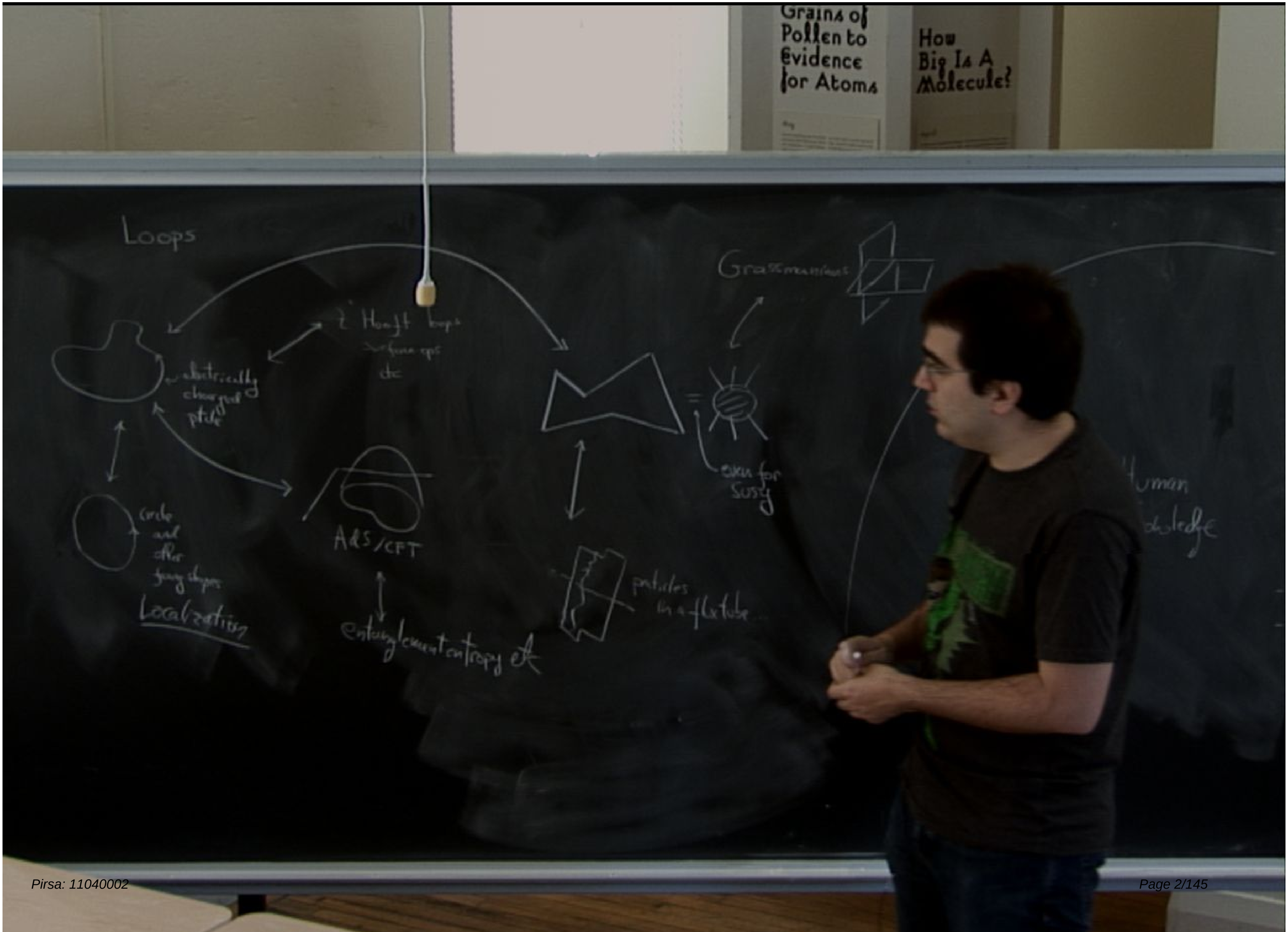
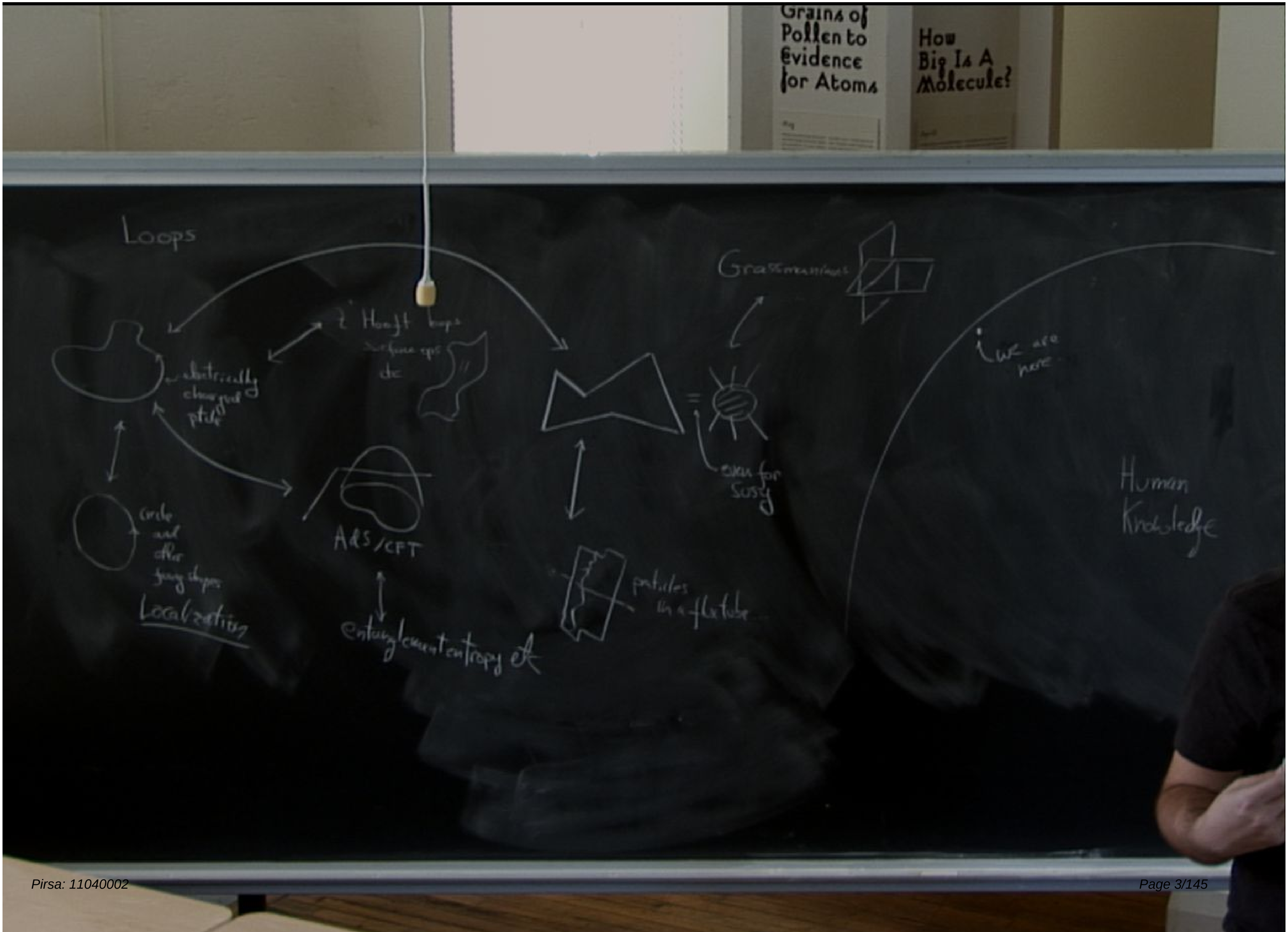
Title: Explorations in String Theory - Lecture 15

Date: Apr 01, 2011 11:30 AM

URL: http://pirsa.org/11040002

Abstract:


Grains of
Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Loops
Localization
AdS/CFT
particles
in a flux tube...


Grains of Pollen to Evidence for Atoms
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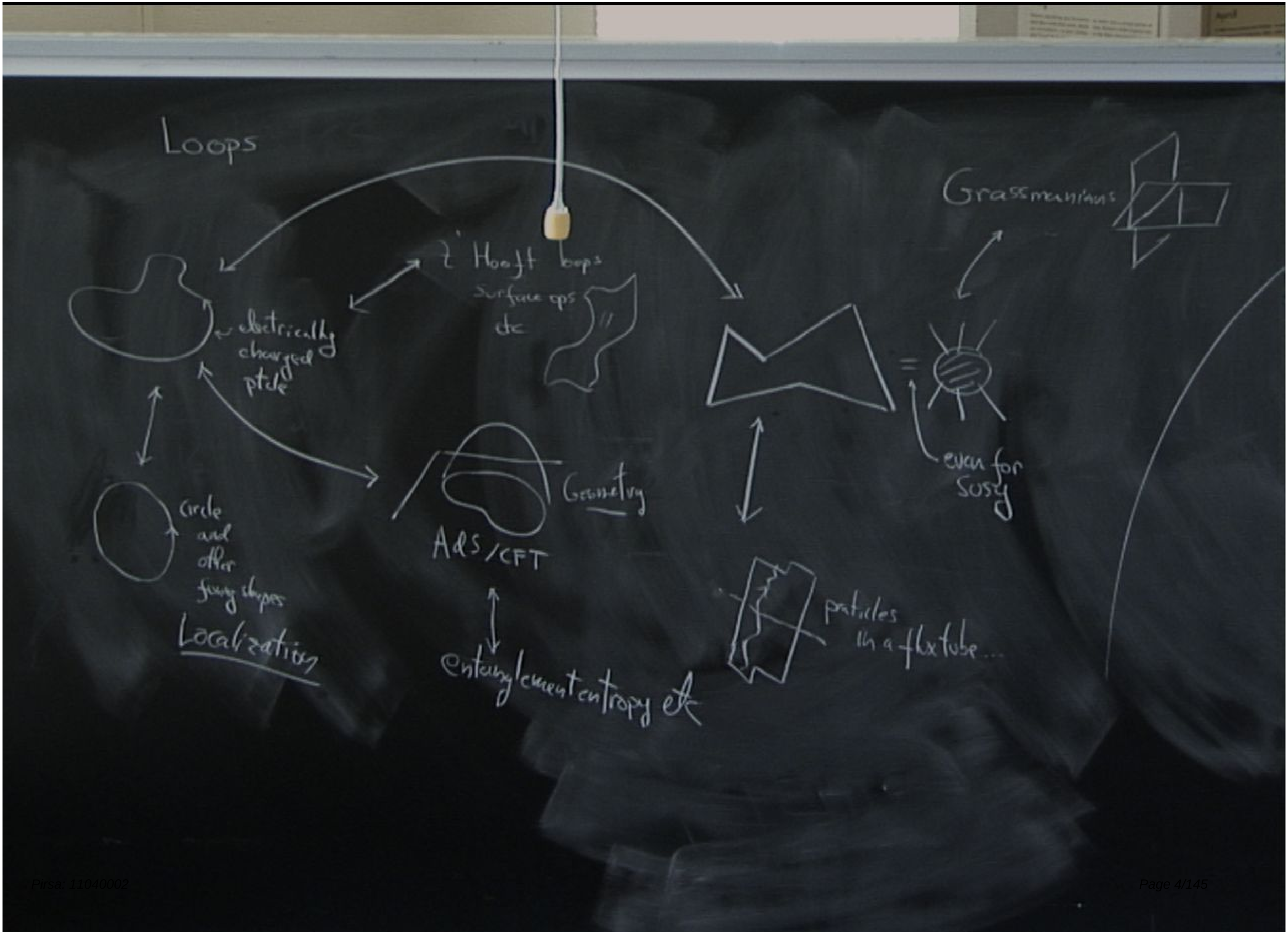
Loops
Grassmannians
AdS/CFT
Localization
we are here
Human Knowledge


Loops
't Hooft loops
surface ops
etc
electrically
charged
ptcle
Grassmanians
even for
SUSY
Geometry
AdS/CFT
circle
and
other
funny shapes
Localization
entanglement entropy etc
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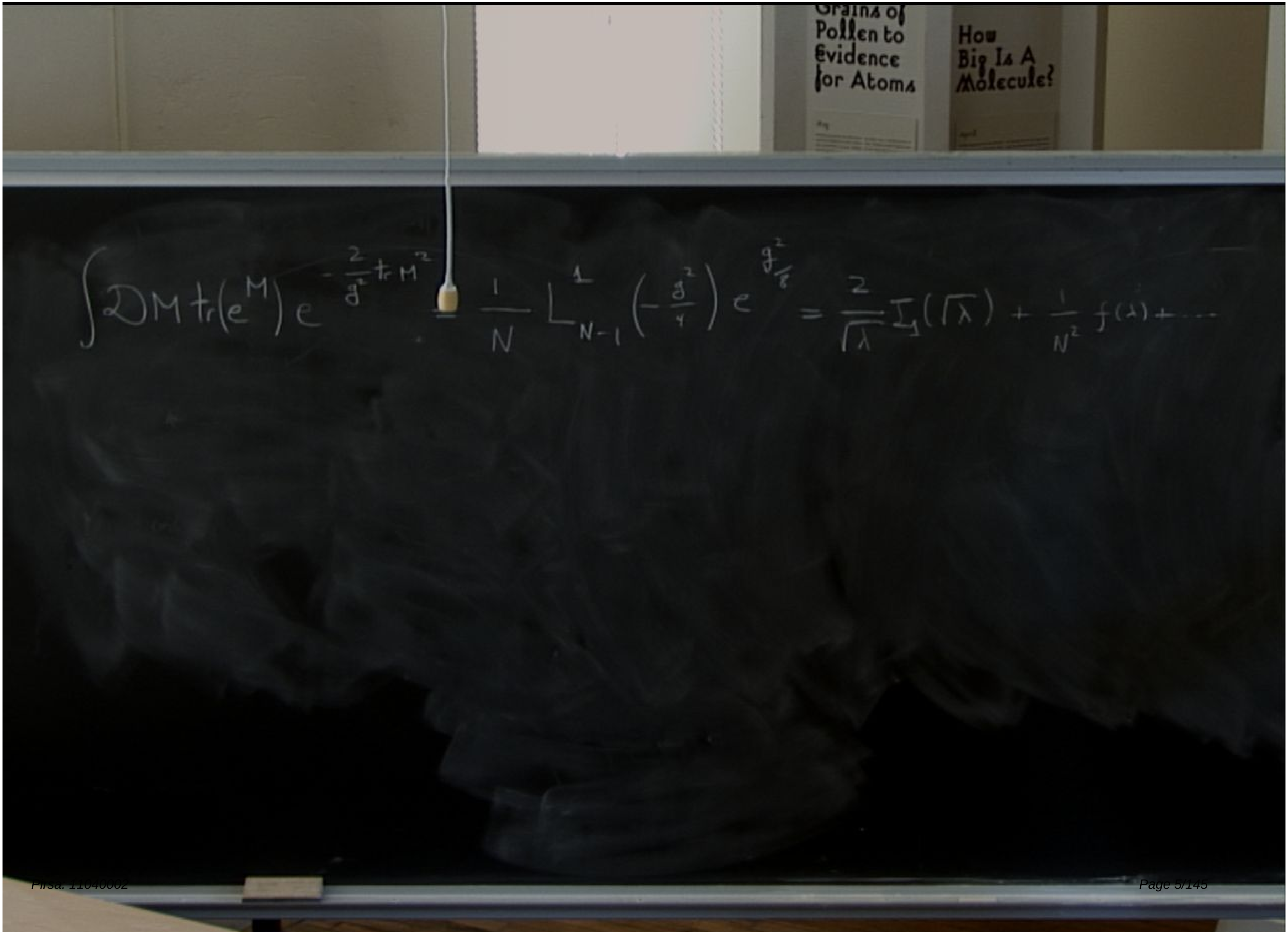
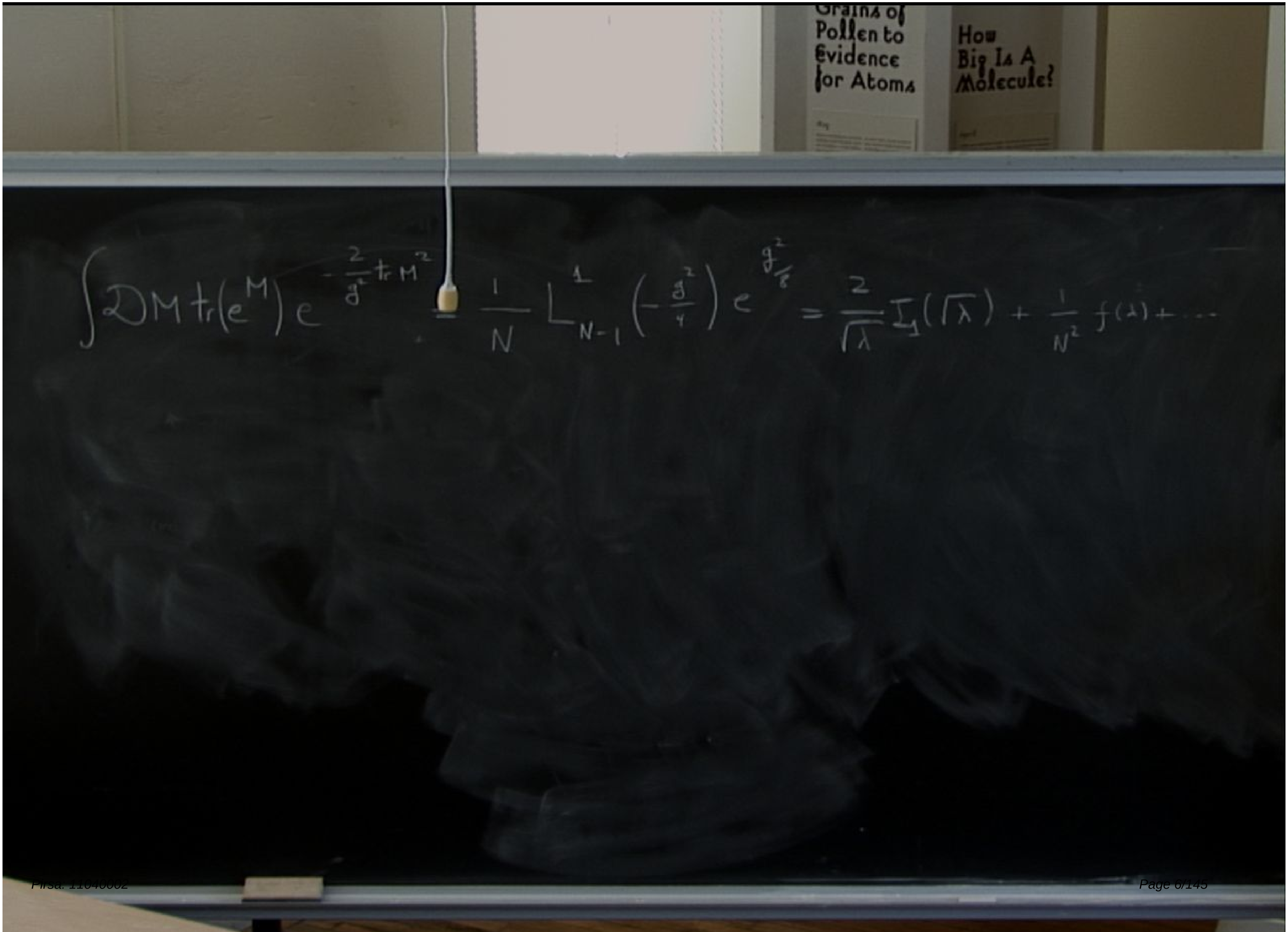

