Title: Explorations in Cosmology - Lecture 4

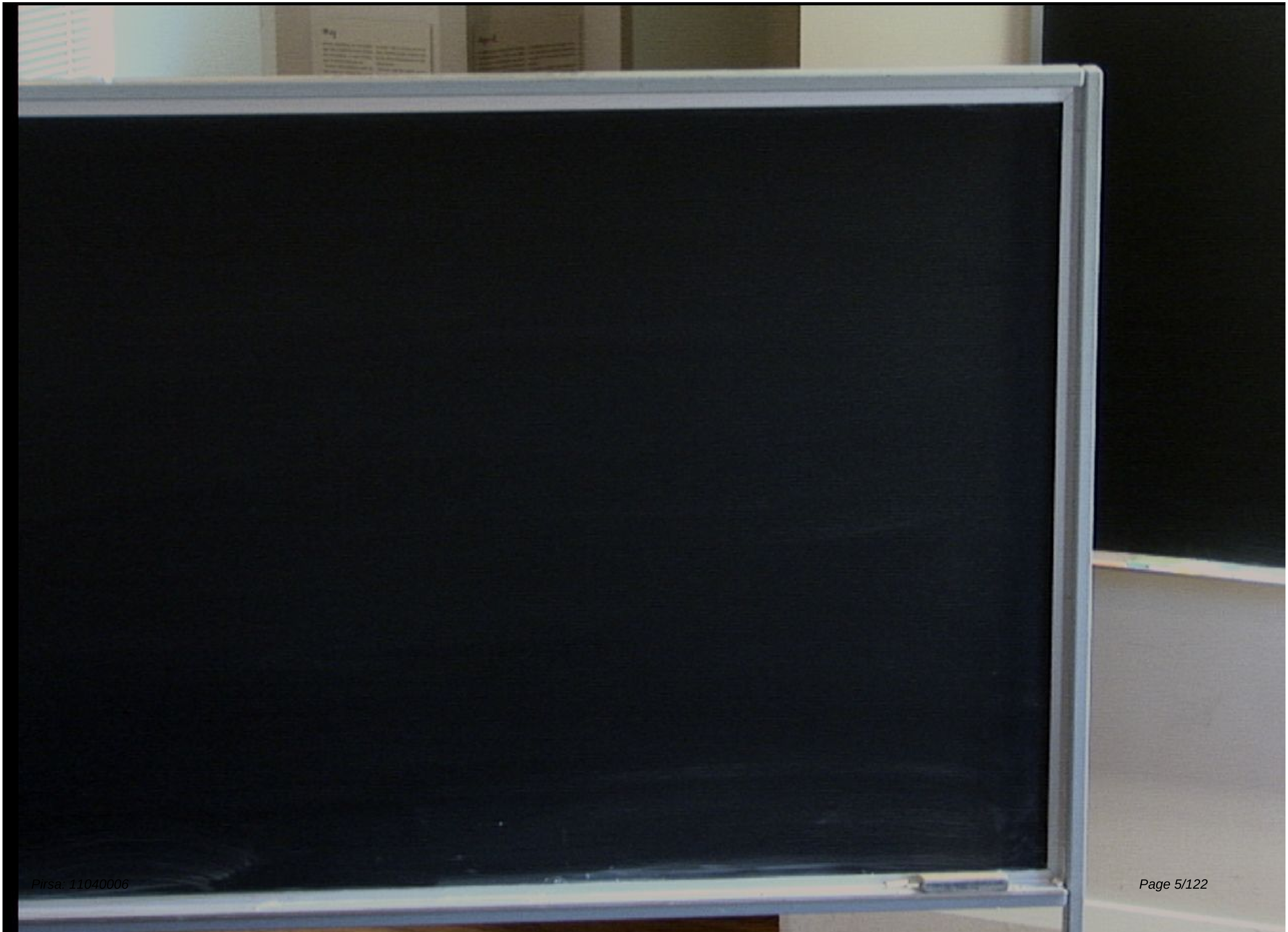
Date: Apr 07, 2011 09:00 AM

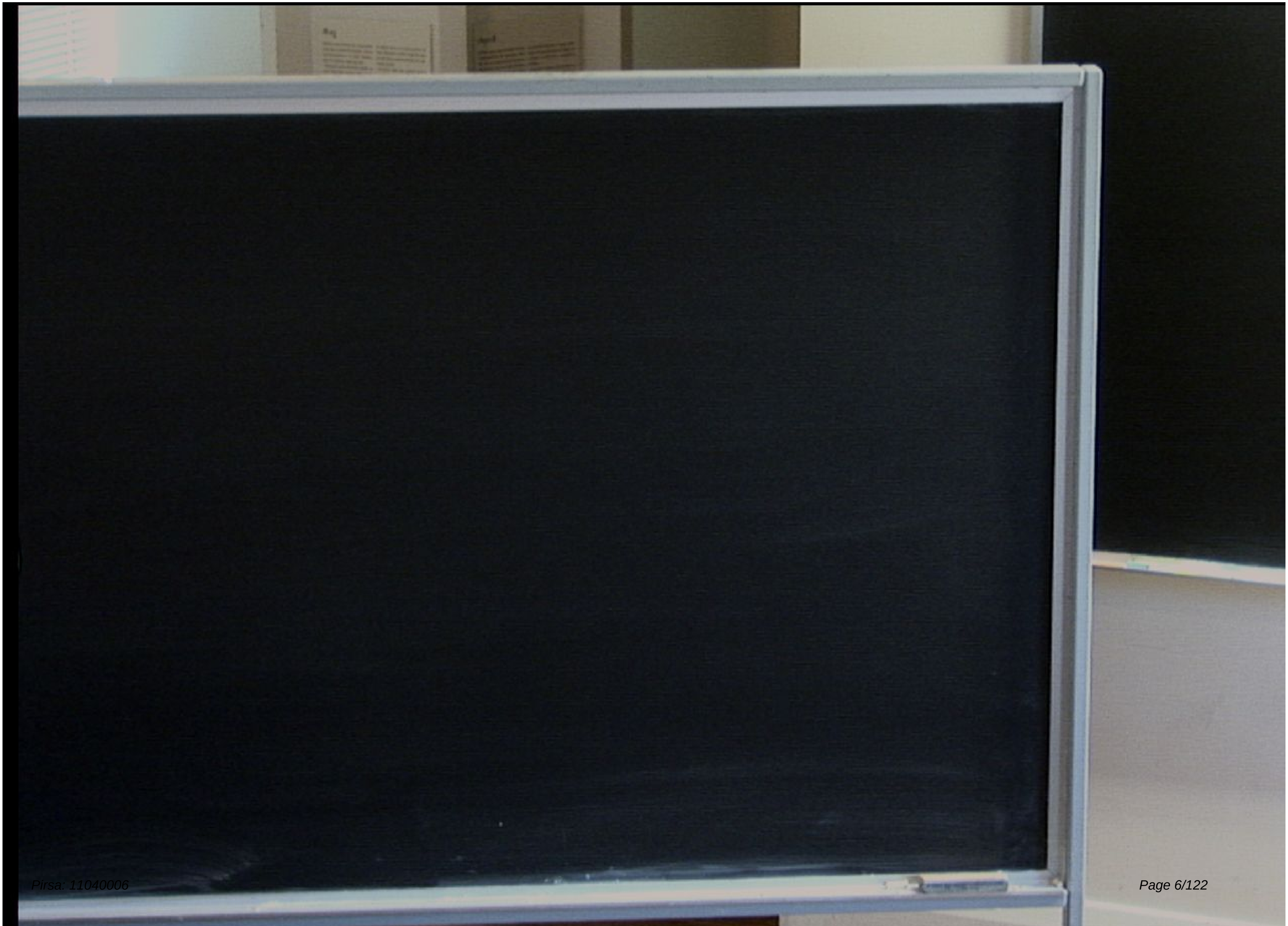
URL: http://pirsa.org/11040006

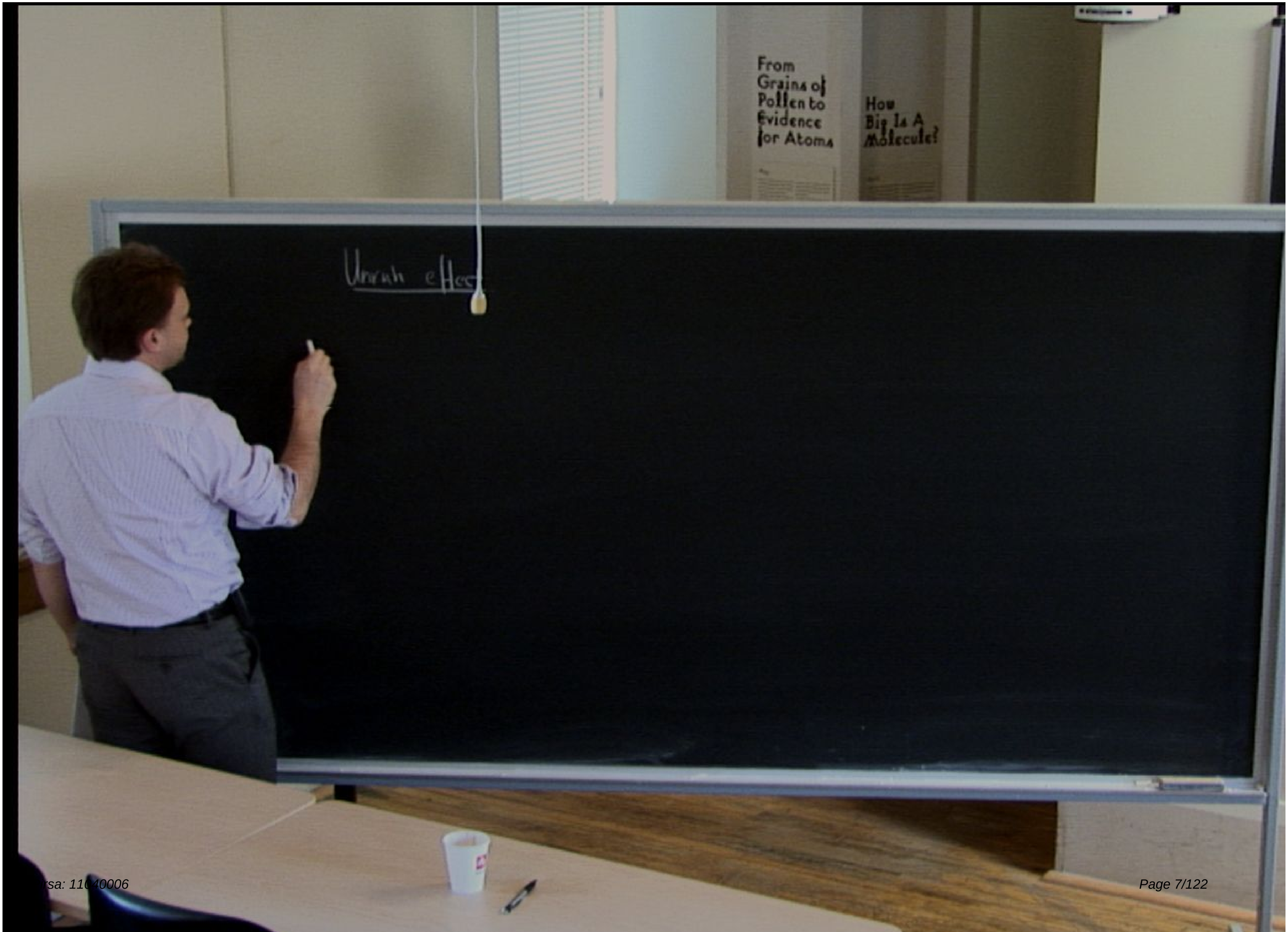
Abstract:

perimeter scholars
international

perimeter scholars
international



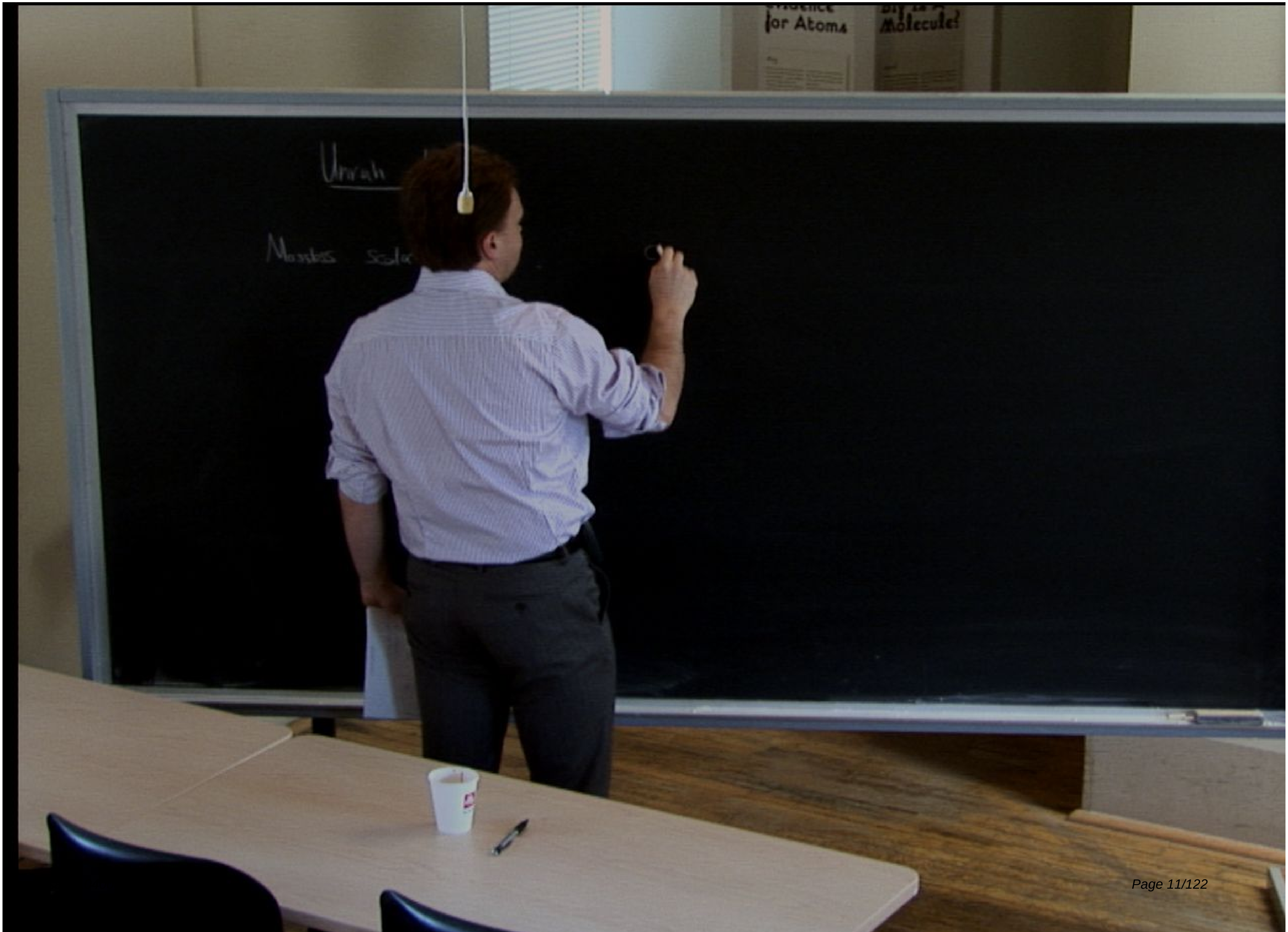


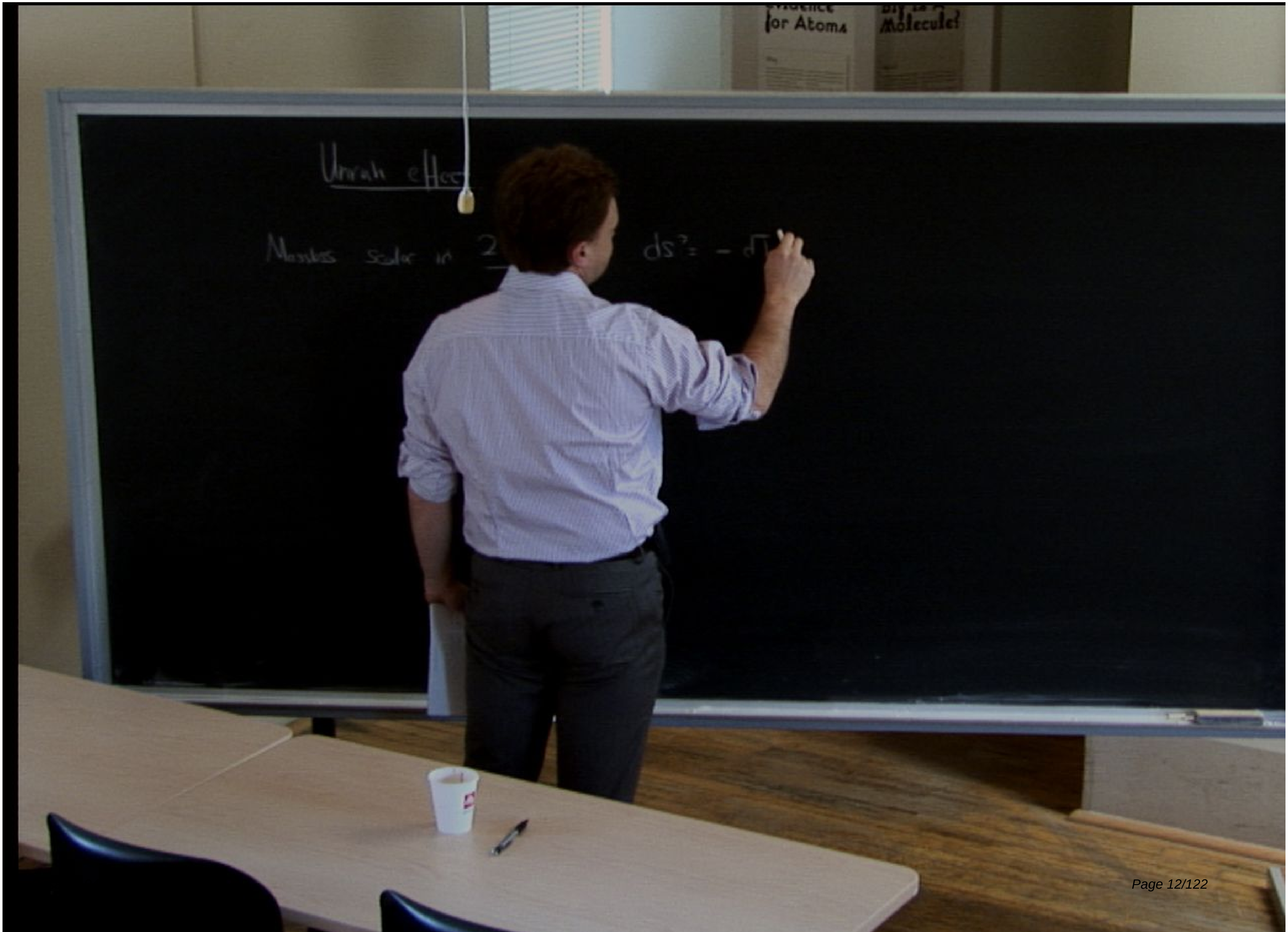

Unruh
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?

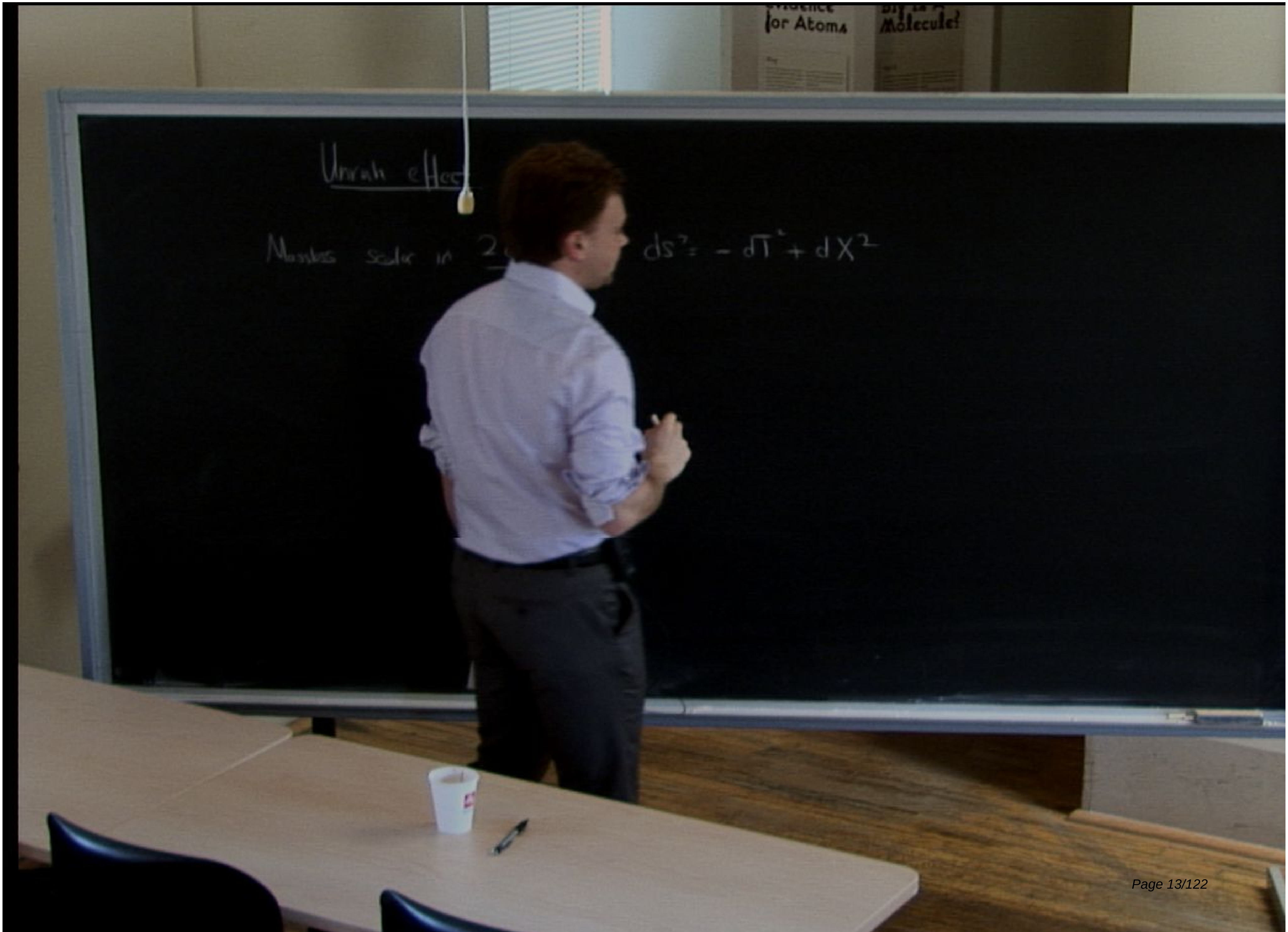

Unruh effect

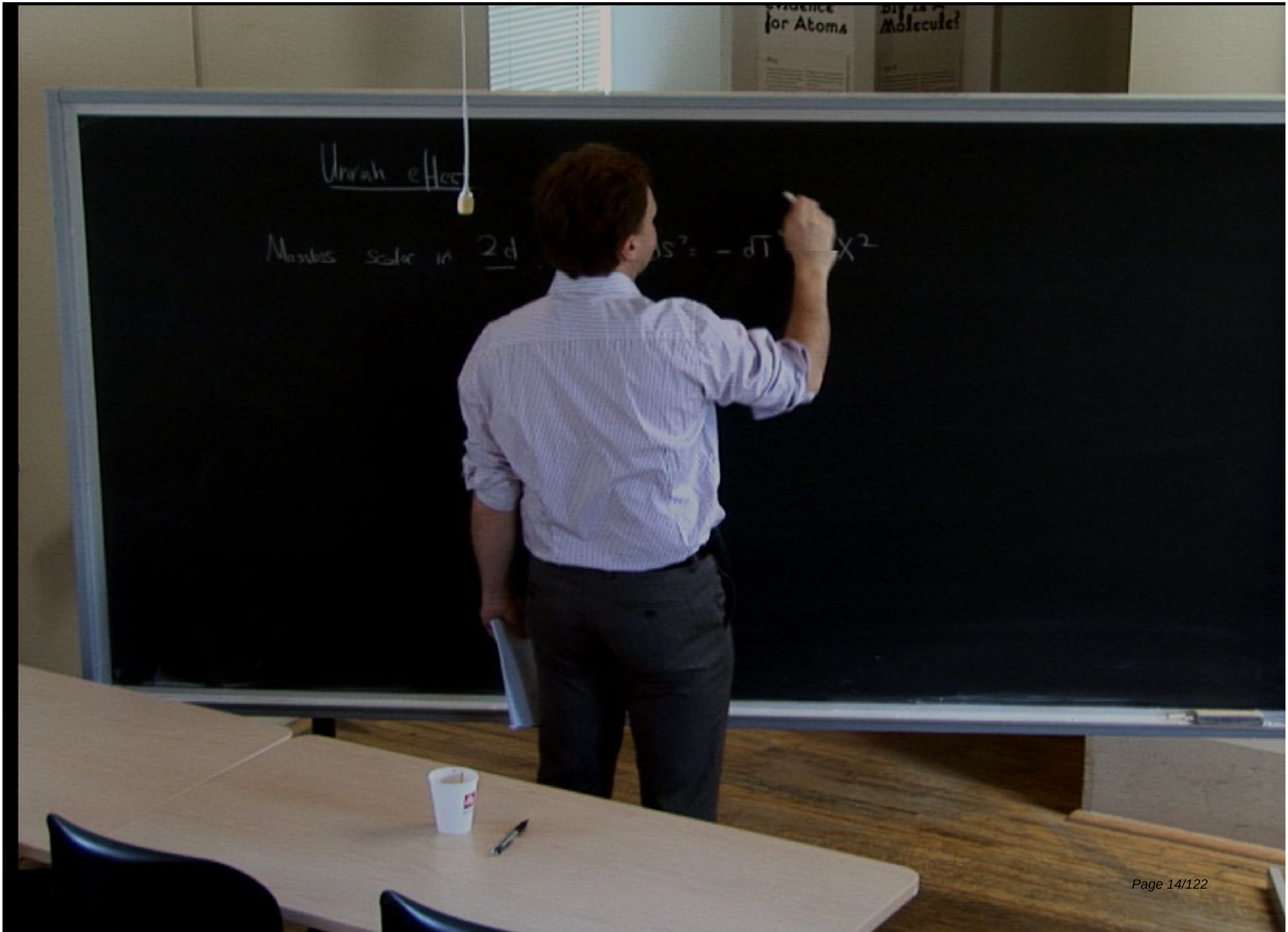

for Atoms
Molecule!

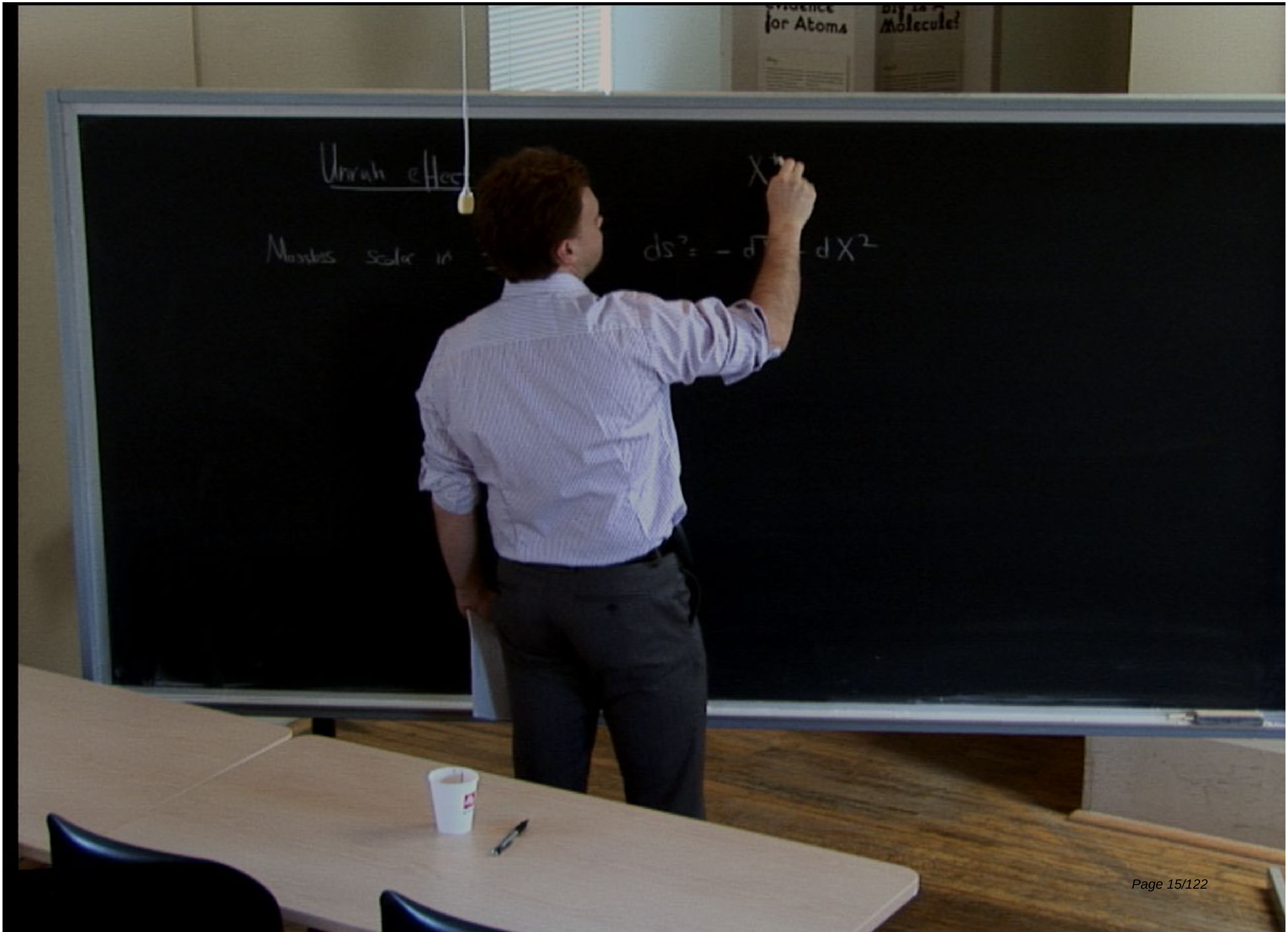

for Atoms
Molecule!
Unruh effect
scalar in 2d

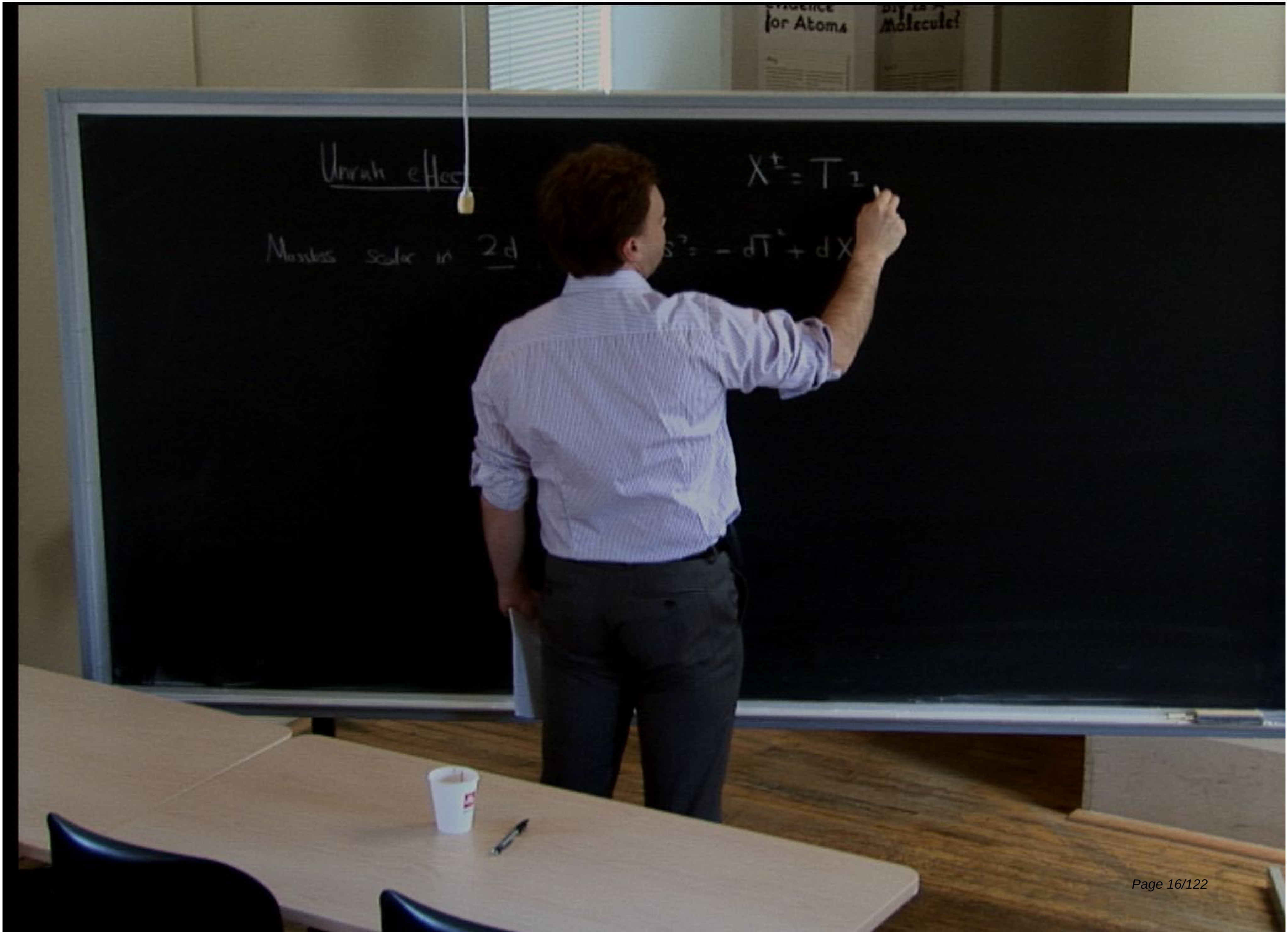

for Atoms
Molecule!

