Title: Explorations in Cosmology - Lecture 2

Date: Apr 05, 2011 09:00 AM

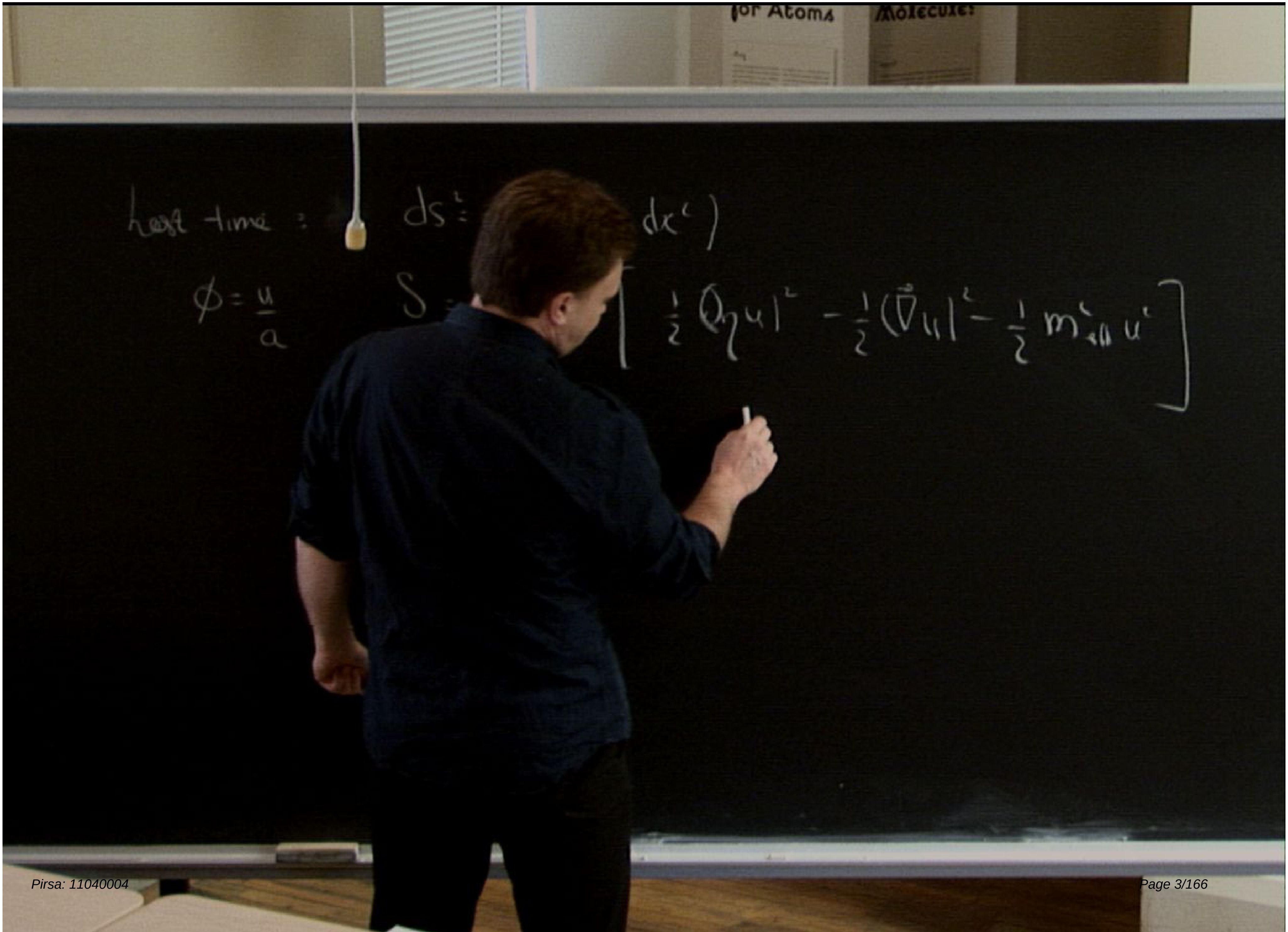
URL: http://pirsa.org/11040004

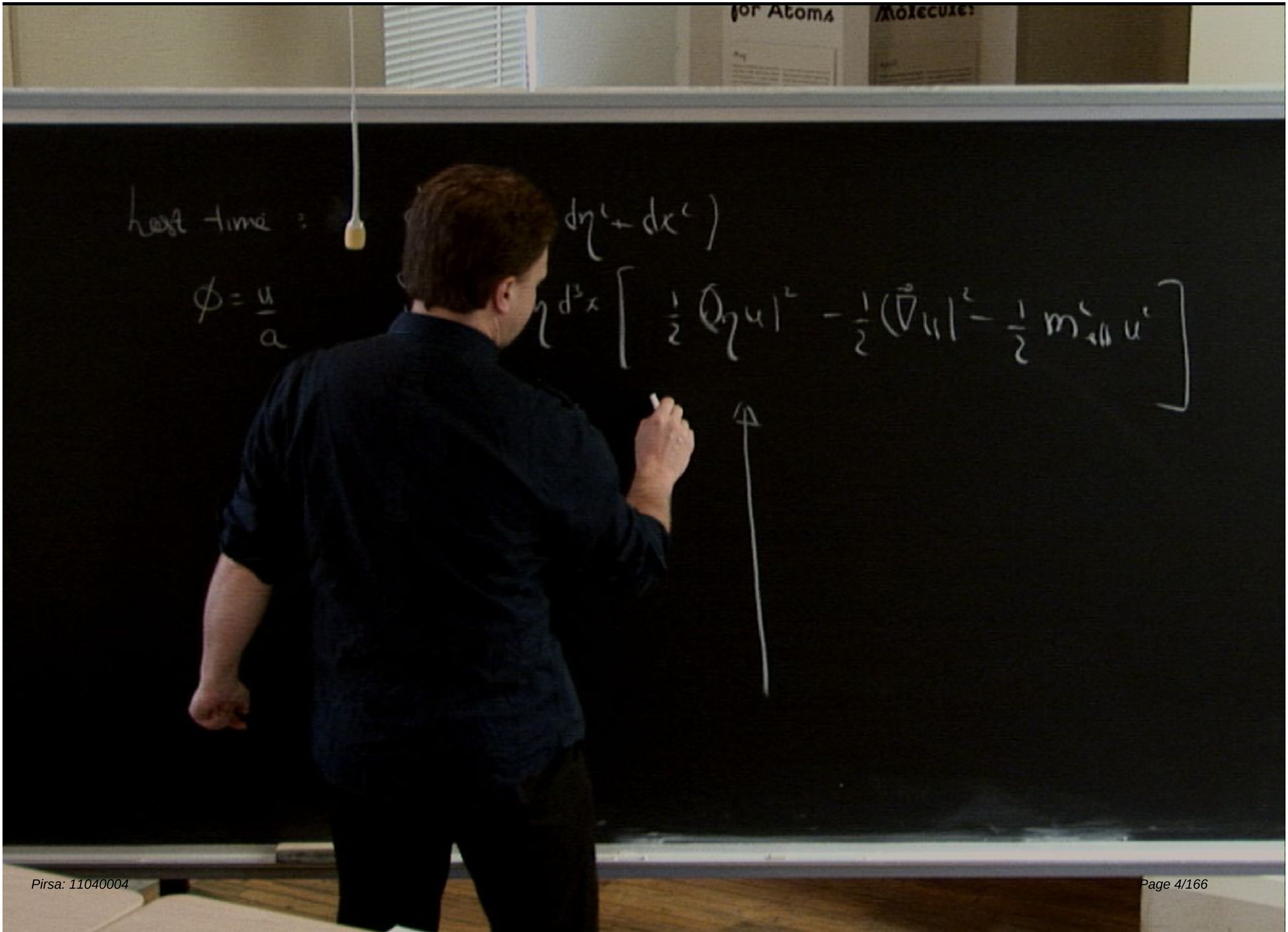
Abstract:

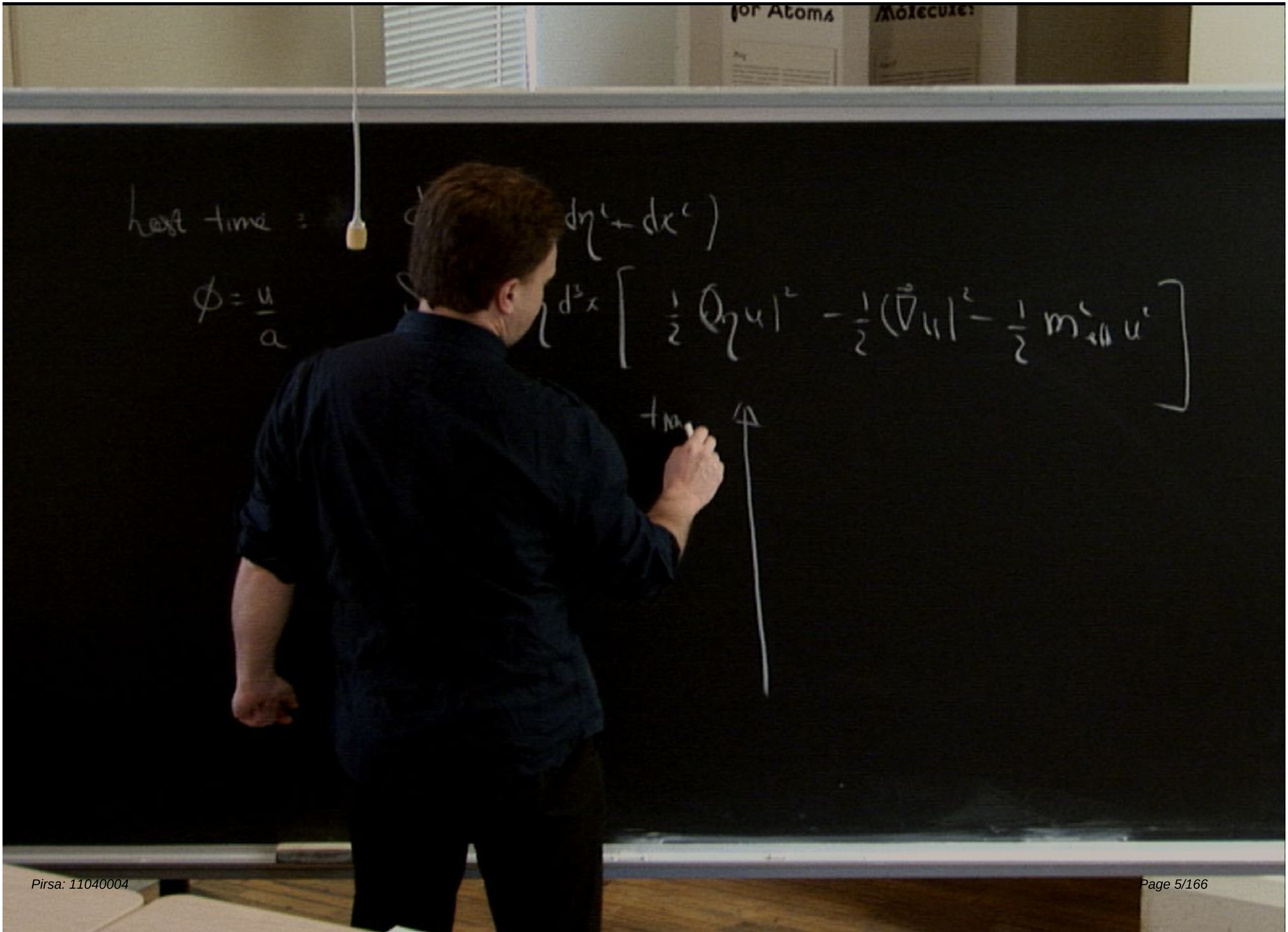
Last time: $ds^2 = a^2(-d\eta^2 + dx^2)$

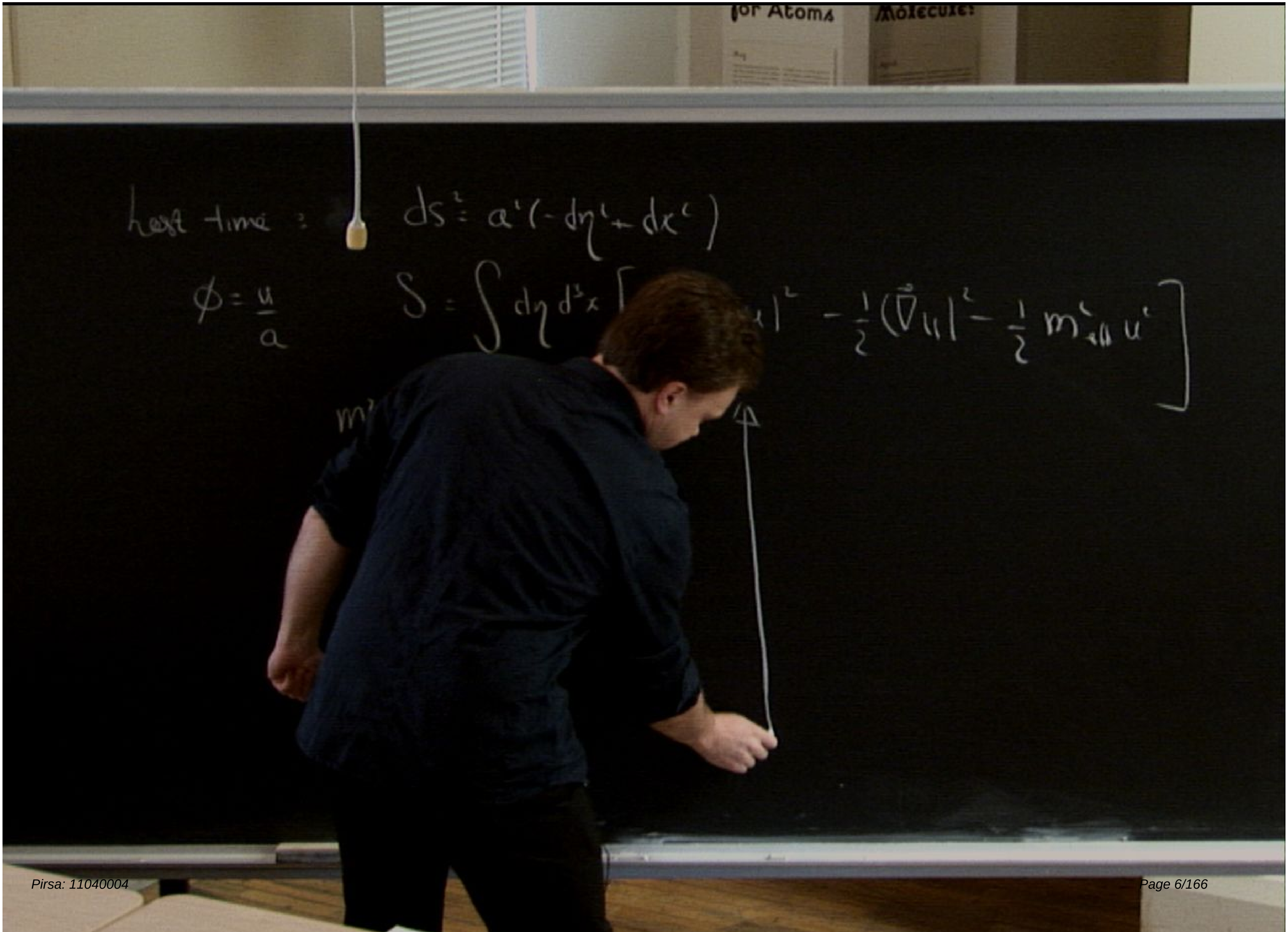
$$\phi = \frac{u}{a} \qquad S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m_{eff}^2 u^2 \right]$$

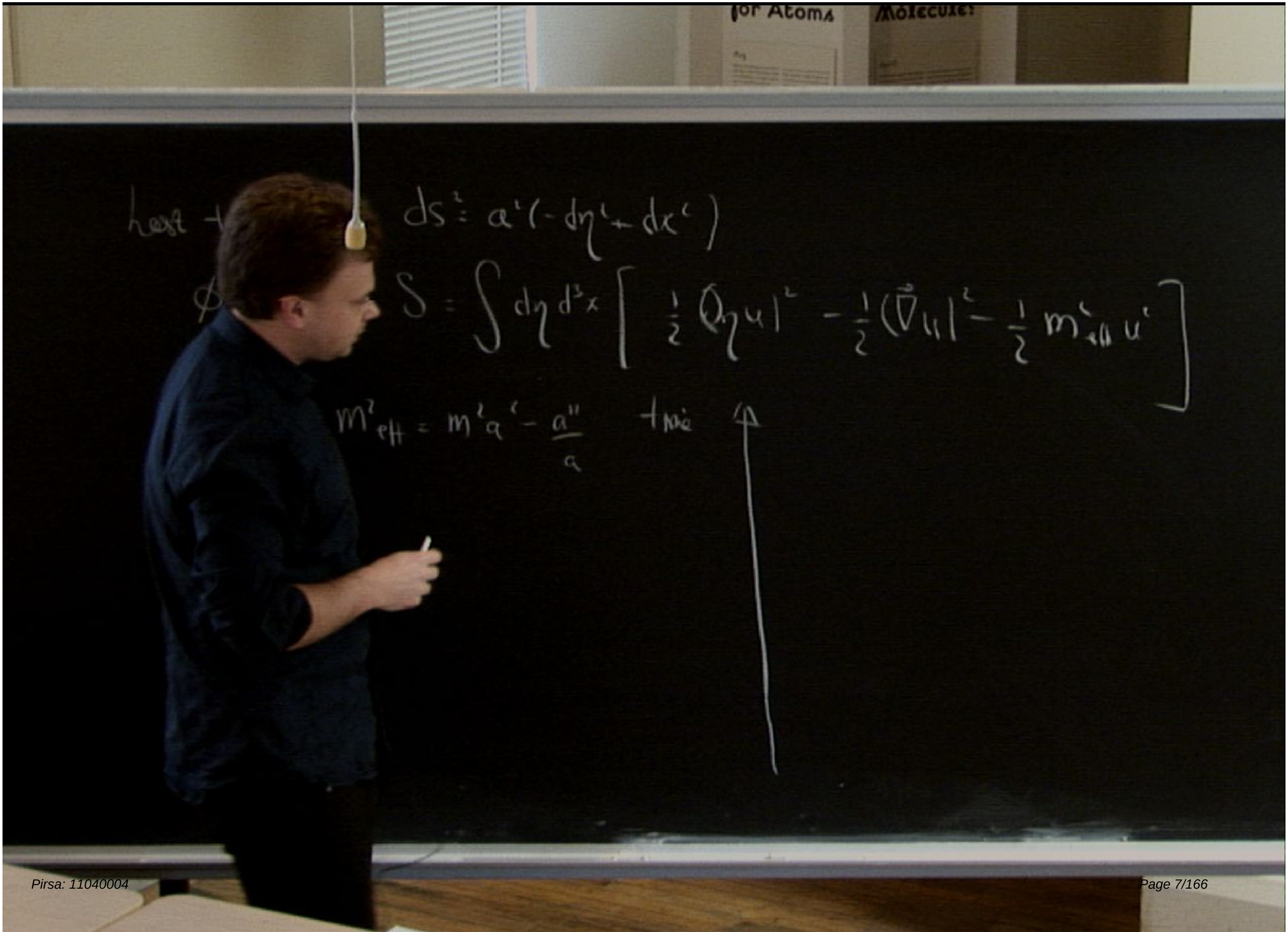
$$m_{eff}^2 = m^2 a^2 - \frac{a''}{a}$$

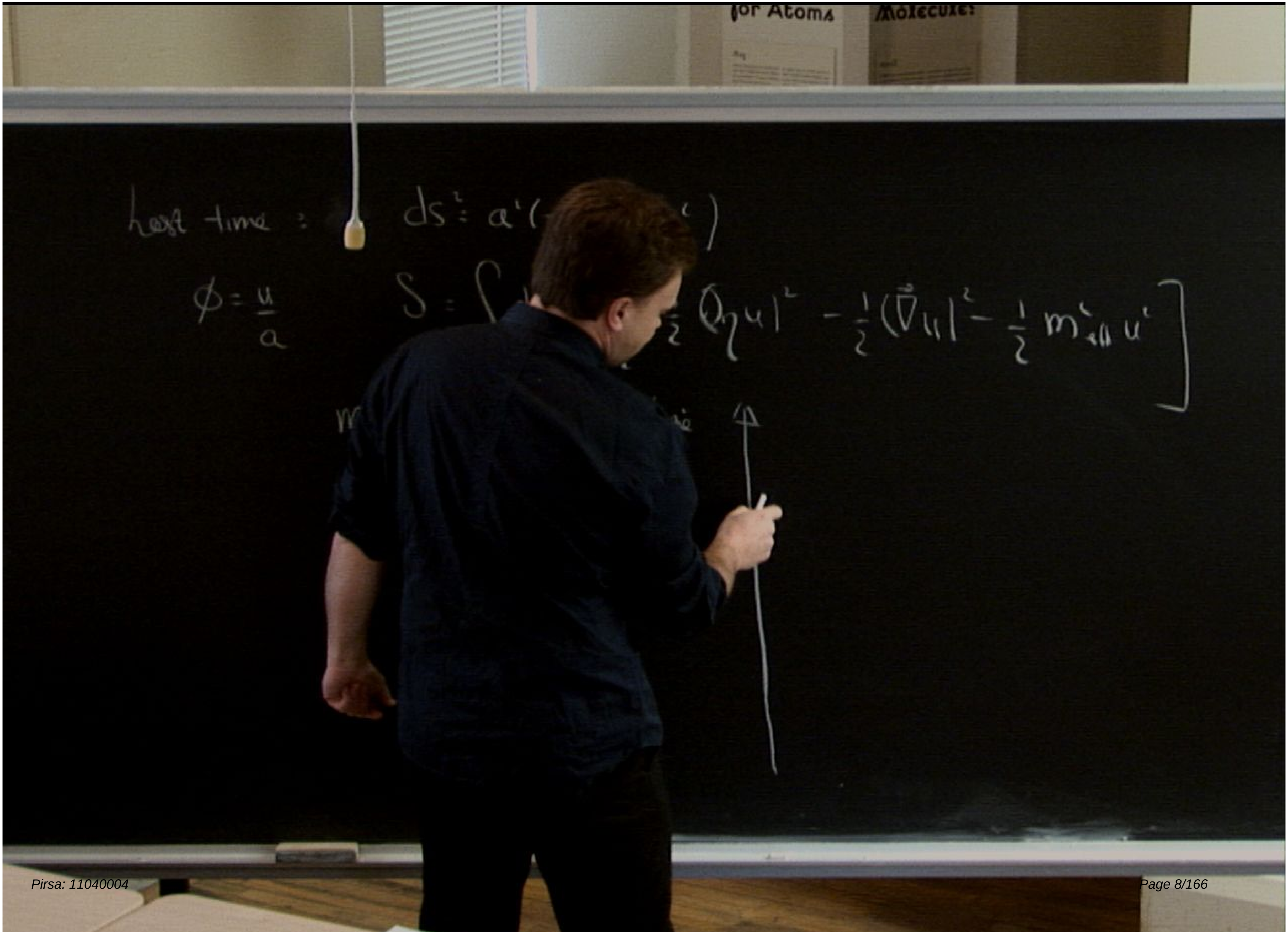

for Atoms
Molecules
Last time :
ds² =
dx²)
ϕ = u/a
S =

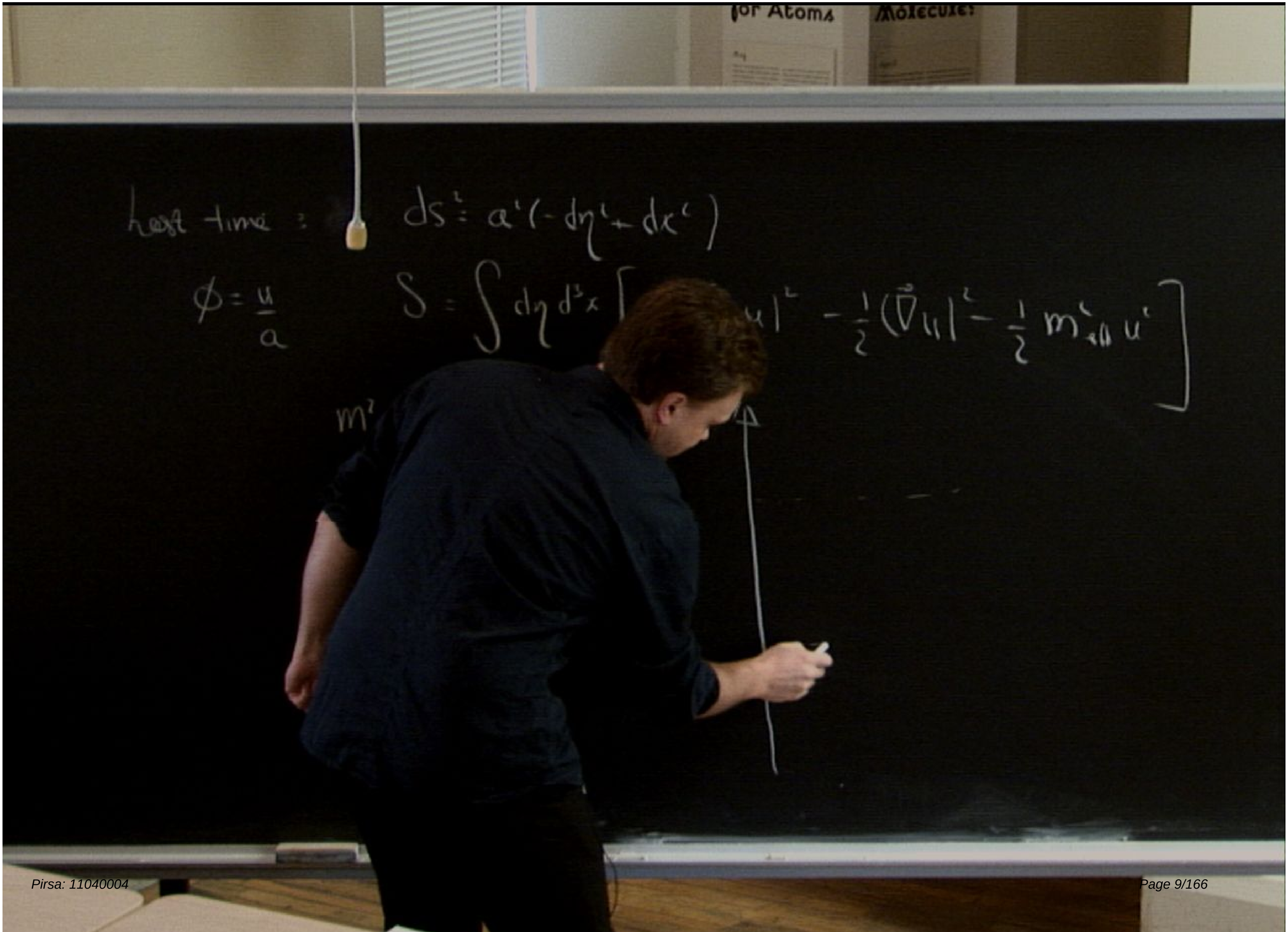

for Atoms
Molecules
Last time :
dη² + dx²)
φ = u/a
d³x [½ (∂η u)² − ½ (∇u)² − ½ m²eff u²]

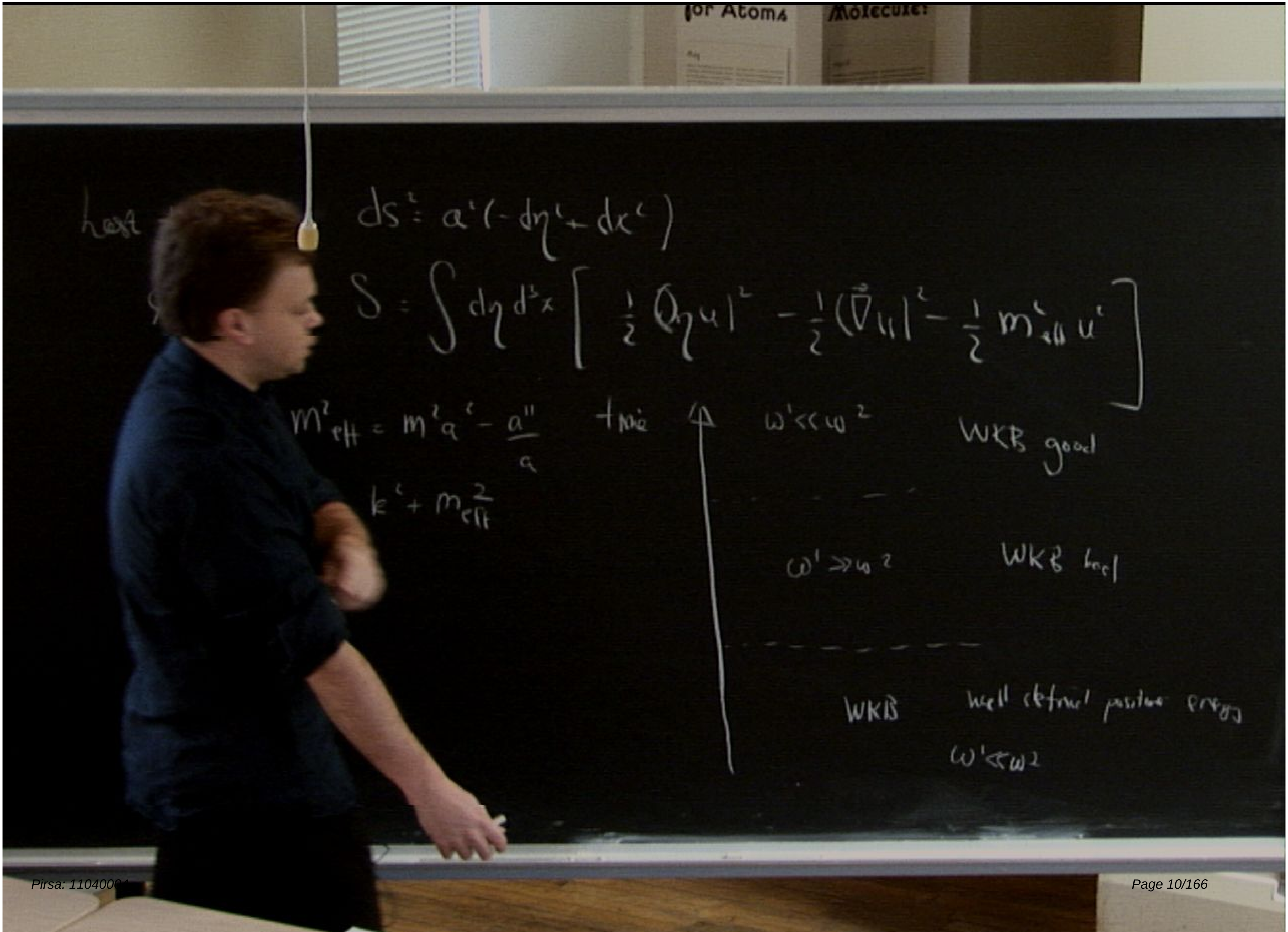

for Atoms
Molecules
Last time :

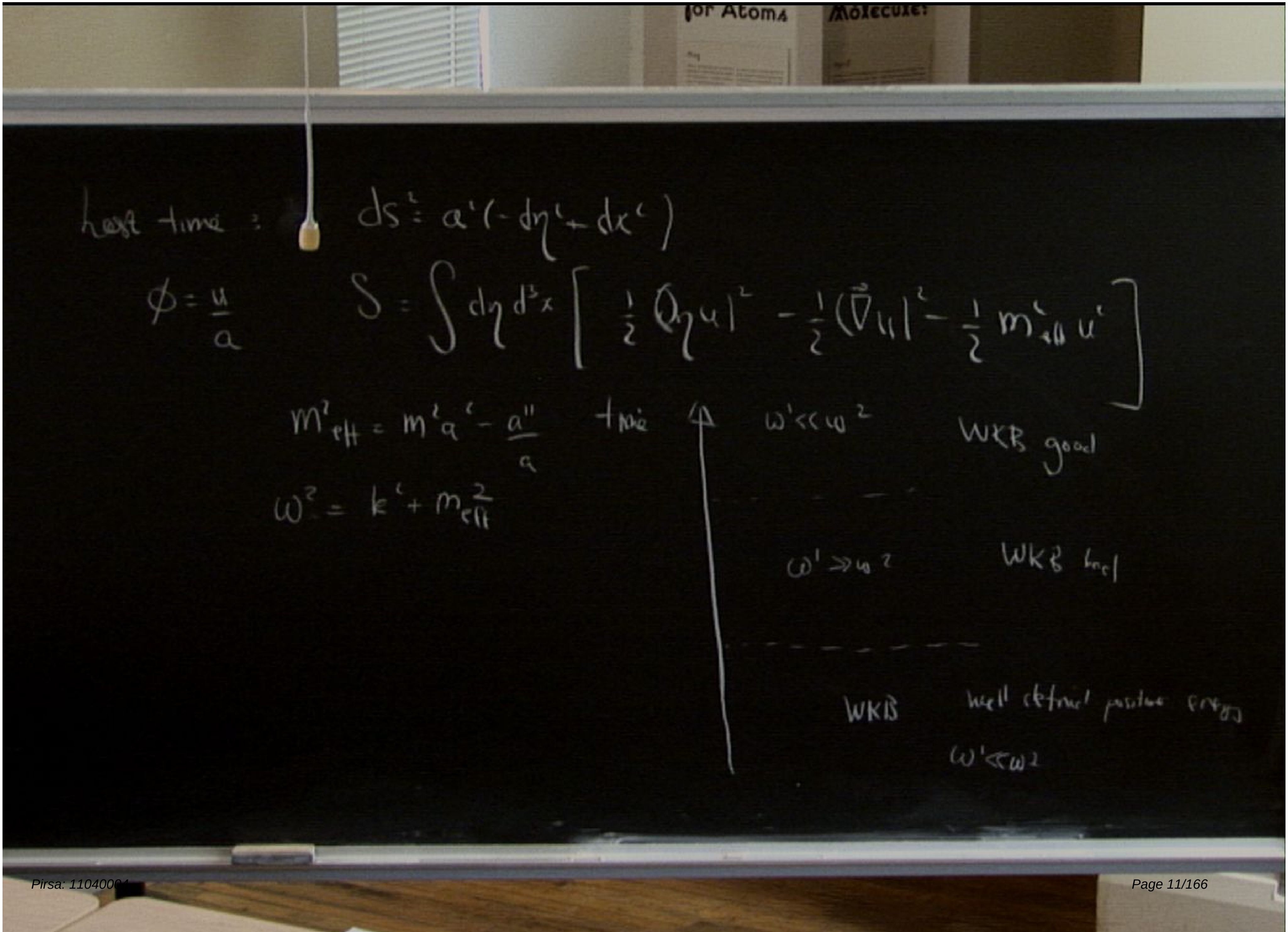

for Atoms
Molecules
Last time :
ds² = a²(-dη² + dx²)
φ = u/a


for Atoms
Molecules
time


for Atoms
Molecules
last time :
φ = u/a


for Atoms
Molecules
Last time :
ds² = a²(-dη² + dx²)
φ = u/a
S = ∫ dη d³x [


for Atoms
Molecules
$ds^2 = a^2(-d\eta^2 + dx^2)$
$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m_{eff}^2 u^2 \right]$
$m_{eff}^2 = m^2 a^2 - \frac{a''}{a}$
time
$k^2 + m_{eff}^2$
$\omega' \ll \omega^2$
WKB good
$\omega' \gg \omega^2$
WKB bad
WKB
well defined positive energy
$\omega' \ll \omega^2$


last time : $ds^2 = a^2(-d\eta^2 + dx^2)$
$\phi = \frac{u}{a}$
$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m_{eff}^2 u^2 \right]$
$m^2_{eff} = m^2 a^2 - \frac{a''}{a}$
$\omega^2 = k^2 + m^2_{eff}$
time
$\omega' \ll \omega^2$
WKB good
$\omega' \gg \omega^2$
WKB bad
WKB
well defined positive energy
$\omega' \ll \omega^2$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k + a_k^\dagger u_k^*$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

$$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

$$u_k'' = -\omega_k^2 u_k \qquad u_k(\eta)$$

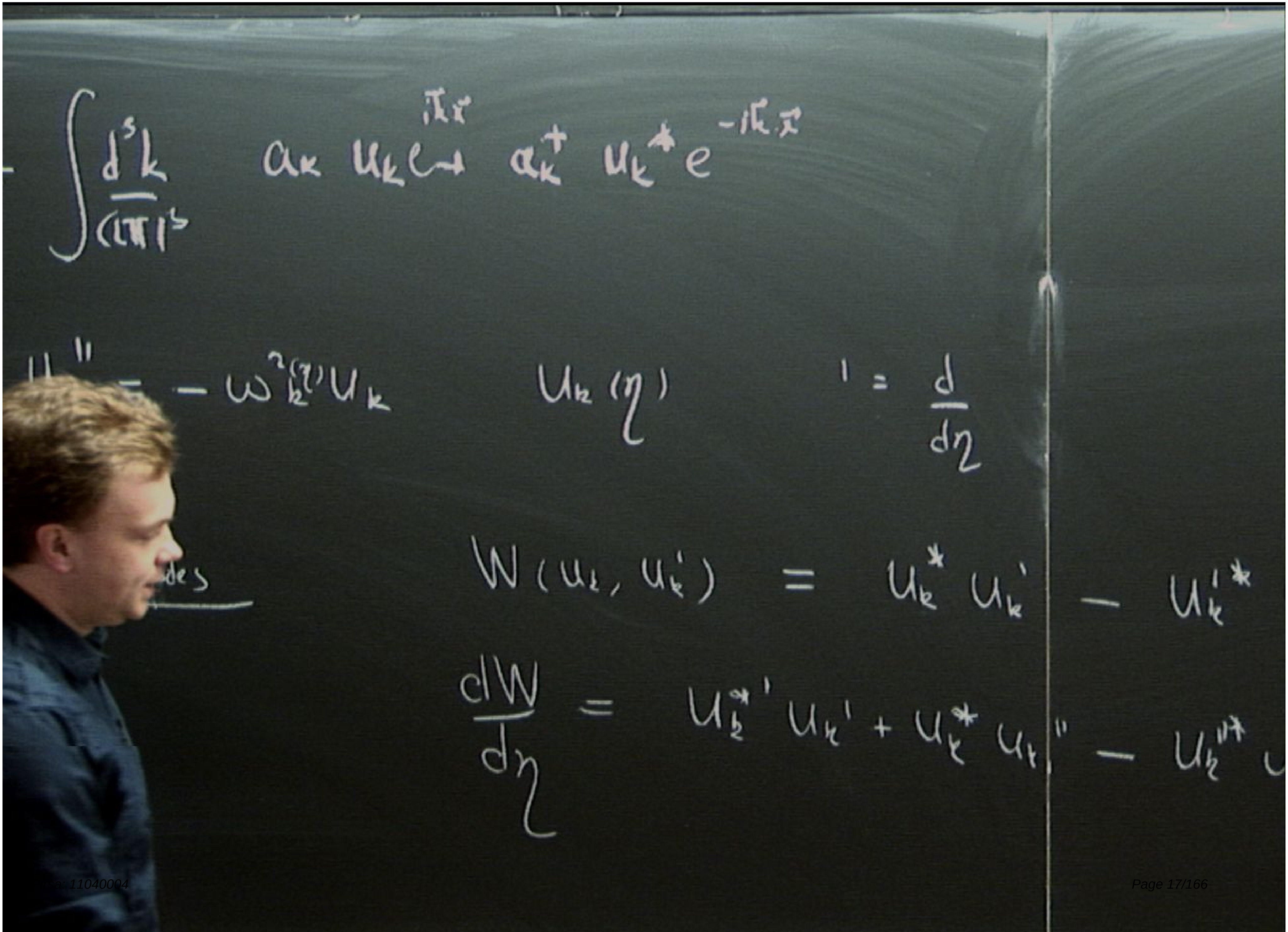
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \quad a_k \, u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