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\int \mathcal{D}M \, \mathrm{tr}(e^M) \, e^{-\frac{2}{g^2}\mathrm{tr}M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{g^2/8} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \cdots$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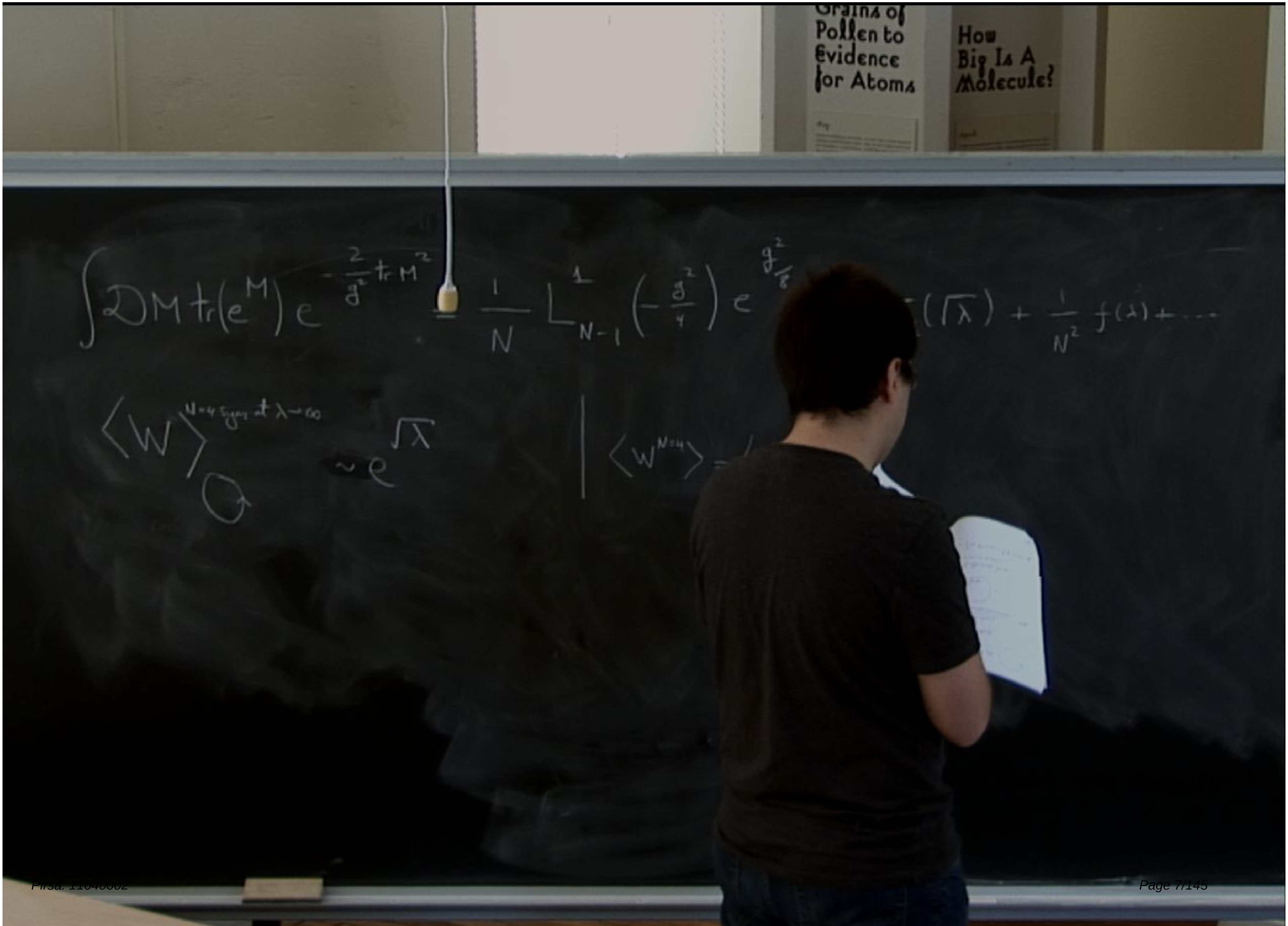
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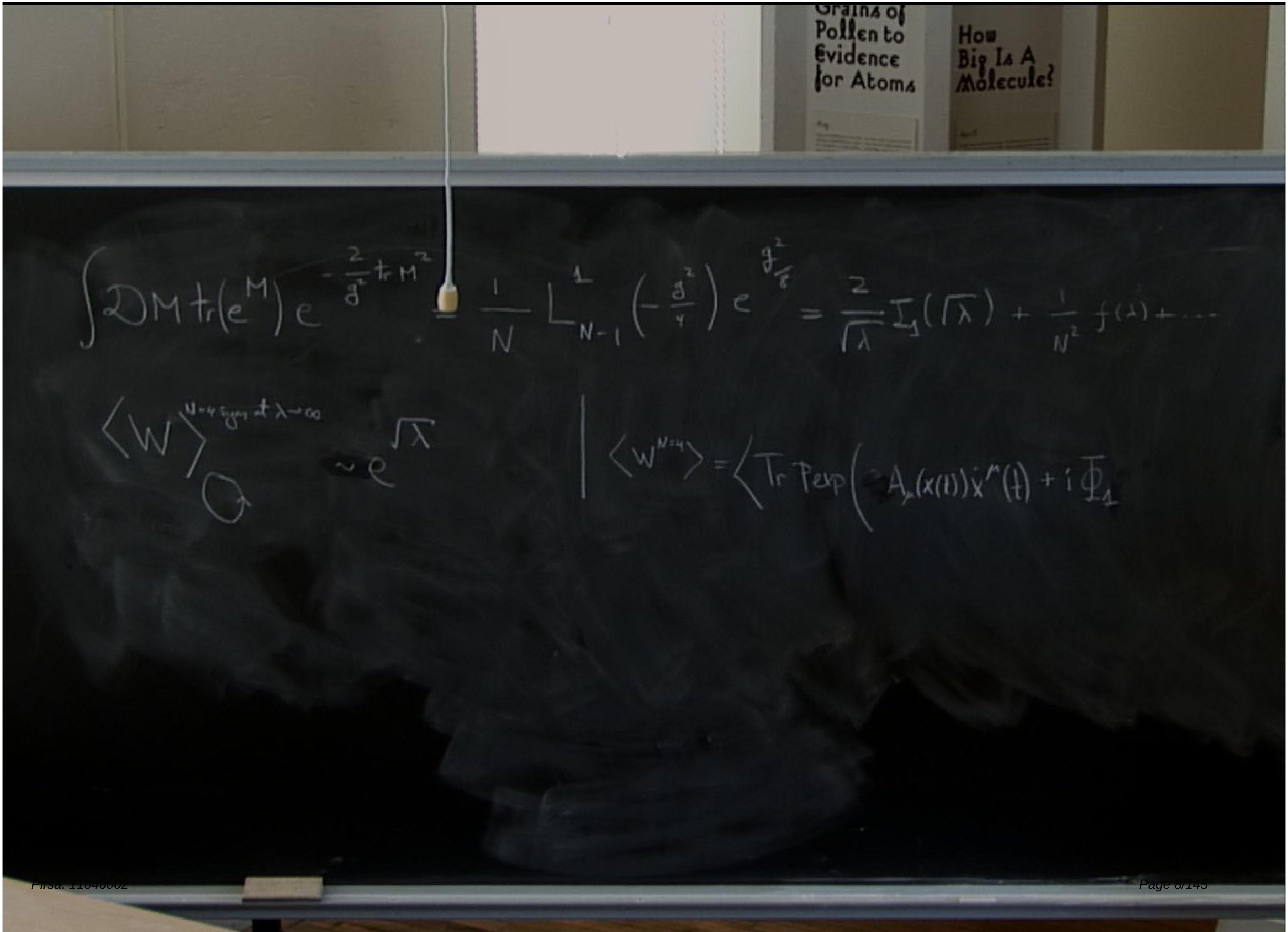
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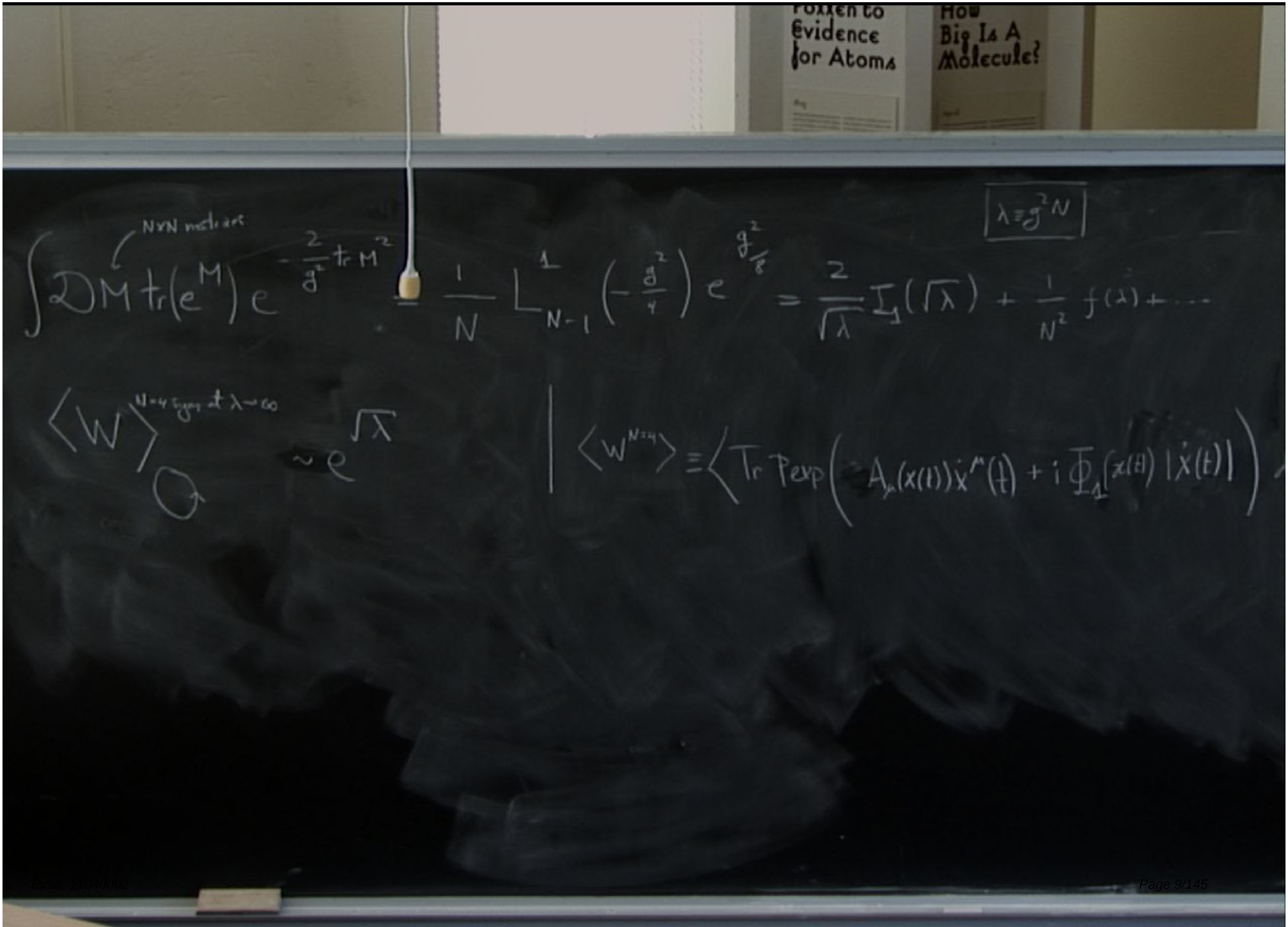
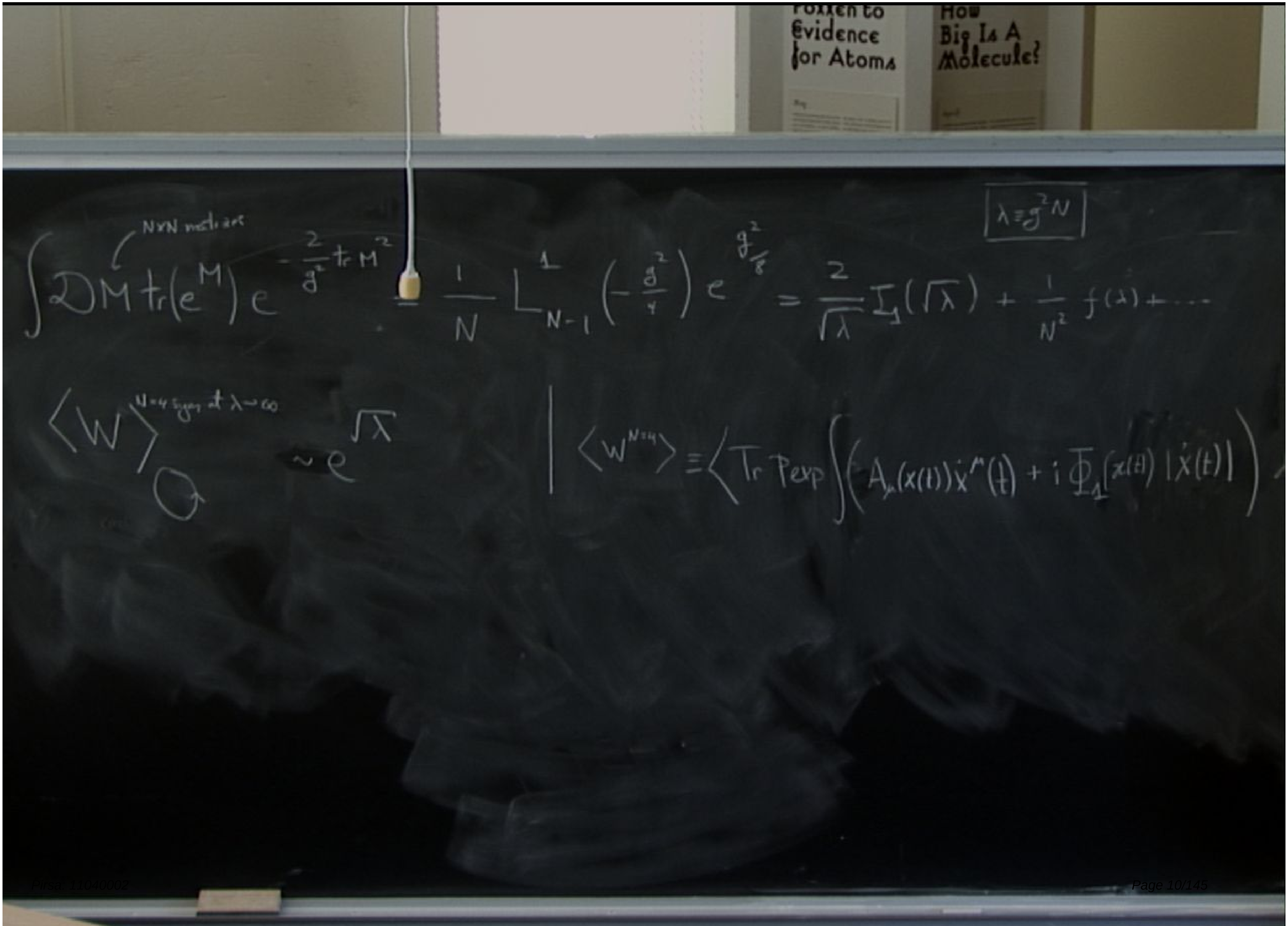
$\langle W \rangle^{N=4 \text{ Sym at } \lambda \to \infty}_{\bigcirc} \sim e^{\sqrt{\lambda}}$
$\langle W^{N=4} \rangle = \langle \mathrm{Tr}\, P\exp\left(\oint A_\mu(x(t)) \dot{x}^\mu(t) + i \Phi_1 \right.$


Evidence for Atoms
How Big Is A Molecule?
$\lambda \equiv g^2 N$
N×N matrices
$\int \mathcal{D}M \, \mathrm{tr}(e^M) \, e^{-\frac{2}{g^2} \mathrm{tr} M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{g^2/8} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \cdots$
N=4 SYM at λ→∞
$\langle W \rangle \sim e^{\sqrt{\lambda}}$
$\langle W^{N=4} \rangle \equiv \langle \mathrm{Tr}\, P\exp\left(A_\mu(x(t)) \dot{x}^\mu(t) + i \Phi_1(x(t)) |\dot{x}(t)| \right)$



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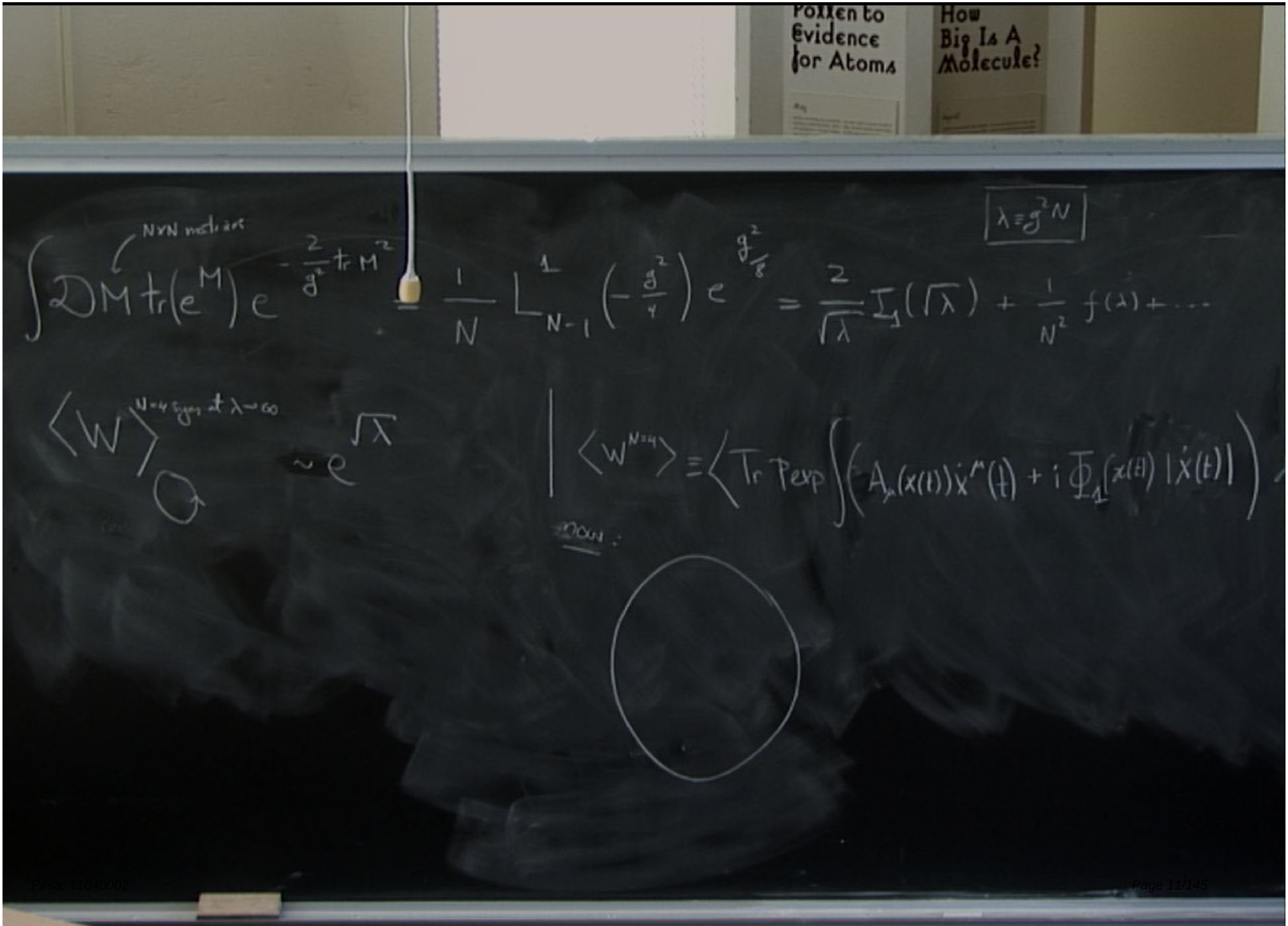
$\langle W^{N=4} \rangle \equiv \left\langle \mathrm{Tr} \, P\exp \int \left(A_\mu(x(t)) \dot{x}^\mu(t) + i \Phi_1(x(t)) |\dot{x}(t)| \right) \right.$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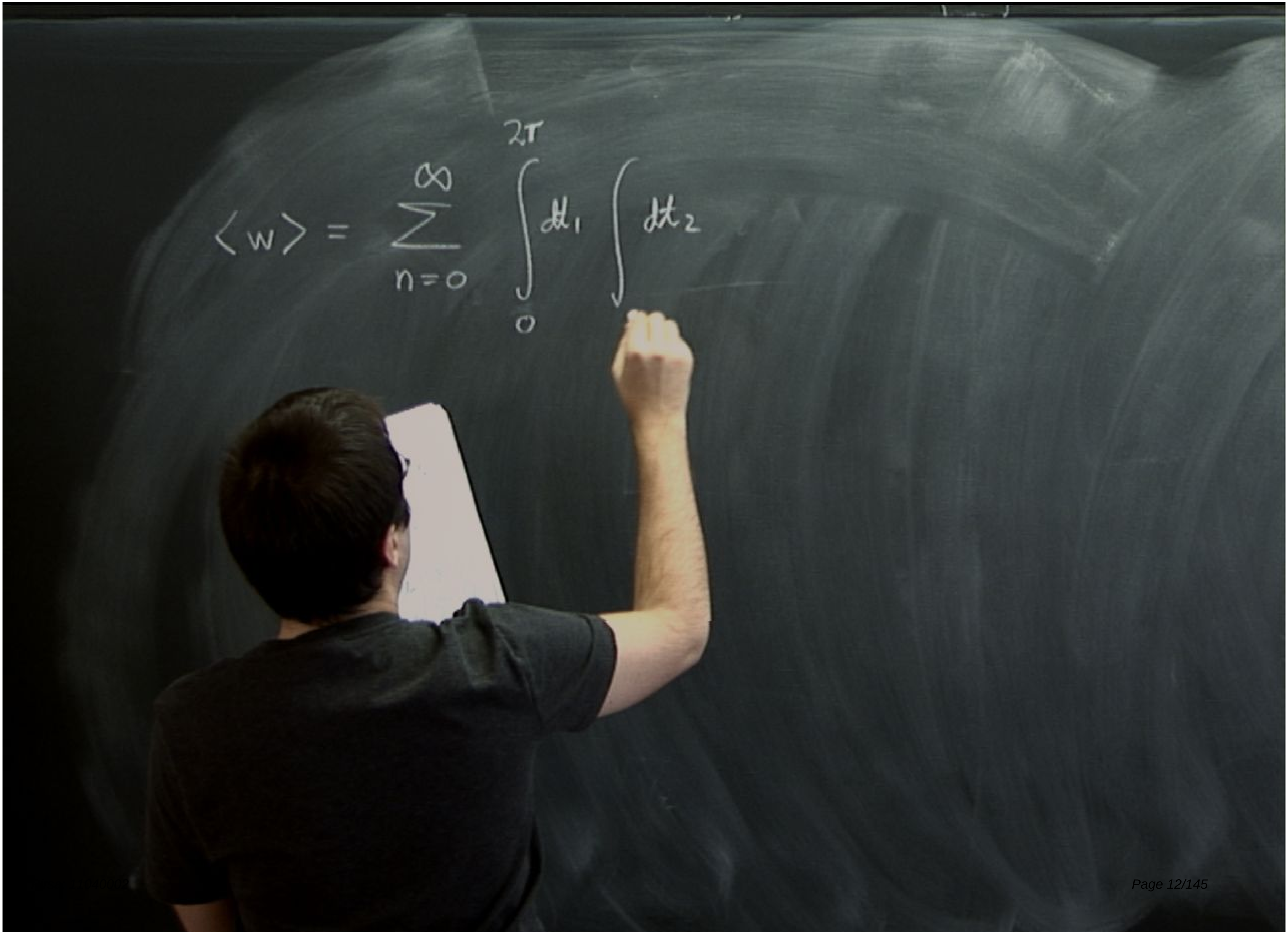
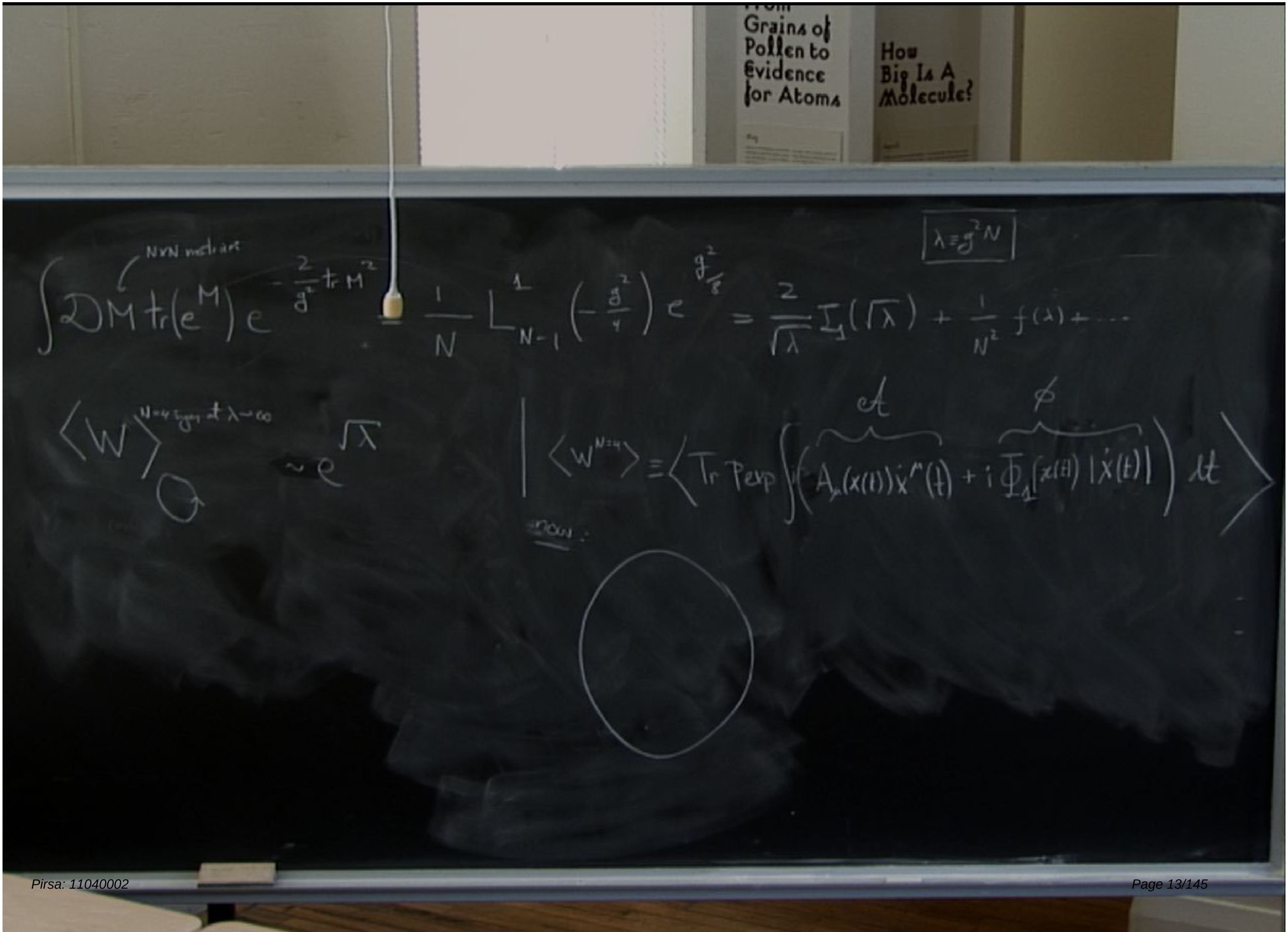
∫DM tr(e^M) e^{-2/g² tr M²} = 1/N L¹_{N-1}(-g²/4) e^{g²/8} = 2/√λ I₁(√λ) + 1/N² f(λ) + ...
⟨W⟩ N=4 SYM at λ→∞ ~ e^{√λ}
⟨W^{N=4}⟩ ≡ ⟨Tr P exp ∫(A_μ(x(t)) ẋ^μ(t) + i Φ₁(x(t)) |ẋ(t)|)⟩
now:


