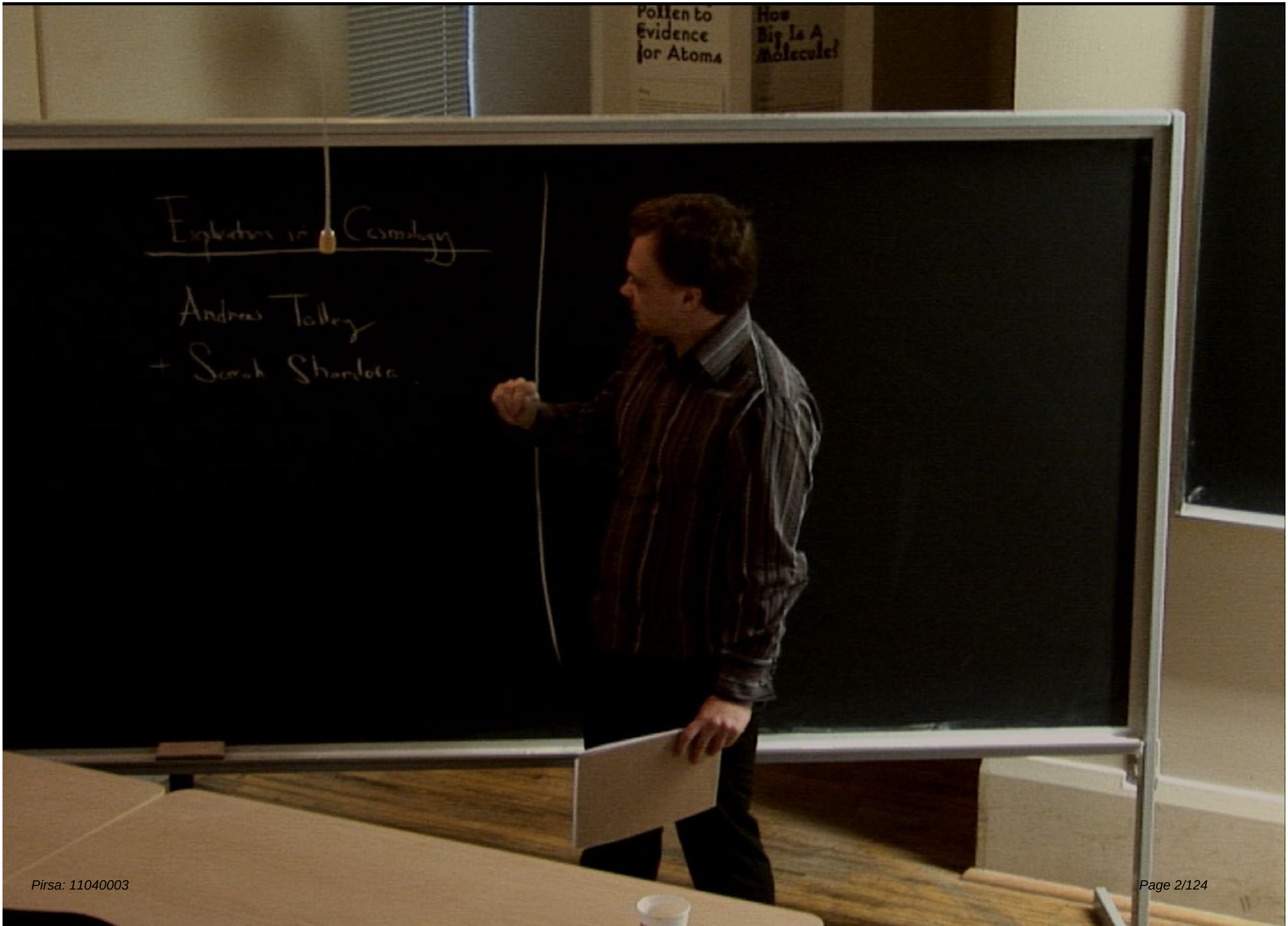
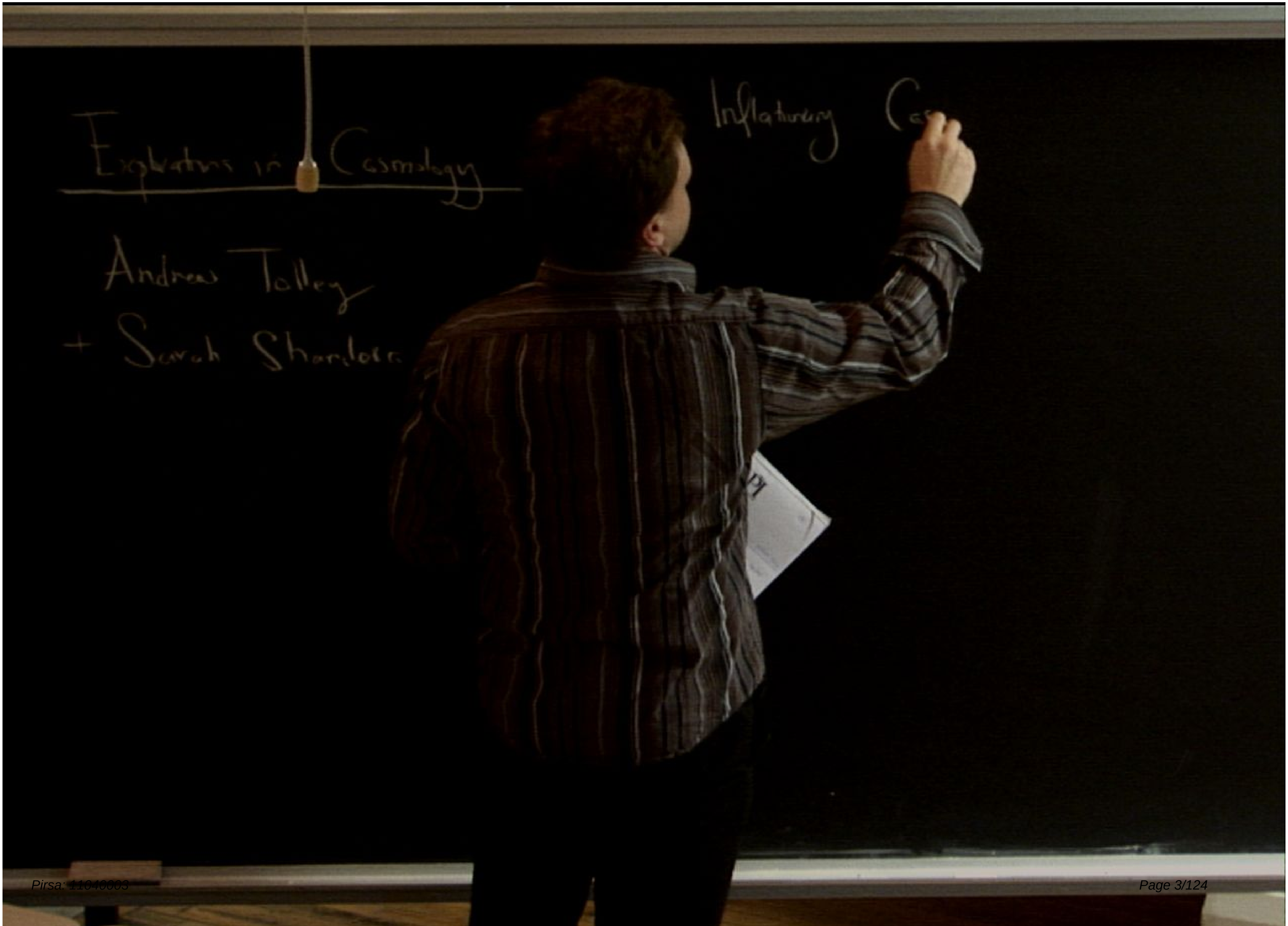
Title: Explorations in Cosmology - Lecture 1

Date: Apr 04, 2011 09:00 AM

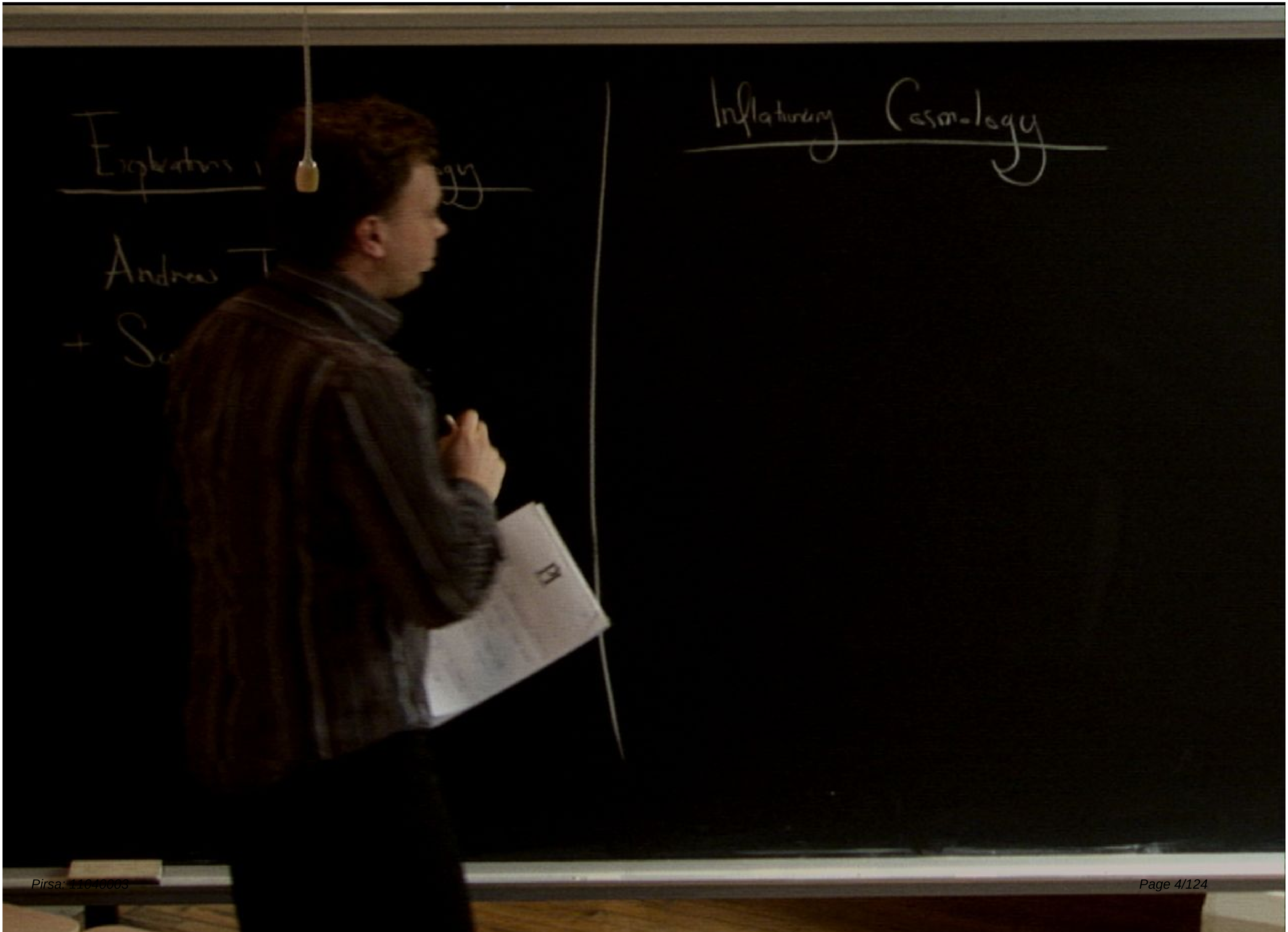
URL: http://pirsa.org/11040003

Abstract:


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Explorations in Cosmology
Andrew Tolley
+ Sarah Shandera



Explorations in Cosmology
Andrew Tolley
+ Sarah Shandera
Inflationary



Inflationary Cosmology
Andrew

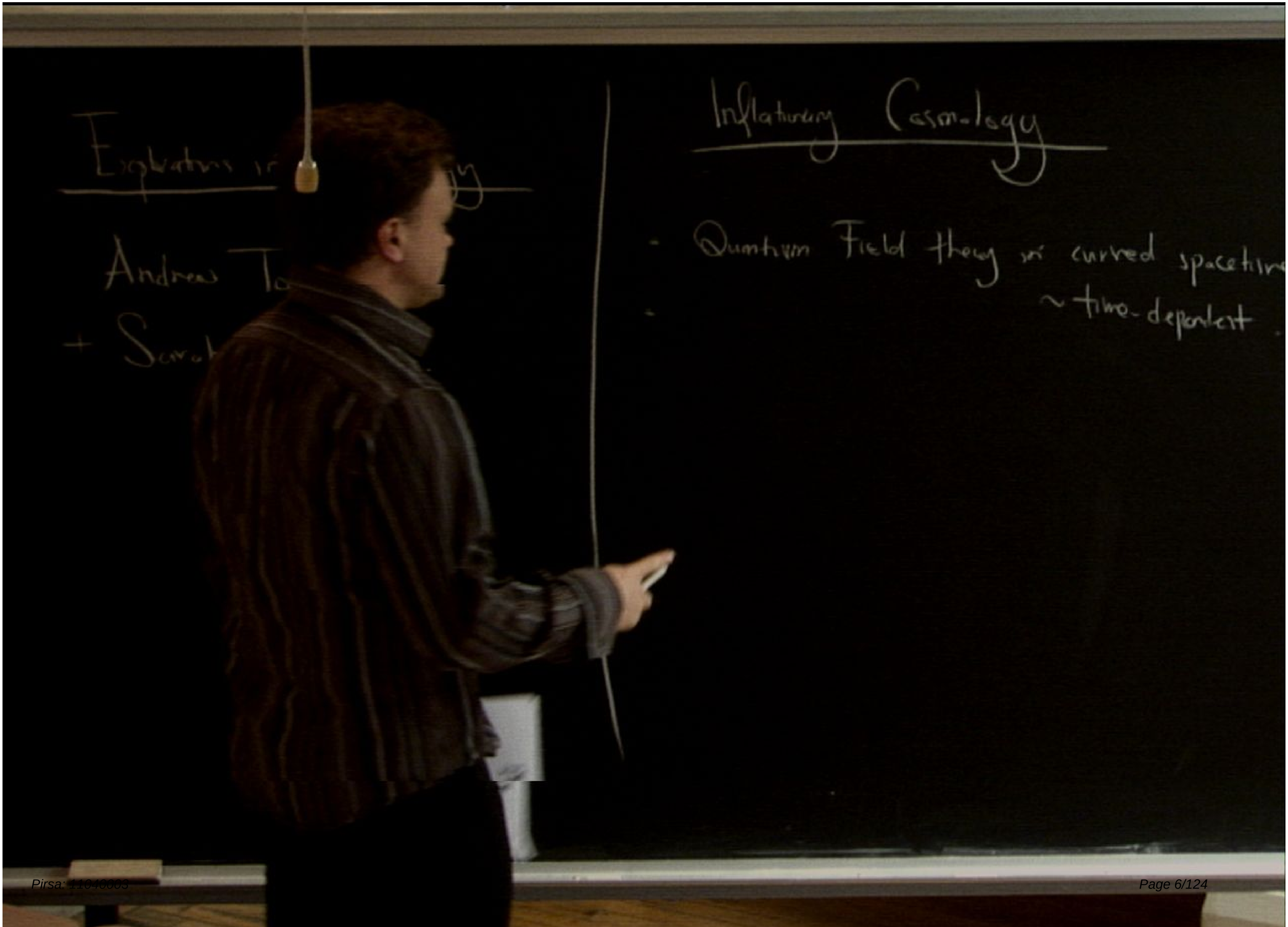
Explorations in Cosmology

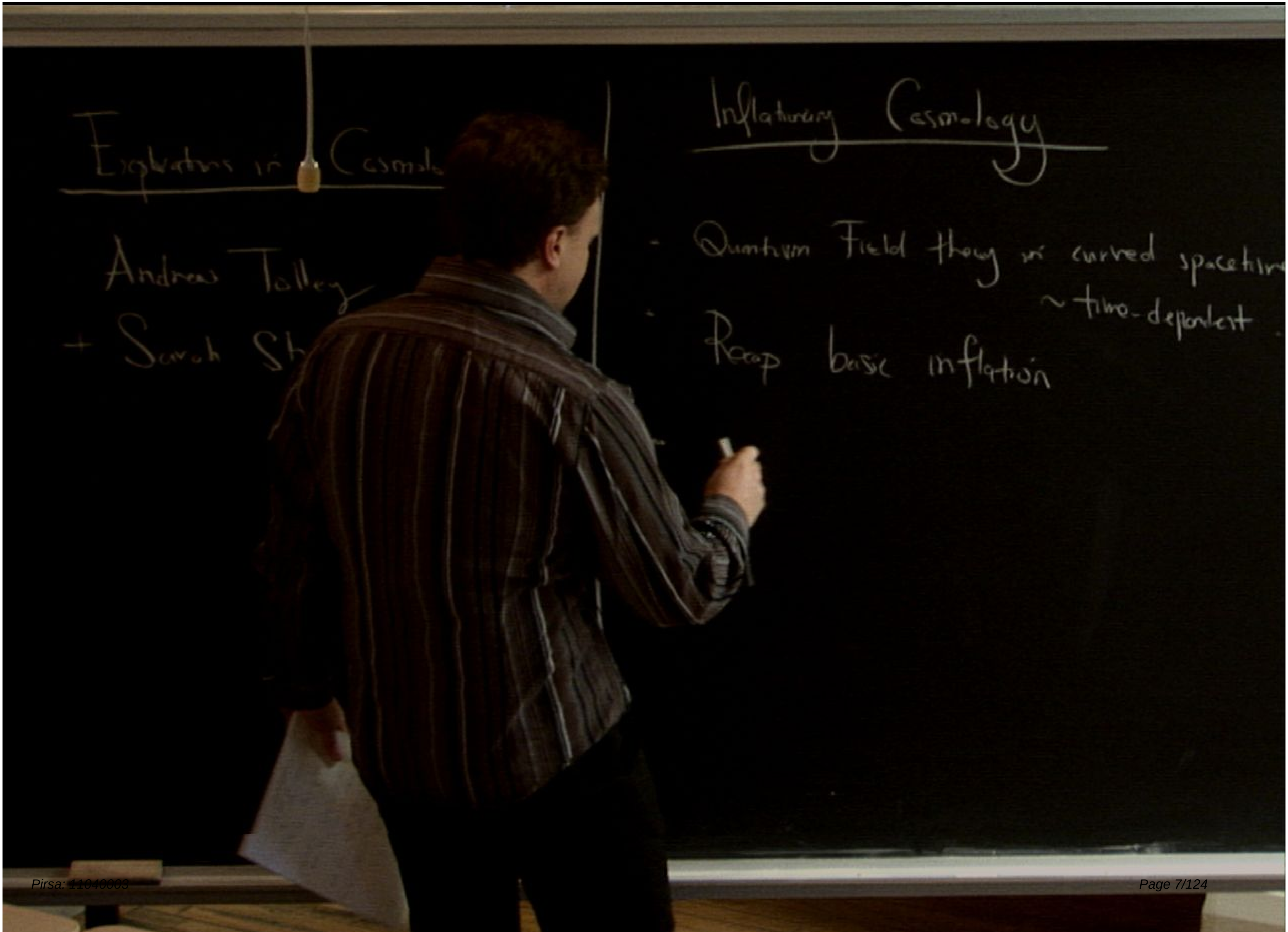
Andrew Tolley

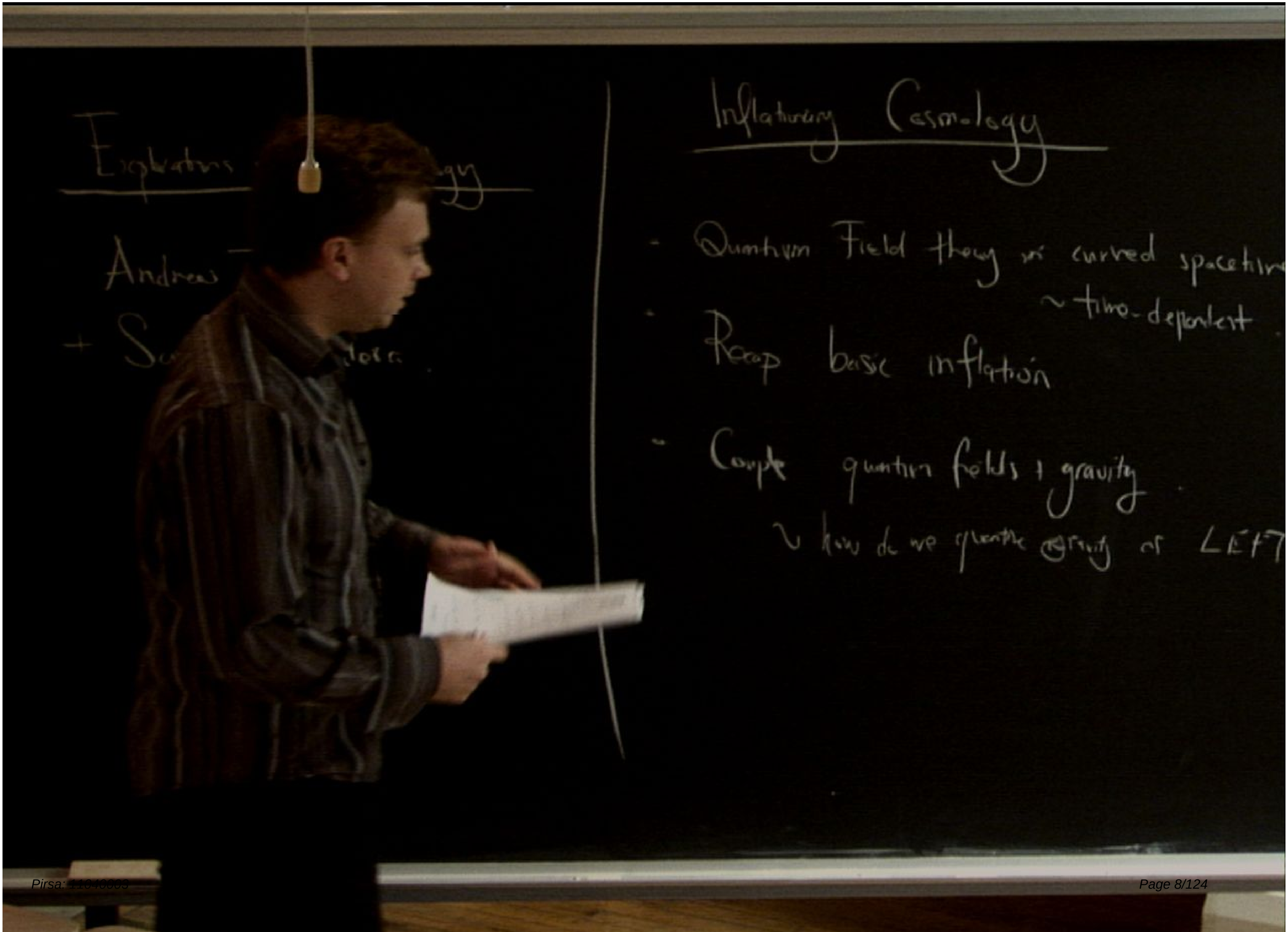
+ Sarah Shandera.

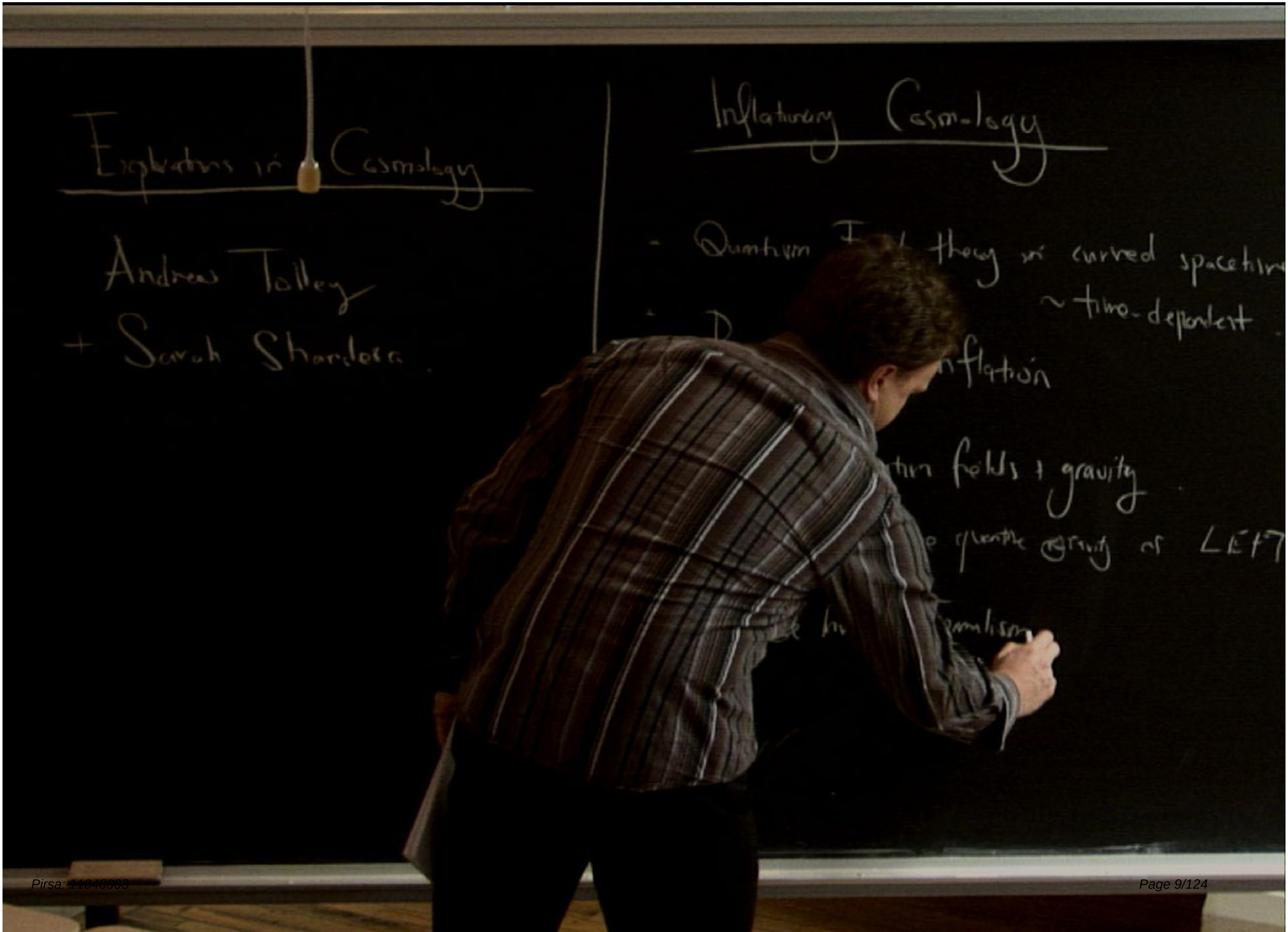
Inflationary Cosmology

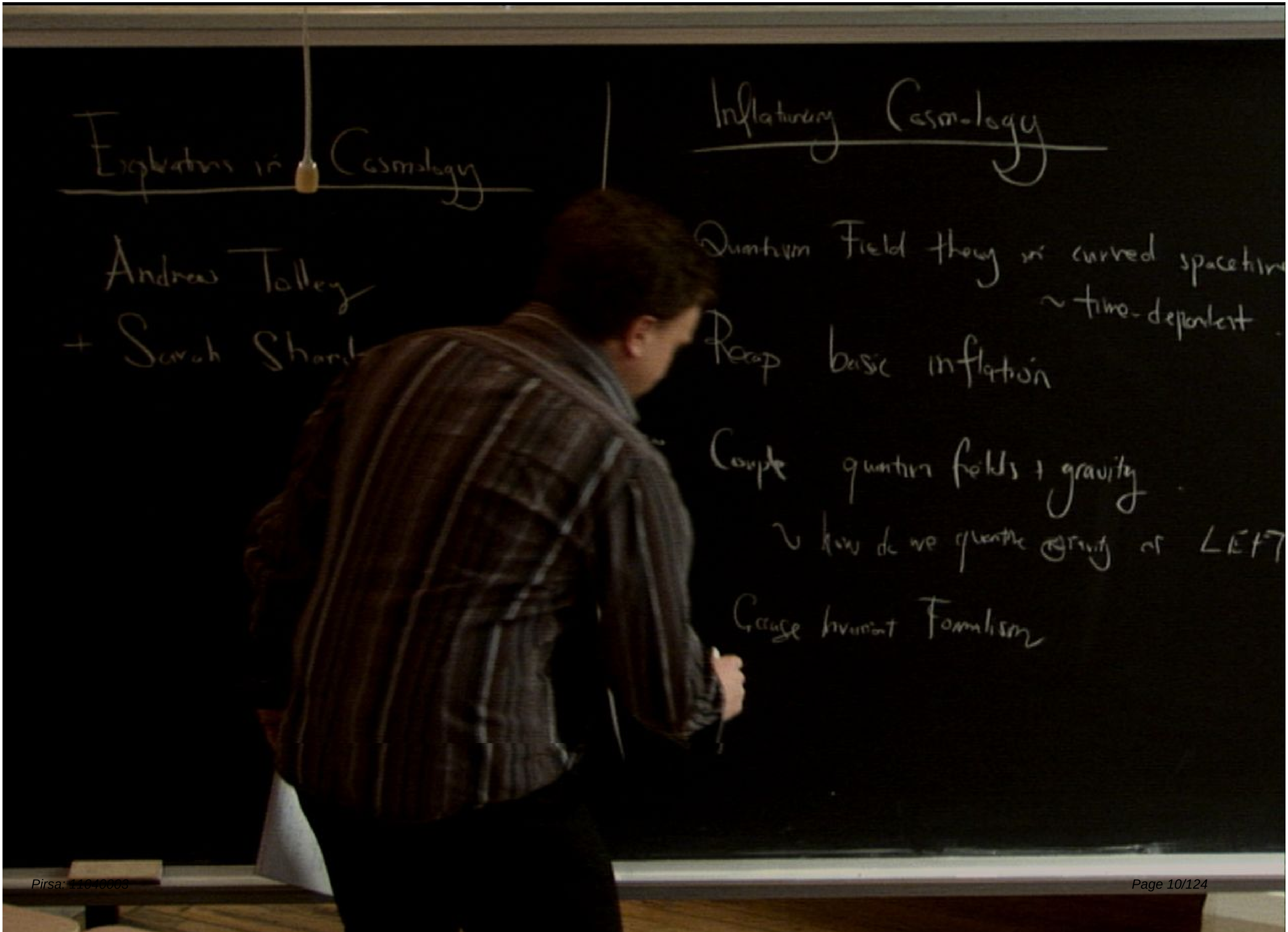
- Quantum Field theory in curved spacetime

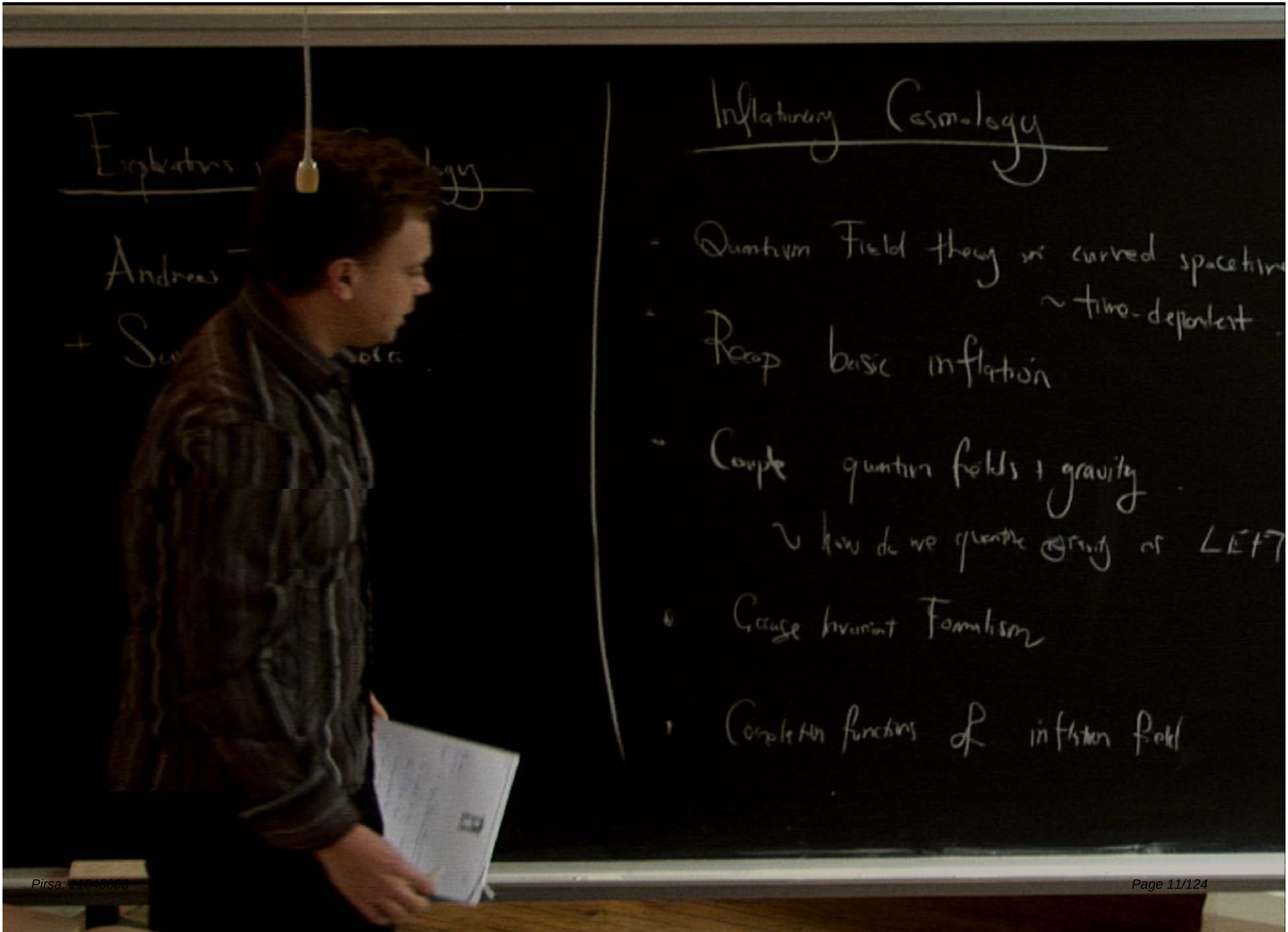

Inflationary Cosmology
- Quantum Field theory in curved spacetime
~ time-dependent
Andrew
+ Sarah

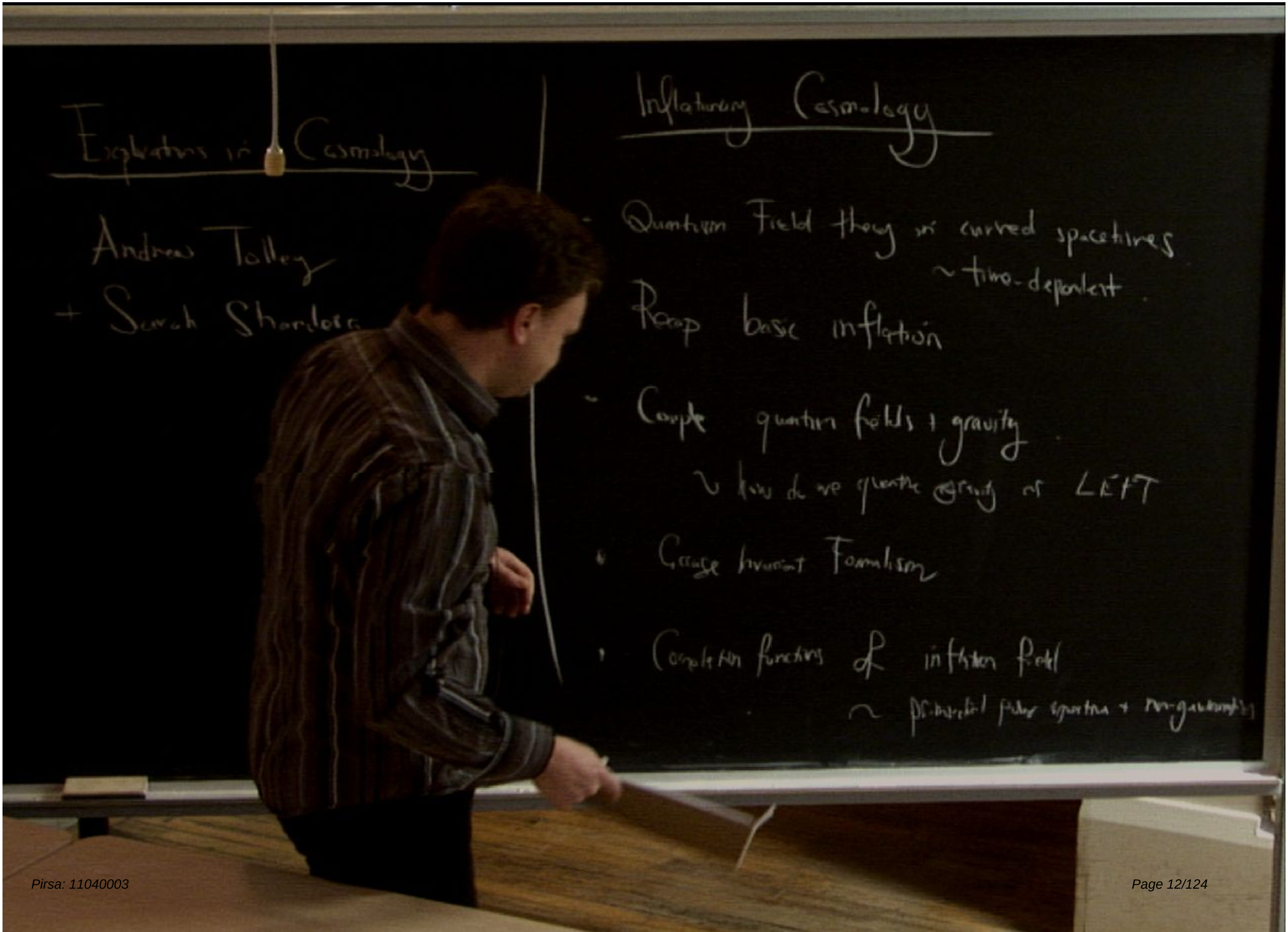

Inflationary Cosmology
- Quantum Field theory in curved spacetime
~ time-dependent
- Recap basic inflation
Andrew Tolley


Inflationary Cosmology
Andrew
- Quantum Field theory in curved
~ time-dependent
- Recap basic inflation
- Couple quantum fields + gravity.
~ how do we quantize gravity or


Explorations in Cosmology
Andrew Tolley
+ Sarah Shandera
Inflationary Cosmology
- Quantum
theory in curved spacetime
~ time-dependent
nflation
fields + gravity


Explorations in Cosmology
Andrew Tolley
+ Sarah Shand
Inflationary Cosmology
Quantum Field theory in curved spacetime
~ time-dependent
Recap basic inflation
Couple quantum fields + gravity
~ how do we quantize gravity or LEFT
Gauge Invariant Formalism


Inflationary Cosmology
Quantum Field theory in curved spacetime
~ time-dependent
Recap basic inflation
Couple quantum fields + gravity
Gauge Invariant Formalism
Correlation functions of inflaton field
Andrew


Explorations in Cosmology
Andrew Tolley
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Inflationary Cosmology
Quantum Field theory in curved spacetimes
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Recap basic inflation
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Correlation functions of inflaton field

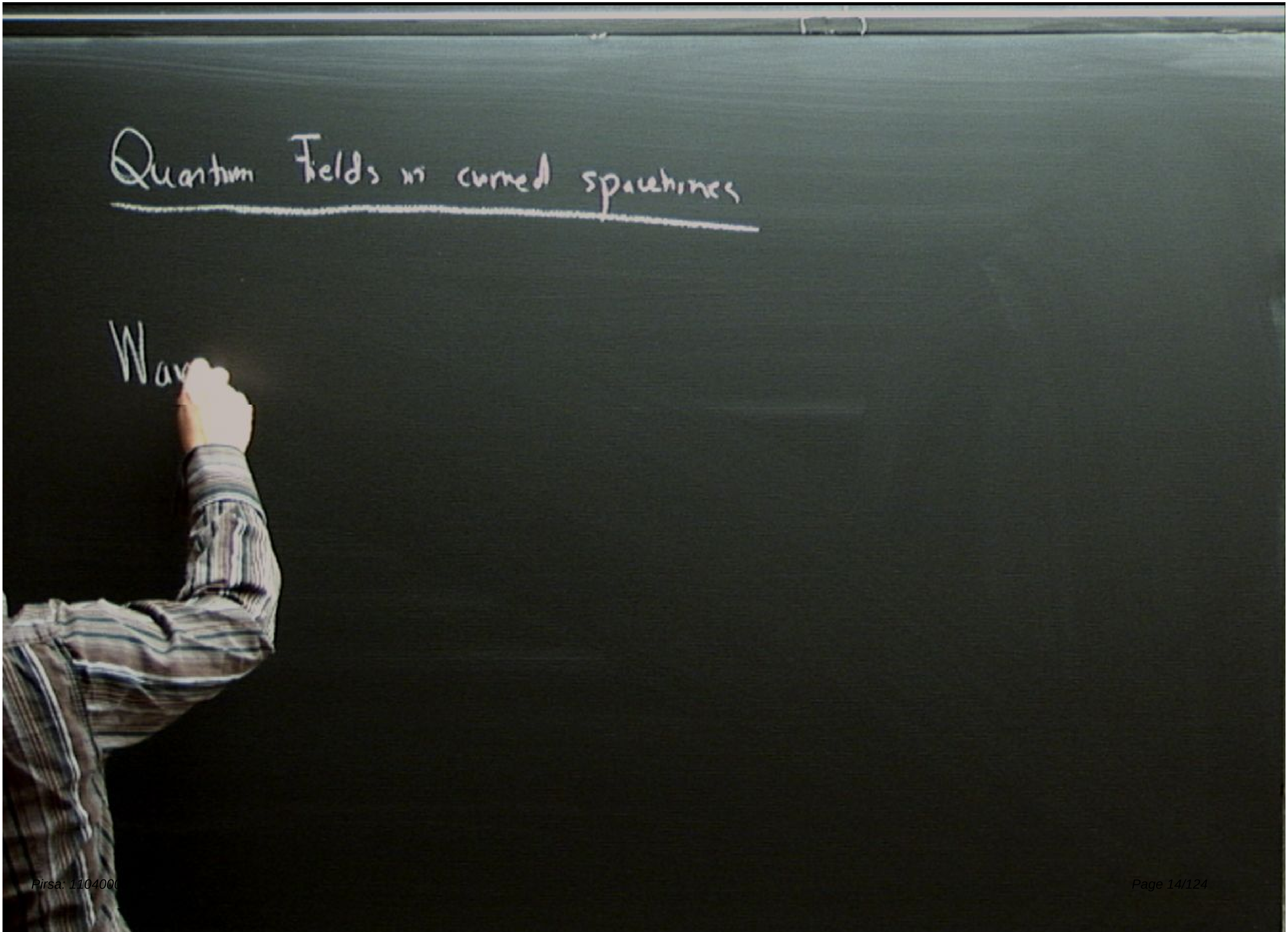
Explorations in Cosmology

Andrew Tolley

+ Sarah Shandera.

