

Title: Explorations in Cosmology - Lecture 1

Date: Apr 04, 2011 09:00 AM

URL: <http://pirsa.org/11040003>

Abstract:

Pollen to  
Evidence  
for Atoms

How  
Big Is A  
Molecule?

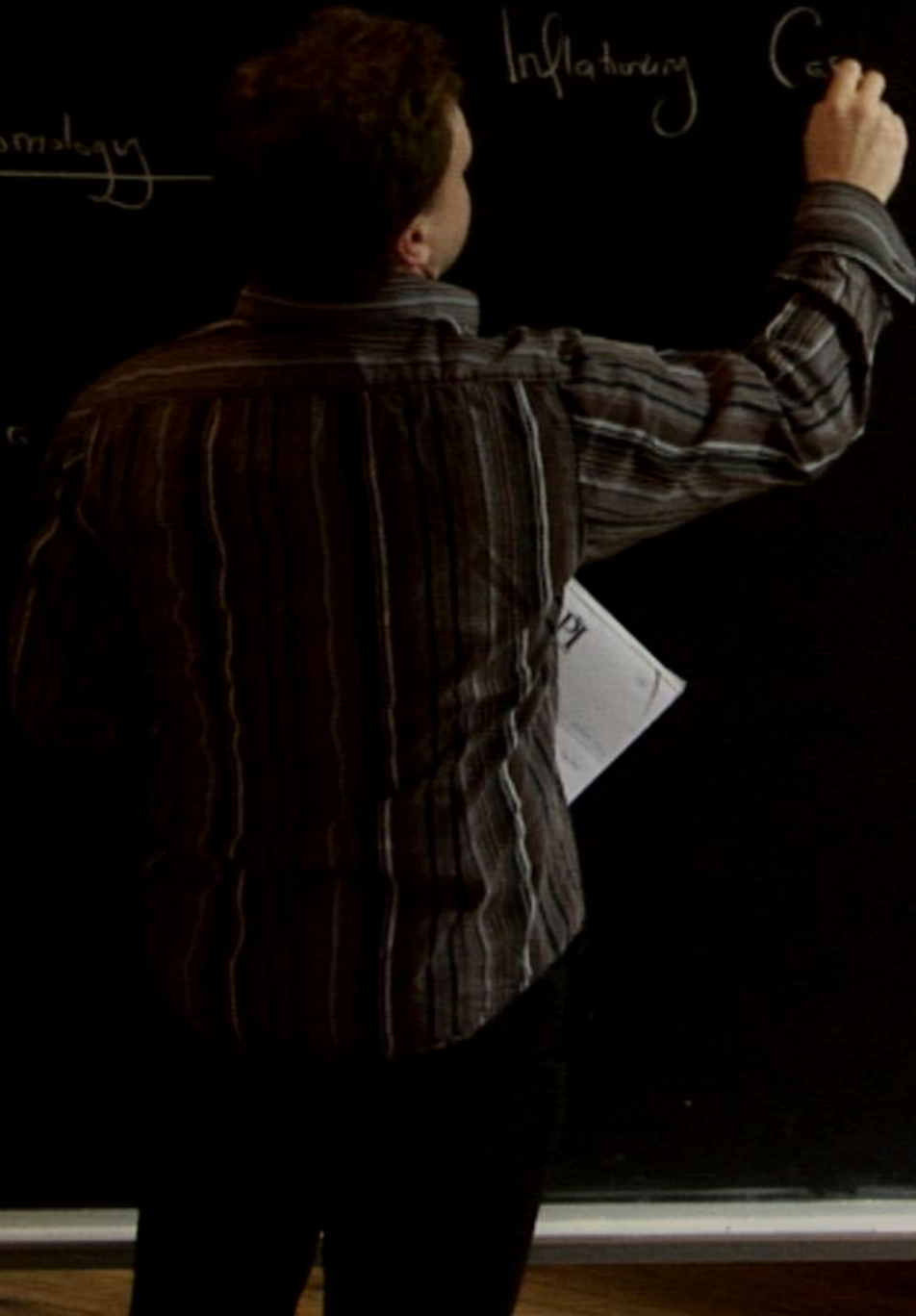
# Explorations in Cosmology

Andreas Tolley  
+ Sarah Shandera

Explorations in Cosmology

Inflating Gas

Andrew Talley  
+ Sarah Shandera



Explanations

ogy

Andrew T

+ S

# Inflationary Cosmology



## Explanations in Cosmology

Andrew Tolley  
+ Sarah Shandera

## Inflationary Cosmology

- Quantum Field theory in curved spacetime

Explanations in

Andreas To  
+ Sarah

## Inflationary Cosmology

- Quantum Field theory in curved spacetime
- $\sim$  time-dependent

## Explanations in Cosmology

Andrew Tolley  
+ Sarah Shell

## Inflationary Cosmology

- Quantum Field theory in curved spacetime  
~ time-dependent
- Recap basic inflation

Explanations

Andreas  
+ S...

## Inflationary Cosmology

- Quantum Field theory in curved spacetime  
~ time-dependent
- Recap basic inflation
- Couple quantum fields + gravity  
~ how do we quantize gravity or LQG?



# Explorations in Cosmology

Andrew Tolley  
+ Sarah Shandera

# Inflationary Cosmology

- Quantum Fields in curved spacetime  
~ time-dependent

Inflation

Scalar fields + gravity

to quantum gravity of LQG

... in cosmology

## Explorations in Cosmology

Andrew Talley  
+ Sarah Shand

## Inflationary Cosmology

Quantum Field theory in curved spacetime  
~ time-dependent

Recap basic inflation

Couple quantum fields + gravity

~ how do we quantize gravity or LQG?

Gauge invariant Formalism

Explanations

Andreas

+ Seminar

## Inflationary Cosmology

- Quantum Field theory in curved spacetime  
~ time-dependent
- Recap basic inflation
- Couple quantum fields + gravity  
~ how do we quantize gravity or LQG?
- Gauge invariant Formalism
- Completion functions of inflation field



# Explorations in Cosmology

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## Inflationary Cosmology

- Quantum Field theory in curved spacetimes  
~ time-dependent
- Reop base inflation
- Couple quantum fields + gravity  
~ how do we quantize gravity or LEFT
- Gauge Invariant Formalism
- Completion functions of inflation field  
~ perturbative field spectra + non-gaussianities



# Explorations in Cosmology

Andrew Tolley  
+ Sarah Shandera

## Inflationary Cosmology

- Quantum Field theory in curved spacetimes  
~ time-dependent
- Recap basic inflation
- Couple quantum fields + gravity  
~ how do we quantize gravity or LEFT
- Gauge Invariant Formalism
- Completion functions of inflation field  
~ perturbative power spectrum + non-gaussianities

# Quantum Fields in curved spacetimes

Wave

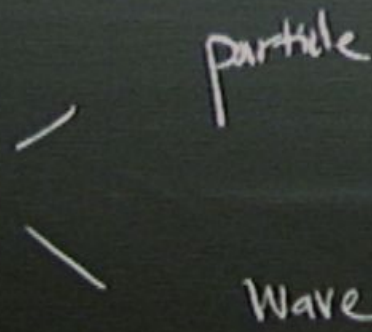


Quantum Fields in curved spacetimes

Wave-particle duality. — particle.

Quantum Fields in curved spacetimes

wave-particle duality.



$$\Psi(x)$$



Quantum  
Field

$$\phi(x)$$

Scalar quantum

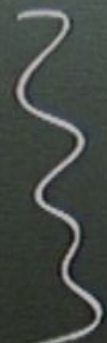


# Quantum Fields in curved spacetimes

Wave-particle duality

particle

Wave  $\psi(x)$



Quantum  
Field

$\phi(x)$

Scalar quantum fields

# Quantum Fields in curved spacetimes

Wave-particle duality

particle

Wave  $\psi(x)$

Quantum Field

$\phi(x)$

Scalar quantum fields

State of quantum field

$|\phi(\vec{x})\rangle$





Wave,

$$\psi(x)$$



Scalar quantum field

State of quantum

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle = |\phi(\vec{x})\rangle$$

Minkowski spacetime.

Quantum  
Field

$$\hat{\phi}(x)$$

Scalar quantum fields.

State of quantum field

$$|\phi(x)\rangle = \phi(x) |\phi(x)\rangle \quad |\phi(\vec{x})\rangle =$$



Multiparticle states



Wave,  $\psi(x)$

Scalar quantum

State of a

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0$$

Wave,

$$\psi(x)$$



Scalar quantum

State of a

$$\phi(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle \quad | \phi$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0$$

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\hbar \delta^{(4)}(x-y)$$



Minkowski spacetime.

$$\hat{\phi}(x)$$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 \sqrt{2\omega_k}}$$

$$e^{-i\vec{k}\cdot\vec{x} - i\omega_k t}$$

$$e^{i\vec{k}\cdot\vec{x} + i\omega_k t} \hat{a}_{\vec{k}} + e$$

$$e^{-i\vec{k}\cdot\vec{x} - i\omega_k t} \hat{a}_{\vec{k}}$$

Scalar quantum fields.

State of quantum field

$$|\psi\rangle = |\phi(\vec{x})\rangle =$$



mult.



Minkowski spacetime.

$$\hat{\phi}(x)$$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}}$$

$$-ik\vec{x} - i\omega t$$

$$-ik\vec{x} + i\omega t$$

$e$

$$\hat{a}_k + e$$

$$a_k^\dagger$$



kellon operaattori.



create operaattori.

quantum fields.

A quantum field.

$$|\phi(\vec{x})\rangle =$$



Multiv

Minkowski spacetime.

$\hat{\phi}(x)$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} e^{i(k\vec{x} - \omega t)}$$

$-i(k\vec{x} - \omega t)$

$\hat{a}_k + e$

$-i(k\vec{x} + \omega t)$

$a_k^\dagger$

↑  
annihilation operator

↑  
creation operator

quantum fields

posit

$\mathcal{L}$  quantum

$|\phi(\vec{x})\rangle$

(particle states)



Morokowski spacetime.

