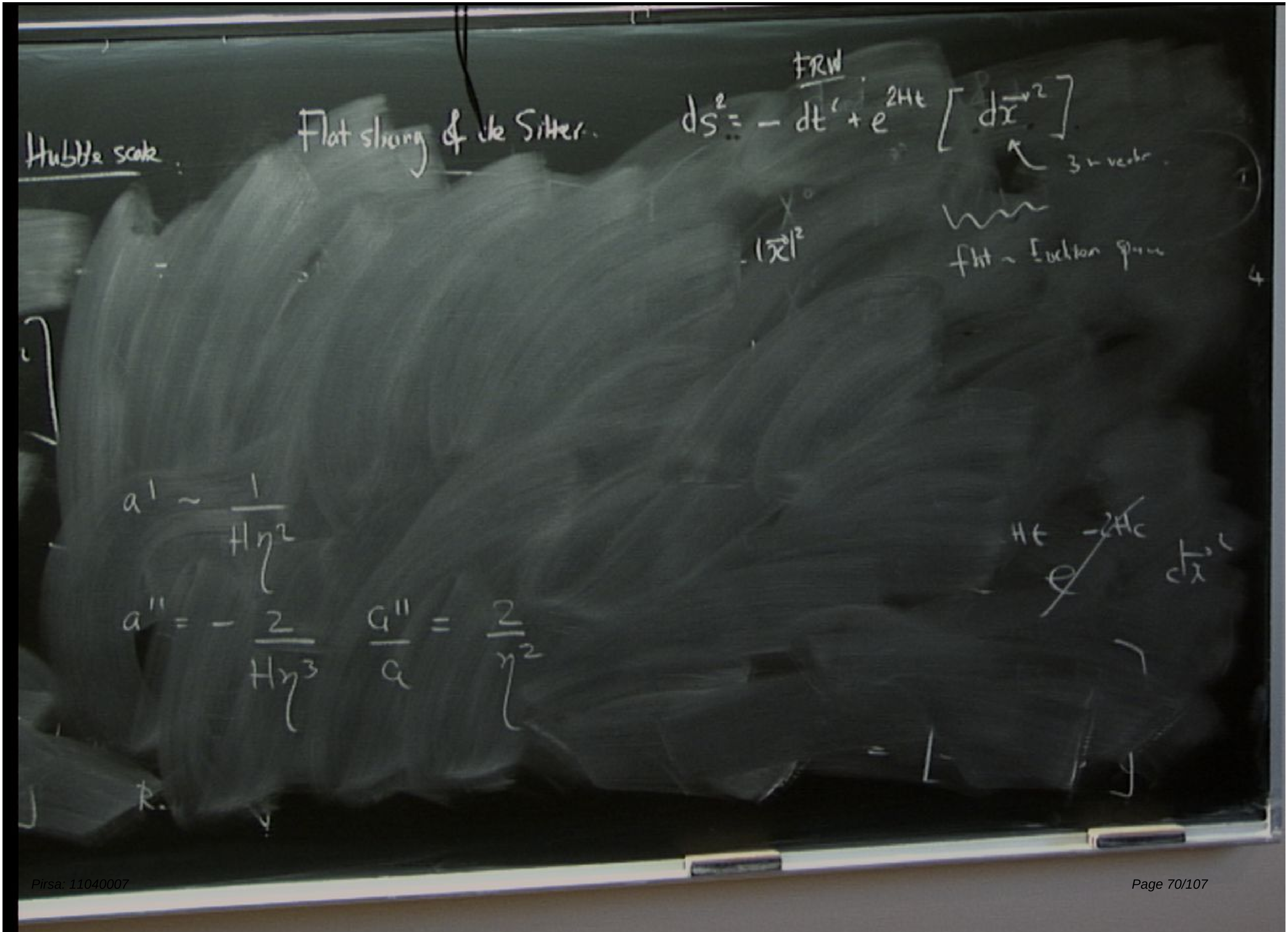
$$ds^2 = \frac{1}{H^2\eta^2}\Big[\underbrace{-d\eta^2 + d\vec{x}^{\,2}}_{\text{Minkowsch.}}\Big] = a^2(\eta)\Big[-d\eta^2 + d$$

$$a(\eta) = \left(-\frac{1}{H\eta}\right)$$

Massless scalar in 4d

$$\phi = \frac{u}{a(\eta)}$$

$$u_k'' = -\omega_k^2 u_k \qquad \omega_k^2 = k^2 - \frac{a''}{a}$$



Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

3-vector

$|\vec{x}|^2$

flat ~ Euclidean space

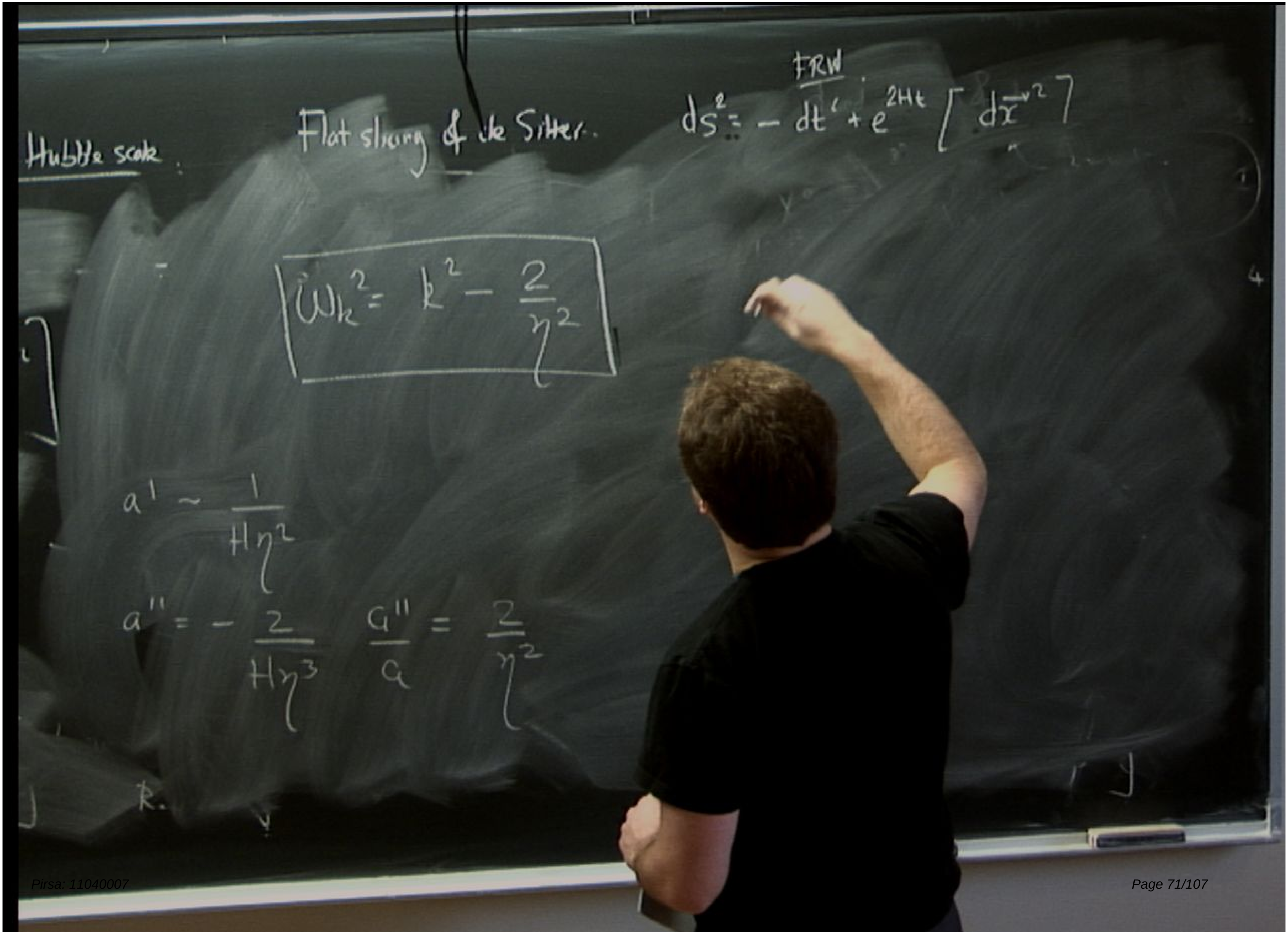
$$a' \sim \frac{1}{H\eta^2}$$

$$a'' = -\frac{2}{H\eta^3} \qquad \frac{a''}{a} = \frac{2}{\eta^2}$$

$\cancel{e^{Ht\ -2Ht}}\ d\vec{x}^2$

$= [\quad]$

R.



Hubble scale :

Flat slicing of de Sitter.

FRW

$$ds^2 = -dt^2 + e^{2Ht}\left[d\vec{x}^2\right]$$

$$\boxed{\omega_k^2 = k^2 - \frac{2}{\eta^2}}$$

$$a' \sim \frac{1}{H\eta^2}$$

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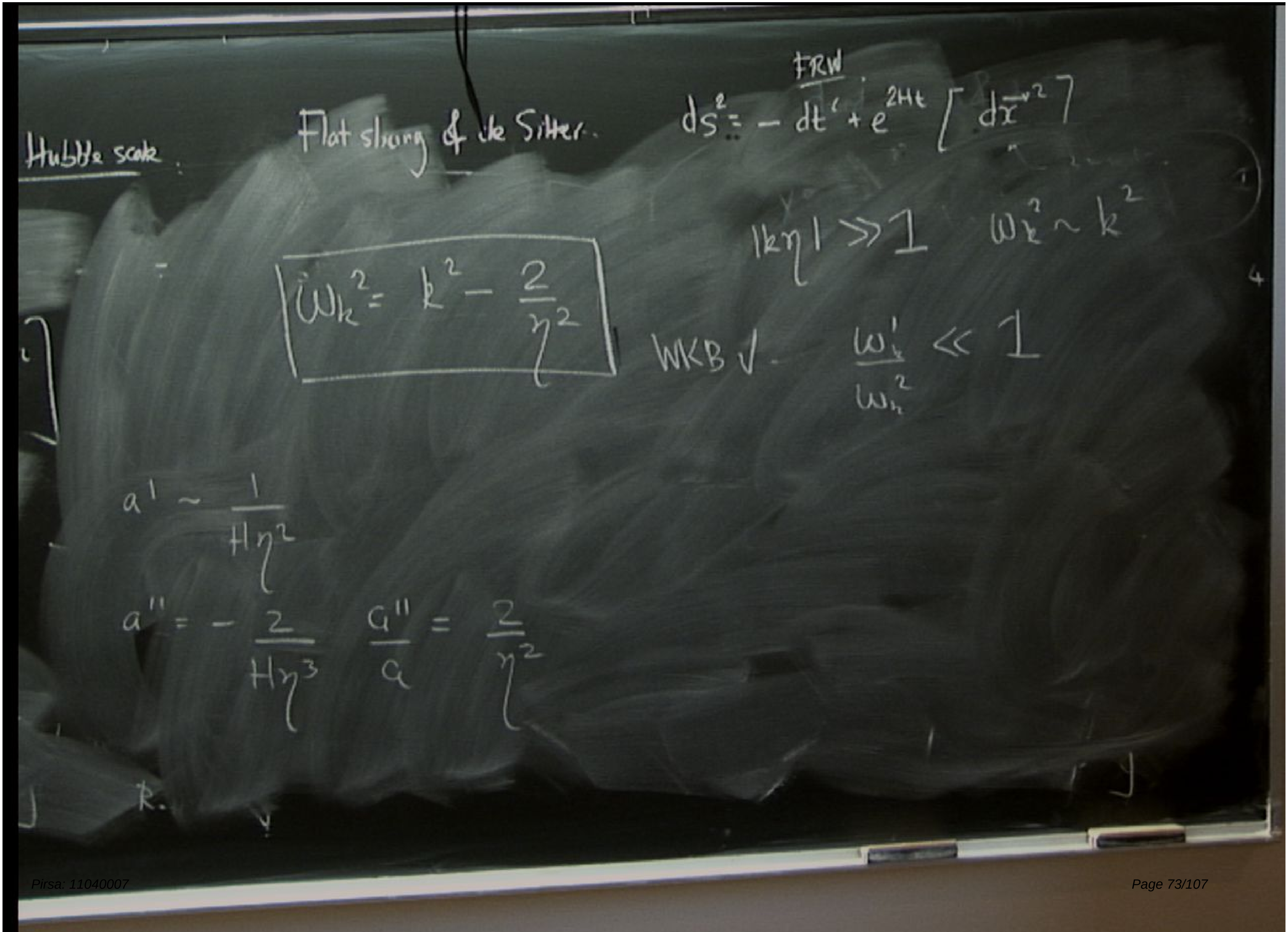