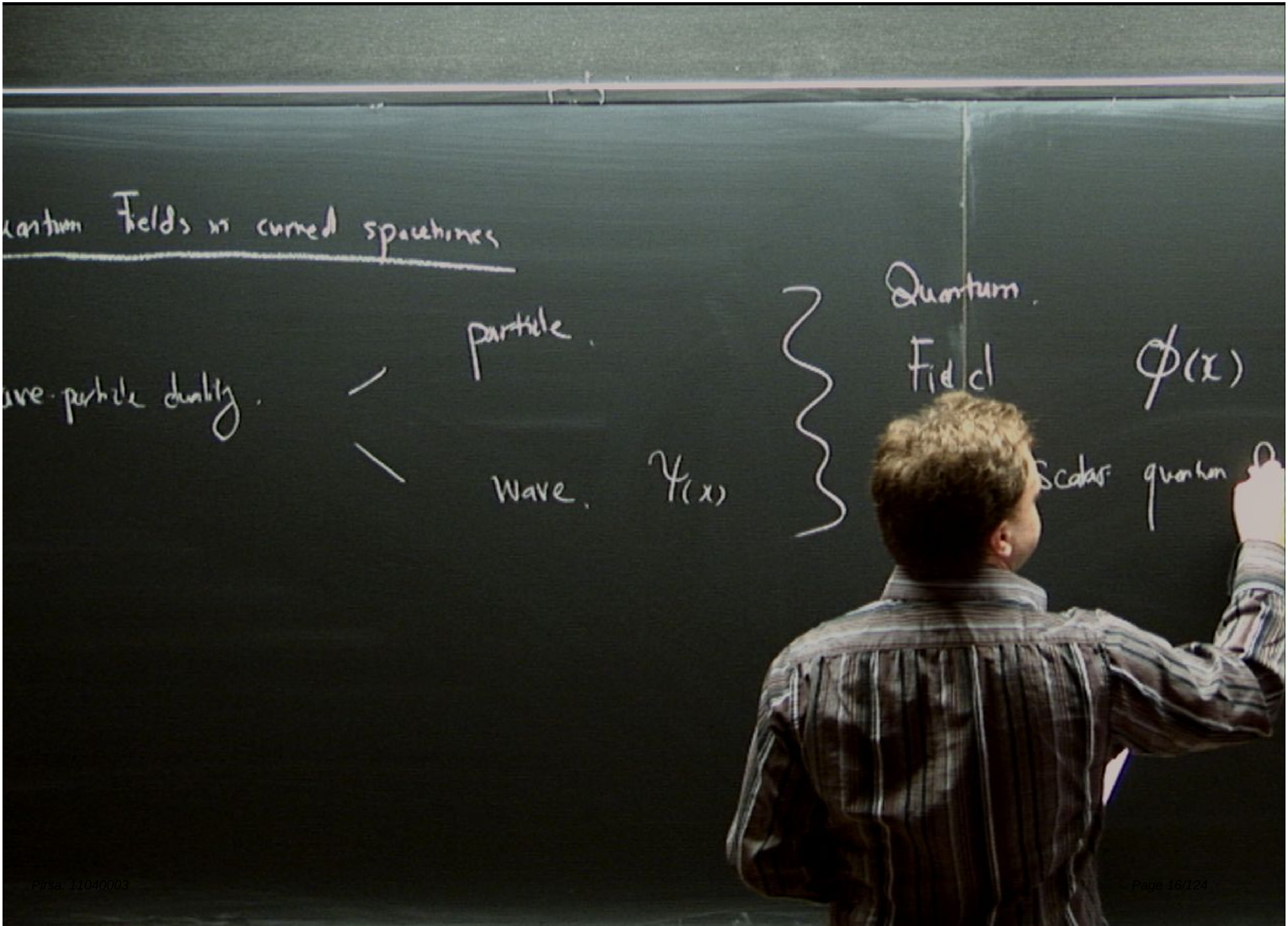
Inflationary Cosmology

- Quantum Field theory in curved spacetimes
  ~ time-dependent.
- Recap basic inflation
- Couple quantum fields + gravity.
  ~ how do we quantize gravity or LEFT
- Gauge Invariant Formalism
- Correlation functions of inflaton field
  ~ primordial power spectrum + non-gaussianity


Quantum Fields in curved spacetimes
Wa

Quantum Fields in curved spacetimes

Wave-particle duality. — particle.


uantum Fields in curved spacetimes
ave-particle duality.
particle.
wave. $\Psi(x)$
Quantum
Field $\phi(x)$
Scalar quantum

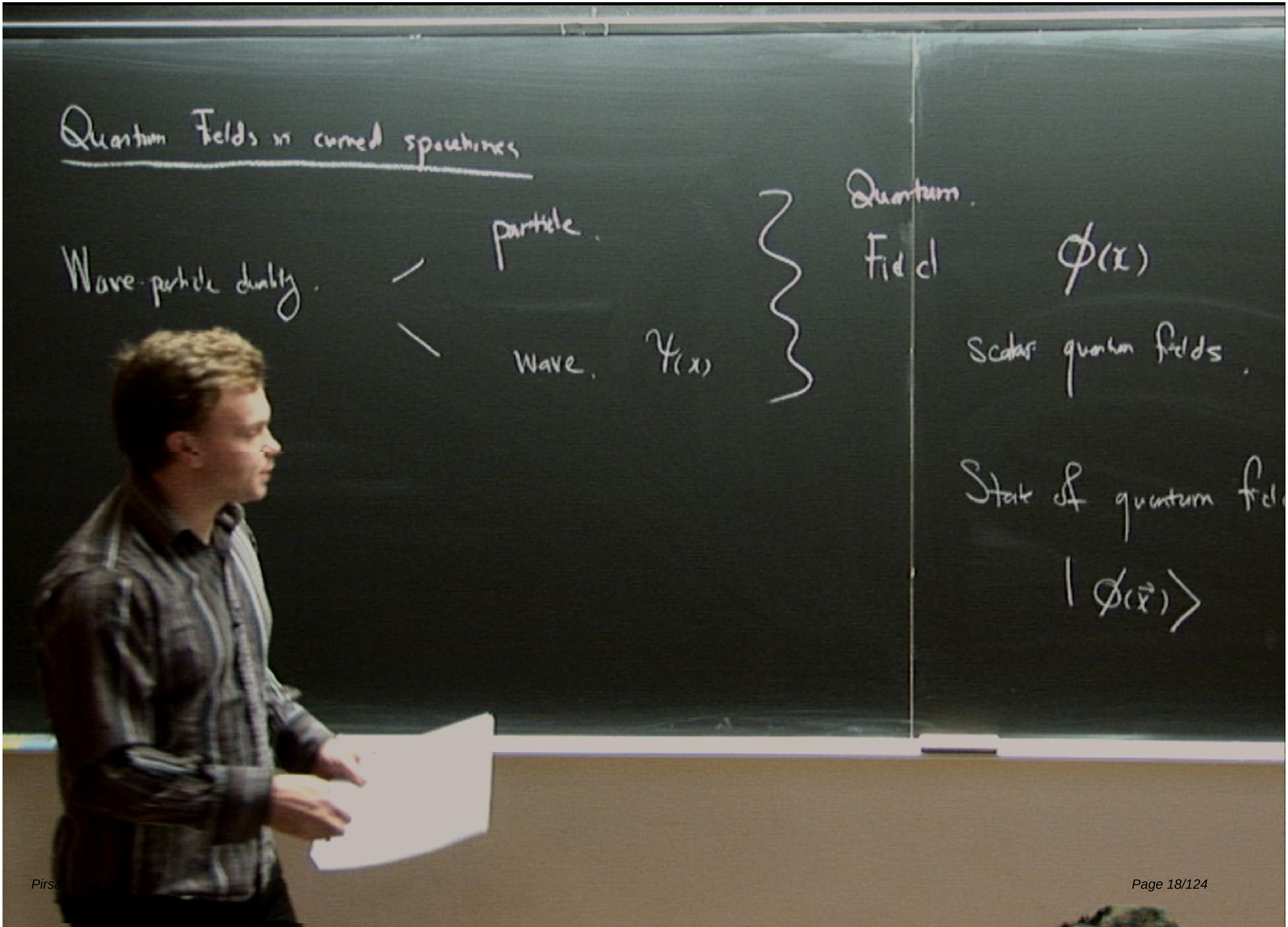
Quantum Fields in curved spacetimes

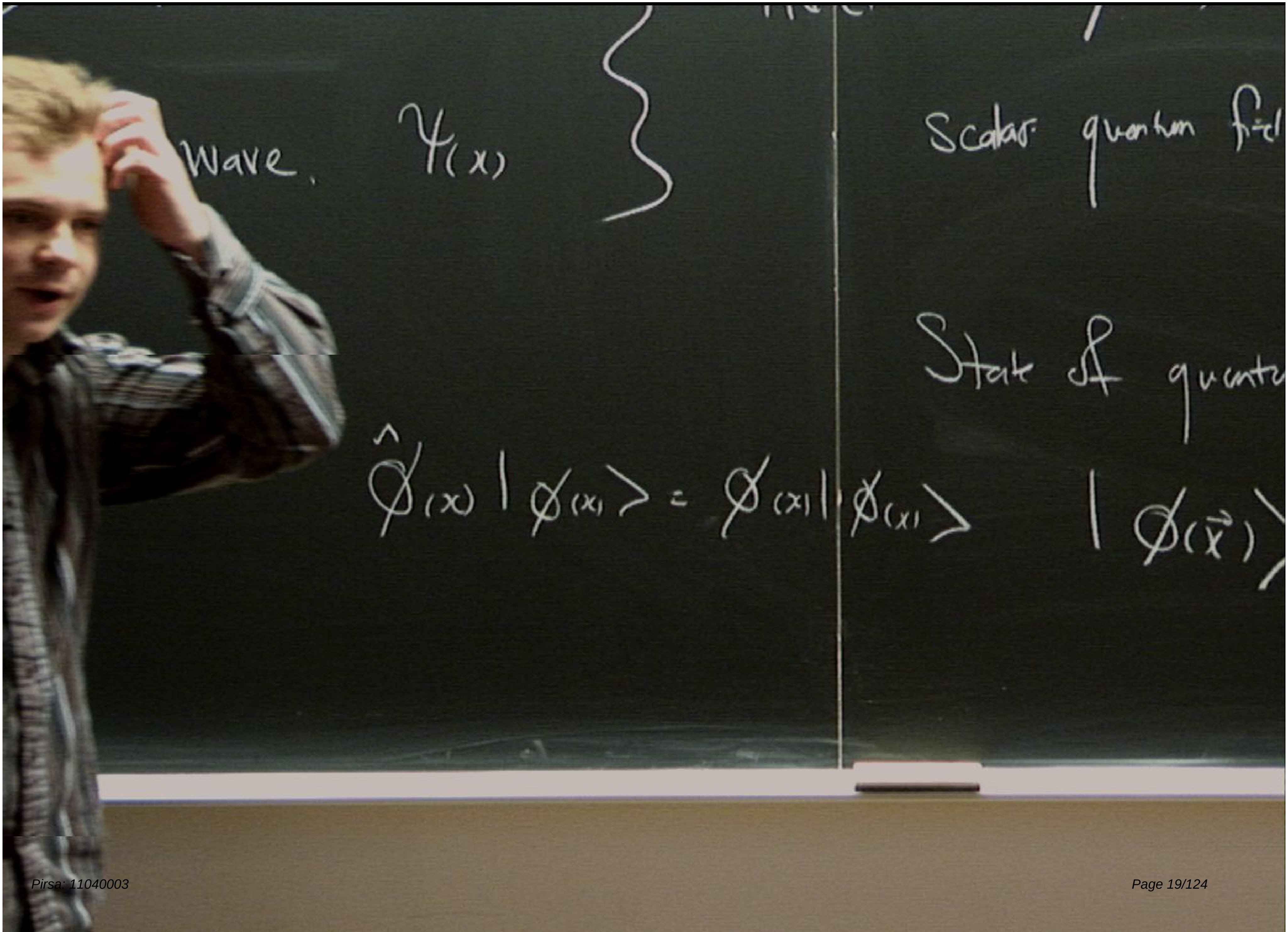
Wave-particle duality.

- particle.
- wave. $\psi(x)$

Quantum Field $\phi(x)$

Scalar quantum fields.


Quantum Fields in curved spacetimes
Wave-particle duality.
particle.
wave. $\Psi(x)$
Quantum Field
$\phi(x)$
Scalar quantum fields.
State of quantum fiel
$|\phi(\vec{x})\rangle$


Wave.
$\psi(x)$
Scalar quantum
State of
$\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle$
$|\phi(\vec{x})\rangle$

Minkowski spacetime.

Quantum.

Field $\hat{\phi}(x)$

Scalar quantum fields.

State of quantum field.

$\phi(x)\rangle = \phi(x)|\phi(x)\rangle$ $\quad |\phi(\vec{x})\rangle = \sum$ multiparticle states

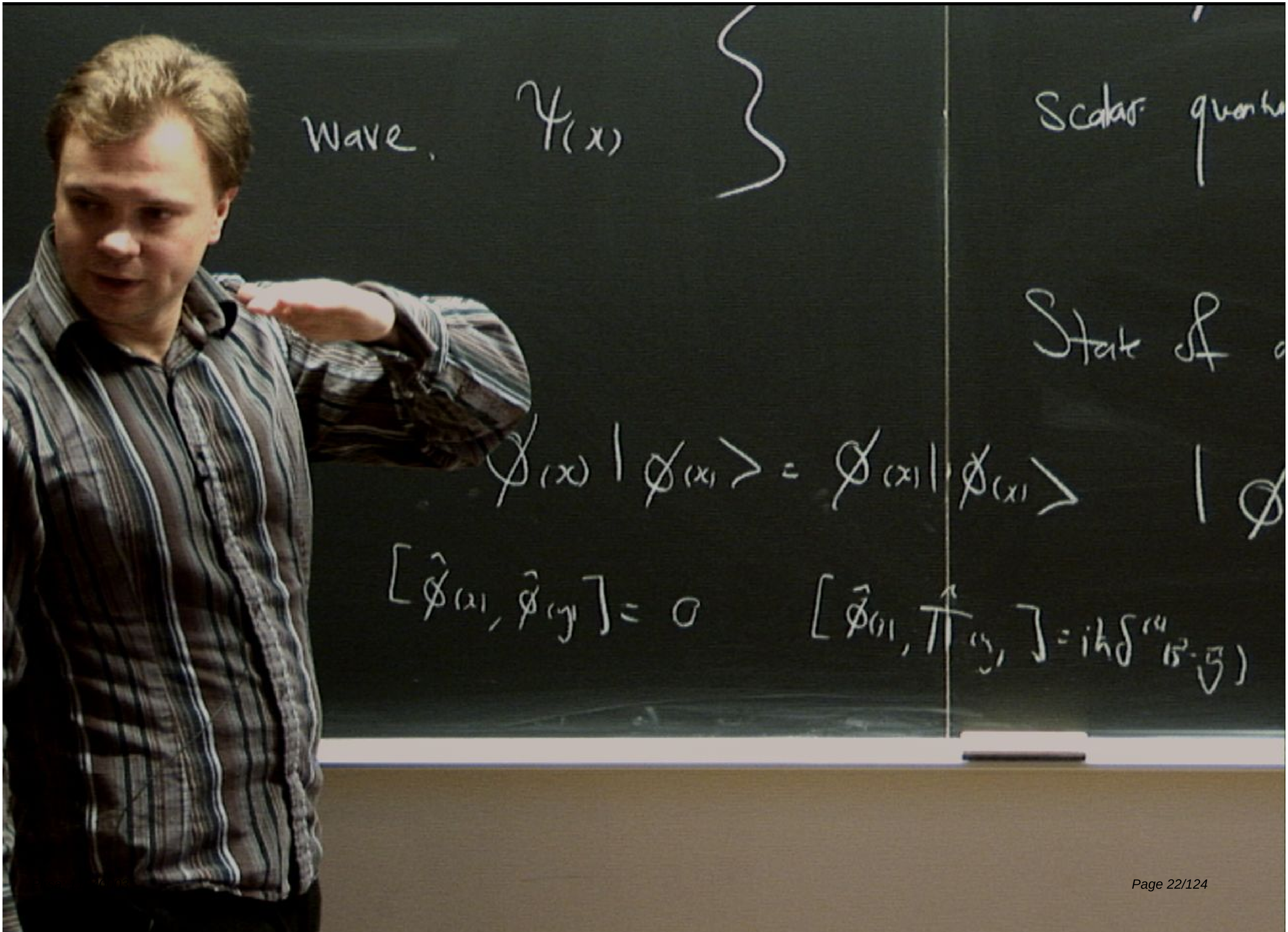
Wave. $\psi(x)$

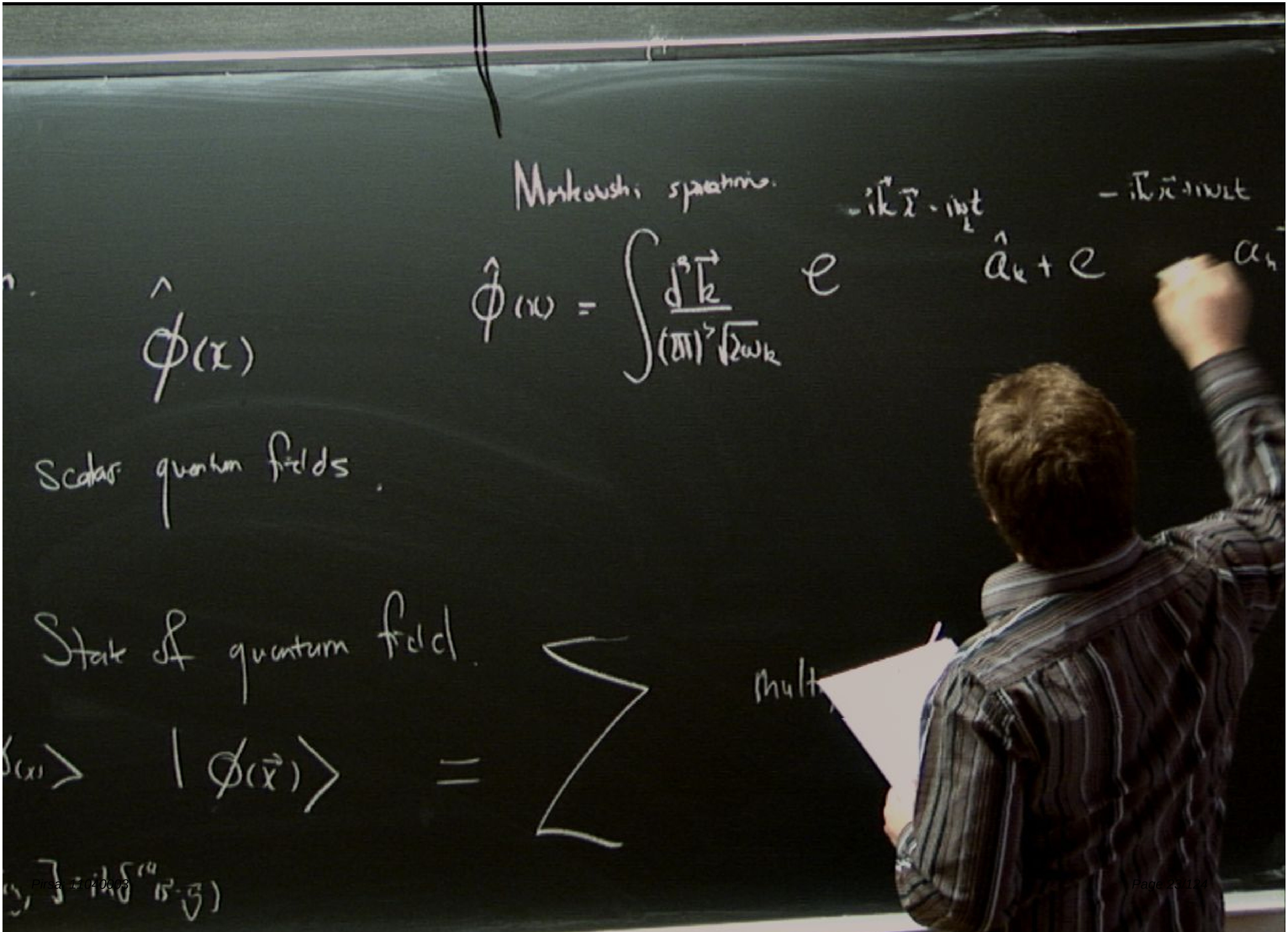
Scalar quantu

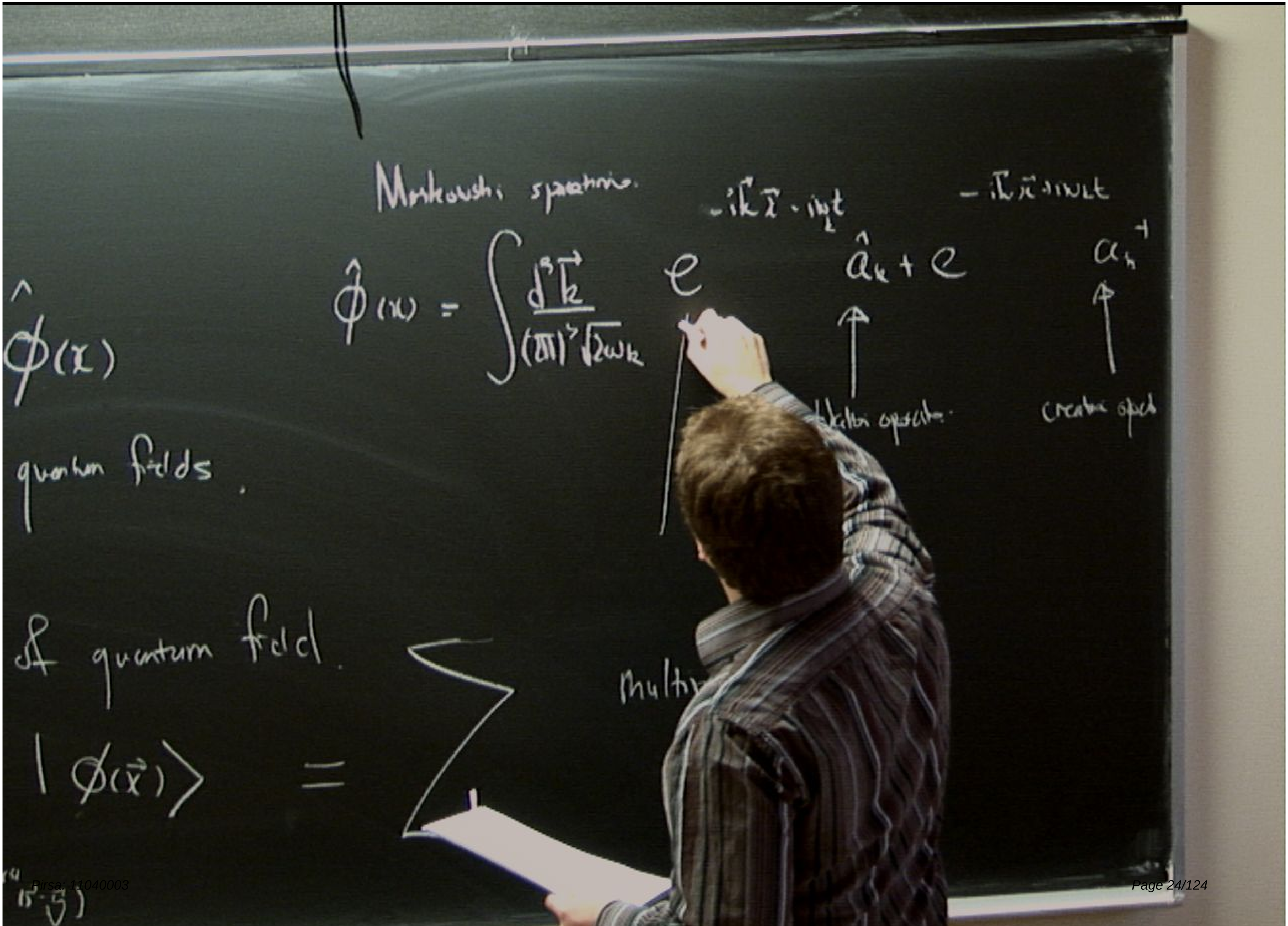
State of a

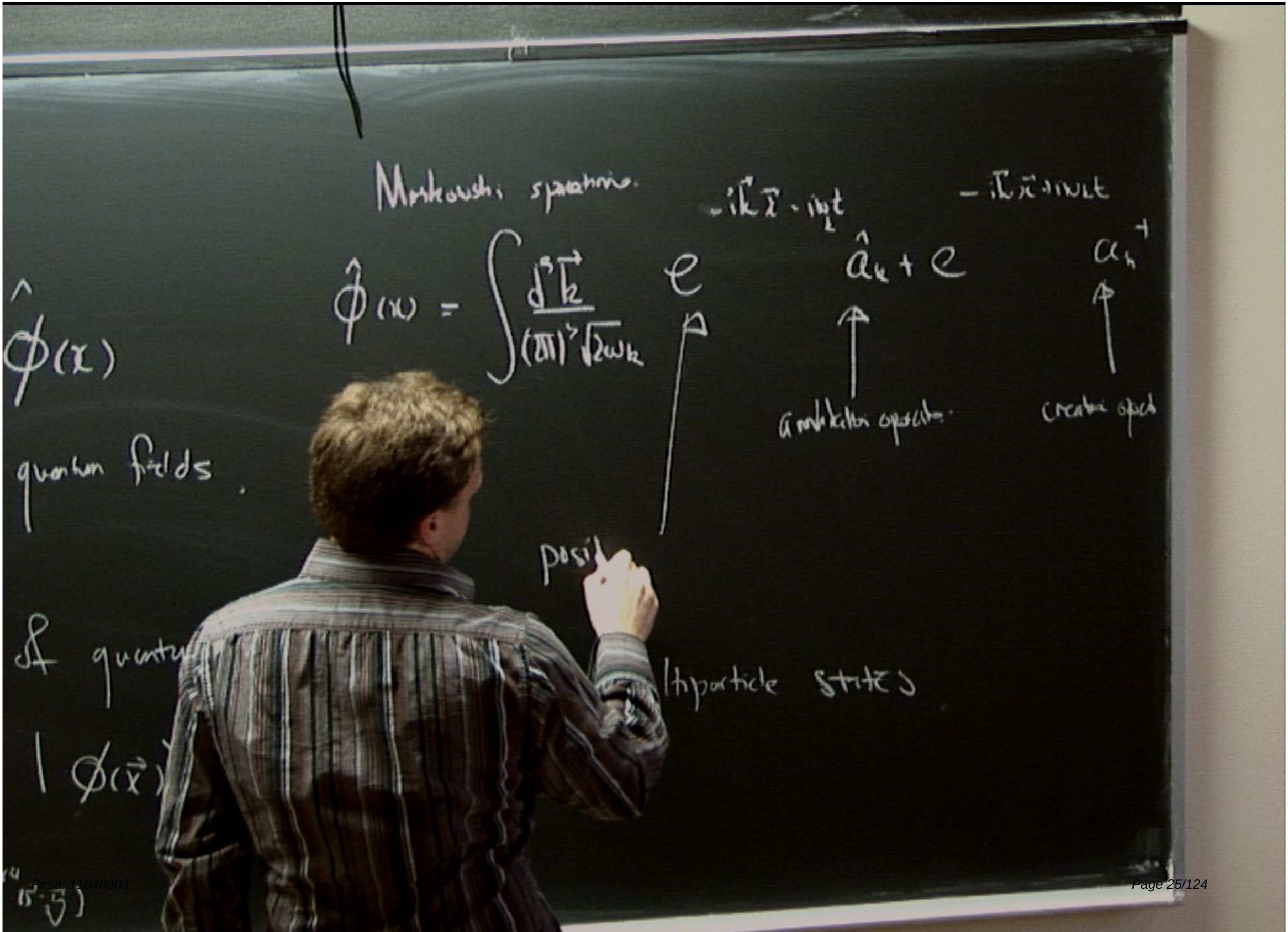
$$\hat{\phi}(x)\,|\phi(x)\rangle = \phi(x)\,|\phi(x)\rangle \qquad |\phi$$

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0$$


Wave. $\Psi(x)$
Scalar
State of
$\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle$
$[\hat{\phi}(x), \hat{\phi}(y)] = 0$
$[\hat{\phi}(x), \hat{\Pi}(y)] = i\hbar\delta^{(3)}(\vec{x}-\vec{y})$


Minkowski spacetime.
$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} e^{-i\vec{k}\vec{x} - i\omega_k t} \hat{a}_k + e^{-i\vec{k}\vec{x} + i\omega_k t} a_k$
$\hat{\phi}(x)$
Scalar quantum fields.
State of quantum field
$|\phi(\vec{x})\rangle = \sum$
mult


Minkowski spacetime
quantum fields
quantum field
$|\phi(\vec x)\rangle =$


Minkowski spacetime.
annihilation operator
quantum fields

Minkowski spacetime.

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}}\, e^{-i\vec{k}\vec{x} - i\omega_k t}\,\hat{a}_k + e^{-i\vec{k}\vec{x} + i\omega_k t}\,a_k^{\dagger}$$

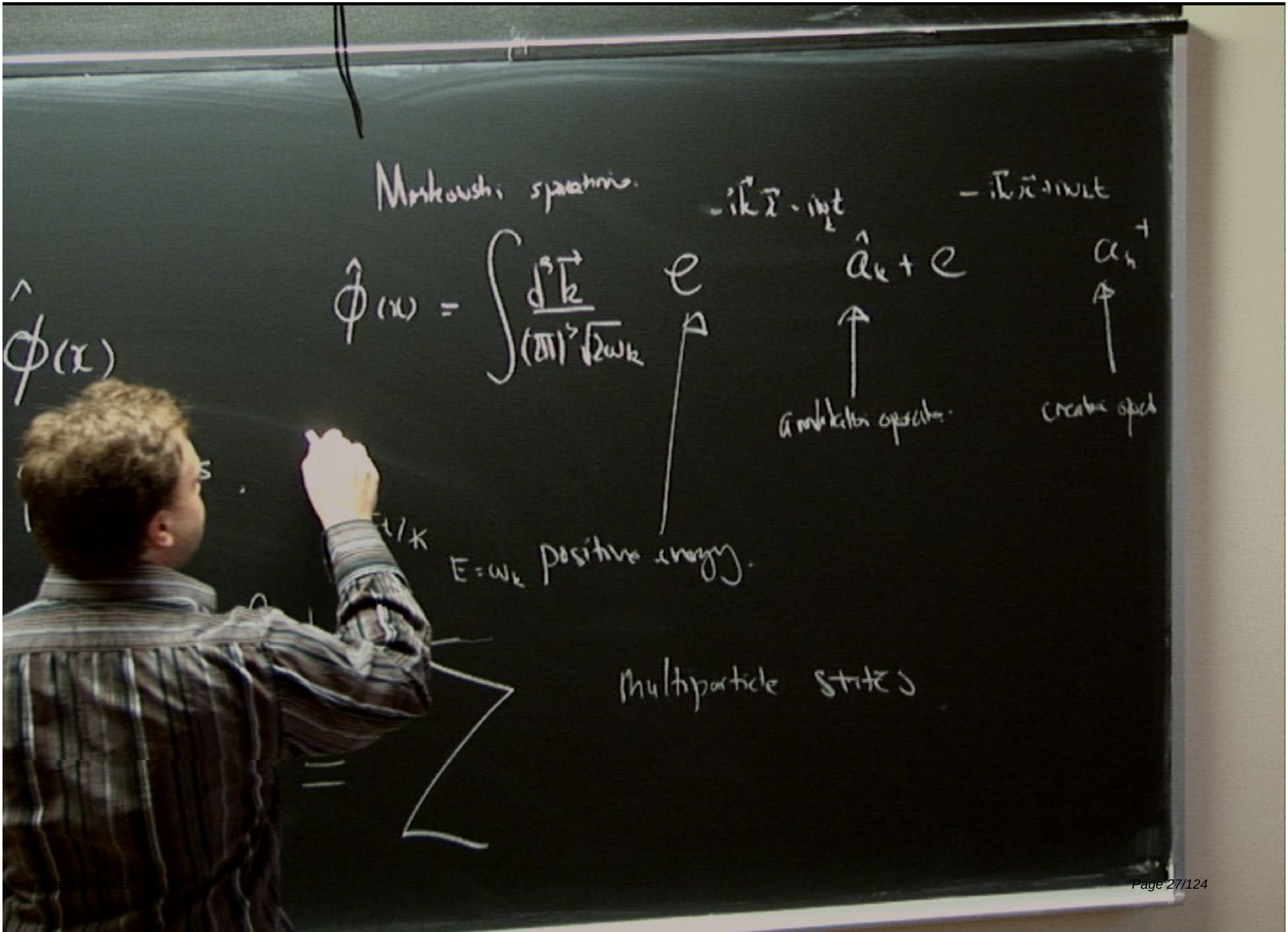
positive energy.

annihilation operator.

creation oper.

$\hat{\phi}(x)$

Multiparticle states


Minkowski spacetime
annihilation operator
creation oper
$E=\omega_k$ positive energy.
multiparticle states

Minkowski spacetime.

$\hat{\phi}(x)$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} \, e^{-i\vec{k}\vec{x} - i\omega_k t} \, \hat{a}_k + e^{-i\vec{k}\vec{x} + i\omega_k t} \, a_k^{+}$$

annihilation operator. creation operator

quantum fields.

$\omega_k^2 = \vec{k}^2 + m^2$

$c = \hbar = 1$

$e^{-iEt/\hbar}$ $E = \omega_k$ positive energy.

of quantum field

$$|\phi(\vec{x})\rangle = \sum$$

multiparticle states

$\hat{\phi}(x)$

quantum fields.

Minkowski spacetime.

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} \left[e^{-i\vec{k}\cdot\vec{x} - i\omega_k t}\,\hat{a}_k + e^{-i\vec{k}\cdot\vec{x} + i\omega_k t}\, a_k^{\dagger} \right]$$

annihilation operator.

creation oper.

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Minkowski spacetime.

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} \left[e^{-i\vec{k}\vec{x} - i\omega_k t}\, \hat{a}_k + e^{-i\vec{k}\vec{x} + i\omega_k t}\, a_k^{\dagger} \right]$$

annihilation operator   creation operator

$\hat{\phi}(x)$

quantum fields.

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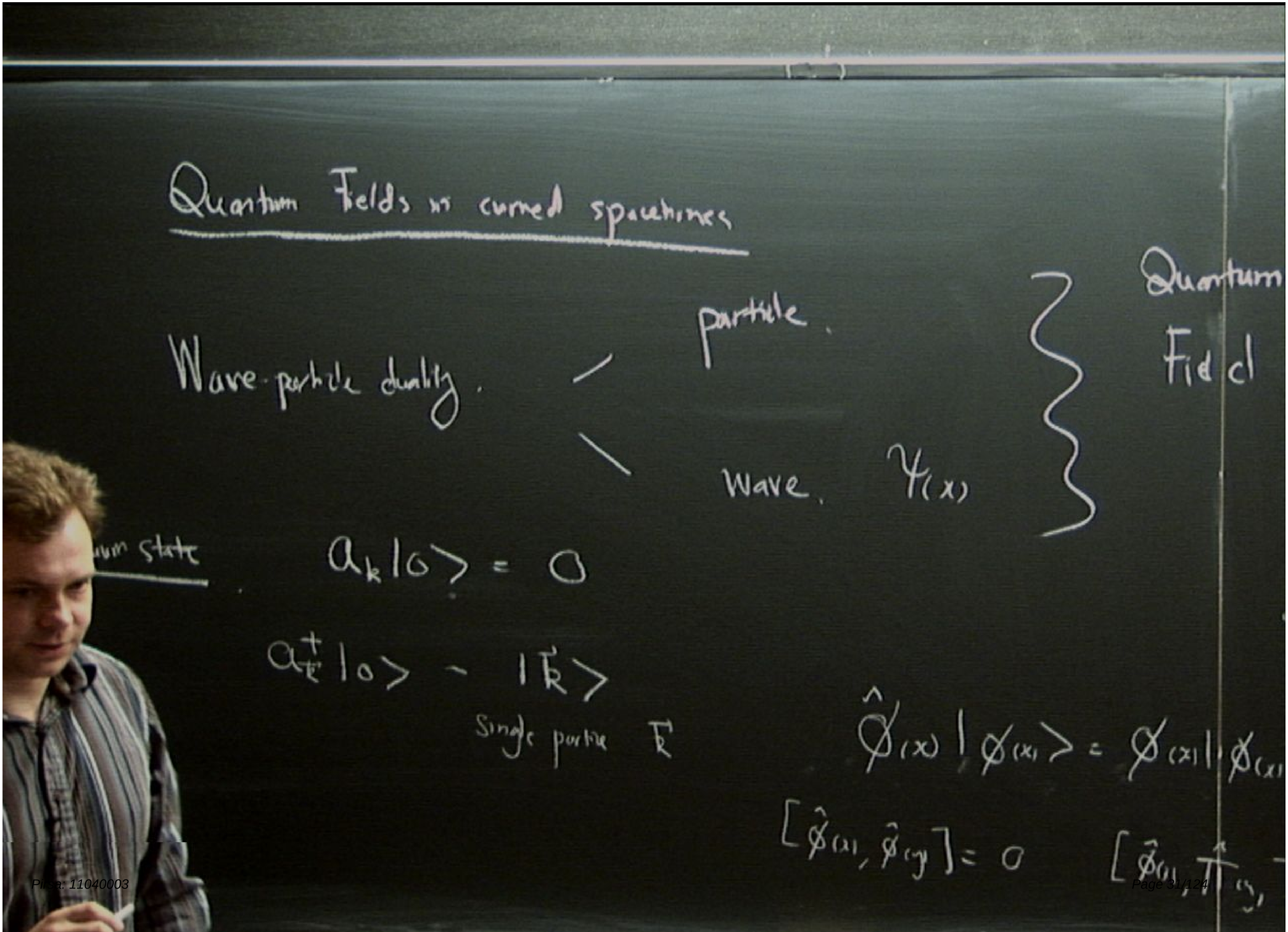
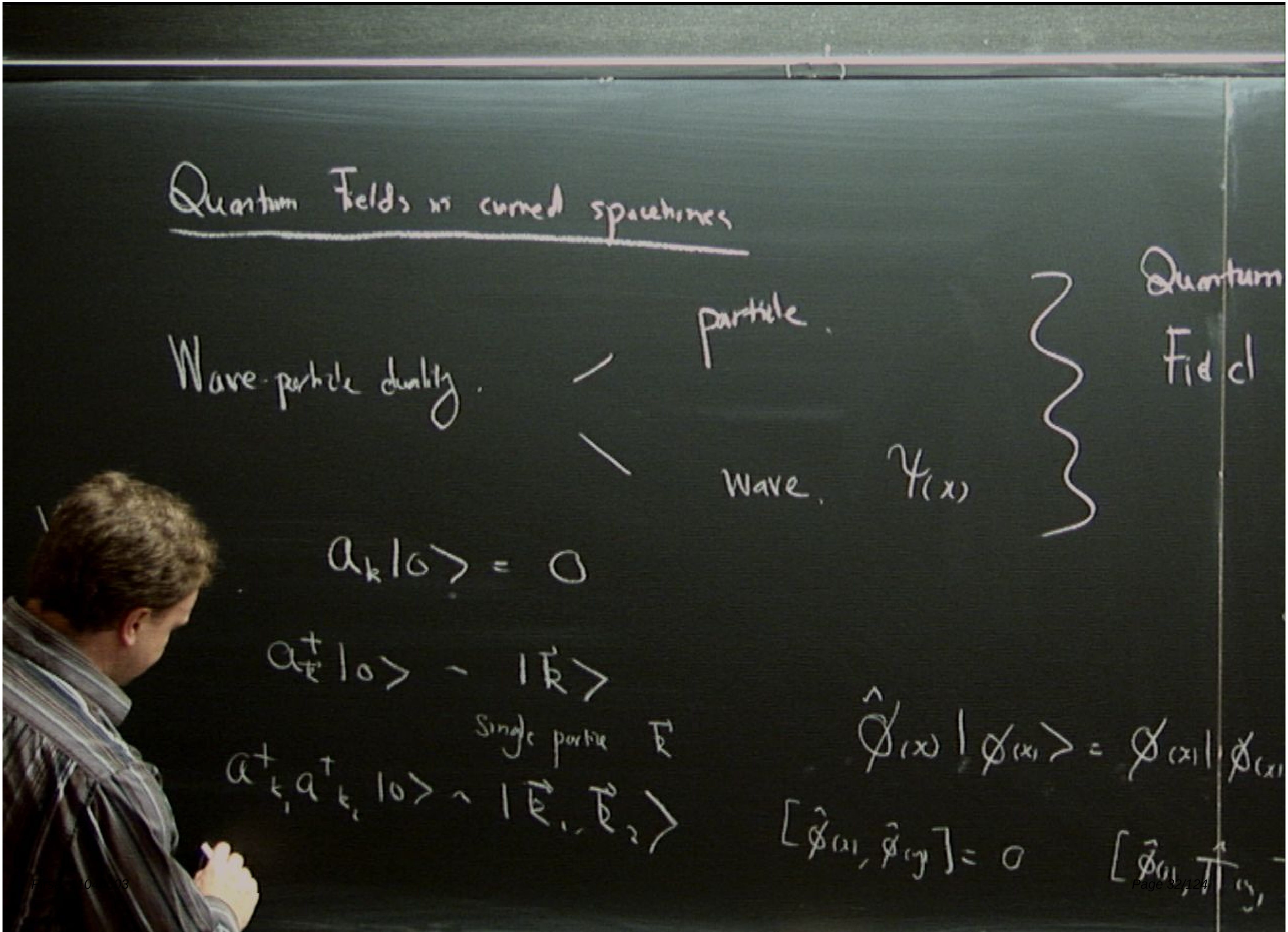

Quantum Fields in curved spacetimes
Wave-particle duality.
particle.
wave. $\Psi(x)$
Quantum
Field
state
$a_k|0\rangle = 0$
$a^\dagger_{\vec{k}}|0\rangle \sim |\vec{k}\rangle$
Single particle $\vec{k}$
$\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle$
$[\hat{\phi}(x), \hat{\phi}(y)] = 0$
$[\hat{\phi}(x), \hat{\Pi}(y)]$


Quantum Fields in curved spacetimes
Wave-particle duality.
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Quantum
Field
$a_k|0\rangle = 0$
$a^\dagger_{\vec{k}}|0\rangle \sim |\vec{k}\rangle$
Single particle $\vec{k}$
$a^\dagger_{k_1} a^\dagger_{k_2} |0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$
$\hat{\phi}(x)\,|\phi(x)\rangle = \phi(x)\,|\phi(x)\rangle$
$[\hat{\phi}(x), \hat{\phi}(y)] = 0$
$[\hat{\phi}(x), \hat{\pi}(y)]$