$\hat{\phi}(x)$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 \sqrt{2\omega_k}} e^{-ik\vec{x} - i\omega_k t}$$

$e$

$-ik\vec{x} - i\omega_k t$

$\hat{a}_k + e$

$-ik\vec{x} + i\omega_k t$

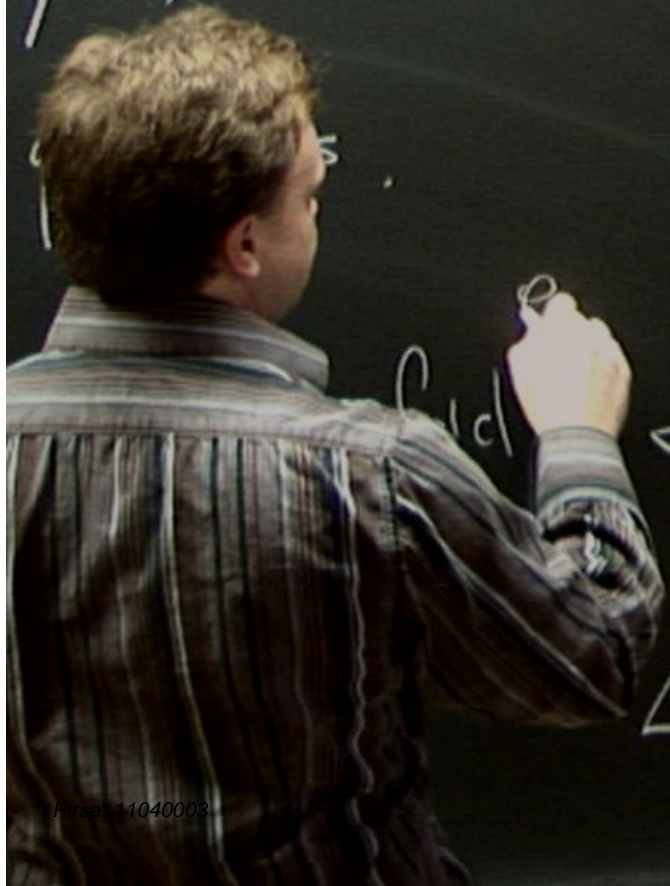
$a_k^\dagger$

annihilation operator

creation operator

positive energy

Multiparticle states





Minkowski spacetime.

$\hat{\phi}(x)$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 \sqrt{2\omega_k}} e^{-ik \cdot x - i\omega_k t}$$

$-ik \cdot x - i\omega_k t$

$-ik \cdot x + i\omega_k t$

$\hat{a}_k + e$

$\hat{a}_k^\dagger$

annihilation operator

creation operator

$E = \omega_k$  positive energy.

Multiparticle states

Minkowski spacetime.

$$\hat{\phi}(x)$$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} e^{i\vec{k}\cdot\vec{x} - i\omega_k t}$$

quantum fields

$$\omega_k^2 = \vec{k}^2 + m^2$$

$c = k = 1$

$$e^{-iEt/\hbar} \quad E = \omega_k \text{ positive energy.}$$

$$-i\vec{k}\cdot\vec{x} - i\omega_k t$$

$$\hat{a}_k + e$$

annihilation operator

$$-i\vec{k}\cdot\vec{x} + i\omega_k t$$

$$a_k^\dagger$$

creation operator

of quantum field



Multiparticle states

$$|\phi(\vec{x})\rangle =$$

=



$\hat{\phi}(x)$

Minkowski spacetime

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} e^{-ik \cdot x - i\omega_k t}$$

$-ik \cdot x - i\omega_k t$

$-ik \cdot x + i\omega_k t$

$\hat{a}_k + e$

$\hat{a}_k^\dagger$

annihilation operator

creation operator

quantum fields

$$\omega_k^2 = \vec{k}^2 + m^2$$
$$c = k = 1$$

$e^{-iEt/\hbar}$   
 $E = \omega_k$  positive energy

of quantum field



Multiparticle states

$|\phi(\vec{x})\rangle$

=



$\hat{\phi}(x)$

Minkowski spacetime

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} e^{-ik \cdot x - i\omega_k t}$$

$-ik \cdot x - i\omega_k t$

$-ik \cdot x + i\omega_k t$

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$\hat{a}_k^\dagger$

annihilation operator

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quantum fields

$$\omega_k^2 = \vec{k}^2 + m^2$$

$c = k = 1$

$e^{-iEt/\hbar}$   
 $E = \omega_k$  positive energy

of quantum field

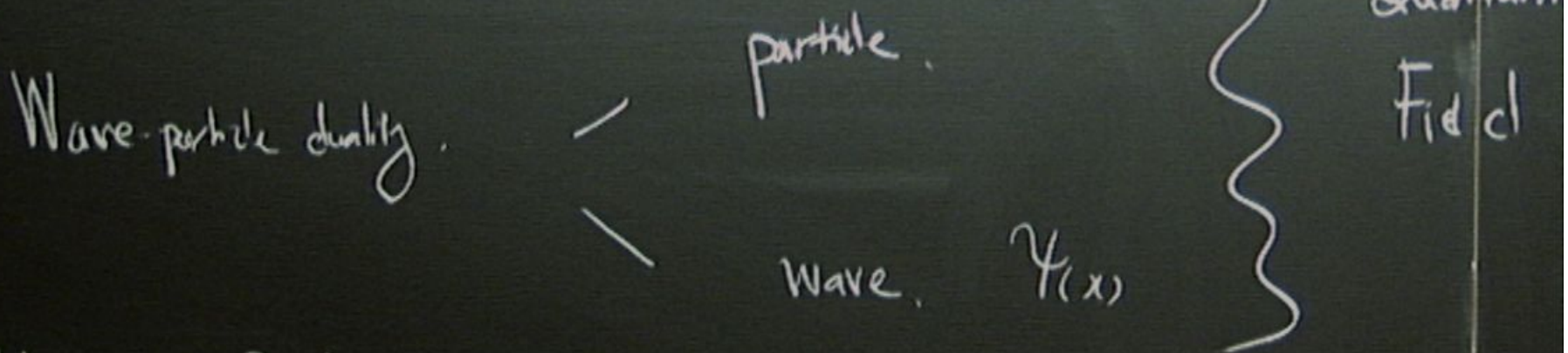


Multiparticle states

$|\phi(\vec{x})\rangle =$

$=$

# Quantum Fields in curved spacetimes



vacuum state

$$a_k |0\rangle = 0$$

$$a_k^\dagger |0\rangle = |\vec{k}\rangle$$

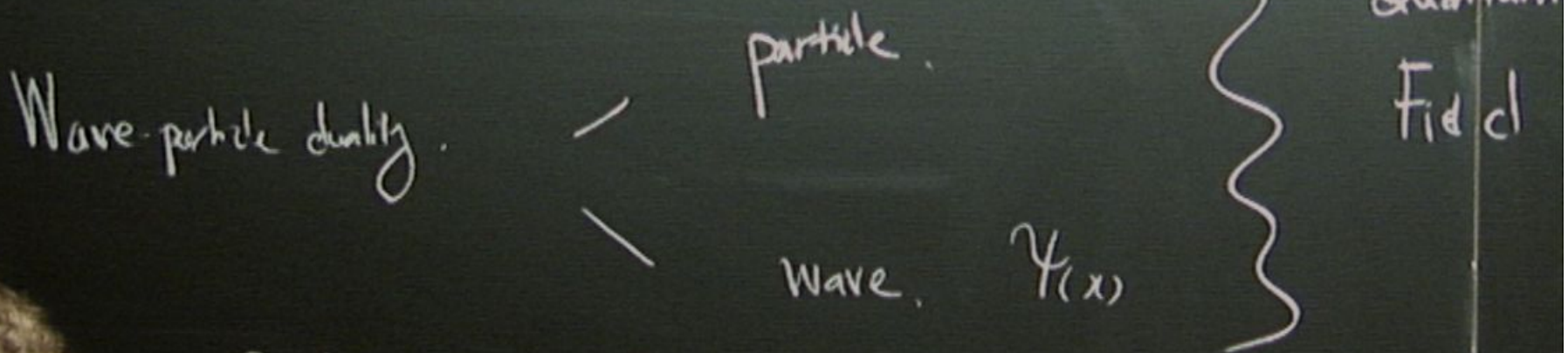
single particle  $\vec{k}$

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0 \quad [\hat{\phi}(x), \hat{\pi}(y)] = i\delta(x-y)$$



# Quantum Fields in curved spacetimes



$$a_k |0\rangle = 0$$

$$a_k^\dagger |0\rangle \sim |\vec{k}\rangle$$

Single particle  $\vec{k}$

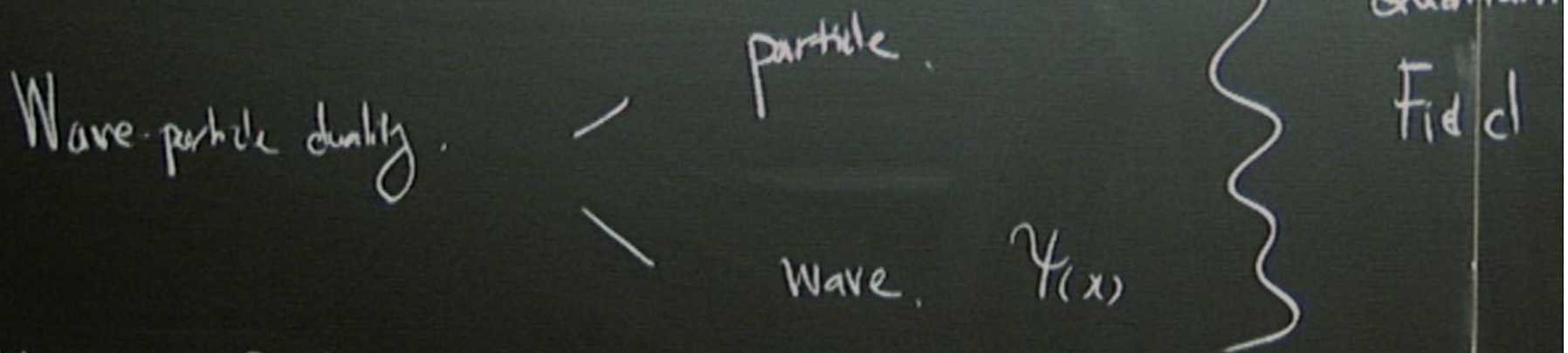
$$a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$$

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0 \quad [\hat{\phi}(x), \hat{\pi}(y)] = i\delta(x-y)$$