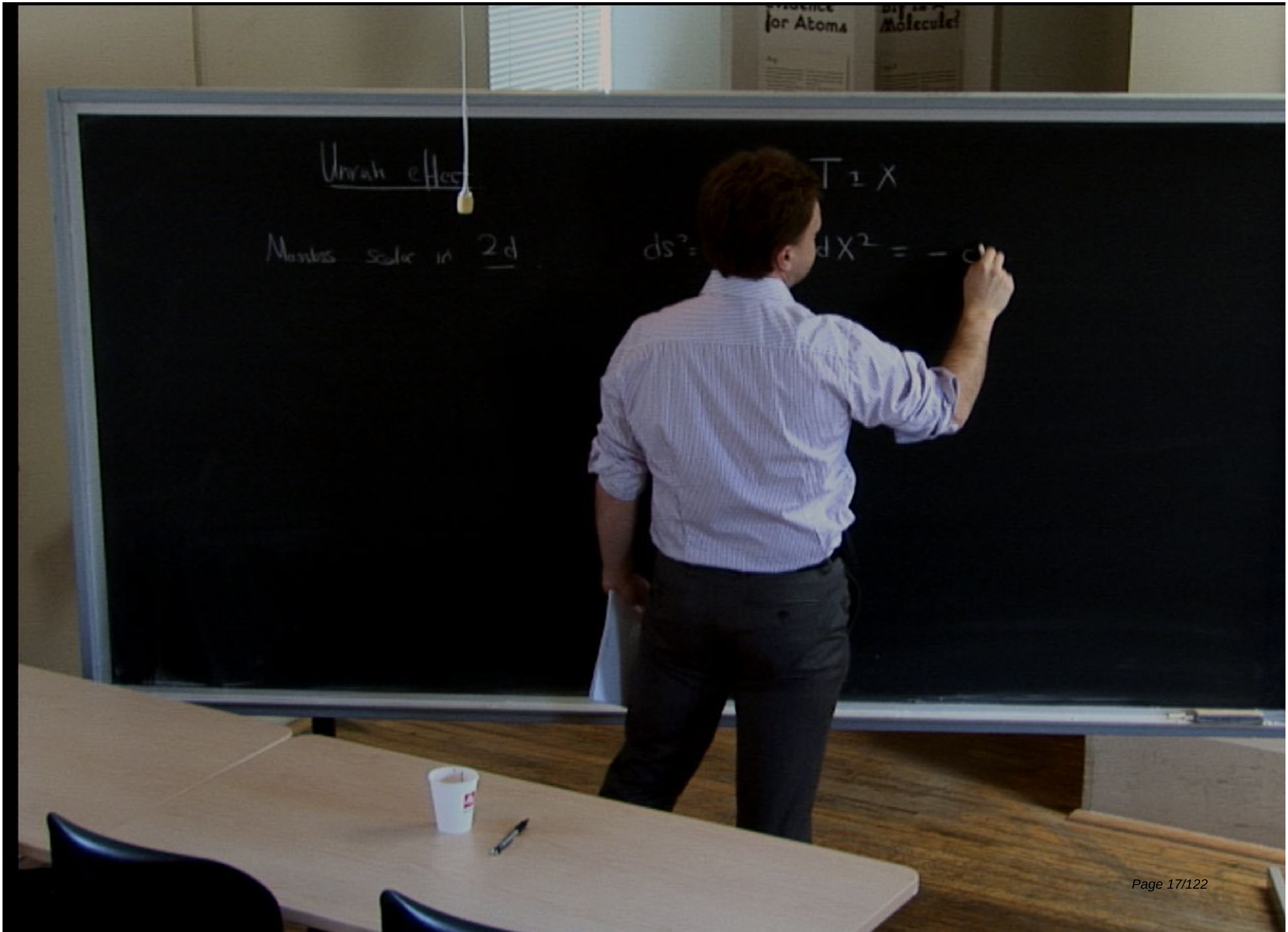

for Atoms
Molecule!
Massless scalar in 2

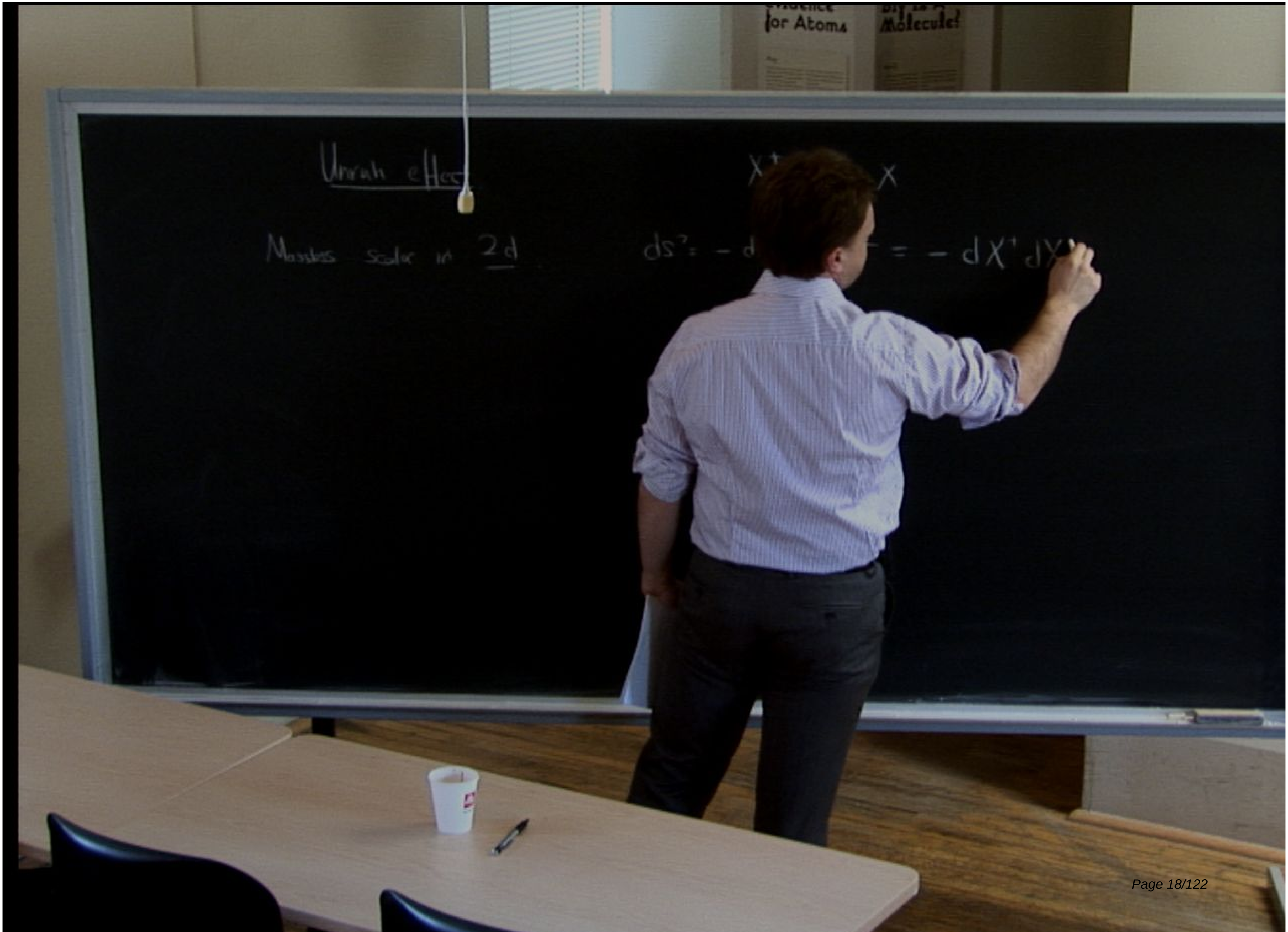

for Atoms
Molecule!
Unruh effect
ds² = - dT² + dX²


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d


for Atoms
Molecule!


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d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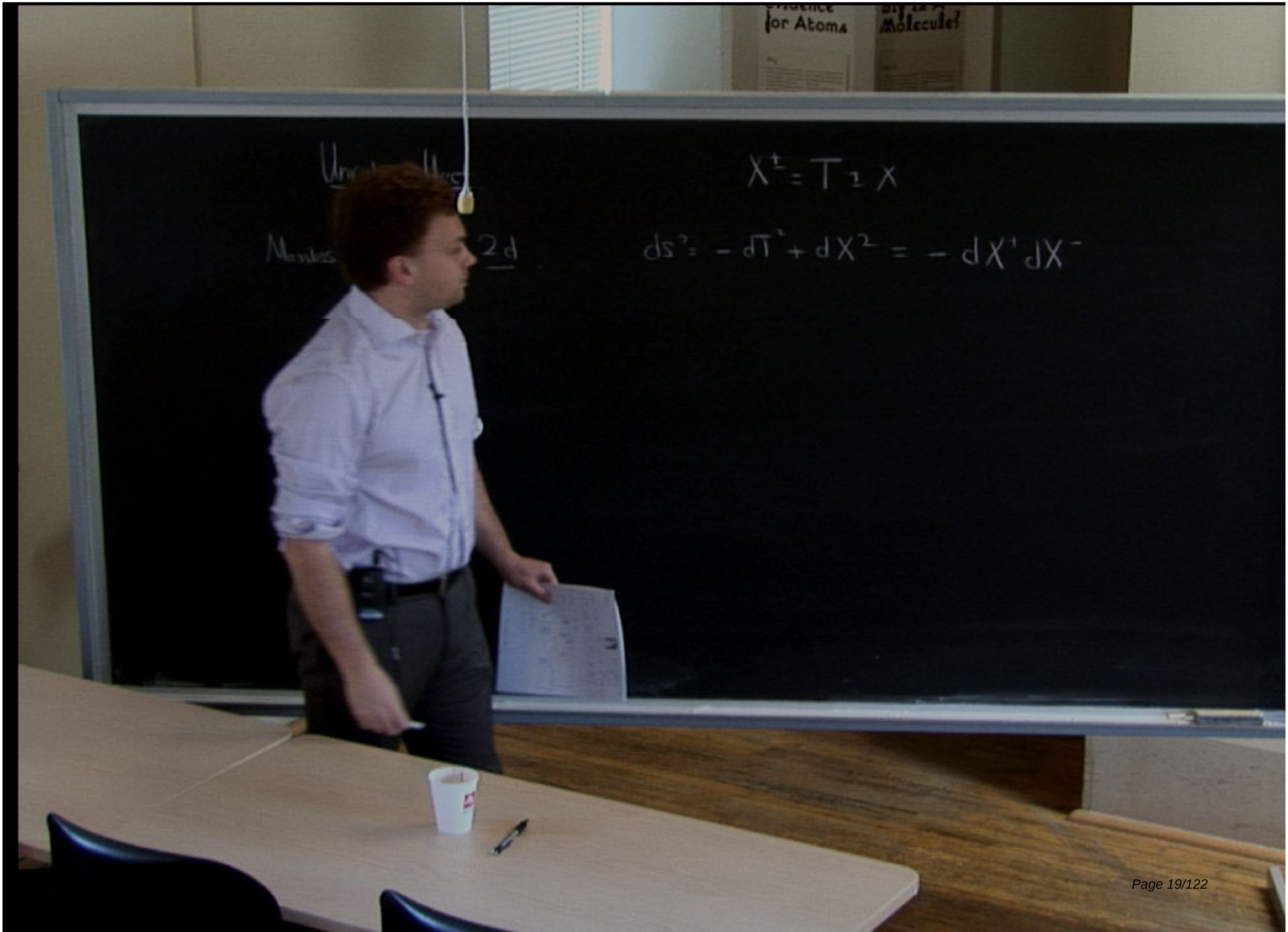
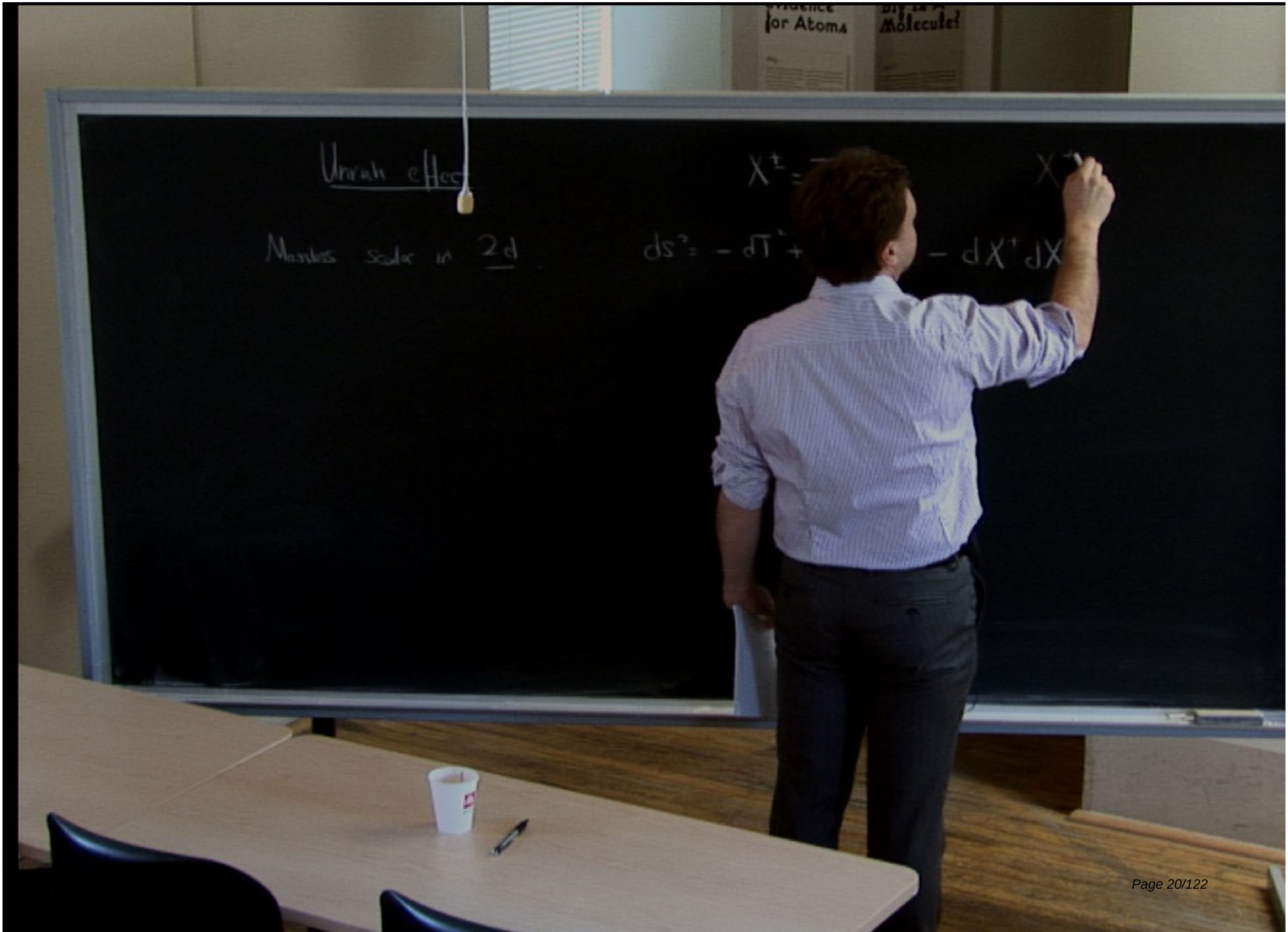
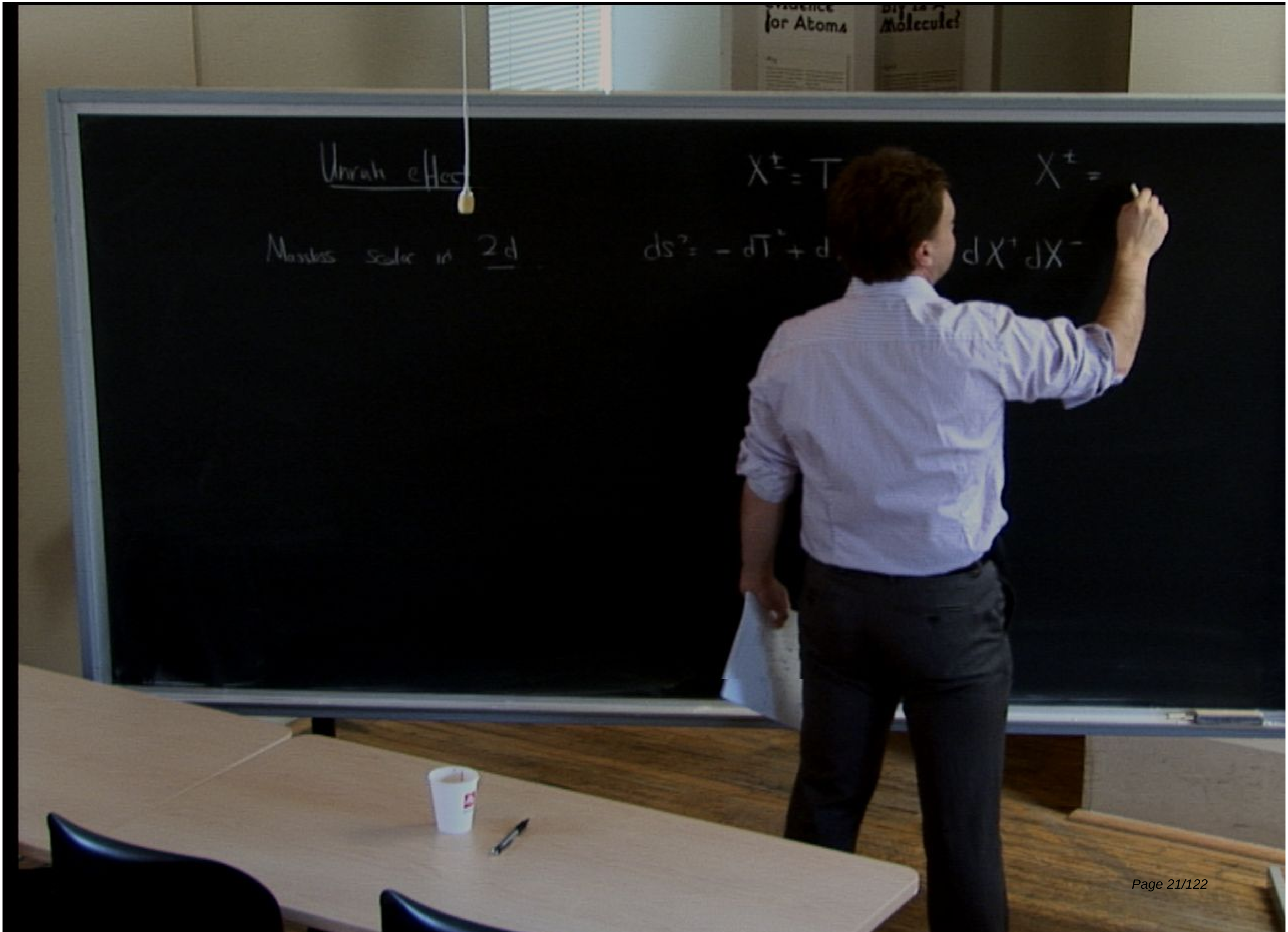
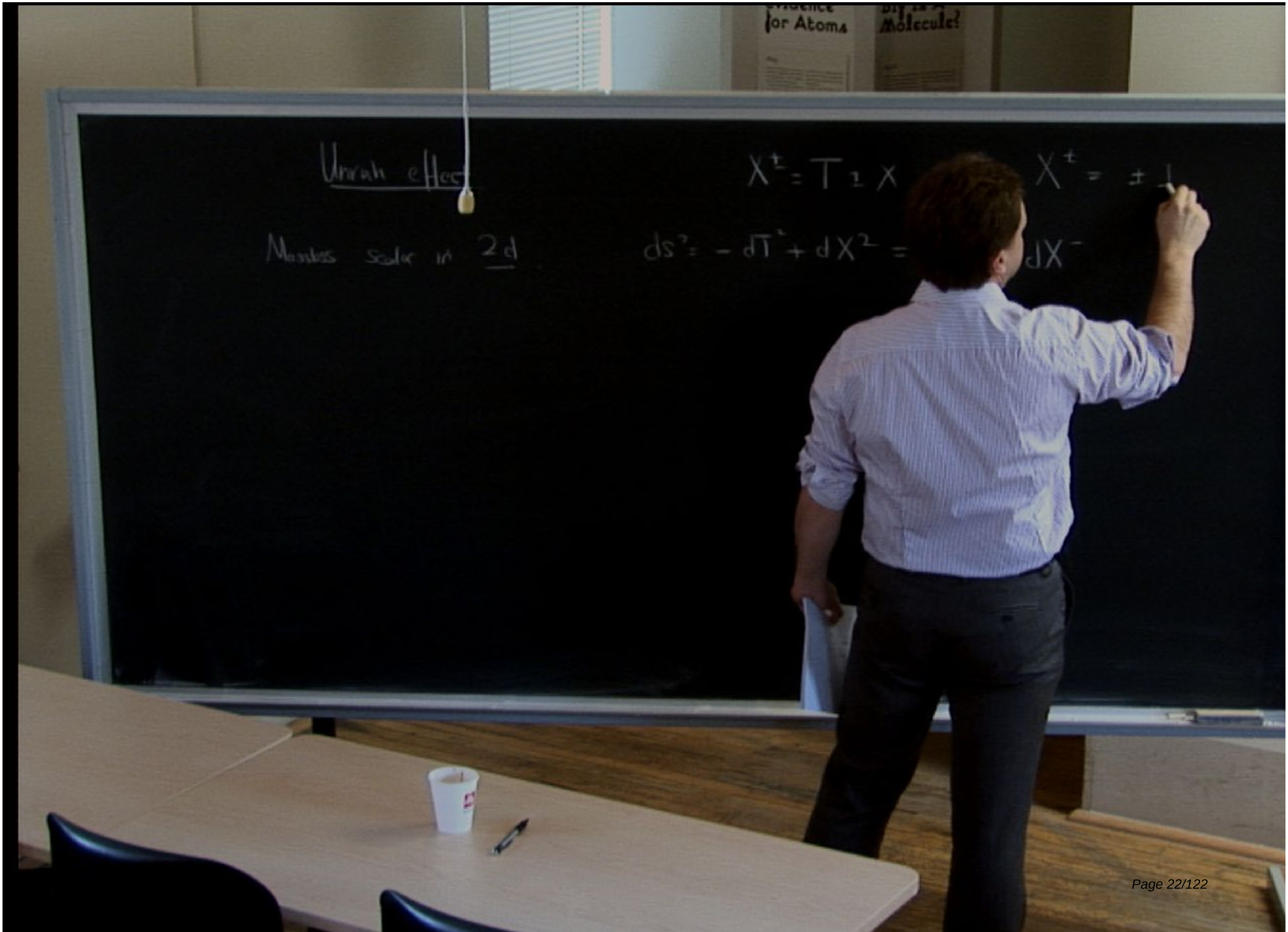
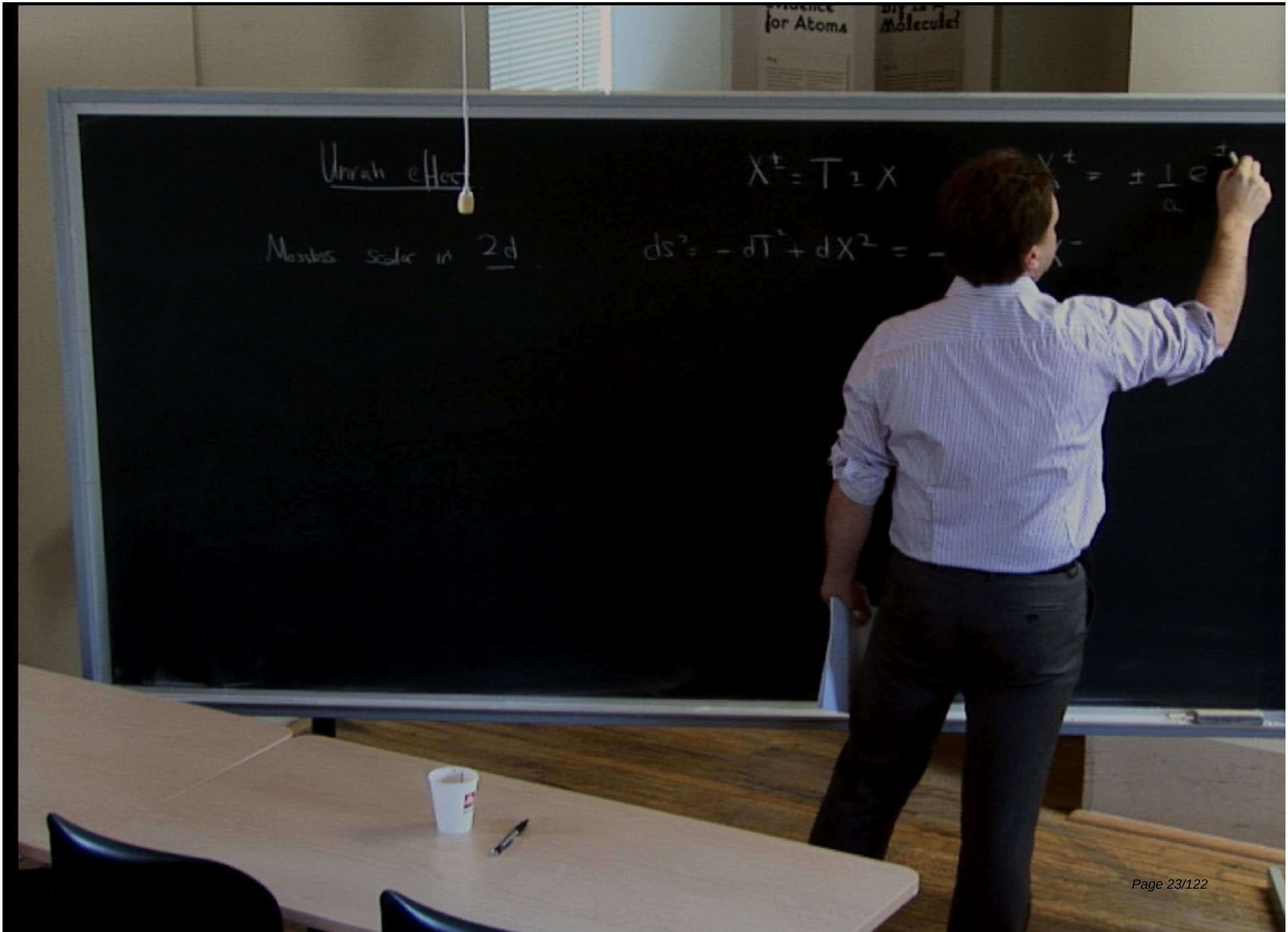
Molecule!
Unruh effect
Massless scalar in 2d

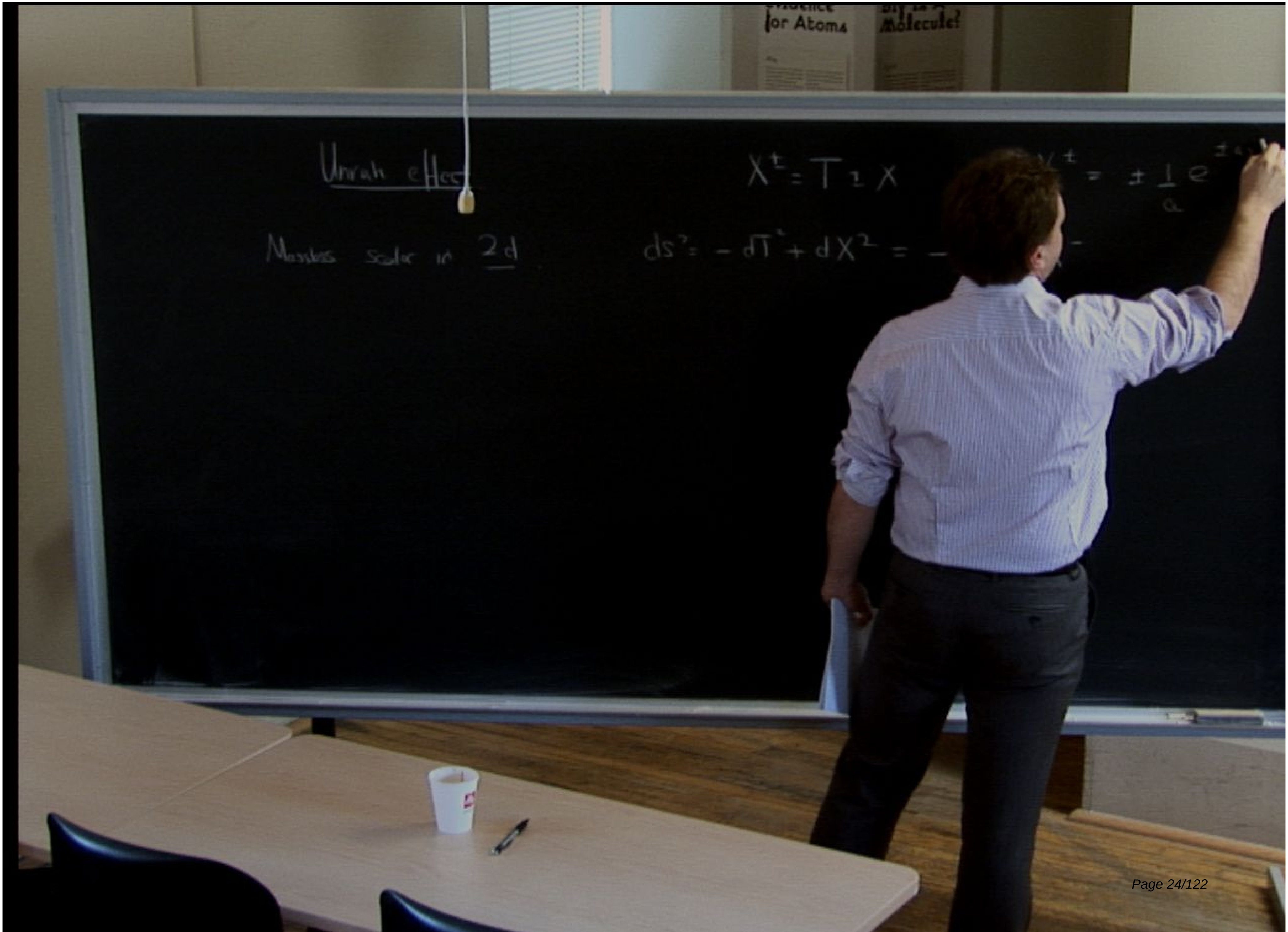

for Atoms
Molecule!
X± = T ± X
ds² = − dT² + dX² = − dX⁺ dX⁻

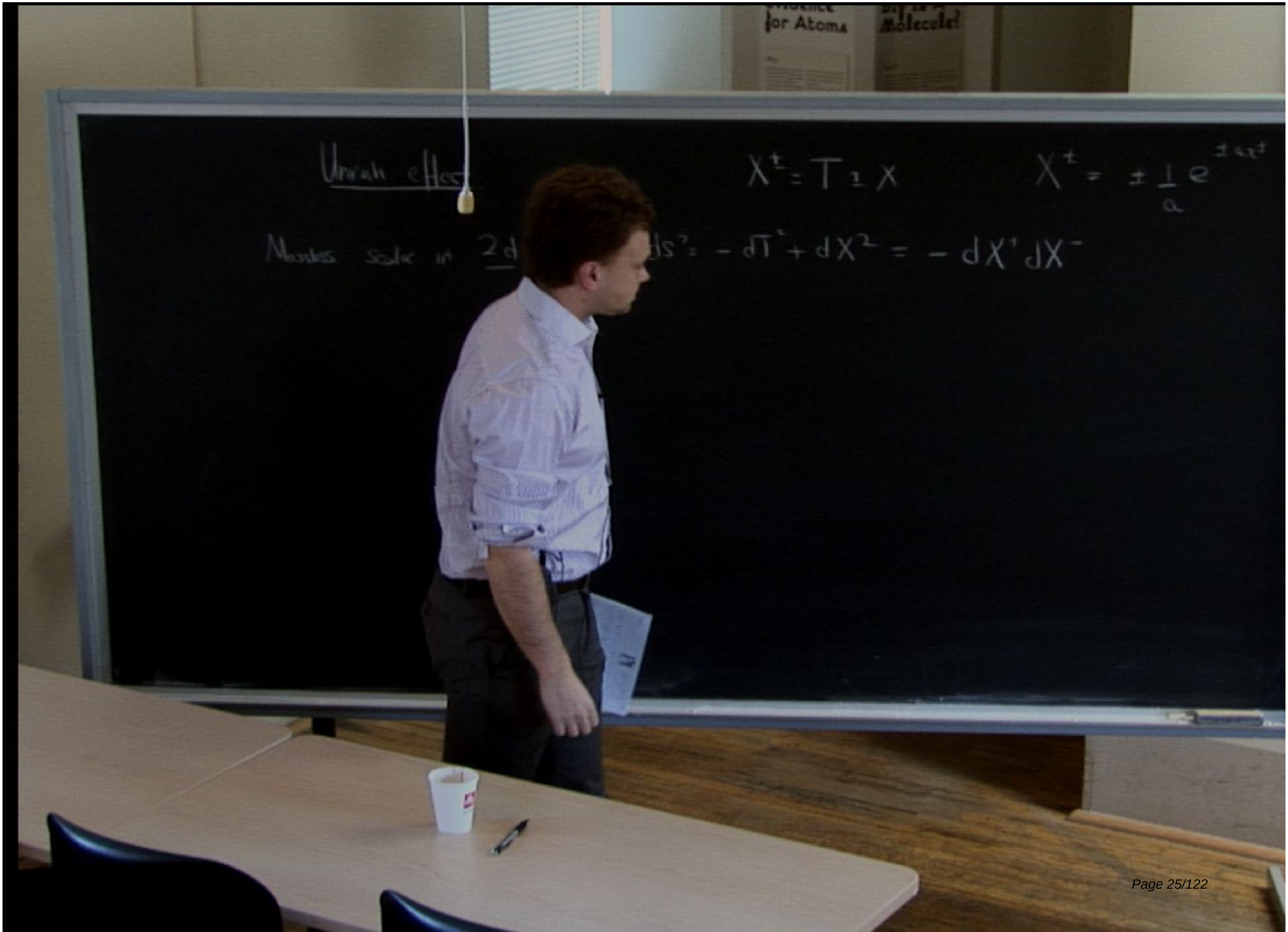

for Atoms
Unruh effect
Massless scalar in 2d

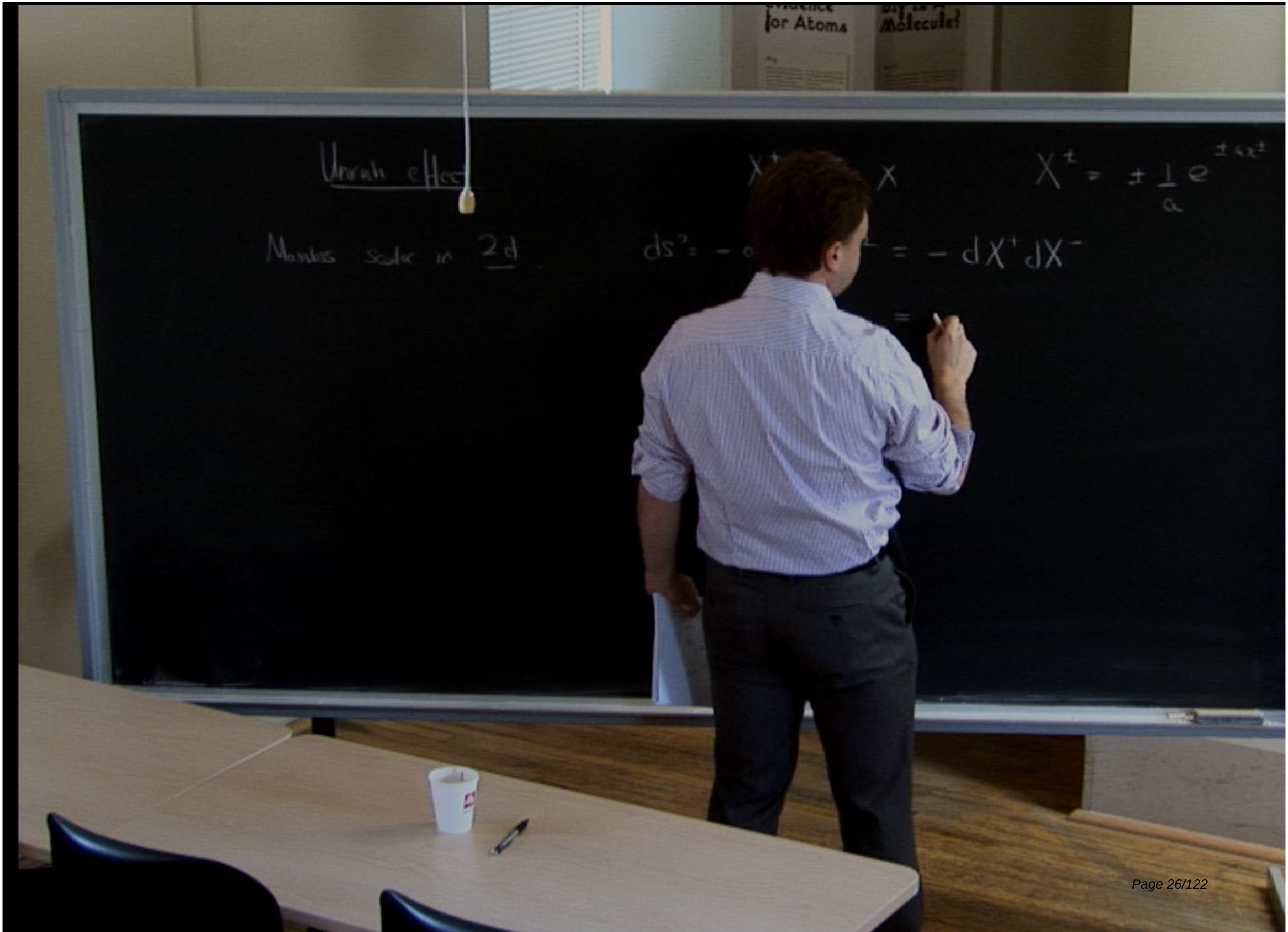

for Atoms
Molecule?
Unruh effect
Massless scalar in 2d
$X^{\pm} = T$
$X^{\pm} =$
$ds^2 = -dT^2 + d$
$dX^+ dX^-$


for Atoms
Molecule!
Unruh effect
$X^{\pm} = T \pm X$
Massless scalar in 2d
$ds^2 = -dT^2 + dX^2 =$


