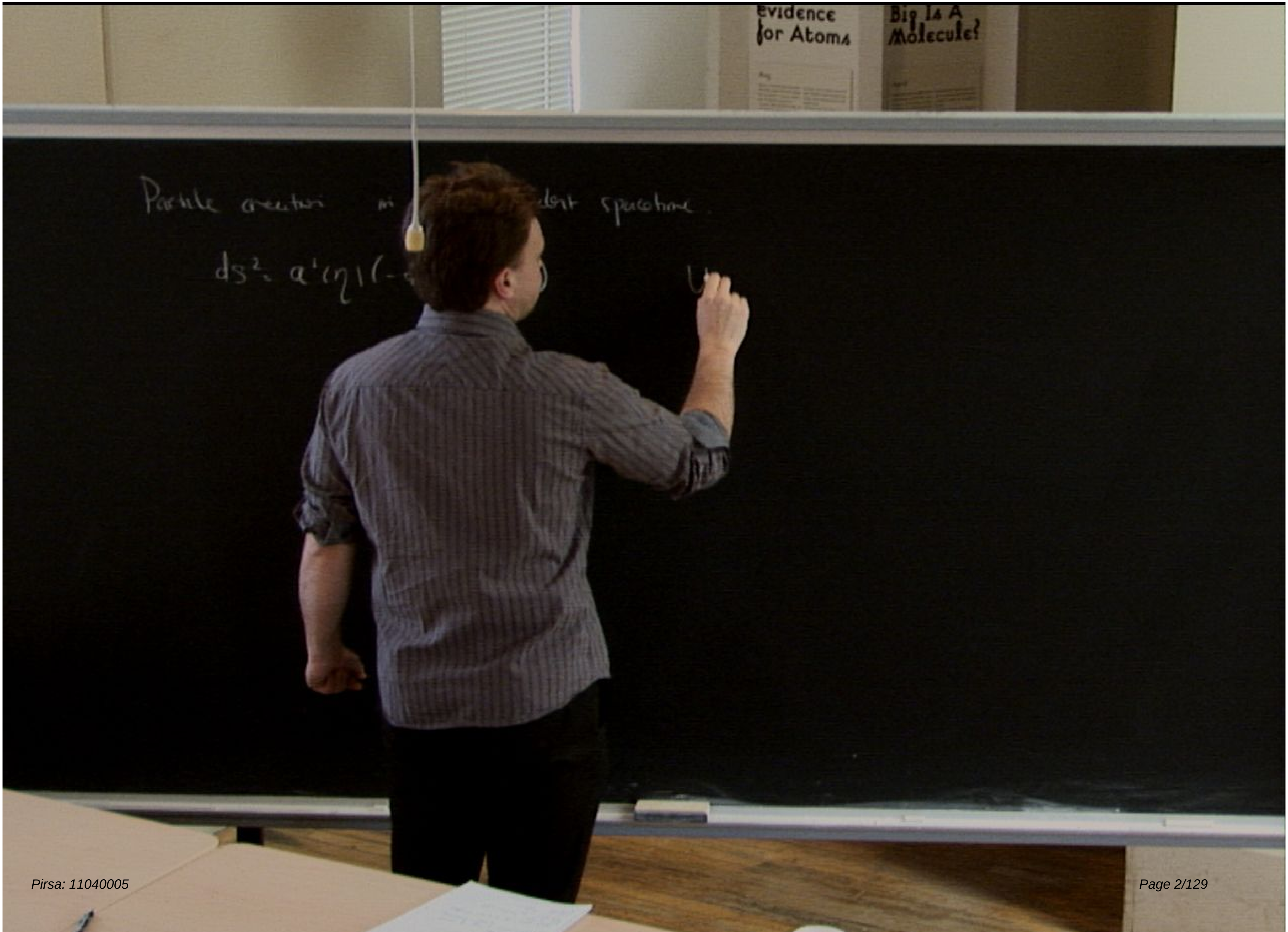
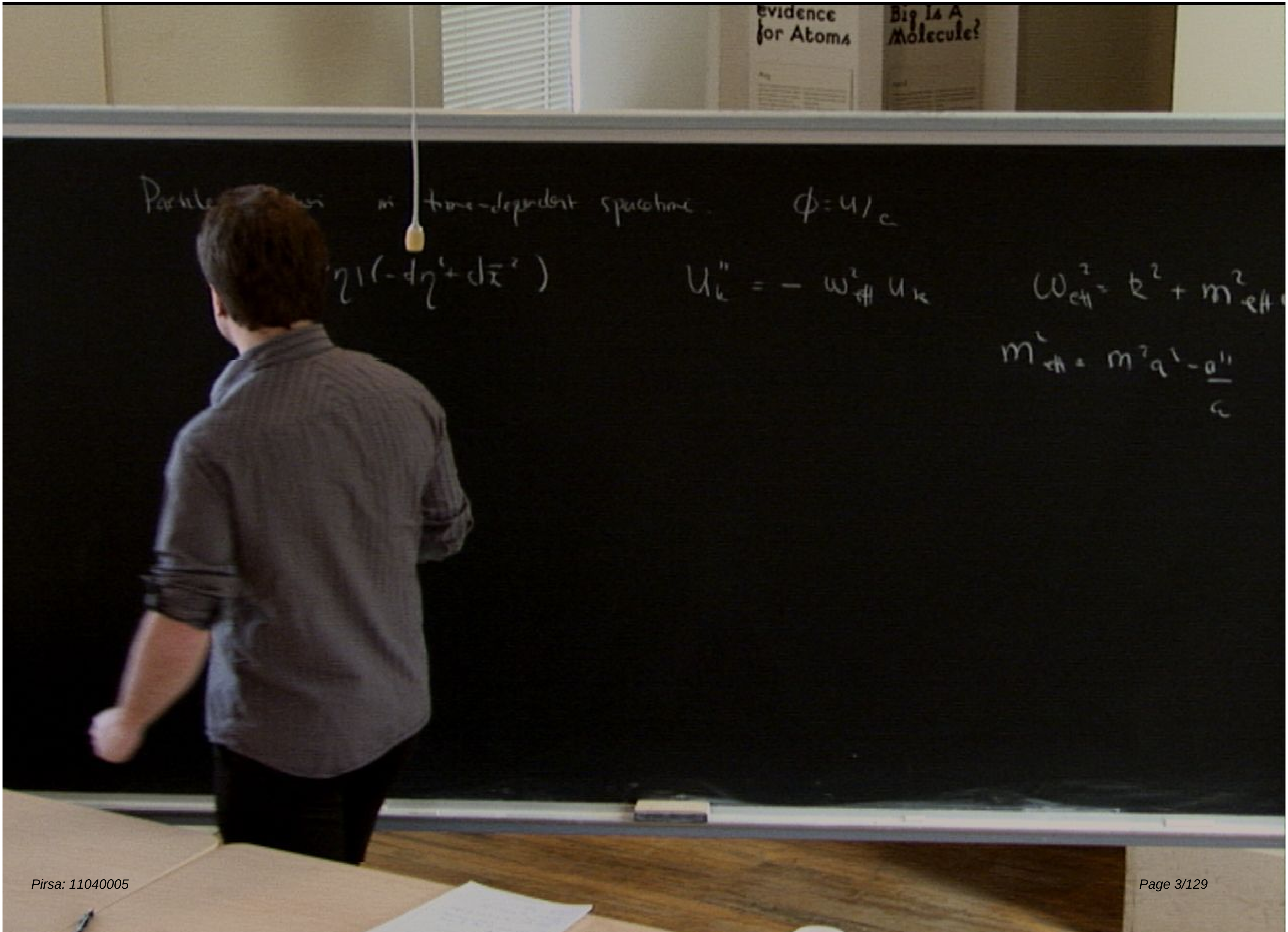
Title: Explorations in Cosmology - Lecture 3

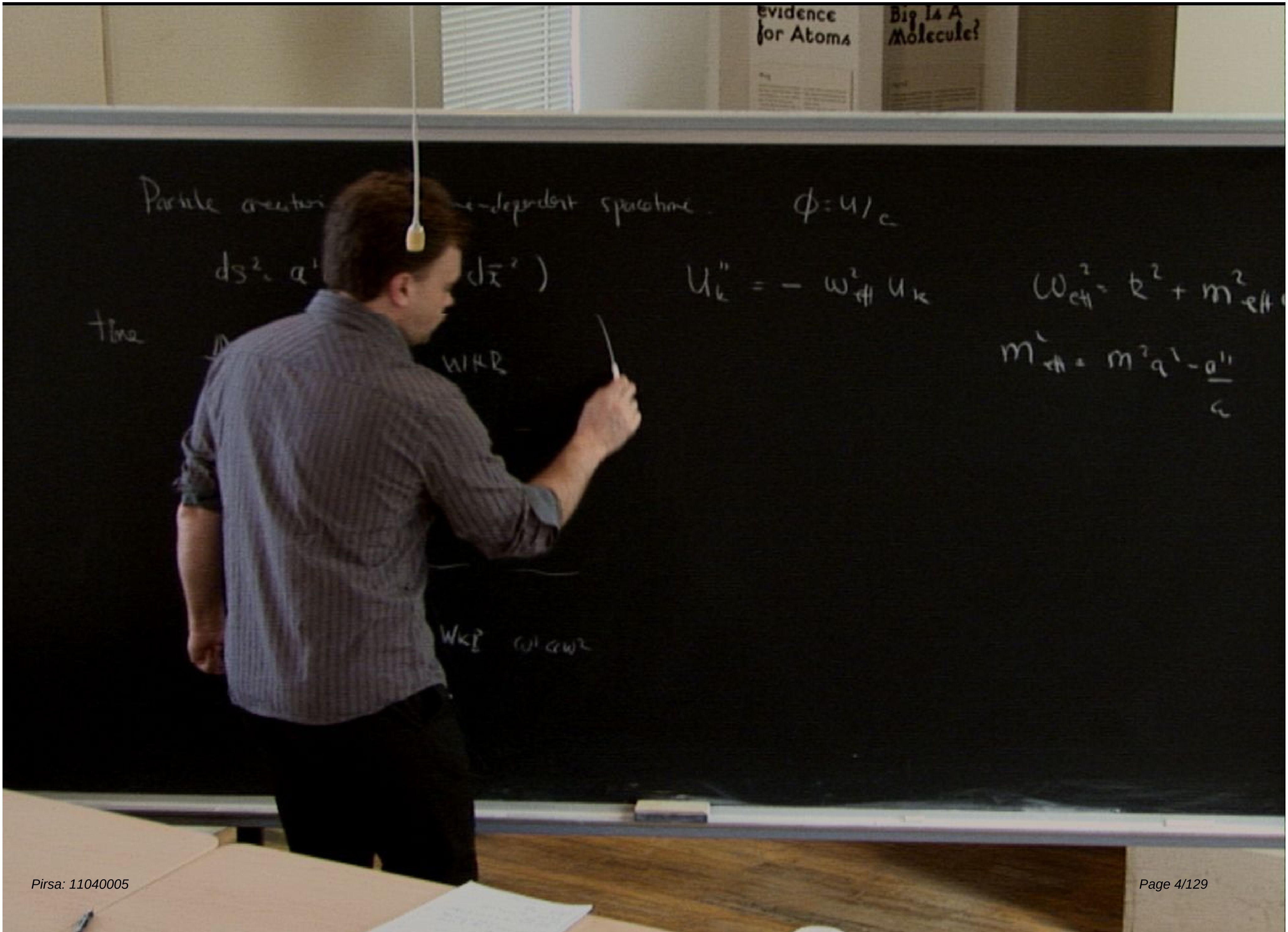
Date: Apr 06, 2011 09:00 AM

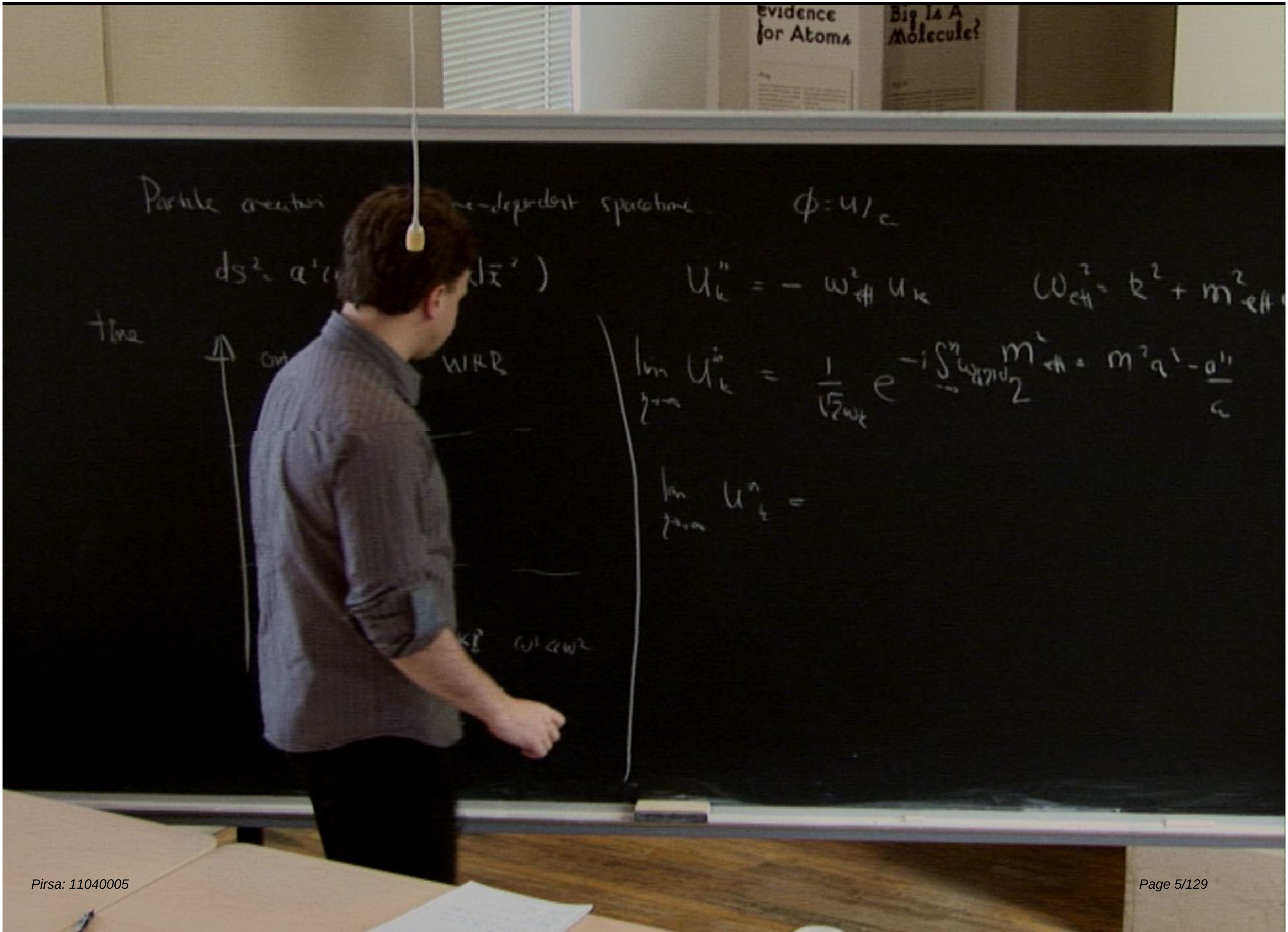
URL: http://pirsa.org/11040005

Abstract:


Evidence
for Atoms
Big Is A
Molecule?


for Atoms
Molecule?
time-dependent spacetime


Evidence for Atoms
Big Is A Molecule?
Particle creation in ...-dependent spacetime
φ = u/a
u_k'' = − ω_eff² u_k
(ω_eff² = k² + m_eff²
m_eff² = m²a² − a''/a
time
WKB


Evidence for Atoms
Big Is A Molecule?
Particle creation
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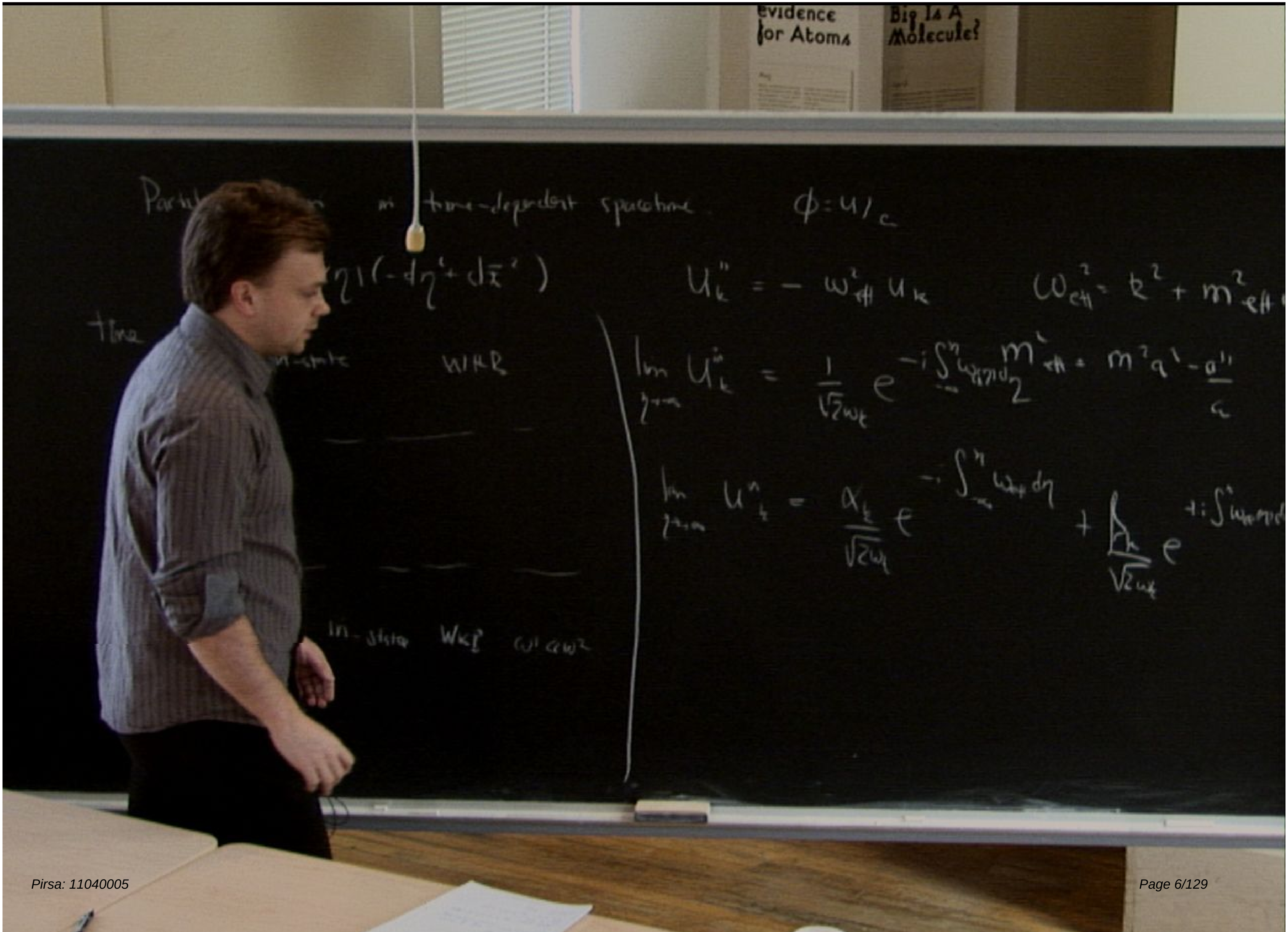
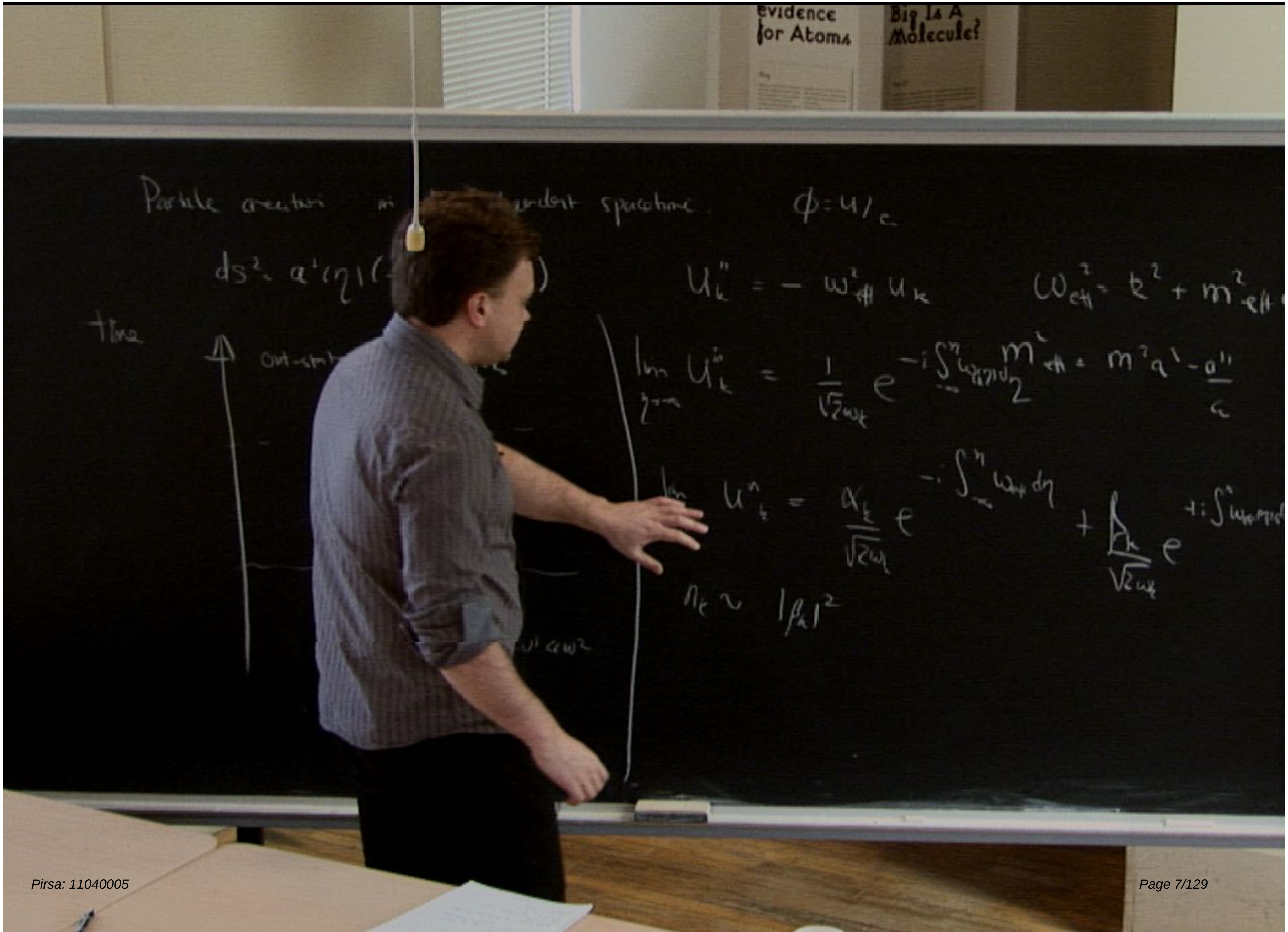
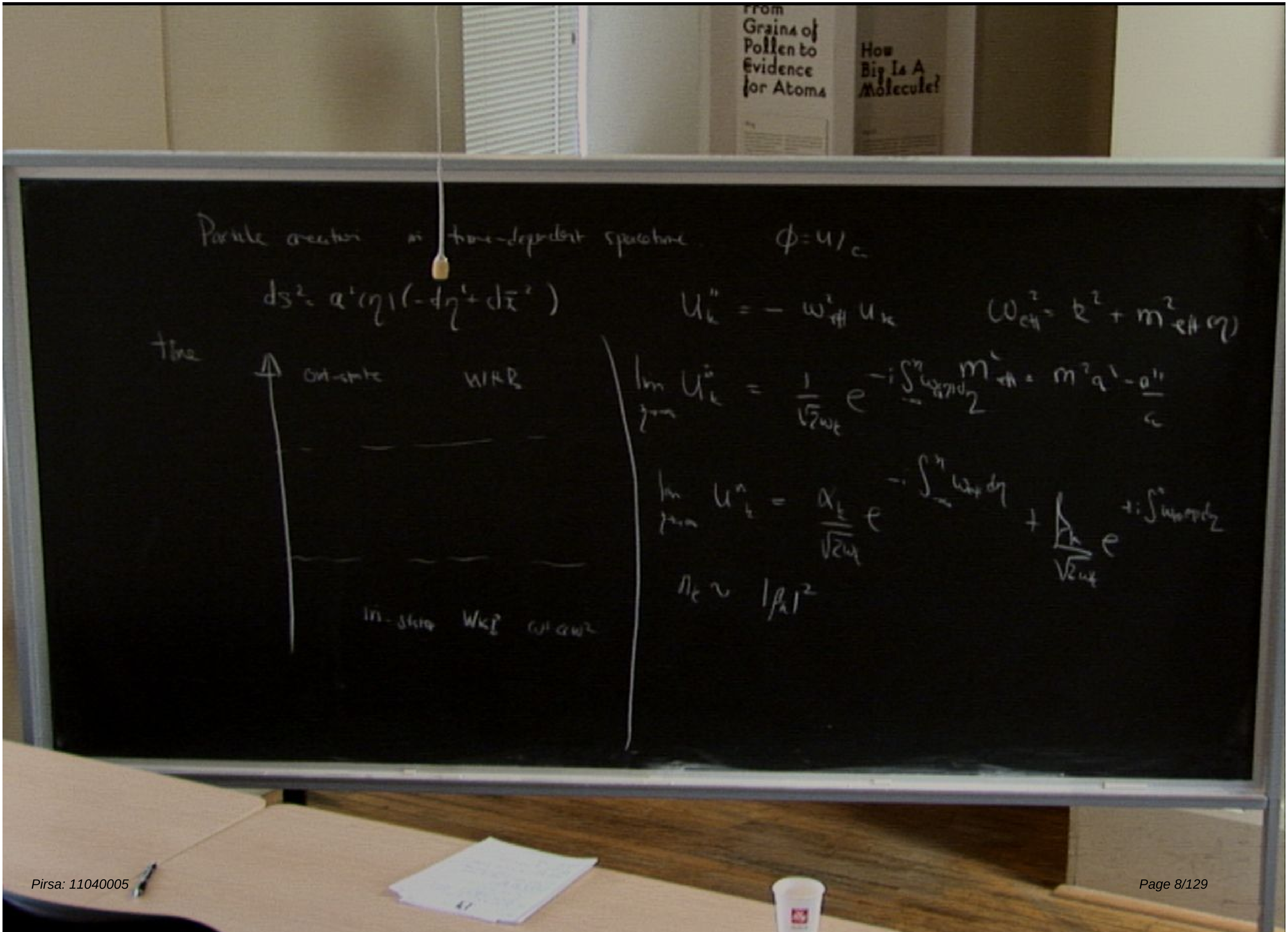
time
WKB


Evidence for Atoms
Big Is A Molecule?
time-dependent spacetime
WKB
in-state WKB



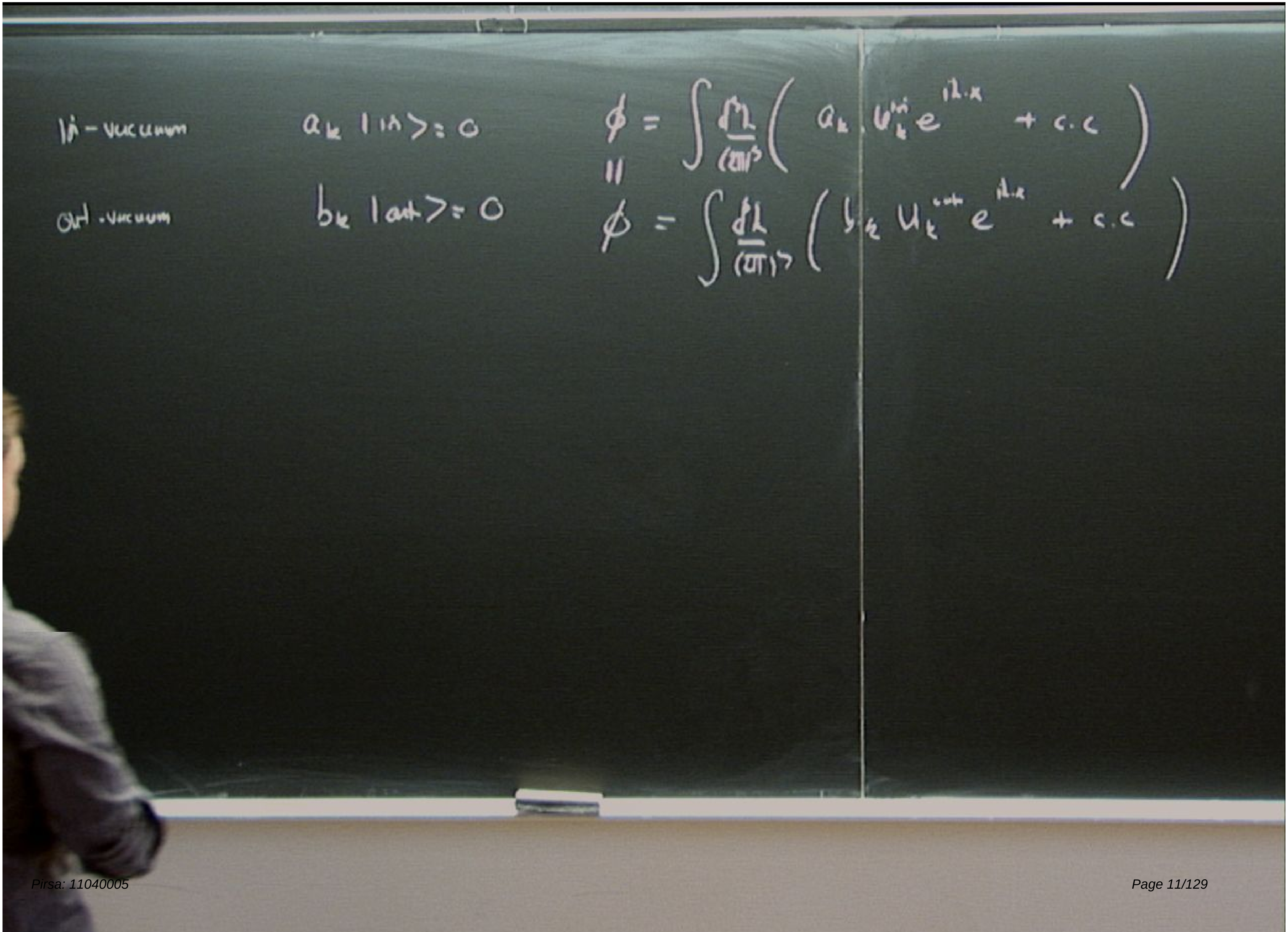
Evidence for Atoms
Big Is A Molecule?
Particle creation in
spacetime
time
out-state


From
Grains of
Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
time
out-state
WKB
in-state WKB

m $a_k |in\rangle = 0$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \left(a_k \, u_k^{in} e^{ik\cdot x} + c.c \right)$$


in - vacuum
a_k |in> = 0
out - vacuum
b_k |out> = 0
φ = ∫ d³k/(2π)³ (a_k v_k^in e^{ik·x} + c.c)
φ =


in - vacuum
$a_k |in\rangle = 0$
out - vacuum
$b_k |out\rangle = 0$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(a_k u_k^{in} e^{ik\cdot x} + c.c \right)$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(b_k u_k^{out} e^{ik\cdot x} + c.c \right)$

in - vacuum $\qquad a_k |in\rangle = 0$

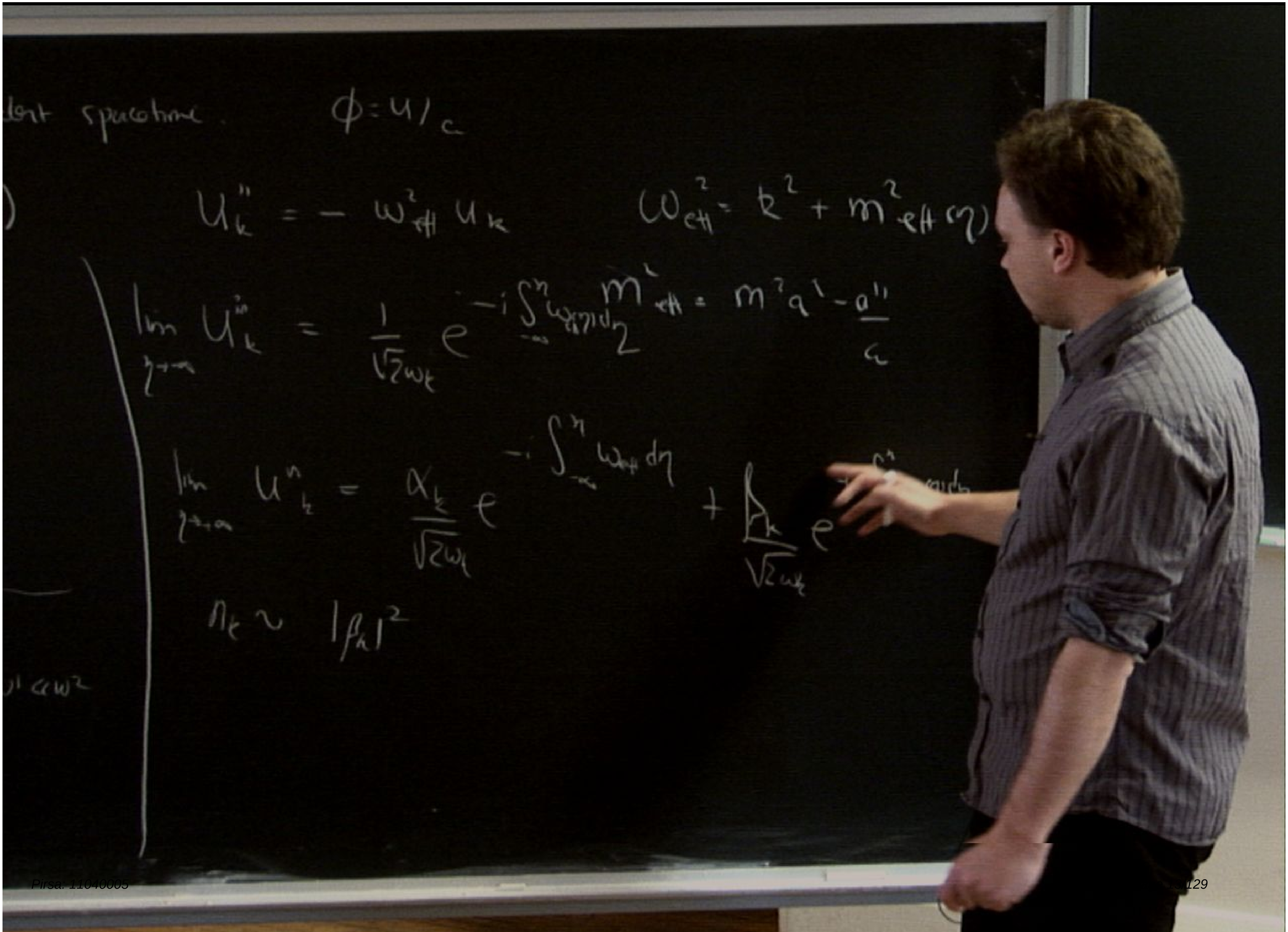
out - vacuum $\qquad b_k |out\rangle = 0$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \left(a_k \, u_k^{in} e^{ik\cdot x} + c.c \right)$$

$$\parallel$$

$$\phi = \int \frac{d^3k}{(2\pi)^3} \left(b_k \, u_k^{out} e^{ik\cdot x} + c.c \right)$$

$$|in\rangle \neq |out\rangle$$



...dent spacetime.   φ = u / a
u''_k = − ω²_eff u_k   ω²_eff = k² + m²_eff(η)
m²_eff = m²a² − a''/a
lim_{η→−∞} u^in_k = 1/√(2ω_k) e^{−i ∫_{−∞}^{η} ω_eff(η) dη}
lim_{η→+∞} u^in_k = α_k/√(2ω_k) e^{−i ∫_{−∞}^{η} ω_eff dη} + β_k/√(2ω_k) e
n_k ~ |β_k|²

- vacuum $a_k |in\rangle = 0$

- vacuum $b_k |out\rangle = 0$

$$\phi = \int \frac{d^3k}{(2\pi)^3}\left(a_k u_k^{in} e^{ik\cdot x} + c.c \right)$$

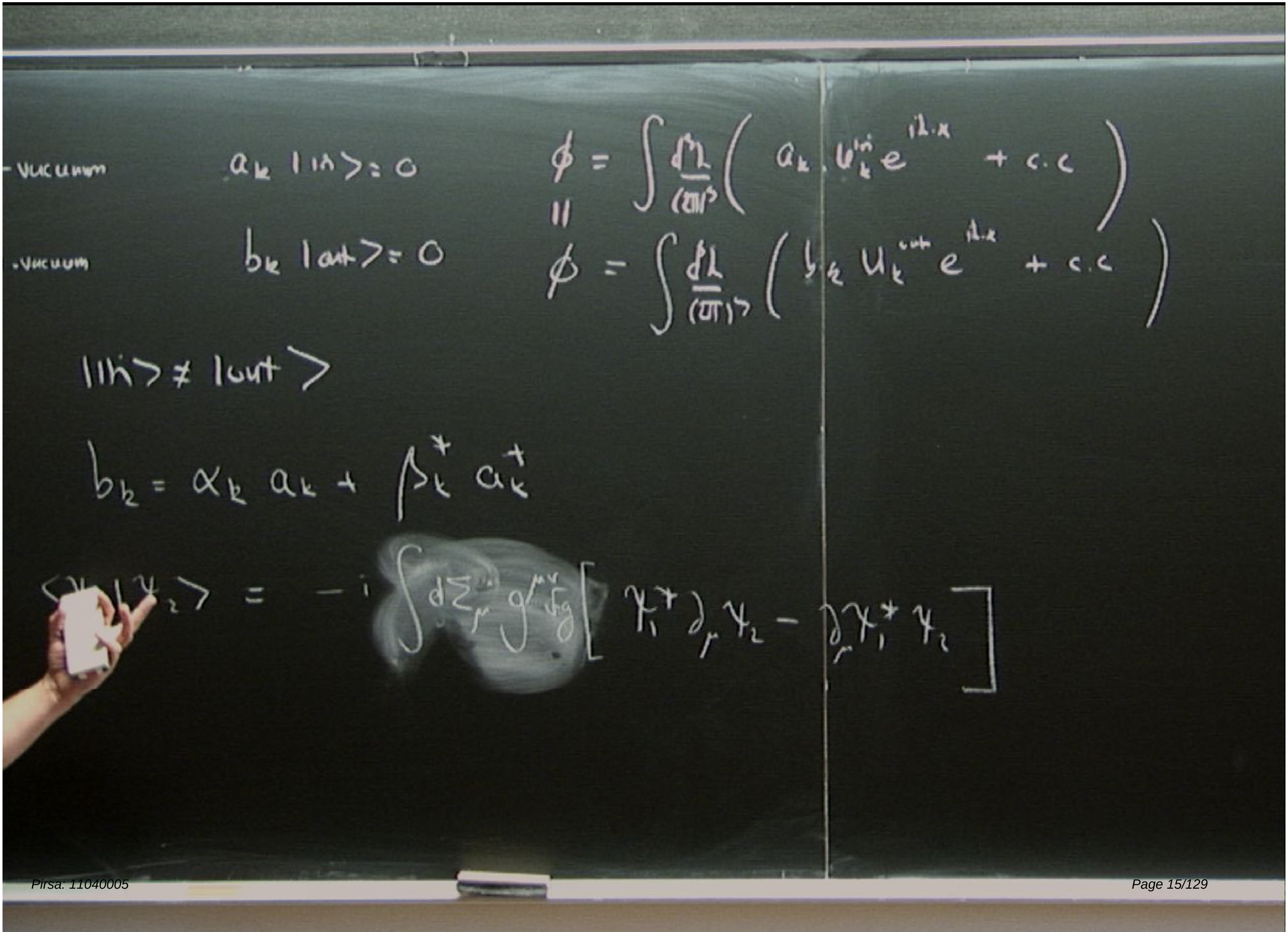
$$\parallel$$

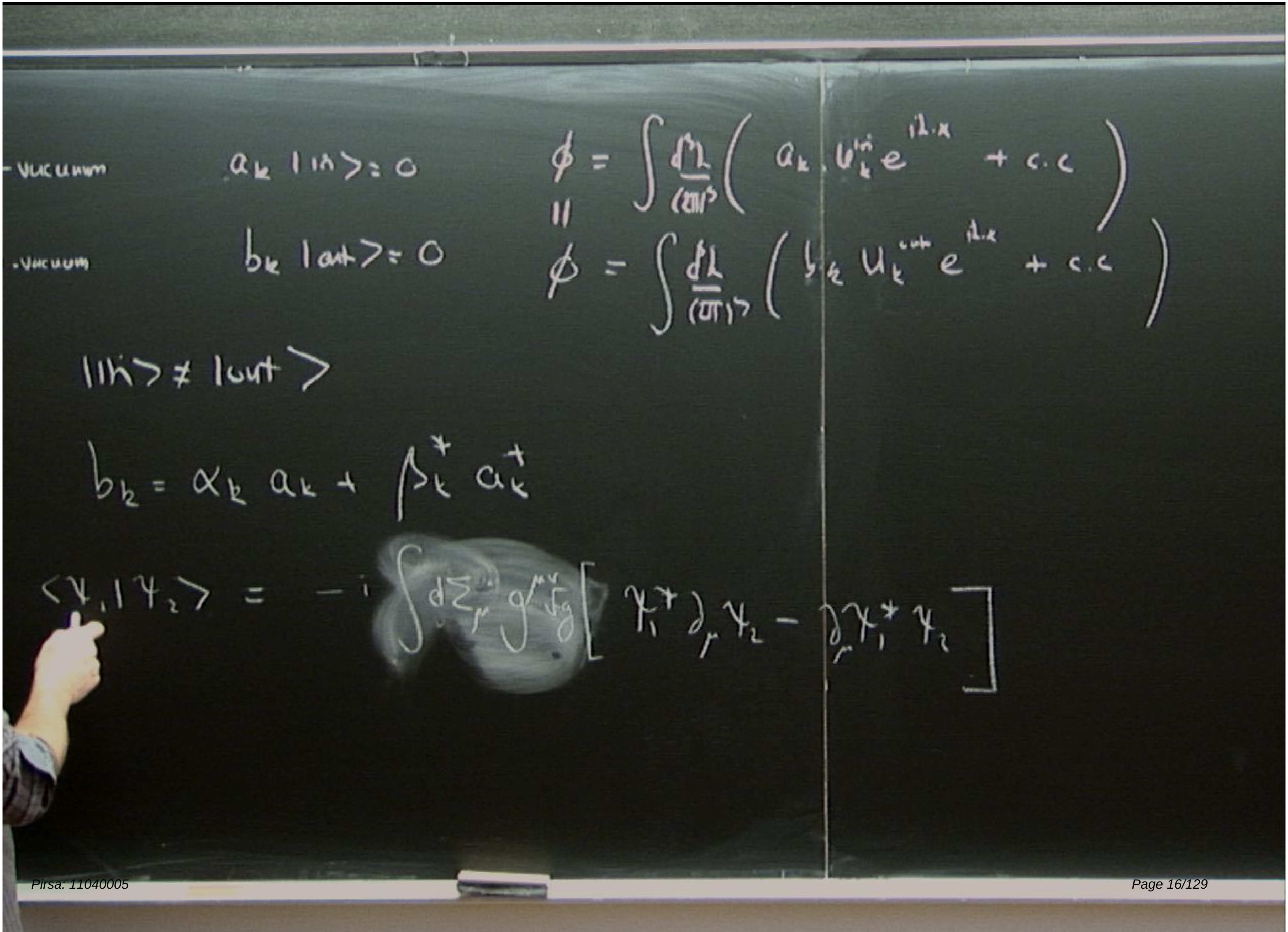
$$\phi = \int \frac{d^3k}{(2\pi)^3}\left(b_k u_k^{out} e^{ik\cdot x} + c.c \right)$$

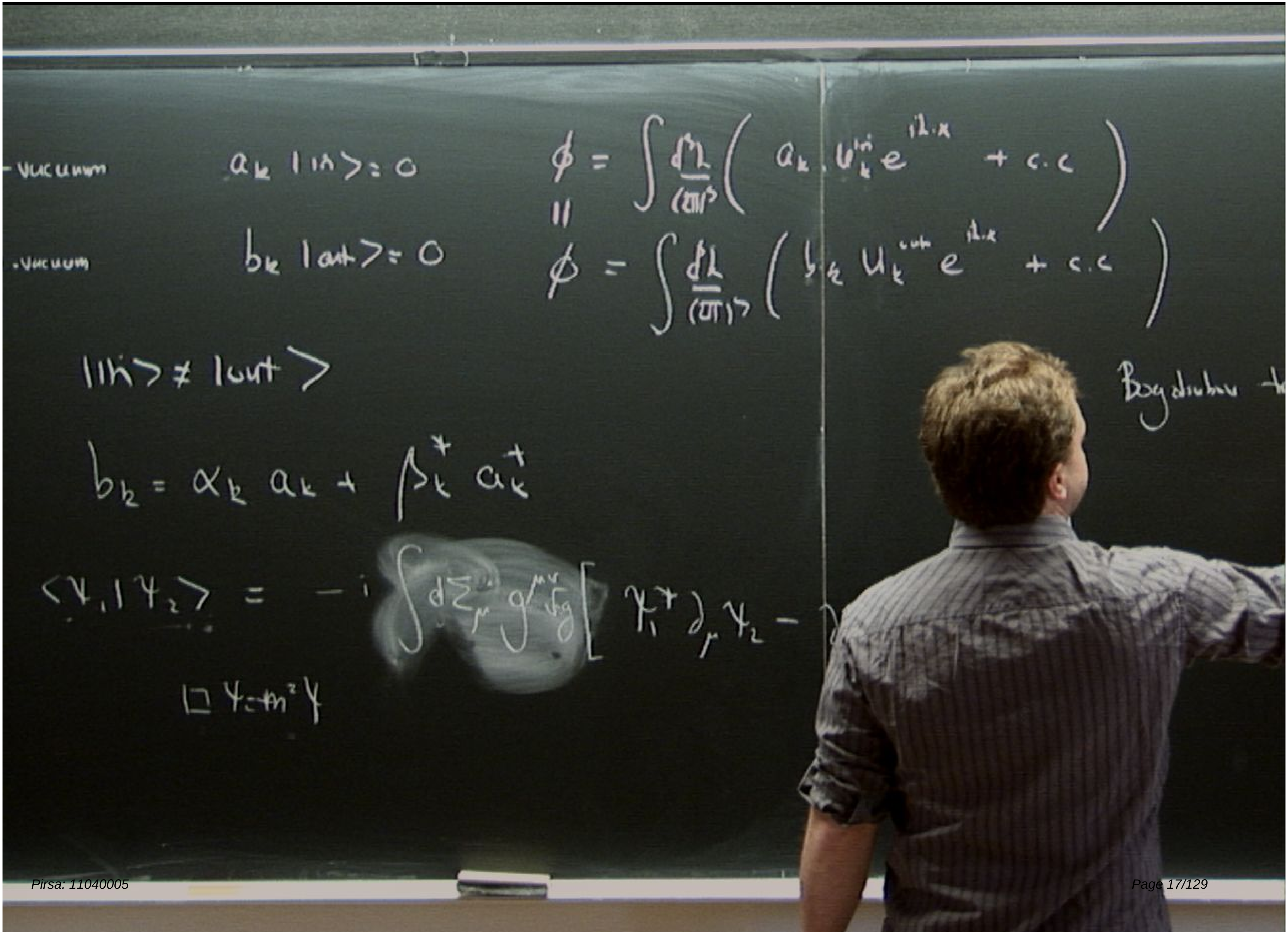
$$|in\rangle \neq |out\rangle$$

$$b_k = \alpha_k a_k + \beta_k^* a_k^\dagger$$

$$\langle \psi_1 | \psi_2$$


vacuum
a_k |in> = 0
φ = ∫ d³k/(2π)³ (a_k u_k^in e^{ik·x} + c.c)
vacuum
b_k |out> = 0
φ = ∫ d³k/(2π)³ (b_k u_k^out e^{ik·x} + c.c)
|in> ≠ |out>
b_k = α_k a_k + β_k* a_k†
<ψ_1|ψ_2> = −i ∫ dΣ_μ g^{μν} √g [ψ_1* ∂_μ ψ_2 − ∂_μ ψ_1* ψ_2]


-vacuum
-vacuum
$a_k |in\rangle = 0$
$b_k |out\rangle = 0$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(a_k u_k^{in} e^{ik\cdot x} + c.c \right)$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(b_k u_k^{out} e^{ik\cdot x} + c.c \right)$
$|in\rangle \neq |out\rangle$
$b_k = \alpha_k a_k + \beta_k^* a_k^\dagger$
$\langle \psi_1 | \psi_2 \rangle = -i \int d\Sigma_\mu g^{\mu\nu} \sqrt{g} \left[\psi_1^* \partial_\mu \psi_2 - \partial_\mu \psi_1^* \psi_2 \right]$


-vacuum
$a_k |in\rangle = 0$
-vacuum
$b_k |out\rangle = 0$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(a_k u_k^{in} e^{ik\cdot x} + c.c \right)$
$\phi = \int \frac{d^3k}{(2\pi)^3} \left(b_k u_k^{out} e^{ik\cdot x} + c.c \right)$
$|in\rangle \neq |out\rangle$
$b_k = \alpha_k a_k + \beta_k^* a_k^+$
$\langle \psi_1 | \psi_2 \rangle = -i \int d\Sigma_\mu g^{\mu\nu} \sqrt{g} \left[\psi_1^* \partial_\mu \psi_2 - \right.$
$\Box \psi = m^2 \psi$

$e^{ik\cdot x} + c.c \Big)$

$e^{i\omega t} e^{ik\cdot x} + c.c \Big)$

Bogoliubov transform α_k, β_k

$\psi_1^* \psi_2 \Big]$

$$e^{ik\cdot x} + c.c \Big)$$

$$e^{ik\cdot x} + c.c \Big)$$

$$^{*}\chi_2 \Big]$$

Bogoliubov transform $\qquad \alpha_k, \ \beta_k$

$$v_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$$

$e^{ik\cdot x} + c.c \,)$

$e^{i\omega t} e^{ik\cdot x} + c.c \,)$

Bogoliubov transform α_k, β_k

$$v_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$$

$|\alpha_k|^2 - |\beta_k|^2$ (modes normalization)

$e^{ik\cdot x} + c.c \Big)$

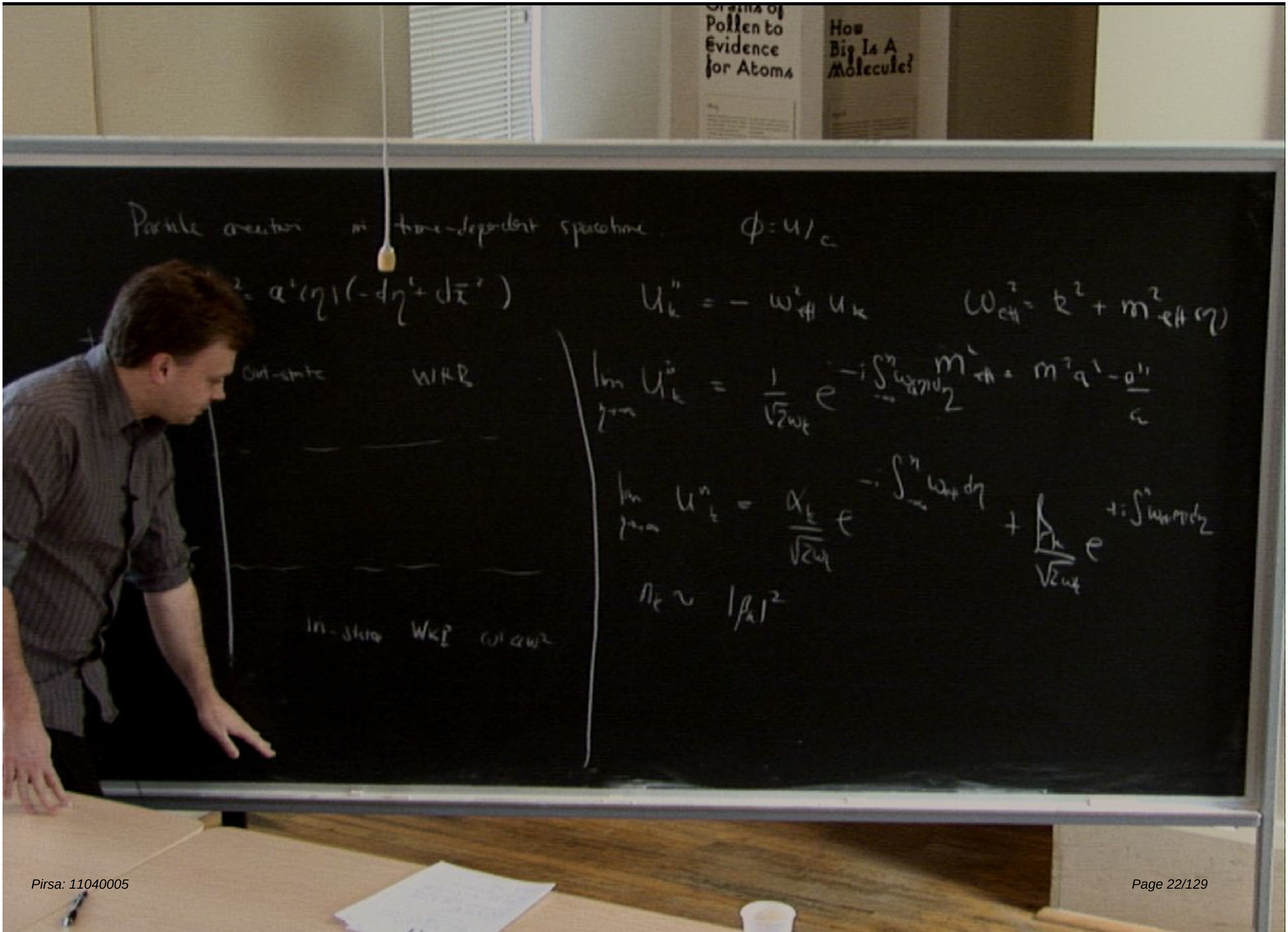
$_k^{-i\omega t} e^{ik\cdot x} + c.c \Big)$