# Quantum Fields in curved spacetimes



Vacuum state

$$a_k |0\rangle = 0$$

$$a_k^+ |0\rangle \sim |\vec{k}\rangle$$

Single particle  $\vec{k}$

$$a_{k_1}^+ a_{k_2}^+ |0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$$

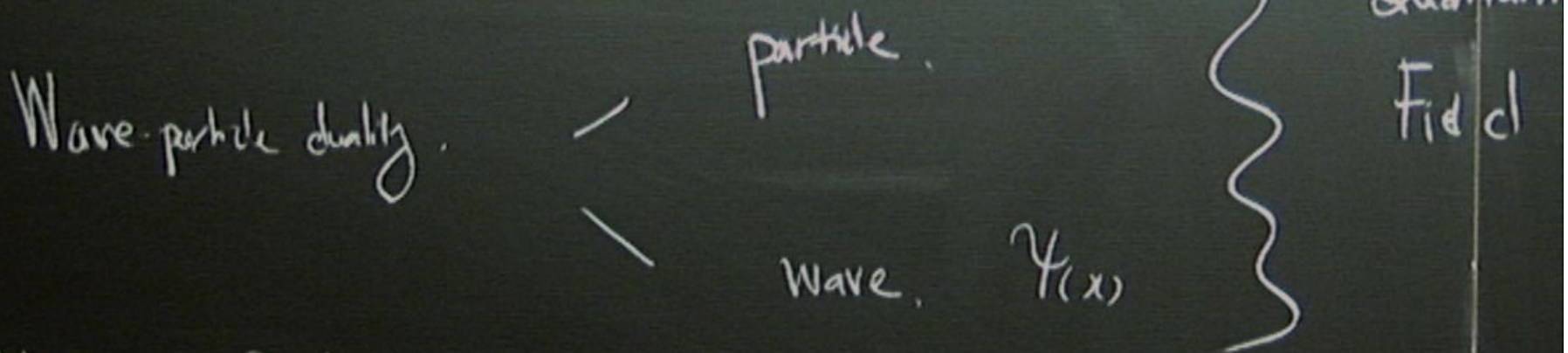
1-particle  $(a^+)^n |0\rangle$

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0$$

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\delta(x-y)$$

# Quantum Fields in curved spacetimes



Vacuum state

$$a_k |0\rangle = 0$$

$$a_k^\dagger |0\rangle \sim |\vec{k}\rangle$$

Single particle  $\vec{k}$

$$a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$$

1-particle  $(a^\dagger)^n |0\rangle$

$$\hat{\phi}(x) |\phi(x)\rangle = \phi(x) |\phi(x)\rangle$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0$$

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\delta(x-y)$$



$$E^2 = \vec{k}^2 + m^2$$

$$c = k = 1$$

$$-iEt/\hbar$$

$E = \omega_k$  positive energy.

$$(\hat{\pi})^2 \sqrt{2\omega_k}$$

annihilation operator.

creation operator

$d/c$

Multiparticle states

$$\alpha |0\rangle + \beta_k a_k^\dagger |0\rangle + \beta_{k_1, k_2} a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle$$

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = k = 1$$

$$e^{-iEt/\hbar} \quad E = \omega_k \text{ positive energy.}$$

annihilation operator

creation operator

Fock space

field

Multiparticle states

$$= \left[ \alpha |0\rangle + \beta_k a_k^\dagger |0\rangle + \beta_{k_1, k_2} a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle + \dots \right]$$



$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = k = 1$$

$-iEt/\hbar$   
 $E = \omega_k$  positive energy.

$$|\langle \pi | \rangle \sqrt{2\omega_k}$$

annihilation operator

creation operator

Fock space

$$\sum f_1 \otimes f_2 \otimes f_3$$

Multiparticle states

$$\sum \oplus (f_1)^n$$

$$\alpha |0\rangle + \beta_k a_k^\dagger |0\rangle + \beta_{k_1, k_2} a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle$$



$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = k = 1$$

$$e^{-iEt/\hbar} \quad E = \omega_k \text{ positive energy}$$

annihilation operator

creation operator

Fock space

$$\sum \mathbb{F} \otimes \mathbb{F} \otimes \mathbb{F}$$

Multiparticle states

$$\sum_n \oplus (\otimes \mathbb{F})^n$$

vacuum field

$$| \rangle$$

$$=$$


$$\alpha |0\rangle + \beta_k a_k^\dagger |0\rangle + \beta_{k_1, k_2}^\dagger a_{k_1}^\dagger a_{k_2}^\dagger |0\rangle + \dots$$



$$\hat{\phi}(x)$$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} e^{i(kx - \omega_k t)}$$

$$\hat{a}_k + e$$

↑  
annihilation operator

as quantum fields

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = k = 1$$

$$e^{-iEt/\hbar}$$

$E = \omega_k$  positive energy

Fock space

state of quantum field

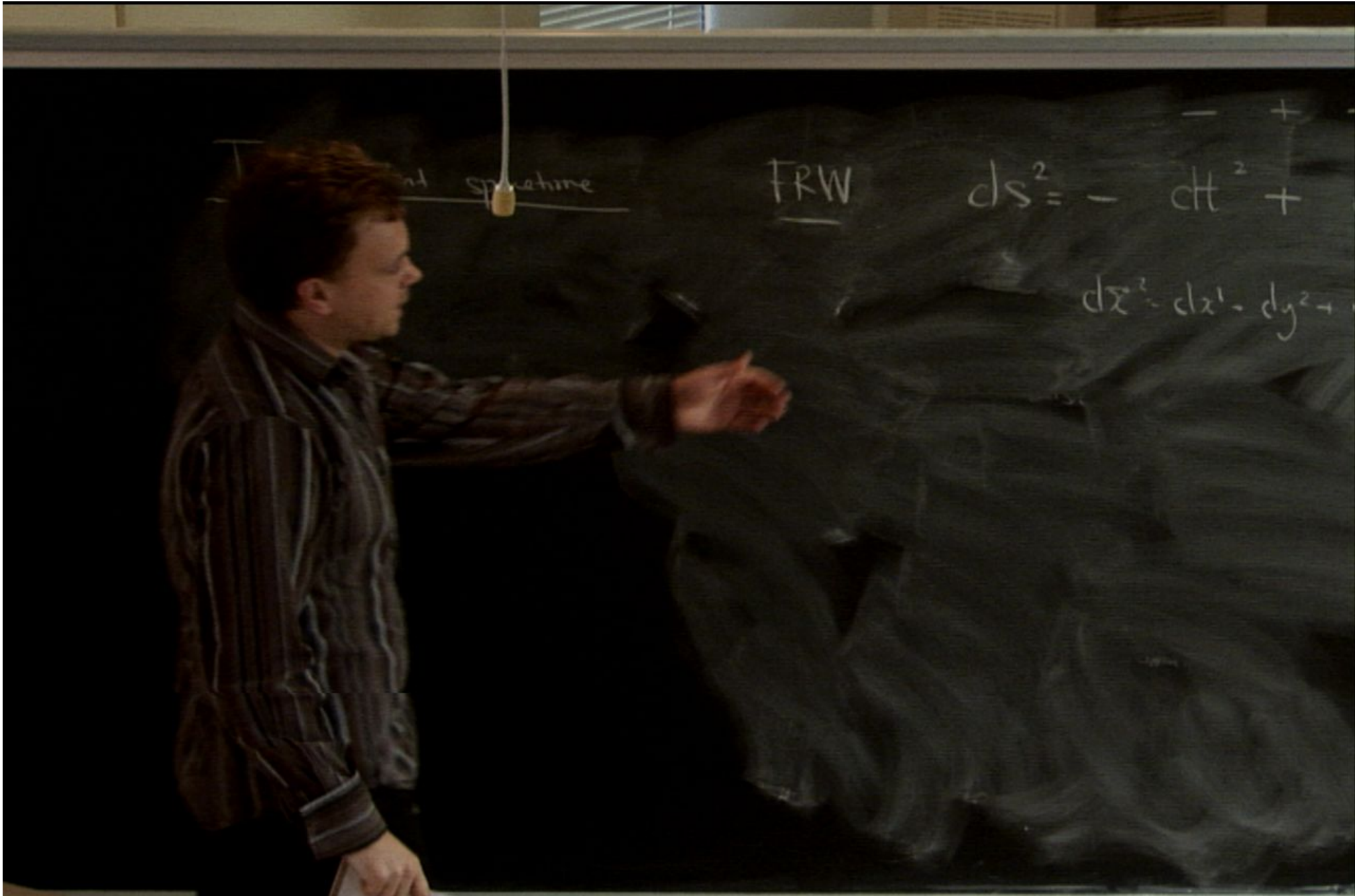
$$\sum f|0\rangle + f|1\rangle + f|2\rangle + \dots$$

$$|\phi(\vec{x})\rangle$$

$$= \alpha |0\rangle + \beta_k a_k^\dagger |0\rangle + \dots$$

Multiparticle states

$$i\hbar \delta^{(4)}(x-y)$$



spacetime

FRW

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$dx^2 + dy^2 + dz^2$$



Time dependent space

FRW

$$ds^2 = - dt^2 + a^2(t) d\vec{x}^2$$

$$d\vec{x}^2 = dx^2 + dy^2 + dz^2$$

conform

Time-dependent spacetime

TPW

$$ds^2 = - dt^2 + a^2(t) d\vec{x}^2$$

$$d\vec{x}^2 = dx^1 + dy^2 + dz^2$$

in time

$$dt = a d\eta \quad \text{conformal time}$$



Time-dependent spacetime

FRW

$$ds^2 = - dt^2 + a^2(t) d\vec{x}^2$$

$$d\vec{x}^2 = dx^1^2 + dy^2 + dz^2$$

conformal time

$$dt = a d\eta \quad \text{conformal time}$$

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + d\vec{x}^2 \right]$$

dependent spacetime

FRW

$$ds^2 = - dt^2 + a^2(t) d\vec{x}^2$$

$$d\vec{x}^2 = dx^1^2 + dy^2 + dz^2$$

conformal time

$$dt = a d\eta \quad \text{conformal time}$$

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + d\vec{x}'^2 \right]$$



Massive scalar field

(Massless  $m=0$ )

Massive scalar field (Massless  $m \rightarrow 0$ )

- + + +

Action

$$S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$$



Massive scalar field (Massless  $m \rightarrow 0$ )  
- + + +  $\hbar = c = 1$

Action  $S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$\phi(\vec{x})$

$\delta^{10} (15-3)$

Massive scalar field (Massless  $m \rightarrow 0$ )  
- + + +  $\text{tr} \epsilon = 1$

Action  $S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\phi(\vec{x})$

$\sqrt{-g}$



Massive scalar field (Massless  $m \rightarrow 0$ )  
- + + +  $\eta_{\mu\nu} = \eta$