Quantum Fields in curved spacetimes

Wave-particle duality. — particle. — wave. $\Psi(x)$ } Quantum Field

Vacuum state

$a_k|0\rangle = 0$

$a^\dagger_{\vec{k}}|0\rangle \sim |\vec{k}\rangle$

Single particle $\vec{k}$

$a^\dagger_{k_1} a^\dagger_{k_2}|0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$

n-particle $(a^\dagger)^n|0\rangle$

$\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle$

$[\hat{\phi}(x), \hat{\phi}(y)] = 0$ $[\hat{\phi}(x), \hat{\pi}(y)$

Quantum Fields in curved spacetimes

Wave-particle duality. particle. wave. $\psi(x)$ Quantum Field

Vacuum state $a_k|0\rangle = 0$

$a^\dagger_{\vec{k}}|0\rangle \sim |\vec{k}\rangle$

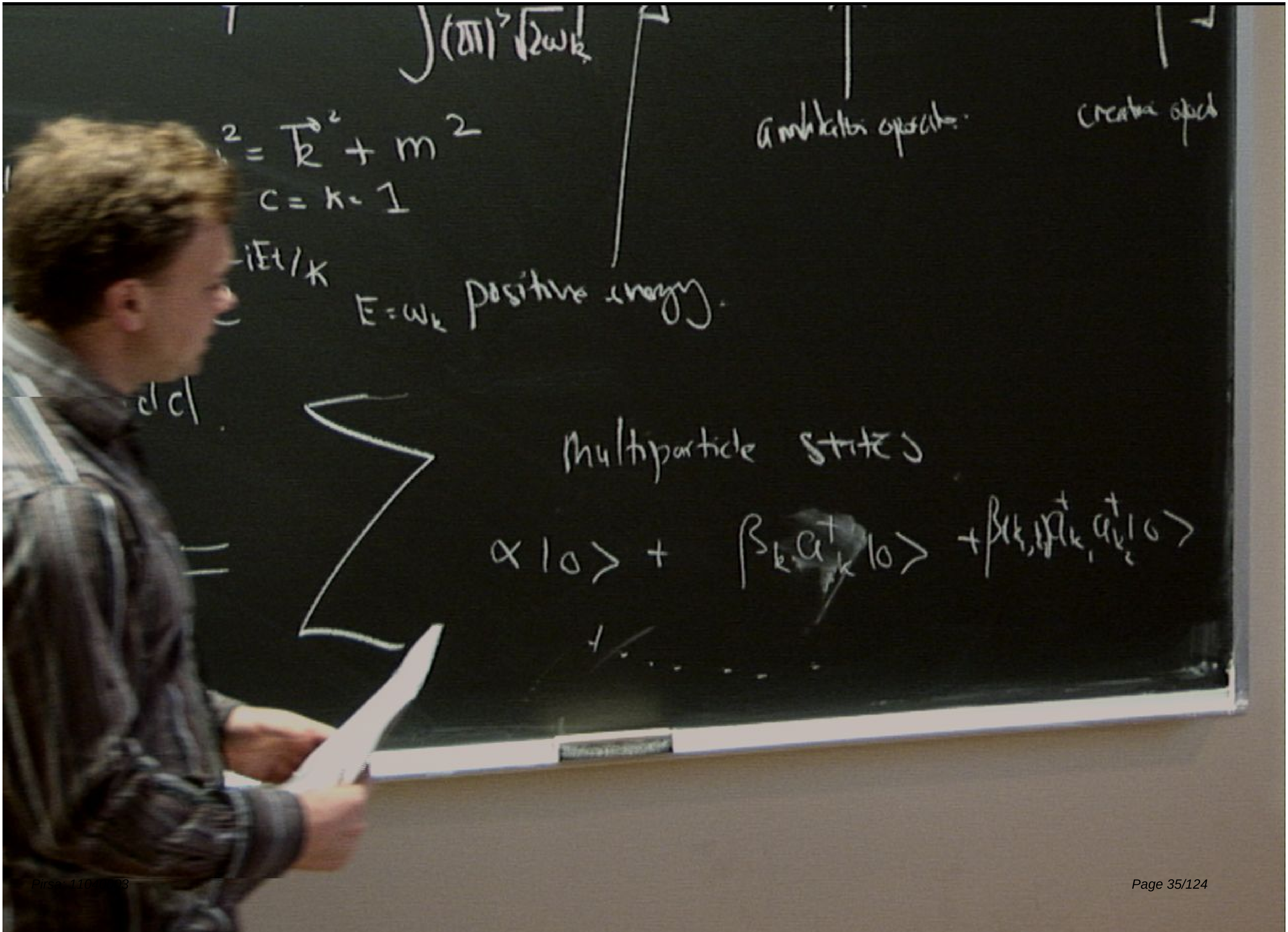
single particle $\vec{k}$

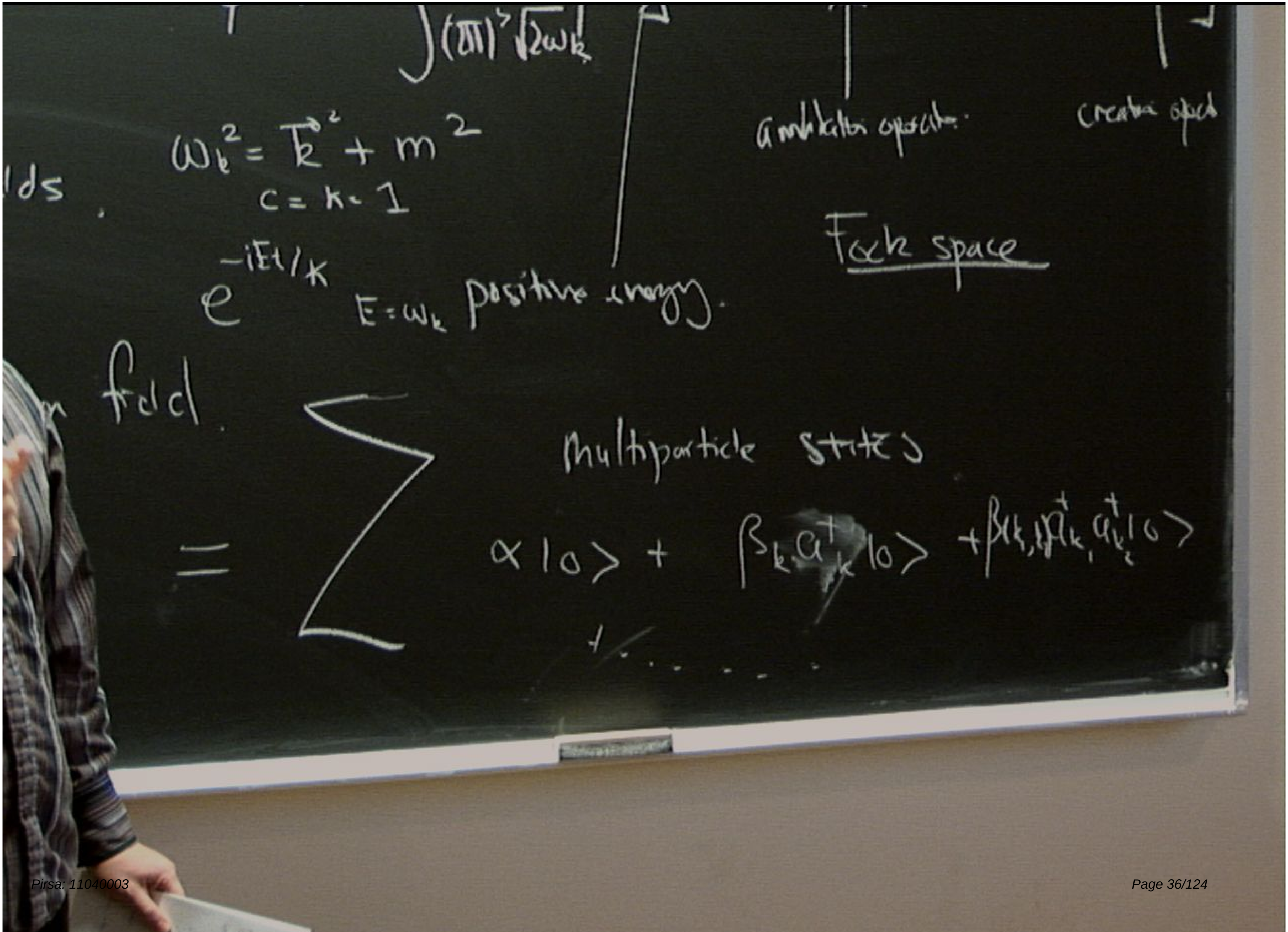
$a^\dagger_{k_1} a^\dagger_{k_2} |0\rangle \sim |\vec{k}_1, \vec{k}_2\rangle$

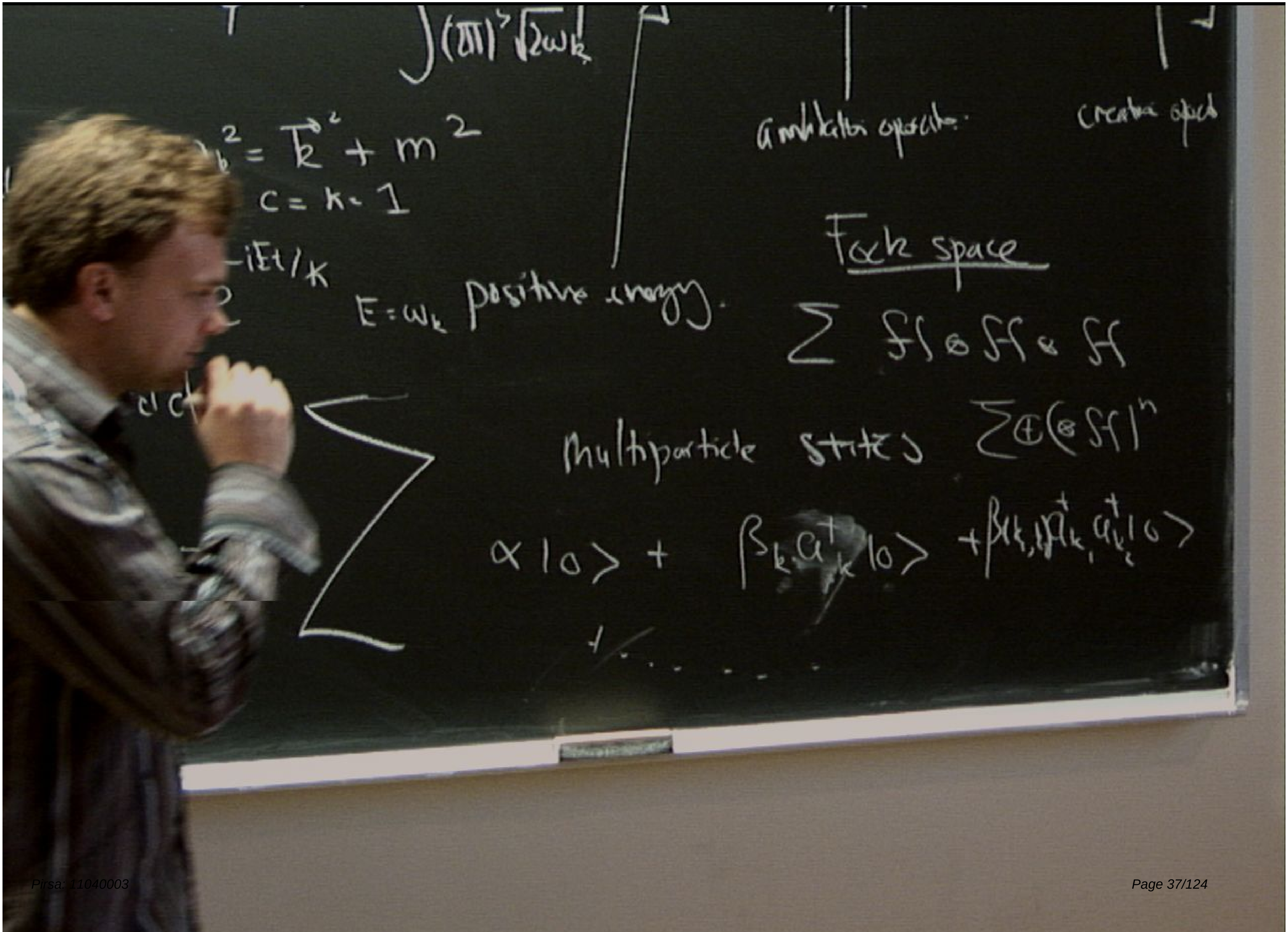
n-particle $(a^\dagger)^n |0\rangle$

$\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle$

$[\hat{\phi}(x), \hat{\phi}(y)] = 0$ $\quad [\hat{\phi}(x), \hat{\pi}(y)$


annihilation operator
creation operator
$c = \hbar = 1$
$E = \omega_k$ positive energy.
Multiparticle states
$\alpha |0\rangle + \beta_k a^\dagger_k |0\rangle + \beta(k_1,k_2) a^\dagger_{k_1} a^\dagger_{k_2} |0\rangle$


annihilation operator
creation operator
$\omega_k^2 = \vec{k}^2 + m^2$
$c = \hbar = 1$
$e^{-iEt/\hbar}$
$E = \omega_k$ positive energy.
Fock space
field
multiparticle states
$= \sum \alpha|0\rangle + \beta_k a_k^\dagger|0\rangle + \beta(k_1,k_2) a_{k_1}^\dagger a_{k_2}^\dagger|0\rangle$
+ . . . . . .


c = k = 1
E = ω_k positive energy
Fock space
Multiparticle states

$$\int \frac{\quad}{(2\pi)^3 \sqrt{2\omega_k}}$$

annihilation operator · creation operator

lds .

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = \hbar = 1$$

$e^{-iEt/\hbar}$ $E = \omega_k$ positive energy.

Fock space

$$\sum \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$$

tum field.

multiparticle states $\sum_{n=0}^{\infty} \oplus (\otimes \mathcal{H})^n$

$$|\;\rangle = \sum \alpha|0\rangle + \beta_k a_k^\dagger|0\rangle + \beta(k_1,k_2) a_{k_1}^\dagger a_{k_2}^\dagger|0\rangle + \dots$$

$\hat{\phi}(x)$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega_k}} \Big|\; e \qquad \hat{a}_k + e$$

annihilation operator

crea

ar quantum fields.

$$\omega_k^2 = \vec{k}^2 + m^2$$

$$c = \hbar = 1$$

$e^{-iEt/\hbar}$ $E = \omega_k$ positive energy.

Fock space

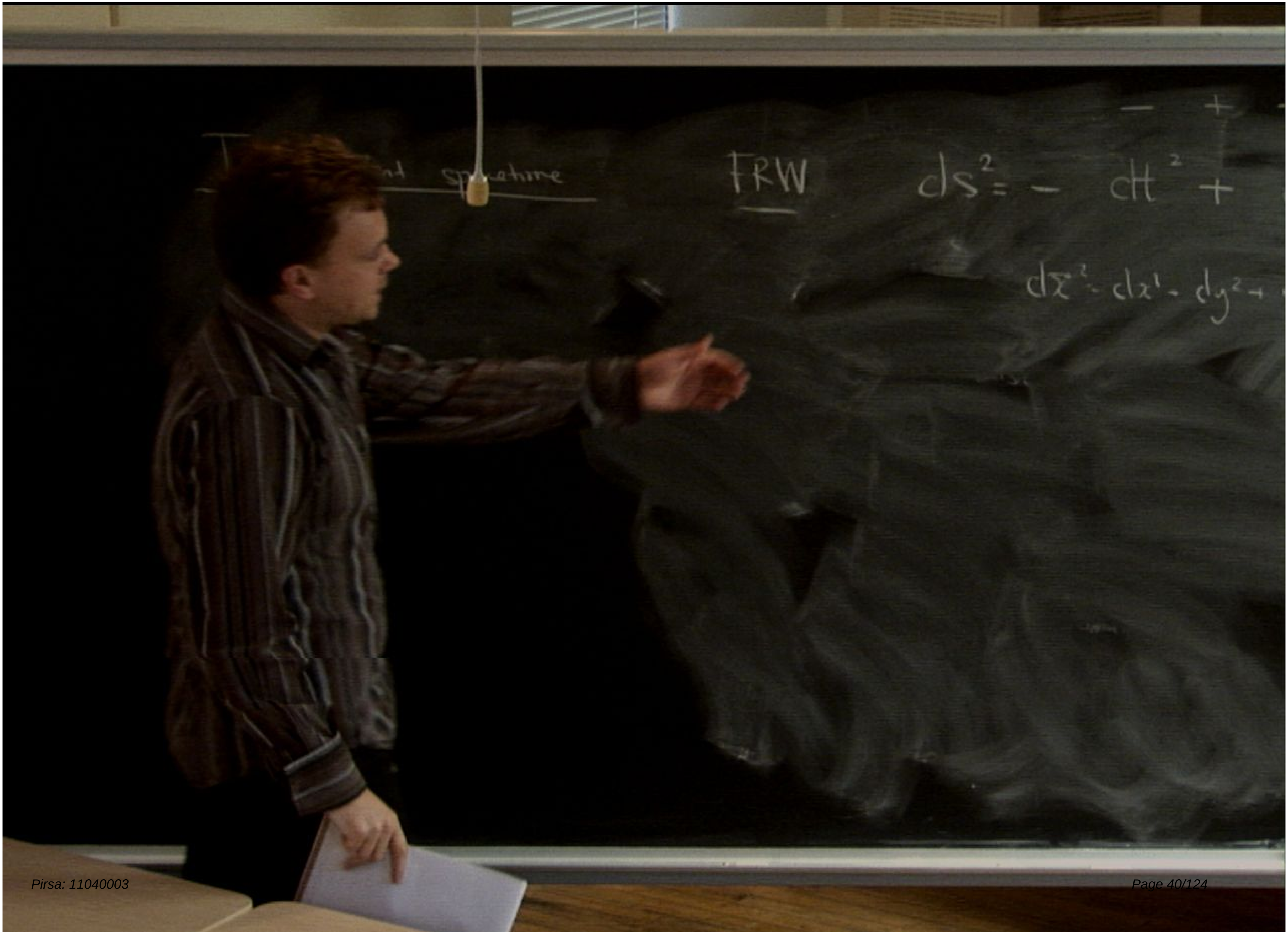
$\sum \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$

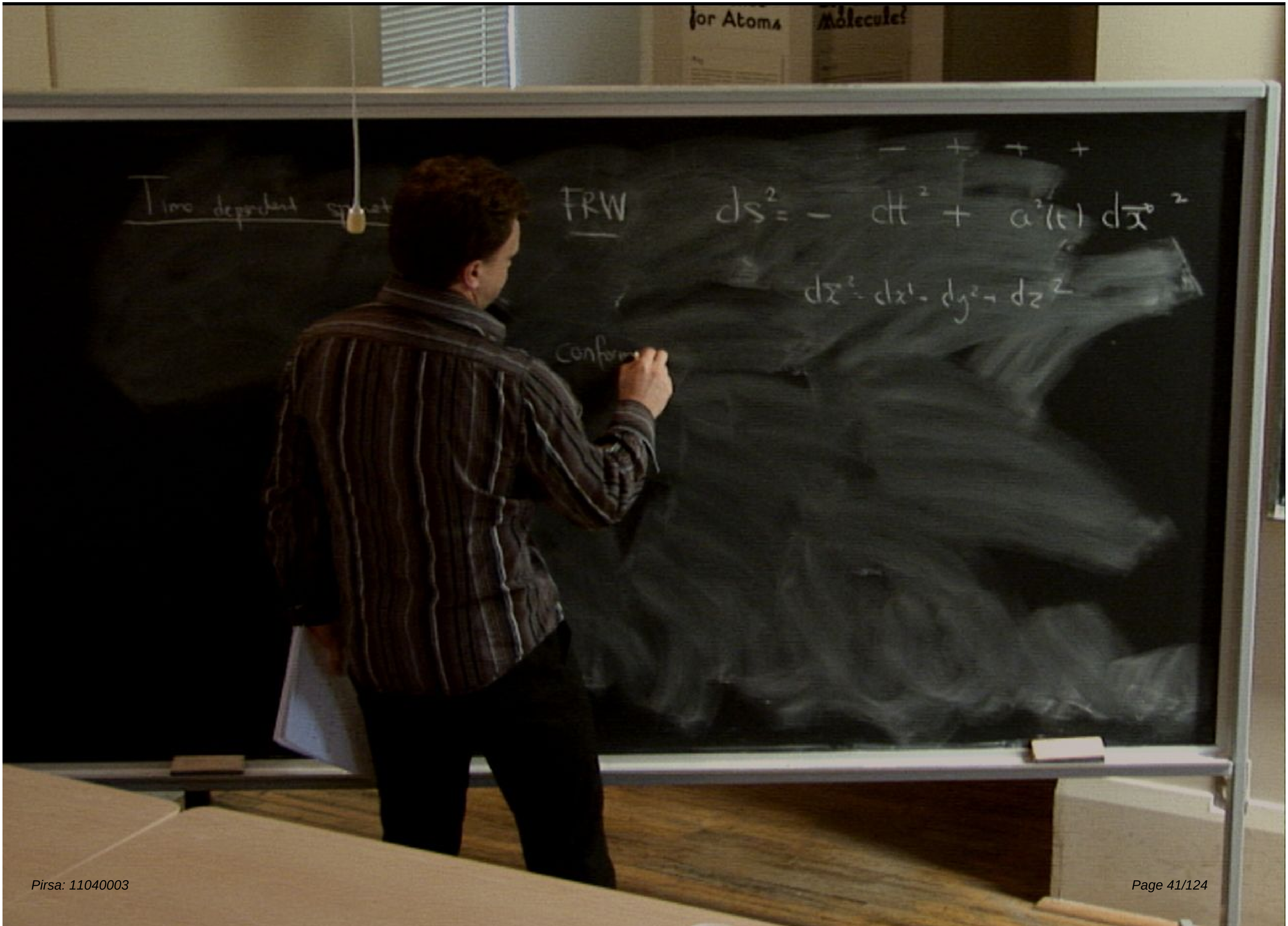
ate of quantum field

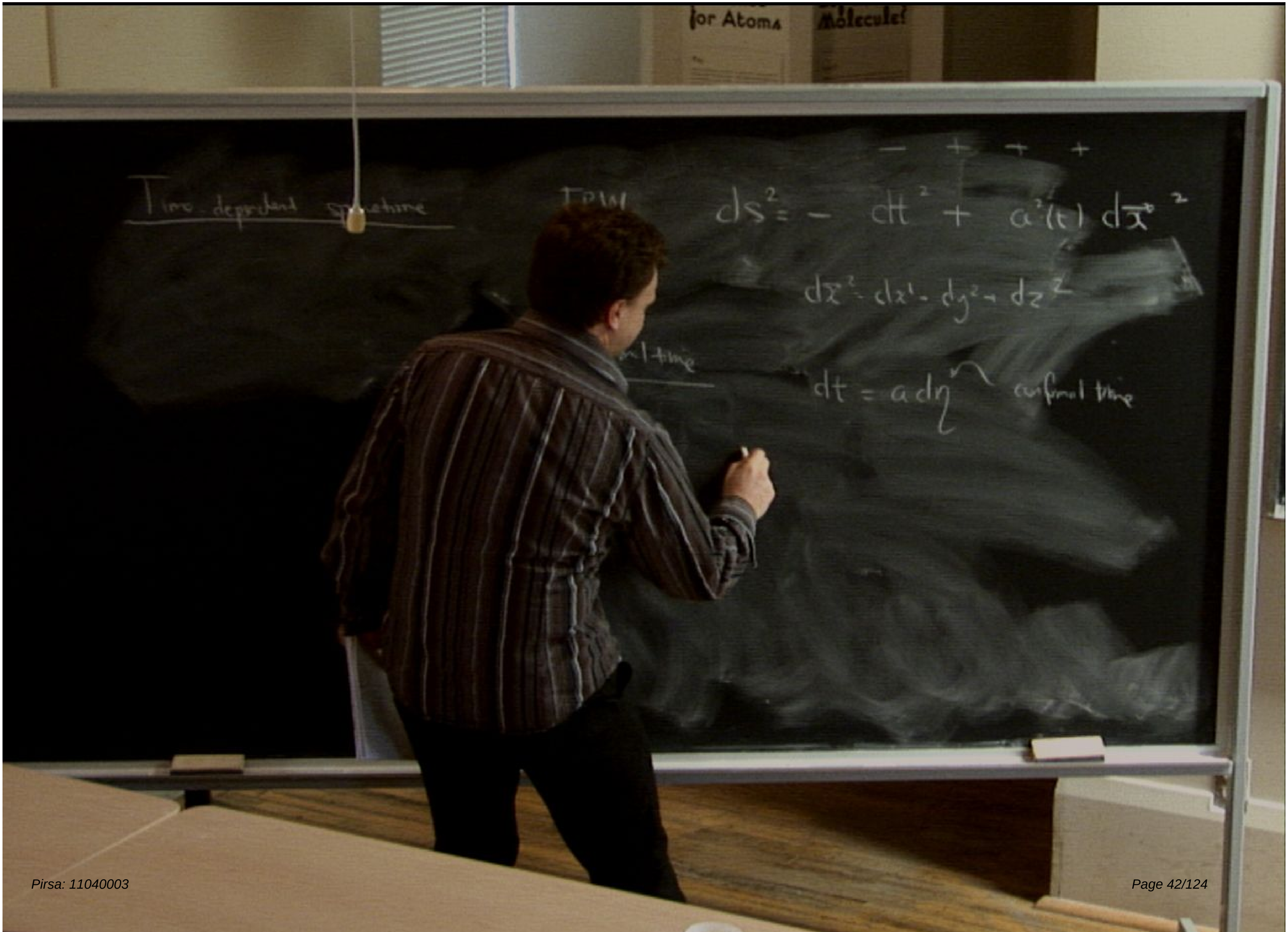
multiparticle states $\sum_{n=0}^{\infty} \oplus (\otimes \mathcal{H}$

$$|\phi(\vec{x})\rangle = \sum \alpha|0\rangle + \beta_k a_k^{\dagger}|0\rangle + \beta(k_1,k_2) a_{k_1}^{\dagger} a_{k_2}^{\dagger} + \ldots$$

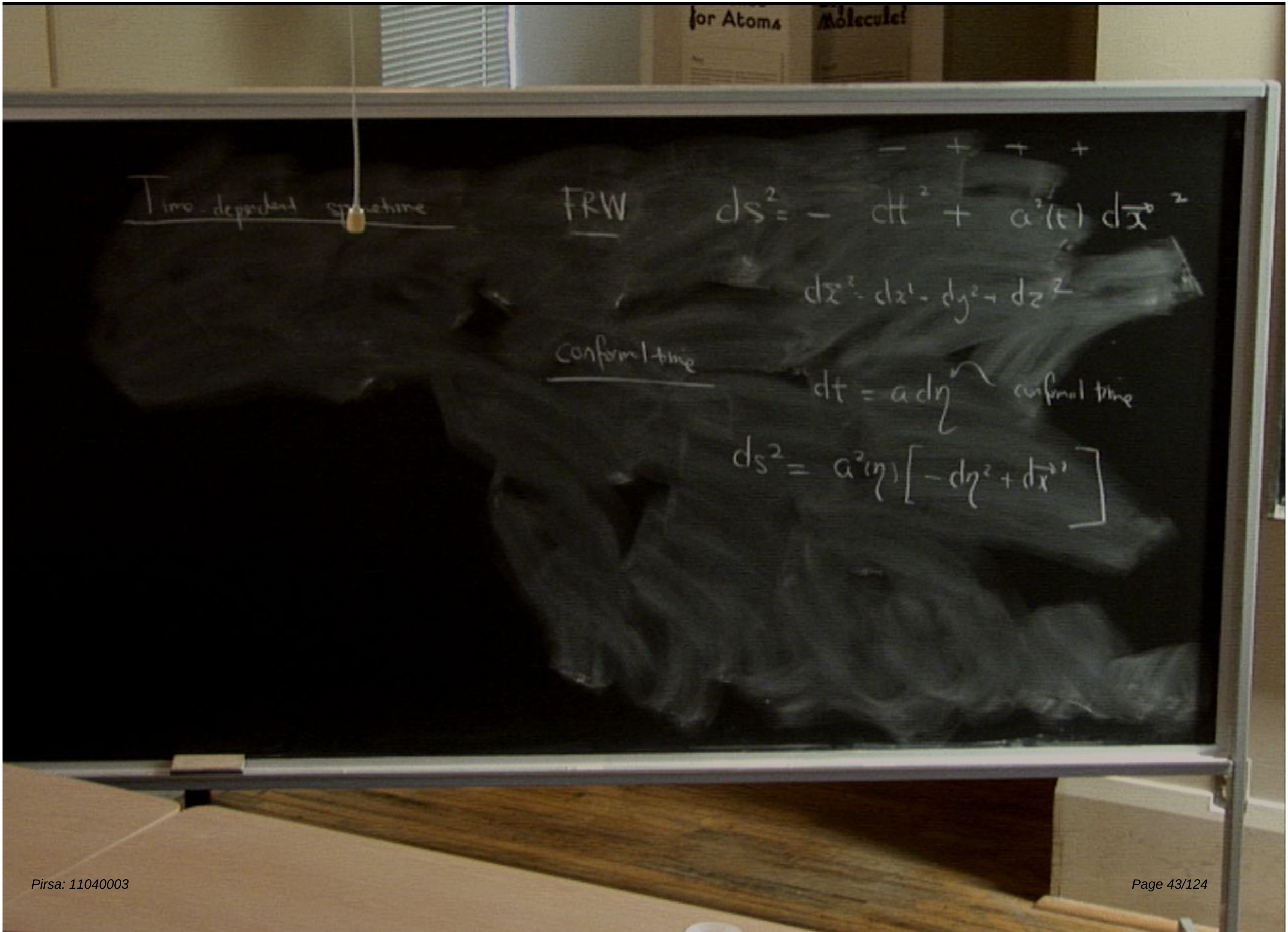
$i\hbar\delta^{(4)}(\vec{x}-\vec{y})$

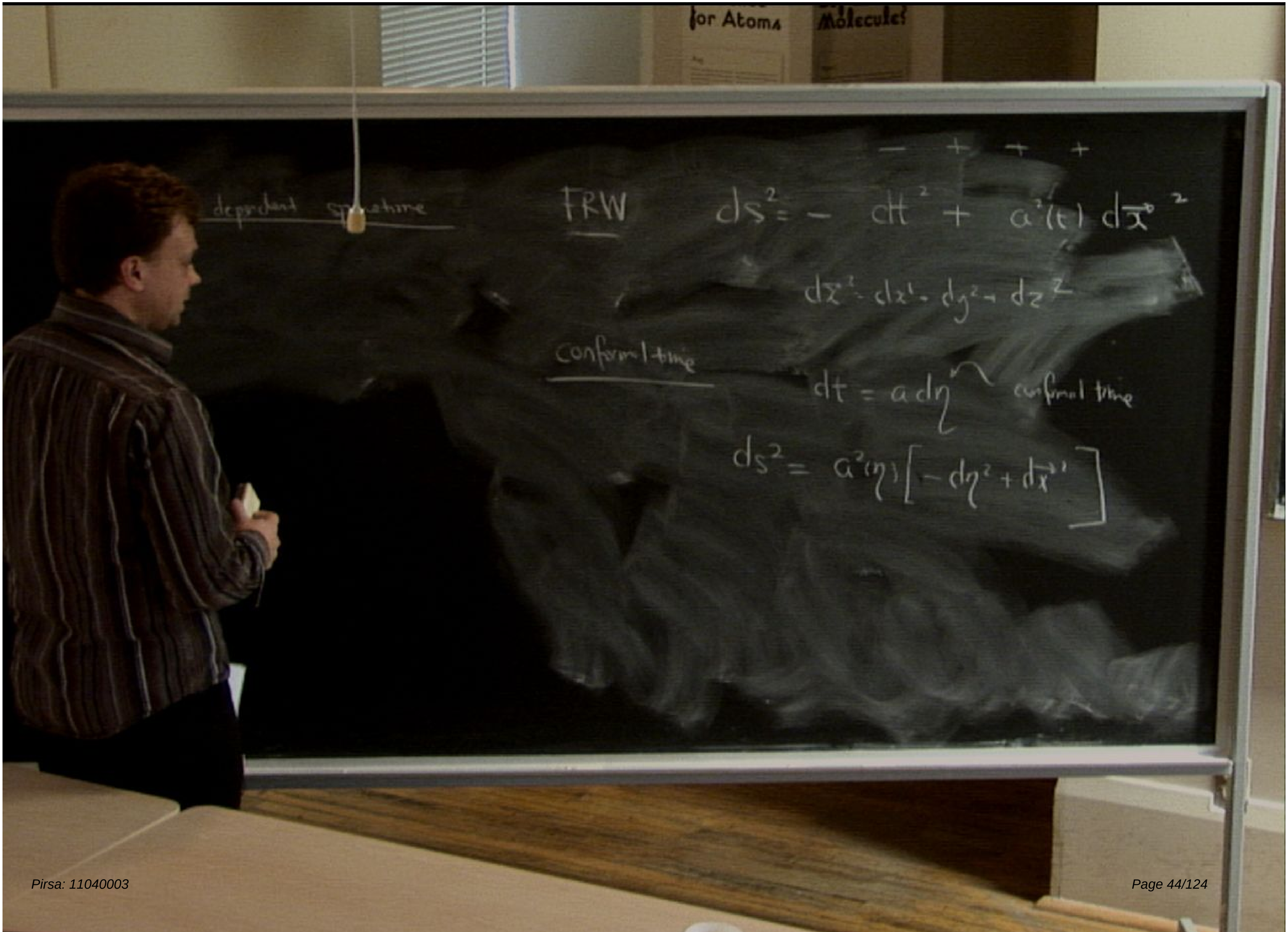

FRW
ds² = − dt² +


for Atoms
Molecules
Time dependent
FRW
− + + +
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$d\vec{x}^2 = dx^2 + dy^2 + dz^2$

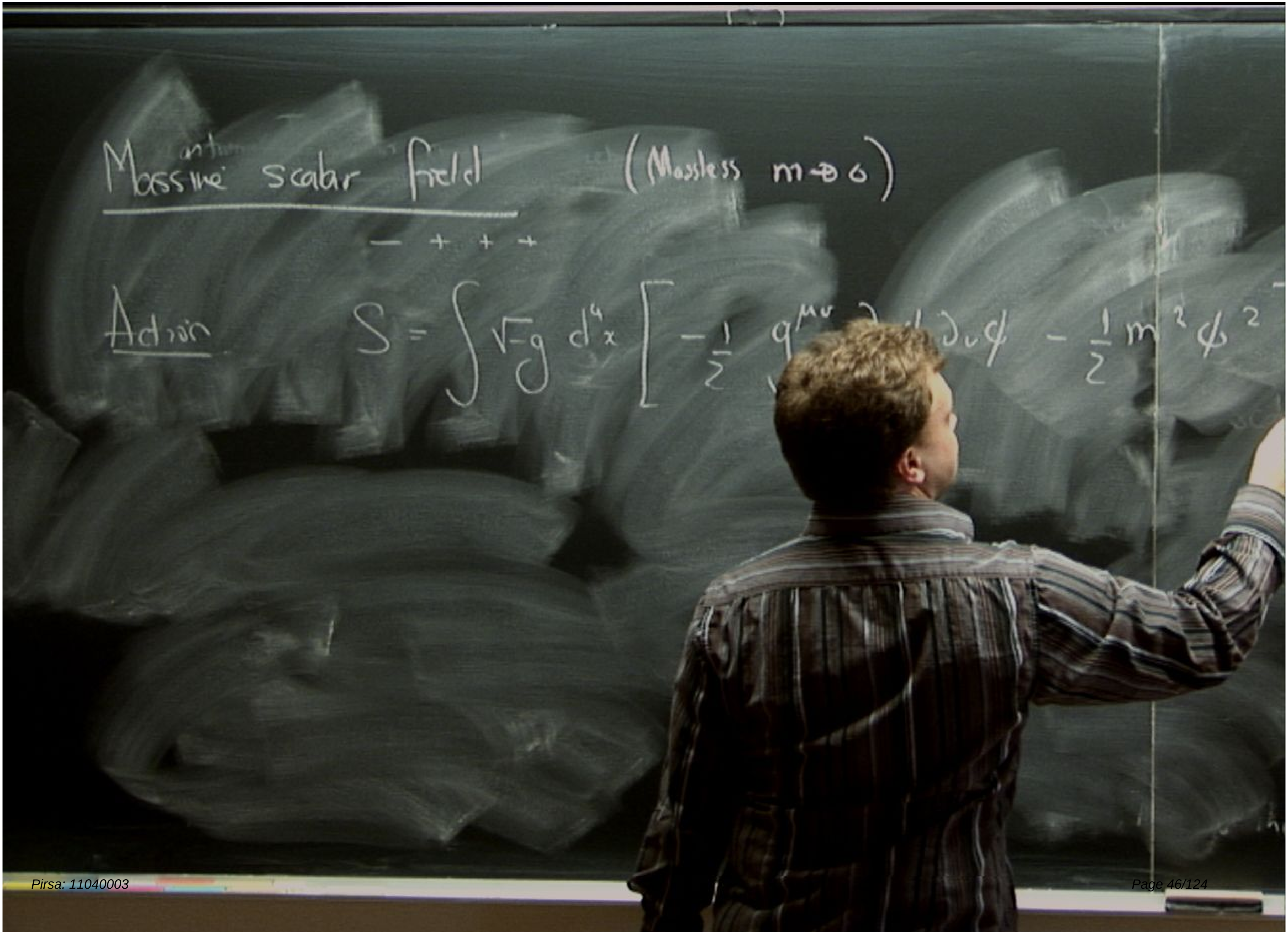


for Atoms
Molecule!
Time dependent spacetime
FRW
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$d\vec{x}^2 = dx^2 + dy^2 + dz^2$
$dt = a\, d\eta$
conformal time


for Atoms
Time dependent spacetime
FRW
$- \; + \; + \; +$
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$d\vec{x}^2 = dx^2 + dy^2 + dz^2$
conformal time
$dt = a\, d\eta$ conformal time
$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$


for Atoms
Molecules
$- \; + \; + \; +$
dependent spacetime
FRW
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$d\vec{x}^2 = dx^2 + dy^2 + dz^2$
conformal time
$dt = a\, d\eta$
conformal time
$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$

Massive scalar field (Massless $m \to 0$)


Massive scalar field
(Massless $m \to 0$)
$- + + +$
Action
$S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \ldots \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right.$

Massive scalar field (Massless $m \to 0$)

$- + + +$ $\quad \hbar = c = 1$

Action $\quad S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$| \phi(\vec{x}) \rangle$

Massive scalar field (Massless $m \to 0$)

$- + + +$ $\quad \hbar = c = 1$

Action $\quad S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$| \phi(\vec{x}) \rangle$

$\hbar \delta^{(3)}(\vec{x} - \vec{y})$

Massive scalar field (Massless $m \to 0$)

$- + + + \quad \hbar = c = 1$

Action $\quad S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sqrt{\det(-g)} = a^4$$

$| \phi(\vec{x}) \rangle$

Massive scalar field (Massless $m \to 0$)

$- + + +$ $\hbar = c = 1$

Action $S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\sqrt{\det(-g)} = a^4$

$| \phi(\vec{x}) \rangle$

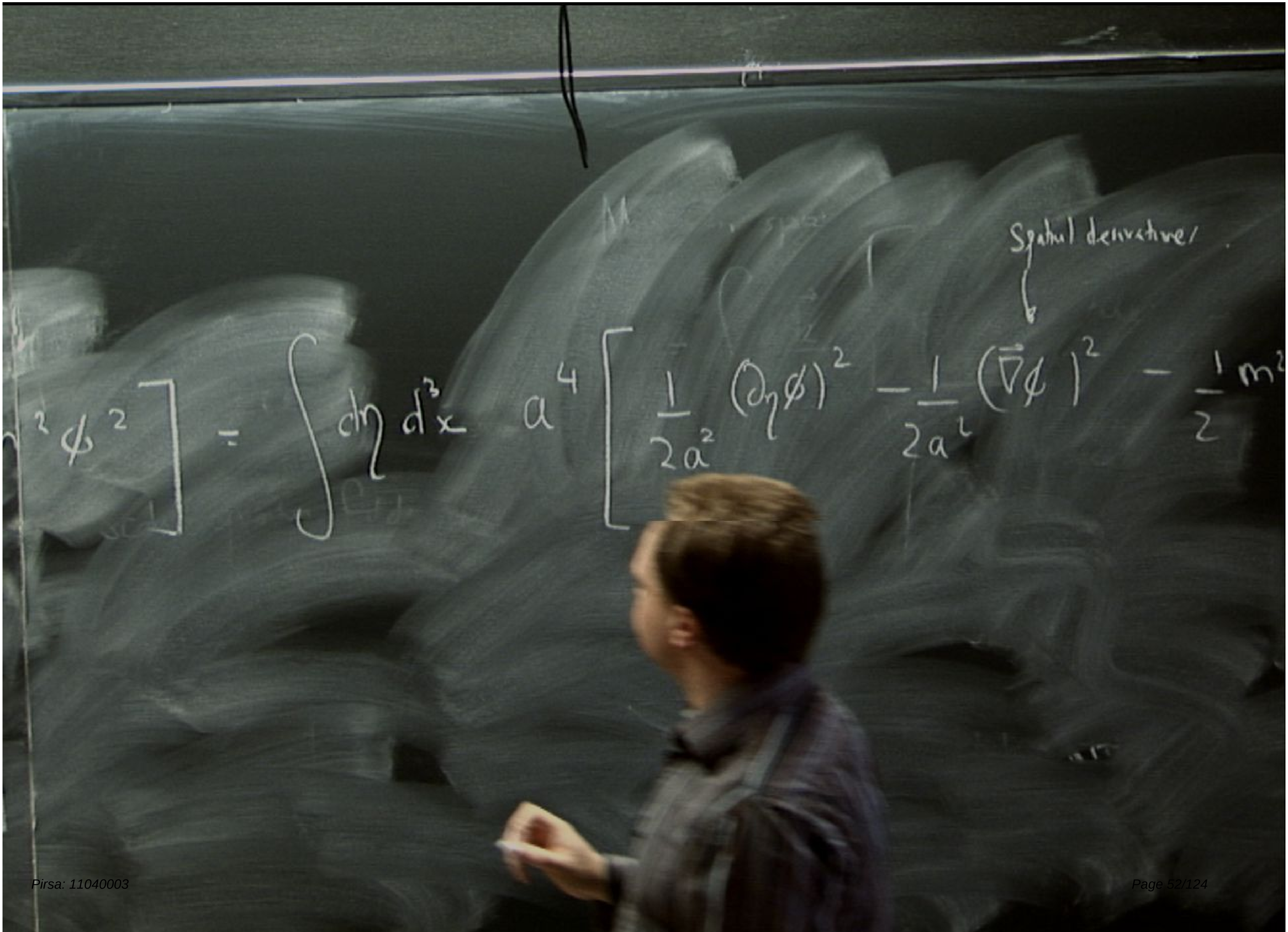
Massive scalar field (Massless $m \to 0$)

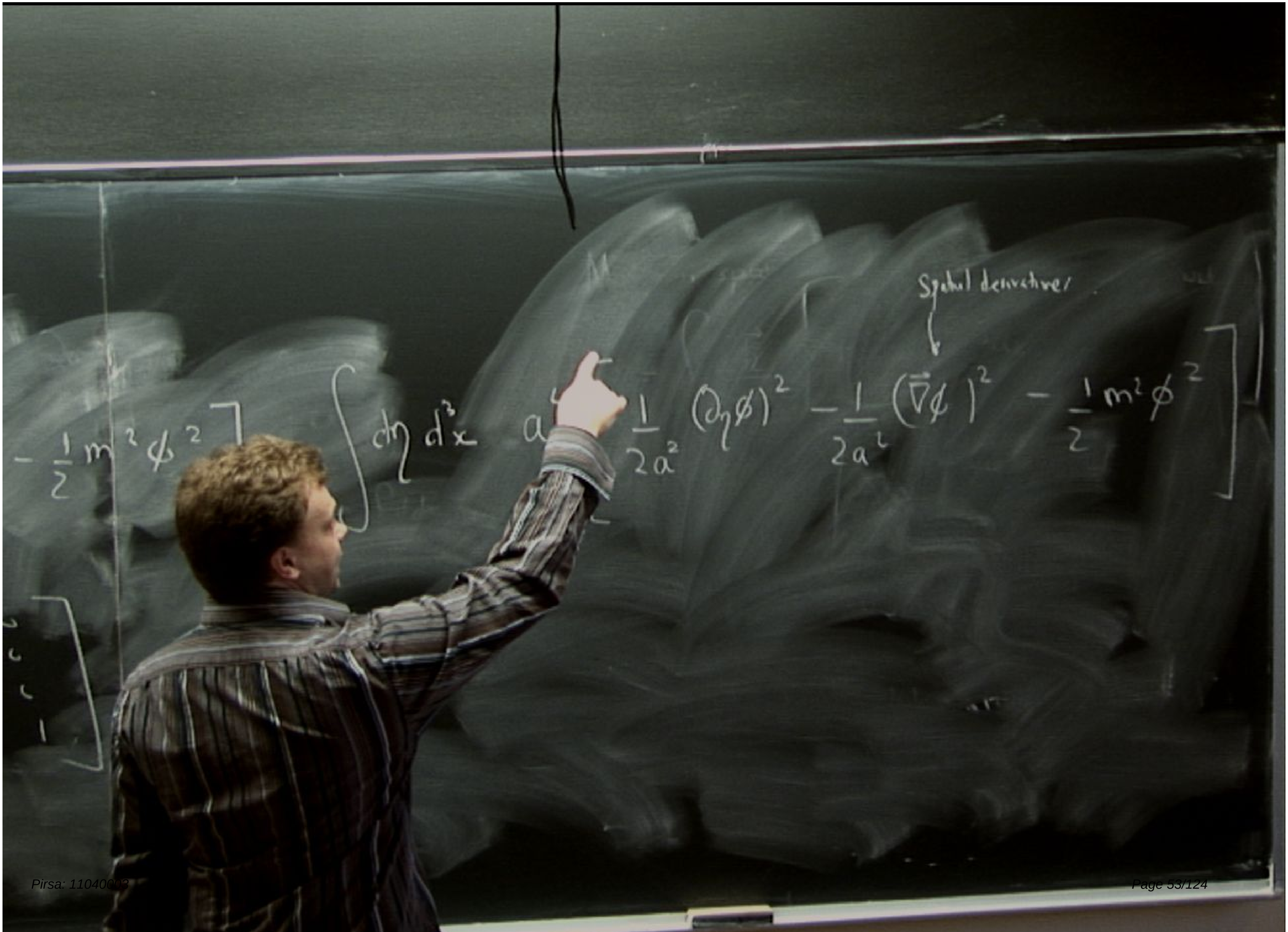
$- + + +$ $\quad \hbar = c = 1$

Action $\quad S = \int \sqrt{-g}\, d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$

$$g = a^2 \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sqrt{\det(-g)} = a^4$$