$_1^* \psi_2 \Big]$

Bogoliubov transform α_k , β_k

$$v_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$$

$|\alpha_k|^2 - |\beta_k|^2$ (modes normalization)


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Particle creation in time-dependent spacetime
out-state WKB
in-state WKB

$+ \; c.c \;)$

$+ \; c.c \;)$

Bogoliubov transform $\qquad \alpha_k \, , \; \beta_k$

$$v_k^+ = \alpha_k u_k^+ + \beta_k \left(u_k^+\right)^*$$

$$|\alpha_k|^2 - |\beta_k|^2 \qquad \text{(modes normalization)}$$

$$N_k = \int$$

$+ \; c.c \Big)$

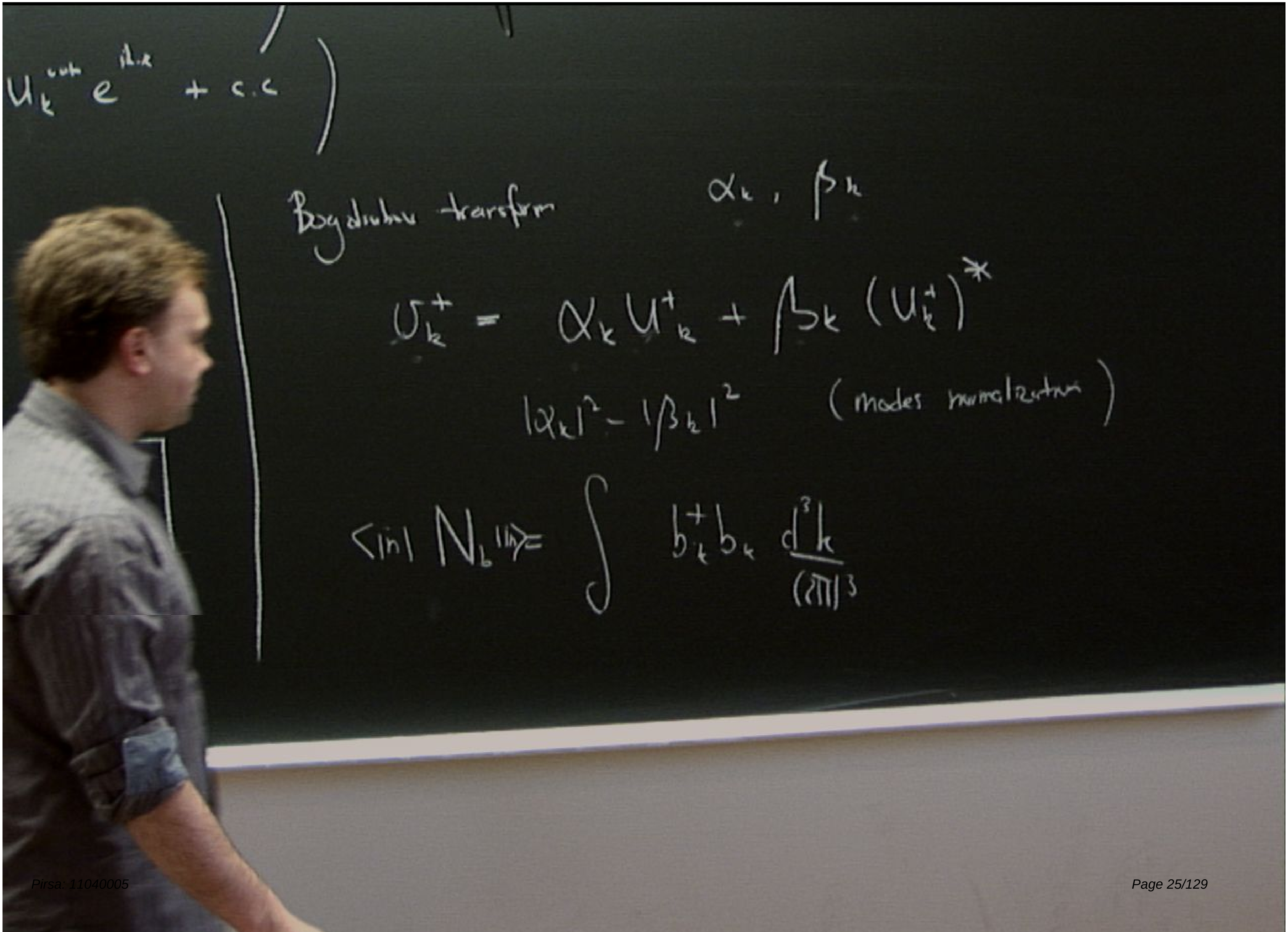
$+ \; c.c \Big)$

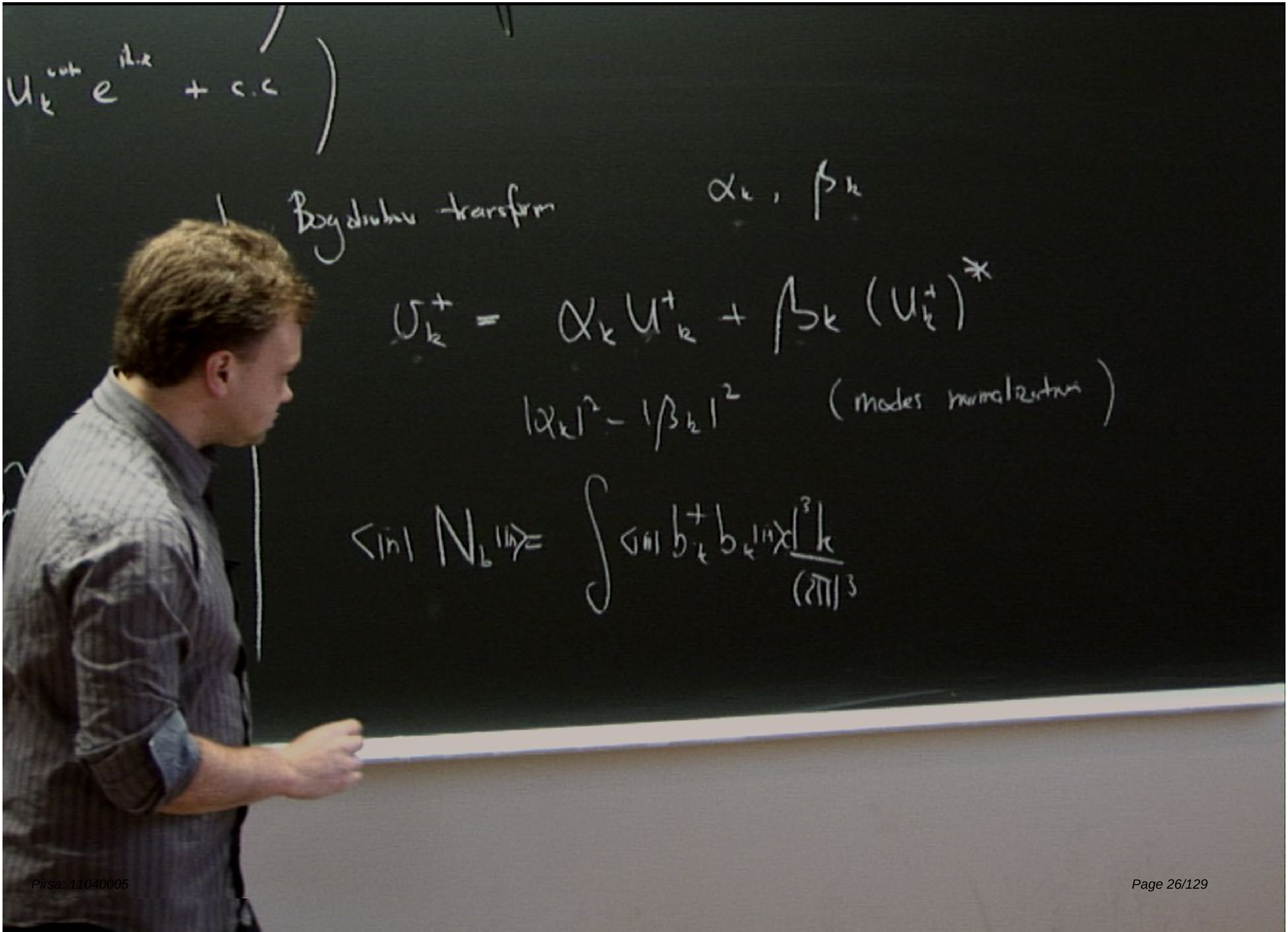
Bogoliubov transform $\quad \alpha_k, \beta_k$

$$U_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$$

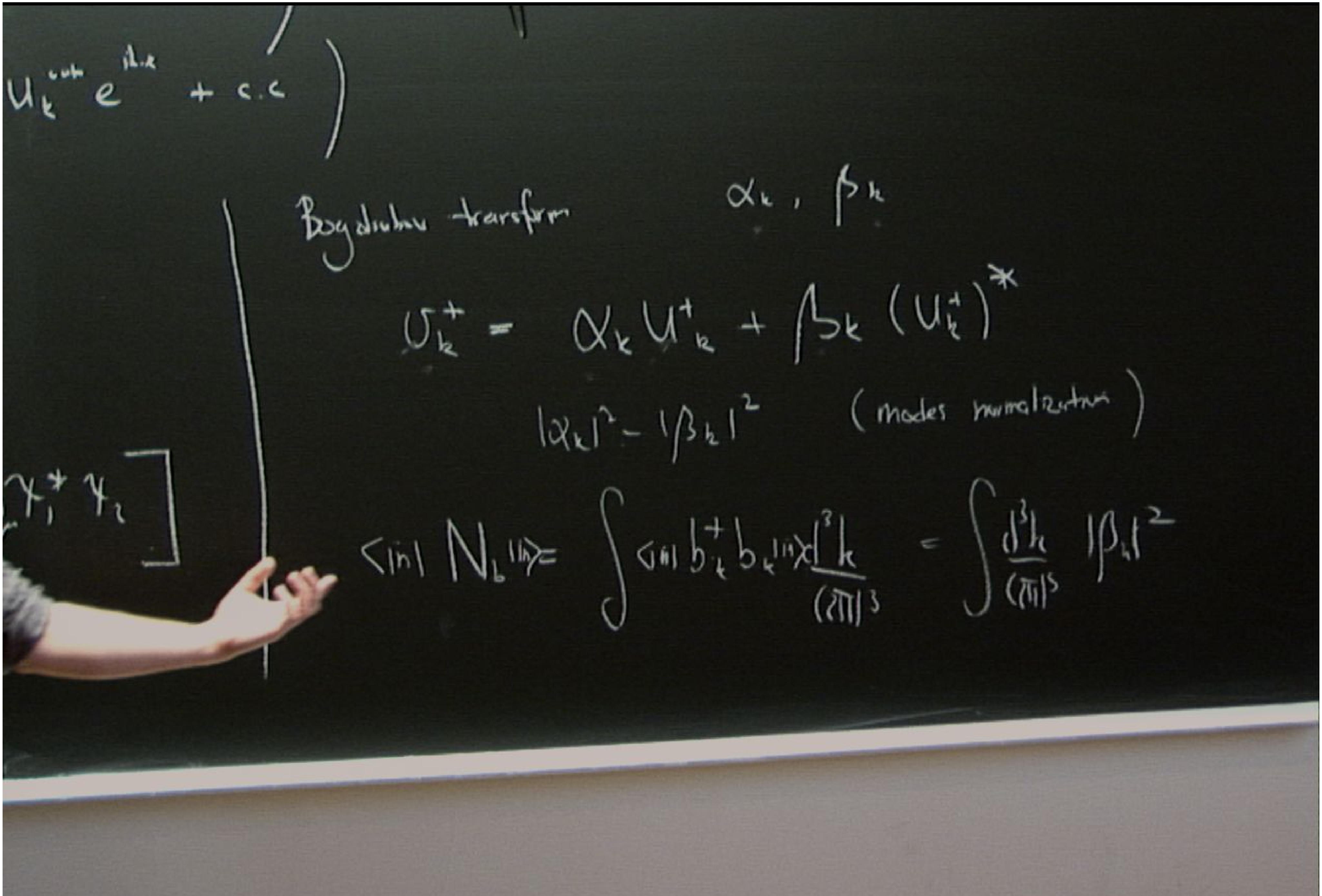
$$|\alpha_k|^2 - |\beta_k|^2 \quad \text{(modes normalization)}$$

$$\langle in | N_k | in \rangle = \int$$


$u_k^{vac} e^{ik \cdot x} + c.c$
Bogoliubov transform
α_k, β_k
$U_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$
$|\alpha_k|^2 - |\beta_k|^2$
(modes normalization)
$\langle in | N_b | in \rangle = \int b_k^+ b_k \frac{d^3k}{(2\pi)^3}$



Bogoliubov transform
α_k , β_k
$U_k^+ = \alpha_k U_k^+ + \beta_k (U_k^+)^*$
$|\alpha_k|^2 - |\beta_k|^2$
(modes normalization)
$\langle in | N_b | in \rangle = \int \langle in | b_k^+ b_k | in \rangle \frac{d^3k}{(2\pi)^3}$


+ c.c
Bogoliubov transform
α_k , β_k
$v_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$
$|\alpha_k|^2 - |\beta_k|^2$ (modes normalization)
$\langle in| N_b |in\rangle = \int \langle in| b_k^+ b_k |in\rangle \frac{d^3k}{(2\pi)^3} = \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2$

$u_k^{out} e^{ik\cdot x} + c.c \Big)$

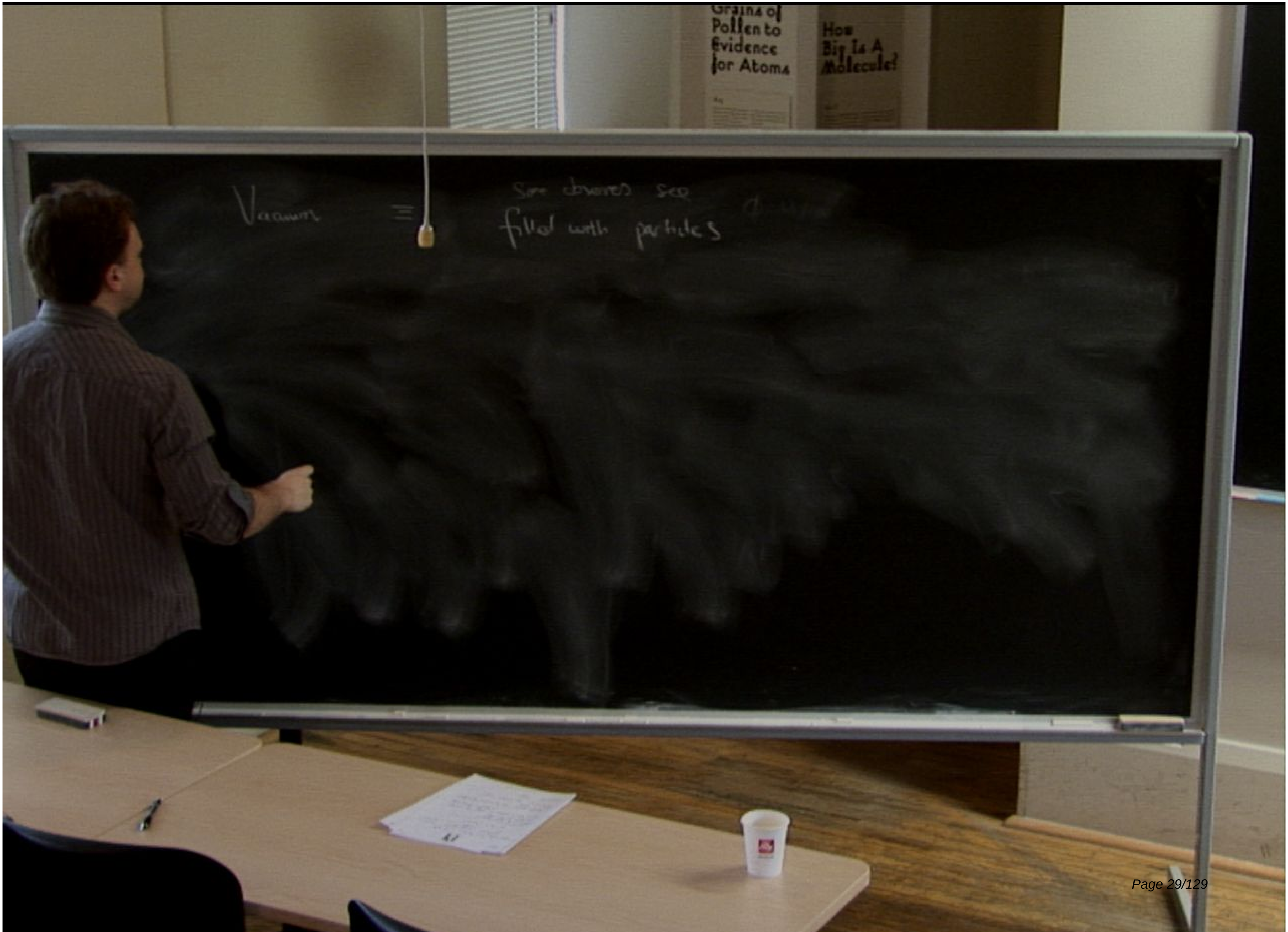
$\chi_1^* \chi_2 \Big]$

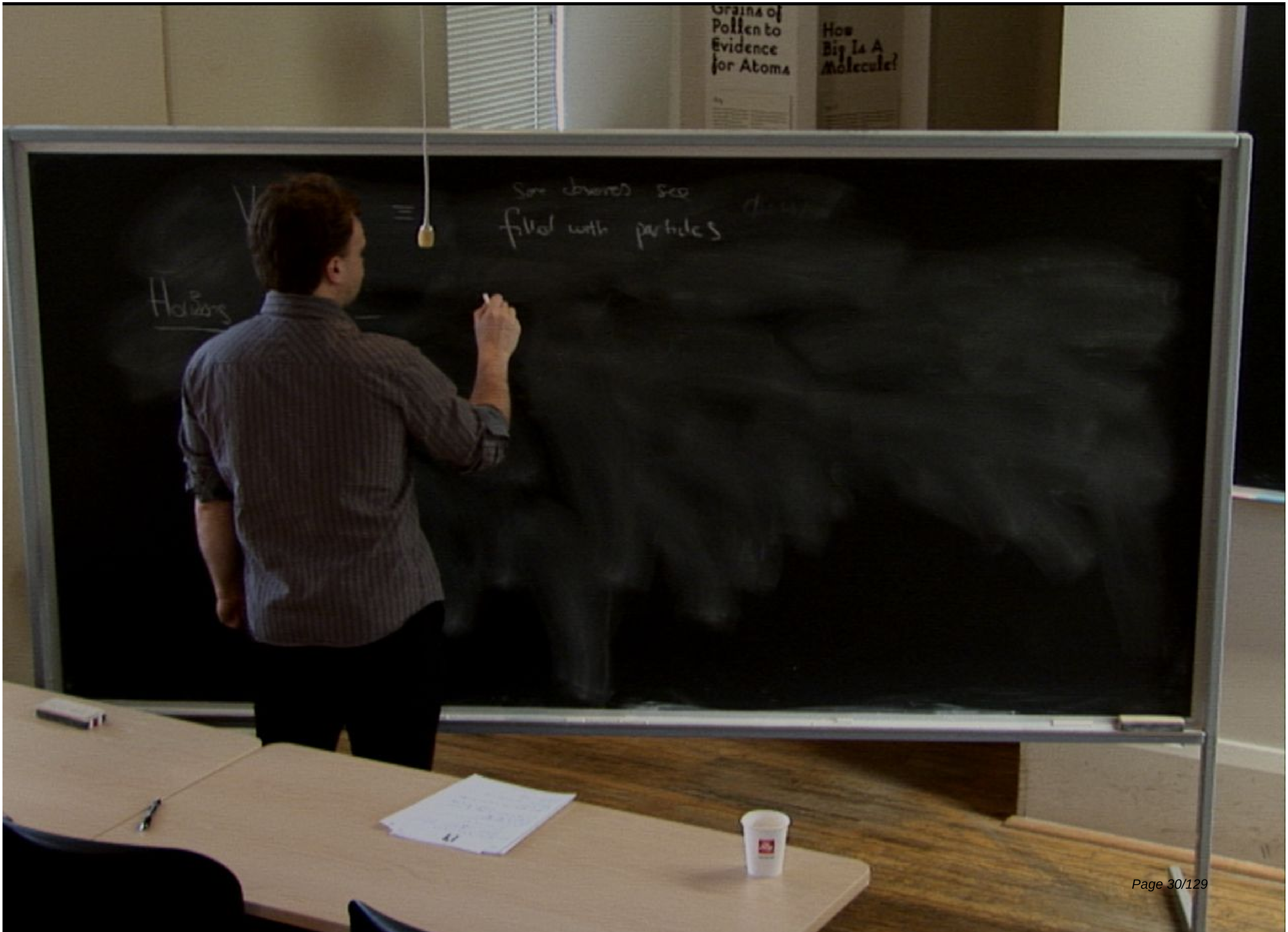
Bogoliubov transform α_k , β_k

$$v_k^+ = \alpha_k u_k^+ + \beta_k (u_k^+)^*$$

$|\alpha_k|^2 - |\beta_k|^2$ (modes normalization)

$$\langle in | N_b | in \rangle = \int \langle in | b_k^+ b_k | in \rangle \frac{d^3k}{(2\pi)^3} = \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2$$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Vacuum ≡
Some observers see
filled with particles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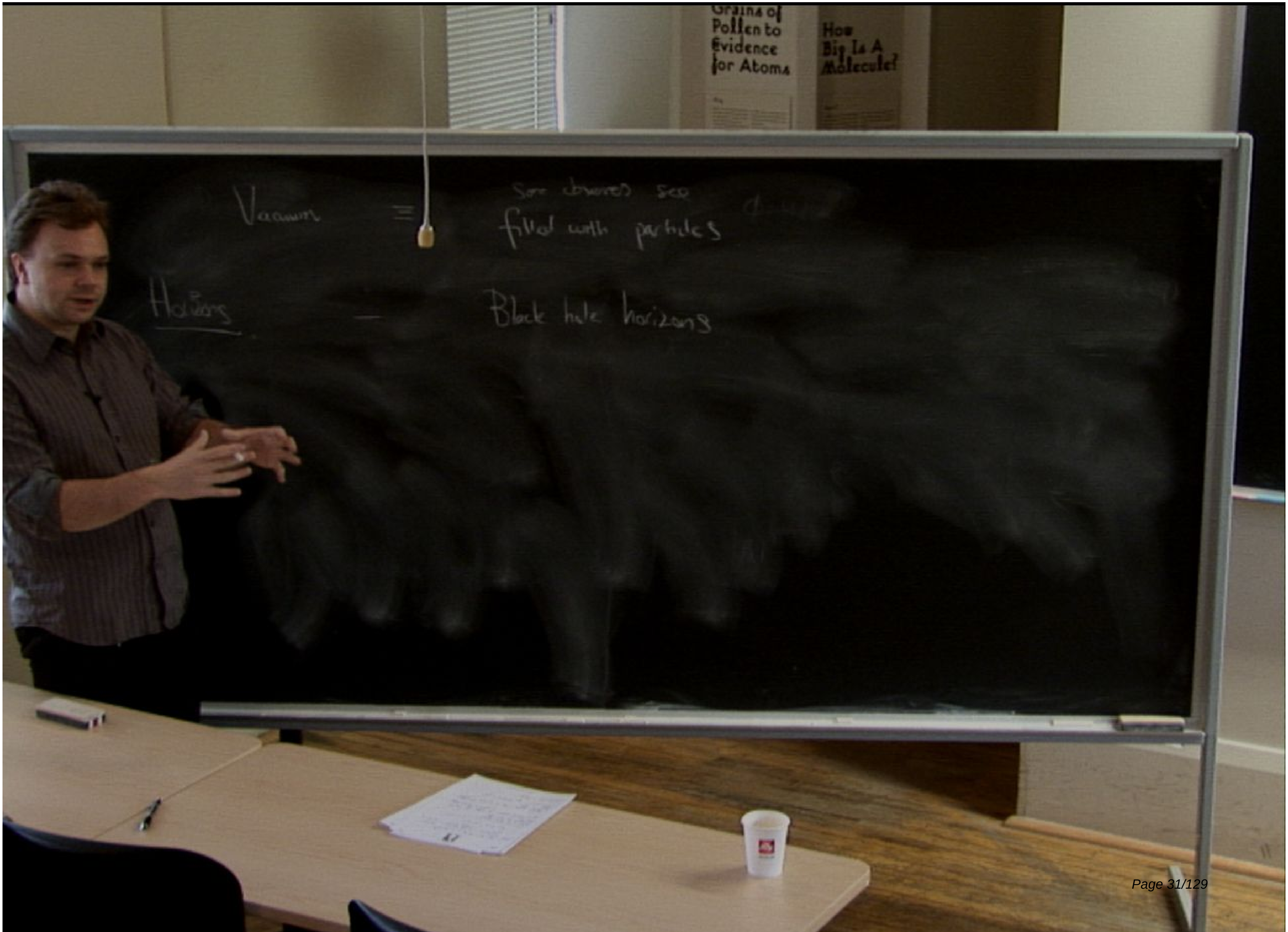
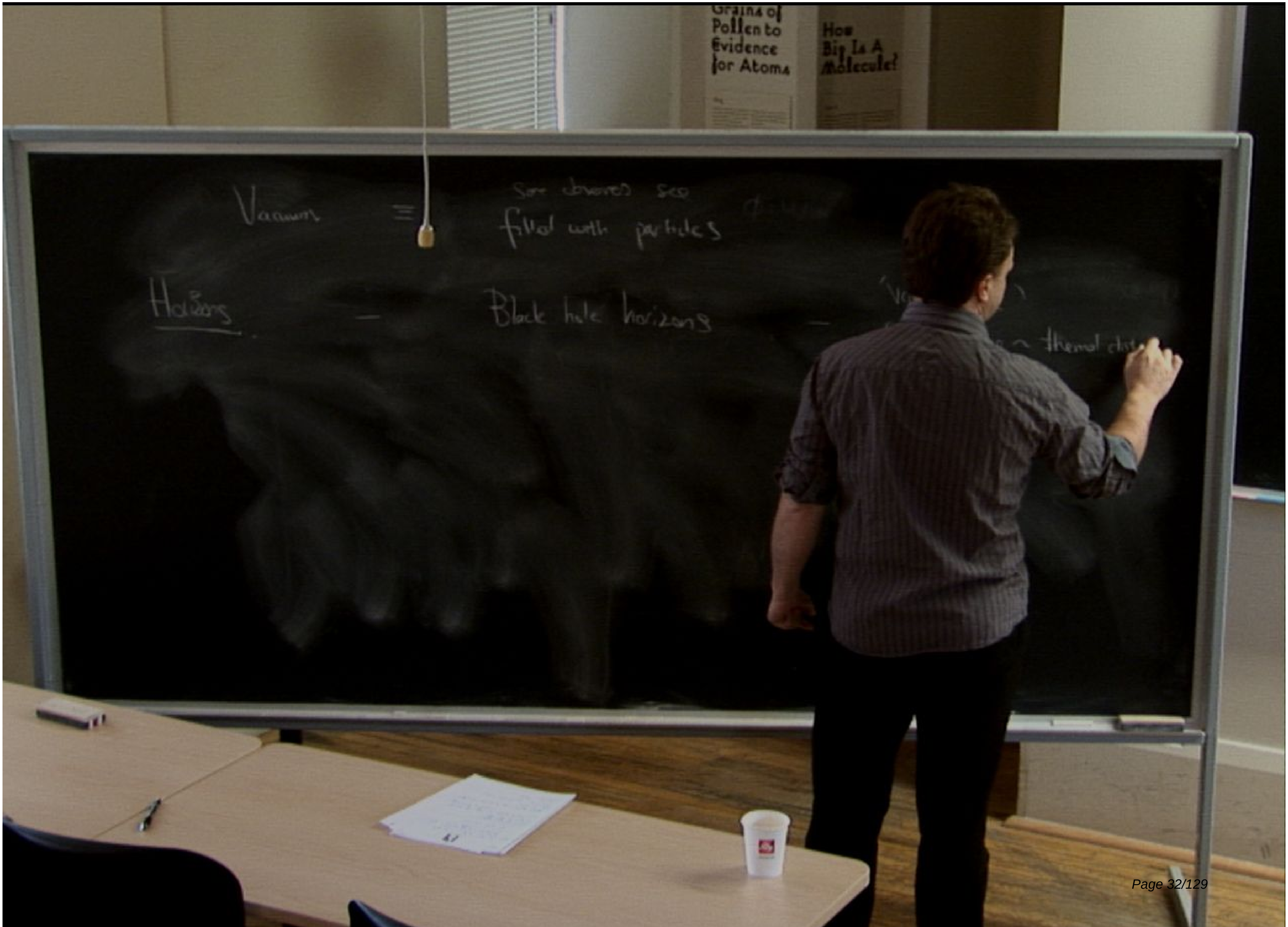
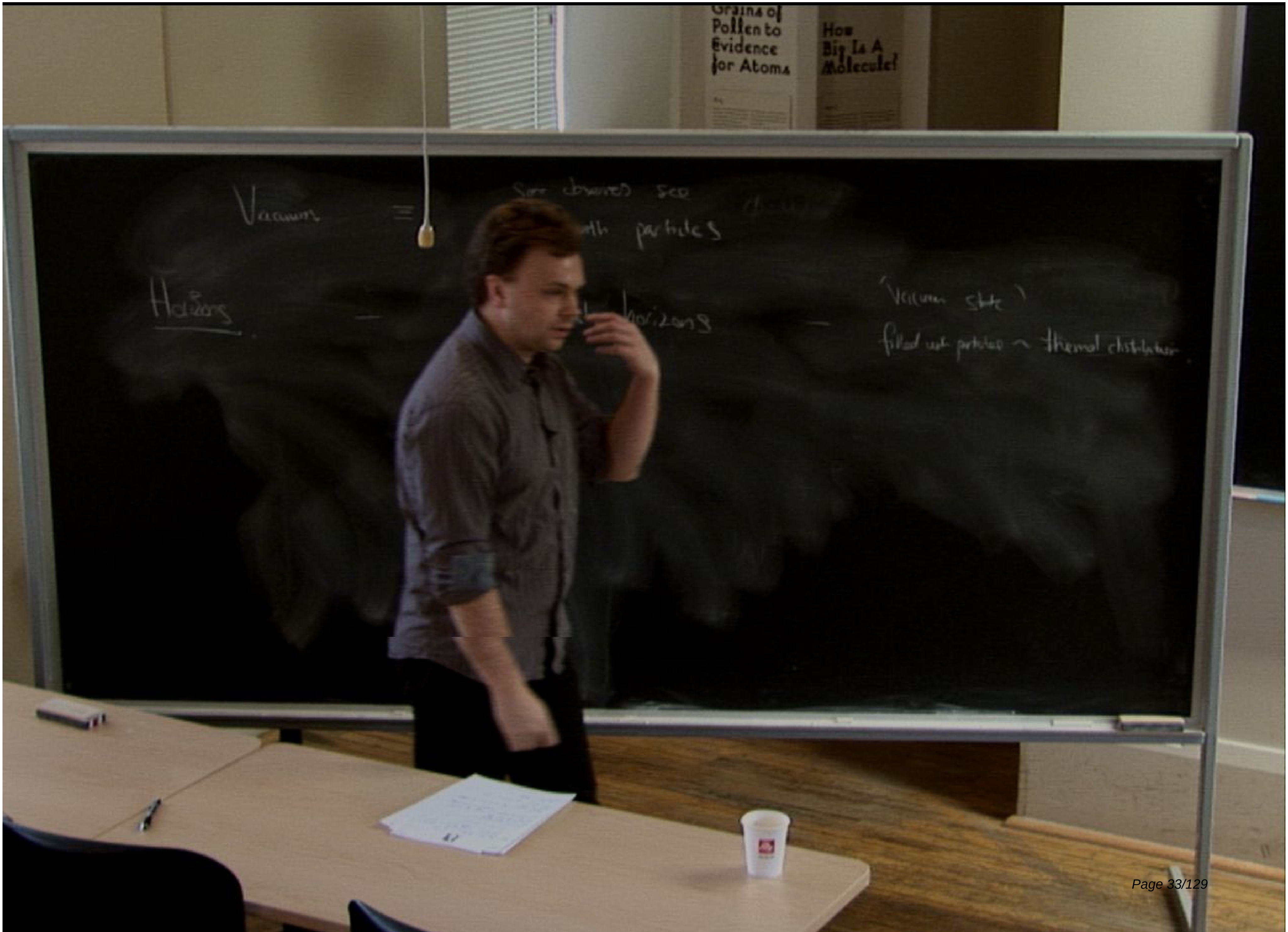
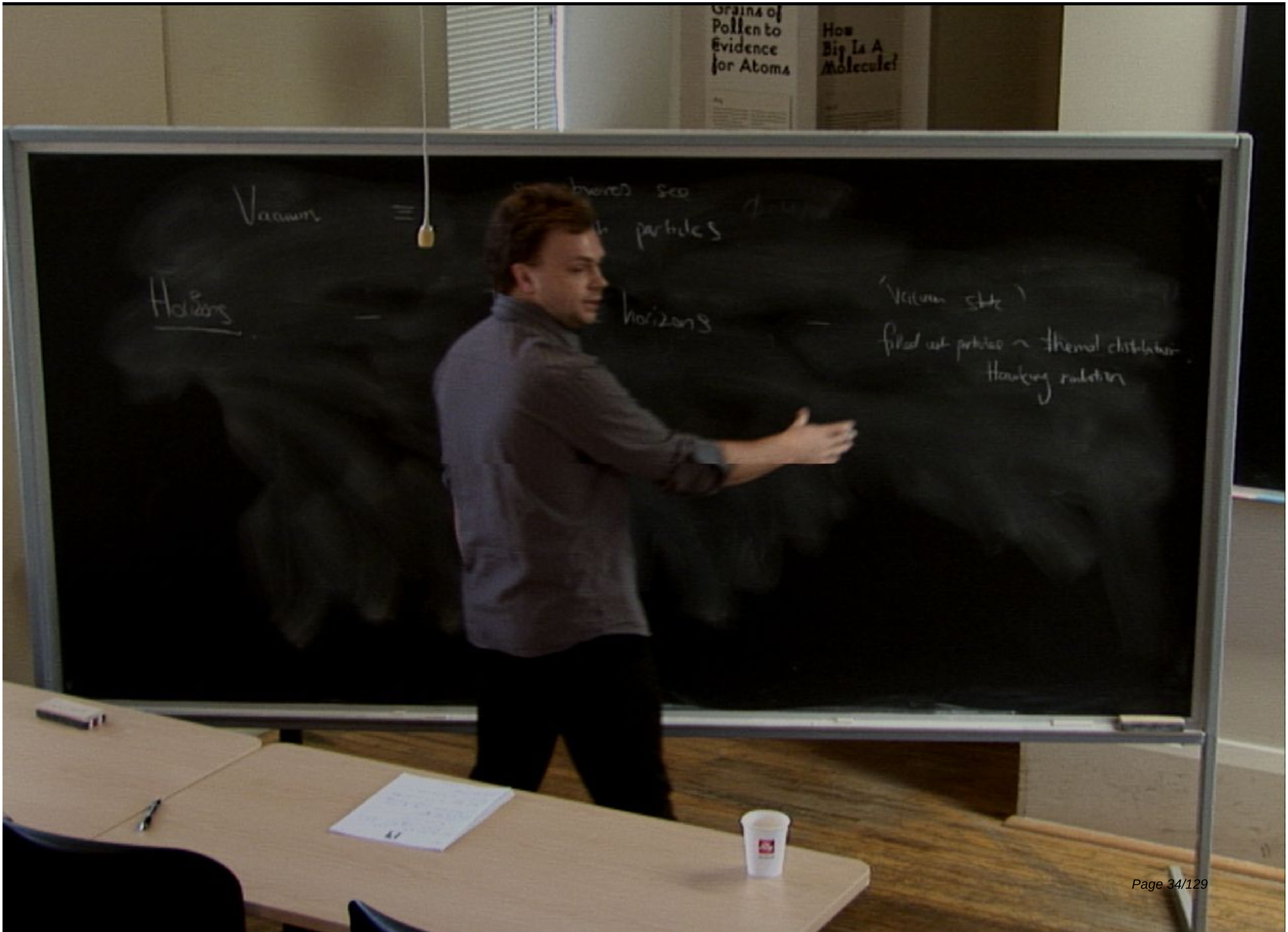

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Vacuum ≡ some observers see filled with particles
Horizons = Black hole horizons


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Vacuum ≡
filled with particles
Horizons
Black hole horizons


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Vacuum
particles
Horizons
horizons
'Vacuum state'
filled with particles ~ thermal distribution


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
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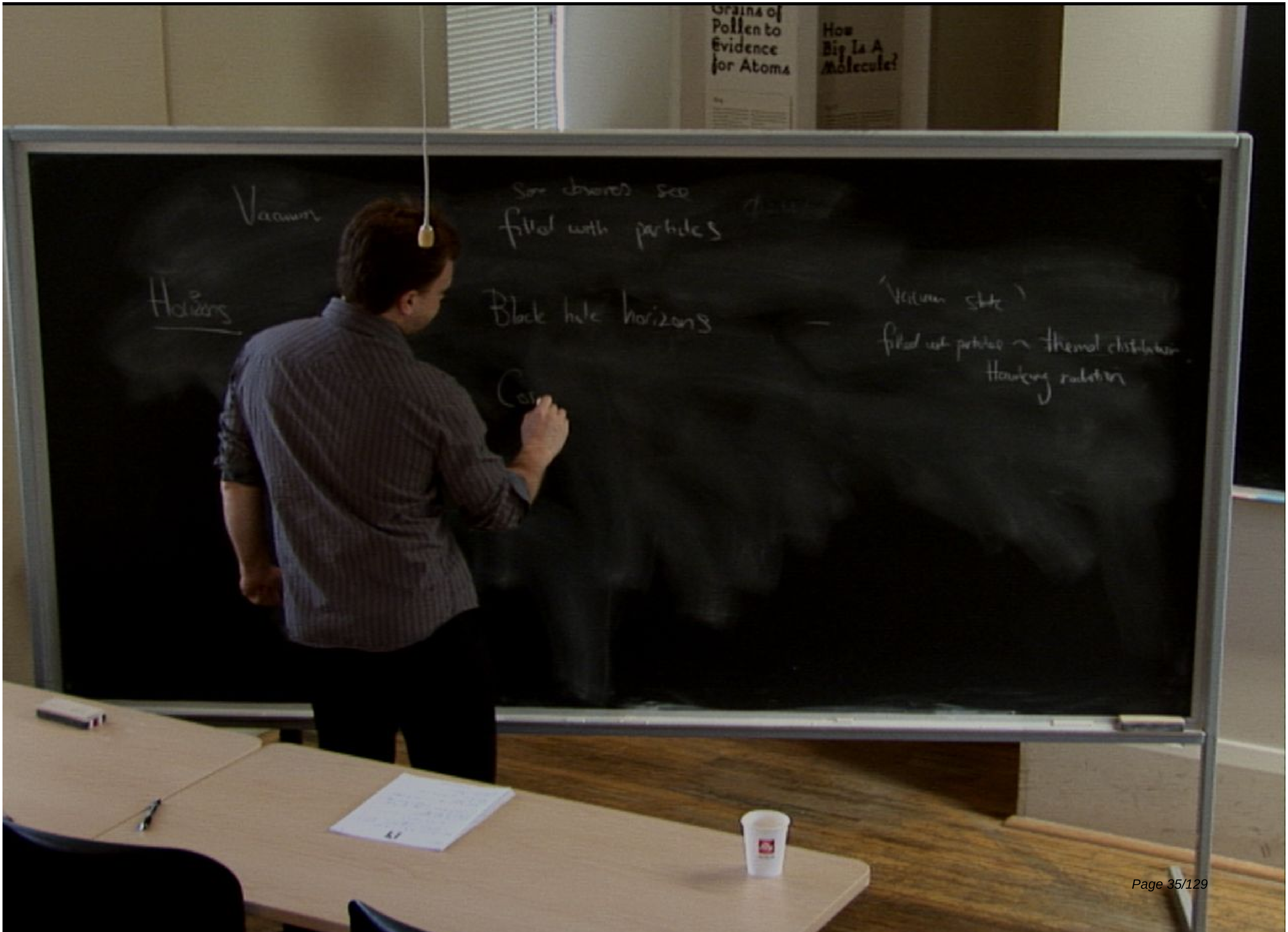
Horizons
horizons
'Vacuum state'
Hawking radiation


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
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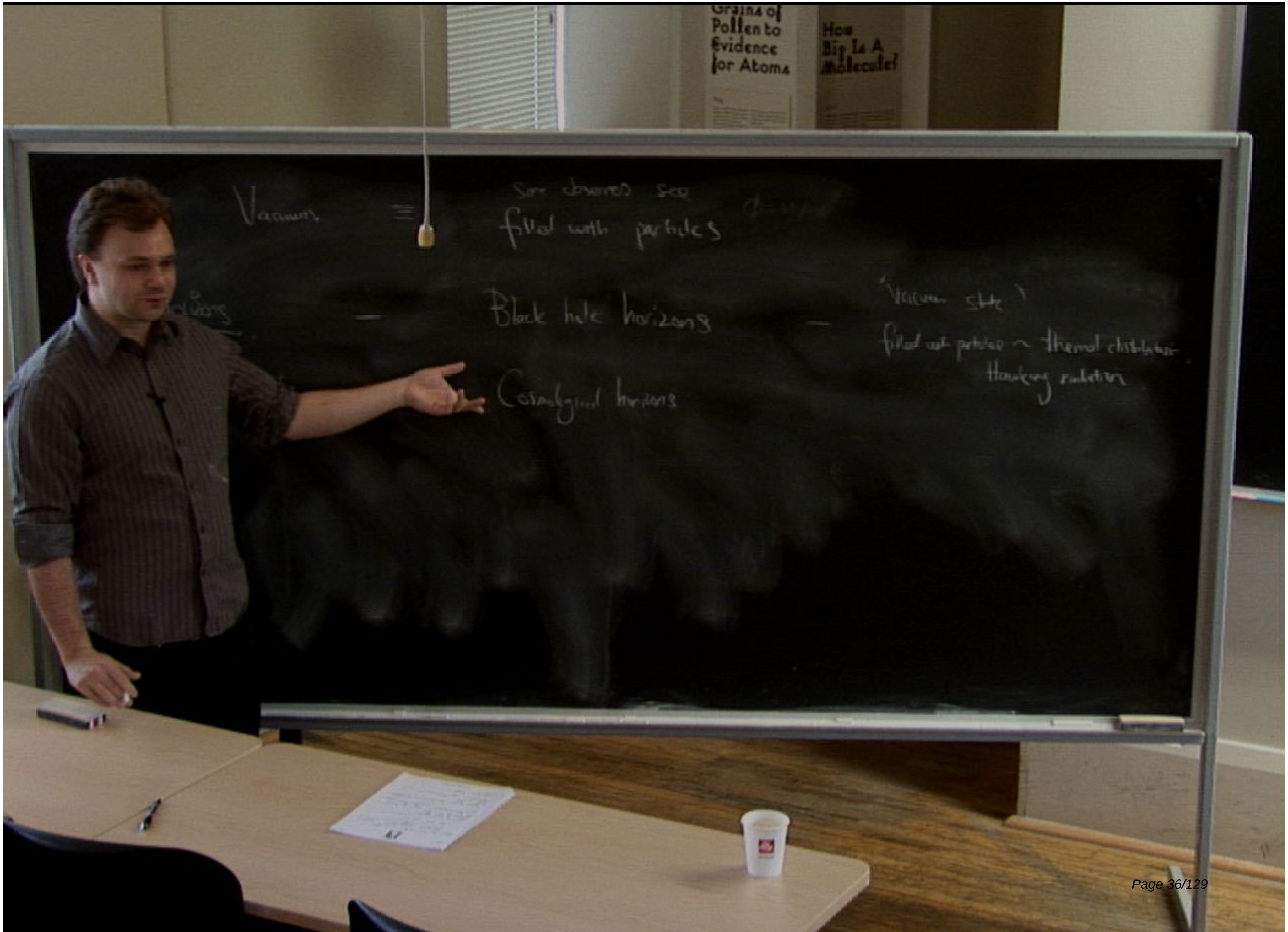
Some observers see
filled with particles
Horizons
Black hole horizons
'Vacuum state'
filled with particles ~ thermal distribution
Hawking radiation


Grains of
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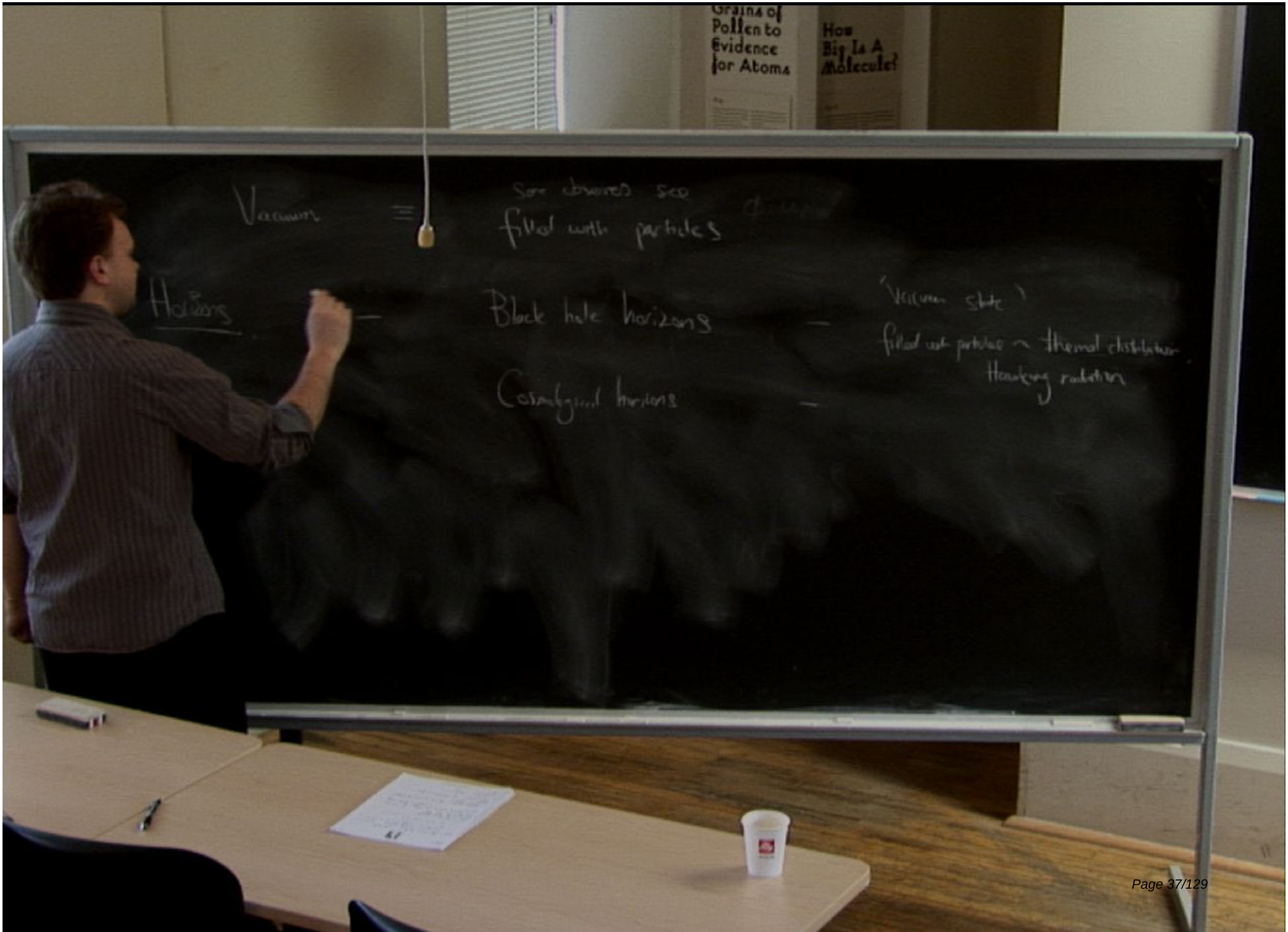
Hawking radiation
Cosmological horizons


Grains of Pollen to Evidence for Atoms
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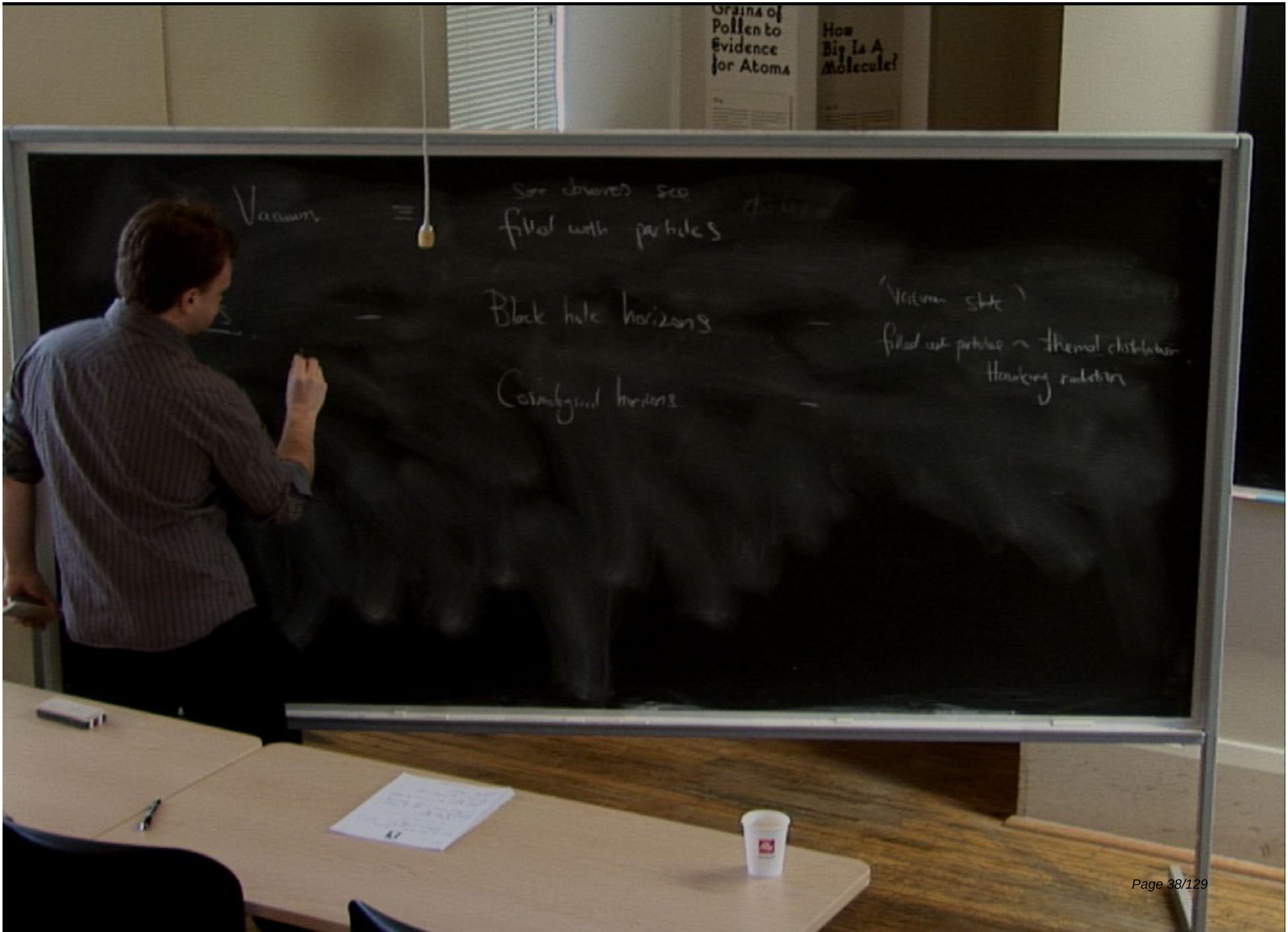
Hawking radiation