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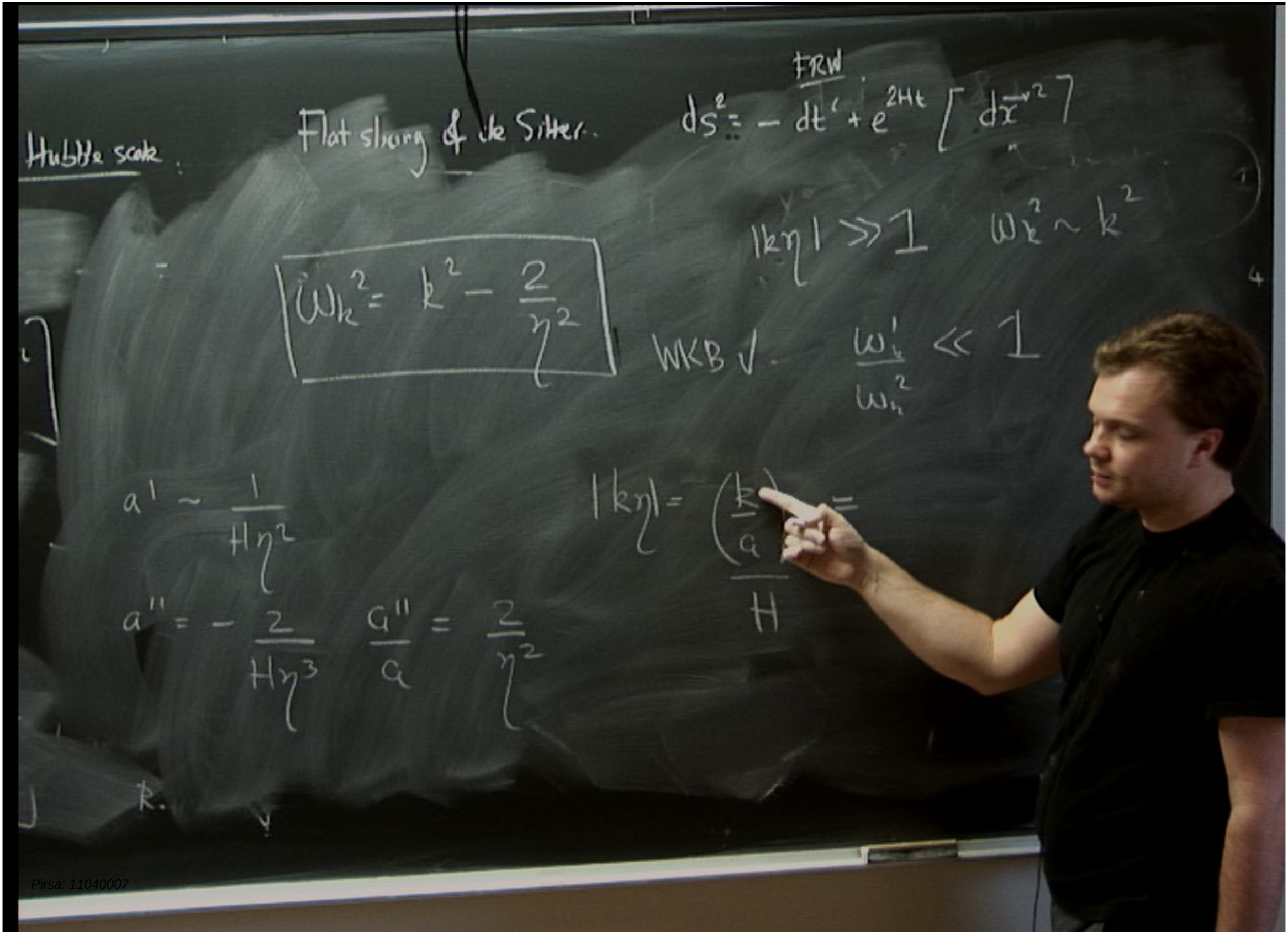

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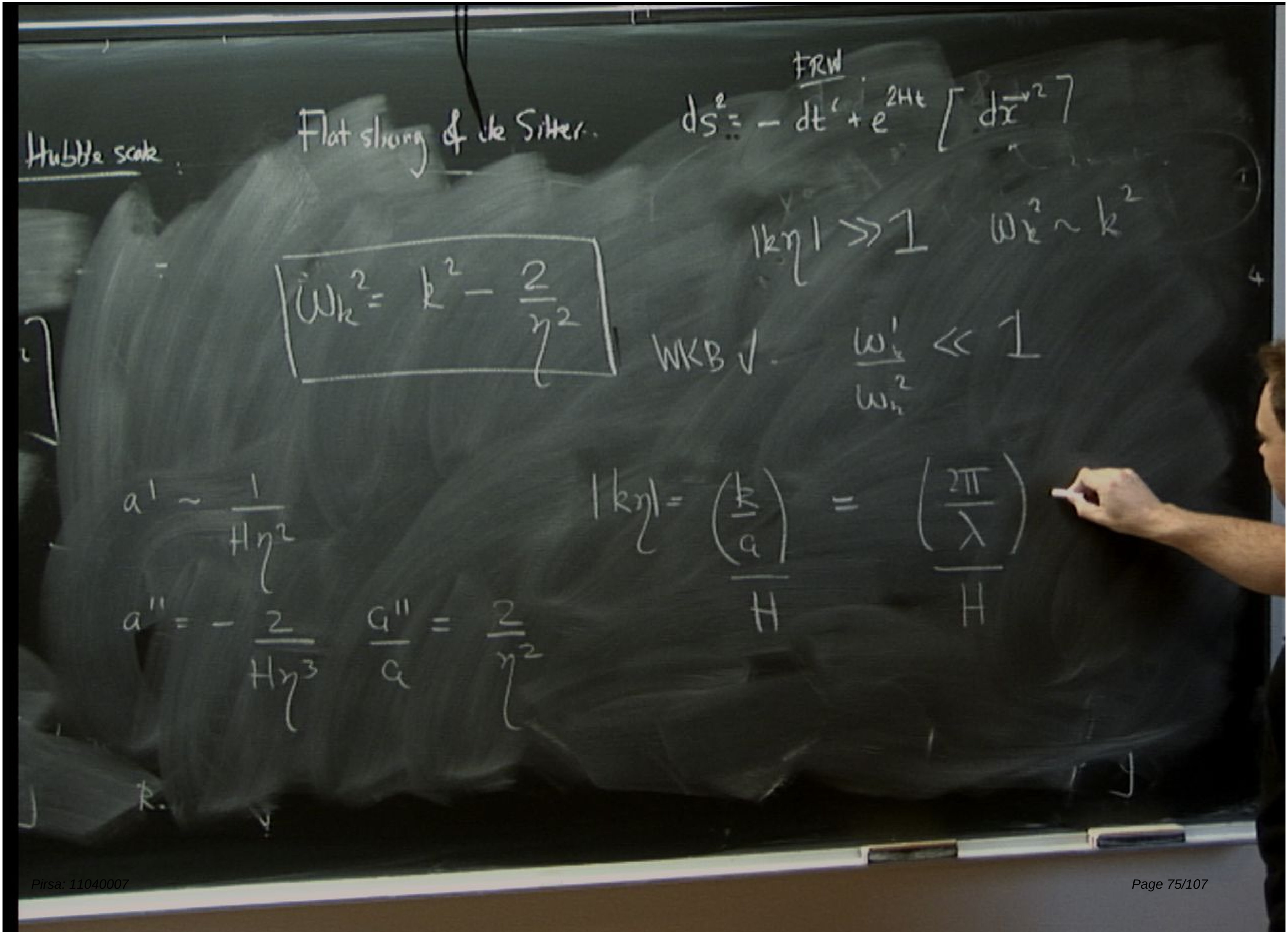

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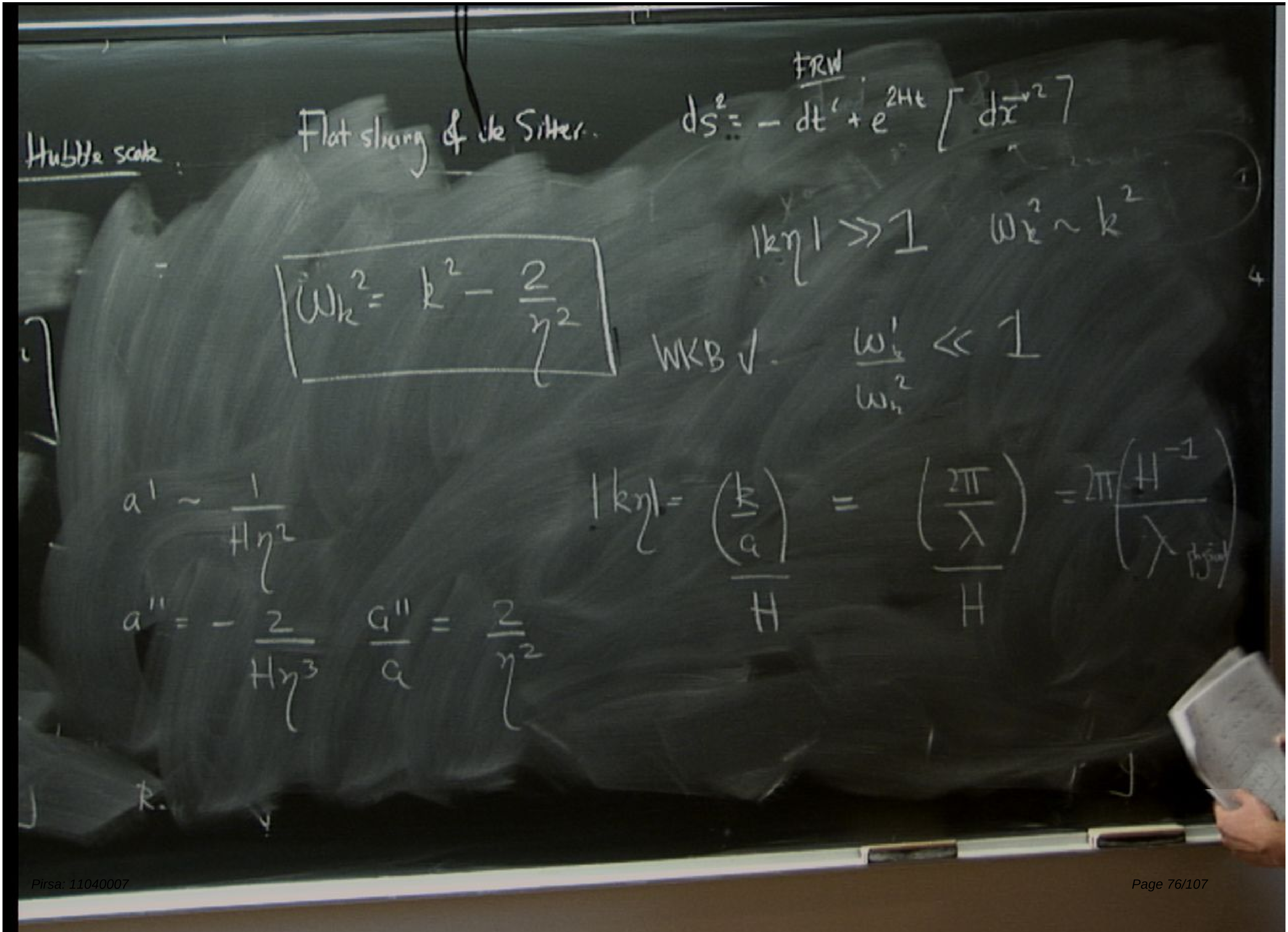

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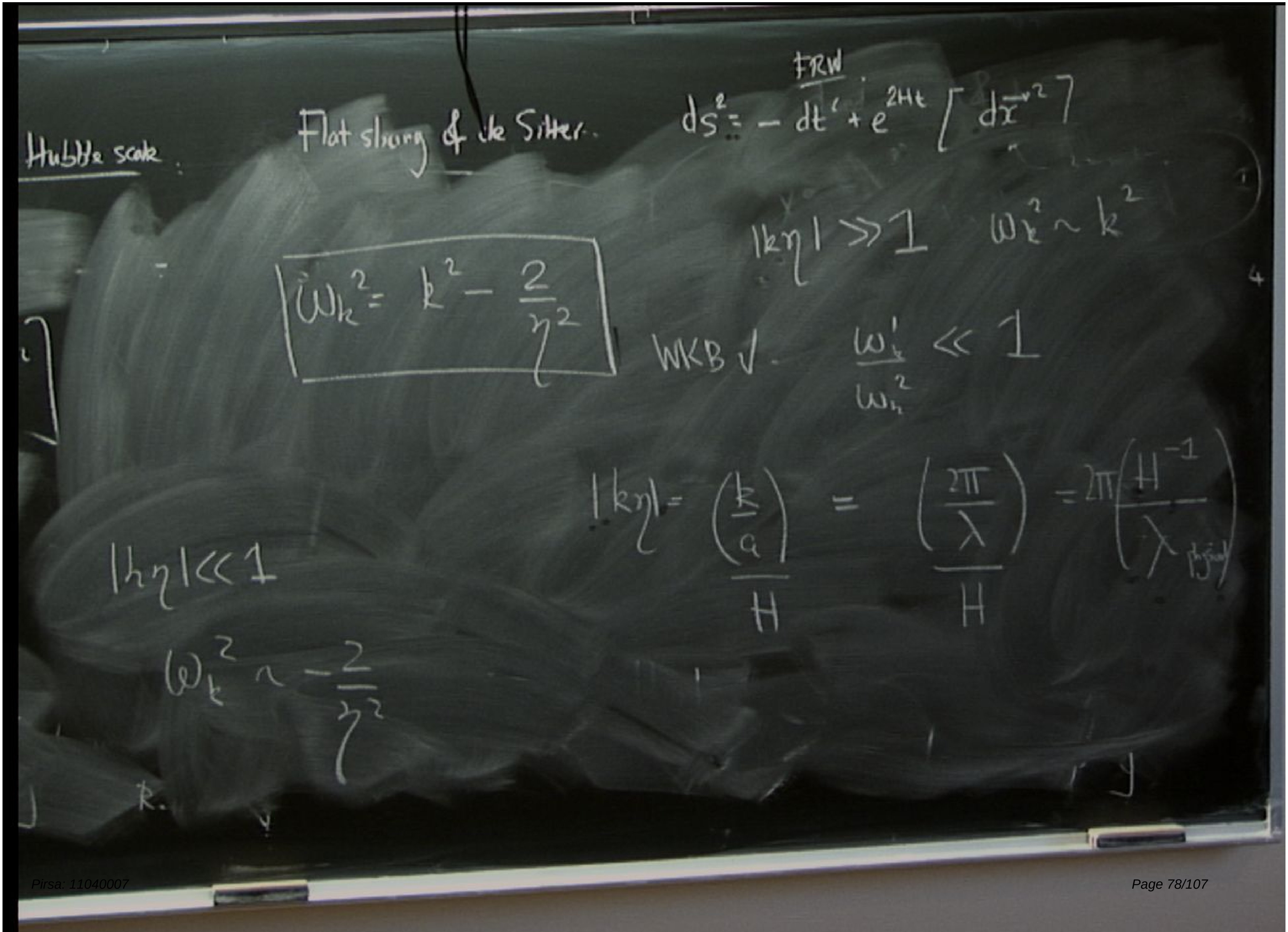

