

Title: Explorations in Cosmology - Lecture 4

Date: Apr 07, 2011 09:00 AM

URL: <http://pirsa.org/11040006>

Abstract:



perimeter scholars
INTERNATIONAL



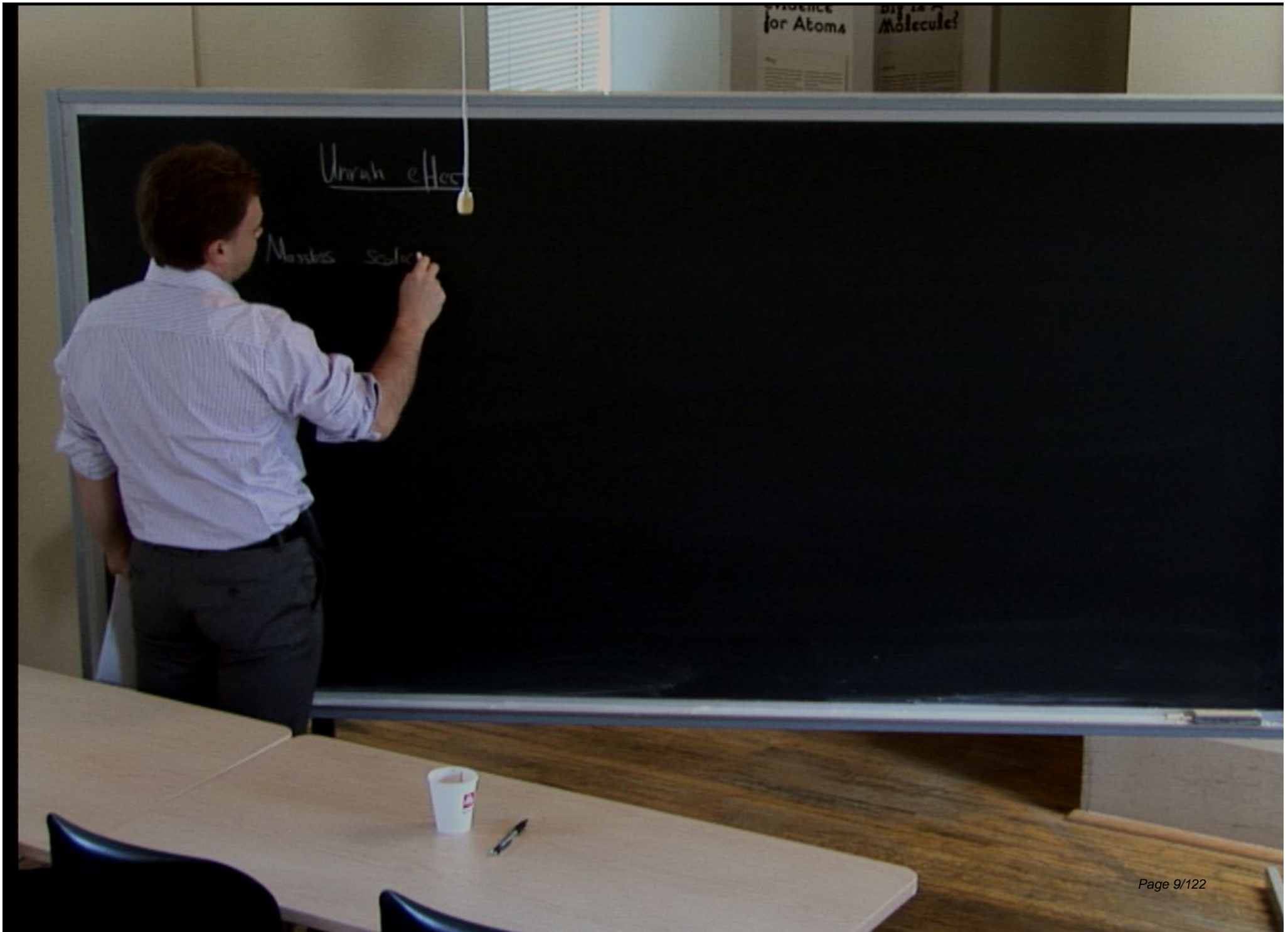
perimeter scholars
INTERNATIONAL

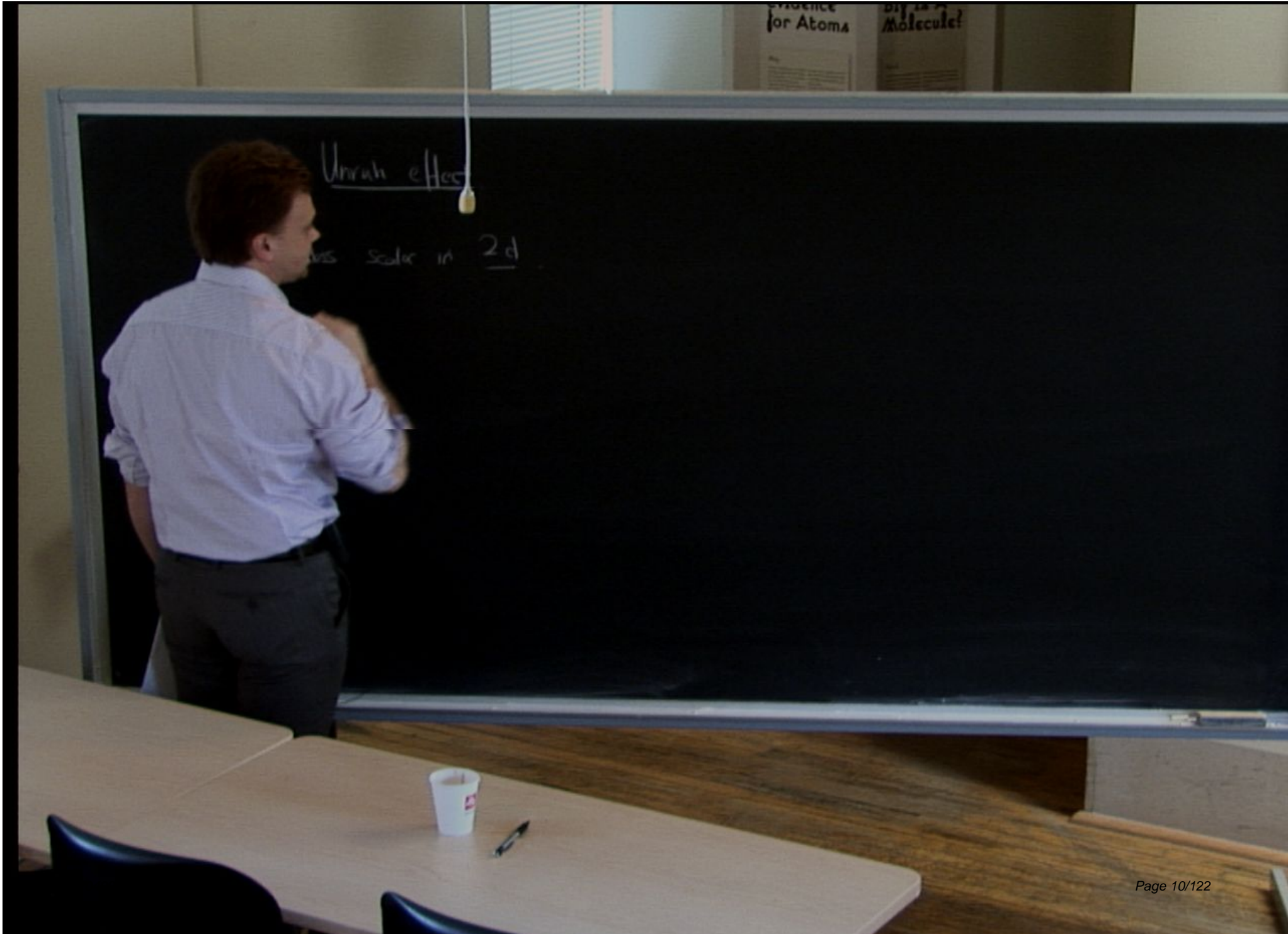
Uranium effect

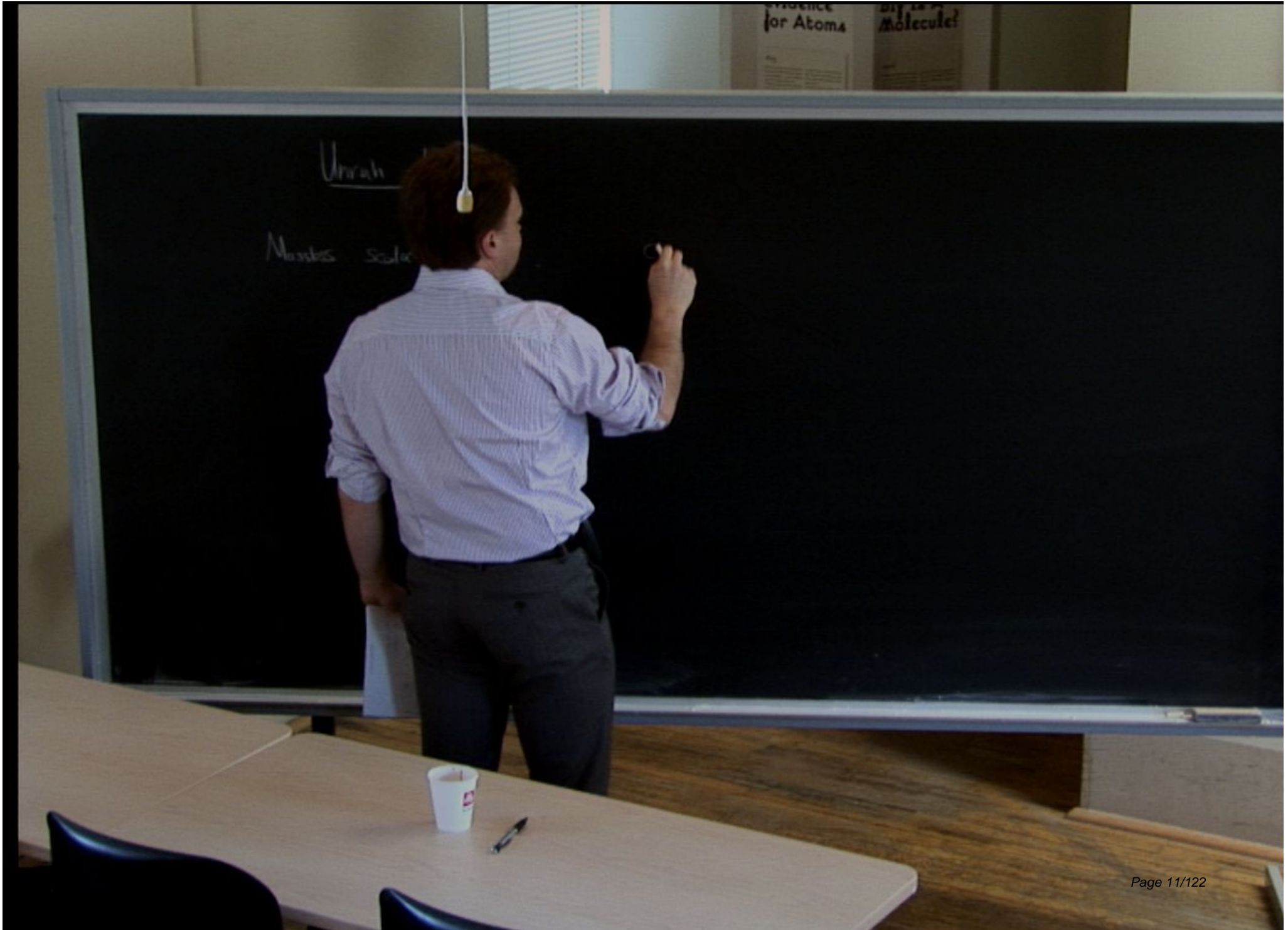
From
Grains of
Pollen to
Evidence
for Atoms

How
Big Is A
Molecule?