Grains of Pollen to Evidence for Atoms
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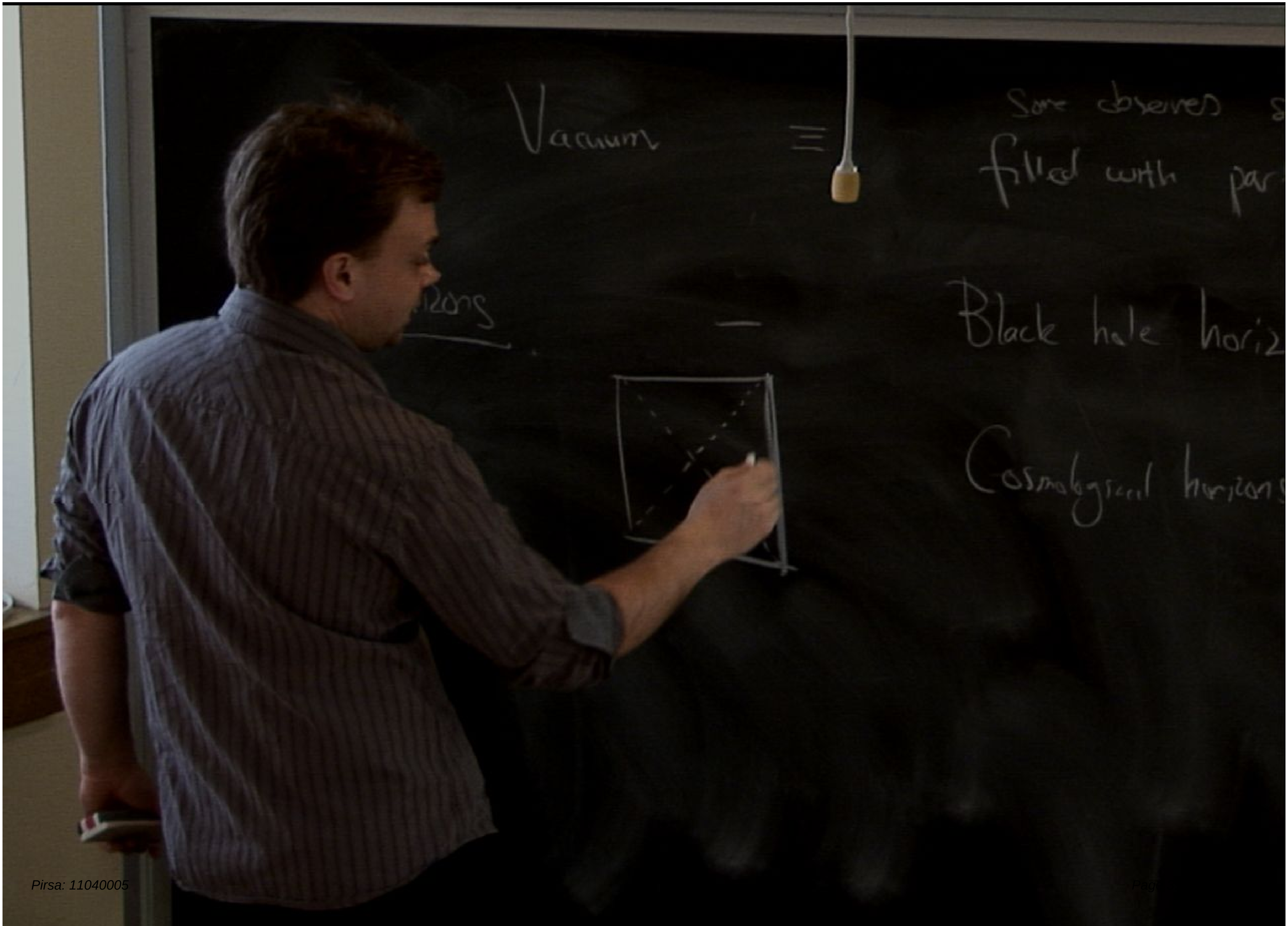
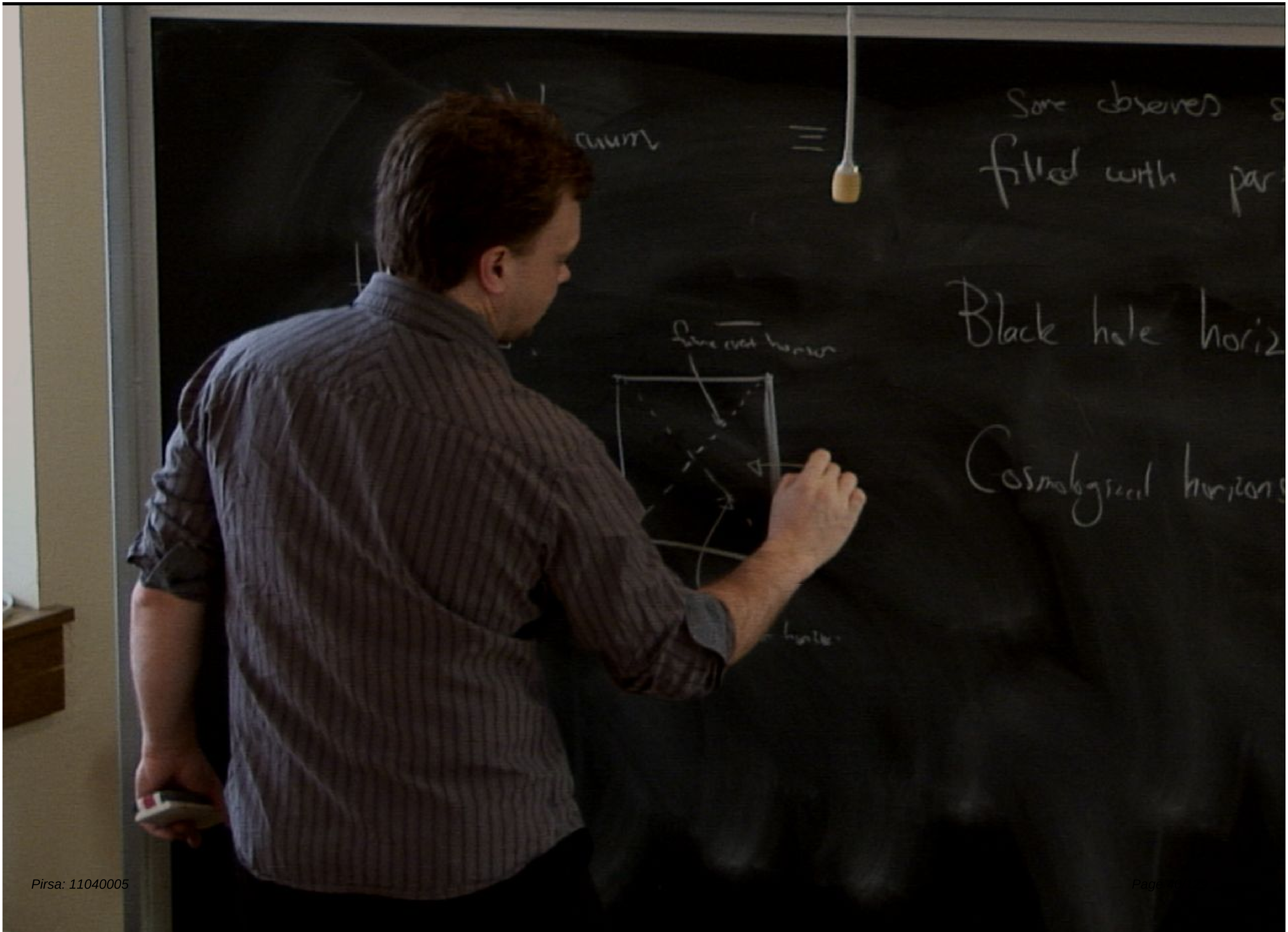

Vacuum ≡
filled with
Black hole
Cosmological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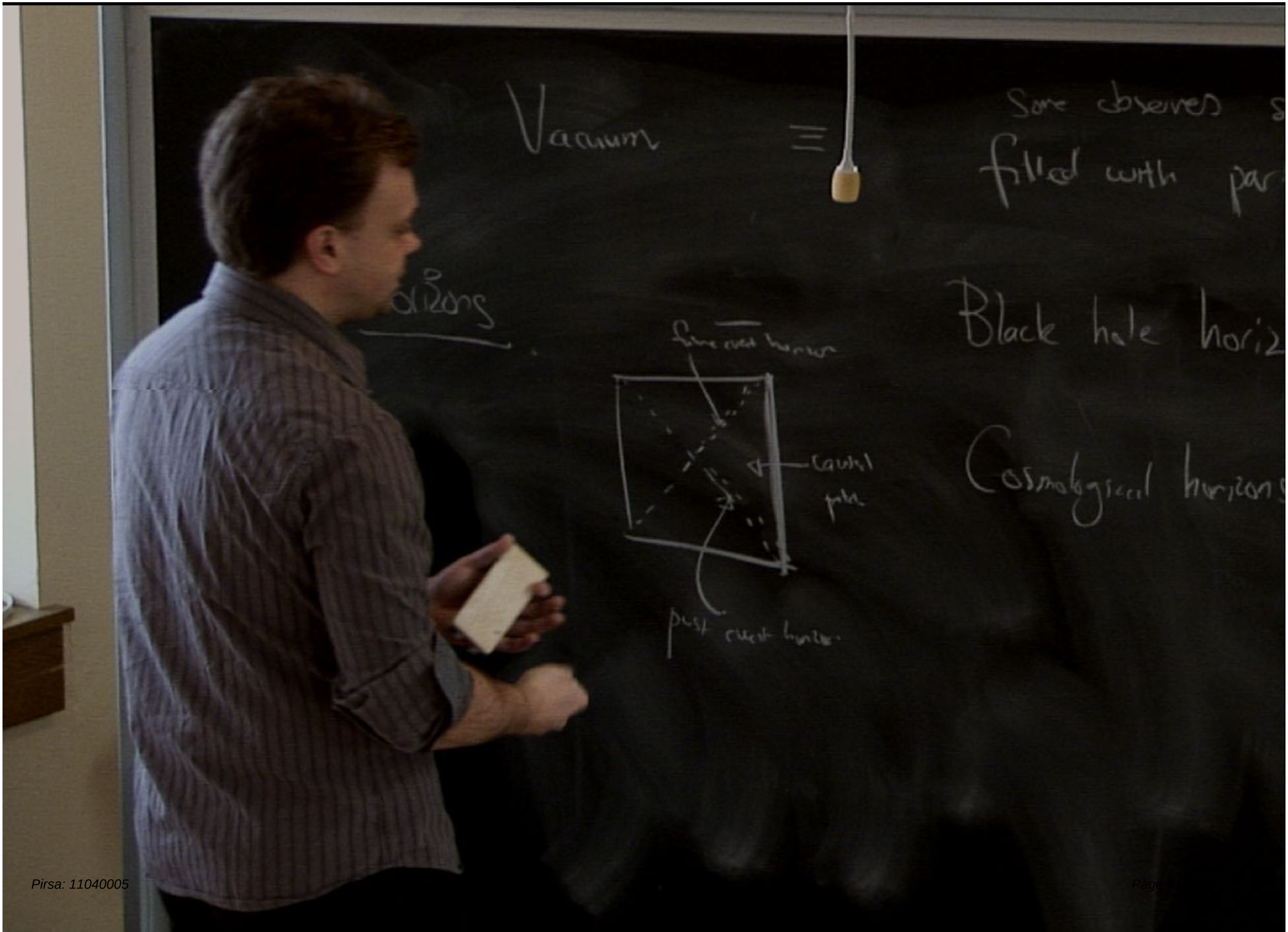
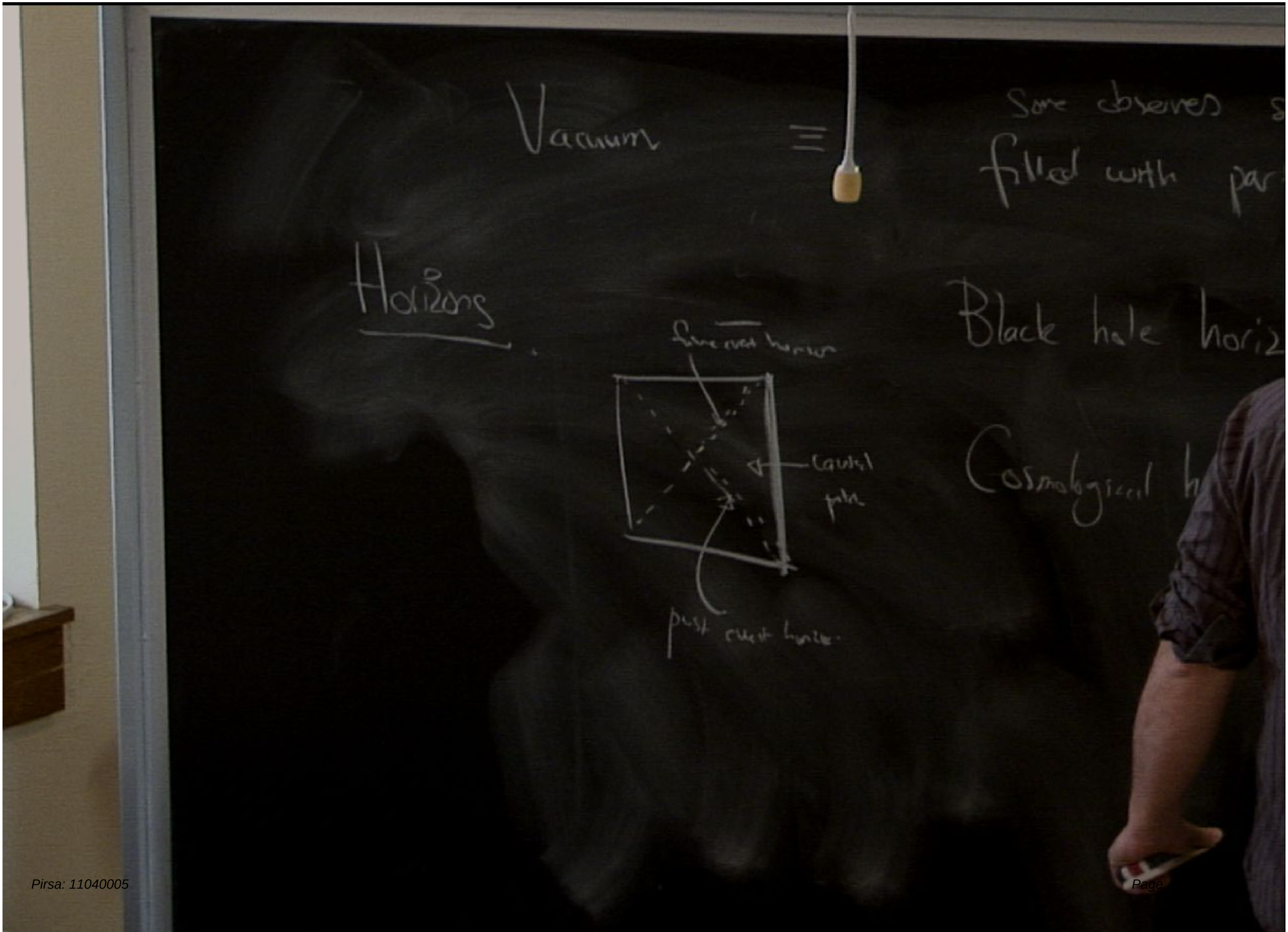

Vacuum ≡
Some observers s
filled with par
Black hole horiz
Cosmological horizons
causal
patch


Vacuum ≡
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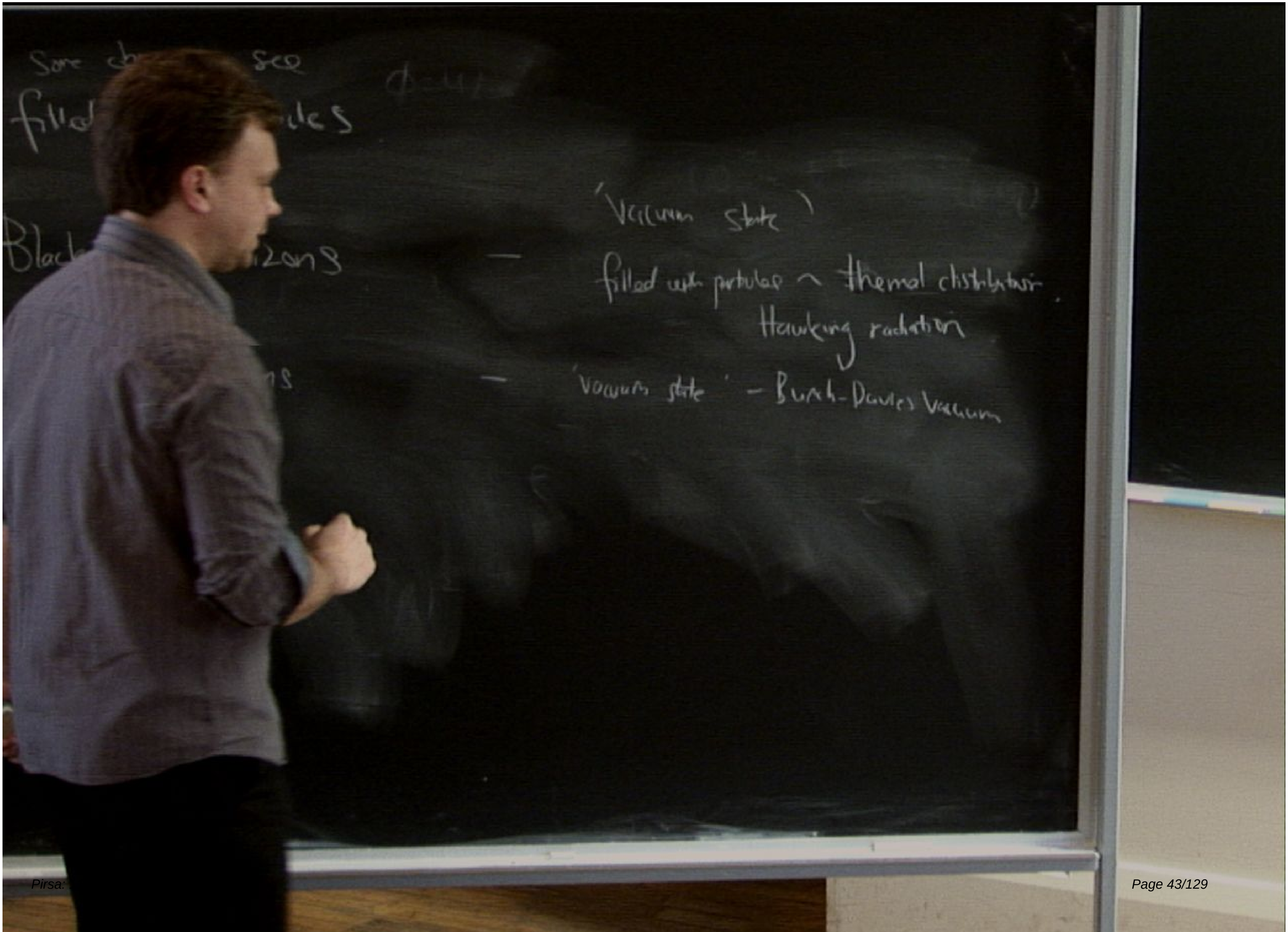
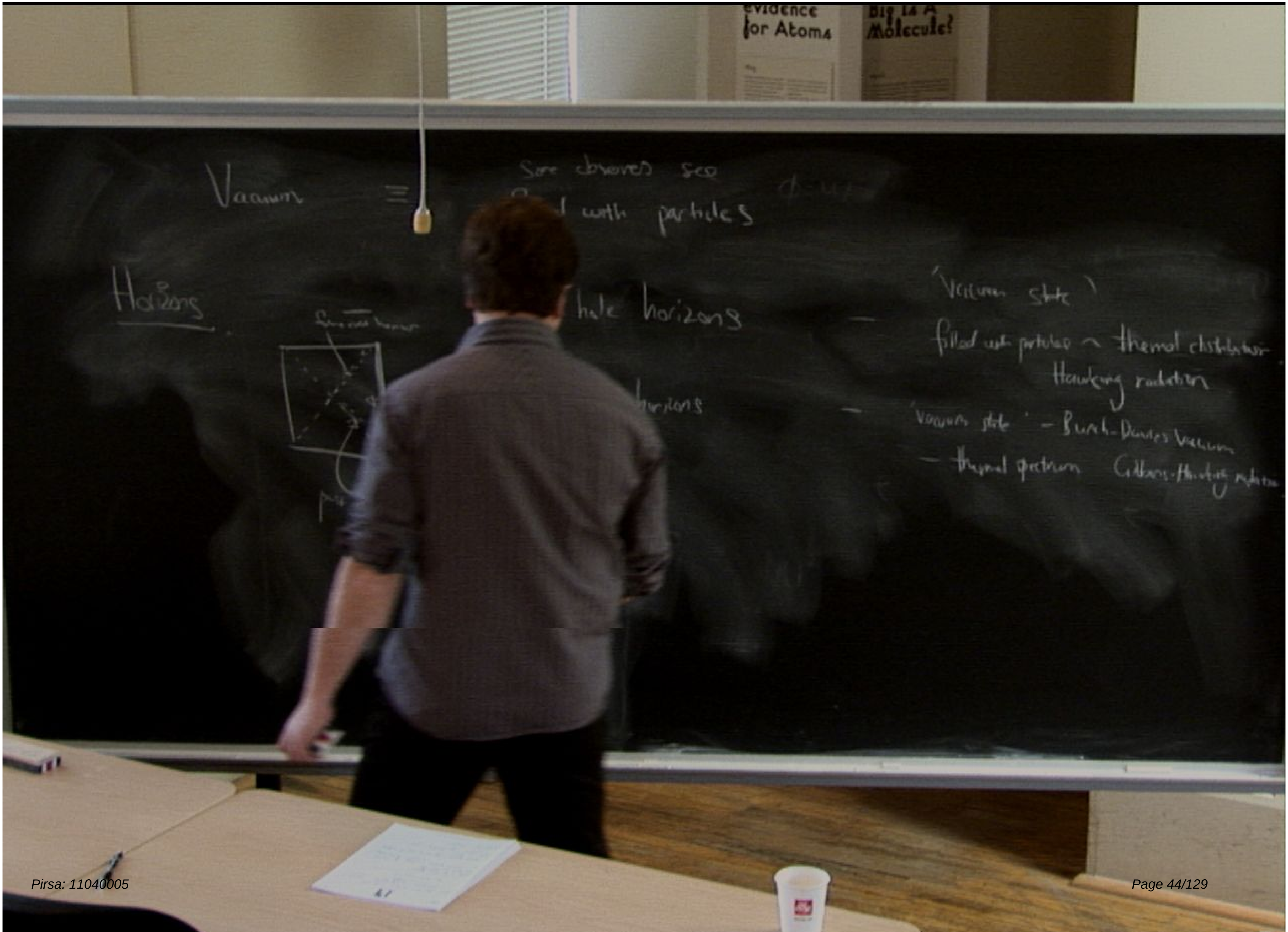
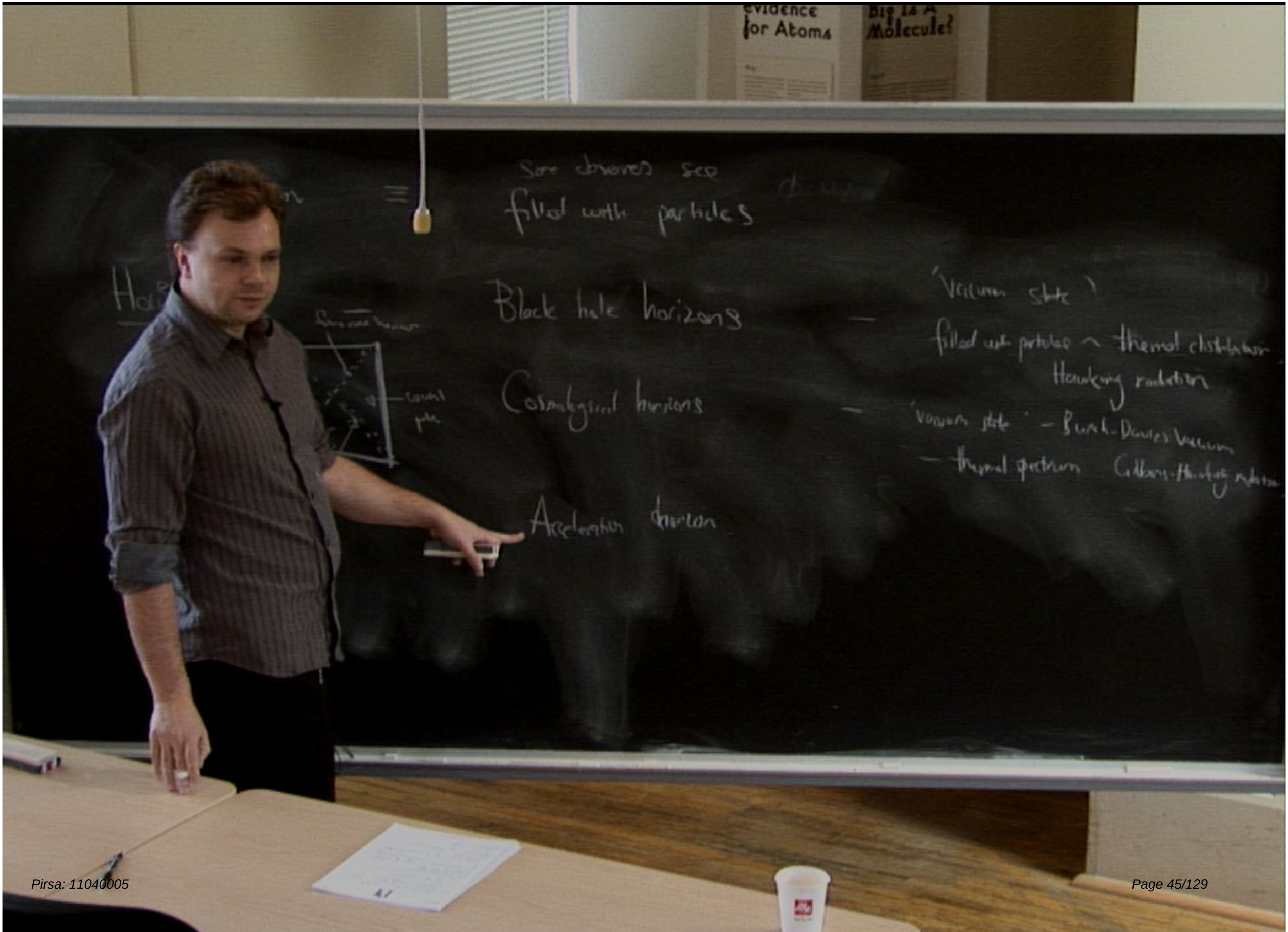
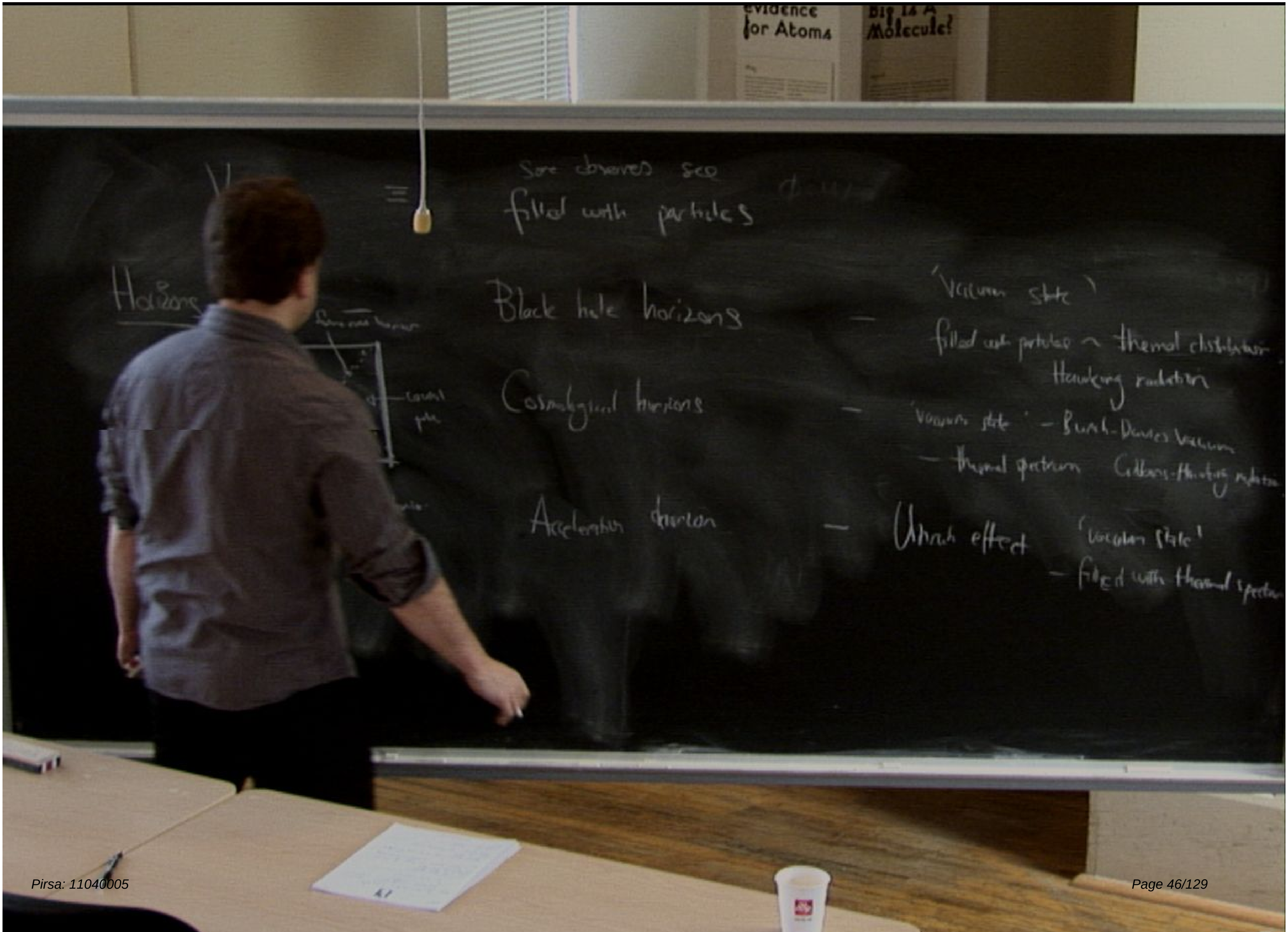
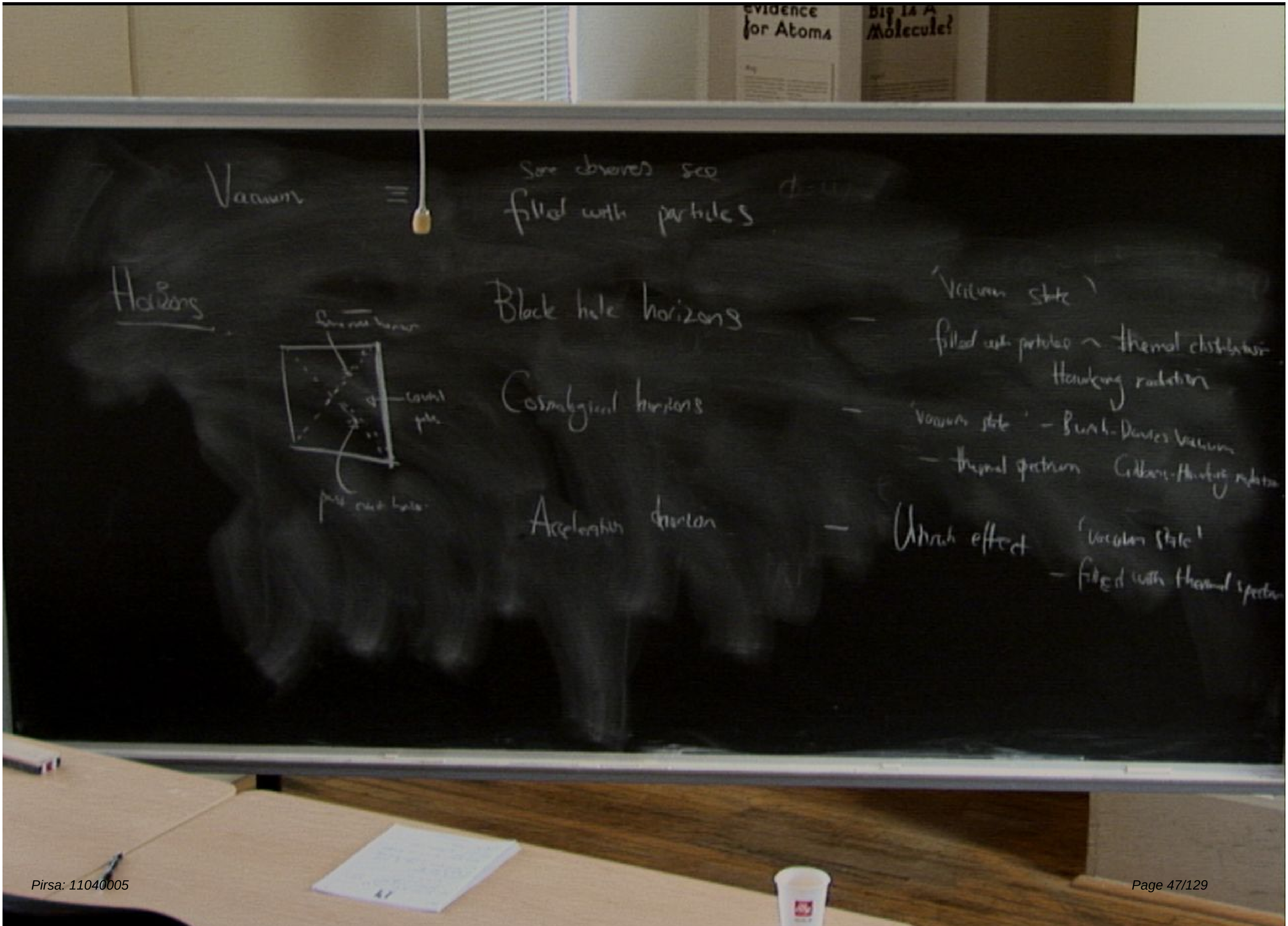
Horizons
Black hole
Cosmological


'Vacuum state'
filled with particles ~ thermal distribution
Hawking radiation
'vacuum state' – Bunch-Davies vacuum


for Atoms
Molecule?
Vacuum
Some observers see
with particles
Horizons
hole horizons
'Vacuum state'
filled with particles ~ thermal distribution
Hawking radiation
'Vacuum state' — Bunch-Davies vacuum
— thermal spectrum


for Atoms
Molecule?
filled with particles
Black hole horizons
Cosmological horizons
Acceleration horizon
'Vacuum state'
Hawking radiation


Some observers see
filled with particles
Horizons
Black hole horizons
'Vacuum state'
filled with particles ~ thermal distribution
Hawking radiation
Cosmological horizons
'Vacuum state' - Bunch-Davies vacuum
thermal spectrum Gibbons-Hawking radiation
Acceleration horizon
Unruh effect
'vacuum state'
filled with thermal spectrum


Vacuum
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Bunch-Davies vacuum
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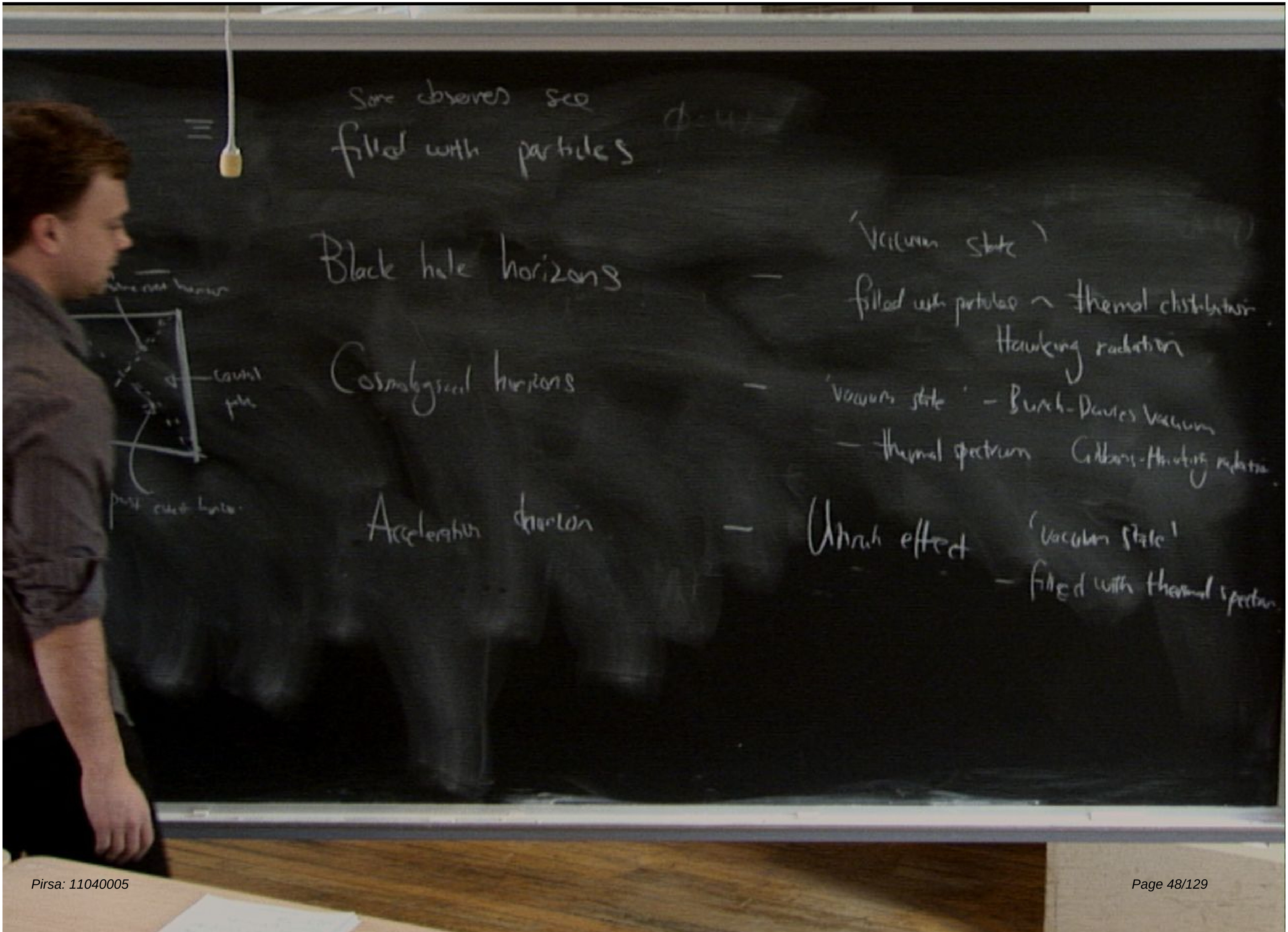
Some observers see
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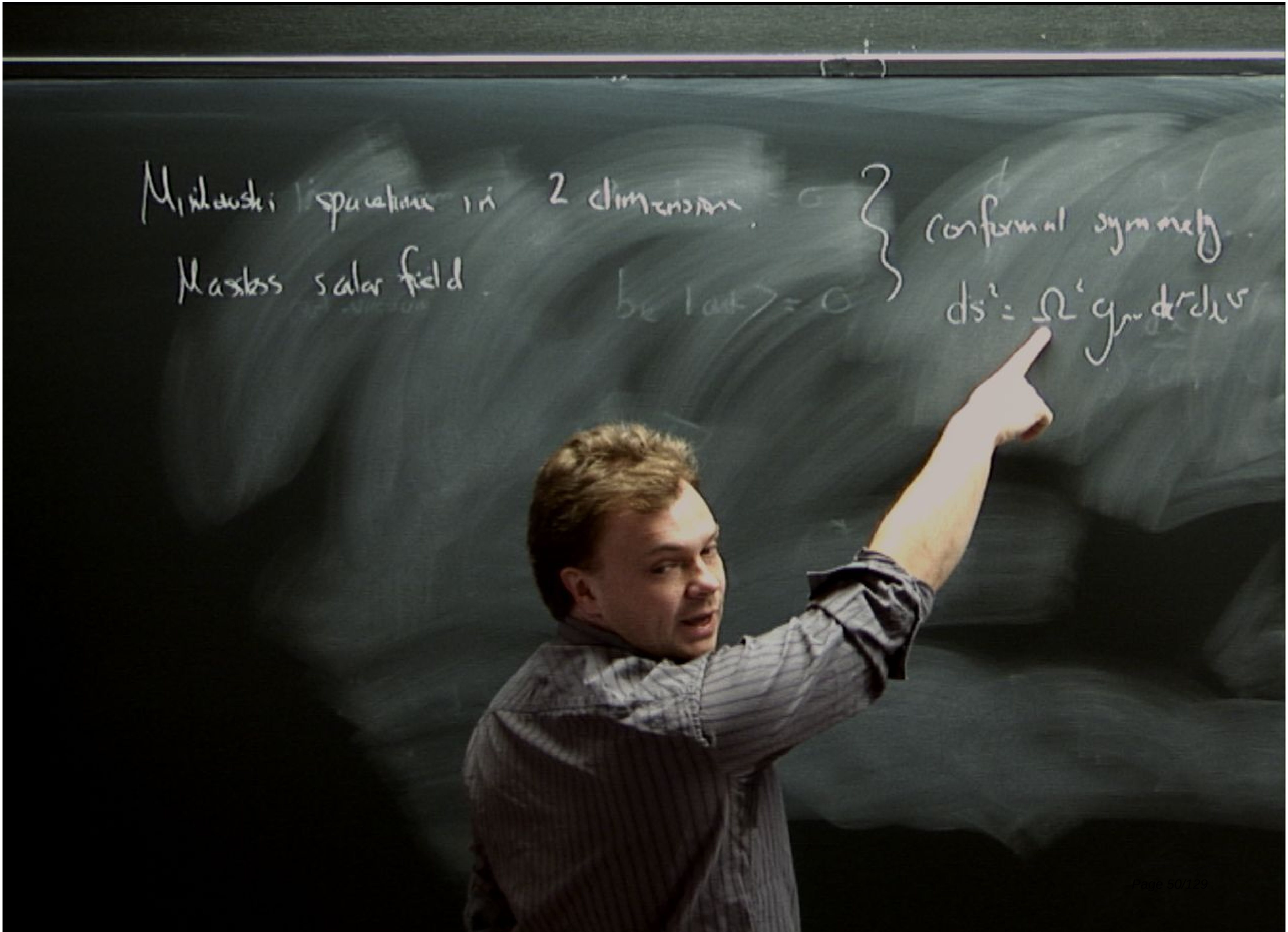
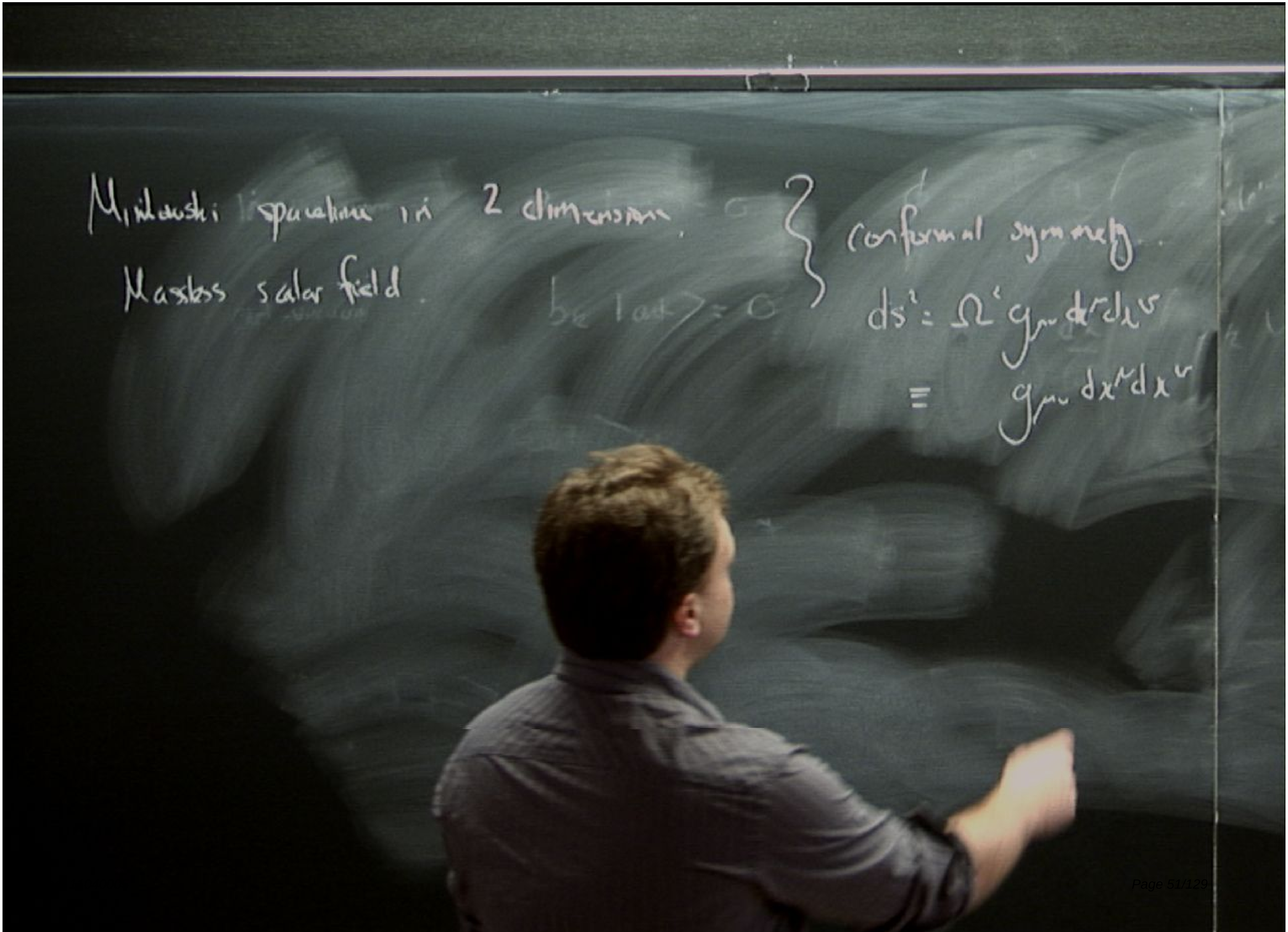
Black hole horizons — 'Vacuum state'
filled with particles ~ thermal distribution
Hawking radiation
Cosmological horizons — 'vacuum state' — Bunch-Davies vacuum
— thermal spectrum Gibbons-Hawking radiation
Acceleration horizon — Unruh effect 'vacuum state'
— filled with thermal spectrum

Minkowski spacetime in 2 dimensions.

Massless scalar field.


Minkowski spacetime in 2 dimension.
Massless scalar field
conformal symmetry
$ds^2 = \Omega^2 g_{\mu\nu} dx^\mu dx^\nu$


Minkowski spacetime in 2 dimensions
Massless scalar field
conformal symmetry.
ds^2 = Ω^2 g_μν dx^μ dx^ν
≡ g_μν dx^μ dx^ν

Minkowski spacetime in 2 dimensions.
Massless scalar field.

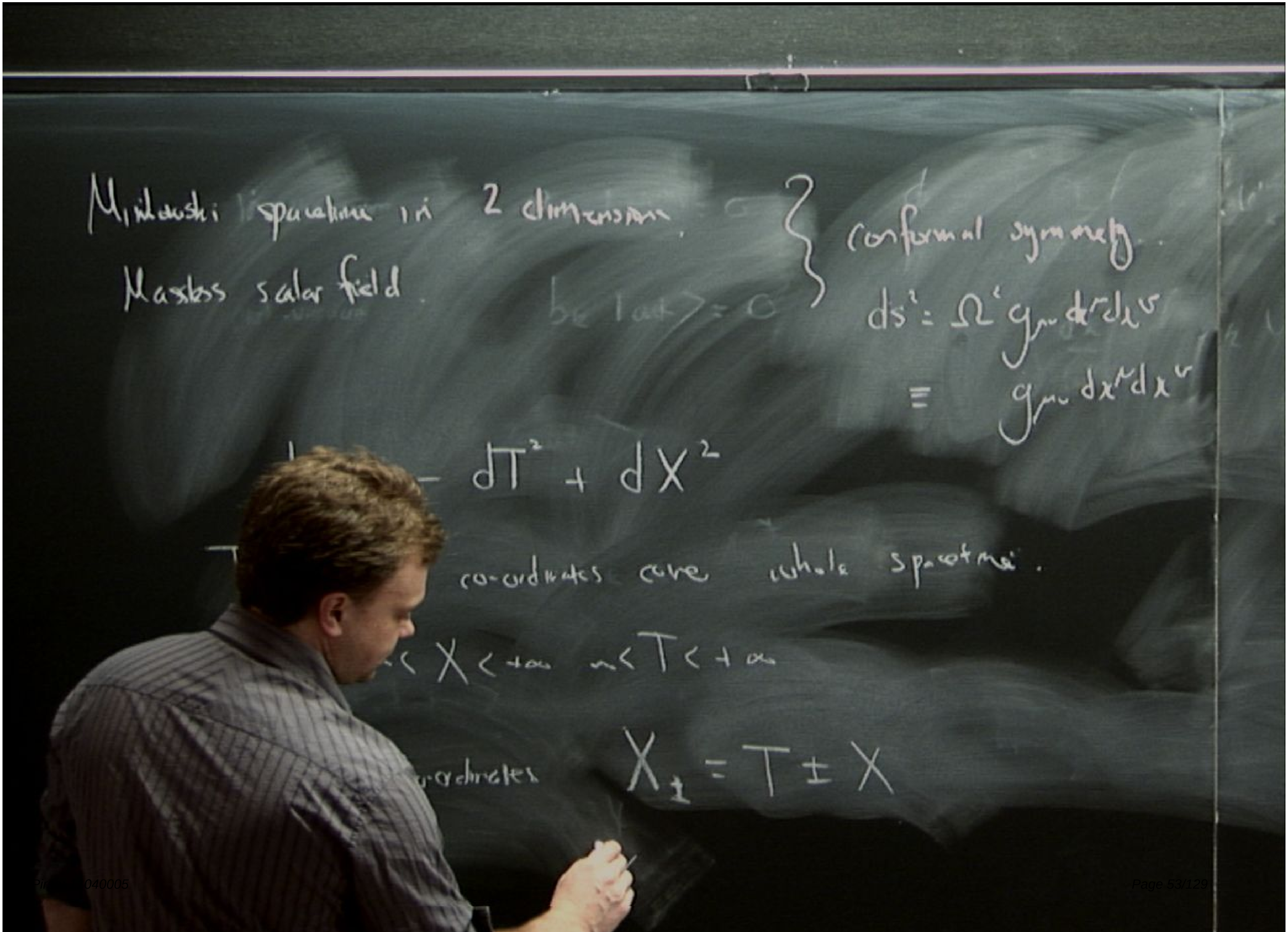
conformal symmetry.