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$\omega_k^2 \sim -\frac{2}{\eta^2}$ ~~WKB~~

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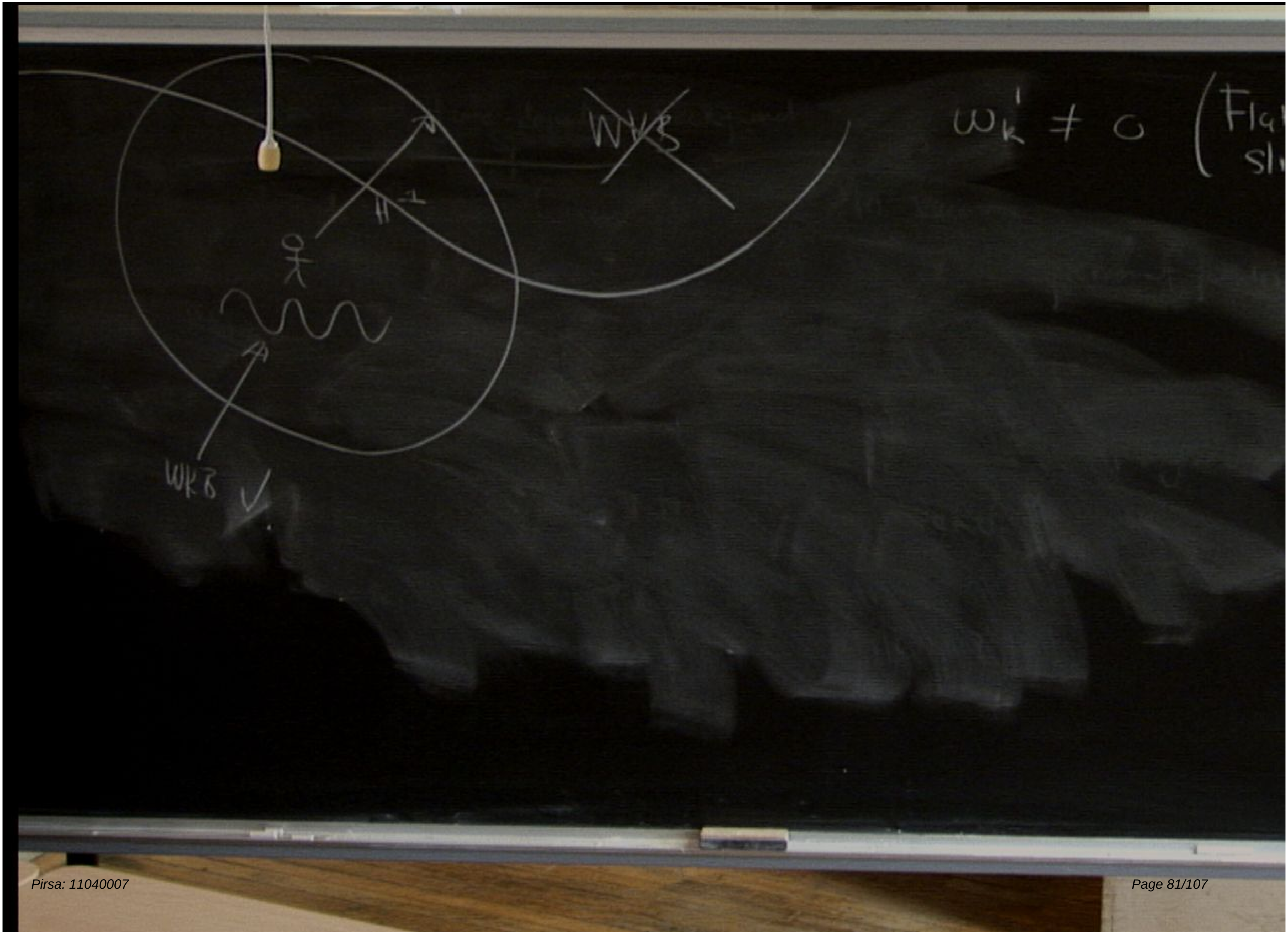
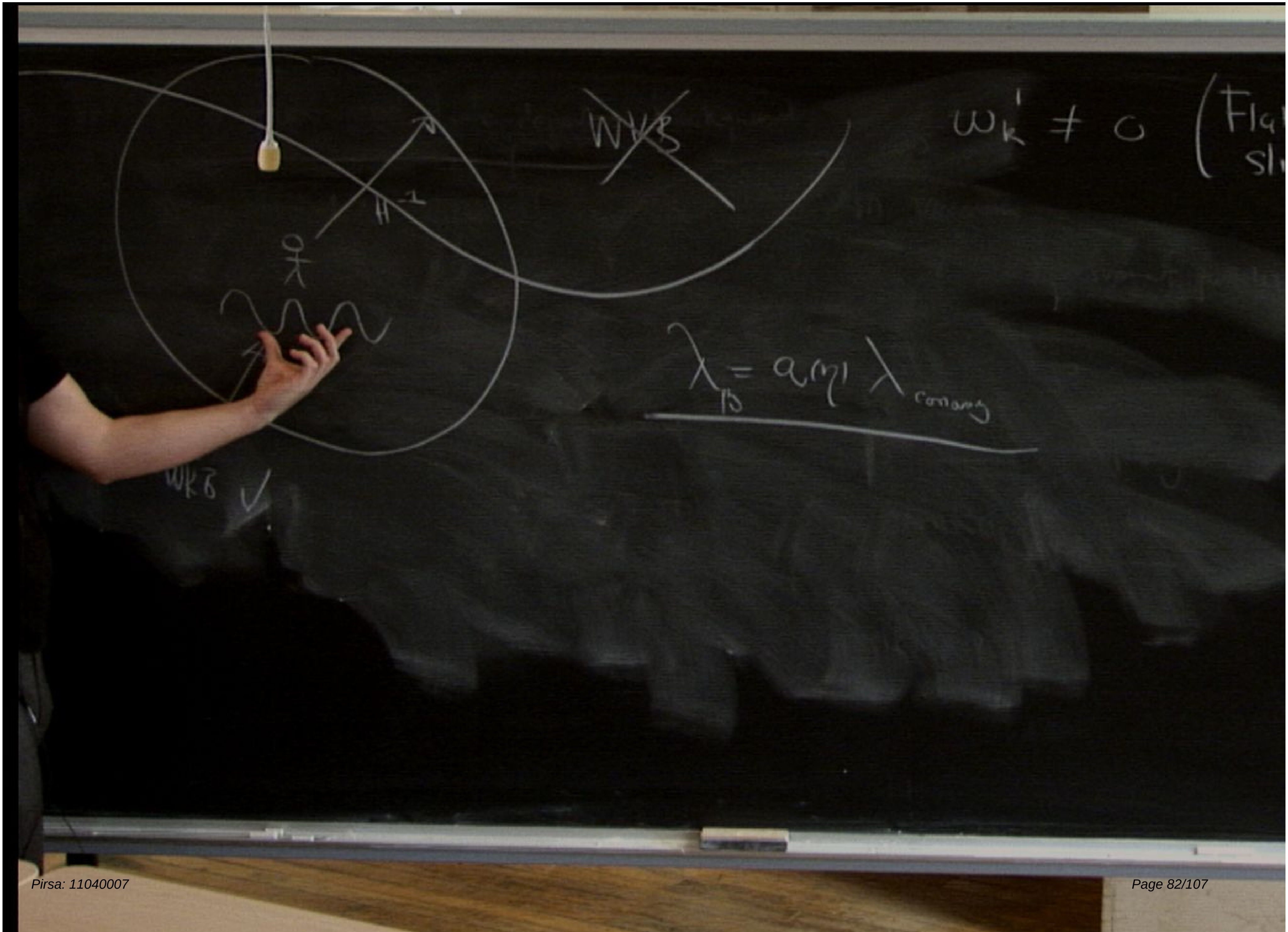
$|k\eta| \ll 1$