Unruh effect

Maximal scalar is $\frac{2}{\hbar}$

$$ds^2 = -dt^2$$

EVIDENCE
for Atoms

DIS LA P
Molecules?

Unruh effect

Number scale in $2g$

$$ds^2 = -dT^2 + dX^2$$

EVIDENCE
for Atoms

DIS LA 74
Molecule!

Unruh effect

Masses scale in $2d$ $S^2 = -dT^2 + X^2$

EVIDENCE
for Atoms

DIY LA 74
Molecule!

Unruh effect

Minkowski scalar in

$$ds^2 = -dt^2 - dX^2$$

X^{μ}

EVIDENCE
for Atoms

DIS 18-74
Molecule!

Unruh effect

Massless scalar in 2d

$$X^\pm = T_\pm$$

$$s^2 = -dT^2 + dX^2$$

EVIDENCE
for Atoms

DIY IN THE
MOLECULE?

Unruh effect

Minkowski scalar in 2d

$$ds^2 =$$

$$T \pm X$$

$$dX^2 = -c$$

EVIDENCE
for Atoms

DISCRETE
Molecules!

Unruh effect

Massless scalar in 2d

X^+ X

$$ds^2 = -dt^2 - dx^2 = -dX^+ dX^-$$

EVIDENCE for Atoms
DIY LA 7
Molecule!

Unit 11

Monday 2d

$$X^{\pm} = T_{\pm} X$$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$



Unruh effect

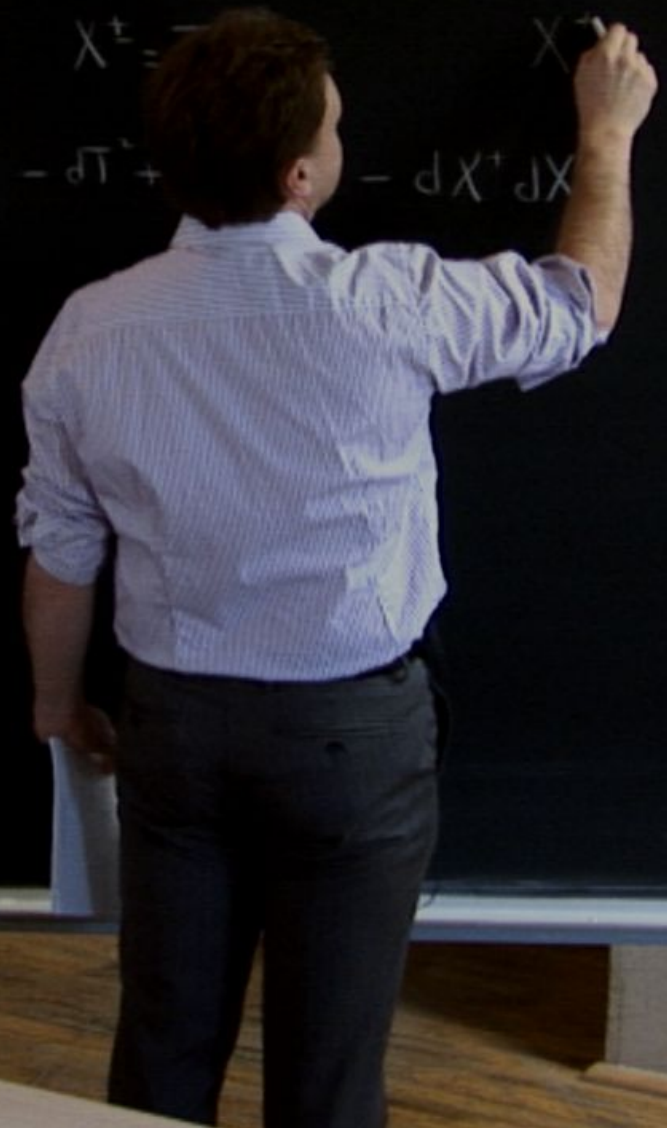
Massless scalar in 2d

$$ds^2 = -dT^2 +$$

$$X^{\pm} =$$

$$X^{\pm}$$

$$-dX^+dX^-$$



EVIDENCE
for Atoms

DIY LA-
Molecule?

Unruh effect

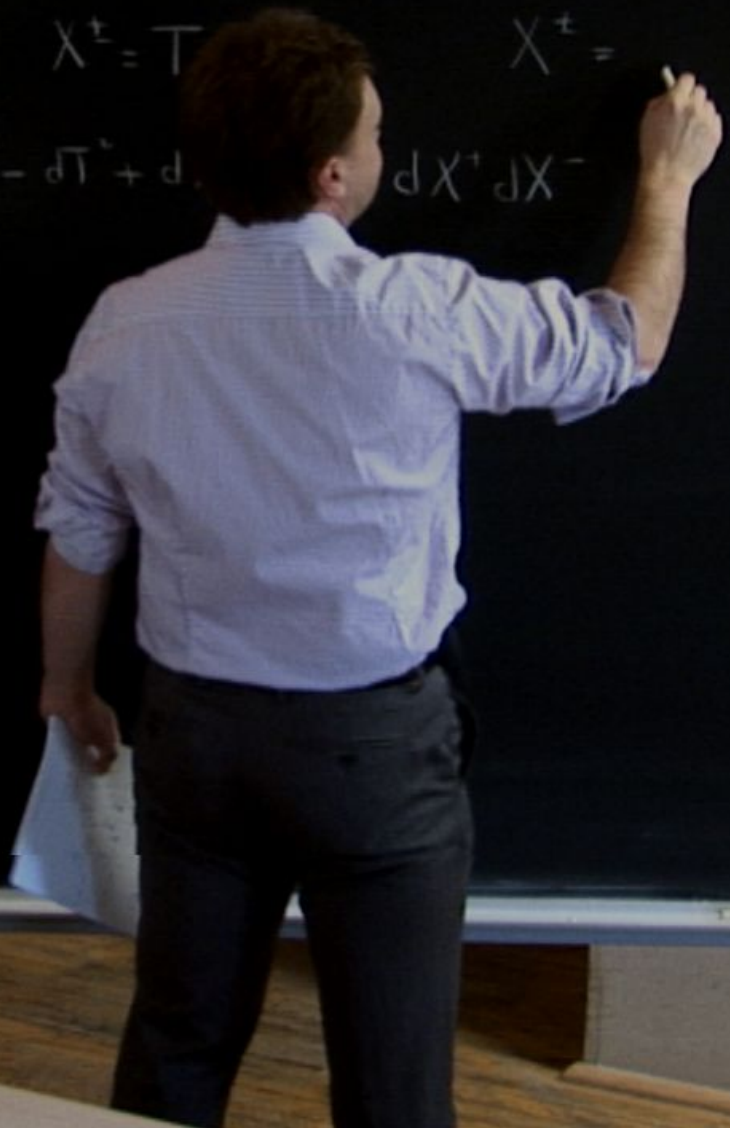
Massless scalar in 2d

$$X^{\pm} = T$$

$$ds^2 = -dT^2 + dx^2$$

$$X^{\pm} =$$

$$dX^+ dX^-$$



Unruh effect

Massless scalar in 2d

$$X^\pm = T_\pm X$$

$$X^\pm = \pm 1$$

$$ds^2 = -dT^2 + dX^2 = dX^-$$

EVIDENCE
for Atoms

DISPERSED
MOLECULES!

Unruh effect

Massless scalar in 2d

$$X^\pm = T_\pm X$$

$$ds^2 = -dT^2 + dX^2 = -$$

$$X^\pm = \pm \frac{1}{a} e^{\pm}$$



Unruh effect

Massless scalar in 2d

$$X^\pm = T_\pm X$$

$$ds^2 = -dT^2 + dX^2 = -$$

$$X^\pm = \pm \frac{1}{a} e^{\pm \dots}$$

Unruh effect

$$X^{\pm} = T_{\pm} X$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^2}$$

Massless scalar in $2d$ $ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$

Unruh effect

Massless scalar in 2d

X^+ X^-

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^2}$$

$$ds^2 = -c^2 dt^2 = -dX^+ dX^-$$



EVIDENCE
for Atoms

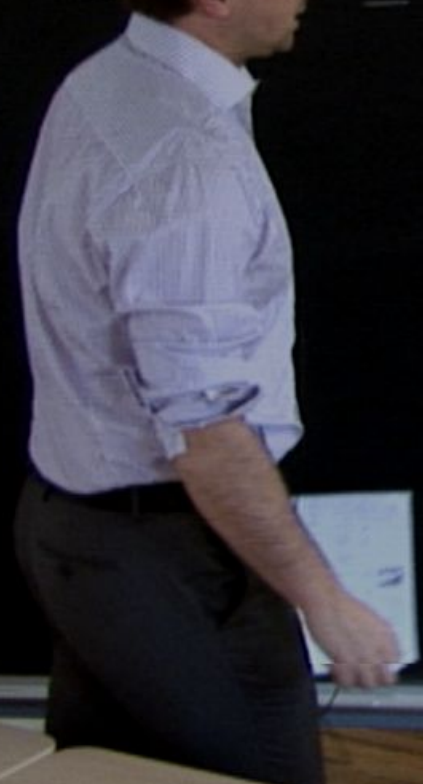
DISCRETE
Molecules!