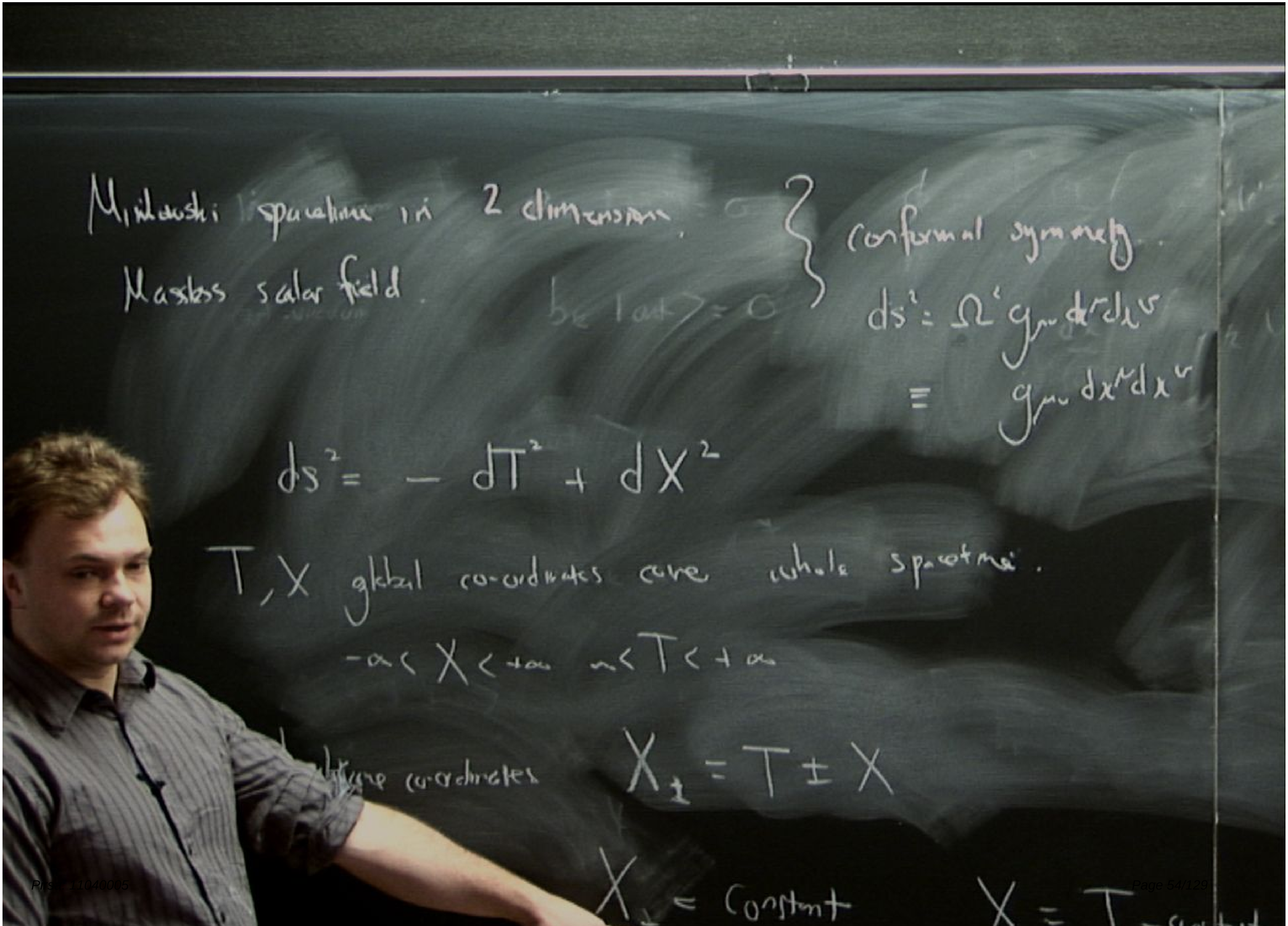
$$ds^2 = \Omega^2 g_{\mu\nu} dx^\mu dx^\nu \equiv \tilde{g}_{\mu\nu} dx^\mu dx^\nu$$

$$ds^2 = -dT^2 + dX^2$$

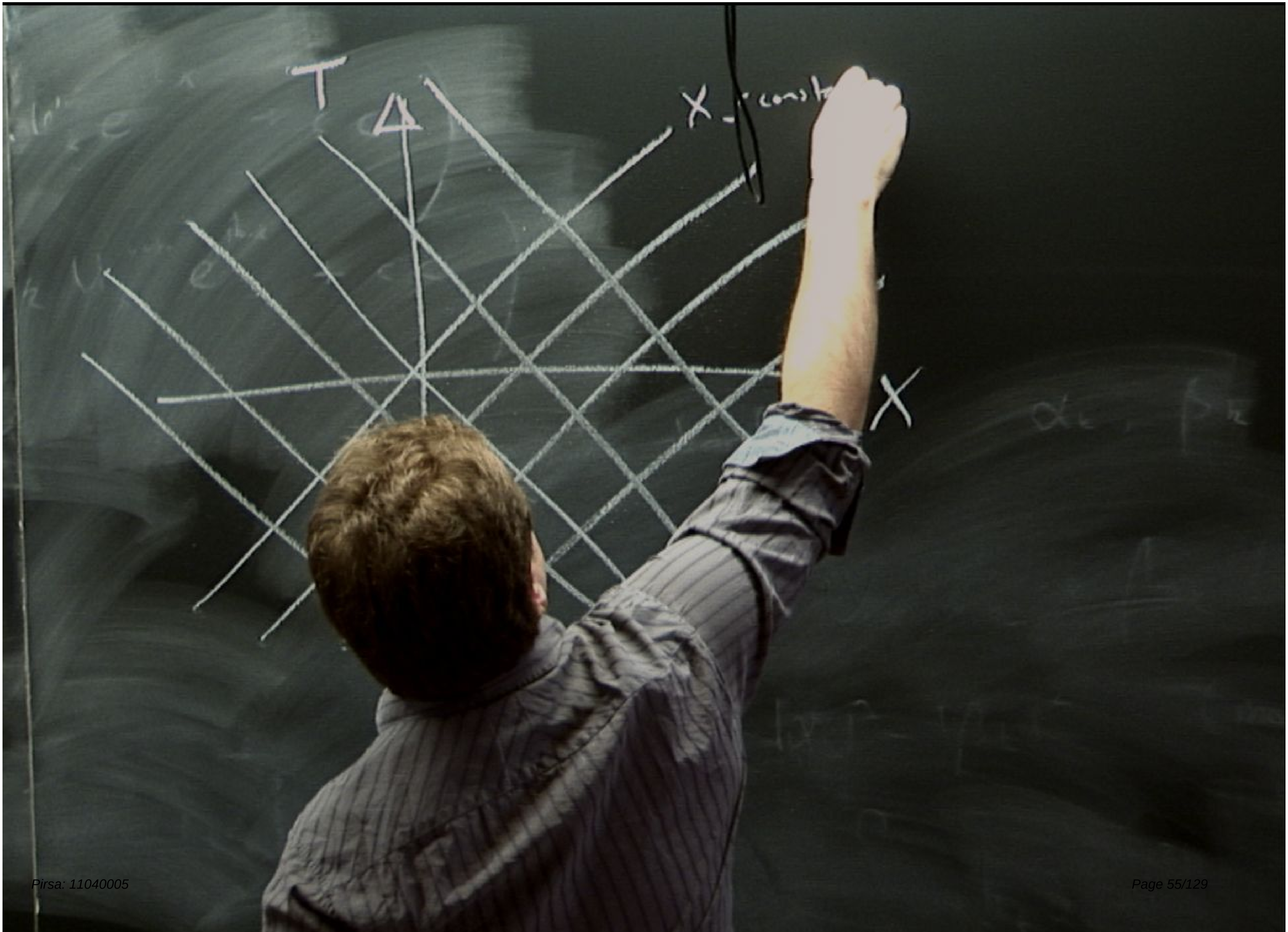
T, X global co-ordinates cover whole spacetime.

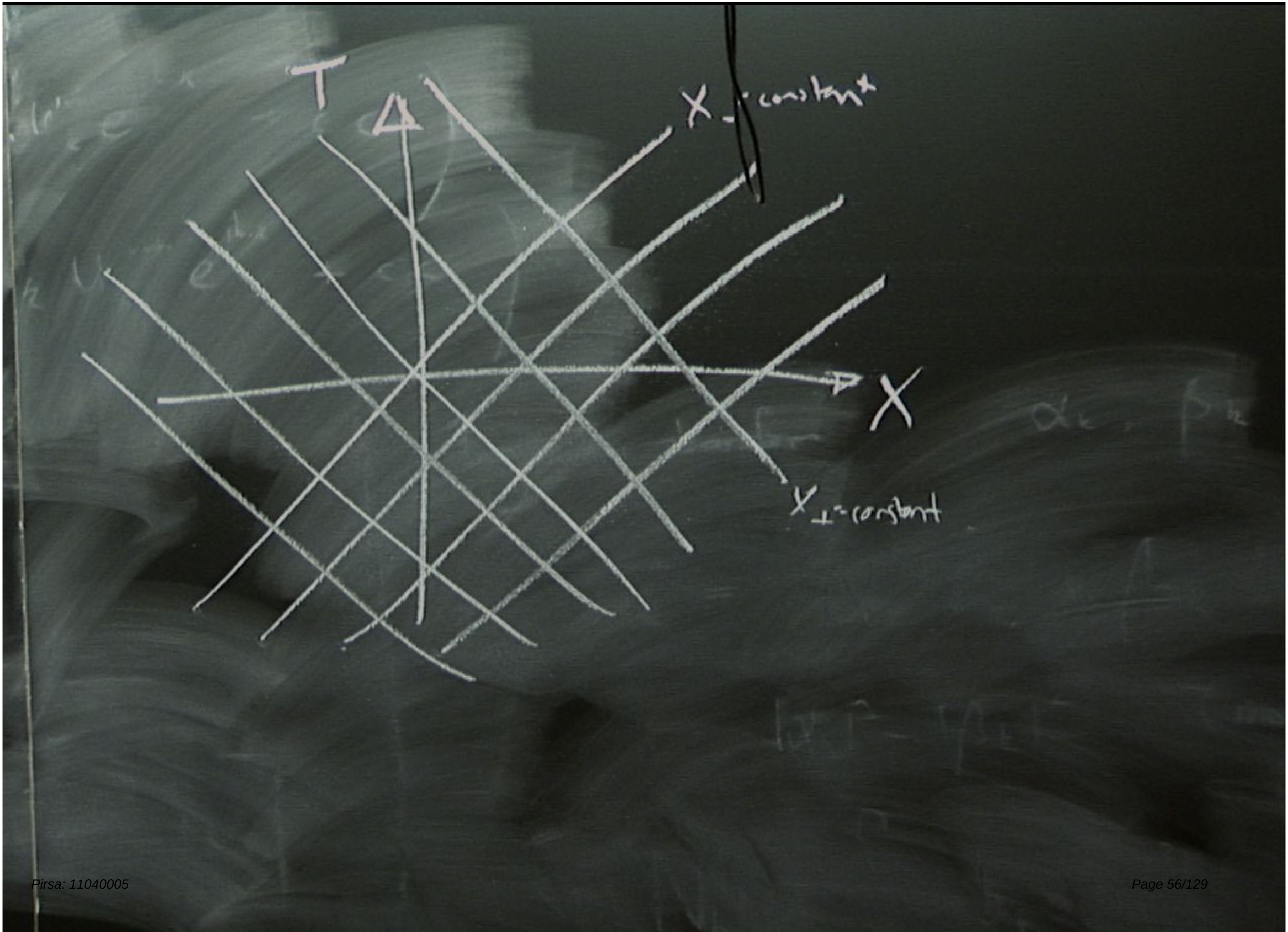
$-\infty < X < +\infty \quad -\infty < T < +\infty$


Minkowski spacetime in 2 dimensions.
Massless scalar field
conformal symmetry.
ds² = Ω² gμν dxμ dxν
≡ gμν dxμ dxν
− dT² + dX²
co-ordinates cover whole spacetime.
X < +∞
< T < +∞
co-ordinates
X± = T ± X

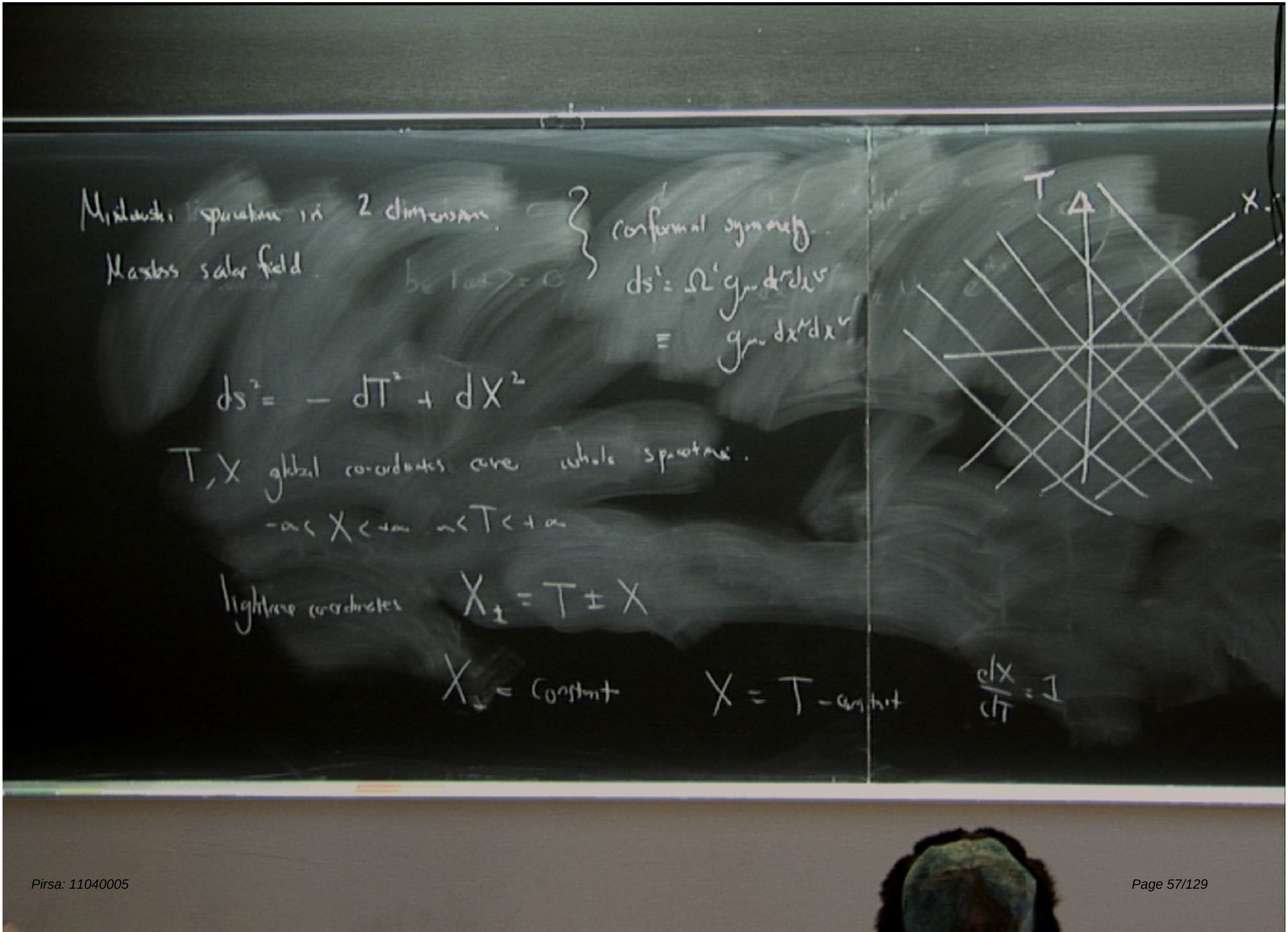


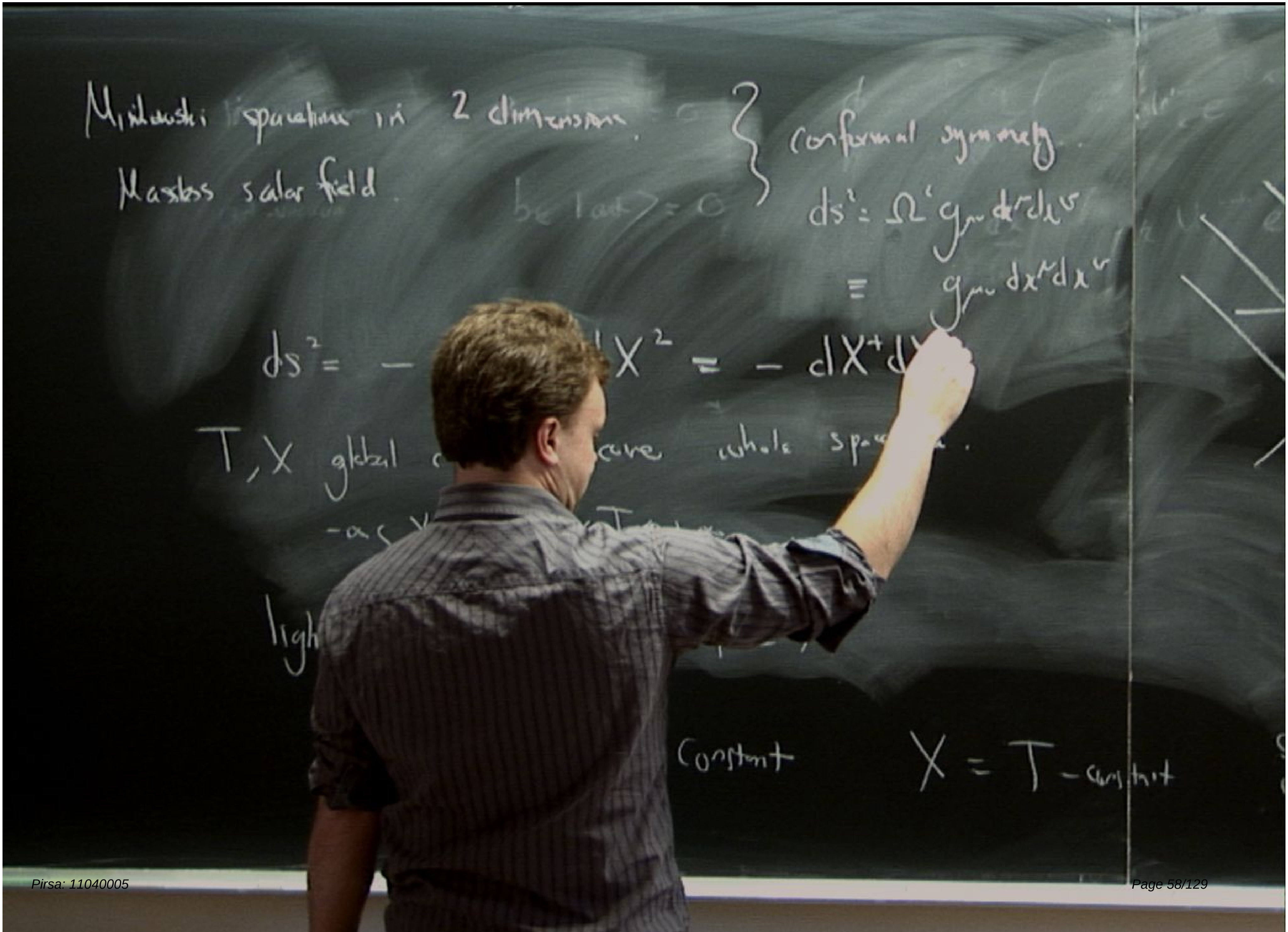
Minkowski spacetime in 2 dimensions
Massless scalar field
conformal symmetry
ds² = Ω² g_μν dx^μ dx^ν
≡ g̃_μν dx^μ dx^ν
ds² = − dT² + dX²
T, X global co-ordinates cover whole spacetime.
−∞ < X < +∞ −∞ < T < +∞
lightcone co-ordinates X± = T ± X
X± = Constant
X = T −

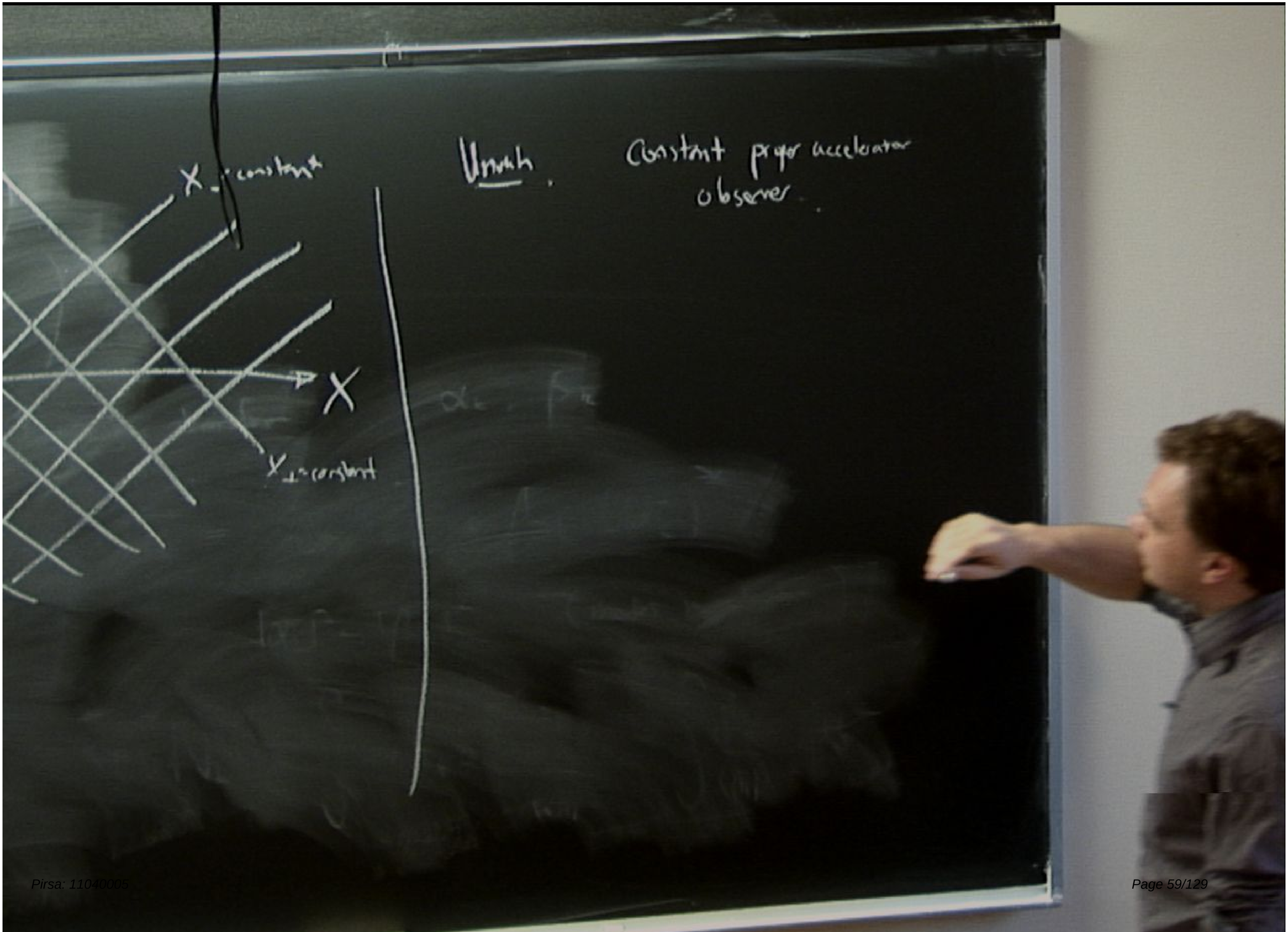

T
X_ = const
X

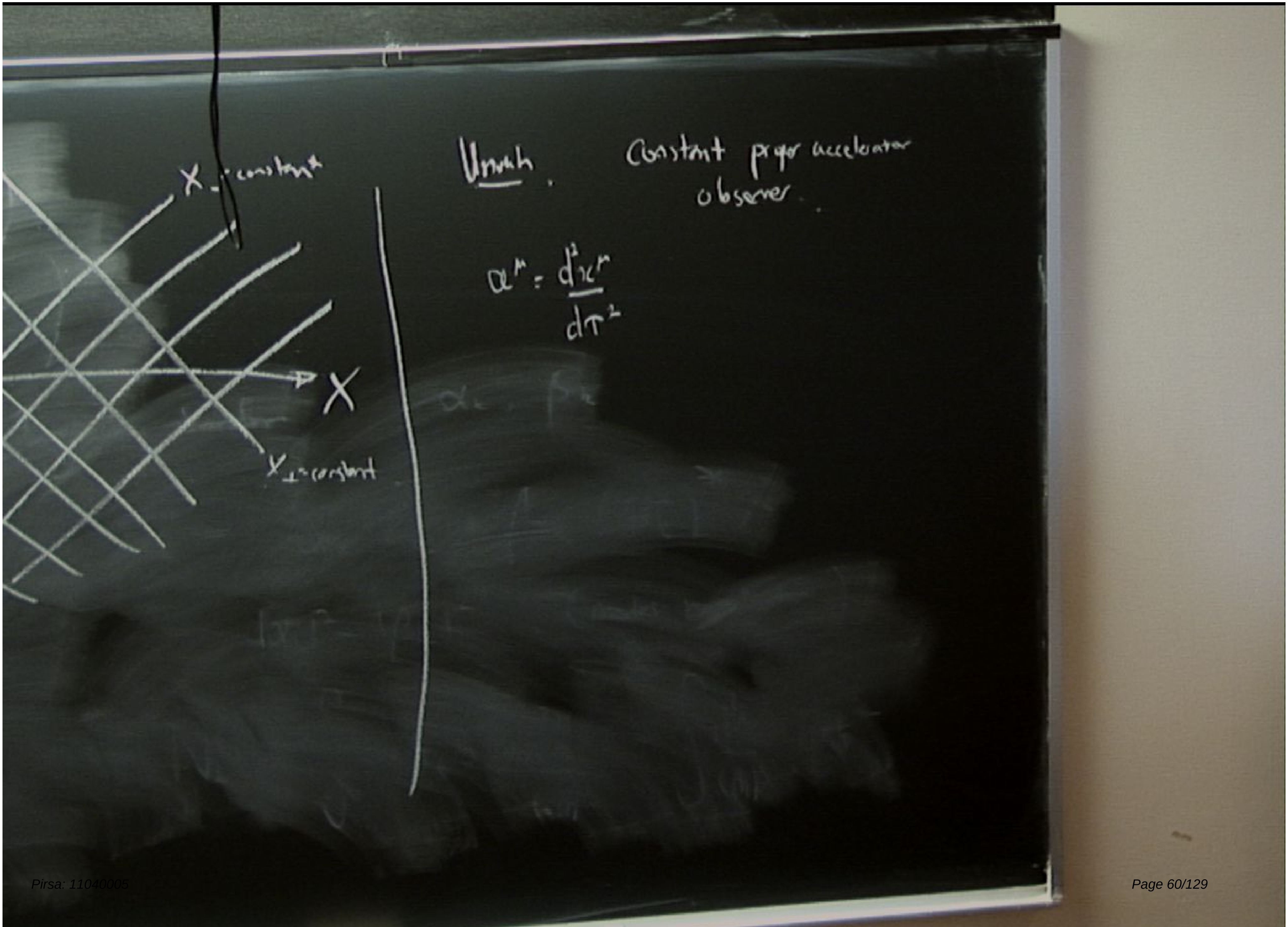


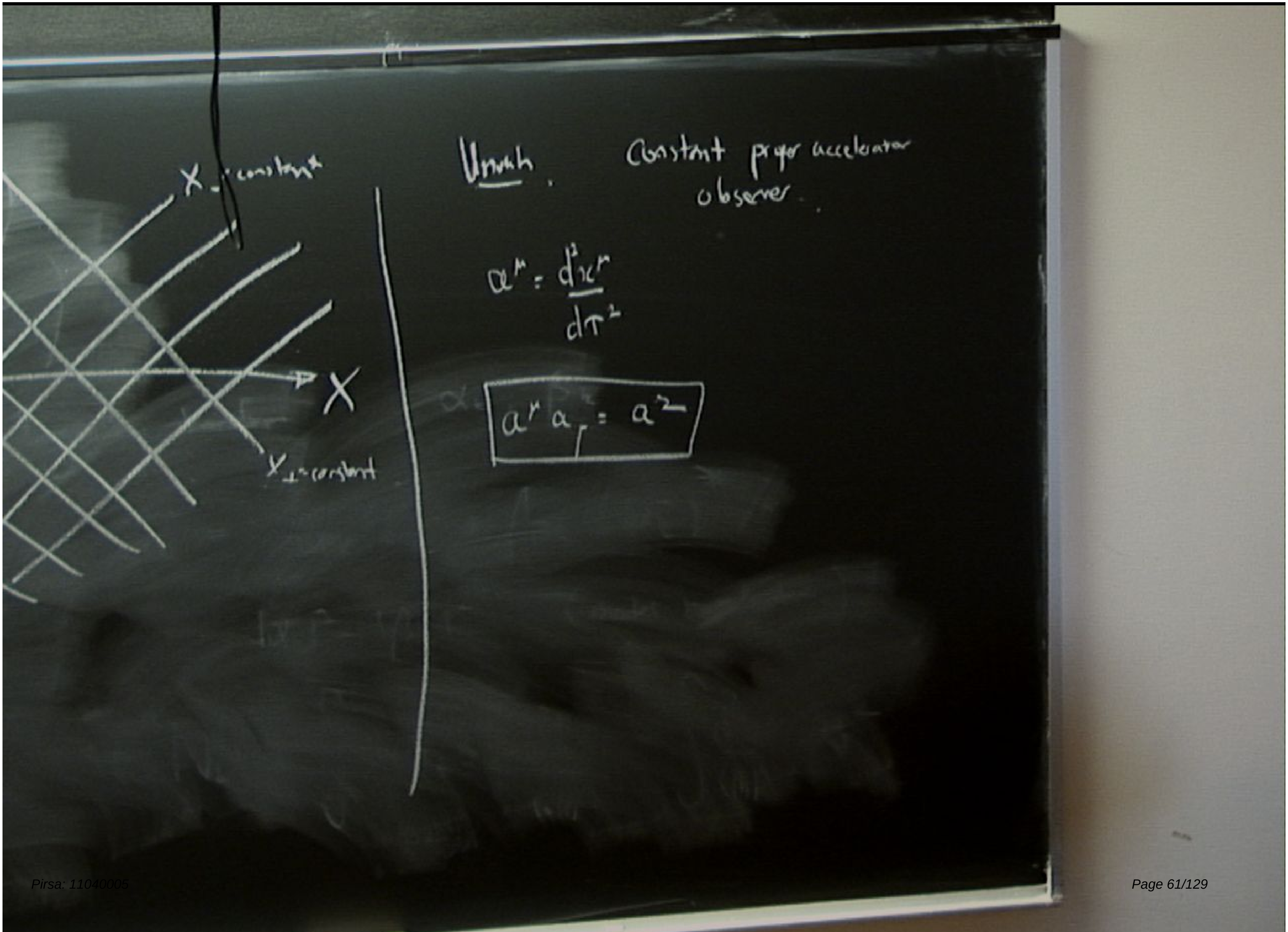
T
$X_- = $ constant
X
$X_+ = $ constant

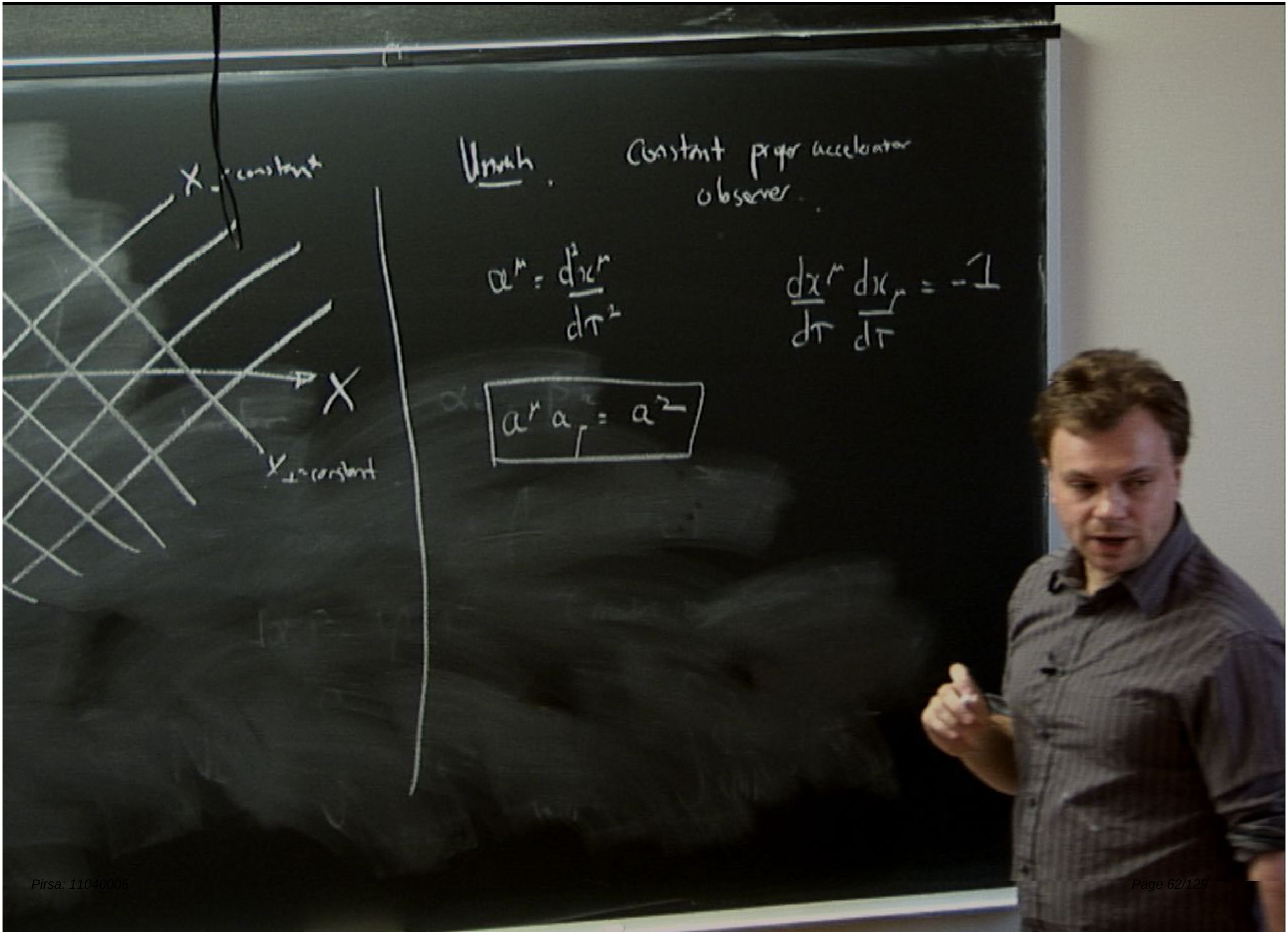

Minkowski spacetime in 2 dimensions
Massless scalar field
conformal symmetry
ds² = Ω² gμν dxμ dxν
≡ gμν dxμ dxν
ds² = − dT² + dX²
T, X global co-ordinates cover whole spacetime.
−∞ < X < +∞ −∞ < T < +∞
lightcone coordinates X± = T ± X
X₋ = Constant
X = T − constant
dX/dT = 1
T
X

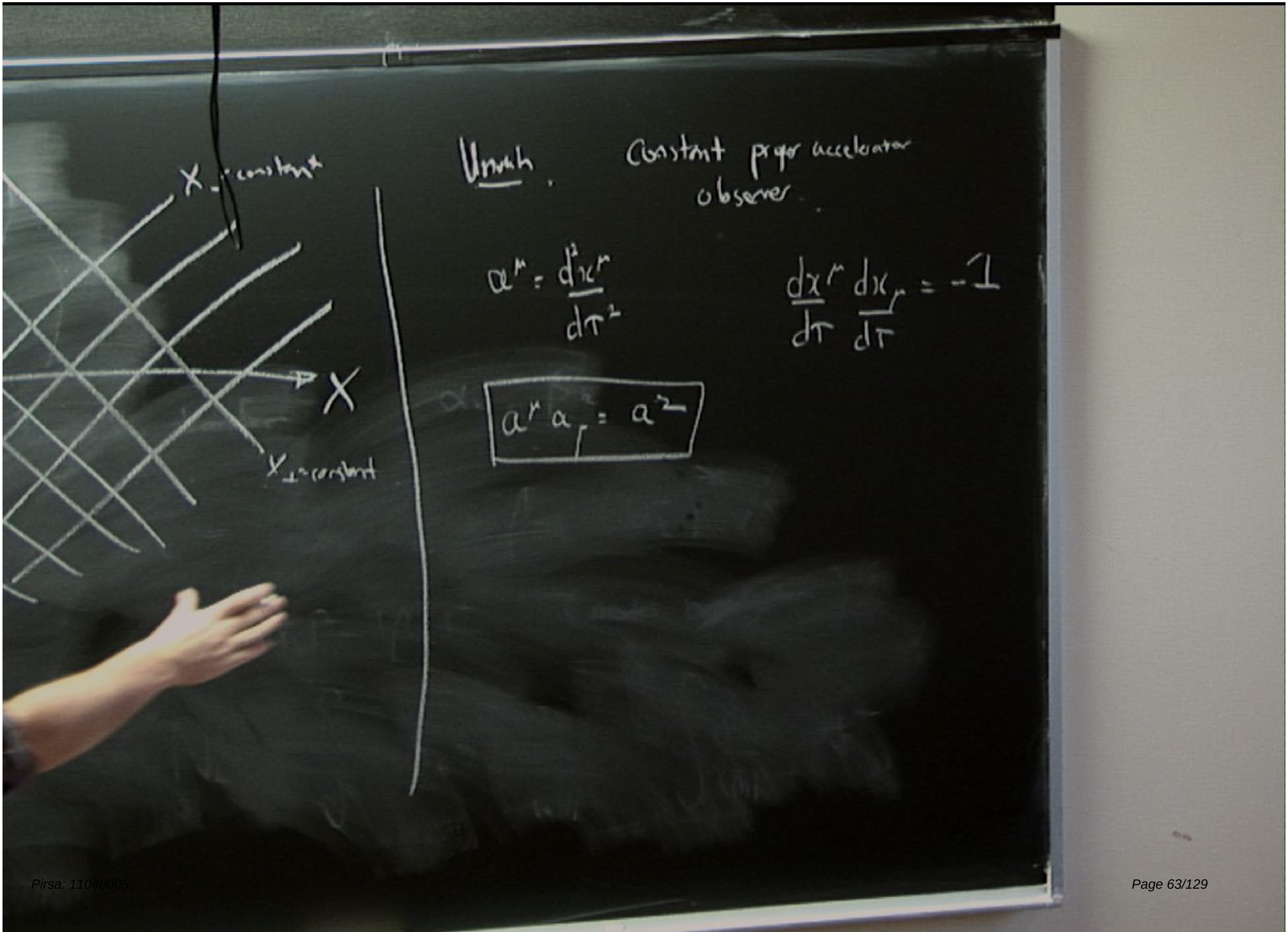

Minkowski spacetime in 2 dimensions
Massless scalar field
conformal symmetry
ds² = Ω² g dx dx
= g dx dx
whole space
Constant
X = T - constant

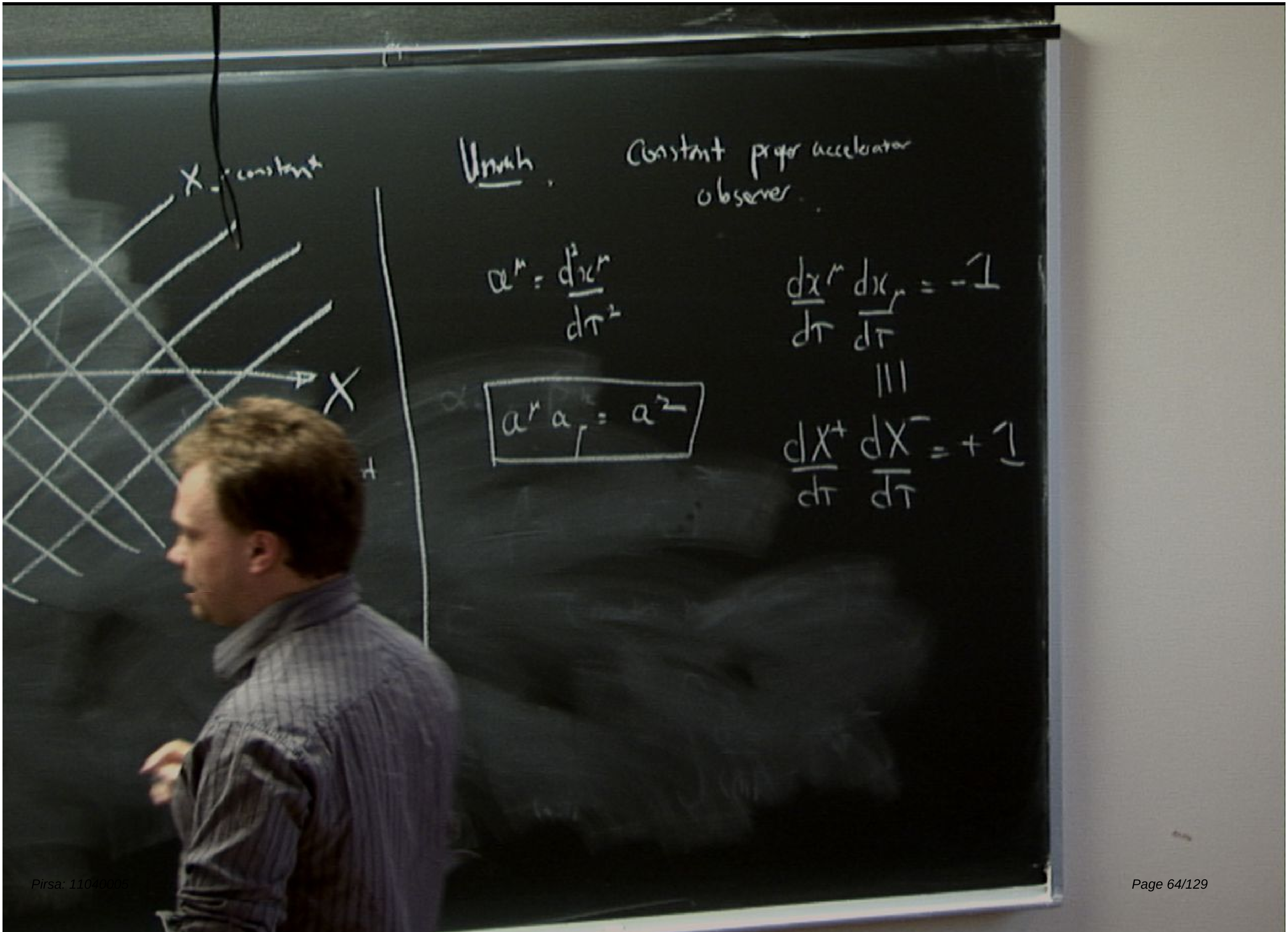

Unruh.
Constant proper accelerator
observer.
X

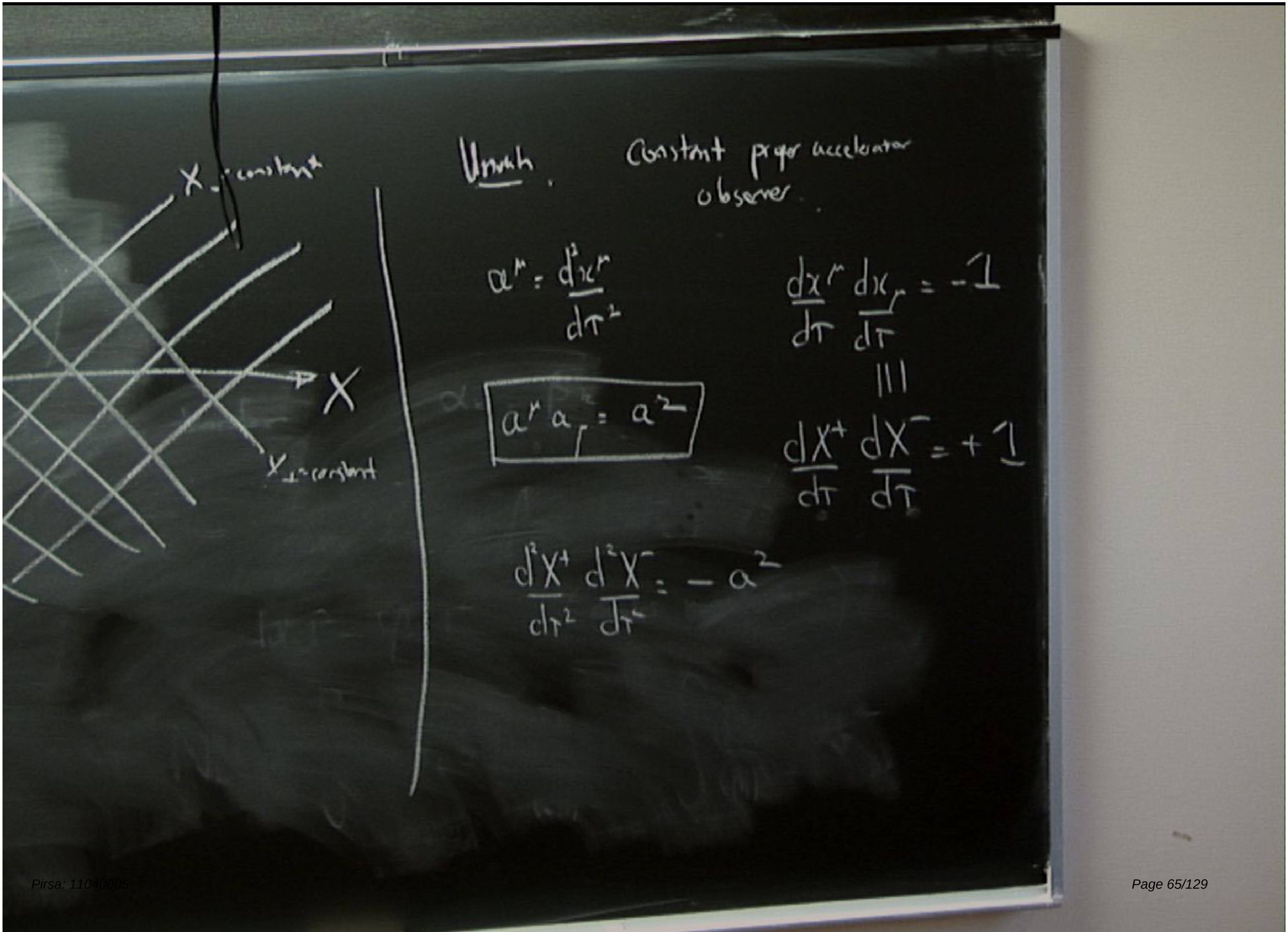

Unruh
Constant proper accelerator
observer
a^μ = d²x^μ / dτ²
X_- = constant
X
X_+ = constant

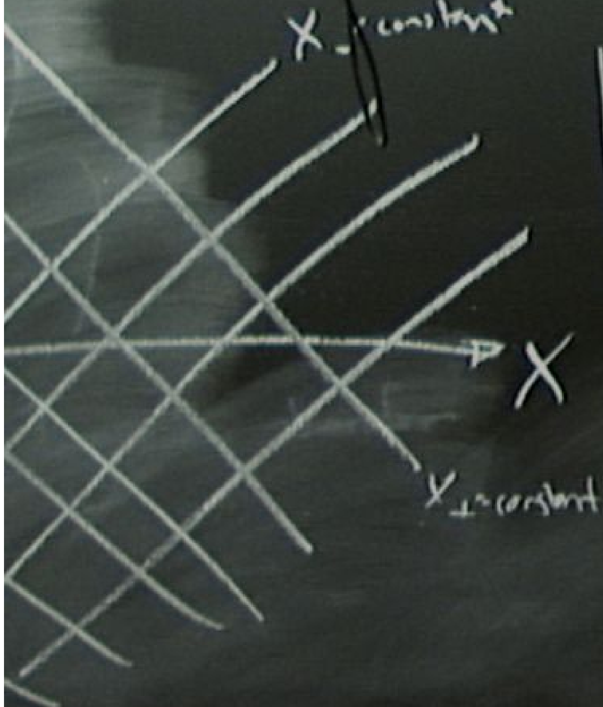

Unruh
Constant proper acceleration observer
X_+ = constant
X
X_⊥ = constant
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$


Unruh.
Constant proper accelerator observer.
X_- = constant
X
X_+ = constant
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = -1$
$a^\mu a_\mu = a^2$


Unruh.
Constant proper accelerator observer
X_ = constant
X
X_+ = constant
a^μ = d²x^μ / dτ²
dx^μ/dτ dx_μ/dτ = -1
a^μ a_μ = a²


Unruh.
Constant proper acceleration observer.
X_ = constant
X
$a^\mu = \frac{d^2 x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$
$\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = -1$
$\frac{dX^+}{d\tau}\frac{dX^-}{d\tau} = +1$


Unruh.
Constant proper accelerator observer.
X_+ = constant
X
X_- = constant
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$
$\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = -1$
$\frac{dX^+}{d\tau}\frac{dX^-}{d\tau} = +1$
$\frac{d^2X^+}{d\tau^2}\frac{d^2X^-}{d\tau^2} = -a^2$



Unruh. Constant proper acceleration observer.

$$a^\mu = \frac{d^2x^\mu}{d\tau^2}$$

$$\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = -1$$

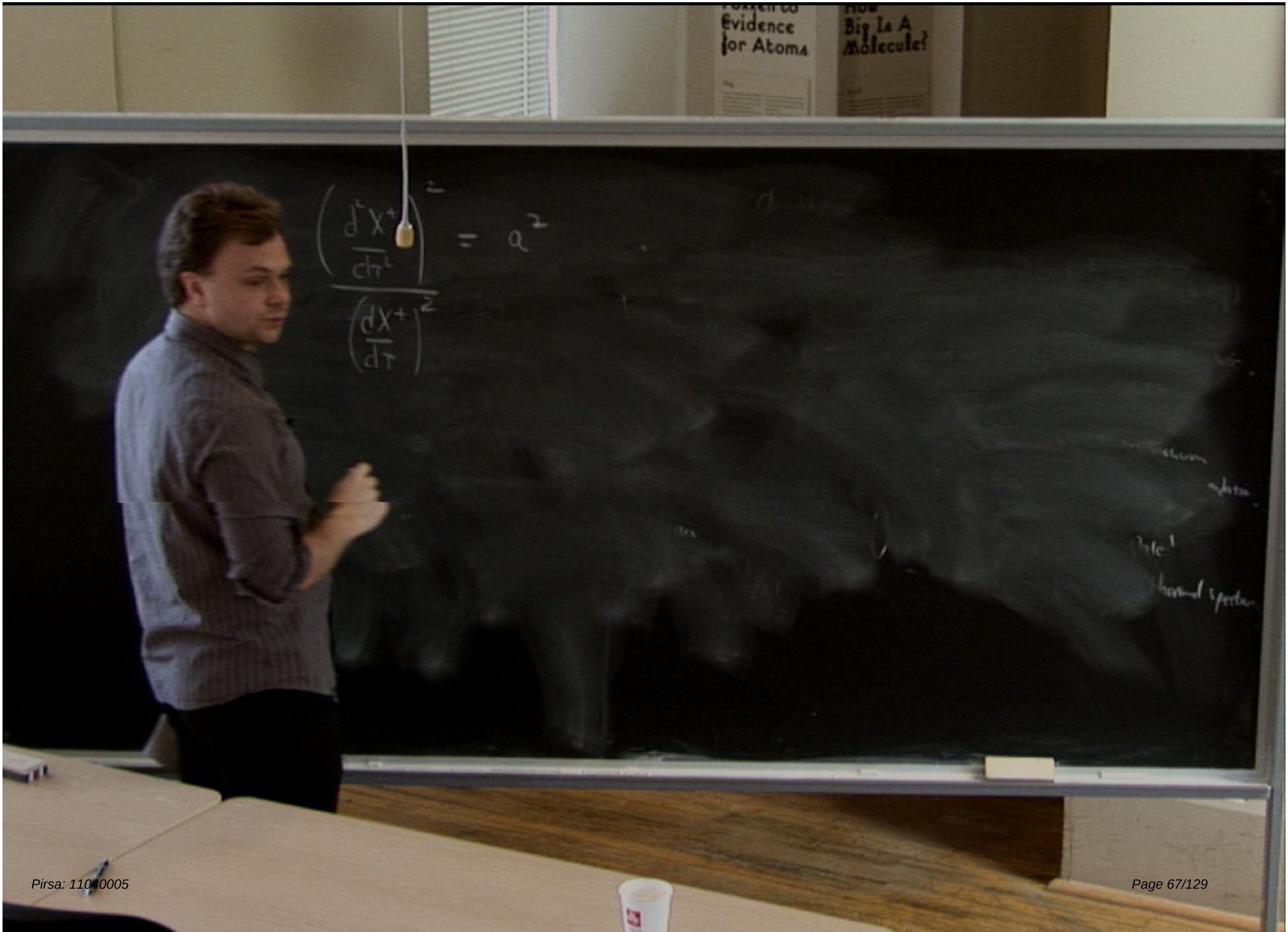
$$\equiv$$

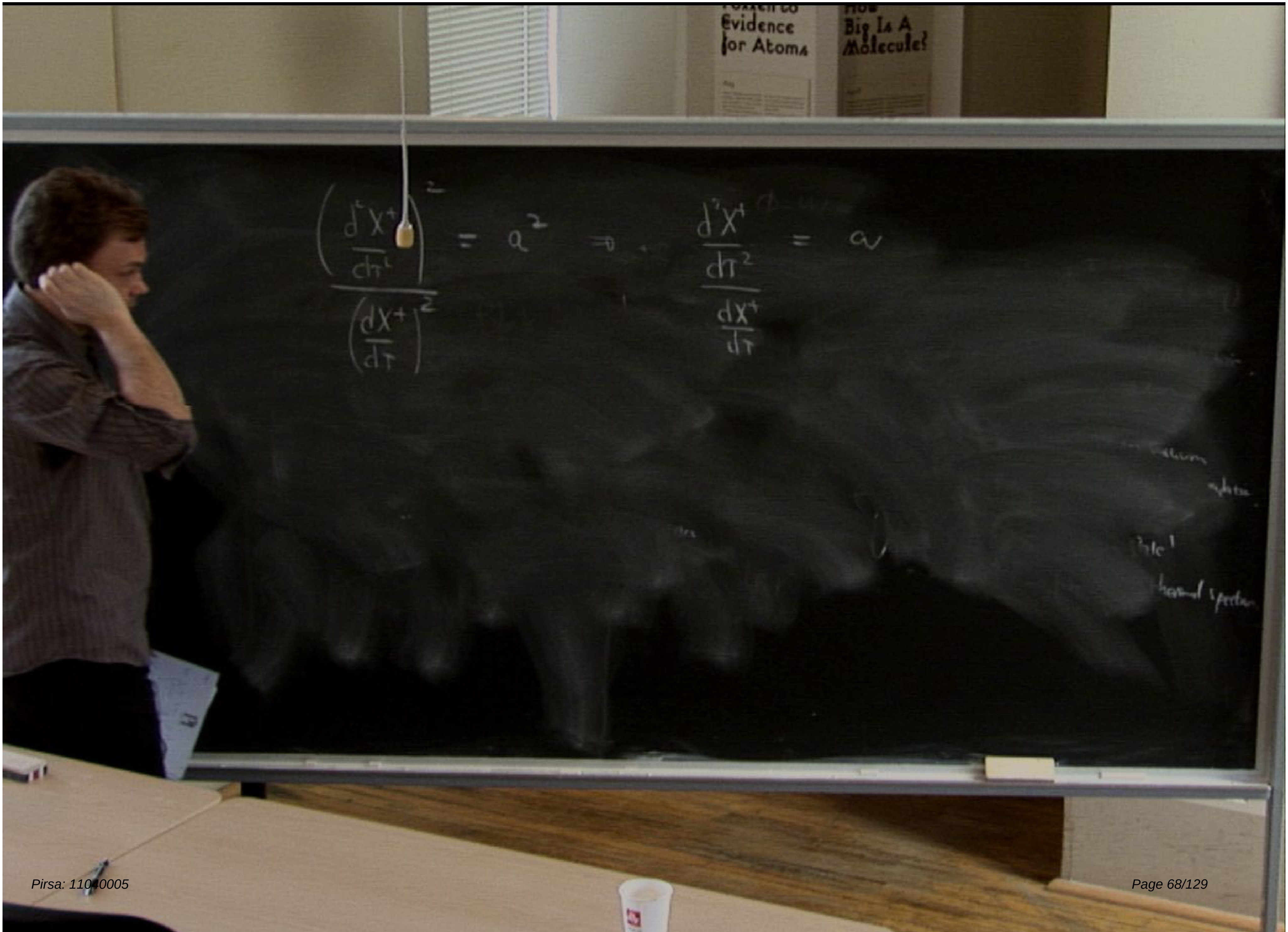
$$\frac{dX^+}{d\tau}\frac{dX^-}{d\tau} = +1$$