Action  $S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sqrt{\det(-g)} = a^4$$

$|\phi(\vec{x})\rangle$

$\delta^{\mu\nu} \eta_{\mu\nu}$

Massive scalar field (Massless  $m \rightarrow 0$ )  
- + + + trace = 1

Action  $S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sqrt{\det(-g)} = a^4$$

$|\phi(\vec{x})\rangle$

$\int \sqrt{-g} d^4x$



Massive scalar field (Massless  $m \rightarrow 0$ )

$- + + + \quad \hbar = c = 1$

Action  $S = \int \sqrt{-g} d^4x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sqrt{\det(-g)} = a^4$$

$$\left[ \int d\eta d^3x \right] = \int d\eta d^3x a^4 \left[ \frac{1}{2a^2} (\partial_\eta \phi)^2 - \frac{1}{2a^2} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Spatial derivative





$$\left[ -\frac{1}{2} m^2 \phi^2 \right] \int d\eta d^3x \left[ a^4 \left( \frac{1}{2a^2} (\partial_\eta \phi)^2 - \frac{1}{2a^2} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 \right) \right]$$

Spatial derivatives



$$-\frac{1}{2} m^2 \phi^2] = \int d\eta d^3x \quad a^4 \left[ \frac{1}{2a^2} (\partial_\eta \phi)^2 - \frac{1}{2a^4} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Spatial derivatives  
↓

$$= \int d\eta d^3x \left\{ \frac{a^2}{2} \left[ (\partial_\eta \phi)^2 - (\vec{\nabla} \phi)^2 \right] - \frac{1}{2} m^2 a^4 \phi^2 \right\}$$

Moments conjugate      $\Pi = \frac{\partial \mathcal{L}}{\partial (\partial_\eta \phi)}$



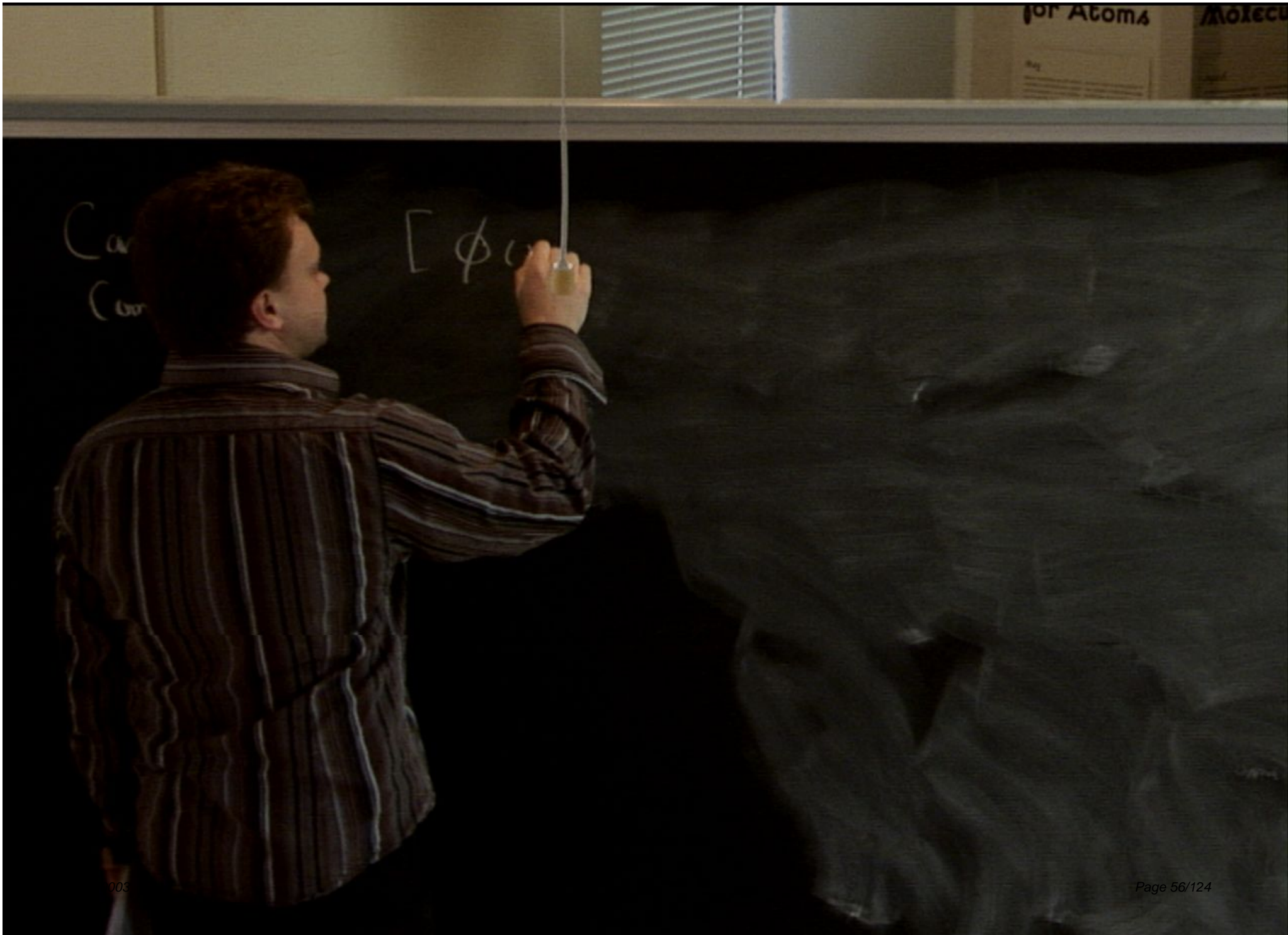
$$-\frac{1}{2} m^2 \phi^2] = \int d\eta d^3x \quad a^4 \left[ \frac{1}{2a^2} (\partial_\eta \phi)^2 - \frac{1}{2a^4} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Spatial derivatives  
↓

$$= \int d\eta d^3x \left\{ \frac{a^2}{2} \left[ (\partial_\eta \phi)^2 - (\vec{\nabla} \phi)^2 \right] - \frac{1}{2} m^2 a^4 \phi^2 \right\}$$

Moments conjugate

$$\Pi = \frac{\partial \mathcal{L}}{\partial(\partial_\eta \phi)} = a^2 \partial_\eta \phi$$





Canonical  
Commutation

$$[\phi(\vec{x}, \eta), \phi(\vec{x}')] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}')] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta(\vec{x} - \vec{x}')$$

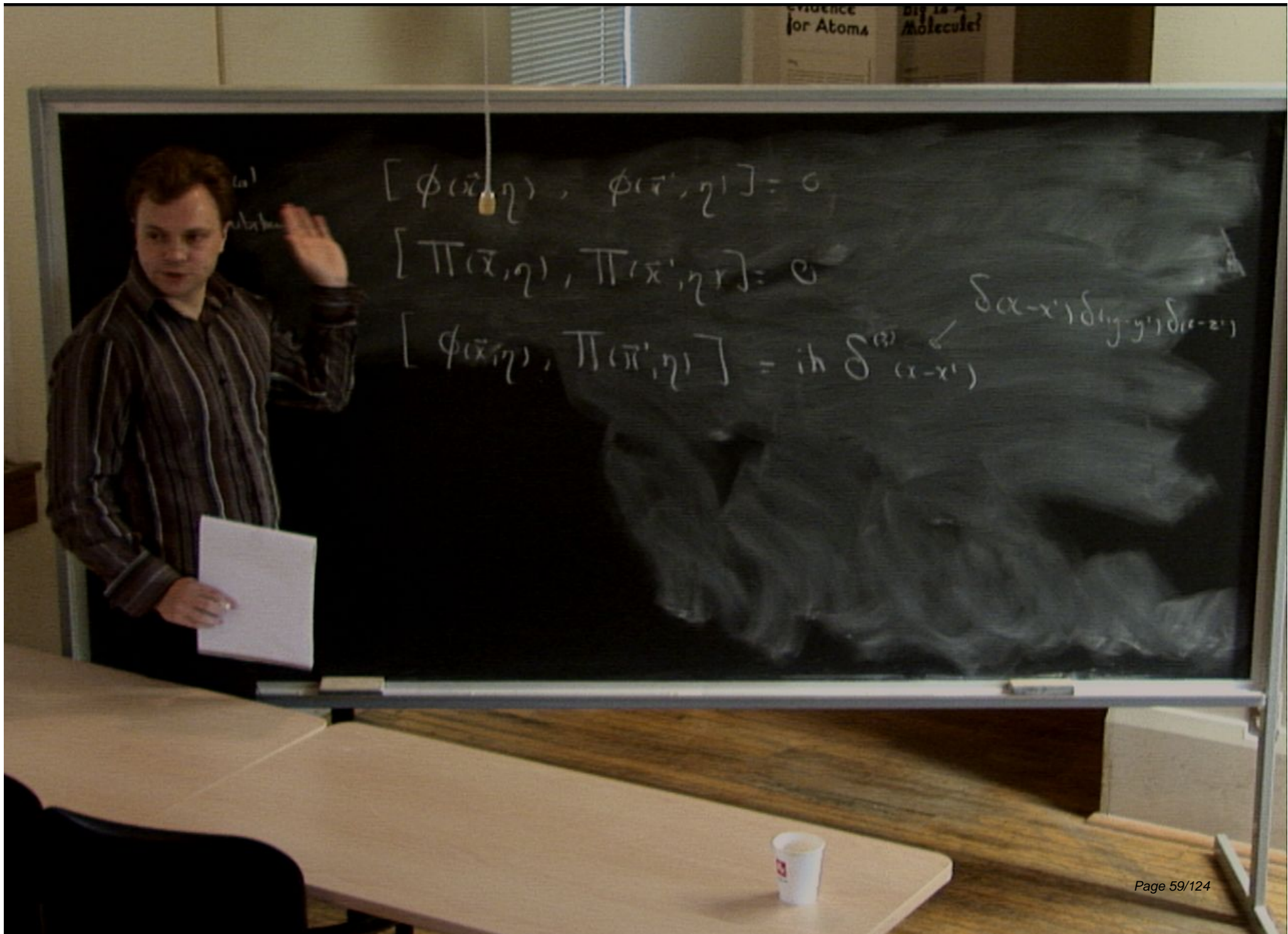
Canonical  
Commutation

$$[\phi(\vec{x}', \eta), \pi(\vec{x}, \eta)] = 0$$

$$[\phi(\vec{x}', \eta), \phi(\vec{x}, \eta)] = 0$$

$$[\pi(\vec{x}', \eta), \pi(\vec{x}, \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$