U
2d

$$X^{\pm} = T_{\pm} X$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

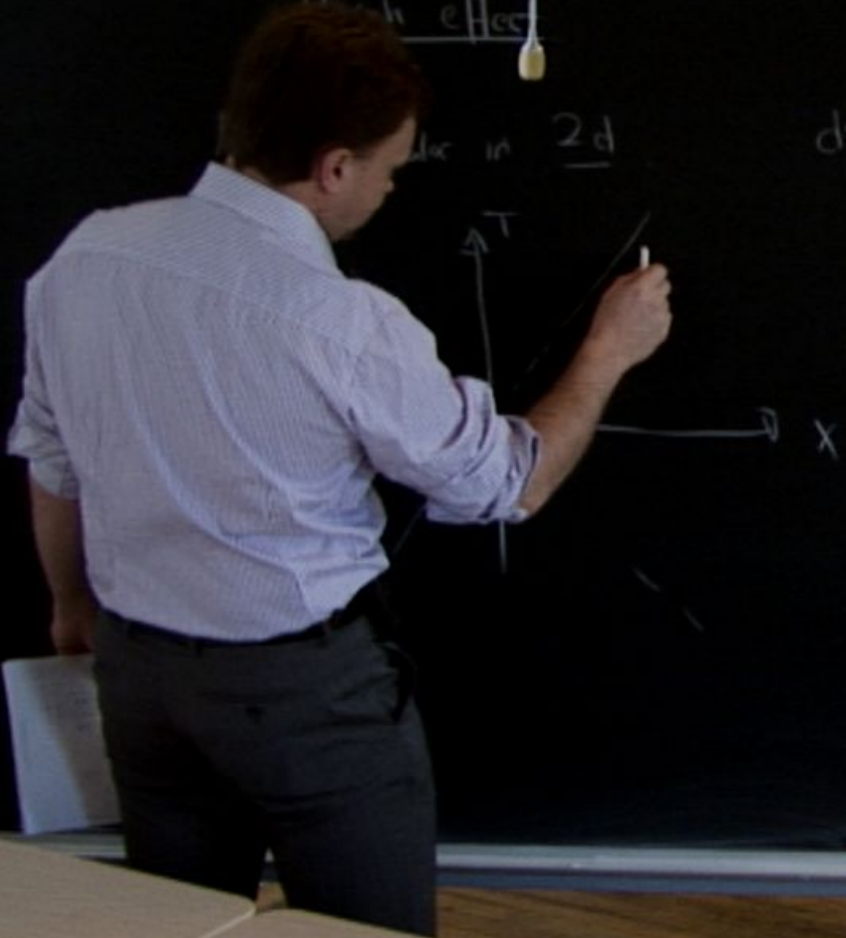


EVIDENCE
for Atoms

Did it
Molecule?

length effect

dx in 2d



$$X^{\pm} = T \pm X$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$x^{\pm} = t \pm x$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

$$= -e^{2ax} (-dt^2 + dx^2)$$

$$= -e^{a(x^+ - x^-)} dx^+ dx^-$$

Evidence for Atoms
 Discrete Molecules!

How effect
 Motion in 2d

$$X^\pm = T \pm X$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^2}$$

$x^\pm = t \pm x$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

$$= -e^{2ax} (-dt^2 + dx^2)$$

$$= -e^{a(x^+x^-)} dx^+ dx^-$$



constant x
 Trajectories of constant acceleration



Unruh effect

$$X^{\pm} = T \pm X$$

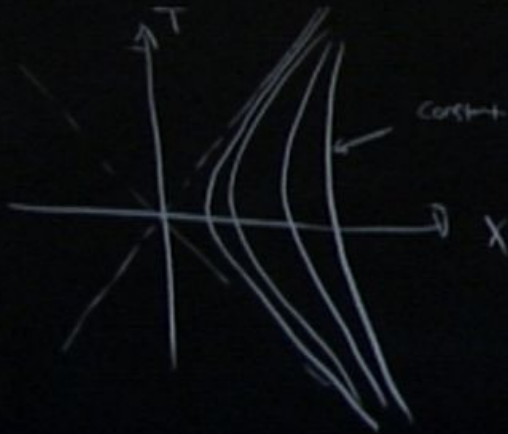
$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$x^{\pm} = t \pm x$

Massless scalar in 2d

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

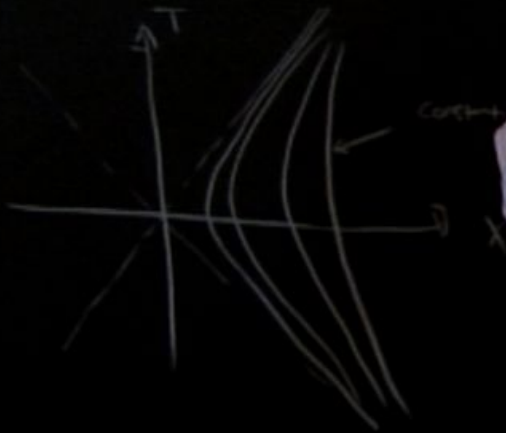
$$= -e^{2ax} (-dt^2 + dx^2)$$
$$= -e^{a(x^+ - x^-)} dx^+ dx^-$$



constant x
Trajectories of
constant acceleration

Unruh effect

Massless scalar in 2d



$$ds^2 = -$$

$$X^2 = -dX^+ dX^-$$

$$= -e^{2ax} (-dt^2 + dx^2)$$

$$= -e^{ax^+} dx^+ dx^-$$

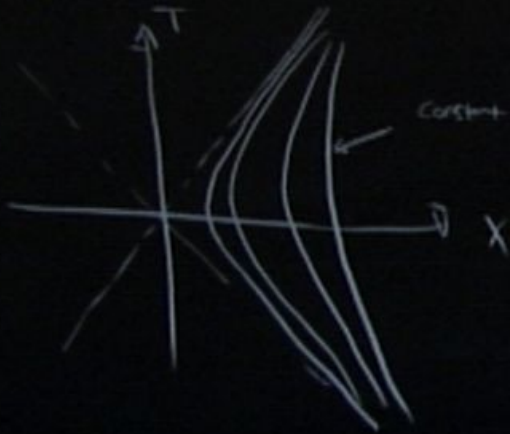
Real Velocity Multiple velocity

EVIDENCE
for Atoms

DISCRETE
Molecules!

Unruh effect

Massless scalar in 2d



trajectories of constant acceleration

$$X^{\pm} = T \pm X$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

$$= -e^{2ax} (-dt^2 + dx^2)$$

$$= -e^{a(x^+ - x^-)} dx^+ dx^-$$

$$T \rightarrow T + c$$

$$t \rightarrow t + c$$

Rindler vacuum

Minkowski vacuum

Unruh effect

$$X^{\pm} = T \pm X$$

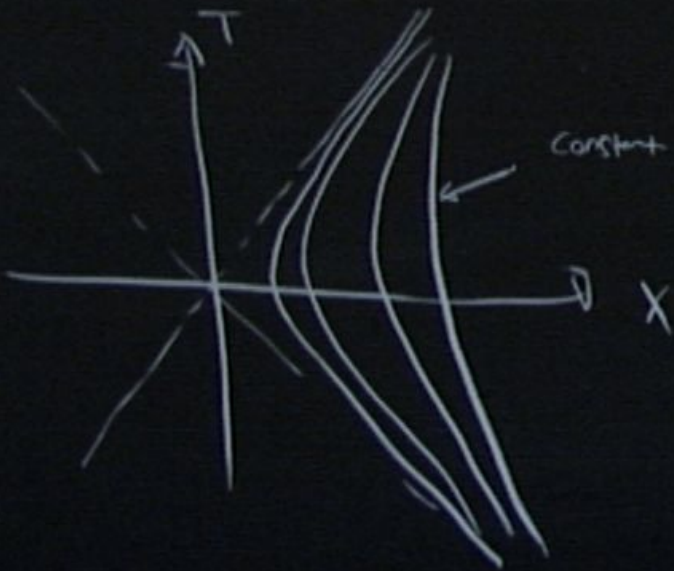
$$X^{\pm} = \pm \frac{1}{a} e^{\pm a x}$$

Massless scalar in 2d

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

$$= + e^{2ax} (-dt^2 + dx^2)$$

$$= - e^{a(x^+ - x^-)} dx^+ dx^-$$



constant x trajectories of constant acceleration

$$T \rightarrow T + c$$

$$t \rightarrow t + c$$

Real vacuum Minkowski vacuum

Rindler vacuum

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$$

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g_{\mu\nu}'$$

$$S = \int \sqrt{g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g_{\mu\nu}'$$

$$S = \int \sqrt{-g} d^2x \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$= \int dX dT \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial T} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial X} \right)^2 \right]$$

Massless scalar in 2d - conformally invariant

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \quad \sqrt{g} = \Omega^2 \sqrt{g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$S = \int \sqrt{g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$= \int dX dT \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial T} \right)^2 - \left(\frac{\partial \phi}{\partial X} \right)^2 \right]$$

Massless scalar in 2d - conformally invariant

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$$

$$\sqrt{g} = \Omega^2 \sqrt{g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$S = \int \sqrt{g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{g} g'^{\mu\nu} = \sqrt{g'} g^{\mu\nu}$$

$$= \int dX dT \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial T} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial X} \right)^2 \right]$$

multiply interaction

$$\Omega^2 g_{\mu\nu} \quad \sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$\left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$\left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right] = \int dt dx \left[+ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$-\sqrt{-g} \quad g^{\mu\nu} = \frac{1}{g^{\mu\nu}}$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$-\sqrt{-g} \quad g^{\mu\nu} = \frac{1}{a^2} g^{m'}$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0$$

$$\phi(x, T) = \phi_R(T-x) + \phi_L(T+x)$$

$$-\sqrt{-g'} \quad g'^{\mu\nu} = \frac{1}{a^2} g^{\mu\nu}$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 \quad = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-x) + \phi_L(T+x)$$

$$-\sqrt{-g} \quad g^{\mu\nu} = \frac{1}{a^2} g^{\mu\nu'}$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-x) + \phi_L(T+x)$$