$\langle w \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int dt_2$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A
φ
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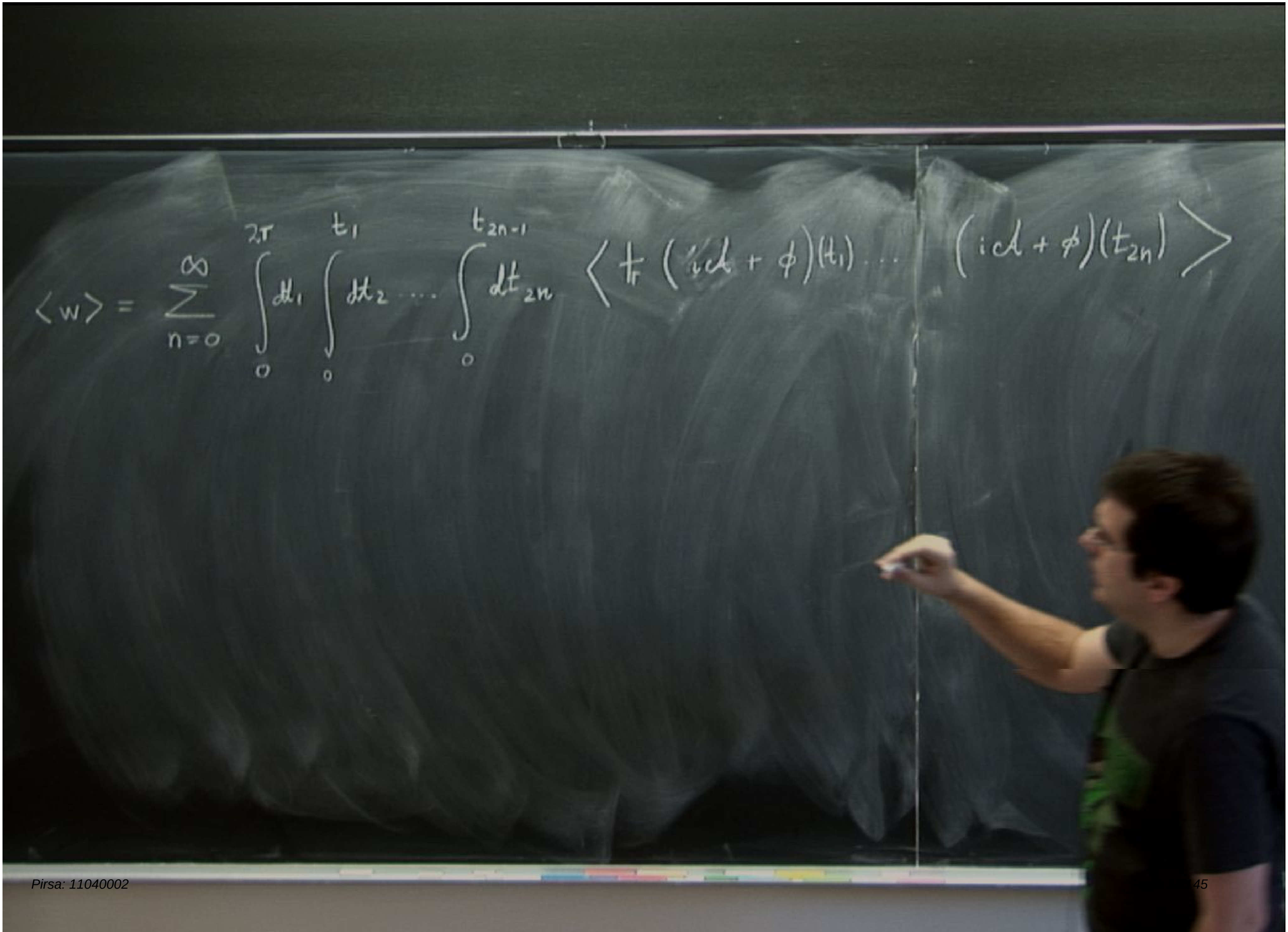
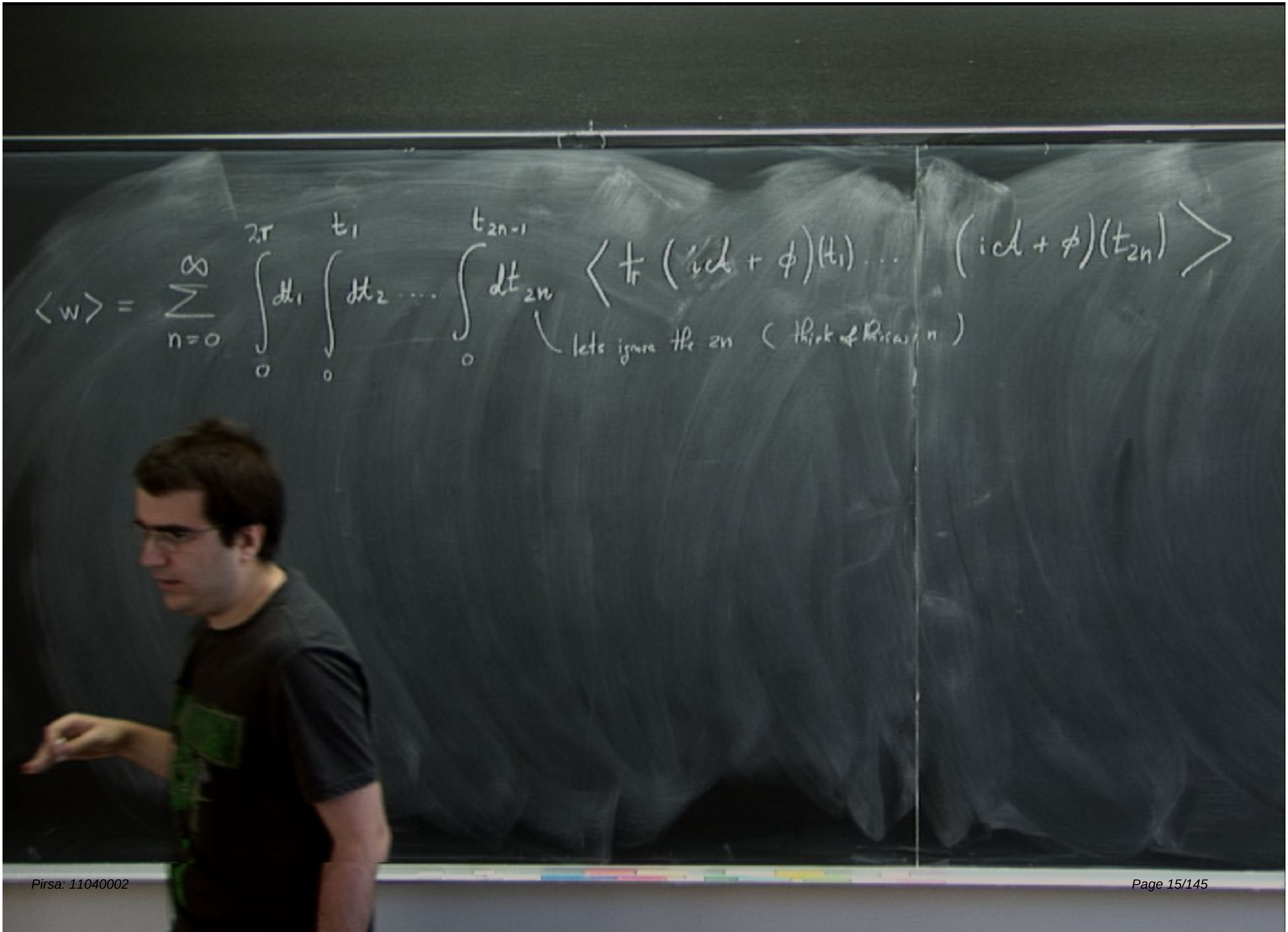
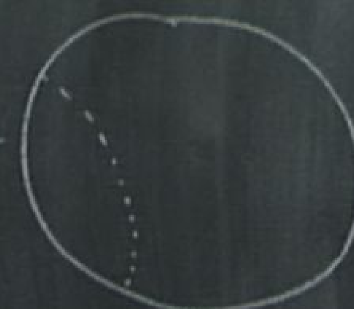

$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr} \, (iA + \phi)(t_1) \cdots (iA + \phi)(t_{2n}) \right\rangle$


$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\, (i\dot{x} + \phi)(t_1) \cdots (i\dot{x} + \phi)(t_{2n}) \right\rangle$
lets ignore the 2n (think of this as n)

$i\mathcal{A} + \phi)(t_{2n})\rangle$

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr} \left(iA + \phi \right. \right.$$

↖ lets ignore the 2n (th

$$= 1 +$$  $$+$$ 

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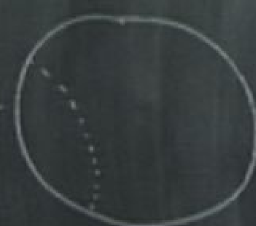
$$= 1 +$$  $$+$$ 

$$\mathrm{Tr}\, \left\langle (iA + \phi)(t_1)\, (iA + \phi)(t_2) \right\rangle = \frac{-g^2 N}{4\pi^2} \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - |\dot{x}(t_1)|\,|\dot{x}(t_2)|}{|x(t_1) - x(t_2)|^2}$$

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$$i A + \phi)(t_{2n}) \rangle$$

$$X(t) = R(\cos t, \sin t, 0, 0)$$

$$\dot{X}(t) = R(-\sin t, \cos t, 0, 0)$$

$$|x(t_1) - x(t_2)|^2 =$$

$$\frac{(t_1)|\,|\dot{x}(t_2)|}{\quad} = \frac{\lambda}{16\pi^2}$$

$$\ldots i c A + \phi)(t_{2n}) \Big\rangle$$

$$\ldots(t_1)| \, |\dot{x}(t_2)| \ldots = \frac{\lambda}{16\pi^2}$$

$$\begin{cases} X(t) = R(\cos t, \sin t, 0, 0) \\ \dot{X}(t) = R(-\sin t, \cos t, 0, 0) \\ |x(t_1) - x(t_2)|^2 = \ldots\ldots \quad \text{etc} \end{cases}$$

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr} \left(i\mathcal{A} + \phi \right)(t_1) \cdots \left(i\mathcal{A} + \phi \right)(t_{2n}) \right\rangle$$

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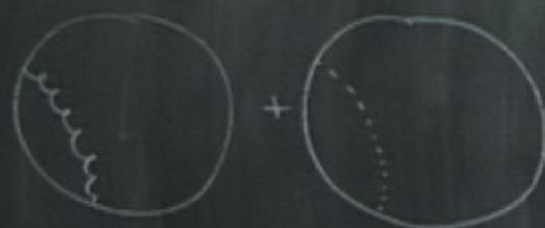
$$= 1 +$$ 

$$\mathrm{Tr} \left\langle (i\mathcal{A} + \phi)(t_1)\, (i\mathcal{A} + \phi)(t_2) \right\rangle = \frac{-\overbrace{g^2 N}^{\lambda}}{4\pi^2} \frac{\dot{x}(t_1) \cdot \dot{x}(t_2) - |\dot{x}(t_1)|\,|\dot{x}(t_2)|}{|x(t_1) - x(t_2)|^2} = \frac{\lambda}{16\pi^2}$$

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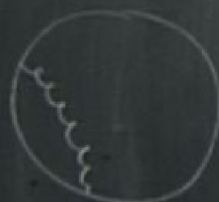
$$= 1 +$$  $$+$$  $$= 1 + \frac{\lambda}{16\pi^2} \underbrace{2\pi^2}_{\iint} + \cdots = 1 + \frac{\lambda}{8} + \cdots$$

$$\mathrm{Tr}\,\langle (iA + \phi)(t_1)\,(iA + \phi)(t_2) \rangle = \frac{-\overbrace{g^2 N}^{\lambda}}{4\pi^2} \frac{\dot{x}(t_1) \cdot \dot{x}(t_2) - |\dot{x}(t_1)|\,|\dot{x}(t_2)|}{|x(t_1) - x(t_2)|^2} = \frac{\lambda}{16\pi^2}$$

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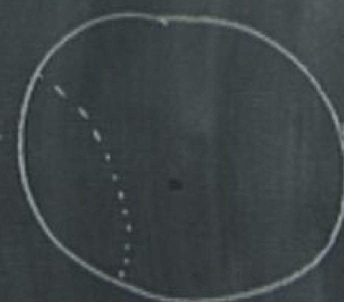
$$= 1 +$$  $$+$$  $$+ \cdots = 1 + \frac{\lambda}{16\pi^2} \underbrace{2\pi^2}_{\int\int} + \cdots = 1 + \cdots$$

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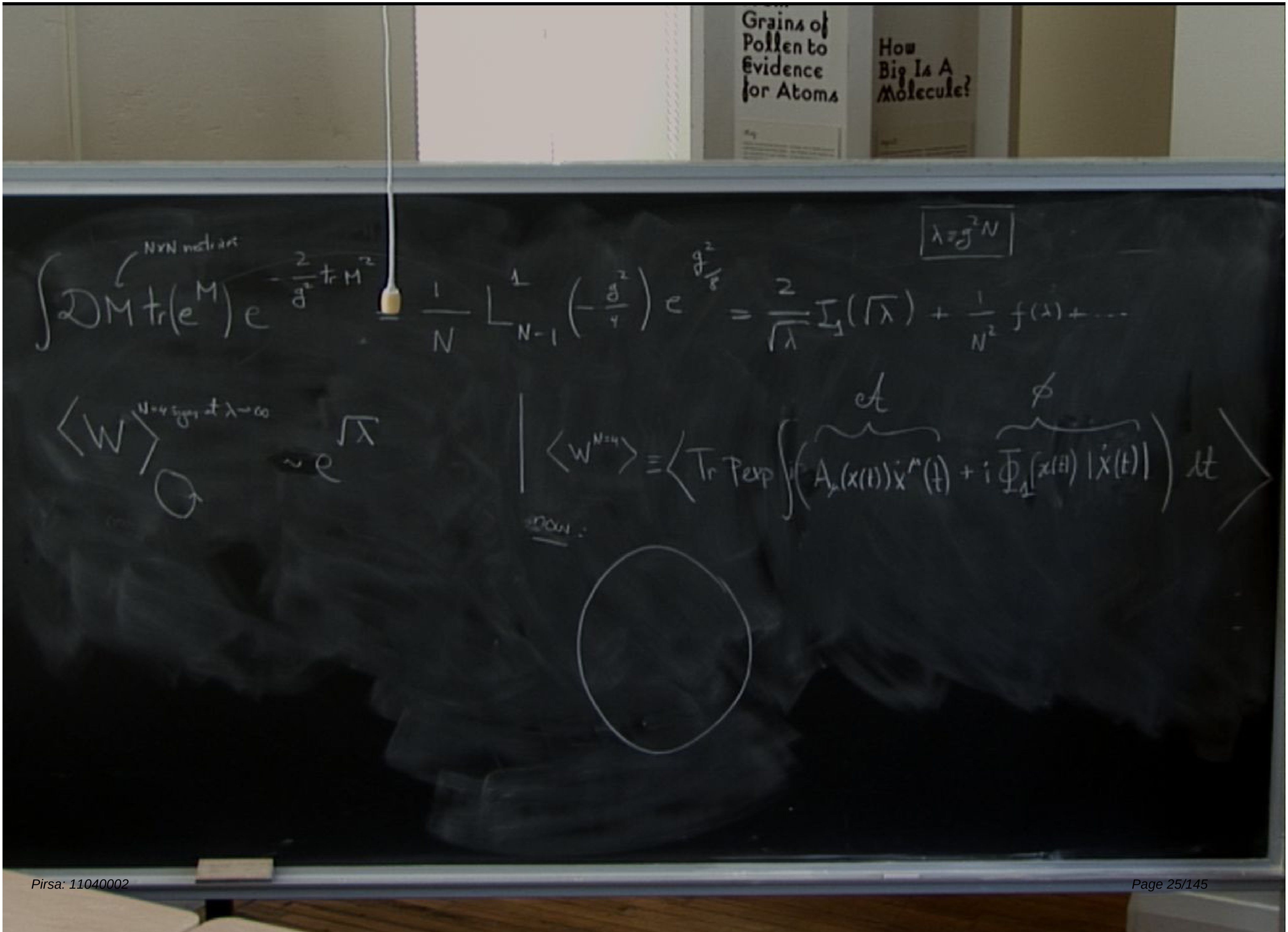
 $$= \frac{\lambda}{16\pi^2}$$ (a constant!)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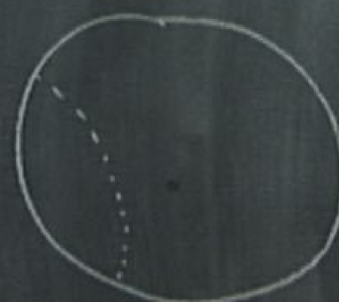
A
ϕ