$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}') \delta(\eta - \eta')$$

$$\delta(\alpha - \alpha') \delta(\eta - \eta') \delta(\vec{x} - \vec{x}')$$

Canonical  
Coordinates

$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

$$\delta(\alpha - \alpha') \delta(\eta - \eta') \delta(\vec{r} - \vec{r}')$$



massless  $m \rightarrow 0$ )

$$c = 1$$

$$\left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right] = \int dt d^3x$$

$$g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$V = \frac{1}{2} m^2 \phi^2 = \int dt d^3x$$

Momentum conjugate

II

canonical  
commutation

$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

$$\delta(\alpha - \alpha') \delta(\eta - \eta') \delta(\vec{c} - \vec{c}')$$



canonical  
commutation

$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

$$\delta(\alpha - \alpha') \delta(\eta - \eta') \delta(\vec{x} - \vec{x}')$$

canonical  
commutation

$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

$$\delta_{\alpha-\alpha'} \delta_{\eta-\eta'} \delta_{\vec{x}-\vec{x}'}$$

$$\partial_{\eta} \phi(\vec{x}, \eta) = \frac{i\hbar}{c^2} \nabla^2 \phi(\vec{x}, \eta)$$



canonical  
commutation

$$[\phi(\vec{x}, \eta), \phi(\vec{x}', \eta)] = 0$$

$$[\Pi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = 0$$

$$[\phi(\vec{x}, \eta), \Pi(\vec{x}', \eta)] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

$$\delta(\alpha - \alpha') \delta(\eta - \eta') \delta(\vec{c} - \vec{c}')$$

$$[\phi(\vec{x}, \eta), \partial_\eta \phi(\vec{x}', \eta)] = \frac{i\hbar}{c^2} \delta^{(3)}(\vec{x} - \vec{x}')$$

Canonically normalizing

Defn

$$\phi = \frac{u}{\sqrt{2m}}$$

$$= \int d\eta d^3x$$

Vergleichen

$$\int d\eta d^3x$$

Messwert  $\Pi =$



# Canonically normalizing

Defn

$$\phi = \frac{u}{\alpha \eta}$$

Quantization procedure

$K \rightarrow 1$

## Canonically normalizing

Defn

$$\phi = \frac{u}{\alpha m}$$

Quantization procedure  
is invariant under field  
redefinitions



## Canonically normalizing

Defn

$$\phi^{(\eta, \vec{x})} = \frac{u(\eta, \vec{x})}{\alpha(\eta)}$$

Quantization procedure  
is invariant under field  
redefinitions

## Canonically normalizing

Defn

$$\phi^{(\eta, \vec{x})} = \frac{u(\eta, \vec{x})}{\alpha(\eta)}$$

Quantization procedure  
is invariant under field  
redefinitions



## Canonically normalizing

Defn

$$\phi^{(\eta, \vec{x})} = \frac{u(\eta, \vec{x})}{\alpha \eta}$$

Quantization procedure  
is invariant under field  
redefinitions

$$\begin{aligned} \partial_\eta \phi &= \frac{\partial_\eta u}{\alpha} - \frac{\alpha'}{\alpha^2} u \\ &= \frac{1}{\alpha} \left[ \partial_\eta u - \frac{\alpha'}{\alpha} u \right] \end{aligned}$$

$$V = \frac{1}{2} m^2 \phi^2$$

Moments (energy)

## Canonically normalizing

Defn

$$\phi(\eta, \vec{x}) = \frac{u(\eta, \vec{x})}{\alpha(\eta)}$$

Quantization procedure  
is invariant under field  
redefinitions

$$\partial_\eta \phi = \frac{\partial_\eta u}{\alpha} - \frac{\alpha'}{\alpha^2} u$$

$$= \frac{1}{\alpha} \left[ \partial_\eta u - \frac{\alpha'}{\alpha} u \right]$$

$$\vec{\nabla} \phi = \frac{\vec{\nabla} u}{\alpha}$$

$$V = \frac{1}{2} m^2 \phi^2$$

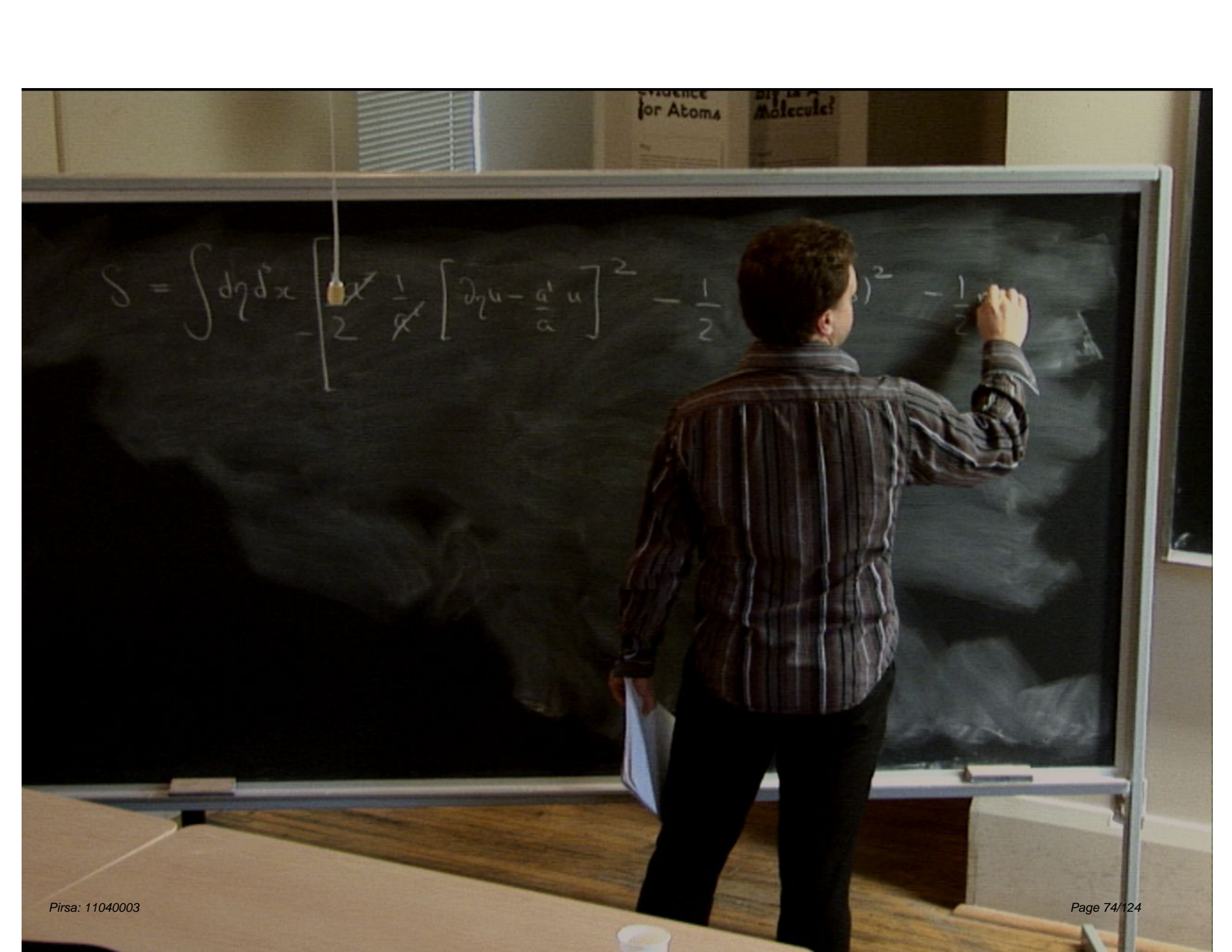
Momentum conjugate



$$S = \int d\eta d^3x$$

EVIDENCE  
for Atoms

BY ERN  
Molecule!



The image shows a man from behind, wearing a striped shirt and dark pants, writing on a chalkboard. The chalkboard contains a mathematical equation for the action  $S$ . The equation is written in white chalk and includes a double integral over  $y$  and  $x$ , a square root in the denominator, and a squared term in the numerator. The man is currently writing the coefficient  $-\frac{1}{2}$  at the end of the equation.

$$S = \int dy dx \sqrt{\frac{1}{2} \left[ \partial_y u - \frac{a'}{a} u \right]^2 - \frac{1}{2}}$$



EVIDENCE  
for Atoms  
Molecules

$$\int d\tau d^3x \left[ \frac{1}{2} \left( \partial_t u - \frac{a'}{a} u \right)^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 a^4 \frac{u^2}{a^2} \right]$$

$$\begin{aligned}
 S &= \int d\eta d^3x \left[ \frac{1}{2} \left[ \partial_\eta u - \frac{a'}{a} u \right]^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right] \\
 &= \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \partial_\eta u u \frac{a'}{a} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]
 \end{aligned}$$



Pollen to Evidence for Atoms

How Big Is A Molecule?

$$S = \int \left[ \cancel{\frac{1}{2}} \left[ \cancel{\partial_t u - \frac{a'}{a} u} \right]^2 - \frac{1}{2} \cancel{\frac{1}{a}} \frac{1}{\cancel{a}} (\nabla u)^2 - \frac{1}{2} m^2 a^2 \frac{u^2}{\cancel{a}} \right]$$

$$\frac{1}{2} (\partial_t u)^2 - \underbrace{\partial_t u u \frac{a'}{a}} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 a^2 u^2$$

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Evidence  
for Atoms

How  
Big Is A  
Molecule?