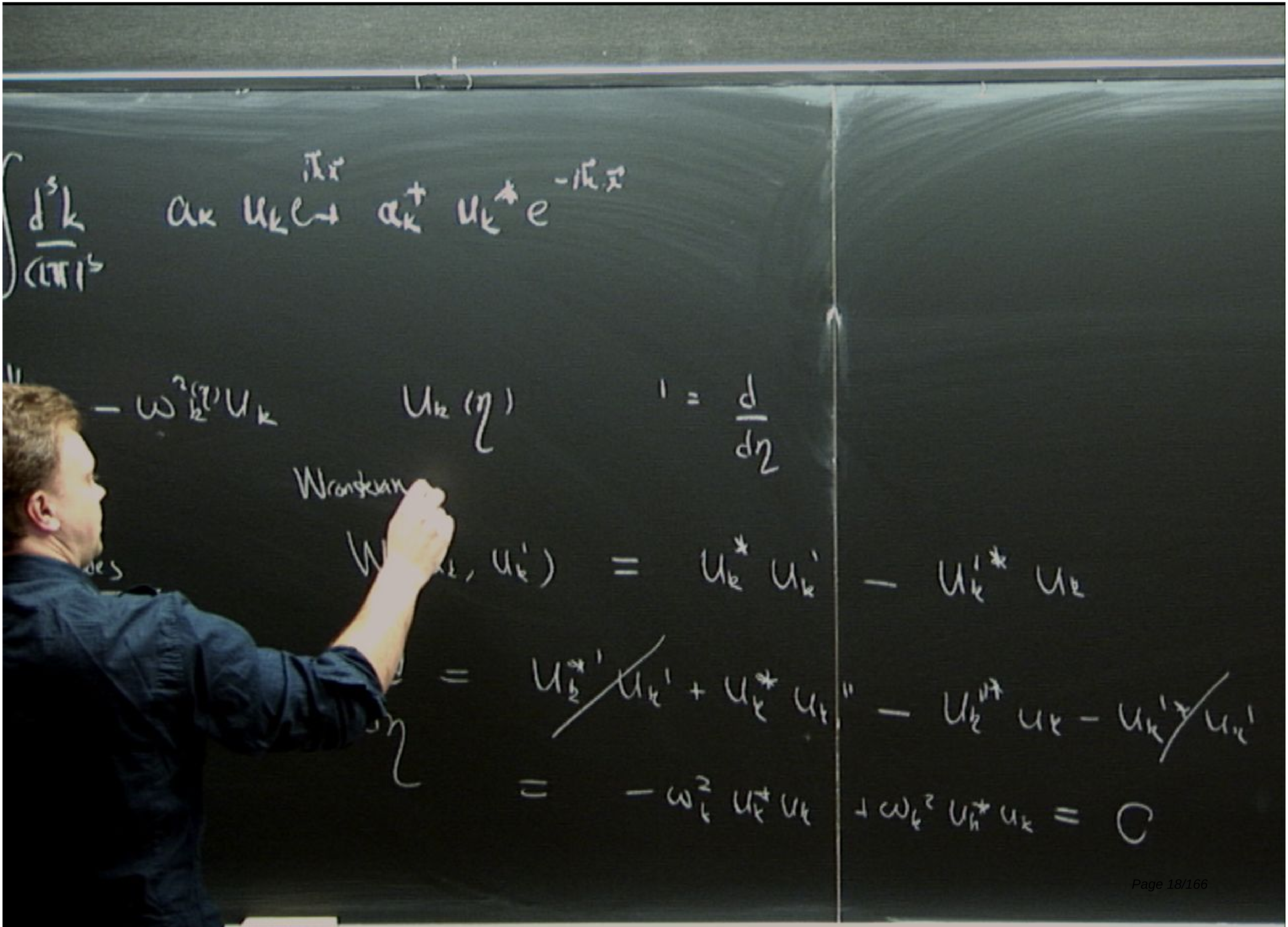
$$u_k'' = -\omega_k^2(\eta) u_k \qquad u_k(\eta)$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$u_k'' = -\omega_k^2(\eta) u_k \qquad u_k(\eta)$$

Normalization of modes


∫ d³k/(2π)³ a_k u_k e^{ik·x} a_k^† u_k^* e^{-ik·x}
= −ω_k²(η) u_k
u_k(η)
' = d/dη
W(u_k, u_k') = u_k^* u_k' − u_k'^*
dW/dη = u_k^*' u_k' + u_k^* u_k'' − u_k''^*


∫ d³k/(2π)³ a_k u_k e^{ik·x} + a_k^† u_k^* e^{-ik·x}
− ω_k²(η) u_k
u_k(η)
' = d/dη
Wronskian
W(u_k, u_k') = u_k^* u_k' − u_k'^* u_k
= u_k^*' u_k' + u_k^* u_k'' − u_k''^* u_k − u_k'^* u_k'
= −ω_k² u_k^* u_k + ω_k² u_k^* u_k = 0

$$\int \frac{d^3k}{(2\pi)^3} \quad a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$u_k'' = -\omega_k^2(\eta) u_k \qquad u_k(\eta) \qquad ' = \frac{d}{d\eta}$$

Wronskian.

...modes

$$W(u_k, u_k') = u_k^* u_k' - u_k'^* u_k$$

$$\frac{dW}{d\eta} = u_k^{*\prime} u_k' + u_k^* u_k'' - u_k''^* u_k - u_k'^* u_k'$$

$$= -\omega_k^2 u_k^* u_k + \omega_k^2 u_k^* u_k = 0$$

$iW = 1$

$\boxed{W = -i}$

$u_k'^* \, u_k$

$u_k''^* \, u_k - u_k'^* u_k'$

$k^2 u_k^* u_k = C$

$$iW = 1$$

$$\boxed{W = -i}$$

$$u_k = \frac{1}{\sqrt{2\omega}} e^{-i\omega\eta}$$

u_k

$u_k - u_k'{}^* / u_k'$

$_k = C$

$$iW = 1$$

$$\boxed{W = -i}$$

$$u_k = \frac{1}{\sqrt{2\omega}} e^{-i\omega\eta}$$

$$W = \frac{1}{\sqrt{2\omega}} e^{+i\omega\eta} \; -i\omega \frac{1}{\sqrt{2\omega}} e^{-i\omega\eta} \times 2$$

$$= \frac{1}{2\omega} \times (2 \times -i\omega) = -i$$

$iW = 1$

$\boxed{W = -i}$

$u_k = \frac{1}{\sqrt{2\omega}} e^{-i\omega\eta}$

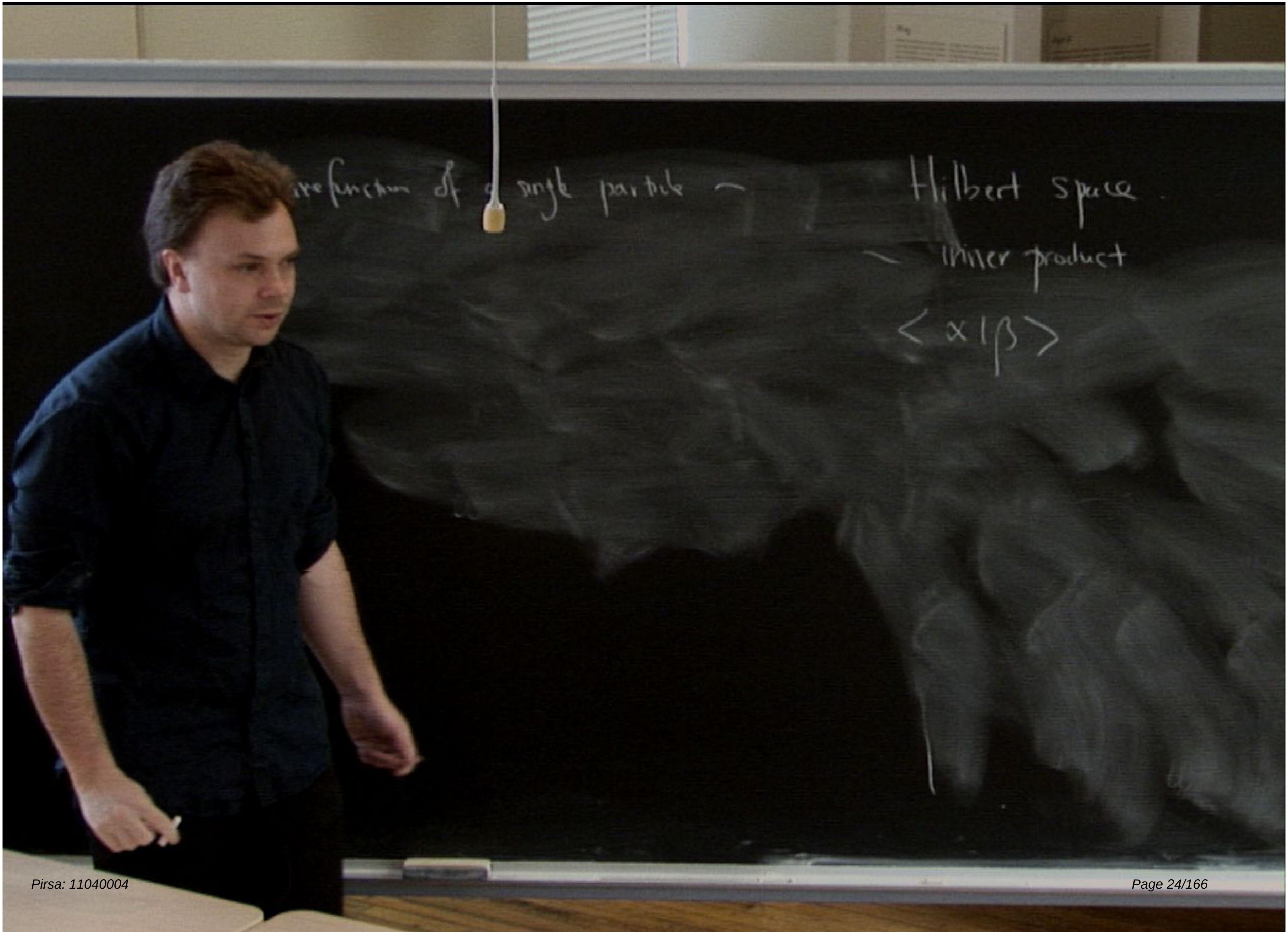
$W = \frac{1}{\sqrt{2\omega}} e^{+i\omega\eta} \; -i\omega \frac{1}{\sqrt{2\omega}} e^{-i\omega\eta} \times 2$

$= \frac{1}{2\omega} \times (2 \times -i\omega) = -i$

u_k

$u_k - u_k' \not{\times} u_k'$

$= C$


function of single particle ~
Hilbert space.
~ inner product
< α|β >

Wavefunction of a single particle ~ Hilbert space.

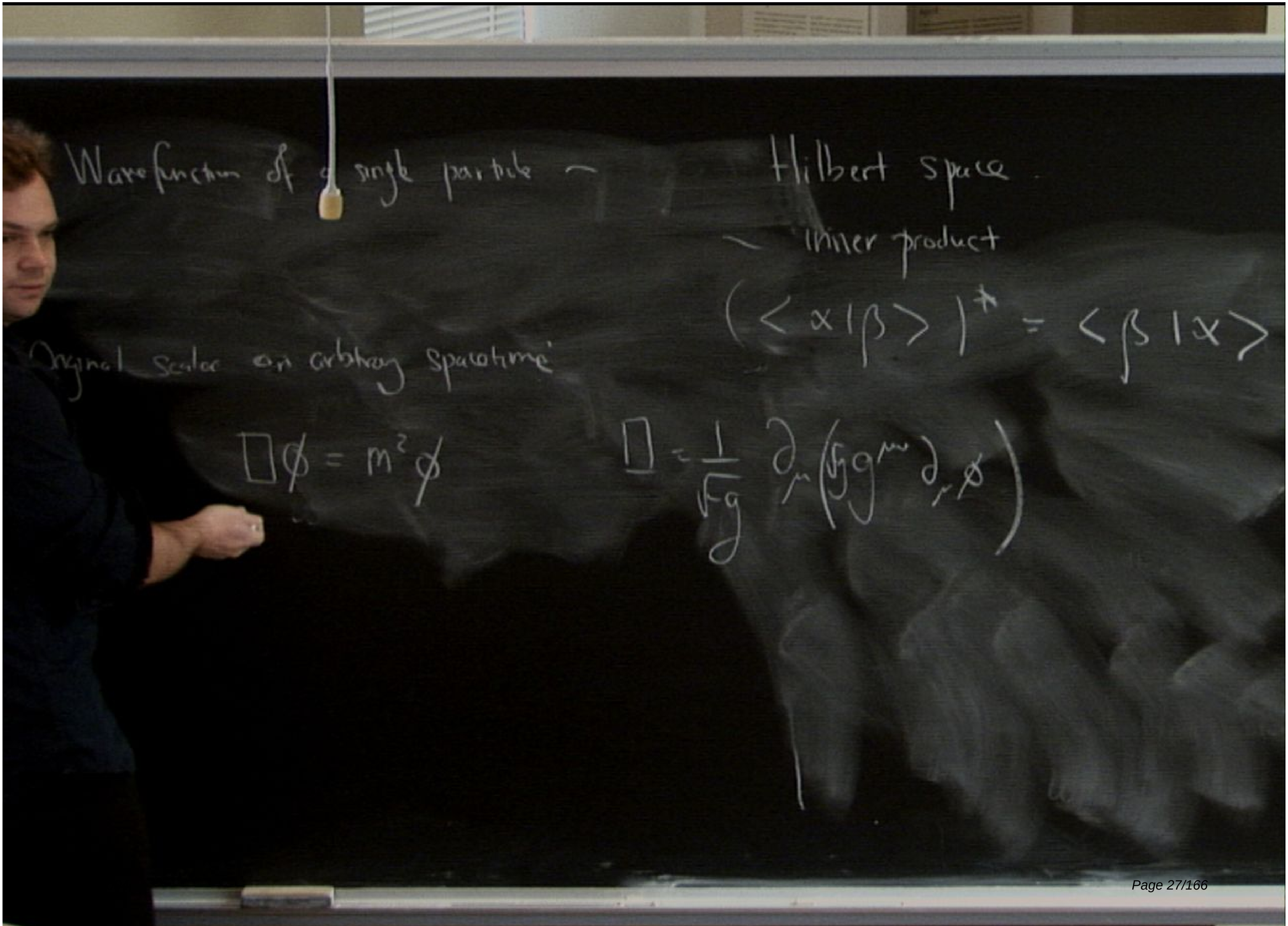
~ inner product

$$(\langle \alpha | \beta \rangle)^* = \langle \beta | \alpha \rangle$$

Wavefunction of a single particle ~ Hilbert space.

~ inner product

$$(\langle \alpha | \beta \rangle)^* = \langle \beta | \alpha \rangle$$


Wavefunction of a single particle ~
Hilbert space.
~ inner product
$(\langle \alpha | \beta \rangle)^* = \langle \beta | \alpha \rangle$
Original scalar on arbitrary spacetime
$\Box \phi = m^2 \phi$
$\Box = \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \phi)$

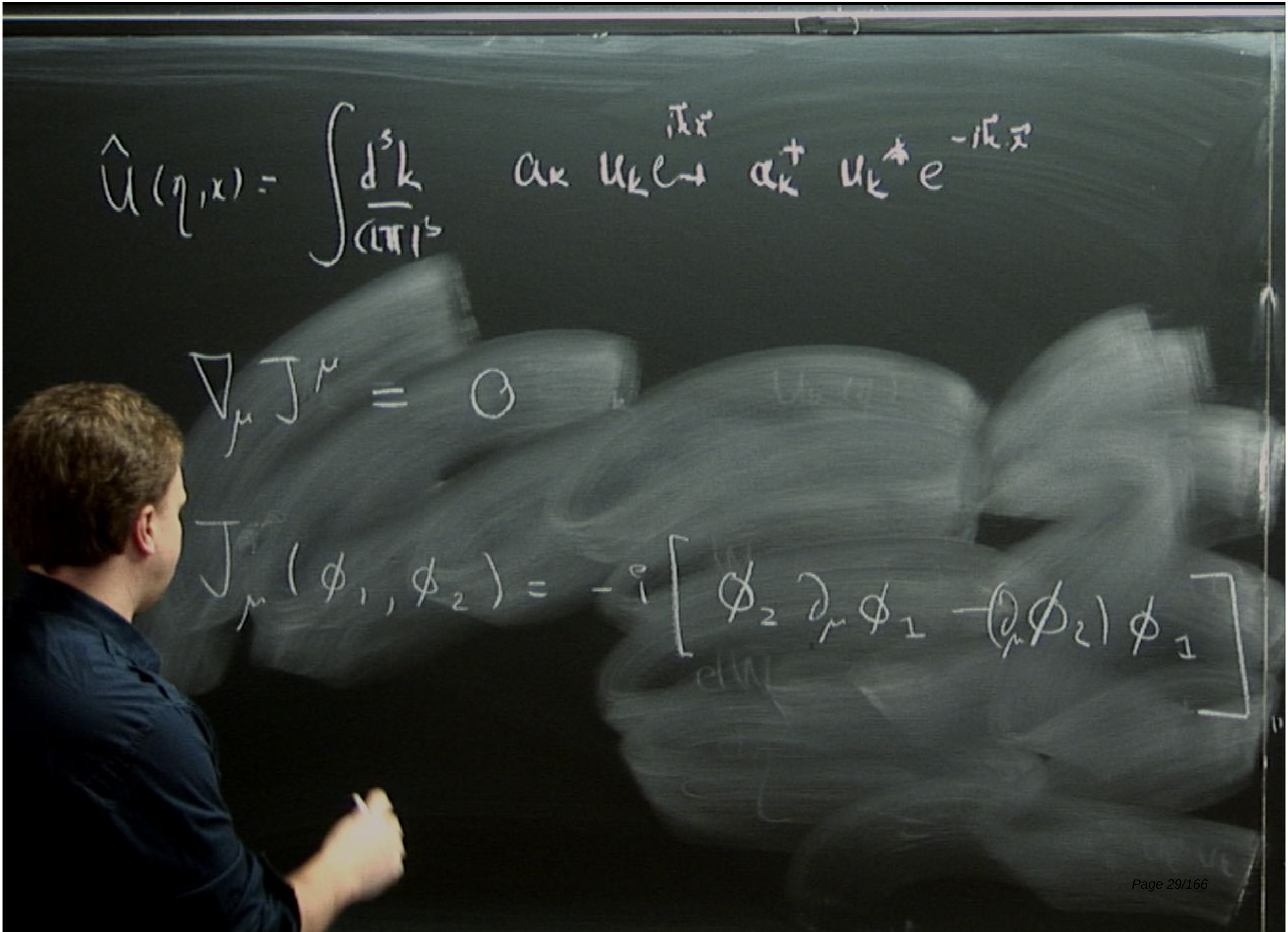
Wavefunction of a single particle — Hilbert space.

~ inner product

$$(\langle \alpha | \beta \rangle)^* = \langle \beta | \alpha \rangle$$

Original scalar on arbitrary spacetime

$$\Box \phi = m^2 \phi \qquad \Box = \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g}\, g^{\mu\nu} \partial_\nu \phi \right)$$


$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$
$\nabla_\mu J^\mu = 0$
$J_\mu(\phi_1, \phi_2) = -i\left[\phi_2 \partial_\mu \phi_1 - (\partial_\mu \phi_2)\phi_1\right]$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\nabla_\mu J^\mu = 0$$

$$J_\mu(\phi_1, \phi_2) = -i\left[\phi_2 \partial_\mu \phi_1 - (\partial_\mu \phi_2)\phi_1\right]$$

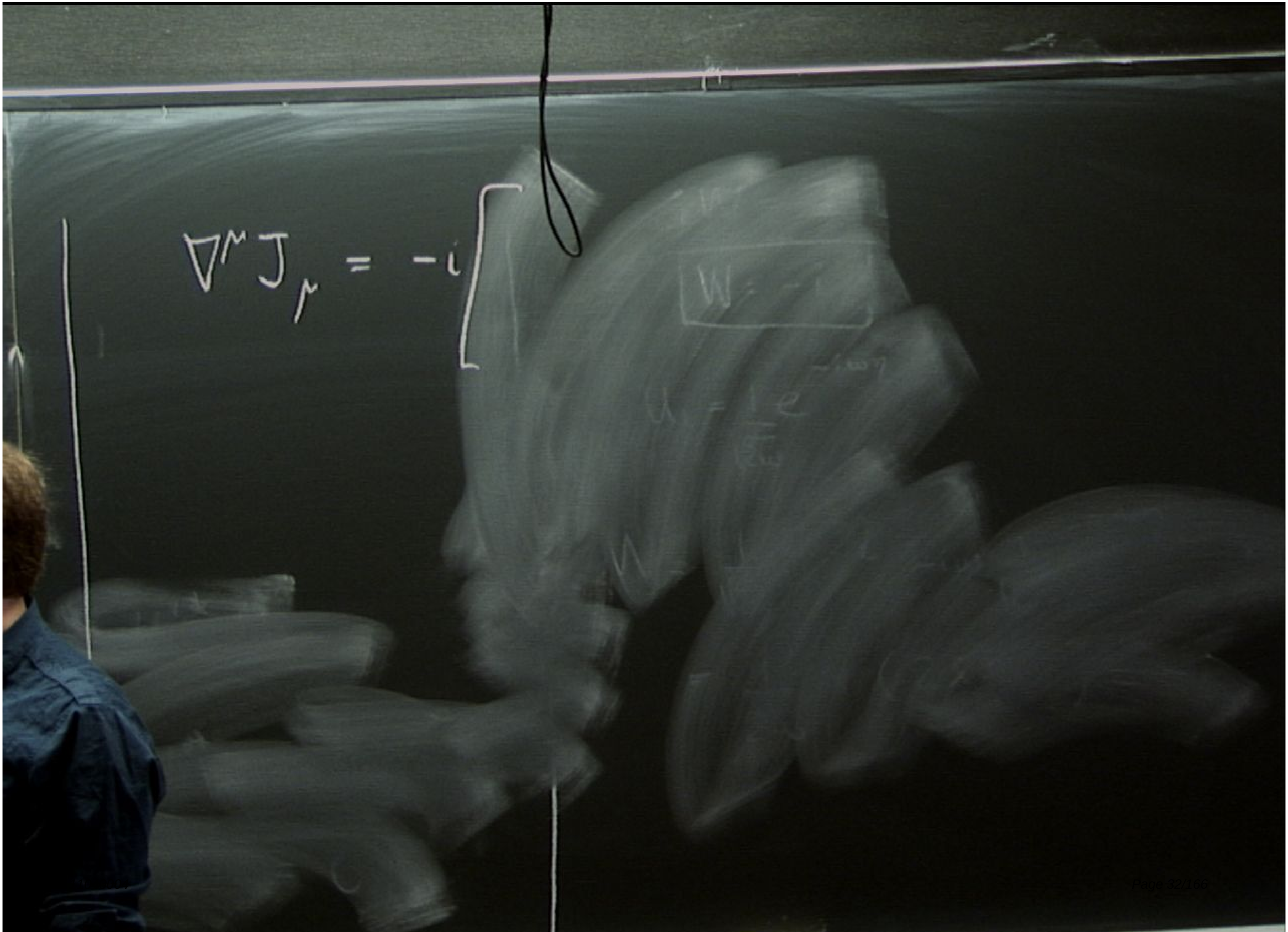
Klein-Gordon inner product

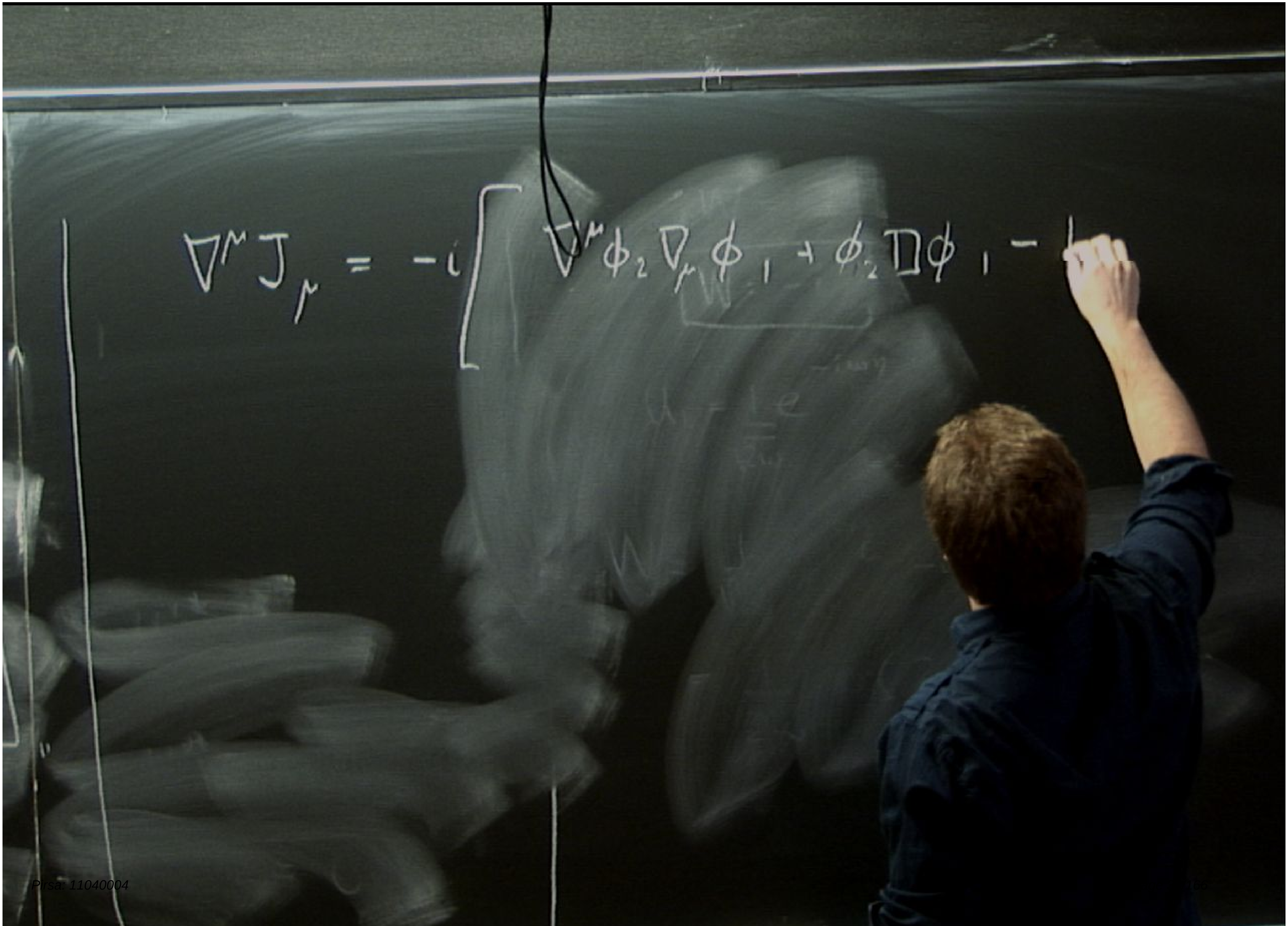
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \, a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\nabla_\mu J^\mu = 0$$

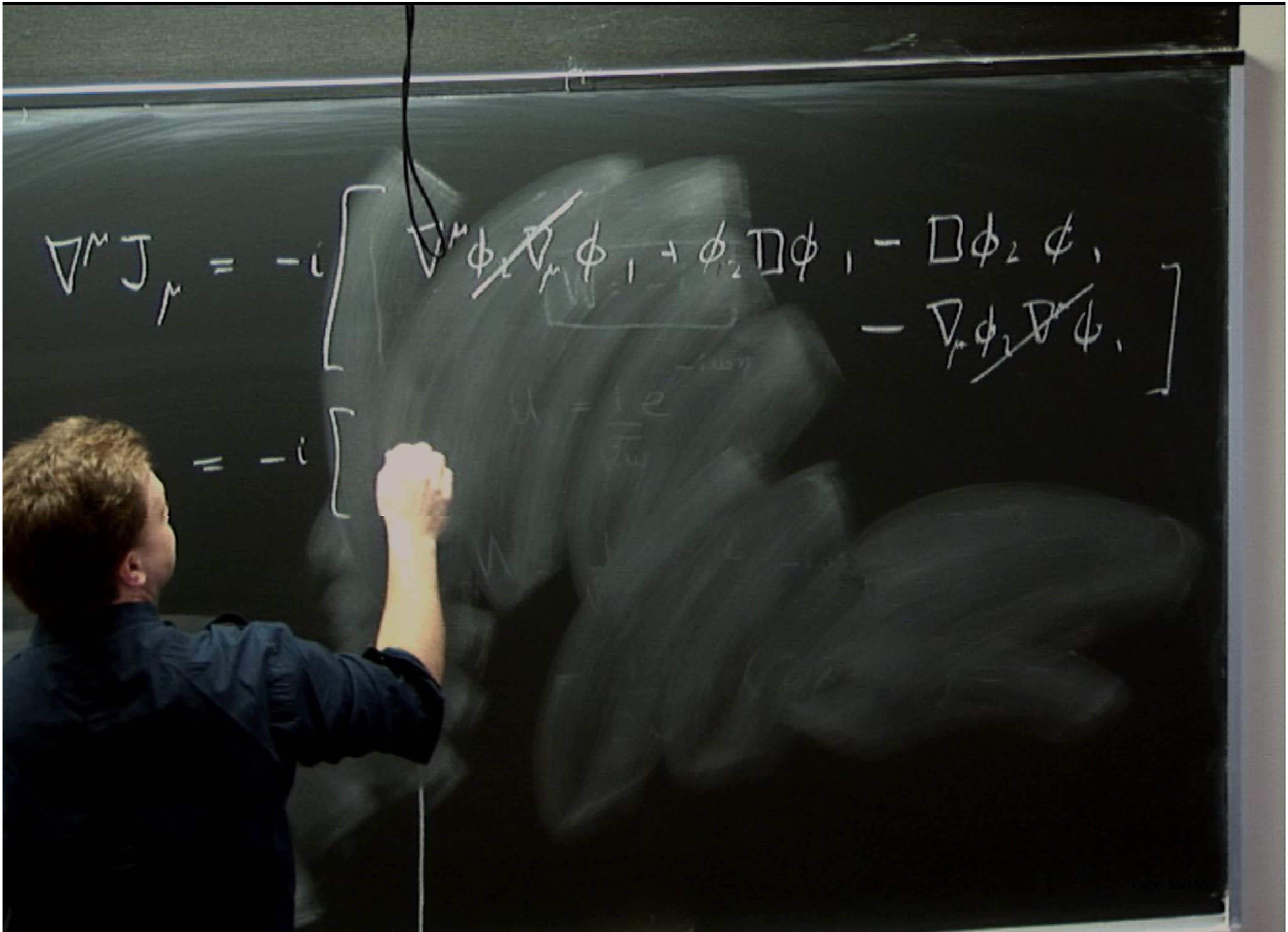
$$J_\mu(\phi_1, \phi_2) = -i\left[\phi_2 \partial_\mu \phi_1 - (\partial_\mu \phi_2)\phi_1\right]$$

Klein-Gordon inner product


$\nabla^\mu J_\mu = -i\,[$


∇^μ J_μ = −i ∫ ∇^μ φ_2 ∇_μ φ_1 + φ_2 □φ_1 −

$$\nabla^\mu J_\mu = -i \left[\nabla^\mu \phi_2 \nabla_\mu \phi_1 + \phi_2 \Box \phi_1 - \Box \phi_2 \phi_1 - \nabla_\mu \phi_2 \nabla^\mu \phi_1 \right.$$


$\nabla^\mu J_\mu = -i [\nabla^\mu \phi_2 \nabla_\mu \phi_1 + \phi_2 \Box \phi_1 - \Box \phi_2 \phi_1 - \nabla_\mu \phi_2 \nabla^\mu \phi_1]$
$= -i [$

$$\nabla^\mu J_\mu = -i\left[\cancel{\nabla^\mu \phi_2 \nabla_\mu \phi_1} + \phi_2 \Box \phi_1 - \Box \phi_2\, \phi_1 - \cancel{\nabla_\mu \phi_2 \nabla^\mu \phi_1}\right]$$

$$= -i\left[m^2 \phi_2 \phi_1 - m^2 \phi_2 \phi_1\right] = 0$$

$$\nabla^\mu J_\mu = -i\left[\nabla^\mu \phi_2 \nabla_\mu \phi_1 + \phi_2 \Box \phi_1 - \Box \phi_2 \phi_1 - \nabla_\mu \phi_2 \nabla^\mu \phi_1\right]$$

$$= -i\left[m^2 \phi_2 \phi_1 - m^2 \phi_2 \phi_1\right] = 0$$

$$J_\mu(\phi_1, \phi_2) = -J_\mu(\phi_2, \phi_1)$$

$$\nabla^\mu J_\mu = -i\left[\nabla^\mu \phi_2 \nabla_\mu \phi_1 + \phi_2 \Box \phi_1 - \Box \phi_2 \phi_1 - \nabla_\mu \phi_2 \nabla^\mu \phi_1\right]$$

$$= -i\left[m^2 \phi_2 \phi_1 - m^2 \phi_2 \phi_1\right] = 0$$

$$J_\mu(\phi_1, \phi_2) = -J_\mu(\phi_2, \phi_1)$$

Problem: Define inner product on Hilbert space $\langle \alpha | \alpha \rangle \geq 0$