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{Tr}\, (i\mathcal{A} + \phi)(t_1) \cdots \right.$$

lets ignore the 2n (think of this as

$$= 1 +$$  $$+$$  $$+ \cdots = 1 + \frac{\lambda}{16\pi^2} \underbrace{2\pi^2}_{\iint} + \cdots = 1 + \cdots$$

$$\mathrm{Tr}\, \langle (i\mathcal{A}+\phi)(t_1)\,(i\mathcal{A}+\phi)(t_2) \rangle = -\frac{\overbrace{g^2 N}^{\lambda}}{4\pi^2} \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - \cdots}{|x(t_1) - x(t_2)|^2}$$

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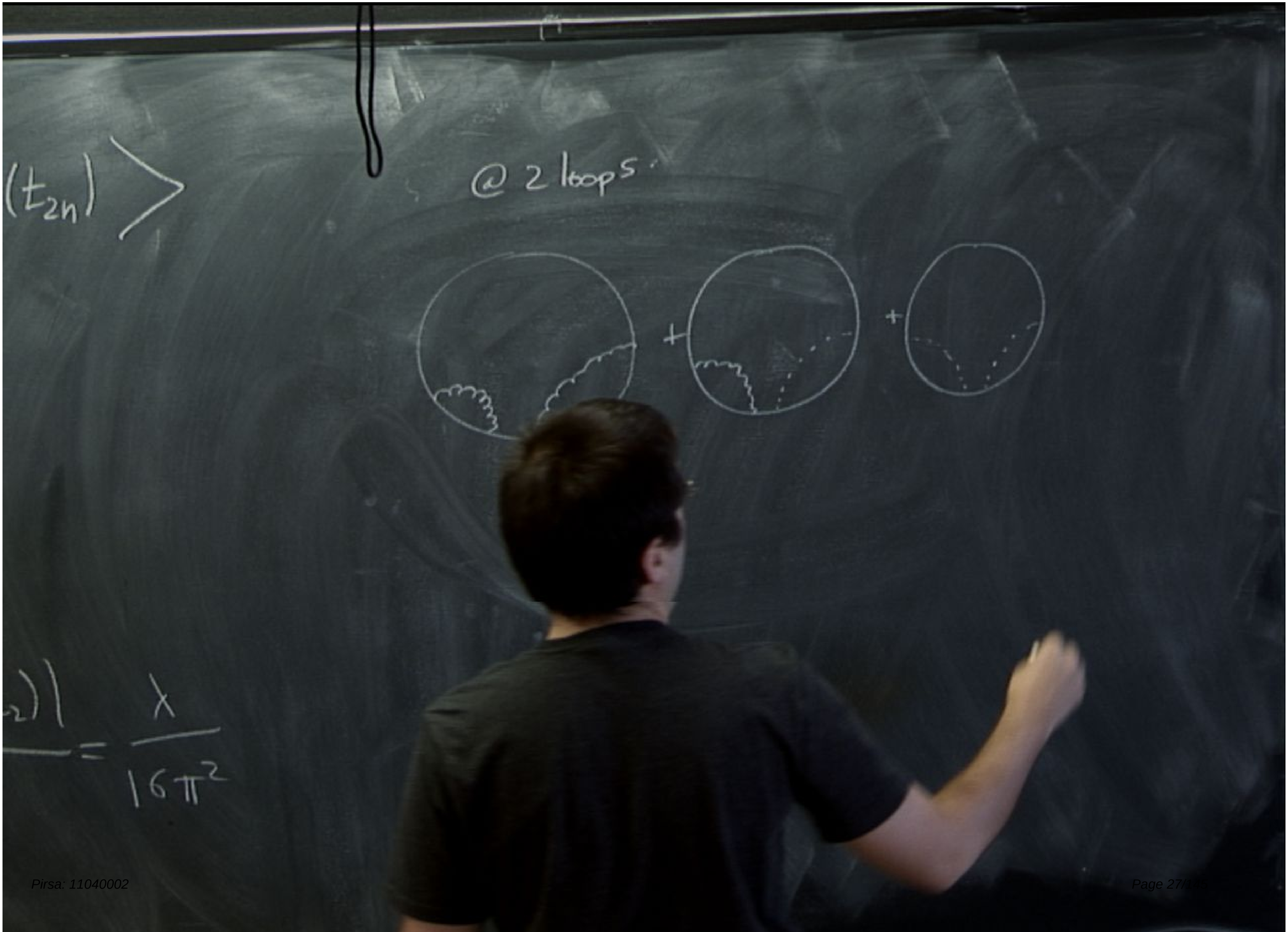
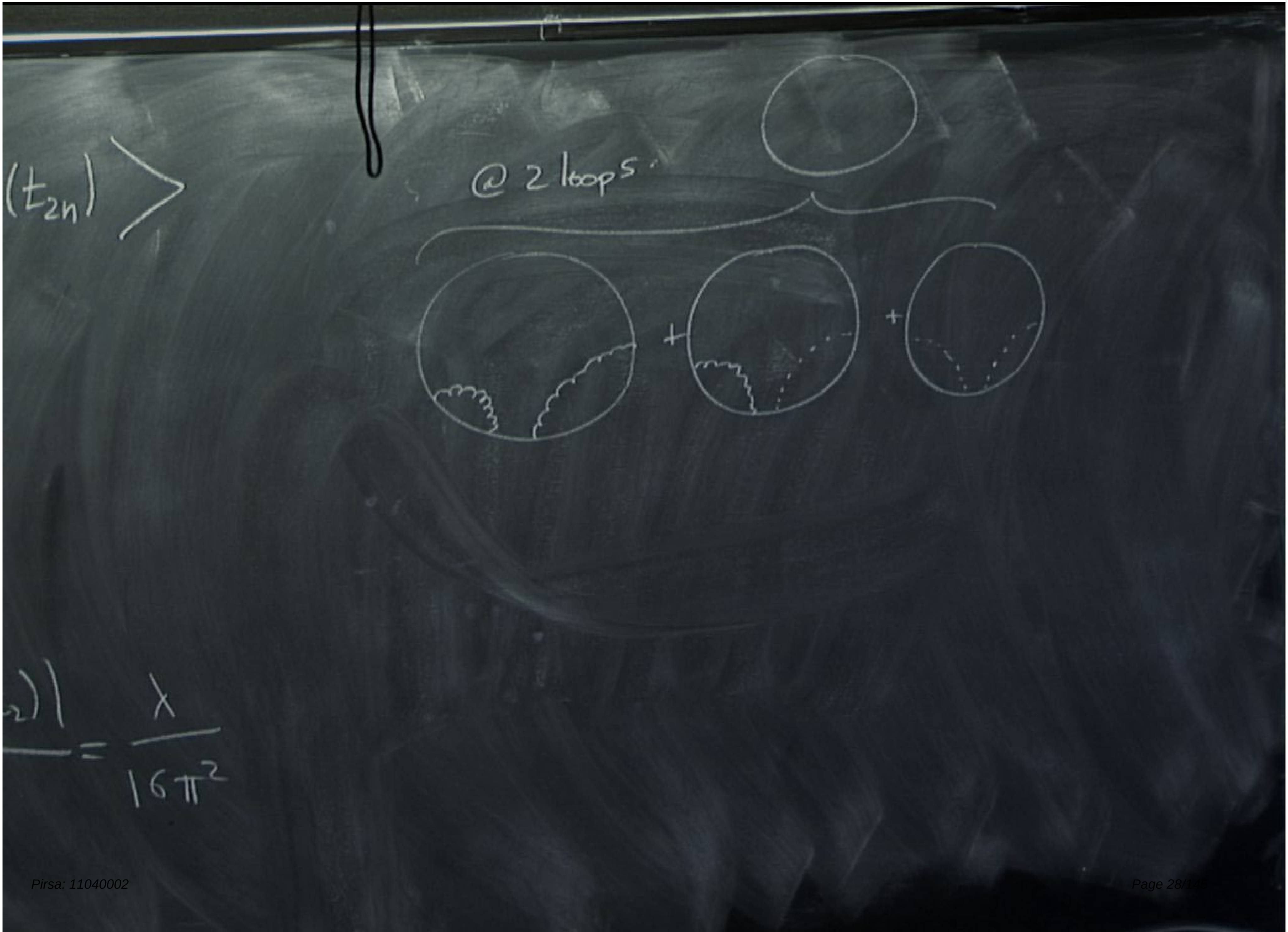
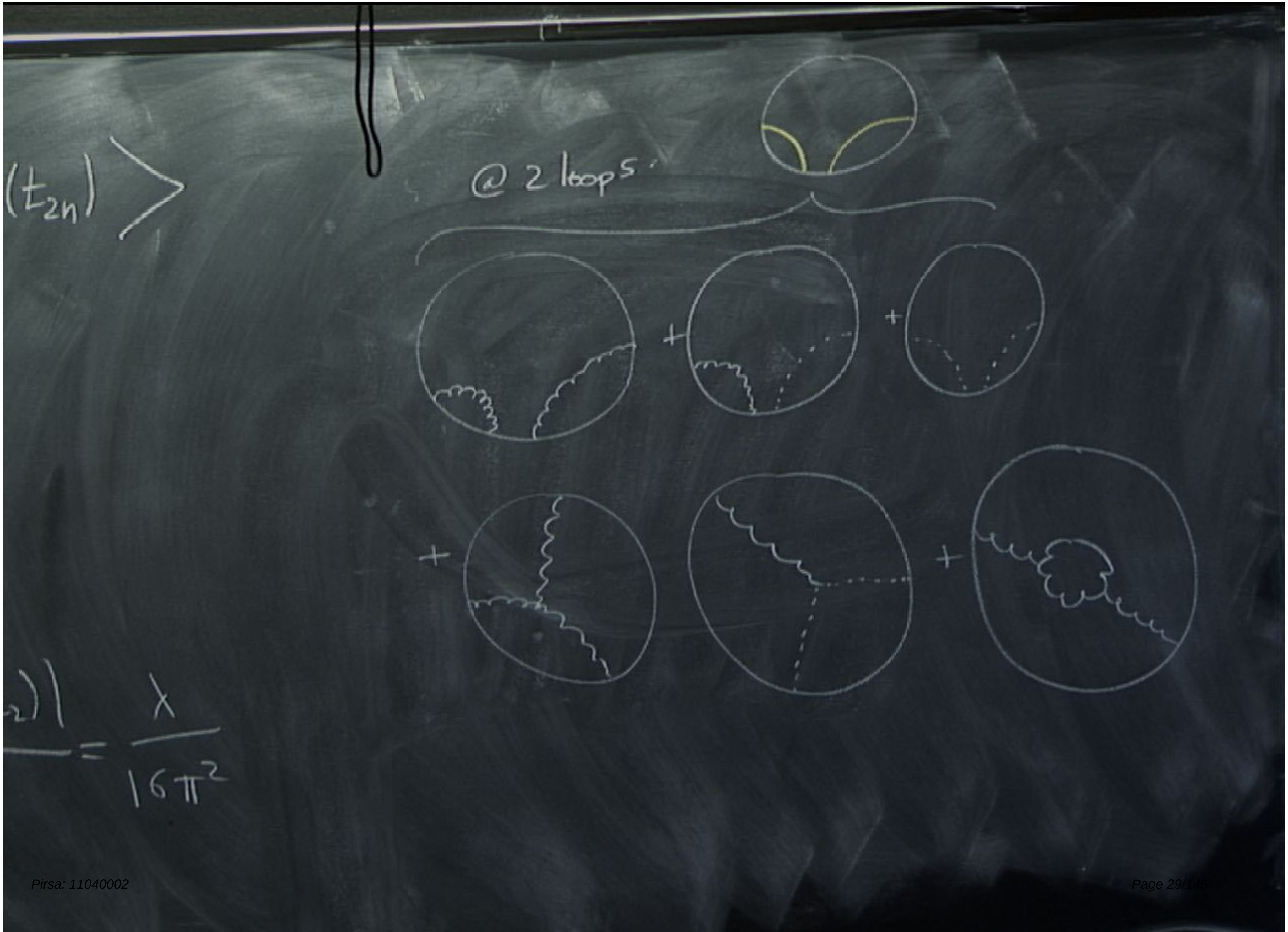
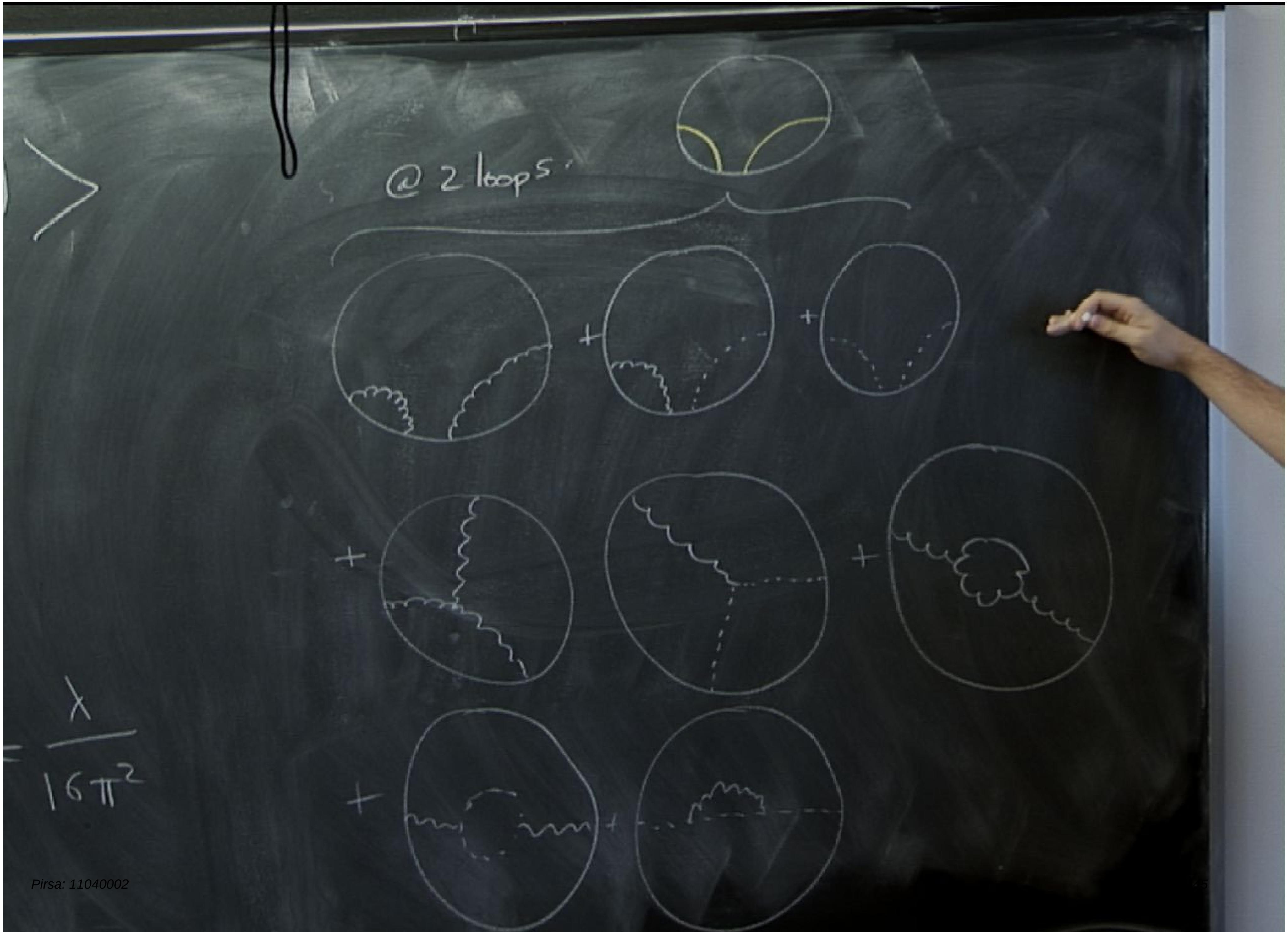
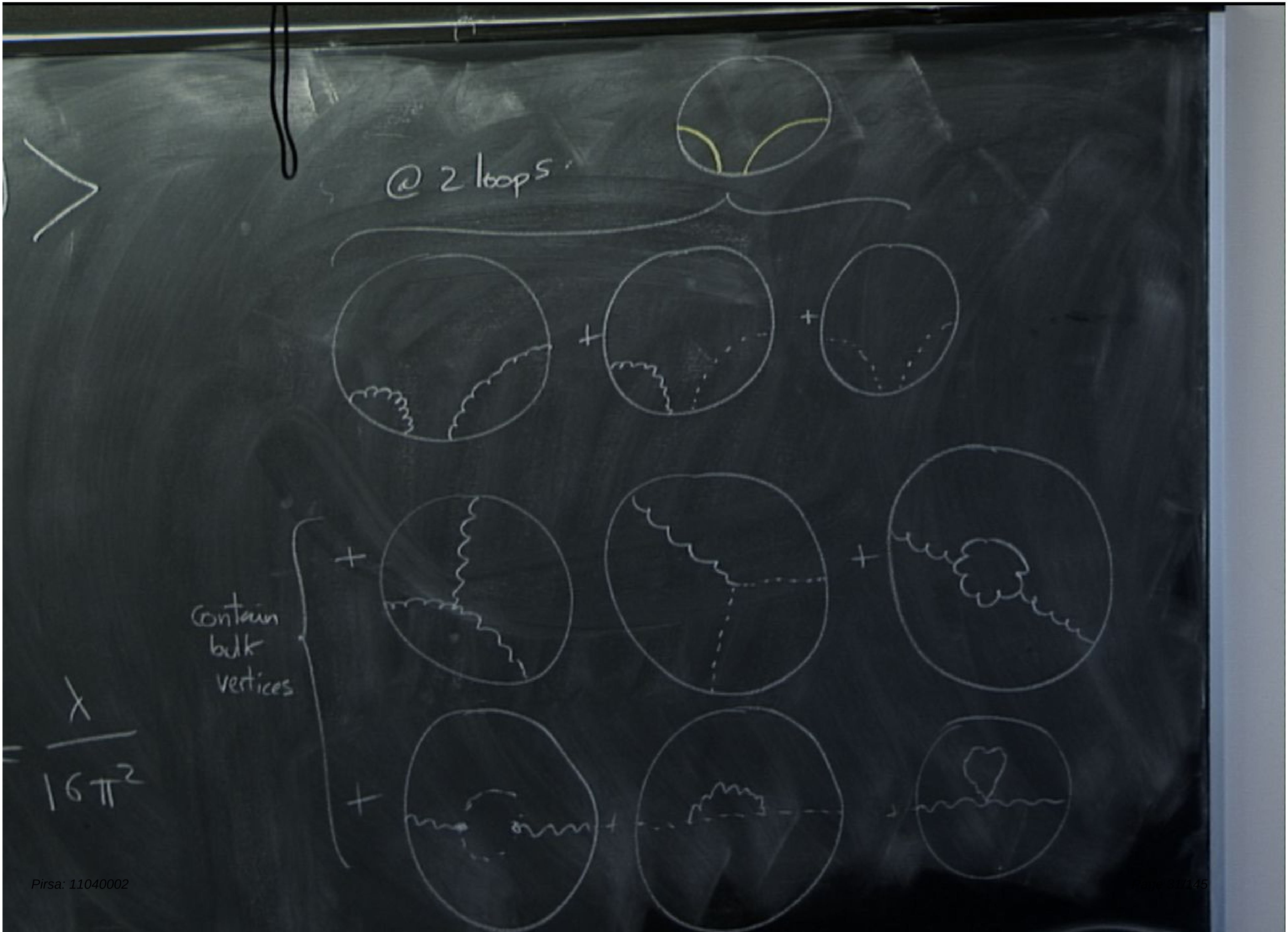
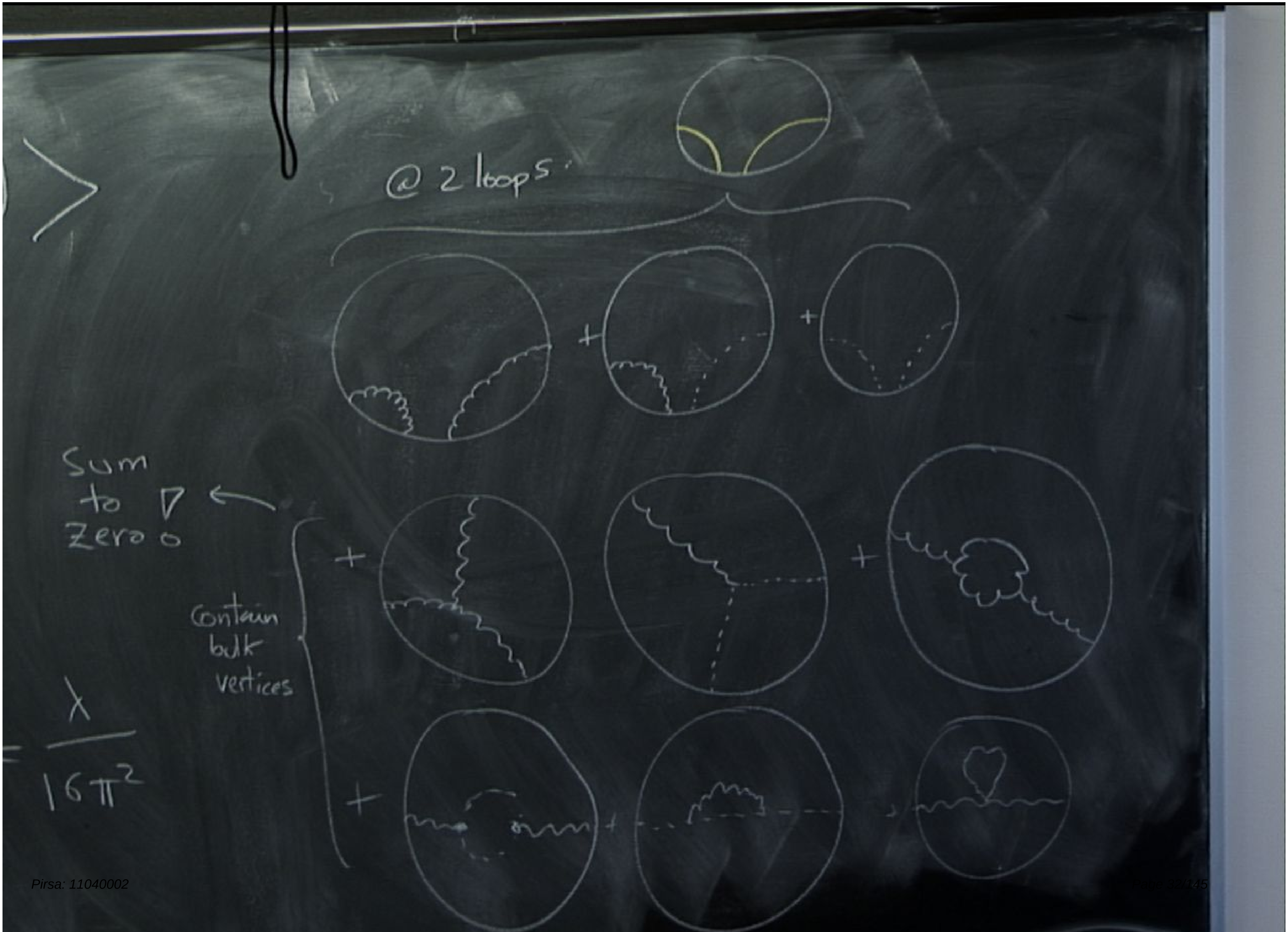