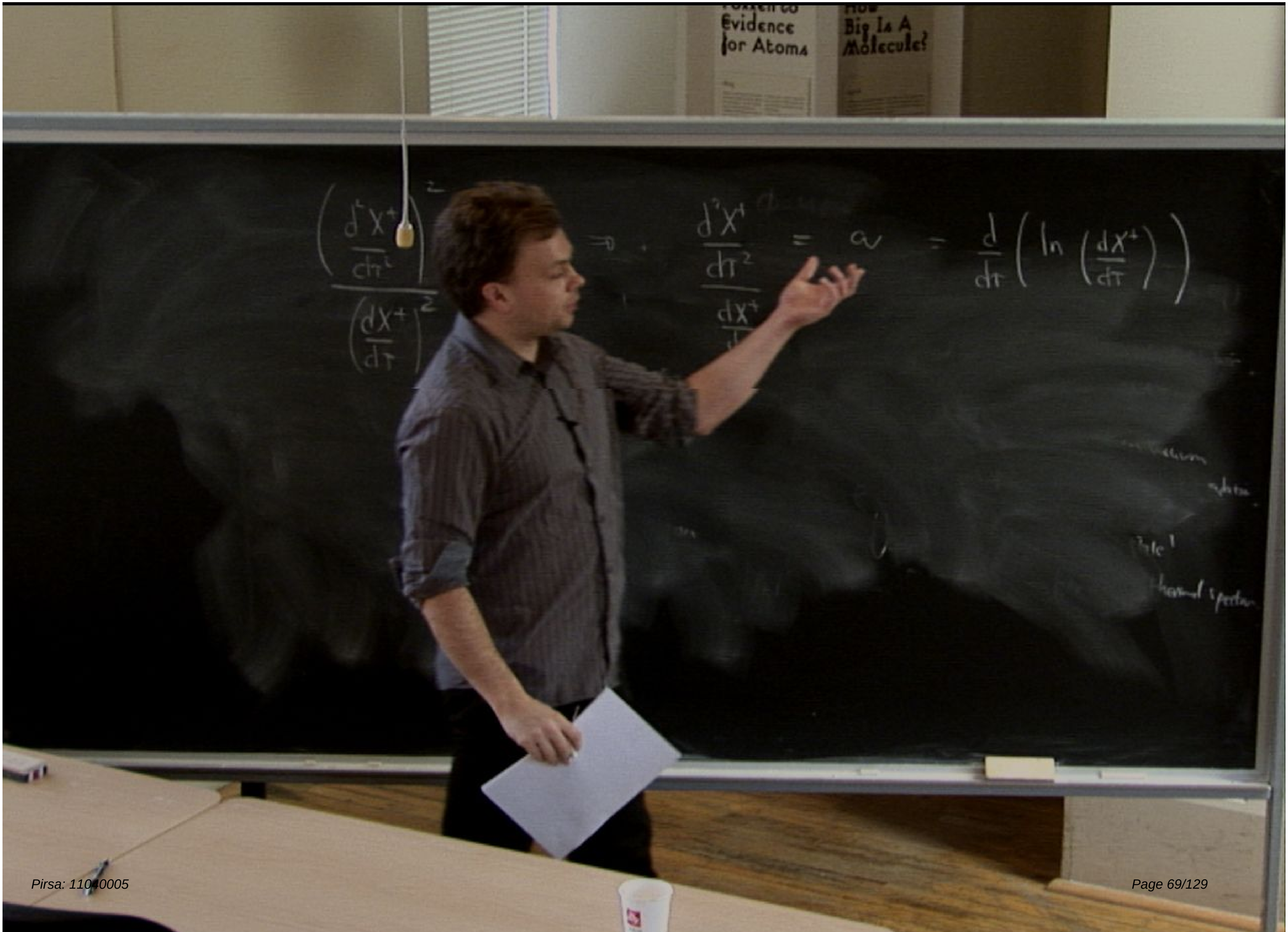
$$\boxed{a^\mu a_\mu = a^2}$$

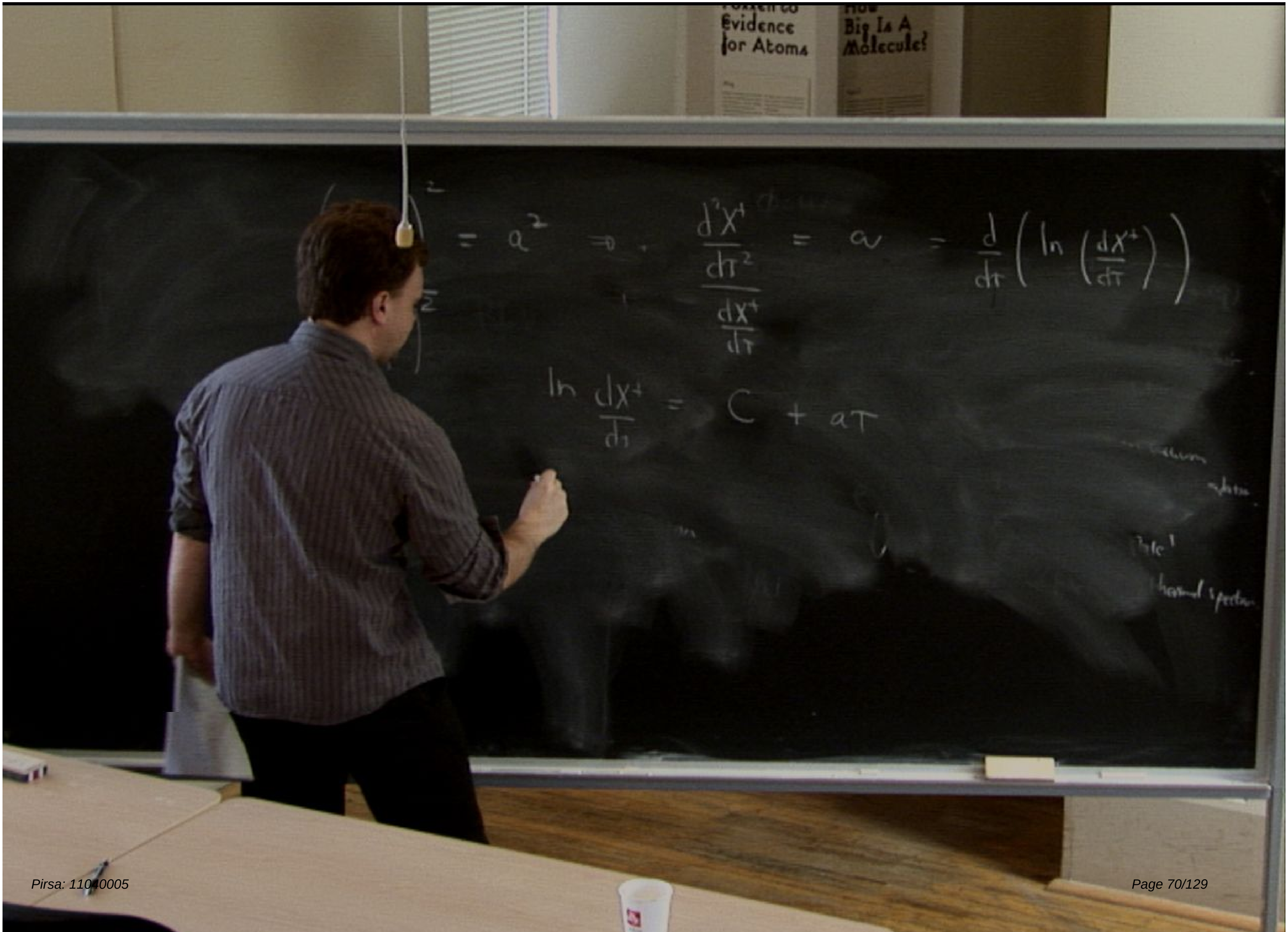
$$\frac{d^2X^+}{d\tau^2}\frac{d^2X^-}{d\tau^2} = -a^2$$

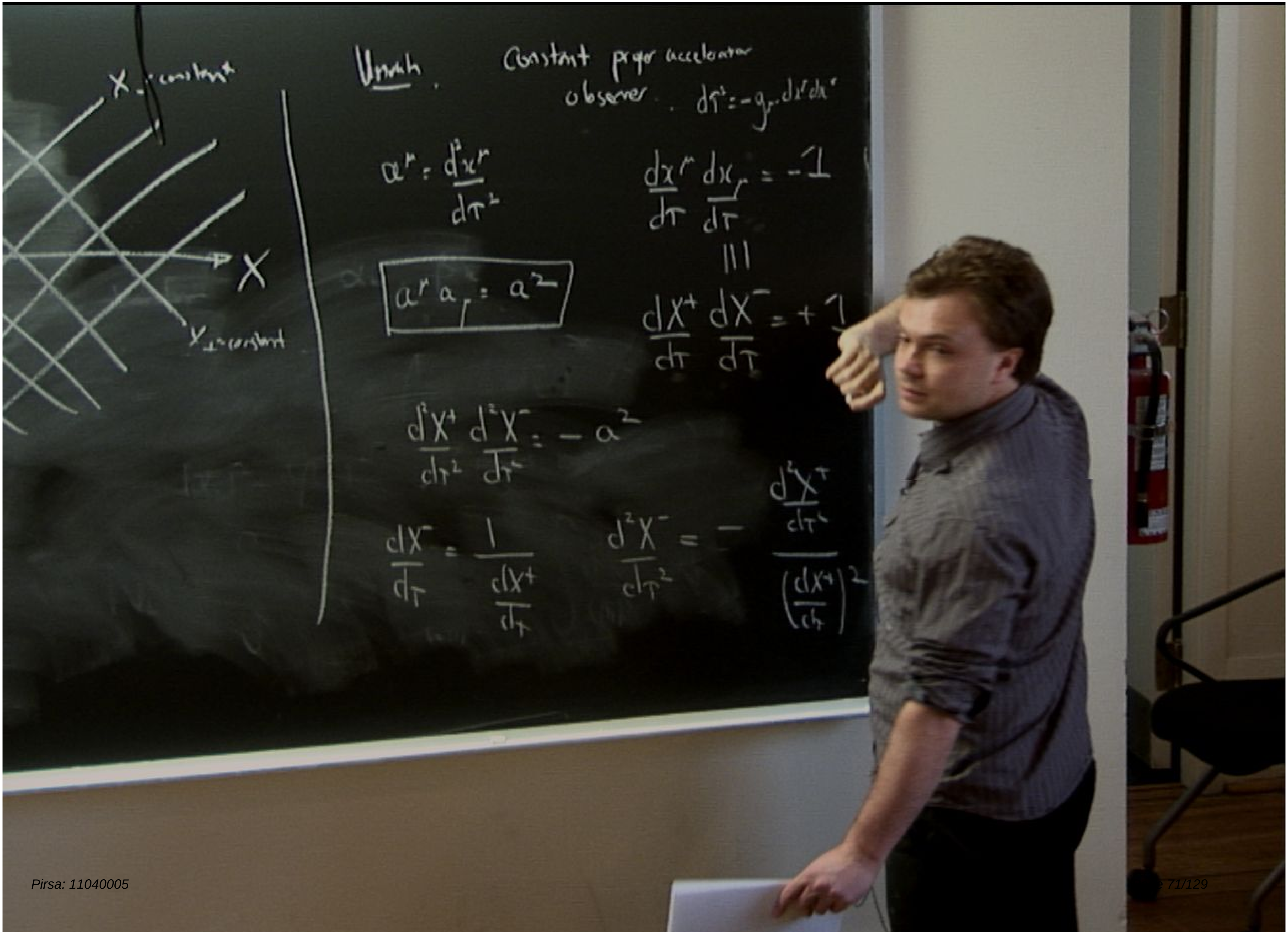
$$\frac{dX^-}{d\tau} = \frac{1}{\frac{dX^+}{d\tau}} \qquad \frac{d^2X^-}{d\tau^2} = -\frac{\frac{d^2X^+}{d\tau^2}}{\left(\frac{dX^+}{d\tau}\right)^2}$$


Evidence
for Atoms
Big Is A
Molecule?
$$\frac{\left(\frac{d^2X^+}{d\tau^2}\right)^2}{\left(\frac{dX^+}{d\tau}\right)^2} = a^2$$

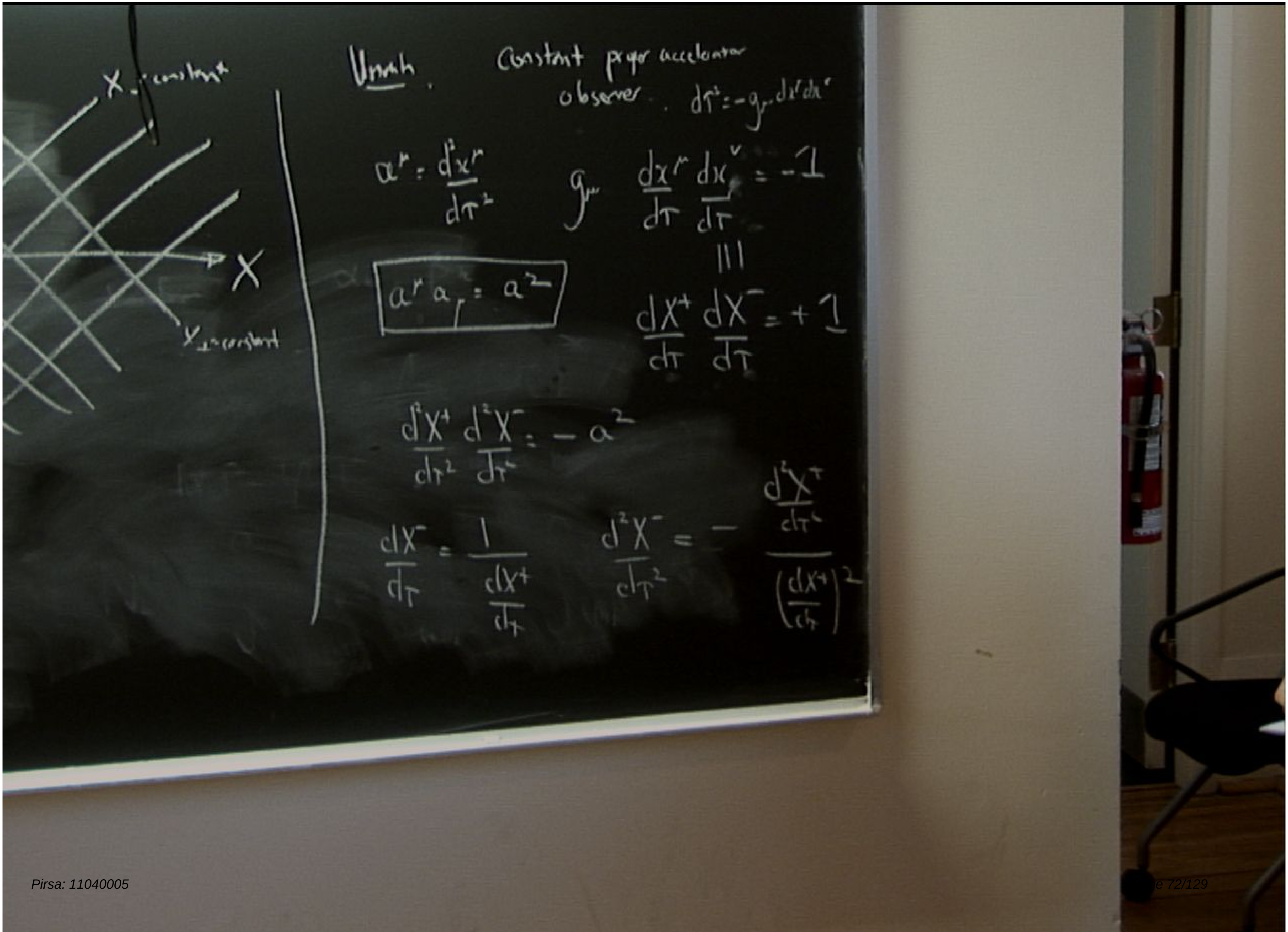

Evidence
for Atoms
Big Is A
Molecule?


Evidence
for Atoms
Big Is A
Molecule?

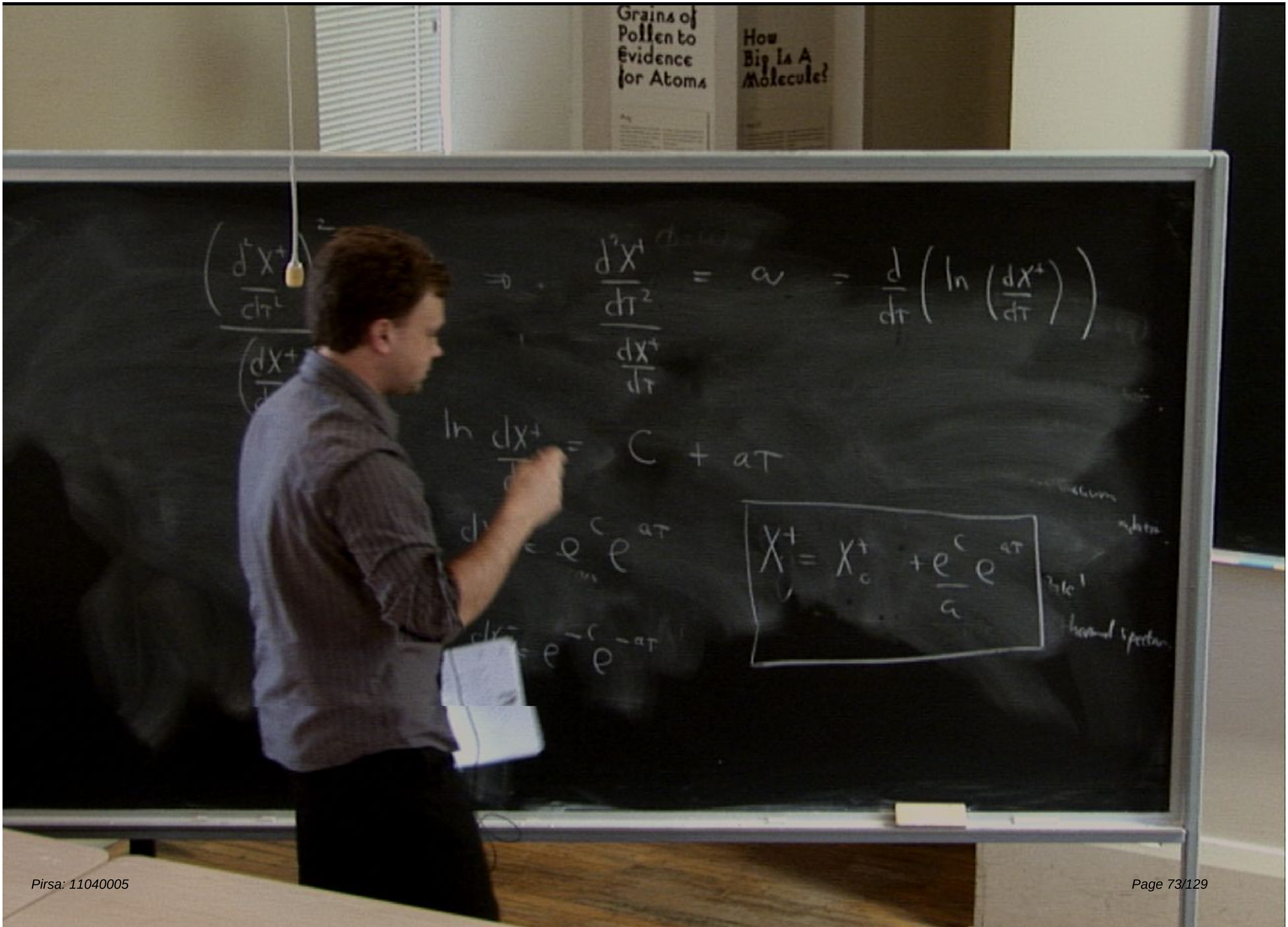

Evidence
for Atoms
Big Is A
Molecule?

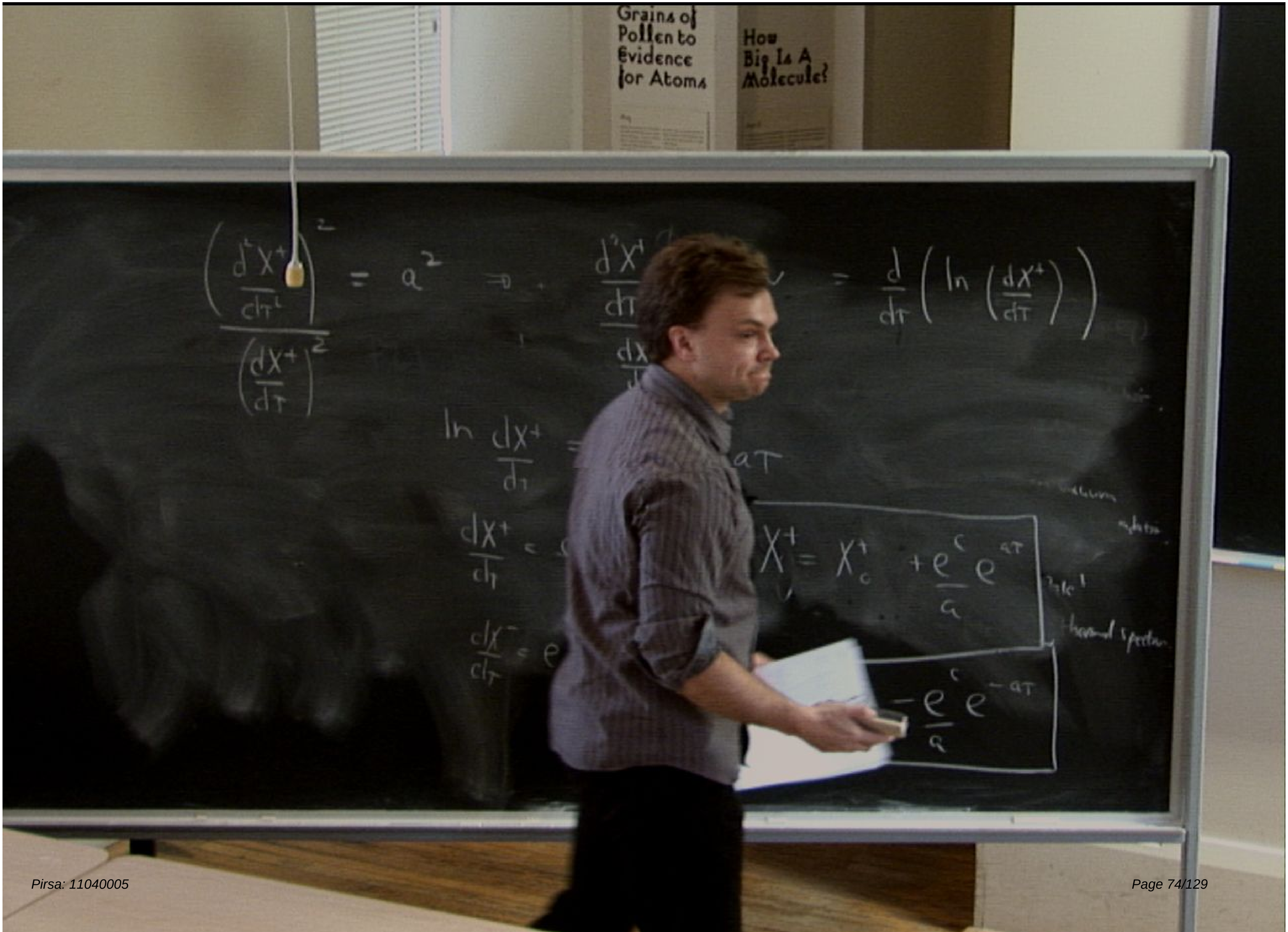


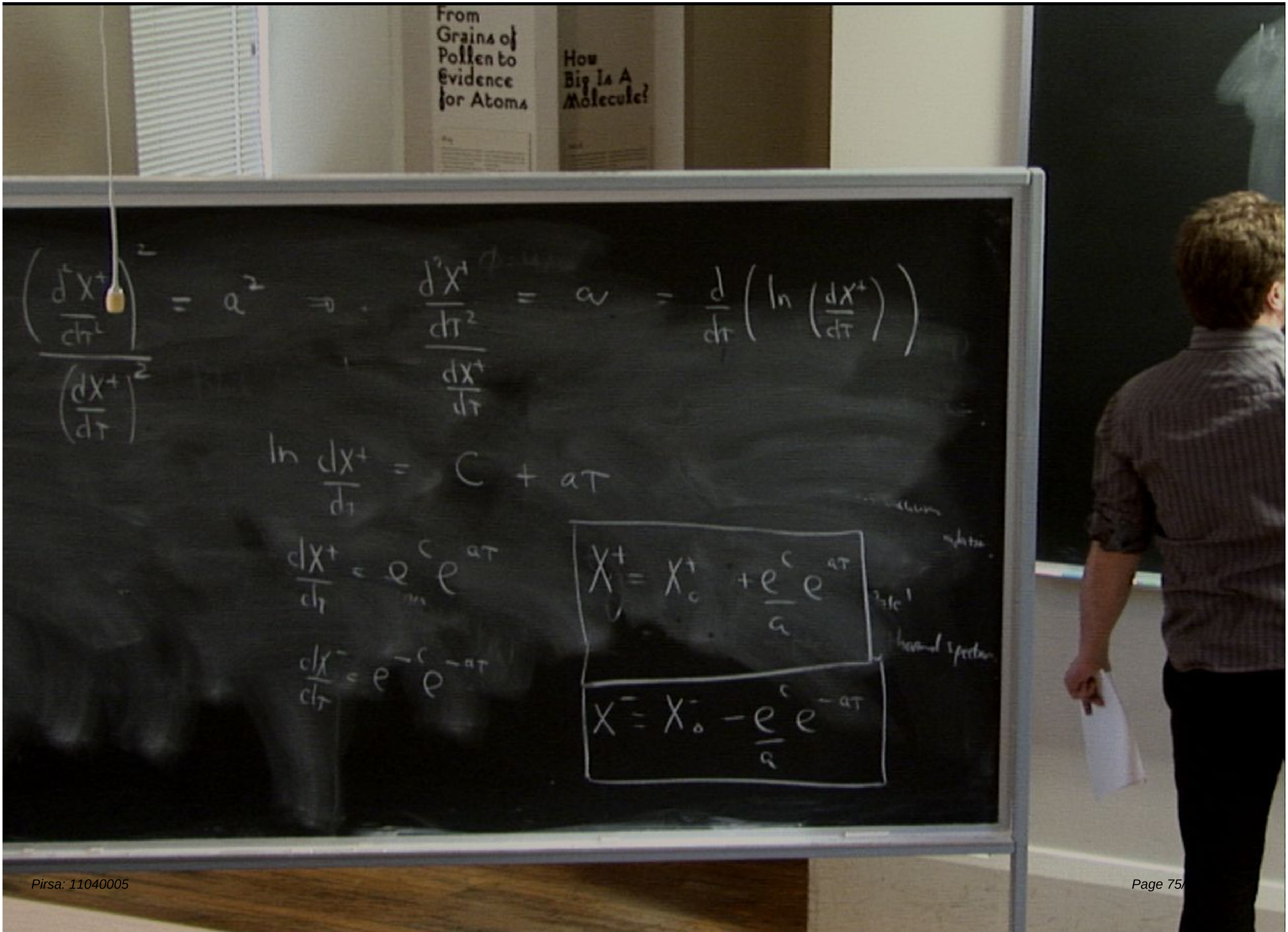
Unruh.
Constant proper acceleration observer.
$d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu$
$X_- = $ constant
$X_+ = $ constant
X
$a^\mu = \frac{d^2 x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$
$\frac{dx^\mu}{d\tau} \frac{dx_\mu}{d\tau} = -1$
$\frac{dX^+}{d\tau} \frac{dX^-}{d\tau} = +1$
$\frac{d^2X^+}{d\tau^2} \frac{d^2X^-}{d\tau^2} = -a^2$
$\frac{dX^-}{d\tau} = \frac{1}{\frac{dX^+}{d\tau}}$
$\frac{d^2X^-}{d\tau^2} = -\frac{\frac{d^2X^+}{d\tau^2}}{\left(\frac{dX^+}{d\tau}\right)^2}$

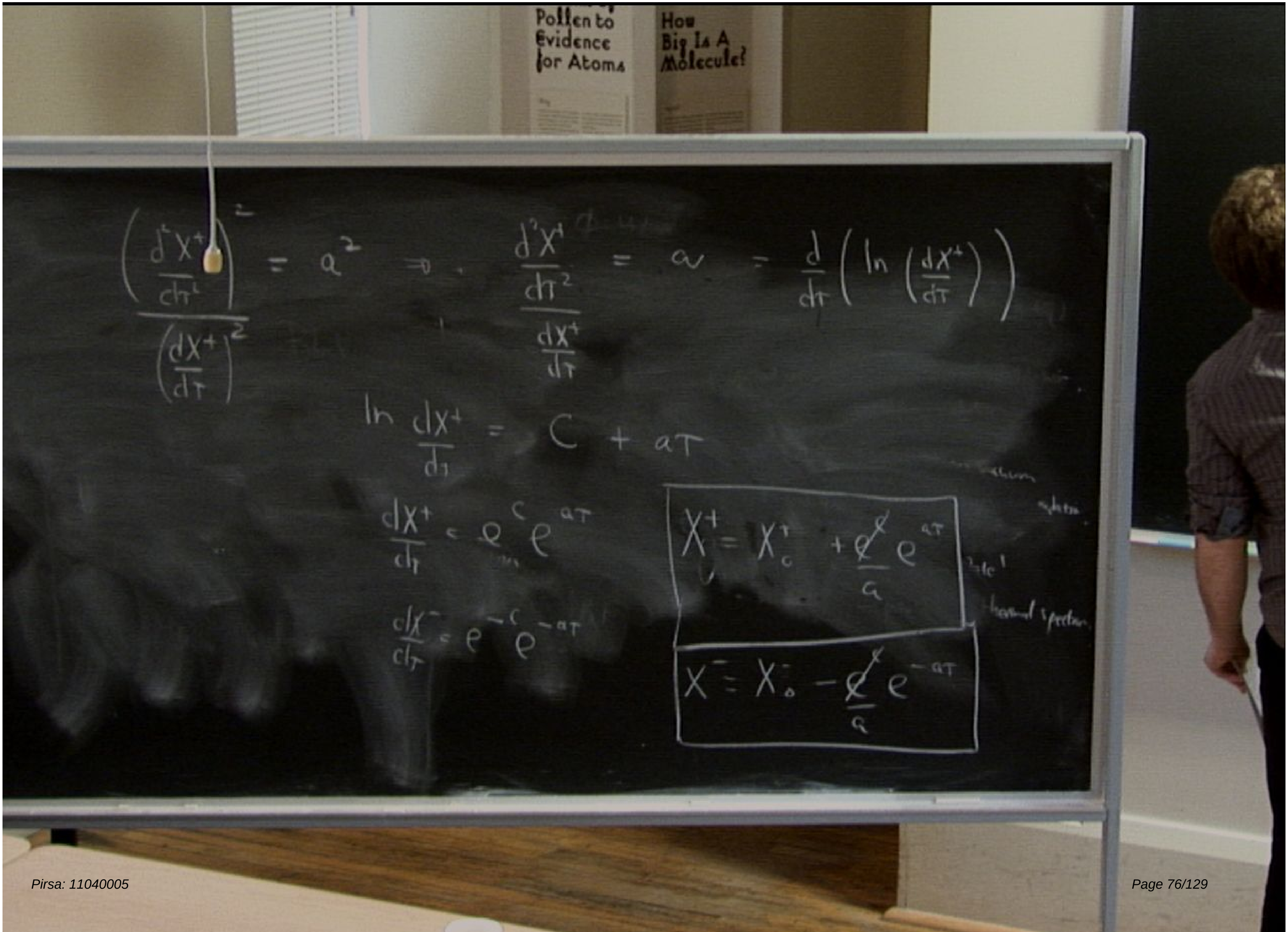


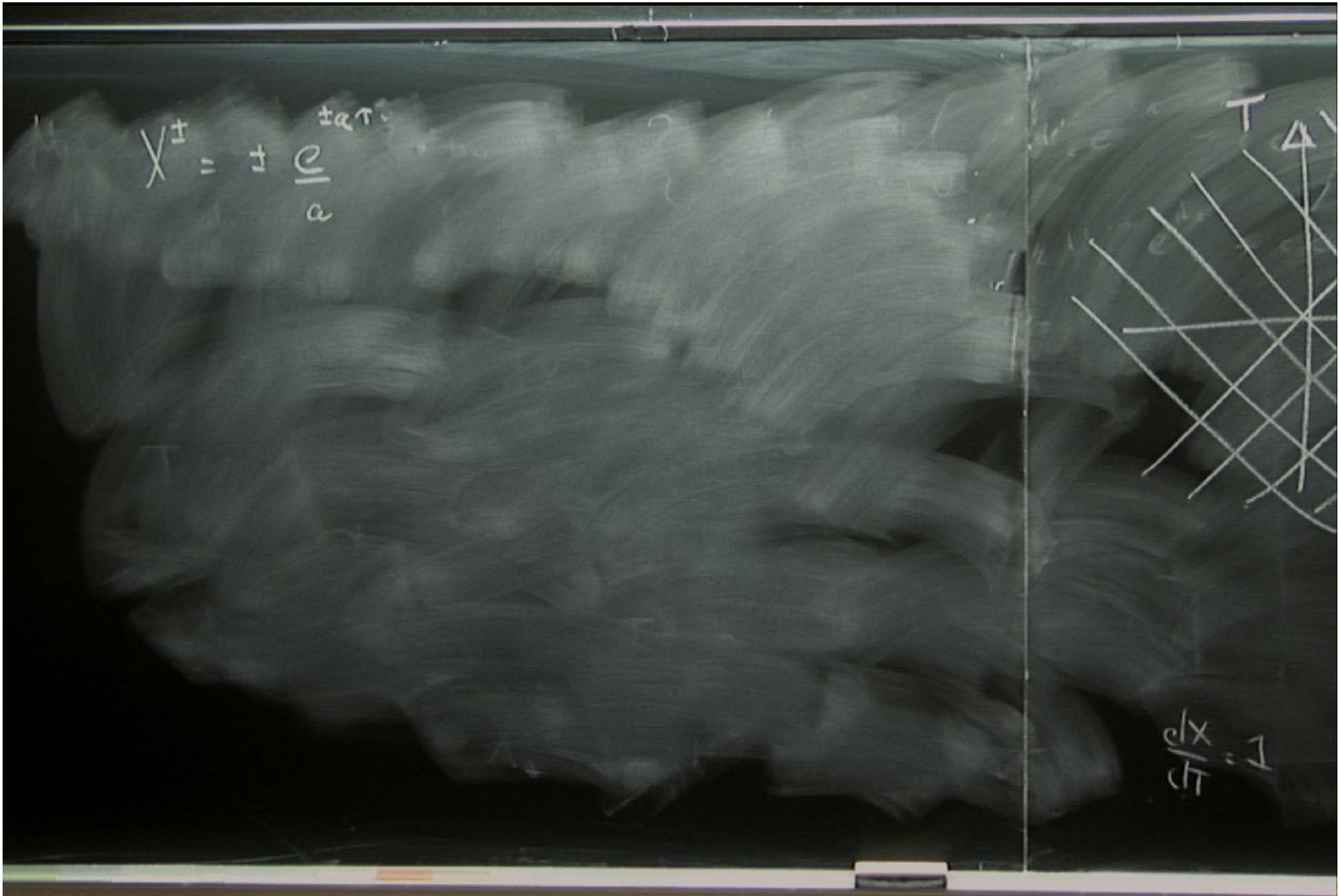
$X_- =$ constant
X
$X_+ =$ constant
Unruh.
Constant proper acceleration observer.
$d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu$
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -1$
|||
$\frac{dX^+}{d\tau}\frac{dX^-}{d\tau} = +1$
$a^\mu a_\mu = a^2$
$\frac{d^2X^+}{d\tau^2}\frac{d^2X^-}{d\tau^2} = -a^2$
$\frac{dX^-}{d\tau} = \frac{1}{\frac{dX^+}{d\tau}}$
$\frac{d^2X^-}{d\tau^2} = -\frac{\frac{d^2X^+}{d\tau^2}}{\left(\frac{dX^+}{d\tau}\right)^2}$

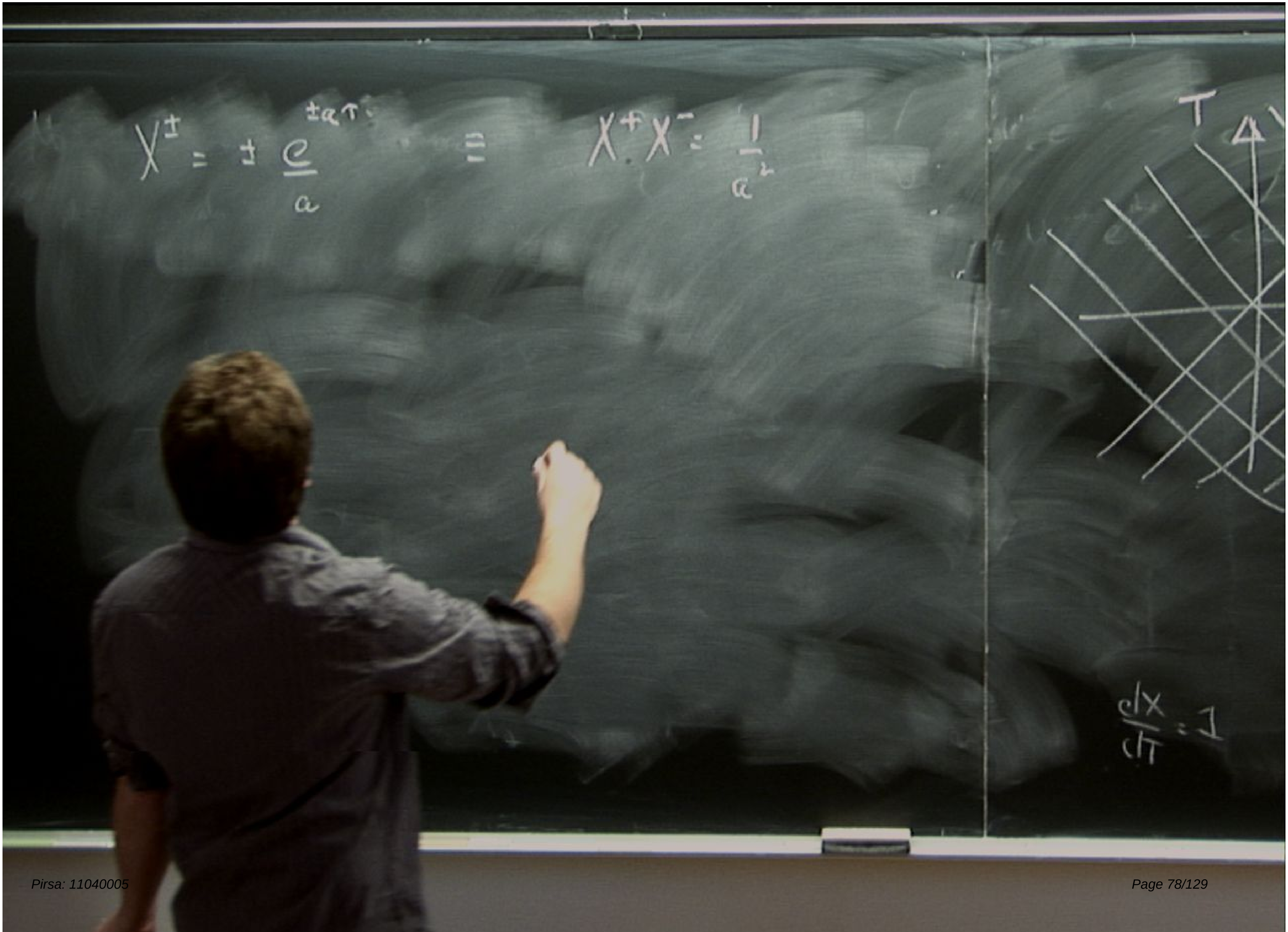

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\frac{d^2X^+}{d\tau^2} = a = \frac{d}{d\tau}\left(\ln\left(\frac{dX^+}{d\tau}\right)\right)$
$\ln\frac{dX^+}{d\tau} = C + a\tau$
$X^+ = X^+_c + \frac{e^c}{a}e^{a\tau}$

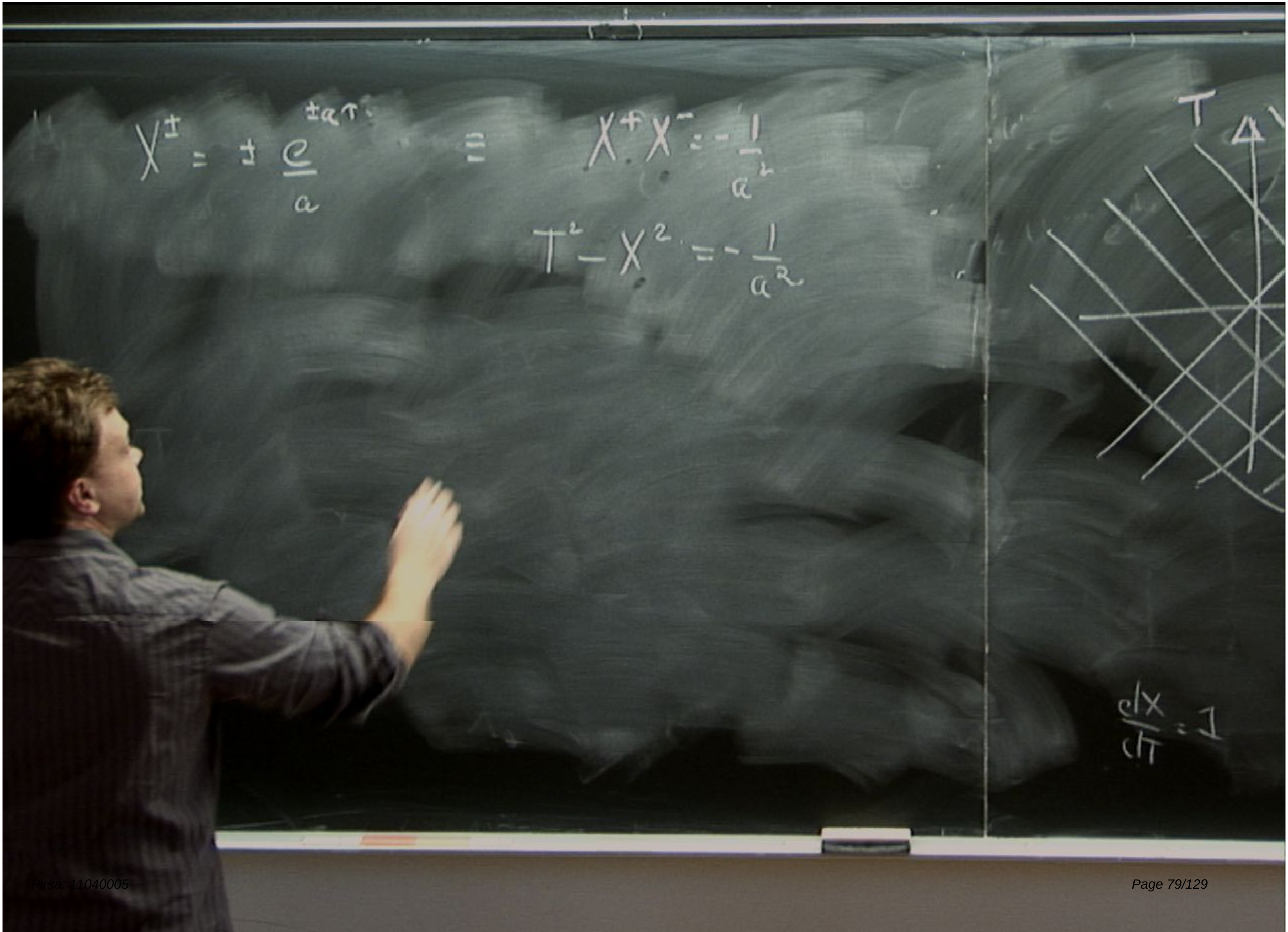

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
ln dX+/dτ = C + aτ
dX+/dτ = e^C e^{aτ}
dX−/dτ = e^{−C} e^{−aτ}
X+ = X+₀ + e^C/a e^{aτ}
X− = X−₀ − e^C/a e^{−aτ}

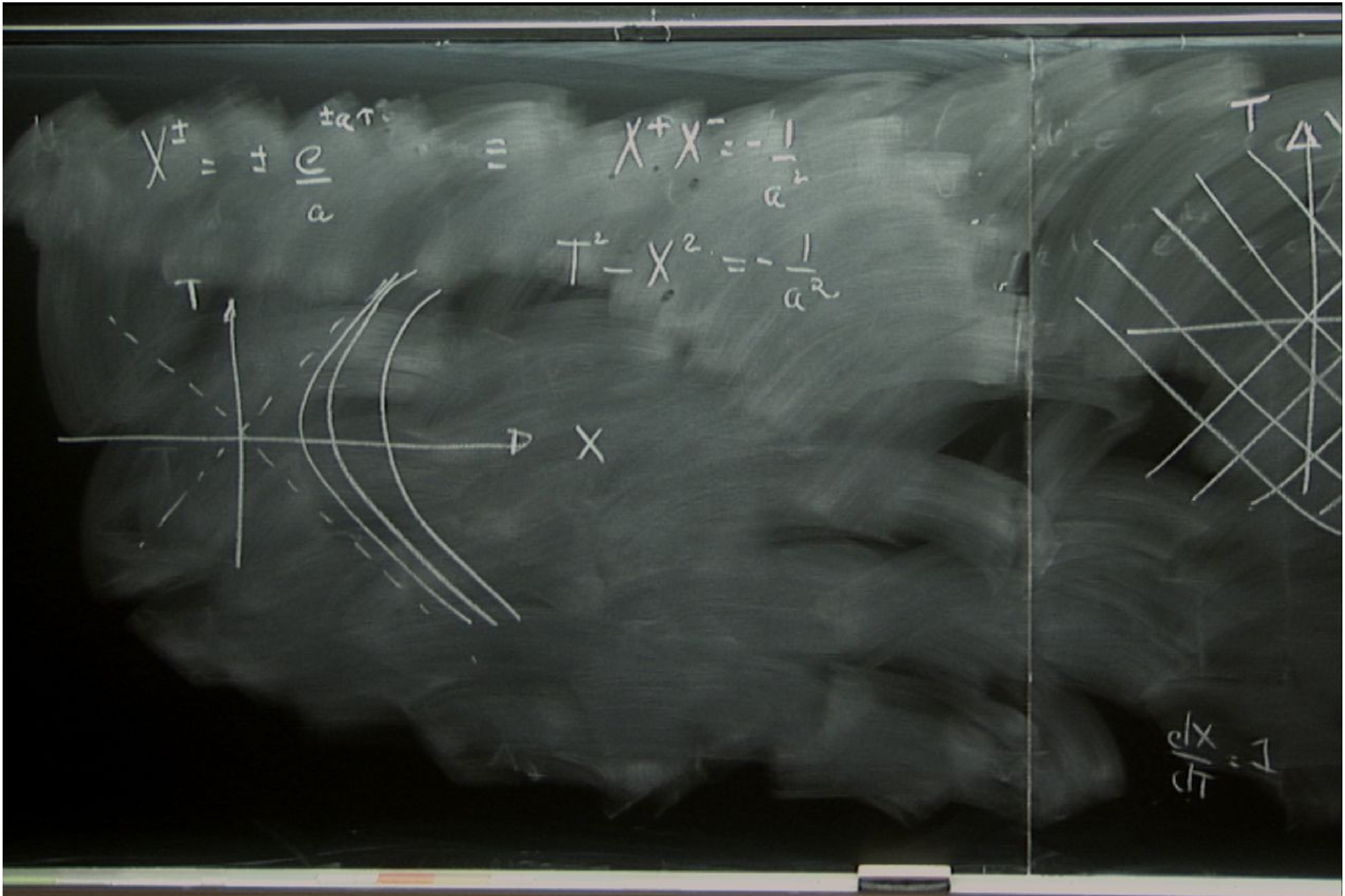

Pollen to Evidence for Atoms
How Big Is A Molecule?
$\frac{\left(\frac{d^2X^+}{d\tau^2}\right)^2}{\left(\frac{dX^+}{d\tau}\right)^2} = a^2 \Rightarrow \frac{\frac{d^2X^+}{d\tau^2}}{\frac{dX^+}{d\tau}} = a = \frac{d}{d\tau}\left(\ln\left(\frac{dX^+}{d\tau}\right)\right)$
$\ln \frac{dX^+}{d\tau} = C + a\tau$
$\frac{dX^+}{d\tau} = e^C e^{a\tau}$
$\frac{dX^-}{d\tau} = e^{-C} e^{-a\tau}$
$X^+ = X_0^+ + \frac{e^C}{a} e^{a\tau}$
$X^- = X_0^- - \frac{e^C}{a} e^{-a\tau}$
thermal spectrum