Unruh effect

$$X^\pm = T \pm X$$

$$X^\pm = \pm \frac{1}{a}$$

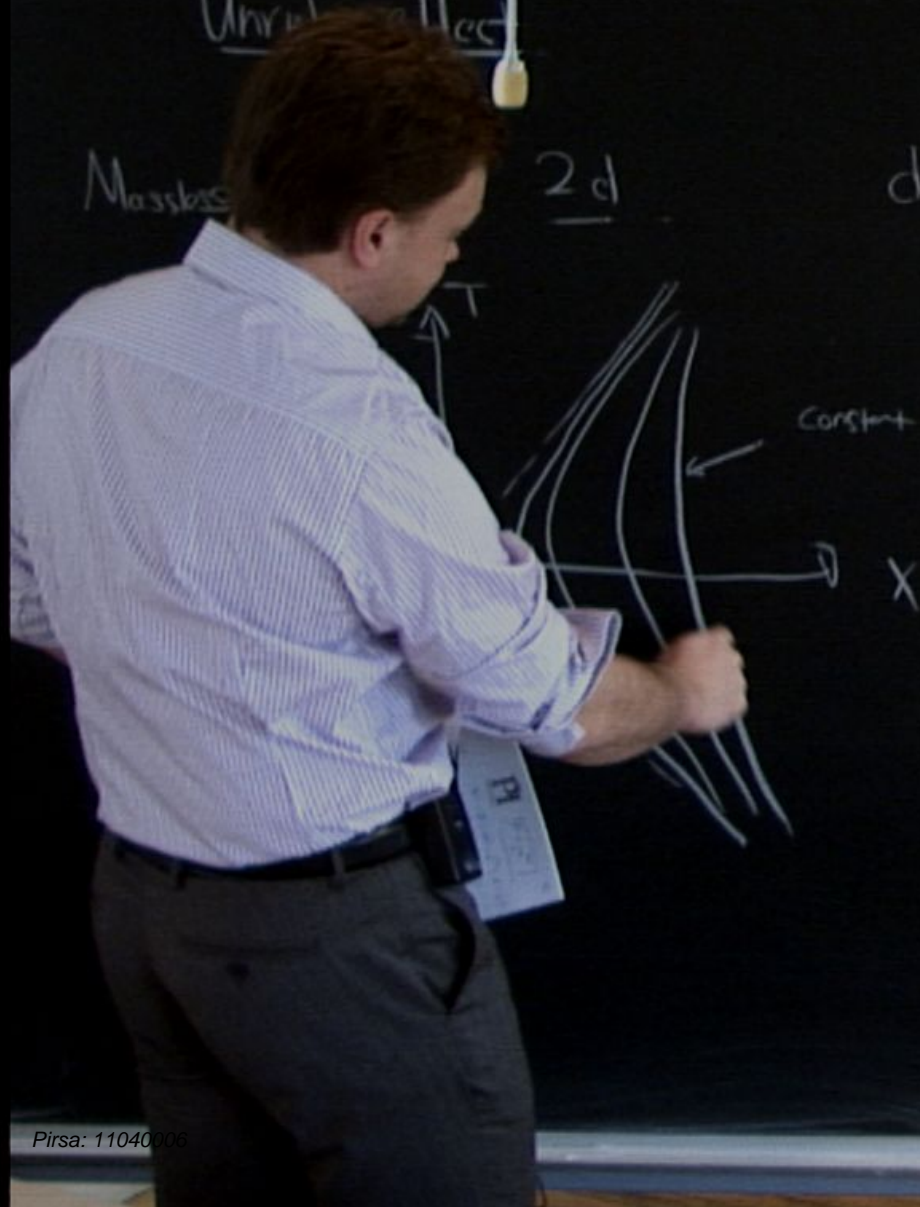
Massless

$\frac{2d}{c}$

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^- =$$

$$= + e^{2ax} (-dt^2 + dx^2)$$

$$= - e^{a(x^+ - x^-)} dx^+ dx^-$$



constant x

trajectories of constant acceleration

$$T \rightarrow T + c$$

$$t \rightarrow t + c$$

Real vacuum. Minkowski vacuum

Rindler vacuum

Unruh effect

$$X^{\pm} = T \pm X$$

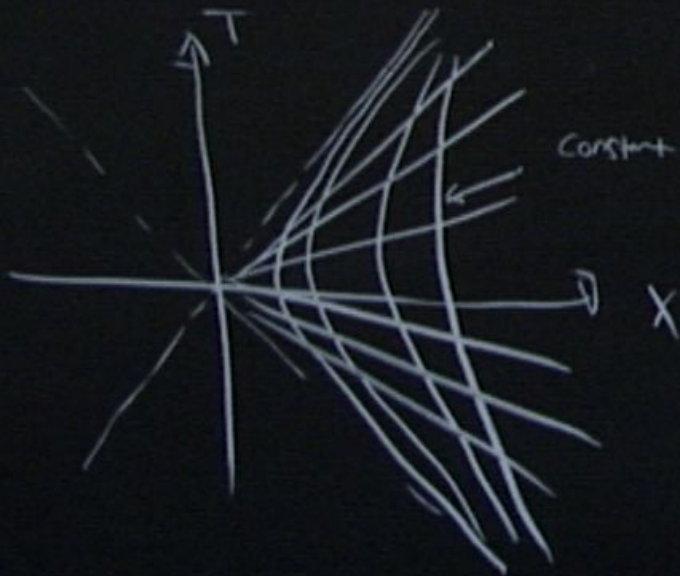
$$X^{\pm} = \pm \frac{1}{a}$$

Massless scalar in 2d

$$ds^2 = -dT^2 + dX^2 = -dX^+ dX^-$$

$$= + e^{2ax} (-dt^2 + dx^2)$$

$$= - e^{a(x^+ - x^-)} dx^+ dx^-$$



constant x
 trajectories of
 constant acceleration

$$T \rightarrow T + c$$

$$t \rightarrow t + c$$

real vacuum. Minkowski vacuum

Rindler vacuum

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 \quad = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\begin{aligned} \phi(X, T) &= \phi_R(T-X) + \phi_L(T+X) \\ &= \underbrace{\phi_R(X^-)} + \phi_L(X^+) \end{aligned}$$

$$\left[\frac{1}{2} \left(\frac{\partial \phi}{\partial X} \right)^2 \right]$$

$$\sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$\left[\frac{\partial \phi}{\partial x^\mu} \right] \sqrt{-g} g'^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu} \left[\frac{\partial \phi}{\partial x^\nu} \right]$$

$$\left[\left(\frac{\partial \phi}{\partial x^\mu} \right)^2 \right] = \int dt dx \left[+ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \dots \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(X, T) = \phi_R(T-X) + \phi_L(T+X)$$

$$= \underbrace{\phi_R(X^-)} + \phi_L(X^+)$$

$$\frac{\partial' \phi}{\partial X^+} - \frac{\partial' \phi}{\partial X^-} = 0$$

$$\sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$\left[\frac{\partial \phi}{\partial x^\mu} \right] \sqrt{-g} g'^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu} \left[\frac{\partial \phi}{\partial x^\nu} \right]$$

$$\left[\left(\frac{\partial \phi}{\partial x^\mu} \right)^2 \right] = \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-X) + \phi_L(T+X)$$

$$= \phi_R(X^-) + \phi_L(X^+)$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, t) = \phi_R(x-t) + \phi_L(x+t)$$

$$\sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$\partial_\nu \phi$

$$\sqrt{-g} g'^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$\left(\frac{\partial\phi}{\partial x^\nu}\right)^2$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2} \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$\square\phi = 0$$

$$\frac{\partial^2\phi}{\partial T^2} - \frac{\partial^2\phi}{\partial X^2} = 0 = \frac{\partial^2\phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-x) + \phi_L(T+x)$$

$$= \phi_R(X^-) + \phi_L(X^+)$$

$$\frac{\partial^2\phi}{\partial X^2} - \frac{\partial^2\phi}{\partial T^2} = 0 = \frac{\partial^2\phi}{\partial X^+ \partial X^-}$$

$$\phi(x, t) = \phi_R(x-t) + \phi_L(x+t)$$

$$g^{\mu\nu} = \frac{1}{a^2} g^{mn}$$

$$\phi_R(x^+) = \phi_R\left(\frac{1}{a} e^{ax^+}\right)$$

$$\bar{g} g^{\mu\nu} = \sqrt{-g} g^{\mu\nu}$$

$$= \tilde{\phi}_R$$

$$\int dt dx \left[+ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 \right]$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0$$

$$\frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-X) + \phi_L(T+X)$$

$$= \phi_R(X^-) + \phi_L(X^+)$$

$$\frac{\partial^2 \phi}{\partial X^2} - \frac{\partial^2 \phi}{\partial T^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, t) = \phi_R(x^-) + \phi_L(x^+)$$

$$g^{\mu\nu} = \frac{1}{c^2} g^{mn} \phi_R(x^+)$$

$$= \frac{1}{c^2} g^{mn} \left(\frac{1}{a} e^{ax^+} \right)$$

$$= \tilde{\phi}_R(x^+)$$

$$\left[+ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\square \phi = 0$$

$$\frac{\partial^2 \phi}{\partial T^2} - \frac{\partial^2 \phi}{\partial X^2} = 0 = \frac{\partial^2 \phi}{\partial X^+ \partial X^-}$$

$$\phi(x, T) = \phi_R(T-x) + \phi_L(T+x)$$

$$= \underbrace{\phi_R(x^-)} + \phi_L(x^+)$$

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = 0 = \frac{\partial^2 \phi}{\partial x^+ \partial x^-}$$

$$\phi(x, t) = \tilde{\phi}_R(x^-) + \tilde{\phi}_L(x^+)$$

Unruh effect

Minkowski / Global coordinate

$$\phi_R(X^-) = \int dp$$

$$X^\pm = \pm \frac{1}{a} e^{\pm a \tau}$$

Unruh effect

$$X^- = T - X$$

$$T = X$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm \dots}$$

Unruh / Gibbani coordinate

$$\phi_R(X^-) = \int dp a_p e^{-ipX^-}$$

evidence
for atoms

is it a
molecule?

Unruh effect

$$X^- = T - X$$

$$X^+ = \pm \frac{1}{a}$$

Generalize construction

$$\phi_R(X^-) = \int_0^\infty \frac{dp}{(2\pi)} a_p e^{-ipX^-}$$

Unruh effect

$$X^- = T - X$$

pos. in eqg



$$X^+ = \pm \frac{1}{a}$$

Minkowski / Global coordinate

ϕ

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^+ e^{+ipX^-} \right]$$

$$[a_p, a_p] = 0$$

EVIDENCE
for Atoms

DIS IS A
MOLECULE?