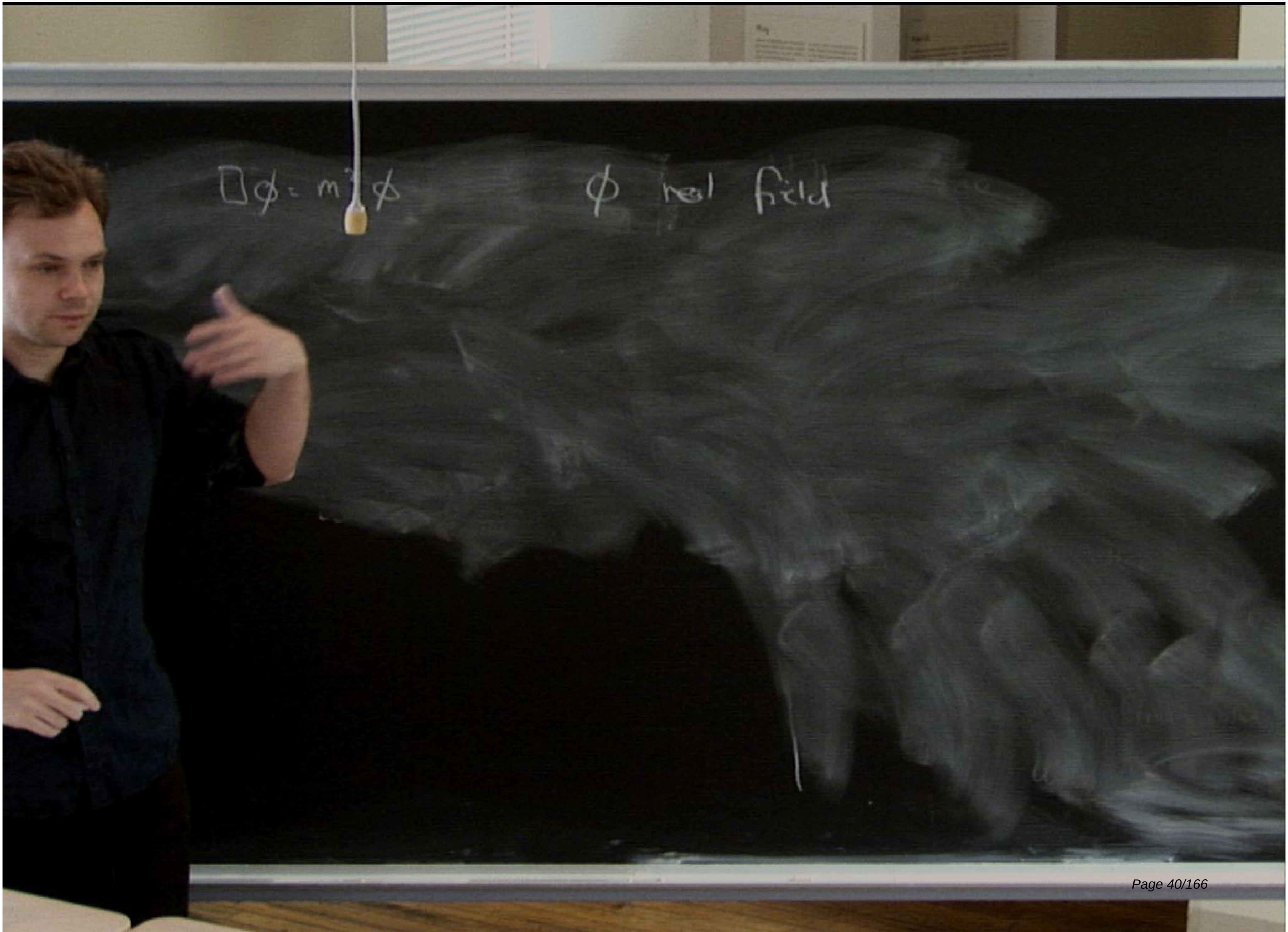
$$\nabla^\mu J_\mu = -i\Big[\nabla^\mu\phi_2\nabla_\mu\phi_1 + \phi_2\Box\phi_1 - \Box\phi_2\,\phi_1 - \nabla_\mu\phi_2\nabla^\mu\phi_1\Big]$$

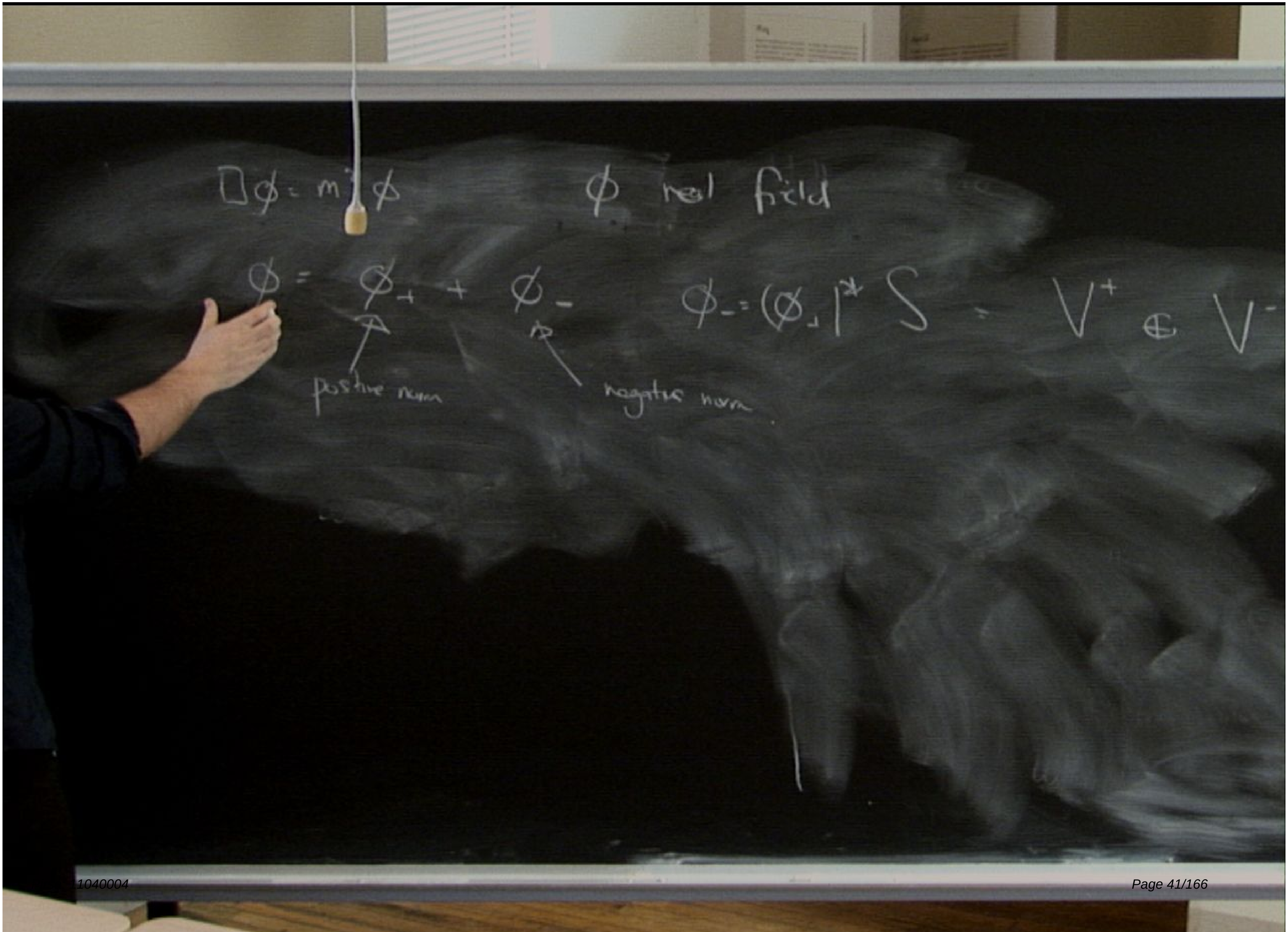
$$= -i\left[m^2\phi_2\phi_1 - m^2\phi_2\phi_1\right] = 0$$

$$J_\mu(\phi_1,\phi_2) = -J_\mu(\phi_2,\phi_1)$$

Problem: Define inner product on Hilbert space $\langle\alpha|\alpha\rangle > 0$

$$J_\mu(\phi_1,\phi_1) = 0$$


□φ = m²φ
φ real field


□φ = m²φ
φ real field
φ = φ₊ + φ₋
φ₋ = (φ₊)*
S = V⁺ ⊕ V⁻
positive norm
negative norm

$\Box \phi = m^2 \phi$ ϕ real field

$\phi = \phi_+ + \phi_-$ $\phi_- = (\phi_+)^*$ $S = V^+ \oplus V^-$

(ϕ_+: positive norm; ϕ_-: negative norm)

freedom to split into complex solutions $\equiv$ freedom in choice of vacuum state

$\equiv$ choice of defn particles.

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

$$\nabla_\mu J^\mu = 0$$

$$(\phi_1, \phi_2) = -i\left[\phi_2 \partial_\mu \phi_1 - (\partial_\mu \phi_2) \phi_1 \right]$$

$$G_\mu(\phi_1, \phi_2) = -i\left[\phi_2^- \partial_\mu \phi_1^+ - \partial_\mu \phi_2^- \phi_1^+ \right]$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k \, e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger \, u_k^* \, e^{-i\vec{k}\cdot\vec{x}}$$

$$\nabla_\mu J^\mu = 0$$

$$J_\mu(\phi_1, \phi_2) = -i\left[\phi_2 \partial_\mu \phi_1 - (\partial_\mu \phi_2)\phi_1\right]$$

$$G_\mu(\phi_1, \phi_2) = -i\left[\phi_2^- \partial_\mu \phi_1^+ - \partial_\mu \phi_2^- \, \phi_1^+\right]$$

ϕ_1^+ is really wavefunction of single particle state

$$\phi_1^- = (\phi_1^+)^*$$

ϕ_1^+ is really wavefunction of single particle state $\quad \psi$

$\phi_1^- = (\phi_1^+)^*$ ~ complex conjugate of wavefunction $\quad \psi^*$

ϕ_1^+ is really wavefunction of single particle state ψ

$\phi_1^- = (\phi_1^+)^*$ ~ complex conjugate of wavefunction ψ^*

$$G_\mu = -i\left[\psi_2^* \partial_\mu \psi_1 - \partial_\mu \psi_2^* \psi_1\right]$$

ϕ_1^+ is really wavefunction of single particle state ψ

$\phi_1^- = (\phi_1^+)^*$ ~ complex conjugate of wavefunction ψ^*

$$G_\mu = -i\left[\psi_2^* \partial_\mu \psi_1 - \partial_\mu \psi_2^* \psi_1\right]$$

$$\nabla_\mu J^\mu = 0$$

ϕ_1^+ is really wavefunction of single particle state ψ

$\phi_1^- = (\phi_1^+)^*$ ~ complex conjugate of wavefunction ψ^*

$$G_\mu = -i\left[\psi_2^* \partial_\mu \psi_1 - \partial_\mu \psi_2^* \psi_1 \right]$$

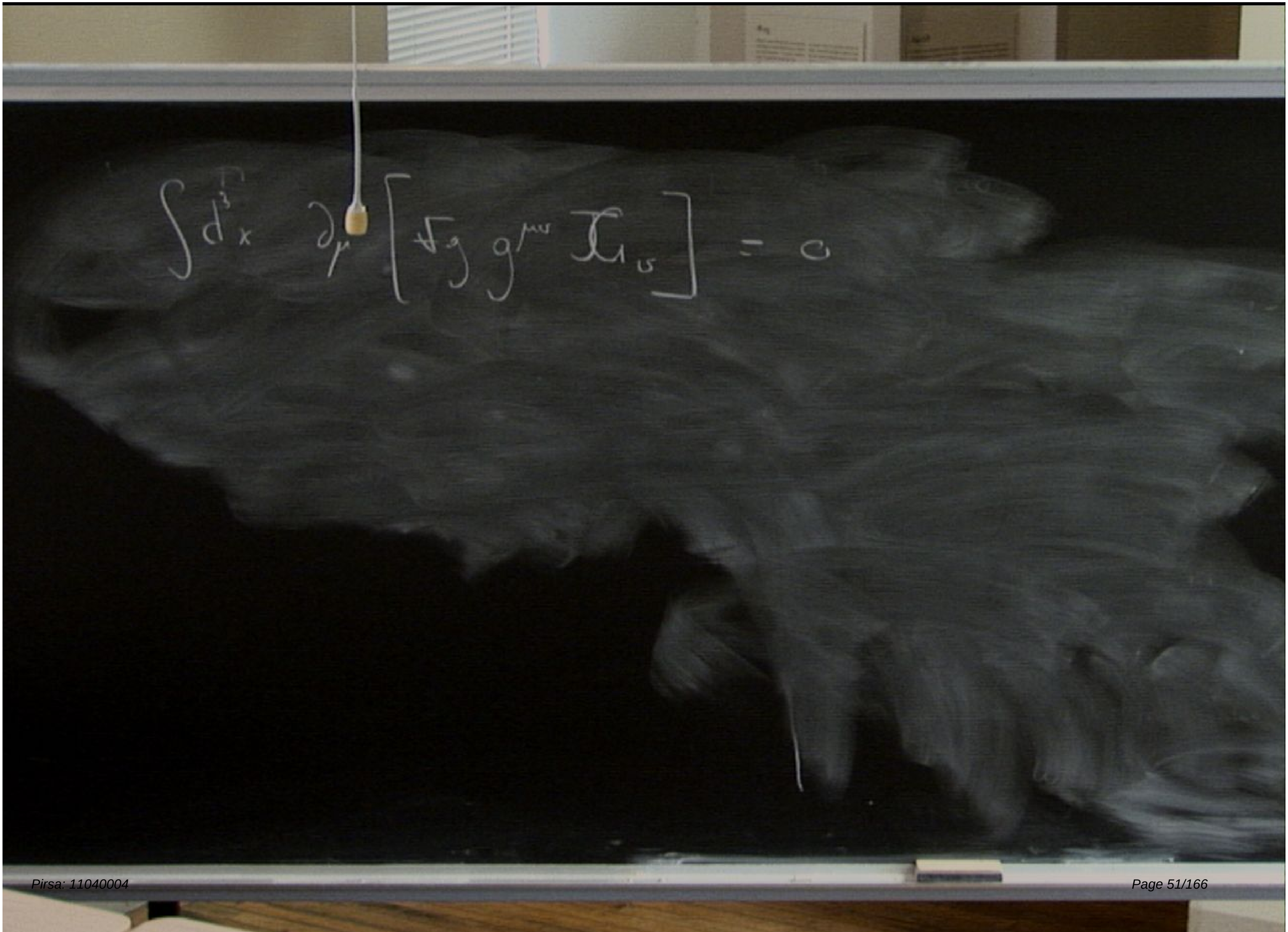
$$\nabla_\mu J^\mu = 0 \qquad \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} J_\nu \right) = 0$$

ϕ_1^+ is really wavefunction of single particle state $\quad \psi$

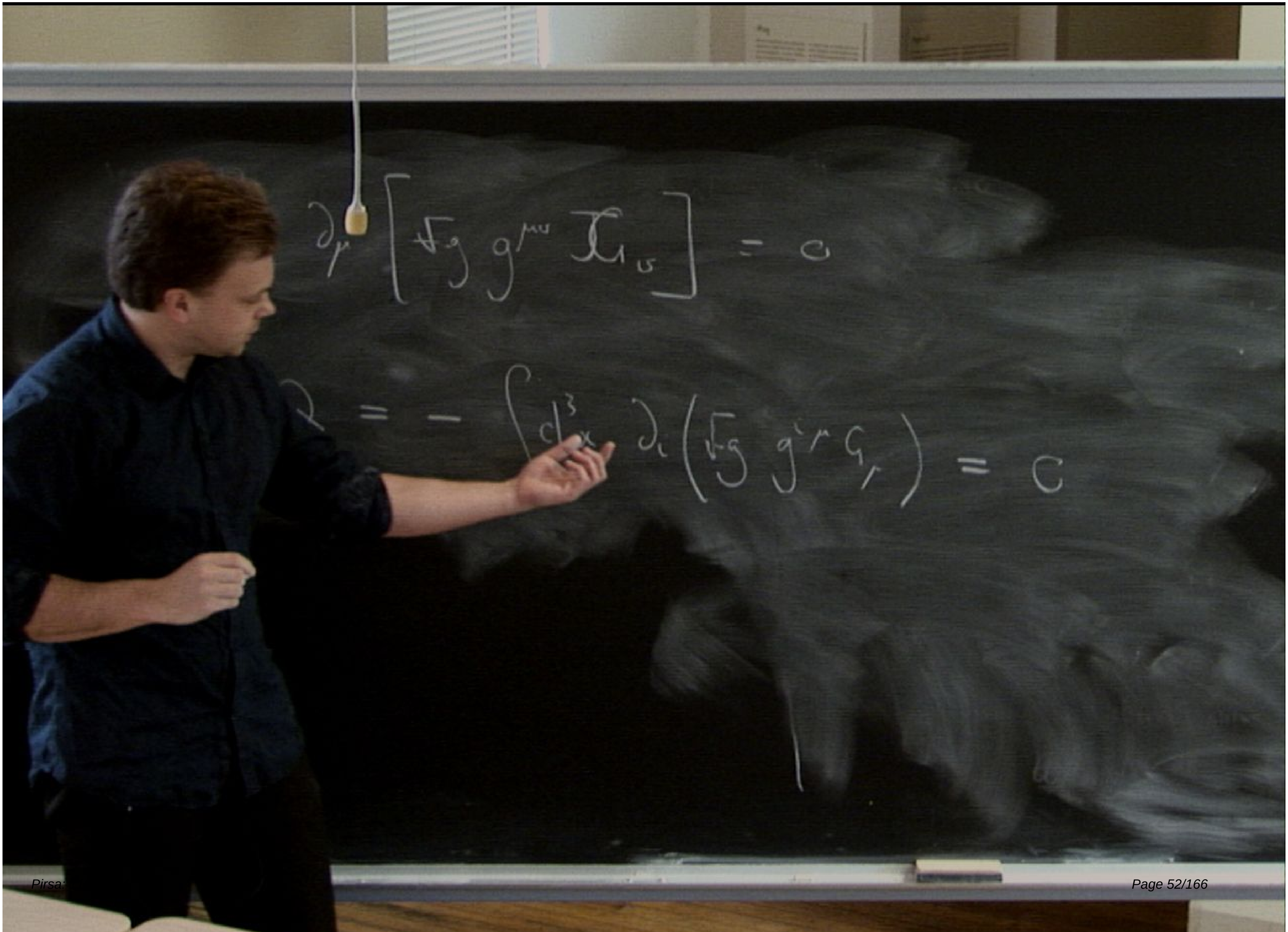
$\phi_1^- = (\phi_1^+)^*$ ~ complex conjugate of wavefunction $\quad \psi^*$

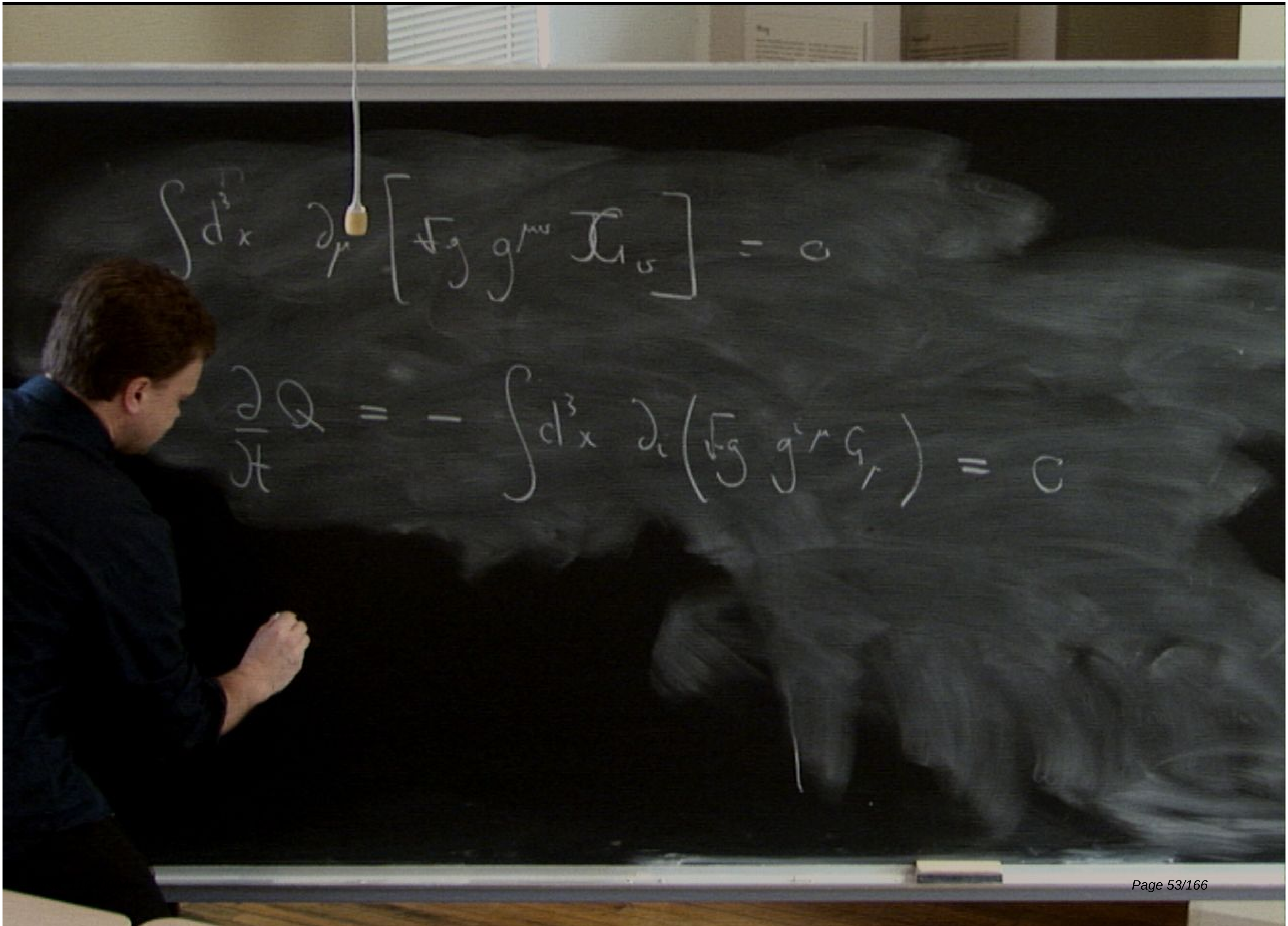
$$G_\mu = -i\left[\psi_2^* \partial_\mu \psi_1 - \partial_\mu \psi_2^* \psi_1\right]$$

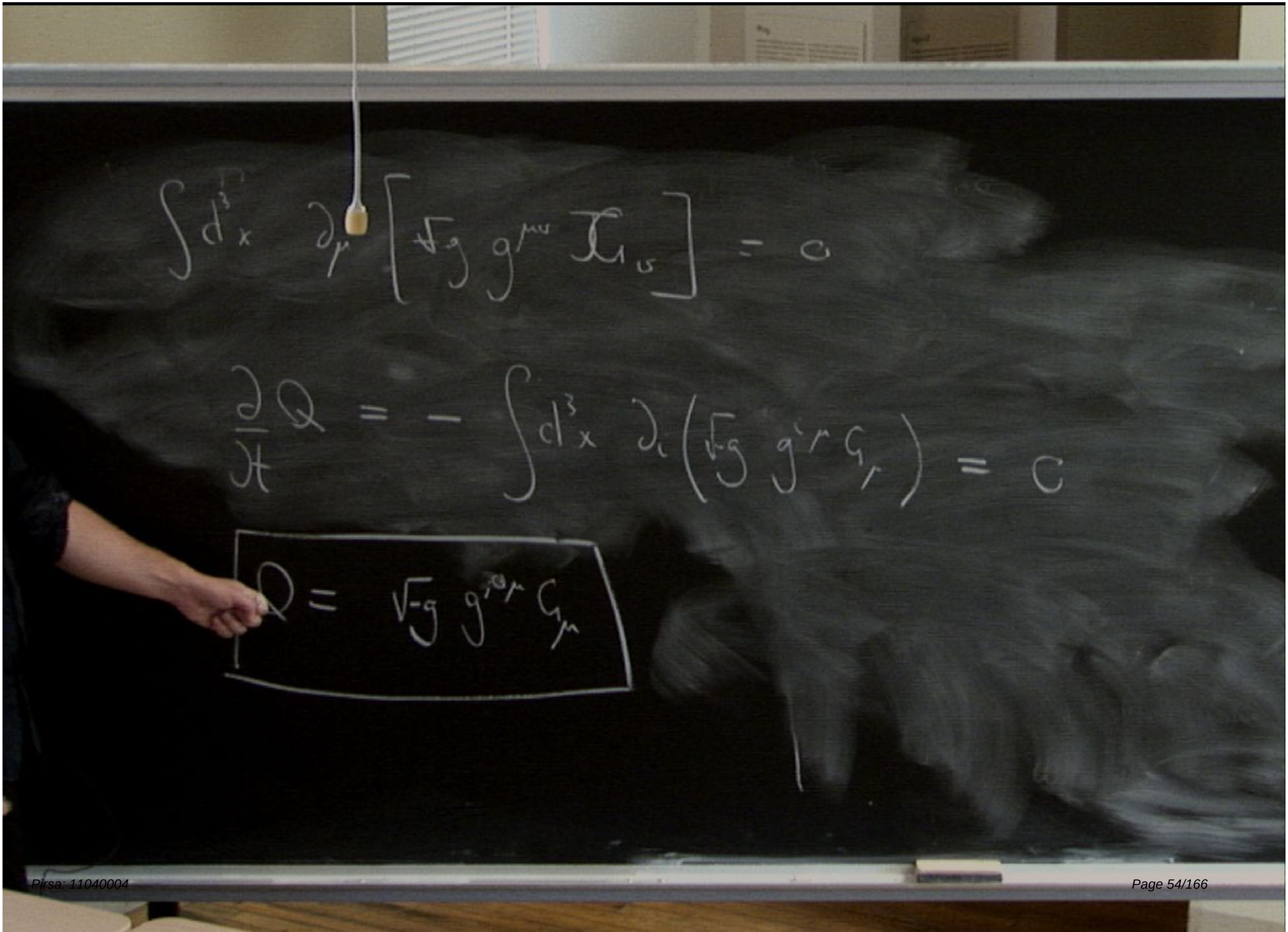
$$\nabla_\mu J^\mu = 0 \qquad \partial_\mu\left(\sqrt{-g}\, g^{\mu\nu} J_\nu\right) = 0$$



$$\int d^3x \; \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu} \mathcal{X}_{\nu} \right] = 0$$




$\int d^3x \; \partial_\mu \left[\sqrt{g}\, g^{\mu\nu} J_\nu \right] = 0$
$\frac{\partial}{\partial t} Q = - \int d^3x \; \partial_i \left(\sqrt{g}\, g^{i\mu} J_\mu \right) = 0$


$\int d^3x \; \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu} J_\nu \right] = 0$
$\frac{\partial}{\partial t} Q = - \int d^3x \; \partial_i \left(\sqrt{-g}\, g^{i\mu} J_\mu \right) = 0$
$Q = \sqrt{-g}\, g^{0\mu} J_\mu$

$$\int d^3x \; \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu} \mathfrak{J}_{\nu} \right] = 0$$

$$\frac{\partial}{\partial t} Q = - \int d^3x \; \partial_i \left(\sqrt{-g}\, g^{i\mu} G_\mu \right) = 0$$

$$\boxed{Q = \sqrt{-g}\, g^{0\mu} G_\mu}$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \quad a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \Psi_2 | \Psi_1 \rangle$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \quad a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \psi_2 | \psi_1 \rangle = -i \int d^3x \sqrt{-g}\, g^{0\mu} \left[\psi_2^* \partial_\mu \psi_1 - (\partial_\mu \psi_2^*) \psi_1 \right]$$

$$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \psi_2 | \psi_1 \rangle = -i \int d^3x \, \sqrt{-g} \, g^{0\mu} \left[\psi_2^* \partial_\mu \psi_1 - (\partial_\mu \psi_2^*) \psi_1 \right]$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \psi_2 | \psi_1 \rangle = -i \int d^3x \sqrt{-g}\, g^{0\mu} \left[\psi_2^* \partial_\mu \psi_1 - (\partial_\mu \psi_2^*) \psi_1 \right]$$

not guaranteed positive definite

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \psi_2 | \psi_1 \rangle = -i \int d^3x \, \sqrt{-g} \, g^{0\mu} \left[\psi_2^* \partial_\mu \psi_1 - (\partial_\mu \psi_2^*) \psi_1 \right]$$

not guaranteed positive definite

$$\phi = \phi^+ + \phi^- = \psi + \psi^*$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

$$\langle \psi_2 | \psi_1 \rangle = -i \int d^3x \, \sqrt{-g} \, g^{0\mu} \left[\psi_2^{*} \partial_\mu \psi_1 - (\partial_\mu \psi_2^{*}) \psi_1 \right]$$

not guaranteed positive definite

$$\phi = \phi^{+} + \phi^{-} = \psi + \psi^{*}$$

In Minkowski spacetime.

$$\langle \Psi_2 | \Psi_1 \rangle = i \int d^3x \left[\Psi_2^* \partial_t \Psi_1 - (\partial_t \Psi_2^*) \Psi_1 \right]$$

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

split into positive and negative frequency solutions.

in present case

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

in present case — split into positive and negative frequency solutions.

$$\phi = \sum \quad e^{-i\omega t} \quad + \quad e^{i\omega t}$$

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

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Pirsa: 11040004

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

in present case - split into positive and negative frequency solutions.

$$\phi = \sum \underbrace{e^{-i\omega t}}_{\psi} + \underbrace{e^{i\omega t}}_{\psi^*}$$

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

in present case — split into positive and negative frequency solutions.

$$\phi = \sum \underbrace{e^{-i\omega t}}_{\psi} + \underbrace{e^{i\omega t}}_{\psi^*}$$

$$\boxed{\langle \psi_2 | \psi_1 \rangle = 2\omega \int d^3x \; \psi_2^* \psi_1}$$

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

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$$\phi = \sum \underbrace{e^{-i\omega t}}_{\psi} + \underbrace{e^{i\omega t}}_{\psi^*}$$

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(ω positive)

In Minkowski spacetime.

$$\langle \psi_2 | \psi_1 \rangle = i \int d^3x \left[\psi_2^* \partial_t \psi_1 - (\partial_t \psi_2^*) \psi_1 \right]$$

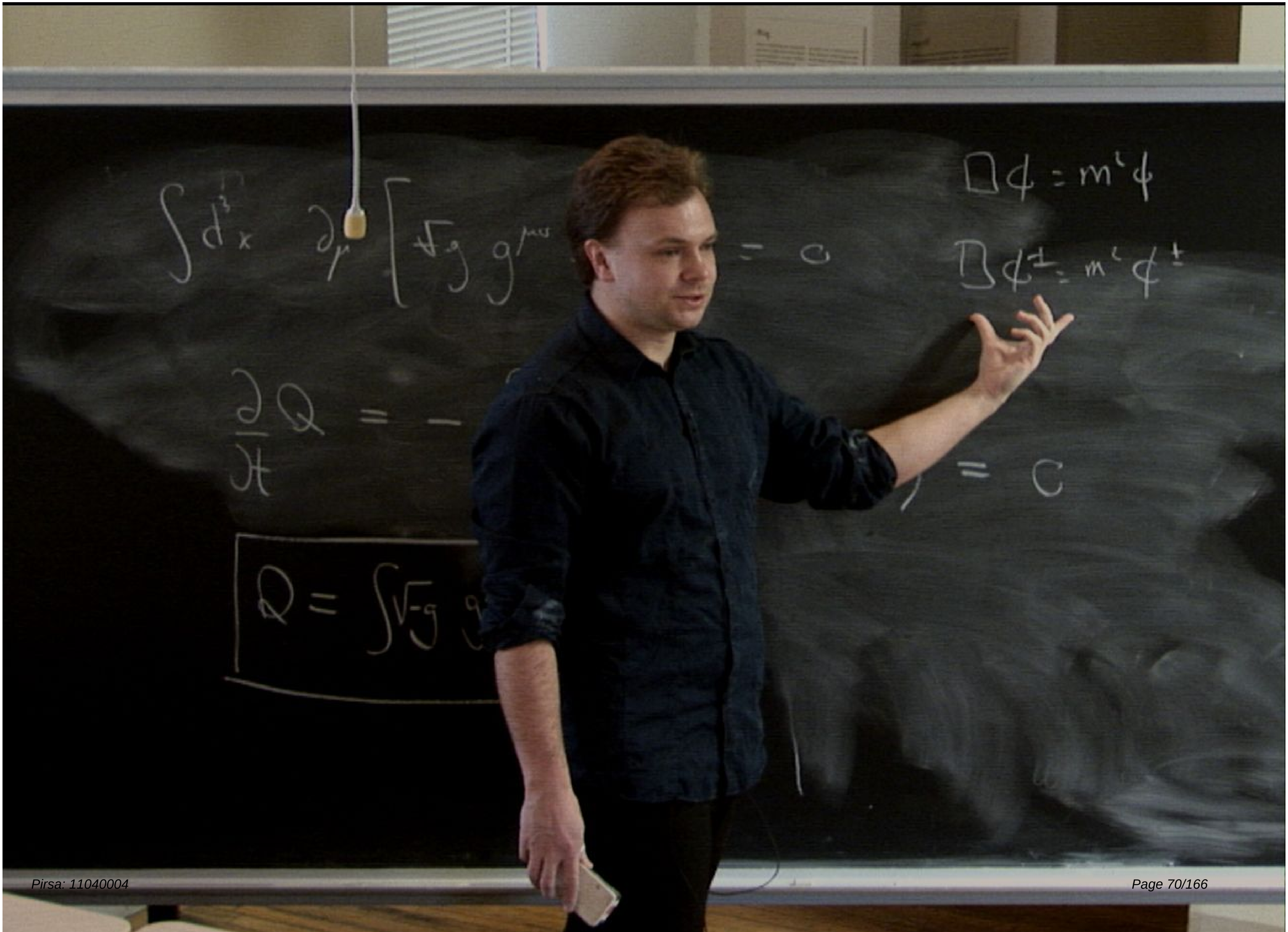
In present case — split into positive and negative frequency solutions.