(t2n) >
@ 2 loops.
+
+
λ
16π²

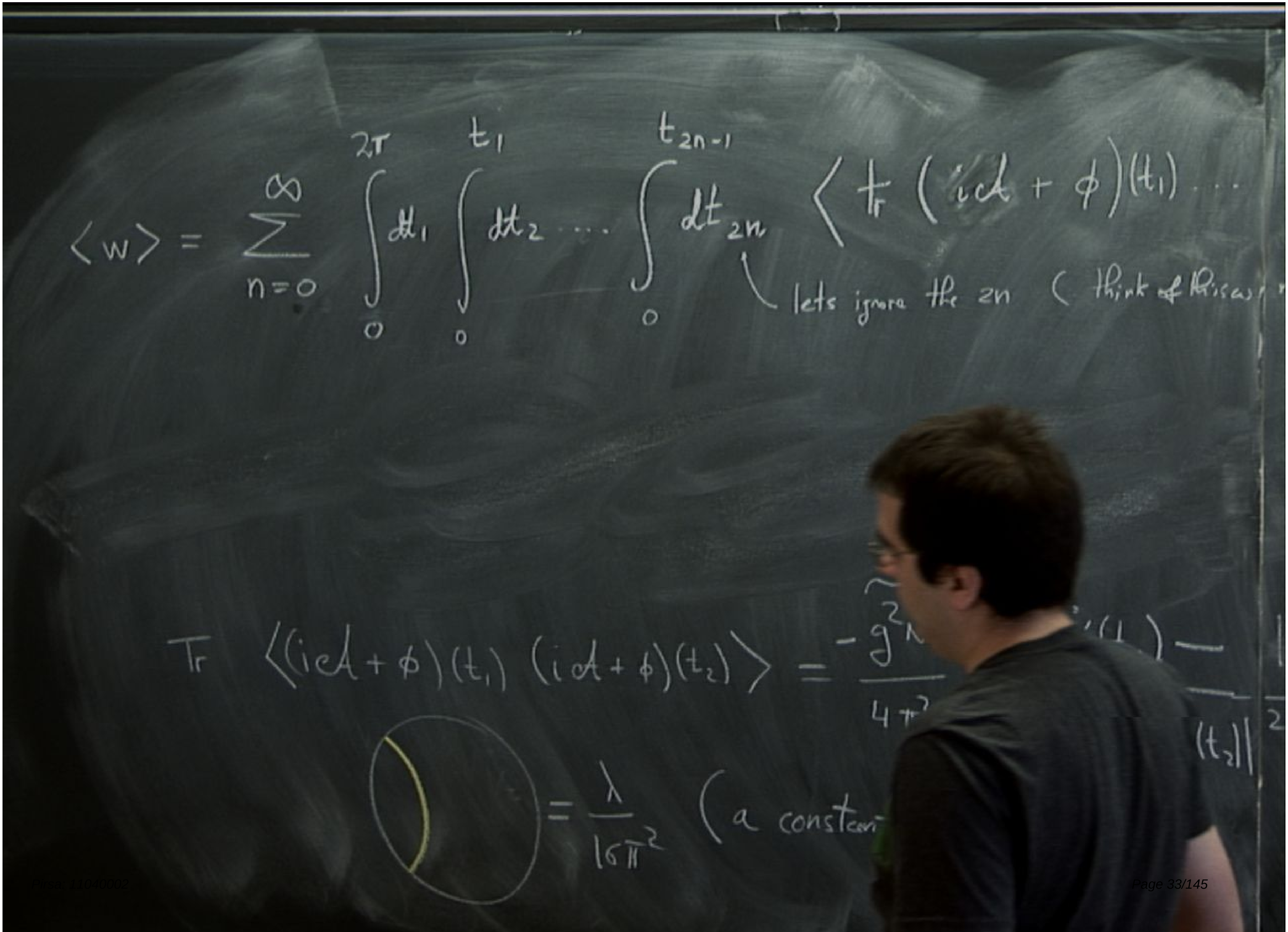

(t2n) >
@ 2 loops:
+
+
= λ / 16π²


(t2n) >
@ 2 loops:
= λ / 16π2


@ 2 loops
λ / 16π²


@ 2 loops:
contain bulk vertices
$\frac{\lambda}{16\pi^2}$


@ 2 loops:
Sum to ▽ zero!
contain bulk vertices
λ
16π²



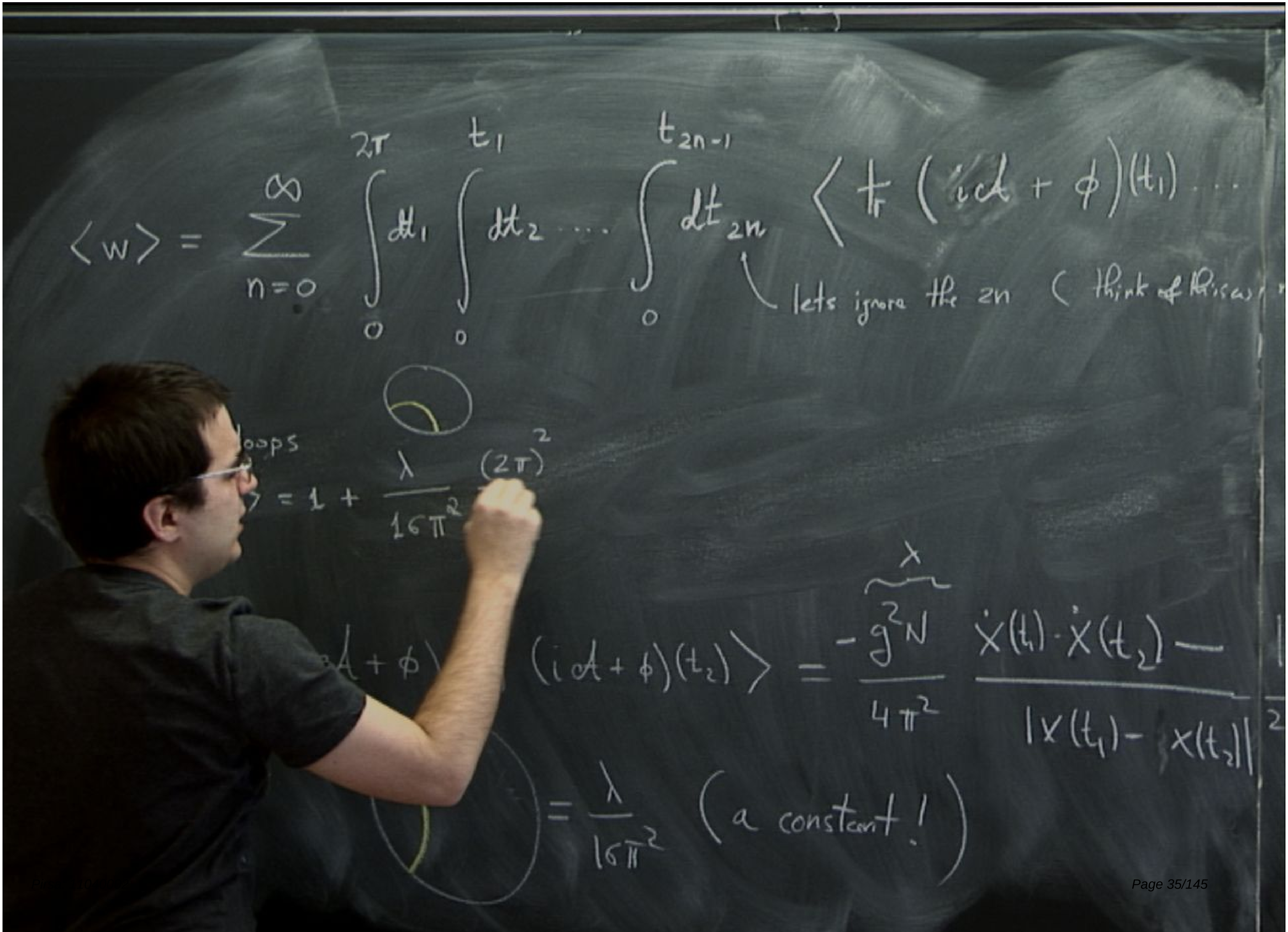
$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr} (iA + \phi)(t_1) \ldots$
lets ignore the 2n (think of this as
$\mathrm{Tr} \langle (iA + \phi)(t_1)\,(iA + \phi)(t_2) \rangle = \frac{-g^2}{4\pi^2}$
$= \frac{\lambda}{16\pi^2}$ (a constant

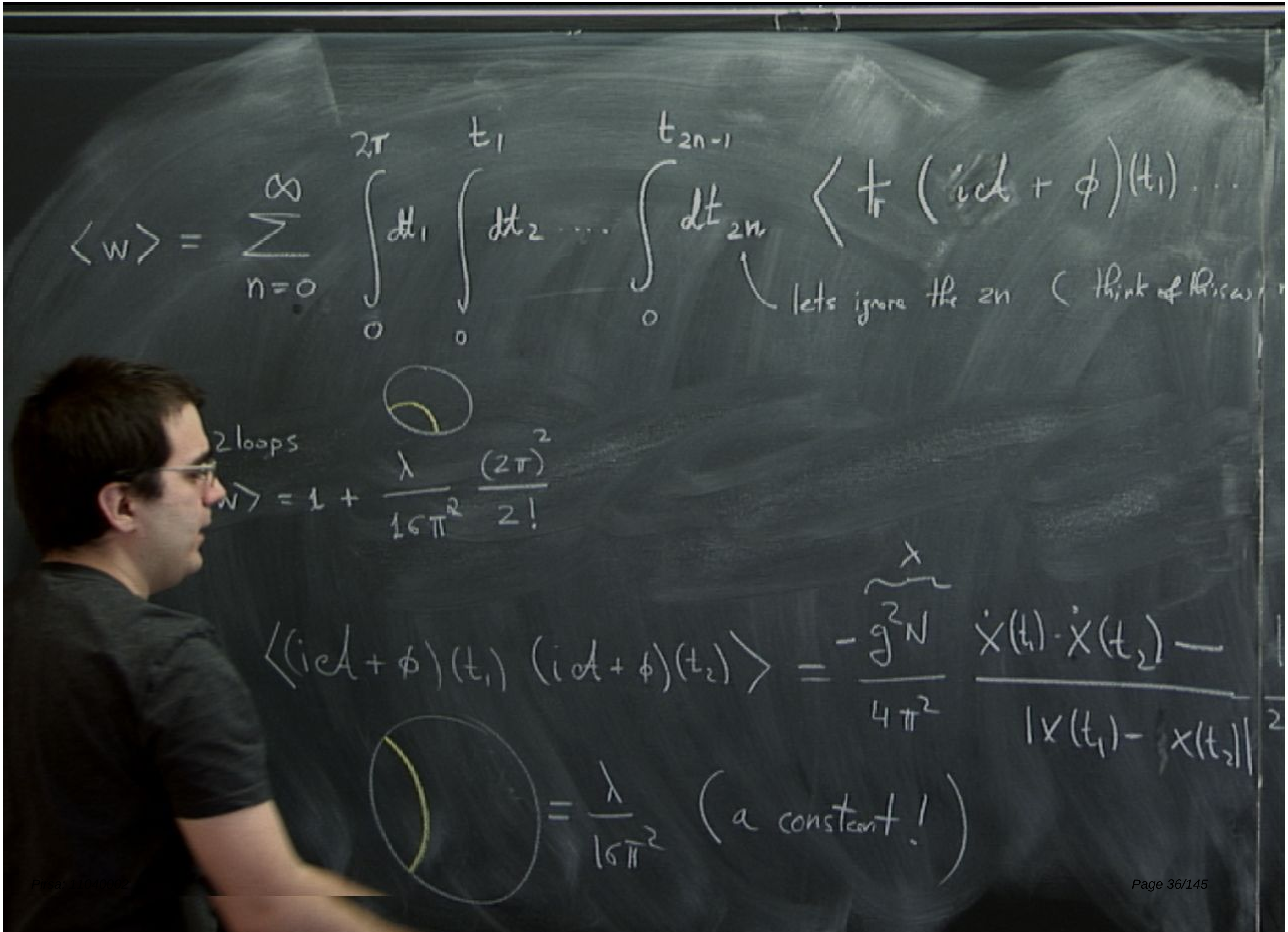
$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(iA + \phi \right)(t_1) \cdots \right.$$

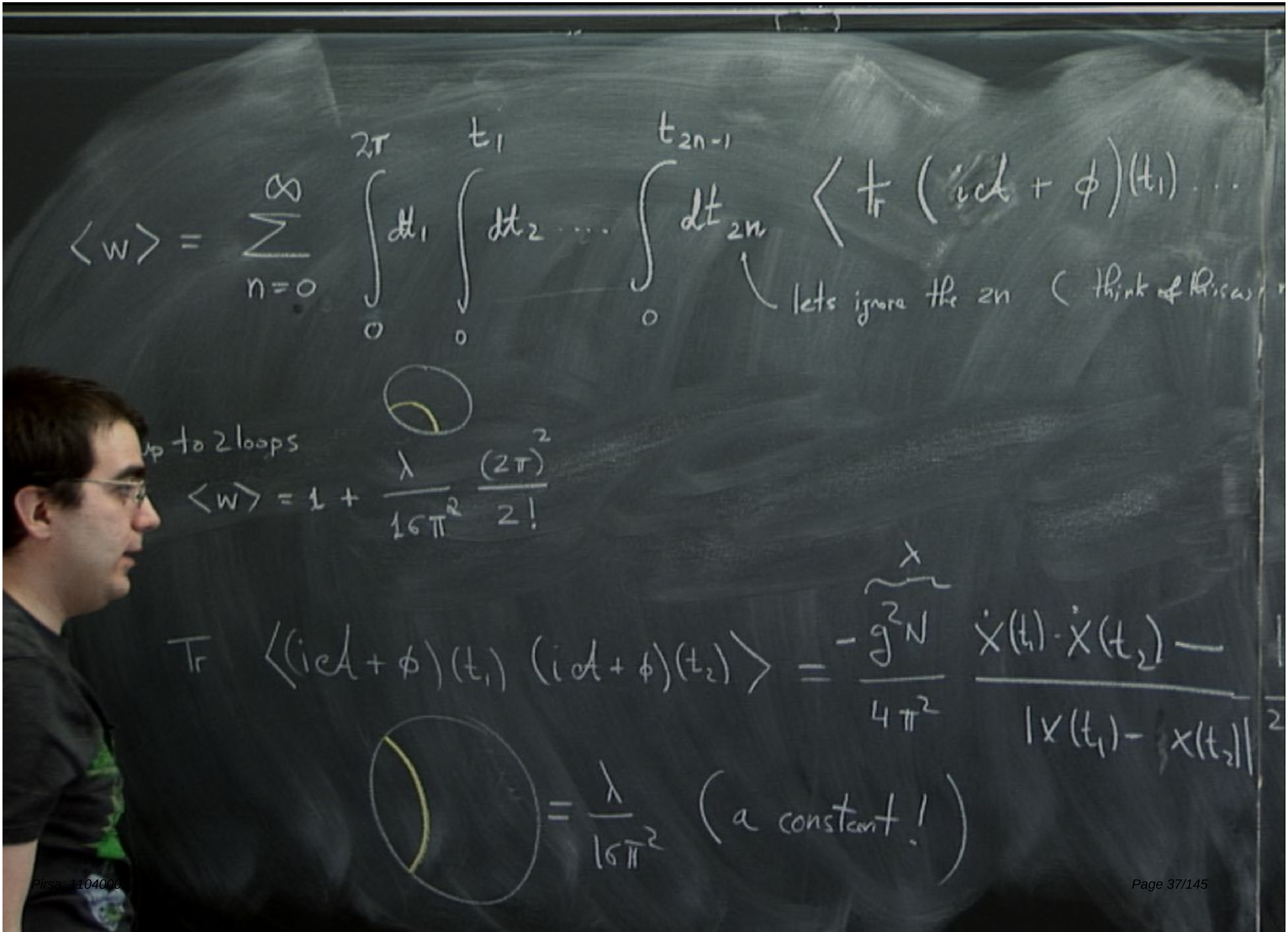
lets ignore the 2n (think of this as [illegible]

$$\langle (iA+\phi)(t_1)\,(iA+\phi)(t_2) \rangle = \underbrace{-\frac{g^2 N}{4\pi^2}}_{\lambda} \, \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - [illegible]}{|x(t_1) - x(t_2)|^2}$$

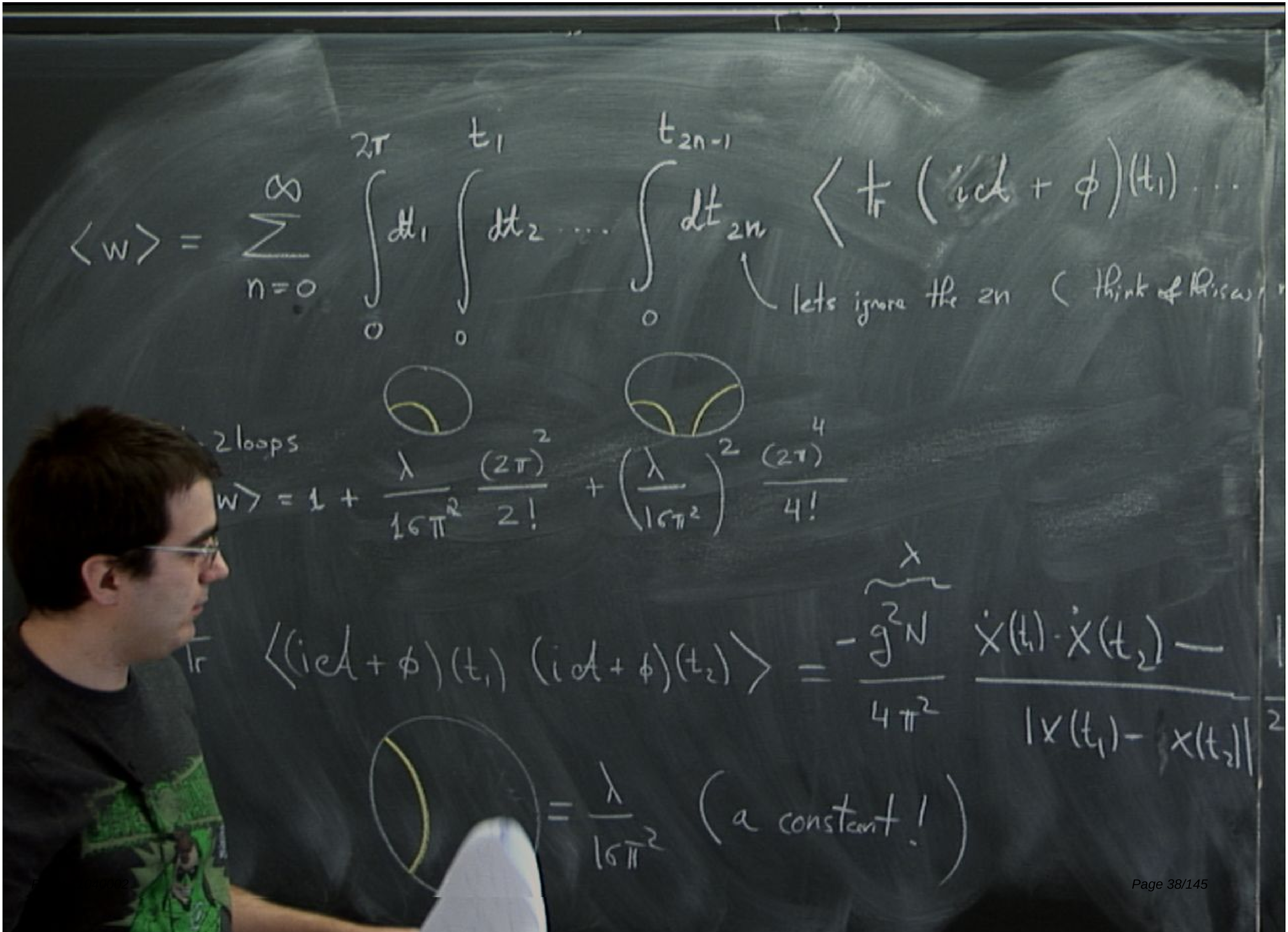
$$= \frac{\lambda}{16\pi^2} \quad (\text{a constant!})$$


⟨W⟩ = Σ ∫dt1 ∫dt2 .... ∫dt2n ⟨tr (iA + φ)(t1) ...
n=0 ∞ 0 2π 0 t1 0 t2n-1
lets ignore the 2n (think of this as
loops
= 1 + λ / 16π² (2π)²
(iA + φ)(t2)⟩ = -g²N / 4π² ẋ(t1)·ẋ(t2) — / |x(t1) - x(t2)|²
λ
= λ / 16π² (a constant!)