Heisenberg effect

$$X^- = T - X$$

position



$$X^+ = \pm \frac{1}{a}$$

General coordinate

$$\phi_R(X^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^+ e^{+ipX^-} \right]$$
$$[a_p, a_{p'}^+] = (2\pi i) \delta(p-p')$$

Unruh effect

$$X^- = T - X$$

positive energy



$$X^+ = \pm \frac{1}{a}$$

Minkowski / Gibbons coordinates

$$\phi_R(X^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^+ e^{+ipX^-} \right]$$

$$[a_p, a_{p'}] = 0$$

$$[a_p, a_{p'}^+] = (\pi i \delta(p-p')) \alpha$$

Rindler coordinates

evidence
for atoms

Dirac's
Molecule?

eHc

$$X^- = T - X \quad \text{pos. in energy}$$

$$X^+ = \pm \frac{1}{a}$$

M.

concentrate

$\gamma = 0$

times

$$\phi_R(X^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^+ e^{+ipX^-} \right]$$

$$[a_p, a_{p'}^+] = (2\pi) \delta(p-p')$$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^+ e^{+ikx^-} \right]$$

evidence for atoms

big is a molecule?

Heisenberg effect

$$X^- = T - X$$

positive energy

$$X^+ = \pm \frac{1}{a}$$

liberal construction

$$\phi_R(X^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^+ e^{+ipX^-} \right]$$

$T=0$

$$[a_p, a_{p'}^+] = (\pi i) \delta(p-p')$$

$$x = t - x$$

$t=0$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^+ e^{+ikx^-} \right]$$

positive energy mode

negative energy mode

Unruh effect

$$X^- = T - X$$

positive energy



$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$x^\pm = t \pm x$

Minkowski / Global coordinates

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipx^-} + a_p^\dagger e^{+ipx^-} \right]$$

$$[a_p, a_{p'}] = 0$$

$$[a_p, a_{p'}^\dagger] = (2\pi) \delta(p-p')$$

$$x = t - x$$

Rindler coordinates

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^\dagger e^{+ikx^-} \right]$$

positive energy mode

negative energy mode

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu}$$

$$\sqrt{-g} = \Omega^4 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$

$$S = \int d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$\int d^4x dT \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial T} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial X} \right)^2 \right]$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$\frac{\partial \phi}{\partial T}$

$$[\phi(x, T), \pi(x', T)] = i \delta(x - x')$$

$$p = \frac{\partial \phi}{\partial t}$$

$[\phi$

scalar in 2d - conformally invariant

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \quad \sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu} \quad \phi_R(x') = \phi_R\left(\frac{1}{a} e^{ax^+}\right)$$

$$S = \int \sqrt{-g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{-g} g'^{\mu\nu} = \sqrt{-g'} g^{\mu\nu}$$

$$= \tilde{\phi}_R(x^+)$$

$$= \int dx d\tau \left[\frac{1}{2} \left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2} \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$= \int dx d\tau \left[\frac{1}{2} \left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2} \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$\pi = \frac{\partial\phi}{\partial\tau}$$

$$[\phi(x, \tau), \pi(x', \tau)]$$

$$= i\delta(x-x')$$

$$\pi$$

$$= i\delta(x-x')$$

scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \quad \sqrt{-g} = \Omega^2 \sqrt{-g'}$$

$$S = \int \sqrt{-g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g'} g'^{\mu\nu}$$

$$g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu} \quad \phi_R(x')$$

$$= \phi_R\left(\frac{1}{a} e^{ax^+}\right)$$

$$= \tilde{\phi}_R(x^+)$$

$$= \int dX dT \left[\frac{1}{2} \left(\frac{\partial\phi}{\partial T}\right)^2 - \frac{1}{2} \left(\frac{\partial\phi}{\partial X}\right)^2 \right] = \int dt dx \left[+\frac{1}{2} \left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2} \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

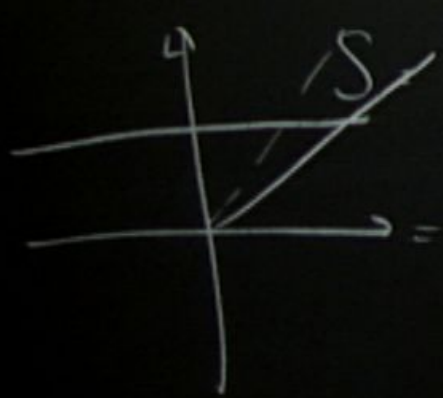
$$\pi = \frac{\partial\phi}{\partial T} \quad [\phi(x, T), \pi(x', T)] = i\delta(x-x')$$

$$P = \frac{\partial\phi}{\partial t} \neq \pi$$

$$[\phi(x, t), P(x', t)] = i\delta(x-x')$$

Massless scalar in 2d - conformally invariant.

$$g_{\mu\nu} = \Omega^2 g'_{\mu\nu} \quad \sqrt{g} = \Omega^2 \sqrt{g'} \quad g'^{\mu\nu} = \frac{1}{\Omega^2} g^{\mu\nu}$$



$$\int \sqrt{g} d^2x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\sqrt{g} g'^{\mu\nu} = \sqrt{g'} g^{\mu\nu}$$

$$= \int dx dt \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$= \int dt dx \left[+\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

$$P = \frac{\partial \mathcal{L}}{\partial \dot{x}} \neq \dots$$

$$[\phi(x,t), P(x',t)]$$

Unruh effect

$$X^- = T - X$$

positive energy

$$X^\pm = \pm \frac{1}{a}$$

Minkowski / Global coordinates

$$\phi_R(X^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^\dagger e^{+ipX^-} \right]$$

$$[a_p, a_{p'}] = 0$$

$$[a_p, a_{p'}^\dagger] = (2\pi) \delta(p-p')$$

$$x^- = t - x$$

Rindler coordinates

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^\dagger e^{+ikx^-} \right]$$

$$[b_k, b_{k'}^\dagger] = (2\pi) \delta(k-k')$$

positive energy mode

negative energy mode

X^{\pm} positive energy

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipX^-} + a_p^{\dagger} e^{+ipX^-} \right]$$

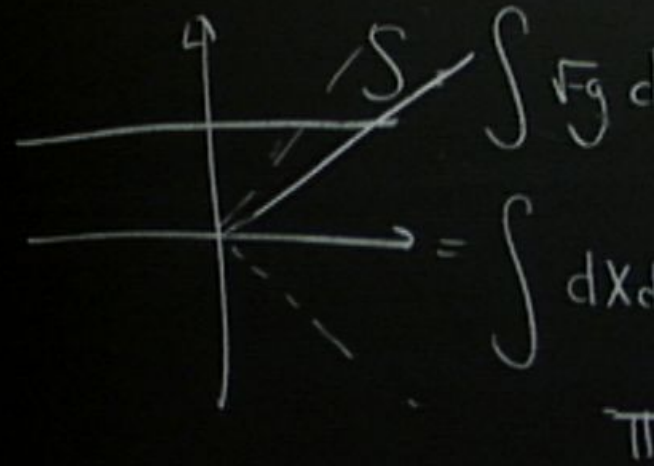
$x = t - x$

$$\int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^{\dagger} e^{+ikx^-} \right]$$

positive energy mode

negative energy mode

$$\Pi) \delta(x - t')$$



Two different minima \Rightarrow for $a_{ip} |Mink\rangle = 0$ for all

$b_k |Kindler\rangle = 0$ for all

$|Mink\rangle \neq$

Two different modes

$$a_{\alpha p} |Mink\rangle = 0$$

f. all p .

$$b_k |Rindler\rangle = 0$$

for all k .

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U |Rindler\rangle$$

$$\rho = |Mink\rangle \langle Mink|$$

$$= \text{Tr} \rho$$

dof. ignore

Two different modes

$$a_{kp} |Mink\rangle = 0 \quad \text{for all } p.$$

$$b_k |Rindler\rangle = 0 \quad \text{for all } k.$$

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U |Rindler\rangle$$

$$\rho = |Mink\rangle \langle Mink|$$

$$\rho_{Rindler} = \text{Tr}_{\text{dof. lightlike}} \rho = Z^{-1} e^{-\beta H_{Rindler}}$$

Two different vacua

$$a_{kp} |Mink\rangle = 0 \quad \text{for all } p.$$

$$b_k |Rindler\rangle = 0 \quad \text{for all } k.$$

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U |Rindler\rangle$$

$$\rho = |Mink\rangle \langle Mink|$$

$$\rho_{Rindler} = \text{Tr}_{\text{dof. left side}} \rho = Z^{-1} e^{-\beta H_{Rindler}}$$

Two different vacua?

$$a_{k,p} |Mink\rangle = 0 \quad \text{for all } p.$$

$$b_k |Rindler\rangle = 0 \quad \text{for all } k.$$

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U |Rindler\rangle$$

$$\rho = |Mink\rangle \langle Mink|$$

$$\rho_{Rindler} = \text{Tr}_{\text{dof. lightlike}} \rho = Z^{-1} e^{-\beta H_{Rindler}}$$

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$$x^\pm = t \pm x$$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p e^{-ipx^-} + a_p^+ e^{+ipx^-} \right]$$

$x^- = t - x$

$$[a_p, a_{p'}^+] = (2\pi) \delta(p - p')$$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[b_k e^{-ikx^-} + b_k^+ e^{+ikx^-} \right]$$

$$[b_k, b_{k'}^+] = (2\pi) \delta(k - k')$$

positive energy mode

negative energy mode

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

0. f. all p.
= 0 for all k

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^{\infty} \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Boğulmuş Co-efficients

Two different ways

$$a_k |Mink\rangle = 0 \quad \text{for all } k$$

$$b_k |Rindler\rangle = 0 \quad \text{for all } k$$

$$|Mink\rangle \neq |Rindler\rangle$$

$$|Mink\rangle \neq U |Rindler\rangle$$

$$\rho = |Mink\rangle \langle Mink|$$

$$\rho_{Rindler} = \text{Tr}_{\text{dof.}} \rho = Z^{-1} e^{-\beta H_{Rindler}}$$

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$$x^\pm = t \pm x$$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[a_p \frac{e^{-ipx^-}}{\sqrt{2\pi}} + a_p^\dagger \frac{e^{+ipx^-}}{\sqrt{2\pi}} \right]$$

$$[a_p, a_{p'}^\dagger] = (2\pi) \delta(p-p')$$

$$x^- = t - x$$

$$\phi_R(x^-) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} \left[\frac{b_k}{\sqrt{2\pi}} e^{-ikx^-} + \frac{b_k^\dagger}{\sqrt{2\pi}} e^{+ikx^-} \right]$$

$$[b_k, b_{k'}^\dagger] = (2\pi) \delta(k-k')$$

positive energy mode

negative energy mode

$$b_k = \int_0^{\infty} \frac{dp}{2\pi} \left[a_{kp} a_p \right]$$

$$\sqrt{2p}$$

$$(2\pi)^{-1}$$



$$\sqrt{2k}$$

B_0

$$e^{-ipX^-} = \int_0^\infty \frac{dk}{2\pi} \left[\alpha_{kp} e^{-ikx^-} + \beta_{kp} e^{+ikx^-} \right] \frac{1}{\sqrt{2k}}$$