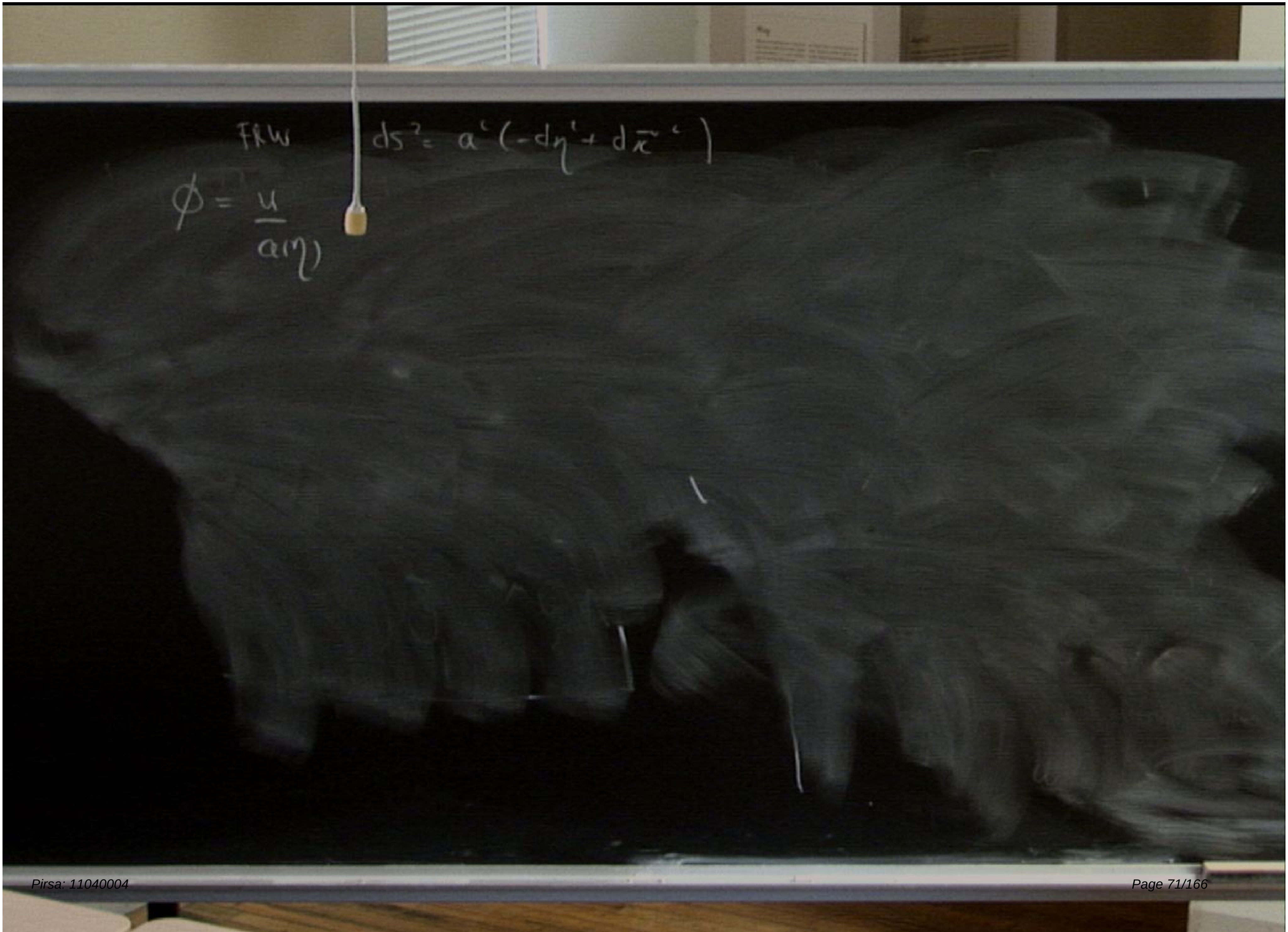
$$\phi = \sum \underbrace{e^{-i\omega t}}_{\psi} + \underbrace{e^{i\omega t}}_{\psi^*}$$

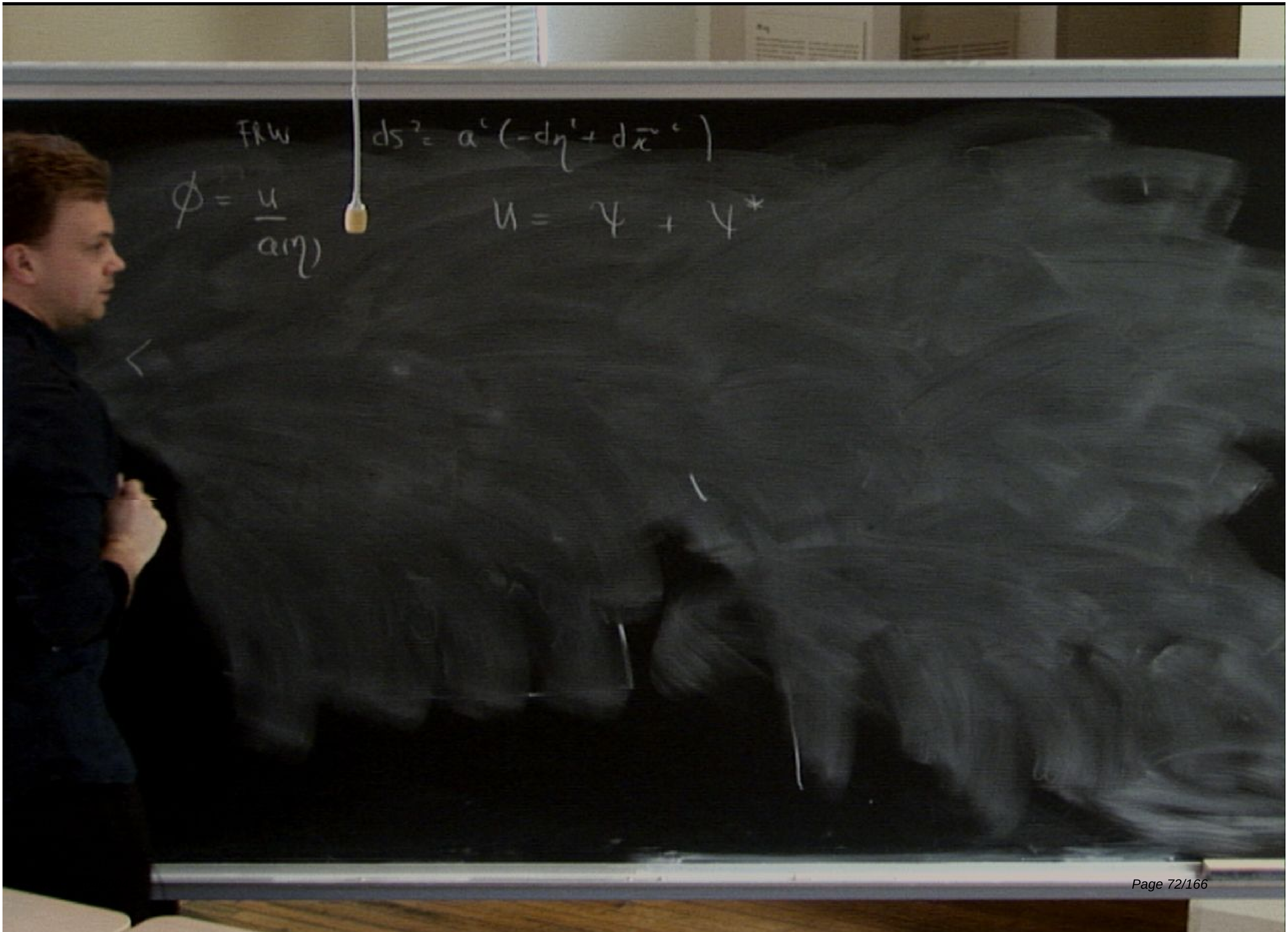
$$\boxed{\langle \psi_2 | \psi_1 \rangle = 2\omega \int d^3x \, \psi_2^* \psi_1}$$

(ω positive)

$$\boxed{\langle \psi_1 | \psi_1 \rangle \geq 0}$$


□ϕ = m²ϕ
□ϕ† = m²ϕ†

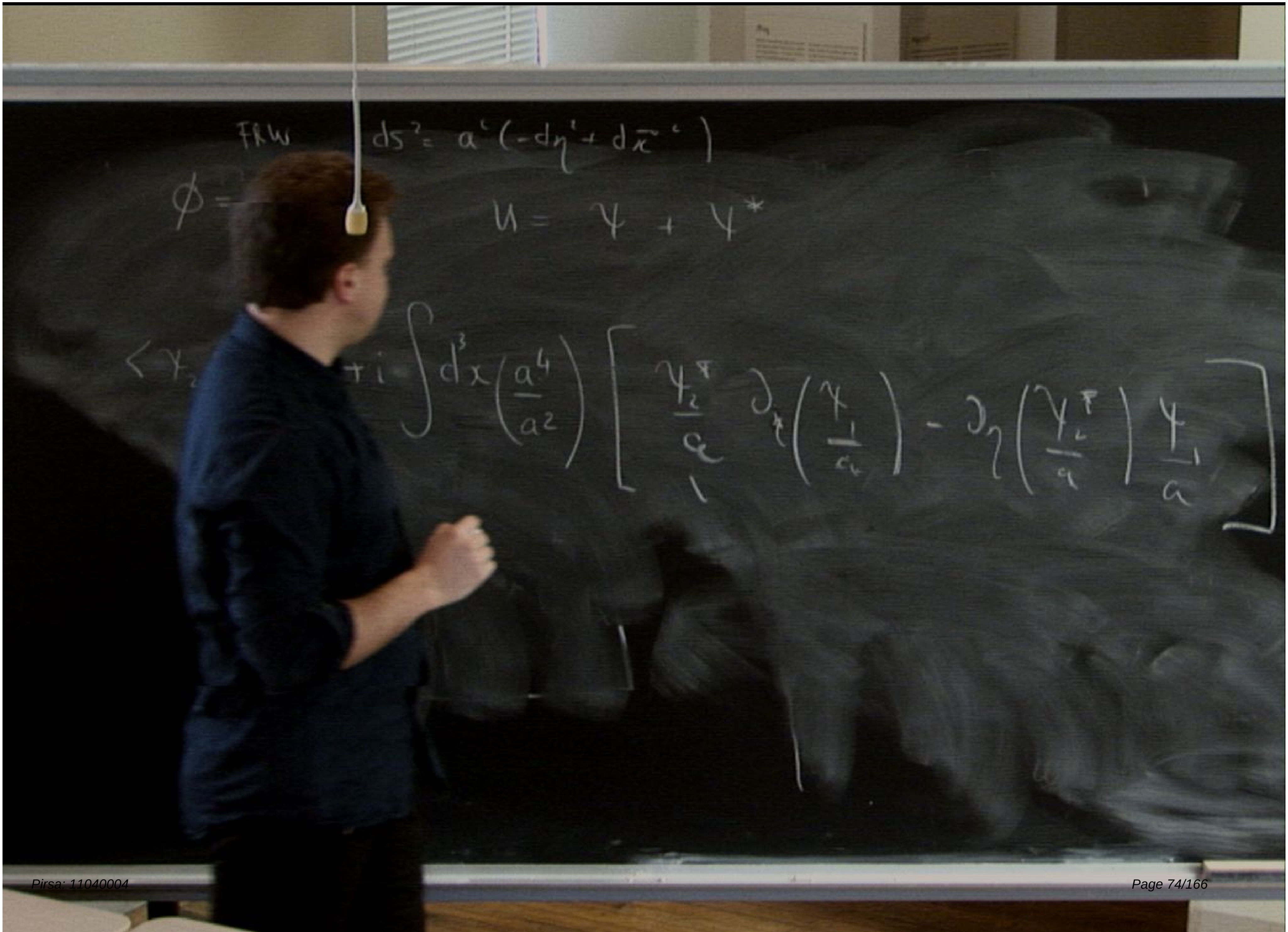

FRW
$ds^2 = a^2(-d\eta^2 + d\vec{x}^2)$
$\phi = \frac{u}{a(\eta)}$

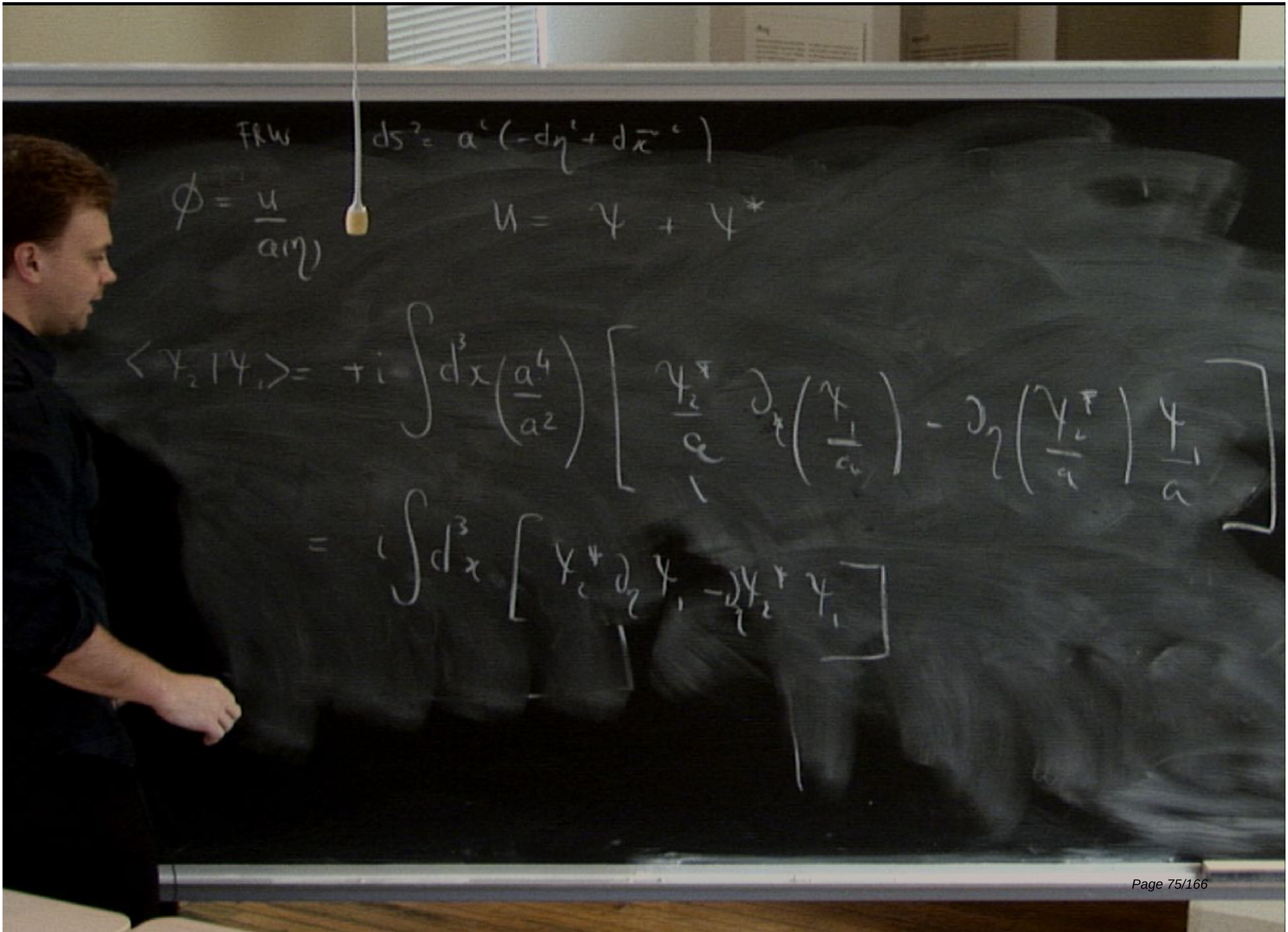

FRW
ds² = a²(-dη² + dx²)
φ = u / a(η)
u = ψ + ψ*

FRW $ds^2 = a^2(-d\eta^2 + d\vec{x}^2)$

$$\phi = \frac{u}{a(\eta)} \qquad u = \psi + \psi^*$$

$$\langle \psi_2 | \psi_1 \rangle = +i \int d^3x \left(\frac{a^4}{a^2}\right) \Big[$$


FRW
ds² = a²(-dη² + dx²)
φ =
u = ψ + ψ*


FRW $ds^2 = a^2(-d\eta^2 + d\vec{x}^2)$
$\phi = \frac{u}{a(\eta)}$
$u = \psi + \psi^*$
$\langle \psi_2 | \psi_1 \rangle = +i \int d^3x \left(\frac{a^4}{a^2}\right) \left[\frac{\psi_2^*}{a} \partial_\eta \left(\frac{\psi_1}{a}\right) - \partial_\eta \left(\frac{\psi_2^*}{a}\right) \frac{\psi_1}{a} \right]$
$= i \int d^3x \left[\psi_2^* \partial_\eta \psi_1 - \partial_\eta \psi_2^* \psi_1 \right]$

FRW $ds^2 = a^2(-d\eta^2 + d\vec{x}^2)$

$$\phi = \frac{u}{a(\eta)} \qquad u = \psi + \psi^*$$

$$\langle \psi_2 | \psi_1 \rangle = +i \int d^3x \left(\frac{a^4}{a^2}\right) \left[\frac{\psi_2^*}{a} \partial_\eta \left(\frac{\psi_1}{a}\right) - \partial_\eta \left(\frac{\psi_2^*}{a}\right) \frac{\psi_1}{a} \right]$$

$$= i \int d^3x \left[\psi_2^* \partial_\eta \psi_1 - \partial_\eta \psi_2^* \psi_1 \right]$$

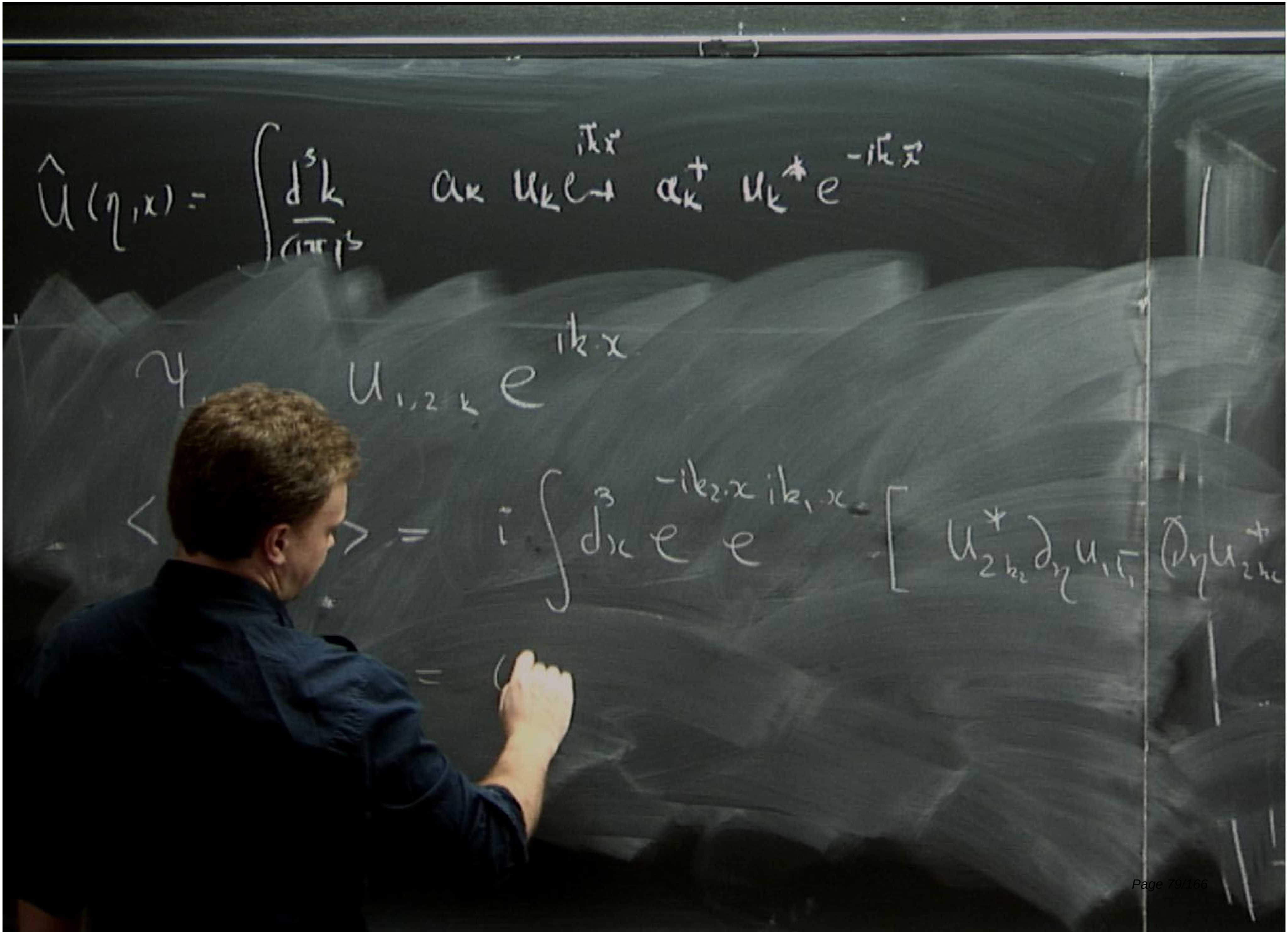
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

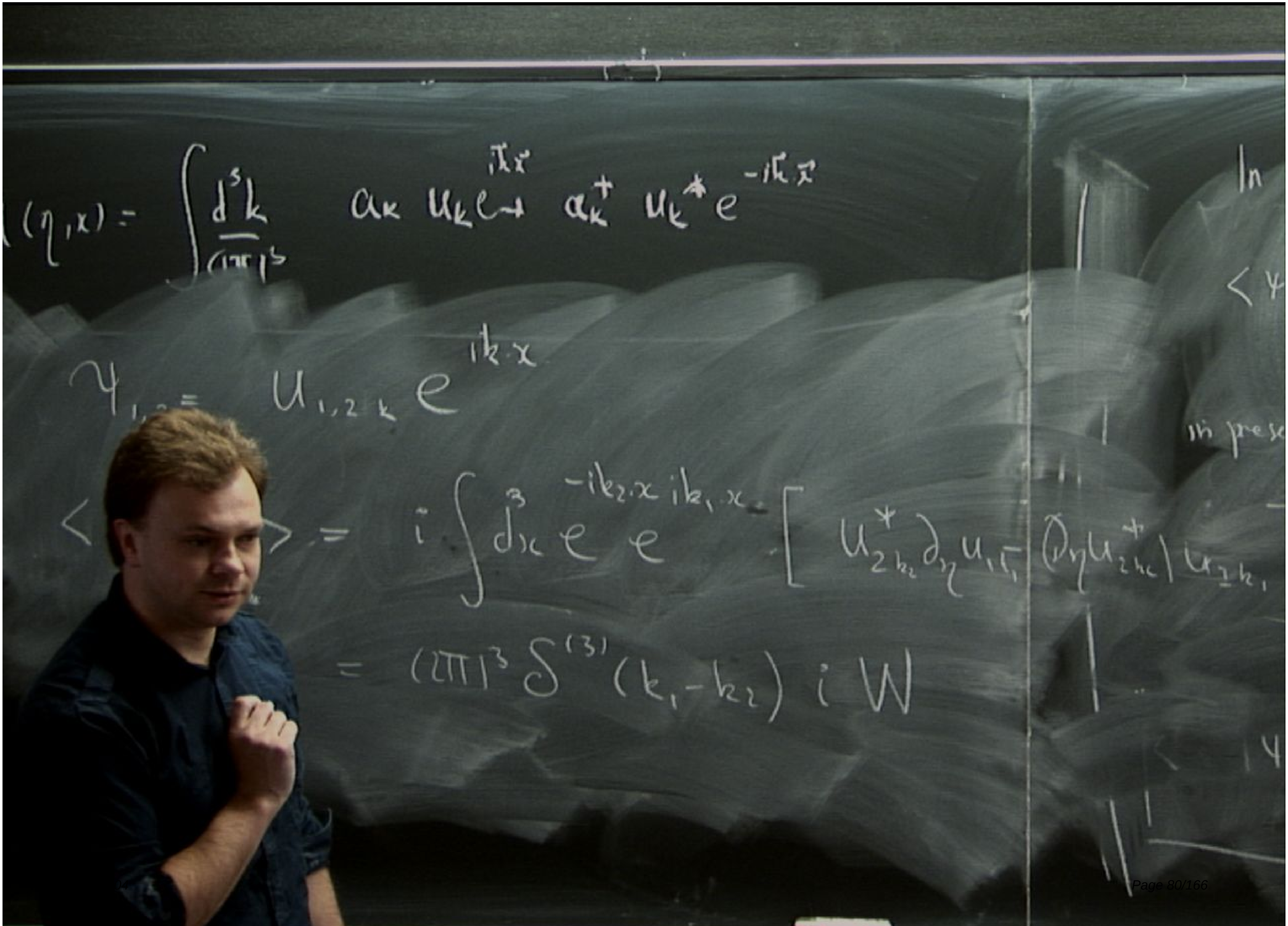
$$\psi_{1,2} = u_{1,2\,k}\, e^{ik\cdot x}$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

$$\psi_{1,2} = u_{1,2\,k}\, e^{ik\cdot x}$$

$$\langle k_2 | k_1 \rangle = i \int d^3x \quad e^{ik_1 \cdot x_1}$$


$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$
$\psi \quad u_{1,2\,k}\, e^{ik\cdot x}$
$\langle \quad \rangle = i \int d^3x \; e^{-ik_2\cdot x} e^{ik_1\cdot x} \left[u^*_{2k_2} \partial_\eta u_{1k_1} - \partial_\eta u^*_{2k_2} \right.$
$= ($


$(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\vec{x}} + a_k^+ u_k^* e^{-i\vec{k}\vec{x}}$
$\psi_{1,2} = u_{1,2\,k} e^{ik\cdot x}$
$\langle \;\; \rangle = i \int d^3x \, e^{-ik_2\cdot x} e^{ik_1\cdot x} \left[u^*_{2k_2} \partial_\eta u_{1k_1} - (\partial_\eta u^*_{2k_2}) u_{1k_1} \right.$
$= (2\pi)^3 \delta^{(3)}(k_1 - k_2) \, i W$
In
in prese

$$\phi(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\psi_{1,2} = u_{1,2\,k}\, e^{ik\cdot x}$$

$$\langle k_2 | k_1 \rangle = i \int d^3x \, e^{-ik_2\cdot x} e^{ik_1\cdot x} \left[u^*_{2k_2} \partial_\eta u_{1k_1} - (\partial_\eta u^*_{2k_2}) u_{2k_1} \right.$$

$$= (2\pi)^3 \delta^{(3)}(k_1 - k_2)\, i W$$

$$\boxed{\langle k_2 | k_1 \rangle = (2\pi)^3 \delta^{(3)}(k_1 - k_2)}$$

In

$\langle \psi$

in prese

$| \psi$

$$\phi(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

$$\psi_{1,2} = u_{1,2\,k}\, e^{ik\cdot x}$$

$$\langle k_2 | k_1 \rangle = i \int d^3x\, e^{-ik_2\cdot x} e^{ik_1\cdot x} \left[u^*_{2k_2} \partial_\eta u_{1k_1} - (\partial_\eta u^*_{2k_2})\, u_{2k_1} \right.$$

$$= (2\pi)^3 \delta^{(3)}(k_1 - k_2)\, i W$$

$$\boxed{\langle k_2 | k_1 \rangle = (2\pi)^3 \delta^{(3)}(k_1 - k_2)}$$

In

$\langle \psi$

in prese

$| \psi$

$$\phi(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

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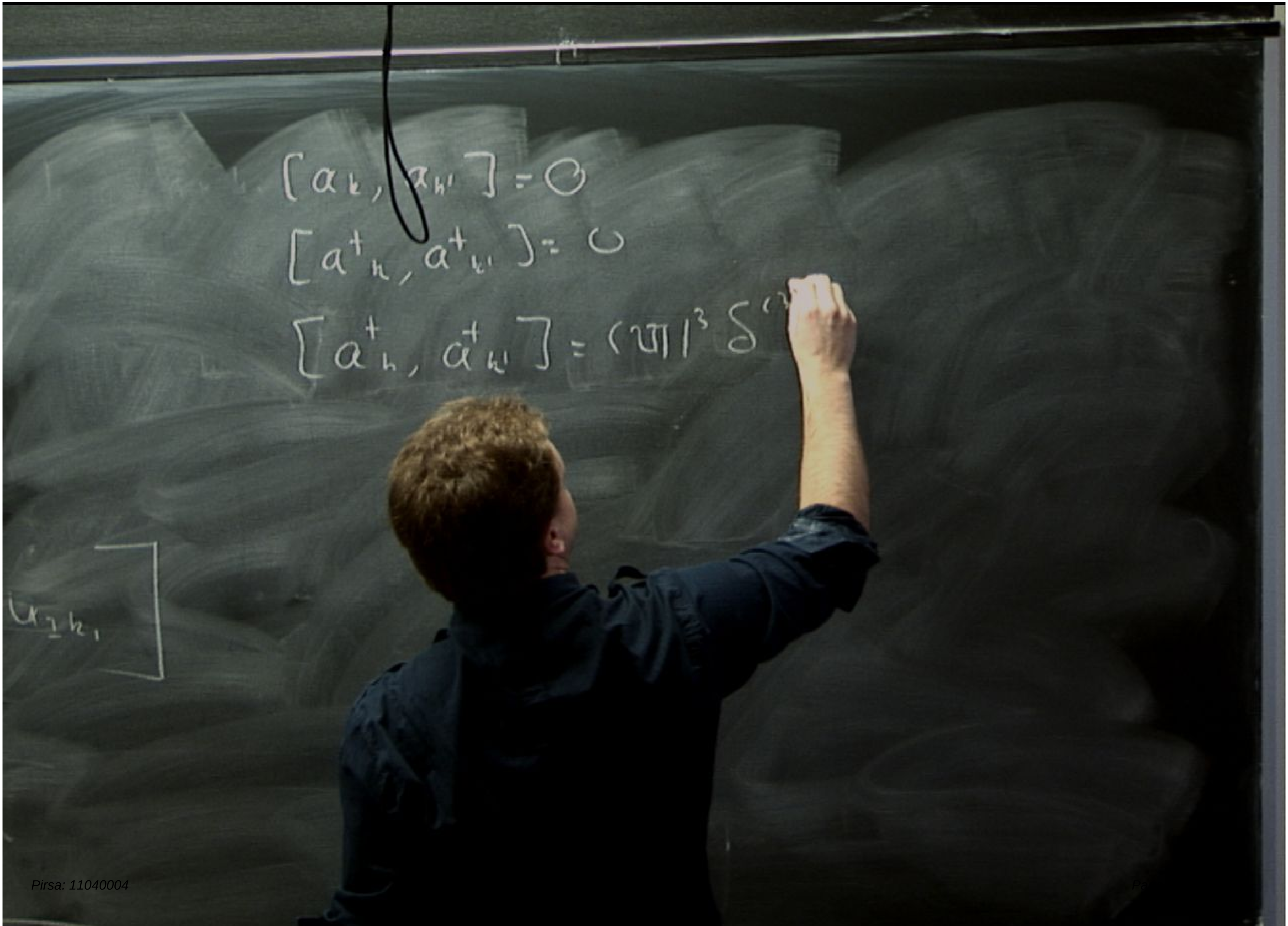
$$= (2\pi)^3 \delta^{(3)}(k_1 - k_2)\, i W$$

$$\boxed{\langle k_2 | k_1 \rangle = (2\pi)^3 \delta^{(3)}(k_1 - k_2)}$$

In

$\langle \psi$

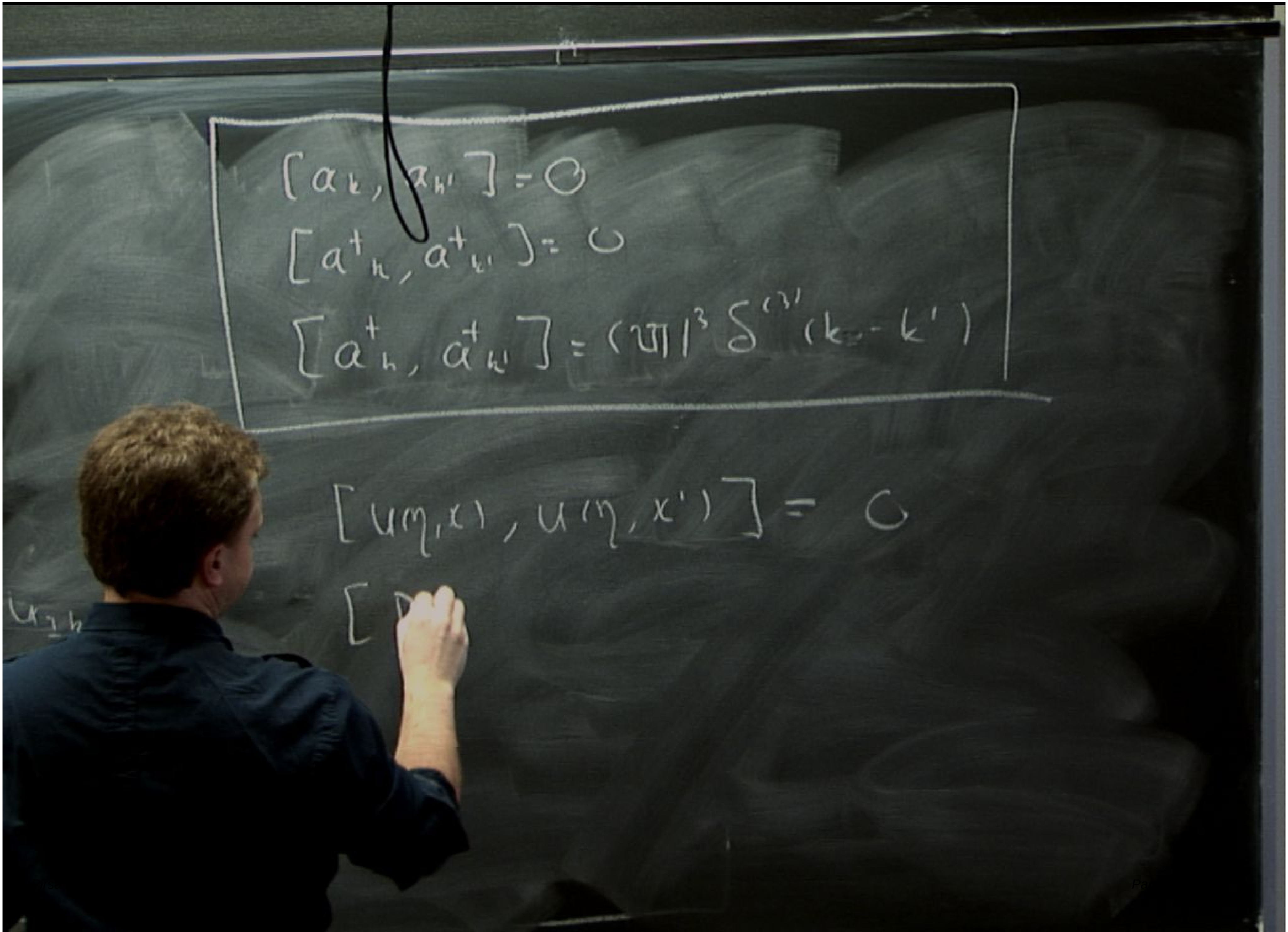

[a_k, a_k'] = 0
[a+_k, a+_k'] = 0
[a+_k, a+_k'] = (2π)^3 δ^(3

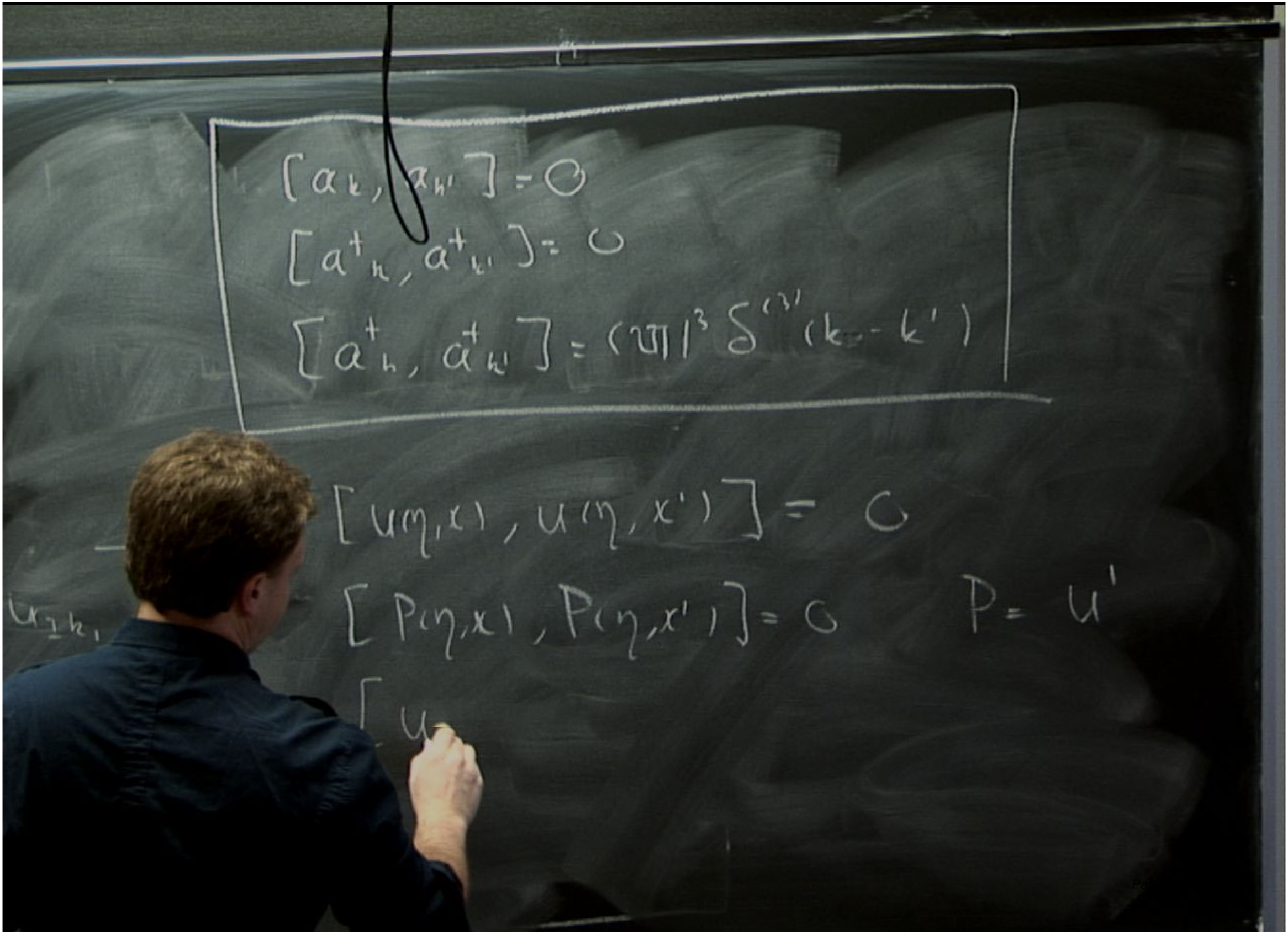
$$[a_k, a_{k'}] = 0$$

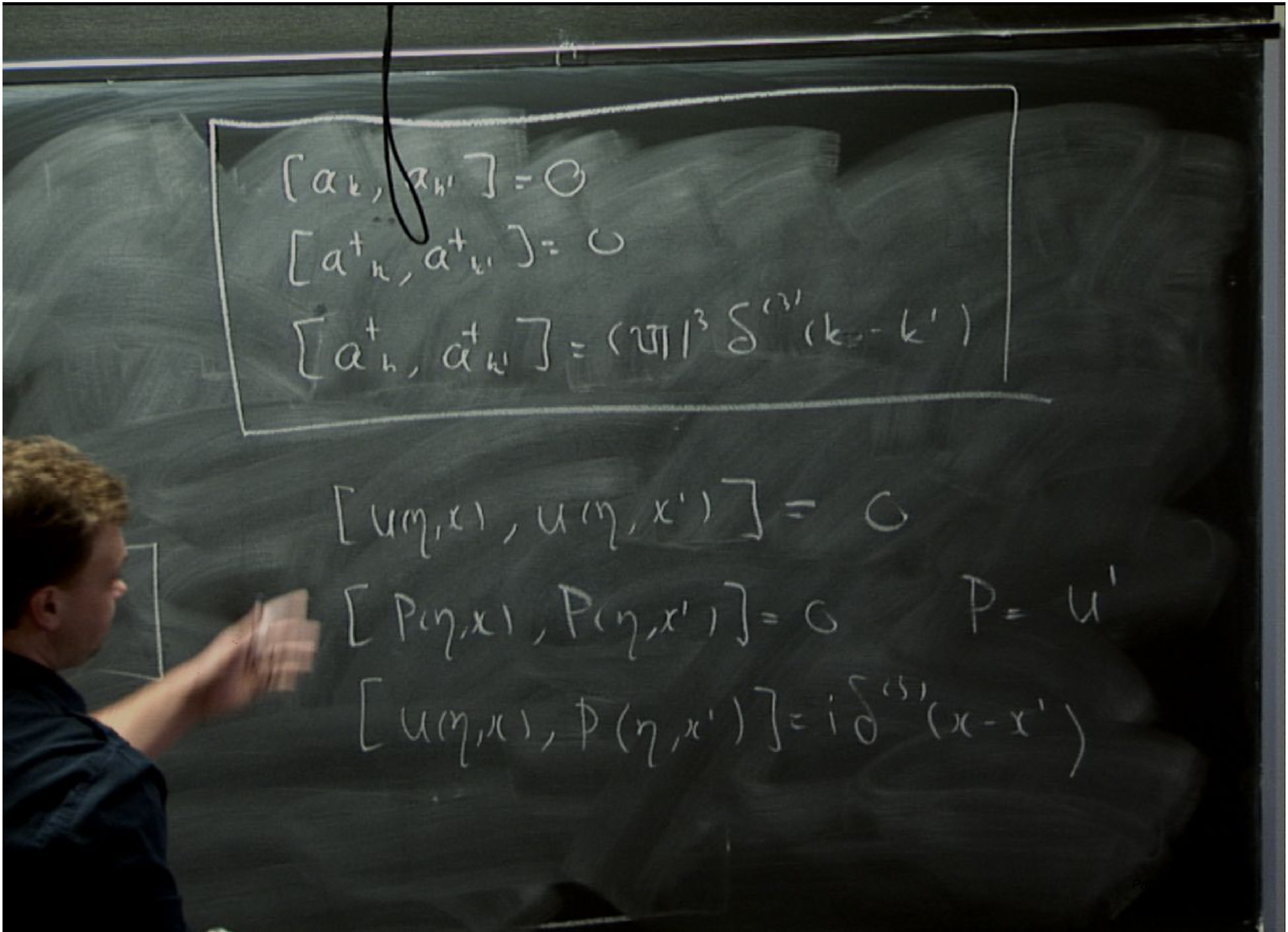
$$[a^\dagger_k, a^\dagger_{k'}] = 0$$

$$[a^\dagger_k, a^\dagger_{k'}] = (2\pi)^3 \delta^{(3)}(k - k')$$

$\omega_{\underline{k}},$


[a_k, a_k'] = 0
[a†_k, a†_k'] = 0
[a_k, a†_k'] = (2π)^3 δ^(3)(k - k')
[u(η, x), u(η, x')] = 0
[