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d
X± = T ± X


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d
$X^\pm = T \pm X$
$ds^2 = -dT^2 + dX^2 = -$


for Atoms
Molecule!
Unruh effect
X± = T ± X
Massless scalar in 2d


for Atoms
Molecule!
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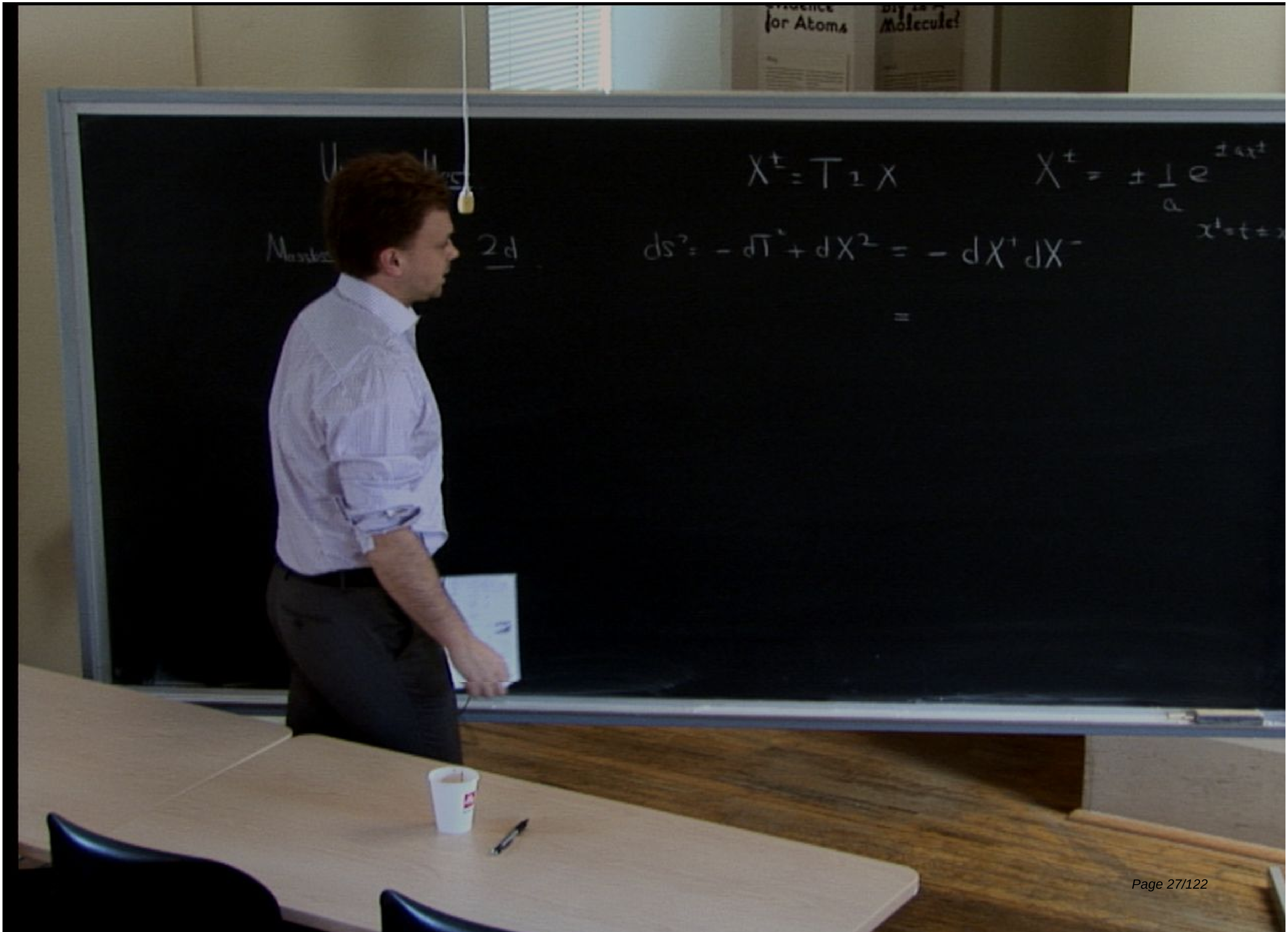
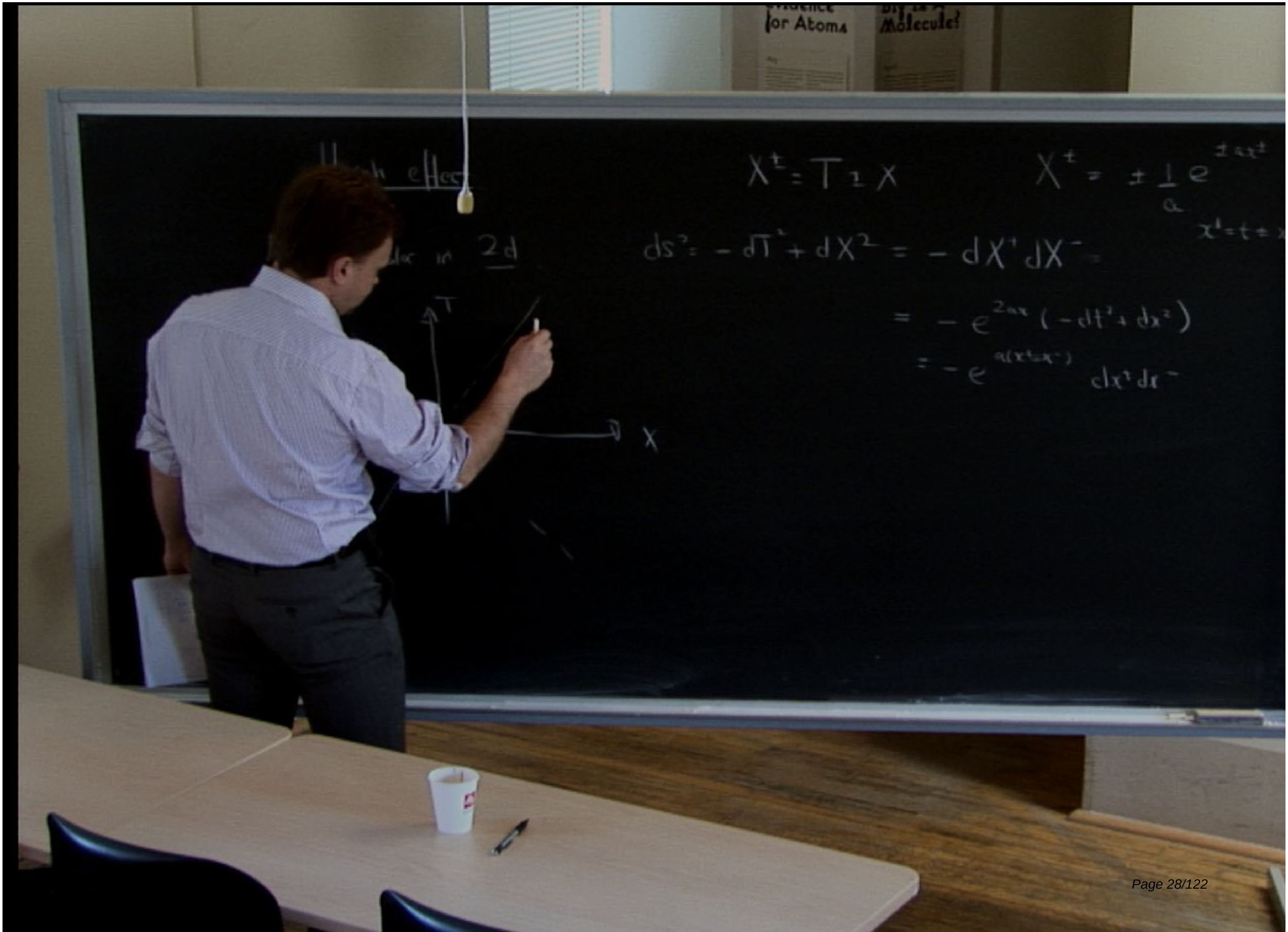
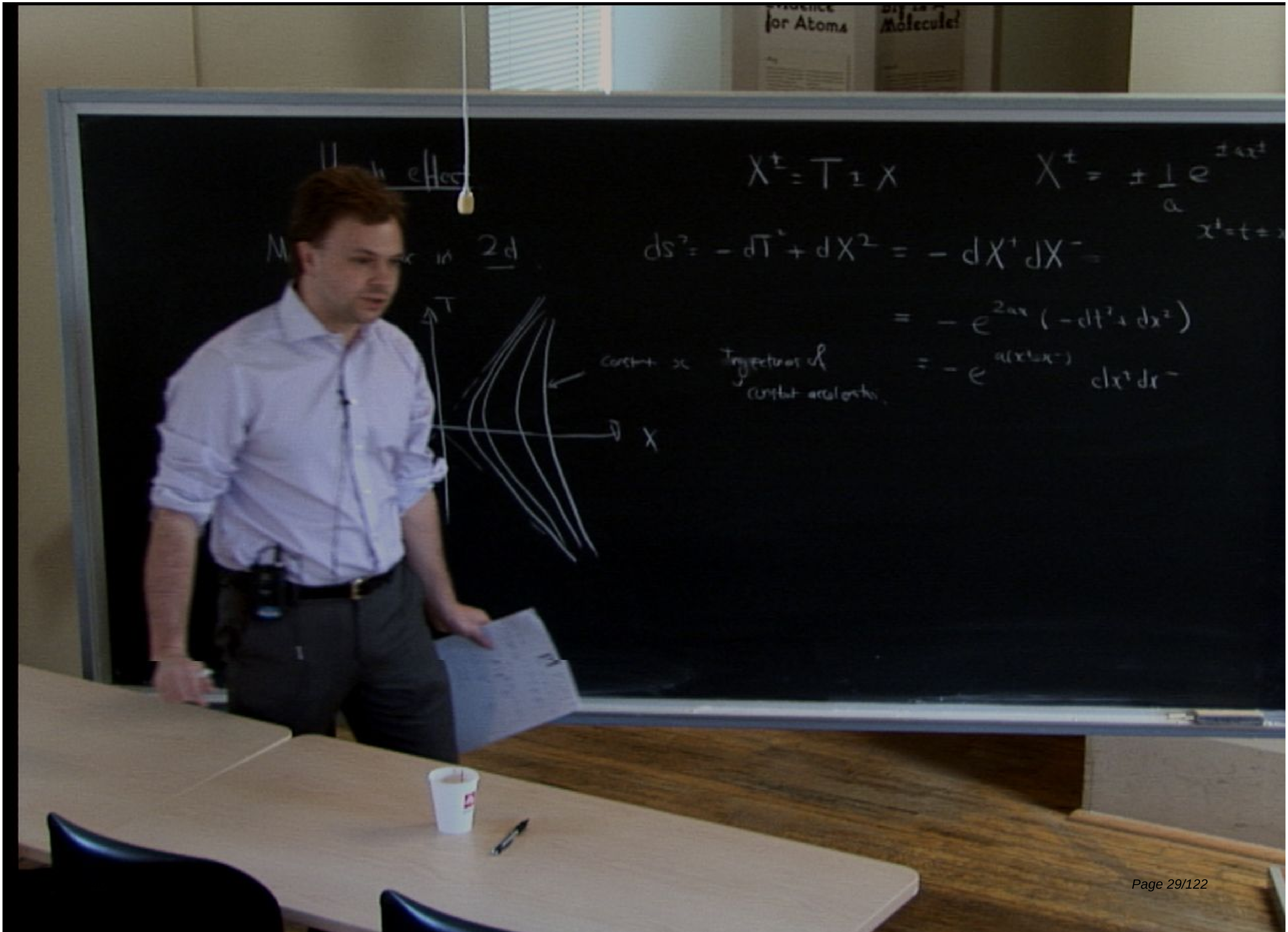
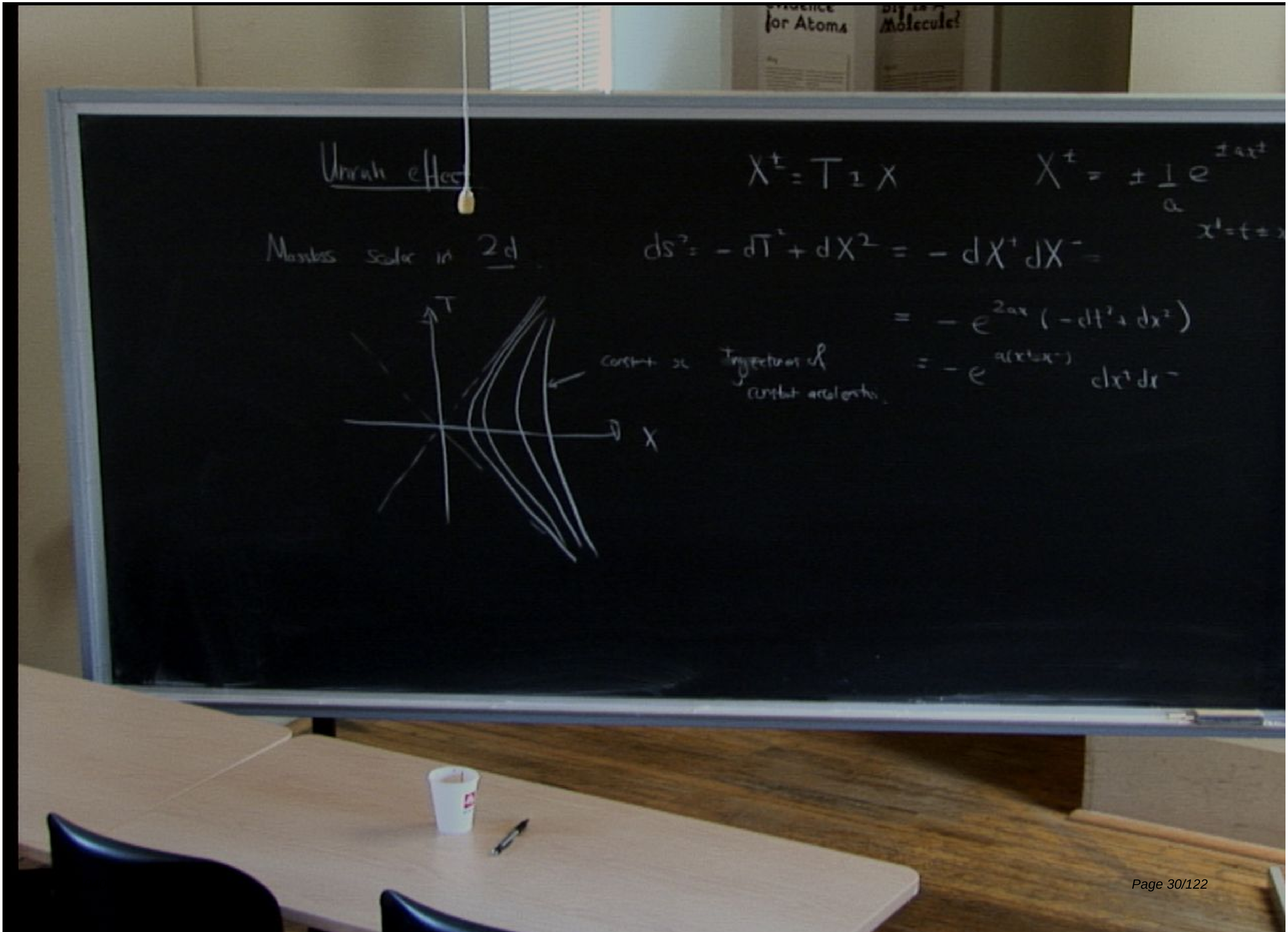
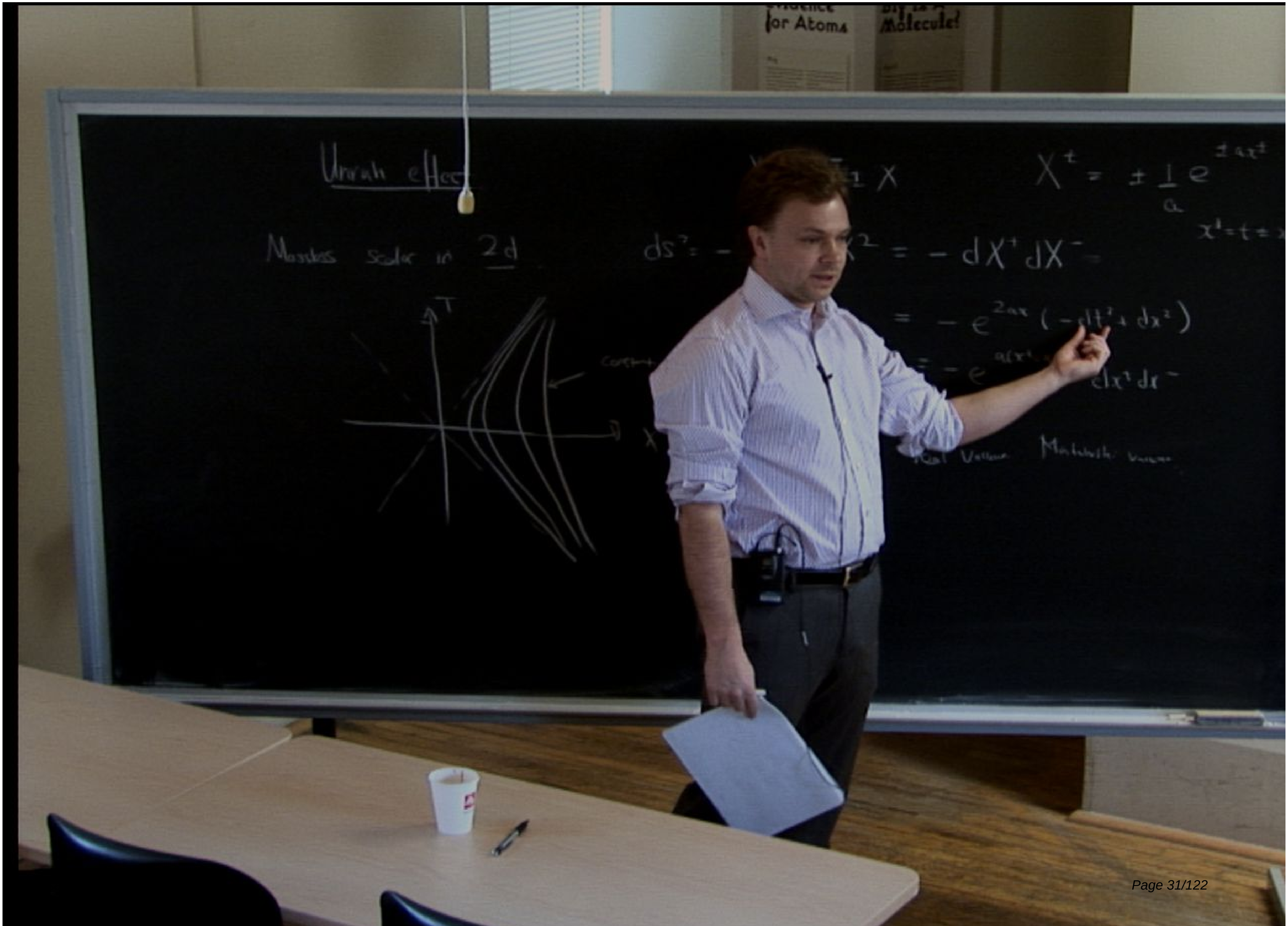
Massless scalar in 2d

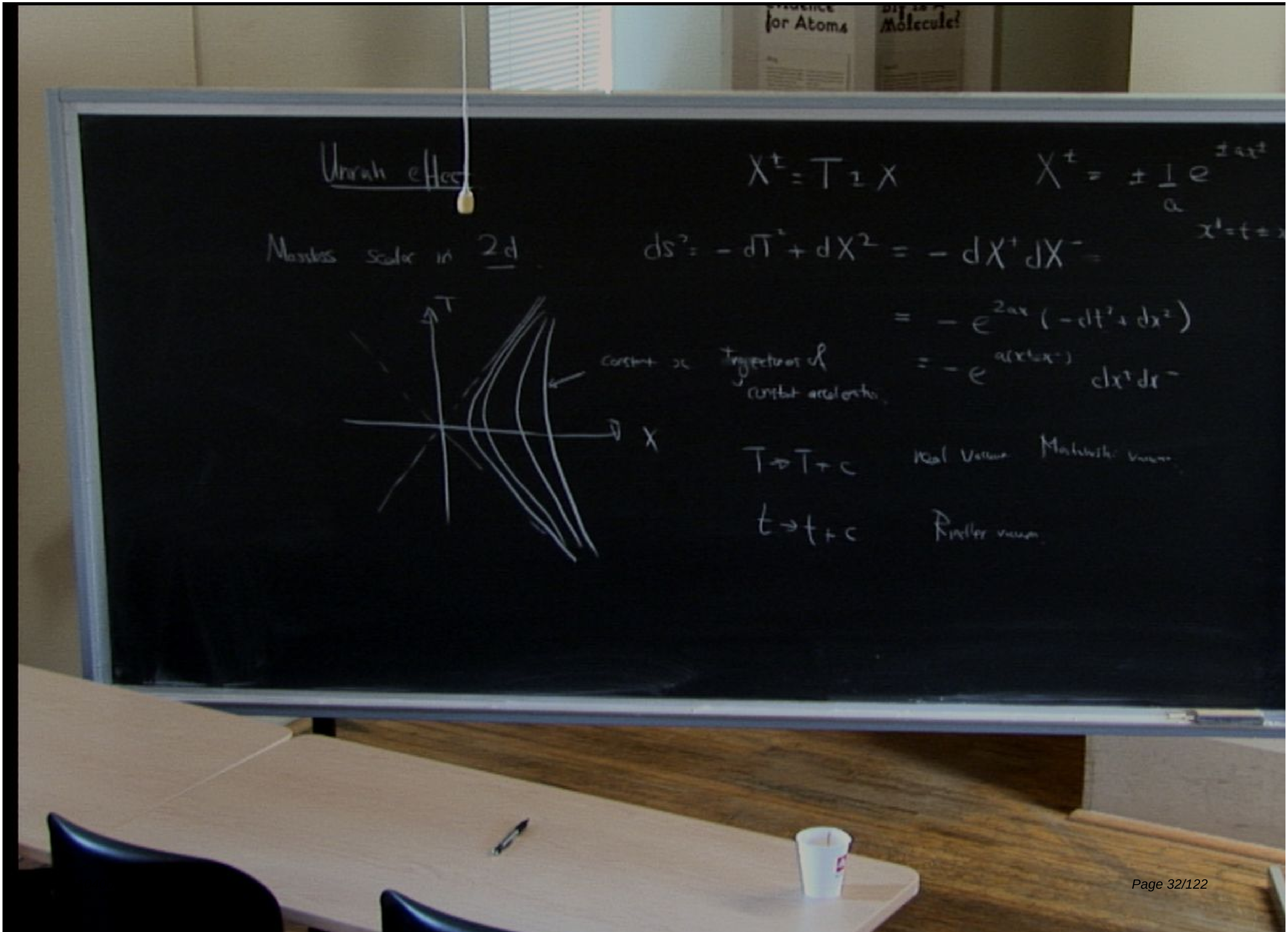

for Atoms
Molecule!
2d
X± = T ± X
ds² = − dT² + dX² = − dX⁺ dX⁻


for Atoms
Molecule!


for Atoms
Molecule!
in 2d
$X^\pm = T \pm X$
$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$
$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$
$= -e^{2ax}(-dt^2 + dx^2)$
$= -e^{a(x^+ - x^-)} dx^+ dx^-$
T
X
constant x trajectories of
constant acceleration


Unruh effect
Massless scalar in 2d
X^± = T ± X
X^± = ± 1/a e^{±ax^±}
x^± = t ± x
ds² = − dT² + dX² = − dX⁺ dX⁻ =
= − e^{2ax} (− dt² + dx²)
= − e^{a(x⁺−x⁻)} dx⁺ dx⁻
const x trajectory of constant acceleration
T
X
for Atoms
Molecule!


for Atoms
Molecule!
Unruh effect
Massless scalar in 2d


Unruh effect
Massless scalar in 2d
X± = T ± X
ds² = −dT² + dX² = −dX⁺dX⁻ =
= −e^{2ax} (−dt² + dx²)
constant x trajectories of constant acceleration
T → T + c
Real Vacuum Minkowski vacuum
t → t + c
Rindler vacuum

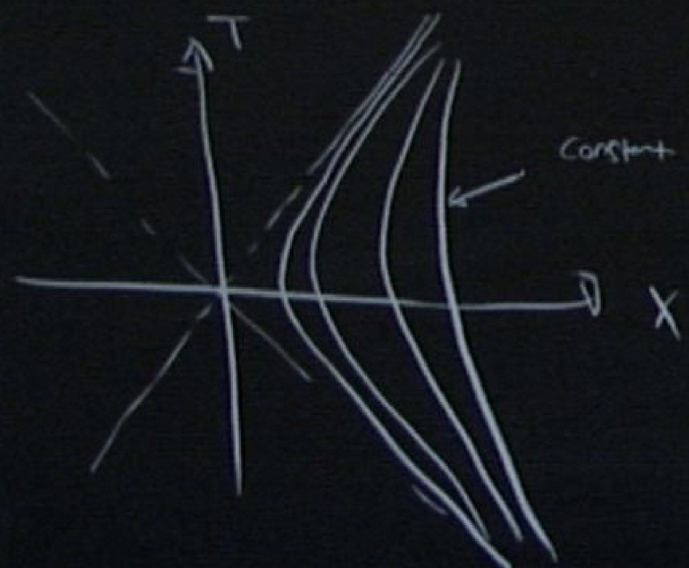
Unruh effect

Massless scalar in 2d

$$X^{\pm} = T \pm X \qquad X^{\pm} = \pm \frac{1}{a} e^{\pm a \ldots}$$

$$x^{\pm} = \ldots$$

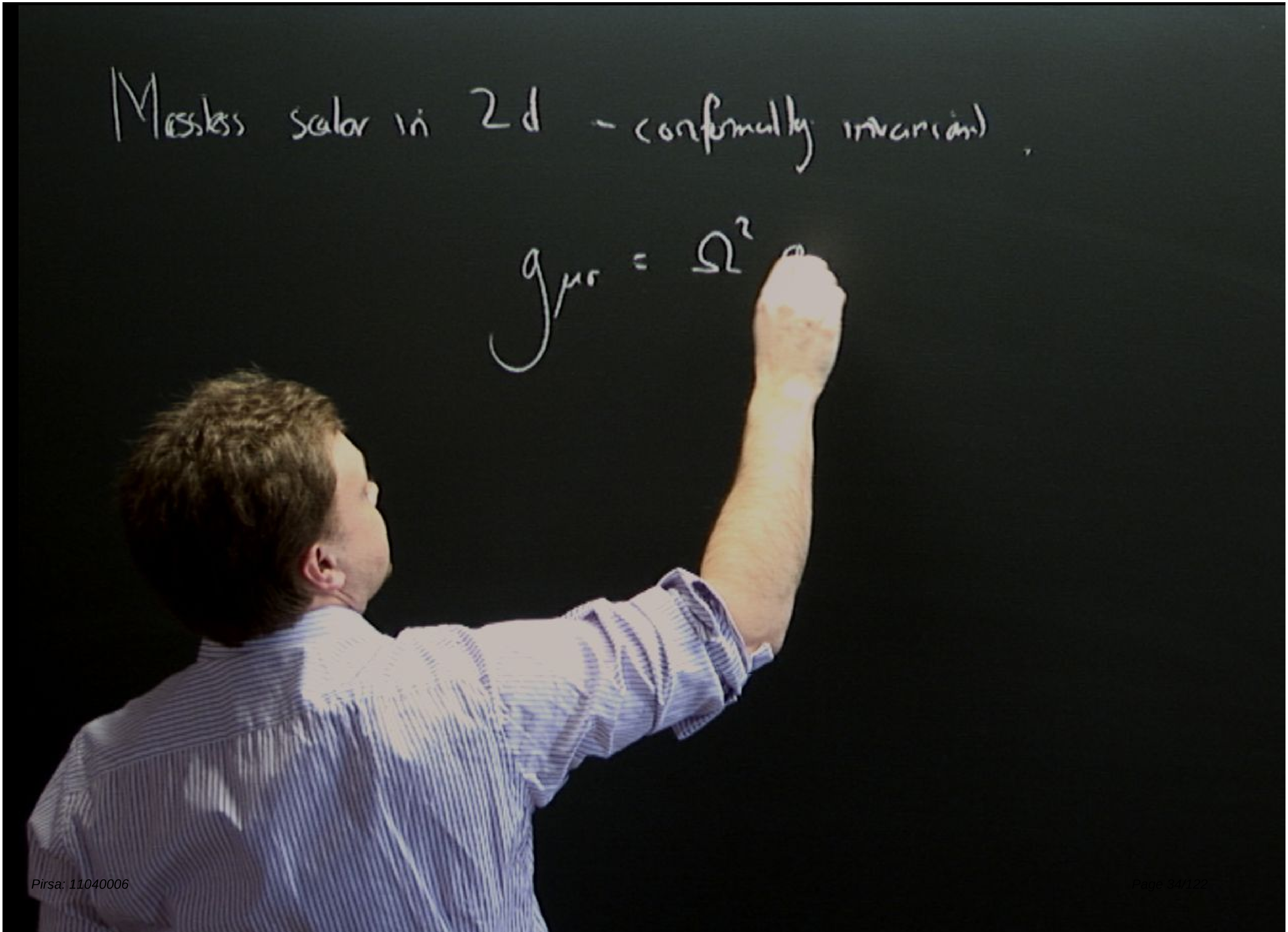
$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$
$$= +e^{2ax}(-dt^2 + dx^2)$$
$$= -e^{a(x^+ - x^-)} dx^+ dx^-$$



constant x trajectories of constant acceleration

$T \to T + c$ real vacuum Minkowski vacuum

$t \to t + c$ Rindler vacuum


Massless scalar in 2d - conformally invariant.
$g_{\mu\nu} = \Omega^2$

Massless scalar in 2d – conformally invariant.

$$g_{\mu\nu} = \Omega^2 g_{\mu\nu}'$$

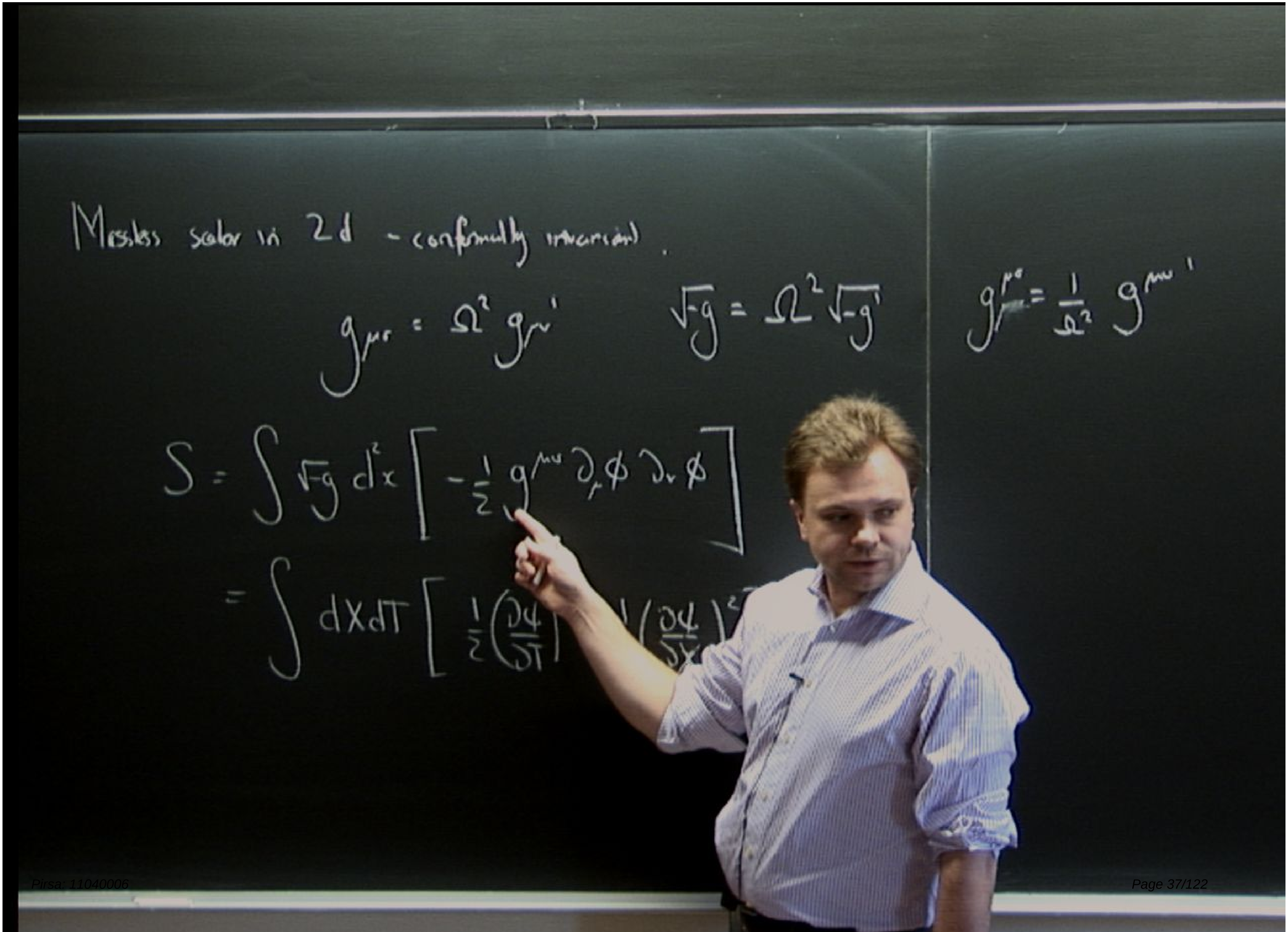
$$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right]$$

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$$

$$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi \right]$$

$$= \int dX\, dT \left[\frac{1}{2}\left(\frac{\partial \phi}{\partial T}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial X}\right. \right.$$


Massless scalar in 2d - conformally invariant.
$g_{\mu\nu} = \Omega^2 g_{\mu\nu}'$
$\sqrt{-g} = \Omega^2 \sqrt{-g'}$
$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$
$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi \right]$
$= \int dX\, dT \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial T}\right)^2 \quad \left(\frac{\partial\phi}{\partial X}\right)^2$

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \qquad \sqrt{-g} = \Omega^2 \sqrt{-g'} \qquad g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$$

$$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right] \qquad \sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$$

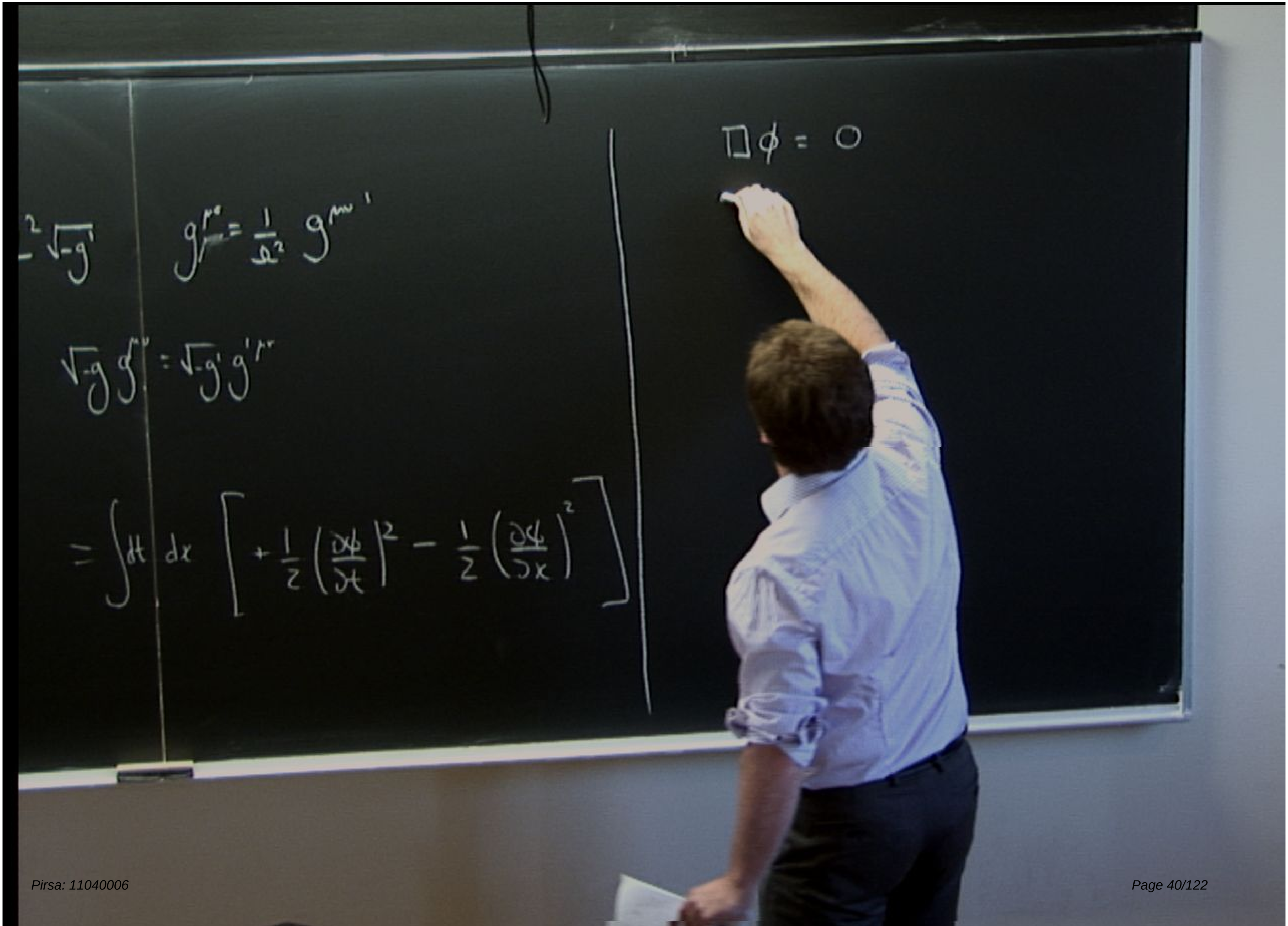
$$= \int dX\, dT \left[\frac{1}{2}\left(\frac{\partial \phi}{\partial T}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial X}\right)^2 \right]$$

rmally invariant

$$\Omega^2 g_{\mu\nu}' \qquad \sqrt{-g} = \Omega^2 \sqrt{-g'} \qquad g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$$

$$\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \Big] \qquad \sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$$

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 \Bigg] = \int dt\, dx \left[+ \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 \right]$$