Spatial derivative
$^2\phi^2] = \int d\eta\, d^3x \;\; a^4 \Big[\frac{1}{2a^2}(\partial_\eta\phi)^2 - \frac{1}{2a^2}(\vec{\nabla}\phi)^2 - \frac{1}{2}m^2$


$-\frac{1}{2}m^2\phi^2\Big]$
Spatial derivatives
$\int d\eta\, d^3x \;\; a^4\Big[\frac{1}{2a^2}(\partial_\eta\phi)^2 - \frac{1}{2a^2}(\vec{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2\Big]$

$$-\frac{1}{2}m^2\phi^2\Big] = \int d\eta\, d^3x\; a^4\left[\frac{1}{2a^2}(\partial_\eta\phi)^2 - \frac{1}{2a^2}(\vec{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2\right]$$

Spatial derivatives

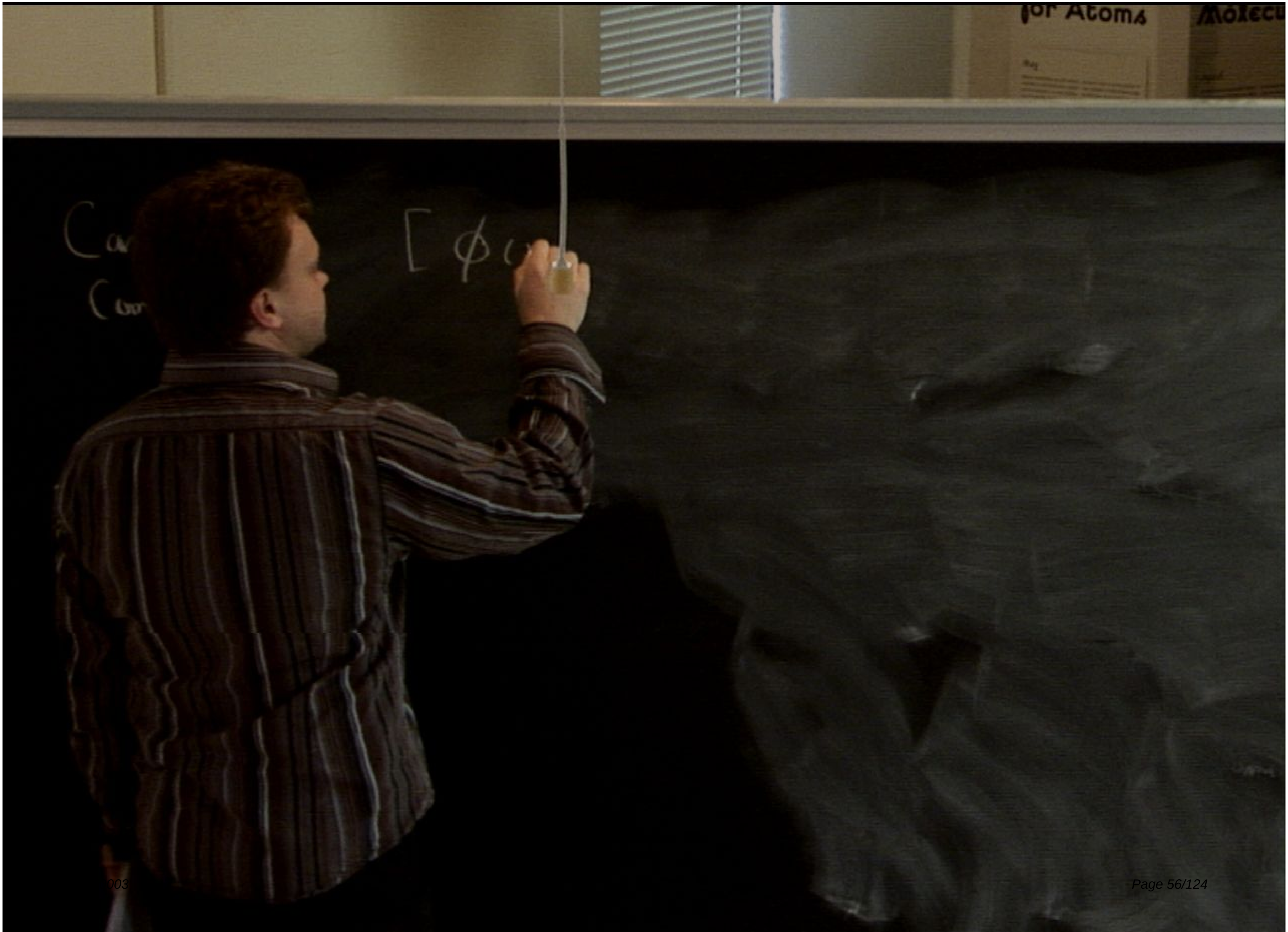
$$= \int d\eta\, d^3x \left\{\frac{a^2}{2}\left[(\partial_\eta\phi)^2 - (\vec{\nabla}\phi)^2\right] - \frac{1}{2}m^2a^4\phi^2\right\}$$

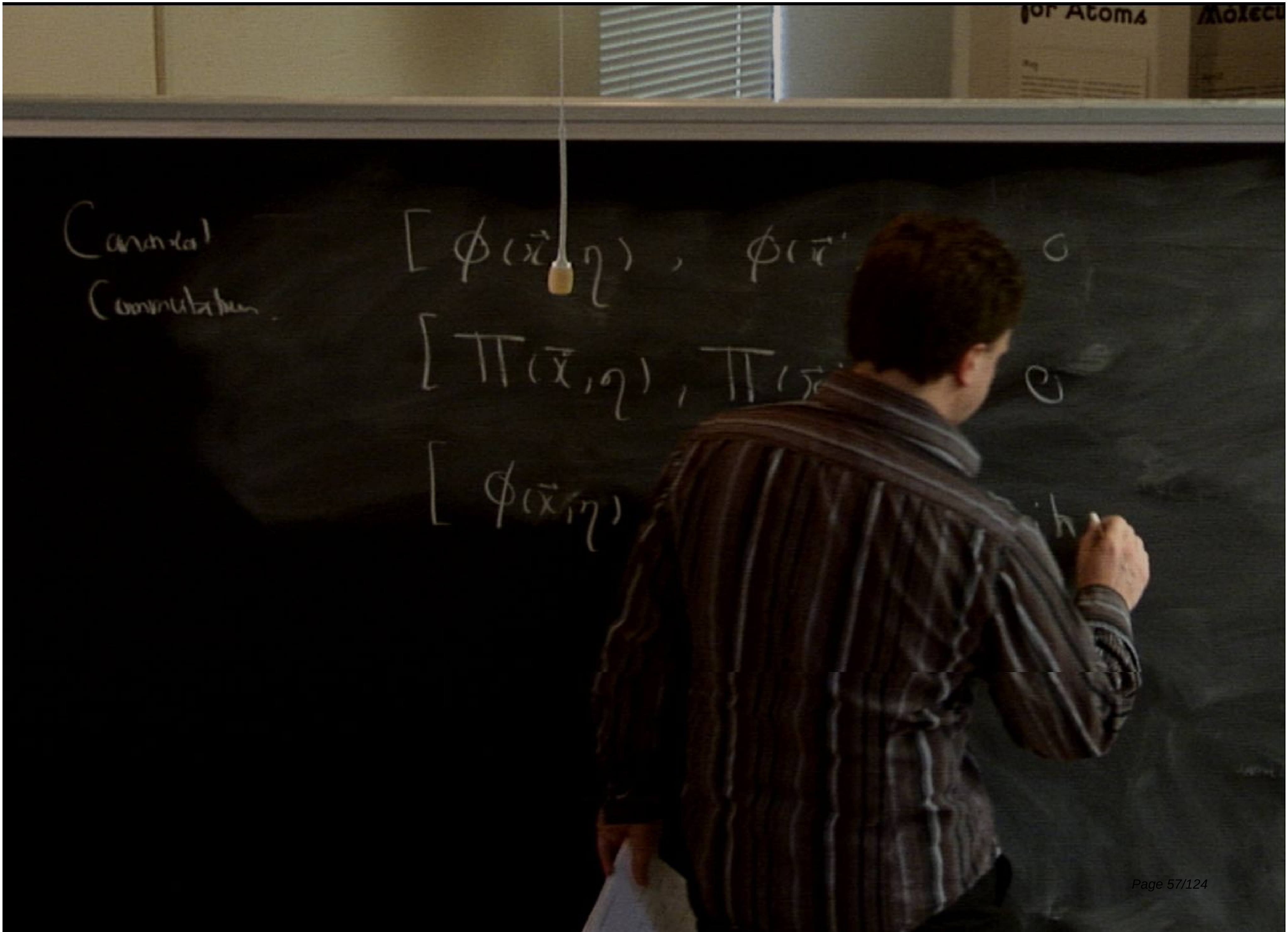
Momentum conjugate $\quad \Pi = \frac{\partial \mathcal{L}}{\partial(\partial_\eta\phi)}$

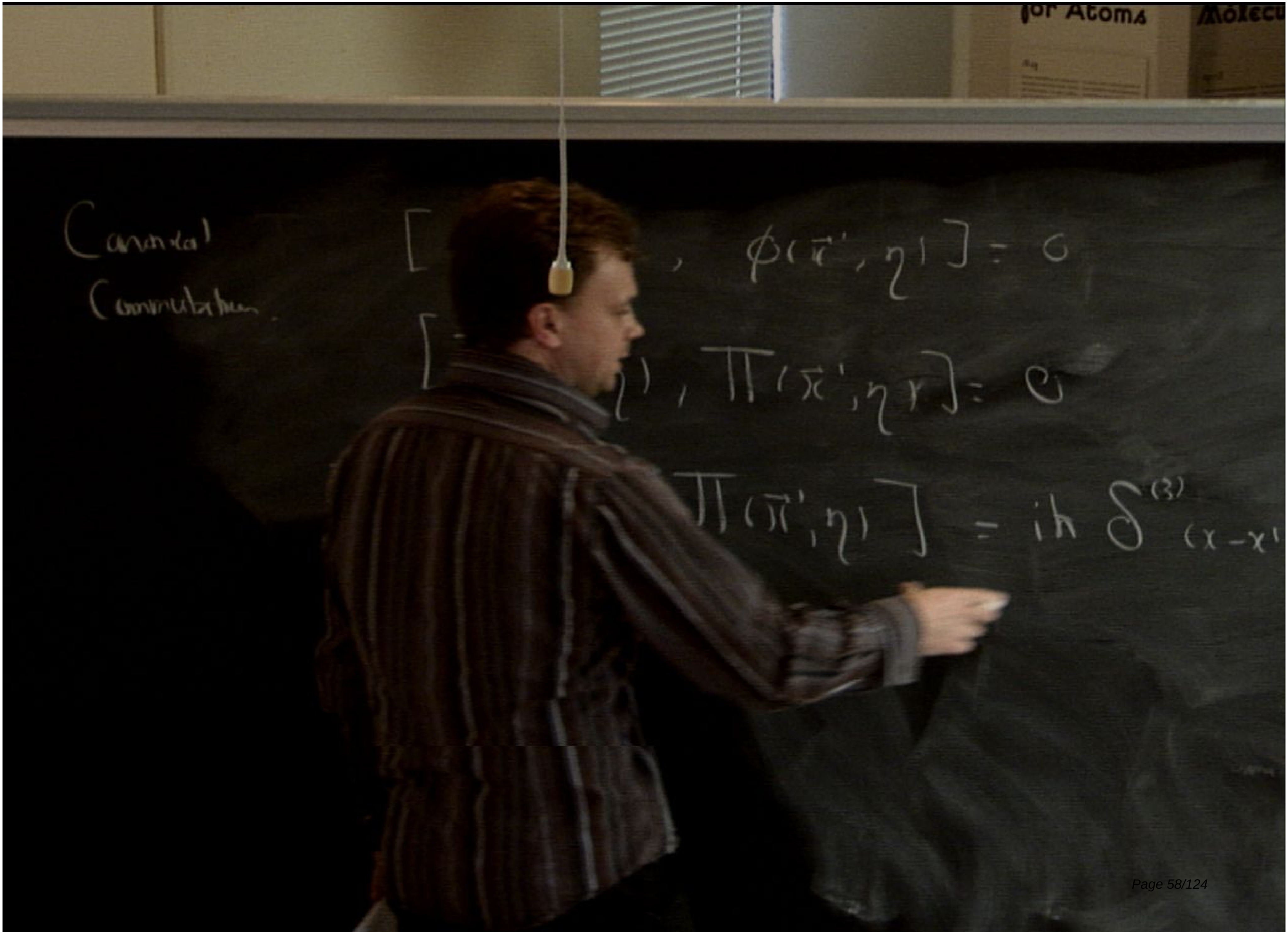
$$- \frac{1}{2} m^2 \phi^2 \Big] = \int d\eta\, d^3x \; a^4 \left[\frac{1}{2a^2} (\partial_\eta \phi)^2 - \frac{1}{2a^2} \underset{\text{Spatial derivatives}}{(\vec{\nabla}\phi)^2} - \frac{1}{2} m^2 \phi^2 \right]$$

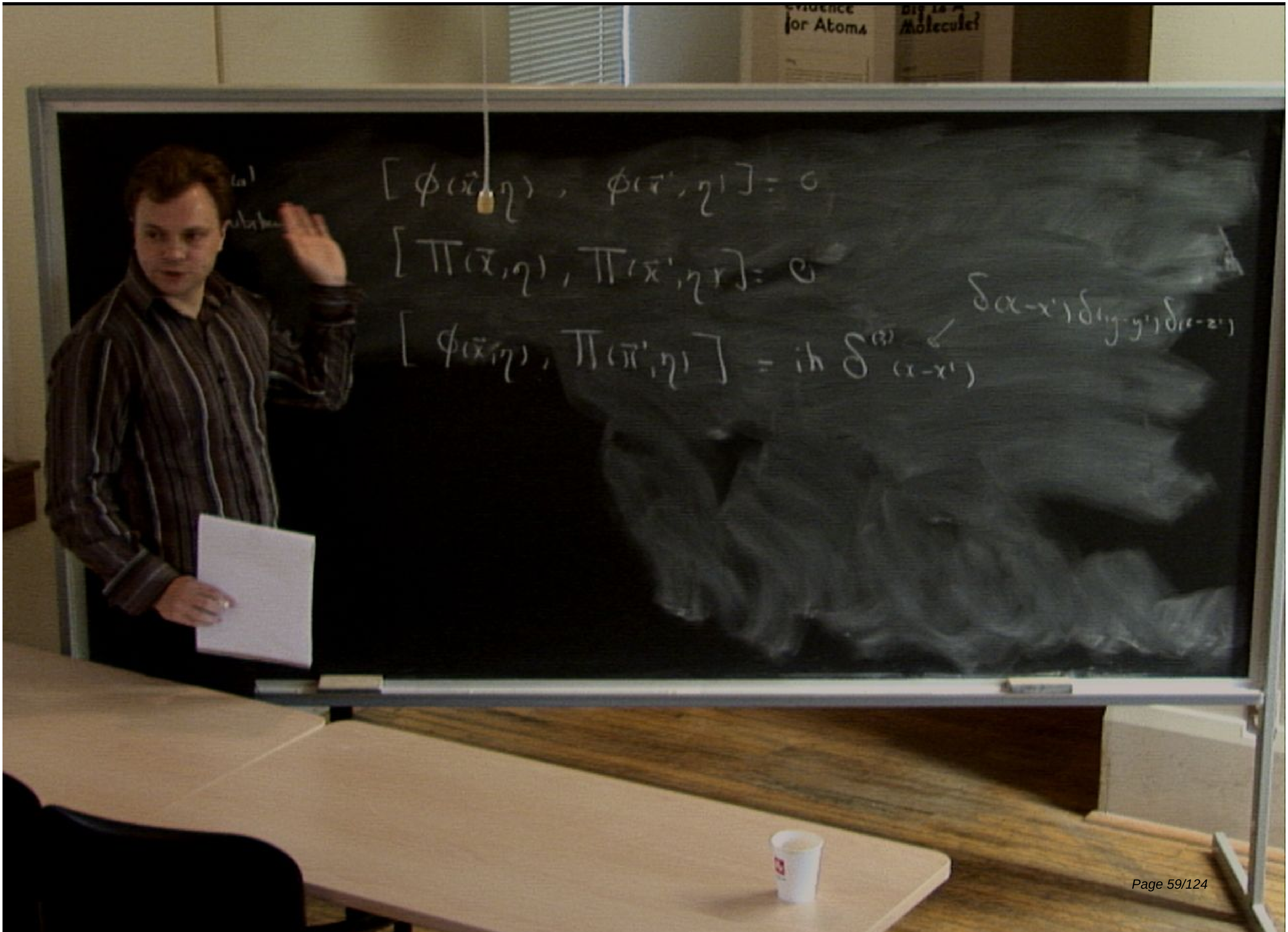
$$= \int d\eta\, d^3x \left\{ \frac{a^2}{2} \left[(\partial_\eta \phi)^2 - (\vec{\nabla}\phi)^2 \right] - \frac{1}{2} m^2 a^4 \phi^2 \right\}$$

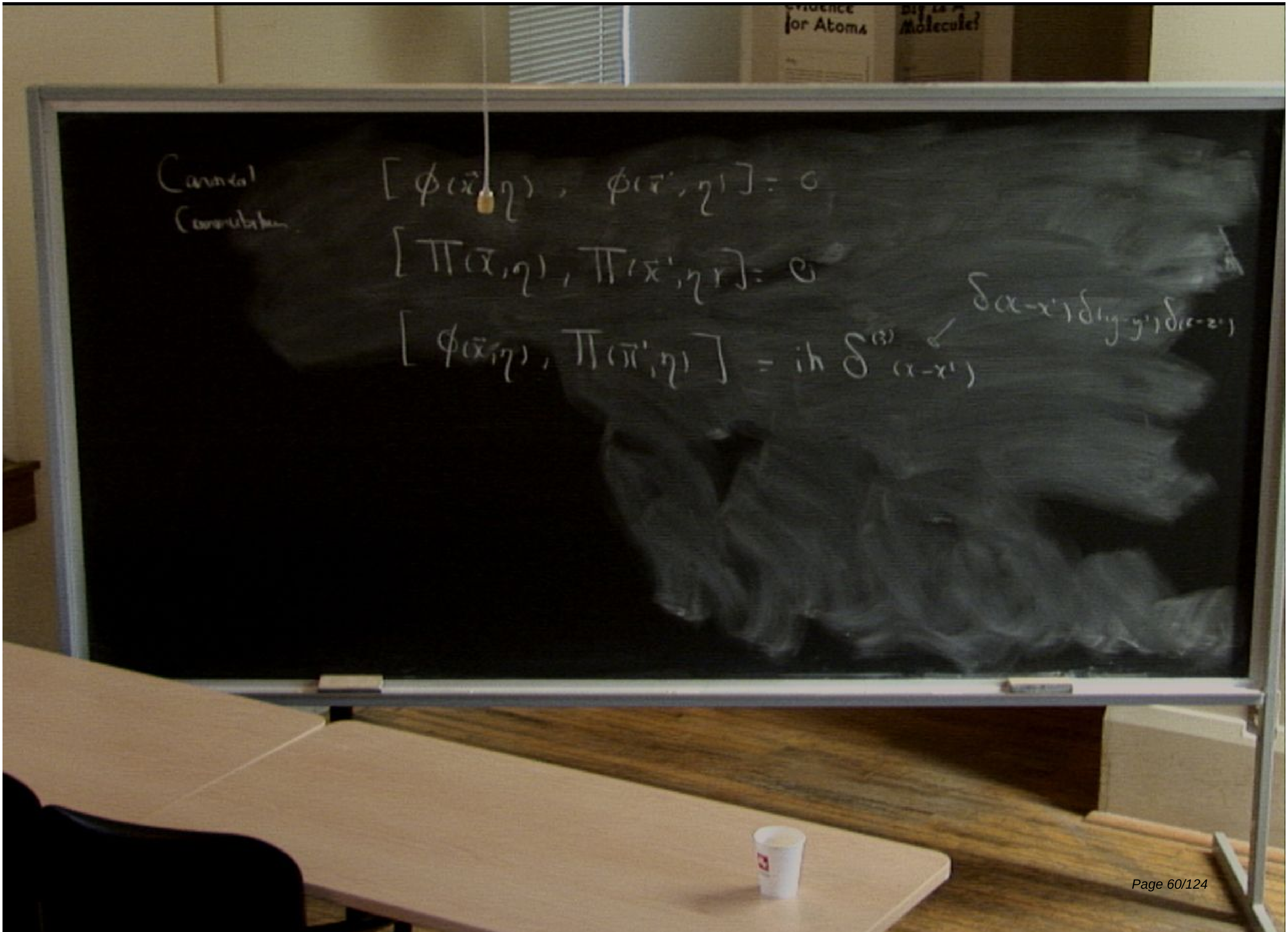
Momentum conjugate $\quad \Pi = \dfrac{\partial \mathcal{L}}{\partial(\partial_\eta \phi)} = a^2 \partial_\eta \phi$




for Atoms
Canonical
Commutation


for Atoms
Canonical
Commutation


for Atoms
Molecule?
$[\phi(\vec{x},\eta), \phi(\vec{x}',\eta')] = 0$
$[\Pi(\vec{x},\eta), \Pi(\vec{x}',\eta')] = 0$
$[\phi(\vec{x},\eta), \Pi(\vec{x}',\eta')] = i\hbar\,\delta^{(3)}(x-x')$
$\delta(x-x')\delta(y-y')\delta(z-z')$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$\delta(x-x')\delta(y-y')\delta(z-z')$

massless $m \to 0$)

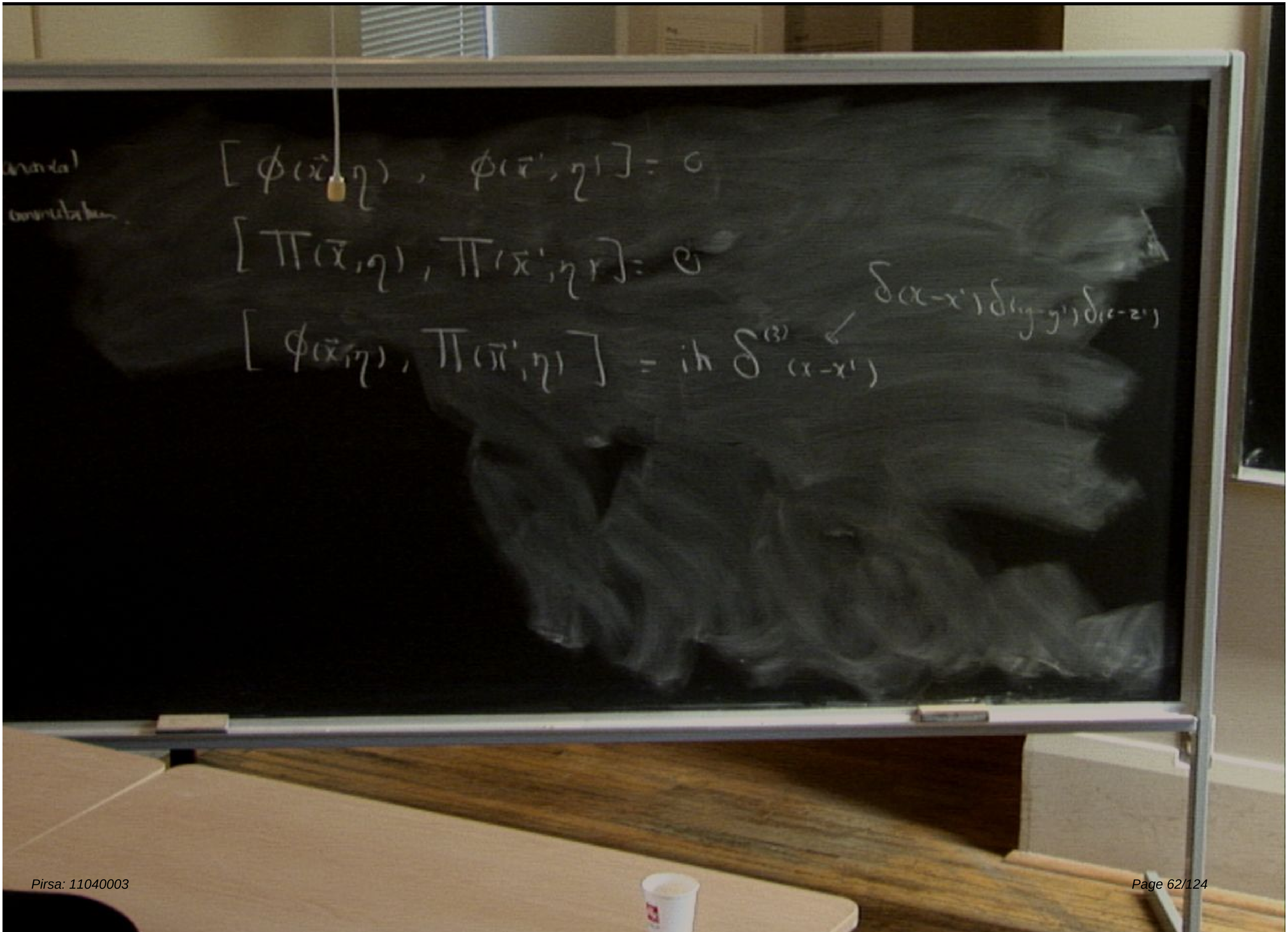
$c = 1$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \Big] = \int d\eta \, d^3x$$

$$g^{-1} = \frac{1}{a^2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$V = \frac{1}{2} m \omega^2 (\eta) x^2 = \int d\eta \, d^3x$

Momentum conjugate Π


$[\phi(\vec{x},\eta), \phi(\vec{x}',\eta')] = 0$
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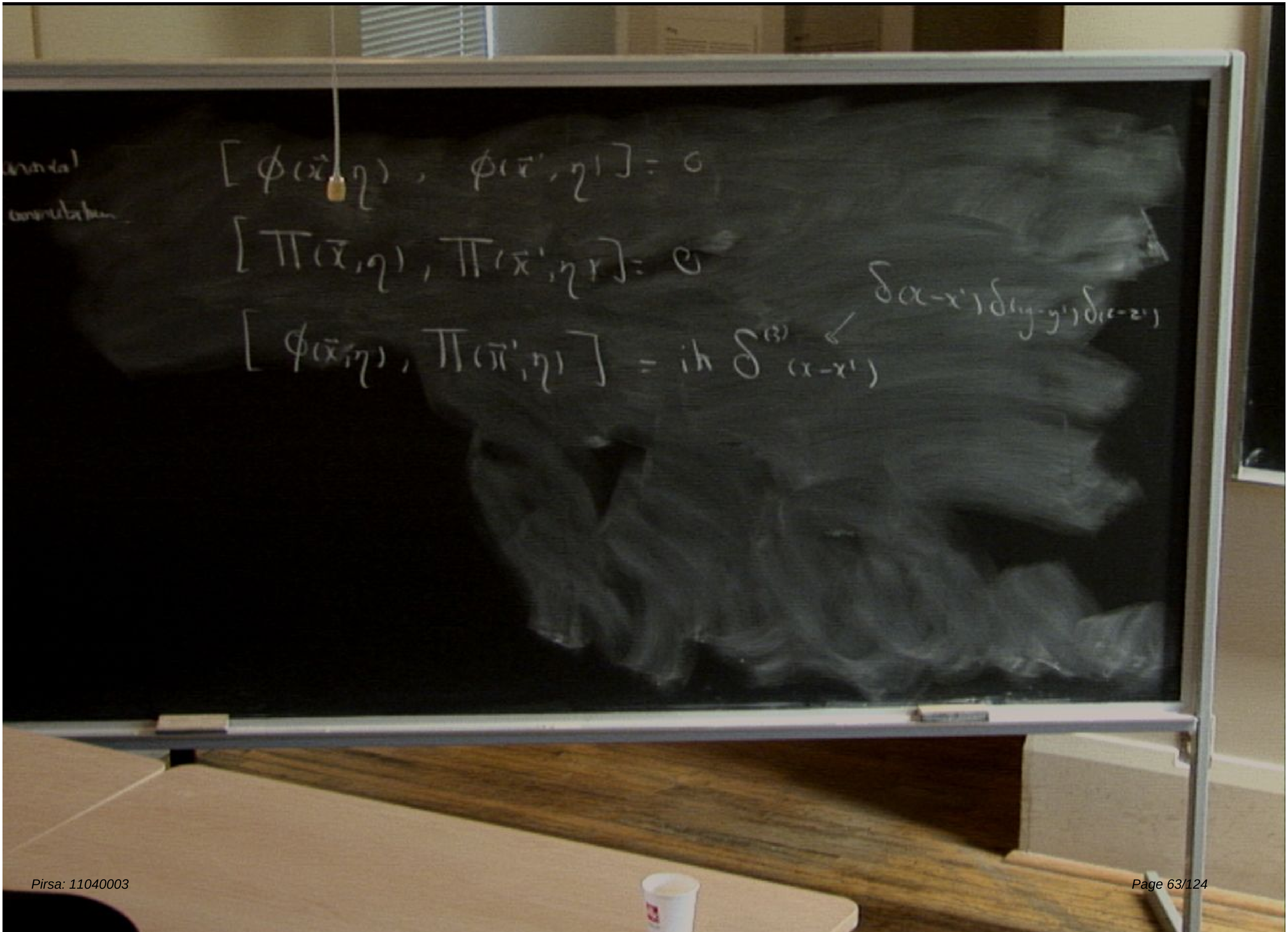
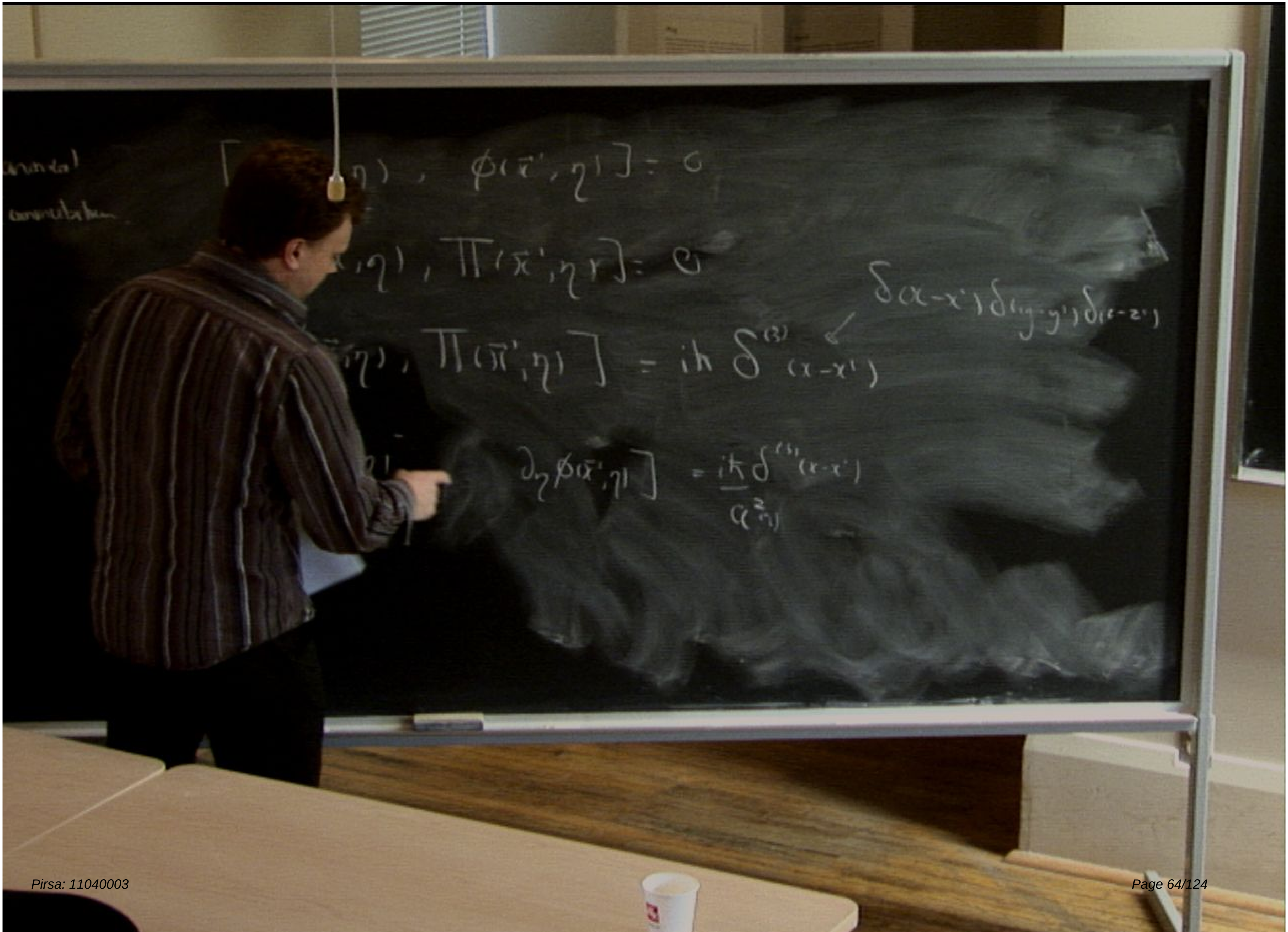
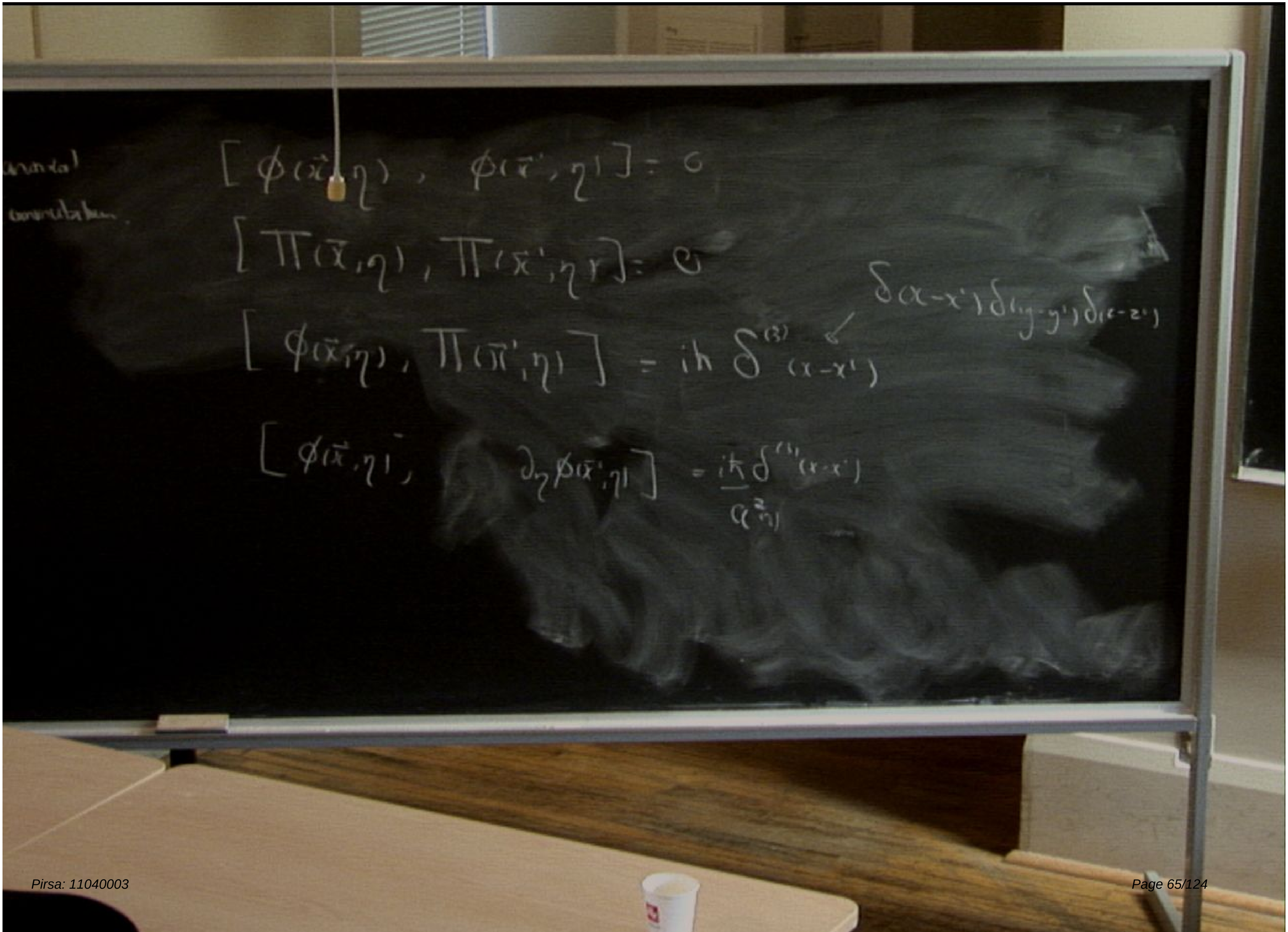
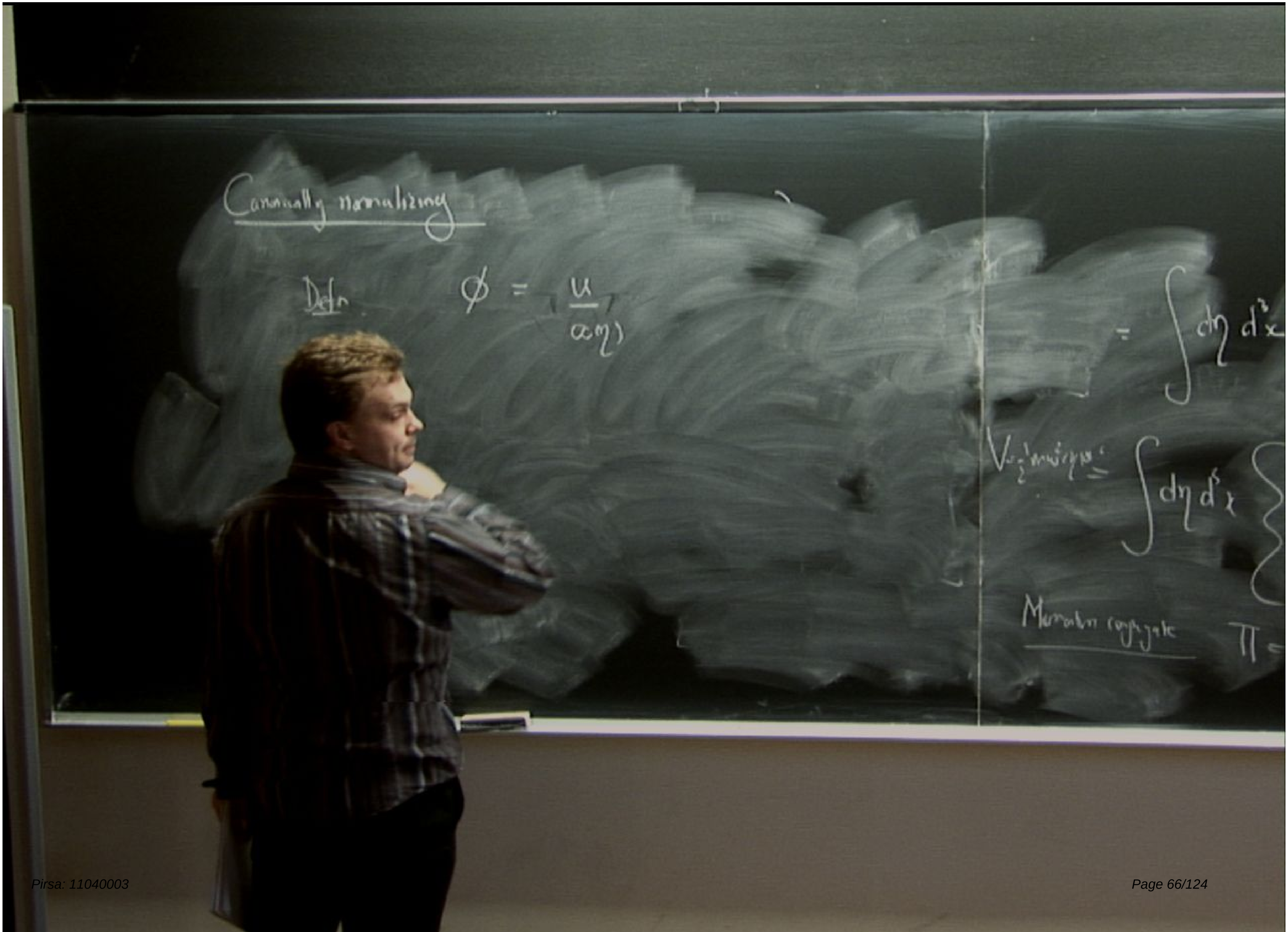
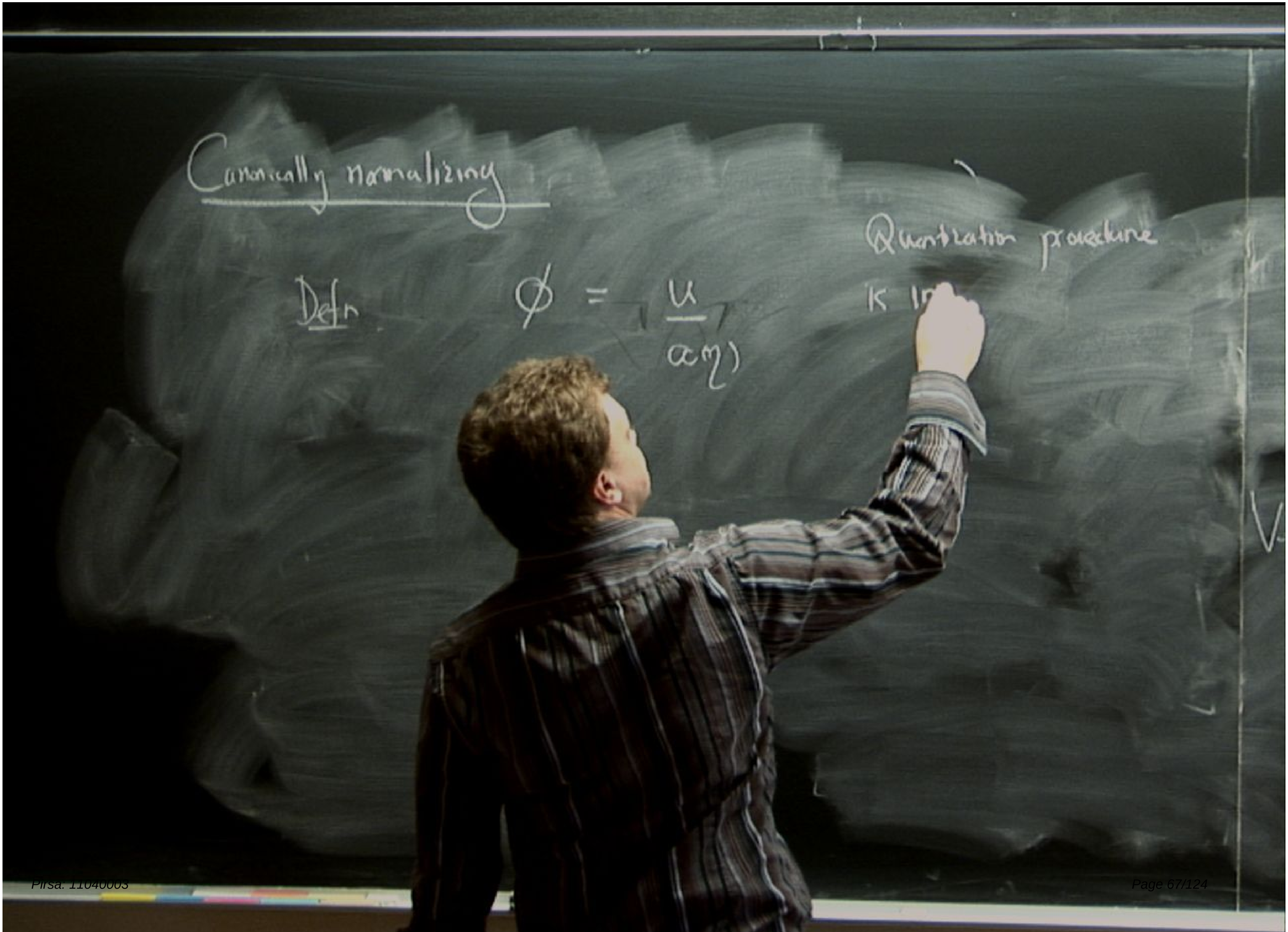

commutation
$[\phi(\vec{x},\eta),\ \phi(\vec{x}',\eta')] = 0$
$[\Pi(\vec{x},\eta),\ \Pi(\vec{x}',\eta')] = 0$
$\delta(x-x')\delta(y-y')\delta(z-z')$
$[\phi(\vec{x},\eta),\ \Pi(\vec{x}',\eta)] = i\hbar\,\delta^{(3)}(x-x')$