$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr}\,(iA + \phi)(t_1) \ldots$
lets ignore the 2n (think of this as
2 loops
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2} \frac{(2\pi)^2}{2!}$
$\langle (iA+\phi)(t_1)\,(iA+\phi)(t_2) \rangle = \frac{-g^2 N}{4\pi^2} \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) -}{|x(t_1) - x(t_2)|^2}$
λ
$= \frac{\lambda}{16\pi^2}$ (a constant!)



$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr}(iA + \phi)(t_1) \cdots$
lets ignore the 2n (think of this as
up to 2 loops
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2} \frac{(2\pi)^2}{2!}$
$\mathrm{Tr} \langle (iA + \phi)(t_1)(iA + \phi)(t_2) \rangle = \frac{-g^2 N}{4\pi^2} \frac{\dot{x}(t_1) \cdot \dot{x}(t_2) -}{|x(t_1) - x(t_2)|^2}$
λ
$= \frac{\lambda}{16\pi^2}$ (a constant!)



$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr}\,(iA + \phi)(t_1) \ldots$
lets ignore the $2n$ (think of this as
2 loops
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + \left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!}$
$\mathrm{Tr}\,\langle (iA+\phi)(t_1)\,(iA+\phi)(t_2) \rangle = \frac{-g^2 N}{4\pi^2} \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) -}{|x(t_1)-x(t_2)|^2}$
λ
$= \frac{\lambda}{16\pi^2}$ (a constant!)

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\, (iA + \phi)(t_1) \cdots \right.$$

lets ignore the 2n (think of this as ...

up to 2 loops

$$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2} \frac{(2\pi)^2}{2!} + \left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \cdots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$

$$\langle (iA+\phi)(t_1)\,(iA+\phi)(t_2) \rangle = \frac{-\overbrace{g^2 N}^{\lambda}}{4\pi^2} \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - }{|x(t_1) - x(t_2)|^2}$$

$$= \frac{\lambda}{16\pi^2} \quad (\text{a constant!})$$

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(i\dot{A} + \phi \right)(t_1) \cdots \right.$$

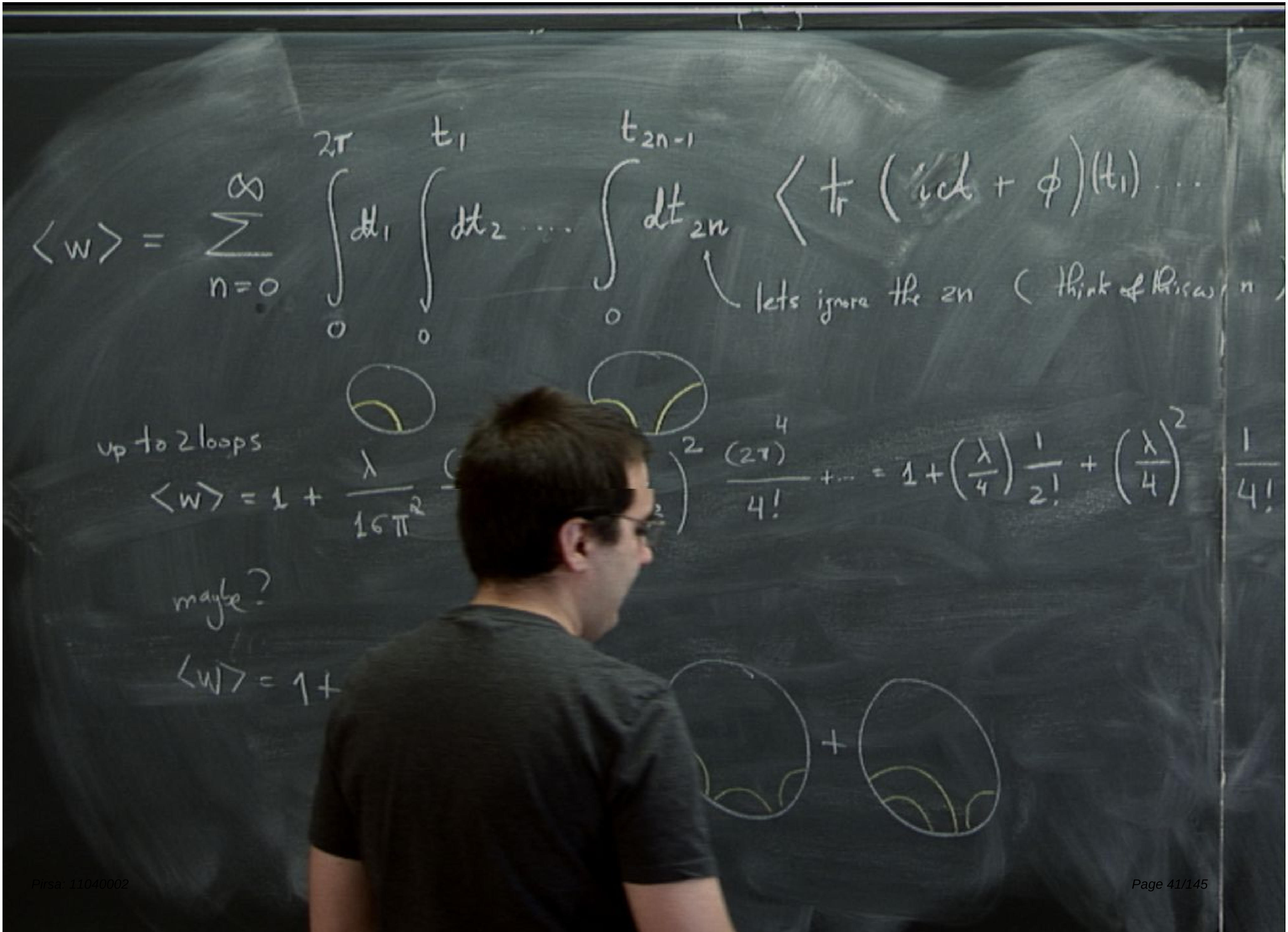
lets ignore the $2n$ (think of this as ...

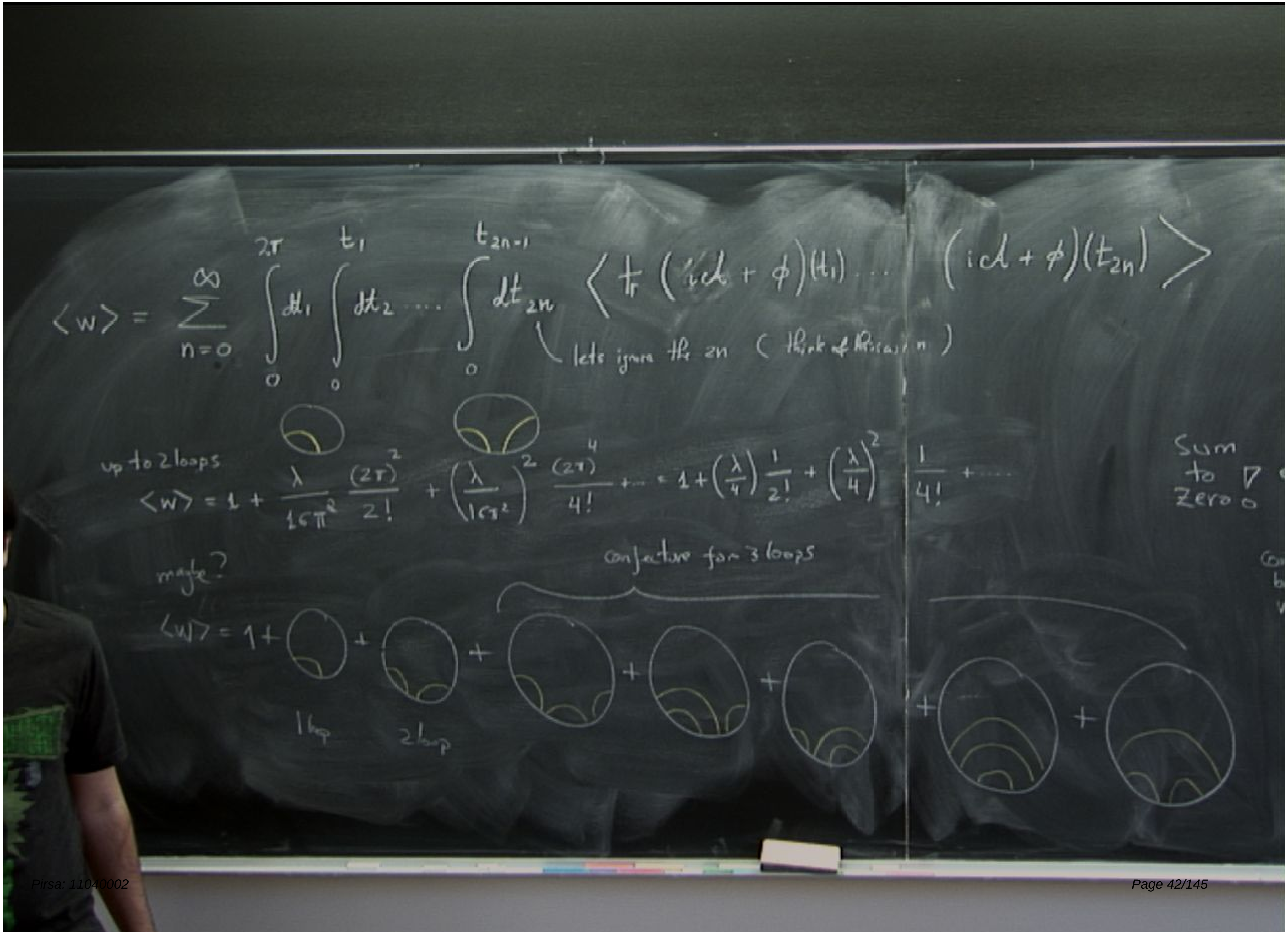
up to 2 loops

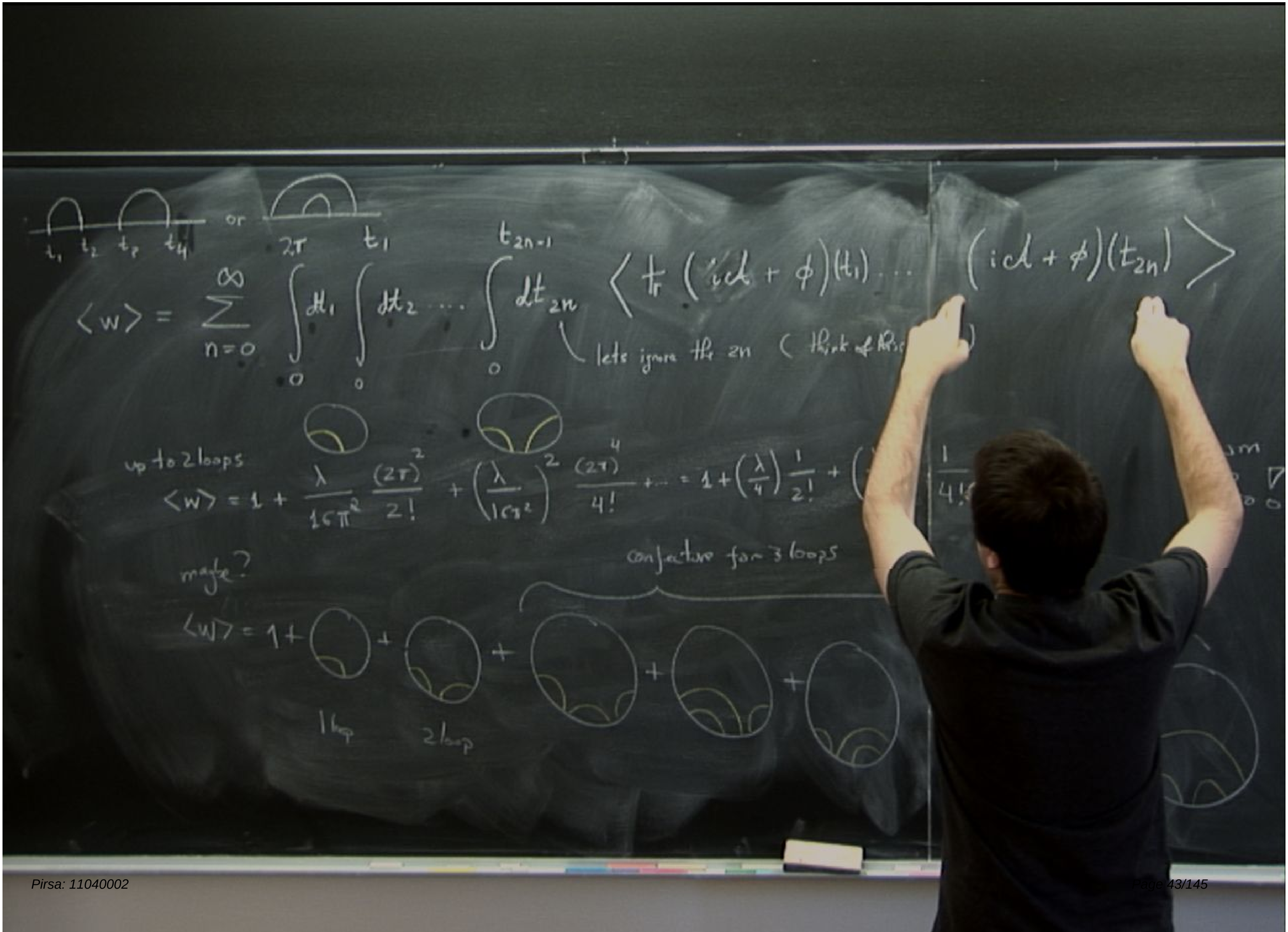
$$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + \left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \cdots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$

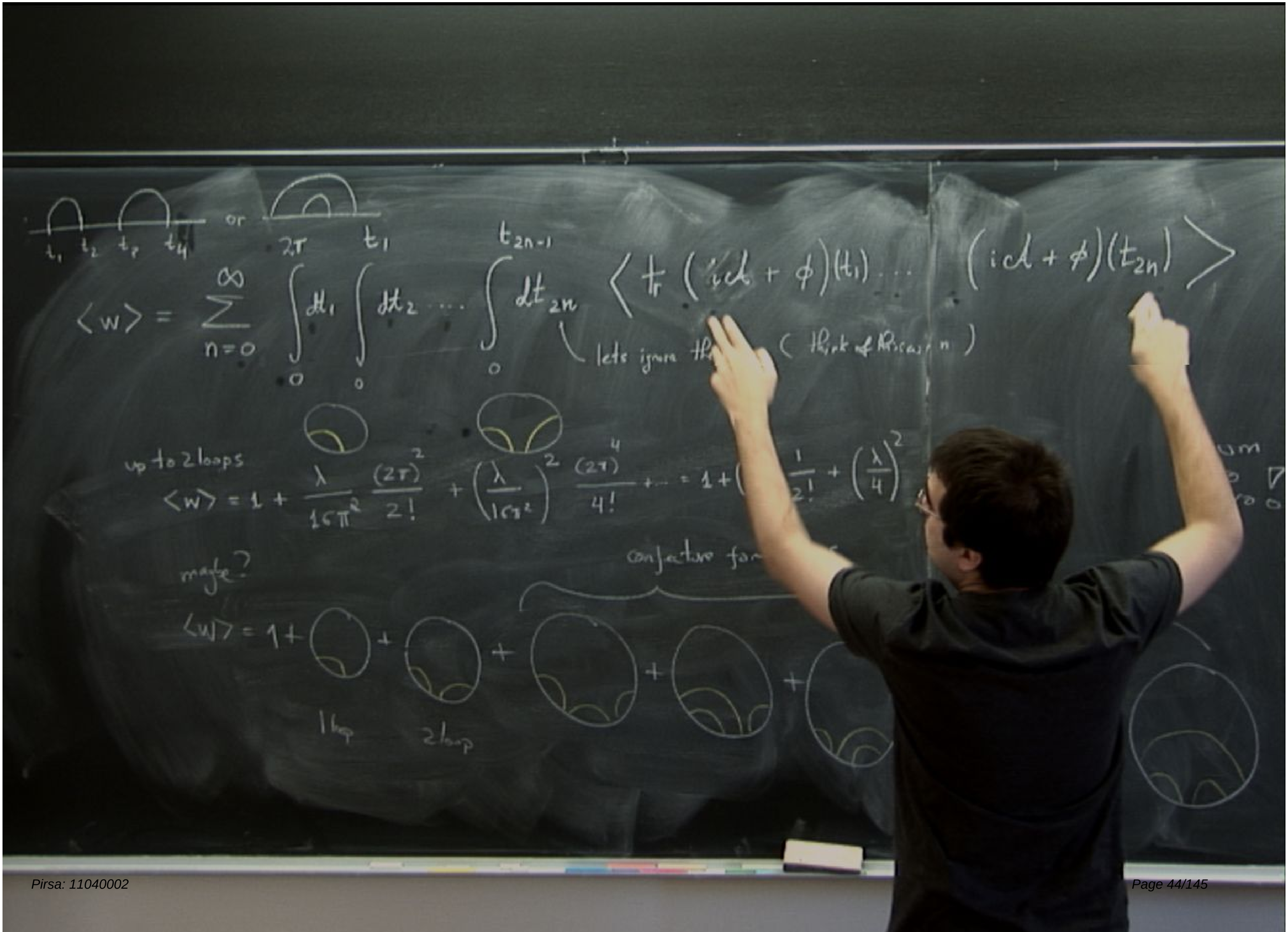
$$\mathrm{Tr}\left\langle (i\dot{A} + \phi)(t_1)\,(i\dot{A}+\phi)(t_2) \right\rangle = \frac{-\overbrace{g^2 N}^{\lambda}}{4\pi^2}\,\frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - }{|x(t_1) - x(t_2)|^2}$$

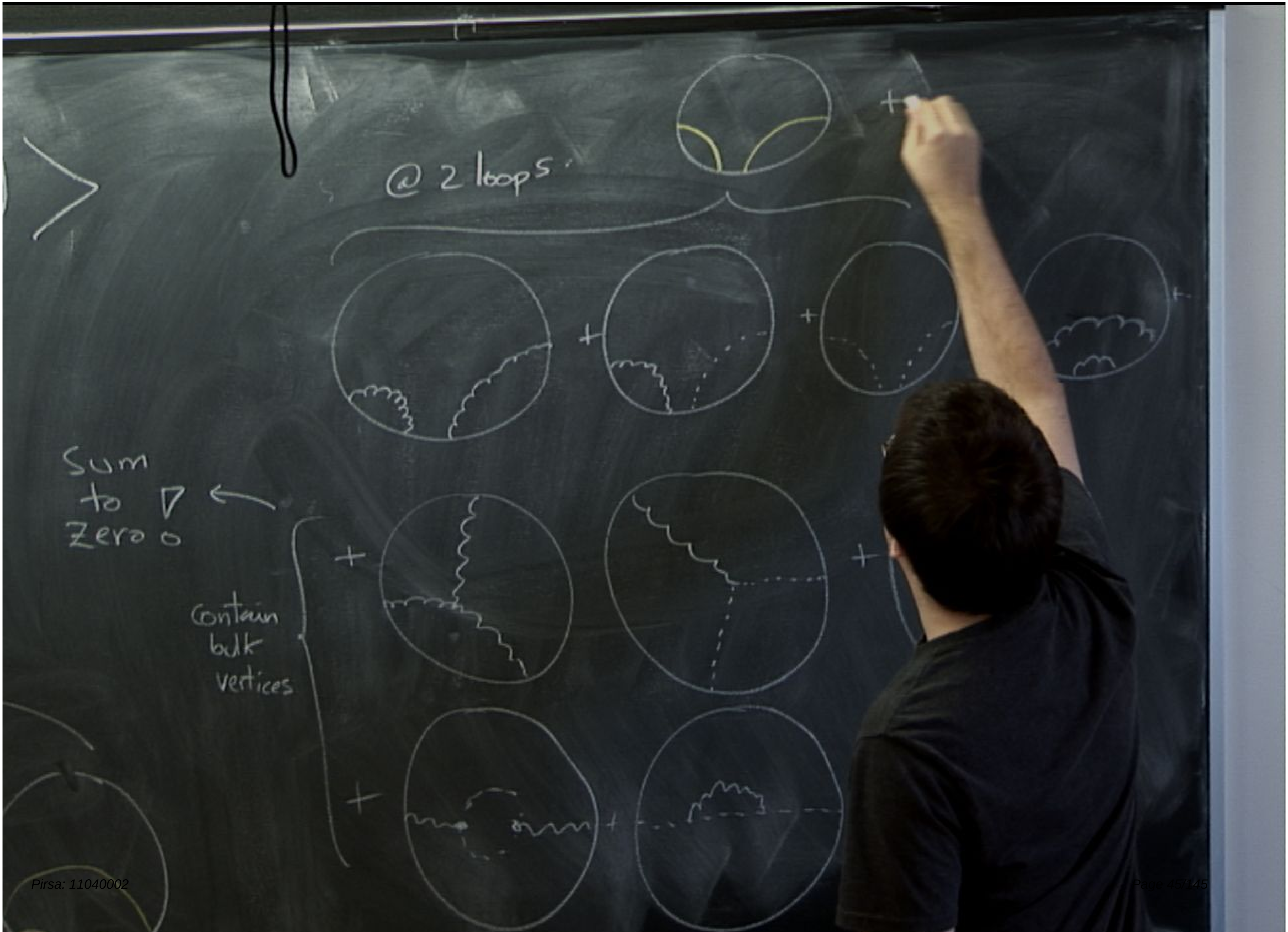
$$= \frac{\lambda}{16\pi^2} \quad \text{(a constant!)}$$