$$\omega_k^2 \sim -\frac{2}{\eta^2}$$

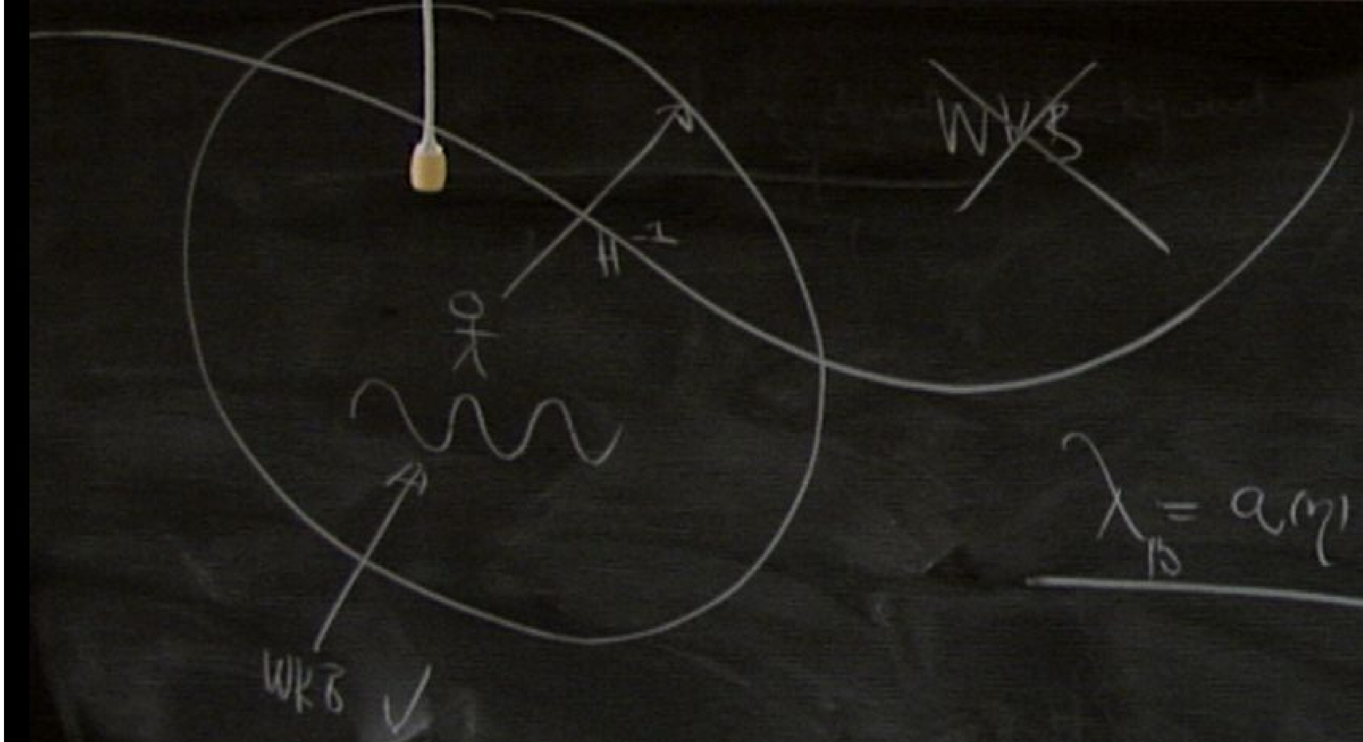
~~WKB~~


~~WKB~~
$\omega_k' \neq 0$ (Fla
sl
H^{-1}
WKB ✓



$\omega_k' \neq 0$ (Fla
sl

$\lambda_p = a(\eta)\,\lambda_{\text{comoving}}$



$\omega_k' \neq 0$ (Fla
sl

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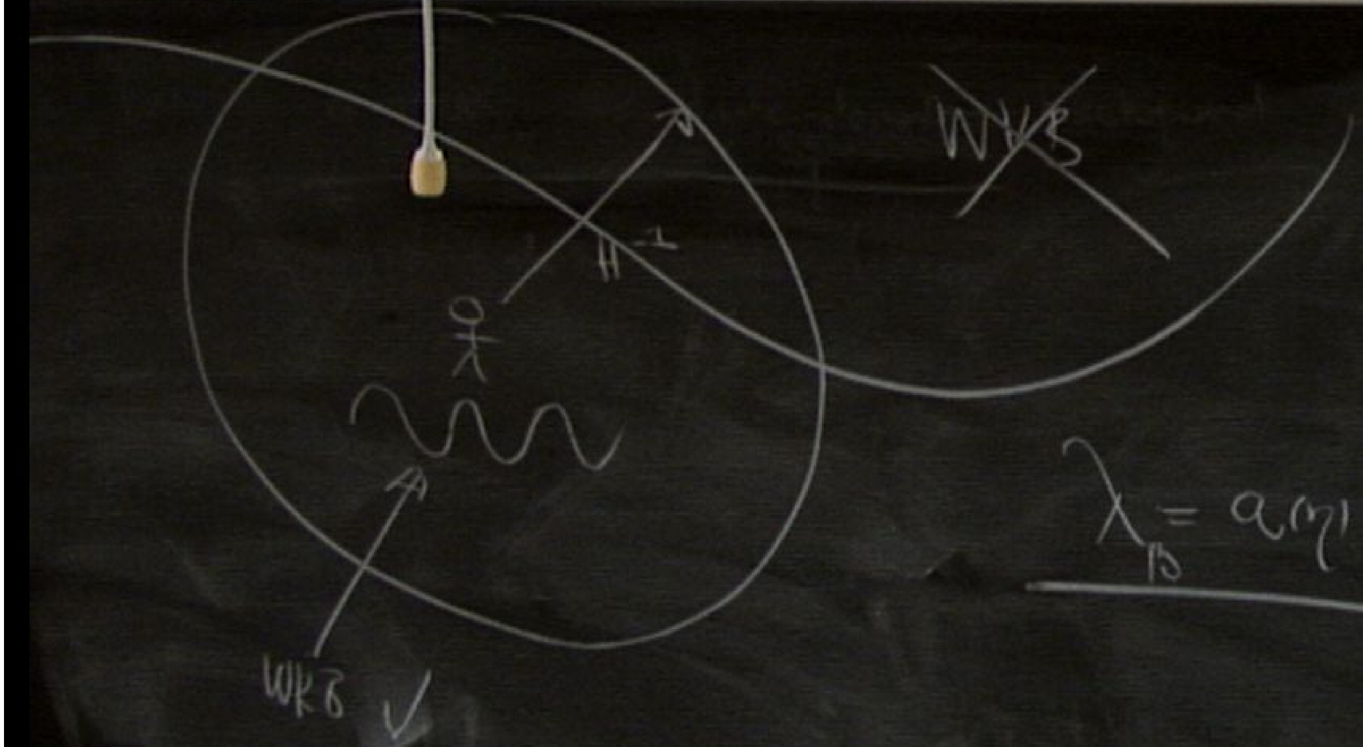
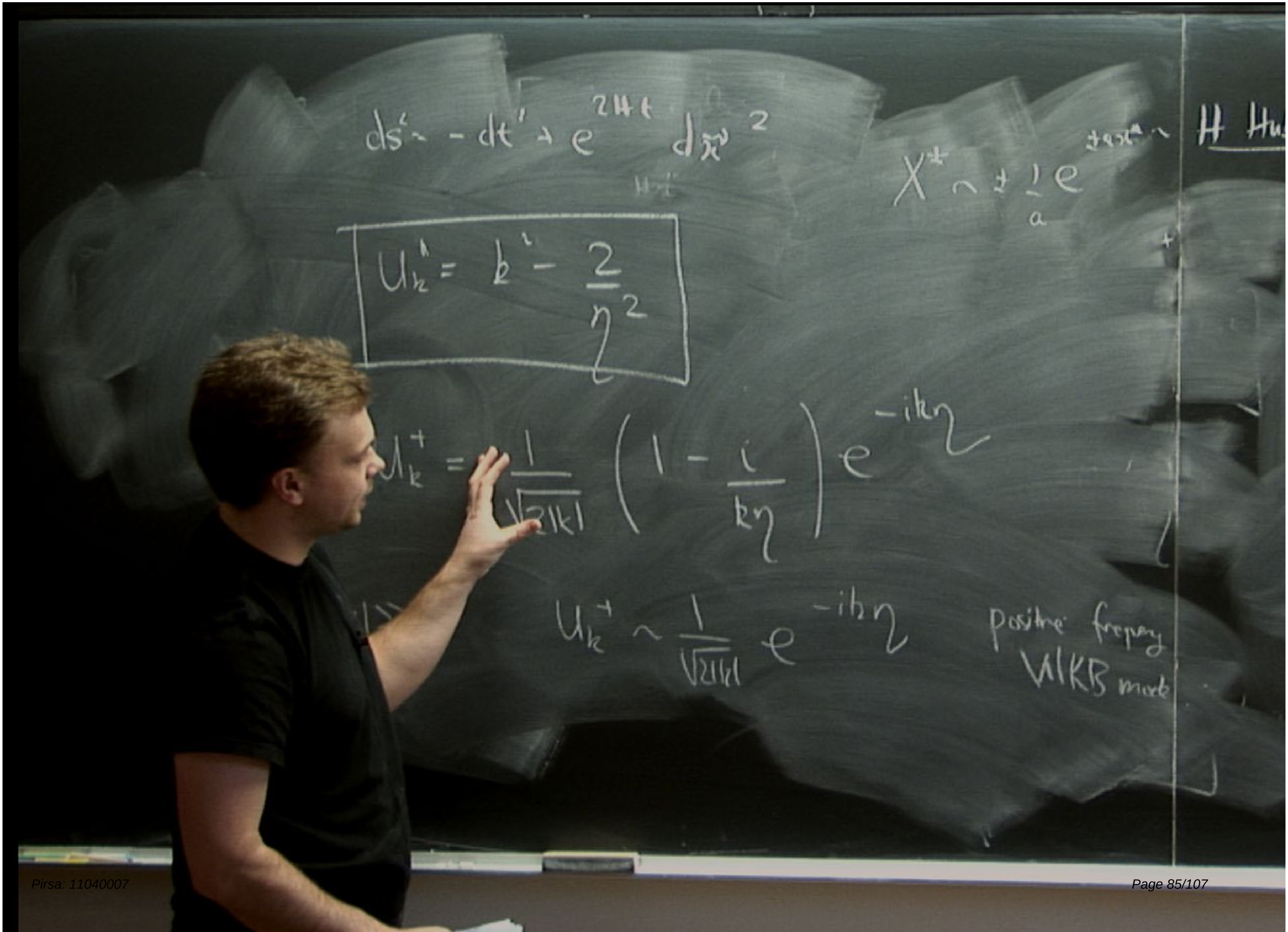


$\omega_k' \neq 0$ (Fla
sl

$$\lambda_{p} = a(\eta)\,\lambda_{\text{comoving}}$$



$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

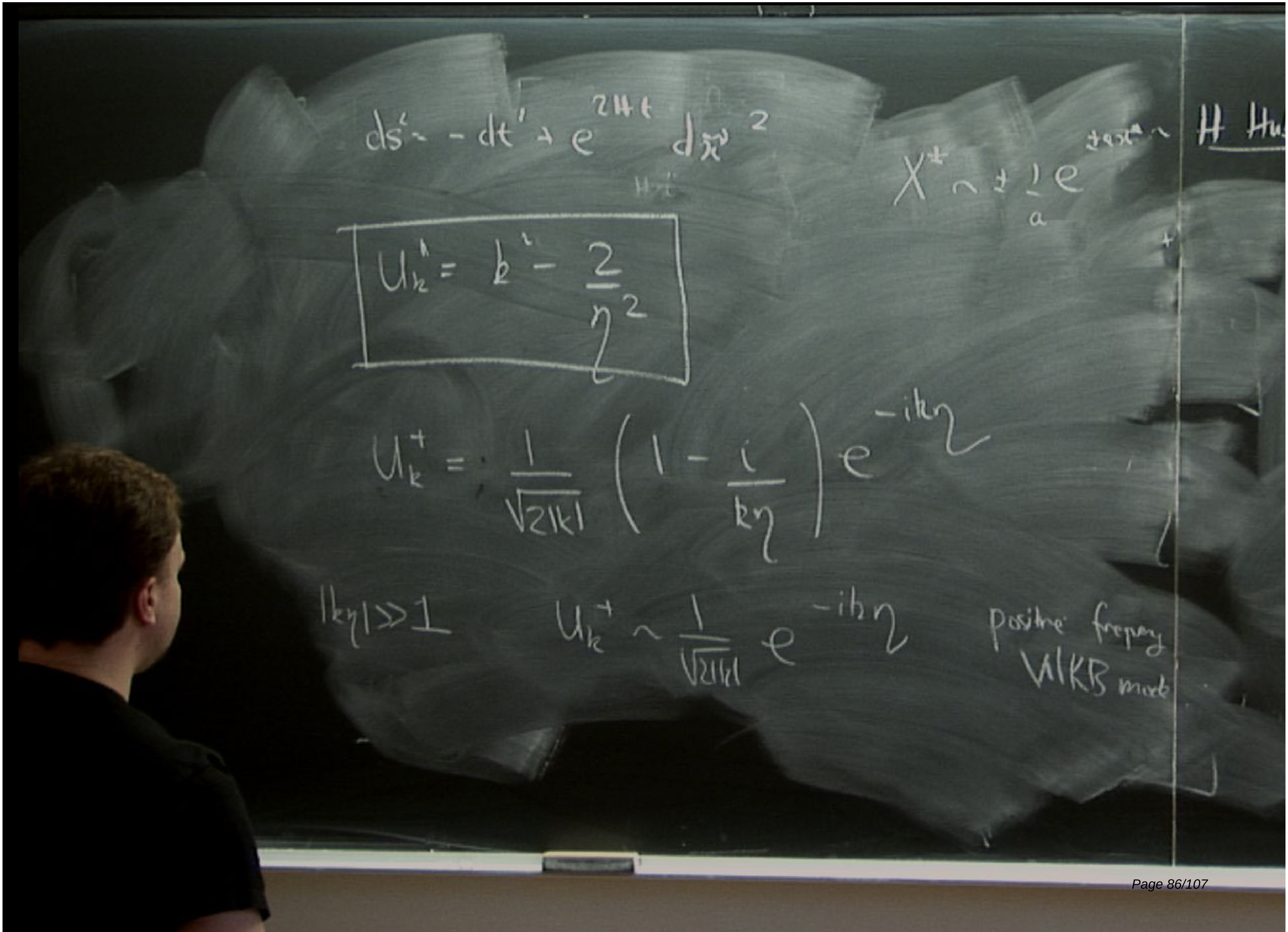
$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{\pm}}$$

$$\boxed{u_k'' = k^2 - \frac{2}{\eta^2}}$$

$$u_k^{+} = \frac{1}{\sqrt{2|k|}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$$u_k^{+} \sim \frac{1}{\sqrt{2|k|}}\, e^{-ik\eta}$$

positive frequency WKB mode



$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^2$$

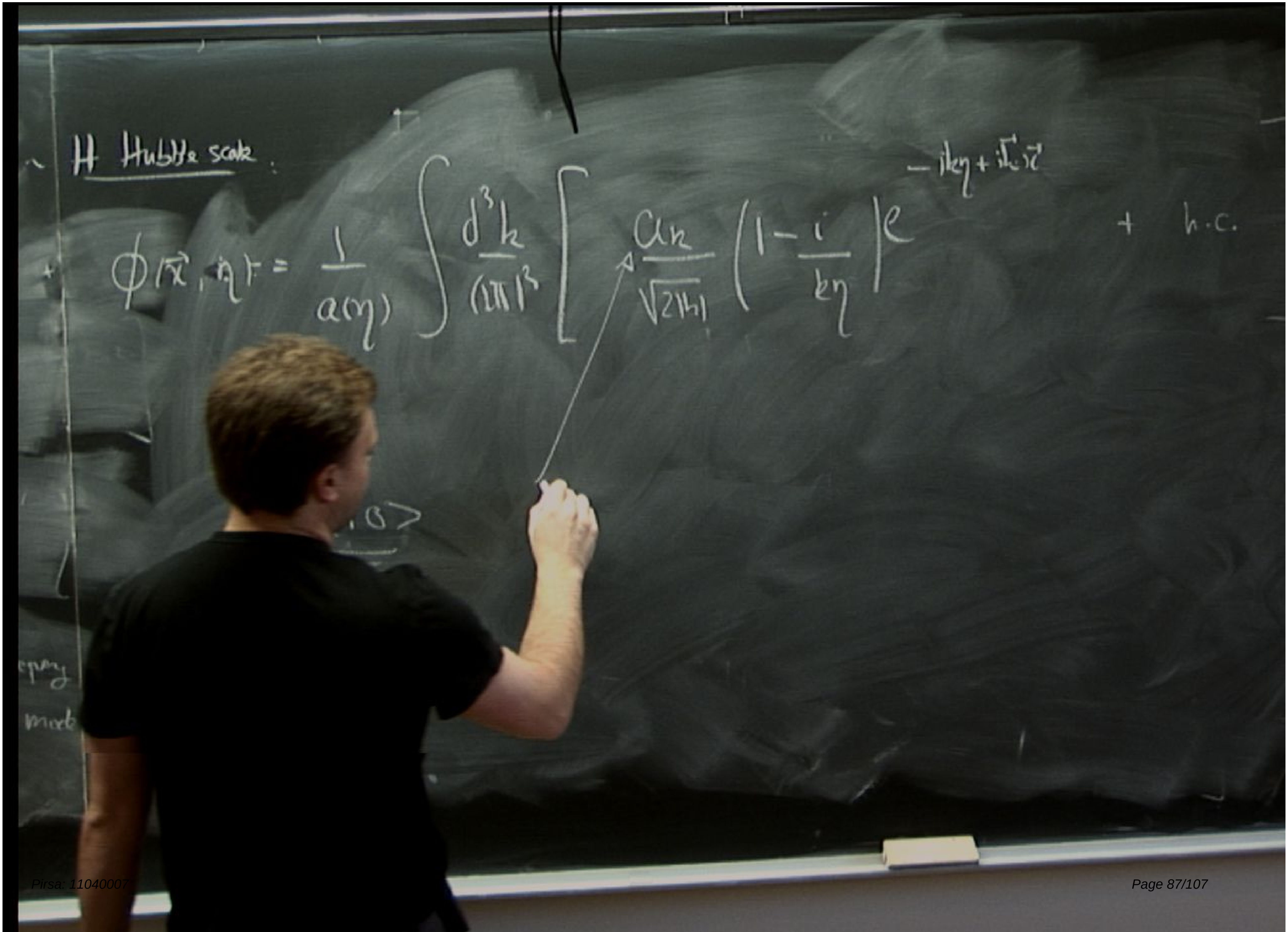
$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm \dots}$$

H Hu

$$\boxed{u_k'' = k^2 - \frac{2}{\eta^2}}$$

$$u_k^+ = \frac{1}{\sqrt{2|k|}}\left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

$|k\eta| \gg 1$ $$u_k^+ \sim \frac{1}{\sqrt{2|k|}} e^{-ik\eta}$$ positive frequency WKB mode



H Hubble scale:

$$\phi(\vec{x},\eta) = \frac{1}{a(\eta)}\int \frac{d^3k}{(2\pi)^3}\left[\frac{a_k}{\sqrt{2|k|}}\left(1-\frac{i}{k\eta}\right)e^{-ik\eta+i\vec{k}\cdot\vec{x}} \qquad + \quad h.c.\right.$$