$\Box\phi = 0$
$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$
$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$= \int dt\, dx \left[+ \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 \right]$

$$\sqrt{-g'}$$

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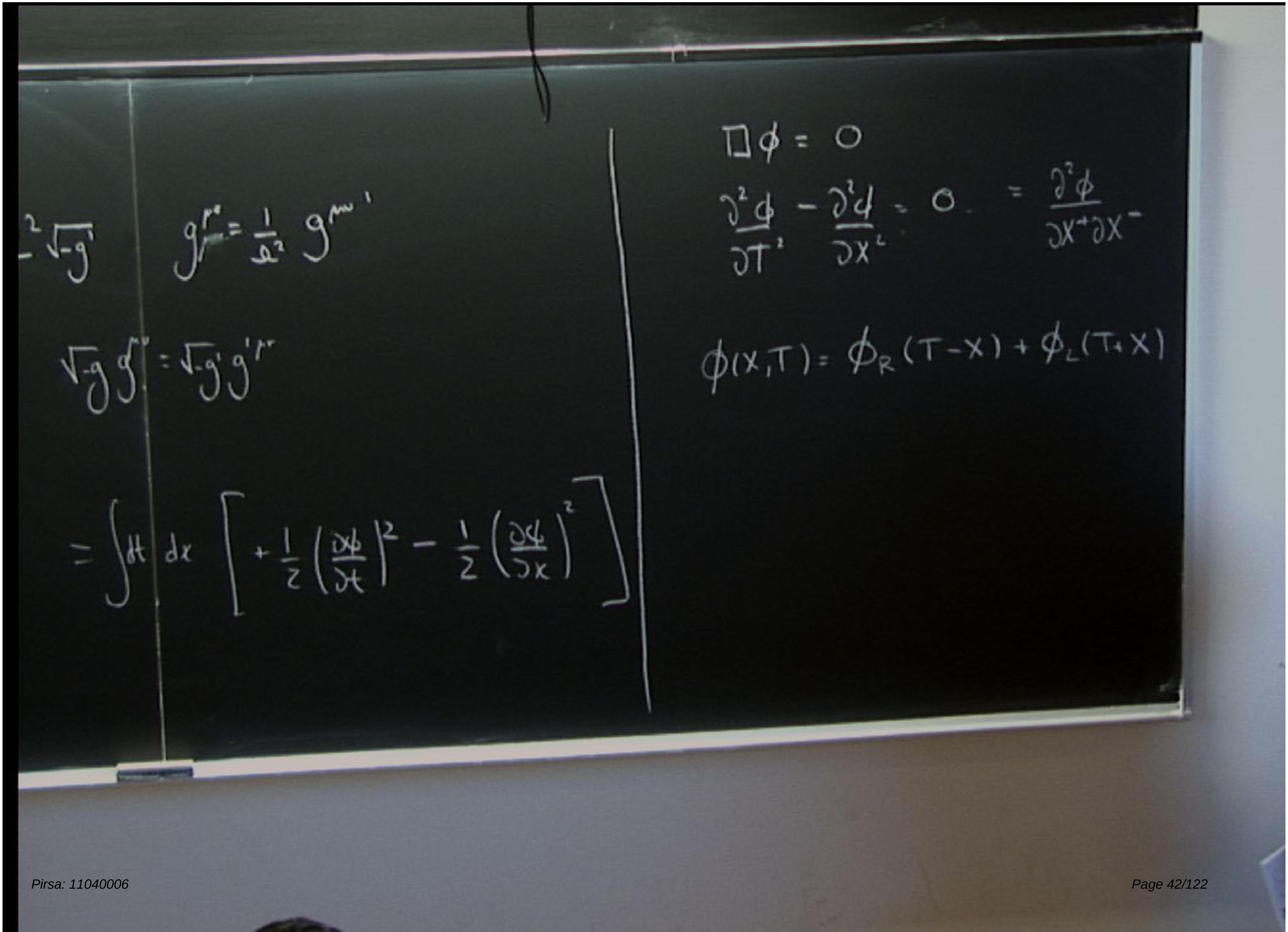
$$\Box\phi = 0$$

$$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0$$

$$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$$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$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0 = \frac{\partial^2\phi}{\partial X^+\partial X^-}$
$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$
$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$
$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$= \int dt\, dx \left[+\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 \right]$

$$\ldots^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$$

$$= \int dt\, dx \left[+\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

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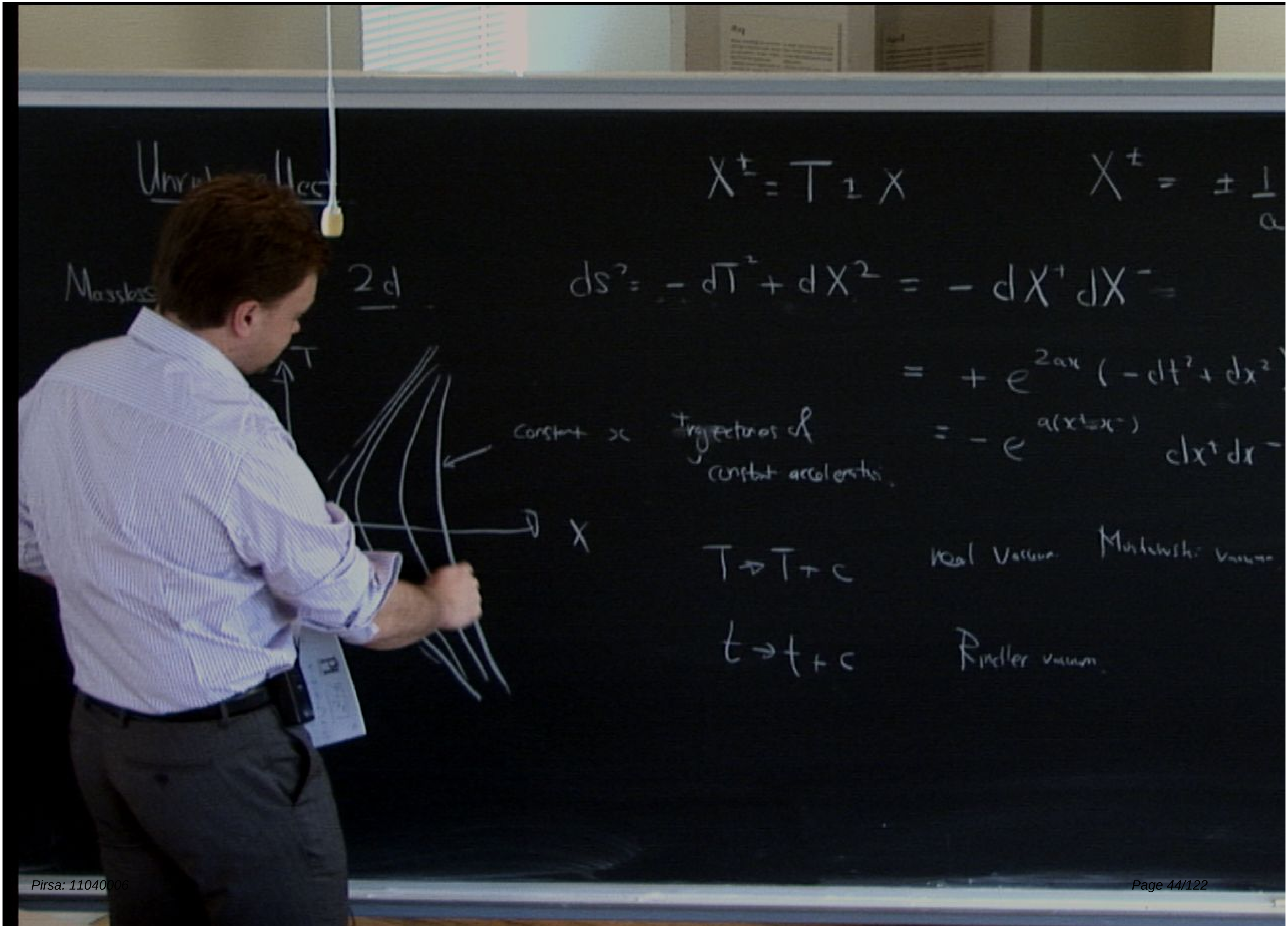
$$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$$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= + e^{2ax} (−dt² + dx²
= − e^{a(x⁺−x⁻)} dx⁺dx⁻
constant x
trajectories of constant acceleration
X
T → T + c
real vacuum
Minkowski vacuum
t → t + c
Rindler vacuum

Unruh effect

Massless scalar in 2d

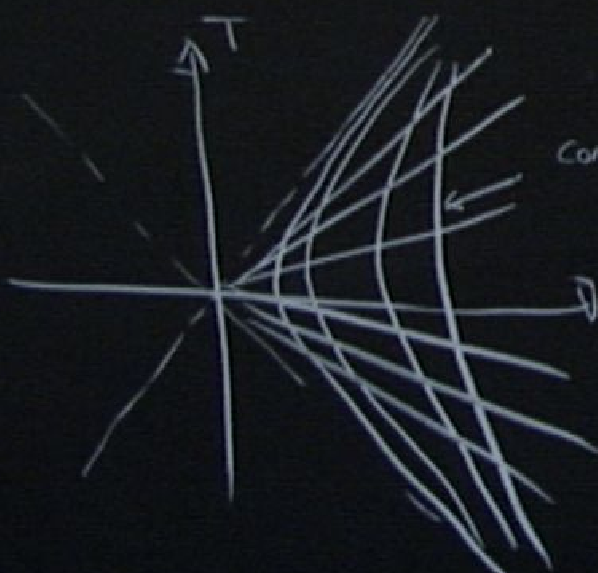
$X^{\pm} = T \pm X$

$X^{\pm} = \pm \frac{1}{a}$

$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$

$= + e^{2ax}(-dt^2 + dx^2)$

$= -e^{a(x^+ - x^-)} dx^+ dx^-$



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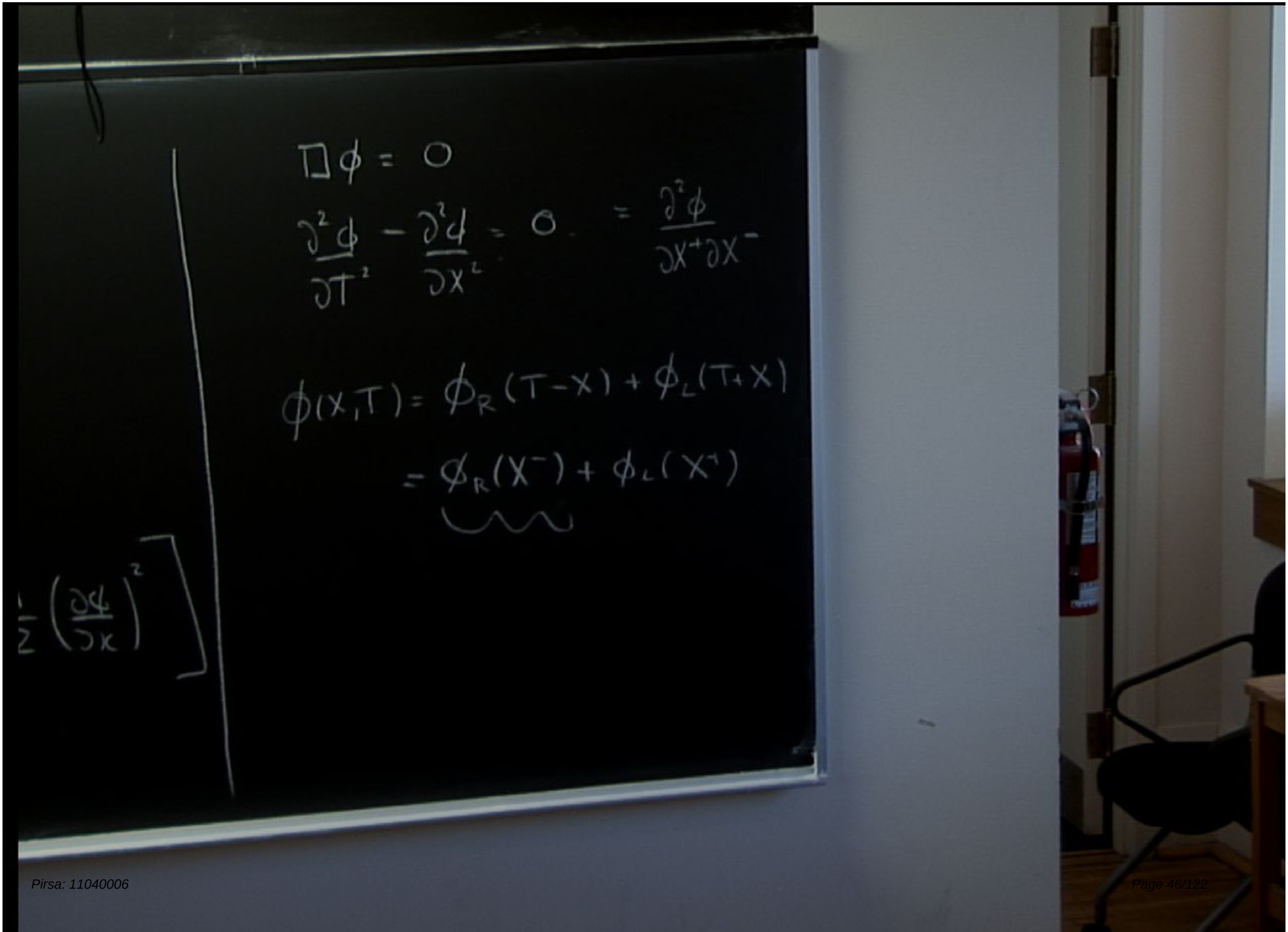
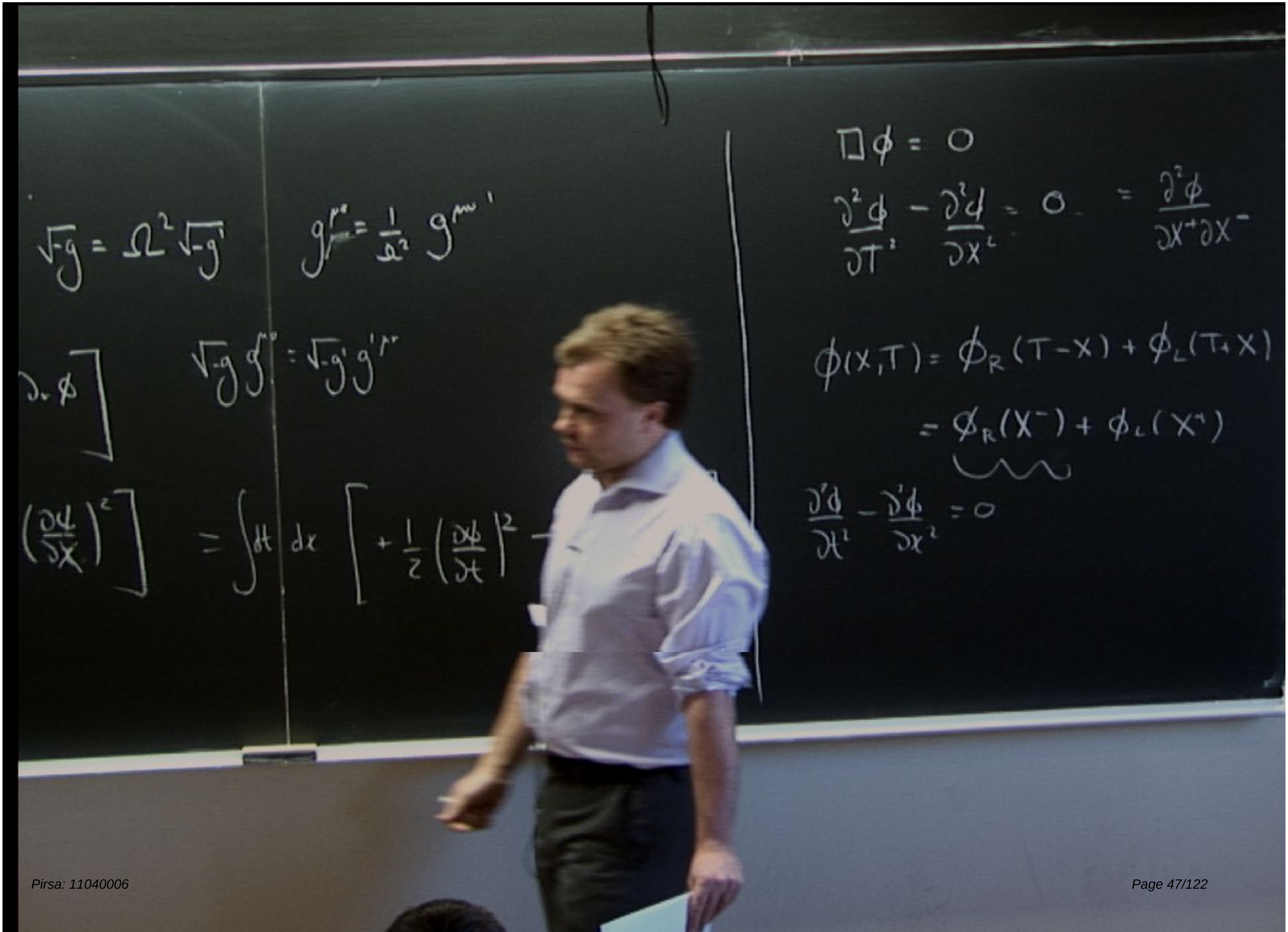


$\Box \phi = 0$
$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$
$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$
$= \phi_R(X^-) + \phi_L(X^+)$
$\frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 \Big]$



$\sqrt{-g} = \Omega^2 \sqrt{-g'}$
$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\prime}$
$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$= \int dt\, dx \left[+ \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 -$
$\Box\phi = 0$
$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0 = \frac{\partial^2\phi}{\partial X^+ \partial X^-}$
$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$
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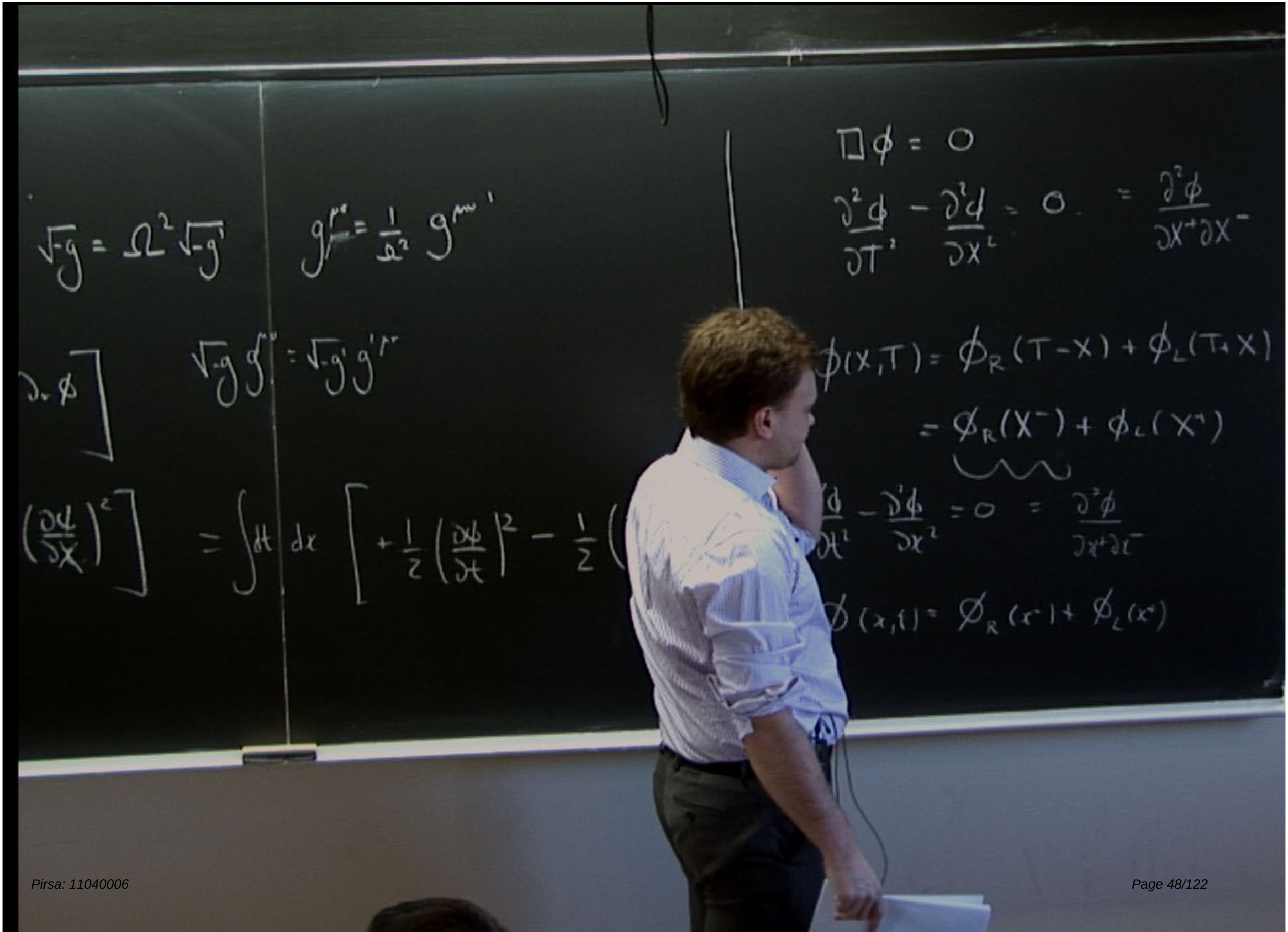
$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0$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$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$= \int dt\, dx \left[+ \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\right. \right.$
$\Box\phi = 0$
$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0 = \frac{\partial^2\phi}{\partial X^+ \partial X^-}$
$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$
$= \phi_R(X^-) + \phi_L(X^+)$
$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0 = \frac{\partial^2\phi}{\partial x^+ \partial x^-}$
$\phi(x,t) = \phi_R(x^-) + \phi_L(x^+)$

$$\sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime}$$

$$\partial_\nu \phi \Big]$$

$$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$$

$$\left(\frac{\partial\phi}{\partial X}\right)^2 \Big] = \int dt\, dx \left[+ \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$\Box\phi = 0$$

$$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0 = \frac{\partial^2\phi}{\partial X^+ \partial X^-}$$

$$\phi(X,T) = \phi_R(T-X) + \phi_L(T+X)$$

$$= \underbrace{\phi_R(X^-)}_{} + \phi_L(X^+)$$

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0 = \frac{\partial^2\phi}{\partial x^+ \partial x^-}$$

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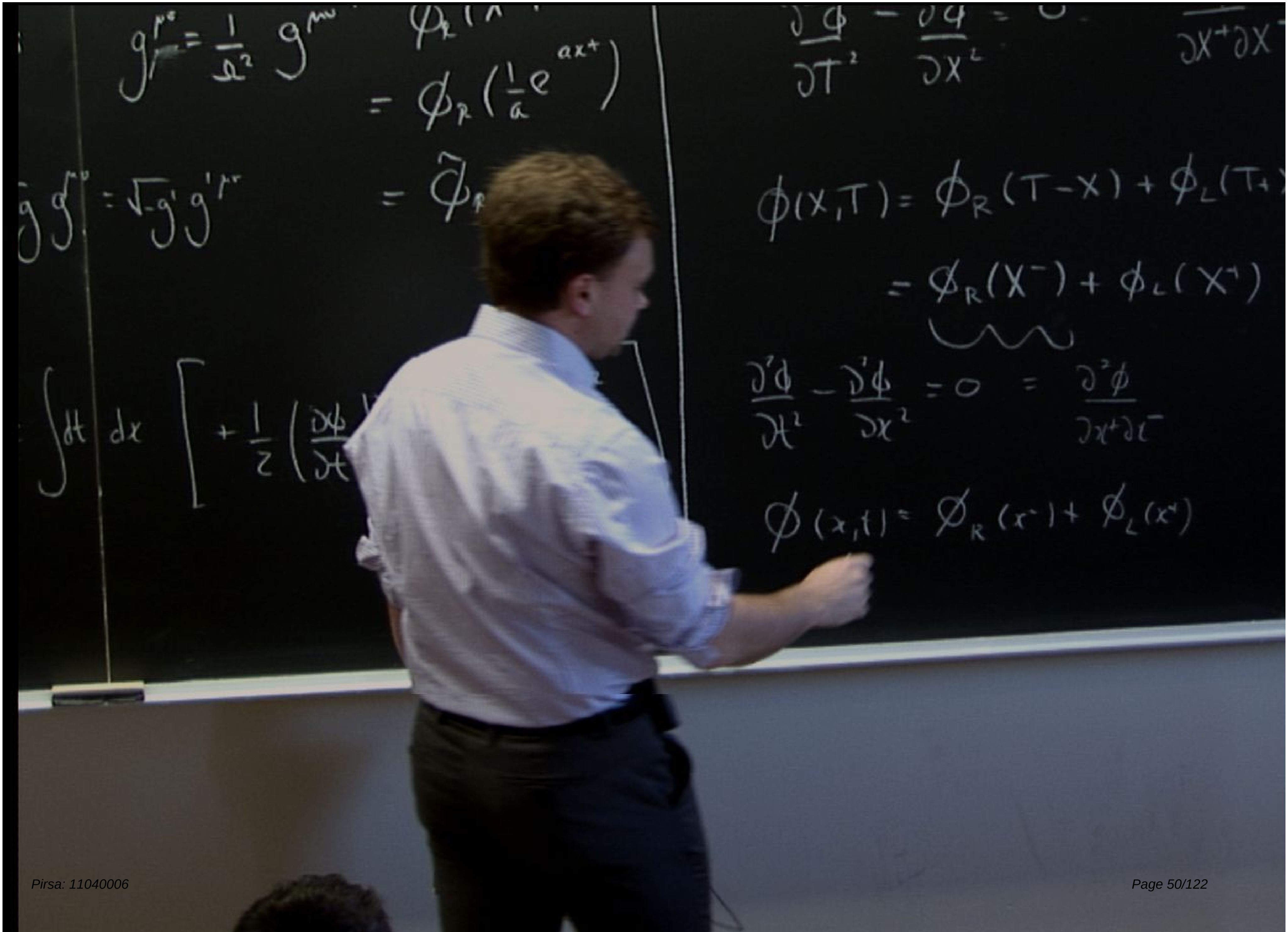
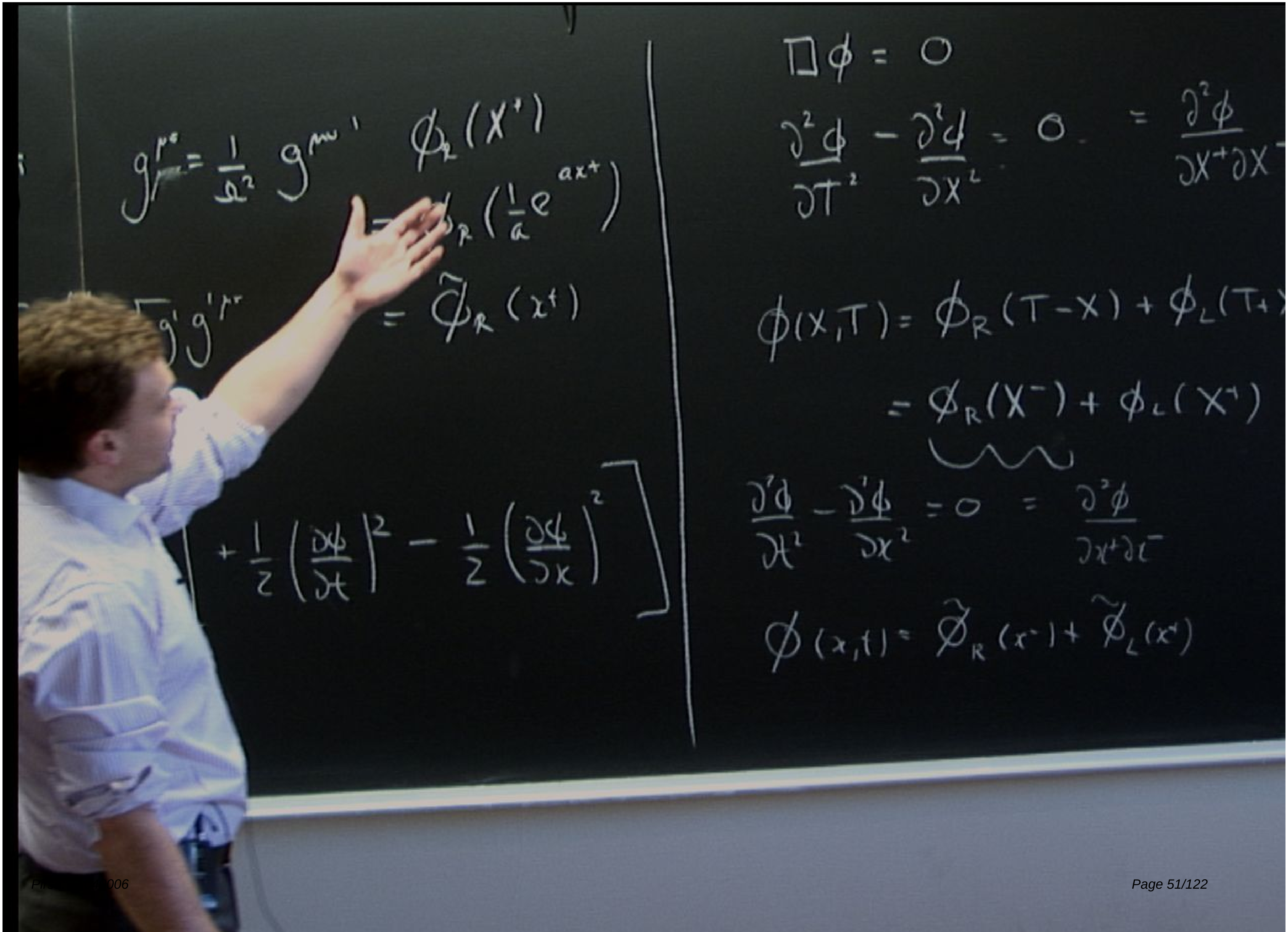
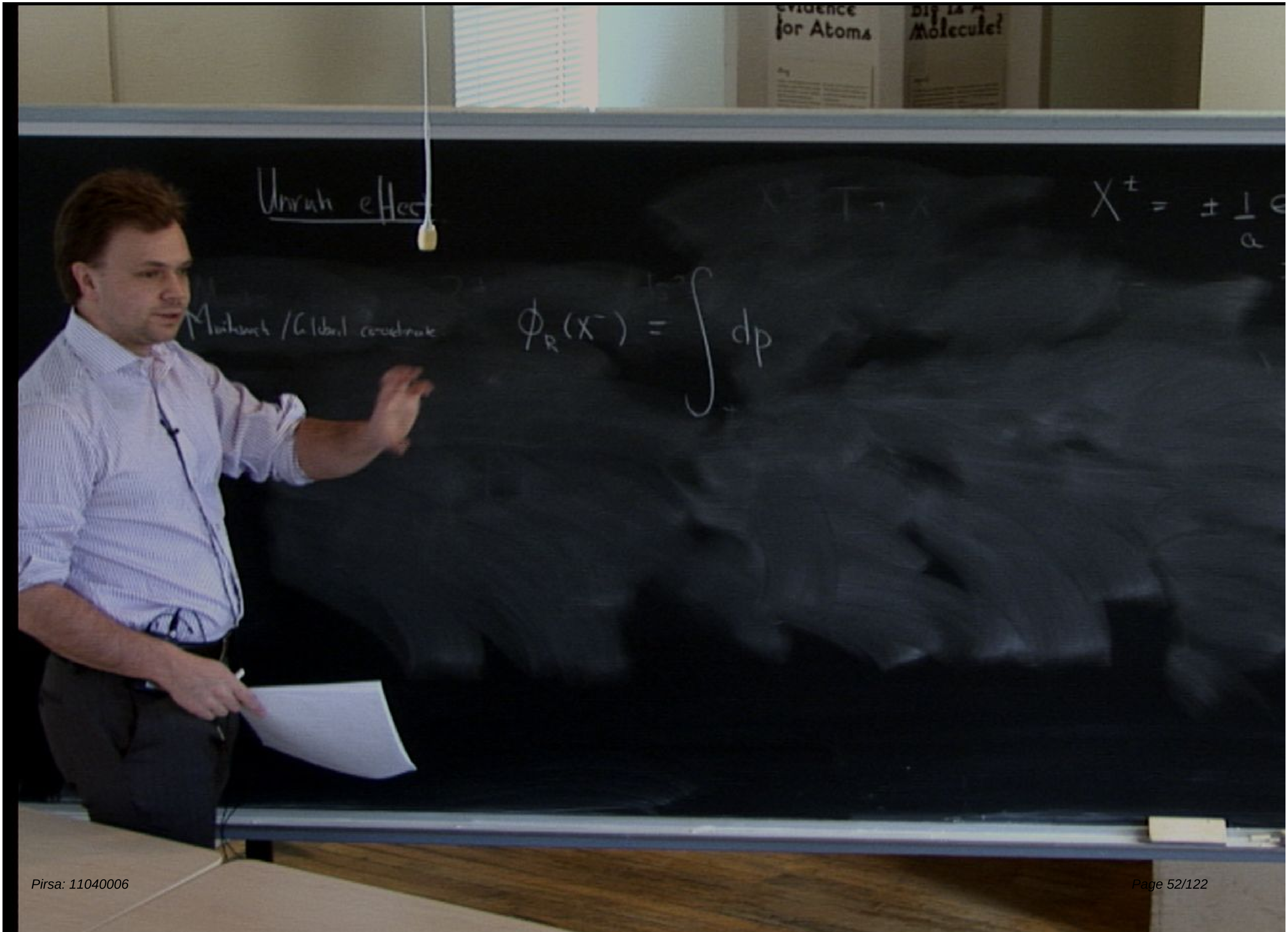
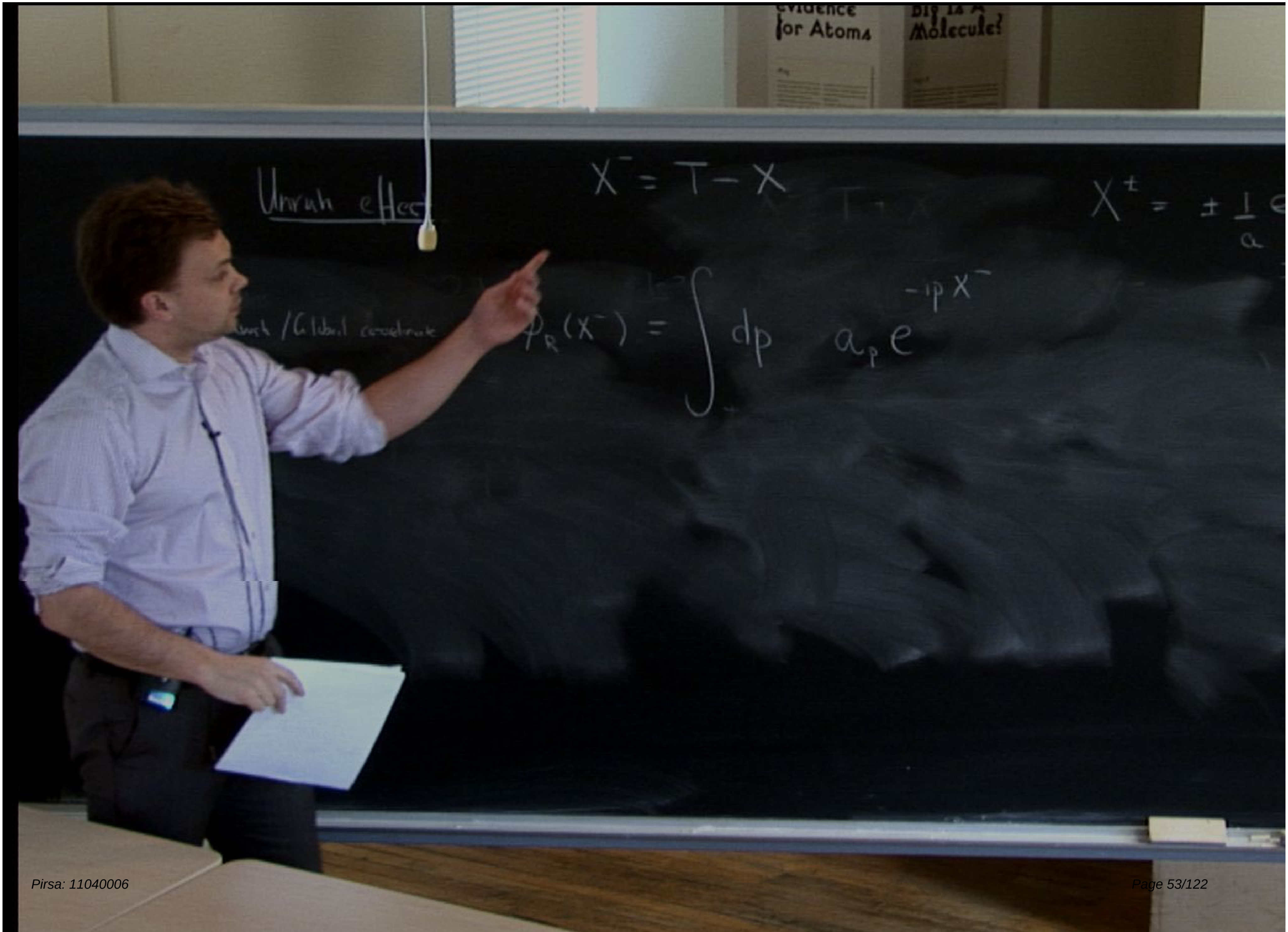