$[\phi(\vec{x},\eta), \phi(\vec{x}',\eta')] = 0$
$[\pi(\vec{x},\eta), \pi(\vec{x}',\eta')] = 0$
$[\phi(\vec{x},\eta), \pi(\vec{x}',\eta')] = i\hbar\,\delta^{(3)}(x-x')$
$\delta(x-x')\delta(y-y')\delta(z-z')$
$[\phi(\vec{x},\eta), \partial_\eta \phi(\vec{x}',\eta')] = \frac{i\hbar}{a^2(\eta)}\delta^{(3)}(x-x')$

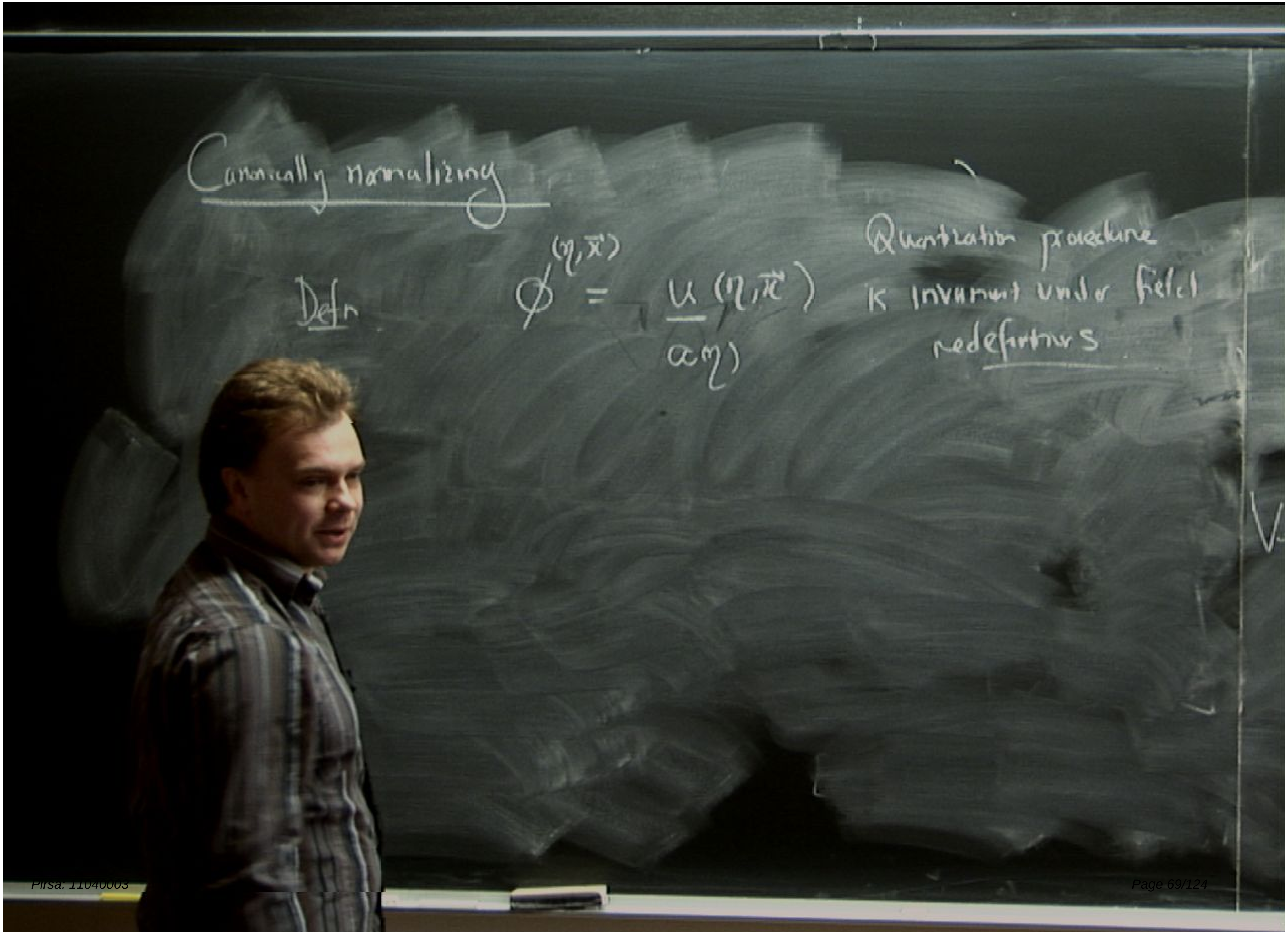

Canonically normalizing
Defn
ϕ = u / a(η)
Momentum conjugate
Π =


Canonically normalizing
Defn
φ = u / a(η)
Quantization procedure

Canonically normalizing

Defn $\phi = \frac{u}{\alpha(\eta)}$

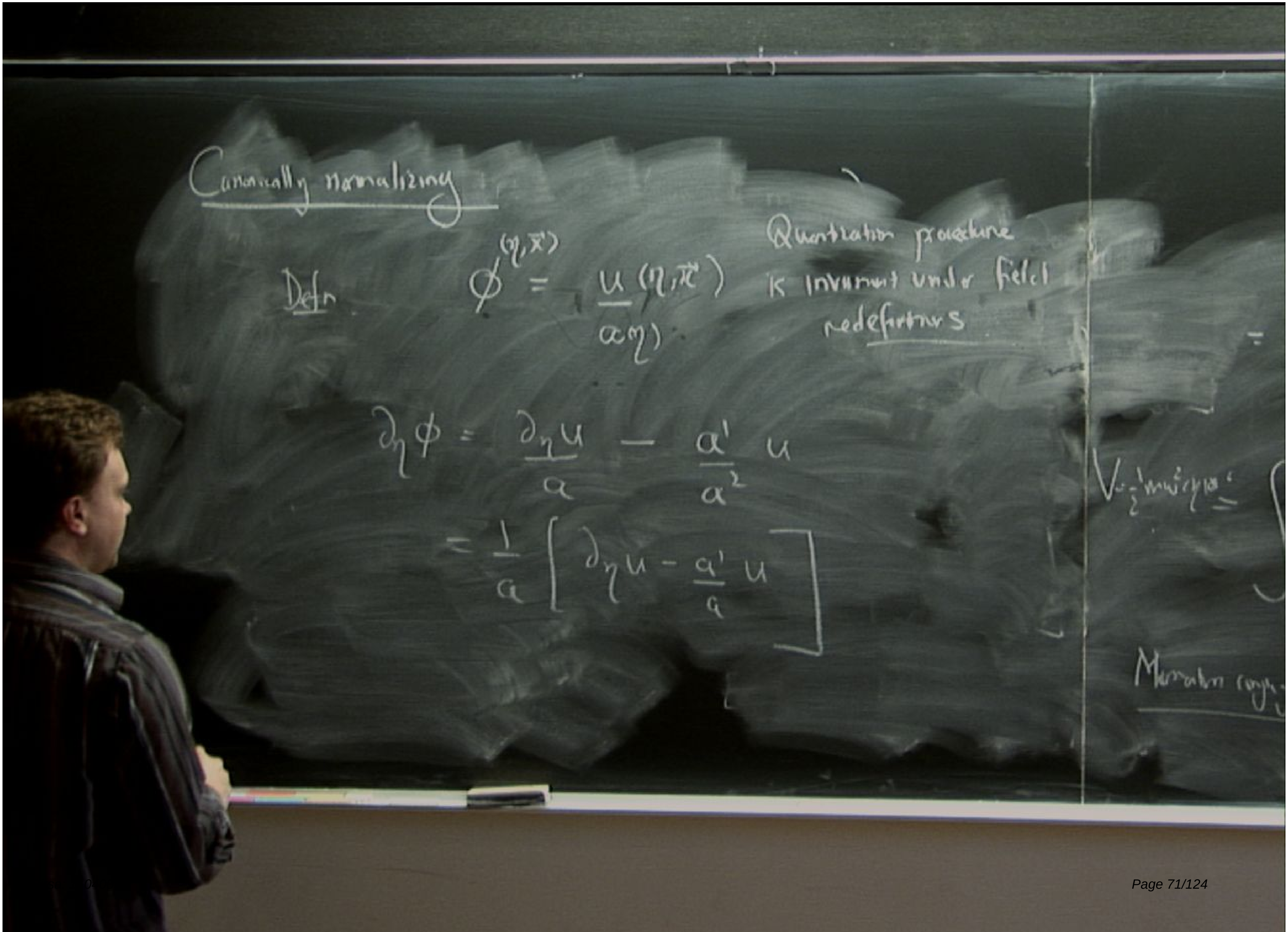
Quantization procedure is invariant under field redefinitions


Canonically normalizing
Defn
$\phi(\eta,\vec{x}) = \frac{u(\eta,\vec{x})}{a(\eta)}$
Quantization procedure
is invariant under field
redefinitions

Canonically normalizing

Defn $\phi(\eta,\vec{x}) = \dfrac{u(\eta,\vec{x})}{a(\eta)}$

Quantization procedure is invariant under field redefinitions


Canonically normalizing
Defn $\phi(\eta,\vec{x}) = \frac{u(\eta,\vec{x})}{a(\eta)}$
Quantization procedure is invariant under field redefinitions
$\partial_\eta \phi = \frac{\partial_\eta u}{a} - \frac{a'}{a^2} u$
$= \frac{1}{a}\left[\partial_\eta u - \frac{a'}{a} u\right]$

Canonically normalizing

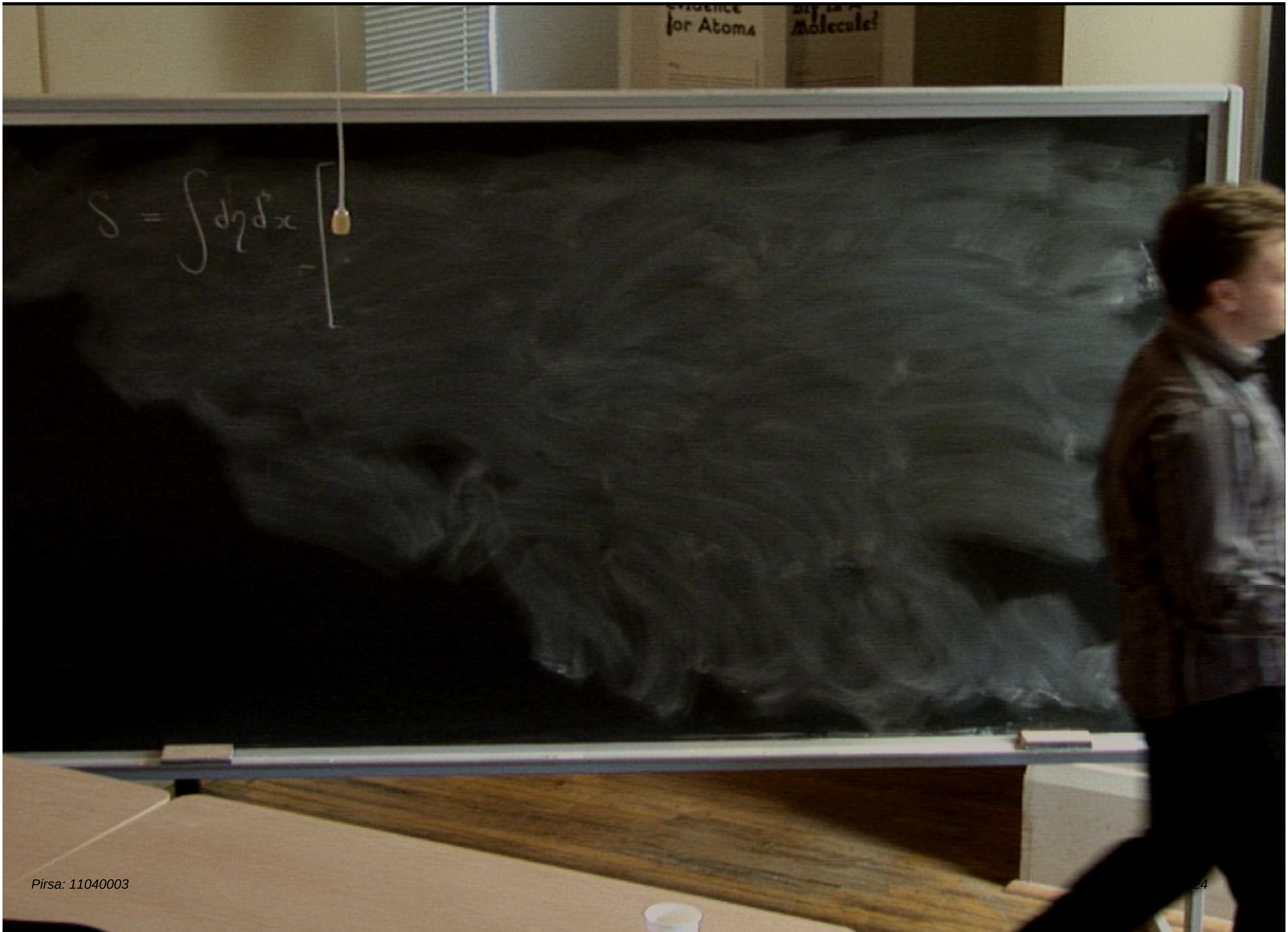
Defn $\phi^{(\eta,\vec{x})} = \dfrac{u(\eta,\vec{x})}{a(\eta)}$

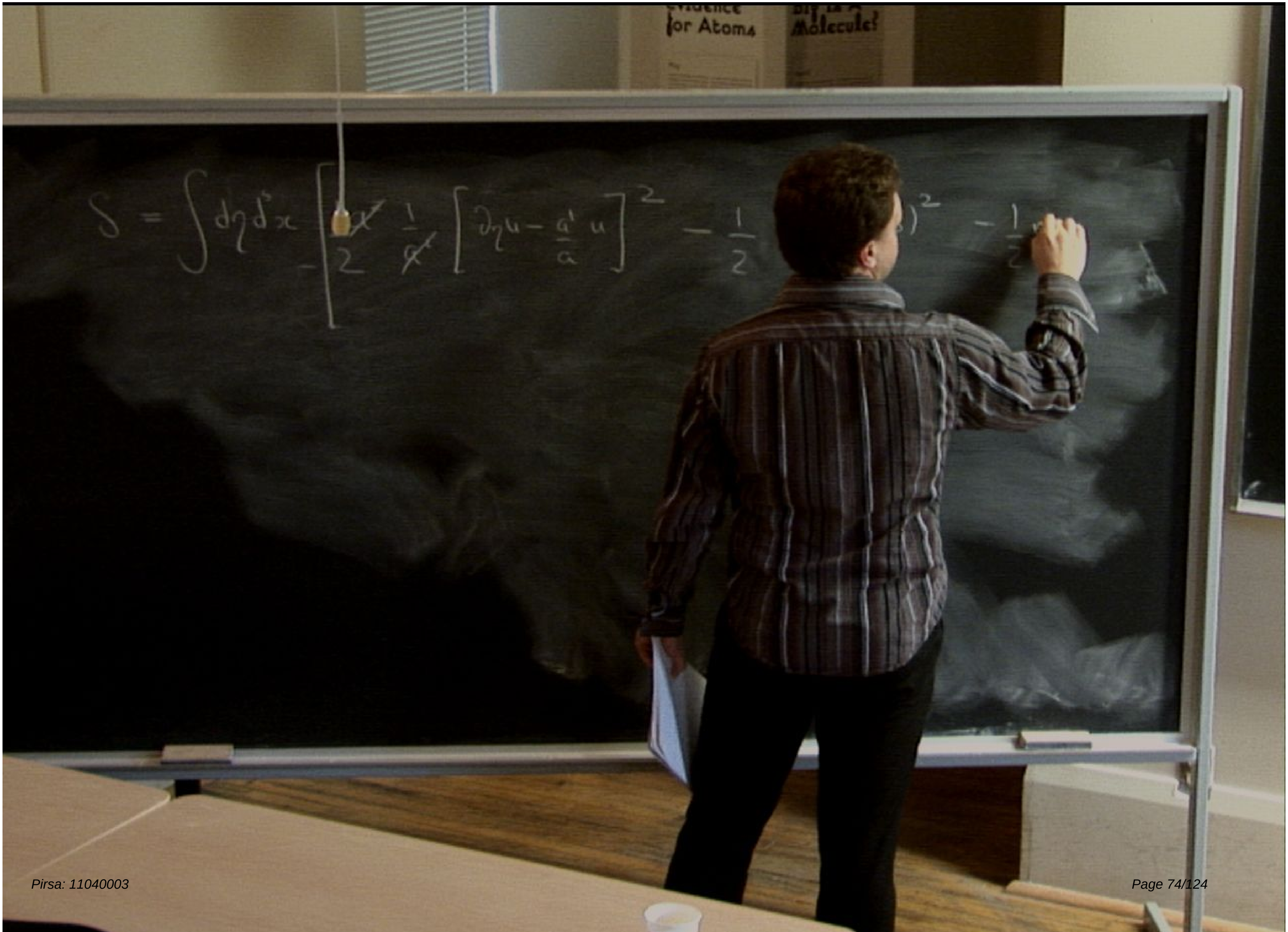
Quantization procedure is invariant under field redefinitions

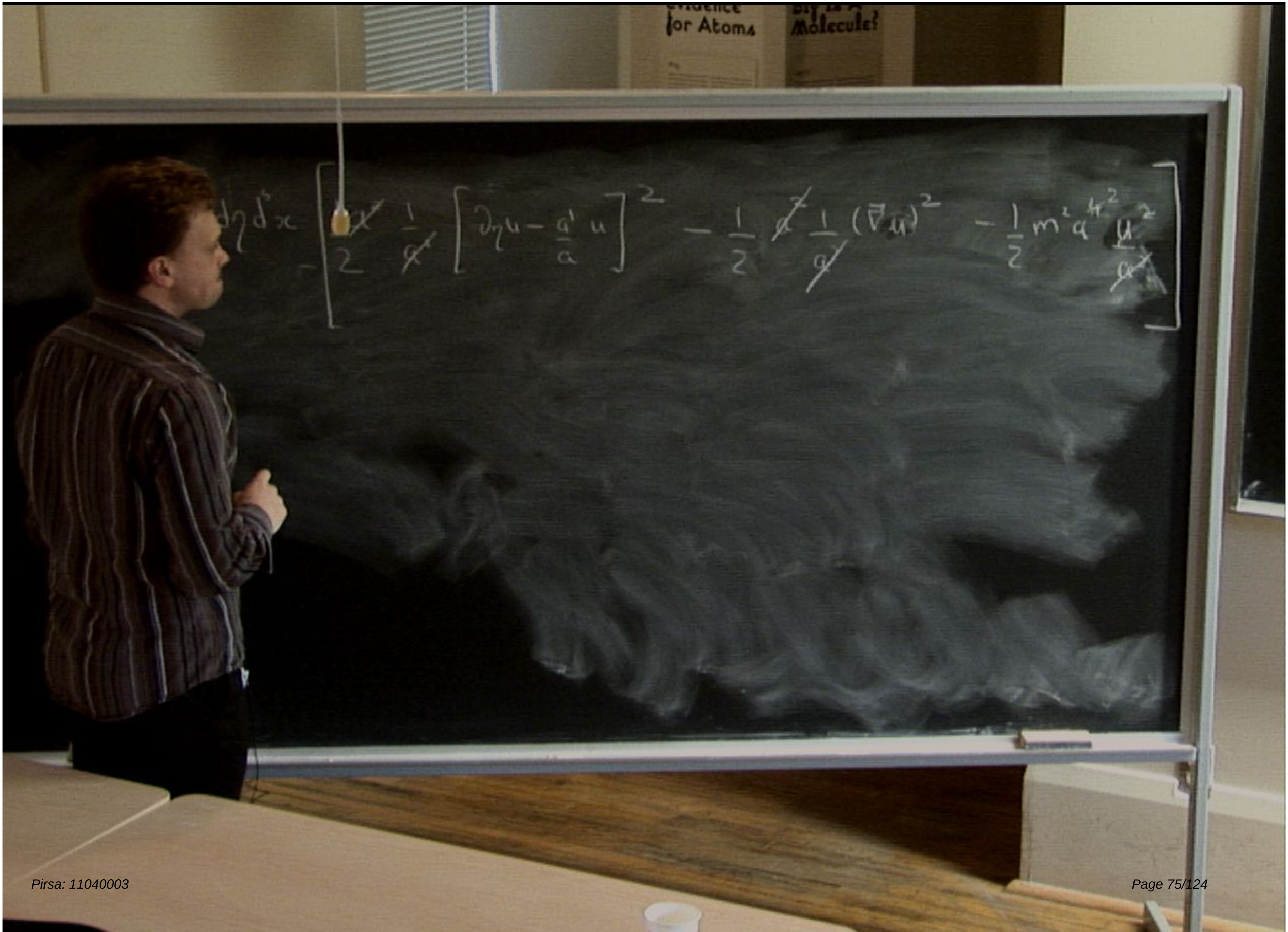
$$\partial_\eta \phi = \frac{\partial_\eta u}{a} - \frac{a'}{a^2} u$$

$$= \frac{1}{a}\left[\partial_\eta u - \frac{a'}{a} u\right]$$

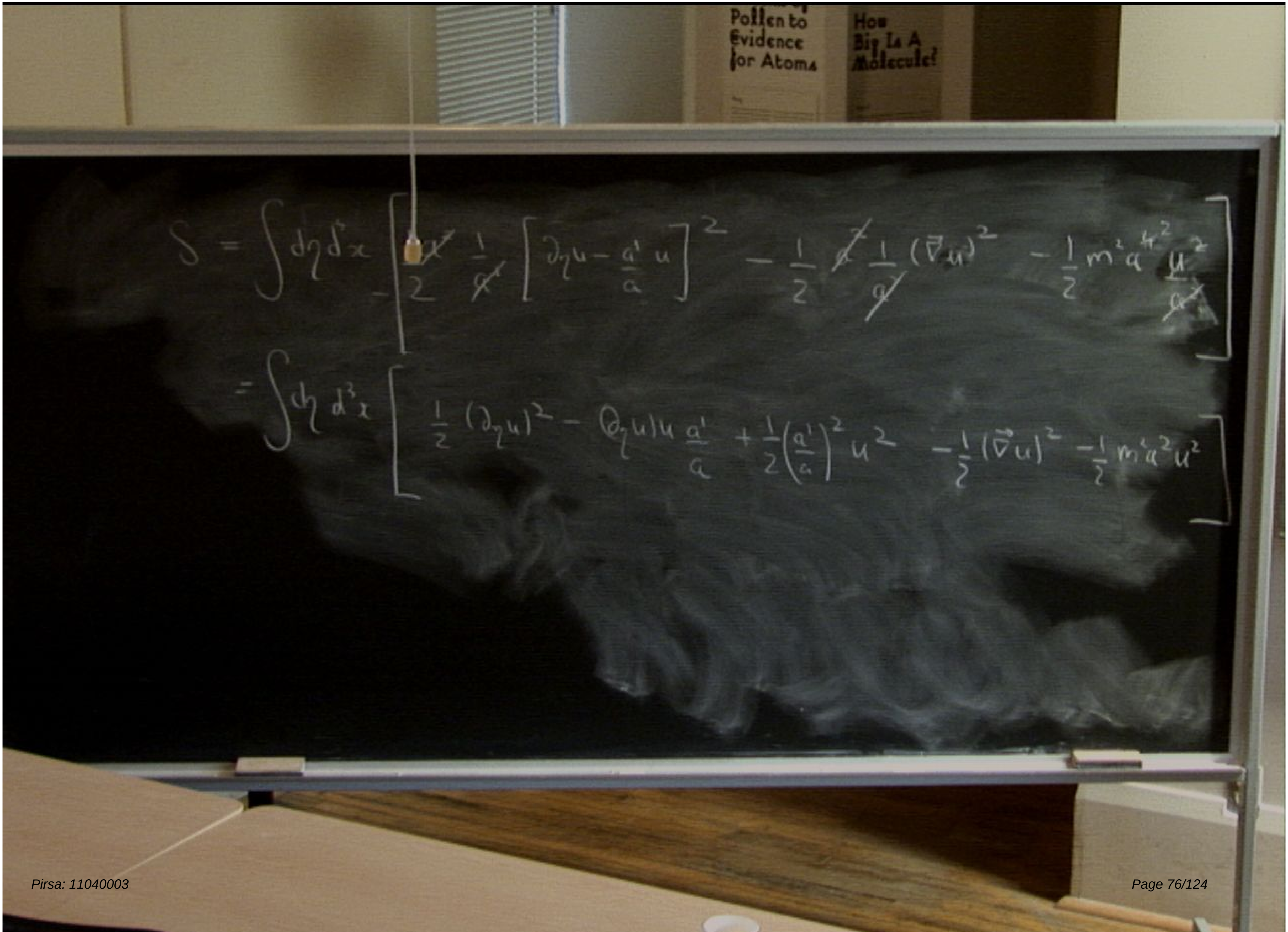
$$\vec{\nabla}\phi = \frac{\vec{\nabla} u}{a}$$

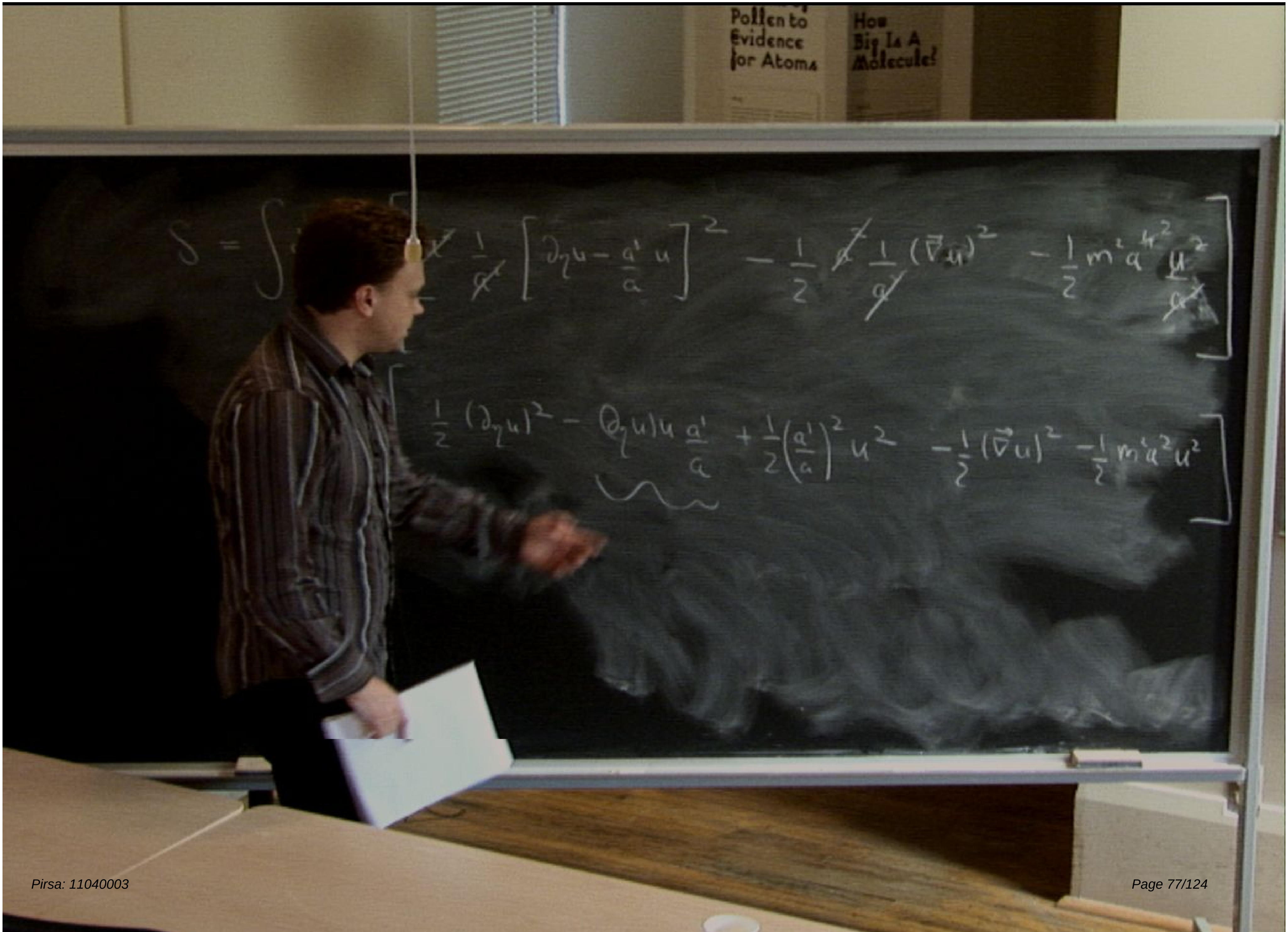

$S = \int d\eta \, d^4x \, \Big[$
for Atoms

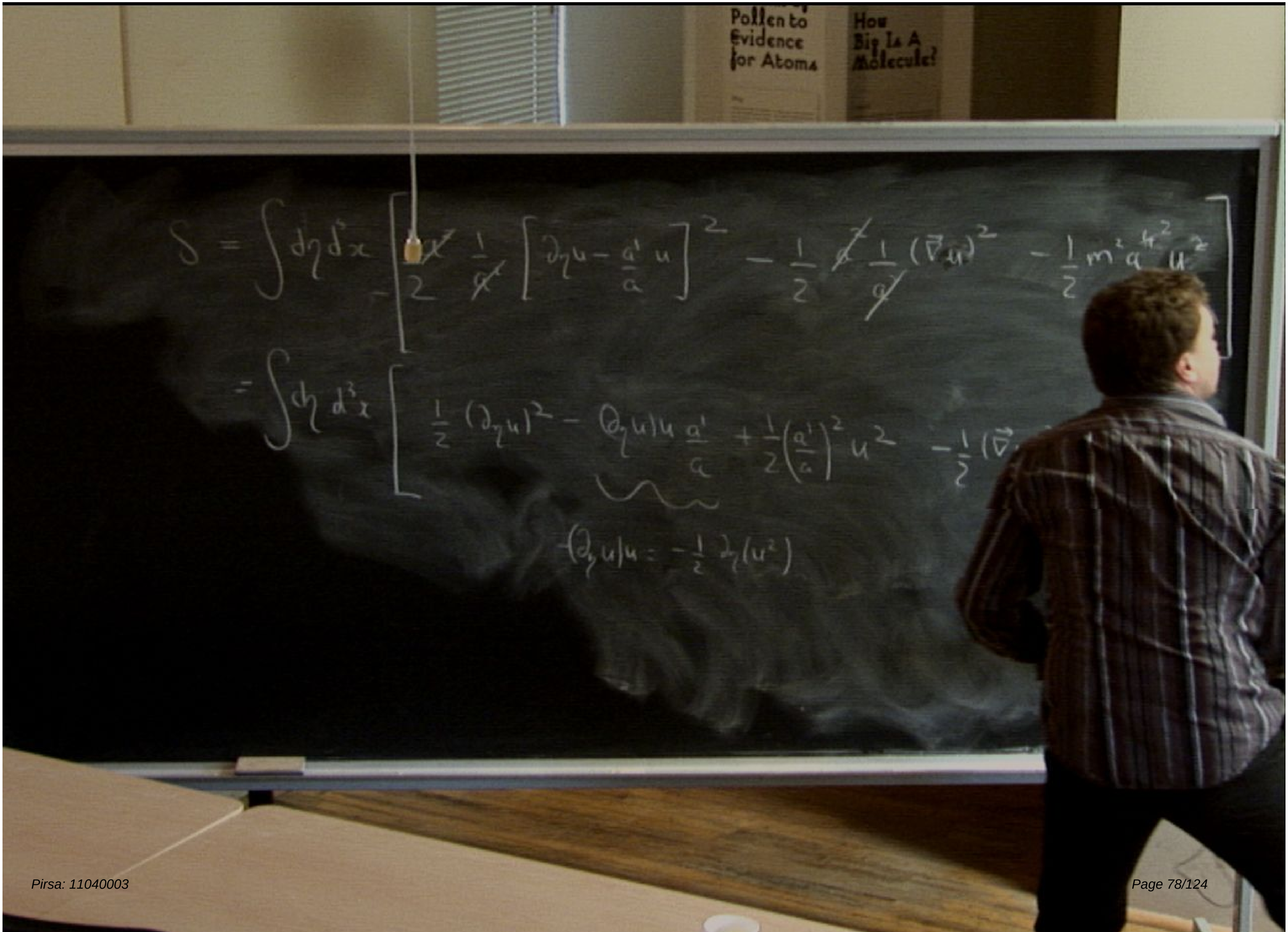

for Atoms

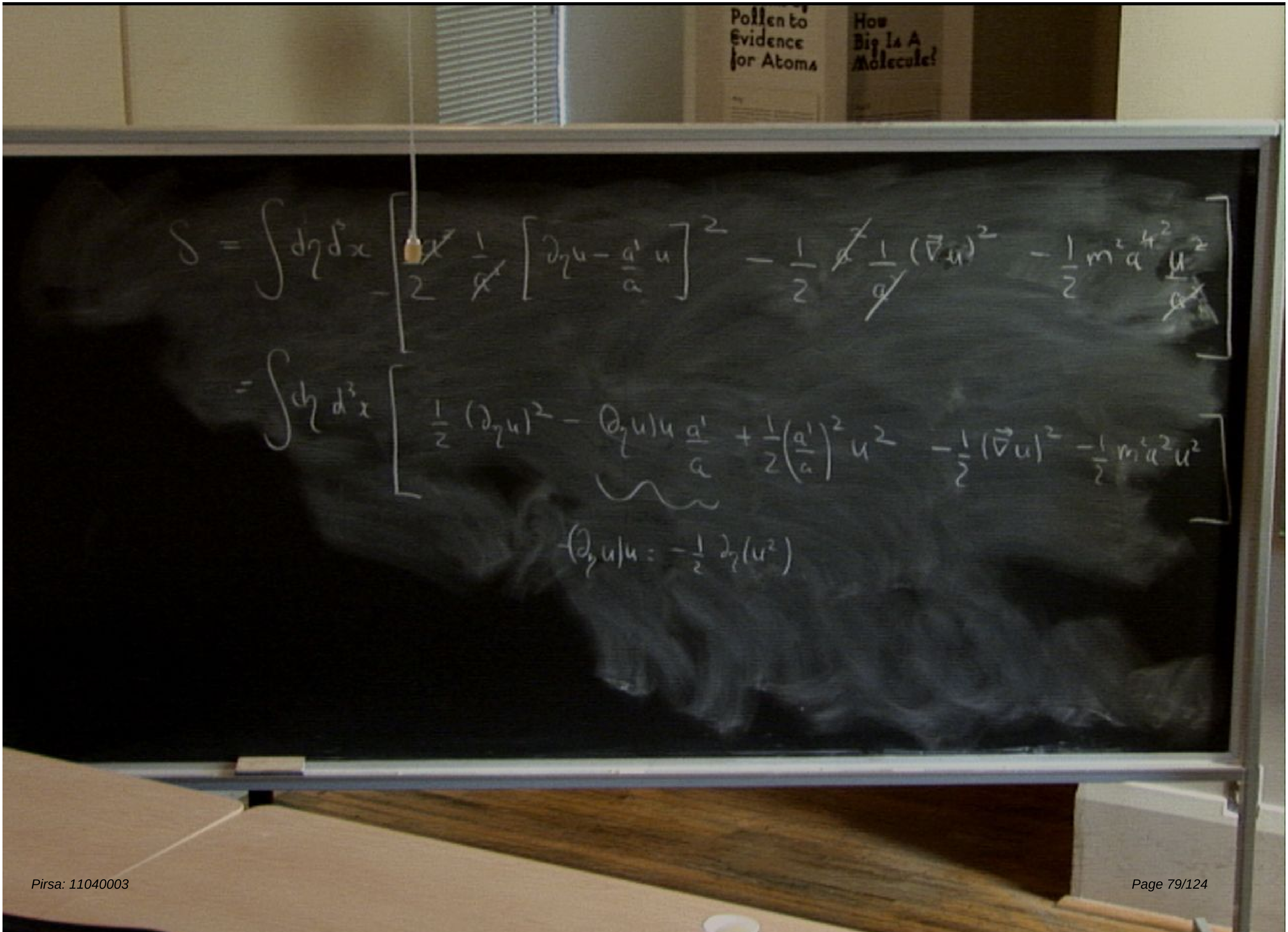


for Atoms
Molecule?
$\int d\eta\, d^3x \left[\frac{\cancel{a^2}}{2} \frac{1}{\cancel{a^2}} \left[\partial_\eta u - \frac{a'}{a} u \right]^2 - \frac{1}{2} \cancel{a^2} \frac{1}{\cancel{a^2}} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^4 \frac{u^2}{\cancel{a^2}} \right]$

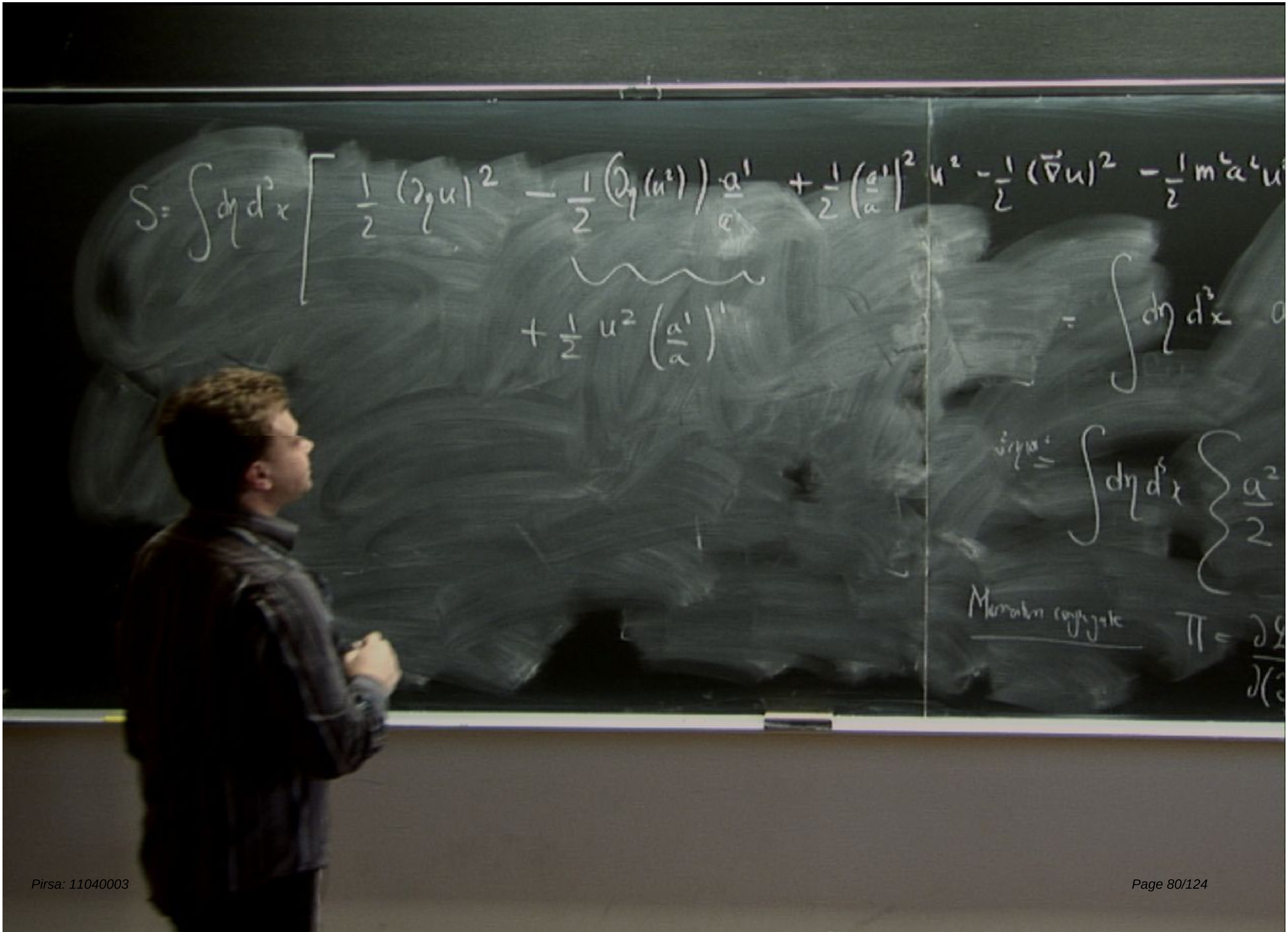

Pollen to Evidence for Atoms
How Big Is A Molecule?
$S = \int d\eta\, d^3x \left[\frac{\cancel{a^2}}{2} \frac{1}{\cancel{a^2}} \left[\partial_\eta u - \frac{a'}{a} u\right]^2 - \frac{1}{2} \cancel{a^2} \frac{1}{\cancel{a^2}} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^{4^2} \frac{u^2}{\cancel{a^2}} \right]$
$= \int d\eta\, d^3x \left[\frac{1}{2} (\partial_\eta u)^2 - (\partial_\eta u) u \frac{a'}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right]$

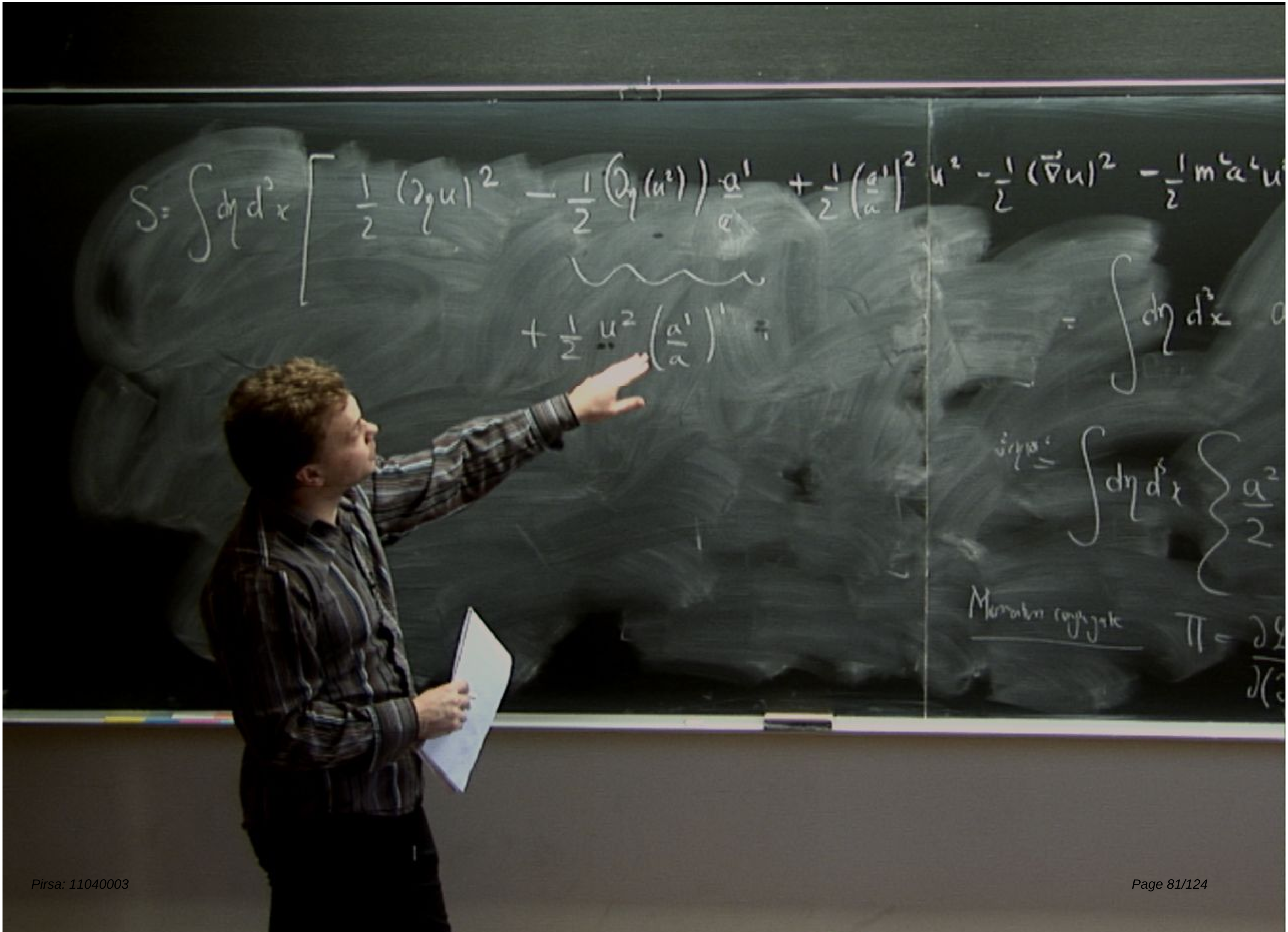

Pollen to Evidence for Atoms
How Big Is A Molecule?
$\frac{1}{2}(\partial_\eta u)^2 - (\partial_\eta u)u\frac{a'}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2}m^2a^2u^2$