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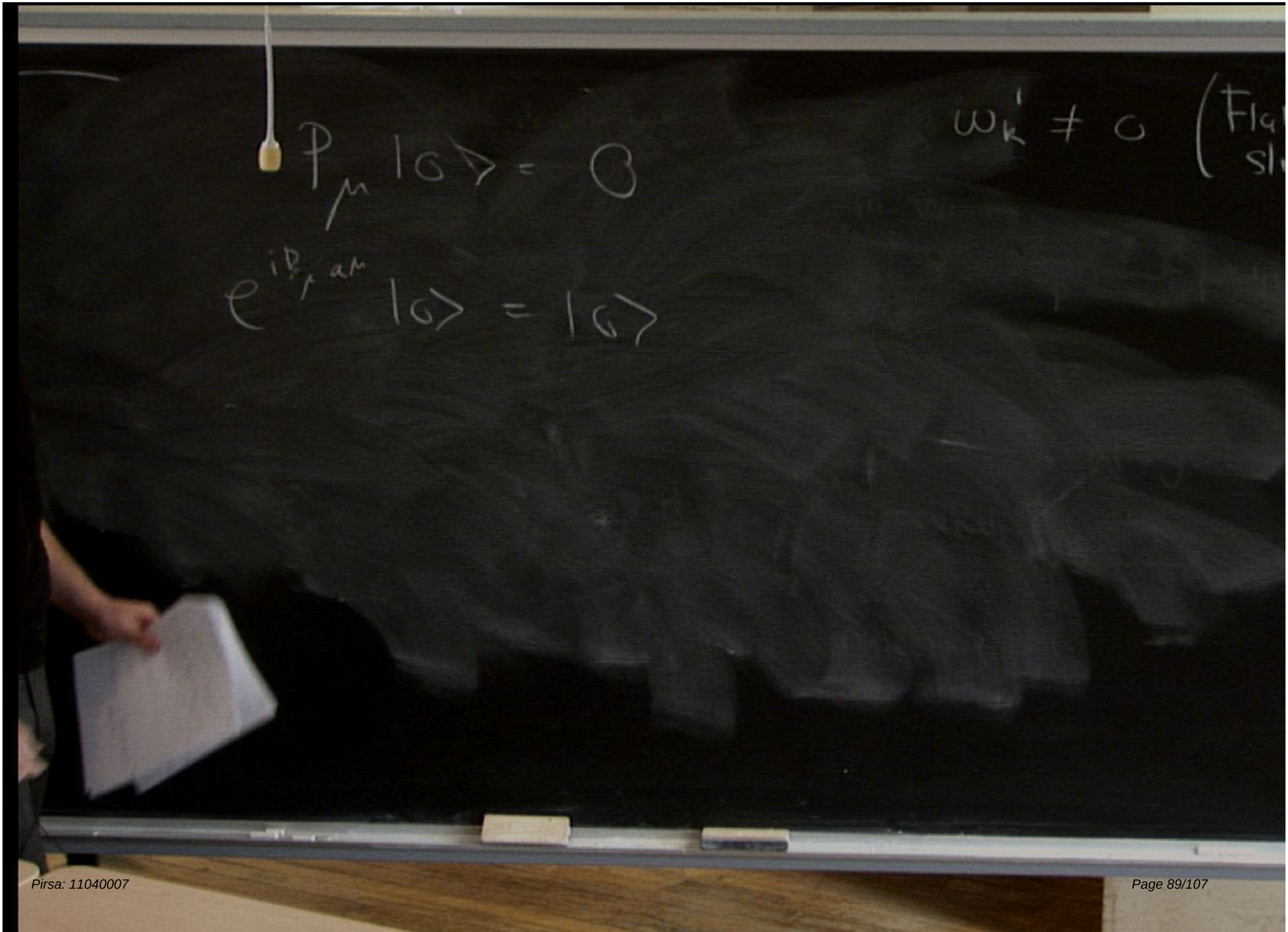
$a_k|0>$

Bunch-Davies Vacuum.

'natural vacuum'.

Invariant under de Sitter transformation $SO(4,1)$

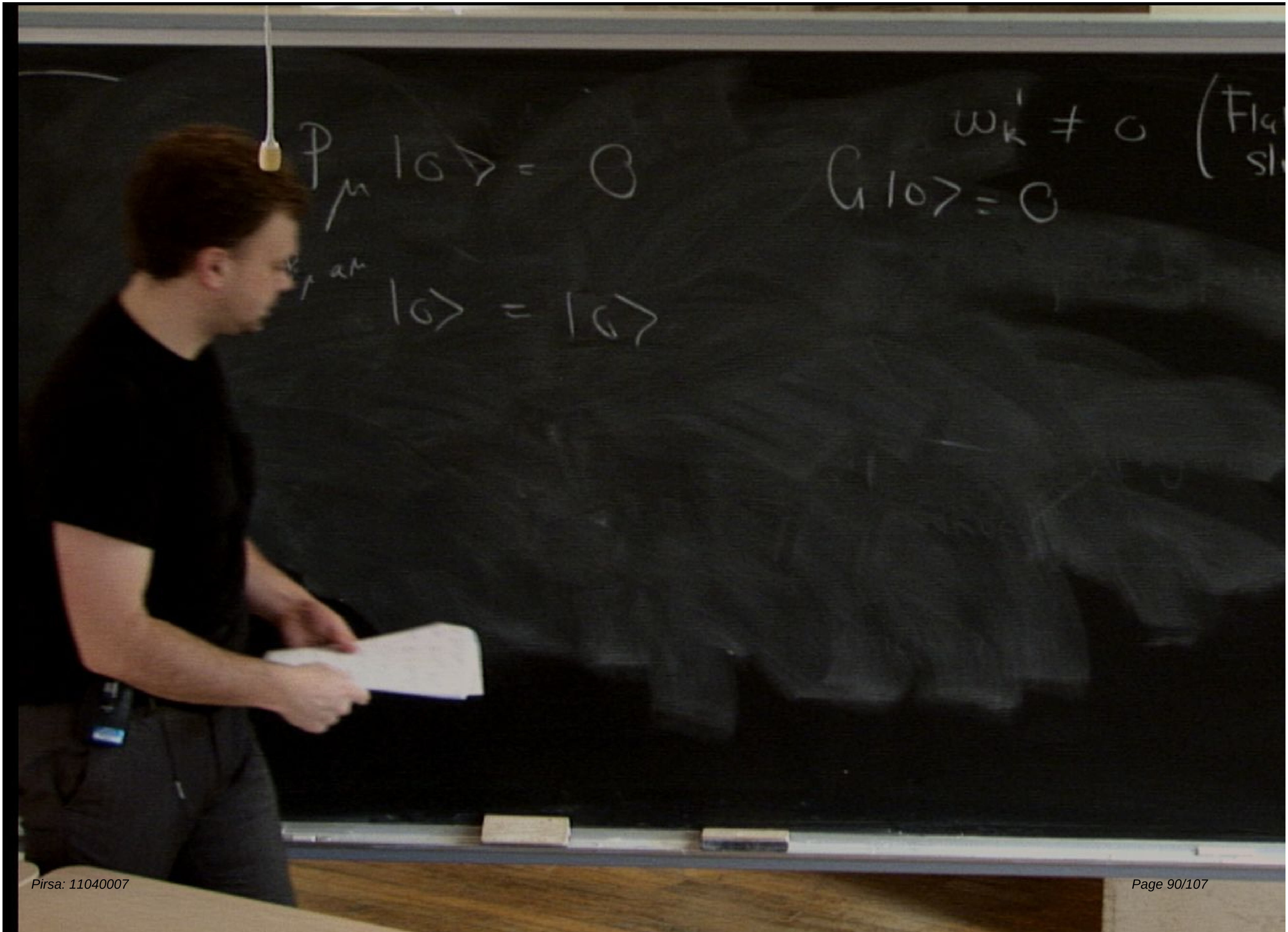


$P_\mu |G\rangle = 0$

$e^{iP_\mu a^\mu} |G\rangle = |G\rangle$

$\omega_k' \neq 0$ (Fla

sl



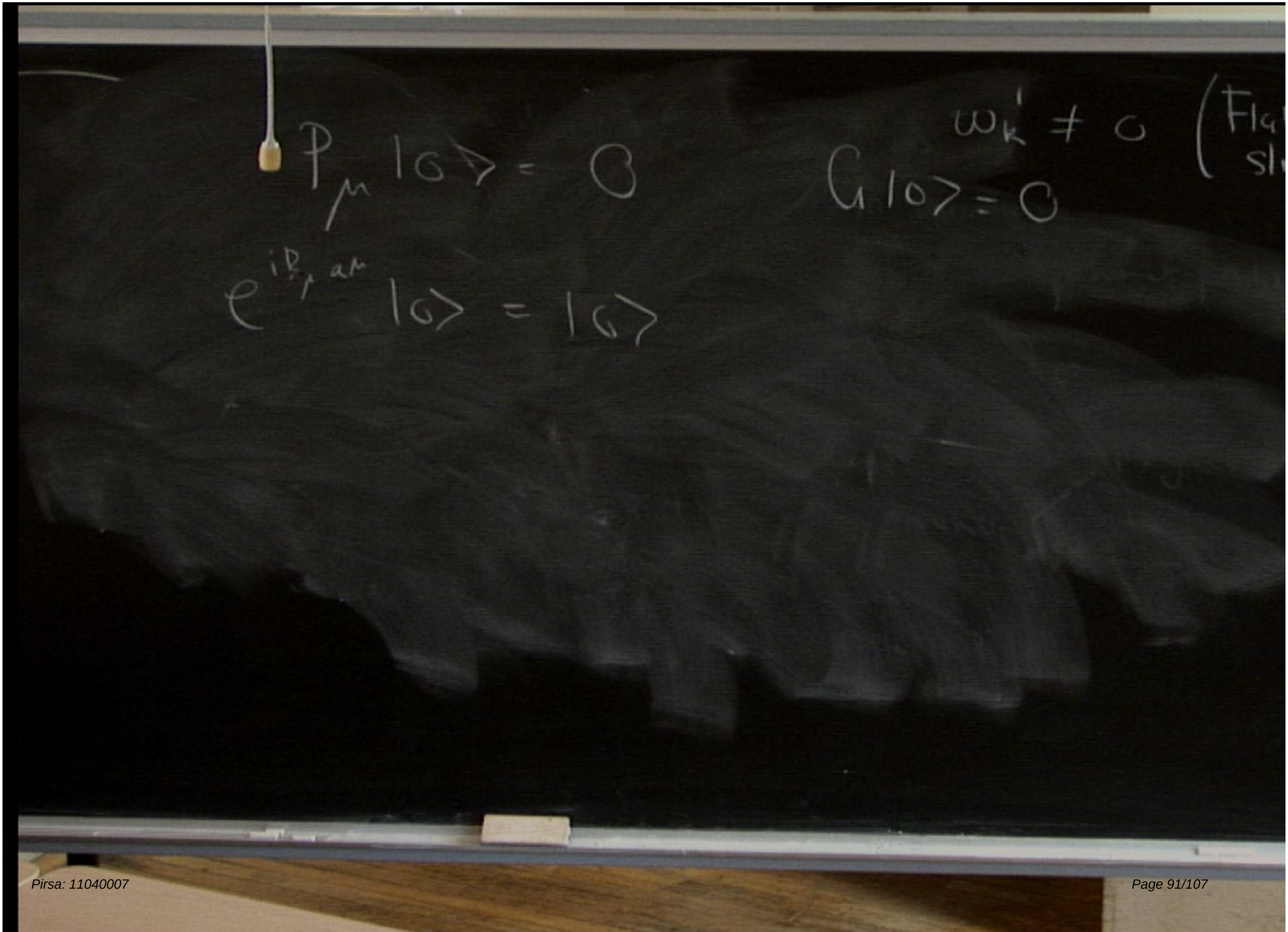
$P_\mu |0\rangle = 0$

$a^\mu |0\rangle = |0\rangle$

$\omega_k' \neq 0$ (Fla

sl

$G|0\rangle = 0$



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$\omega_k' \neq 0$ (Fig

sl

$G|0\rangle = 0$

Hubble scale :

$$\phi(\vec{x},\eta) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[\frac{a_k}{\sqrt{2|k|}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta + i\vec{k}\cdot\vec{x}} + h.c. \right]$$

$a_k|0\rangle$

Bunch-Davies Vacuum.