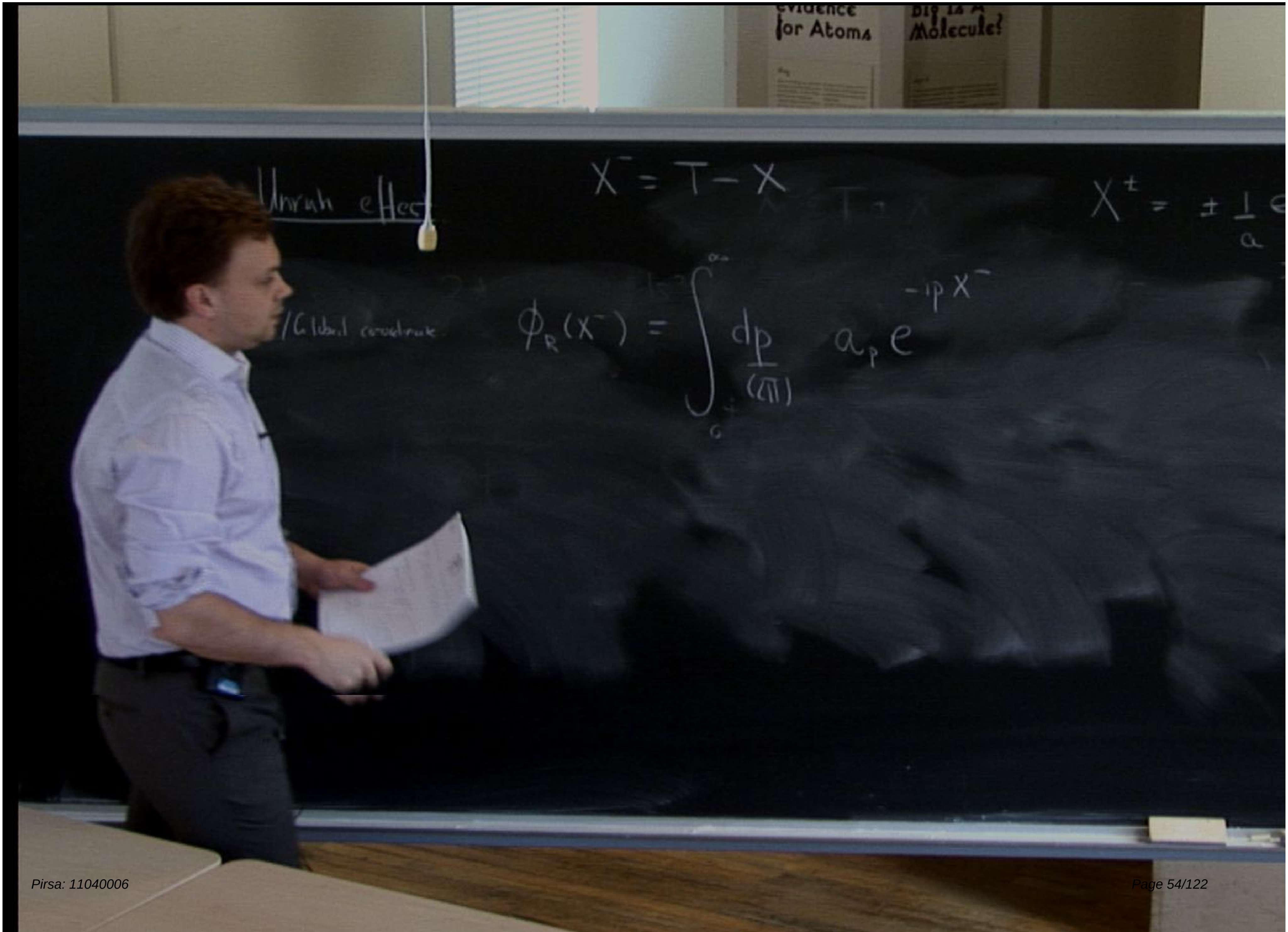
g'^{μσ} = 1/Ω² g^{μν}
= φ_R (1/a e^{ax+})
√-g g^{μν} = √-g' g'^{μν}
∫dt dx [+ 1/2 (∂φ/∂t
φ(X,T) = φ_R(T−X) + φ_L(T+
= φ_R(X⁻) + φ_L(X⁺)
∂²φ/∂t² − ∂²φ/∂x² = 0 = ∂²φ/∂x⁺∂x⁻
φ(x,t) = φ_R(x⁻) + φ_L(x⁺)

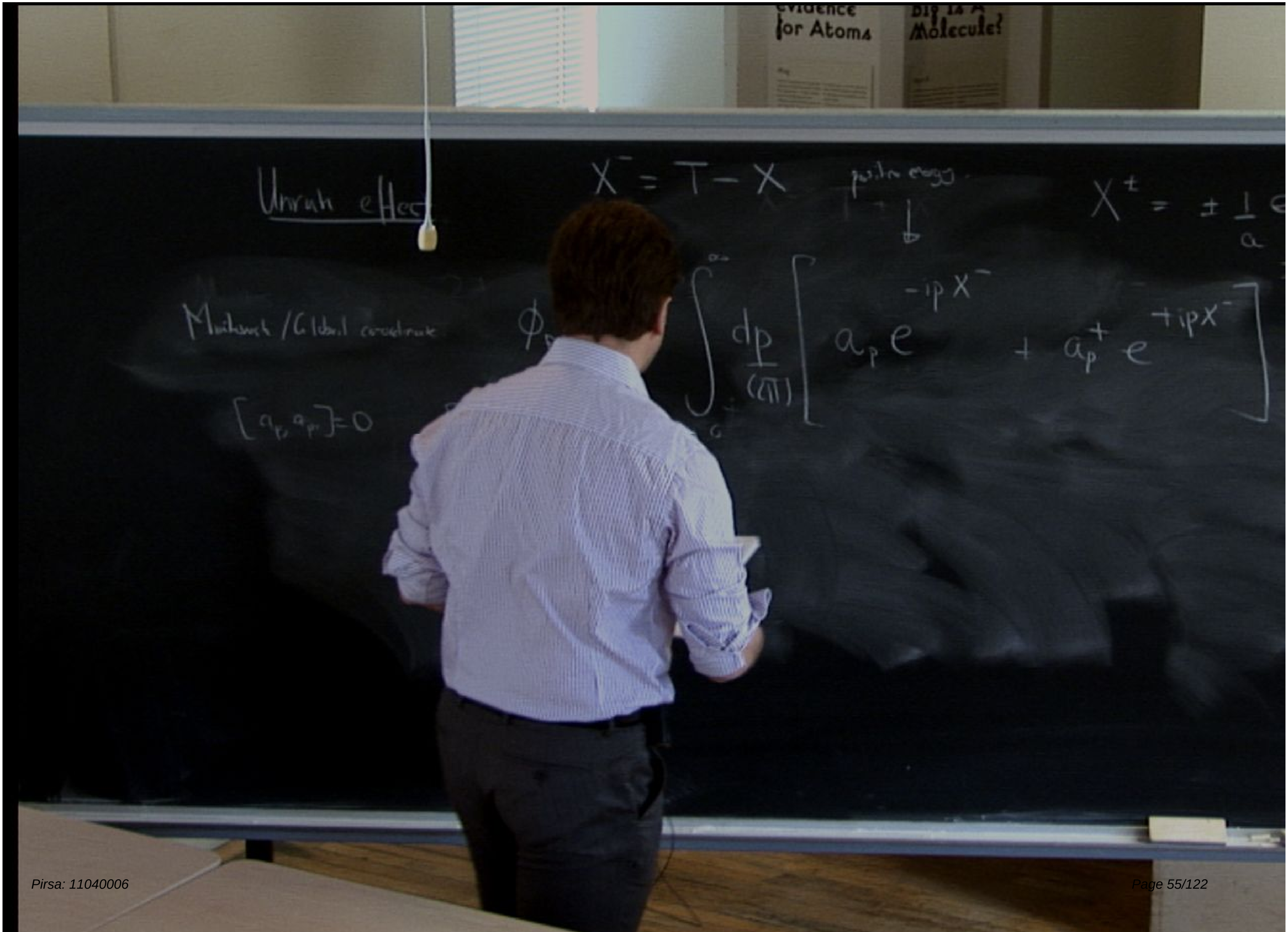

□φ = 0
∂²φ/∂T² − ∂²φ/∂X² = 0 = ∂²φ/∂X⁺∂X⁻
φ(X,T) = φ_R(T−X) + φ_L(T+X)
= φ_R(X⁻) + φ_L(X⁺)
∂²φ/∂t² − ∂²φ/∂x² = 0 = ∂²φ/∂x⁺∂x⁻
φ(x,t) = φ̃_R(x⁻) + φ̃_L(x⁺)
+ 1/2 (∂φ/∂t)² − 1/2 (∂φ/∂x)²

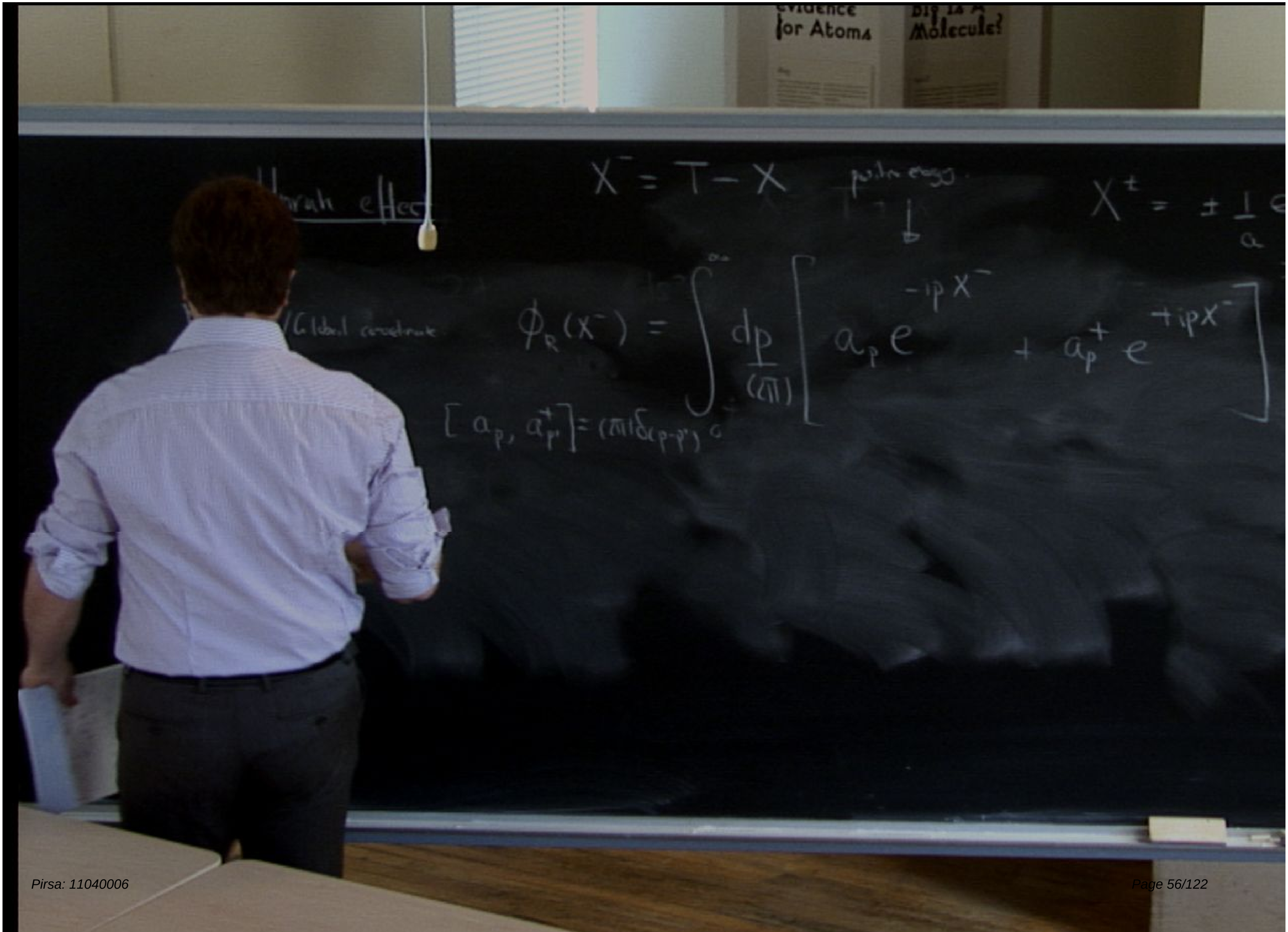

for Atoms
Molecule!
Unruh effect
$\phi_R(x^-) = \int dp$

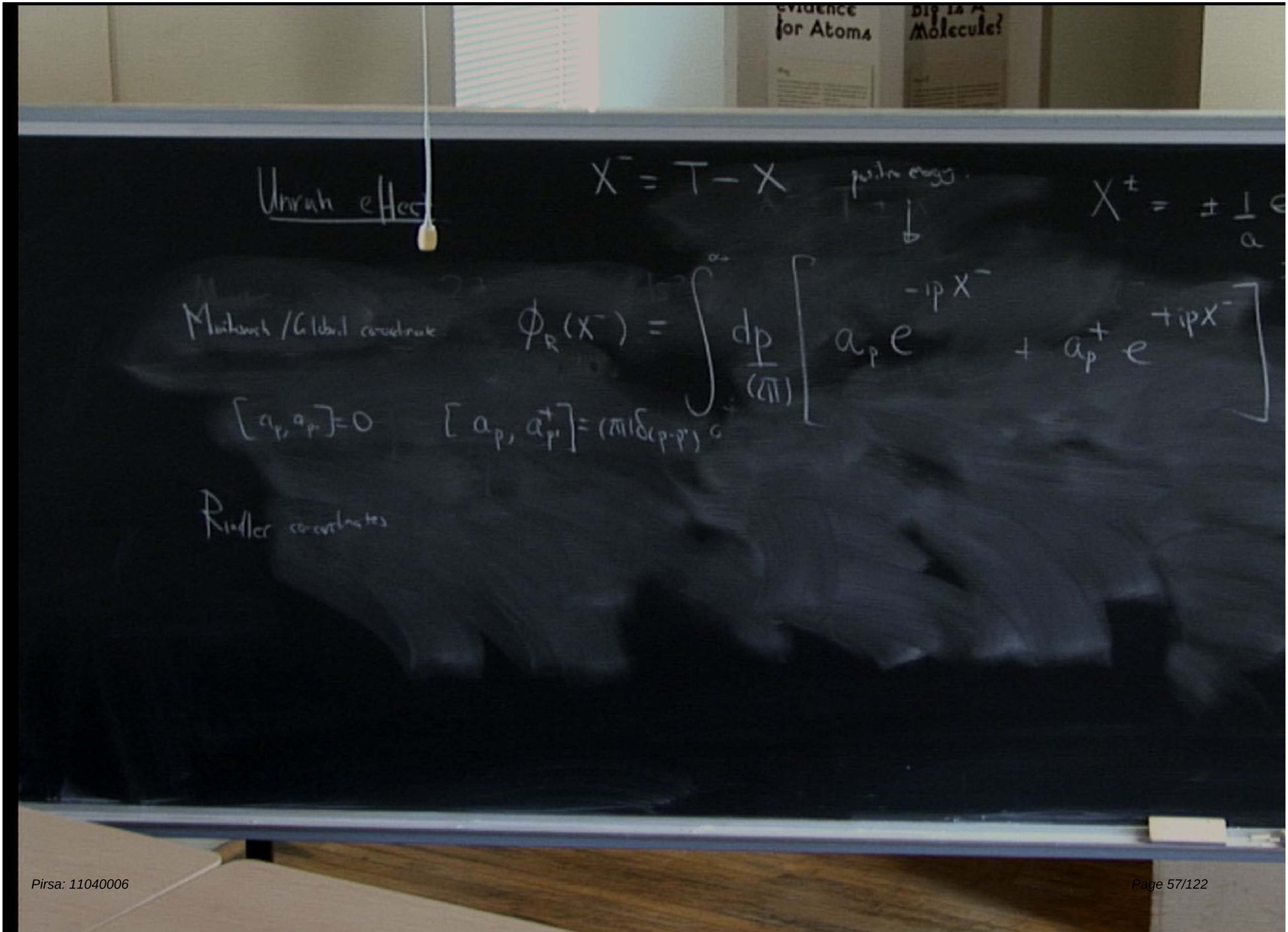


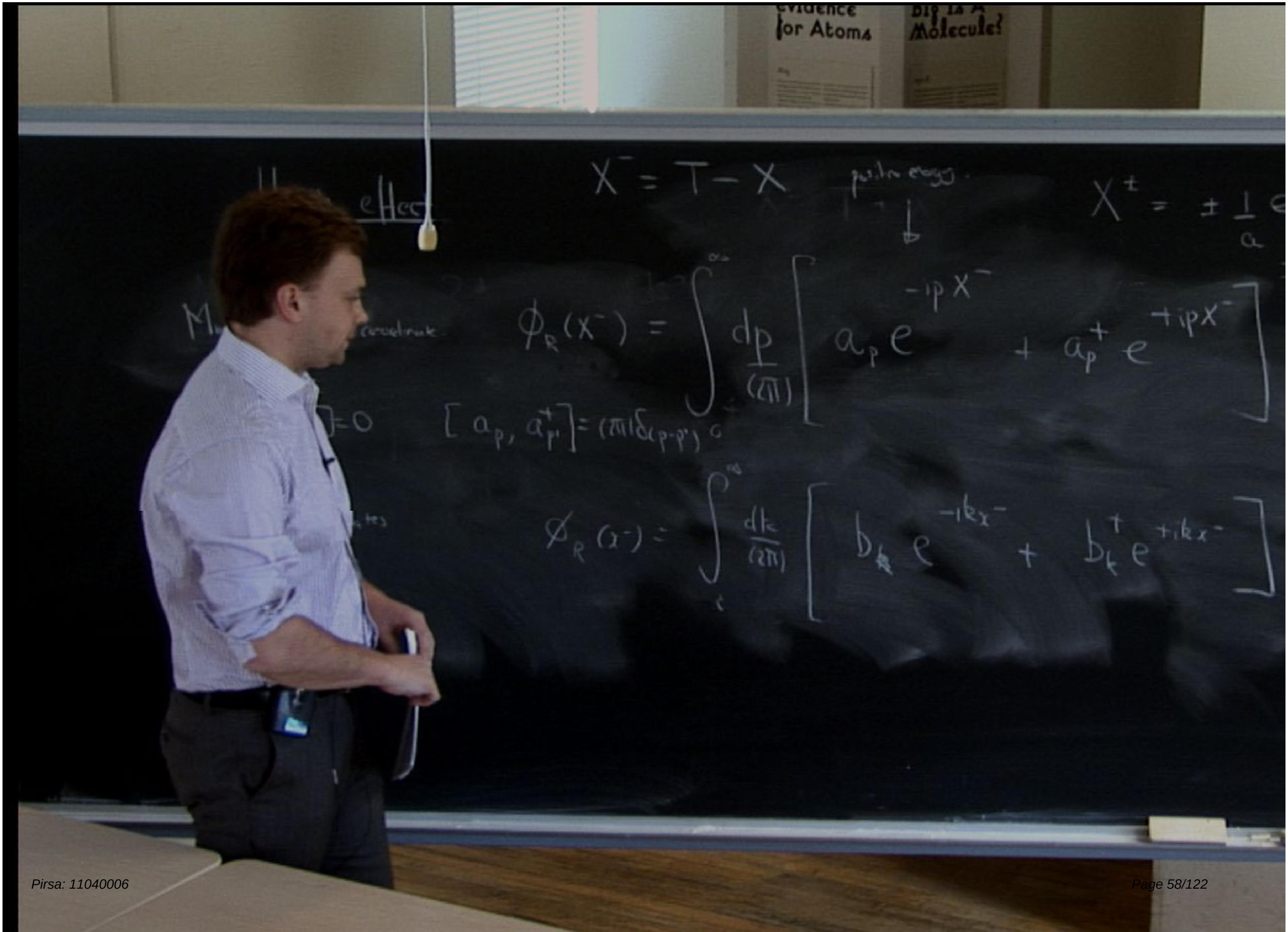
Unruh effect
$X^- = T - X$
$X^\pm = \pm \frac{1}{a} e$
$\phi_R(X^-) = \int dp \; a_p e^{-ipX^-}$
Evidence for Atoms
Molecule?


for Atoms
Molecule!
Unruh effect
$X^- = T - X$
$X^\pm = \pm \frac{1}{a} e$
Global coordinate
$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} a_p e^{-ipX^-}$

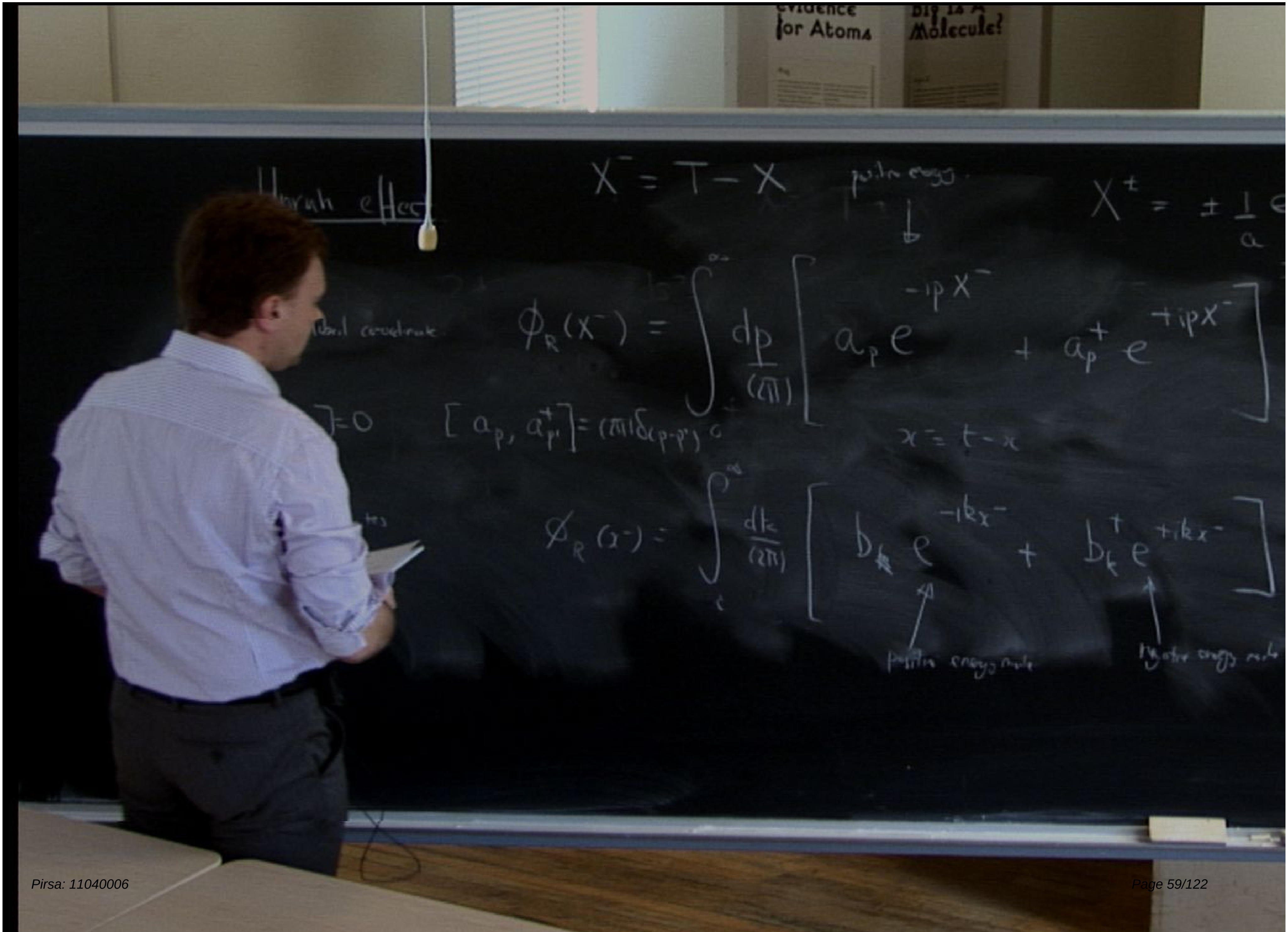

for Atoms
Unruh effect
X⁻ = T − X
positive energy
Minkowski / Global coordinate
[a_p, a_p'] = 0


for Atoms
Molecule?
X⁻ = T − X
φ_R(X⁻) = ∫ dp/(2π) [a_p e^{−ipX⁻} + a_p^† e^{+ipX⁻}]
[a_p , a_{p'}^†] = (2π) δ(p − p')


evidence
for Atoms
Molecule?
Unruh effect
X⁻ = T − X
positive energy
Minkowski / Global coordinate
φ_R(X⁻) = ∫ dp/(2π) [a_p e^{−ipX⁻} + a_p^† e^{+ipX⁻}]
[a_p, a_p'] = 0
[a_p, a_p'^†] = (2π) δ(p−p')
Rindler coordinates



for Atoms
Molecule?
X⁻ = T − X
positive energy
[a_p , a†_p'] = (2π) δ(p − p')
φ_R(x⁻) = ∫₀^∞ dp/(2π) [a_p e^{−ipx⁻} + a†_p e^{+ipx⁻}]
φ_R(x⁻) = ∫₀^∞ dk/(2π) [b_k e^{−ikx⁻} + b†_k e^{+ikx⁻}]


for Atoms
Molecule?
X⁻ = T − X
x⁻ = t − x
Positive energy mode

Unruh effect

$X^- = T - X$

positive energy

$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$

$x^\pm = t \pm$

Minkowski / Global coordinate

$$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^\dagger e^{+ipX^-} \right]$$

$[a_p, a_{p'}] = 0 \qquad [a_p, a_{p'}^\dagger] = (2\pi)\delta(p-p')$

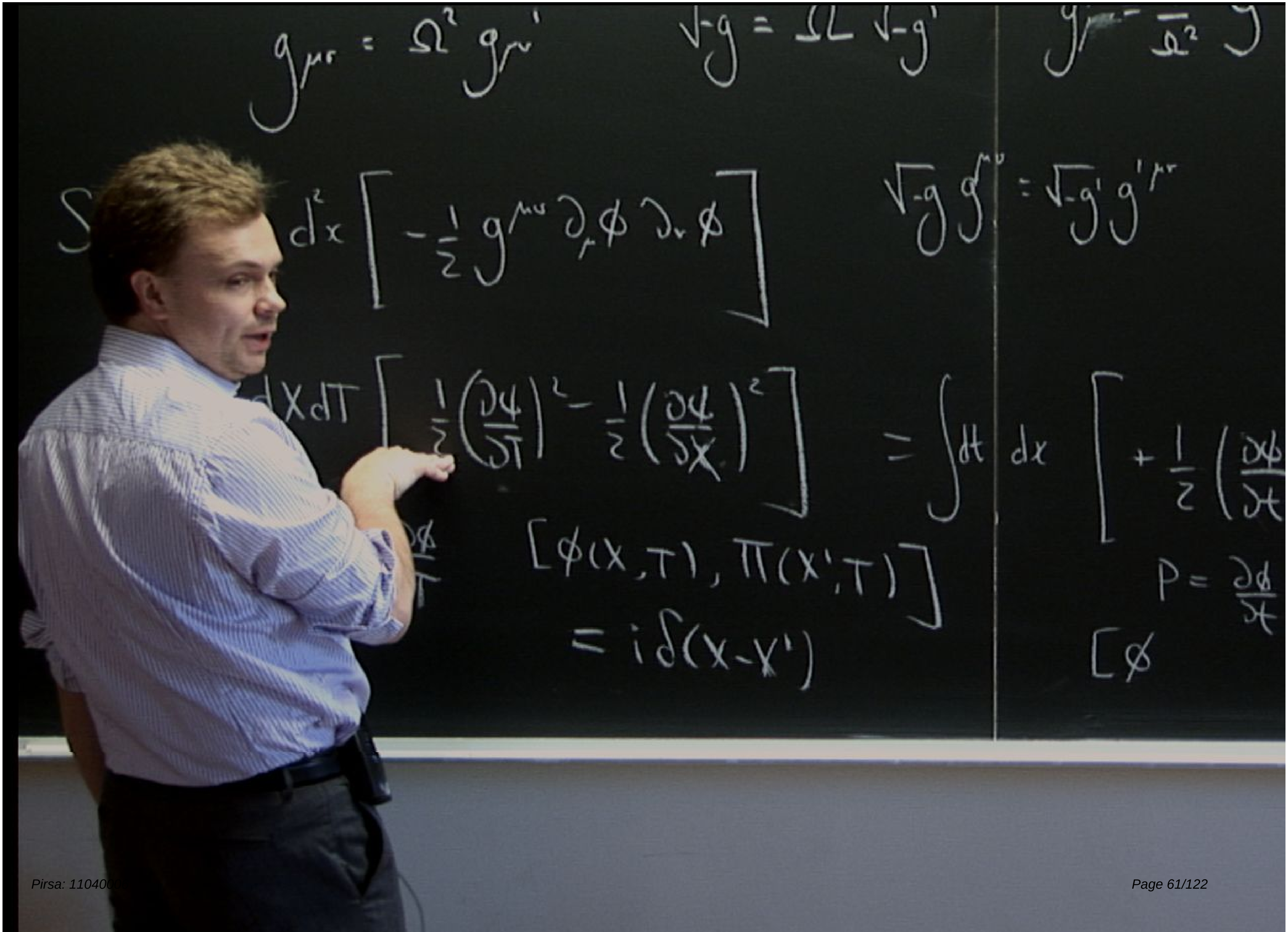
$x^- = t - x$

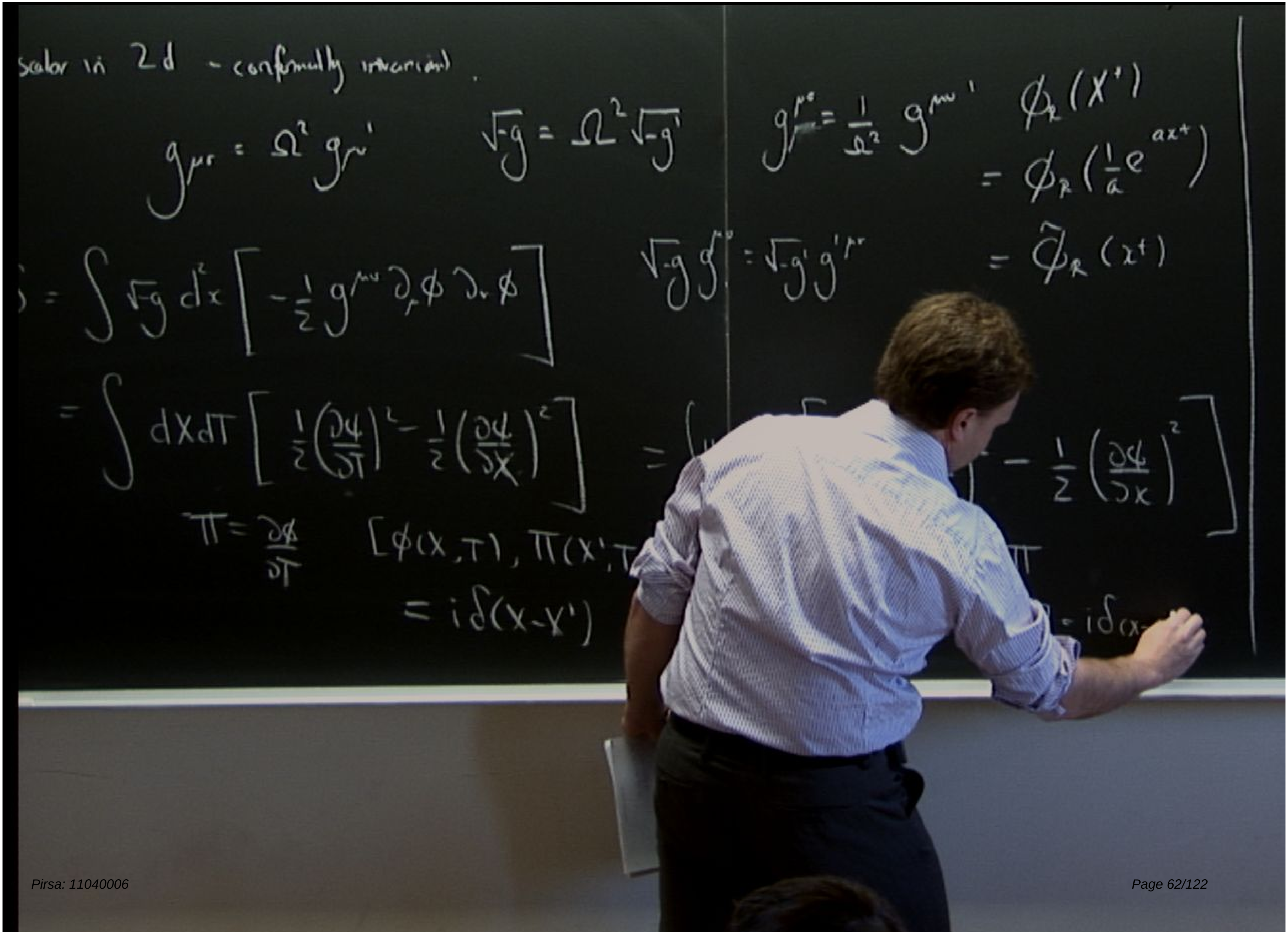
Rindler co-ordinates

$$\phi_R(x^-) = \int_0^\infty \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^\dagger e^{+ikx^-} \right]$$

positive energy mode

negative energy mode.