[a_k , a_k'] = 0
[a†_k , a†_k'] = 0
[a_k , a†_k'] = (2π)^3 δ^(3)(k - k')
[u(η,x) , u(η,x')] = 0
[P(η,x) , P(η,x')] = 0
P = u'
[u


$[a_k, a_{k'}] = 0$
$[a^\dagger_k, a^\dagger_{k'}] = 0$
$[a^\dagger_k, a^\dagger_{k'}] = (2\pi)^3 \delta^{(3)}(k - k')$
$[u(\eta,x), u(\eta,x')] = 0$
$[p(\eta,x), p(\eta,x')] = 0$
$p = u'$
$[u(\eta,x), p(\eta,x')] = i\delta^{(3)}(x - x')$

$$[a_k, a_{k'}] = 0$$

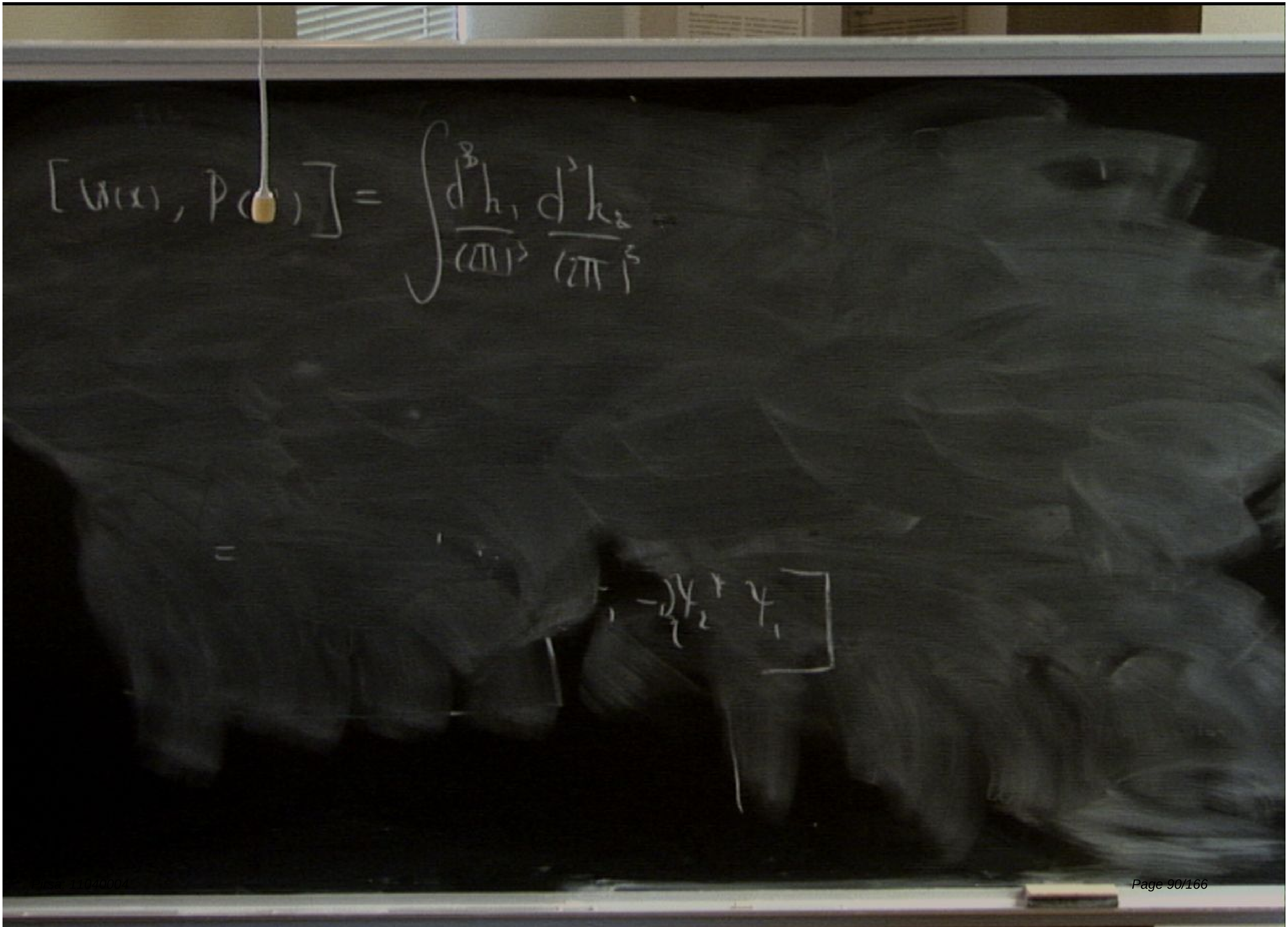
$$[a^{+}_{k}, a^{+}_{k'}] = 0$$

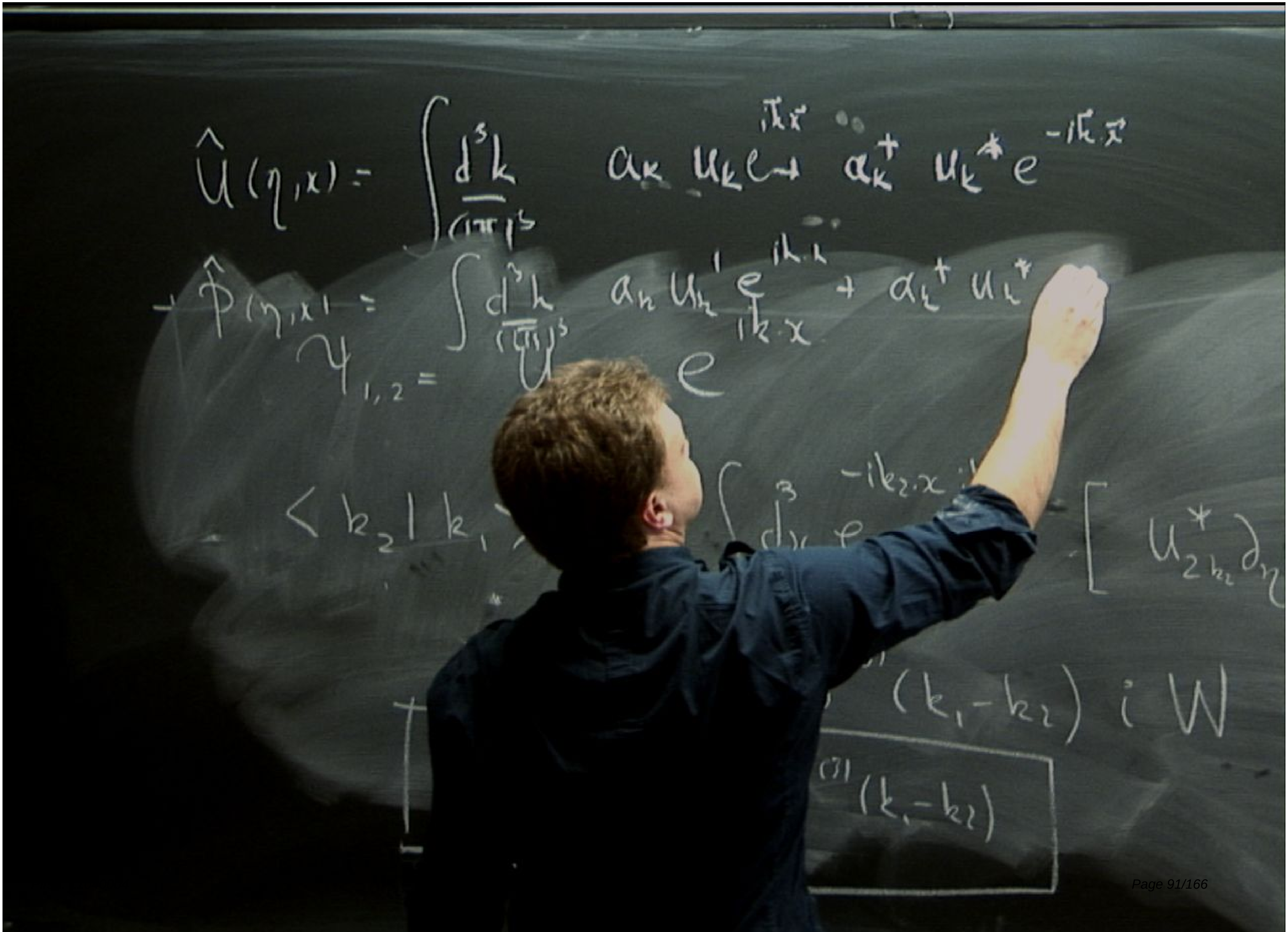
$$[a^{+}_{k}, a^{+}_{k'}] = (2\pi)^3 \delta^{(3)}(k - k')$$

$$[u(\eta, x), u(\eta, x')] = 0$$

$$[P(\eta, x), P(\eta, x')] = 0 \qquad P = u'$$

$$[u(\eta, x), P(\eta, x')] = i\delta^{(3)}(x - x')$$


$[\varphi(x), P(\ )] = \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3}$


$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$
$\hat{p}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k' e^{ik\cdot x} + a_k^{+} u_k^{*}$
$\psi_{1,2} = u \; e^{ik\cdot x}$
$\langle k_2 | k_1 \rangle$
$\left[u^{*}_{2 k_2} \partial_\eta \right.$
$(k_1 - k_2)\, i W$
$(k_1 - k_2)$

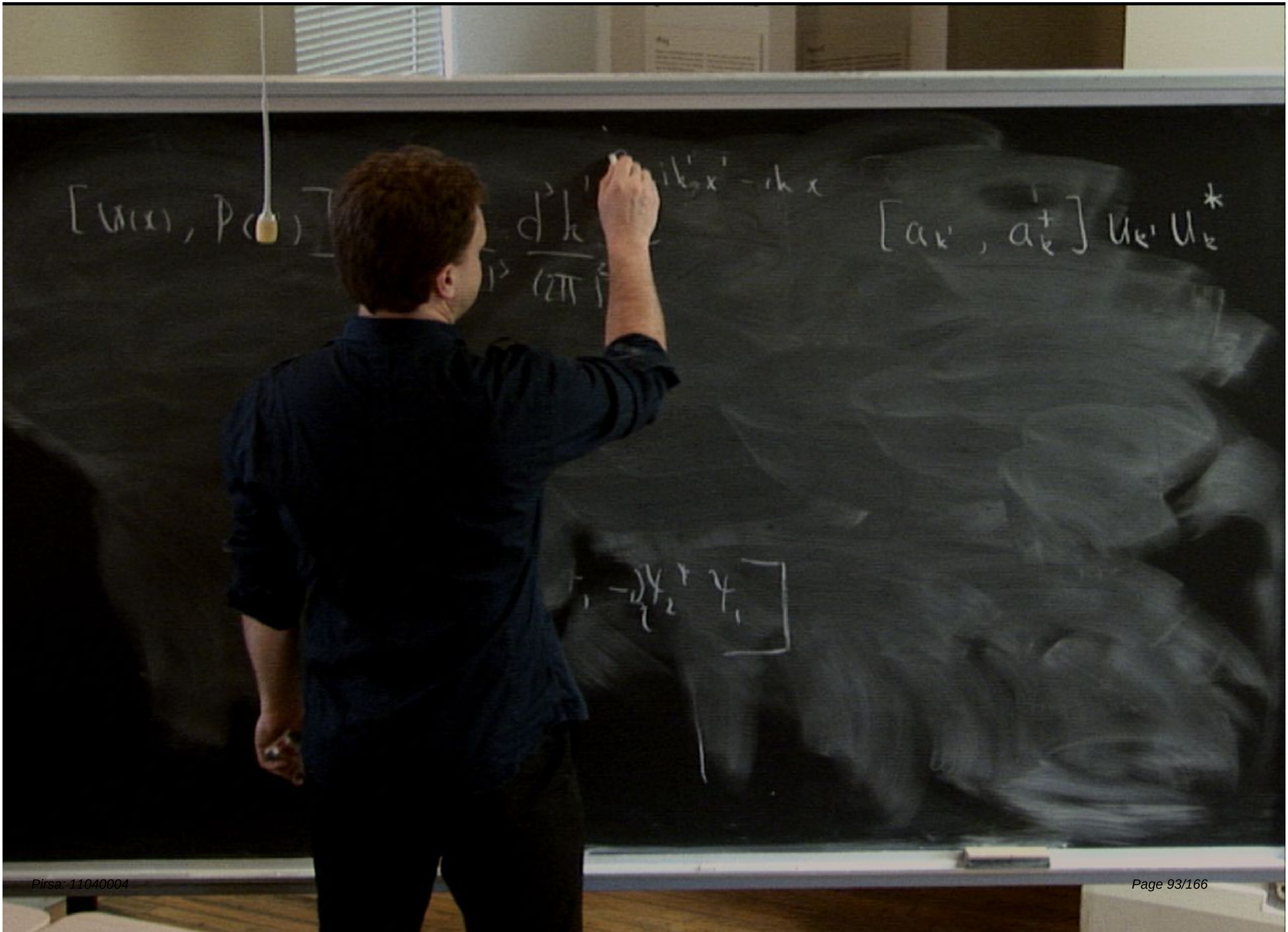
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

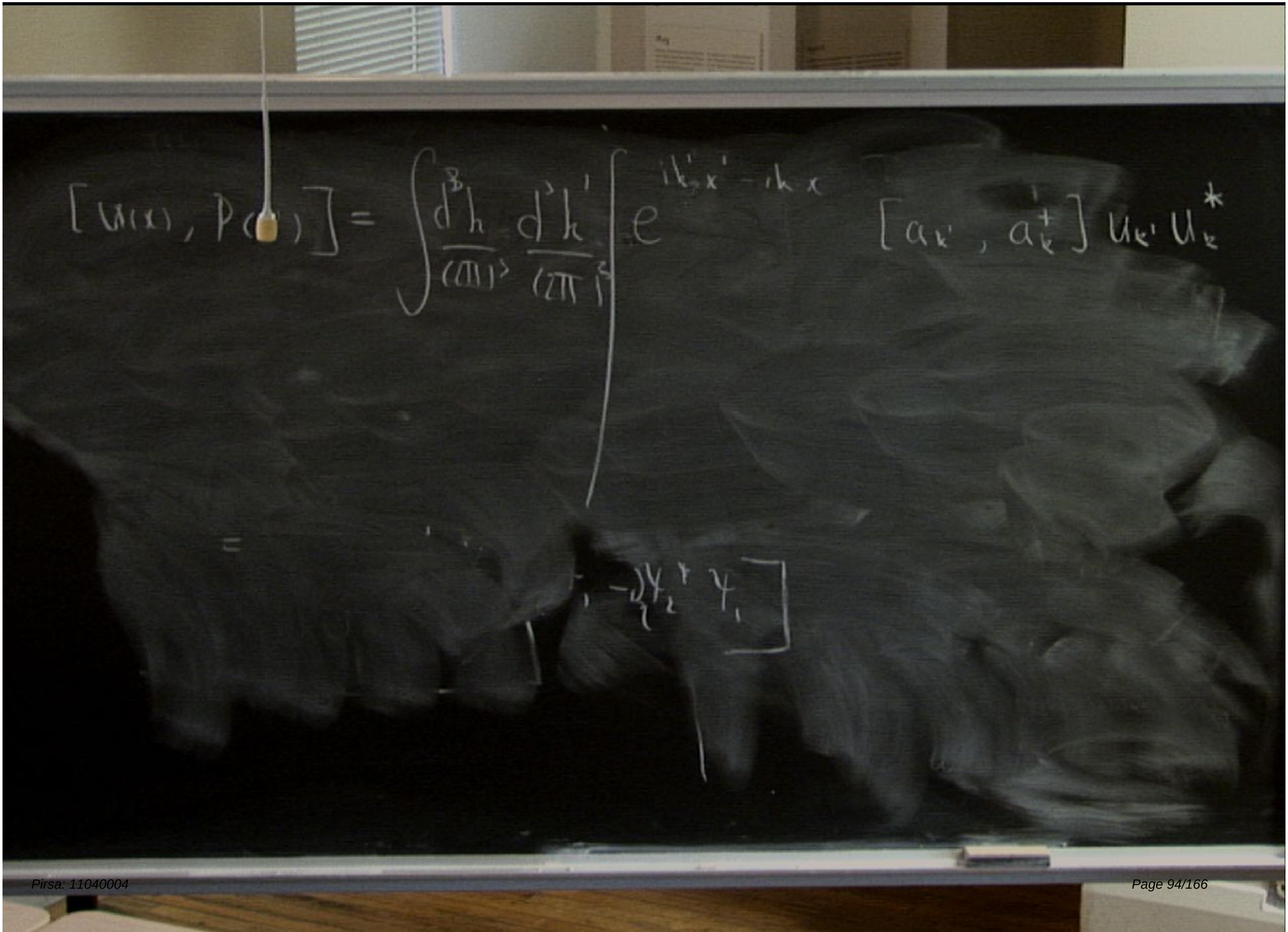
$$\hat{p}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k' e^{ik\cdot x} + a_k^\dagger u_k'^* e^{-ik\cdot x}$$

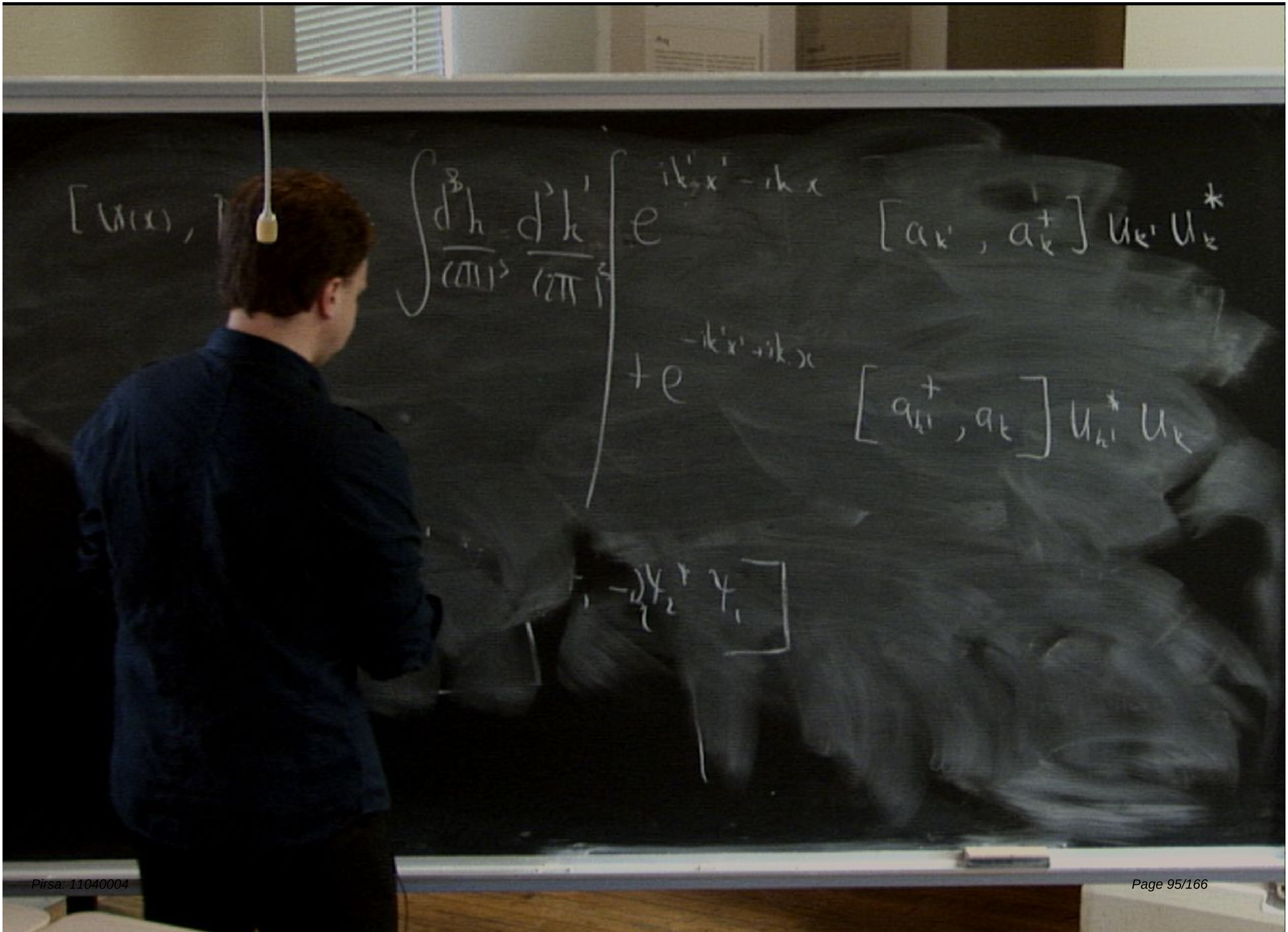
$$= i \int d^3x \, e^{-ik_2\cdot x} e^{ik_1\cdot x} \left[u_{2\,k_2}^* \partial_\eta \right.$$

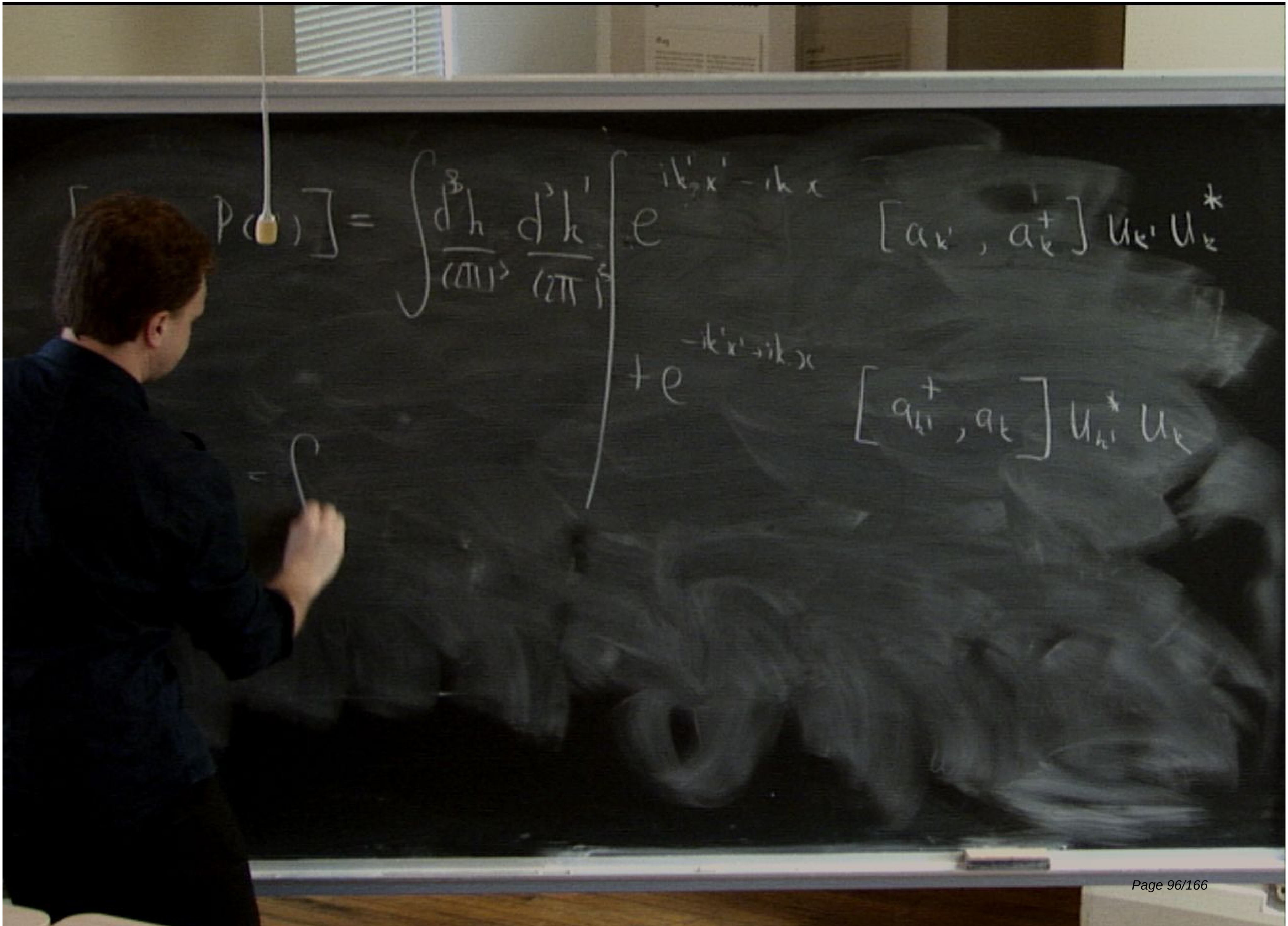
$$= (2\pi)^3 \delta^{(3)}(k_1 - k_2) \, i W$$

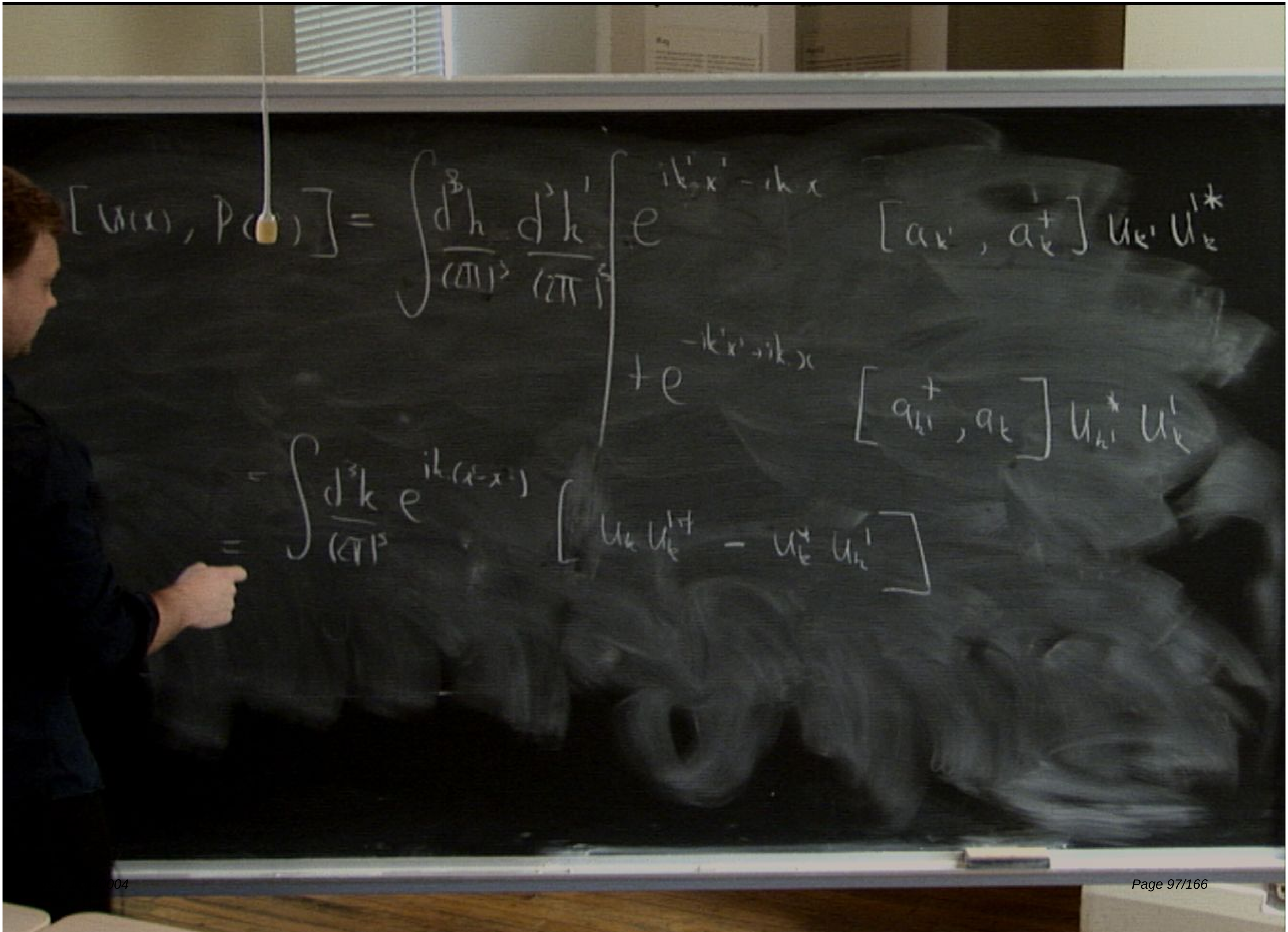
$$\boxed{\langle k_2 | k_1 \rangle = (2\pi)^3 \delta^{(3)}(k_1 - k_2)}$$

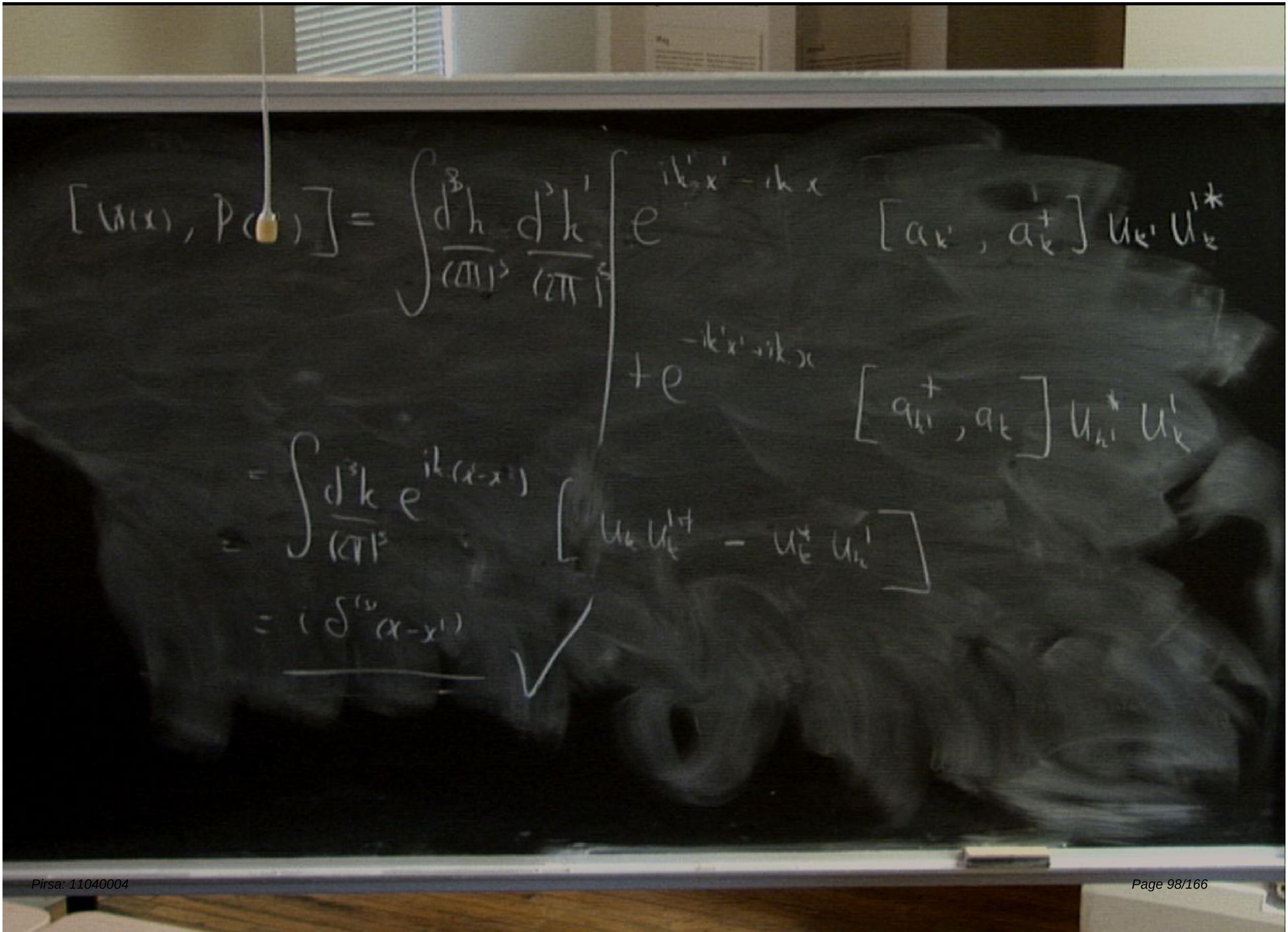



$[\phi(x), P(x')] = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} e^{ik' \cdot x' - ik \cdot x} [a_{k'}, a_k^\dagger] u_{k'} u_k^*$