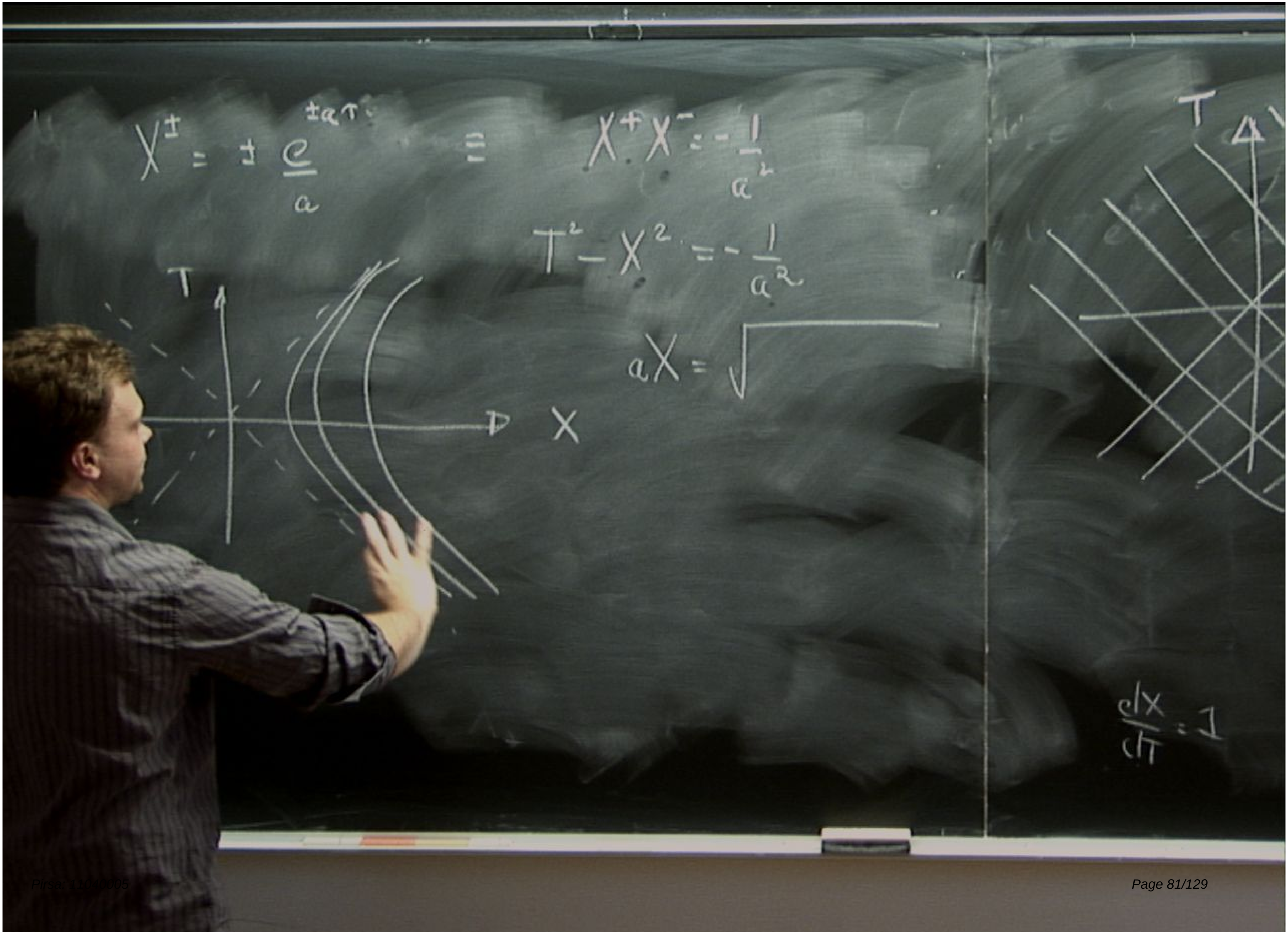

$X^{\pm} = \pm \frac{e^{\pm a\tau}}{a}$
T
$\frac{dx}{d\tau} = 1$


$X^{\pm} = \pm \frac{e^{\pm a\tau}}{a}$
$X^{+}X^{-} = \frac{1}{a^{2}}$
T
$\frac{dx}{d\tau} = 1$

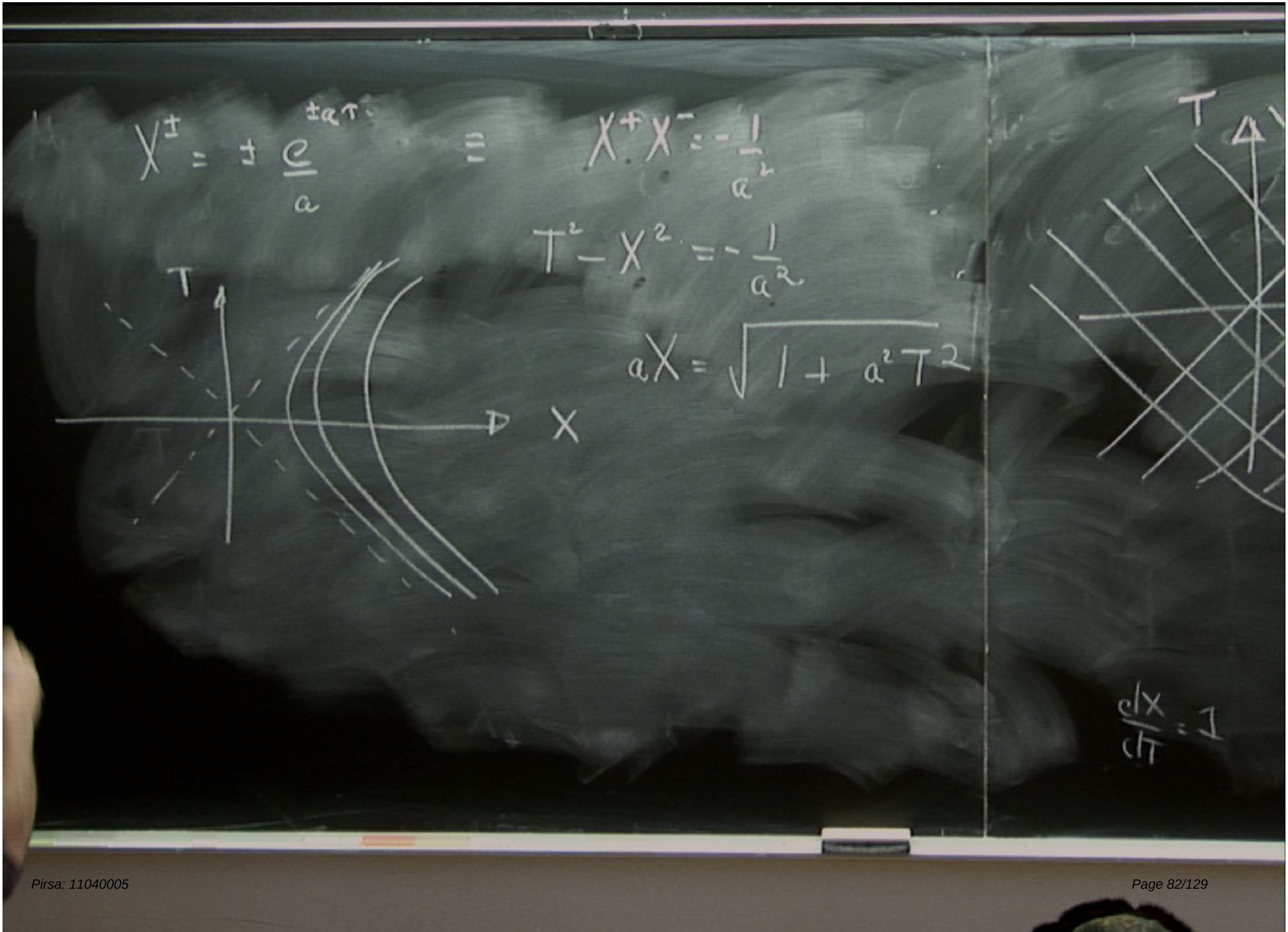


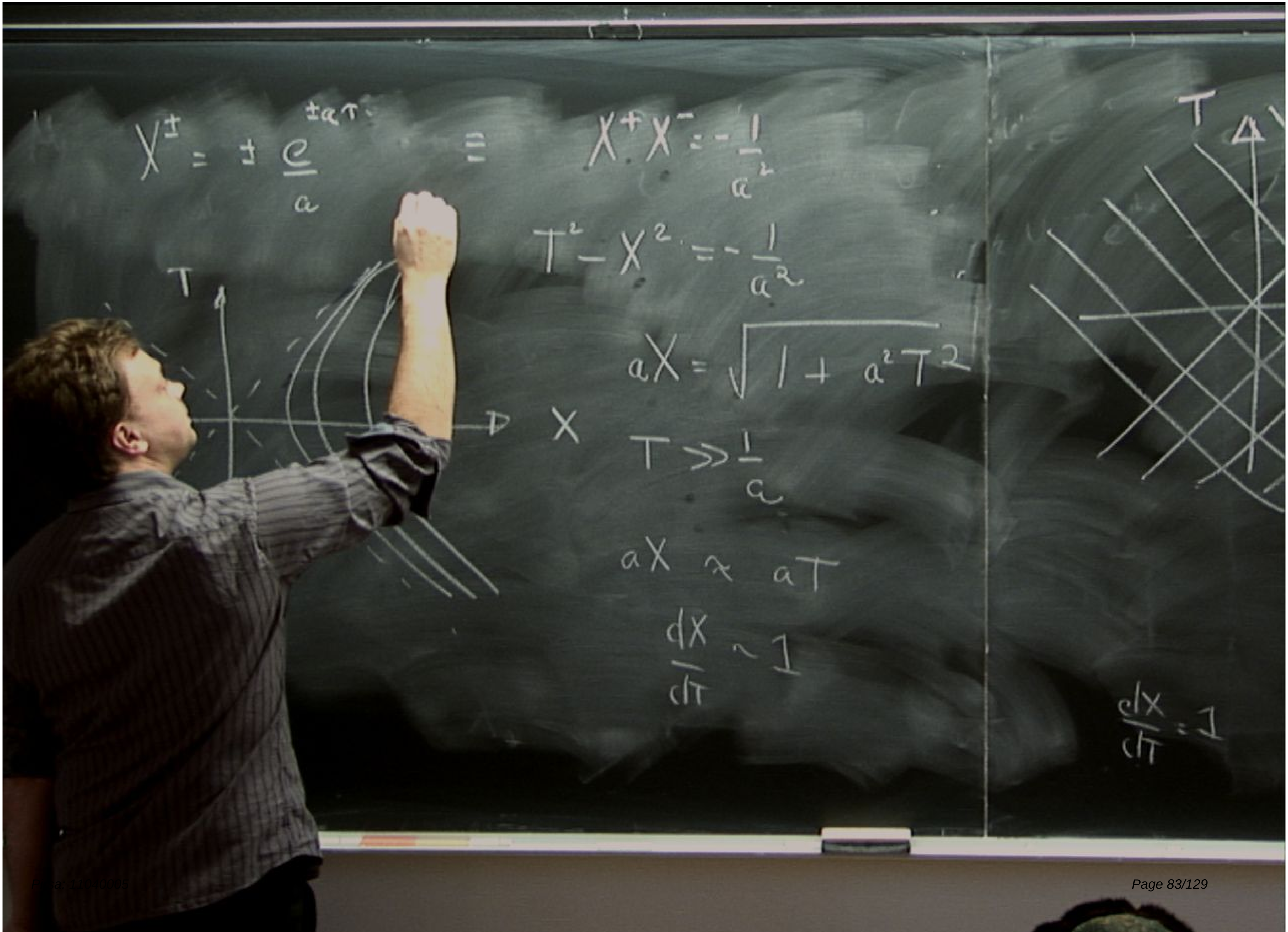
X± = ± e^{±aτ}/a ≡ X+X− = −1/a²
T² − X² = −1/a²
T
dX/dT = 1

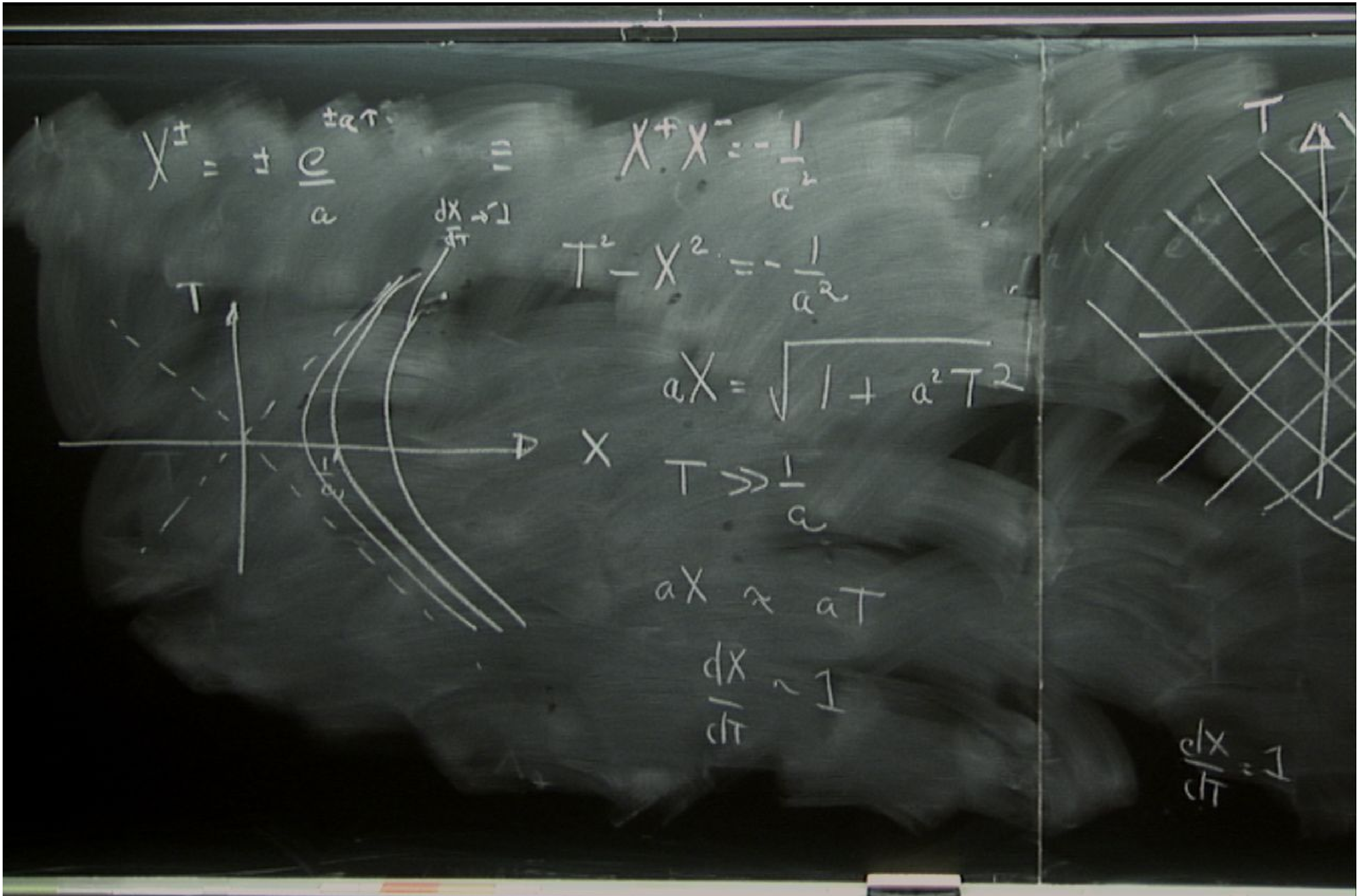

X± = ± e^{±aτ} / a
X+X− = −1/a²
T² − X² = −1/a²
T
X
T
dx/dT = 1



X± = ± e^{±aτ} / a
X+X− = −1/a²
T² − X² = −1/a²
aX = √
T
X
T
dx/dT = 1


X^± = ± e^{±aτ} / a
X^+ X^- = -1/a^2
T^2 - X^2 = -1/a^2
aX = √(1 + a^2 T^2)
T
X
dx/dT = 1

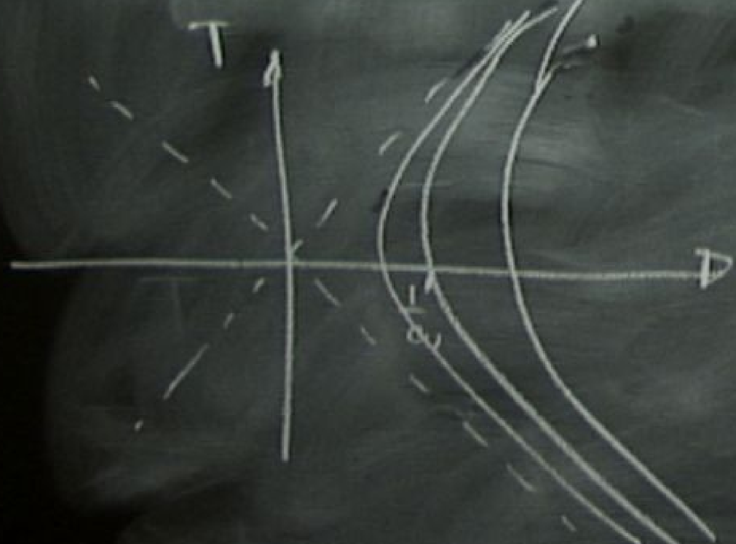

X^± = ± e^{±aτ}/a
X^+ X^- = -1/a^2
T^2 - X^2 = -1/a^2
T
X
aX = √(1 + a^2T^2)
T >> 1/a
aX ≈ aT
dX/dT ~ 1
T
dX/dT = 1


X± = ± e^{±aτ}/a
dX/dT → 1
X+X− = −1/a²
T² − X² = −1/a²
aX = √(1 + a²T²)
X
T
1/a
T ≫ 1/a
aX ≈ aT
dX/dT ~ 1
T
dX/dT = 1

$$X^{\pm} = \pm \frac{e^{\pm a\tau}}{a} \qquad \frac{dX}{d\tau} \to 1$$

$$X^{+}X^{-} = -\frac{1}{a^2}$$

$$T^2 - X^2 = -\frac{1}{a^2}$$

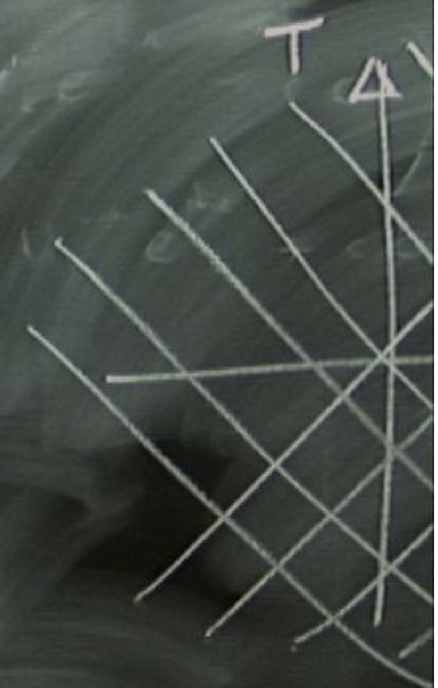


$$aX = \sqrt{1 + a^2T^2}$$

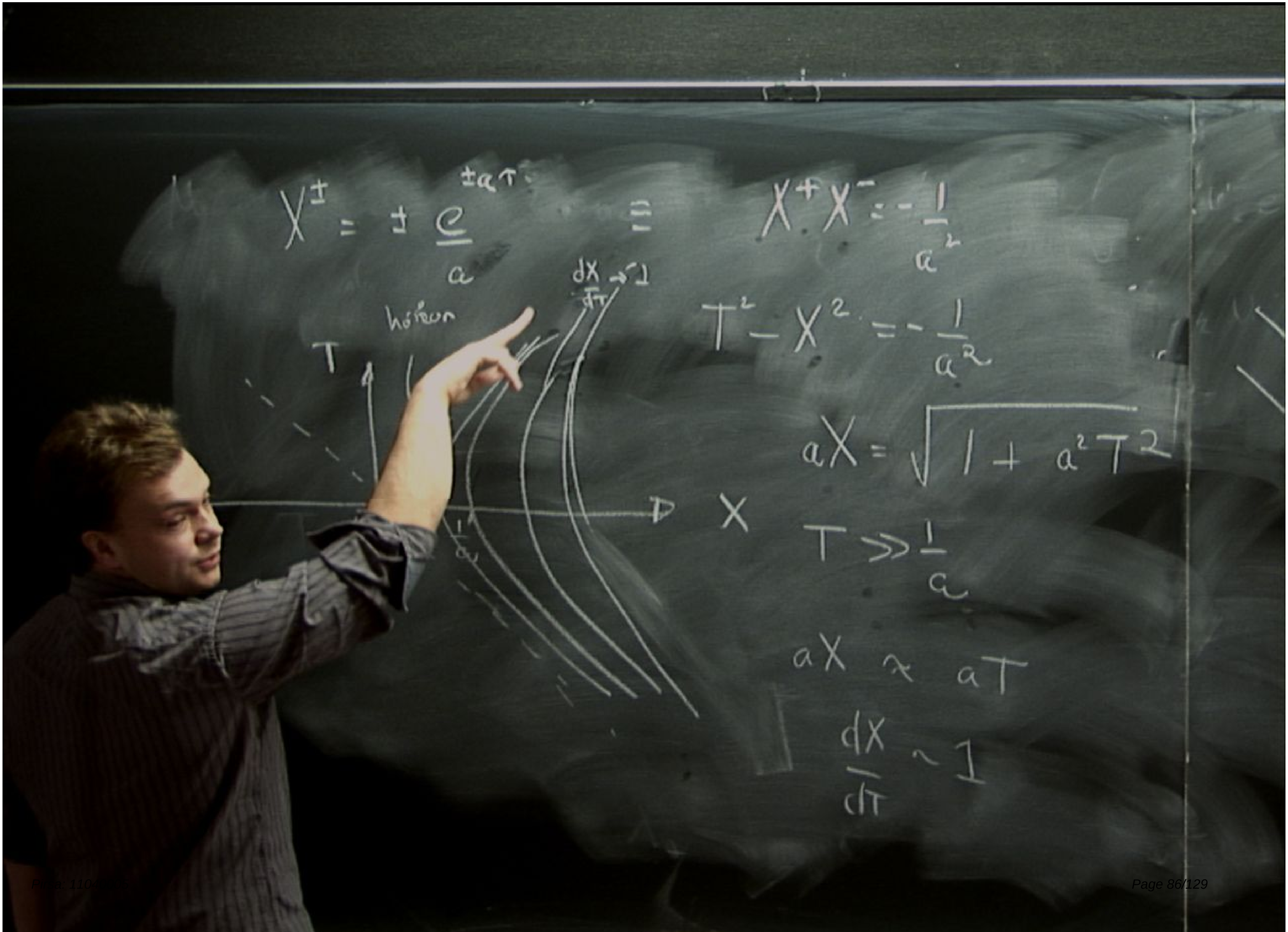
$$T \gg \frac{1}{a}$$

$$aX \approx aT$$

$$\frac{dX}{dT} \sim 1$$

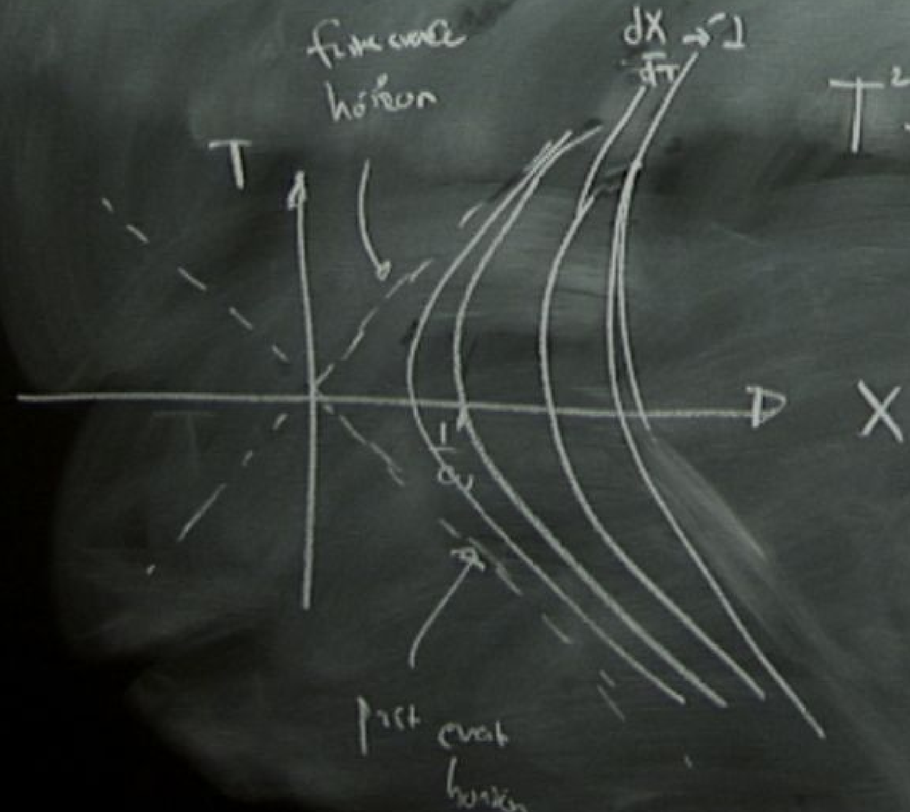


$$\frac{dX}{dT} = 1$$


X± = ± e^{±aτ} / a
X+X− = −1/a²
dX/dτ → 1
horizon
T
T² − X² = −1/a²
aX = √(1 + a²T²)
X
1/a
T >> 1/a
aX ≈ aT
dX/dT ~ 1

$$X^{\pm} = \pm \frac{e^{\pm a\tau}}{} \equiv$$

$$X^+X^- = -\frac{1}{a^2}$$



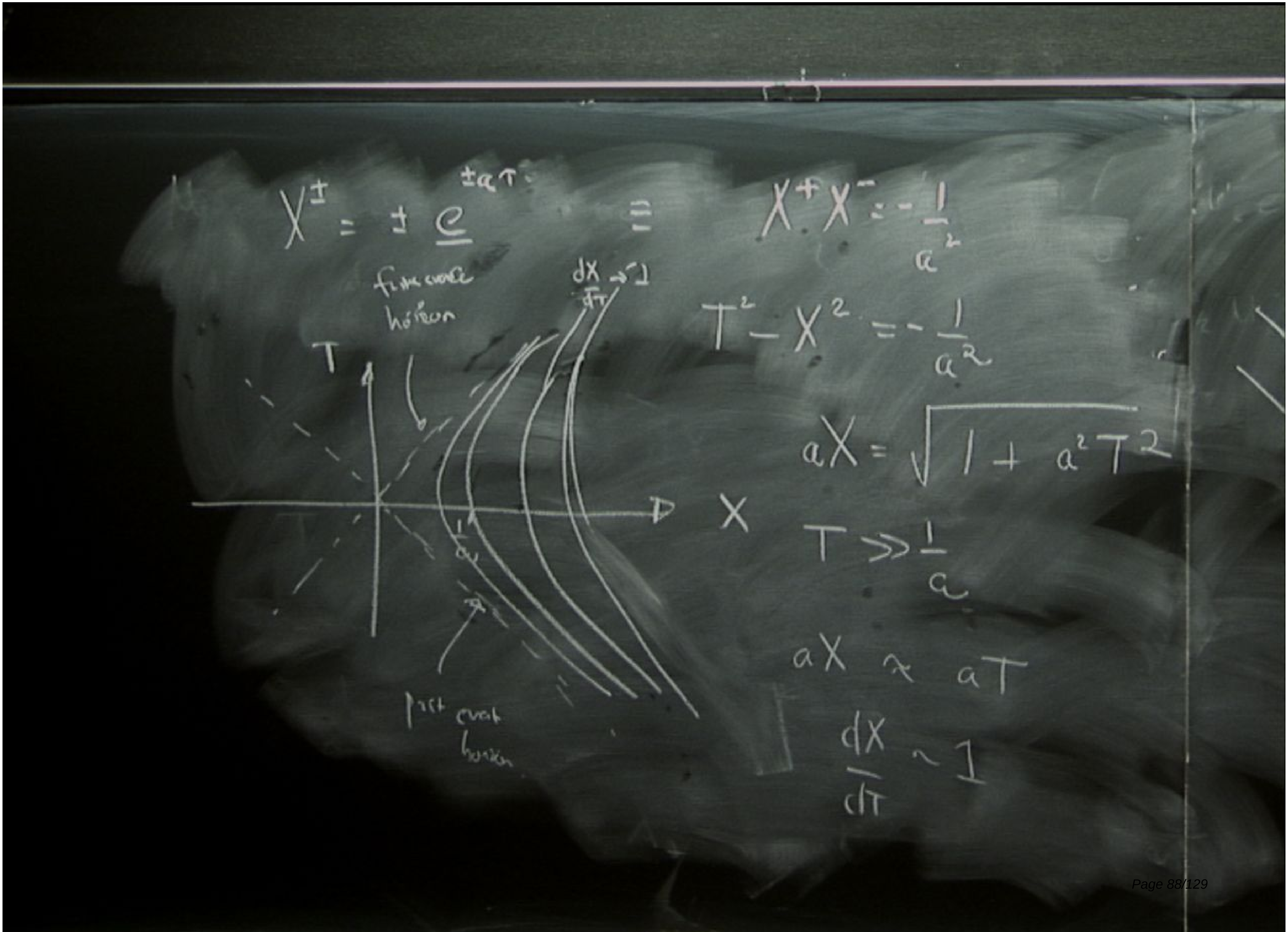
$$T^2 - X^2 = -\frac{1}{a^2}$$

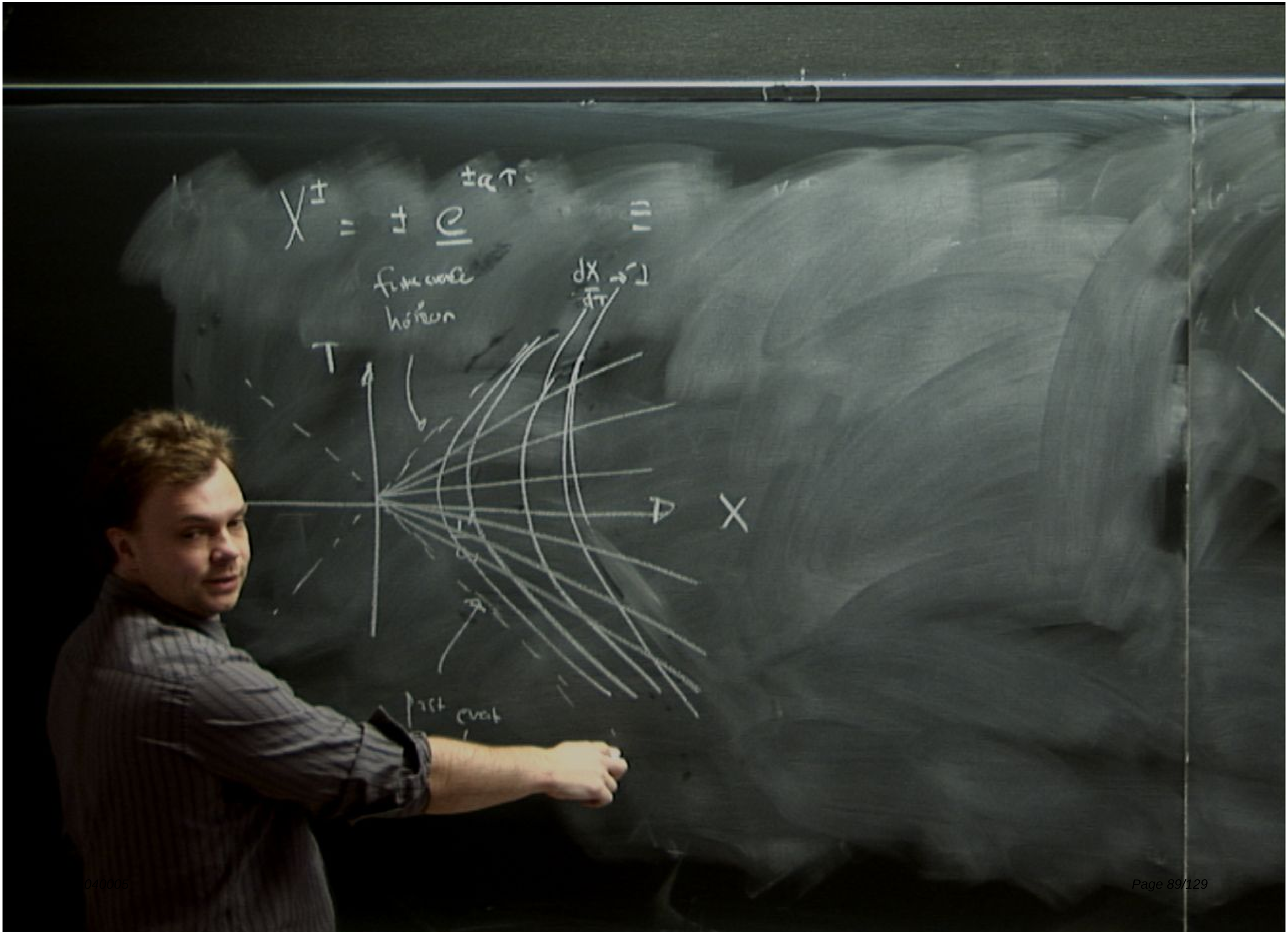
$$aX = \sqrt{1 + a^2T^2}$$

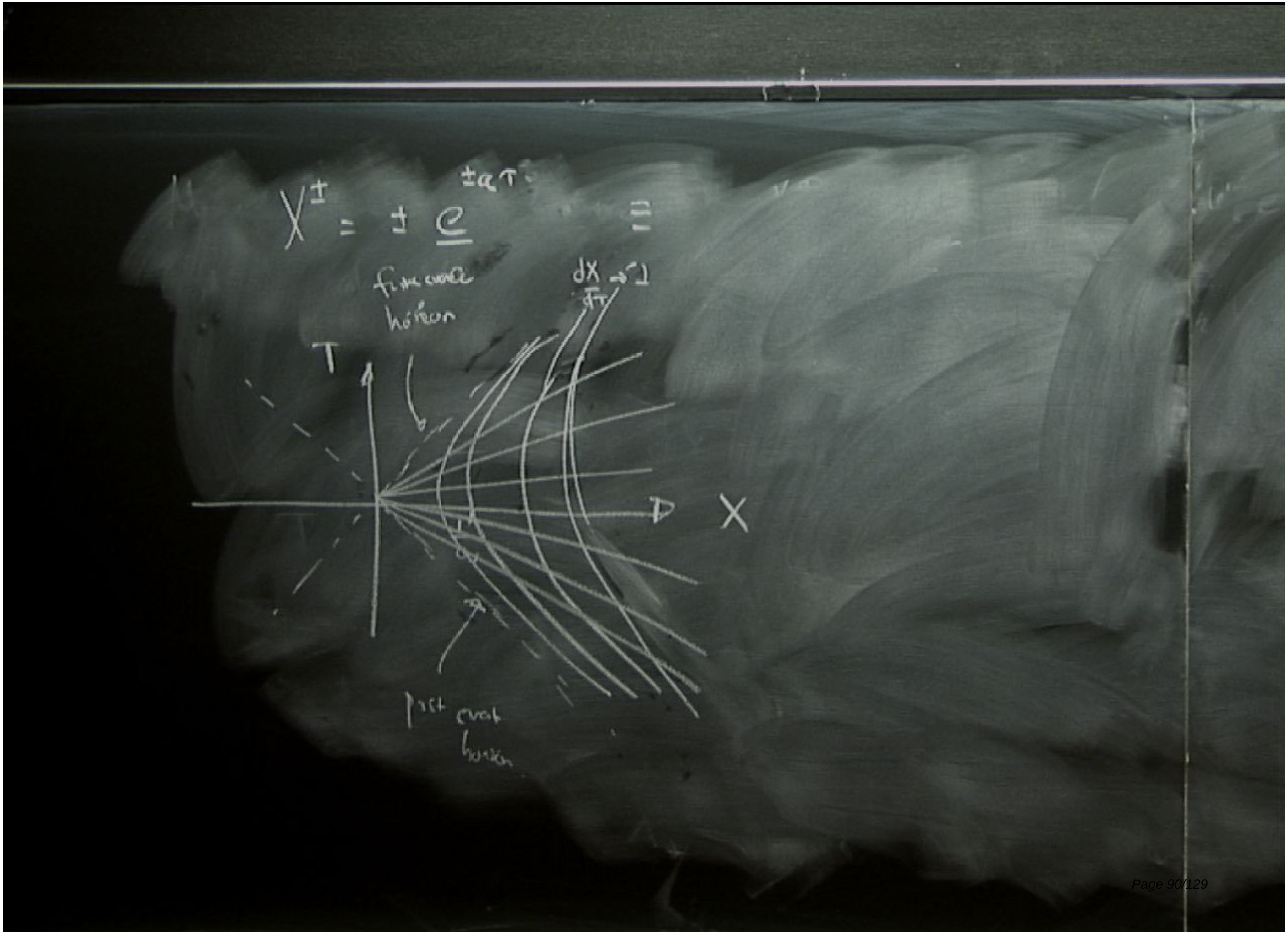
$$T \gg \frac{1}{a}$$

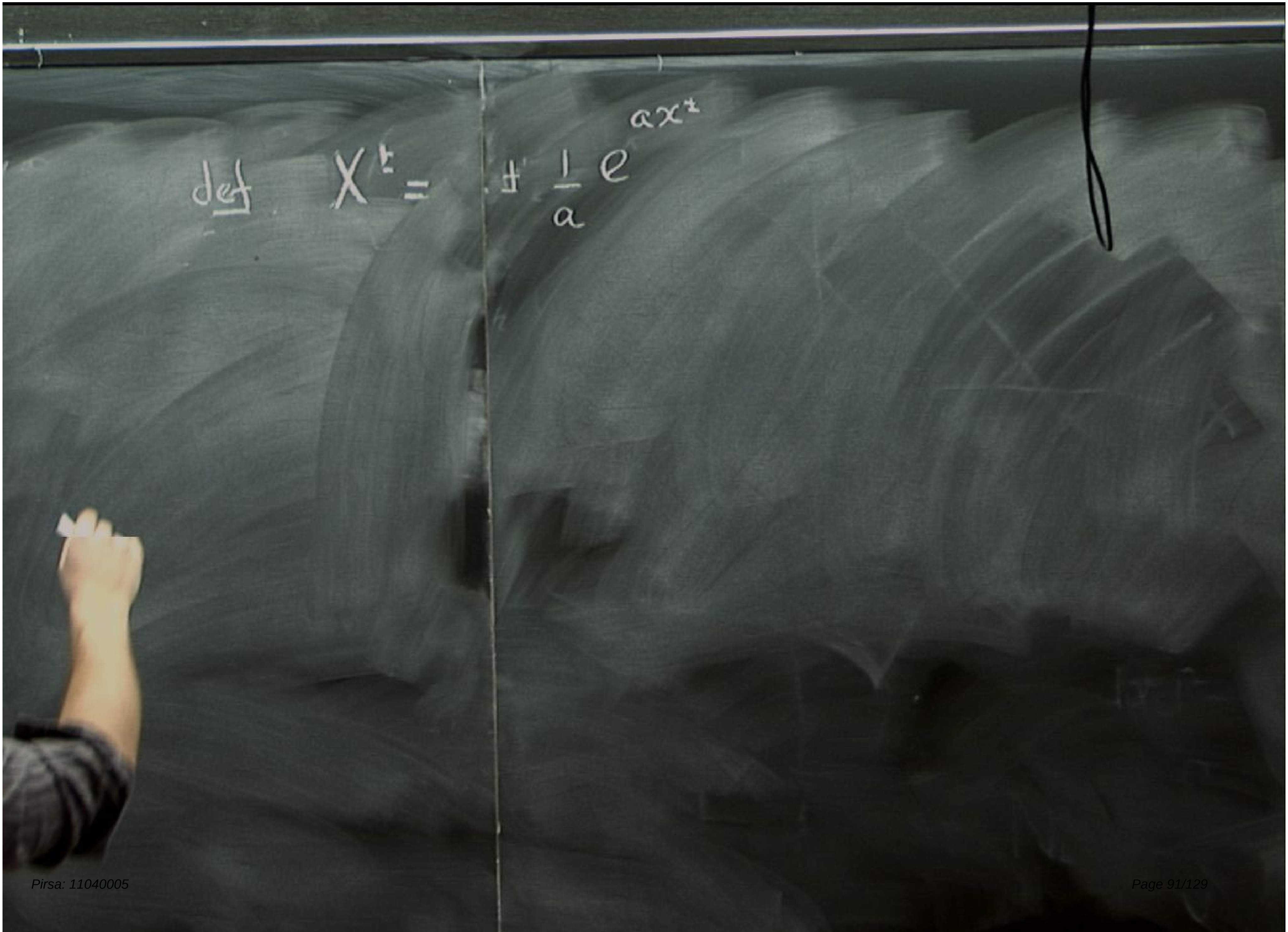
$$aX \approx aT$$

$$\frac{dX}{dT} \sim 1$$


$X^{\pm} = \pm e^{\pm a\tau}$
$X^{+}X^{-} = -\frac{1}{a^2}$
future event horizon
$\frac{dX}{d\tau} \to 1$
$T^2 - X^2 = -\frac{1}{a^2}$
$aX = \sqrt{1 + a^2T^2}$
T
X
$\frac{1}{a}$
$T \gg \frac{1}{a}$
$aX \approx aT$
past event horizon
$\frac{dX}{dT} \sim 1$


T
X


$X^{\pm} = \pm e^{\pm a\tau}$
$\frac{dX}{d\tau} \to 1$
T
X
past event horizon


def $X^{\pm} = \pm \frac{1}{a} e^{a x^{\pm}}$

def $X^{\pm} = \pm \frac{1}{a} e^{a x^{\pm}}$

$X^{\pm} = T \pm X$

$x^{\pm} = t \pm x$

def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$ $dX^{\pm} =$

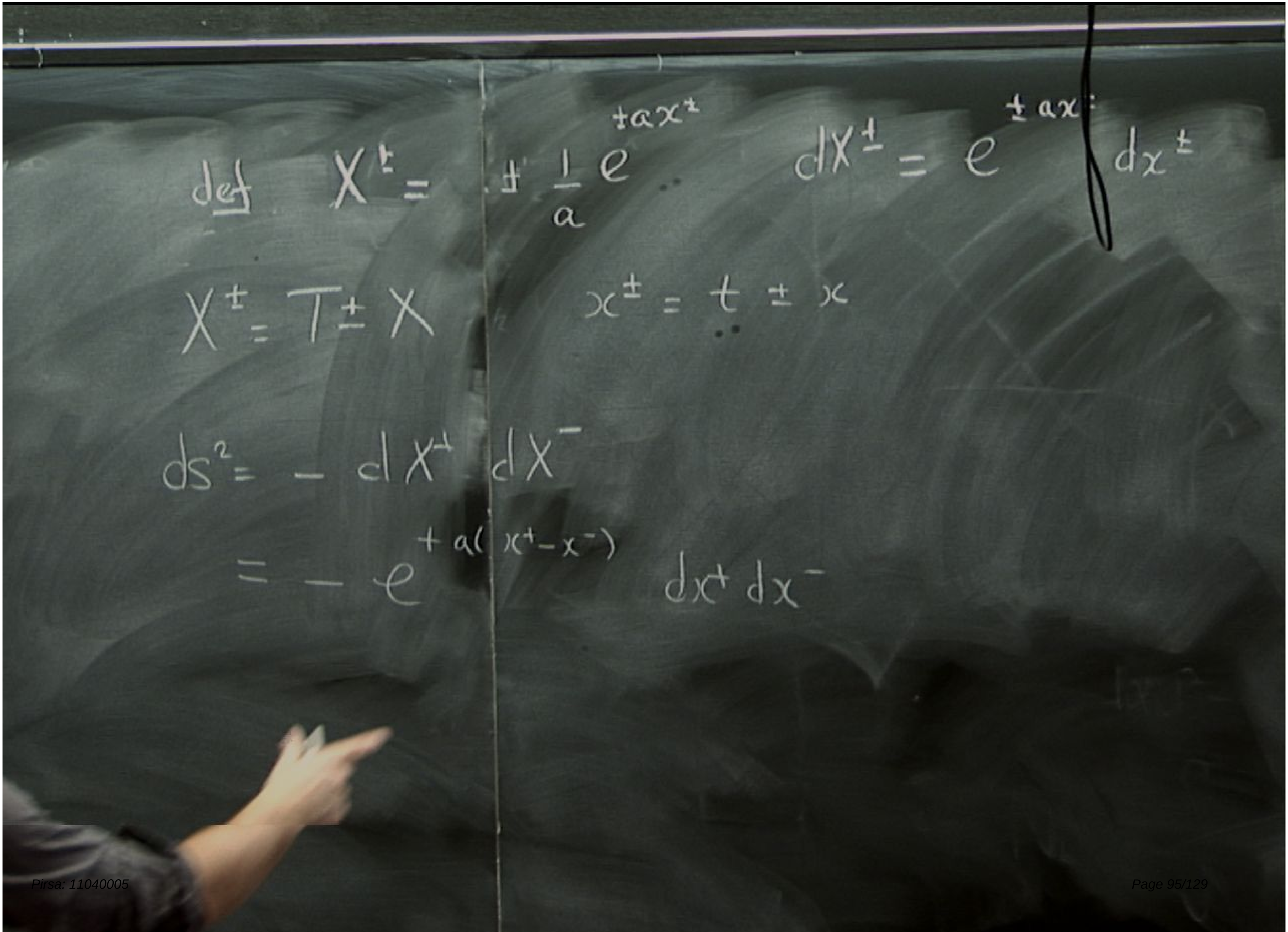
$X^{\pm} = T \pm X$ $x^{\pm} = t \pm x$

$ds^2 = -\, dX^{+} dX^{-}$

def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$ $\qquad dX^{\pm} = e^{\pm a x^{\pm}} dx^{\pm}$

$X^{\pm} = T \pm X$ $\qquad x^{\pm} = t \pm x$

$$ds^2 = - dX^+ dX^-$$


def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$
$dX^{\pm} = e^{\pm a x^{\pm}} dx^{\pm}$
$X^{\pm} = T \pm X$
$x^{\pm} = t \pm x$
$ds^2 = - dX^{+} dX^{-}$
$= - e^{+a(x^{+} - x^{-})} dx^{+} dx^{-}$

def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$ $\qquad dX^{\pm} = e^{\pm a x^{\pm}} dx^{\pm}$