$$S = \int d\eta dx \left[ \frac{1}{2} \left[ \partial_\eta u - \frac{a'}{a} u \right]^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$= \int d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \underbrace{\partial_\eta u u \frac{a'}{a}} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\nabla u)^2 \right]$$

$$(\partial_\eta u) u = -\frac{1}{2} \partial_\eta (u^2)$$



$$S = \int d\eta d^3x \left[ \frac{1}{2} \left[ \partial_\eta u - \frac{a'}{a} u \right]^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$= \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \underbrace{\partial_\eta u u \frac{a'}{a}}_{(\partial_\eta u) u} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$$

$$(\partial_\eta u) u = -\frac{1}{2} \partial_\eta (u^2)$$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$$

$$= \int d\eta d^3x$$

$$\int d\eta d^3x \left\{ \frac{a^2}{2} \right\}$$

Murray's conjugate  $\Pi = \frac{\partial S}{\partial(\dot{u})}$



$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left( \frac{a'}{a} \right)' =$$



$$= \int d\eta d^3x$$

$$\int d\eta d^3x \left\{ \frac{a^2}{2} \right.$$

Metric tensor conjugate  $\Pi = \frac{\partial \mathcal{L}}{\partial (\dot{\phi})}$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)''$$

$$S = \int d\eta d^3x \left[ (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 \text{eff}(\eta) u^2 \right]$$

$$= \int d\eta d^3x \left\{ \frac{a^2}{2} \right\}$$

Momentum conjugate  $\Pi = \frac{\partial S}{\partial(\dot{\phi})}$



$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left( \frac{a'}{a} \right)'$$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} m^2_{\text{eff}}(\eta) u^2 \right]$$

$$= \int d\eta d^3x \left\{ \frac{a^2}{2} \right\}$$

$$m^2_{\text{eff}} = m^2 a^2$$

Momentum conjugate

$$\Pi = \frac{\partial \mathcal{L}}{\partial (\partial_\eta u)}$$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$$

$$S = \int d\eta d^3x \left[ \partial_\eta u^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 e^{4\eta} u^2 \right]$$

$$\left(\frac{a'}{a}\right)^2 - \left(\frac{a''}{a}\right) - \left(\frac{a'}{a}\right)' \frac{a''}{a} - \left(\frac{a'}{a}\right)^2$$

Memoire conjugate  $\Pi = \frac{\partial \mathcal{L}}{\partial (\dot{\phi})}$



$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left( \frac{a'}{a} \right)^2 u^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left( \frac{a'}{a} \right)'$$

$$= \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\nabla u)^2 - \frac{1}{2} m_{\text{eff}}^2(\eta) u^2 \right]$$

$$= \int d\eta d^3x \left\{ \frac{a^2}{2} \right\}$$

$$m_{\text{eff}}^2 = m^2 a^2 - \left( \frac{a'}{a} \right)^2 - \left( \frac{a'}{a} \right)'$$

$$m_{\text{eff}}^2 = m^2 a^2 - \frac{a''}{a}$$

Metric tensor conjugate

$$\Pi = \frac{\partial \mathcal{L}}{\partial (\dot{\phi})}$$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 u^2 \right]$$

$$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)''$$

$$S = \int d\eta d^3x \left[ \frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m_{\text{eff}}^2(\eta) u^2 \right]$$

$$m_{\text{eff}}^2 = m^2 a^2 - \left(\frac{a'}{a}\right)'' - \left(\frac{a'}{a}\right)'$$

$$m_{\text{eff}}^2 = m^2 a^2 - \frac{a''}{a}$$

Murray's conjecture

$$\Pi = \frac{\partial \mathcal{L}}{\partial (\dot{\phi})}$$



Pollen to  
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How  
Big Is A  
Molecule?

$$[\psi(\vec{x}, \eta), u(\vec{x}', \eta)] = 0$$

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for Atoms

How  
Big Is A  
Molecule?

$$[u(\vec{x}, \eta), u(\vec{x}', \eta)] = 0$$

$$p = \frac{\partial \mathcal{L}}{\partial(\partial_\eta u)} =$$



Pollen to  
Evidence  
for Atoms

How  
Big Is A  
Molecule?

$$[u(x, \eta), u(x', \eta)] = 0$$

$$p = \frac{\partial \mathcal{L}}{\partial (\partial_\eta u)}$$

$$[p(x, \eta), p(x', \eta)] = 0$$

$$[u(x, \eta), p(x', \eta)] = i \delta^{(3)}(x - x')$$

$$\Downarrow$$
$$[p(x, \eta), \partial_\eta u(x', \eta)] = i \delta^{(3)}(x - x')$$

$$[u(\bar{x}, \eta), u(\bar{x}', \eta)] = 0$$

$$P = \frac{\partial \mathcal{L}}{\partial(\partial_\eta u)} = \partial_\eta u$$

$$[P(\bar{x}, \eta), P(\bar{x}', \eta)] = 0$$

$$[u(\bar{x}, \eta), P(\bar{x}', \eta)] = \delta^{(3)}(\bar{x} - \bar{x}')$$

$$\boxed{[u(\bar{x}, \eta), \partial_\eta u(\bar{x}', \eta)] = i \delta^{(3)}(\bar{x} - \bar{x}')} \quad \Downarrow$$



$$[\hat{u}(\bar{x}, \eta), \hat{u}(\bar{x}', \eta)] = 0$$

$$P = \frac{\partial \mathcal{L}}{\partial (\partial_\eta u)} = \partial_\eta u$$

$$[\hat{P}(\bar{x}, \eta), \hat{P}(\bar{x}', \eta)] = 0$$

$$[\hat{u}(\bar{x}, \eta), \hat{P}(\bar{x}', \eta)] = i \delta^{(3)}(\bar{x} - \bar{x}')$$

$$\boxed{[\hat{u}(\bar{x}, \eta), \partial_\eta \hat{u}(\bar{x}', \eta)] = i \delta^{(3)}(\bar{x} - \bar{x}')} \quad \Downarrow$$

$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3}$$

$$ds^2 = \alpha' \eta_{\mu\nu} [-d\eta^\mu + d\tilde{\kappa}^\nu]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}$$



$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3} a_k +$$

$$ds^2 = \alpha' \eta_{\mu\nu} [-d\eta^\mu + d\tilde{\kappa}^\mu]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}$$

$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3} \hat{a}_k U_k(\eta) e^{i\vec{k} \cdot \vec{x}}$$

$$ds^2 = \alpha^i \eta_i \left[ -d\eta^4 + d\vec{\kappa}^2 \right]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}$$



$$c_k U_k(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$a_k^{\dagger} e^{-i\vec{k}\cdot\vec{x}} U_k^{\dagger}(\eta)$$

$$U_k^\dagger(\eta)$$

Spatial derivative

Quantum field satisfies the operator version of classical equations of motion.

$$-\frac{1}{2} m^2 \phi^2$$

$$m^2 a^4 \phi^2$$

$$a^2 \partial_\eta \phi$$



$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3} \hat{a}_k U_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger e^{-i\vec{k}\cdot\vec{x}} U_k^*(\eta)$$

$$ds^2 = \alpha^2 \eta_1 [-d\eta^2 + d\vec{x}^2]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}$$

$$S = \int d\eta d^3 x \left[ \frac{1}{2} \partial_\eta \psi^2 - \frac{1}{2} (\nabla \psi)^2 - \frac{1}{2} m_{eff}^2 \psi^2 \right]$$

$$m_{eff}^2 = m^2 a^2 - \frac{a''}{a}$$

Heisenberg representation.

$$\psi'' - \nabla^2 \psi = 0$$

presentation.

Quantum field satisfies the operator  
of classical equations of

$$\psi'' - \nabla^2 \psi = -m_{eff}^2(\eta) \psi$$

$$a^2 \partial_\eta \phi$$



$$\hat{U}^+ = \hat{U}$$

$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3} \hat{a}_k U_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^+ e^{-i\vec{k}\cdot\vec{x}}$$

$$ds^2 = \alpha^2 \eta_1 [-d\eta^2 + d\vec{x}^2]$$

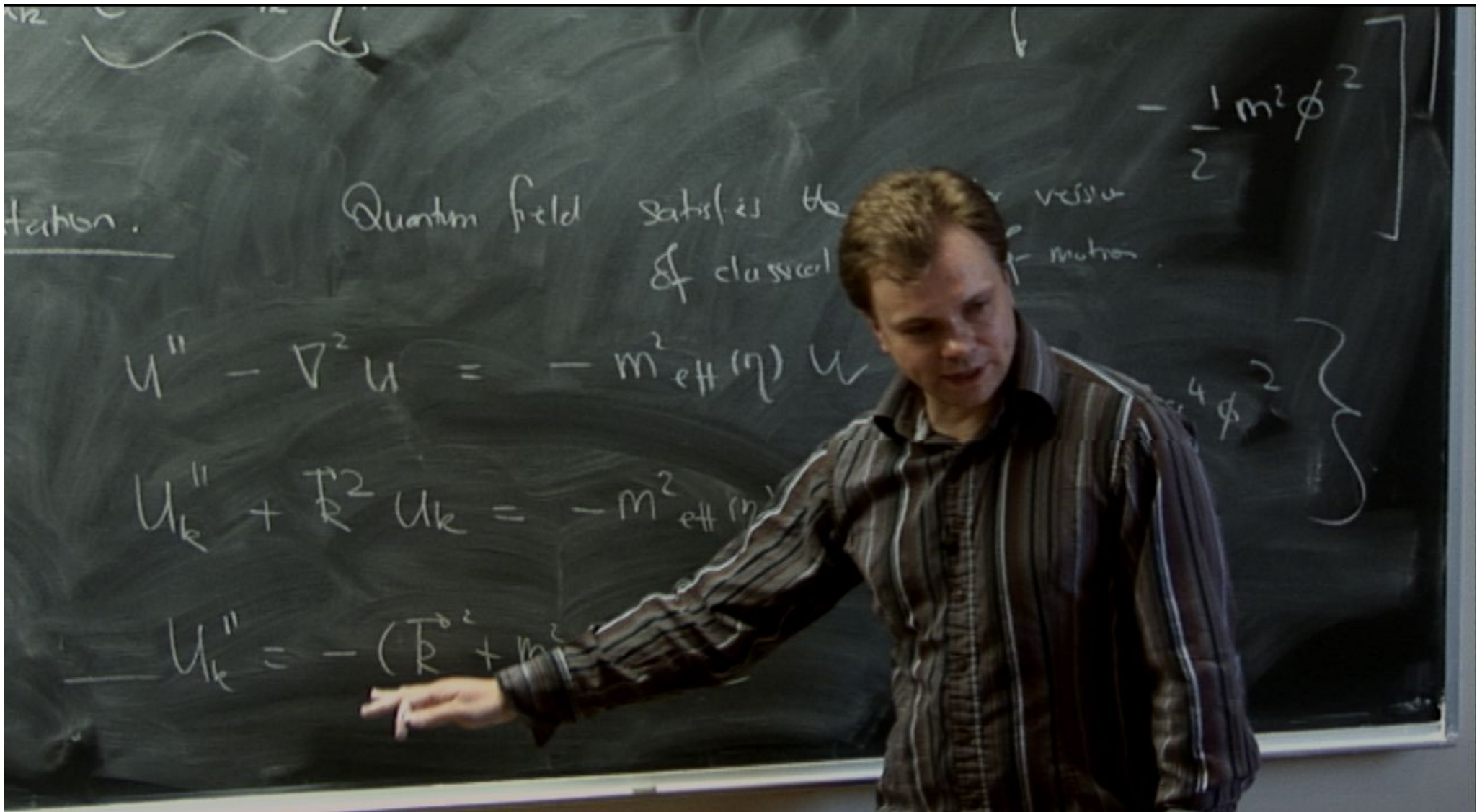
$$\vec{x} \Rightarrow \vec{x} + \vec{a}$$

$$S = \int d\eta d^3 x \left[ \frac{1}{2} (\partial_\eta \psi)^2 - \frac{1}{2} (\nabla \psi)^2 - \frac{1}{2} m^2 \psi^2 \right]$$

$$m_{\text{eff}}^2 = m^2 a^2 - \frac{a''}{a}$$

Heisenberg station.