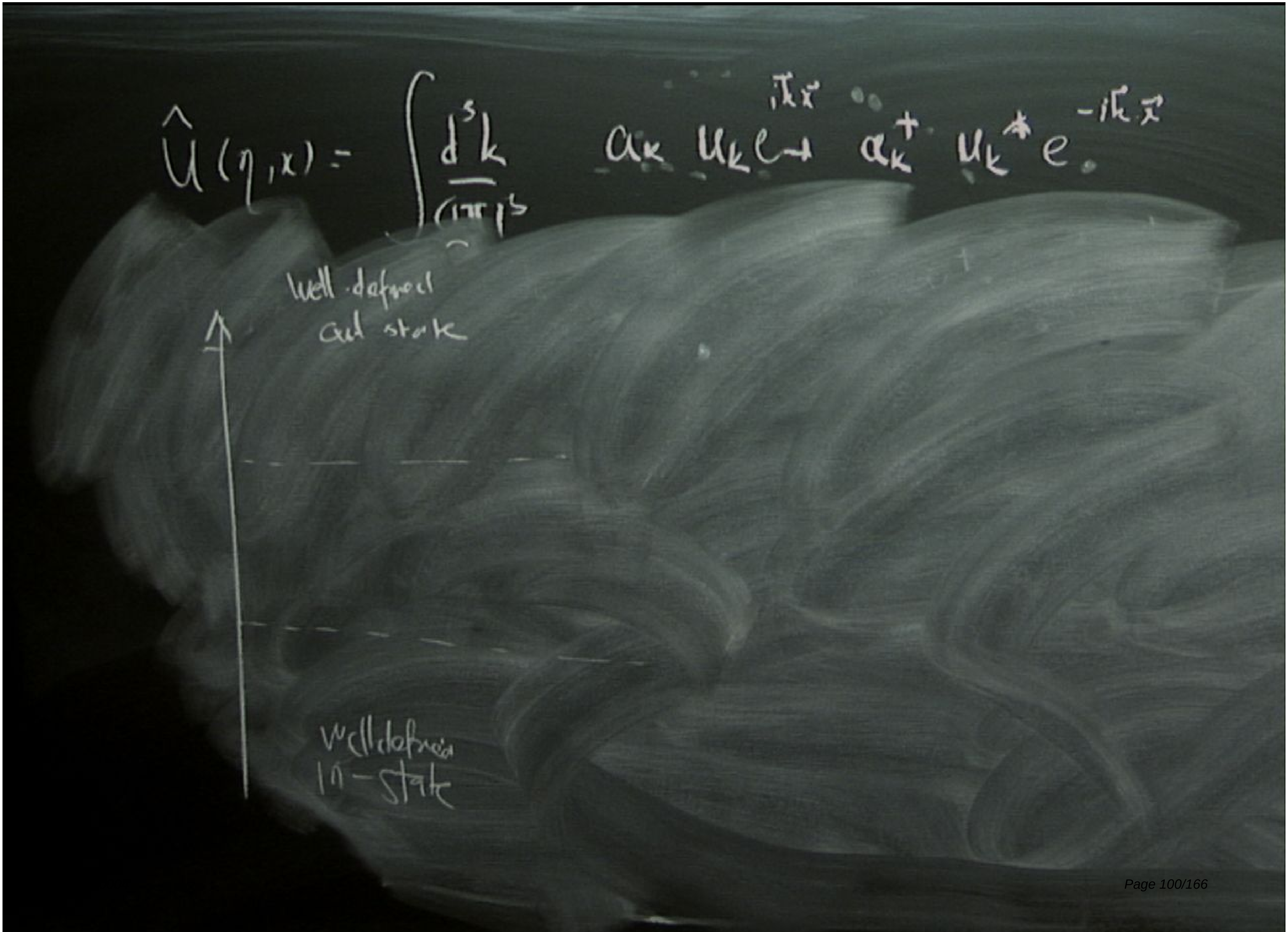

[a_k' , a_k^†] u_k' u_k^*
[a_k'^† , a_k] u_k'^* u_k

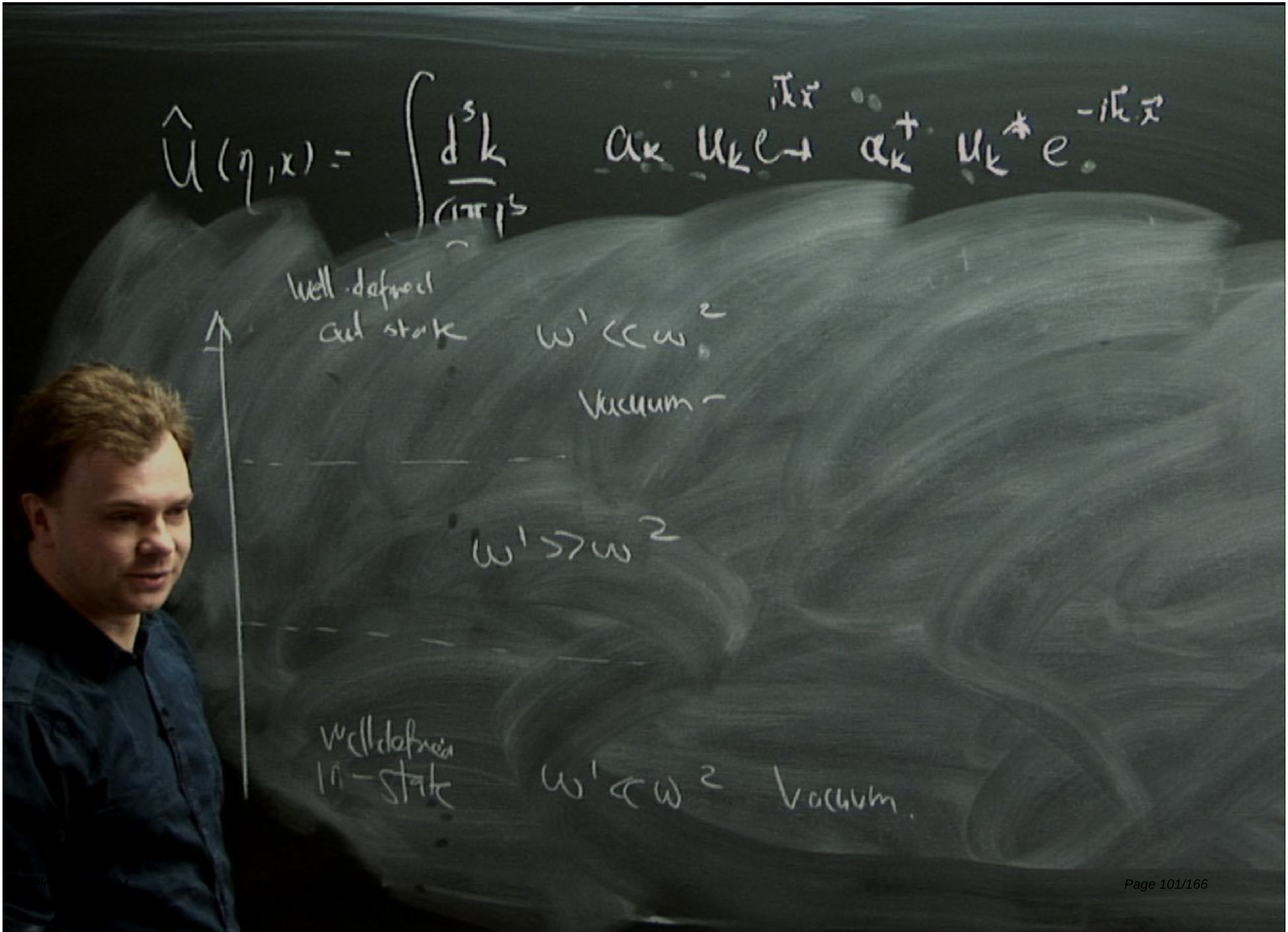

[ak' , a+k] uk' u*k
[a+k' , ak] u*k' uk


$[\phi(x), p(x')] = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Big\{ e^{ik'\cdot x' - ik\cdot x} [a_{k'}, a_k^\dagger]\, u_{k'} u_k'^*$
$+ e^{-ik'x' + ik\cdot x} [a_{k'}^\dagger, a_k]\, u_{k'}^* u_k' \Big\}$
$= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} \big[u_k u_k'^* - u_k^* u_k' \big]$


$[\phi(x), p(x')] = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Big\{ e^{ik'x' - ikx} [a_{k'}, a_k^\dagger] u_{k'} u_k'^* + e^{-ik'x' + ikx} [a_{k'}^\dagger, a_k] u_{k'}^* u_k' \Big\}$
$= \int \frac{d^3k}{(2\pi)^3} e^{ik(x-x')} [u_k u_k'^* - u_k^* u_k']$
$= i\delta^{(3)}(x-x')$
✓

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$


$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$
well defined
out state
well defined
in-state


$\hat{U}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$
Well defined
out state
$\omega' \ll \omega^2$
Vacuum
$\omega' \gg \omega^2$
Well defined
in-state
$\omega' \ll \omega^2$ Vacuum.

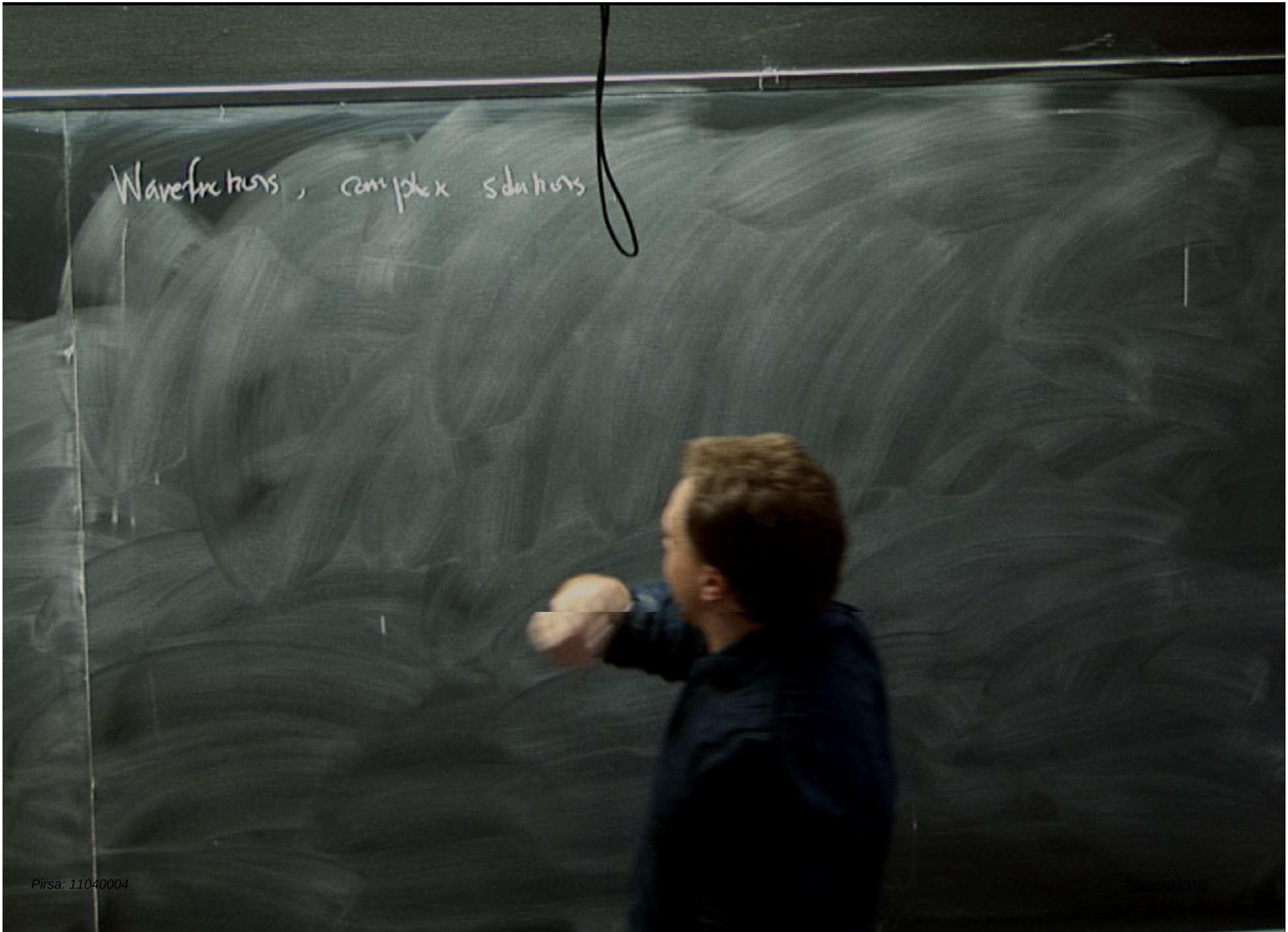
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

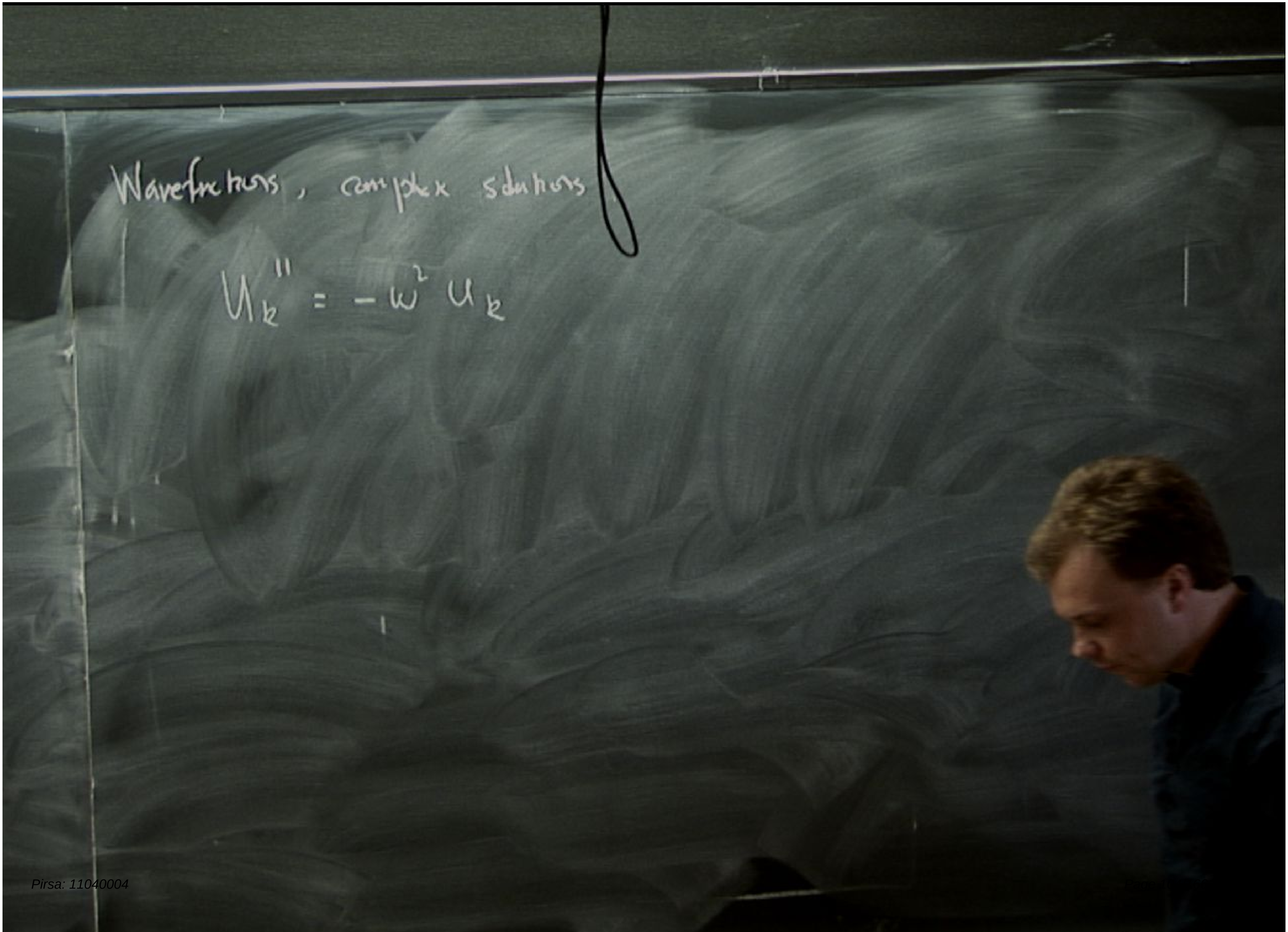
well defined out state $\omega' \ll \omega^2$

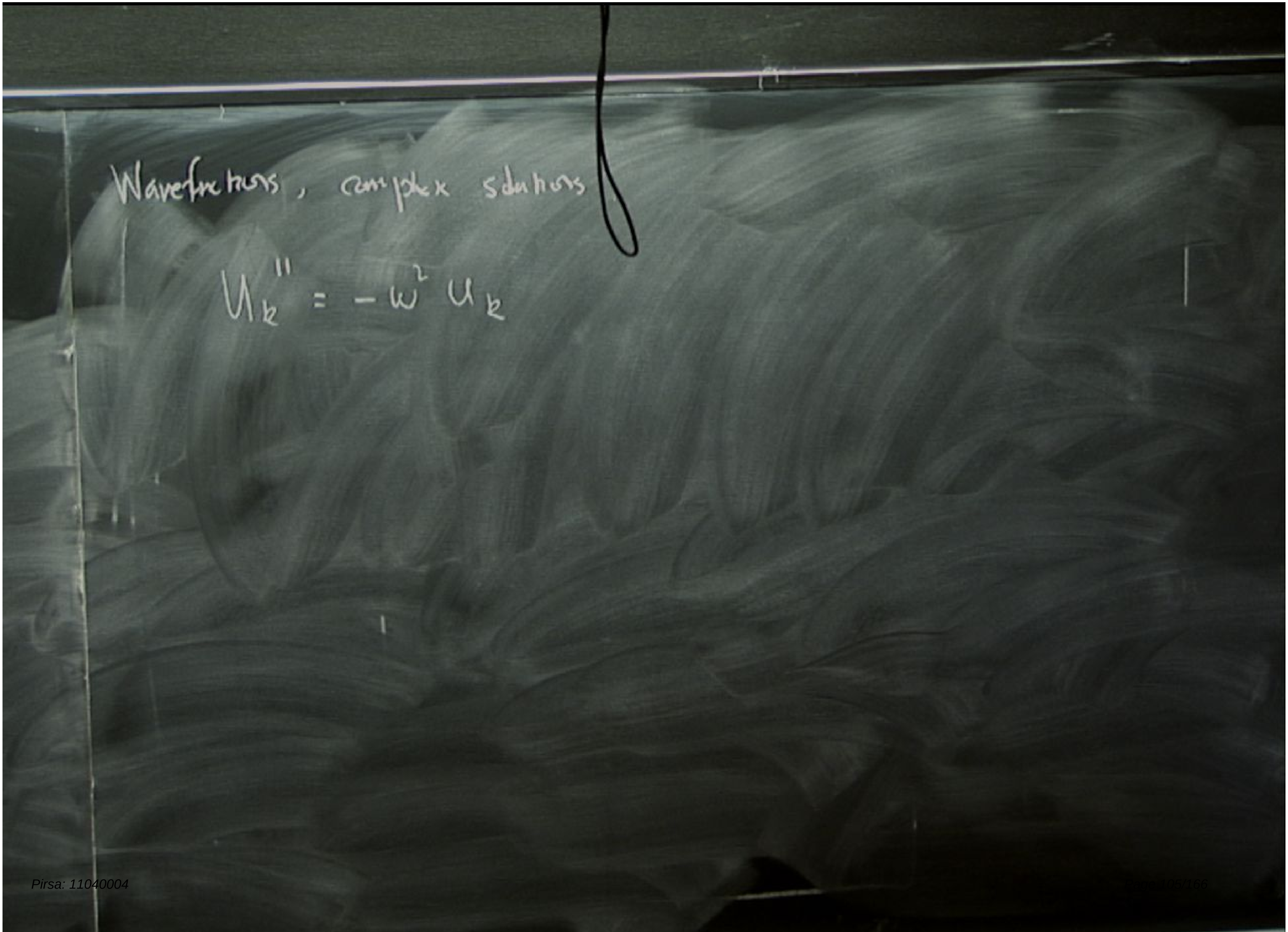
Vacuum – filled with particles!

$\omega' \gg \omega^2$

Well defined in-state $\omega' \ll \omega^2$ Vacuum.


Wavefunctions, complex solutions


Wavefunctions, complex solutions
$u_k'' = -\omega^2 u_k$


Wavefunctions, complex solutions
$u_k'' = -\omega^2 u_k$

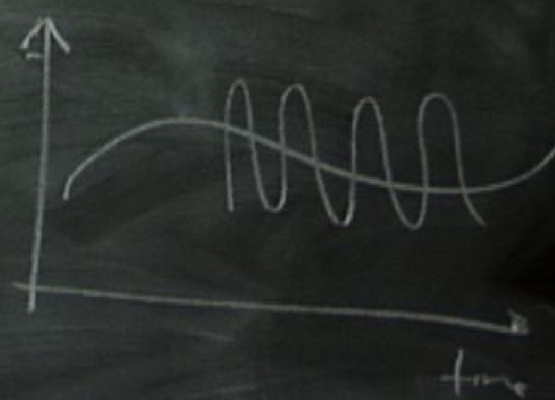
Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm states.

adiabatic

WKB $\omega_k' \ll \omega_k^2$



Wavefunctions, complex solutions

$u_k'' = -\omega_k^2(\eta)\, u_k$

u_k positive norm states.

adiabatic

WKB $\omega_k' \ll \omega_k^2$

$u_k = A e^{-i\int \omega_k(\eta)\, d\eta}$

Wavefunctions, complex solutions

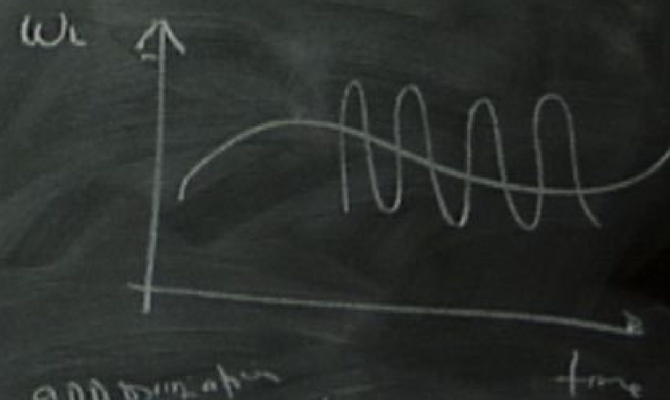
$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm states.

adiabatic

WKB $\quad \omega_k' \ll \omega_k^2$

$$u_k = A\, e^{-i\int \omega_k(\eta')\, d\eta'}$$



WKB approximation.

Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm states.

WKB $\quad \omega_k' \ll \omega_k^2$

adiabatic

$$u_k = A e^{-i\int \omega_k(\eta)\, d\eta}$$

WKB approximation.

Wavefunctions, complex solutions

$$U_k'' = -\omega_k^2(\eta) U_k$$

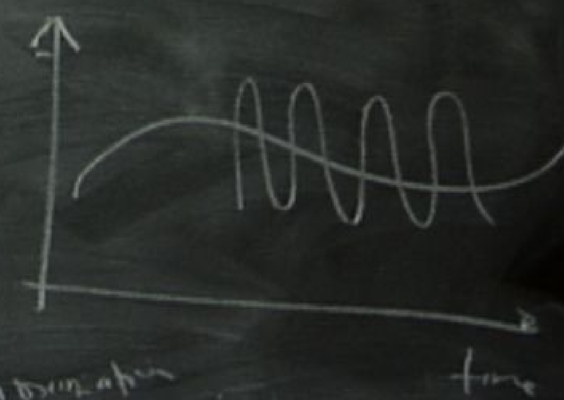
U_k positive norm states.

adiabatic

WKB $\quad \omega_k' \ll \omega_k^2$

$$U_k = A^{(\eta)} e^{-i\int \omega_k(\eta')\, d\eta'}$$

WKB approximation.



$u_k^* e^{-i\vec{k}\cdot\vec{x}}$

...th particles !

Wronskian / normalization

Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm

WKB $\quad \omega_k' \ll \omega_k^2 \quad$ adiabatic

$$u_k = A^{(\eta)} e^{-i\int \omega_k(\eta)\, d\eta}$$

WKB

Wronskian / normalization

$$i\left[u_k^* u_k' - u_k^{*\prime} u_k\right] = 1$$

$u_k^* e^{-i\vec{k}\cdot\vec{x}}$

...ticles !

Wronskian / normalisation

Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm

WKB $\omega_k' \ll \omega_k^2$ adiabatic

$$u_k = A(\eta)\, e^{-i\int \omega_k(\eta)\, d\eta}$$ ✓ WKB

$$i\left[u_k^* u_k' - u_k^{*\prime} u_k\right] = 1$$

$$i|A|^2\, 2\omega_k$$

$u_k^* e^{-i\vec{k}\cdot\vec{x}}$

... particles !

Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm

WKB $\quad \omega_k' \ll \omega_k^2$ adiabatic

$$u_k = A(\eta)\, e^{-i\int \omega_k(\eta)\, d\eta}$$

WKB

Wronskian / normalization

$$i\left[u_k^* u_k' - u_k^{*\prime} u_k\right] = 1$$

$$-i\,i\,|A|^2\, 2\omega_k = 1 \qquad |A| = \frac{1}{\sqrt{2\omega_k}}$$

Wavefunctions, complex solutions

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

u_k positive norm states

adiabatic

WKB $\omega_k' \ll \omega_k^2$

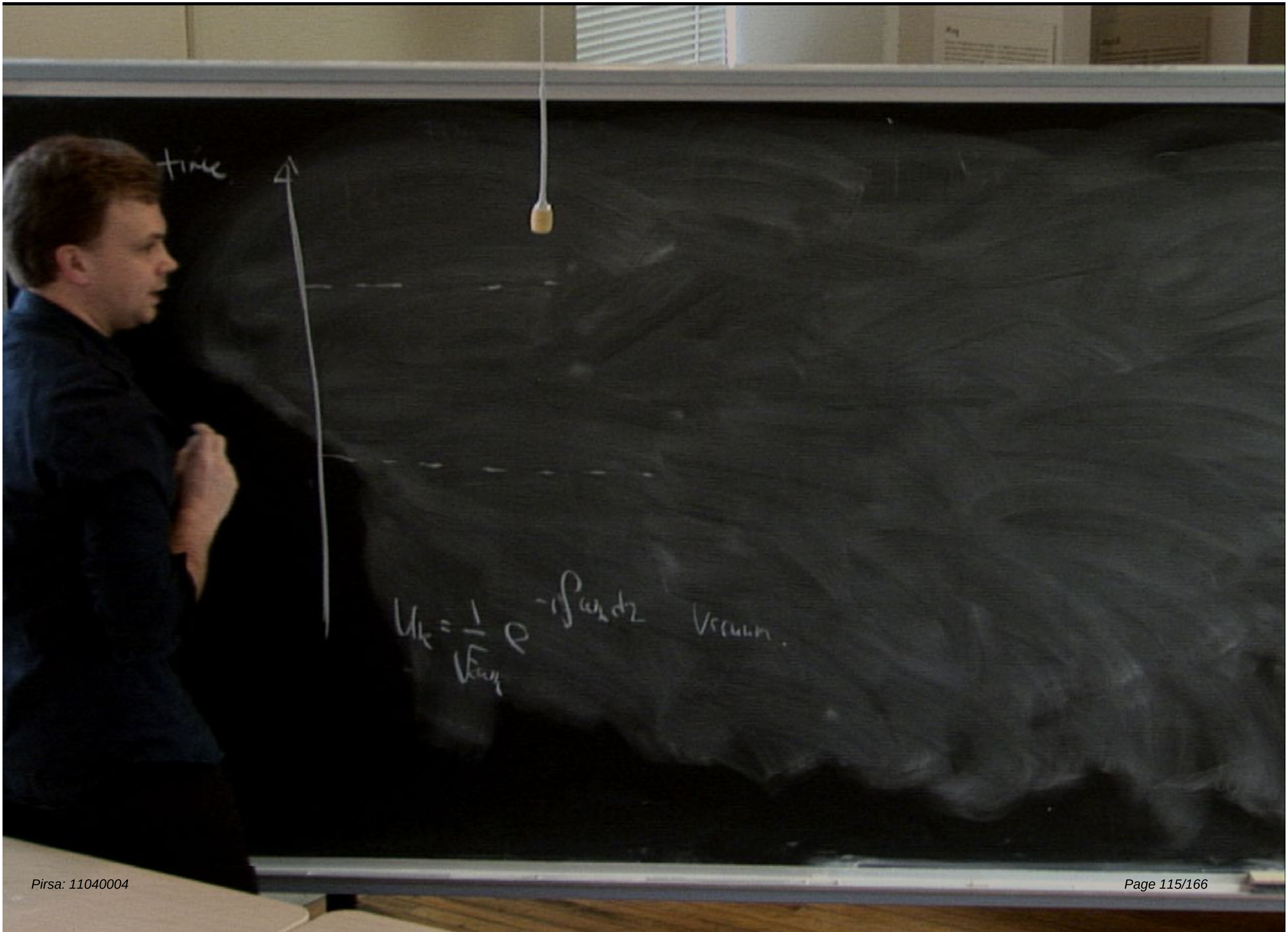
$$u_k = \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k(\eta)\, d\eta}$$

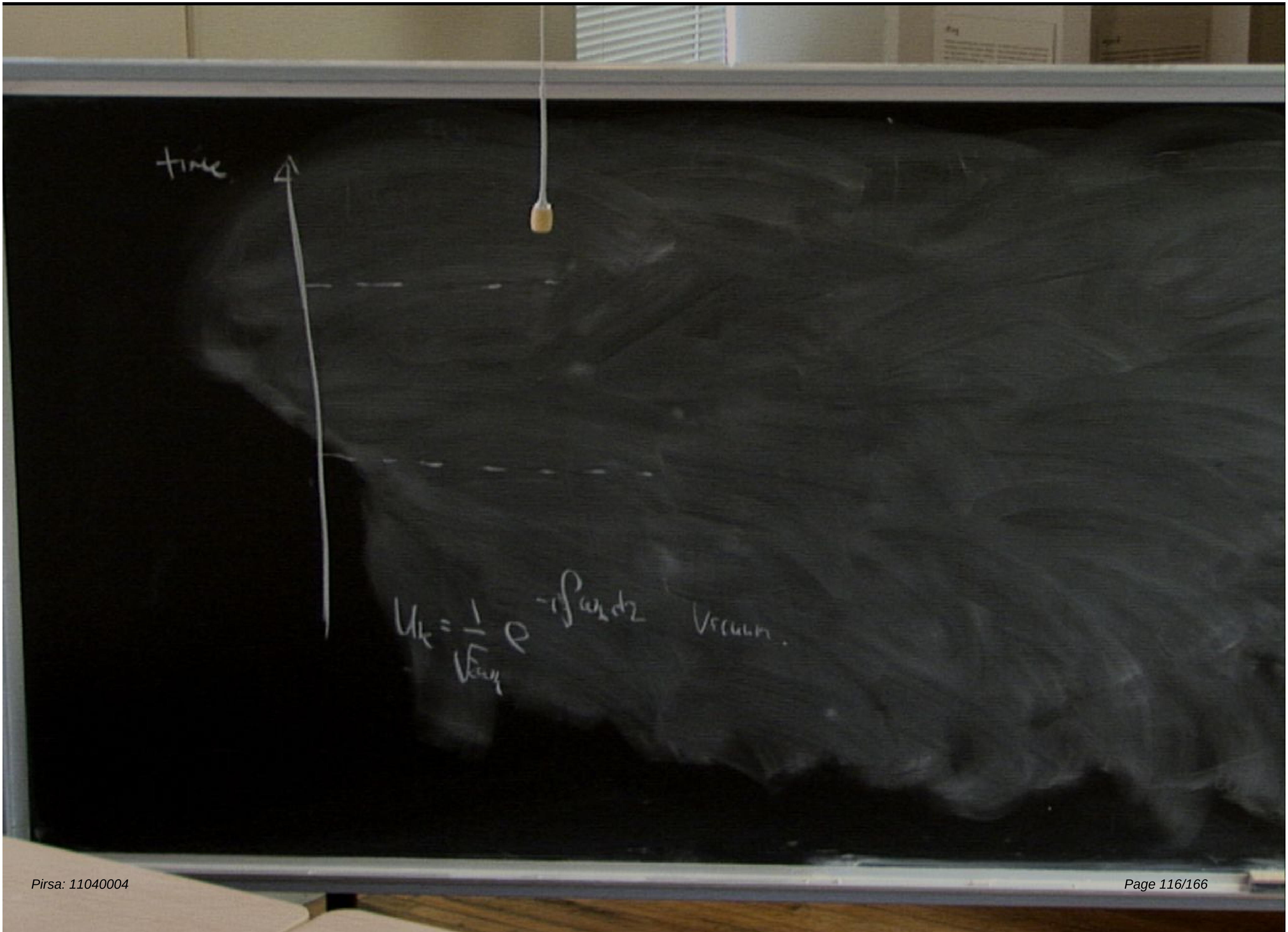
WKB approximation

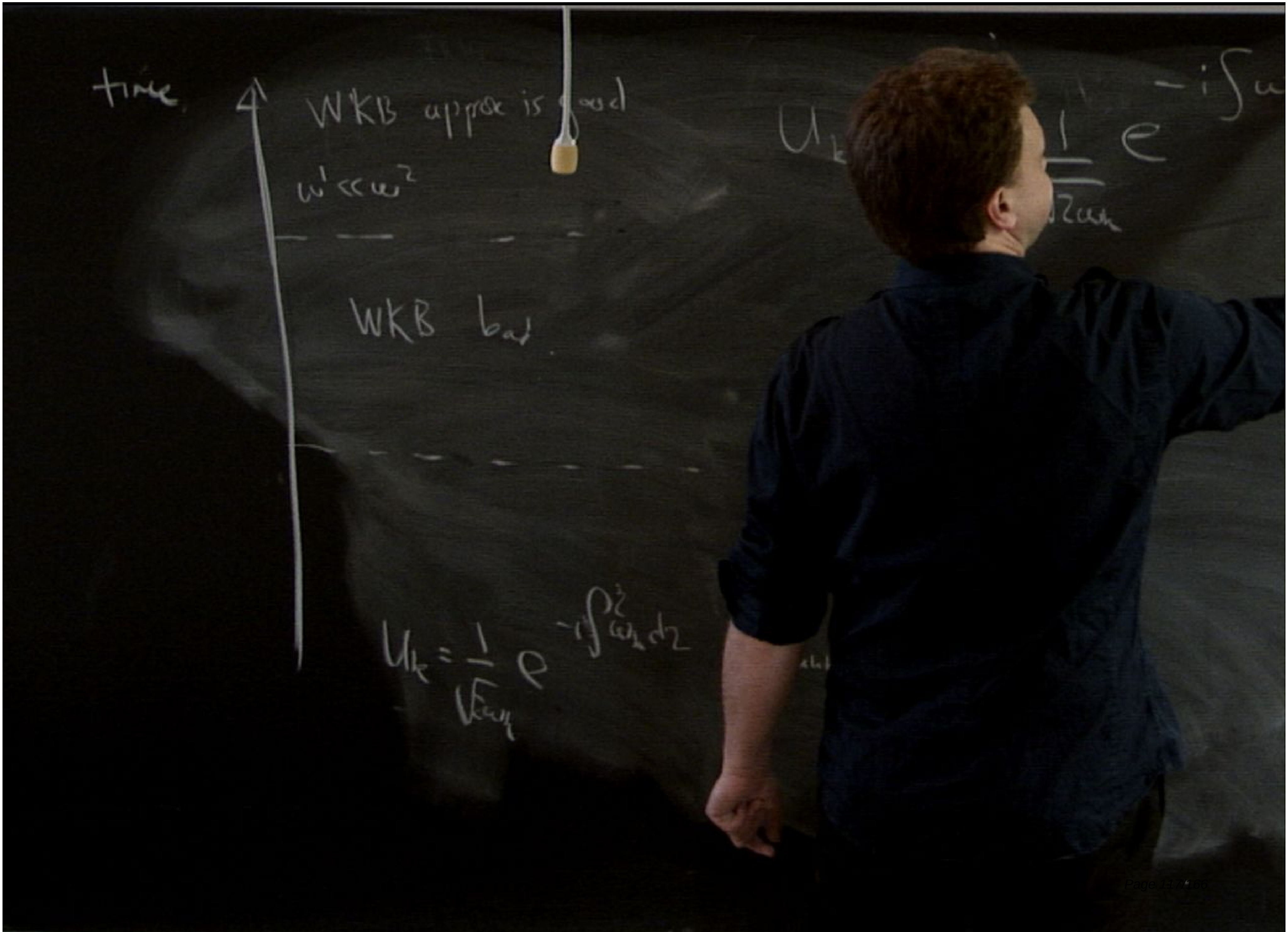
Wronskian / normalization

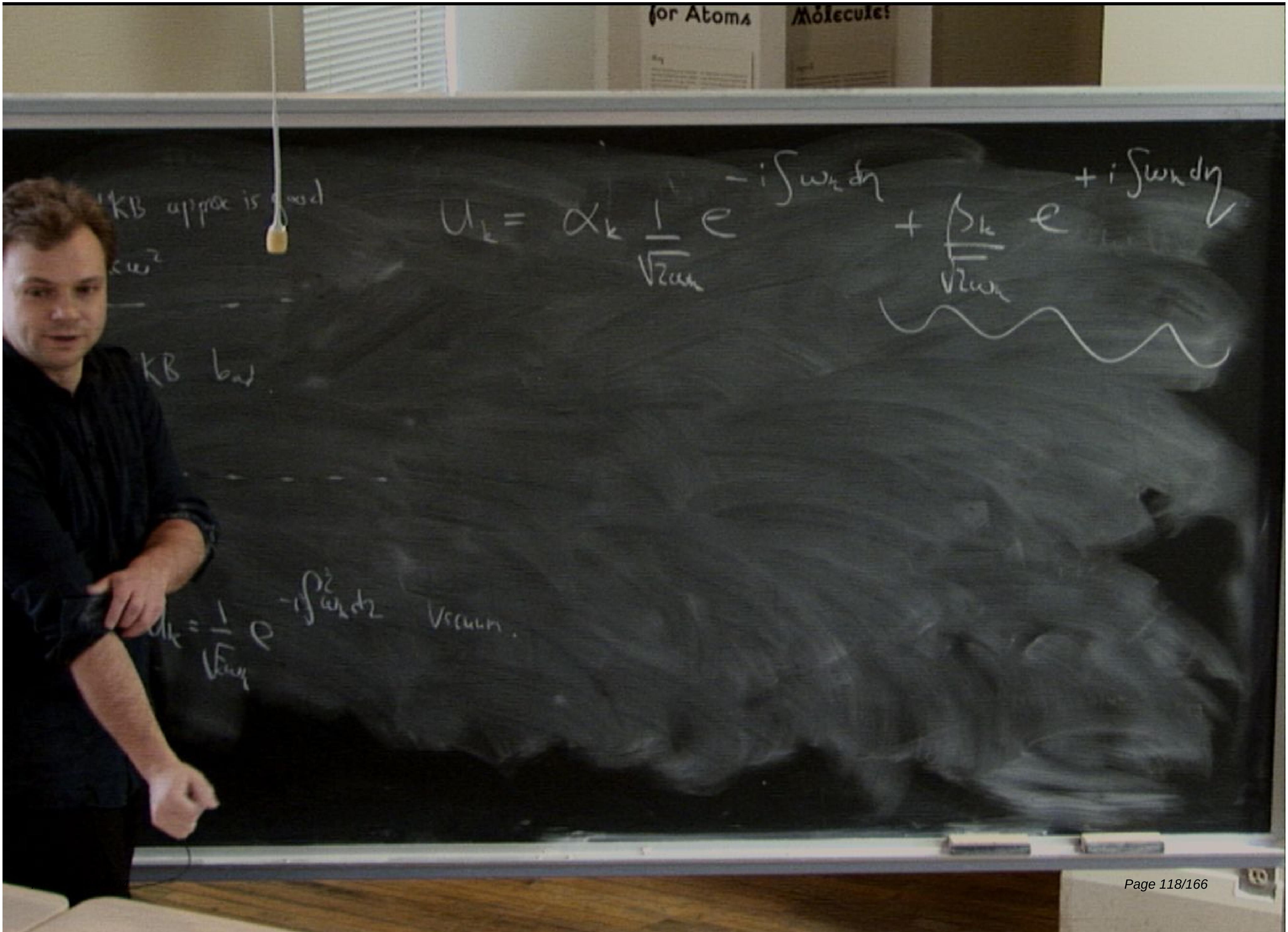
$$i\left[u_k^{*}\, u_k' - u_k^{*\prime}\, u_k\right] = 1$$