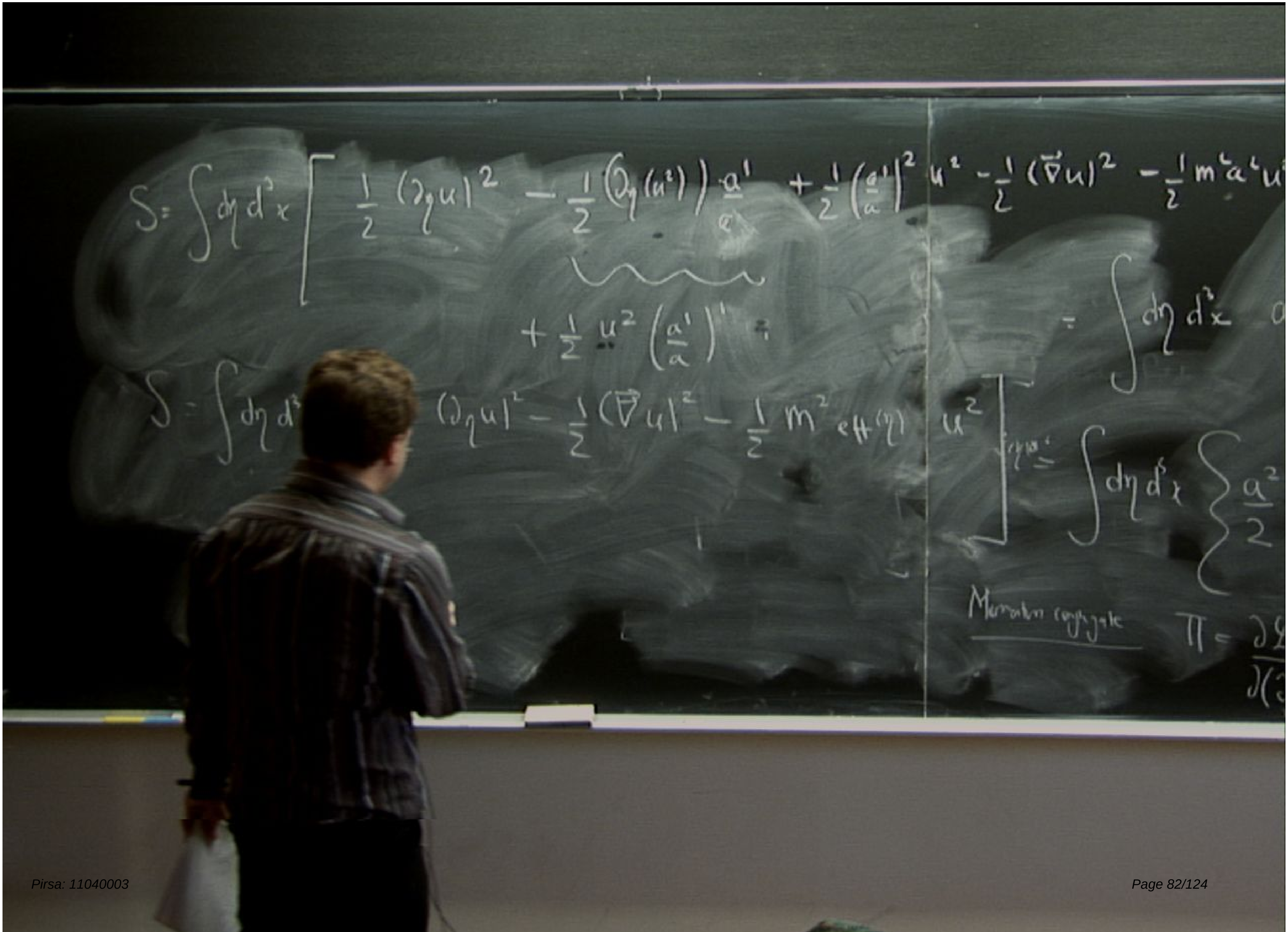

Pollen to Evidence for Atoms
How Big Is A Molecule?
$S = \int d\eta\, d^3x \left[\frac{1}{2}\frac{\cancel{a^2}}{\cancel{a^2}}\left[\partial_\eta u - \frac{a'}{a}u\right]^2 - \frac{1}{2}\frac{\cancel{a^2}}{\cancel{a^2}}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^{4} u^2 \right]$
$= \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \underbrace{(\partial_\eta u) u \frac{a'}{a}} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla}\right.$
$-(\partial_\eta u)u = -\frac{1}{2}\partial_\eta(u^2)$

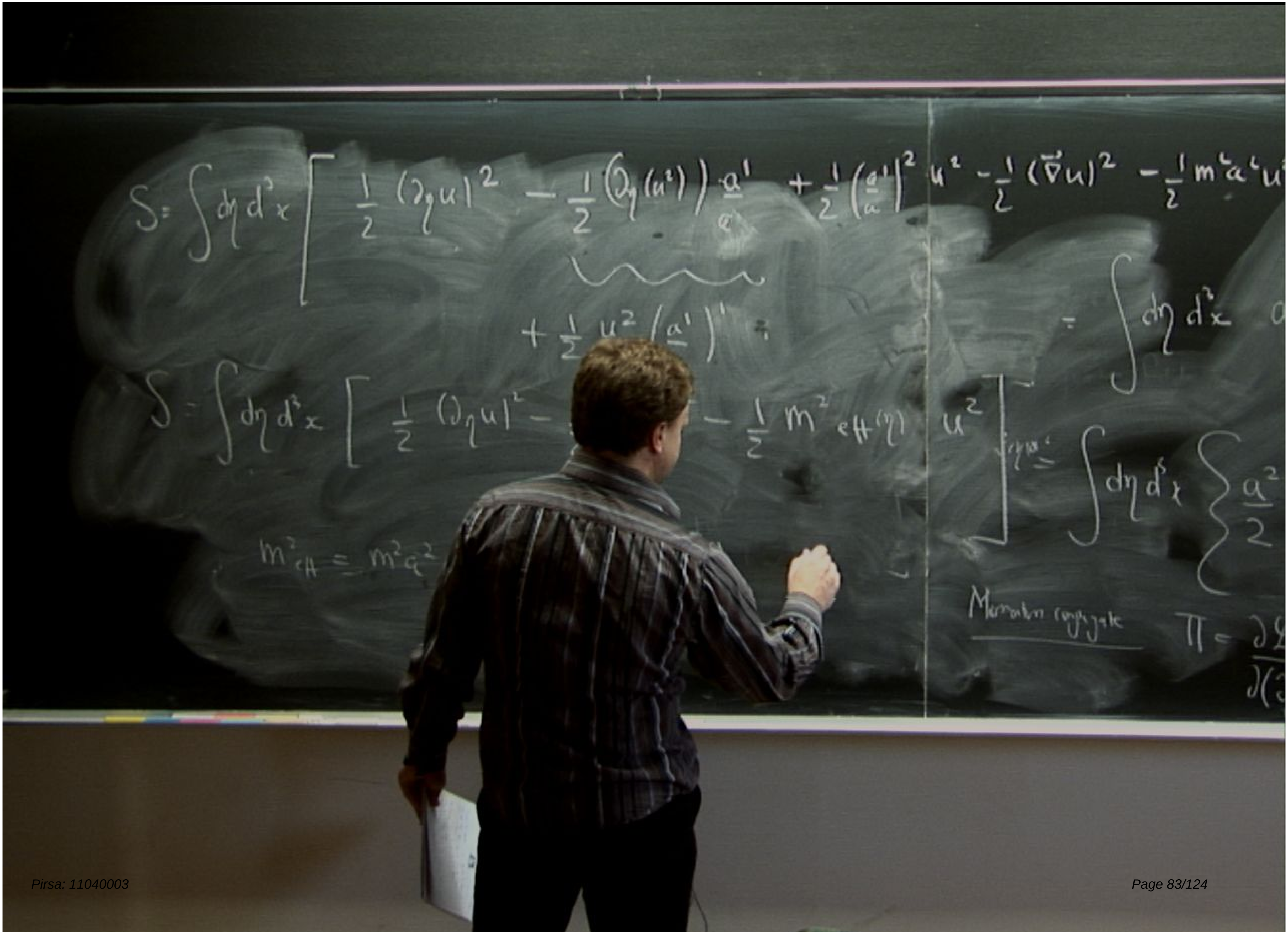


Pollen to Evidence for Atoms
How Big Is A Molecule?
$S = \int d\eta\, d^3x \left[\frac{\cancel{a^2}}{2}\frac{1}{\cancel{a^2}}\left[\partial_\eta u - \frac{a'}{a}u\right]^2 - \frac{1}{2}\cancel{a^2}\frac{1}{\cancel{a^2}}(\vec{\nabla} u)^2 - \frac{1}{2}m^2 a^4 \frac{u^2}{\cancel{a^2}}\right]$
$= \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - (\partial_\eta u)u\frac{a'}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2}m^2 a^2 u^2\right]$
$-(\partial_\eta u)u = -\frac{1}{2}\partial_\eta(u^2)$

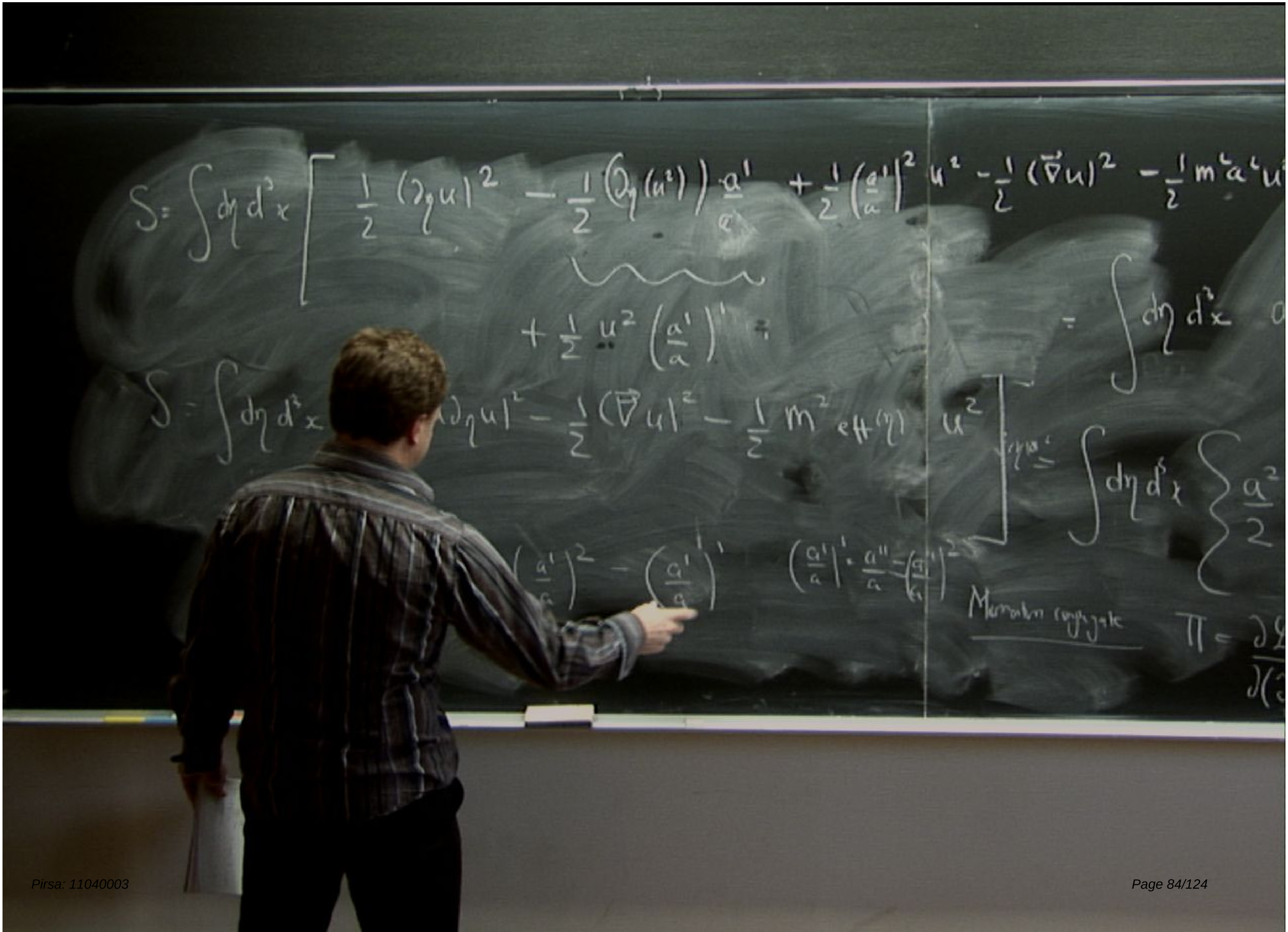

$S = \int d\eta\, d^3x \Big[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}\big(\partial_\eta(u^2)\big)\frac{a'}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2$
$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$
$= \int d\eta\, d^3x$
$= \int d\eta\, d^3x \Big\{ \frac{a^2}{2}$
Momentum conjugate $\Pi = \frac{\partial \mathcal{L}}{\partial(}$


S = ∫dη d³x [1/2 (∂η u)² − 1/2 (∂η(u²)) a'/a + 1/2 (a'/a)² u² − 1/2 (∇u)² − 1/2 m²a²u
+ 1/2 u² (a'/a)'
= ∫dη d³x
∫dη d³x { a²/2
Momentum conjugate
Π = ∂ℒ/∂(

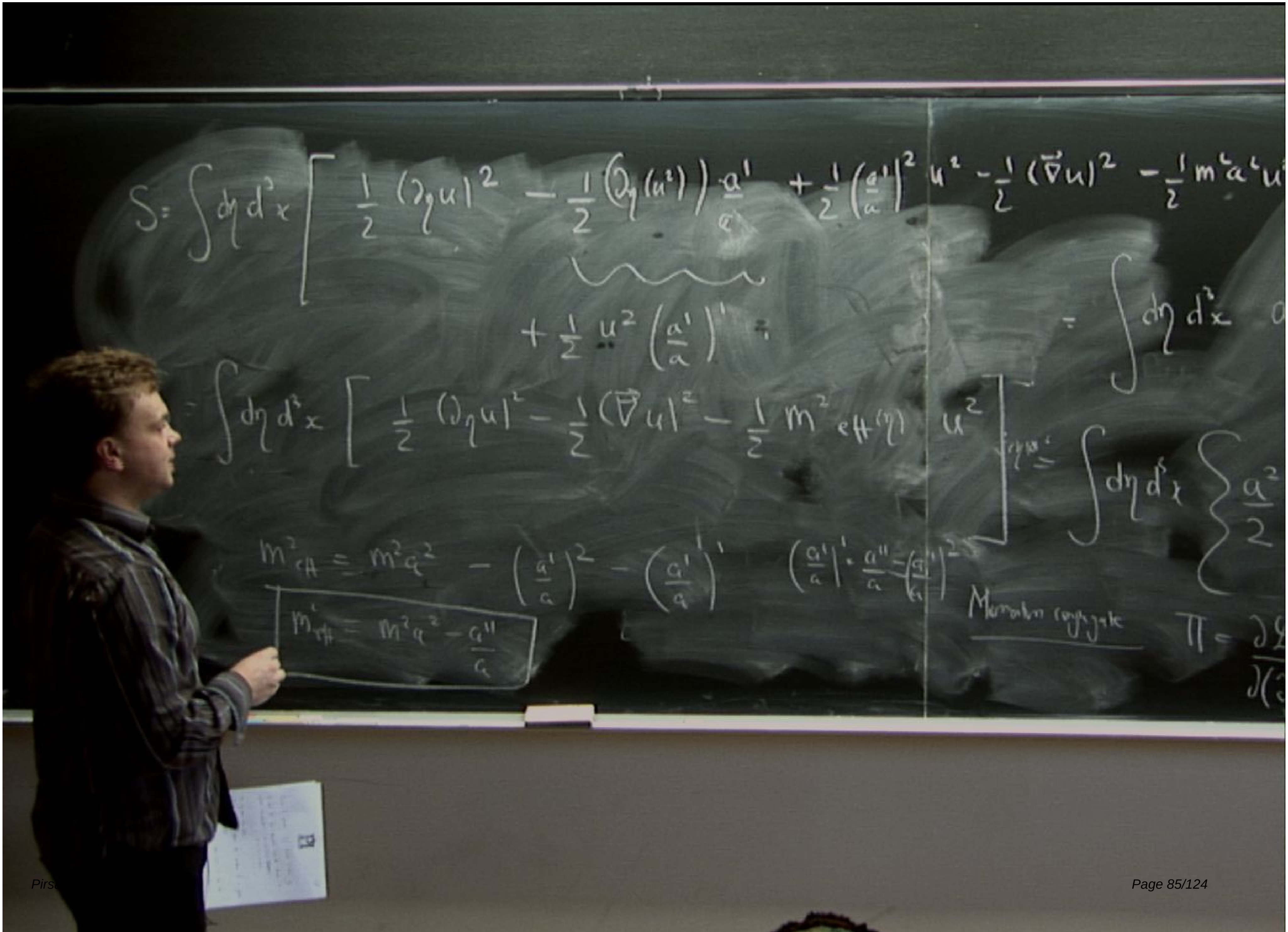

$S = \int d\eta\, d^3x \left[\frac{1}{2} (\partial_\eta u)^2 - \frac{1}{2} (\partial_\eta (u^2)) \frac{a'}{a} + \frac{1}{2} \left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$
$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$
$S = \int d\eta\, d^3x \; (\partial_\eta u)^2 - \frac{1}{2} (\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta) \; u^2$
$= \int d\eta\, d^3x$
$\int d\eta\, d^3x \left\{ \frac{a^2}{2} \right.$
Momentum conjugate
$\Pi = \frac{\partial \mathcal{L}}{\partial (}$



S = ∫dη d³x [1/2 (∂η u)² − 1/2 (∂η(u²)) a'/a + 1/2 (a'/a)² u² − 1/2 (∇u)² − 1/2 m²a²u²
+ 1/2 u² (a'/a)'
= ∫dη d³x
S = ∫dη d³x [1/2 (∂η u)² − 1/2 m²eff(η) u²]
= ∫dη d³x { a²/2
m²eff = m²a²
Momentum conjugate
Π = ∂L/∂(



$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}\left(\partial_\eta (u^2)\right)\frac{a'}{a} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$
$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$
$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{eff}(\eta)\, u^2 \right]$
$\left(\frac{a'}{a}\right)' = \frac{a''}{a} - \left(\frac{a'}{a}\right)^2$
Momentum conjugate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+ 1/2 u² (a'/a)'
= ∫dη d³x [1/2 (∂η u)² − 1/2 (∇u)² − 1/2 m²eff(η) u²]
m²eff = m²a² − (a'/a)² − (a'/a)'
(a'/a)' = a''/a − (a'/a)²
m²eff = m²a² − a''/a
= ∫dη d³x
= ∫dη d³x { a²/2
Momentum conjugate
Π = ∂ℒ/∂(

$$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \underbrace{\frac{1}{2}\left(\partial_\eta (u^2)\right)\frac{a'}{a}} + \frac{1}{2}\left(\frac{a'}{a}\right)^2 u^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2 a^2 u^2 \right.$$

$$+ \frac{1}{2} u^2 \left(\frac{a'}{a}\right)'$$

$$S = \int d\eta\, d^3x \left[\frac{1}{2}(\partial_\eta u)^2 - \frac{1}{2}(\vec{\nabla} u)^2 - \frac{1}{2} m^2_{\mathrm{eff}}(\eta)\, u^2 \right]$$

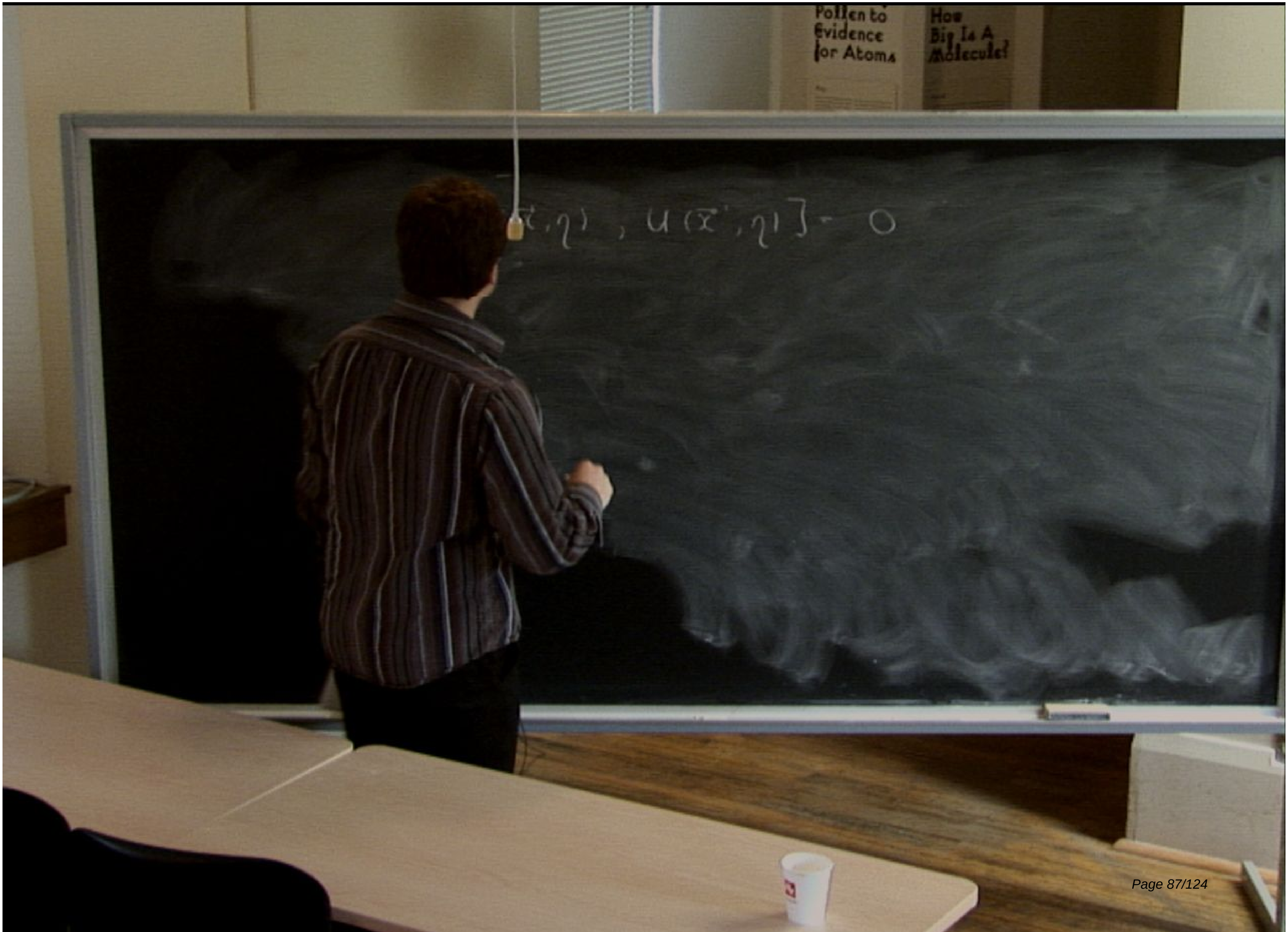
$$m^2_{\mathrm{eff}} = m^2 a^2 - \left(\frac{a'}{a}\right)^2 - \left(\frac{a'}{a}\right)' \qquad \left(\frac{a'}{a}\right)' = \frac{a''}{a} - \left(\frac{a'}{a}\right)^2$$

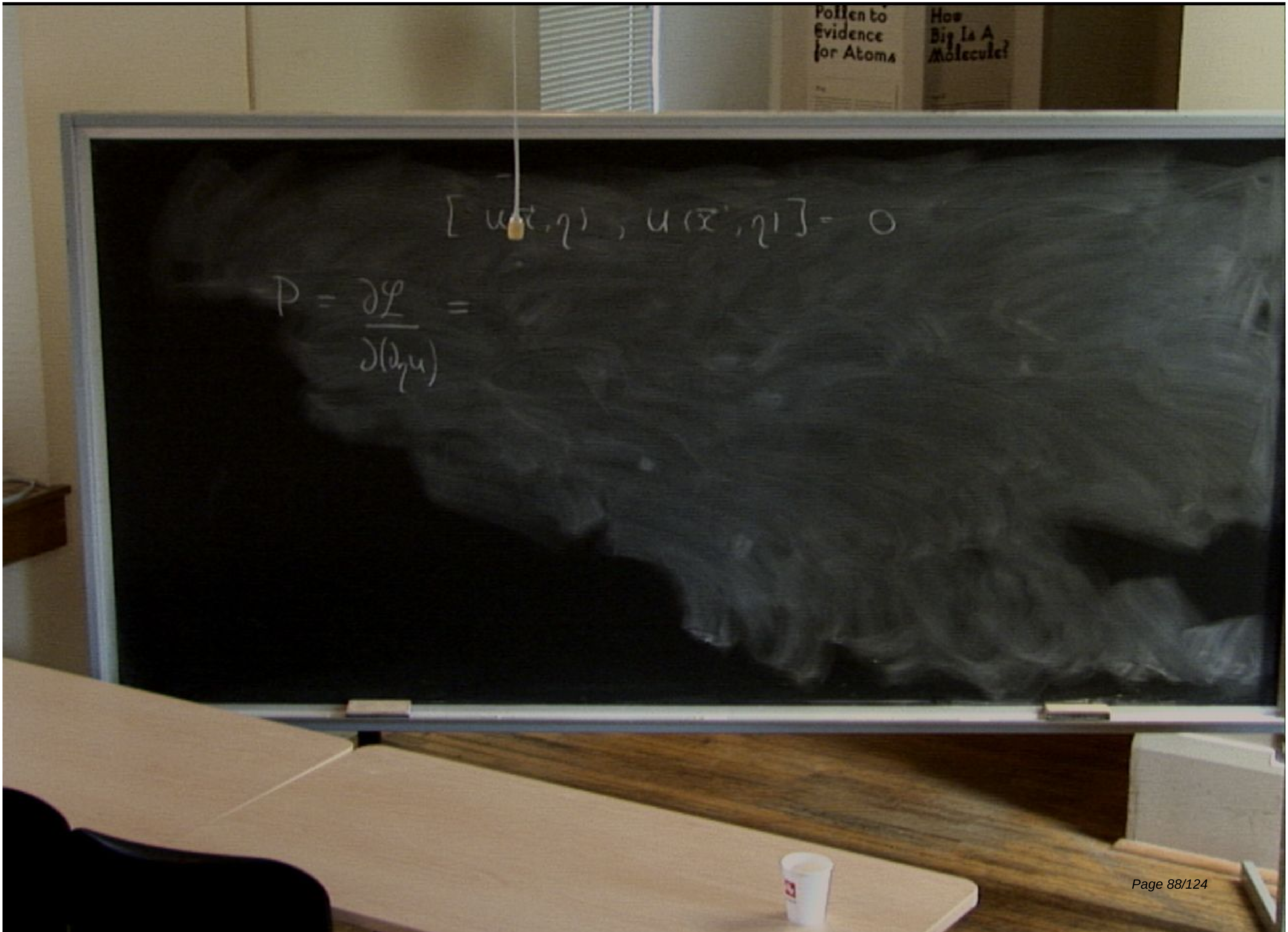
$$\boxed{m^2_{\mathrm{eff}} = m^2 a^2 - \frac{a''}{a}}$$

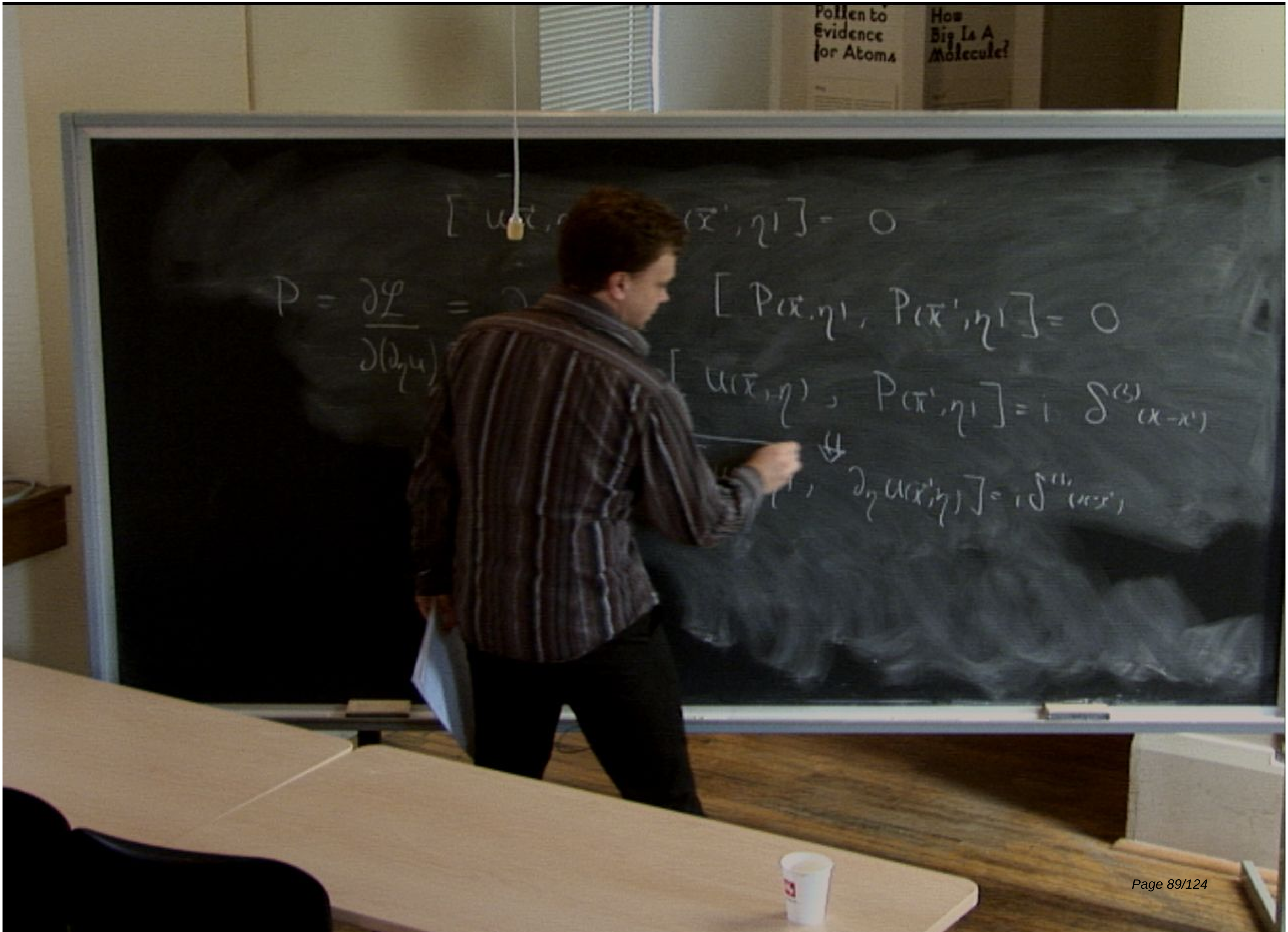
$$= \int d\eta\, d^3x \quad a \ldots$$

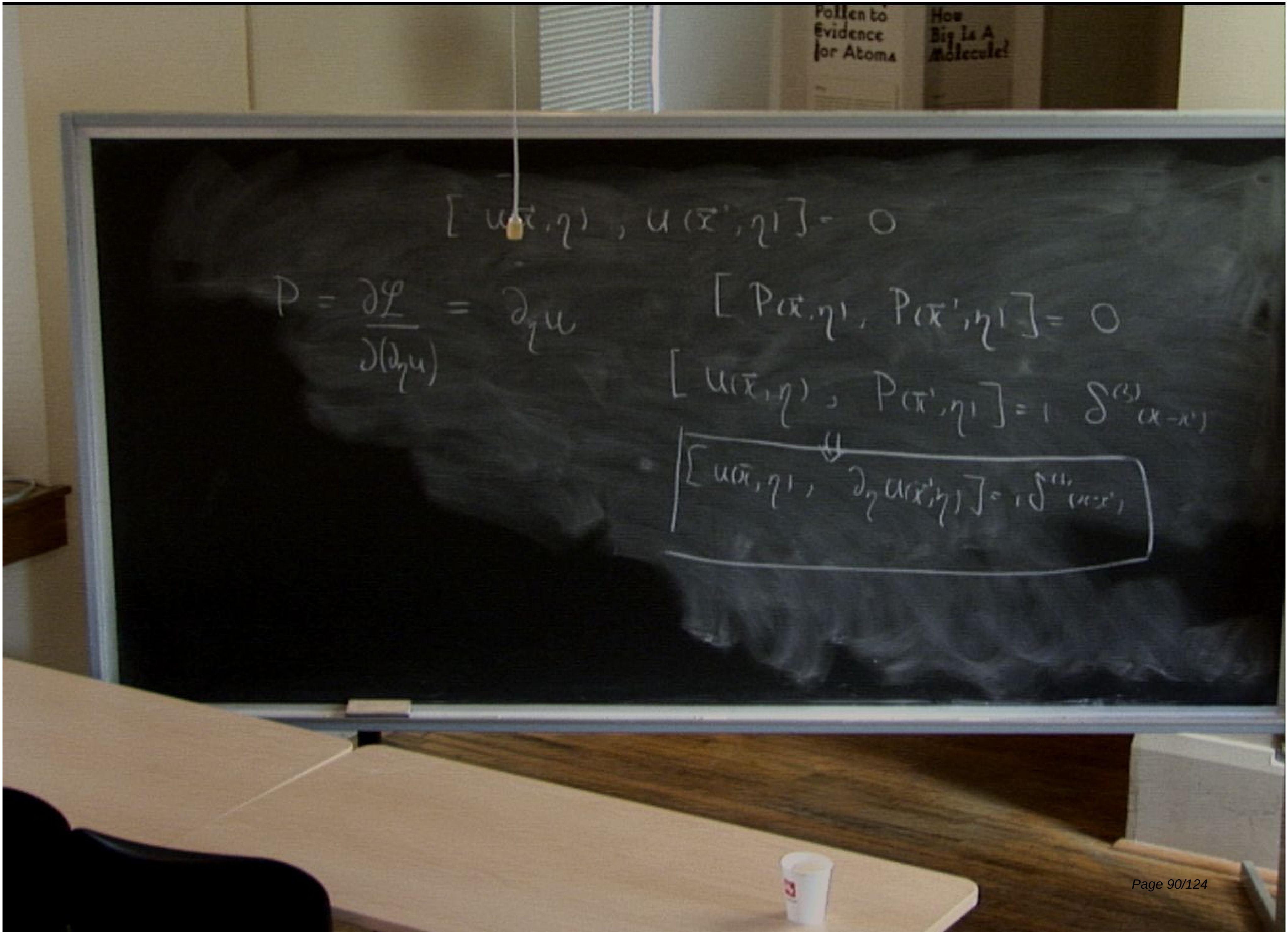
$$\stackrel{\text{[illegible]}}{=} \int d\eta\, d^3x \left\{ \frac{a^2}{2} \ldots \right.$$

Momentum conjugate $\quad \Pi = \frac{\partial \mathcal{L}}{\partial(\ldots}$

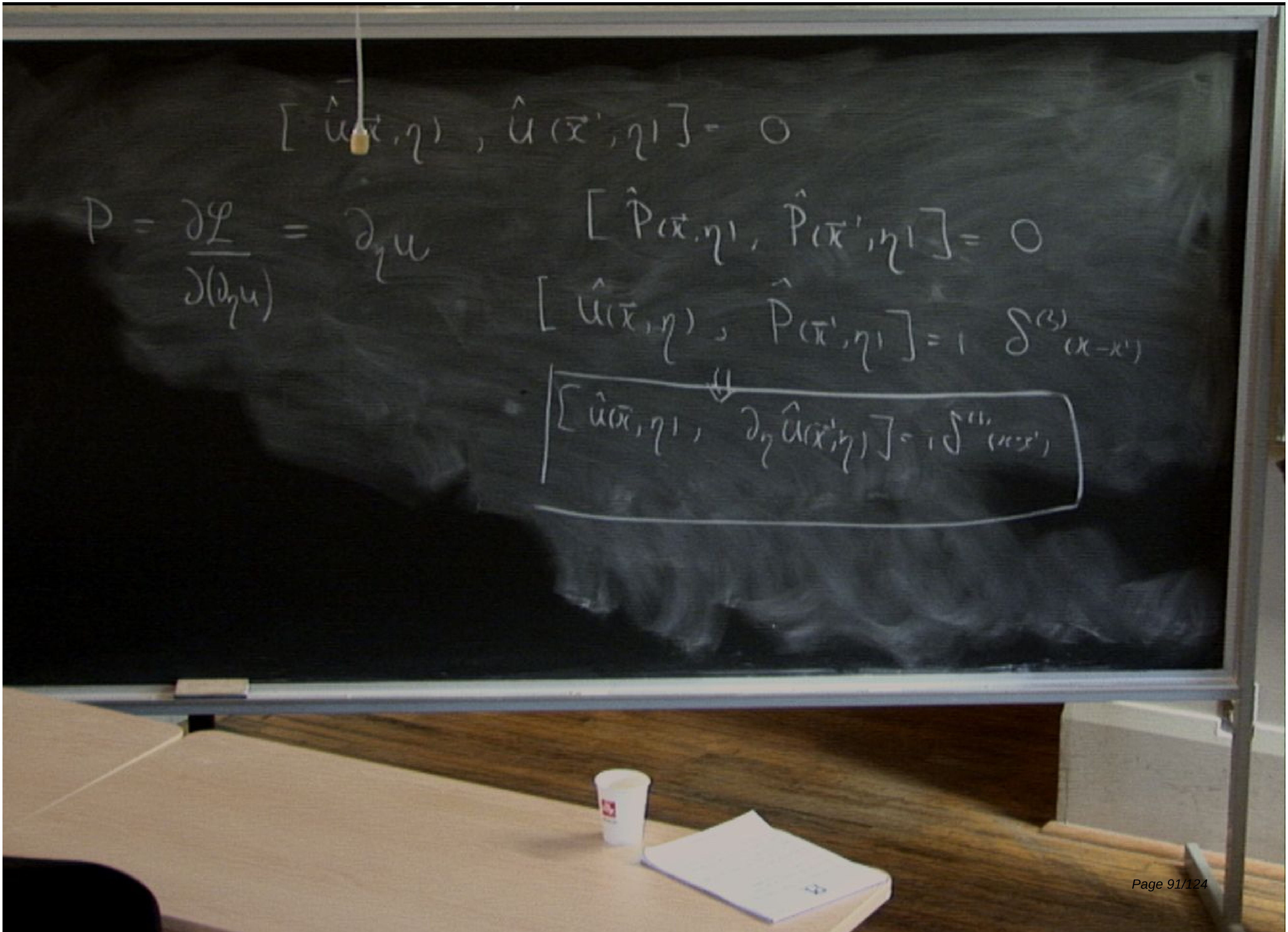

Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
$\vec{x},\eta)\ ,\ u(\vec{x}',\eta)] = 0$

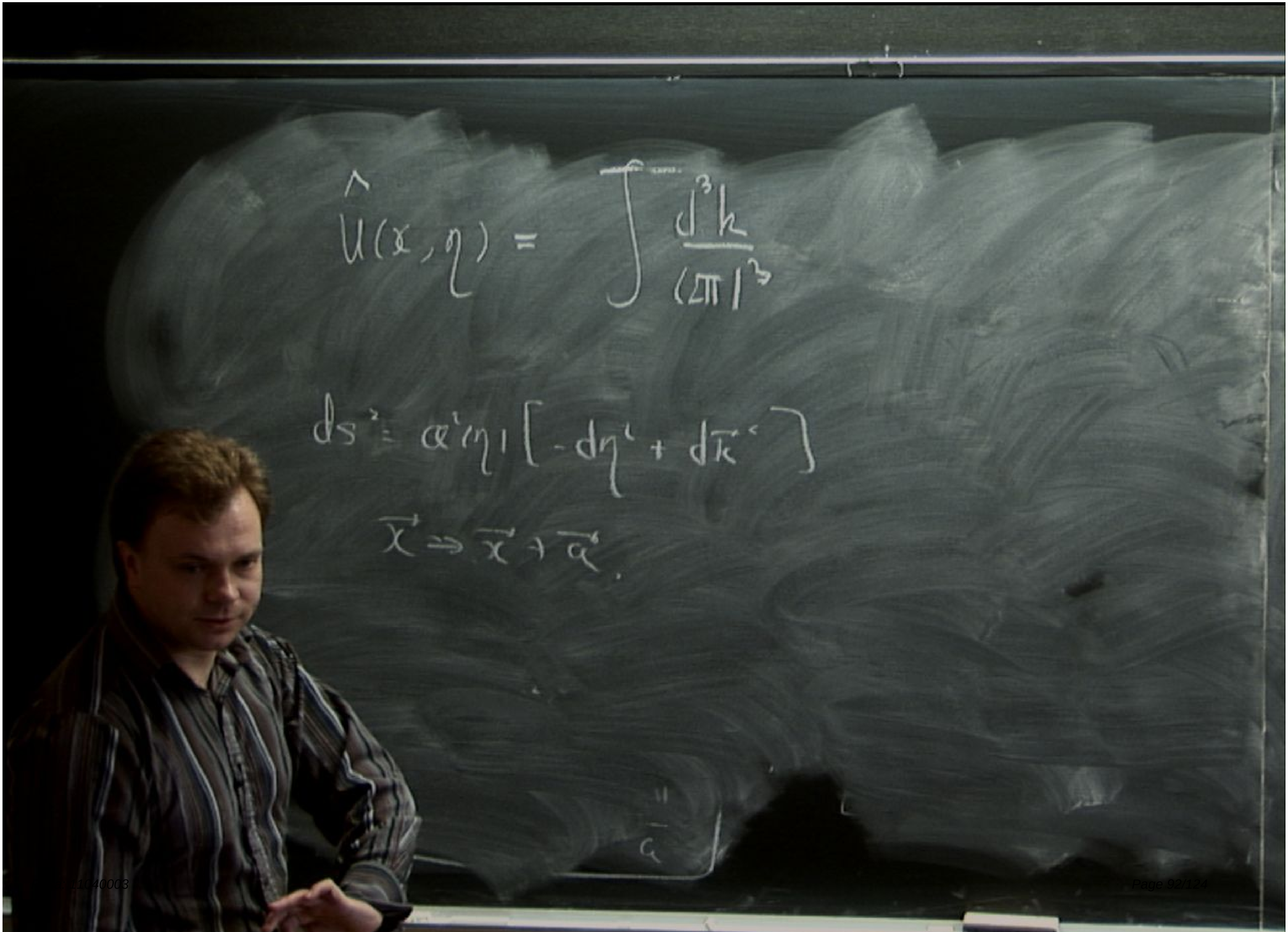

Pollen to Evidence for Atoms
How Big Is A Molecule?
$[u(\vec{x},\eta), u(\vec{x}',\eta)] = 0$
$P = \frac{\partial \mathcal{L}}{\partial(\partial_\eta u)} =$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?