Bogelubov Co-efficients

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{2\pi} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogelubov coefficients

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

$$(b_k = \alpha_k a_k + \beta_k^* a_k^\dagger)$$

$$e^{-ipX^-} = \int_0^\infty \frac{dk}{2\pi} \left[\alpha_{kp} e^{-ikx^-} + \beta_{kp} e^{+ikx^-} \right] \frac{1}{\sqrt{2k}}$$

Bogoliubov coefficients

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

$$b_{k'}^\dagger = (2\pi) \delta(k-k') \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

$$e^{-ipX^-} = \int_0^\infty \frac{dk}{2\pi} \left[\alpha_{kp} e^{-ikx^-} + \beta_{kp} e^{+ikx^-} \right] \frac{1}{\sqrt{2k}}$$

Bogelubov coefficients

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

analogue $|\alpha|^2 - |\beta|^2 = 1$

$$b_{k'}^\dagger b_k = (2\pi) \delta(k-k') \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^\infty \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogelubov co-efficients

$$= \int_0^\infty \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

analogue $|\alpha|^2 - |\beta|^2 = 1$

$$[a_k, a_{k'}^\dagger] = (2\pi) \delta(k-k') \quad \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

f. all p.

for all k

$$\frac{e^{-ipX}}{\sqrt{2p}} = \int_0^{\infty} \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx}}{\sqrt{2k}} \right]$$

Bogelubov co-efficients

$$b_k = \int_0^{\infty} \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

analyse $|\alpha|^2 - |\beta|^2$

$$[b_k, b_{k'}^\dagger] = (2\pi) \delta(k-k') \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

f. all p.

for all k

$$\frac{e^{-ipX}}{\sqrt{2p}} = \int_0^{\infty} \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx}}{\sqrt{2k}} \right]$$

Bogetubor co-efficients

$$b_k = \int_0^{\infty} \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^{\dagger} \right]$$

$$[b_k, b_{k'}^{\dagger}] = (2\pi) \delta(k-k') \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* \right]$$

analyse

$$\frac{e^{-ipX^-}}{\sqrt{2p}} = \int_0^{\infty} \frac{dk}{(2\pi)} \left[\alpha_{kp} \frac{e^{-ikx^-}}{\sqrt{2k}} + \beta_{kp} \frac{e^{+ikx^-}}{\sqrt{2k}} \right]$$

Bogubov co-efficients

$$= \int_0^{\infty} \frac{dp}{2\pi} \left[\alpha_{kp} a_p + \beta_{kp}^* a_p^\dagger \right]$$

analyse $|\alpha|^2 - |\beta|^2 = 1$

$$b_{kt}^\dagger = (2\pi) \delta(k-k') \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

$$\int_{-a}^a dx^- e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

x^\pm

$t = x$

$$\int_{-a}^a dx^- e^{+ikx^-} e^{-ipX^-} = \frac{e}{\sqrt{2p}}$$

x^\pm

$t \pm x$

$$\frac{\alpha_{kp}}{\sqrt{2k}}$$

$$= \int_{-\infty}^{\infty} dx^- e^{+ikx^-} e^{-ipx^-} / \sqrt{2p}$$

k positive.

$$\int_{-\infty}^{\infty} dx^- e^{ik_1 x^-} e^{-ik_2 x^-} = 2\pi \delta(k_1 - k_2)$$

x^\pm

$t = x$

$$\frac{\delta_{kp}}{\sqrt{2k}}$$

$$= \int_{-a}^a dx e^{+ikx} e^{-ipx}$$

\downarrow k positive.

f. d. p.
for all k

$$\frac{e^{-ipx}}{\sqrt{2p}}$$

$$\int_{-a}^a dx e^{ik_1 x} e^{-ik_2 x} = (2\pi) \delta(k_1 - k_2)$$

$$[b_k, b_{k'}] = (2\pi) \delta(k - k')$$

$$= \int_0^\infty \frac{dp}{2\pi} \left[\dots \right]$$

$$\frac{\delta_{kp}}{\sqrt{2k}}$$

$$= \int_{-a}^a dx e^{+ikx} e^{-ipx}$$

k positive.

f. d. p.
for all k

$$\frac{e^{-ipx}}{\sqrt{2p}}$$

$$\frac{e^{-ipx}}{\sqrt{2p}}$$

$$= \int_{-\infty}^{\infty} \frac{dp}{2\pi} \left[\dots \right]$$

$$b_{\frac{1}{2}} = (2\pi) \delta(c)$$

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-a}^a dx^- e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

k positive.

f. d. p.
for all k.

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-a}^a dx^- e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}}$$

$$= \int_{-\infty}^{\infty} \frac{dp}{2\pi} \left[\frac{e^{-ipX}}{\sqrt{2p}} \right] b_{k+} = (2\pi) \delta(k)$$

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_a^{\infty} \frac{dx^-}{\sqrt{2k}} \left[e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

k positive.

f. d. p.
for all k

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$X^- = -\frac{1}{a} e^{-ax^-}$

$$= \int_0^{\infty} \frac{dp}{2\pi} \left[e^{-ipX} \frac{e^{-ipX}}{\sqrt{2p}} \right]$$

$$b_{\frac{1}{2}+} = (2\pi) \delta(c)$$

$$\frac{\alpha_{kp}}{\sqrt{2k}} = \int_{-a}^a dx^- \left[e^{+ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

k positive.

f. d. p.
for all k

$$\frac{\beta_{kp}}{\sqrt{2k}} = \int_{-\infty}^{\infty} dx^- \left[e^{-ikx^-} \frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$X^- = -\frac{1}{a} e^{-ax^-}$

$$= \int_{-\infty}^{\infty} \frac{dp}{2\pi} \left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$b_{\frac{1}{2}+} = (2\pi) \delta(\dots)$

Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$x^1 = t \pm x$

$$X^- \rightarrow -\infty$$

$p > 0$

$$e^{-ipX^-} \rightarrow e^{pX^-}$$

Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm} \quad x^\pm = t \pm x$$

$$X^- \rightarrow -a$$

$$e^{-ipX^-}$$

$p > 0$

\rightarrow

$$e^{pX^-}$$

$$X^- \rightarrow iX^-$$

$$\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$

$$e^{-ax^-} \rightarrow e^{\frac{i\pi}{2}} e^{-ax^-} = i e^{-ax^-}$$

Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}} \quad x^{\pm} = t \pm x$$

$$X^- \rightarrow -\infty$$

$$e^{-ipX^-} \quad p > 0$$

$$e^{pX^-}$$

$$X^- \rightarrow iX^-$$

$$X^- = -\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$

$$e^{-ax^-} \rightarrow e^{-ax^-} e^{\frac{i\pi}{2}} = i e^{-ax^-}$$

Unruh effect

$$X^- = T - X \quad \text{path eqs.}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$$X^- \rightarrow -\infty$$

$$e^{-ipX^-} \quad p > 0$$

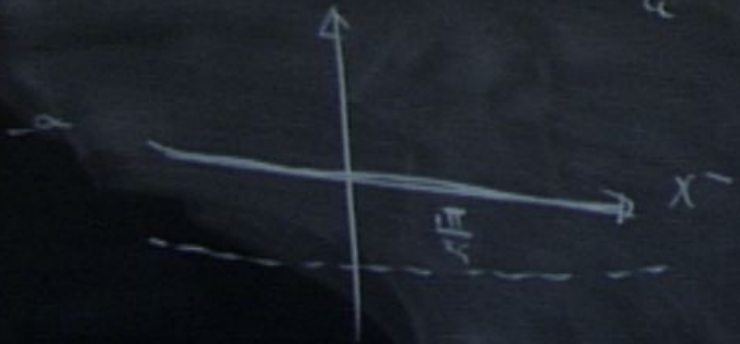
$$\rightarrow e^{pX^-}$$

$$X^- \rightarrow iX^-$$

$$X^- = -\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$

$$e^{-ax^-} \rightarrow e^{\frac{i\pi}{2}} e^{-ax^-} = i e^{-ax^-}$$



Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}} \quad x^{\pm} = t \pm x$$

$$X^- \rightarrow -\infty$$

$$e^{-ipX^-} \quad p > 0$$

$$\rightarrow e^{pX^-}$$

$$X^- \rightarrow iX^-$$

$$X^- = -\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$



$$e^{-ax^-} \rightarrow e^{\frac{i\pi}{2}} e^{-ax^-} = i e^{-ax^-}$$

$$\rightarrow e^{\frac{i\pi}{2a}} e^{-ax^-}$$

Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}} \quad x^{\pm} = t \pm x$$

$$X^- \rightarrow -\infty$$

$$e^{-ipX^-} \quad p > 0$$

$$\rightarrow e^{pX^-}$$

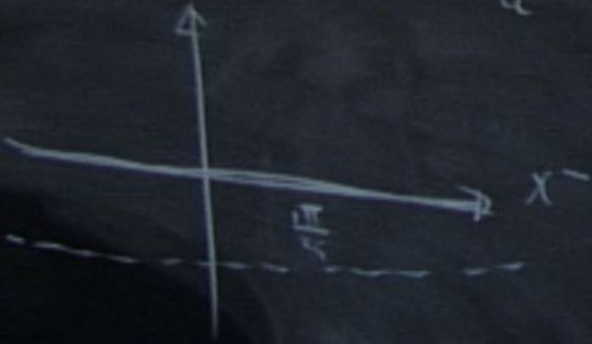
$$X^- \rightarrow iX^-$$

$$X^- = -\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$

$$e^{-ax^-} \rightarrow e^{-ax^-} e^{\frac{i\pi}{2}} = i e^{-ax^-}$$

$$e^{ikx^-} \rightarrow e^{\frac{k\pi}{2a}} e^{ikx^-}$$



Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

$x^\pm = t \pm x$

$$X^- \rightarrow -\infty$$

$$e^{-ipX^-} \quad p > 0$$

$$\rightarrow e^{pX^-}$$

$$X^- \rightarrow iX^-$$

$$X^- = -\frac{1}{a} e^{-ax^-}$$

$$x^- \rightarrow x^- - \frac{1}{a} \left(\frac{i\pi}{2} \right)$$

$$e^{-ax^-} \rightarrow e^{-ax^-} e^{\frac{i\pi}{2}} = i e^{-ax^-}$$

$$e^{ikx^-} \rightarrow e^{ikx^-} e^{\frac{k\pi}{2a}}$$



Side.