$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$
$\sqrt{-g} = \Omega^2 \sqrt{-g'}$
$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
$dX\,dT \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial T}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial X}\right)^2 \right] = \int dt\, dx \left[+\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right.\right.$
$[\phi(X,T), \Pi(X',T)]$
$= i\delta(X-X')$
$P = \frac{\partial\phi}{\partial t}$


scalar in 2d - conformally invariant
$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$
$\sqrt{-g} = \Omega^2 \sqrt{-g'}$
$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,'}$
$\phi_R(X^+)$
$= \phi_R(\frac{1}{a} e^{ax^+})$
$= \tilde{\phi}_R(x^+)$
$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$
$= \int dX dT \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial T}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial X}\right)^2 \right]$
$- \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2$
$\Pi = \frac{\partial\phi}{\partial T}$
$[\phi(X,T), \Pi(X',T$
$= i\delta(X-X')$
$= i\delta(x$

scalar in 2d - conformally invariant

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \qquad \sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$S = \int \sqrt{-g}\, d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$= \int dX\, dT \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial T}\right)^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial X}\right)^2 \right]$$

$$\Pi = \frac{\partial \phi}{\partial T} \qquad [\phi(X,T), \Pi(X',T)] = i\delta(X-X')$$

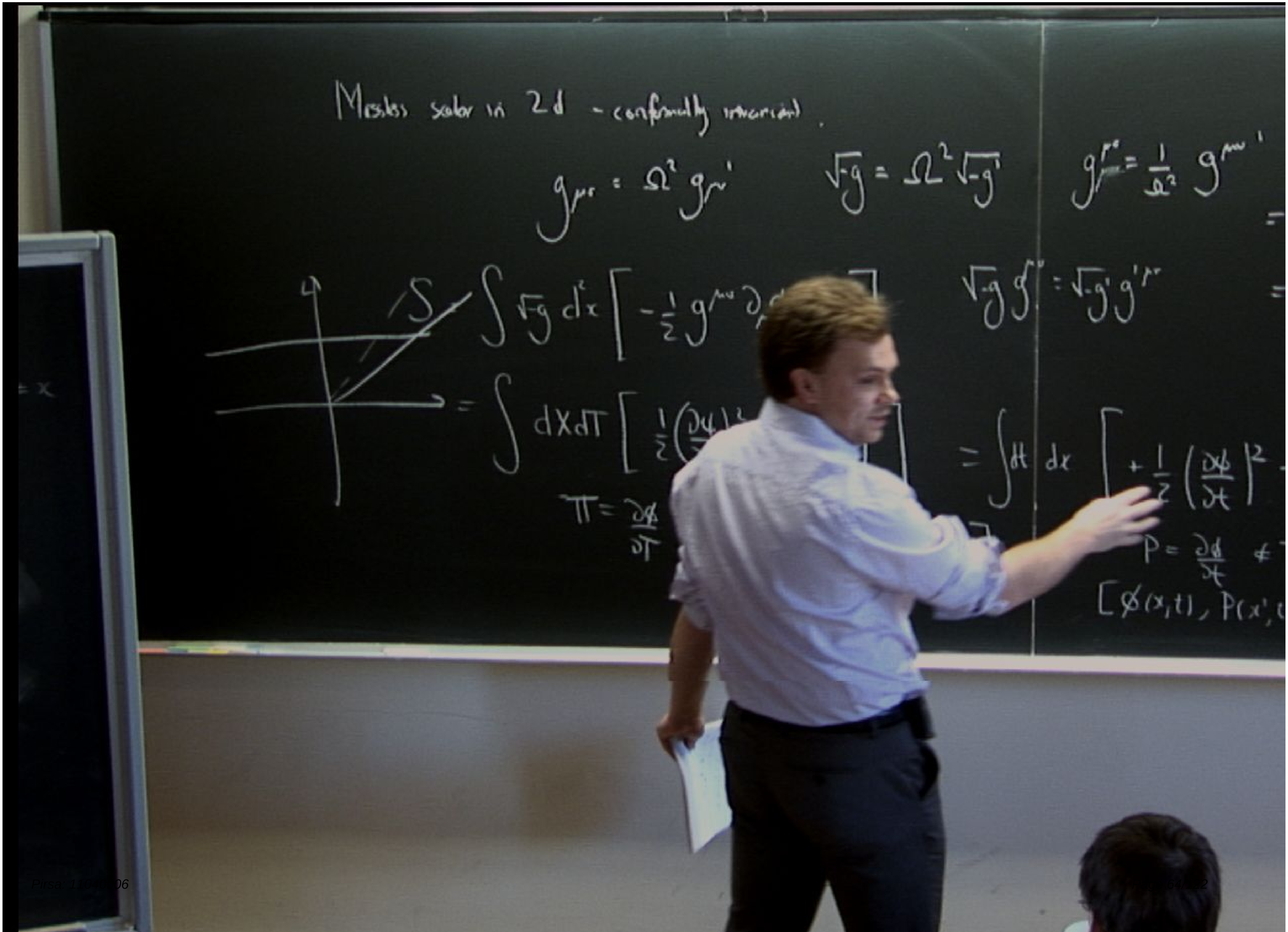
$$g^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu\,\prime} \qquad \phi_R(X^+) = \phi_R\left(\frac{1}{a} e^{a x^+}\right) = \tilde{\phi}_R(x^+)$$

$$\sqrt{-g}\, g^{\mu\nu} = \sqrt{-g'}\, g'^{\mu\nu}$$

$$= \int dt\, dx \left[+\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$P = \frac{\partial\phi}{\partial t} \neq \Pi$$

$$[\phi(x,t), P(x',t)] = i\delta(x-x')$$


Massless scalar in 2d - conformally invariant
$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$
$\sqrt{-g} = \Omega^2 \sqrt{-g'}$

Unruh effect

$$X^- = T - X$$ positive energy

$$X^{\pm} = \pm \frac{1}{a} e$$

Minkowski / Global coordinate

$$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^\dagger e^{+ipX^-} \right]$$

$$[a_p, a_{p'}] = 0 \qquad [a_p, a_{p'}^\dagger] = (2\pi)\delta(p-p')$$

$$x^- = t - x$$

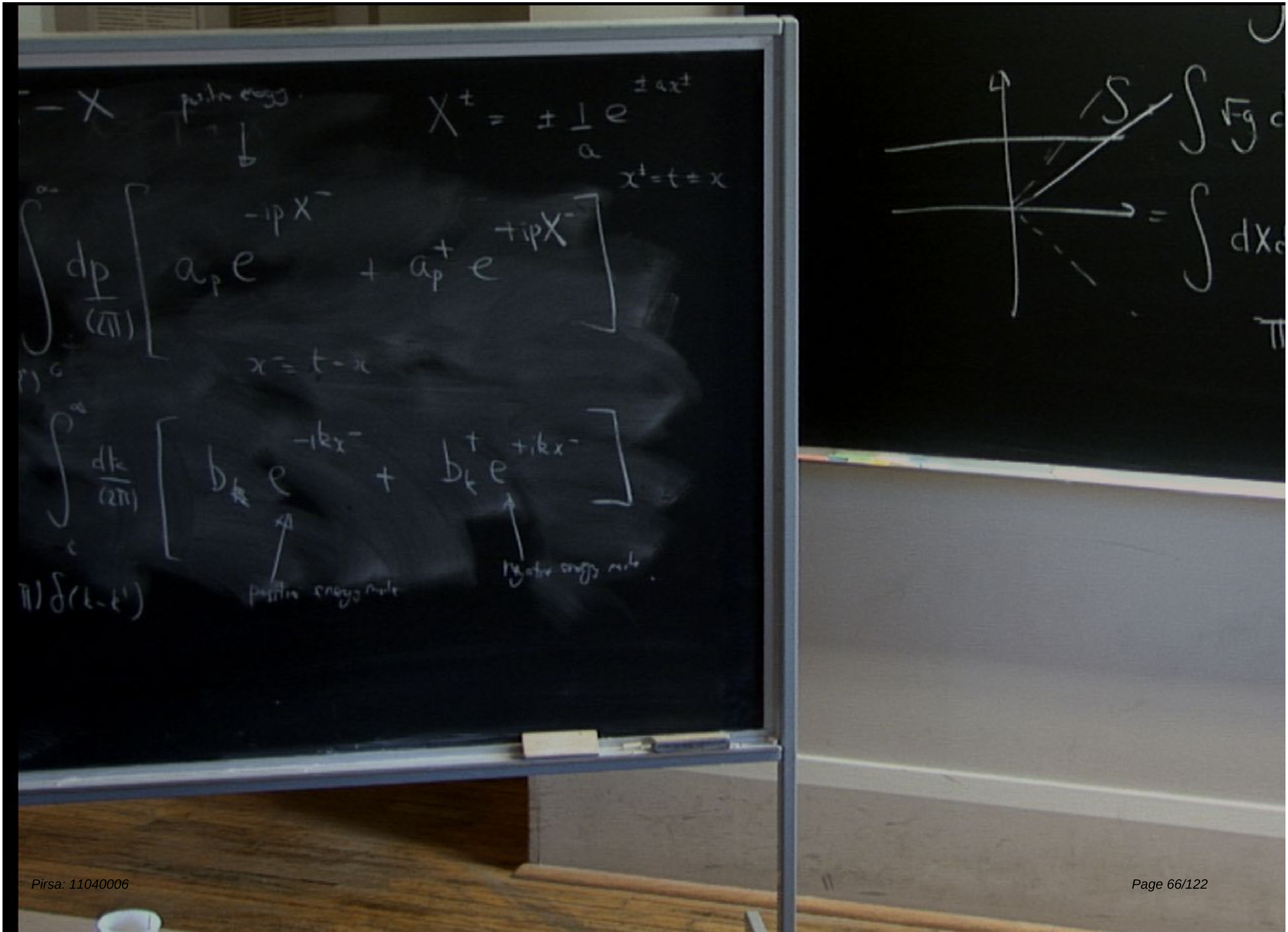
Rindler co-ordinates

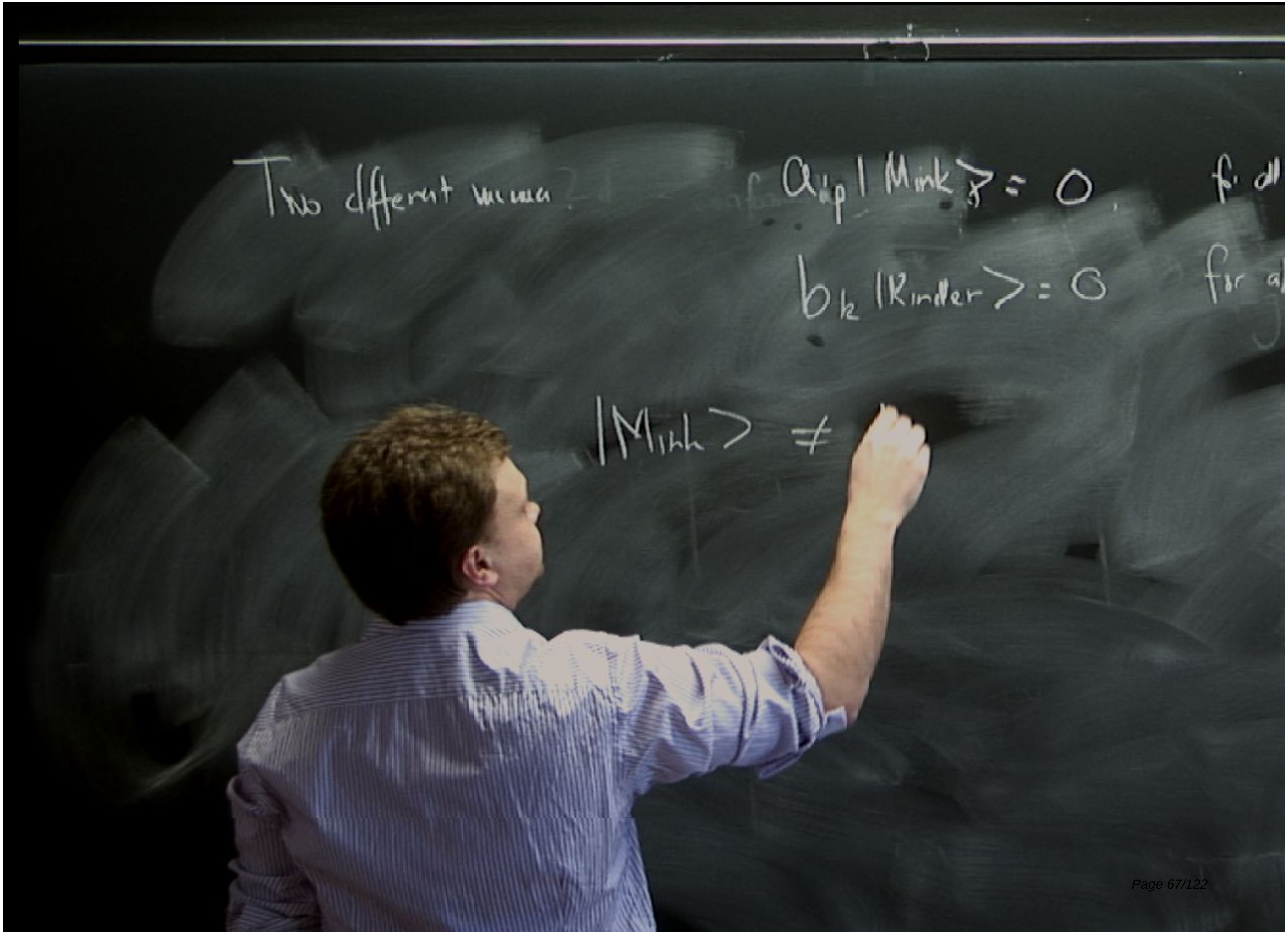
$$\phi_R(x^-) = \int_0^\infty \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^\dagger e^{+ikx^-} \right]$$

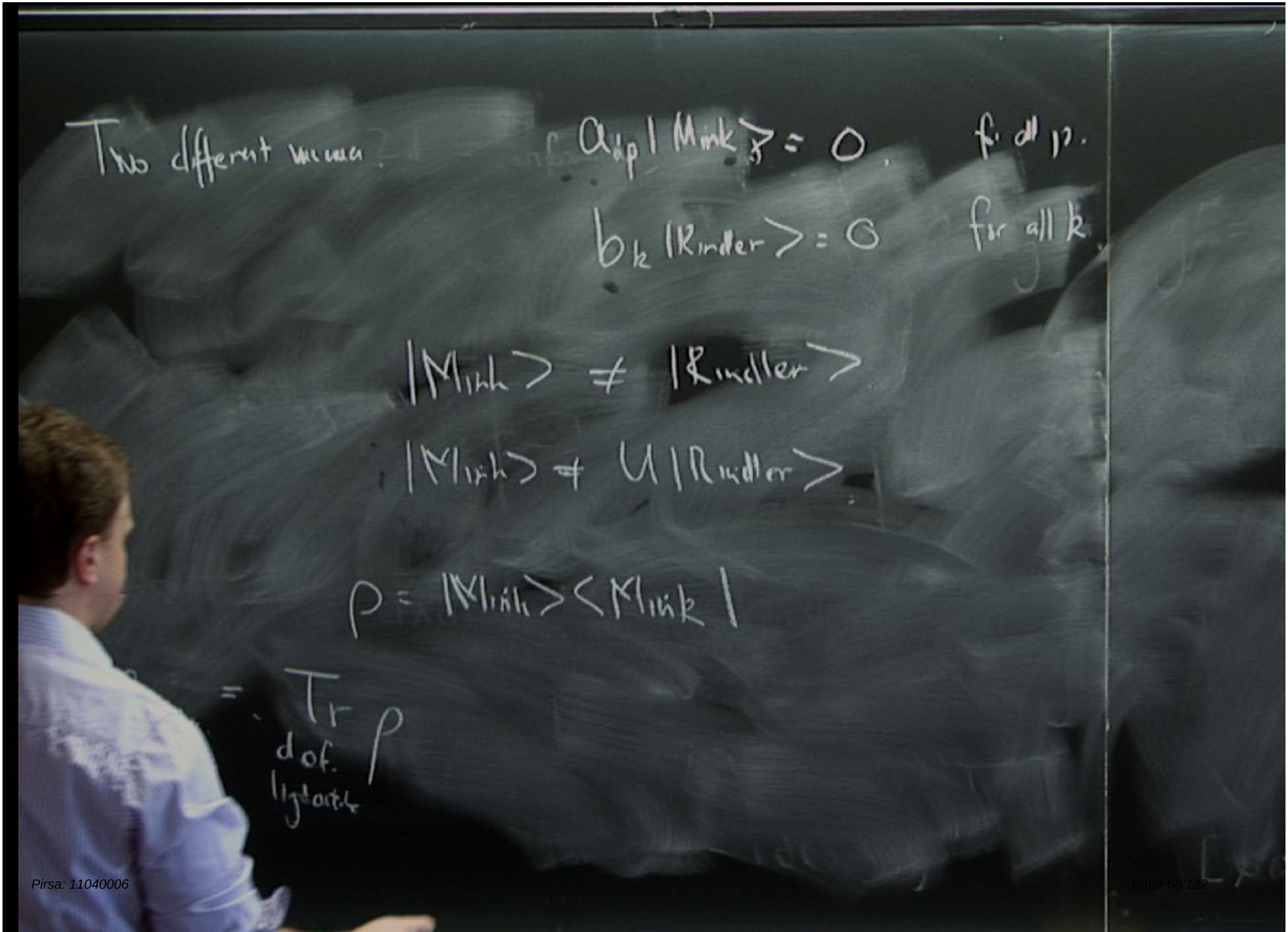
$$[b_k, b_{k'}^\dagger] = (2\pi)\delta(k-k')$$

positive energy mode

negative energy mode


positive energy
X± = ± 1/a e^{±ax±}
x± = t ± x
x⁻ = t − x
positive energy mode
negative energy mode


Two different
a_p |Mink> = 0
b_k |Rindler> = 0
|Mink> ≠


Two different
a_p |Mink> = 0 f. all p.
b_k |Rindler> = 0 for all k
|Mink> ≠ |Rindler>
|Mink> ≠ U|Rindler>
ρ = |Mink><Mink|
= Tr ρ
dof.

Two different vacua

$a_p |\text{Mink}\rangle = 0$ for all p.

$b_k |\text{Rindler}\rangle = 0$ for all k

$$|\text{Mink}\rangle \neq |\text{Rindler}\rangle$$

$$|\text{Mink}\rangle \neq U |\text{Rindler}\rangle$$

$$\rho = |\text{Mink}\rangle\langle\text{Mink}|$$

$$\rho_{\text{Rindler}} = \operatorname{Tr}_{\substack{\text{d.o.f.}\\ \text{lightcone}}} \rho = Z^{-1} e^{-\beta H_{\text{Rindler}}}.$$

Two different vacua

$a_p |Mink\rangle = 0$ for all p.

$b_k |Rindler\rangle = 0$ for all k

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U|Rindler\rangle$$

$$\rho = |Mink\rangle\langle Mink|$$

$$\rho_{Rindler} = \underset{\substack{\text{d.o.f.}\\ \text{lightcone}}}{\mathrm{Tr}} \rho = Z^{-1} e^{-\beta H_{Rindler}}.$$

Two different vacua?

$$a_{\omega p}|\text{Mink}\rangle = 0 \quad \text{for all } p.$$

$$b_k|\text{Rindler}\rangle = 0 \quad \text{for all } k$$

$$|\text{Mink}\rangle \neq |\text{Rindler}\rangle$$

$$|\text{Mink}\rangle \neq U|\text{Rindler}\rangle$$

$$\rho = |\text{Mink}\rangle\langle\text{Mink}|$$

$$\rho_{\text{Rindler}} = \operatorname{Tr}_{\substack{\text{d.o.f.}\\ \text{lightcone}}} \rho = Z^{-1} e^{-\beta H_{\text{Rindler}}}.$$

$X^- = T - X$

positive energy

$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$

$x^\pm = t \pm x$

$$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} \left[\frac{a_p \, e^{-ipX^-}}{\sqrt{2p}} + \frac{a_p^\dagger \, e^{+ipX^-}}{\sqrt{2p}} \right]$$

$[a_p, a^\dagger_{p'}] = (2\pi)\delta(p-p')$

$x^- = t - x$

$$\phi_R(x^-) = \int_0^\infty \frac{dk}{(2\pi)} \left[\frac{b_k \, e^{-ikx^-}}{\sqrt{2k}} + \frac{b_k^\dagger \, e^{+ikx^-}}{\sqrt{2k}} \right]$$

$[b_k, b_{k'}^\dagger] = (2\pi)\delta(k-k')$

positive energy mode

negative energy mode.

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

0 for all p.

$= 0$ for all k

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogoliubov co-efficients

$x^{\pm}$

$t \pm x$

Two different vacua

$a_{k_p} |\text{Mink}\rangle = 0$ for all p.

$b_k |\text{Rindler}\rangle = 0$ for all k

$$|\text{Mink}\rangle \neq |\text{Rindler}\rangle$$

$$|\text{Mink}\rangle \neq U |\text{Rindler}\rangle$$

$$\rho = |\text{Mink}\rangle\langle \text{Mink}|$$

$$\rho_{\text{Rindler}} = \mathrm{Tr}_{\text{d.o.f.}}\, \rho = Z^{-1} e^{-\beta H_{\text{Rindler}}}.$$

$X^- = T - X$

positive energy

$$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$$

$$x^\pm = t \pm x$$

$$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} \left[a_p \frac{e^{-ipX^-}}{\sqrt{2p}} + a_p^\dagger \frac{e^{+ipX^-}}{\sqrt{2p}} \right]$$

$$[a_p, a_{p'}^\dagger] = (2\pi)\delta(p-p')$$

$$x^- = t - x$$

$$\phi_R(x^-) = \int_0^\infty \frac{dk}{(2\pi)} \left[b_k \frac{e^{-ikx^-}}{\sqrt{2k}} + b_k^\dagger \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

$$[b_k, b_{k'}^\dagger] = (2\pi)\delta(k-k')$$

positive energy mode

negative energy mode

$$b_k = \int_0^\infty \frac{dp}{2\pi} \Big[\alpha_{kp} \, a_p$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogolubov co-efficients.

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta^*_{kp} a^\dagger_p \right]$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)}\left[\alpha_{kp}\frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp}\frac{e^{+ikx^-}}{\sqrt{2k}}\right]$$

Bogolubov co-efficients

$$= \int_0^\infty \frac{dp}{2\pi}\left[\alpha_{kp}\, a_p + \beta^*_{kp}\, a^\dagger_p\right]$$

$$\left(b_k = \alpha_k a_k + \beta^*_k a^\dagger_k\right)$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogoliubov co-efficients

$$b_k = \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp}\, a_p + \beta^*_{kp}\, a^\dagger_p \right]$$

$$[b_k, b_{k'}^\dagger] = (2\pi)\,\delta(k-k') \qquad \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha^*_{k'p} - \beta_{k'p} \beta^*_{kp'} \right] = (2\pi)\,\delta(k-k')$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)}\left[\alpha_{kp}\,\frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp}\,\frac{e^{+ikx^-}}{\sqrt{2k}}\right]$$

Bogolubov co-efficients.

$$= \int_0^\infty \frac{dp}{2\pi}\left[\alpha_{kp}\,a_p + \beta^*_{kp}\,a_p^\dagger\right]$$

analogue $|\alpha|^2 - |\beta|^2 = 1$

$$[b_k, b_{k'}^\dagger] = (2\pi)\,\delta(k-k')$$

$$\int_0^\infty \frac{dp}{(2\pi)}\left[\alpha_{kp}\,\alpha^*_{k'p} - \beta_{k'p}\,\beta^*_{kp'}\right] = (2\pi)\,\delta(k-k')$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogoliubov co-efficients.

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp}\, a_p + \beta^*_{kp}\, a^\dagger_p \right]$$

analogue $|\alpha|^2 - |\beta|^2 = 1$

$[b_k, b_{k'}^\dagger] = (2\pi)\,\delta(k-k')$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha^*_{k'p} - \beta_{k'p}\beta^*_{kp} \right] = (2\pi)\,\delta(k-k')$$

f. all p.

for all k

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

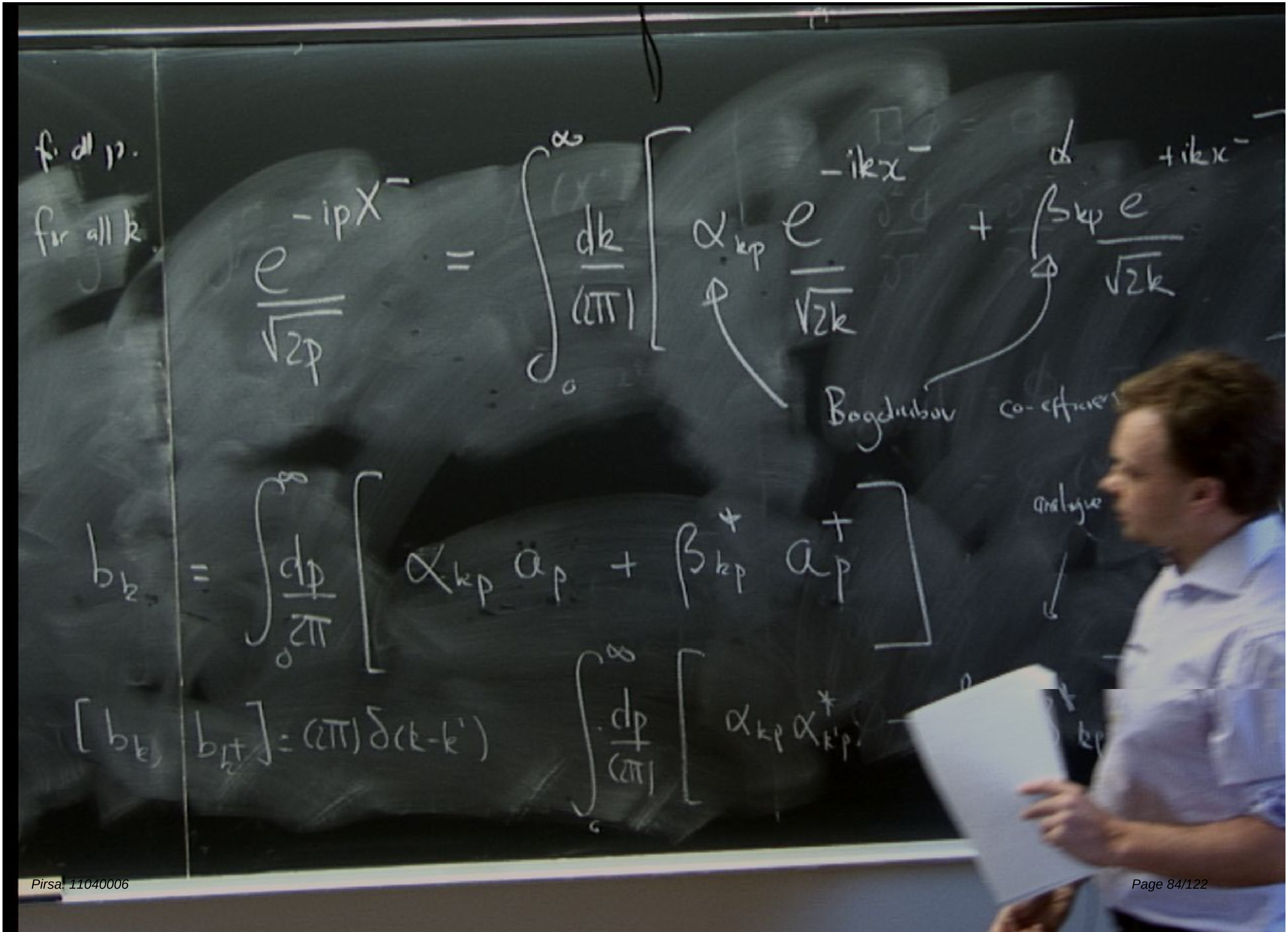
Bogoliubov co-efficients.

$$b_k = \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta^*_{kp} a^\dagger_p \right]$$

analogue $|\alpha|^2 - |\beta|$

$$[b_k, b^\dagger_{k'}] = (2\pi)\delta(k-k')$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha^*_{k'p} - \beta_{k'p}\beta^*_{kp} \right] = (2\pi)\delta$$



for all p.
for all k
$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$
Bogoliubov co-efficients
analogue
$b_k = \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta^*_{kp} a^\dagger_p \right]$
$[b_k, b^\dagger_{k'}] = (2\pi)\delta(k-k')$
$\int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha^*_{k'p} \right.$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)}\left[\alpha_{kp}\frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp}\frac{e^{+ikx^-}}{\sqrt{2k}}\right]$$

Bogoliubov co-efficients

$$= \int_0^\infty \frac{dp}{2\pi}\left[\alpha_{kp}\, a_p + \beta^*_{kp}\, a^\dagger_p\right]$$

analogue $|\alpha|^2 - |\beta|^2 = 1$

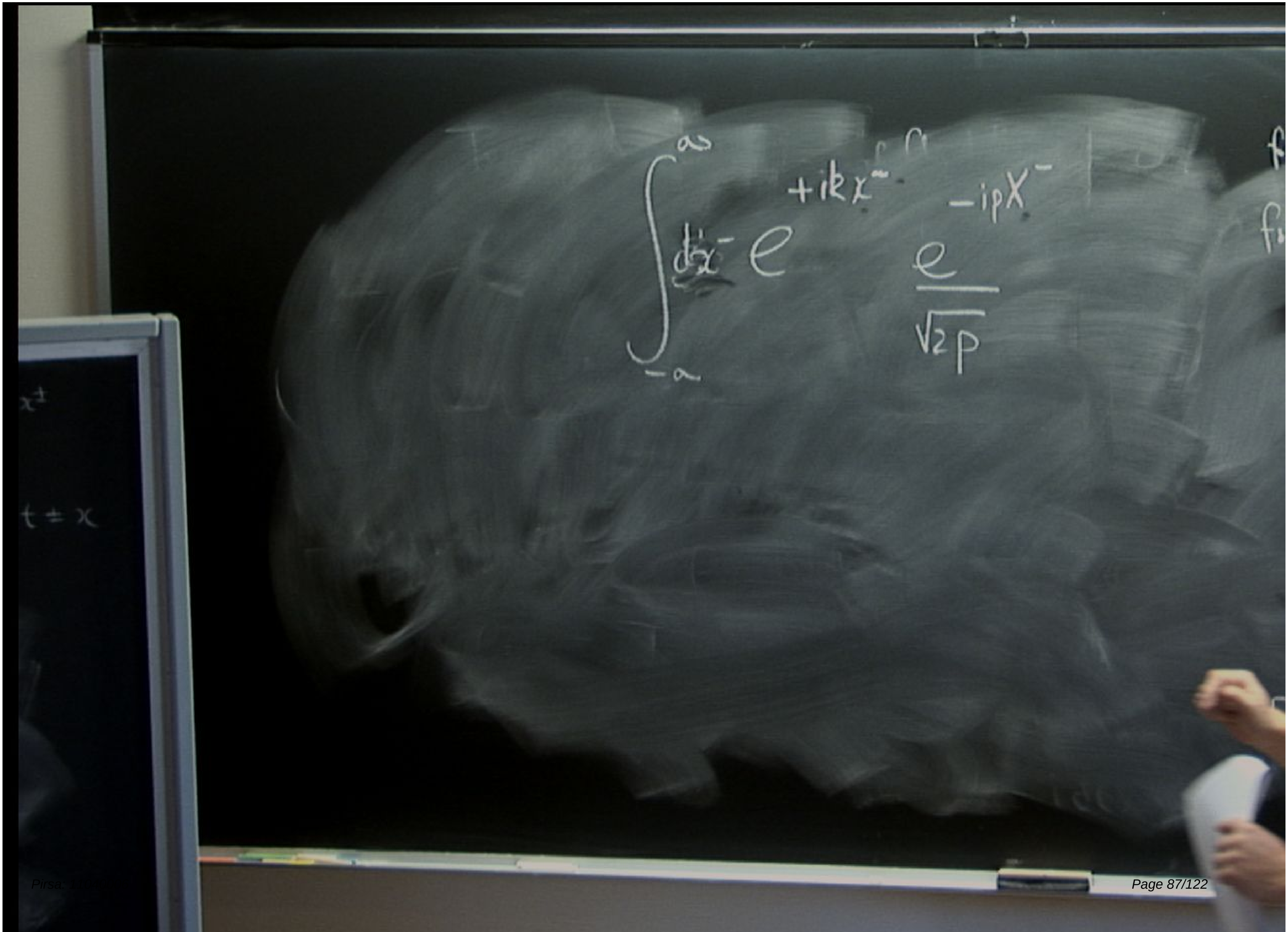
$b_{k}^{+}] = (2\pi)\,\delta(k-k')$

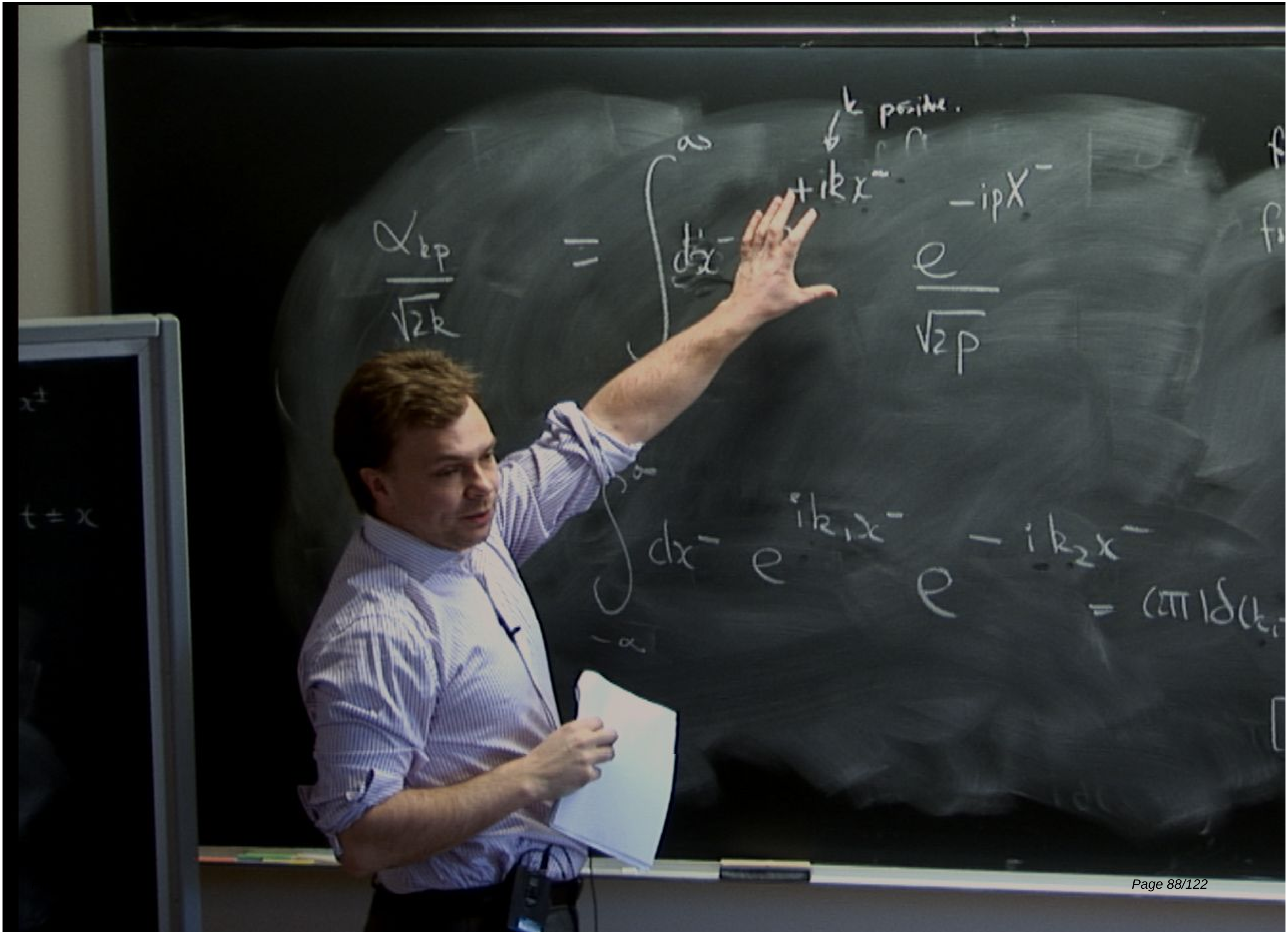
$$\int_0^\infty \frac{dp}{(2\pi)}\left[\alpha_{kp}\alpha^*_{k'p} - \beta_{k'p}\beta^*_{kp}\right] = (2\pi)\,\delta(k-k')$$

$$\int_{-\infty}^{\infty} dx^- \, e^{+ikx^-} \, \frac{e^{-ipX^-}}{\sqrt{2p}}$$

$x^{\pm}$

$t \pm x$


+ikx⁻
−ipX⁻
e
√2p
t ± x


k positive.

k positive.

for all p.

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \, e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

for all k.

$$\frac{e^{-ipX}}{\sqrt{2p}}$$

$$\int_{-\infty}^{\infty} dx^- \, e^{ik_1 x^-} e^{-ik_2 x^-} = (2\pi)\delta(k_1 - k_2)$$

$$b_k = \int_0^\infty \frac{dp}{2\pi} \Big[$$

$$[b_{k_1}, b_{k_2}^+] = (2\pi)\delta($$

k positive.

for all p.

for all k

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \, e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

$$\frac{e^{-ipX}}{\sqrt{2p}}$$

$$= \int_0^{\infty} \frac{dp}{2\pi} \Big[$$

$b_{k}^{+}] = (2\pi)\,\delta($

k positive. for all p.

for all k

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \, e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \, e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

$$\frac{e^{-ipX}}{\sqrt{2p}}$$

$$= \int_0^{\infty} \frac{dp}{2\pi} \Big[$$

$$b_{k}^{+}] = (2\pi)\,\delta($$

k positive.

for all p.

for all k

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$\boxed{X^- = -\frac{1}{a} e^{-ax^-}}$$

$$\frac{e^{-ipX}}{\sqrt{2p}}$$

$$= \int_0^{\infty} \frac{dp}{2\pi} \Big[$$

$$b_k^+] = (2\pi)\,\delta($$

$\uparrow$ k positive.

for all p.