$X^{\pm} = T \pm X \qquad x^{\pm} = t \pm x$

$$ds^2 = -dX^{+} dX^{-}$$

$$= -e^{+a(x^{+} - x^{-})} dx^{+} dx^{-}$$

$$= -e^{2ax}(dt^2 - dx^2)$$

$$ds^2 = -e^{2ax} dt^2 + e^{2ax} dx^2$$

$$\text{def} \quad X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}} \qquad dX^{\pm} = e^{\pm a x^{\pm}} dx^{\pm}$$

$$X^{\pm} = T \pm X \qquad x^{\pm} = t \pm x$$

$$ds^2 = - dX^{+} dX^{-}$$

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$$= - e^{2ax} (dt^2 - dx^2)$$

$$\boxed{ds^2 = - e^{2ax} dt^2 + e^{2ax} dx^2}$$

$$x^{\pm} = t \pm x$$

$$dX^{-}$$

$$x^{+} - x^{-})$$

$$dx^{+}\, dx^{-}$$

$$\left(dt^2 - dx^2 \right)$$

$$\boxed{dt^2 + e^{2ax}\, dx^2}$$

Rindler Coordinates

$$a^{\mu} = \frac{d^2 x^{\mu}}{d\tau^2}$$

$$\boxed{a^{\mu} a_{\mu} = a}$$

$$\frac{d^2X^{+}}{d\tau^2}\,\frac{d^2X^{-}}{d\tau^2} =$$

$$\frac{dX^{-}}{d\tau} = \frac{1}{\frac{dX^{+}}{d\tau}}$$

X

$$ds^2 = -\,dX^+\,dX^-$$

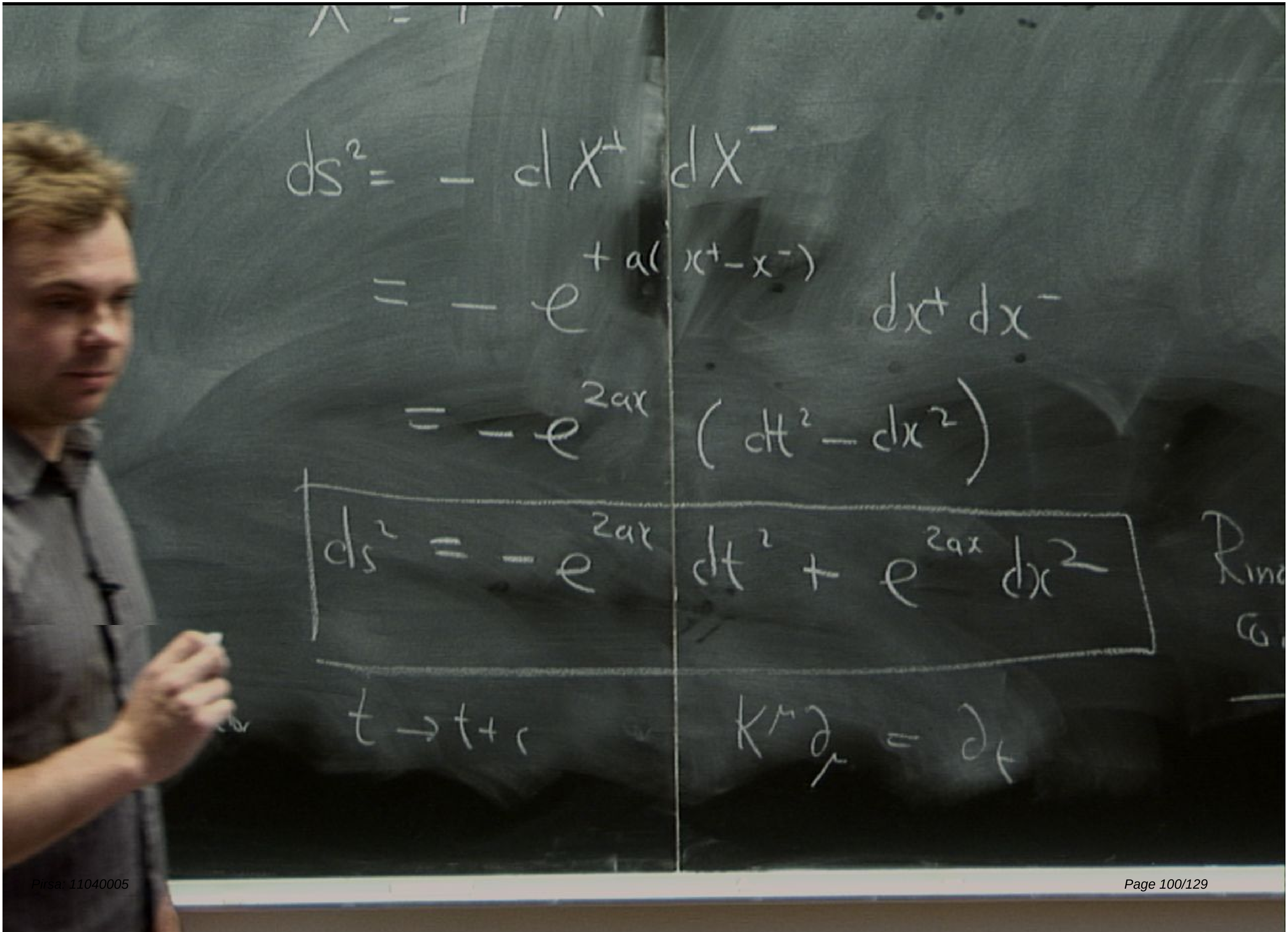
$$= -\,e^{+a(x^+ - x^-)}\,dx^+\,dx^-$$

$$= -\,e^{2ax}\left(dt^2 - dx^2\right)$$

$$\boxed{ds^2 = -\,e^{2ax}\,dt^2 + e^{2ax}\,dx^2}$$

Rin…

Killing vector $t \to t + c$


$ds^2 = - dX^+ dX^-$
$= - e^{+a(x^+ - x^-)} dx^+ dx^-$
$= - e^{2ax} (dt^2 - dx^2)$
$ds^2 = - e^{2ax} dt^2 + e^{2ax} dx^2$
$t \to t + c$
$k^\mu \partial_\mu = \partial_t$

$$ds^2 = -dX^+ dX^-$$

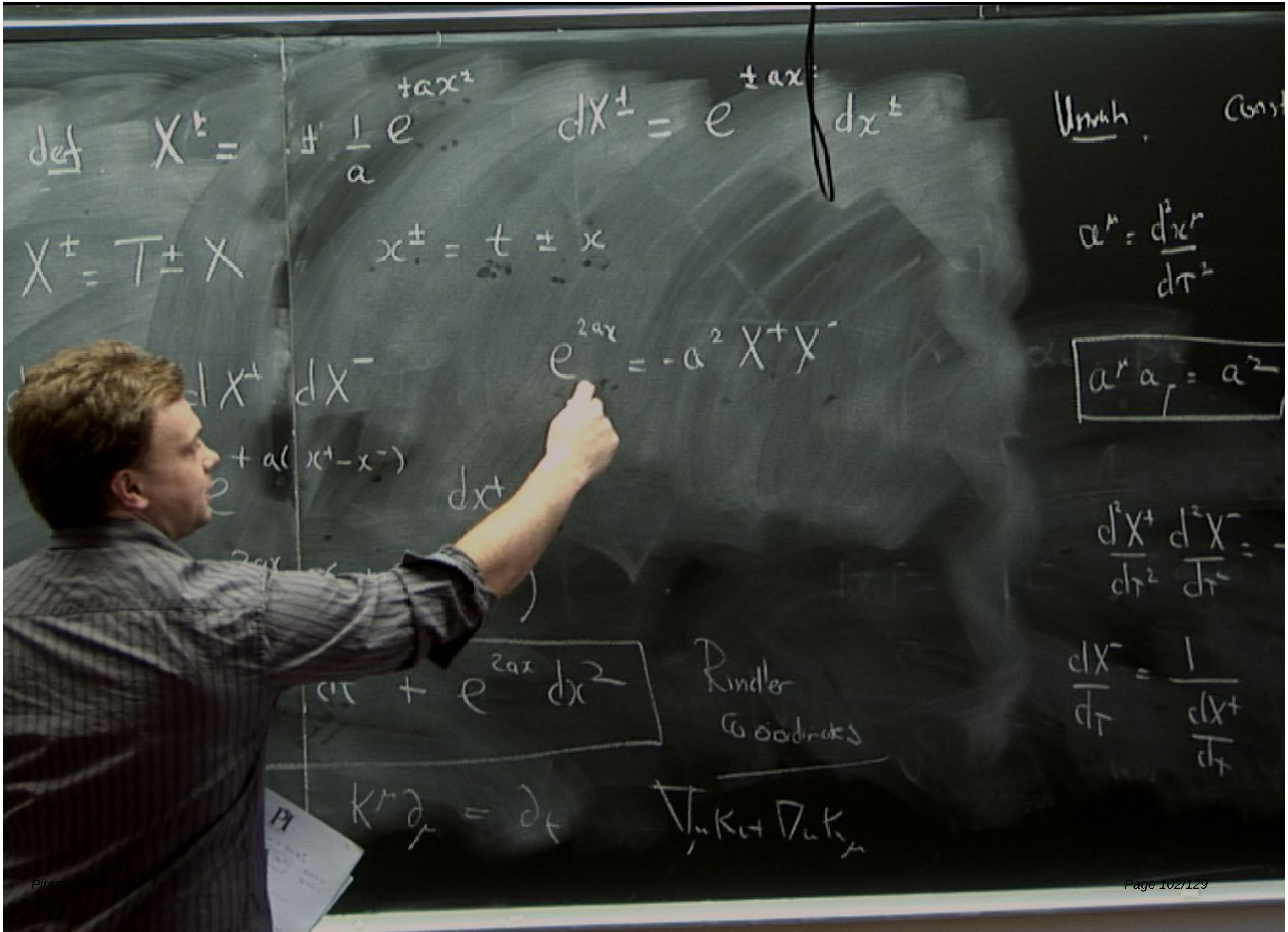
$$= -e^{+a(x^+ - x^-)} dx^+ dx^-$$

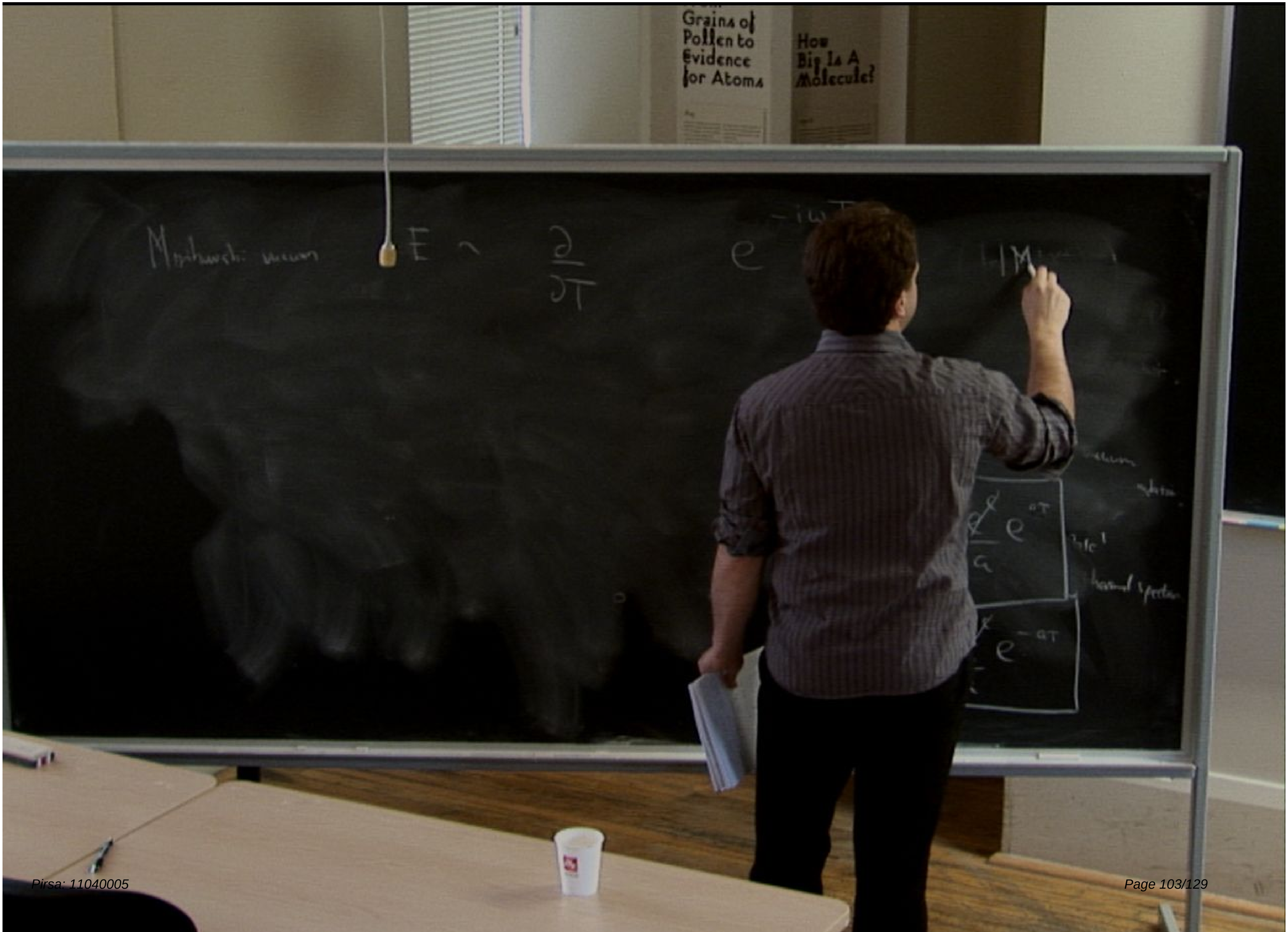
$$= -e^{2ax}(dt^2 - dx^2)$$

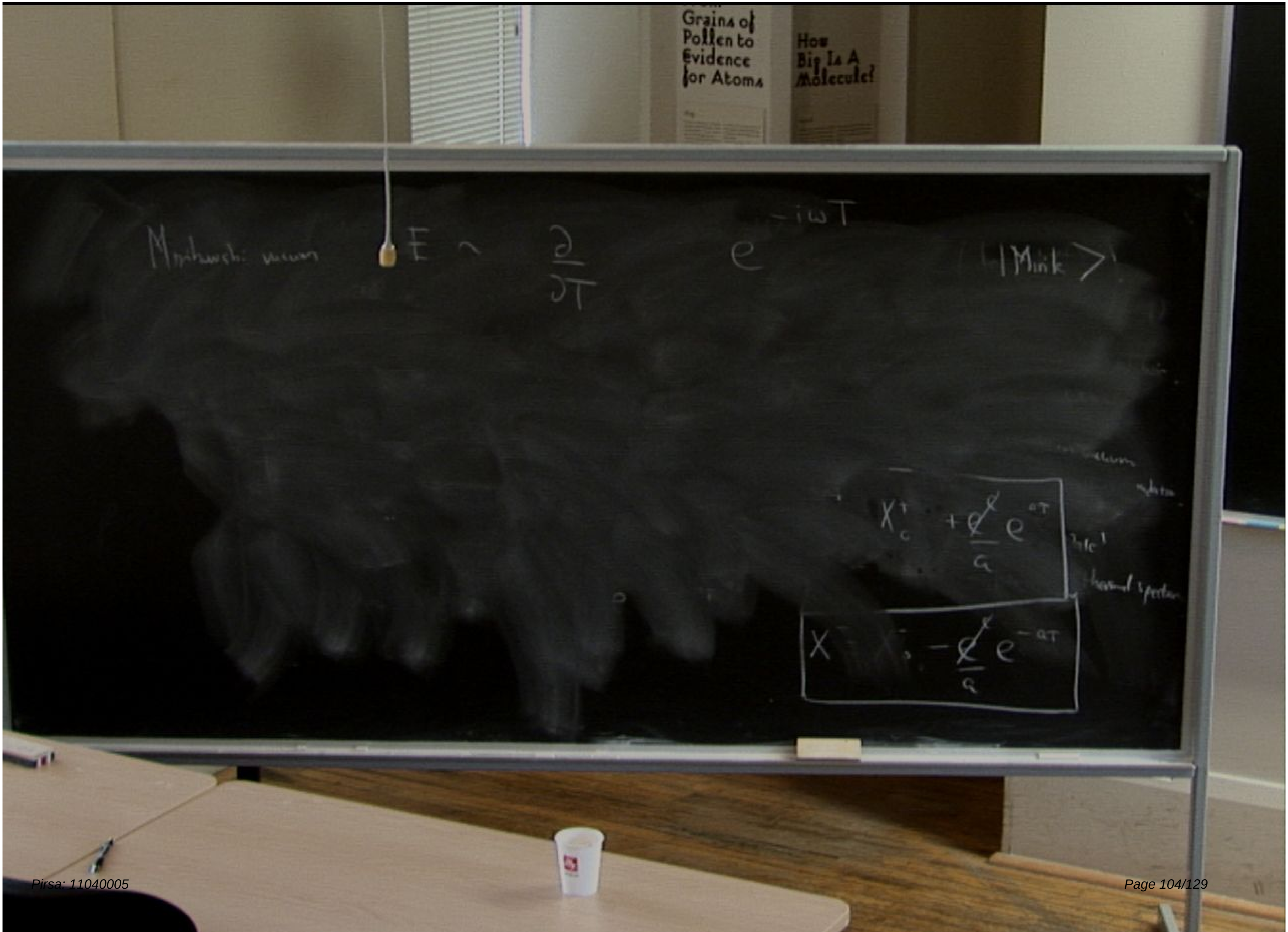
$$ds^2 = -e^{2ax} dt^2 + e^{2ax} dx^2$$

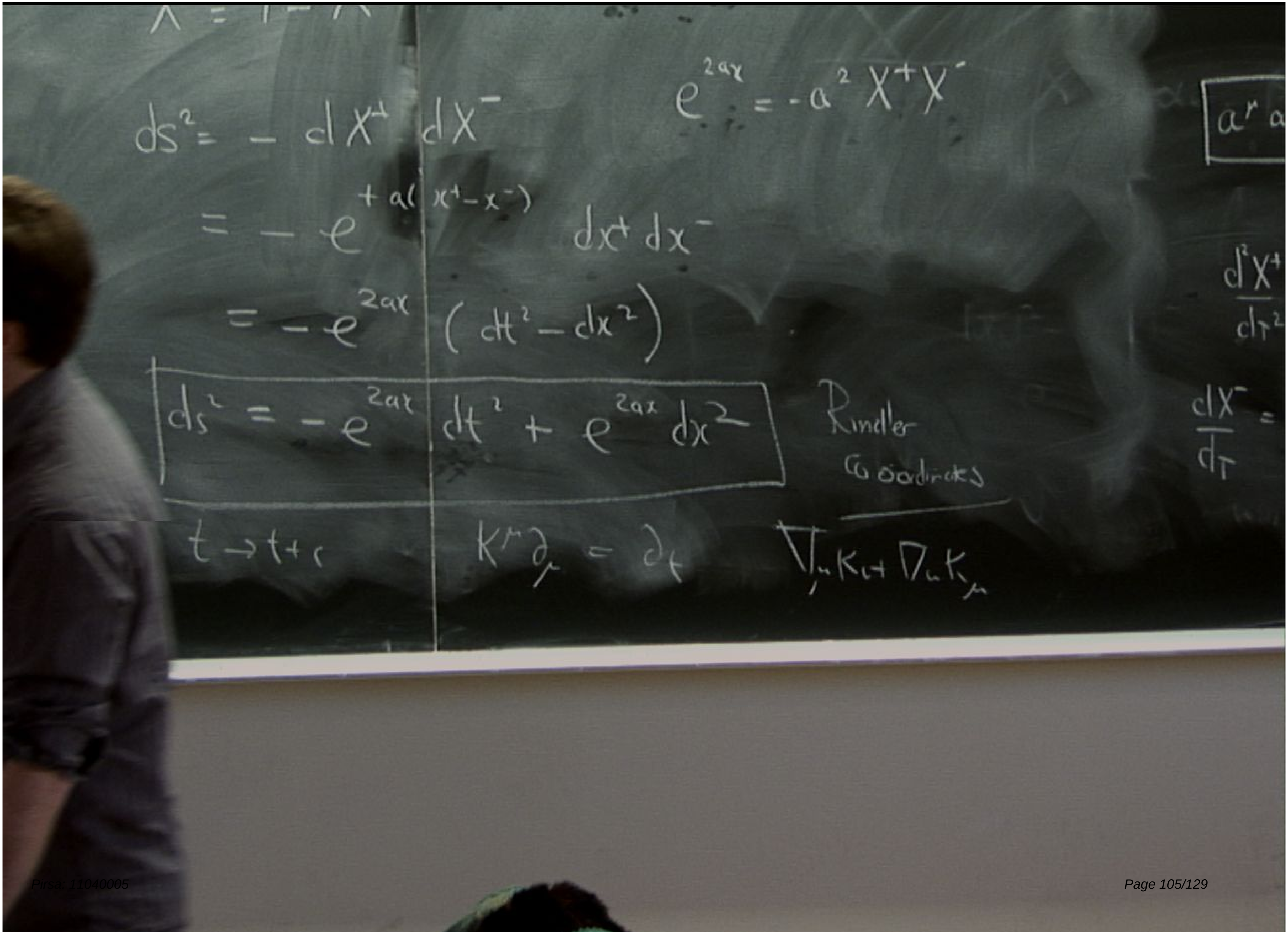
Rindler Coordinates

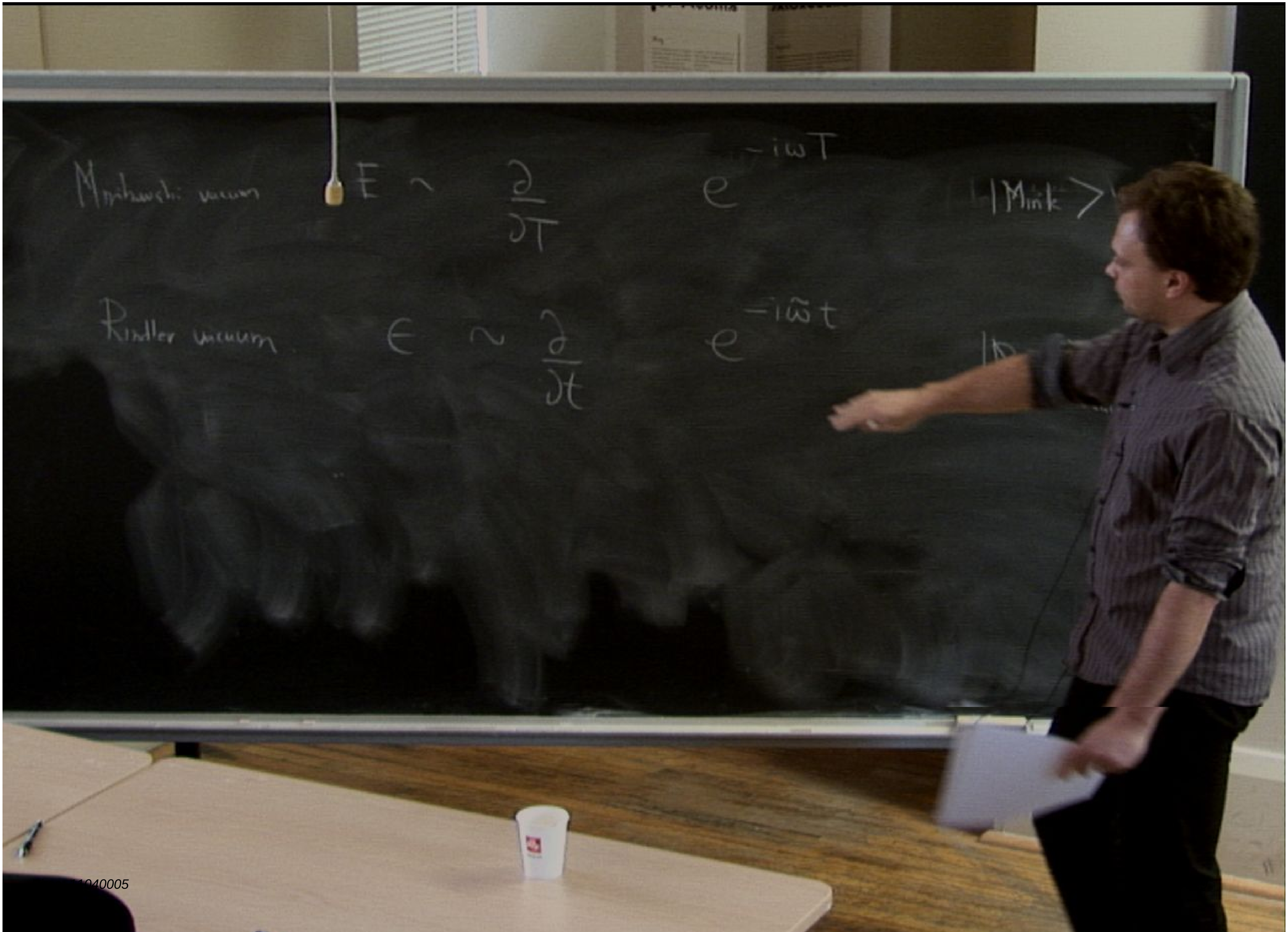
$t \to t + c$ $\quad K^\mu \partial_\mu = \partial_t$ $\quad \nabla_\mu K_\nu + \nabla_\nu K_\mu$

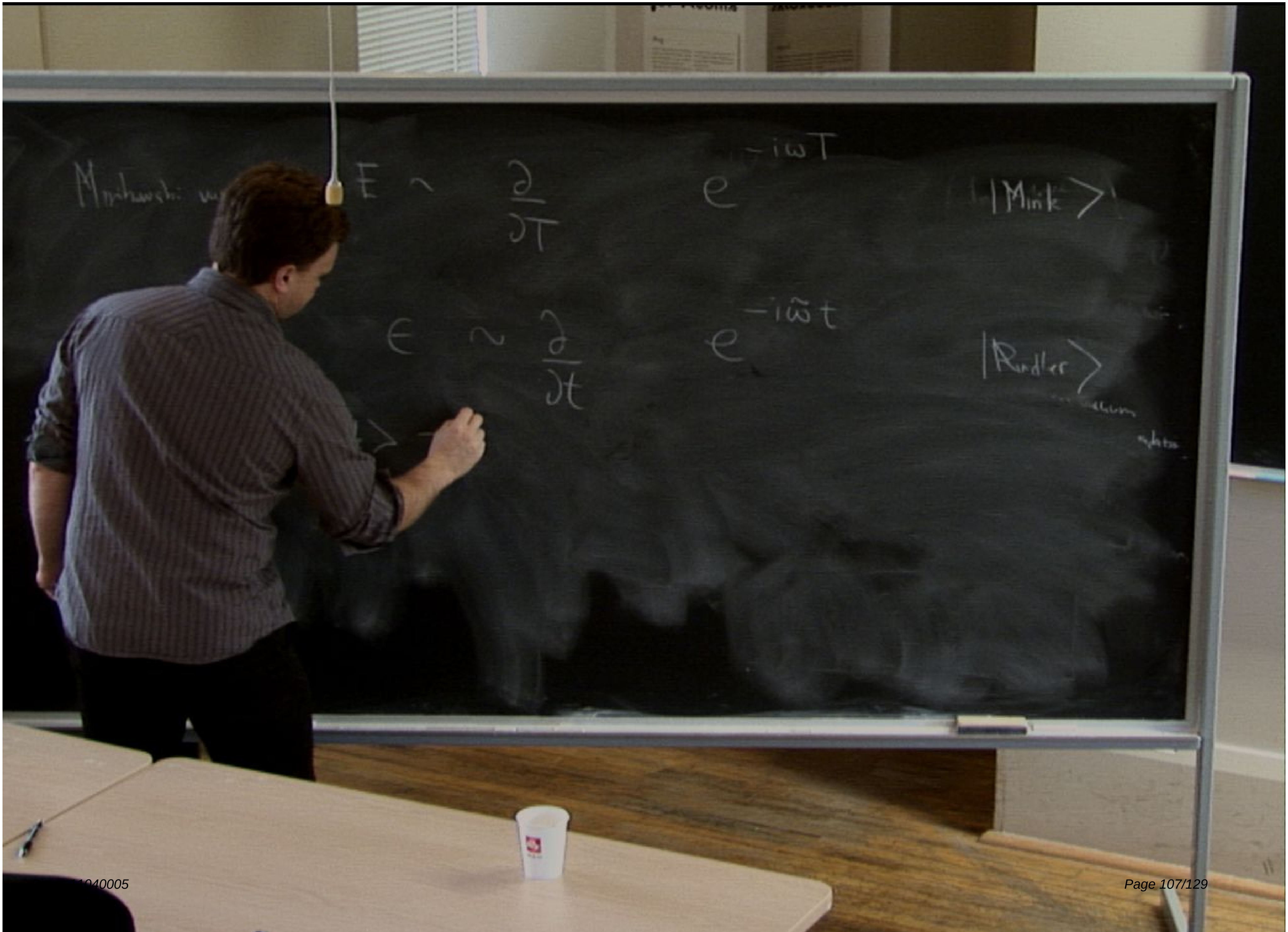

def
Unruh
Rindler
Coordinates

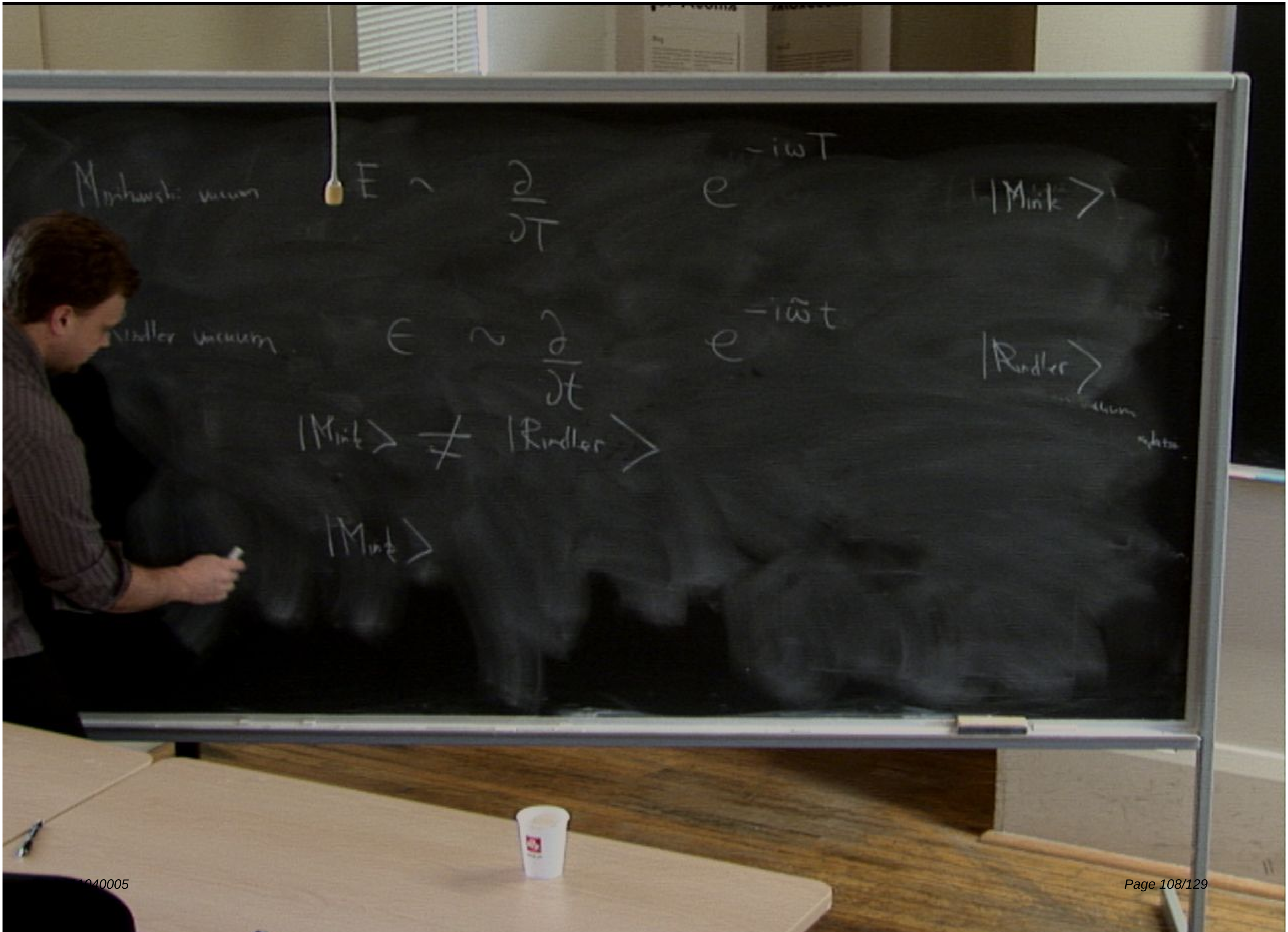

Grains of
Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?

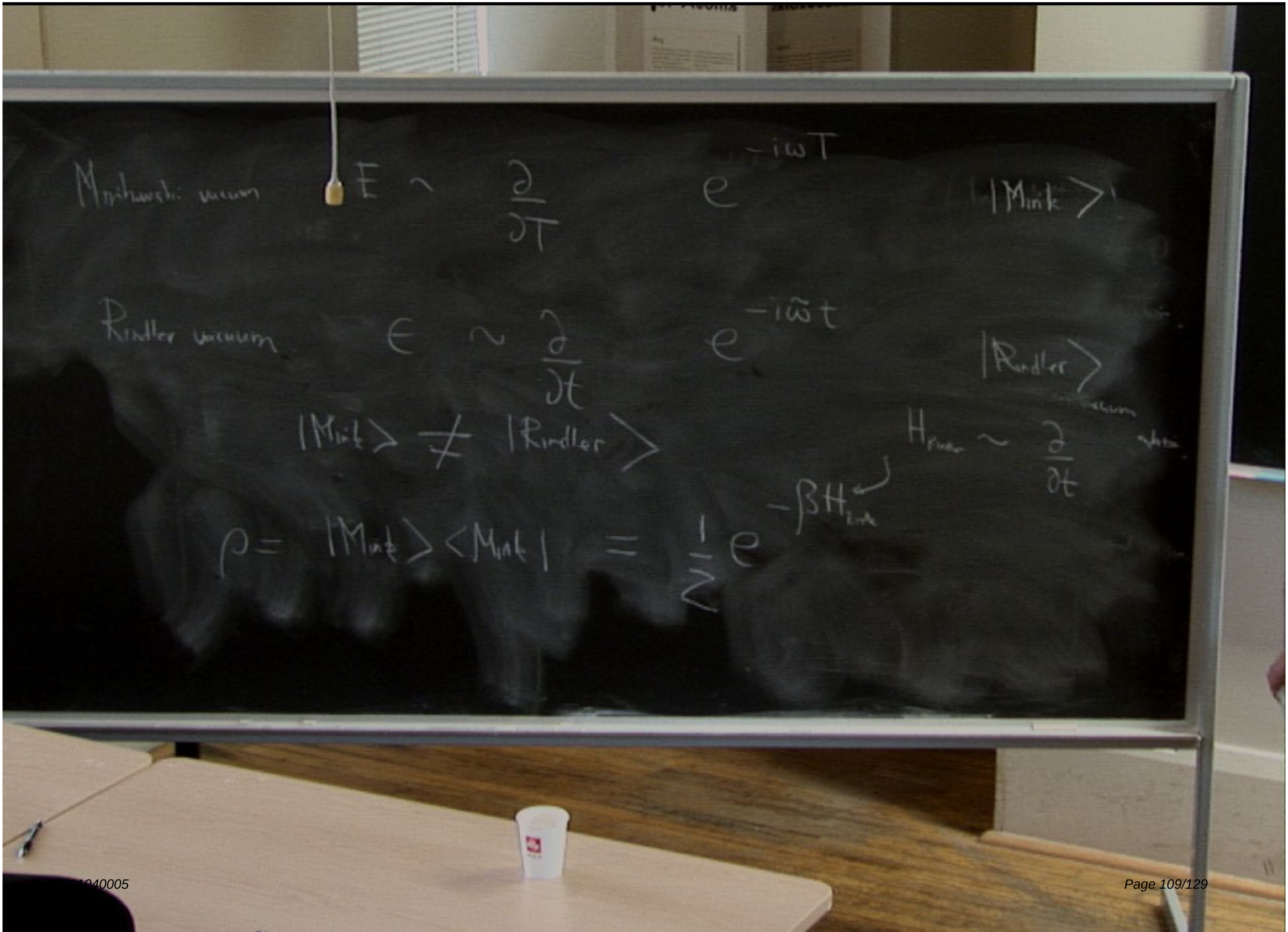

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Minkowski vacuum
|Mink>

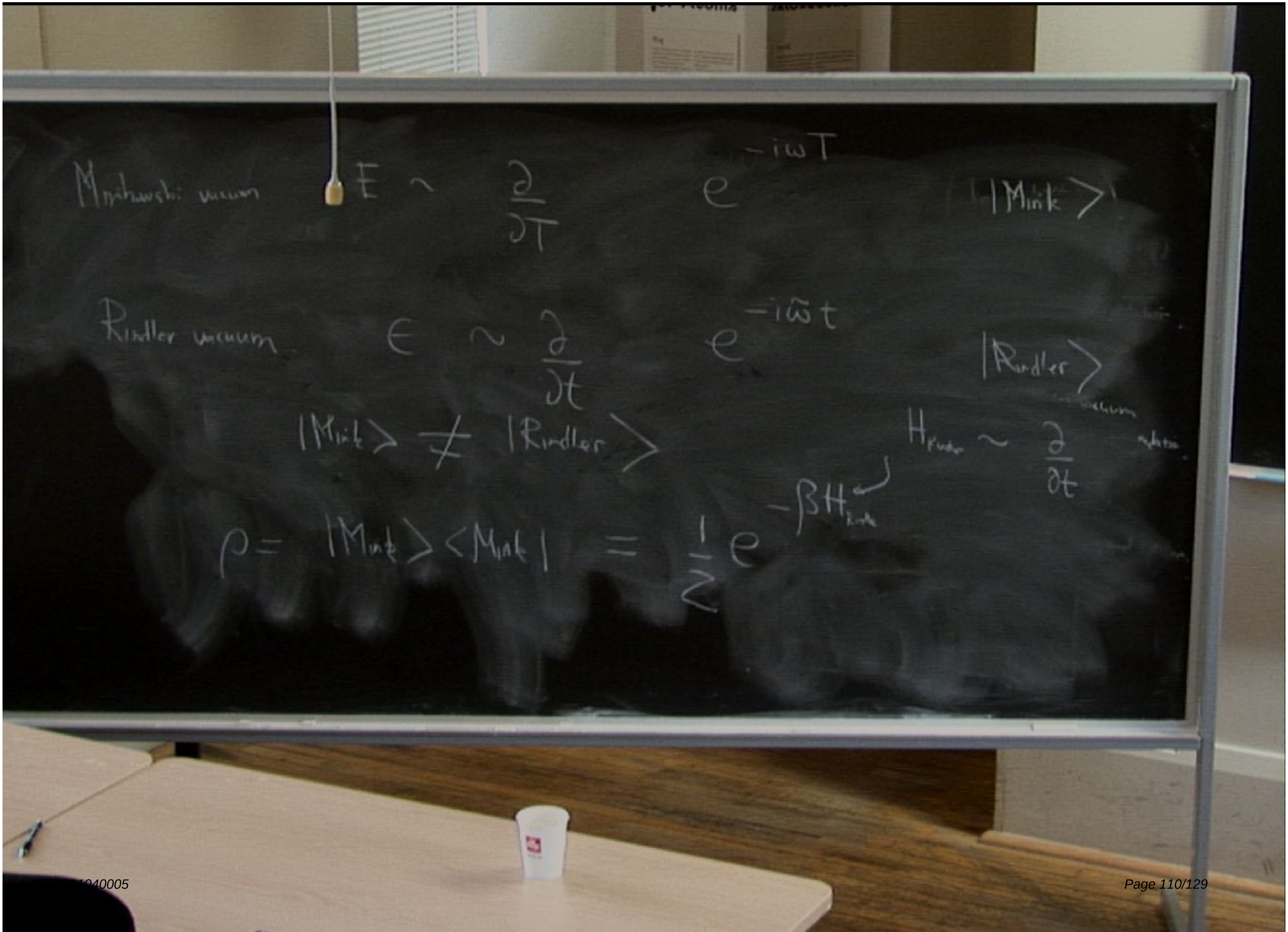

e^{2ax} = -a^2 X^+ X^-
ds^2 = - dX^+ dX^-
= - e^{+a(x^+ - x^-)} dx^+ dx^-
= - e^{2ax} (dt^2 - dx^2)
ds^2 = - e^{2ax} dt^2 + e^{2ax} dx^2
Rindler
Coordinates
t → t + c
K^μ ∂_μ = ∂_t
∇_μ K_ν + ∇_ν K_μ

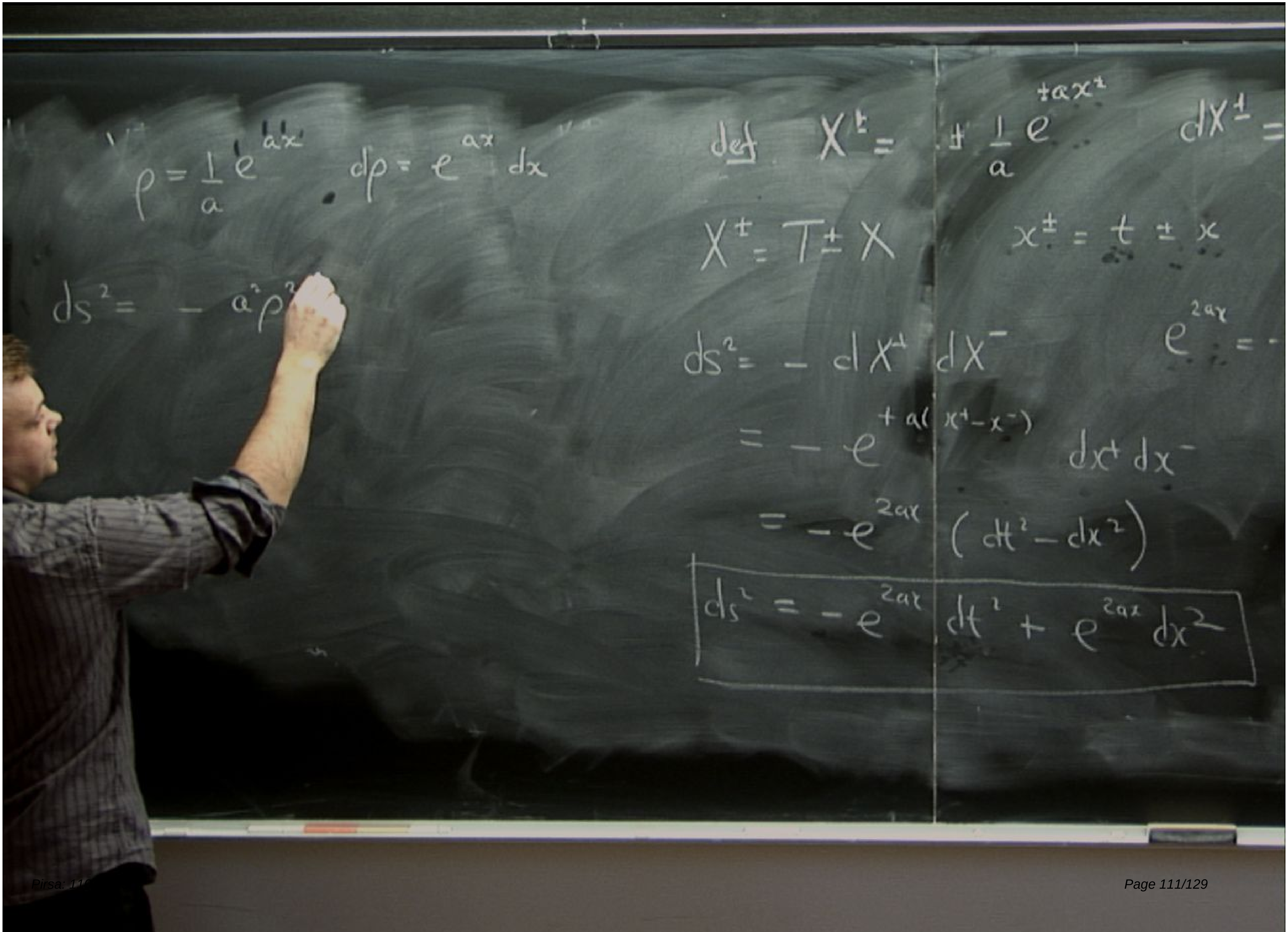

Minkowski vacuum E ~ ∂/∂T e^{-iωT} |Mink>
Rindler vacuum ∈ ~ ∂/∂t e^{-iω̃t} |R

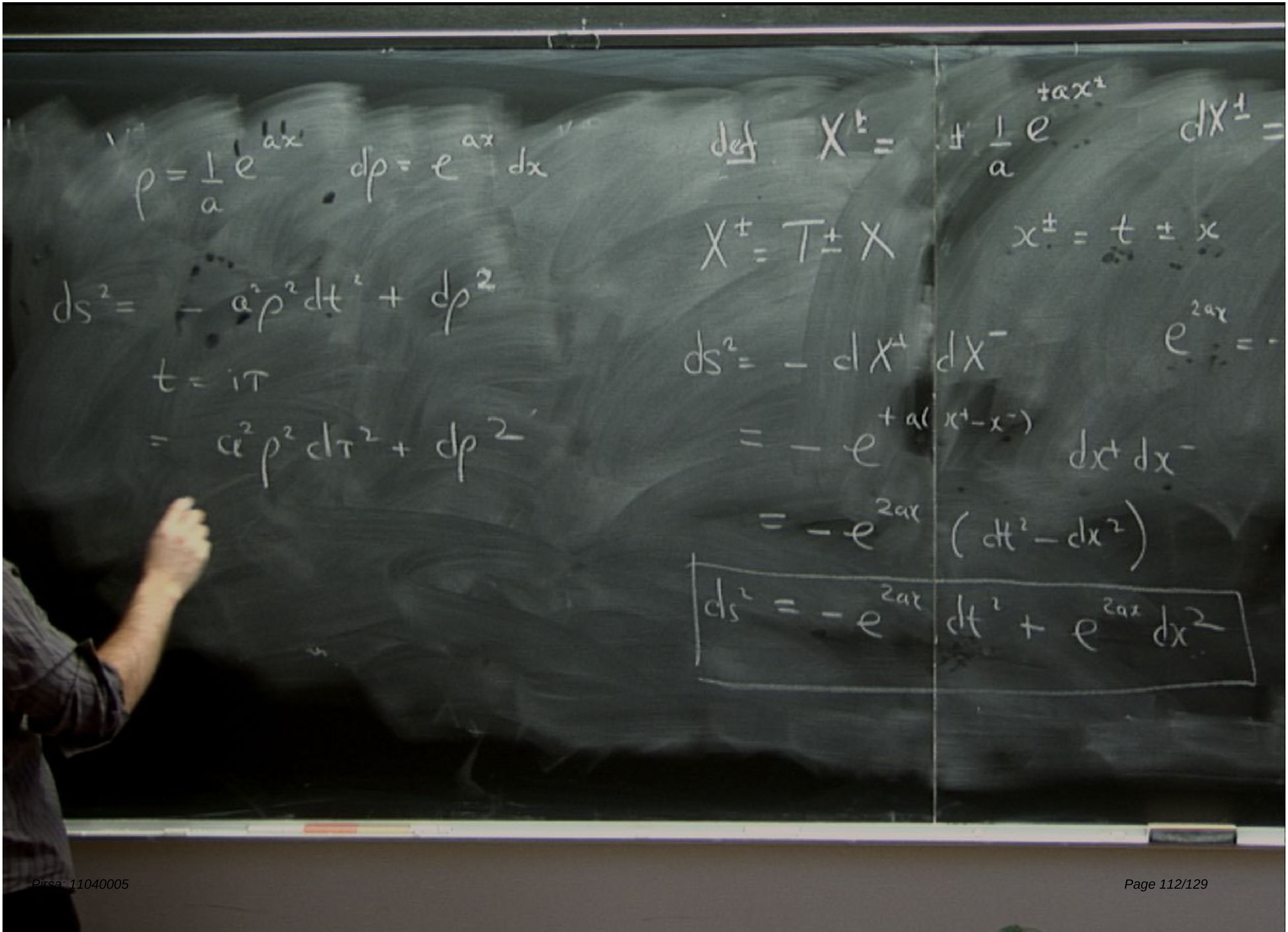

$E \sim \frac{\partial}{\partial T}$
$e^{-i\omega T}$
$|\text{Mink}\rangle$
$\epsilon \sim \frac{\partial}{\partial t}$
$e^{-i\tilde{\omega} t}$
$|\text{Rindler}\rangle$

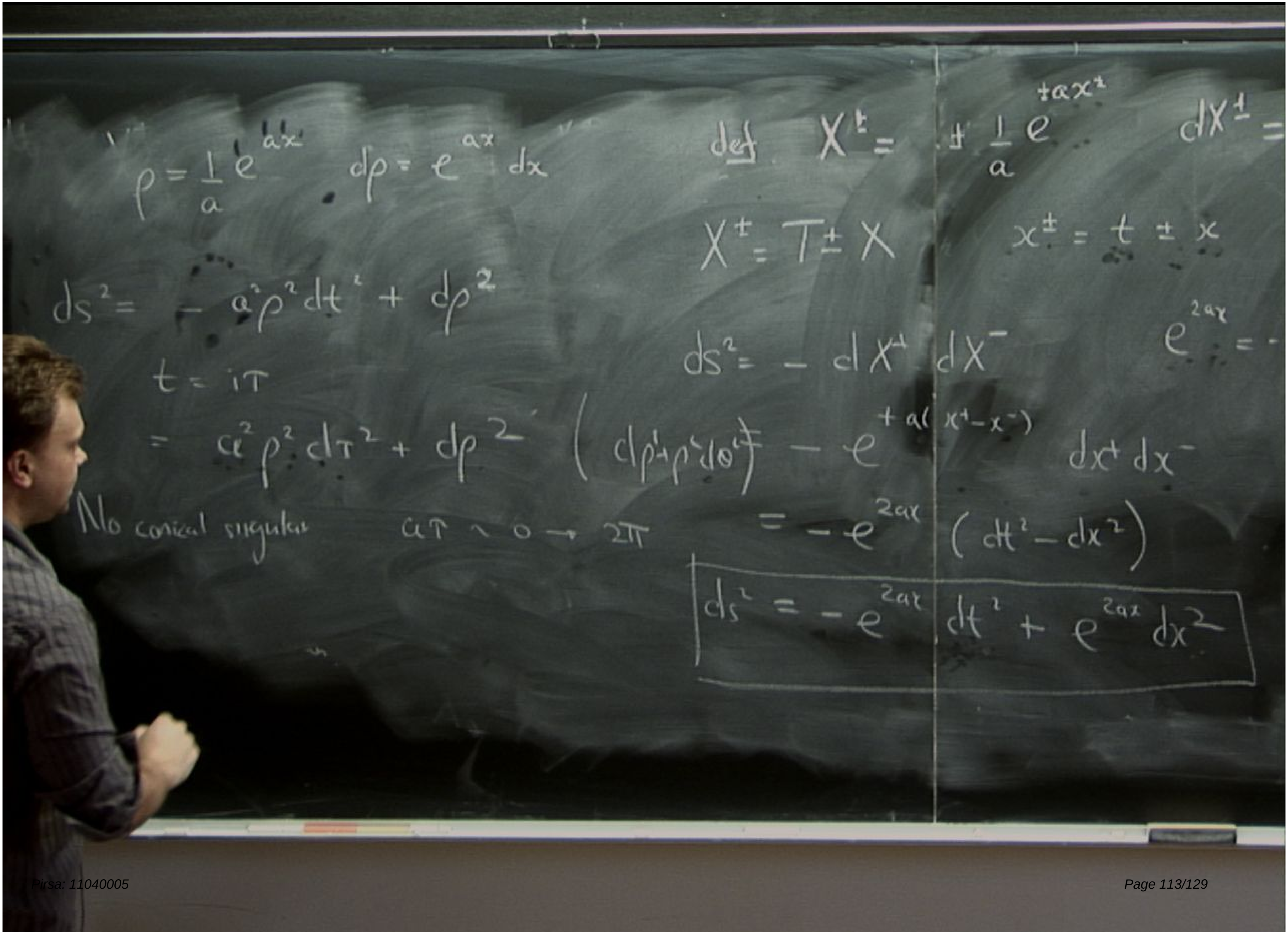

Minkowski vacuum
$E \sim \frac{\partial}{\partial T}$
$e^{-i\omega T}$
$|Mink\rangle$
indler vacuum
$\epsilon \sim \frac{\partial}{\partial t}$
$e^{-i\tilde{\omega} t}$
$|Rindler\rangle$
$|Mink\rangle \neq |Rindler\rangle$
$|Mink\rangle$

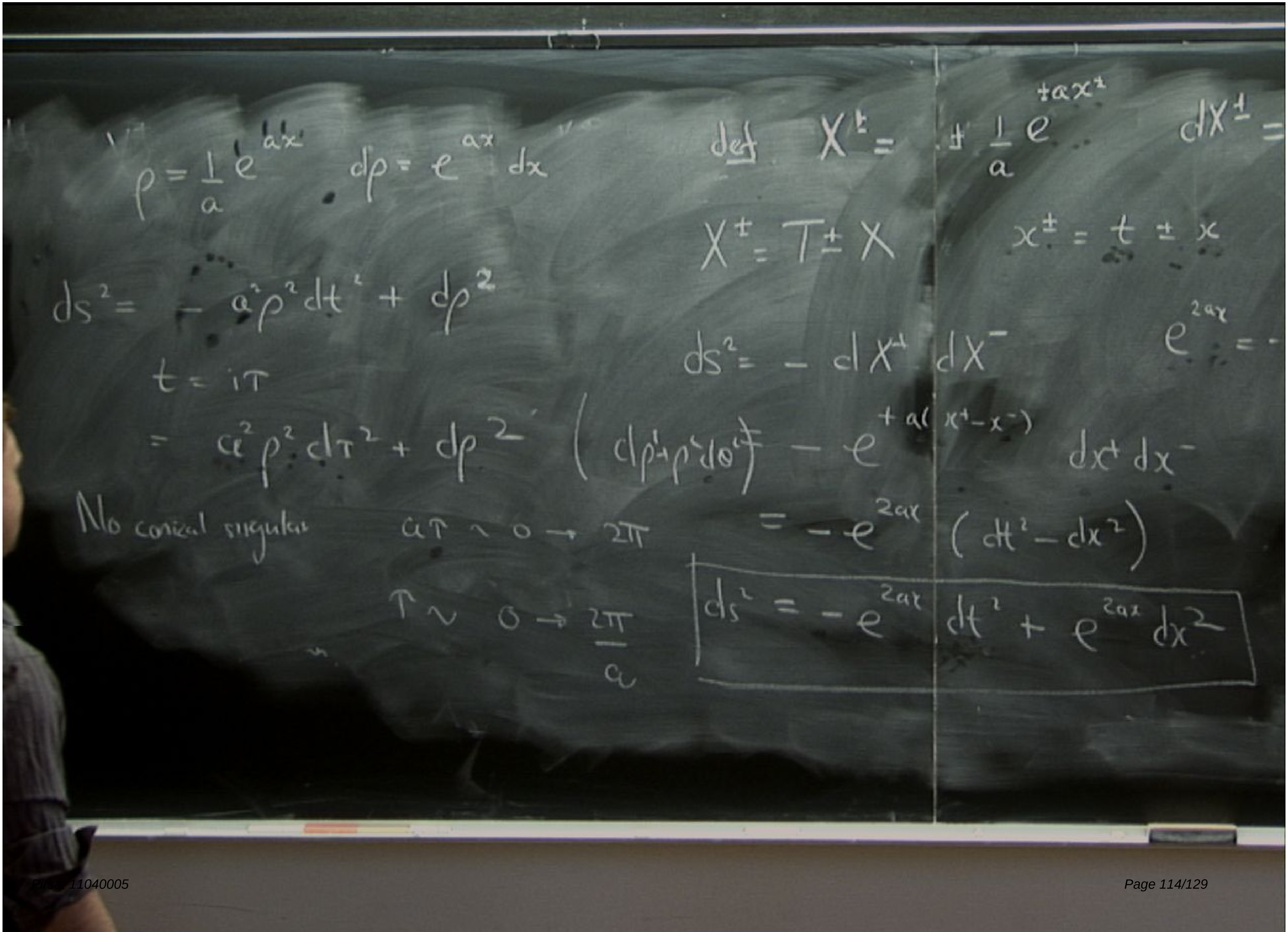

Minkowski vacuum $E \sim \frac{\partial}{\partial T}$ $e^{-i\omega T}$ $|Mink\rangle$
Rindler vacuum $\epsilon \sim \frac{\partial}{\partial t}$ $e^{-i\tilde{\omega} t}$ $|Rindler\rangle$
$|Mink\rangle \neq |Rindler\rangle$
$H_{Rindler} \sim \frac{\partial}{\partial t}$
$\rho = |Mink\rangle\langle Mink| = \frac{1}{Z} e^{-\beta H_{Rind}}$


Minkowski vacuum
E ~ ∂/∂T
e^{-iωT}
|Mink>
Rindler vacuum
E ~ ∂/∂t
e^{-iω̃t}
|Rindler>
|Mink> ≠ |Rindler>
H_Rindler ~ ∂/∂t
ρ = |Mink><Mink| = 1/Z e^{-βH_Rindler}


$\rho = \frac{1}{a} e^{ax}$ $d\rho = e^{ax} dx$
$ds^2 = - a^2\rho^2$
def $X^\pm = \pm \frac{1}{a} e^{\pm a x^\pm}$ $dX^\pm =$
$X^\pm = T \pm X$ $x^\pm = t \pm x$
$ds^2 = - dX^+ dX^-$ $e^{2ax} =$
$= - e^{+a(x^+ - x^-)} dx^+ dx^-$
$= - e^{2ax} (dt^2 - dx^2)$
$ds^2 = - e^{2ax} dt^2 + e^{2ax} dx^2$


ρ = 1/a e^{ax}   dρ = e^{ax} dx
ds² = − a²ρ² dt² + dρ²
t = iτ
= a²ρ² dτ² + dρ²
def X^± = ± 1/a e^{±ax^±}   dX^± =
X^± = T ± X   x^± = t ± x
ds² = − dX⁺ dX⁻   e^{2ax} =
= − e^{+a(x⁺−x⁻)} dx⁺ dx⁻
= − e^{2ax} (dt² − dx²)
ds² = − e^{2ax} dt² + e^{2ax} dx²


ρ = 1/a e^{ax}   dρ = e^{ax} dx
ds² = − a²ρ² dt² + dρ²
t = iT
= a²ρ² dT² + dρ²   (dρ² + ρ² dθ²)
No conical singular   aT ~ 0 → 2π
def X^± = ± 1/a e^{±aX^±}   dX^± =
X^± = T ± X   x^± = t ± x
ds² = − dX⁺ dX⁻   e^{2ax} = −
− e^{+a(x⁺−x⁻)} dx⁺ dx⁻
= − e^{2ax} (dt² − dx²)
ds² = − e^{2ax} dt² + e^{2ax} dx²