'natural vacuum'

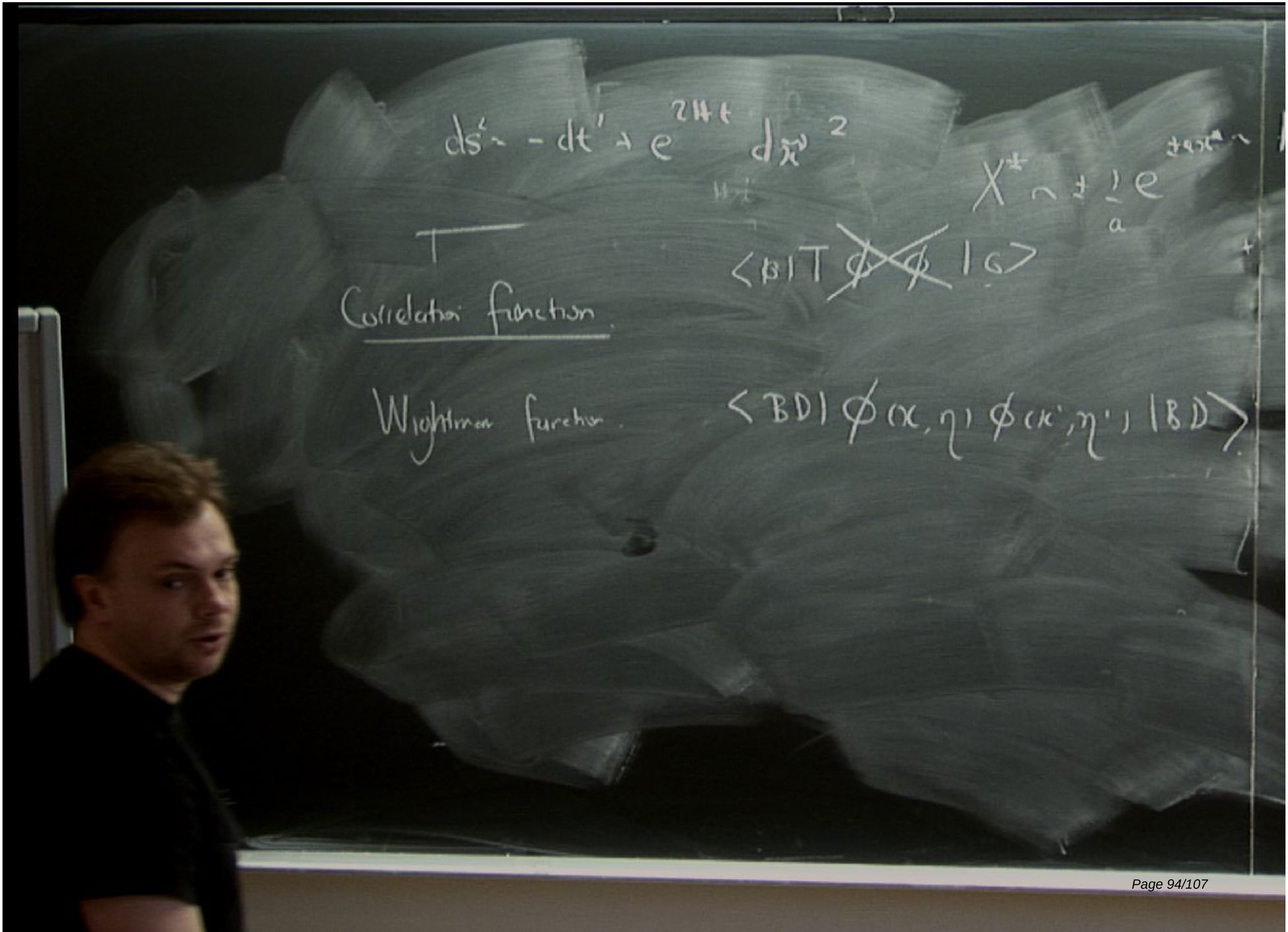
Invariant under de Sitter transformation $SO(4,1)$

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{a}}$$

Correlation function

Wightman function. $\langle BD|\, \phi(x,\eta)\, \phi(x',\eta')\, |BD\rangle$



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

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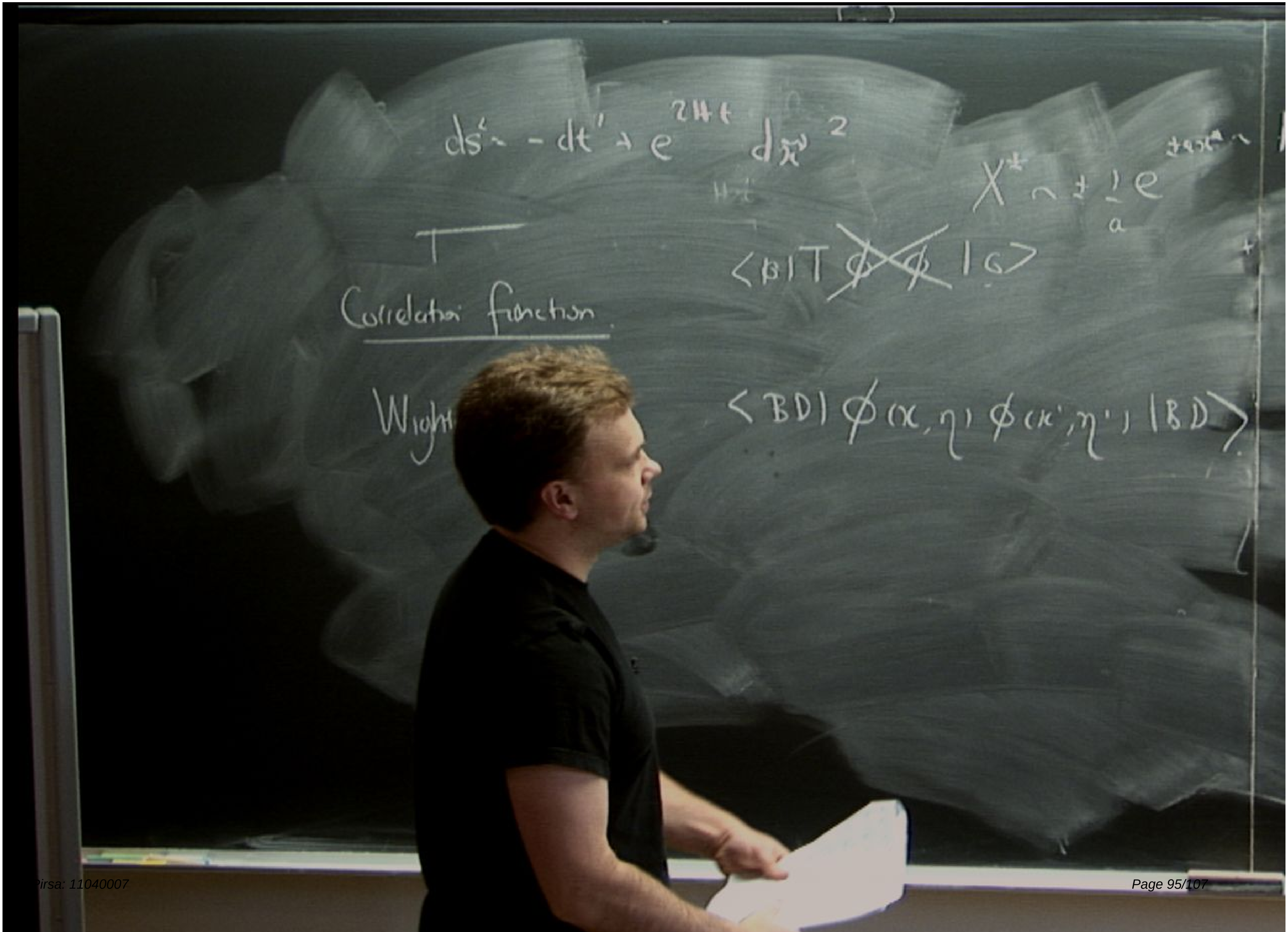
$$\langle \beta | T \cancel{\phi \phi} | G \rangle$$

<u>Correlation function</u>.

Wightman function. $\langle BD | \phi(x, \eta) \phi(x', \eta') | BD \rangle$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T
$\langle B | T \phi \phi | G \rangle$
Correlation function
Wight
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$|BD\rangle$

$a_k|0\rangle$

Bunch-Davies Vacuum.

'natural vacuum'.

$$[a_k, a^\dagger_{k'}] = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')$$

Invariant under de Sitter transformation $SO(4,1)$

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$|BD\rangle$

$\underline{\underline{a_k|0\rangle}}$

Bunch-Davies Vacuum.

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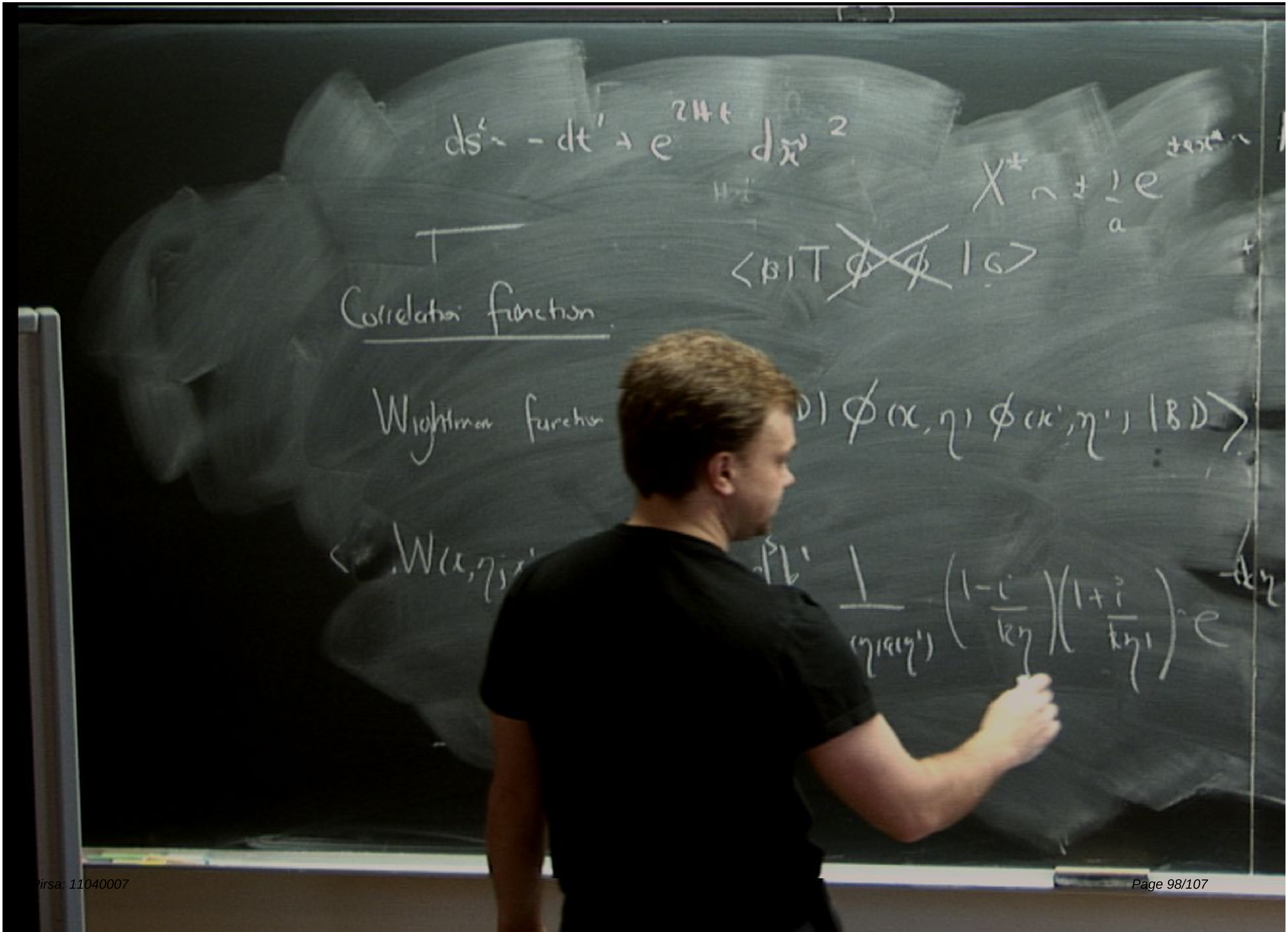


$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$

$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^{\pm}}$

$\langle \beta | T \cancel{\phi \phi} | G \rangle$

Correlation function

Wightman function $\langle BD | \phi(x, \eta) \phi(x', \eta') | BD \rangle$

$\langle W(x, \eta; \quad d^3k \frac{1}{(\eta \eta')} \left(1 - \frac{i}{k\eta}\right)\left(1 + \frac{i}{k\eta'}\right) e^{-ik\eta}$

$$ds^2 = -dt^2 + e^{2Ht}\, d\vec{x}^{\,2}$$

$$X^{\pm} \sim \pm \frac{1}{a} e^{\pm a x^a} \sim$$

$$\langle B | T\, \cancel{\phi\, \phi}\, | G \rangle$$

Correlation function

Wightman function $W = \langle BD | \phi(x,\eta)\, \phi(x',\eta') | BD \rangle$

$$W(x,\eta; x',\eta') = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{1}{a(\eta)a(\eta')} \left(1 - \frac{i}{k\eta}\right)\left(1 + \frac{i}{k\eta'}\right) e^{-ik\eta}$$

Hubble scale :

$$\phi(\vec{x},\eta) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} \left[\frac{a_k}{\sqrt{2|k|}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta + i\vec{k}\cdot\vec{x}} + h.c. \right]$$