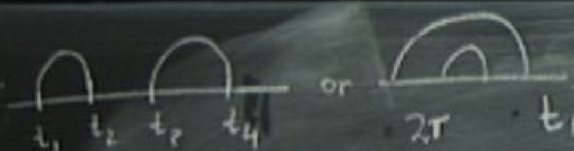

⟨w⟩ = Σ_{n=0}^{∞} ∫_0^{2π} dt_1 ∫_0^{t_1} dt_2 .... ∫_0^{t_{2n-1}} dt_{2n} ⟨ tr (iA + φ)(t_1) ...
lets ignore the 2n (think of this as n)
up to 2 loops
⟨w⟩ = 1 + λ/16π²
(2π)⁴/4! + ... = 1 + (λ/4) 1/2! + (λ/4)² 1/4!
maybe?
⟨w⟩ = 1 +


⟨W⟩ = Σ_{n=0}^{∞} ∫_0^{2π} dt_1 ∫_0^{t_1} dt_2 .... ∫_0^{t_{2n-1}} dt_{2n} ⟨ tr (id + φ)(t_1) ... (id + φ)(t_{2n}) ⟩
lets ignore the 2n (think of this as n)
up to 2 loops
⟨W⟩ = 1 + λ/16π² (2π)²/2! + (λ/16π²)² (2π)⁴/4! + ... = 1 + (λ/4) 1/2! + (λ/4)² 1/4! + ...
Sum to Zero
maybe?
conjecture for 3 loops
⟨W⟩ = 1 +
1 loop
2 loop


or
2π
t₁
t_{2n-1}
⟨w⟩ = Σ_{n=0}^{∞} ∫_0^{2π} dt₁ ∫_0 dt₂ … ∫_0 dt_{2n} ⟨ tr (iẋ + φ)(t₁) … (iẋ + φ)(t_{2n}) ⟩
lets ignore the 2n
up to 2 loops
⟨w⟩ = 1 + λ/16π² (2π)²/2! + (λ/16π²)² (2π)⁴/4! + … = 1 + (λ/4) 1/2! +
4!
maybe?
conjecture for 3 loops
⟨w⟩ = 1 +
1 loop
2 loop

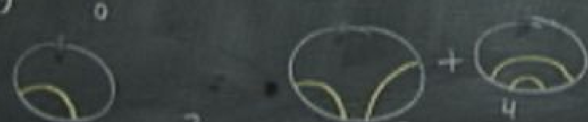

or
up to 2 loops
maybe?
conjecture for
1 loop
2 loop


@ 2 loops:
Sum
to
zero
contain
bulk
vertices



$$\langle W\rangle = \sum_{n=0}^{\infty}\int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\,(iA+\phi)(t_1)\cdots(iA+\phi)(t_{2n})\right\rangle$$

lets ignore the 2n (think of this as n)



number of rainbow diagrams

up to 2 loops

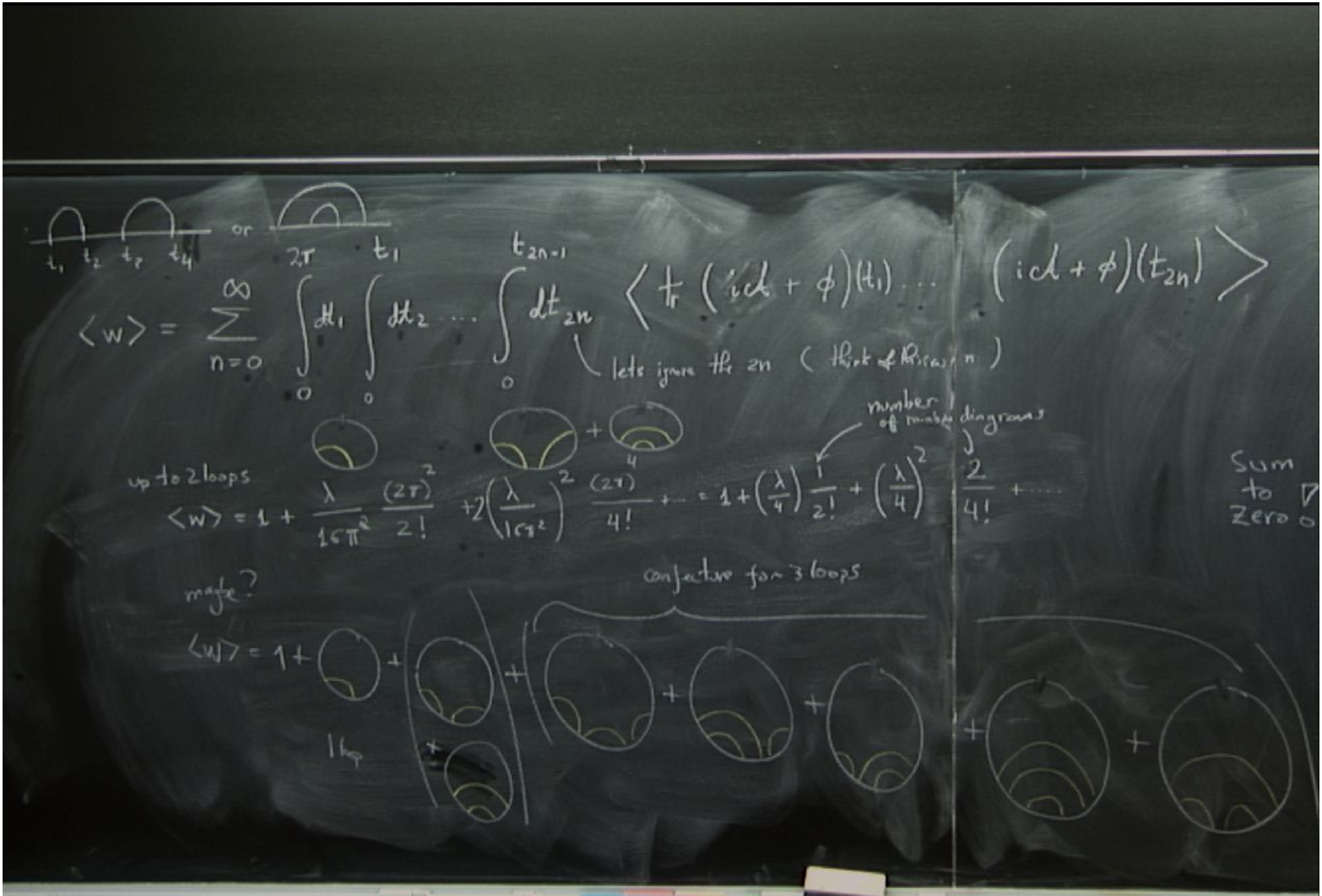
$$\langle W\rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2\frac{(2\pi)}{4!} + \dots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2\frac{2}{4!} + \dots$$

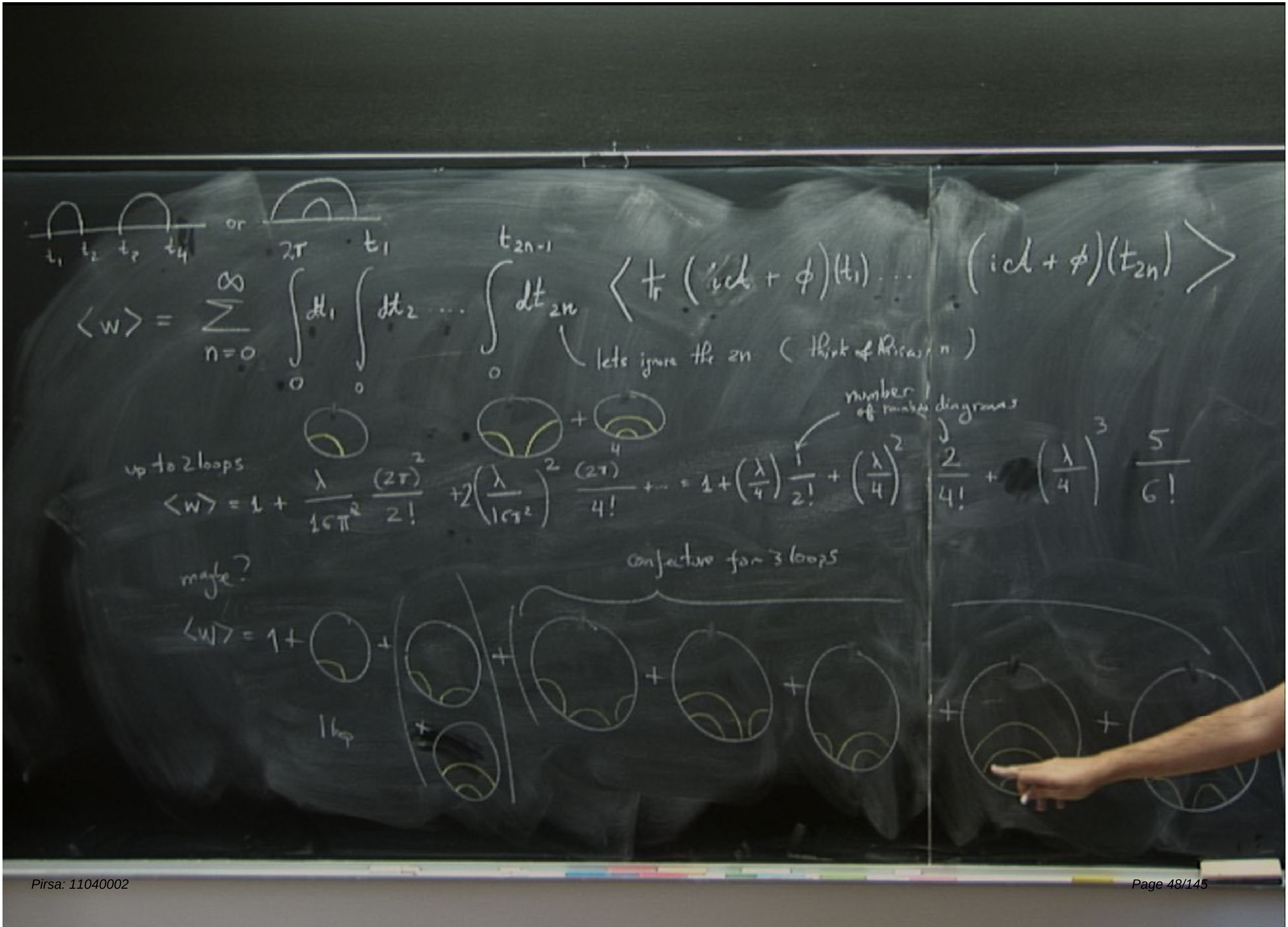
Sum to zero

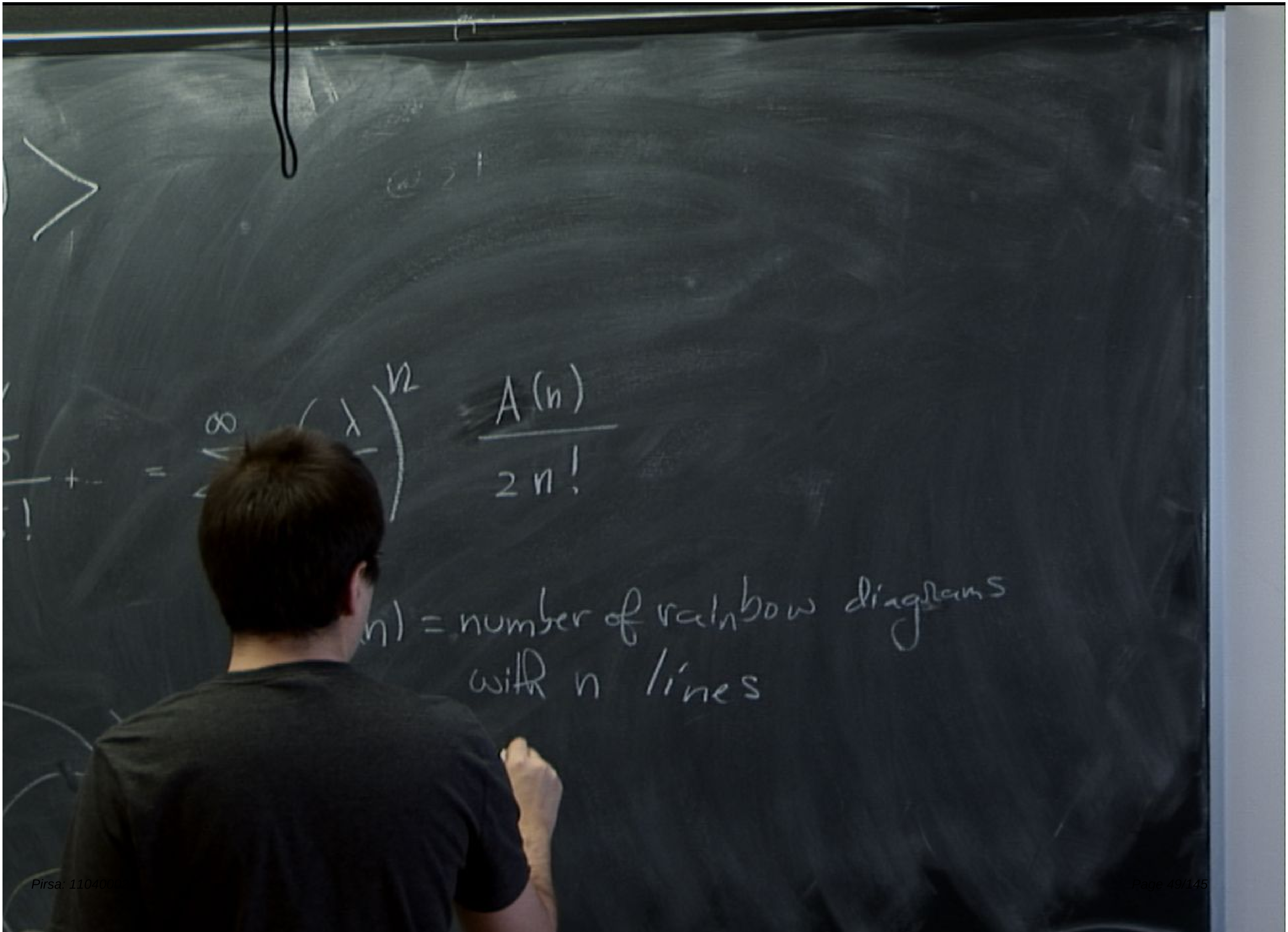
conjecture for 3 loops

maybe?

$\langle W\rangle = 1 +$


or
lets ignore the 2n
number of rainbow diagrams
up to 2 loops
Sum to Zero
maybe?
conjecture for 3 loops
1 loop


or
$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle tr\,(iA + \phi)(t_1) \cdots (iA + \phi)(t_{2n}) \rangle$
lets ignore the 2n
number of rainbow diagrams
up to 2 loops
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \cdots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2 \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!}$
conjecture for 3 loops
maybe?
$\langle W \rangle = 1 +$
1 loop


A(n)
2n!
n) = number of rainbow diagrams
with n lines

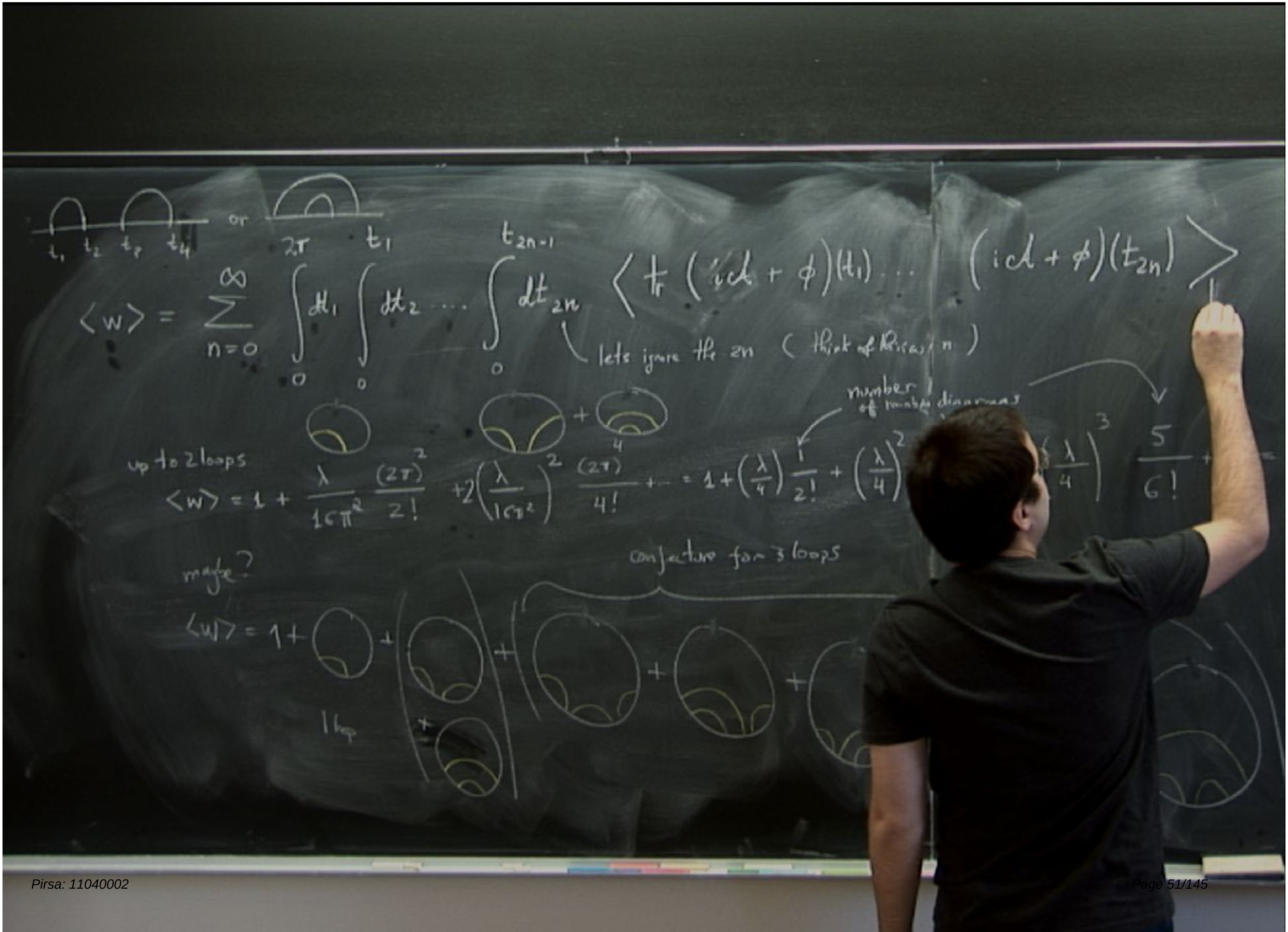
$$\ldots + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^{n} \frac{A(n)}{2n!}$$