$U''$



station.

Quantum field satisfies the  
& classical

$$-\frac{1}{2}m^2\phi^2$$

$$4\phi^2$$

$$U'' - \nabla^2 U = -m_{\text{eff}}^2(\eta) U$$

$$U_k'' + k^2 U_k = -m_{\text{eff}}^2 U_k$$

$$U_k'' = -(k^2 + m^2) U_k$$



ation.

Quantum field satisfies the operator version  
of classical equations of motion.

$$-\frac{1}{2}m^2\phi^2$$

$$U'' - \nabla^2 U = -m^2_{\text{eff}}(\eta) U$$

$$+m^2 a^4 \phi^2$$

$$U_k'' + \vec{k}^2 U_k = -m^2_{\text{eff}}(\eta) U_k$$

$$U_k'' = -(\vec{k}^2 + m^2_{\text{eff}}(\eta)) U_k = -\omega_k^2(\eta) U_k$$

station.

Quantum field satisfies the operator version  
of classical equations of motion.

$$U'' - \nabla^2 U = -m^2_{\text{eff}}(\eta) U$$

$$U_k'' + \vec{k}^2 U_k = -m^2_{\text{eff}}(\eta) U$$

$$U_k'' = -(\vec{k}^2 + m^2_{\text{eff}}(\eta)) U = -\omega_k^2(\eta) U$$

$$-\frac{1}{2} m^2 \phi^2$$

$$m^2 a^4 \phi^2$$



representation.

Quantum field satisfies the operator version of classical equations of motion.

$$-\frac{1}{2}m^2\phi^2$$

$$U'' - \nabla^2 U = -m^2_{eff}(\eta) U$$

$$U_k'' + k^2 U_k = -m^2_{eff}(\eta) U_k$$

$$+ m^2 a^4 \phi^2$$

$$U_k'' = -(\vec{k}^2 + m^2_{eff}(\eta)) U_k = -\omega_k^2(\eta) U_k$$

$$\hat{u}^\dagger = \hat{u}$$

$$\hat{u}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3}$$

$$\hat{a}_k u_k(\eta) e^{i\vec{k}\cdot\vec{x}}$$

Heisenberg

$$u_k'' = -\omega_k^2(\eta) u_k$$

$$\times u_k \sim e^{\pm i\omega_k \eta}$$



positive energy

$$\hat{a}_{\mathbf{k}} \underbrace{U_{\mathbf{k}}(\eta) e^{i\mathbf{k} \cdot \mathbf{x}}}_{e^{-i\omega_{\mathbf{k}}\eta}} +$$

$$\hat{a}_{\mathbf{k}}^{\dagger} \underbrace{e^{-i\mathbf{k} \cdot \mathbf{x}} U_{\mathbf{k}}^{\dagger}(\eta)}_{}$$

Heisenberg representation

$$\frac{d^3k}{(2\pi)^3}$$

$$U_{\mathbf{k}}$$

$$e^{-i\omega_{\mathbf{k}}\eta}$$

$$\frac{d^3k}{(2\pi)^3}$$

positive energy

$$\hat{a}_k U_k(\eta) e^{-i\omega_k \eta}$$

negative energy mode

$$\hat{a}_k^\dagger e^{-i\vec{k}\cdot\vec{x}} U_k^*(\eta) e^{+i\omega_k \eta}$$

Spacelike derivat

Heisenberg representation.

Quantum field satisfies the operator version of classical equations of motion

$$U'' - \nabla^2 U = -m^2_{eff}(\eta) U$$

$$U_k'' + k^2 U_k = -m^2_{eff}(\eta) U_k$$

$$U_k'' = -(k^2 + m^2_{eff}(\eta)) U_k = -\omega_k^2(\eta) U_k$$



$$\hat{u}^\dagger = \hat{u}$$

$$\hat{u}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3}$$

$$\hat{a}_k U_k(\eta) e^{i\vec{k}\cdot\vec{x}} + e^{-i\omega_k \eta}$$

Heisenberg

$$U_k'' = -\omega_k^2(\eta) U_k$$

$$\times U_k \sim e^{\pm i\omega_k \eta}$$

$U_k$  up into positive and negative energy?

$$\hat{U}^\dagger = \hat{U}$$

$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3}$$

$$\hat{a}_k \underbrace{U_k(\eta) e^{-i\vec{k}\cdot\vec{x}}}_{e^{-i\omega_k \eta}}$$

Heisenberg

$$U_k'' = -\omega_k^2(\eta) U_k$$

$$\times U_k \sim e^{\pm i\omega_k \eta}$$

$U_k$  up into positive and negative energy?

$$U_k = \alpha_k U_k^+ + \beta_k U_k^-$$



$$\hat{U}^\dagger = \hat{U}$$

$$\hat{U}(x, \eta) = \int \frac{d^3 k}{(2\pi)^3}$$

$$\hat{a}_k U_k(\eta) e^{i\vec{k}\cdot\vec{x}} e^{-i\omega_k \eta}$$

Heisenberg

$$U_k'' = -\omega_k^2(\eta) U_k$$

$$X \quad U_k \sim e^{\pm i\omega_k \eta}$$

$U_k$  up into positive and negative energy?

$$U_k = \alpha_k U_k^- + \beta_k U_k^+$$

$L$   $J$   $(2\pi)^5$

$$e^{-i\omega_k \eta}$$

Hamiltonian representation

$$U_k'' = -\omega_k^2(\eta) U_k$$

$$X \quad U_k \sim e^{\pm i\omega_k \eta}$$

$U_k$  up into positive quad

$$U_k = \alpha_k U_k^+ + \beta_k U_k^-$$





Adiabatic regime (WKB)

$\omega_k'$  is (adiabatic)

$$|\omega_k''| \ll \omega_k'$$

Adiabatic regime (WKB) app.

$\frac{1}{\omega_k} \sim$  time scale of oscillations

$\omega_k'$  is small (adiabatic)

$$|\omega_k'| \ll \omega_k$$





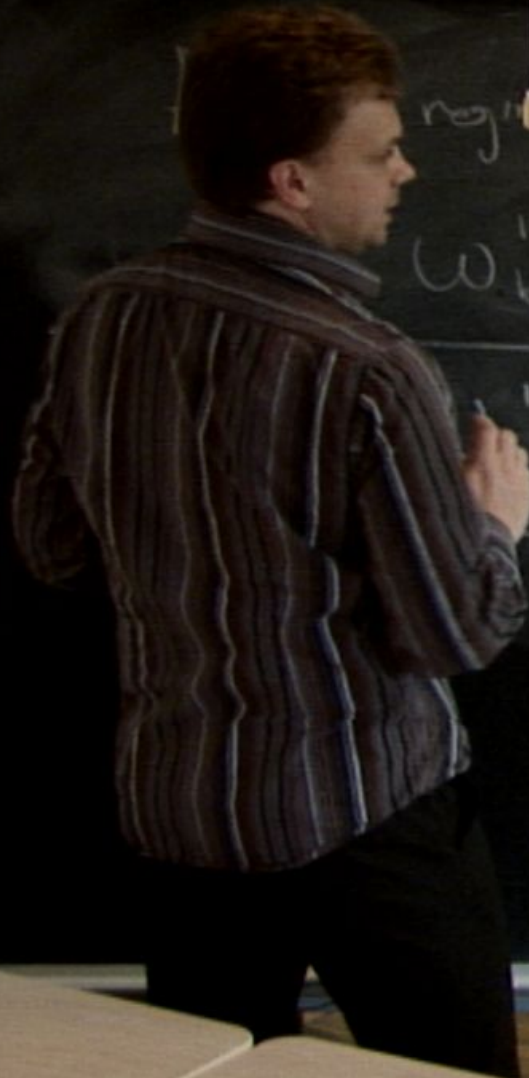
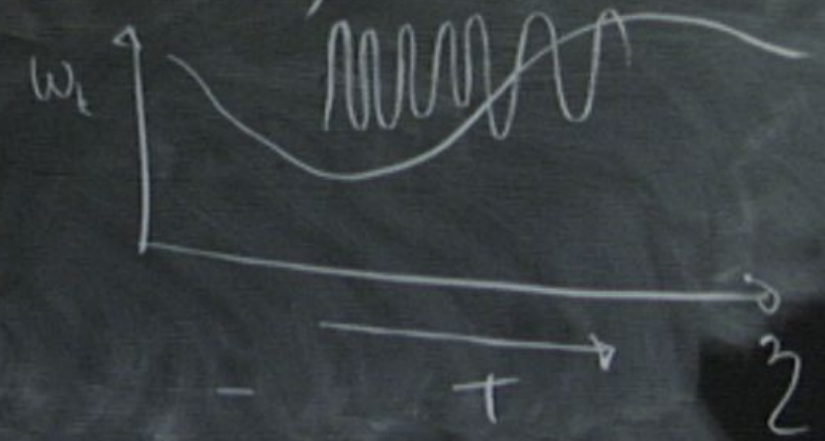
regime (WKB) approximation

$\frac{1}{\omega_k} \sim$  time scale oscillations

$\omega_k'$  is small

(adiabatic)

$\omega_k' \ll \omega_k^2$



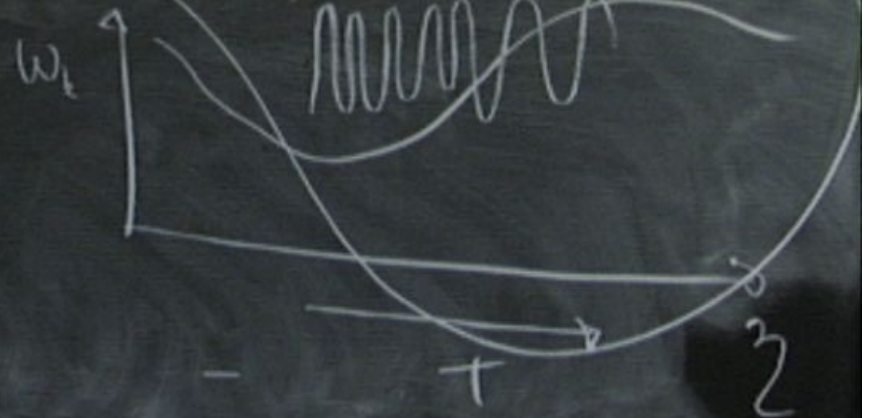
Adiabatic regime (WKB) approximation

$\frac{1}{\omega_k} \sim$  time scale oscillations

$\omega_k'$  is small

(adiabatic)

$$|\omega_k'| \ll \omega_k^2$$





$$\hat{a}_k + e^{-ikx} U_k(\eta)$$

negative energy mode

$$e^{+ikx}$$

Spatial derivative

quantization.