$|BD\rangle$

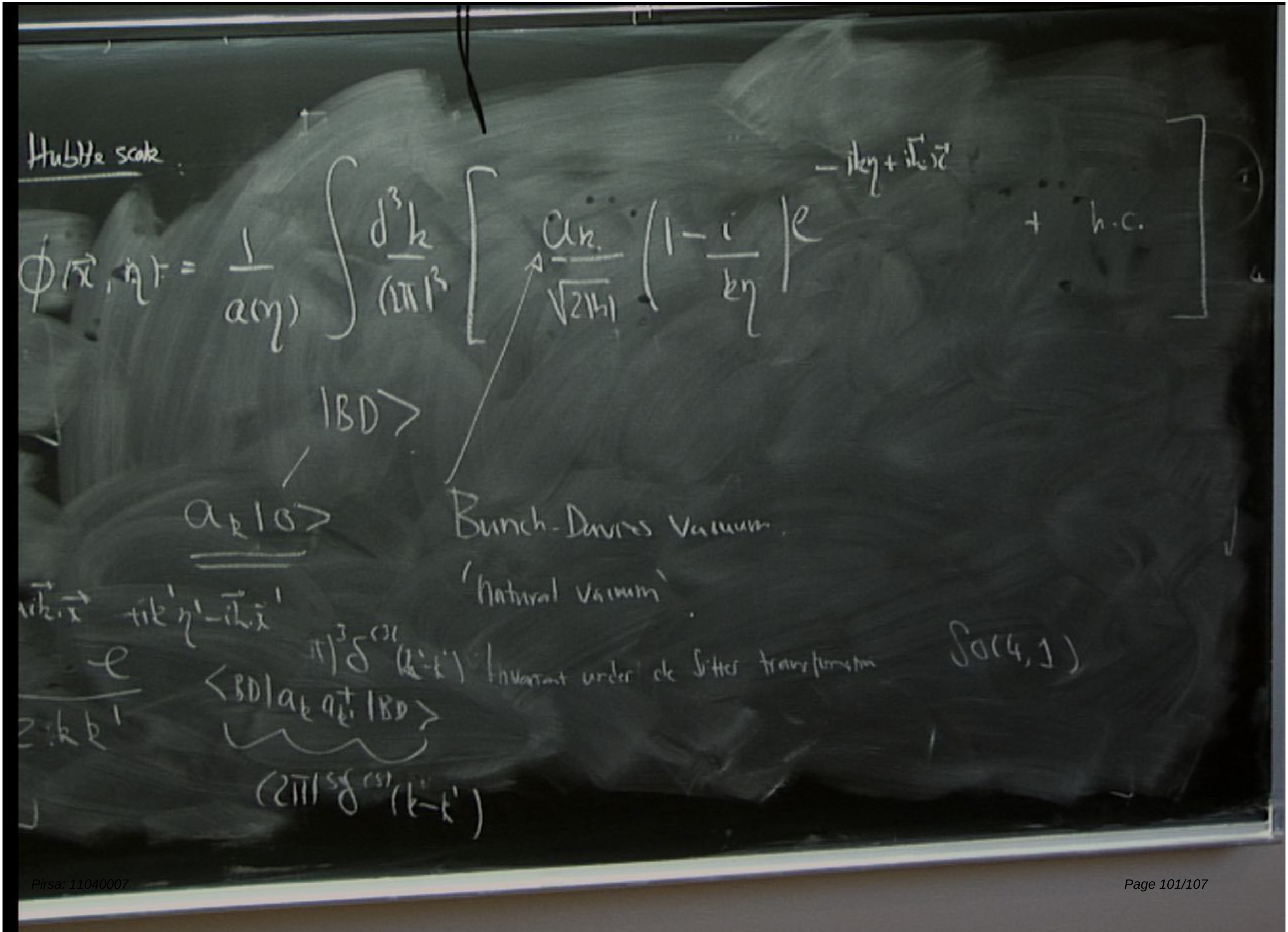
$\underline{\underline{a_k|0\rangle}}$

Bunch-Davies Vacuum.

'natural vacuum'.

$$\frac{e^{i\vec{k}\cdot\vec{x} \quad +ik'\eta' - i\vec{k}'\cdot\vec{x}'}}{2kk'}$$

$\pi)^3 \delta^{(3)}(k'-k')$ Invariant under de Sitter transformation $SO(4,1)$



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$|BD\rangle$

$\underline{\underline{a_k|0\rangle}}$

Bunch-Davies Vacuum.

'natural vacuum'.

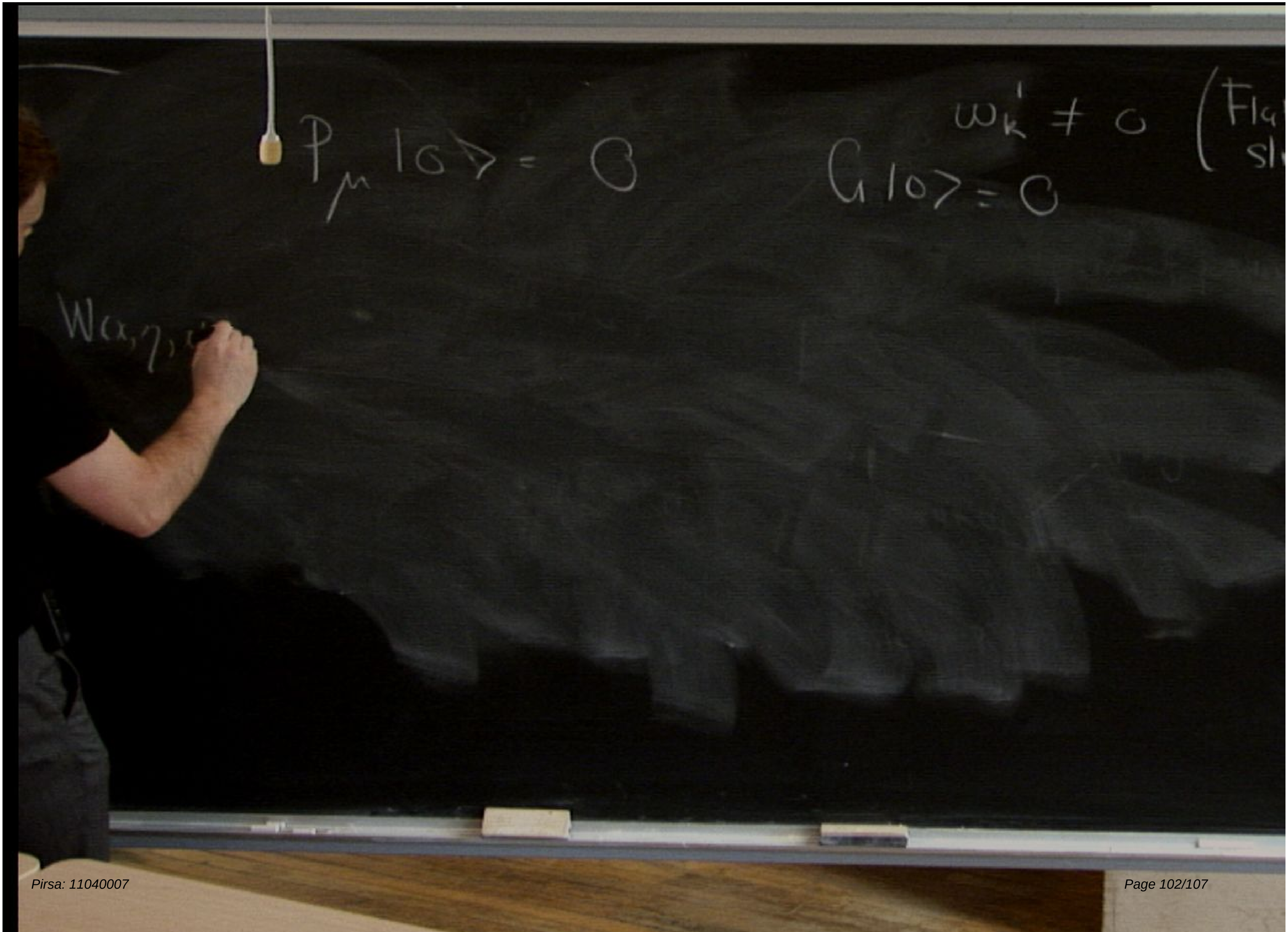
$ik\cdot\vec{x} \quad +ik'\eta' - i\vec{k}\cdot\vec{x}'$

e

$2\,k k'$

$\pi)^3\delta^{(3)}(k-k')$ invariant under de Sitter transformation $SO(4,1)$

$$\underbrace{\langle BD|a_k a^\dagger_{k'}|BD\rangle}_{(2\pi)^3\delta^{(3)}(k-k')}$$