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

for all k

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

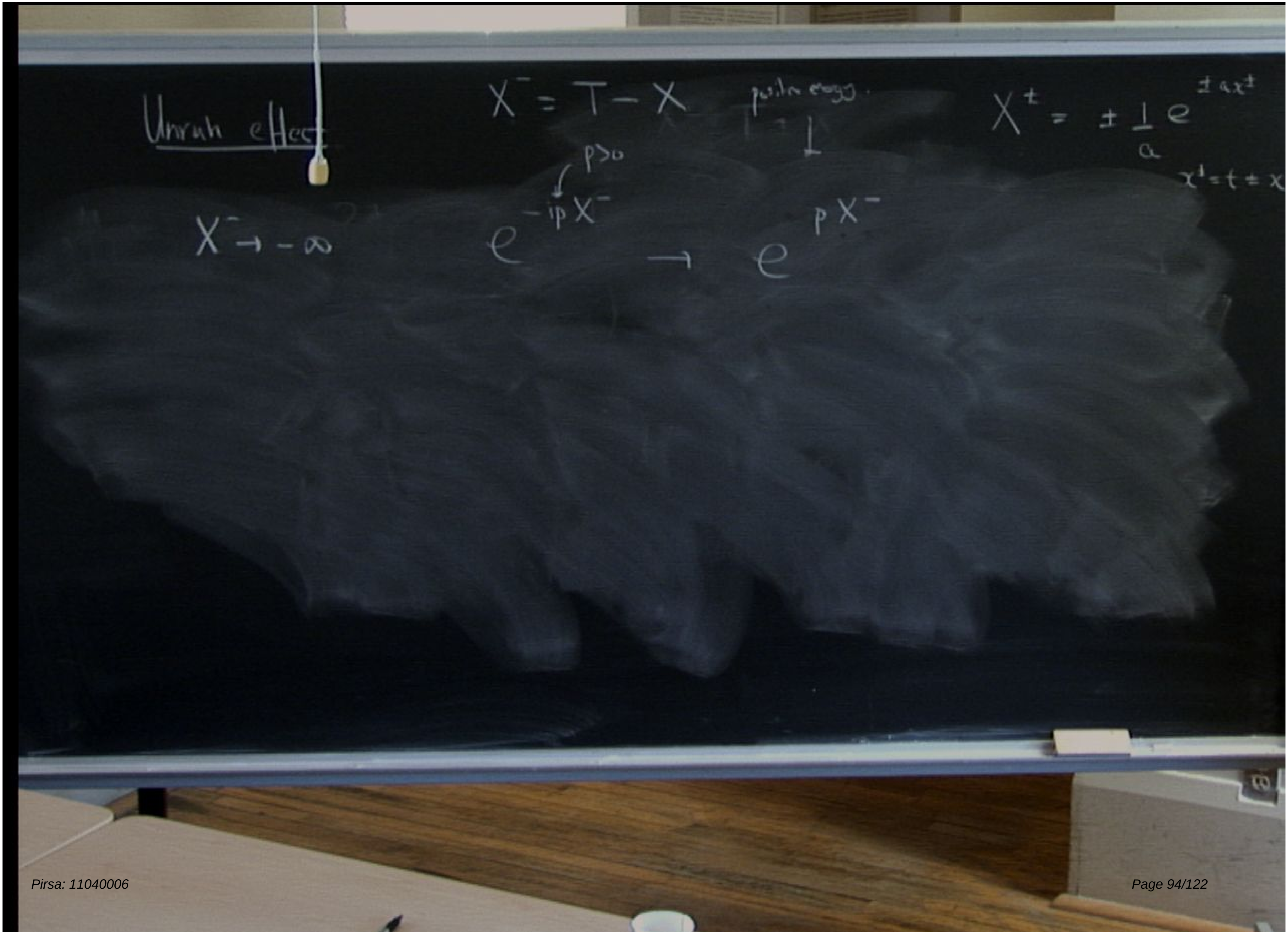
$$\boxed{X^- = -\frac{1}{a} e^{-ax^-}}$$

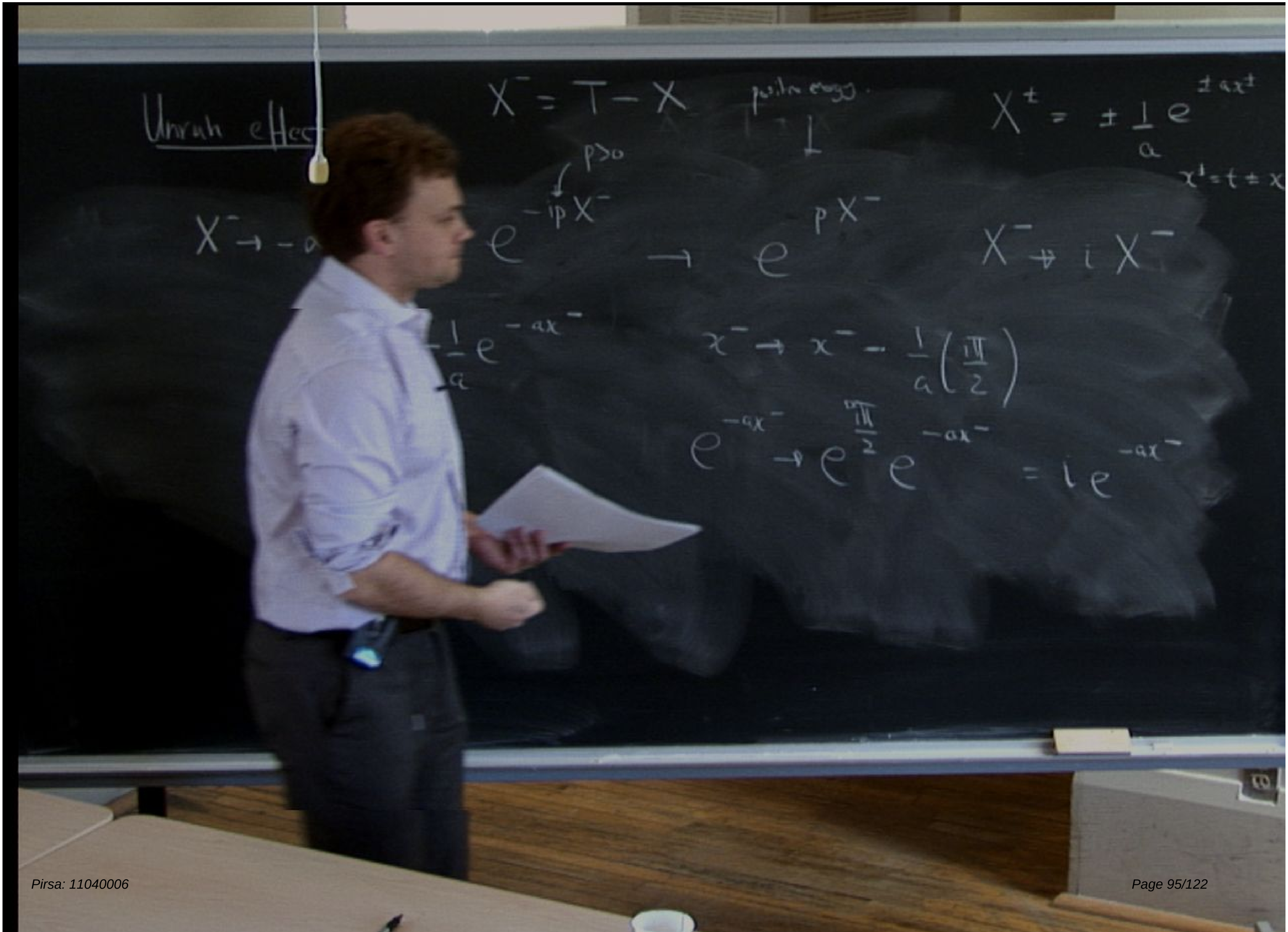
$X^- = 0 \to -\infty$

$$\frac{e^{-ipX}}{\sqrt{2p}}$$

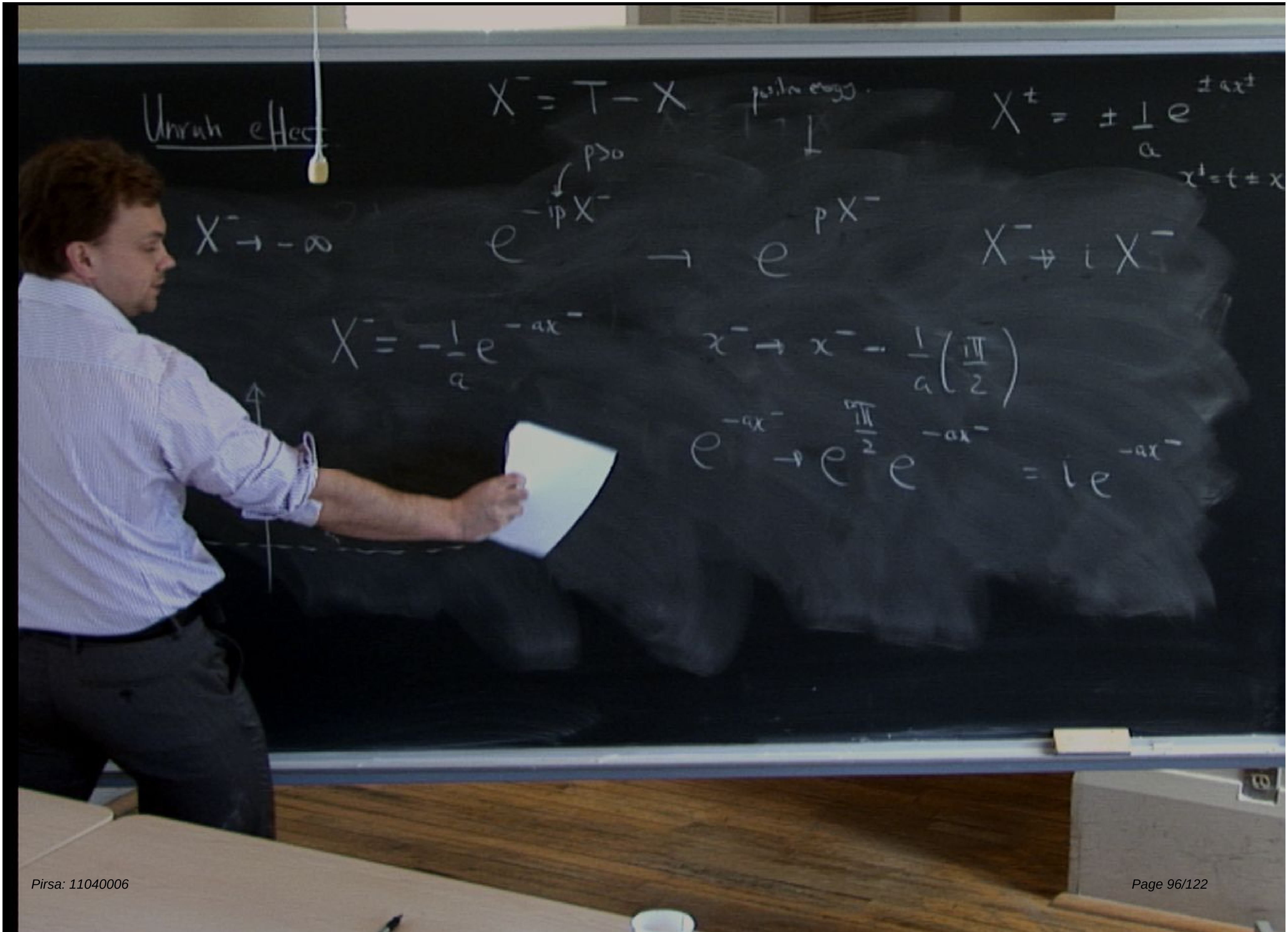
$$= \int_0^{\infty} \frac{dp}{2\pi} \Big[$$

$b_{k}^{+}] = (2\pi)\,\delta($


Unruh effect
$X^- = T - X$
positive energy
$p>0$
$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$
$x^\pm = t \pm x$
$X^- \to -\infty$
$e^{-ipX^-} \to e^{pX^-}$



Unruh effe
X⁻ = T − X
p>0
X⁺ = ± 1/a e^{±ax±}
x± = t ± x


Unruh effect
$X^- = T - X$
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$X^- \to -\infty$
$e^{-ipX^-} \to e^{pX^-}$
$X^- \to iX^-$
$X^- = -\frac{1}{a} e^{-ax^-}$
$x^- \to x^- - \frac{1}{a}\left(\frac{i\pi}{2}\right)$
$e^{-ax^-} \to e^{i\frac{\pi}{2}} e^{-ax^-} = i e^{-ax^-}$

Unruh effect

$X^- = T - X$ positive energy

$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$

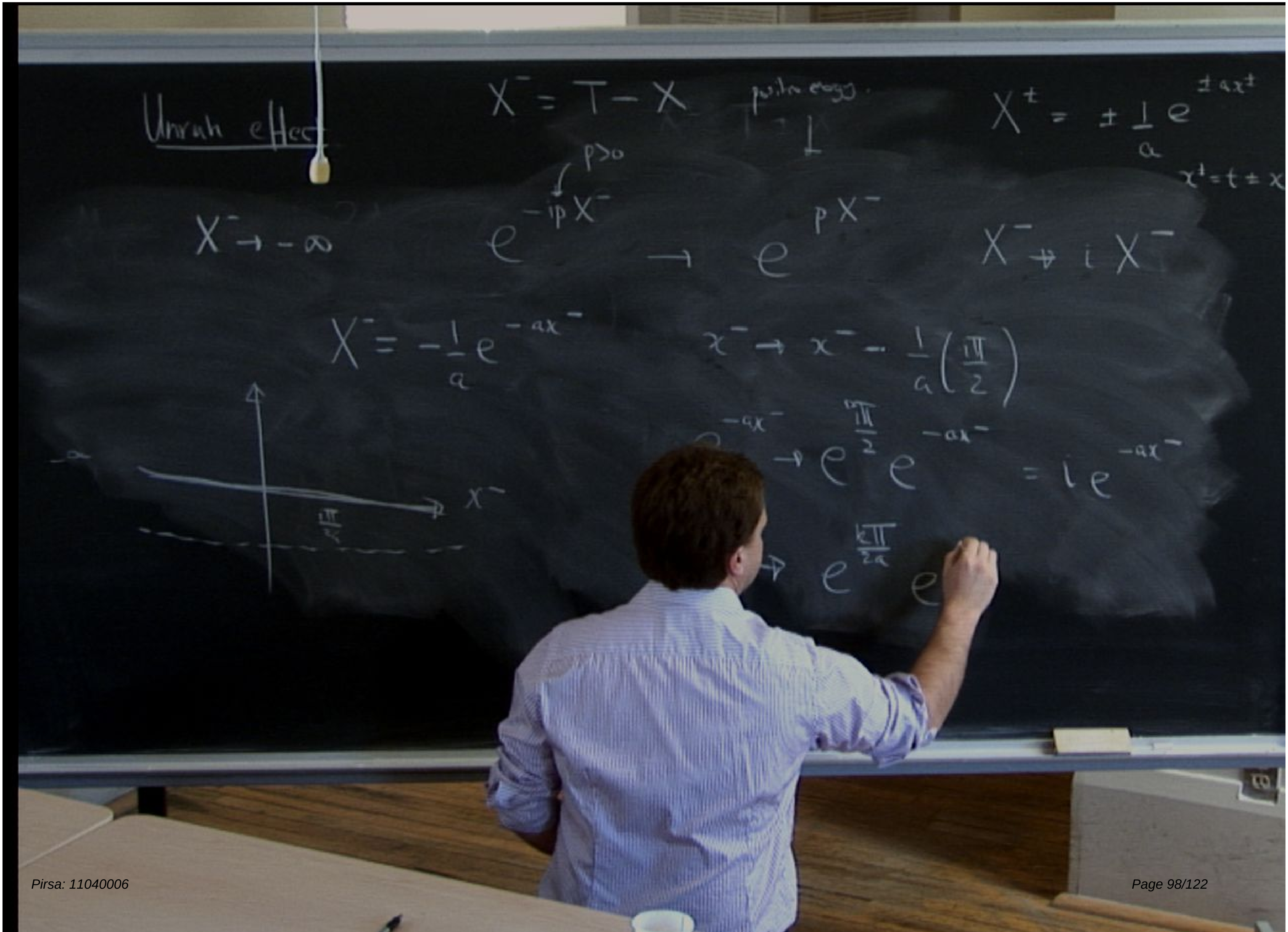
$x^\pm = t \pm x$

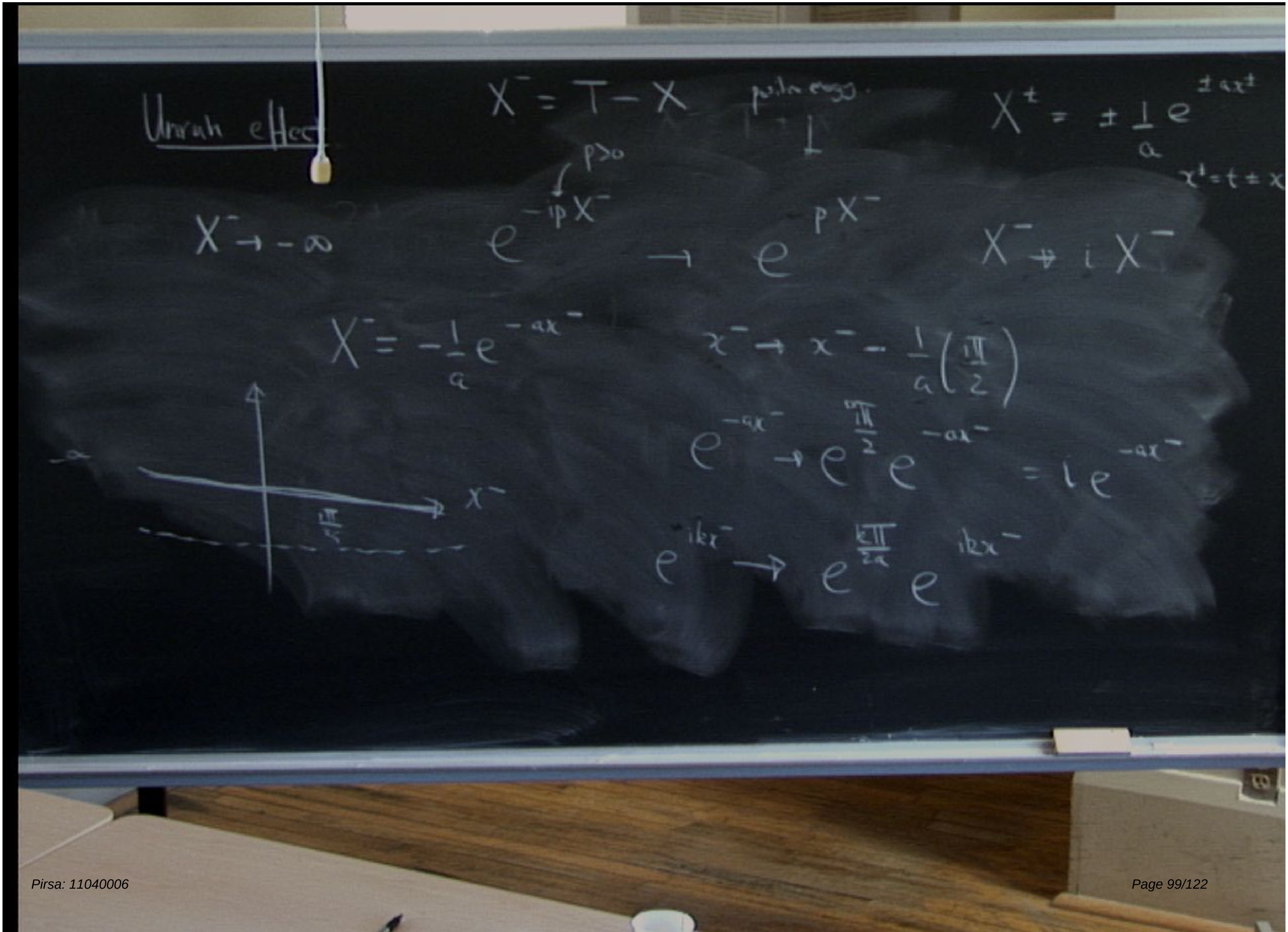
$p > 0$

$X^- \to -\infty$ $e^{-ipX^-} \to e^{pX^-}$ $X^- \to iX^-$

$X^- = -\frac{1}{a} e^{-ax^-}$ $x^- \to x^- - \frac{1}{a}\left(\frac{i\pi}{2}\right)$

$e^{-ax^-} \to e^{\frac{i\pi}{2}} e^{-ax^-} = i e^{-ax^-}$


Unruh effect
X⁻ = T − X
positive energy
X± = ± 1/a e^{±ax±}
x± = t ± x
p>0
X⁻ → −∞
e^{−ipX⁻} → e^{pX⁻}
X⁻ → iX⁻
X⁻ = −1/a e^{−ax⁻}
x⁻ → x⁻ − 1/a (iπ/2)
= i e^{−ax⁻}
x⁻


Unruh effect
X⁻ = T − X
X^± = ± 1/a e^{±ax^±}
x^± = t ± x
p>0
X⁻ → −∞
e^{−ipX⁻} → e^{pX⁻}
X⁻ → iX⁻
X⁻ = −1/a e^{−ax⁻}
x⁻ → x⁻ − 1/a (iπ/2)
e^{−ax⁻} → e^{iπ/2} e^{−ax⁻} = i e^{−ax⁻}
e^{ikx⁻} → e^{kπ/2a} e^{ikx⁻}
x⁻

Unruh effect

$X^- = T - X$ positive energy

$X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$

$x^\pm = t \pm x$

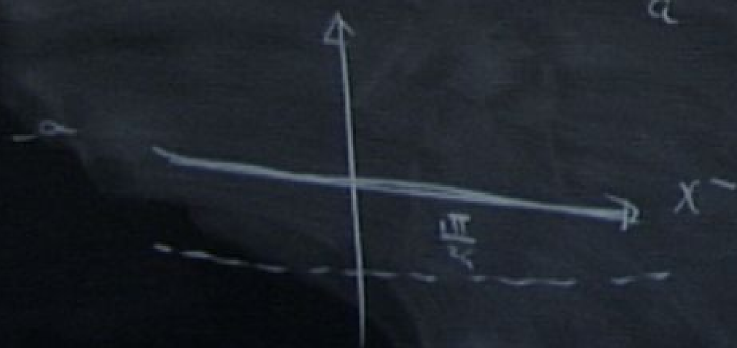
$X^- \to -\infty$

$e^{-ipX^-} \to e^{pX^-}$ ($p>0$)

$X^- \to i X^-$

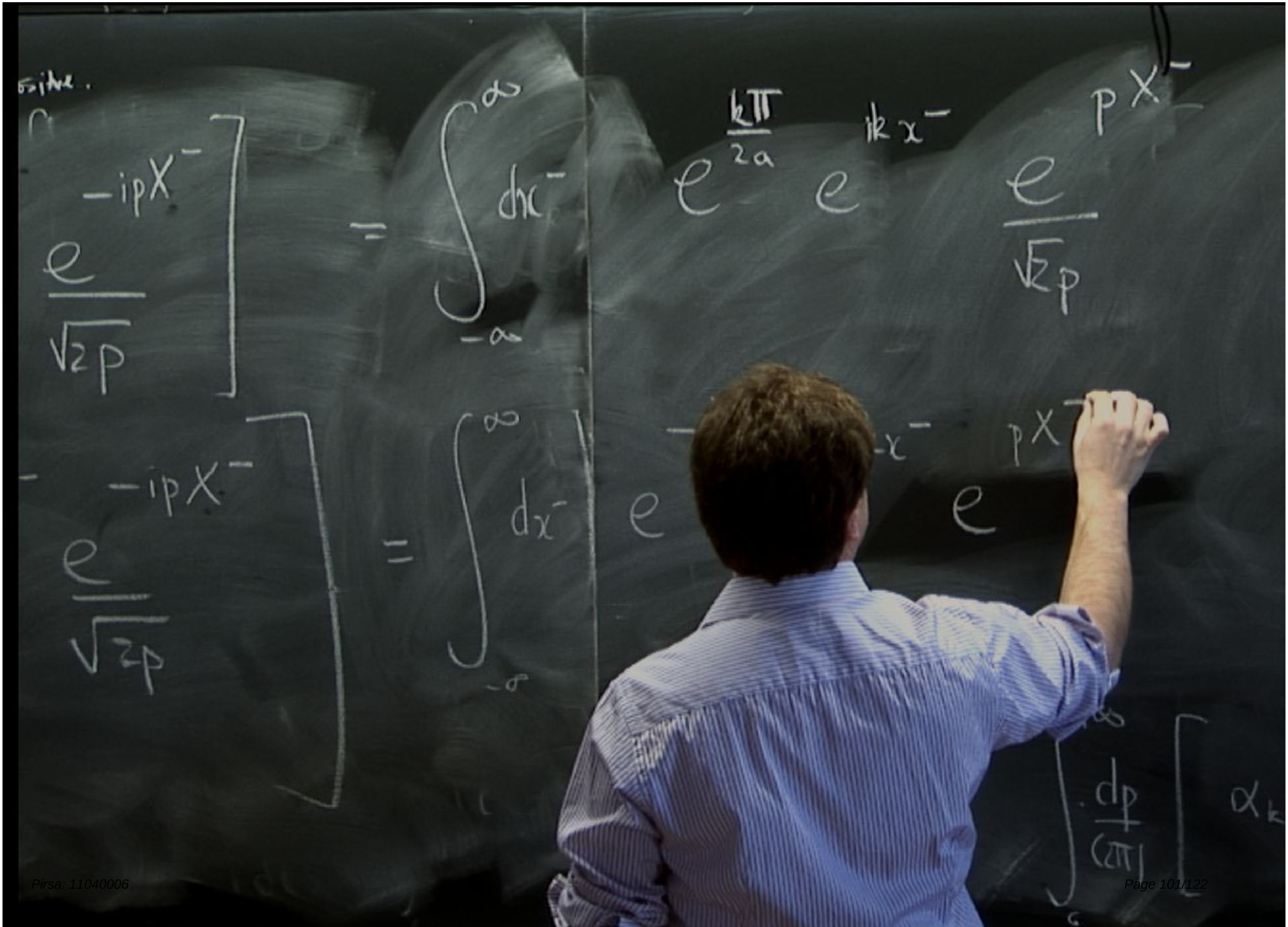
$X^- = -\frac{1}{a} e^{-ax^-}$

$x^- \to x^- - \frac{1}{a}\left(\frac{i\pi}{2}\right)$



$e^{-ax^-} \to e^{\frac{i\pi}{2}} e^{-ax^-} = i e^{-ax^-}$

$e^{ikx^-} \to e^{\frac{k\pi}{2a}} e^{ikx^-}$


$\frac{e^{-ipX^-}}{\sqrt{2p}}$
$= \int_{-\infty}^{\infty} dx^- \, e^{\frac{k\pi}{2a}} \, e^{ikx^-} \, \frac{e^{pX^-}}{\sqrt{2p}}$

sitive.

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right] = \int_{-\infty}^{\infty} dx^- \, e^{\frac{k\pi}{2a}} \, e^{ikx^-} \, \frac{e^{pX^-}}{\sqrt{2p}}$$

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right] = \int_{-\infty}^{\infty} dx^- \, e^{-\frac{k\pi}{2a}} \, e^{ikx^-} \, \frac{e^{pX^-}}{\sqrt{2p}}$$

$$b_{k}^{+}] = (2\pi)\,\delta(k-k')$$

$$\int_{0}^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{k} \right.$$

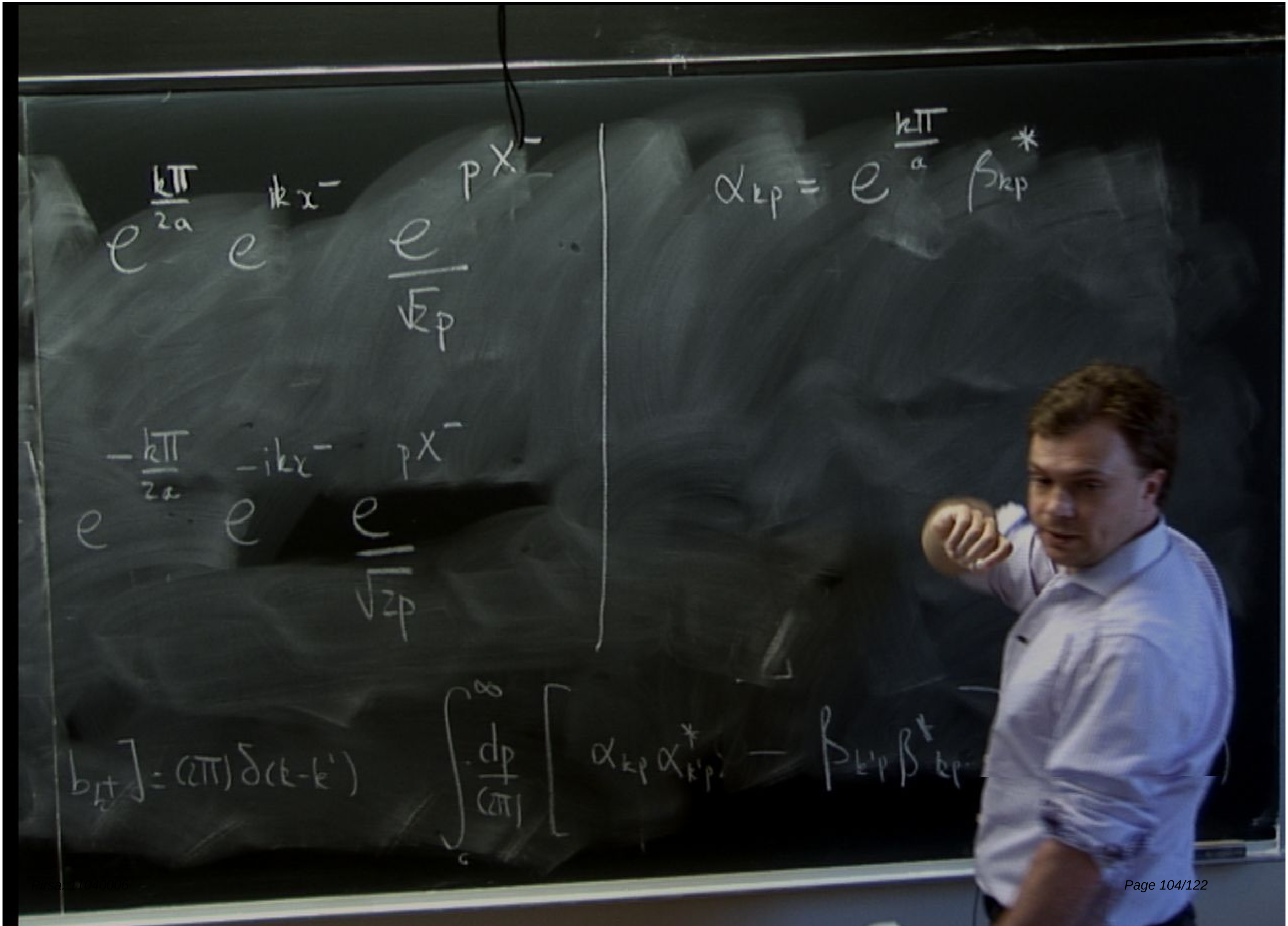
sitive.

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}}\right] = \int_{-\infty}^{\infty} dx^- \, e^{\frac{k\pi}{2a}} \, e^{ikx^-} \, \frac{e^{pX^-}}{\sqrt{2p}}$$

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}}\right] = \int_{-\infty}^{\infty} dx^- \, e^{-\frac{k\pi}{2a}} \, e^{-ikx^-} \, \frac{e^{pX^-}}{\sqrt{2p}}$$

$b_k^{\dagger}] = (2\pi)\,\delta(k-k')$

$$\int_0^{\infty} \frac{dp}{(2\pi)} \Big[$$



$e^{\frac{k\pi}{2a}} e^{ikx^-} \frac{e^{pX^-}}{\sqrt{2p}}$
$\alpha_{kp} = e^{\frac{k\pi}{a}} \beta_{kp}^*$
$e^{-\frac{k\pi}{2a}} e^{-ikx^-} \frac{e^{pX^-}}{\sqrt{2p}}$
$b_{k}^{\dagger}] = (2\pi)\delta(k-k') \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha^*_{k'p} - \beta_{k'p}\beta^*_{kp}\right.$

$$e^{\frac{k\pi}{2a}} \quad e^{ikx^-} \quad \frac{e^{pX^-}}{\sqrt{2p}}$$

$$e^{-\frac{k\pi}{2a}} \quad e^{-ikx^-} \quad \frac{e^{pX^-}}{\sqrt{2p}}$$

$$\boxed{\alpha_{kp} = e^{\frac{k\pi}{a}} \beta_{kp}^*}$$

$$b_{k}^{\dagger}] = (2\pi)\,\delta(k-k') \qquad \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha_{k'p}^* - \beta_{k'p}\beta_{kp}^* \right] = (2\pi)\,\delta(k-k')$$

$$e^{\frac{k\pi}{2a}} \quad e^{ikx^-} \quad \frac{e^{pX^-}}{\sqrt{2p}}$$

$$e^{-\frac{k\pi}{2a}} \quad e^{-ikx^-} \quad \frac{e^{pX^-}}{\sqrt{2p}}$$

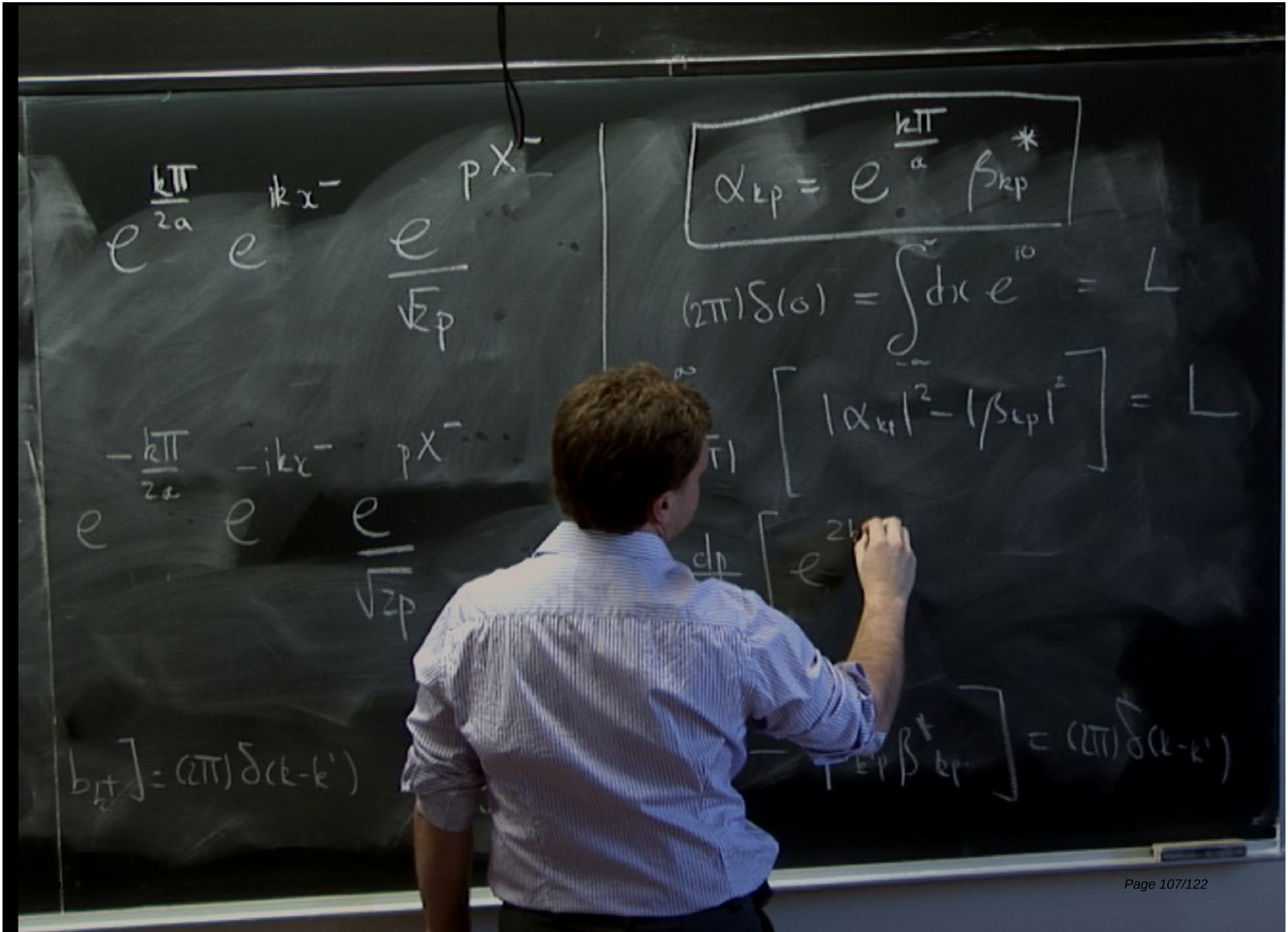
$$\boxed{\alpha_{kp} = e^{\frac{k\pi}{a}} \beta^*_{kp}}$$

$$(2\pi)\delta(0) = \int_{-\infty}^{\infty} dx\, e^{i0} = L$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$b_{k'}^\dagger] = (2\pi)\delta(k-k')$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha^*_{k'p} - \beta_{k'p}\beta^*_{kp} \right] = (2\pi)\delta(k-k')$$