Noether theorem — Conserved energy  
— time translation invariance

exist globally timelike Killing vector

$$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$$

$$\hat{a}_k^\dagger e^{-ikx}$$

$$U_k(\eta)$$

negative energy mode

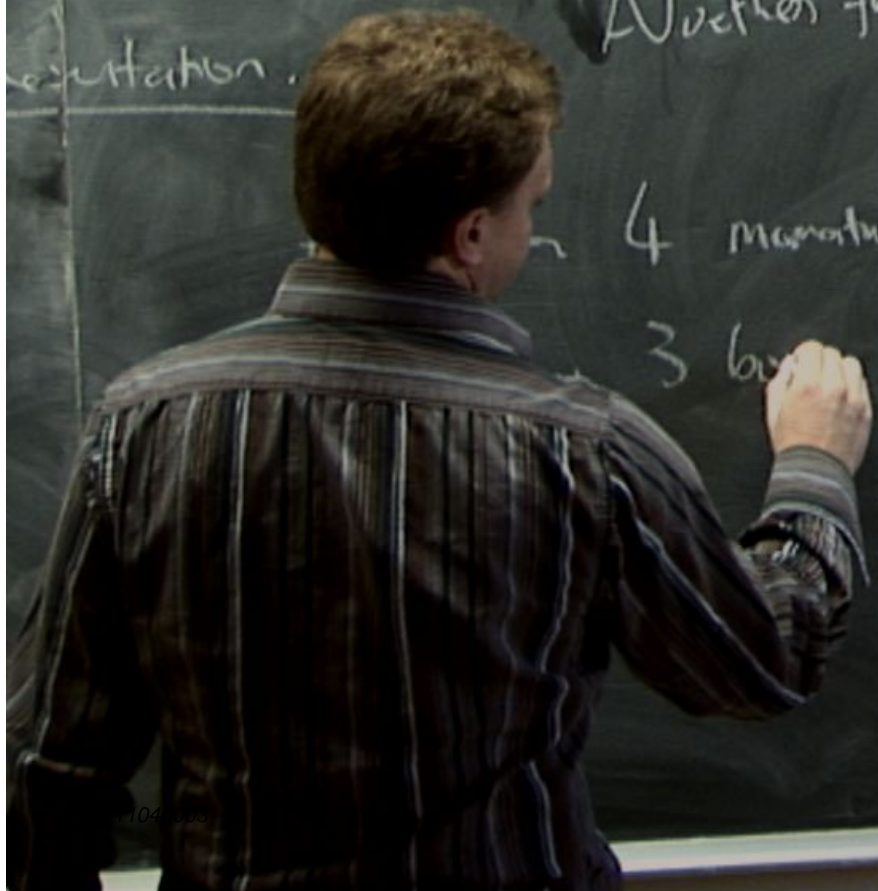
$$e^{+ikx}$$

spatial derivatives

Noether theorem — conserved energy  
 — time translation invariance

4 momentum exist globally timelike Killing vector  
 3 boost

$$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$$





$$\hat{a}_k^\dagger e^{-ikx} U_k(\eta)$$

negative energy mode

$$e^{+i\omega_k \eta}$$

Spatial derivatives

excitation.

Noether theorem — conserved energy  
— time translation invariance

Poincaré ~ 4 momentum exist globally timelike Killing vector  
~ 3 boosts  
~ 3 rotations

$$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$$

10

$$\hat{a}_k^{\dagger} e^{-ikx} U_k(\eta)$$

negative energy mode

$$e^{+i\omega\eta}$$

spatial derivatives

quantization.

Noether theorem — conserved energy  
— time translation invariance

Poincaré — 4 exist globally timelike Killing vector

$$\nabla_{\mu} K_{\nu} + \nabla_{\nu} K_{\mu} = 0$$



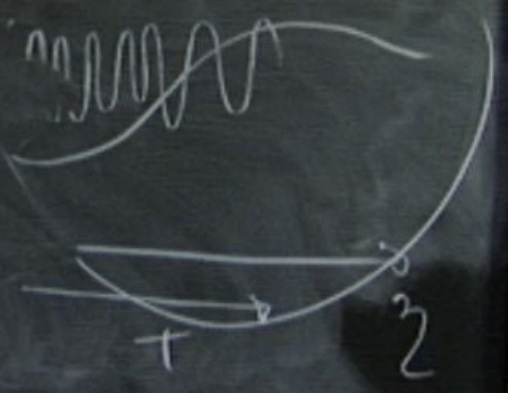
Evidence  
for Atoms

Big Is A  
Molecule!

Inflation



$\frac{1}{\omega_h}$  ~ time scale  
oscillations.

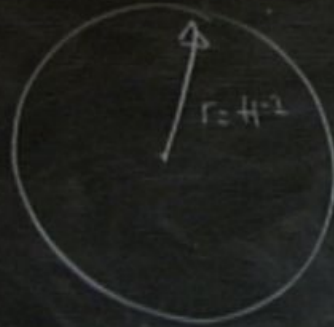


Evidence  
for Atoms

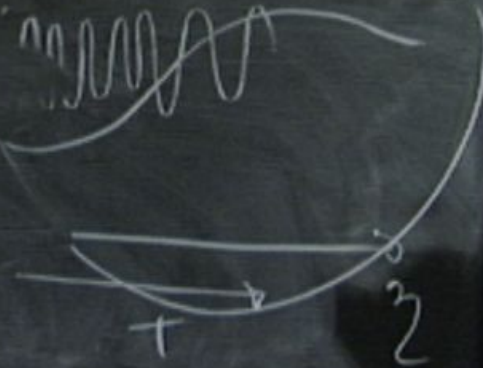
Big Is A  
Molecule?

Inflation

$$d\vec{r} = e^{i\vec{k}\cdot\vec{r}} d\vec{r}$$
$$(-i\vec{k} = d\vec{k})$$



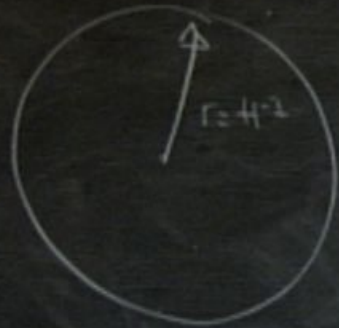
$\frac{1}{\omega_{\vec{k}}} \sim$  time scale  
oscillations.





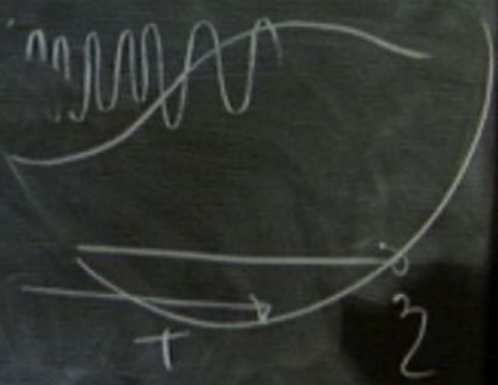
Inflation

$$ds^2 = -dt^2 + e^{int} da^2$$
$$= \frac{1}{(H_0)^2} (-dy^2 + dz^2)$$



$$R \sim H^2$$

$\frac{1}{H_0} \sim$  time scale oscillation.



Evidence for Atoms  
Big Is A Molecule!

u+

Evidence  
for Atoms

Big Is A  
Molecule!

Inflator

No. of  
positive and  
negative

$$R \sim H^2$$

$\frac{1}{\omega_H} \sim$  time scale  
oscillations

$$ds^2 = -dt^2 + e^{2\alpha} dx^2$$
$$= \frac{1}{(\omega_H)^2} \left( -d\tau^2 + dx^2 \right)$$





Evidence  
for Atoms

Big Is A  
Molecule!

Inflation

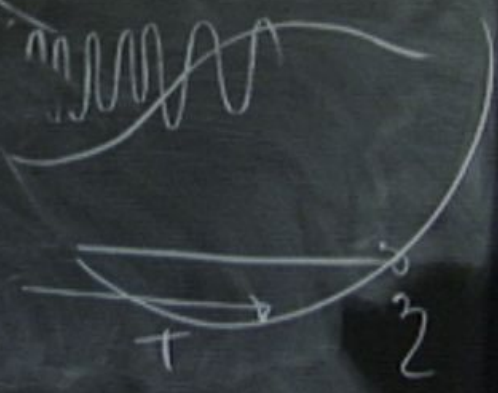
no normal  
level  
negative

$$ds^2 = -dt^2 + e^{2\alpha} dx^2$$
$$= \frac{1}{(H_0)^2} (-dt^2 + dx^2)$$

$$R \sim H^2$$

$\frac{1}{H_0} \sim$  time scale  
oscillations

~~$\omega_{in} \sim H_0$~~

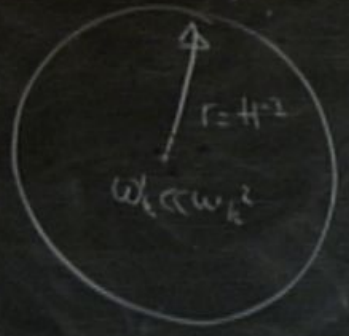


Inflation

$$ds^2 = -dt^2 + e^{Ht} d\vec{x}^2$$

$$= \frac{1}{(H_0)^2} (-dy^0 + dy^i)^2$$

No ratio of positive and negative



$$R \sim H^{-2}$$

$\frac{1}{\omega_k} \sim$  time scale oscillations.