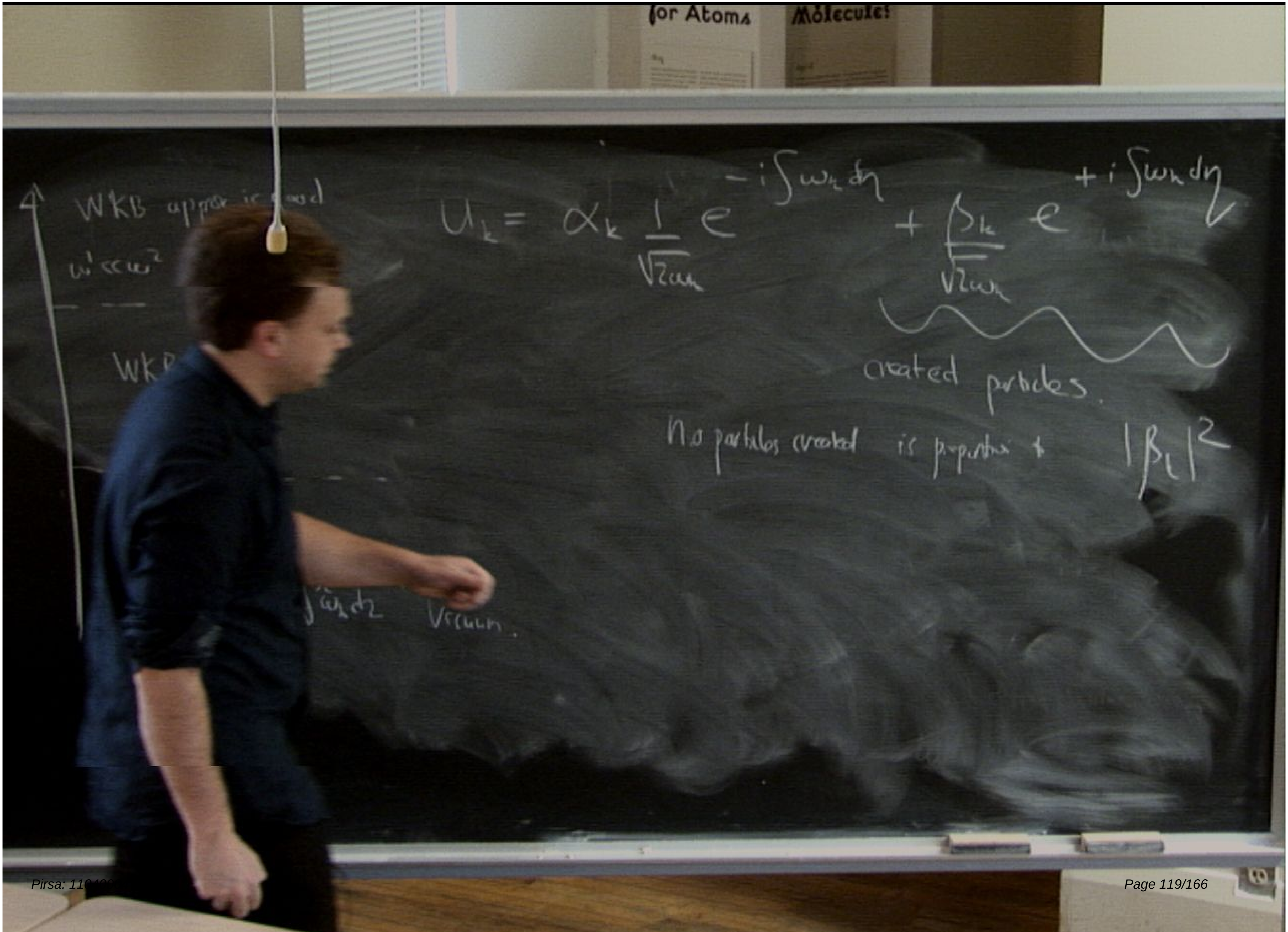
$$-i\, i\, |A|^2\, 2\omega_k = 1 \qquad |A| = \frac{1}{\sqrt{2\omega_k}}$$

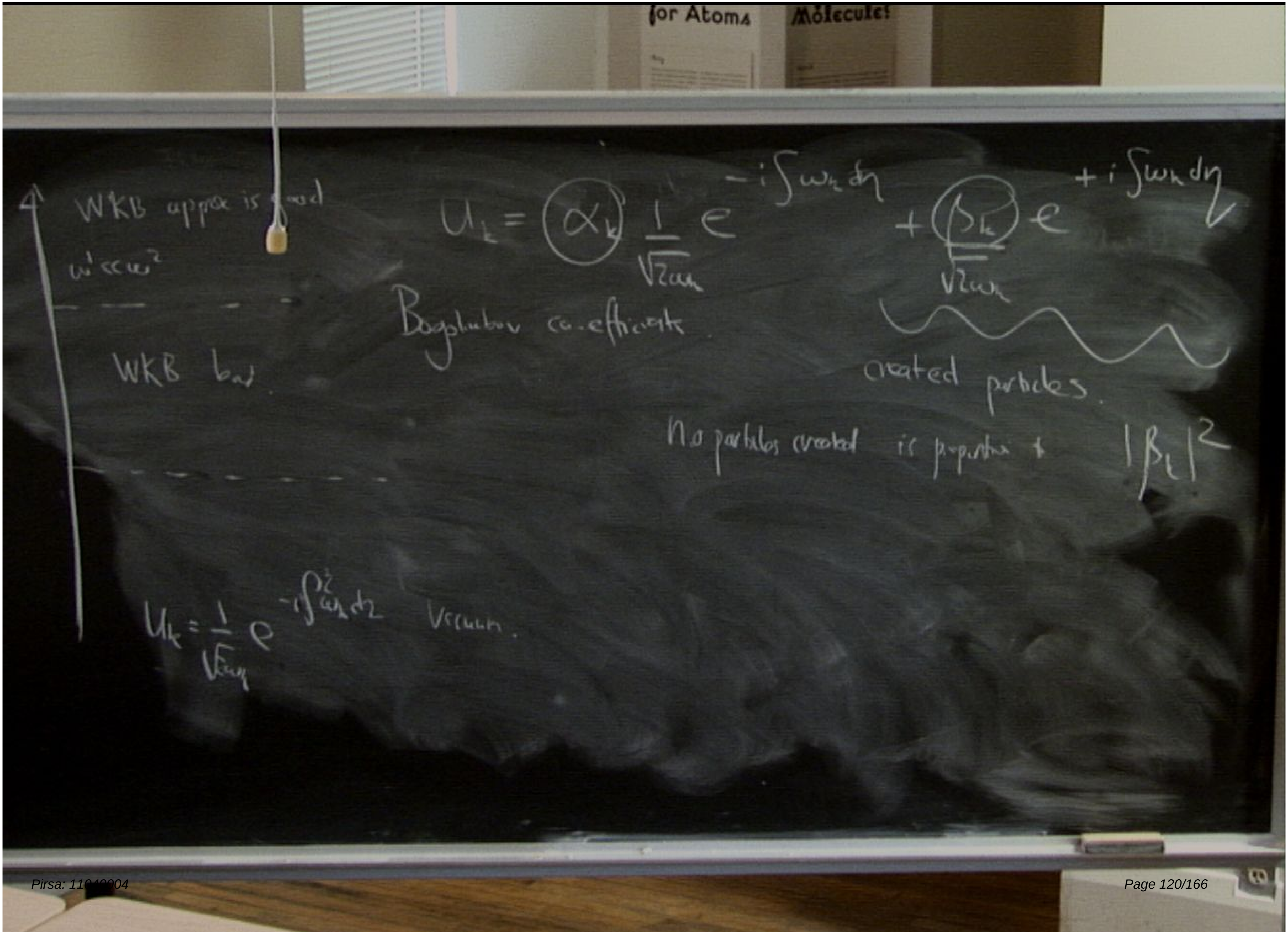

time

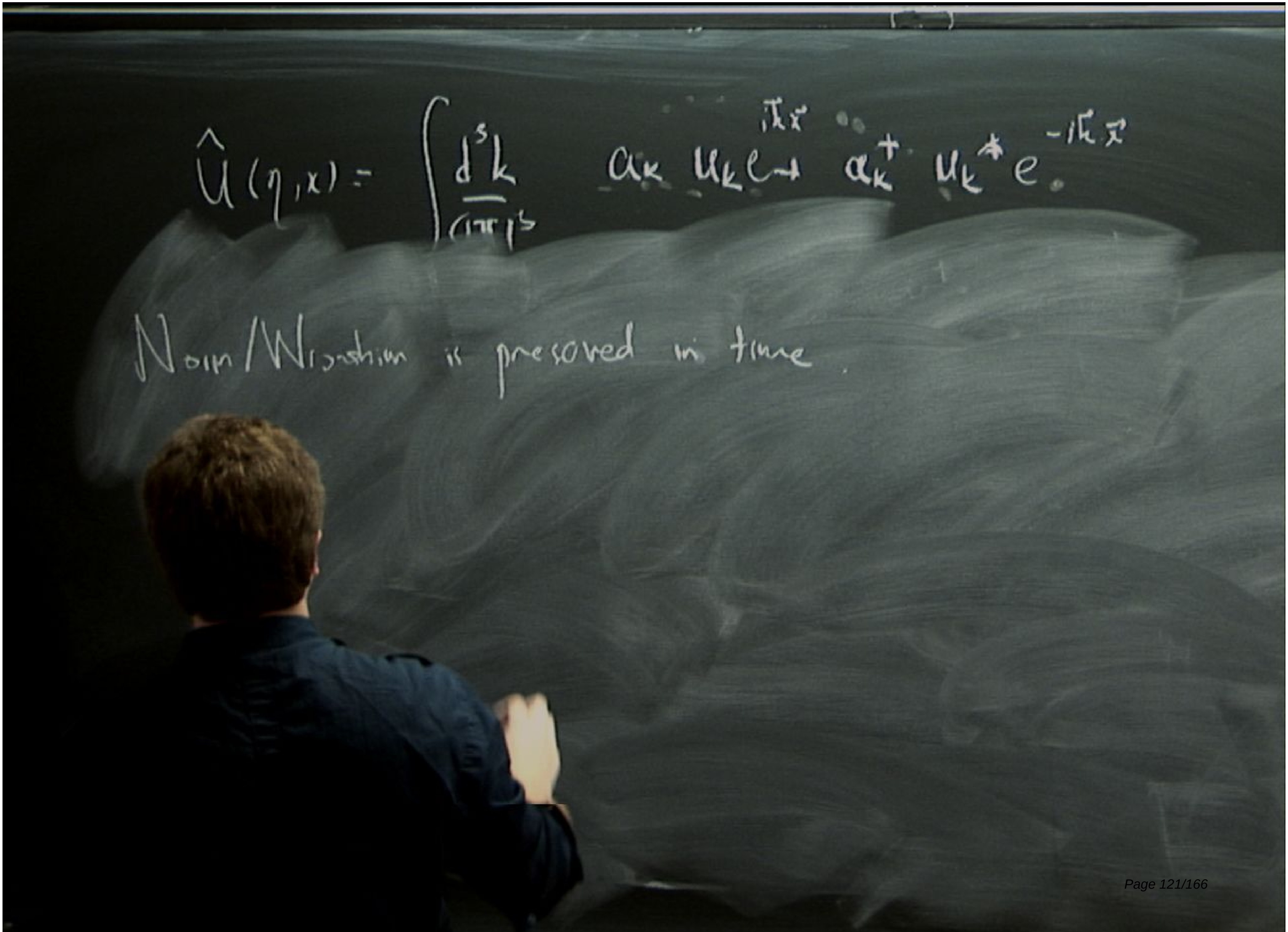

time
$u_k = \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\eta}$ vacuum.


time
WKB approx is good
$\omega' \ll \omega^2$
WKB bad
$u_k = \frac{1}{\sqrt{2\omega_k}} e^{-i\int^{\eta} \omega_k d\eta}$


for Atoms
Molecule!
KB approx is
$U_k = \alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\eta} + \frac{\beta_k}{\sqrt{2\omega_k}} e^{+i\int \omega_k d\eta}$
KB bad
$u_k = \frac{1}{\sqrt{2\omega_k}} e^{-i\int^{\eta} \omega_k d\eta}$ vacuum.


for Atoms
Molecules
WKB approx is good
$U_k = \alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\eta} + \frac{\beta_k}{\sqrt{2\omega_k}} e^{+i\int \omega_k d\eta}$
created particles
No particles created is proportional to $|\beta_k|^2$


for Atoms
Molecules
WKB approx is good
ω' << ω²
WKB bad
u_k = α_k 1/√(2ω_k) e^{-i∫ω_k dη} + β_k 1/√(2ω_k) e^{+i∫ω_k dη}
Bogoliubov co-efficients
created particles
No particles created is proportional to |β_k|²
u_k = 1/√(2ω_k) e^{-i∫^η ω_k dη} vacuum.


$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$
Norm/Wronskian is preserved in time.

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

Norm/Wronskian is preserved in time.

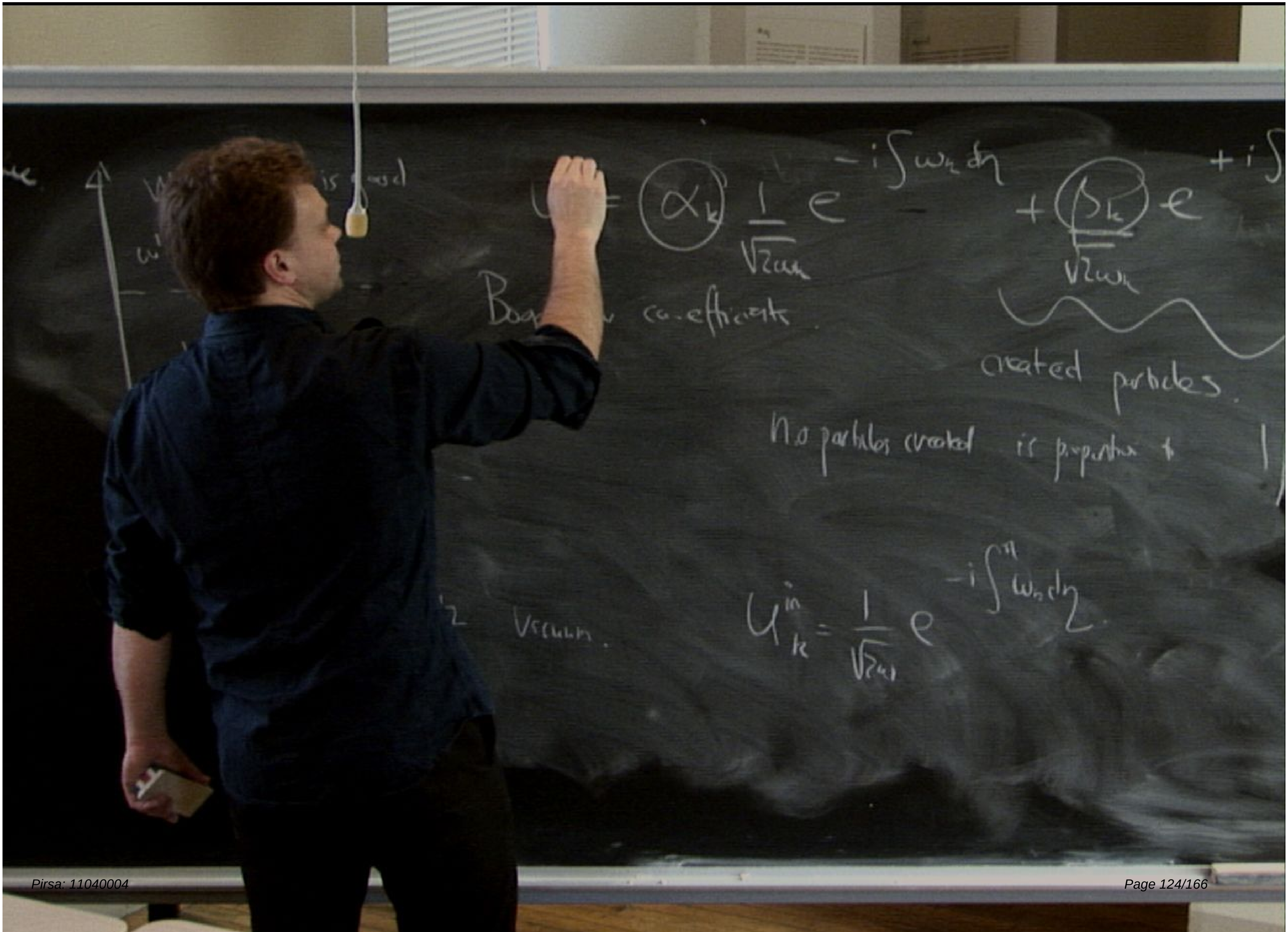
$$-i\left[u_k^* u_k' - u_k'^* u_k\right] = 1$$

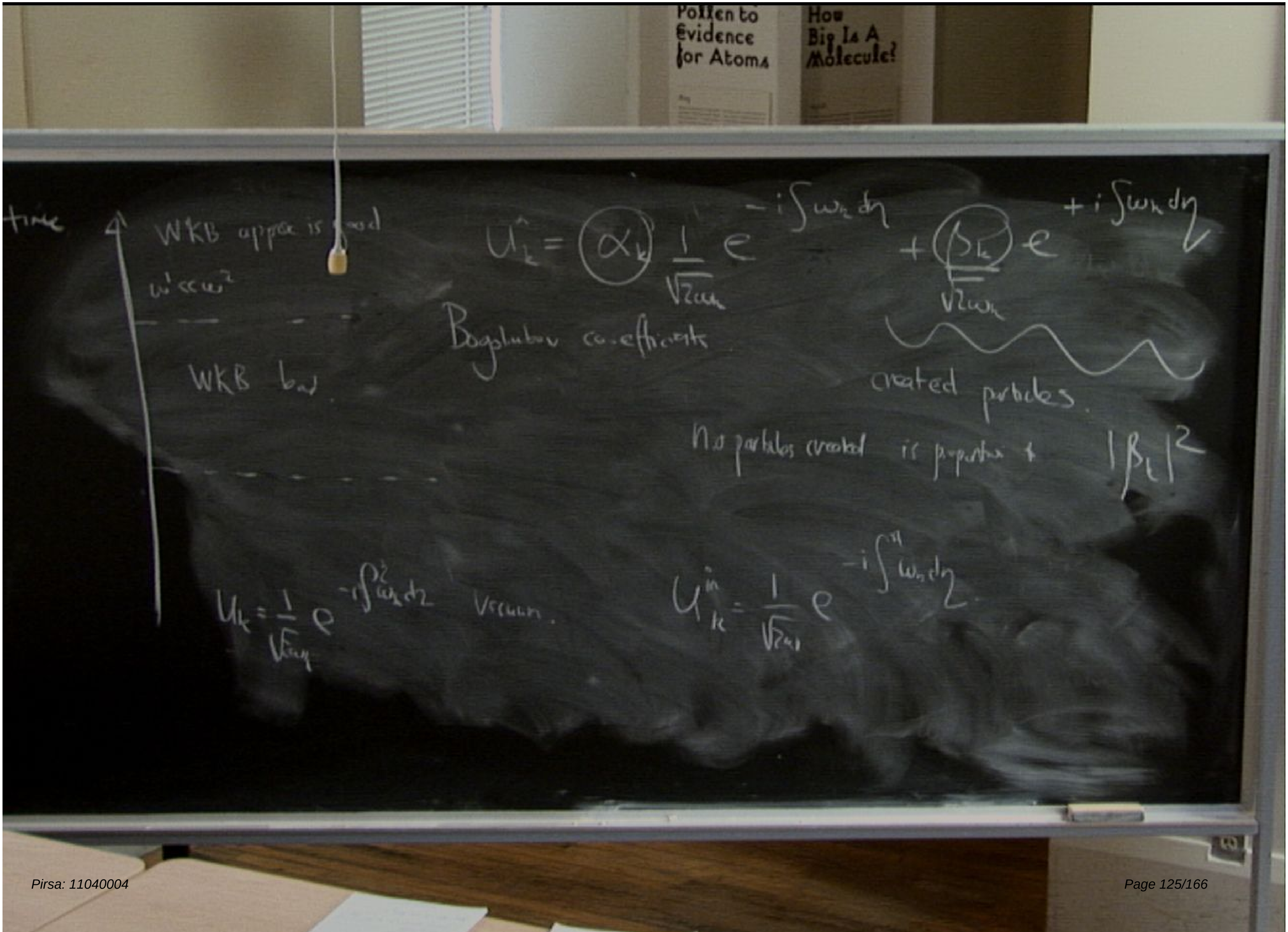
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$$

Norm/Wronskian is preserved in time.

$$-i\left[u_k^{*} u_k' - u_k'^{*} u_k\right] = 1$$

$$-i\left[\right.$$


created particles
vacuum


Pollen to Evidence for Atoms
How Big Is A Molecule?
time
WKB approx is good
Bogolubov co-efficients
WKB bad
created particles
No particles created is proportional to $|\beta_k|^2$
vacuum

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\vec{x}} + a_k^{+} u_k^{*} e^{-i\vec{k}\vec{x}}$$

Norm/Wronskian is preserved in time.

$$-i\left[u_k^{*} u_k' - u_k'^{*} u_k\right] = 1$$

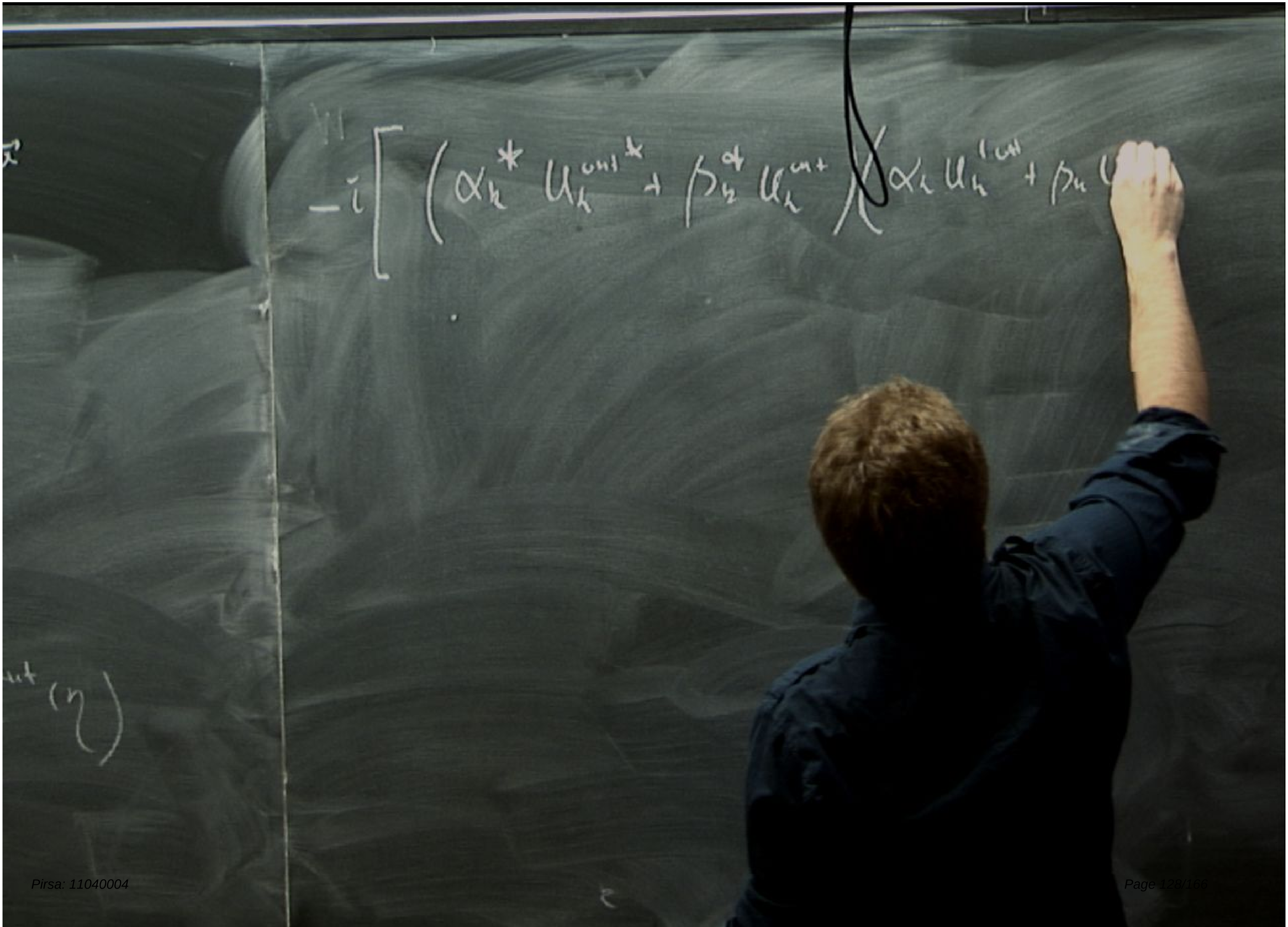
$$\lim_{\eta \to +\infty} u^{in}_k(\eta) = \alpha_k u^{out}_k(\eta) + \beta_k u_k^{out*}(\eta)$$

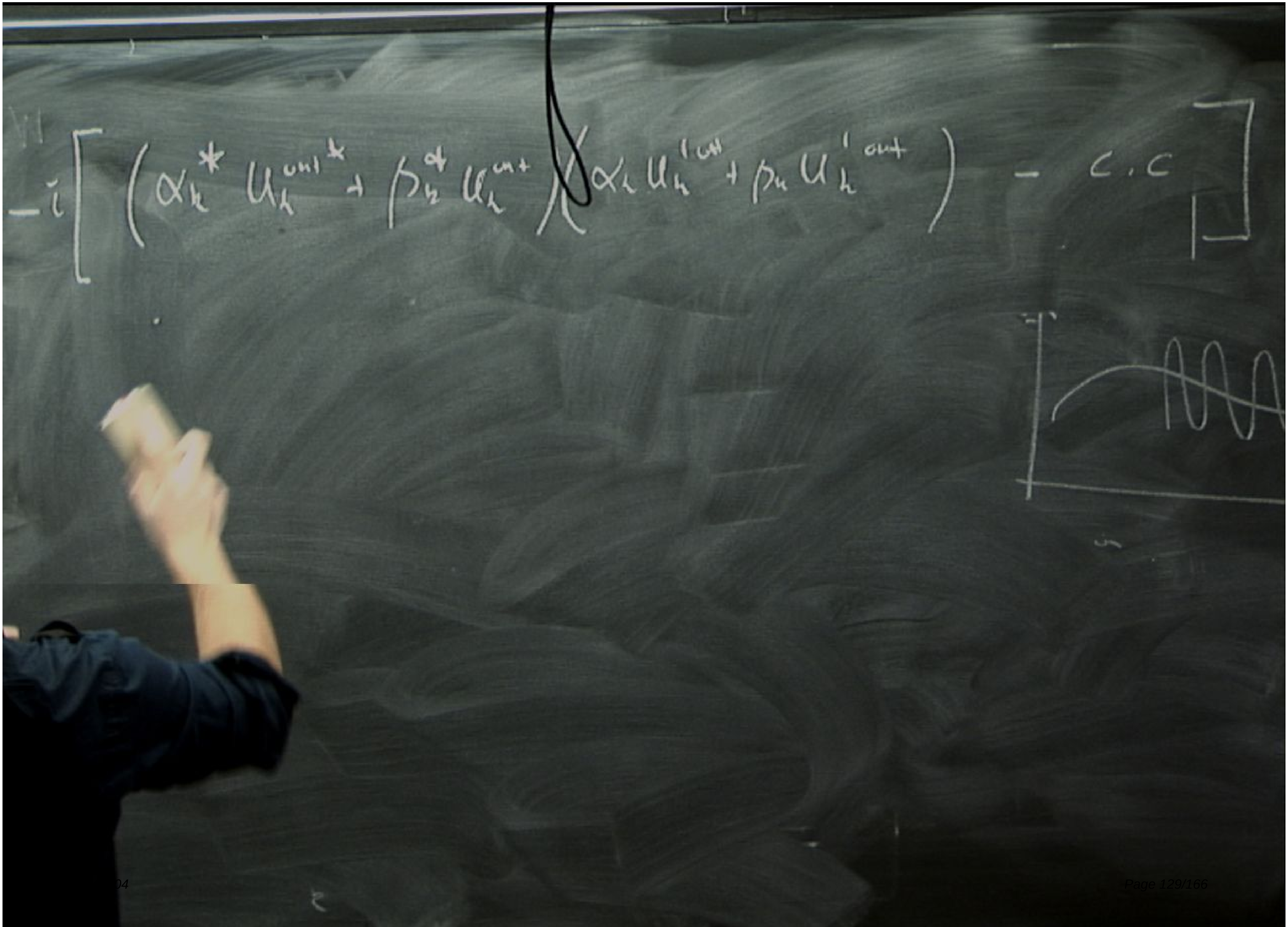
$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger u_k^* e^{-i\vec{k}\cdot\vec{x}}$$

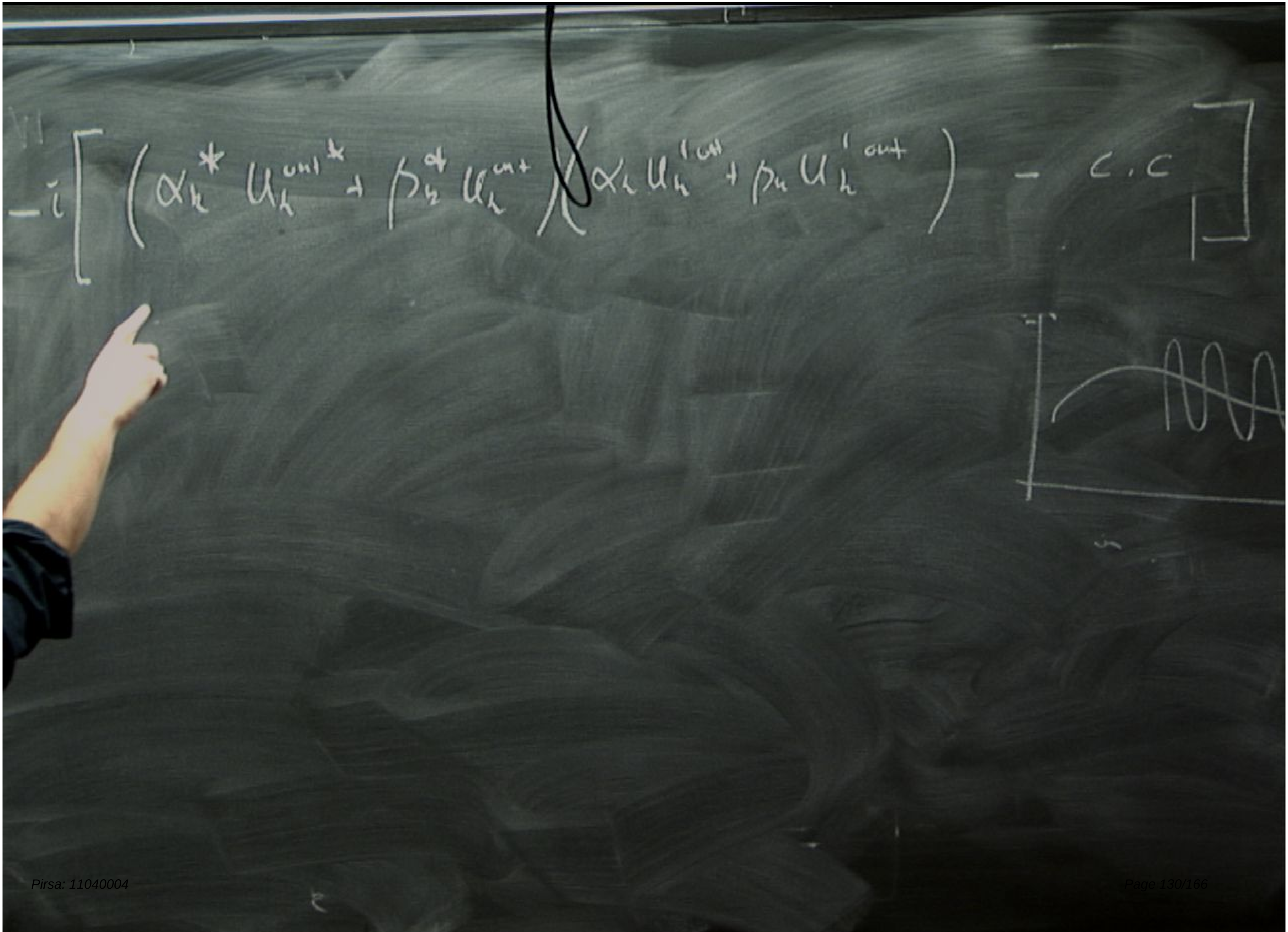
Norm/Wronskian is preserved in time.

$$-i\left[u_k^* u_k' - u_k'^* u_k\right] = 1$$

$$\lim_{\eta \to +\infty} u^{in}_k(\eta) = \alpha_k u^{out}_k(\eta) + \beta_k u^{*out}_k(\eta)$$

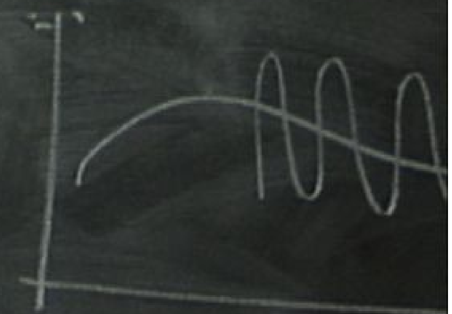



−i [(α_k* u_k^out* + β_k* u_k^out*)(α_k u_k'^out + β_k u_k'^out) − c.c]


$-i\left[\left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out}\right)\left(\alpha_k u_k'^{out} + \beta_k u_k'^{out}\right) - c.c\right]$

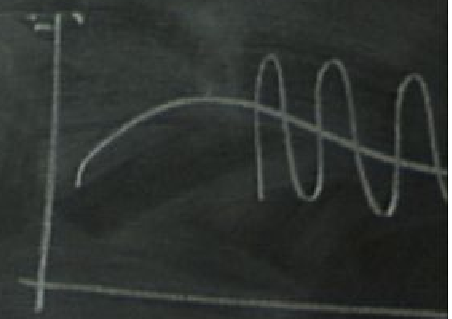
$$-i\left[\left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out}\right)\left(\alpha_k u_k'^{out} + \beta_k u_k'^{out}\right) - c.c\right]$$

$$-i\left[\,|\alpha_k|^2 \, u_k^{out*} u_k'^{out}\right.$$



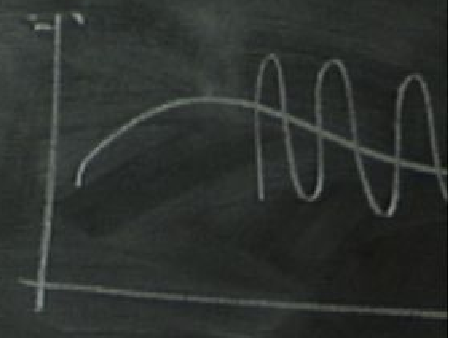
$$-i\left[\left(\alpha_k^{*} u_k^{out\,*} + \beta_k^{*} u_k^{out}\right)\left(\alpha_k u_k'^{\,out} + \beta_k u_k^{*\prime\, out}\right) - c.c\right]$$

$$-i\left[\,|\alpha_k|^2\, u_k^{out\,*}\, u_k'^{\,out}\right.$$



$$-i\left[\left(\alpha_k^{*} u_k^{out\,*} + \beta_k^{*} u_k^{out}\right)\left(\alpha_k u_k'^{\,out} + \beta_k u_k^{*'\,out}\right) - c.c.\right]$$

$$-i\left[\left(|\alpha_k|^2 - |\beta_k|^2\right) u_k^{out\,*}\, u_k'^{\,out}\right]$$