Pollen to Evidence for Atoms
How Big Is A Molecule!
$[u(\vec{x},\eta), u(\vec{x}',\eta)] = 0$
$P = \frac{\partial \mathcal{L}}{\partial(\partial_\eta u)} = \partial_\eta u$
$[P(\vec{x},\eta), P(\vec{x}',\eta)] = 0$
$[u(\vec{x},\eta), P(\vec{x}',\eta)] = i\,\delta^{(3)}(x-x')$
$[u(\vec{x},\eta), \partial_\eta u(\vec{x}',\eta)] = i\,\delta^{(3)}(x-x')$


$[\hat{u}(\vec{x},\eta), \hat{u}(\vec{x}',\eta)] = 0$
$P = \frac{\partial \mathcal{L}}{\partial(\partial_\eta u)} = \partial_\eta u$
$[\hat{P}(\vec{x},\eta), \hat{P}(\vec{x}',\eta)] = 0$
$[\hat{u}(\vec{x},\eta), \hat{P}(\vec{x}',\eta)] = i\,\delta^{(3)}(x-x')$
$\Downarrow$
$\boxed{[\hat{u}(\vec{x},\eta), \partial_\eta \hat{u}(\vec{x}',\eta)] = i\,\delta^{(3)}(x-x')}$


û(x, η) = ∫ d³k/(2π)³
ds² = a²(η)[-dη² + dx²]
x → x + a

$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \quad a_k \quad +$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}.$$

$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \, \hat{a}_k \, u_k(\eta) \, e^{i\vec{k}\cdot}$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$$

$$\vec{x} \Rightarrow \vec{x} + \vec{a}.$$

$$\hat{a}_k\, u_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^{+}\, e^{-i\vec{k}\cdot\vec{x}}\, u_k^{*}(\eta)$$

$u_k^*(\eta)$

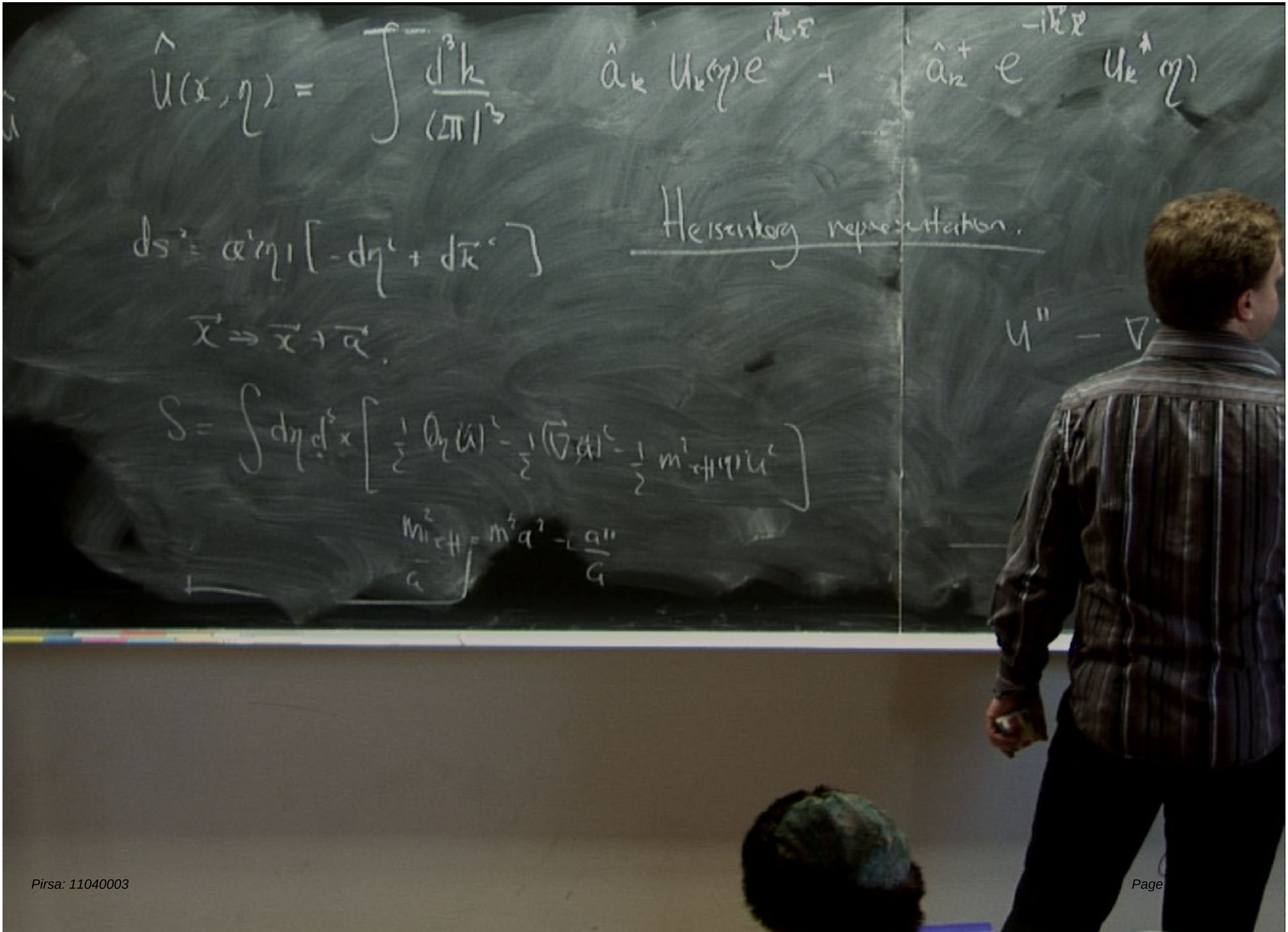
Spatial derivatives

$- \frac{1}{2} m^2 \phi^2 \Big]$

Quantum field satisfies the operator version of classical equations of motion.

$\frac{1}{2} m^2 a^4 \phi^2 \Big\}$

$a^2 \partial_\eta \phi$

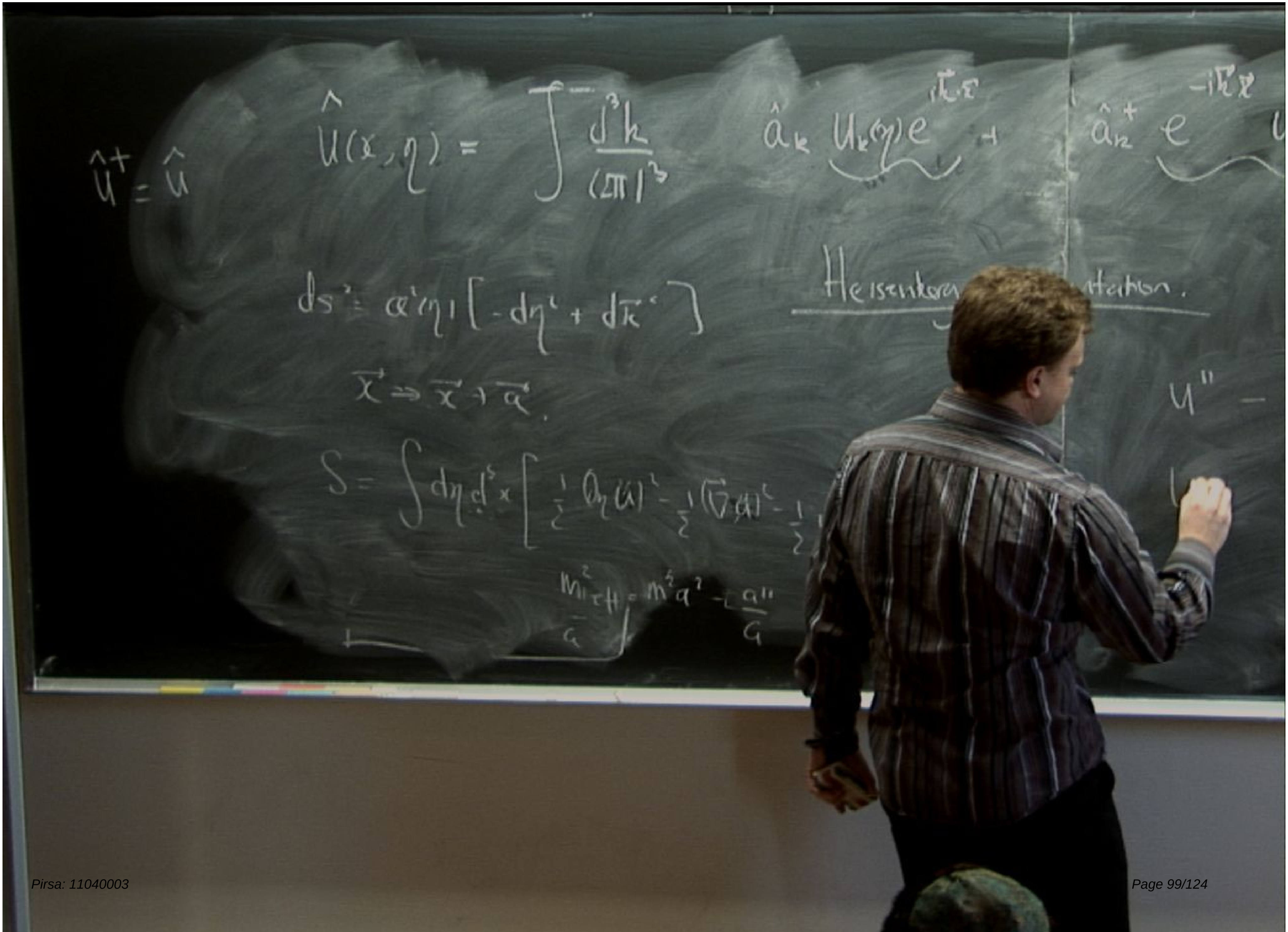

Heisenberg representation.

presentation.

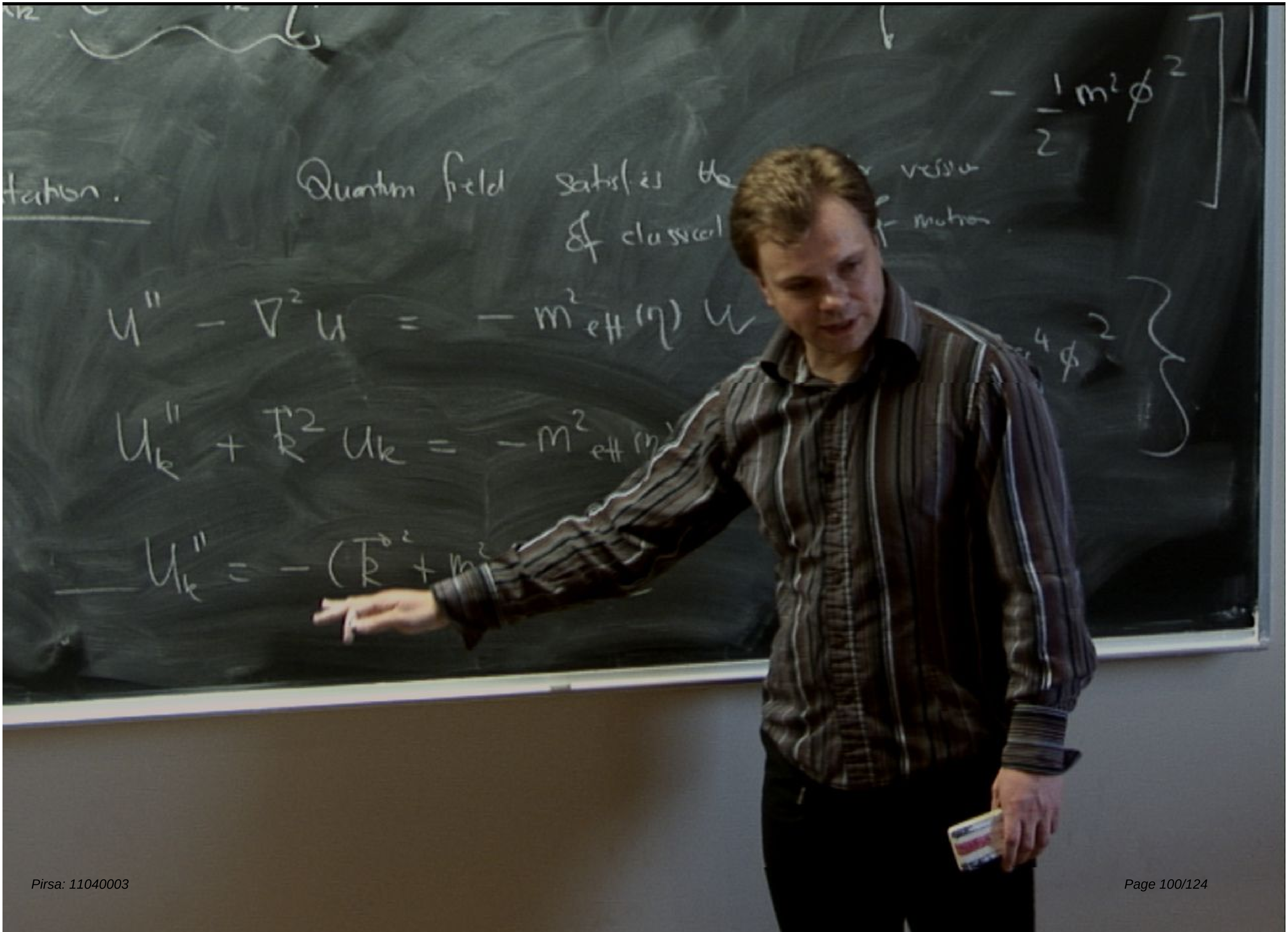
Quantum field satisfies the operator
of classical equations of

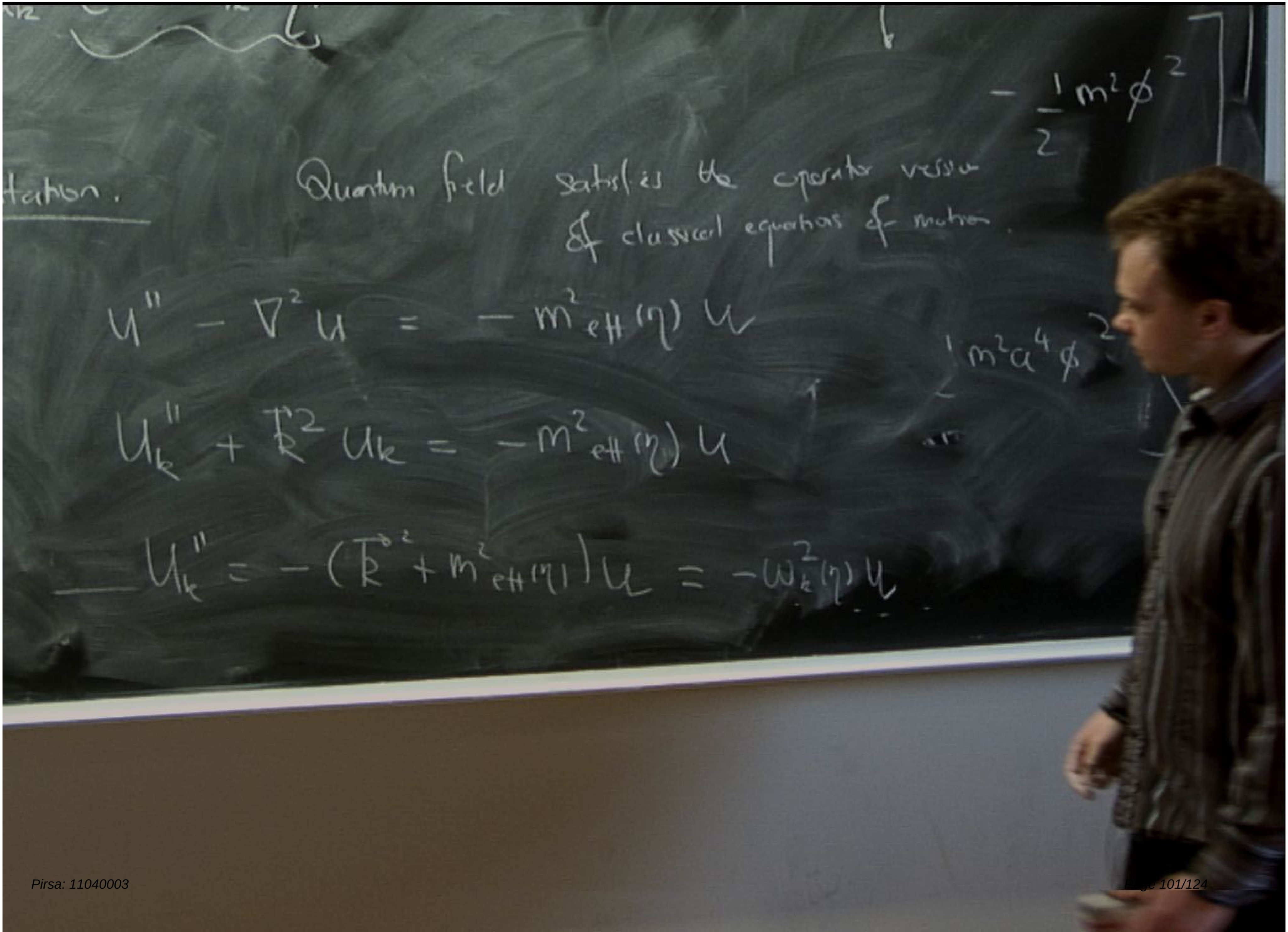
$$u'' - \nabla^2 u = - m^2_{eff}(\eta)\, u$$

$a^2 \partial_\eta \phi$



$\hat{u}^\dagger = \hat{u}$
$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_k u_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger e^{-i\vec{k}\cdot\vec{x}}$
$ds^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$
$\vec{x} \Rightarrow \vec{x} + \vec{a}$
$m^2_{eff} = m^2 a^2 - \frac{a''}{a}$


Quantum field satisfies the
& classical
motion
$-\frac{1}{2} m^2 \phi^2$
$u'' - \nabla^2 u = - m^2_{eff}(\eta)\, u$
$u_k'' + \vec{k}^2 u_k = - m^2_{eff}(\eta)$
$u_k'' = -(\vec{k}^2 + m^2$



$-\frac{1}{2} m^2 \phi^2$
...tation.
Quantum field satisfies the operator version of classical equations of motion.
$u'' - \nabla^2 u = -m^2_{eff}(\eta)\, u$
$\frac{1}{2} m^2 a^4 \phi^2$
$u_k'' + \vec{k}^2 u_k = -m^2_{eff}(\eta)\, u$
$u_k'' = -(\vec{k}^2 + m^2_{eff}(\eta))\, u = -\omega_k^2(\eta)\, u$

$$-\frac{1}{2}m^2\phi^2$$

tation.

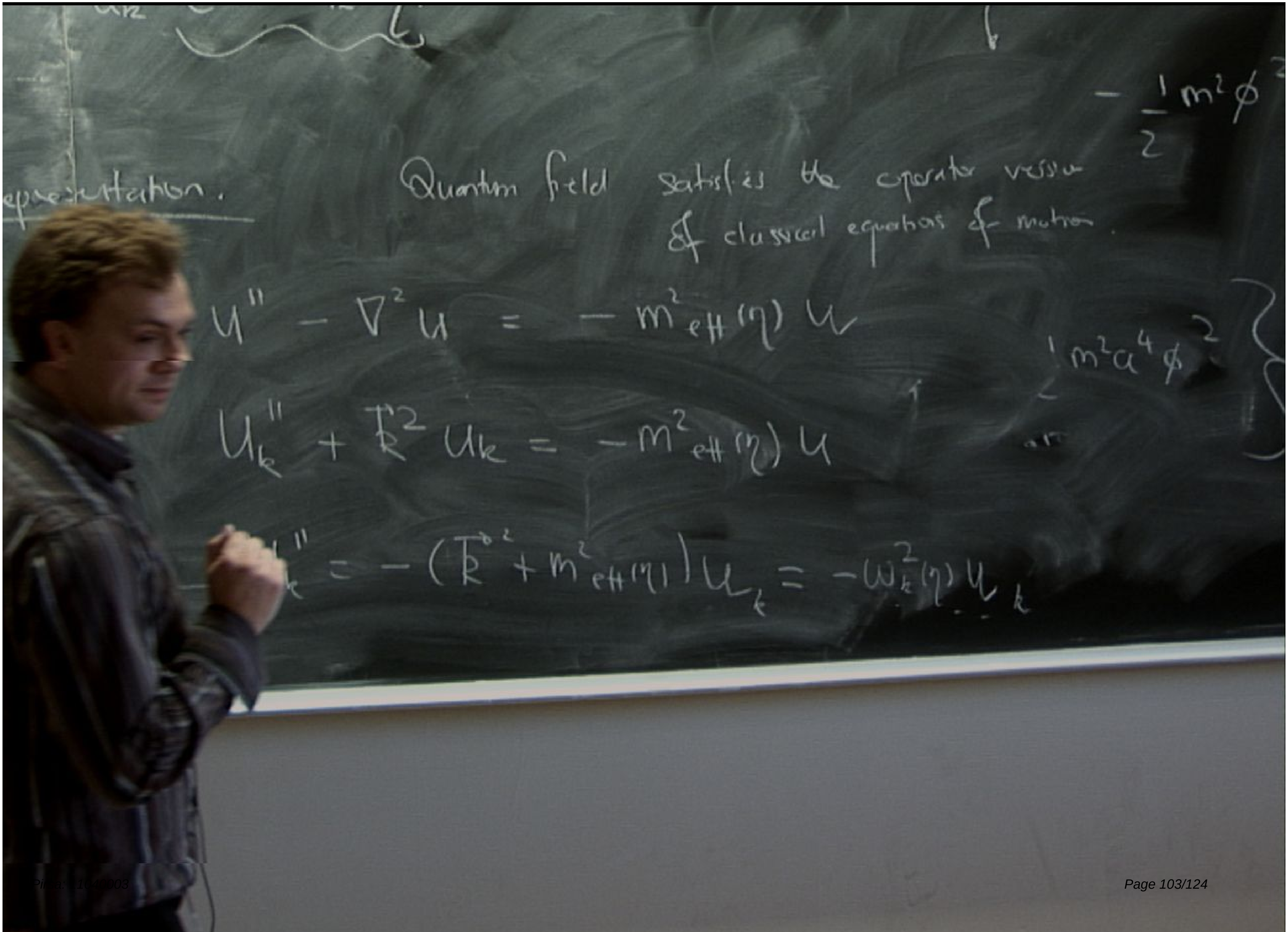
Quantum field satisfies the operator version of classical equations of motion.

$$u'' - \nabla^2 u = -m^2_{eff}(\eta)\, u$$

$$\frac{1}{2} m^2 a^4 \phi^2$$

$$u_k'' + \vec{k}^2 u_k = -m^2_{eff}(\eta)\, u$$

$$u_k'' = -(\vec{k}^2 + m^2_{eff}(\eta))\, u = -\omega_k^2(\eta)\, u$$


epresentation.
Quantum field satisfies the operator version
of classical equations of motion.
$-\frac{1}{2} m^2 \phi^2$
$u'' - \nabla^2 u = -m^2_{eff}(\eta)\, u$
$\frac{1}{2} m^2 a^4 \phi^2$
$u_k'' + \vec{k}^2 u_k = -m^2_{eff}(\eta)\, u$
$u_k'' = -(\vec{k}^2 + m^2_{eff}(\eta))\, u_k = -\omega_k^2(\eta)\, u_k$

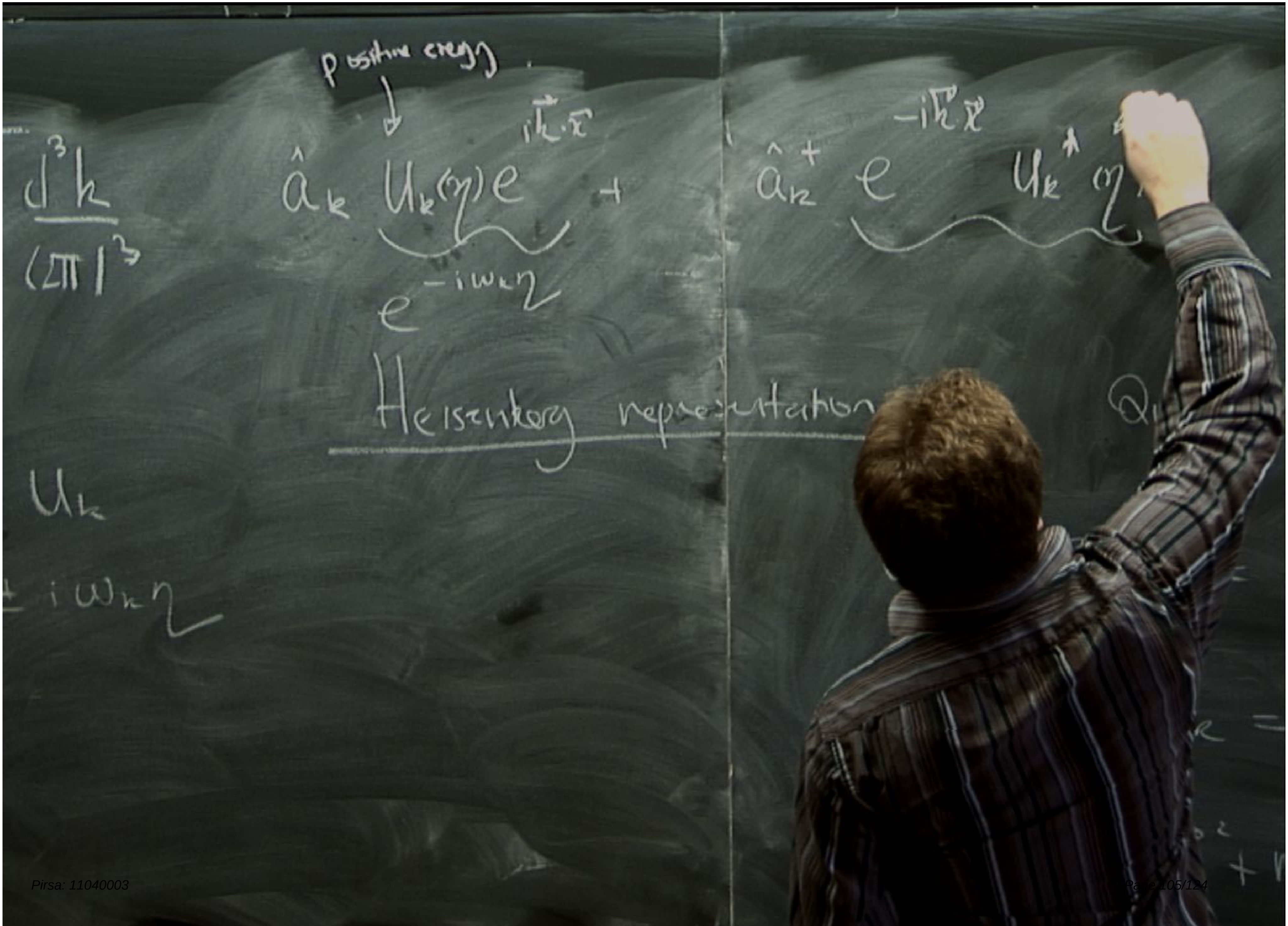
$$\hat{u}^\dagger = \hat{u}$$

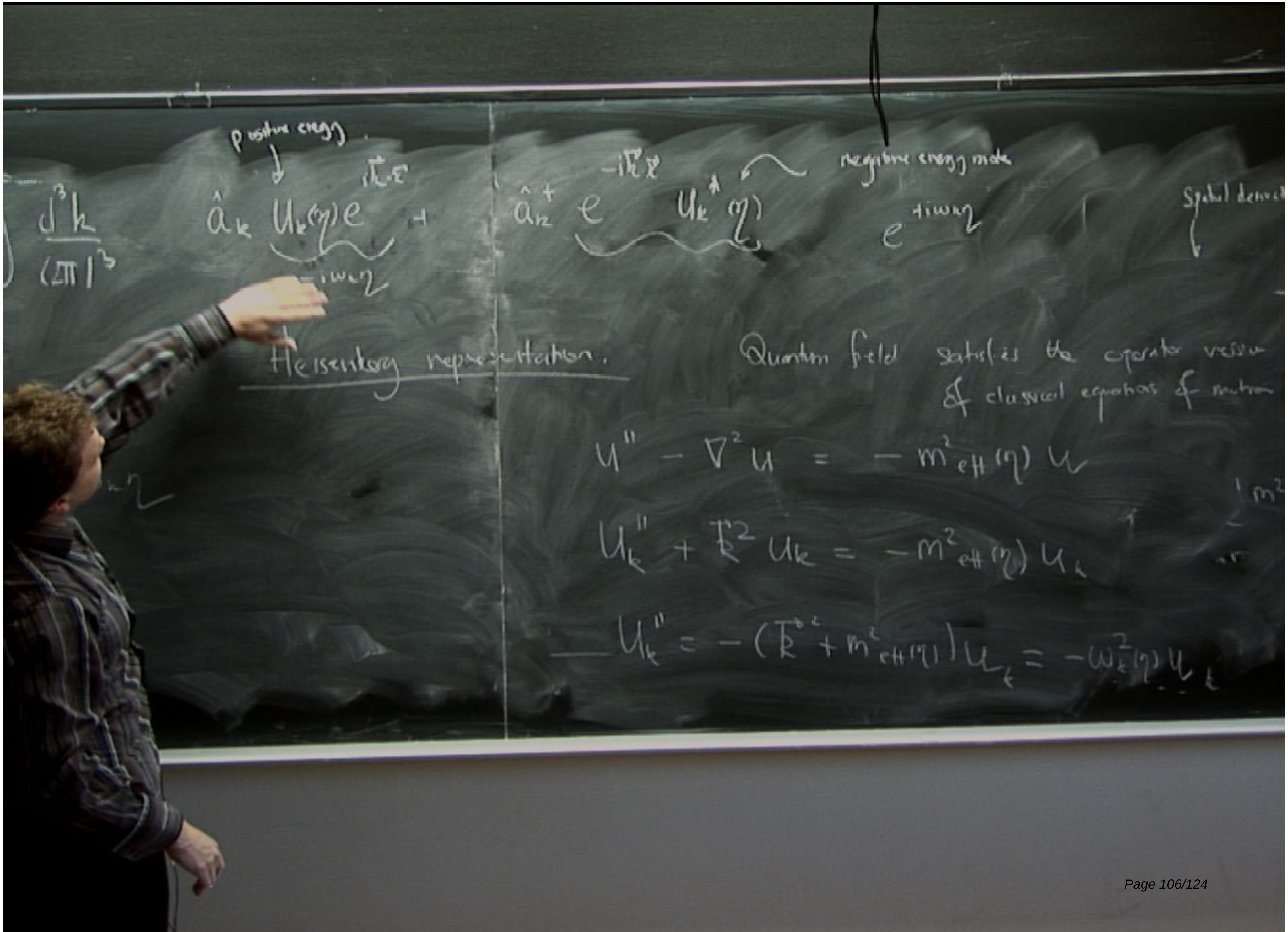
$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3}\, \hat{a}_k\, u_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} + \ldots$$

Heisenberg

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

$$X \quad u_k \sim e^{\pm i \omega_k \eta}$$


positive energy
Heisenberg representation


positive energy
negative energy mode
Spatial derivat
Heisenberg representation.
Quantum field satisfies the operator version
of classical equations of motion

$$\hat{u}^{\dagger} = \hat{u}$$

$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \; \hat{a}_k \underbrace{u_k(\eta)}_{e^{-i\omega_k \eta}} e^{i\vec{k}\cdot\vec{x}} +$$

Heisenberg n

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

$$\times \quad u_k \sim e^{\pm i \omega_k \eta}$$

u_k up into positive and negative energy ?

$$\hat{u}^{\dagger} = \hat{u}$$

$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \; \hat{a}_k \underbrace{u_k(\eta)}_{e^{-i\omega_k\eta}} e^{i\vec{k}\cdot\vec{x}} +$$

Heisenberg

$$u_k'' = -\omega_k^2(\eta)\, u_k$$

$$\times \; u_k \sim e^{\pm i \omega_k \eta}$$

u_k up into positive and negative energy ?

$$u_k = \alpha_k v_k^1 + \beta_k v_k^2$$

$$\hat{u}^{+} = \hat{u}$$

$$\hat{u}(x,\eta) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_k \underbrace{u_k(\eta)}_{e^{-i\omega_k \eta}} e^{i\vec{k}\cdot\vec{x}} + \cdots$$

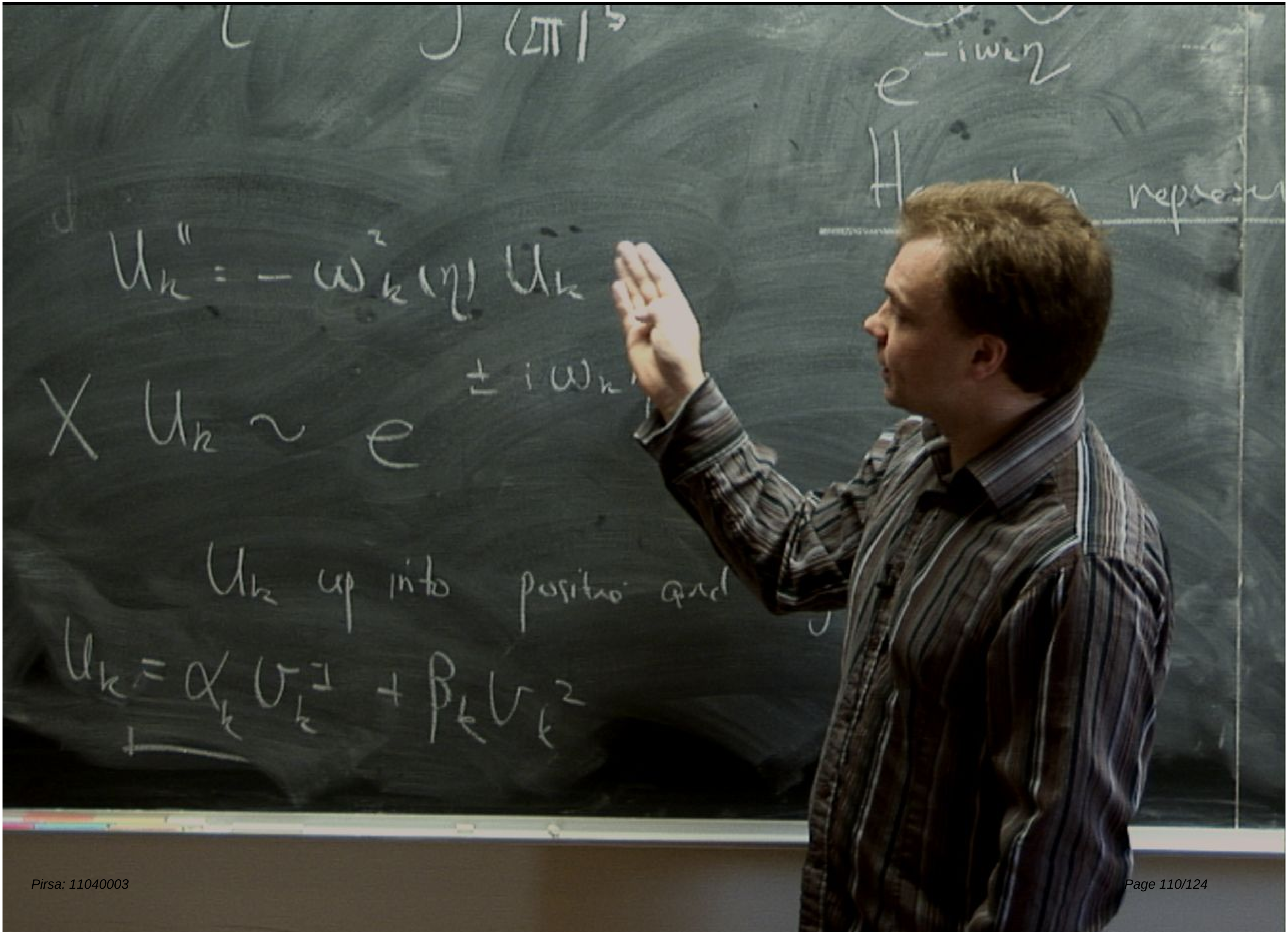
Heisenberg n

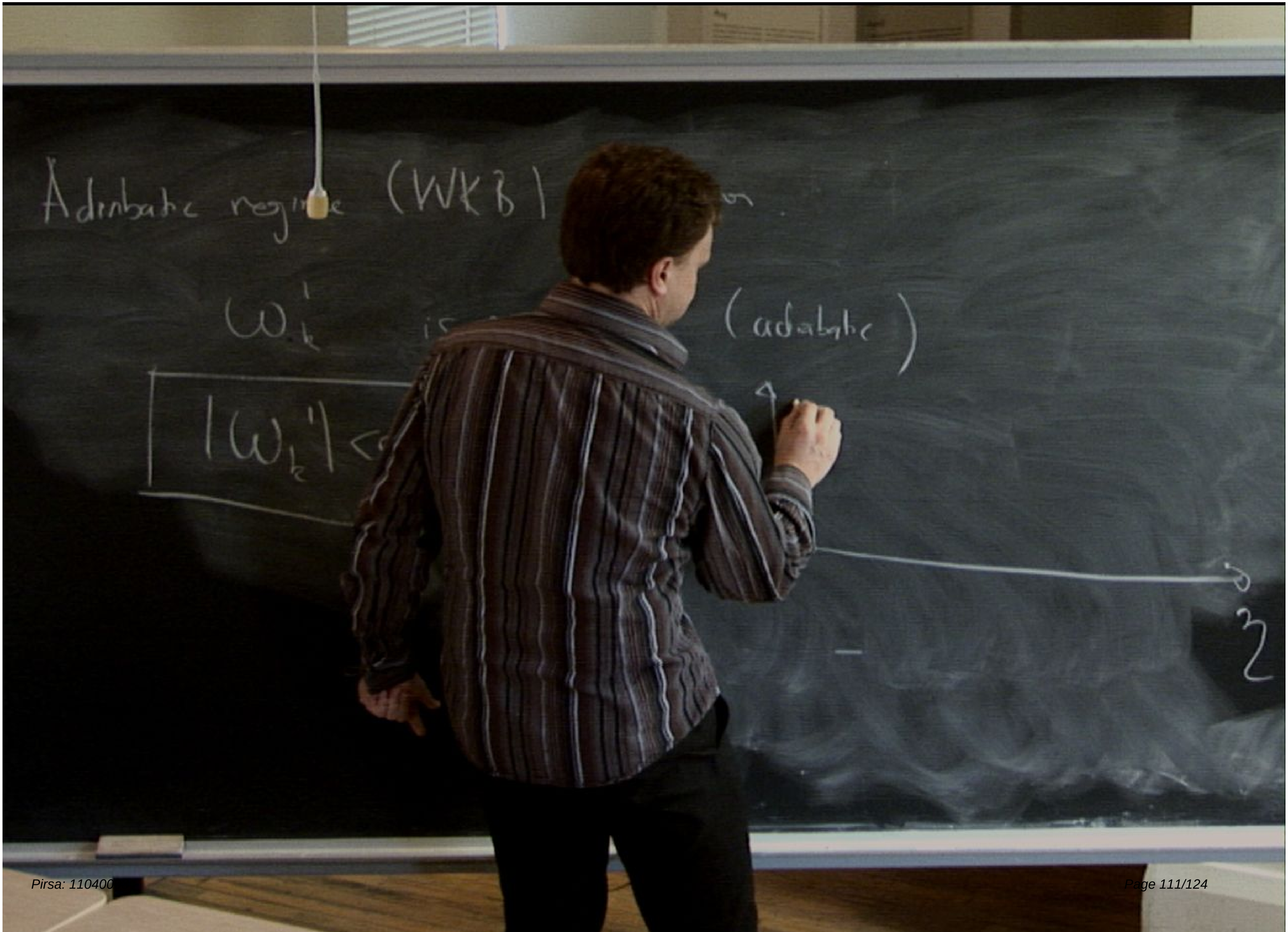
$$u_k'' = -\omega_k^2(\eta)\, u_k$$

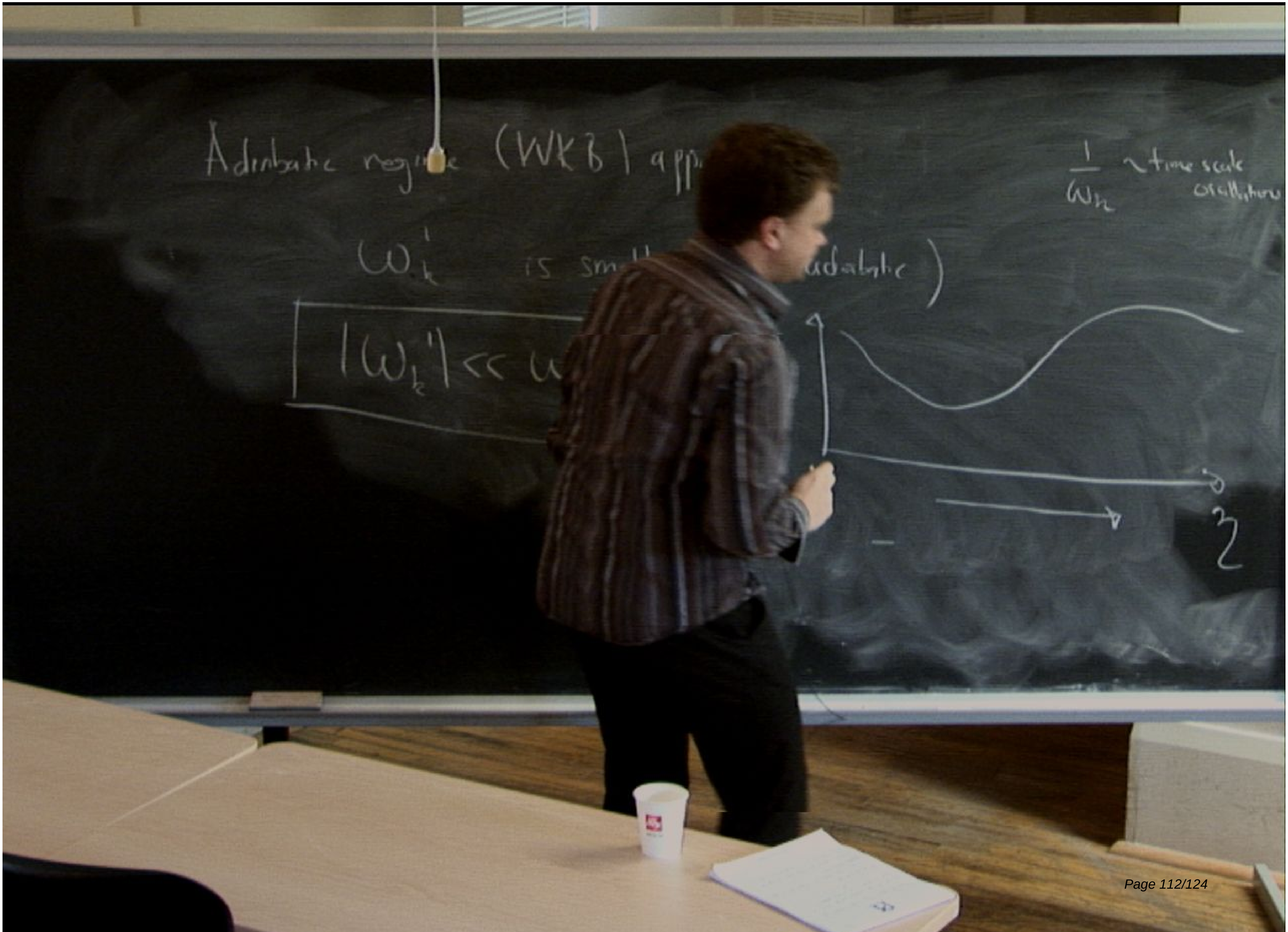
$$\times\ u_k \sim e^{\pm i\omega_k \eta}$$

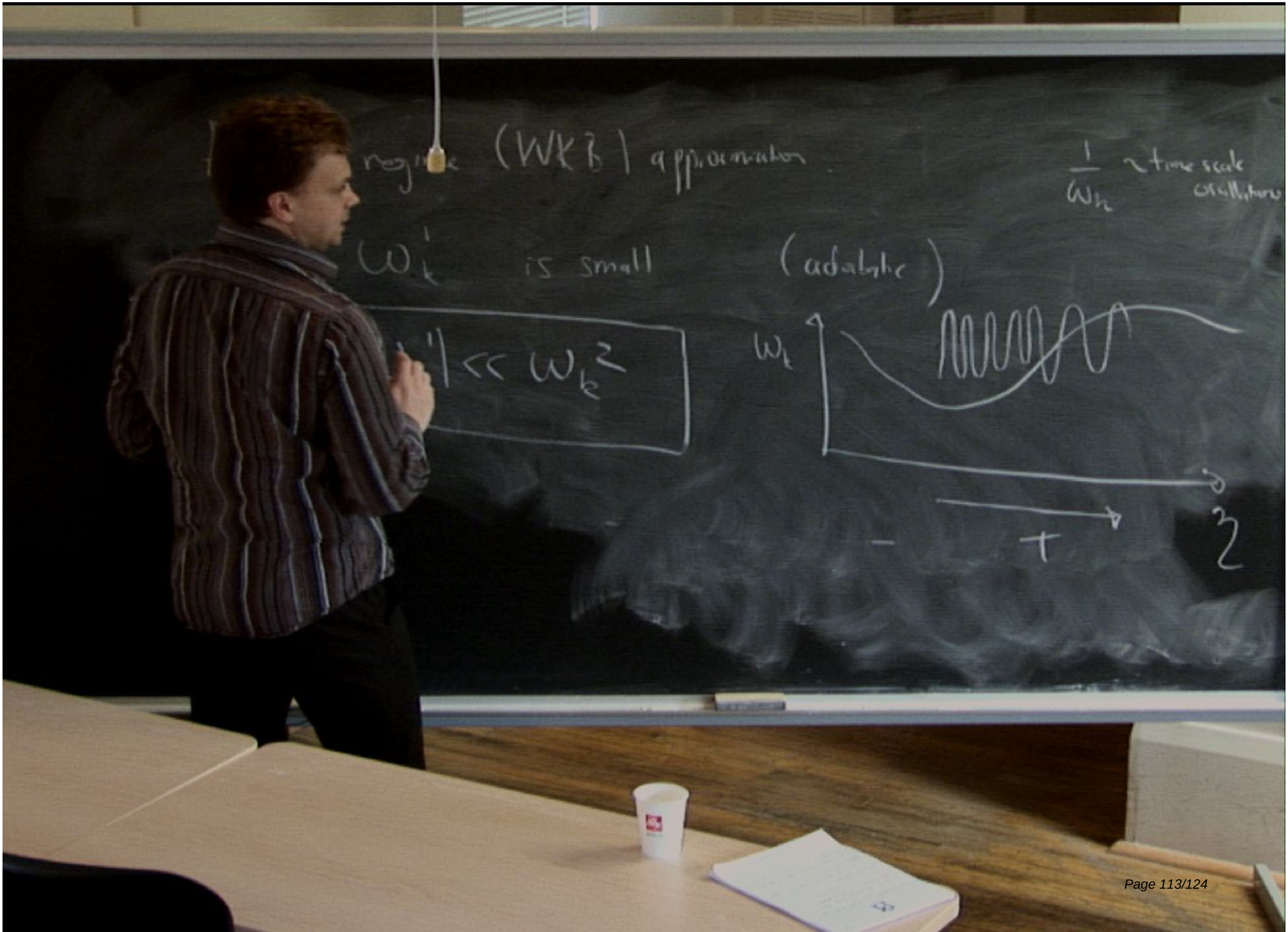
u_k up into positive and negative energy ?