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-a}^{\infty} dc^-$$

$$e^{\frac{k\pi}{2a}} e^{ikx^-}$$

$$\frac{e^{pX^-}}{\sqrt{2p}}$$

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-a}^{\infty} dx^-$$

$$e^{-x^-} e^{pX^-}$$

$$\int \frac{dp}{2\pi} \left[\dots \right] \alpha_k$$

Side.

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-a}^{\infty} dx^-$$

$$e^{\frac{k\pi}{2a}} e^{ikx^-} \frac{e^{-pX^-}}{\sqrt{2p}}$$

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-a}^{\infty} dx^-$$

$$e^{-\frac{k\pi}{2a}} e^{ikx^-} \frac{e^{-pX^-}}{\sqrt{2p}}$$

$$b_{k'} = (2\pi) \delta(k-k')$$

$$\int \frac{dp}{(2\pi)} \alpha_k$$

Side.

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-a}^{\infty} dx^-$$

$$e^{\frac{k\pi}{2a}} e^{ikx^-}$$

$$\frac{e}{\sqrt{2p}} p^-$$

$$\left[\frac{e^{-ipX^-}}{\sqrt{2p}} \right]$$

$$= \int_{-\infty}^{\infty} dx^-$$

$$e^{-\frac{k\pi}{2a}} e^{-ikx^-}$$

$$\frac{e}{\sqrt{2p}} p^-$$

$$b_{k'}^\dagger = (2\pi) \delta(k-k')$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)}$$

$$e^{\frac{kT}{2a}} e^{ikx^-} pX^- \frac{e}{\sqrt{E_p}}$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} pX^- \frac{e}{\sqrt{E_p}}$$

$$\alpha_{kp} = e^{\frac{kT}{a}} \beta_{kp}^*$$

$$b_{kT} = (2\pi) \delta(k-k') \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right]$$

$$e^{\frac{kT}{2a}} e^{ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

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$$e^{\frac{kT}{2a}} e^{ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$\alpha_{kp} = e^{\frac{kT}{a}} \beta_{kp}^*$$

$$(2\pi) \delta(0) = \int dx e^{i0} = L$$

$$\int_0^{\infty} \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$b_{kT} = (2\pi) \delta(k-k') \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

$$e^{\frac{kT}{2a}} e^{ikx^-} \frac{e}{\sqrt{E_p}} pX^-$$

$$\boxed{\alpha_{kp} = e^{\frac{kT}{a}} \beta_{kp}^*}$$

$$(2\pi)\delta(\omega) = \int_{-\infty}^{\infty} dx e^{i\omega x} = L$$

$$\left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} \frac{e}{\sqrt{E_p}} pX^-$$

$$\frac{dp}{2\pi} e^{2ikx^-}$$

$$b_{kT}^\dagger = (2\pi)\delta(k-k')$$

$$\left[\frac{1}{E_p} \beta_{kp}^* \right] = (2\pi)\delta(k-k')$$

$$e^{\frac{kT}{2a}} e^{ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$\alpha_{kp} = e^{\frac{kT}{a}} \beta_{kp}^*$$

$$(2\pi) \delta(\omega) = \int dx e^{i\omega x} = L$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$\int_0^{\infty} \frac{dp}{(2\pi)} \left[\left(e^{\frac{2kT}{a}} - 1 \right) |\beta_{kp}|^2 \right] = L$$

$$b_{kT} = (2\pi) \delta(k-k') \int_0^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

$$e^{\frac{kT}{2a}} e^{ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$e^{-\frac{kT}{2a}} e^{-ikx^-} \frac{e^{pX^-}}{\sqrt{E_p}}$$

$$\alpha_{kp} = e^{\frac{kT}{a}} \beta_{kp}^*$$

$$(2\pi) \delta(\omega) = \int dx e^{i\omega x} = L$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[|\alpha_{kp}|^2 - |\beta_{kp}|^2 \right] = L$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[\left(e^{\frac{2kT}{a}} - 1 \right) |\beta_{kp}|^2 \right] = L$$

$$b_{kT} = (2\pi) \delta(k-k') \int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \left[\alpha_{kp} \alpha_{k'p}^* - \beta_{k'p} \beta_{kp}^* \right] = (2\pi) \delta(k-k')$$

n effect

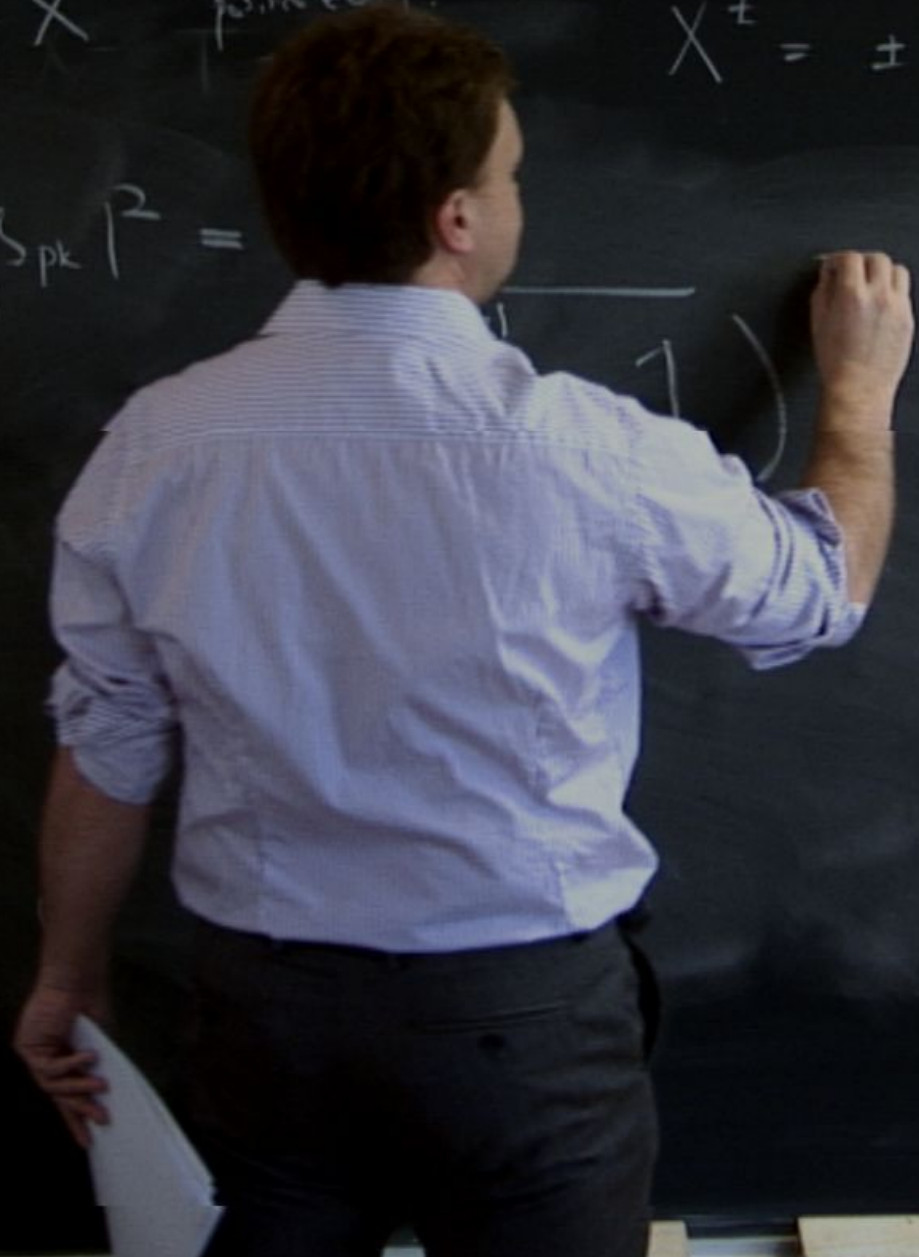
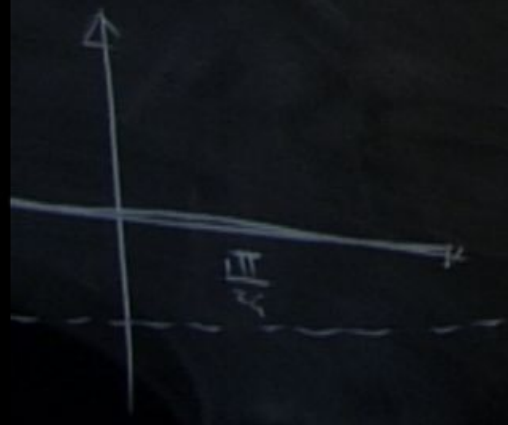
$$X^- = T - X \quad \text{positive energy}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^{\pm}}$$

$x^{\pm} = t \pm$

$\langle \dots \rangle \rightarrow -\infty$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} \quad |S_{pk}|^2 =$$



Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$X^\pm = \pm \frac{1}{a} e^{\pm ax^\pm}$$

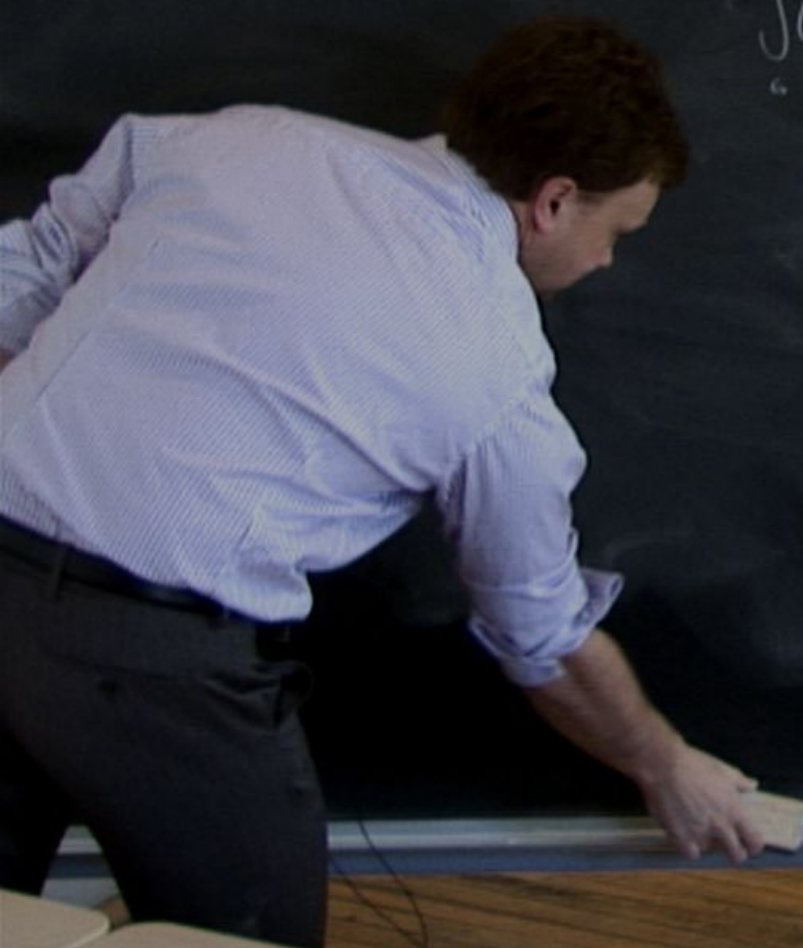
$$\int_{-\infty}^{\infty} \frac{dp}{2\pi} |S_{pk}|^2 = \frac{L}{\left(e^{\frac{2\pi|k|}{a}} - 1\right)} = \frac{L}{e^{\frac{E}{k_B T}} - 1}$$