$\alpha_{kp} = e^{\frac{k\pi}{a}} \beta_{kp}^{*}$
$(2\pi)\delta(0) = \int dx\, e^{i0} = L$
$|\alpha_{kp}|^2 - |\beta_{kp}|^2$
$= (2\pi)\delta(k-k')$

$$e^{\frac{k\pi}{2a}} \, e^{ik x^-} \, \frac{e^{p X^-}}{\sqrt{2p}}$$

$$e^{-\frac{k\pi}{2a}} \, e^{-ik x^-} \, \frac{e^{p X^-}}{\sqrt{2p}}$$

$$\boxed{\alpha_{kp} = e^{\frac{k\pi}{a}} \beta_{kp}^*}$$

$$(2\pi)\delta(0) = \int_{-\infty}^{\infty} dx \, e^{i0} = L$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[\left(e^{\frac{2k\pi}{a}} - 1\right) |\beta_{kp}|^2 \right] = L$$

$$b_{k}^{+}] = (2\pi)\delta(k-k')$$

$$\int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi)\delta(k-k')$$

$$e^{\frac{k\pi}{2a}} \; e^{ik x^-} \; \frac{e^{p X^-}}{\sqrt{E_p}}$$

$$e^{-\frac{k\pi}{2a}} \; e^{-ik x^-} \; \frac{e^{p X^-}}{\sqrt{2p}}$$

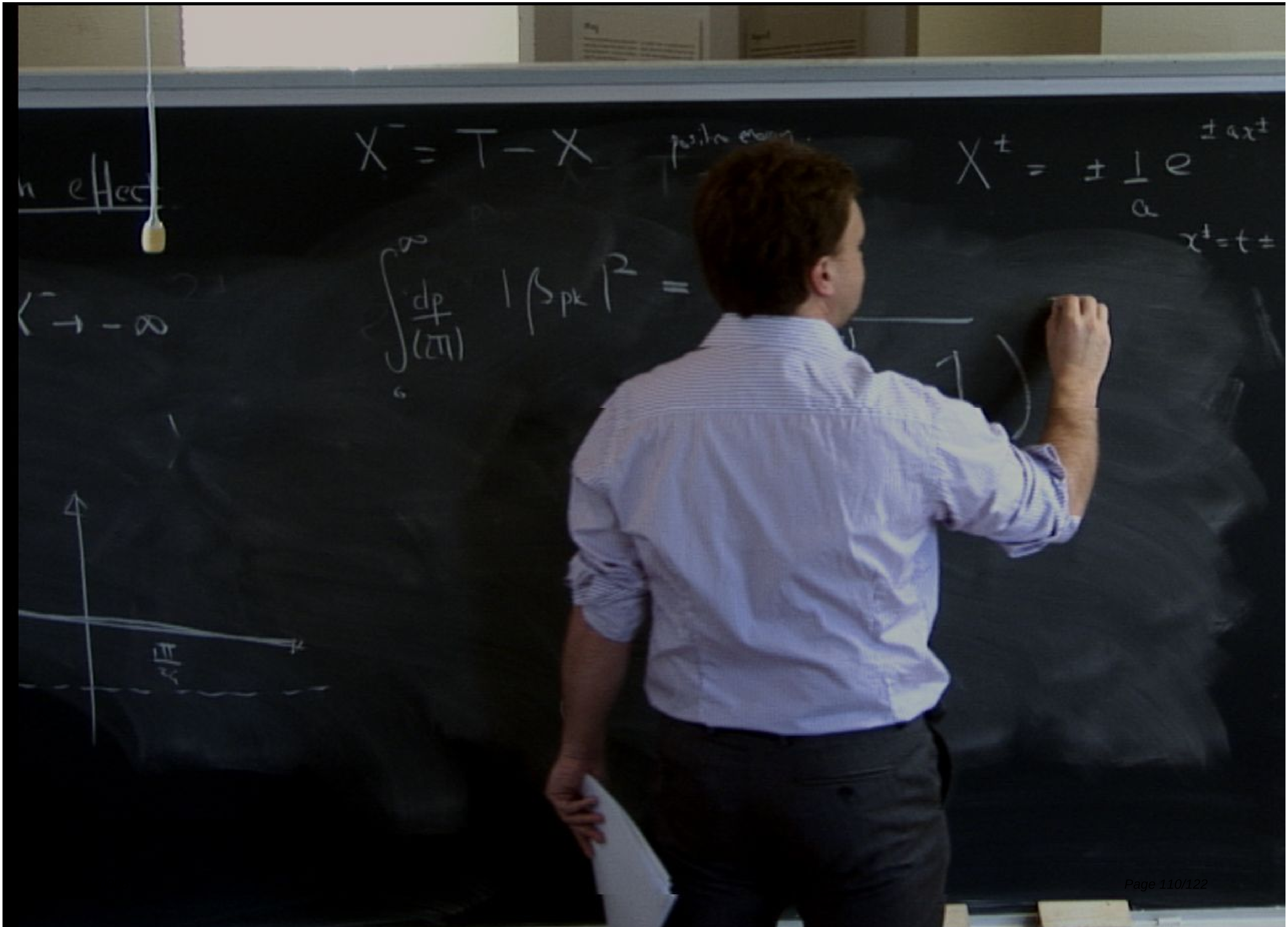
$$\boxed{\alpha_{kp} = e^{\frac{k\pi}{a}} \beta_{kp}^{*}}$$

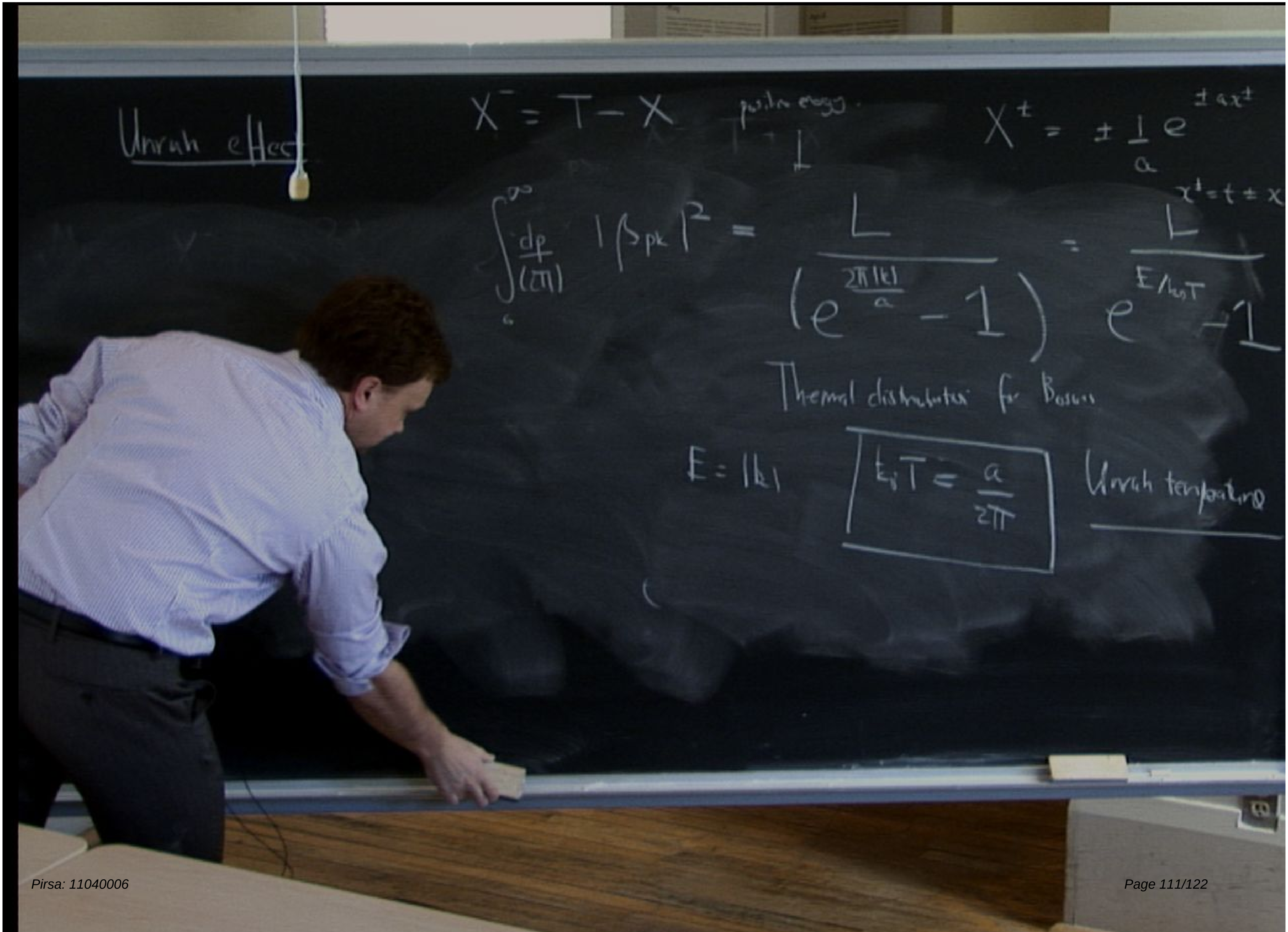
$$(2\pi)\delta(0) = \int_{-\infty}^{\infty} dx\, e^{i0} = L$$

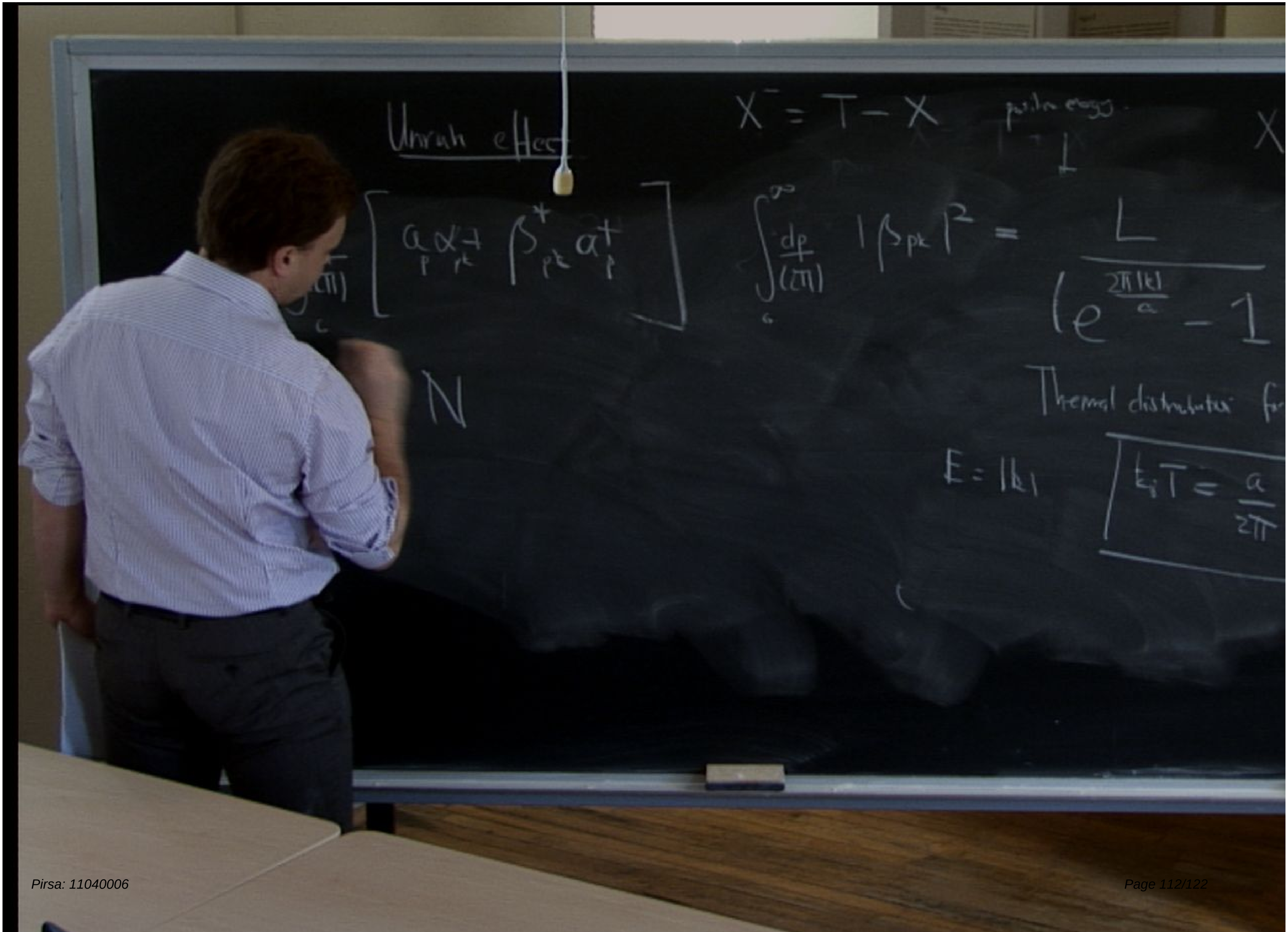
$$\int_0^{\infty} \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$\int_0^{\infty} \frac{dp}{(2\pi)} \left[\left(e^{\frac{2k\pi}{a}} - 1\right) |\beta_{kp}|^2 \right] = L$$

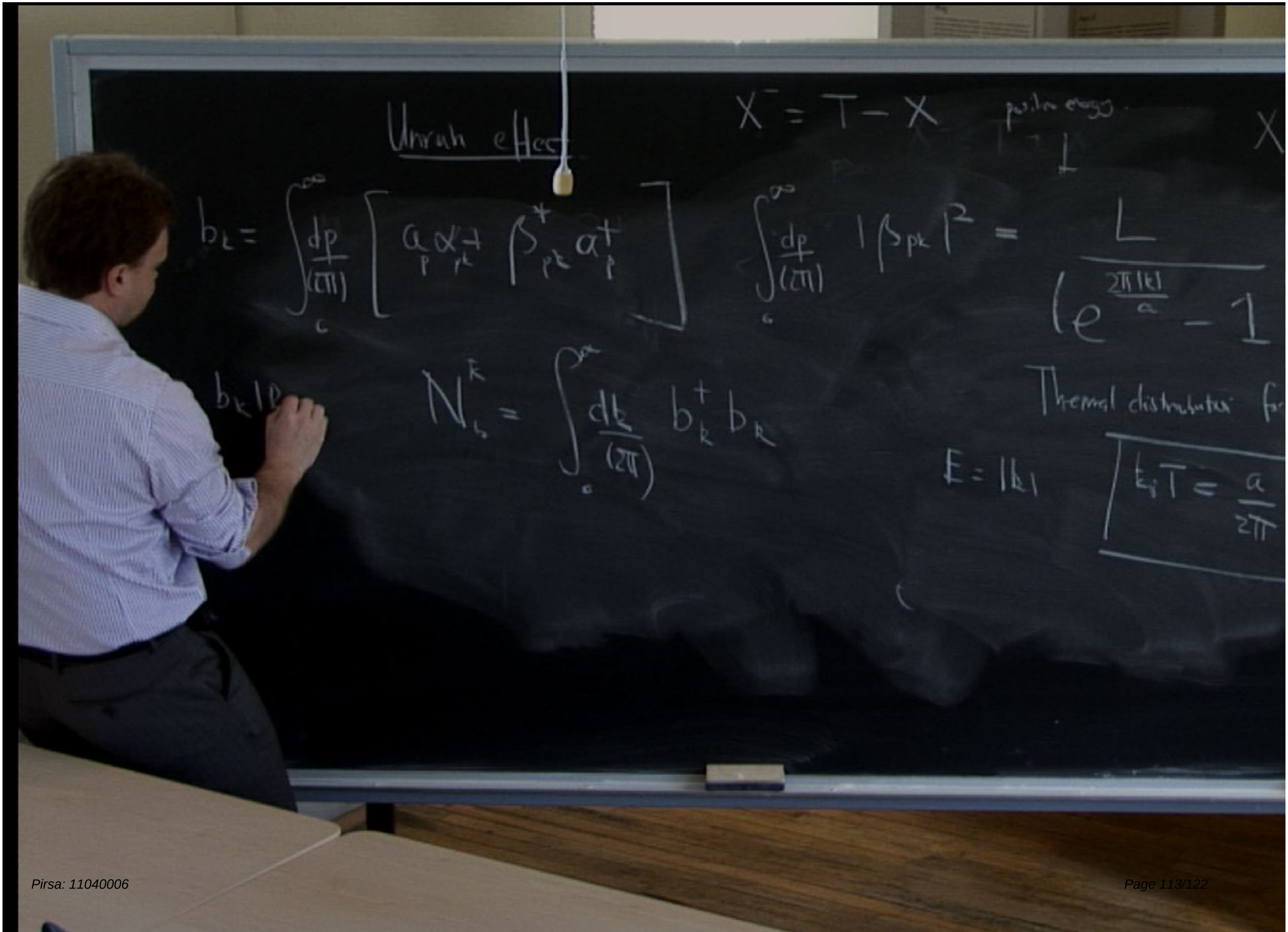
$$b_{k'}^{\dagger}] = (2\pi)\delta(k-k') \qquad \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp}\alpha_{k'p}^{*} - \beta_{k'p}\beta_{kp}^{*} \right] = (2\pi)\delta(k-k')$$


X⁻ = T − X
effect
X± = ± 1/a e^{±ax±}
x± = t ±
X⁻ → −∞


Unruh effect
X⁻ = T − X
positive energy
Thermal distribution for Bosons
E = |k|
Unruh temperature



Unruh effect
X⁻ = T − X
positive energy
N
Thermal distribution
E = |k|
kᵦT = a/2π


Unruh effect
X⁻ = T − X
positive energy
Thermal distribution
E = |k|
kB T = a/2π

Unruh effect

$X^- = T - X$ positive energy

$X^+ =$

$$b_k = \int_0^\infty \frac{dp}{(2\pi)} \left[a_p \alpha_{pk} - \beta^*_{pk} a^\dagger_p \right]$$

$$\int_0^\infty \frac{dp}{(2\pi)} |\beta_{pk}|^2 = \frac{L}{\left(e^{\frac{2\pi|k|}{a}} - 1\right)} =$$

$b_k |R_{vacuum}\rangle = 0$

$$N_b^R = \int_0^\infty \frac{dk}{(2\pi)} b^\dagger_k b_k$$

$N_b^R |R_{vacuum}\rangle = 0$

Thermal distribution for Bosons

$E = |k|$

$$\boxed{k_B T = \frac{a}{2\pi}}$$

k positive

$$\langle in | N_b | in \rangle = \int_{-\infty}^{\infty} dx^- \; e^{\frac{k\pi}{2a}} \; e^{ikx^-}$$

$$\langle Mink | N_b^2 | Mink \rangle$$

$$= \int_0^{\infty} dk \; \langle Mink | b_k^+ b_k | Mink \rangle$$

$$= \int_{-\infty}^{\infty} dx^- \; e^{-\frac{k\pi}{2a}} \; e^{-ikx^-}$$

$$b_{k}^+] = (2\pi)\, \delta(k-k')$$

$$+ \quad \langle in | N_b | in \rangle =$$

$$\langle \mathrm{Mink} | N^R_b | \mathrm{Mink} \rangle$$

$$= \int_0^\infty dk \; \langle \mathrm{Mink} | b_k^\dagger b_k | \mathrm{Mink} \rangle =$$

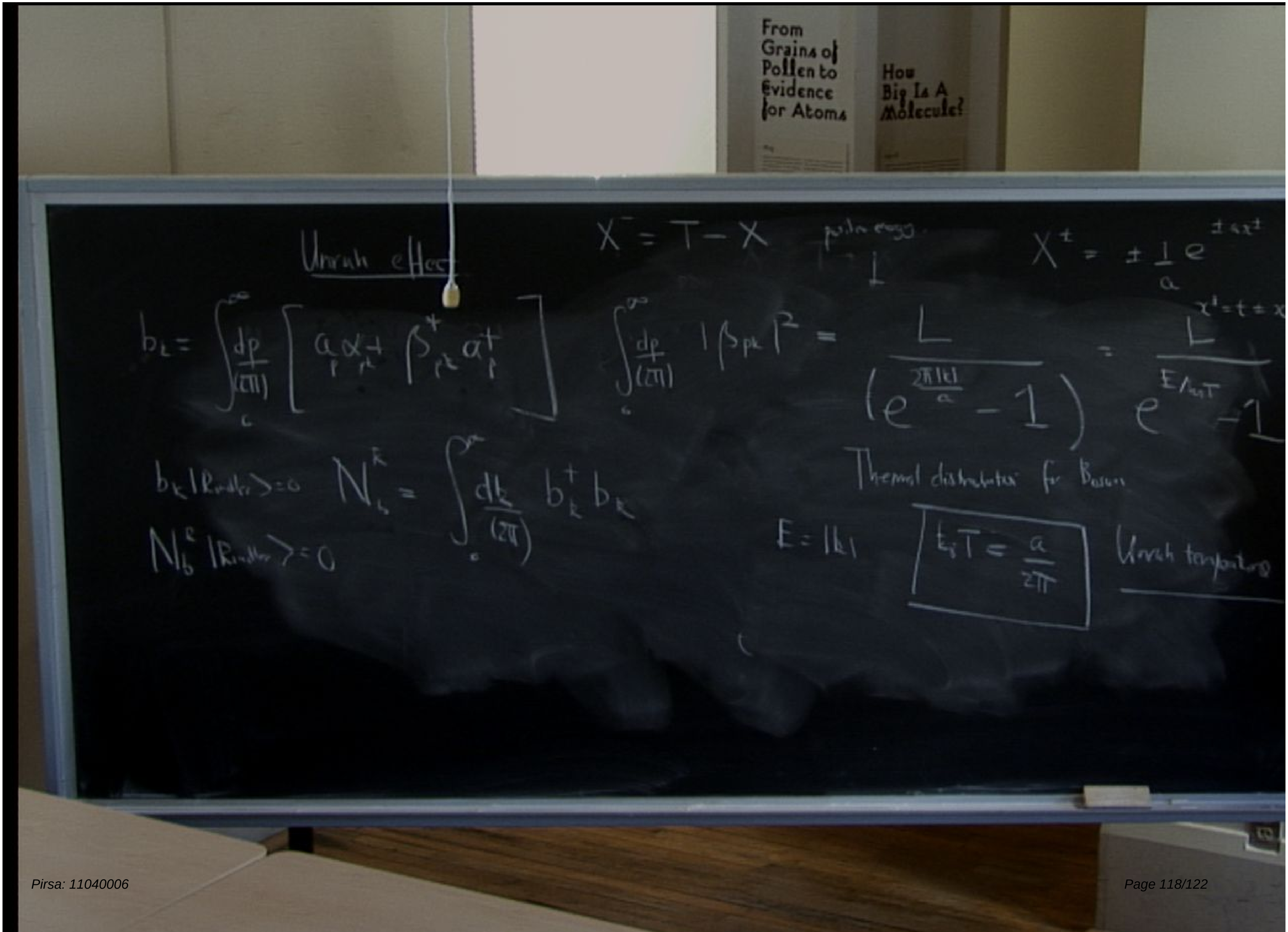
$$= \int dk \, dp \, dp' \qquad \beta^*_{pk} \, a_p^\dagger | \mathrm{Mink} \rangle$$

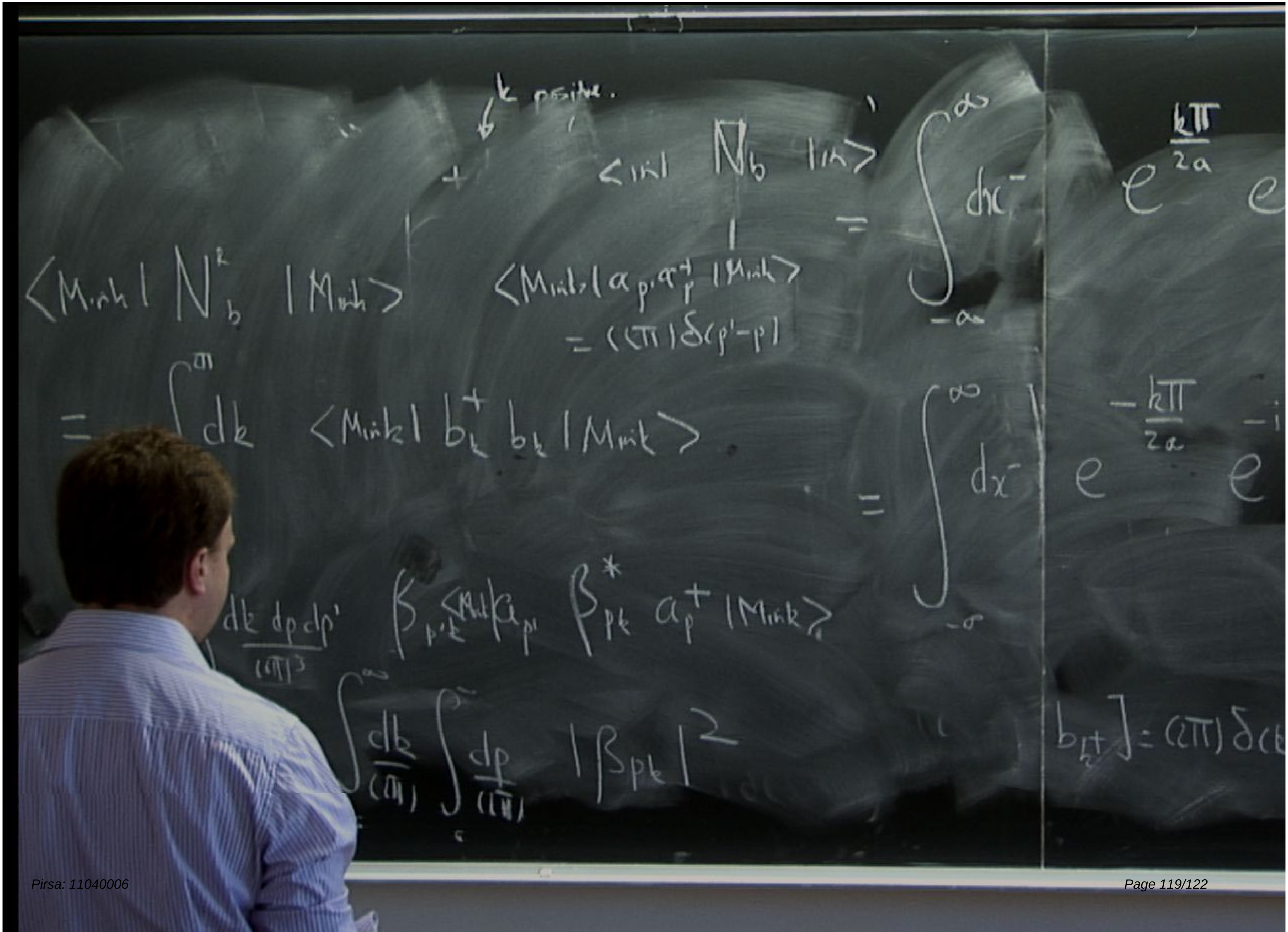
$$+ \quad \langle in | N_b | in \rangle =$$

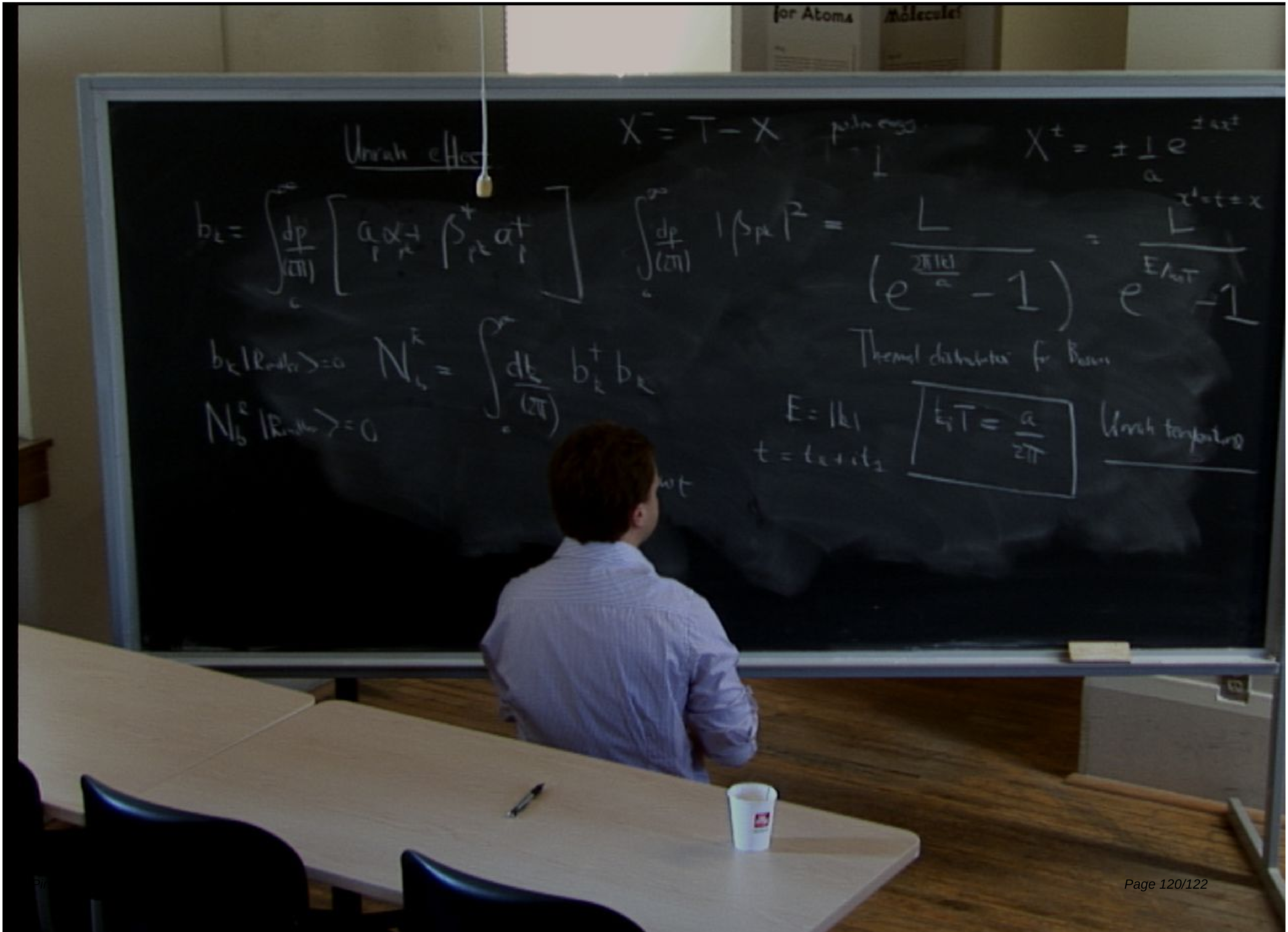
$$\langle Mink | N^R_b | Mink \rangle$$

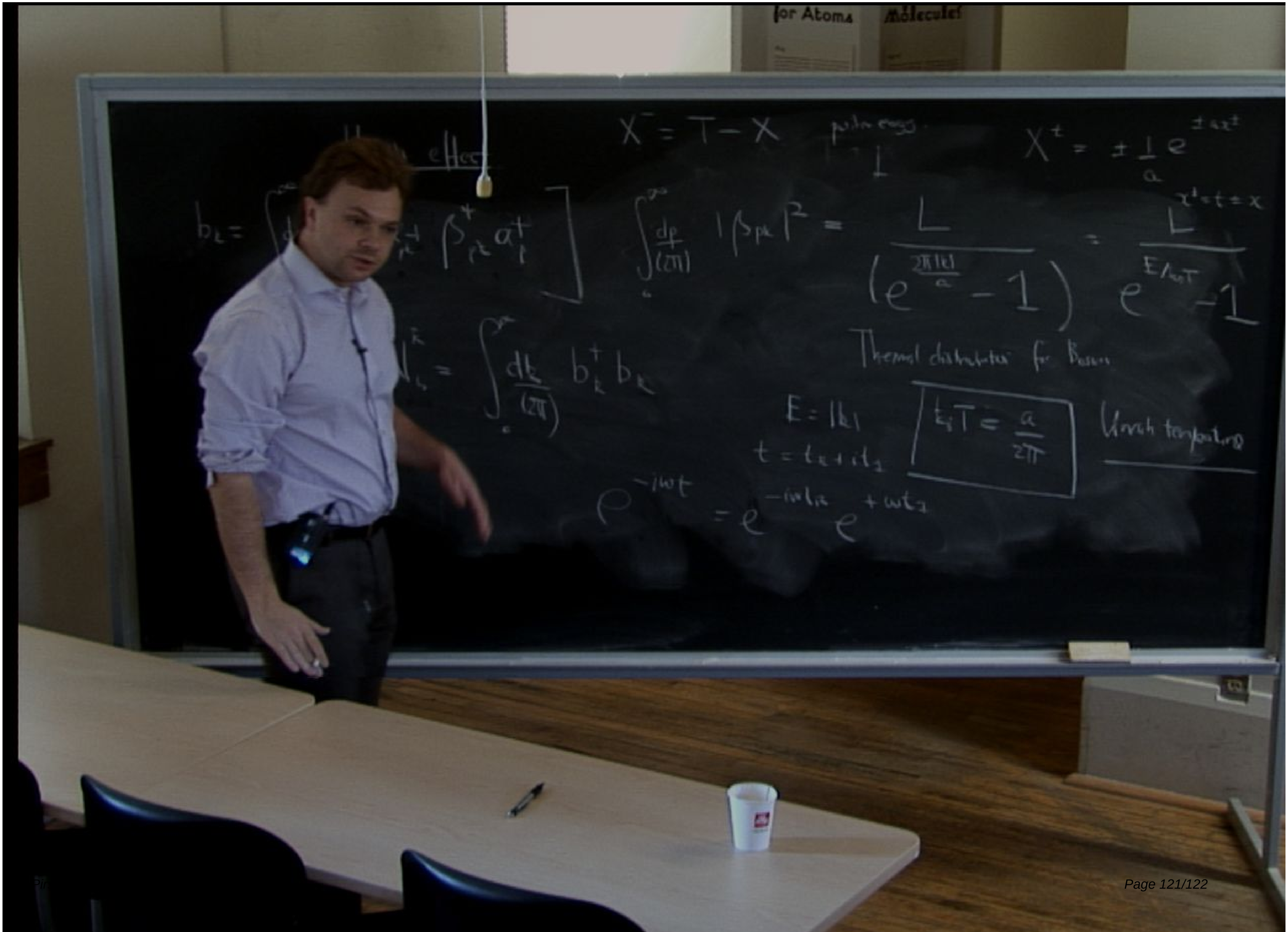
$$= \int_0^\infty dk \, \langle Mink | b_k^\dagger \, b_k | Mink \rangle =$$

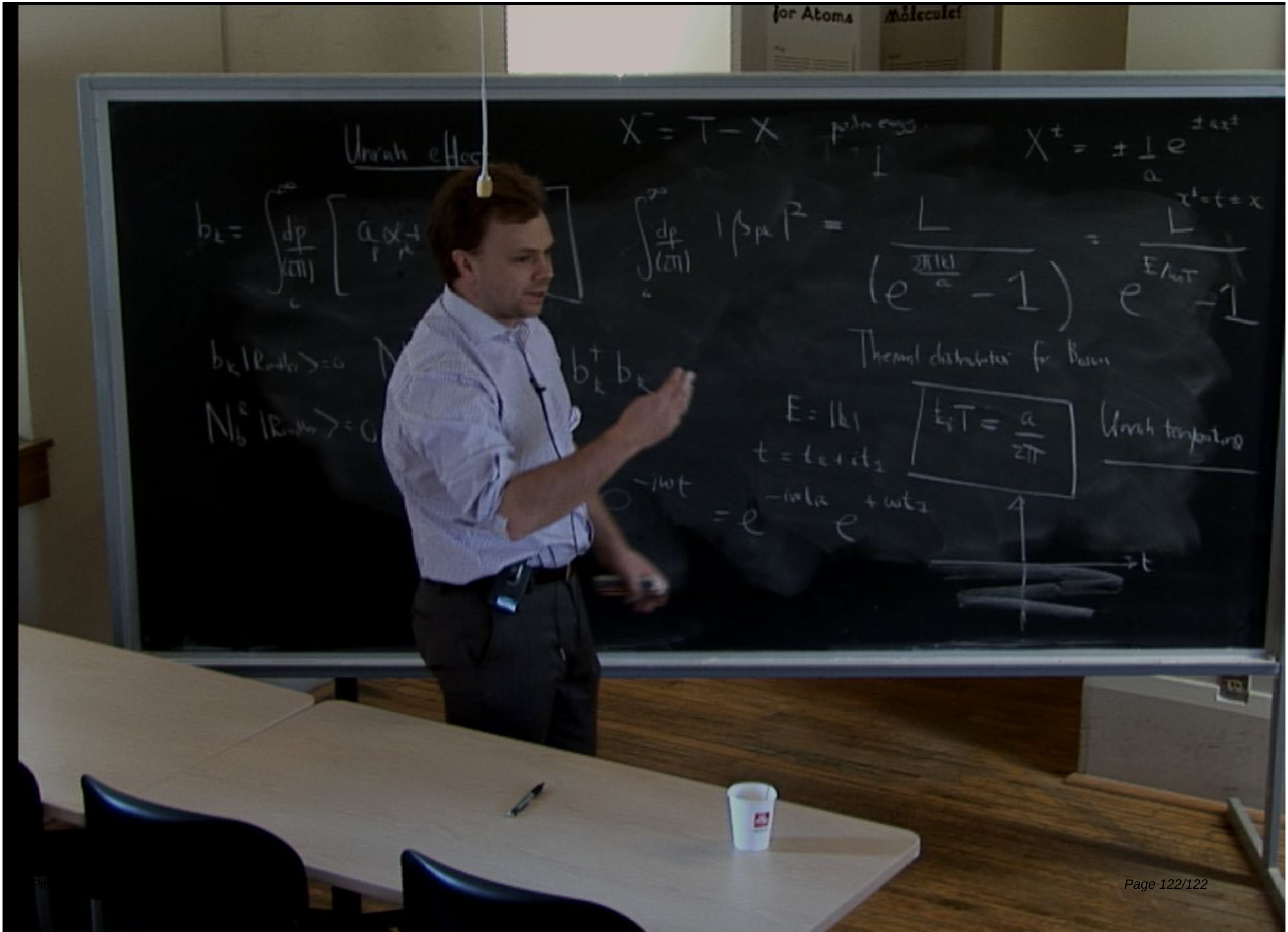
$$= \int dk \, dp \, dp' \; \beta_{p'k} \langle Mink | a_{p'} \; \beta^*_{pk} \; a_p^\dagger | Mink \rangle$$


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Unruh effect
X- = T - X
Thermal distribution for Bosons
E = |k|
Unruh temperature


k positive.
<in| N_b |in>
<Mink| N_b |Mink>
<Mink| a_p' a_p^+ |Mink>
= (2π) δ(p' − p)
= ∫ dk <Mink| b_k^+ b_k |Mink>
dk dp dp' / (2π)^3 β_p'k <Mink| a_p' β*_pk a_p^+ |Mink>
∫ dk/(2π) ∫ dp/(2π) |β_pk|^2
b_k+] = (2π) δ(
dx^-
e^{kπ/2a}
e^{−kπ/2a}


for Atoms
Molecules
Unruh effect
X⁻ = T − X
Thermal distribution for Bosons
Unruh temperature


for Atoms
Molecules
X⁻ = T − X
Thermal distribution for Bosons
E = |k|
k_B T = a/2π
Unruh temperature


for Atoms
Molecules
Unruh effect
Thermal distribution for Bosons
Unruh temperature