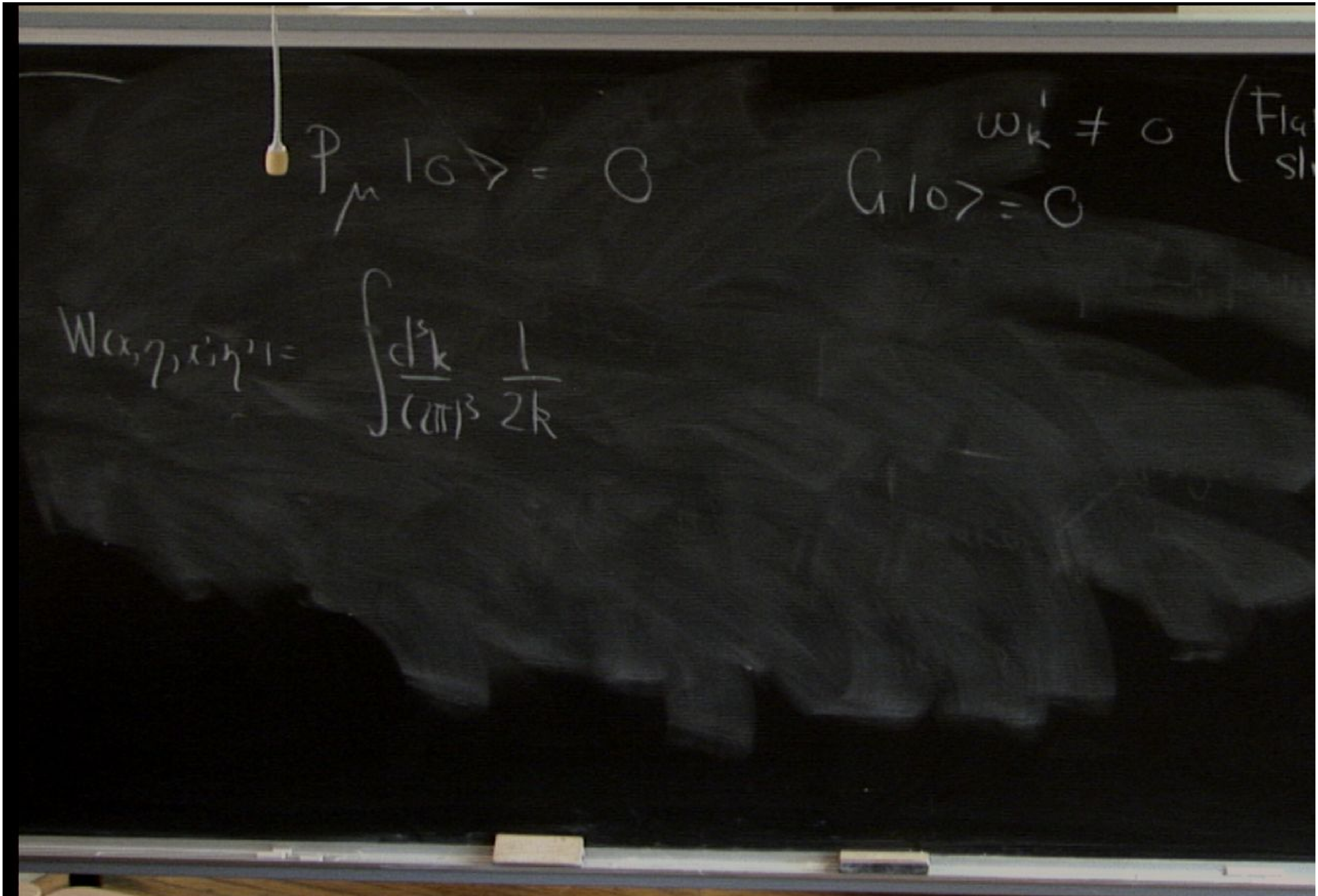


$P_\mu |0\rangle = 0$

$\omega_k' \neq 0$ (Fla
sl

$G|0\rangle = 0$

$W(\alpha,\eta,$

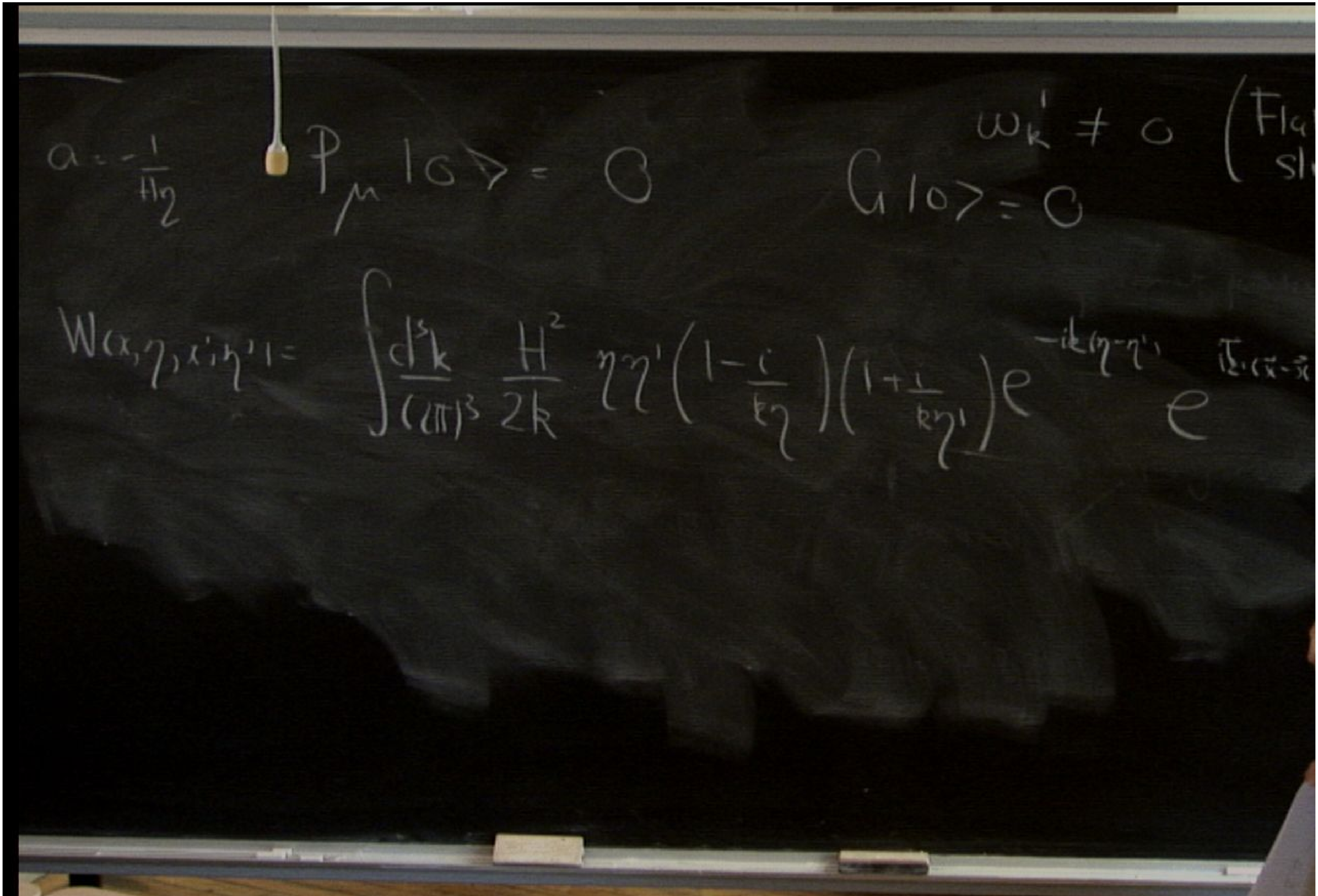


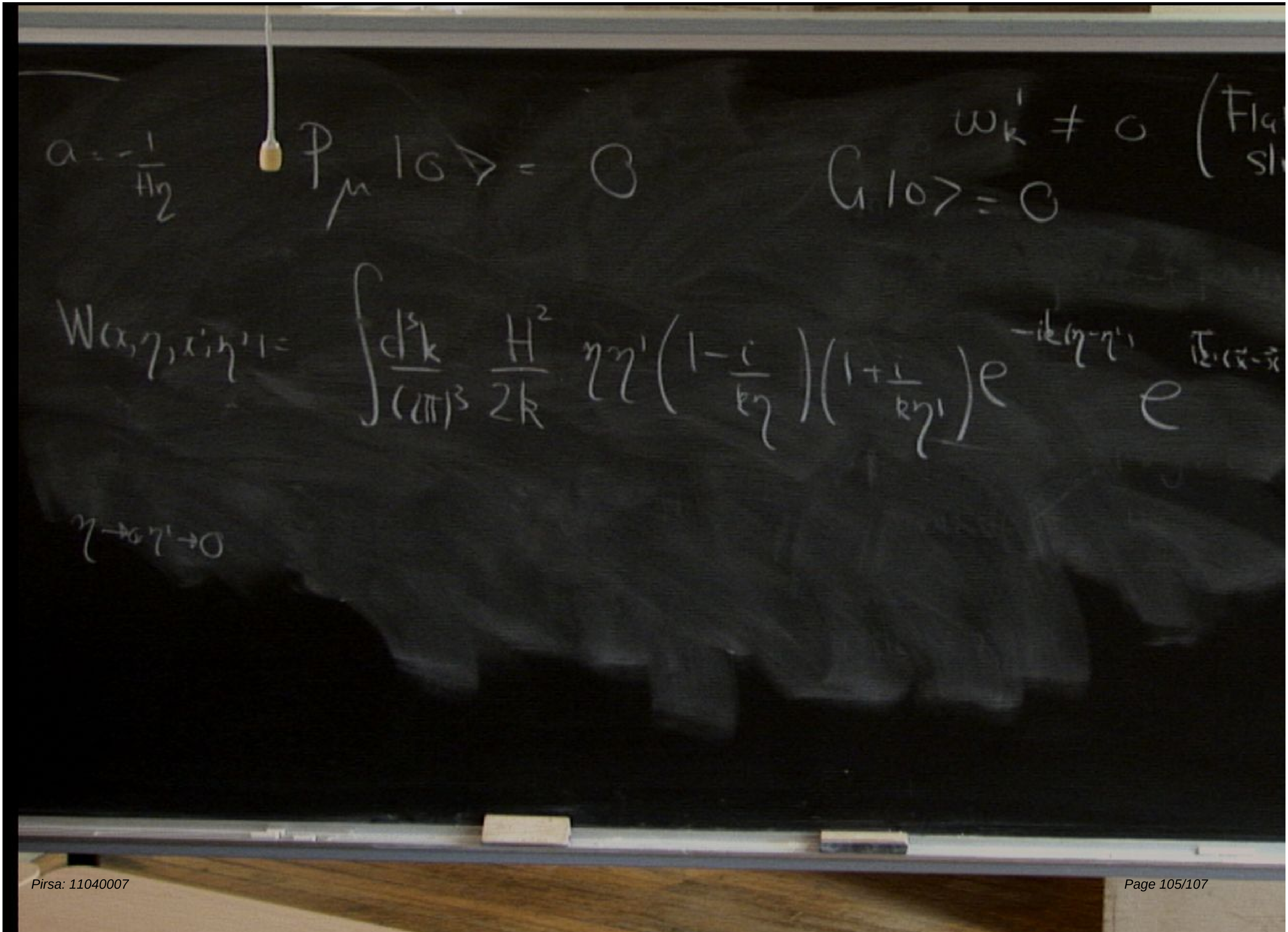
$$P_\mu |0\rangle = 0$$

$$\omega_k' \neq 0 \quad \Big(\text{Fla} \; \text{sl}$$

$$G|0\rangle = 0$$

$$W(x,\eta,x',\eta') = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k}$$





$a = -\frac{1}{H\eta}$

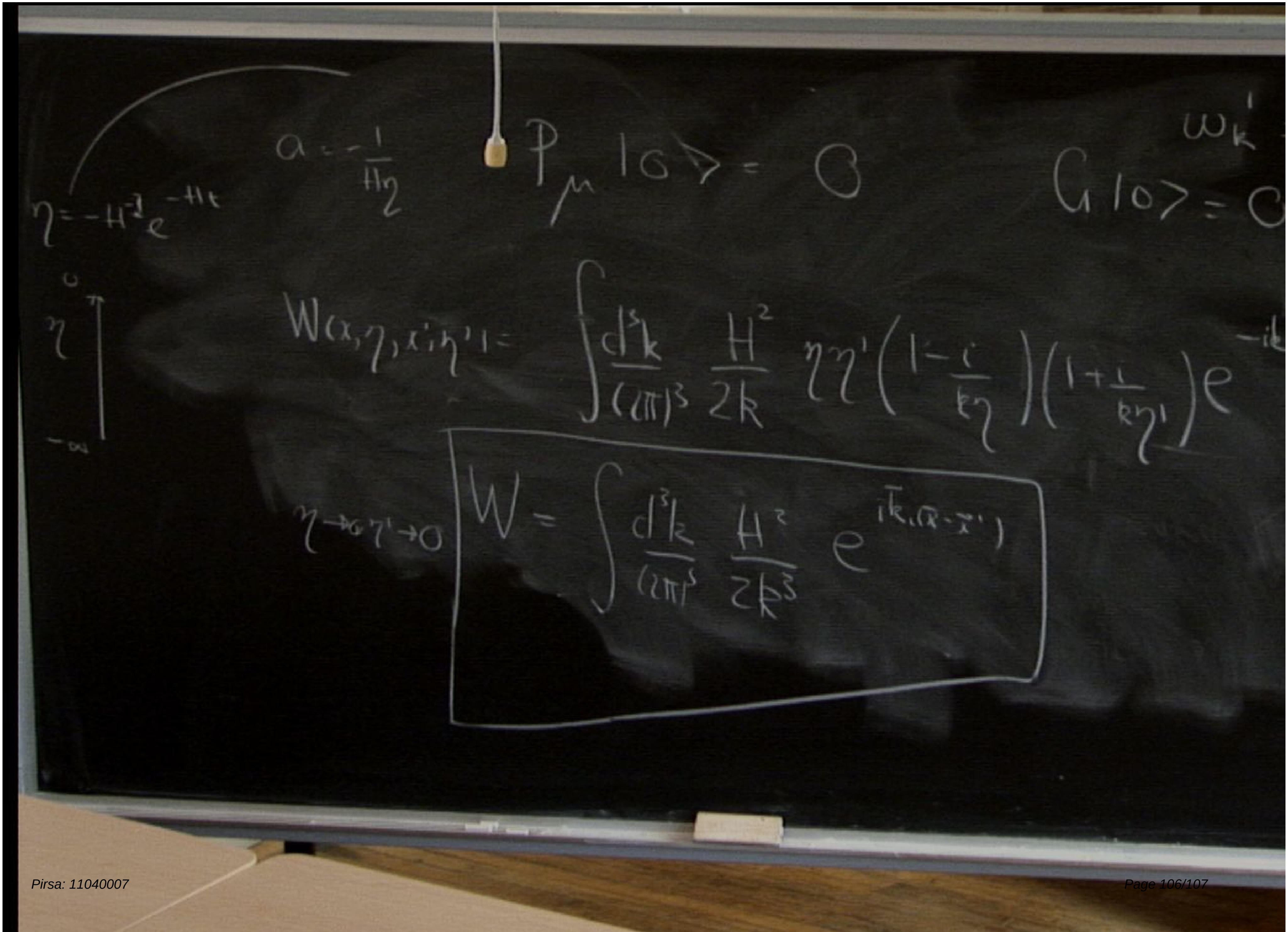
$P_\mu |0\rangle = 0$

$\omega_k' \neq 0$ (Fla
sl

$G|0\rangle = 0$

$$W(x,\eta,x',\eta') = \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k} \eta\eta' \left(1 - \frac{i}{k\eta}\right)\left(1 + \frac{i}{k\eta'}\right) e^{-ik(\eta-\eta')} e^{i\vec{k}\cdot(\vec{x}-\vec{x}}$$

$\eta \to 0 \ \eta' \to 0$



$\eta = -H^{-1} e^{-Ht}$

$a = -\frac{1}{H\eta}$

$P_\mu |0\rangle = 0$

ω_k'

$G|0\rangle = 0$

0

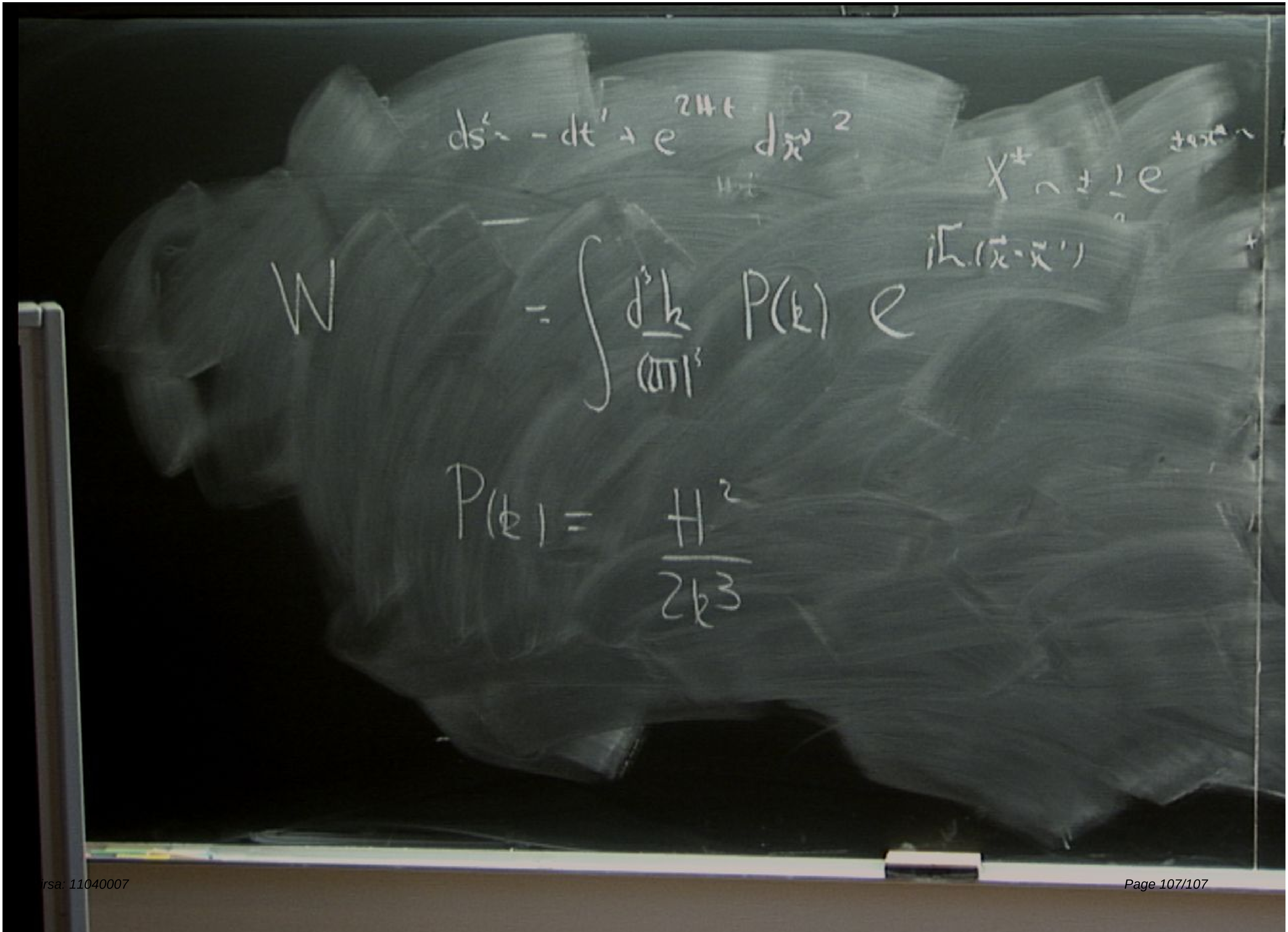
η

$-\infty$

$$W(x,\eta,x',\eta') = \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k} \eta\eta' \left(1 - \frac{i}{k\eta}\right)\left(1 + \frac{i}{k\eta'}\right) e^{-ik}$$

$\eta \to 0 \; \eta' \to 0$

$$\boxed{W = \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}}$$



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$$X^{\pm} \sim \pm \frac{1}{H} e^{\pm H x^{\pm}}$$

$$W = \int \frac{d^3k}{(2\pi)^3} P(k)\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$P(k) = \frac{H^2}{2k^3}$$