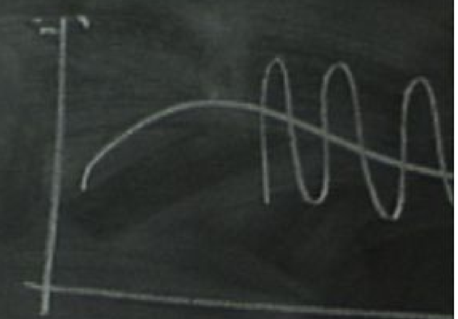
$$-i\left[\left(\alpha_k^* u_k^{out\,*} + \beta_k^* u_k^{out}\right)\left(\alpha_k u_k'^{\,out} + \beta_k u_k^{*\prime\,out}\right) - c.c\right]$$

$$-i\left[\left(|\alpha_k|^2 - |\beta_k|^2\right) u_k^{out\,*}\, u_k'^{\,out} - c.c\right]$$

$$-i\left[\left(\alpha_k^{*} u_k^{out\,*} + \beta_k^{*} u_k^{out}\right)\left(\alpha_k u_k'^{\,out} + \beta_k u_k^{*\prime\,out}\right) - c.c\right]$$

$$-i\left[\begin{pmatrix}|\alpha_k|^2 \\ -|\beta_k|^2\end{pmatrix} u_k^{out\,*}\, u_k'^{\,out} - c.c\right]$$

$$= 1$$



$$-i\left[\left(\alpha_k^{*} u_k^{out\,*} + \beta_k^{*} u_k^{out}\right)\left(\alpha_k u_k'^{\,out} + \beta_k u_k^{*'\,out}\right) - c.c\right]$$

$$-i\left[\left(|\alpha_k|^2 - |\beta_k|^2\right) u_k^{out\,*}\, u_k'^{\,out} - c.c\right]$$

$$= 1$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

$$-i\left[\left(\alpha_k^* u_k^{out\,*} + \beta_k^* u_k^{out}\right)\left(\alpha_k u_k^{\prime\, out} + \beta_k u_k^{*\prime\, out}\right) - c.c\right]$$

$$-i\left[\left(|\alpha_k|^2 - |\beta_k|^2\right) u_k^{out\,*} u_k^{\prime\, out} - c.c\right]$$

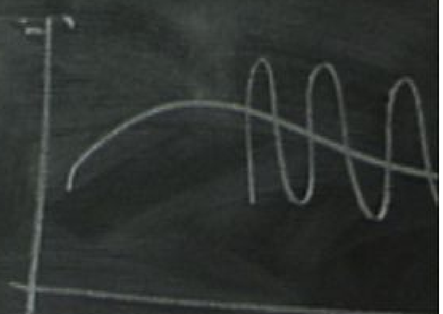
$$= 1$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

Unitarity / conservation of prob.

$$-i\left[\left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out}\right)\left(\alpha_k u_k'^{out} + \beta_k u_k^{*'out}\right) - c.c\right]$$

$$-i\left[\left(|\alpha_k|^2 - |\beta_k|^2\right) u_k^{out*} u_k'^{out} - c.c\right]$$

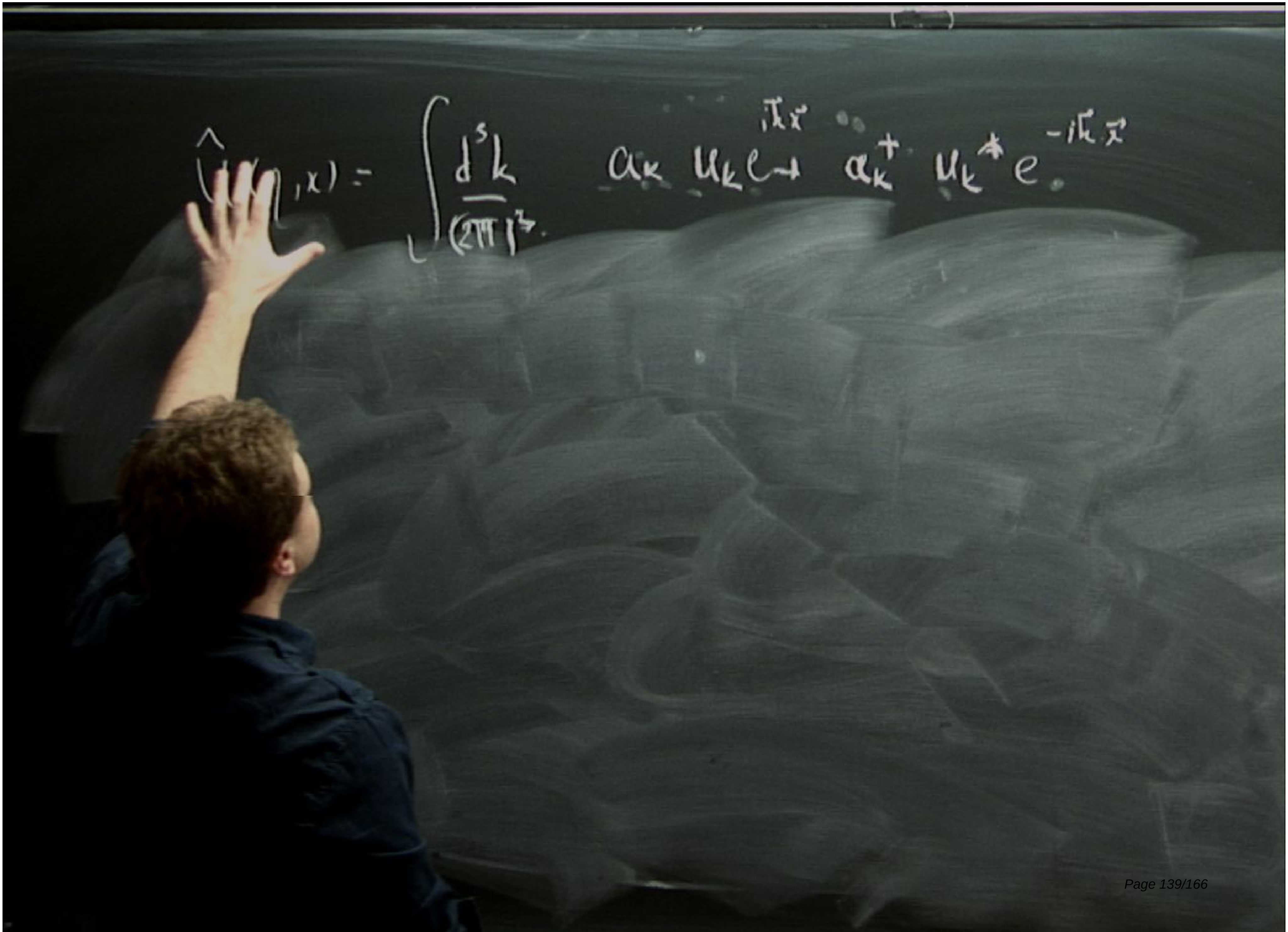


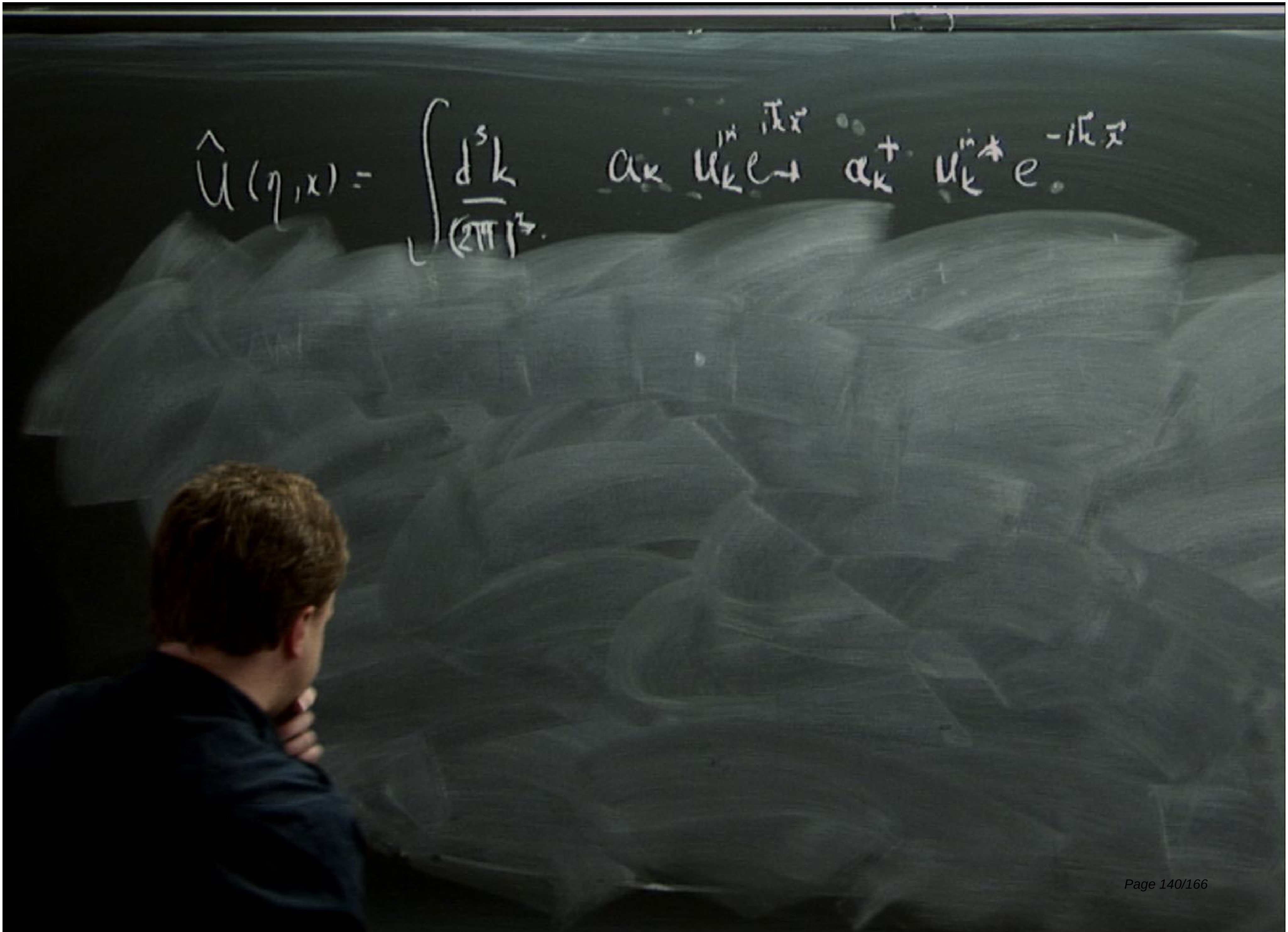
$$= 1$$

$$\boxed{|\alpha_k|^2 - |\beta_k|^2 = 1}$$

Unitarity / conservation of prob

α, β } free constants function of k


$\hat{\phi}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{*} e^{-i\vec{k}\cdot\vec{x}}$


$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{in*} e^{-i\vec{k}\cdot\vec{x}}$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{+} u_k^{in*} e^{-i\vec{k}\cdot\vec{x}}$$

$a_k |in\rangle = 0$. in-vacuum .

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{in *} e^{-i\vec{k}\cdot\vec{x}}$$

$$a_k |in\rangle = 0 \quad . \qquad \text{in-vacuum} .$$

$$\lim_{\eta \to -\infty} u_k^{in}(\eta) =$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{in*} e^{-i\vec{k}\cdot\vec{x}}$$

$$a_k |in\rangle = 0 \qquad \text{in-vacuum}$$

$$\lim_{\eta \to -\infty} u_k^{in}(\eta) = \alpha_k u_k^{out}(\eta) + \beta_k u_k^{out*}(\eta)$$

$^{*} e^{-i\vec{k}\cdot\vec{x}} =$ $\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[a_k u_k^{in} + a_{-k}^{\dagger} u_k^{in*} \right]$

n .

$\beta_k u_k^{out*}(\eta))$

Unitarity / conservation

constants function of k

$$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[a_k u_k^{in} + a_{-k}^\dagger u_k^{in*} \right]$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} \left[a_k \left(\alpha_k u_k^{out} + \beta_k u_k^{out*} \right) + a^\dagger_{\cdot} \left(\right. \right.$$

Unitarity

constants function of k

Pirsa: 11040004

$$\int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \left[a_k u_k^{in} + a_{-k}^\dagger u_k^{in*} \right]$$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{ik\cdot x} \left[a_k \left(\alpha_k u_k^{out} + \beta_k u_k^{out*} \right) + a_{-k}^\dagger \left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out} \right) \right]$$

Unitarity

constants, function of k

$$\int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \left[a_k u_k^{in} + a_{-k}^\dagger u_k^{in*} \right]$$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{ik\cdot x} \left[a_k \left(\alpha_k u_k^{out} + \beta_k u_k^{out*} \right) + a_{-k}^\dagger \left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out} \right) \right]$$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{ik\cdot x} \left[b_k u_k^{out} + b_{-k}^\dagger u_k^{out*} \right]$$

$$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[a_k u_k^{in} + a_{-k}^\dagger u_k^{in*} \right]$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} \left[a_k \left(\alpha_k u_k^{out} + \beta_k u_k^{out*} \right) + a_{-k}^\dagger \left(\alpha_k^* u_k^{out*} + \beta_k^* u_k^{out} \right) \right]$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} \left[b_k u_k^{out} + b_{-k}^\dagger u_k^{out*} \right]$$

$$\boxed{b_k = \alpha_k a_k + \beta_k^* a_{-k}^\dagger}$$

$$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{in*} e^{-i\vec{k}\cdot\vec{x}} \quad =$$

$$a_k |in\rangle = 0 \quad . \qquad \text{in-vacuum}$$

$$\lim_{\eta \to -\infty} u_k^{in}(\eta) = \alpha_k u_k^{out}(\eta) + \beta_k u_k^{out*}(\eta)$$

$$b_k |out\rangle = 0 \qquad b_k, b_k^{\dagger}$$

annihilation + creation operators

def particles $b^{\dagger}$ + ...

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{in\,*} e^{-i\vec{k}\cdot\vec{x}} \quad =$$

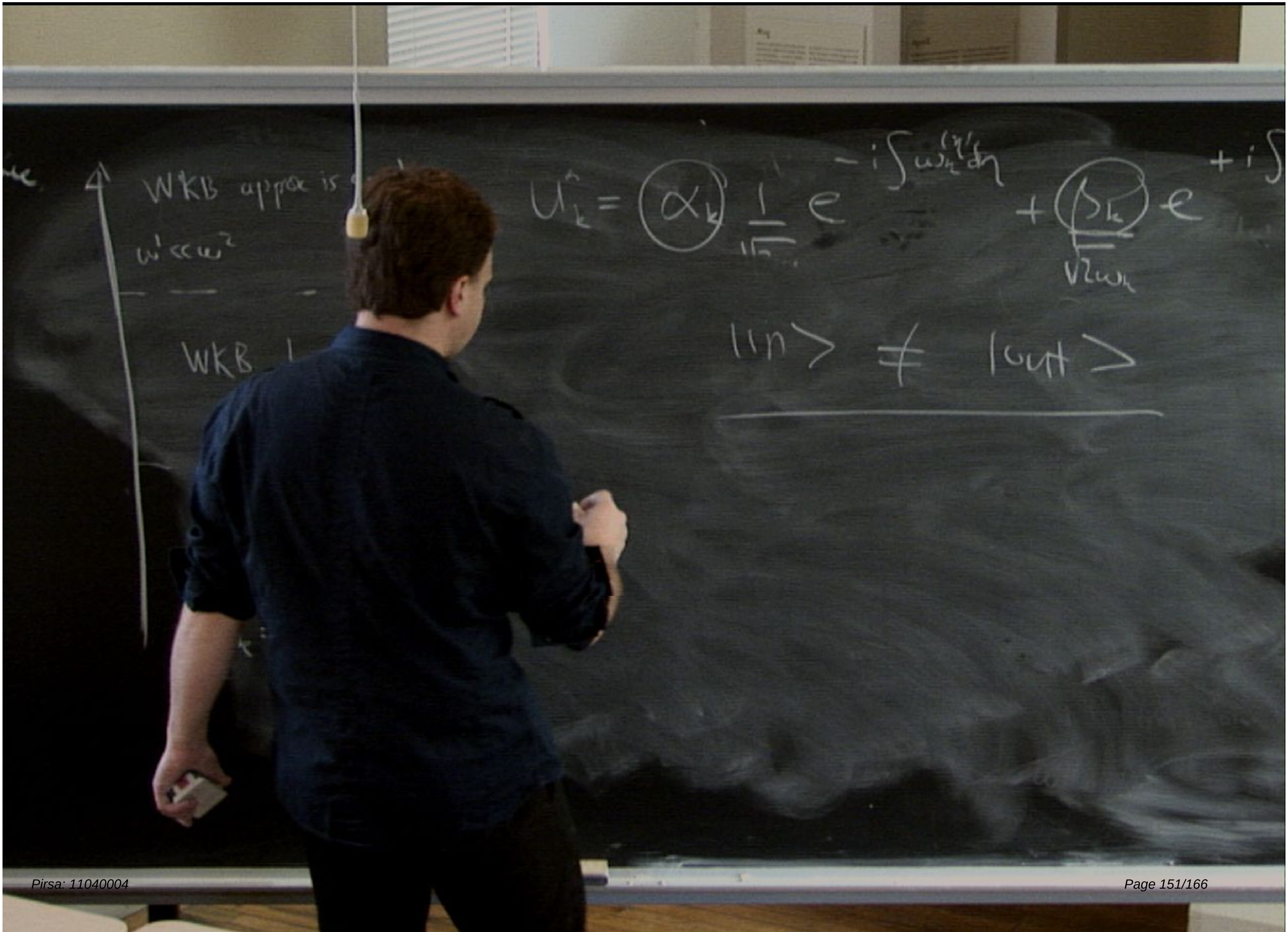
$$a_k |in\rangle = 0 \qquad \text{in-vacuum.}$$

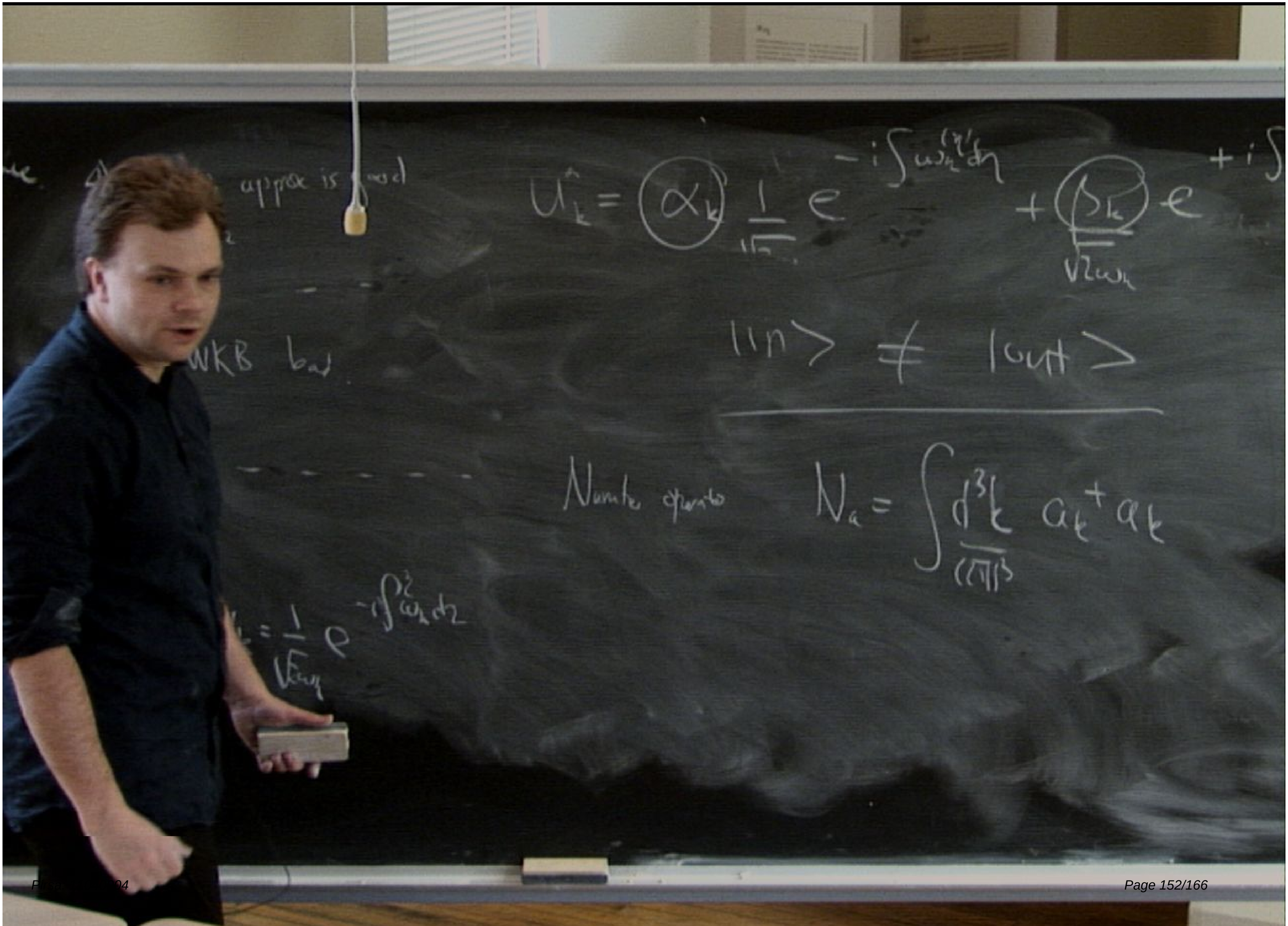
$$\lim_{\eta \to \infty} u_k^{in}(\eta) = \alpha_k u_k^{out}(\eta) + \beta_k u_k^{out\,*}(\eta)$$

$$b_k |out\rangle = 0 \qquad b_k, \, b_k^{\dagger}$$

annihilation + creation operators

def particle ...


WKB approx is
WKB
|in> ≠ |out>


approx is good
WKB bad
$\hat{u}_k = \alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\eta} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{+i\int}$
$|in\rangle \neq |out\rangle$
Number operator
$N_a = \int \frac{d^3k}{(2\pi)^3} a_k^\dagger a_k$

$$u_k^{in} = \alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k(\eta')\,d\eta'} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{+i\int \omega_k\,d\eta}$$

$$|in\rangle \neq |out\rangle$$

Number operator $$N_a = \int \frac{d^3k}{(2\pi)^3}\, a_k^\dagger a_k$$

$$[a_k, N_a] = a_k \qquad N_b = \int \frac{d^3k}{(2\pi)^3}\, b_k^\dagger b_k$$

is good

$$u_k = \alpha_k \frac{1}{\sqrt{2\omega_k}} e^{-i\int^{\eta} \omega_k d\eta} + \beta_k \frac{1}{\sqrt{2\omega_k}} e^{+i\int \omega_k d\eta}$$

$$|in\rangle \neq |out\rangle$$

Number operator $N_a = \int \frac{d^3k}{(2\pi)^3} a_k^+ a_k$

$$[a_k, N_a] = a_k \qquad N_b = \int \frac{d^3k}{(2\pi)^3} b_k^+ b_k$$

$e^{-i\int^{2}\omega_k d\eta}$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} \, u_k^{in\,*} e^{-i\vec{k}\cdot\vec{x}} \quad =$$

N_b

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{in*} e^{-i\vec{k}\cdot\vec{x}} \qquad =$$

$$\langle in | N_k \; | in \rangle$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\vec{x}} + a_k^\dagger u_k^{in*} e^{-i\vec{k}\vec{x}} =$$

$$\langle in | N_b | in \rangle$$

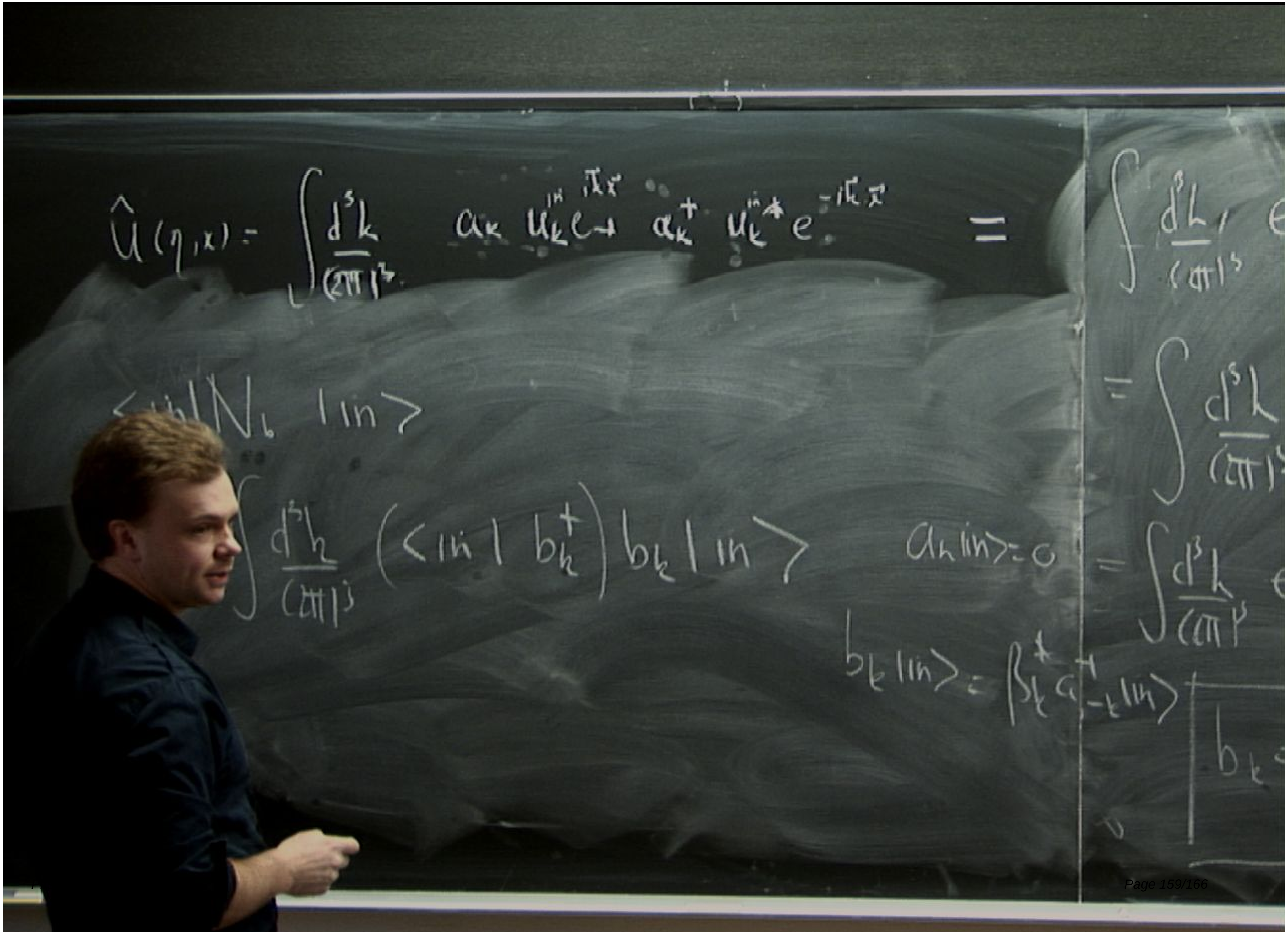
$$= \int \frac{d^3k}{(2\pi)^3} \langle in | b_k^\dagger \, b_k | in \rangle$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{\dagger} u_k^{in*} e^{-i\vec{k}\cdot\vec{x}} =$$

$$\langle in | N_b | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \langle in | b_k^{\dagger} b_k | in \rangle \qquad a_k |in\rangle = 0$$

$$b_k |in\rangle =$$


$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\vec{x}} + a_k^{\dagger} u_k^{in*} e^{-i\vec{k}\vec{x}} =$
<in|N_k |in>
$\int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^{\dagger} \right) b_k | in \rangle$
a_k|in> = 0
$b_k |in\rangle = \beta_k^* a^{\dagger}_{-k} |in\rangle$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger \, u_k^{in*} e^{-i\vec{k}\cdot\vec{x}} \quad =$$

$$\langle in | N_b \; | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | \, b_k^\dagger \right) b_k \, | in \rangle \qquad a_k |in\rangle = 0$$

$$b_k |in\rangle = \beta_k^* \, a_{-k}^\dagger |in\rangle$$

$$\langle in | b_k^\dagger = \langle in | a_{-k} \, \beta_k$$

$$\int \frac{d^3k}{(2\pi)^3}$$

$$= \int \frac{d^3k}{(2\pi)^3}$$

$$= \int \frac{d^3k}{(2\pi)^3}$$

$$b_k$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\vec{x}} + a_k^{+} u_k^{in*} e^{-i\vec{k}\vec{x}} \quad =$$

$$\langle in | N_k | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^{+} \right) b_k | in \rangle \qquad a_k | in \rangle = 0$$

$$b_k | in \rangle = \beta_k^{*} a_{-k}^{+} | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \; \langle in | a_{-k} a_{-k}^{+} | in \rangle \qquad \langle in | b_k^{+} = \langle in | a_{-k} \beta_k$$

$$\int \frac{d^3k}{(2\pi)^3} \; e$$

$$= \int \frac{d^3k}{(2\pi)^3}$$

$$= \int \frac{d^3k}{(2\pi)^3} \; e$$

$$b_k =$$

$$\hat{u}(\eta,x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\vec{x}} + a_k^\dagger u_k^{in*} e^{-i\vec{k}\vec{x}} =$$

$$\int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$\langle in | N_b | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^\dagger \right) b_k | in \rangle \qquad a_k |in\rangle = 0$$

$$b_k |in\rangle = \beta_k^* a_{-k}^\dagger |in\rangle$$

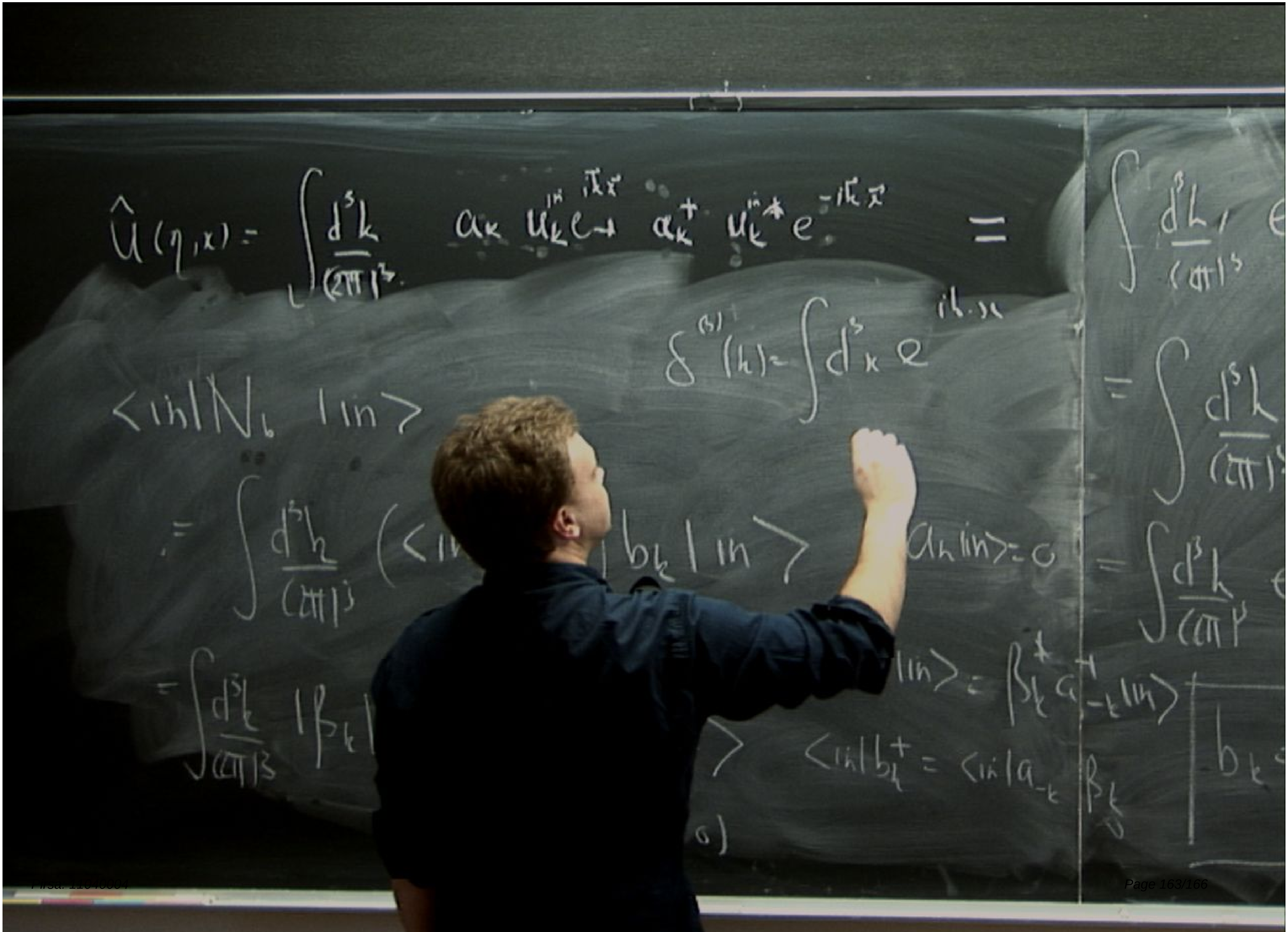
$$= \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \langle in | a_{-k} a_{-k}^\dagger | in \rangle \qquad \langle in | b_k^\dagger = \langle in | a_{-k} \beta_k$$

$$(2\pi)^3 \delta^{(3)}(0)$$

$$= \int \frac{d^3k}{(2\pi)^3}$$

$$= \int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$b_k \ldots$$



û(η,x) = ∫ d³k/(2π)³ a_k u_k^in e^{ik·x} + a_k^† u_k^{in*} e^{-ik·x} =
δ^(3)(k) = ∫ d³x e^{ik·x}
<in|N_k |in>
= ∫ d³k/(2π)³ (<in
b_k | in>
a_k|in> = 0
|in> = β_k^* a^†_{-k}|in>
= ∫ d³k/(2π)³ |β_k|
<in|b_k^† = <in|a_{-k} β_k

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k u_k^{in} e^{i\vec{k}\vec{x}} + a_k^\dagger u_k^{in*} e^{-i\vec{k}\vec{x}} =$$

$$(2\pi)^3 \delta^{(3)}(k) = \int d^3x = V$$

$$\langle in | N_b | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^\dagger\right) b_k | in \rangle \qquad a_k |in\rangle = 0$$

$$b_k |in\rangle = \beta_k^* a_{-k}^\dagger |in\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \langle in | a_{-k} a_{-k}^\dagger | in \rangle \qquad \langle in | b_k^\dagger = \langle in | a_{-k} \beta_k$$

$$(2\pi)^3 \delta^{(3)}(0)$$

$$\int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$= \int \frac{d^3k}{(2\pi)^3}$$

$$= \int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$b_k \ldots$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\cdot\vec{x}} + a_k^{+} \, u_k^{in*} e^{-i\vec{k}\cdot\vec{x}} =$$

$$(2\pi)^3 \delta^{(3)}(k) = \int d^3x = V$$

$$\langle in | N_k | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^{+} \right) b_k | in \rangle \qquad a_k | in \rangle = 0$$

$$b_k | in \rangle = \beta_k^{*} a_{-k}^{+} | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \; V$$

$$\langle in | b_k^{+} = \langle in | a_{-k} \, \beta_k$$

$$= \int \frac{d^3k}{(2\pi)^3} \qquad = \int \frac{d^3k}{(2\pi)^3} \qquad = \int \frac{d^3k}{(2\pi)^3}$$

$$b_k$$

$$\hat{u}(\eta, x) = \int \frac{d^3k}{(2\pi)^3} \; a_k \, u_k^{in} e^{i\vec{k}\vec{x}} + a_k^{+} \, u_k^{in *} e^{-i\vec{k}\vec{x}} =$$

$$(2\pi)^3 \delta^{(3)}(k) = \int d^3x = V$$

$$\langle in | N_k | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} \left(\langle in | b_k^{+} \right) b_k | in \rangle \qquad a_k | in \rangle = 0$$

$$b_k | in \rangle = \beta_k^{*} a_{-k}^{+} | in \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \; V \qquad \langle in | b_k^{+} = \langle in | a_{-k} \, \beta_k$$

$$\int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$= \int \frac{d^3k}{(2\pi)^3} \ldots$$

$$= \int \frac{d^3k}{(2\pi)^3} \; e \ldots$$

$$b_k = \ldots$$