Thermal distribution for Bosons

$$E = |k|$$

$$\boxed{k_B T = \frac{a}{2\pi}}$$

Unruh temperature



Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$\left[\frac{a \alpha^+}{p} \beta_{pk}^+ \alpha_p^+ \right]$$

$$\int_0^\infty \frac{dp}{(2\pi)} |S_{pk}|^2 = \frac{L}{e^{\frac{2\pi|k|}{a}} - 1}$$

N

Thermal distribution for

$$E = |k|$$

$$k_B T = \frac{a}{2\pi}$$

Unruh effect

$$X^- = T - X \quad \text{positive energy}$$

$$b_L = \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{p^+} \alpha_{p^+}^\dagger + \beta_{p^+}^\dagger \alpha_{p^+}^\dagger \right]$$

$$\int_0^\infty \frac{dp}{(2\pi)} \frac{1}{|\beta_{p^+}|^2} = \frac{L}{e^{\frac{2\pi|k|}{a}} - 1}$$

$b_R |D\rangle$

$$N_s^R = \int_0^\infty \frac{dk}{(2\pi)} b_R^\dagger b_R$$

Thermal distribution for

$$E = |k|$$

$$\boxed{k_B T = \frac{a}{2\pi}}$$

Unruh effect

$$X^- = T - X \quad \text{positive}$$

$$X^+ = \dots$$

$$b_L = \int_0^\infty \frac{dp}{(2\pi)} \left[\alpha_{pR} \alpha_{pL}^\dagger + \beta_{pL} \alpha_{pR}^\dagger \right]$$

$$\int_0^\infty \frac{dp}{(2\pi)} |S_{pk}|^2 = \frac{L}{\left(e^{\frac{2\pi|k|}{a}} - 1 \right)}$$

$$b_k |R_{in}(t_r)\rangle = 0 \quad N_b^R = \int_0^\infty \frac{dk}{(2\pi)} b_R^\dagger b_R$$

$$N_b^R |R_{in}(t_r)\rangle = 0$$

Thermal distribution for Bosons

$$E = |k|$$

$$\boxed{k_B T = \frac{a}{2\pi}}$$

+ \downarrow k positive.

$$\langle Mink | N_b^2 | Mink \rangle = \int_{-a}^a dx^- \dots$$

$$= \int_0^a dk \langle Mink | b_k^+ b_k | Mink \rangle = \int_0^a dx^- \dots$$

$$e^{\frac{k\pi}{2a}} e^{ikx^-}$$

$$e^{-\frac{k\pi}{2a}} e^{-ikx^-}$$

$$b_{k\pi}^\dagger = (2\pi) \delta(k-k')$$

$$\langle \text{Mink} | N_b^2 | \text{Mink} \rangle$$

$$= \int_0^\pi dk \langle \text{Mink} | b_k^\dagger b_k | \text{Mink} \rangle$$

$$= \int dk dp dp'$$

$$\beta_{pk}^* a_p^\dagger | \text{Mink} \rangle$$

$$\langle \text{Mink} | N_b^2 | \text{Mink} \rangle$$

$$= \int_0^\infty dk \langle \text{Mink} | b_k^\dagger b_k | \text{Mink} \rangle$$

$$= \int dk dp dp' \beta_{p'k} \langle \text{Mink} | \alpha_{p'k} \beta_{pk}^* \alpha_p^\dagger | \text{Mink} \rangle$$

From Grains of Pollen to Evidence for Atoms

How Big Is A Molecule?

Umrah effect

$$X^- = T - X \quad \text{pulsed}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm kx^2}$$

$$b_L = \int_c^{\infty} \frac{dp}{(2\pi)} \left[\alpha \alpha^{\dagger} + \beta_{R^{\dagger}}^{\dagger} \alpha^{\dagger} \right]$$

$$\int_c^{\infty} \frac{dp}{(2\pi)} |S_{pk}|^2 = \frac{L}{\left(e^{\frac{2\pi|k|}{a}} - 1 \right)} = \frac{L}{e^{\frac{E}{k_B T}} - 1}$$

Thermal distribution for bosons

$$b_K |R_{in}(t) \rangle = 0 \quad N_b^R = \int_c^{\infty} \frac{dk}{(2\pi)} b_K^{\dagger} b_K$$

$$N_b^L |R_{in}(t) \rangle = 0$$

$$E = |k|$$

$$k_B T = \frac{a}{2\pi}$$

Umrah temperature

k positive.

$$\langle Mink | N_b^2 | Mink \rangle$$

$$\langle in | N_b | in \rangle = \int_{-a}^{\infty} dx^-$$

$$\langle Mink | \alpha_p \alpha_p^\dagger | Mink \rangle = (2\pi) \delta(p' - p)$$

$$= \int_{-a}^{\infty} dk \langle Mink | b_k^\dagger b_k | Mink \rangle$$

$$\frac{dk dp dp'}{(2\pi)^3} \beta_{pk} \beta_{pk}^* \langle Mink | \alpha_p \alpha_p^\dagger | Mink \rangle$$

$$\int \frac{dk}{(2\pi)} \int \frac{dp}{(2\pi)} |\beta_{pk}|^2$$

$$e^{\frac{k\pi}{2a}}$$

$$e^{-\frac{k\pi}{2a}}$$

$$b_k^\dagger = (2\pi) \delta(k)$$

Umrah effect

$$X^- = T - X$$

path 030
L

$$X^{\pm} = \pm \frac{1}{a} e^{\pm az^{\pm}}$$

$$z^{\pm} = t \pm x$$

$$b_L = \int_c^{\infty} \frac{dp}{(2\pi)} \left[\alpha \frac{d}{dt} + \beta \frac{d}{dt} \right]$$

$$\int_c^{\infty} \frac{dp}{(2\pi)} |p_{\text{rel}}|^2 = \frac{L}{(e^{\frac{2\pi|k|}{a}} - 1)} = \frac{L}{e^{\frac{E}{k_B T}} - 1}$$

$$b_L |R_{\text{rad}}\rangle = 0 \quad N_L^{\pm} = \int_c^{\infty} \frac{dk}{(2\pi)} b_L^{\pm} b_L$$

$$N_0^{\pm} |R_{\text{rad}}\rangle = 0$$

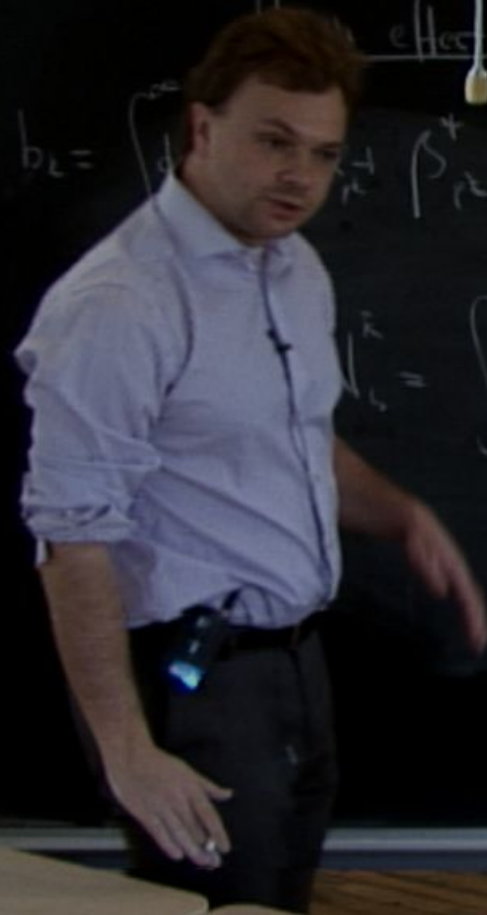
Thermal distribution for bosons

$$E = |k| \quad t = t_1 + it_2$$

$$k_B T = \frac{a}{2\pi}$$

Umrah temperature

$X^- = T - X$ pulsating $X^+ = \pm \frac{1}{a} e^{\pm ax^2}$
 $b_L = \int_{-\infty}^{\infty} dt \left[\beta_{Lk}^+ a_{Lk}^+ \right]$ $\int_{-\infty}^{\infty} \frac{dp}{(2\pi)} |f_{pk}|^2 = \frac{L}{(e^{\frac{2\pi|k|}{a}} - 1)}$ $= \frac{L}{e^{\frac{E_{pk}}{k_B T}} - 1}$
 $\tilde{b}_L = \int_{-\infty}^{\infty} \frac{dt_k}{(2\pi)} b_{Lk}^+ b_{Lk}$ Thermal distribution for bosons
 $E = |k|$ $k_B T = \frac{a}{2\pi}$ Kittel temperature
 $t = t_1 + it_2$ $\rho = e^{-i\omega t} = e^{-i\omega t_1} e^{+\omega t_2}$



Umrah effect

$$X^- = T - X \quad \text{pulsar eggs}$$

$$X^{\pm} = \pm \frac{1}{a} e^{\pm ax^2}$$

$x^2 = t = x$

$$b_L = \int_{-\infty}^{\infty} \frac{dp}{(2\pi\hbar)} \left[\frac{ax + t}{r} \right]$$

$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi\hbar)} |f(p)|^2 = \frac{L}{\left(e^{\frac{2\pi\hbar|a|}{a}} - 1 \right)} = \frac{L}{e^{\frac{E}{k_B T}} - 1}$$

$$b_k |R_{n,l}\rangle = 0$$

$$N_0^e |R_{n,l}\rangle = 0$$

$$b_k^+ b_k$$

Thermal distribution for Bosons

$$E = \hbar\omega$$

$$t = t_1 + it_2$$

$$= e^{-i\omega t_1} e^{+\omega t_2}$$

$$t_1 T = \frac{a}{2\pi}$$

Umrah temperature