$\rho = \frac{1}{a} e^{ax'}$
$d\rho = e^{ax} dx$
def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$
$dX^{\pm} =$
$X^{\pm} = T \pm X$
$x^{\pm} = t \pm x$
$ds^2 = -a^2\rho^2 dt^2 + d\rho^2$
$ds^2 = -dX^+ dX^-$
$e^{2ax} = \cdot$
$t = i\tau$
$= a^2\rho^2 d\tau^2 + d\rho^2$
$(d\rho^2 + \rho^2 d\theta^2)$
$= -e^{+a(x^+ - x^-)} dx^+ dx^-$
No conical singular
$a\tau \sim 0 \to 2\pi$
$= -e^{2ax}(dt^2 - dx^2)$
$\tau \sim 0 \to \frac{2\pi}{a}$
$ds^2 = -e^{2ax} dt^2 + e^{2ax} dx^2$

$\rho = \frac{1}{a} e^{ax}$ $\quad d\rho = e^{ax} dx$

$ds^2 = -a^2\rho^2 dt^2 + d\rho^2$

$t = i\tau$

$= a^2\rho^2 d\tau^2 + d\rho^2 \quad (d\rho^2 + \rho^2 d\theta^2)$

No conical singular $\quad a\tau \sim 0 \to 2\pi$

$\tau \sim 0 \to \frac{2\pi}{a}$

def $X^{\pm} = \pm \frac{1}{a} e^{\pm a x^{\pm}}$ $\quad dX^{\pm} =$

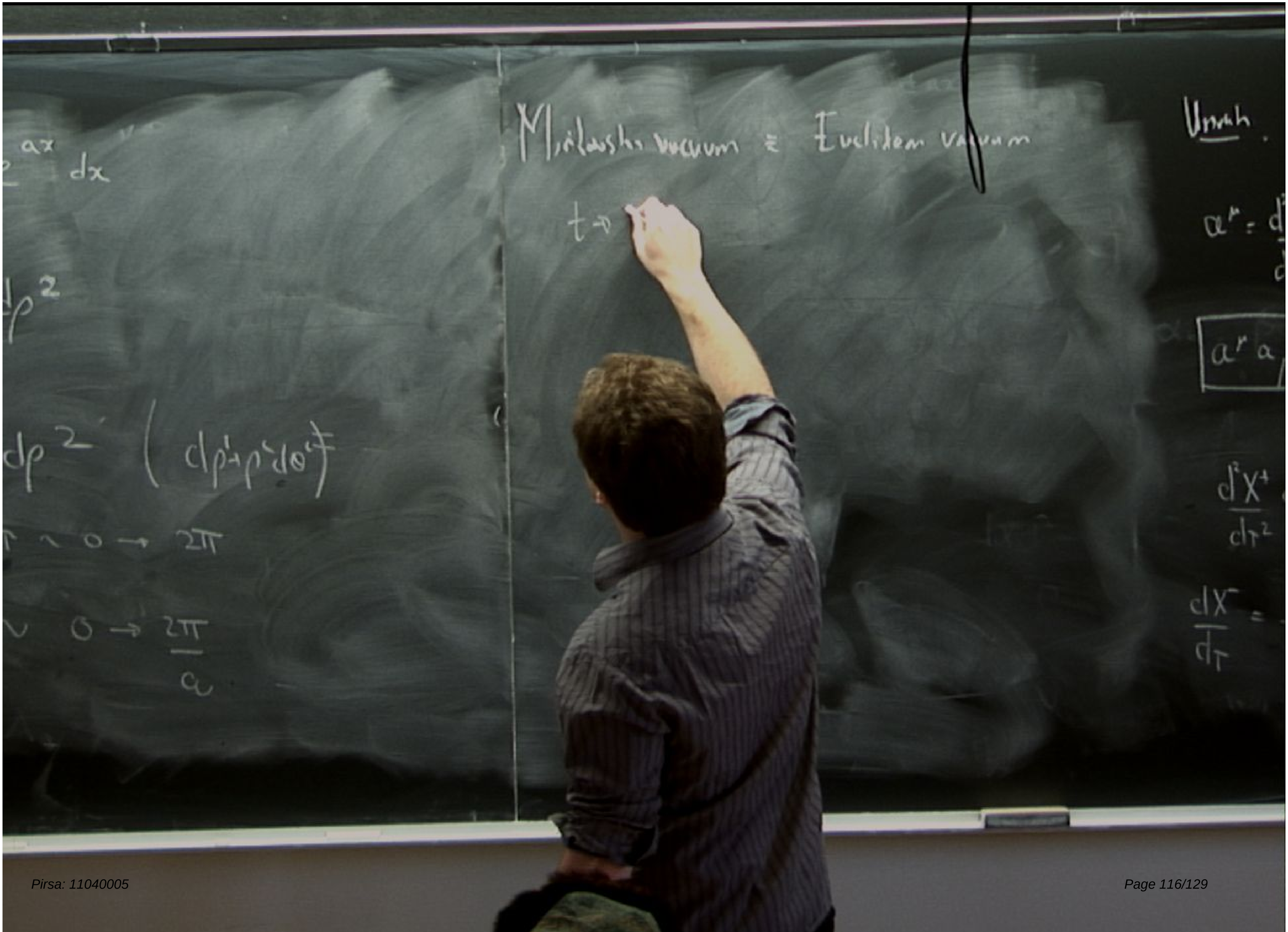
$X^{\pm} = T \pm X$ $\quad x^{\pm} = t \pm x$

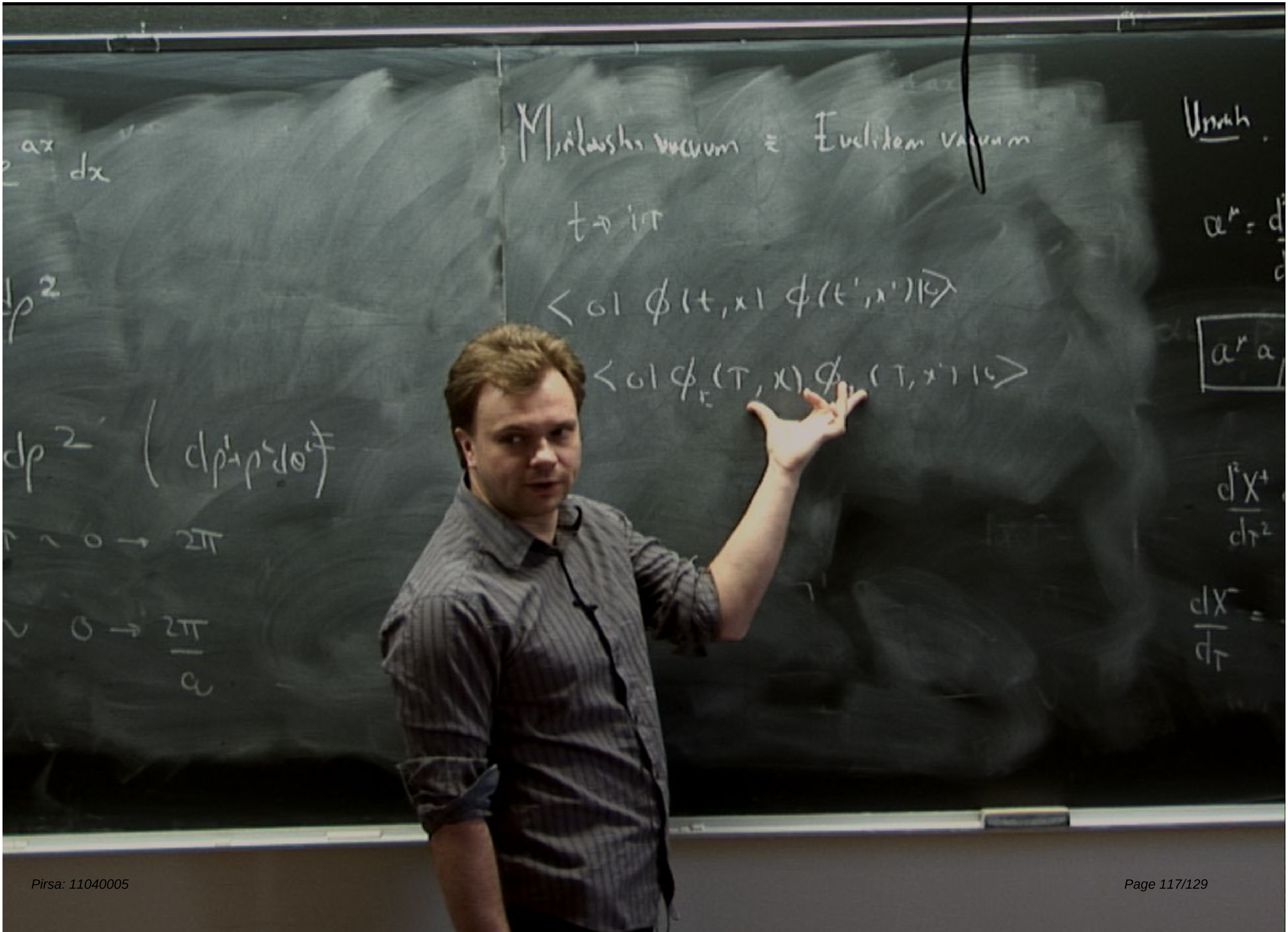
$ds^2 = -dX^{+} dX^{-}$ $\quad e^{2ax} = \cdot$

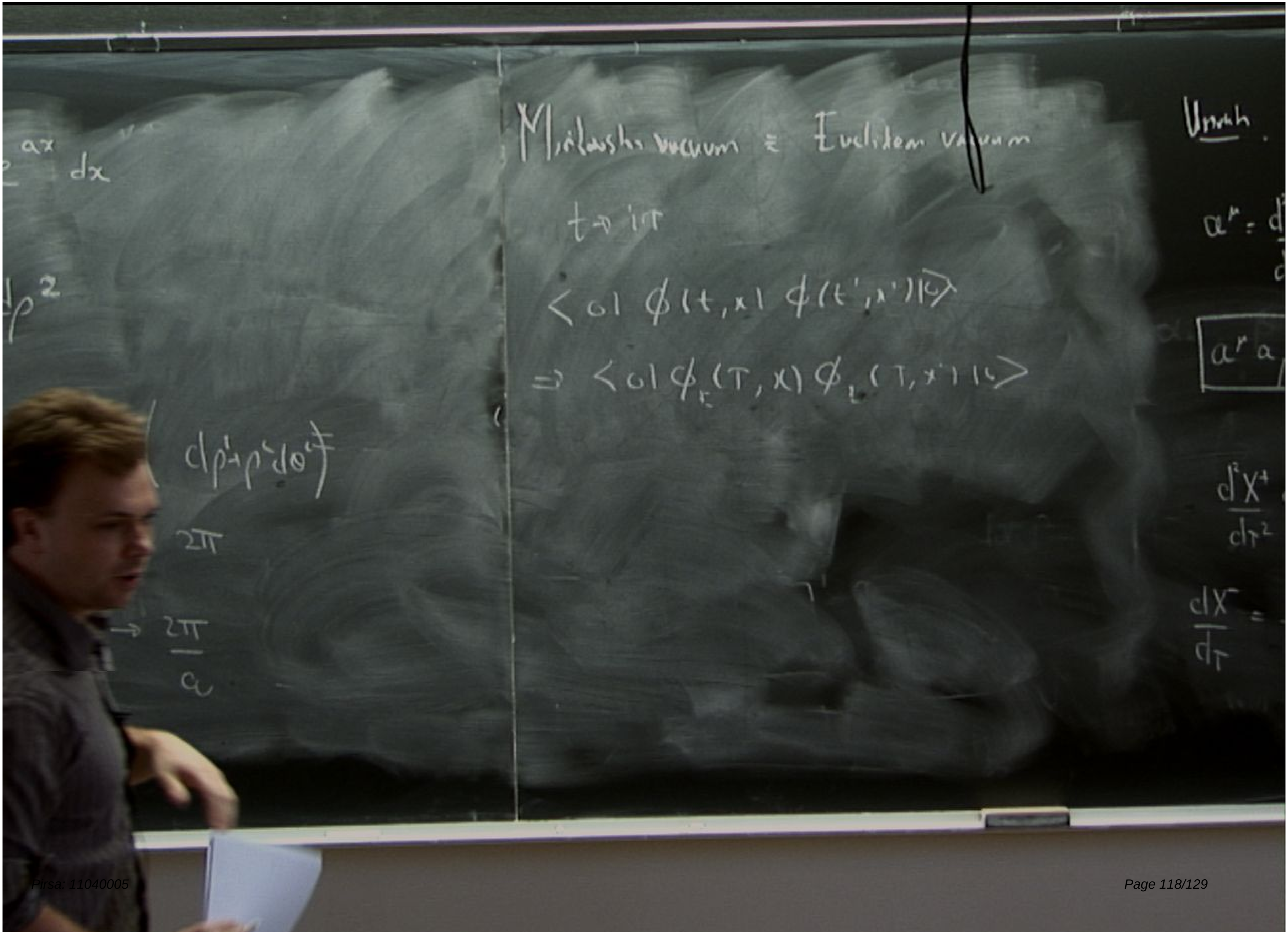
$- e^{+a(x^{+} - x^{-})} dx^{+} dx^{-}$

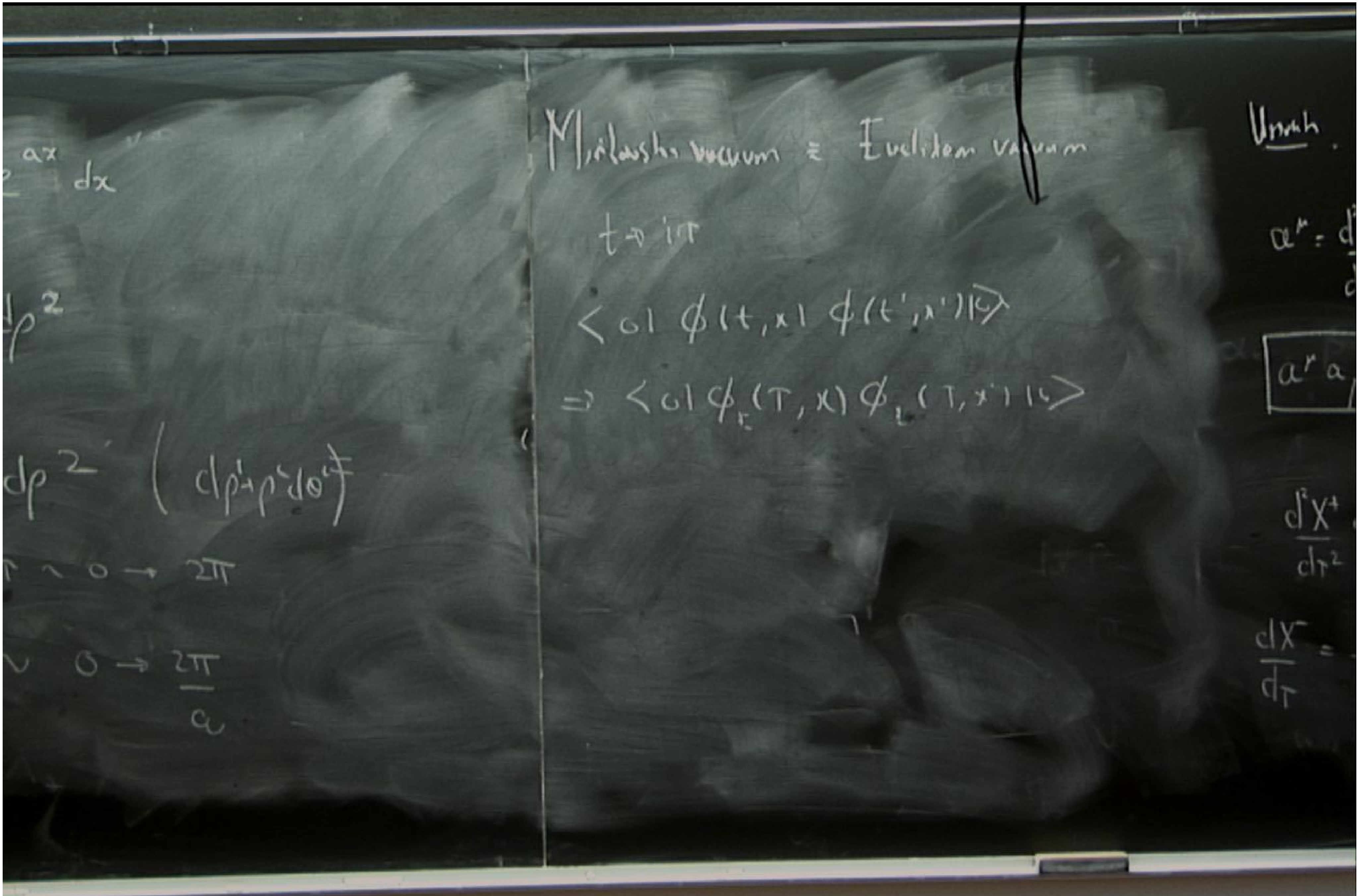
$= -e^{2ax}(dt^2 - dx^2)$

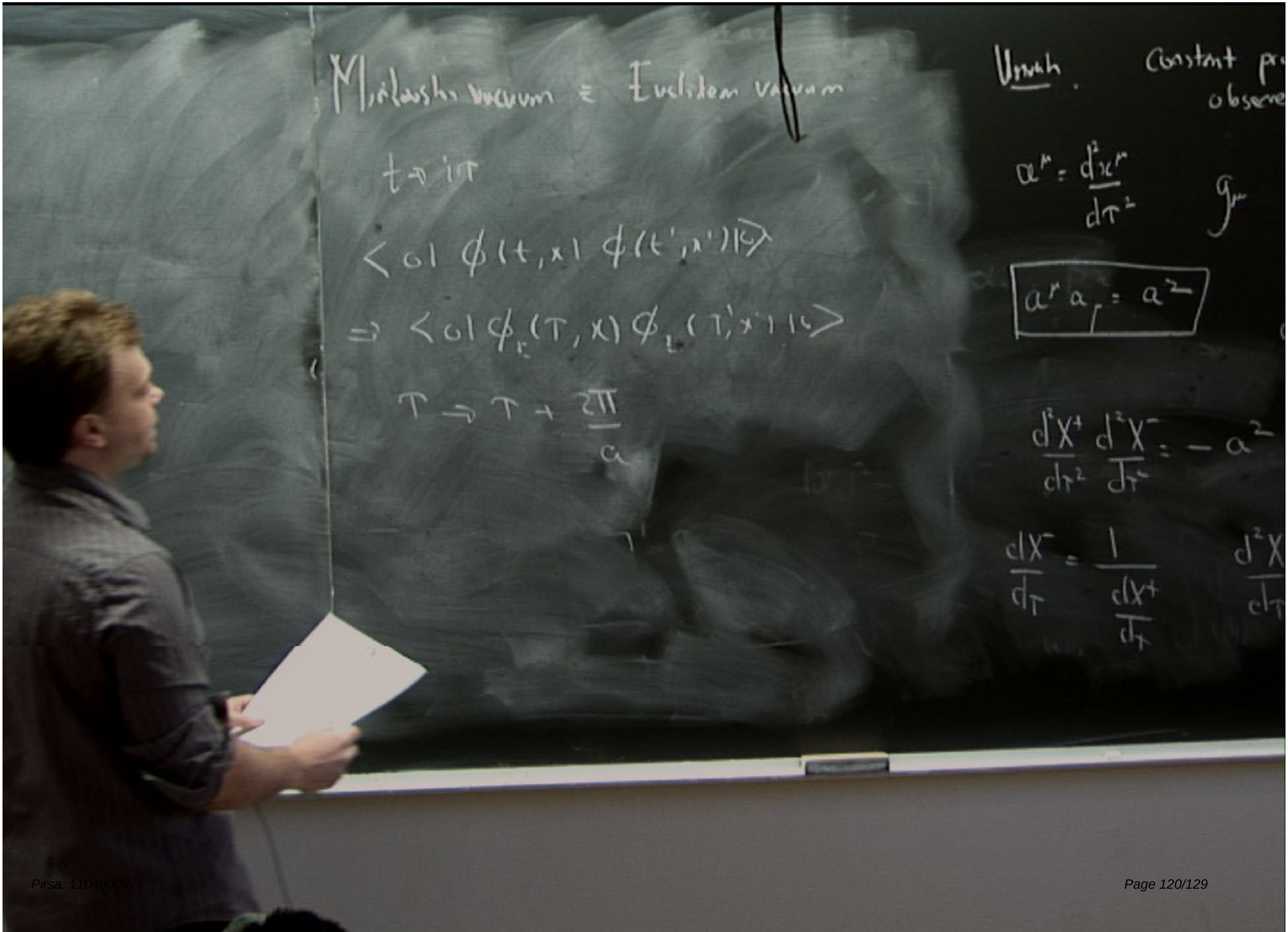
$$ds^2 = -e^{2ax} dt^2 + e^{2ax} dx^2$$

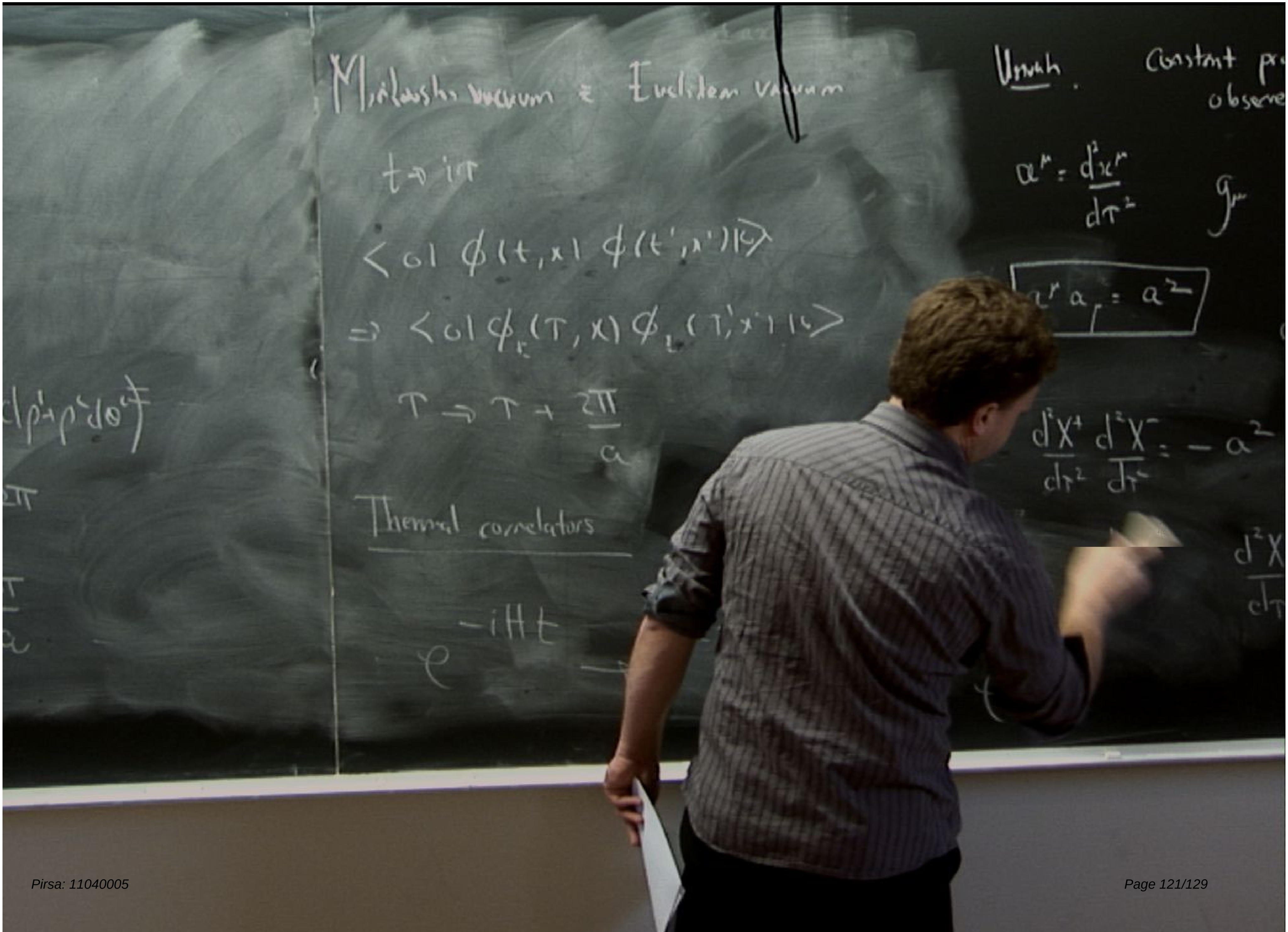

Minkowski vacuum ≈ Euclidean vacuum
Unruh

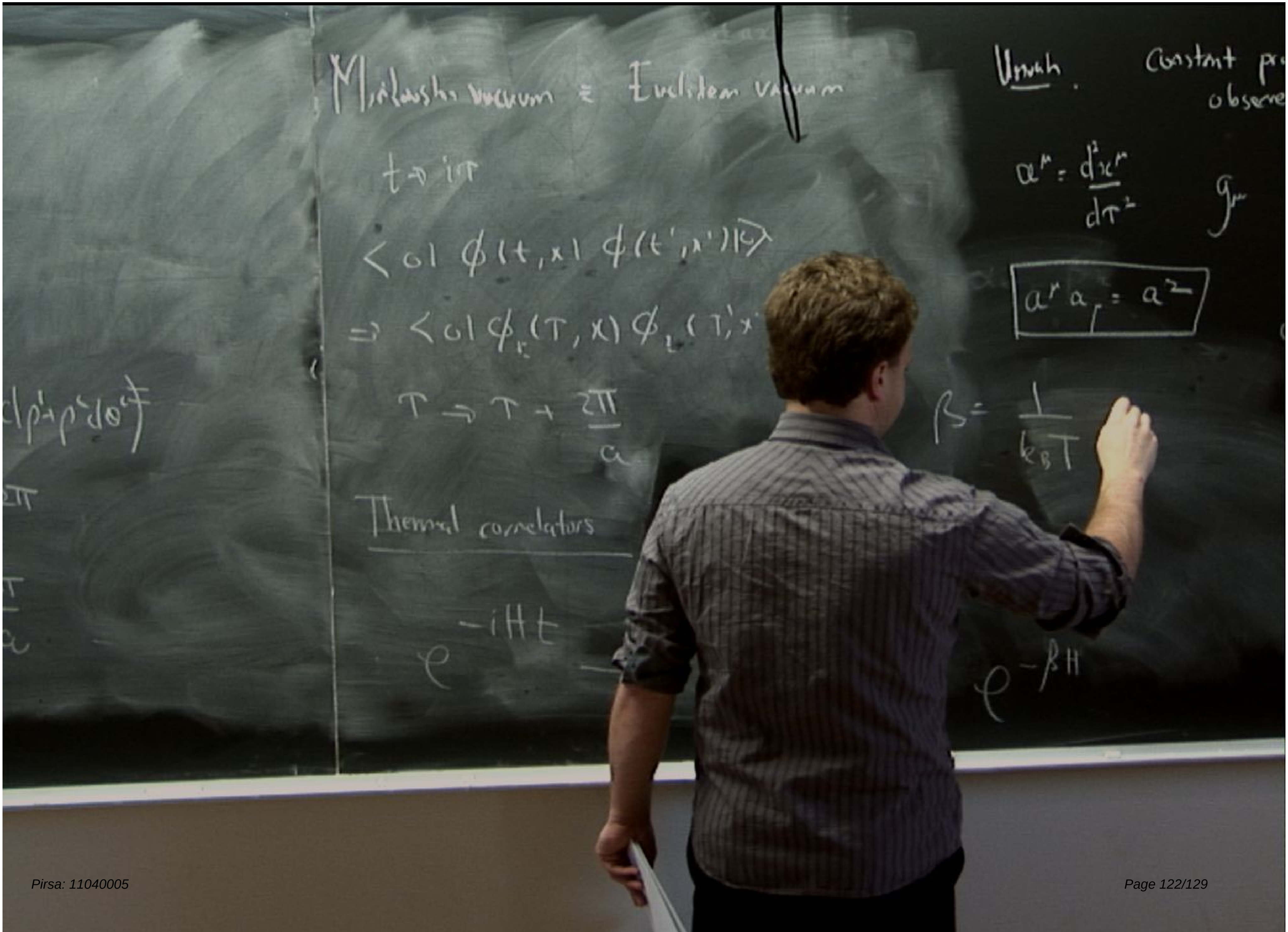

Minkowski vacuum = Euclidean vacuum
t → iτ
Unruh

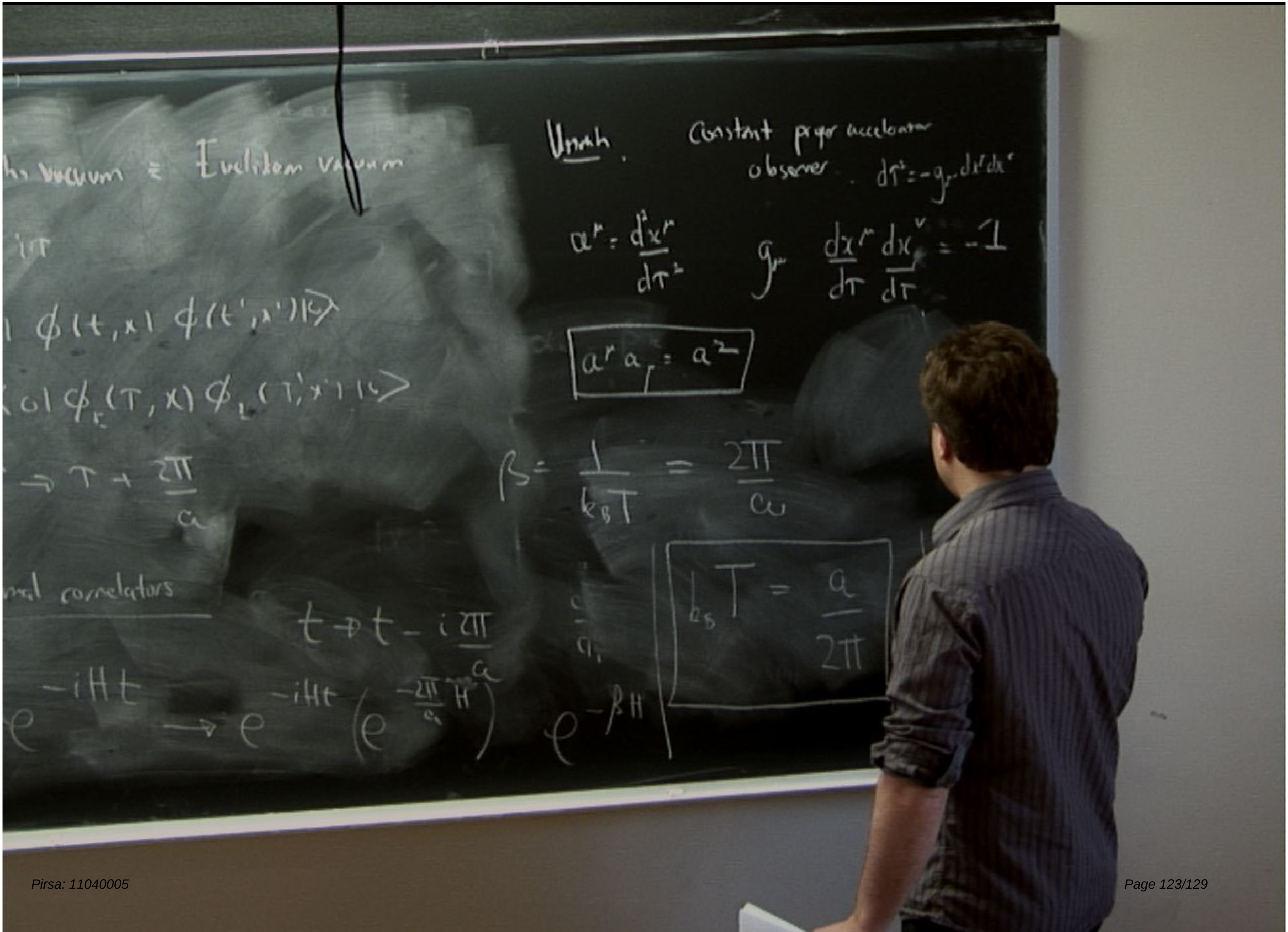

Minkowski vacuum = Euclidean vacuum
t → iτ
<0| φ(t,x) φ(t',x')|0>
⇒ <0| φ_E(τ,x) φ_E(τ,x')|0>
Unruh

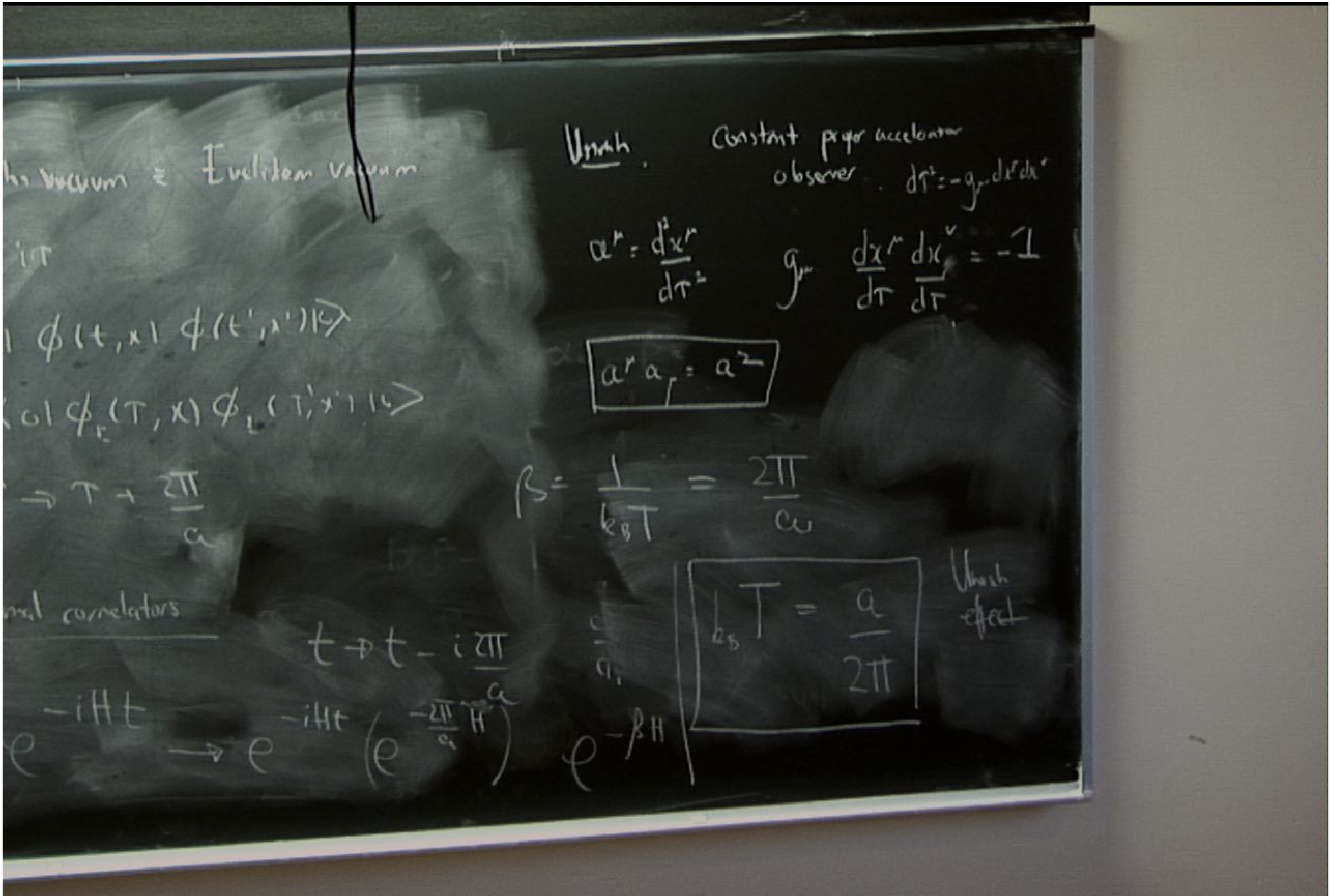

Minkowski vacuum = Euclidean vacuum
t → iτ
<0| φ(t, x) φ(t', x')|0>
⇒ <0| φ_E(τ, x) φ_E(τ, x')|0>
Unruh

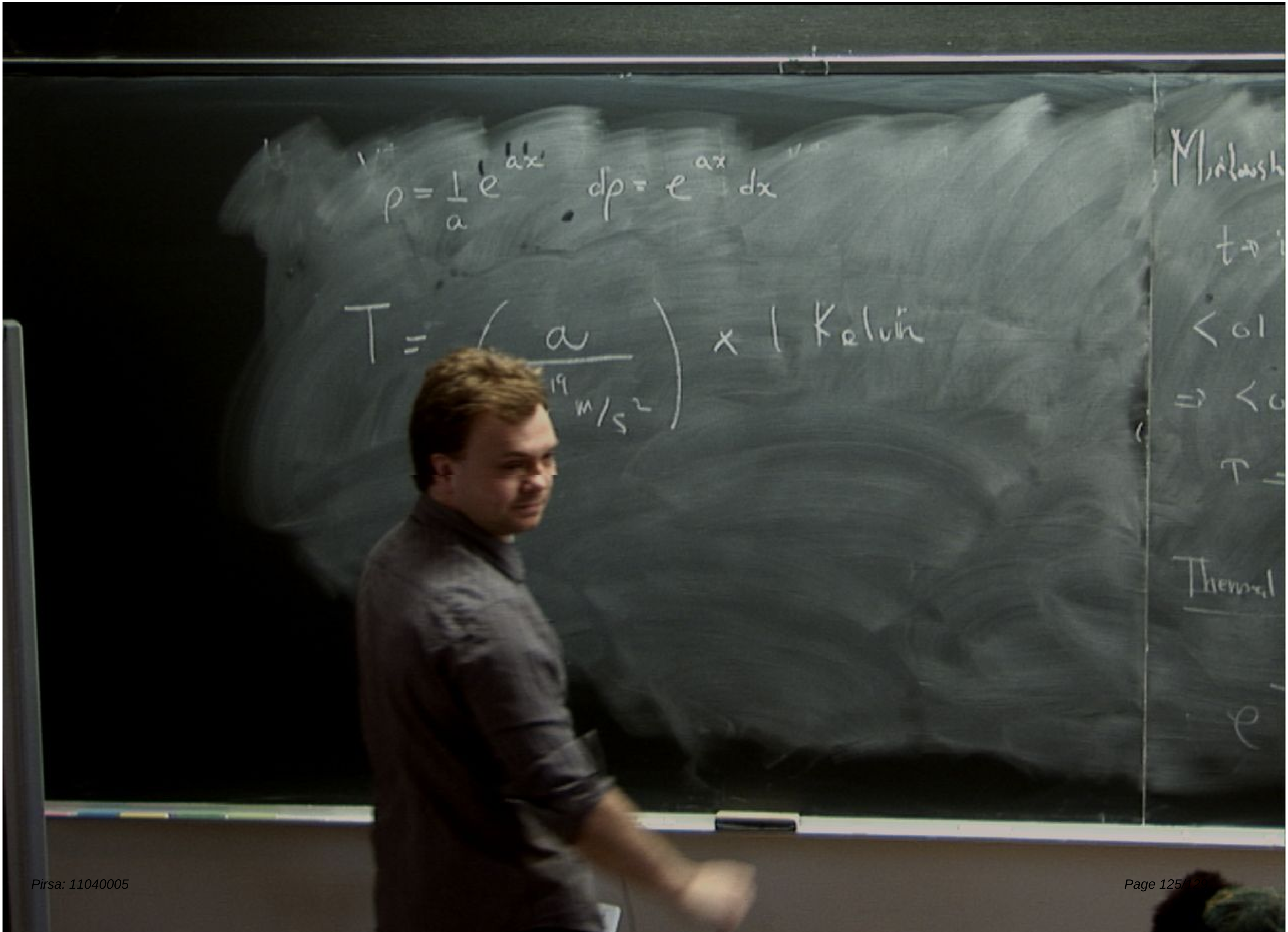

Minkowski vacuum = Euclidean vacuum
t → iτ
Unruh
Constant
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$
$\tau \to \tau + \frac{2\pi}{a}$

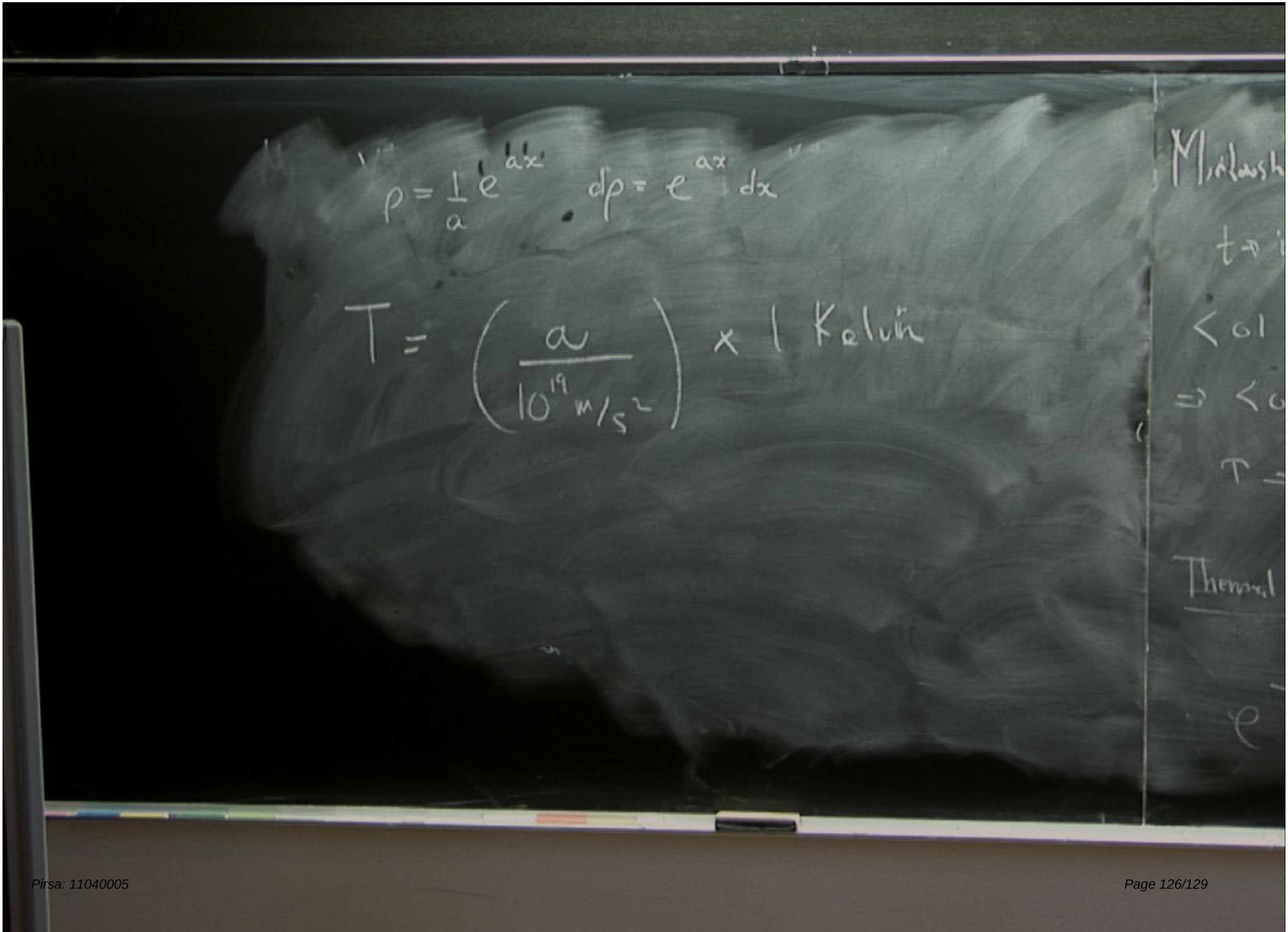

Minkowski vacuum = Euclidean vacuum
t → iτ
Thermal correlators
Unruh


Minkowski vacuum = Euclidean vacuum
$t \to i\tau$
$\tau \to \tau + \frac{2\pi}{a}$
Thermal correlators
e^{-iHt}
Unruh
$a^\mu = \frac{d^2x^\mu}{d\tau^2}$
$a^\mu a_\mu = a^2$
$\beta = \frac{1}{k_B T}$
$e^{-\beta H}$


Unruh
Constant proper acceleration observer
Euclidean vacuum
correlators
$\beta = \frac{1}{k_B T} = \frac{2\pi}{a}$
$k_B T = \frac{a}{2\pi}$
$a^\mu a_\mu = a^2$


Unruh
Constant proper accelerated observer
Euclidean vacuum
correlators
Unruh effect


ρ = 1/a e^{ax}
dρ = e^{ax} dx
T = (a / 19 m/s²) × 1 Kelvin
Thermal

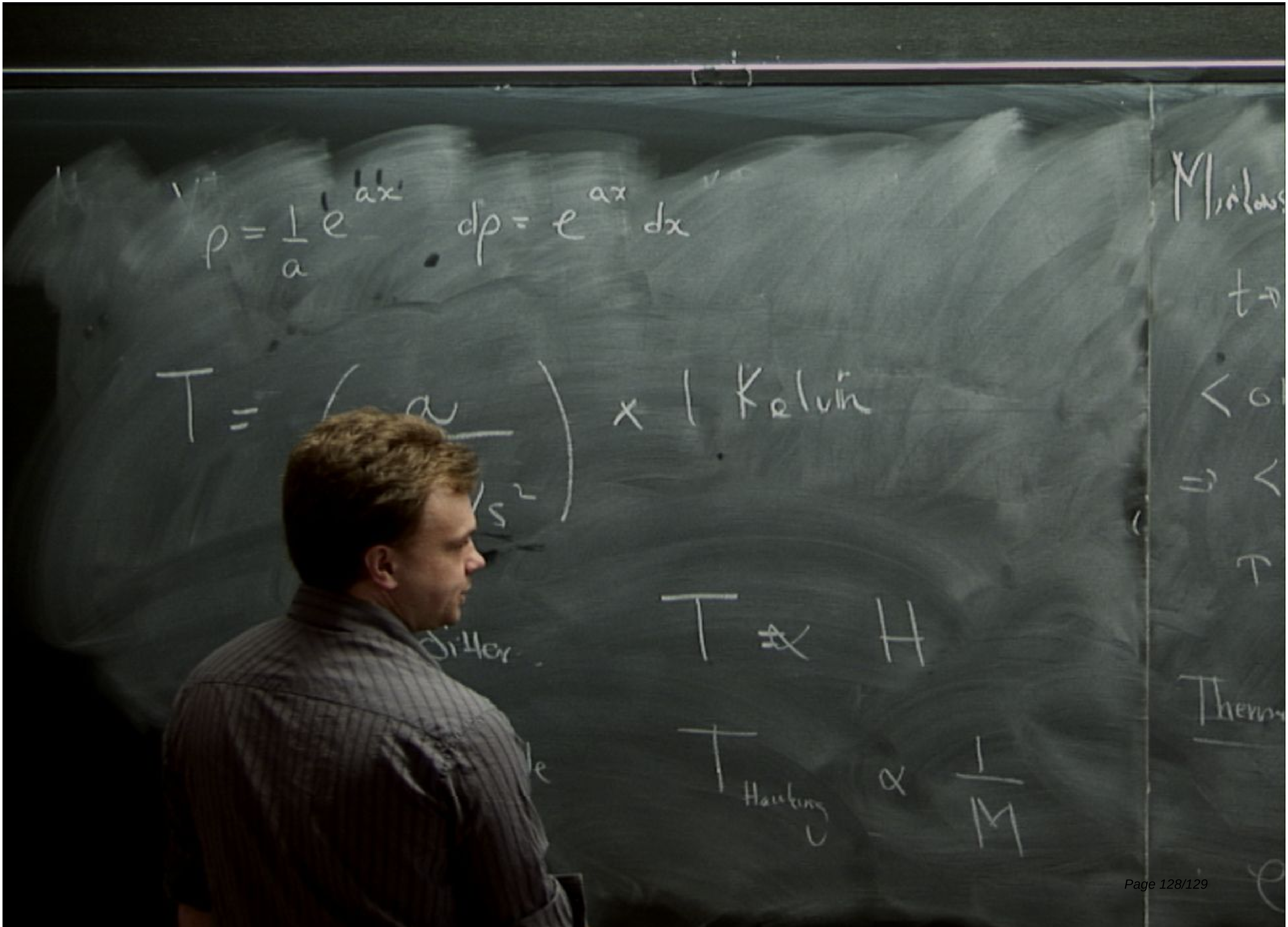


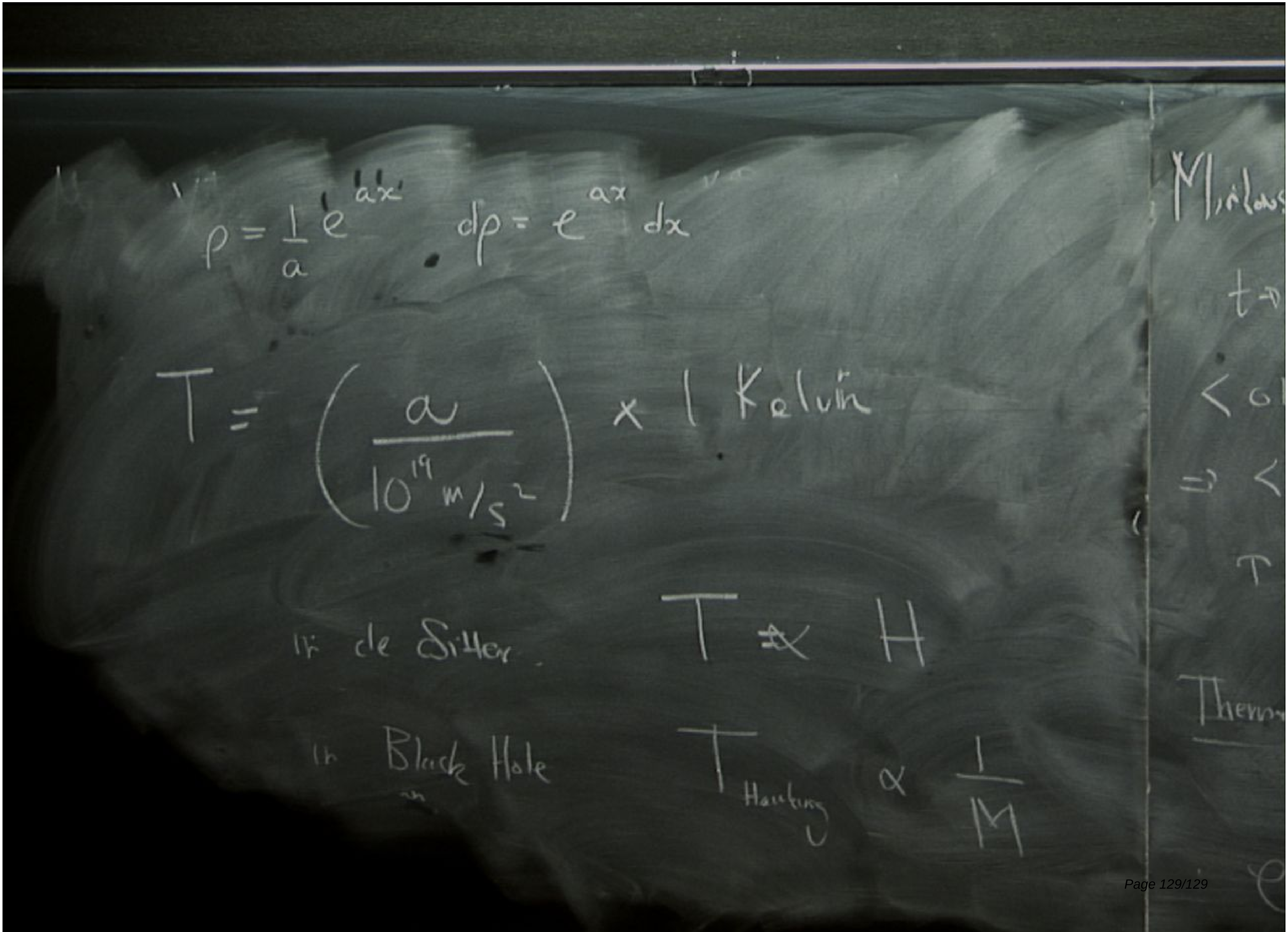
$\rho = \frac{1}{a} e^{ax} \quad d\rho = e^{ax} dx$
$T = \left(\frac{a}{10^{19} m/s^2} \right) \times 1$ Kelvin
Thermal

$$\rho = \frac{1}{a} e^{ax'} \qquad d\rho = e^{ax}\, dx$$

$$T = \left(\frac{a}{10^{19}\ m/s^2} \right) \times 1 \text{ Kelvin}$$

in de Sitter $\quad T =$


ρ = 1/a e^{ax}
dρ = e^{ax} dx
× 1 Kelvin
T ∝ H
T_Hawking ∝ 1/M


ρ = 1/a e^{ax}   dρ = e^{ax} dx
T = (a / 10^19 m/s^2) × 1 Kelvin
in de Sitter   T ∝ H
in Black Hole   T_Hawking ∝ 1/M