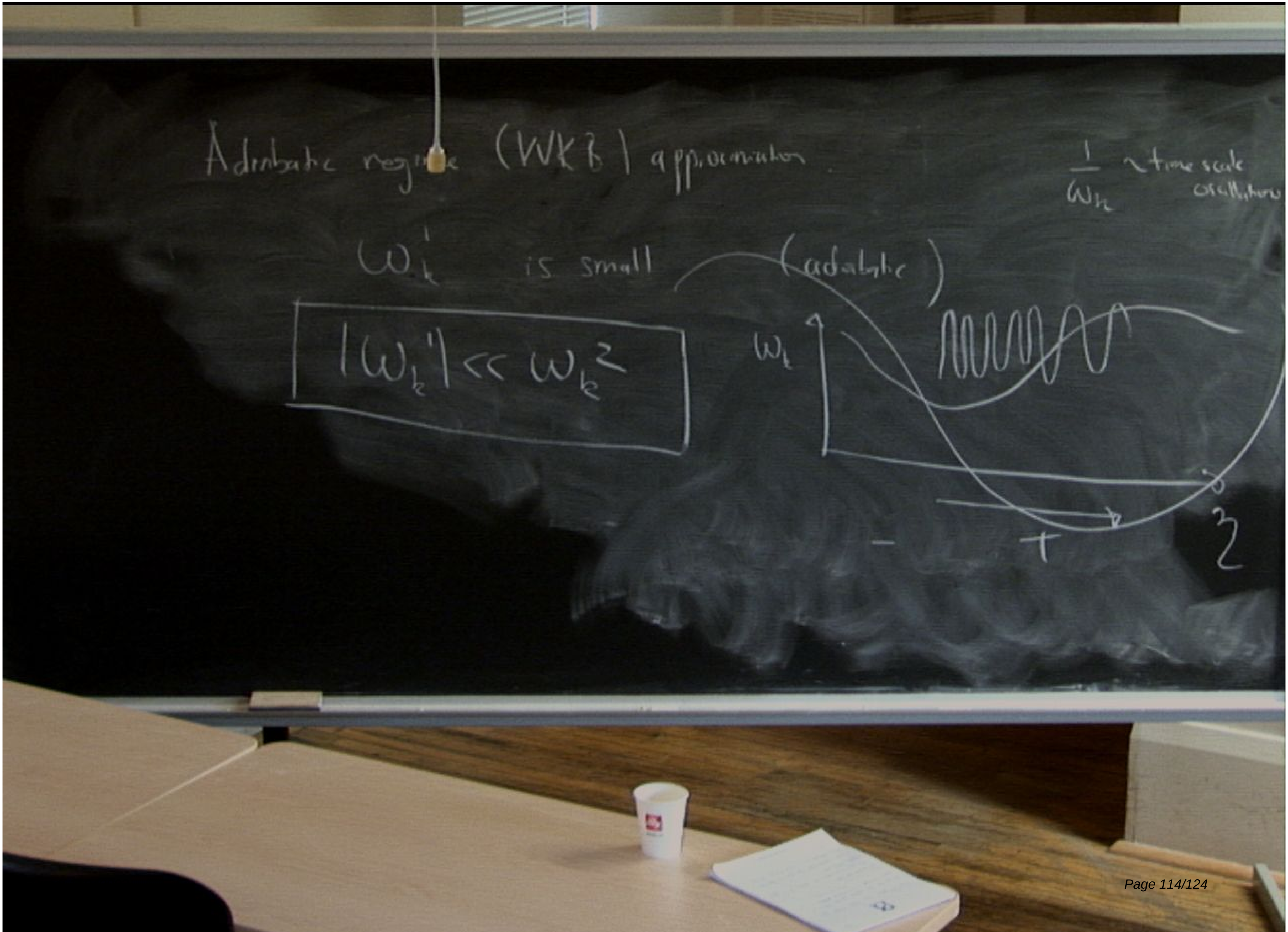
$$u_k = \alpha_k v_k^1 + \beta_k v_k^2$$


e^{-iωkη}
Uk'' = -ω²k(η) Uk
Uk ~ e^{±iωk
Uk up into positive and
Uk = αk vk¹ + βk vk²


Adiabatic regime (WKB)
(adiabatic)


Adiabatic regime (WKB) app
$\frac{1}{\omega_k}$ ~ time scale oscillation
ω_k' is sm adiabatic
$|\omega_k'| \ll \omega$


regime (WKB) approximation
1/ωk ~ time scale oscillations
ω'k is small (adiabatic)
ωk²
ωk
t


Adiabatic regime (WKB) approximation
$\frac{1}{\omega_k}$ ~ time scale oscillations
ω_k' is small (adiabatic)
$|\omega_k'| \ll \omega_k^2$
ω_k

negative energy mode

$\hat{a}_k^{+} e^{-i\vec{k}\vec{x}} u_k^{*}(\eta)$

$e^{+i\omega\eta}$

Spatial derivatives

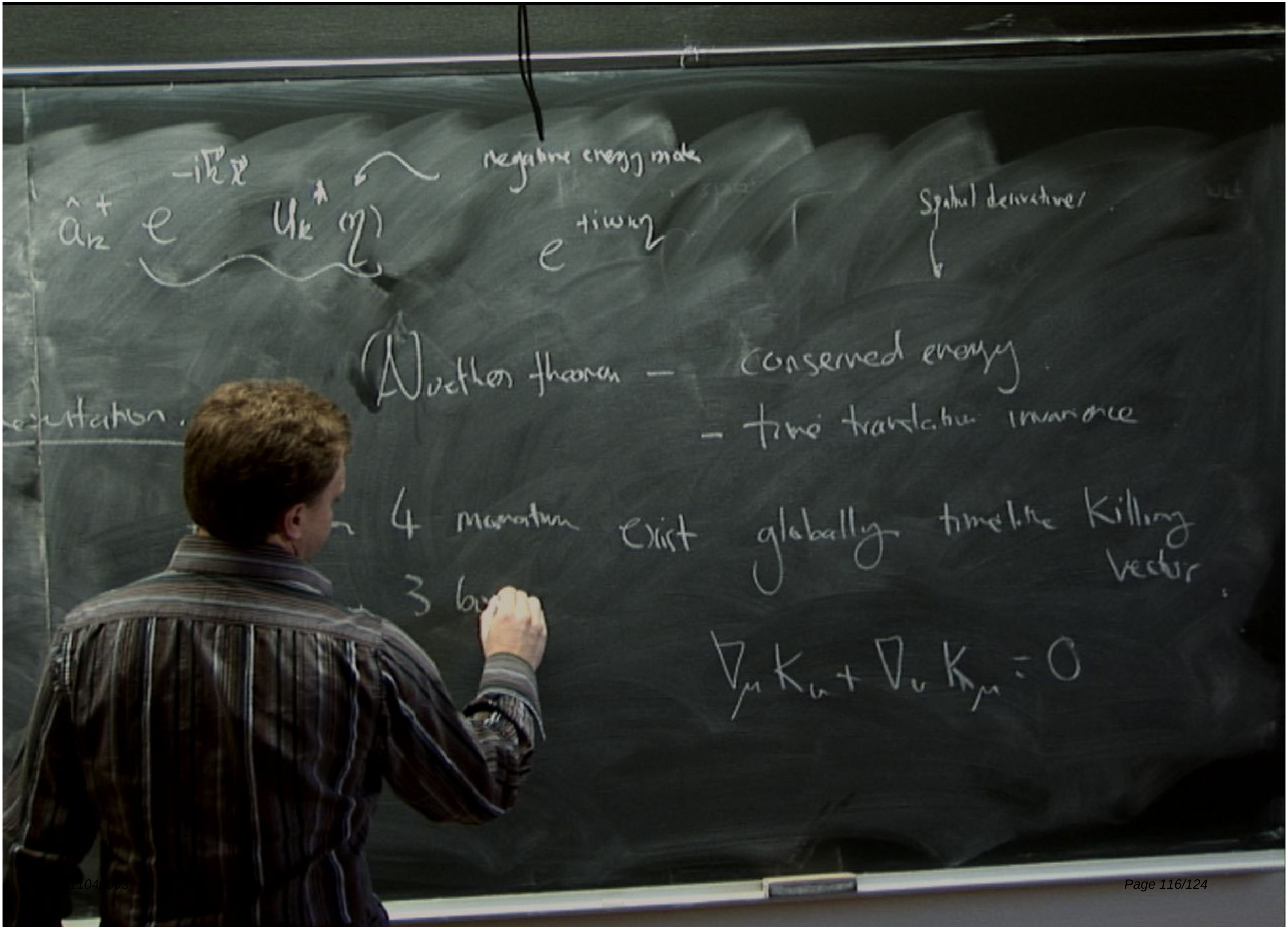
...itation.

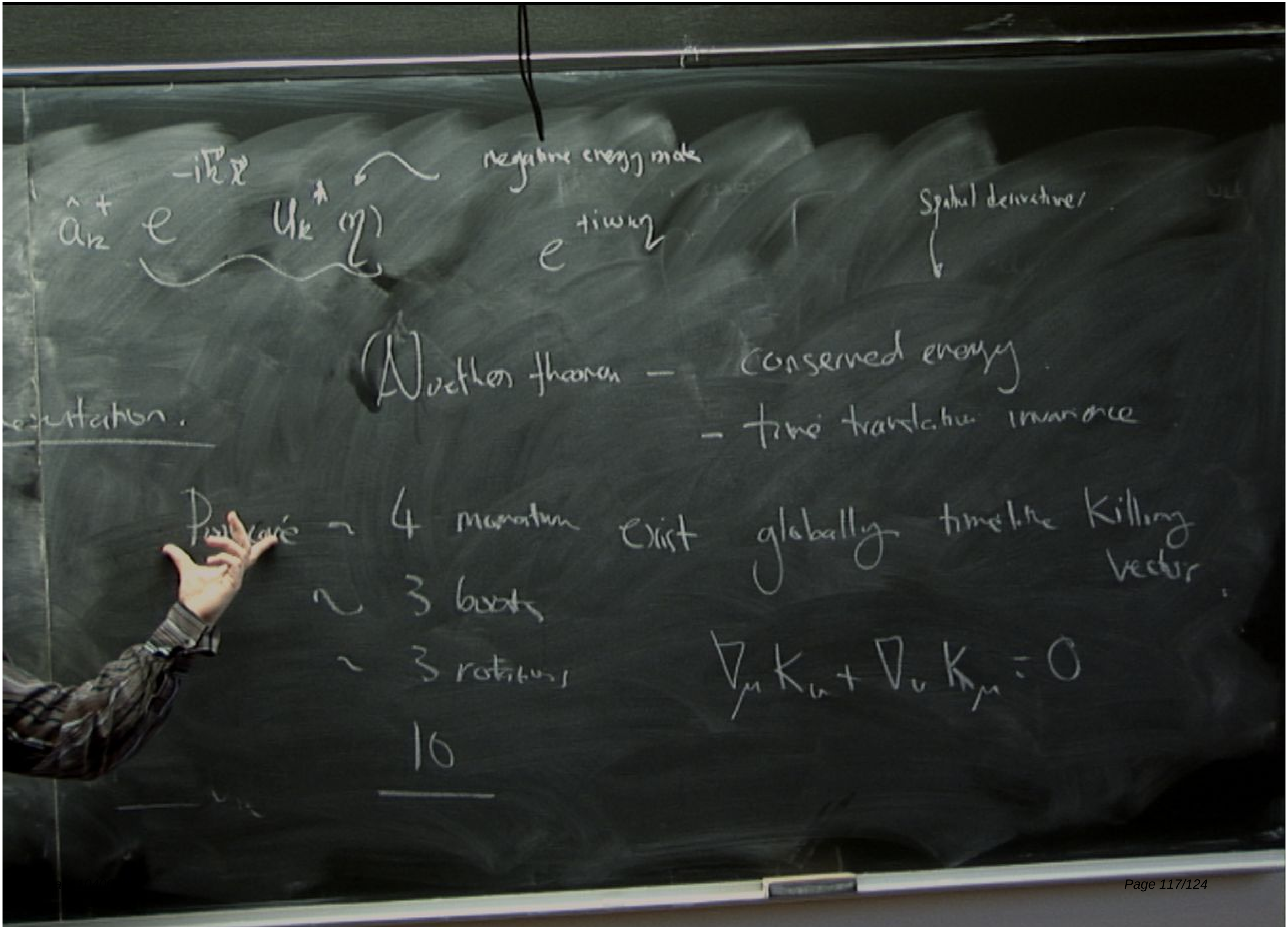
Noether theorem — conserved energy

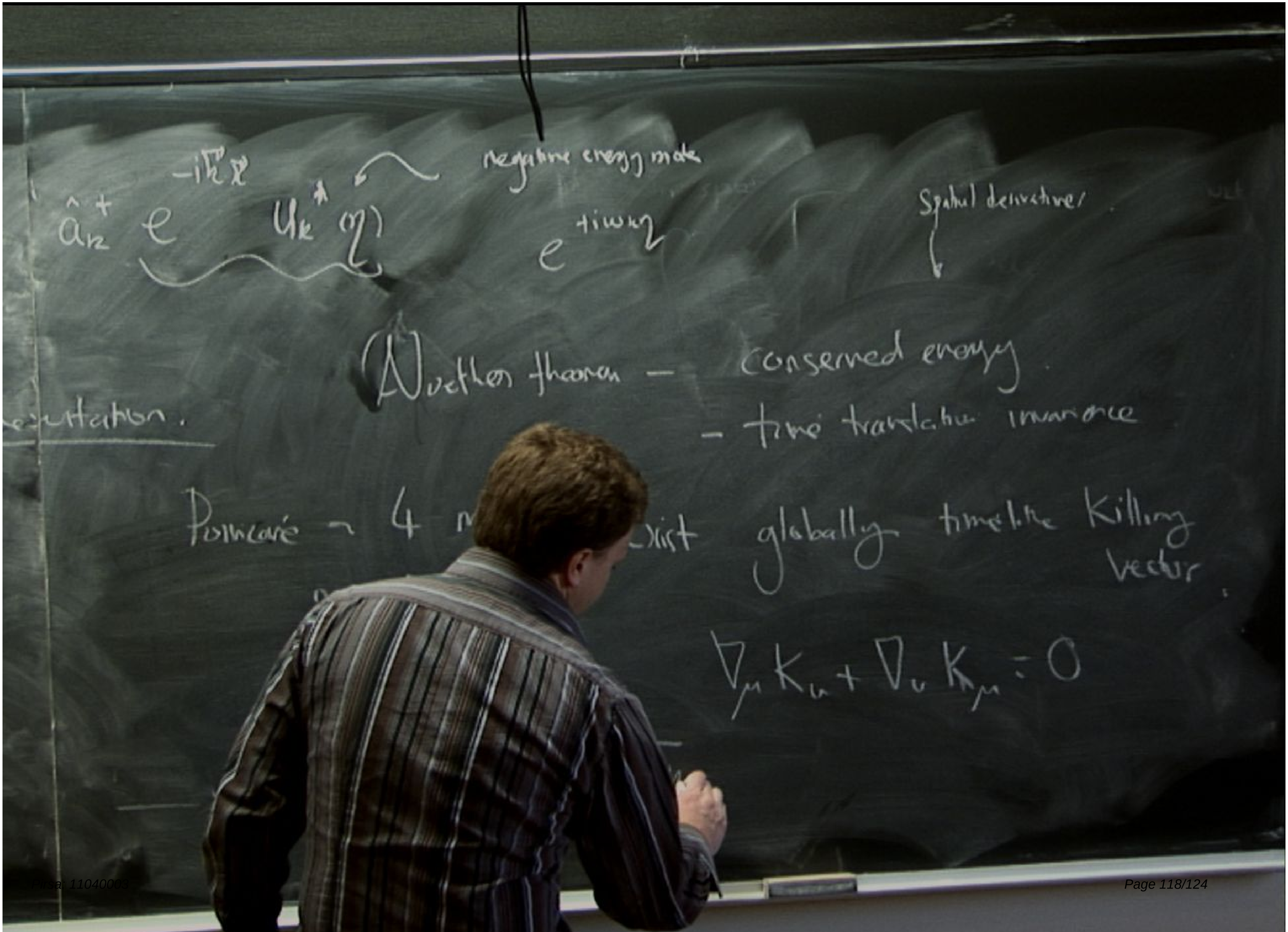
— time translation invariance

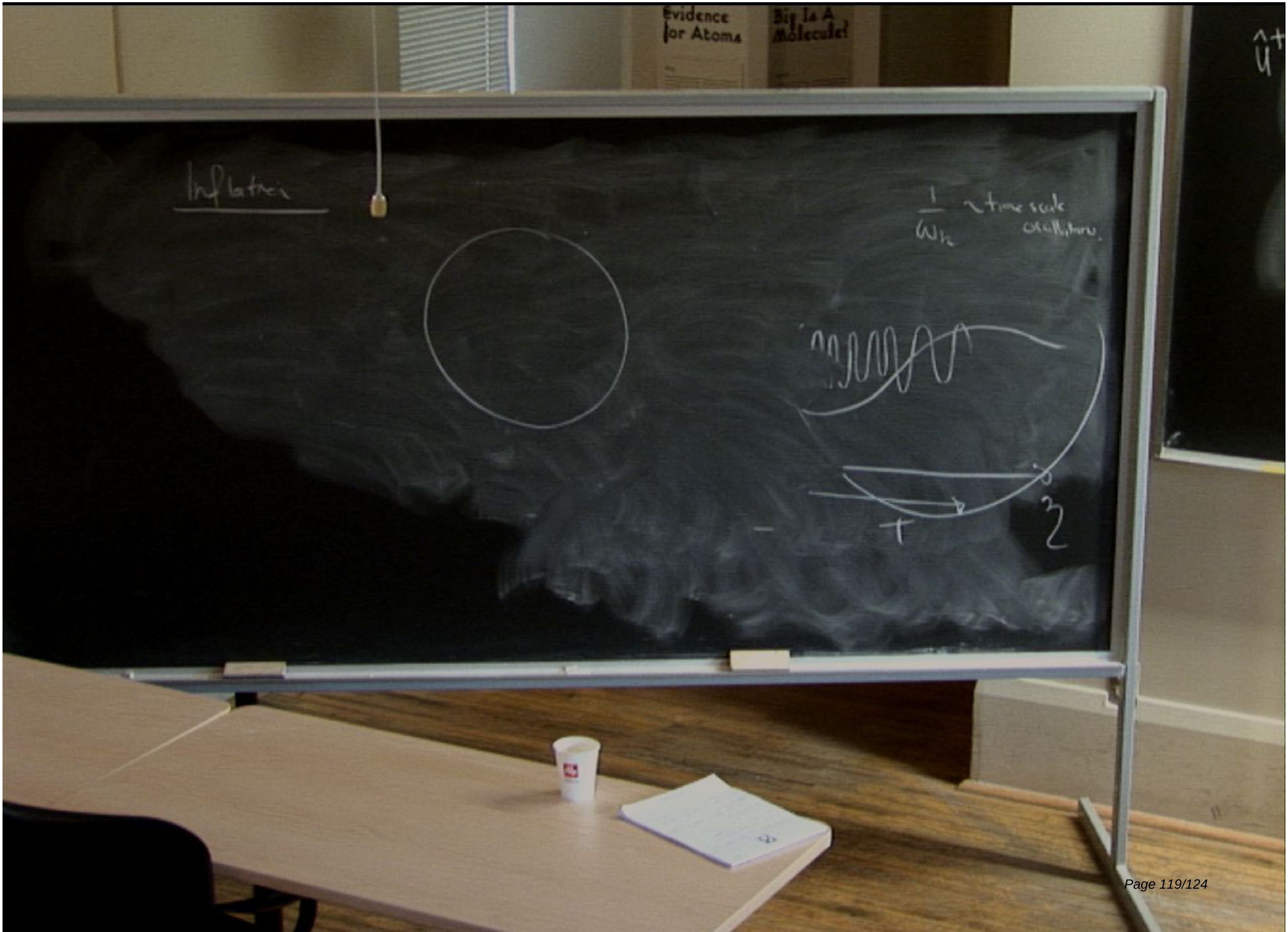
exist globally timelike Killing vector

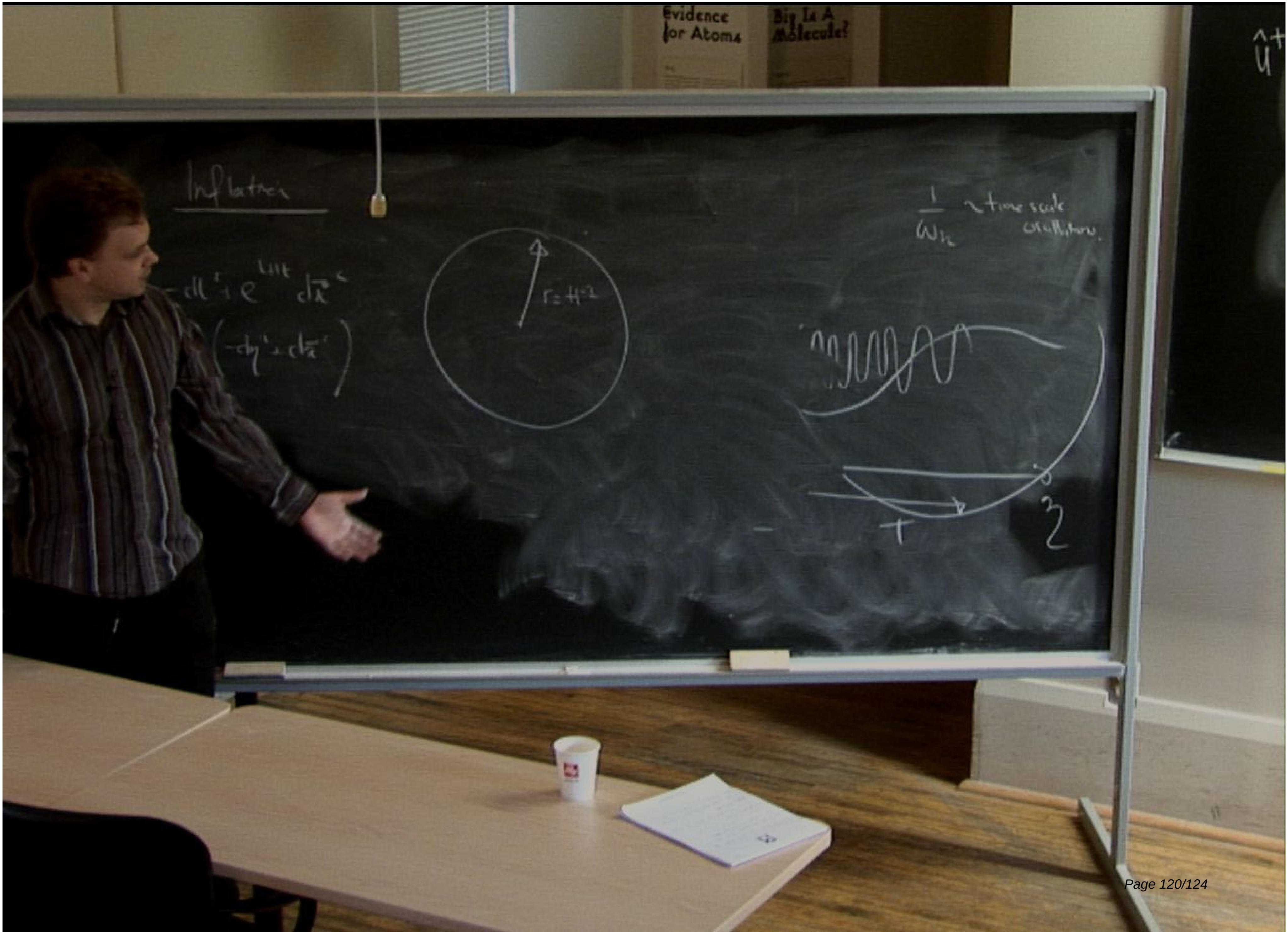
$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$

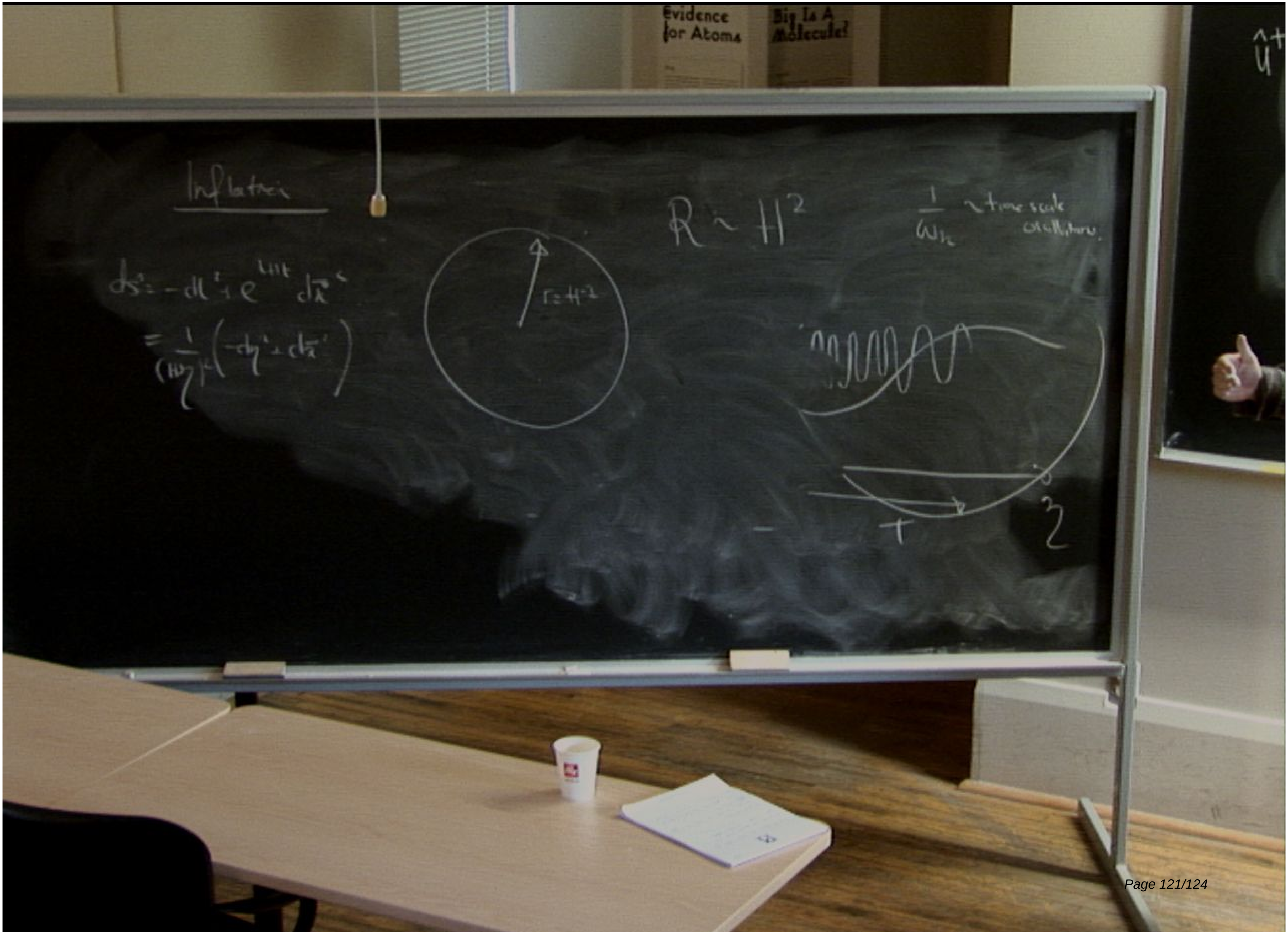

negative energy mode
Spatial derivatives
Noether theorem — conserved energy
— time translation invariance
4 momentum exist globally timelike Killing vector
$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$

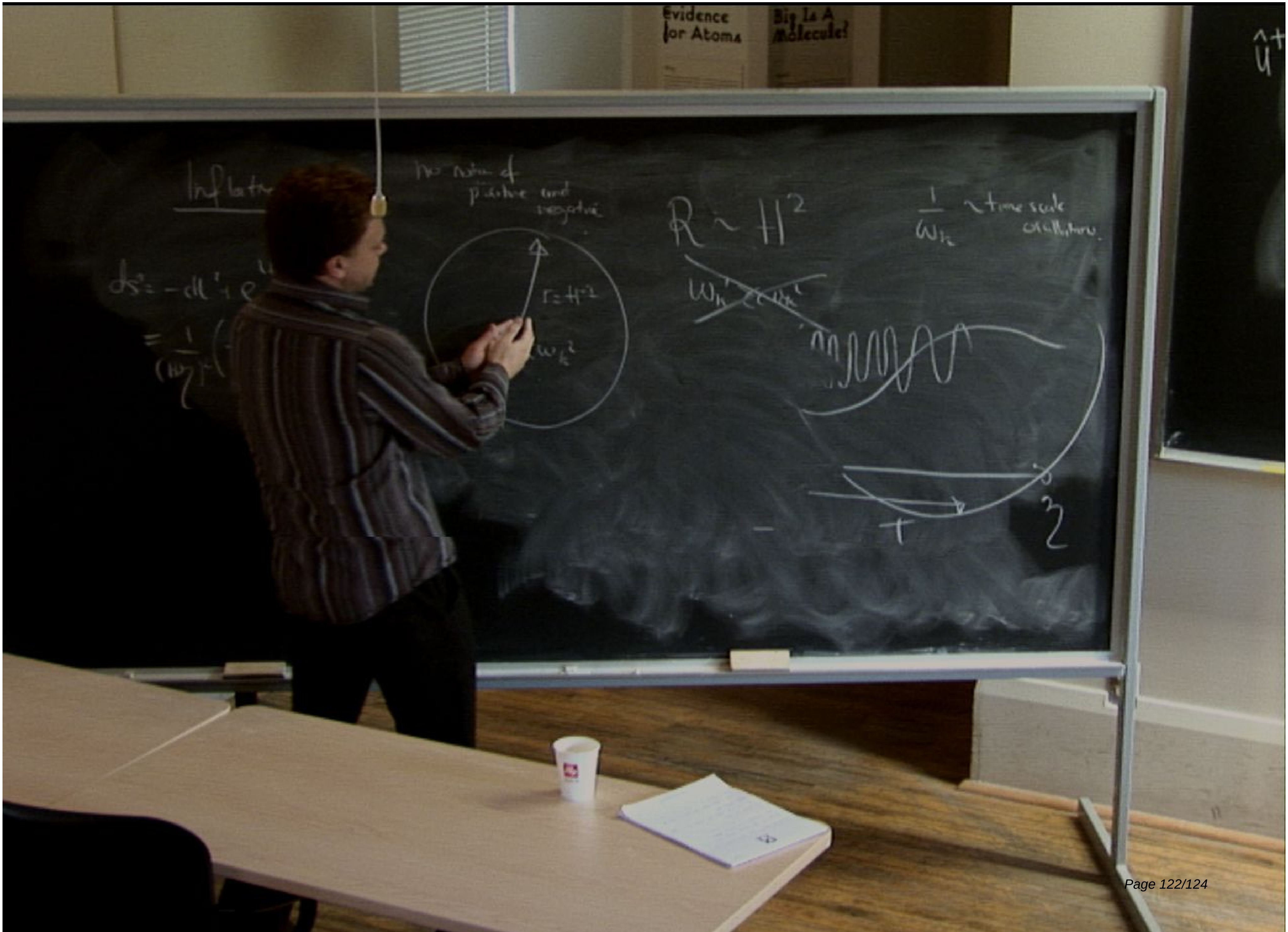

negative energy mode
Spatial derivative
Noether theorem – conserved energy
– time translation invariance
4 momentum
3 boosts
3 rotations
10
exist globally timelike Killing vector
$\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$

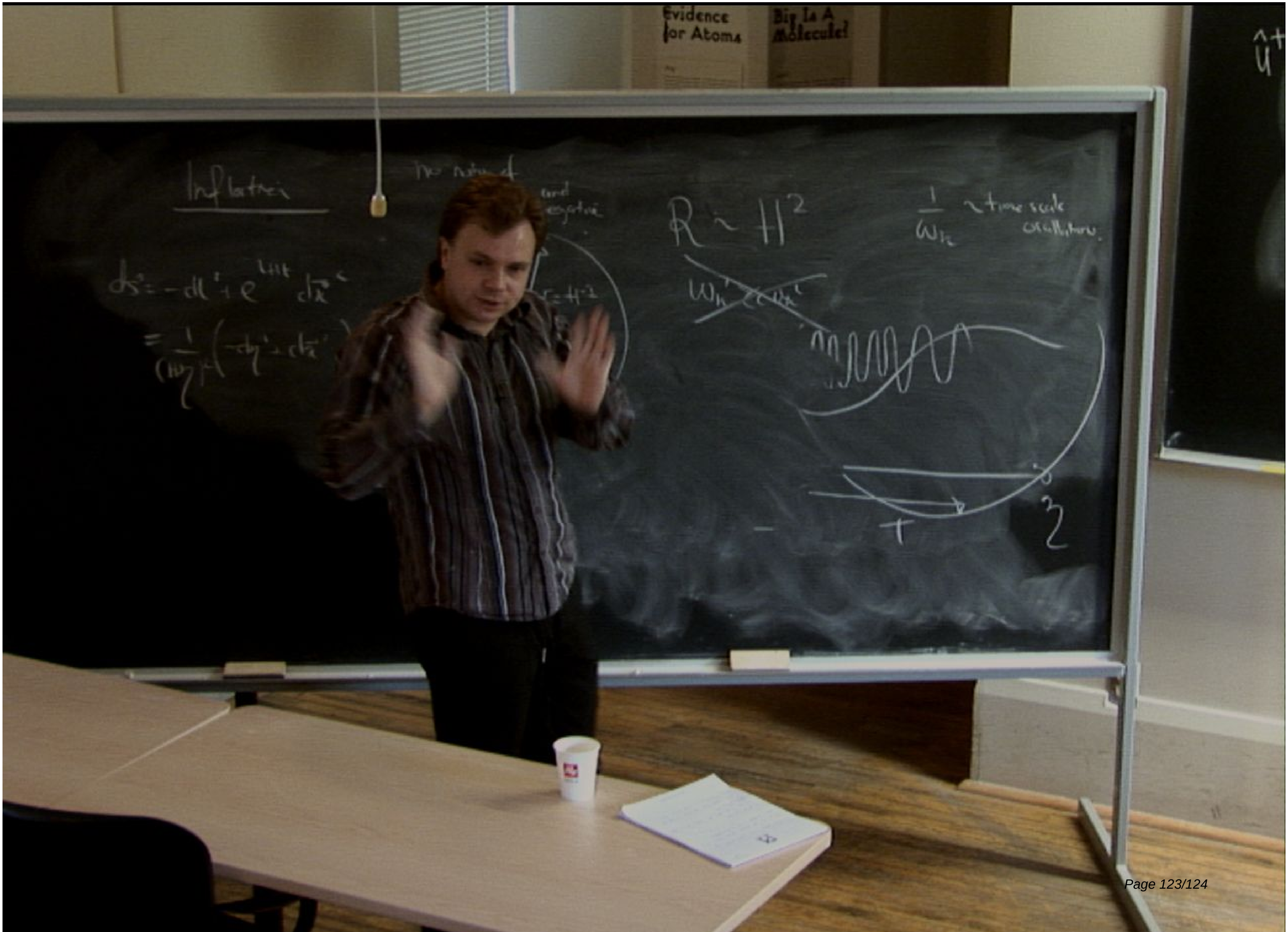

negative energy mode
Spatial derivative/
Noether theorem — conserved energy
— time translation invariance
Poincaré
globally timelike Killing vector
∇μ Kν + ∇ν Kμ = 0

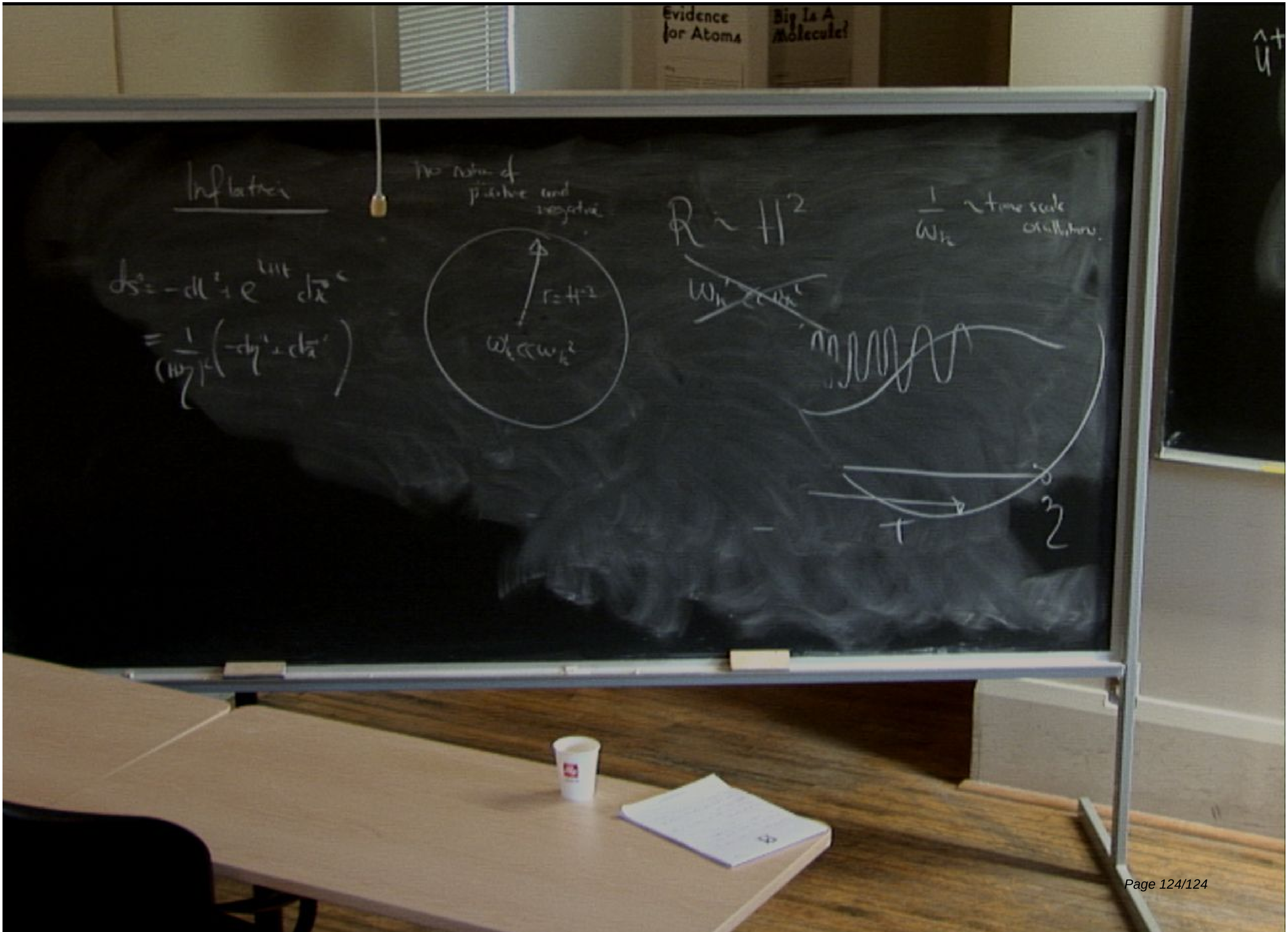

Evidence
for Atoms
Big Is A
Molecule!
Inflation
1/ω_k ~ time scale
oscillation.


Evidence for Atoms
Big Is A Molecule?
Inflation
~ time scale oscillation


Evidence for Atoms
Big Is A Molecule?
Inflation
R ~ H^2
1/ω_k ~ time scale oscillation.


Evidence for Atoms
Big Is A Molecule?
$R \sim H^2$


Evidence
for Atoms
Big Is A
Molecule!
Inflation
R ~ H²


Inflation
Evidence for Atoms
Big Is A Molecule!
$R \sim H^2$
$\frac{1}{\omega_k} \sim$ time scale oscillation.