$A(n)$ = number of rainbow diagrams with n lines

$+ \ldots$

$A(0) = 1 \qquad A(2) = 2$

$A(1) = 1 \qquad A(3) = 5 \qquad$ etc


or
lets ignore the 2n
number
up to 2 loops
maybe?
conjecture for 3 loops

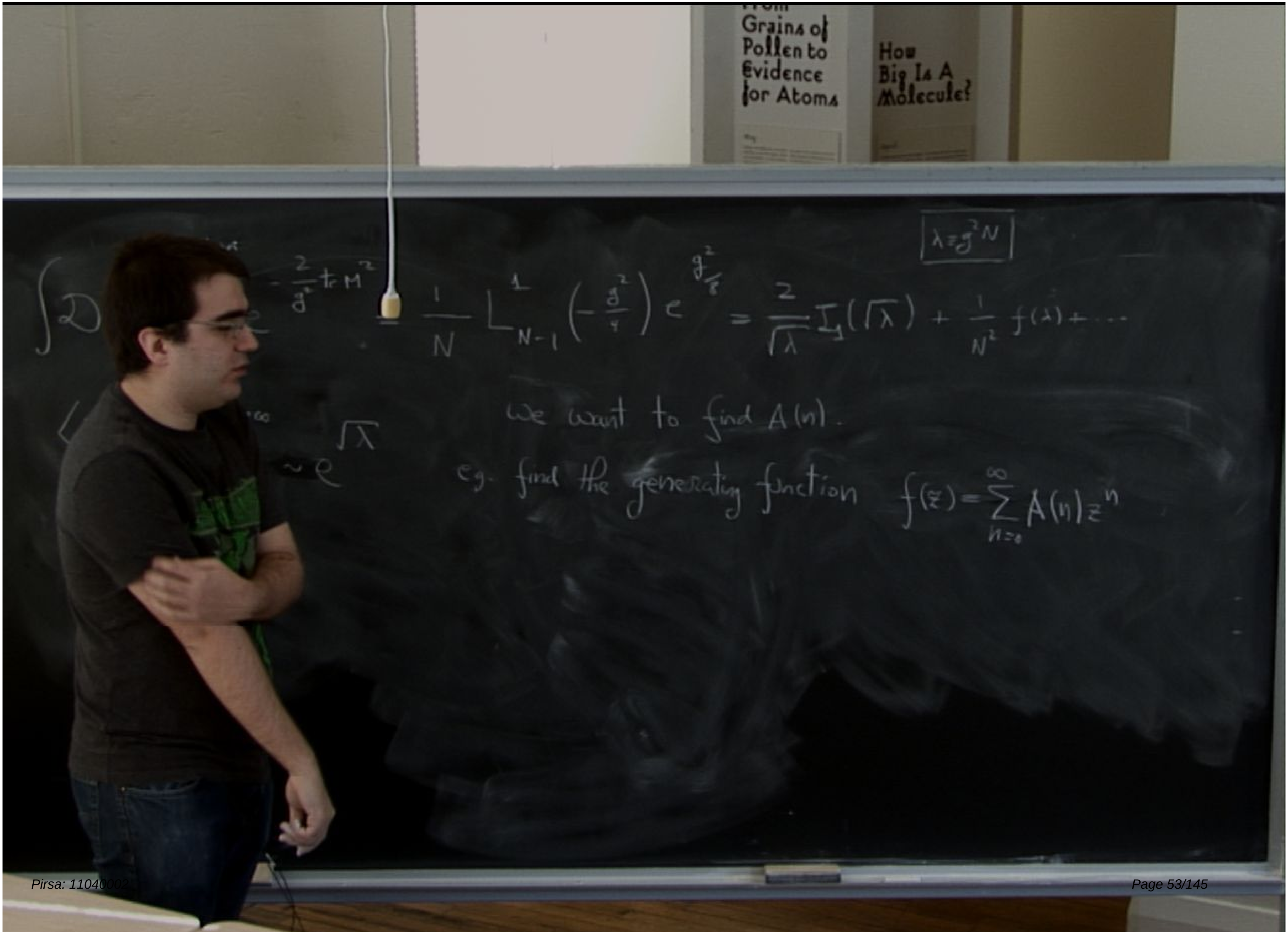
ladder

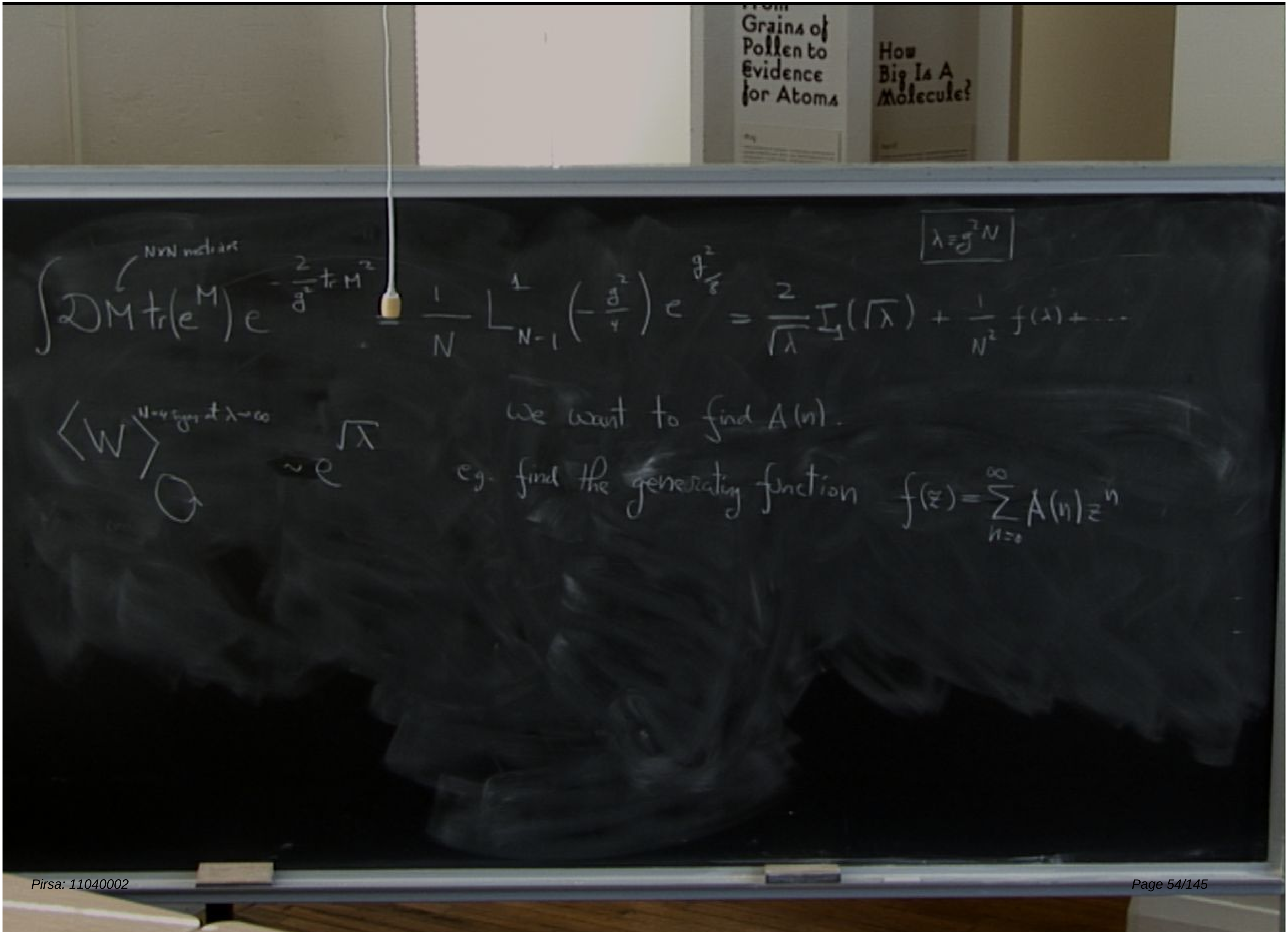
$$\ldots + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^{n} \frac{A(n)}{2n!}$$

$A(n)$ = number of rainbow diagrams with n lines

$A(0)=1$ $A(2)=2$

$+ \ldots$ $A(1)=1$ $A(3)=5$ etc


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\lambda = g^2 N$
$= \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \dots$
We want to find A(n).
e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
λ = g²N
N×N matrices
∫ DM tr(e^M) e^(−(2/g²) tr M²) = (1/N) L¹_{N−1}(−g²/4) e^(g²/8) = (2/√λ) I₁(√λ) + (1/N²) f(λ) + ...
⟨W⟩ N=4 Sym at λ→∞ ~ e^√λ
We want to find A(n).
e.g. find the generating function f(z) = Σ_{n=0}^∞ A(n) zⁿ

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Untitled-11

In[452]:=

```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=

$$\frac{2}{1 + \sqrt{1 - 4 z}}$$

In[454]:=

```
Series[f[z], {z, 0, 14}]
```

Out[454]=

$$1 + z + 2 z^2 + 5 z^3 + 14 z^4 + 42 z^5 + 132 z^6 + 429 z^7 + 1430 z^8 + 4862 z^9 + 16\,796 z^{10} + 58\,786 z^{11} + 208\,012 z^{12} + 742\,900 z^{13} + 2\,674\,440 z^{14} + O[z]^{15}$$

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Untitled-11

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f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
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[452]=

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[454]=

$$1+z+2 z^2+5 z^3+14 z^4+42 z^5+132 z^6+429 z^7+1430 z^8+4862 z^9+16796 z^{10}+58786 z^{11}+208012 z^{12}+742900 z^{13}+2674440 z^{14}+O[z]^{15}$$

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Untitled-11

[452]:=

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f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
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```
SeriesCoefficient[f[z] // Apart, {z,
```

[456]=

$$\frac{1}{2z}-\frac{\sqrt{1-4z}}{2z}$$

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Untitled-11

In[452]:=

```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=

$$\frac{2}{1+\sqrt{1-4\,z}}$$

In[455]:=

```
Series[f[z], {z, 0, 14}]
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Out[455]=

$$1+z+2\,z^2+5\,z^3+14\,z^4+42\,z^5+132\,z^6+429\,z^7+1430\,z^8+4862\,z^9+16\,796\,z^{10}+58\,786\,z^{11}+208\,012\,z^{12}+742\,900\,z^{13}+2\,674\,440\,z^{14}+O[z]^{15}$$

```
SeriesCoefficient[f[z] // Apart, {z, 0, n}]
```

Out[456]=

$$\frac{1}{2\,z}-\frac{\sqrt{1-4\,z}}{2\,z}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-11

In[452]:=

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Out[452]=

$$\frac{2}{1+\sqrt{1-4z}}$$

In[455]:=

```
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Out[455]=

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In[457]:=

```
SeriesCoefficient[f[z] // Apart, {z, 0, n}]
```

Out[457]=

$$\begin{cases} (-1)^n 2^{1+2n}\,\text{Binomial}\left[\frac{1}{2}, 1+n\right] & n > -1 \\ 0 & \text{True} \end{cases}$$

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Untitled-11

In[452]:=

```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=

$$\frac{2}{1+\sqrt{1-4z}}$$

In[455]:=

```
Series[f[z], {z, 0, 14}]
```

Out[455]=

$$1+z+2z^2+5z^3+14z^4+42z^5+132z^6+429z^7+1430z^8+4862z^9+16\,796z^{10}+58\,786z^{11}+208\,012z^{12}+742\,900z^{13}+2\,674\,440z^{14}+O[z]^{15}$$

```
Simplif|SeriesCoefficient[f[z] // Apart, {z, 0, n}]
```

Out[457]=

$$\begin{cases} (-1)^n 2^{1+2n}\,\text{Binomial}\left[\frac{1}{2}, 1+n\right] & n > -1 \\ 0 & \text{True} \end{cases}$$

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Untitled-11

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[452]=

$$\frac{2}{1+\sqrt{1-4\,z}}$$

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Series[f[z], {z, 0, 14}]
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[455]=

$$1+z+2\,z^2+5\,z^3+14\,z^4+42\,z^5+132\,z^6+429\,z^7+1430\,z^8+4862\,z^9+16\,796\,z^{10}+58\,786\,z^{11}+208\,012\,z^{12}+742\,900\,z^{13}+2\,674\,440\,z^{14}+O[z]^{15}$$

```
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
 {n >|
```

[457]=

$$\begin{cases} (-1)^n\, 2^{1+2n}\,\mathrm{Binomial}\left[\frac{1}{2},\, 1+n\right] & n > -1 \\ 0 & \mathrm{True} \end{cases}$$

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Untitled-11

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$$1+z+2z^2+5z^3+14z^4+42z^5+132z^6+429z^7+1430z^8+4862z^9+16\,796z^{10}+58\,786z^{11}+208\,012z^{12}+742\,900z^{13}+2\,674\,440z^{14}+O[z]^{15}$$

In[458]:=

```
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
 n > -1]
```

Out[458]=

$$(-1)^n\,2^{1+2n}\,\mathrm{Binomial}\left[\frac{1}{2},\,1+n\right]$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-11

In[452]:=
```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=
$$\frac{2}{1+\sqrt{1-4 z}}$$

In[455]:=
```
Series[f[z], {z, 0, 14}]
```

Out[455]=
$$1+z+2 z^2+5 z^3+14 z^4+42 z^5+132 z^6+429 z^7+1430 z^8+4862 z^9+16\,796 z^{10}+58\,786 z^{11}+208\,012 z^{12}+742\,900 z^{13}+2\,674\,440 z^{14}+O[z]^{15}$$

In[459]:=
```
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
  n > -1] // FunctionExpand
```

Out[459]=
$$\frac{(-1)^n 2^{2 n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right] \text{Gamma}[2+n]}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-11

In[452]:=
```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=
$$\frac{2}{1+\sqrt{1-4 z}}$$

In[455]:=
```
Series[f[z], {z, 0, 14}]
```

Out[455]=
$$1+z+2 z^2+5 z^3+14 z^4+42 z^5+132 z^6+429 z^7+1430 z^8+4862 z^9+16796 z^{10}+58786 z^{11}+208012 z^{12}+742900 z^{13}+2674440 z^{14}+O[z]^{15}$$

In[459]:=
```
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
  n > -1] // FunctionExpand
```

Out[459]=
$$\frac{(-1)^n 2^{2 n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right] \text{Gamma}[2+n]}$$

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Untitled-11

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```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

[452]=

$$\frac{2}{1 + \sqrt{1 - 4 z}}$$

[455]:=

```
Series[f[z], {z, 0, 14}]
```

[455]=

$$1 + z + 2 z^2 + 5 z^3 + 14 z^4 + 42 z^5 + 132 z^6 + 429 z^7 + 1430 z^8 + 4862 z^9 + 16\,796 z^{10} + 58\,786 z^{11} + 208\,012 z^{12} + 742\,900 z^{13} + 2\,674\,440 z^{14} + O[z]^{15}$$

```
A[Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
   n > -1] // FunctionExpand
```

[459]=

$$\frac{(-1)^n 2^{2 n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-11

In[452]:=
```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=

$$\frac{2}{1+\sqrt{1-4\,z}}$$

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```
Series[f[z], {z, 0, 14}]
```

Out[455]=

$$1+z+2\,z^2+5\,z^3+14\,z^4+42\,z^5+132\,z^6+429\,z^7+1430\,z^8+4862\,z^9+16\,796\,z^{10}+58\,786\,z^{11}+208\,012\,z^{12}+742\,900\,z^{13}+2\,674\,440\,z^{14}+O[z]^{15}$$

```
A[n_] =
 Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
   n > -1] // FunctionExpand
```

Out[459]=

$$(-1)^n\, 2^{2\,n}\,\sqrt{\pi}$$

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Untitled-11

In[452]:=

```
f[z_] = FindGeneratingFunction[{1, 1, 2, 5}, z]
```

Out[452]=

$$\frac{2}{1+\sqrt{1-4z}}$$

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```
Series[f[z], {z, 0, 14}]
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Out[455]=

$$1+z+2z^2+5z^3+14z^4+42z^5+132z^6+429z^7+1430z^8+4862z^9+16796z^{10}+58786z^{11}+208012z^{12}+742900z^{13}+2674440z^{14}+O[z]^{15}$$

In[460]:=

```
A[n_] =
 Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
   n > -1] // FunctionExpand
```

Out[460]=

$$\frac{(-1)^n 2^{2n}\sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right]\text{Gamma}[2+n]}$$

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Untitled-11

In[455]:=

```
Series[f[z], {z, 0, 14}]
```

Out[455]=

$$1+z+2z^2+5z^3+14z^4+42z^5+132z^6+429z^7+1430z^8+4862z^9+16796z^{10}+58786z^{11}+208012z^{12}+742900z^{13}+2674440z^{14}+O[z]^{15}$$

In[460]:=

```
A[n_] =
 Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
   n > -1] // FunctionExpand
```

Out[460]=

$$\frac{(-1)^n\, 2^{2n}\sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right]\,\text{Gamma}[2+n]}$$

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Untitled-11

In[455]:=

```
Series[f[z], {z, 0, 14}]
```

Out[455]=

$$1+z+2z^2+5z^3+14z^4+42z^5+132z^6+429z^7+1430z^8+4862z^9+16\,796\,z^{10}+58\,786\,z^{11}+208\,012\,z^{12}+742\,900\,z^{13}+2\,674\,440\,z^{14}+O[z]^{15}$$

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A[n_] =
 Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
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```
A
```

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Out[460]=

$$\frac{(-1)^n 2^{2n}\sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right]\text{Gamma}[2+n]}$$

In[461]:=

```
A /@ Range[0, 10]
```


Out[461]=

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Untitled-11

```
A[n_] =
 Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
   n > -1] // FunctionExpand
```

[460]=

$$\frac{(-1)^n\, 2^{2 n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

[461]:=

```
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```

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{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16 796}

[454]=

$$1 + z + 2 z^2 + 5 z^3 + 14 z^4 + 42 z^5 + 132 z^6 + 429 z^7 + 1430 z^8 + 4862 z^9 + 16\,796 z^{10} + 58\,786 z^{11} + 208\,012 z^{12} + 742\,900 z^{13} + 2\,674\,440 z^{14} + O[z]^{15}$$

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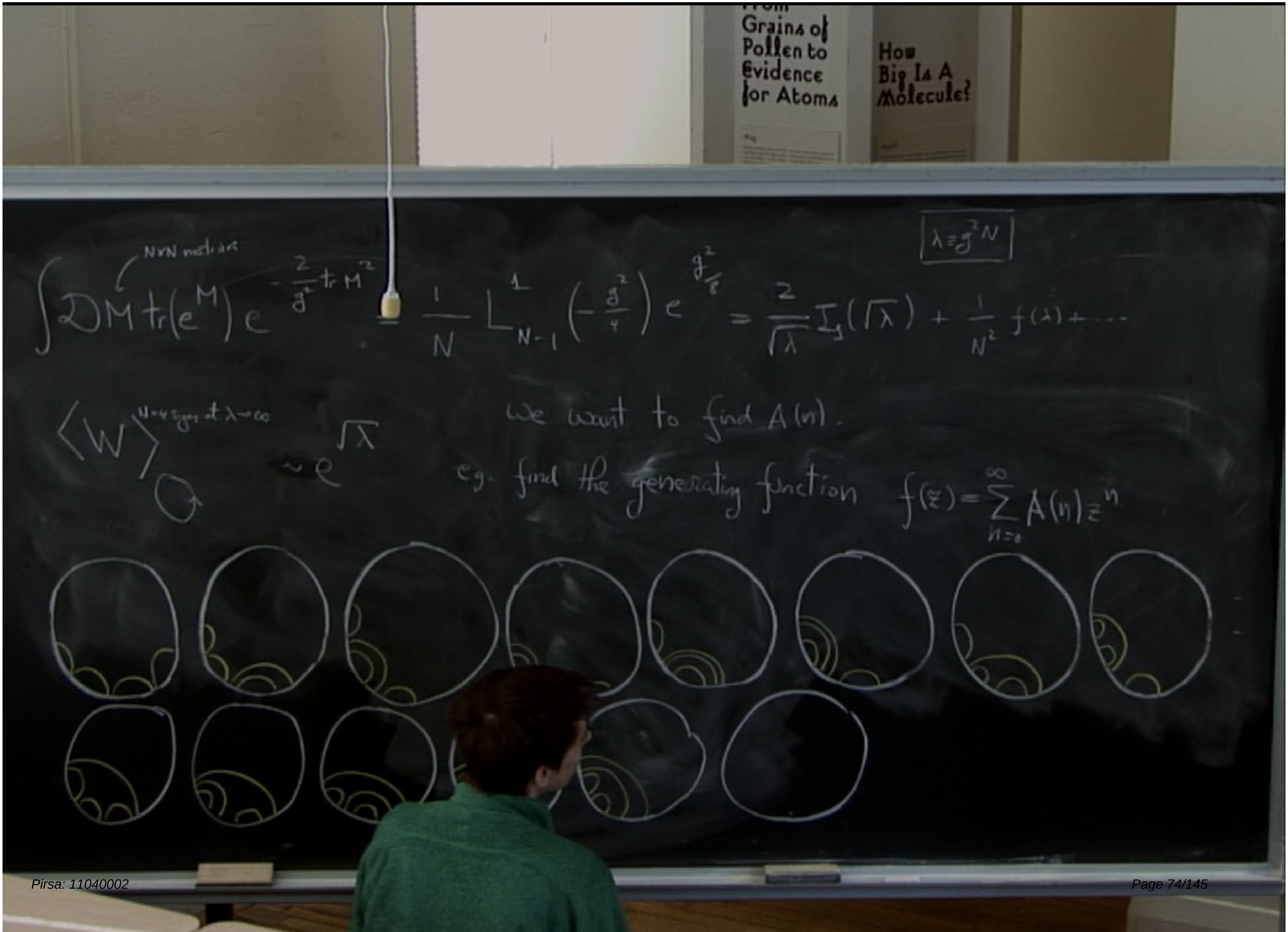
Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\lambda = g^2 N$
N×N matrices
$\int \mathcal{D}M \, \mathrm{tr}(e^M) \, e^{-\frac{2}{g^2}\mathrm{tr}\, M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{\frac{g^2}{8}} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \dots$
N=4 SYM at $\lambda \to \infty$
$\langle W \rangle \sim e^{\sqrt{\lambda}}$
We want to find A(n).
e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$

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```
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A /@ Range[0, 10]
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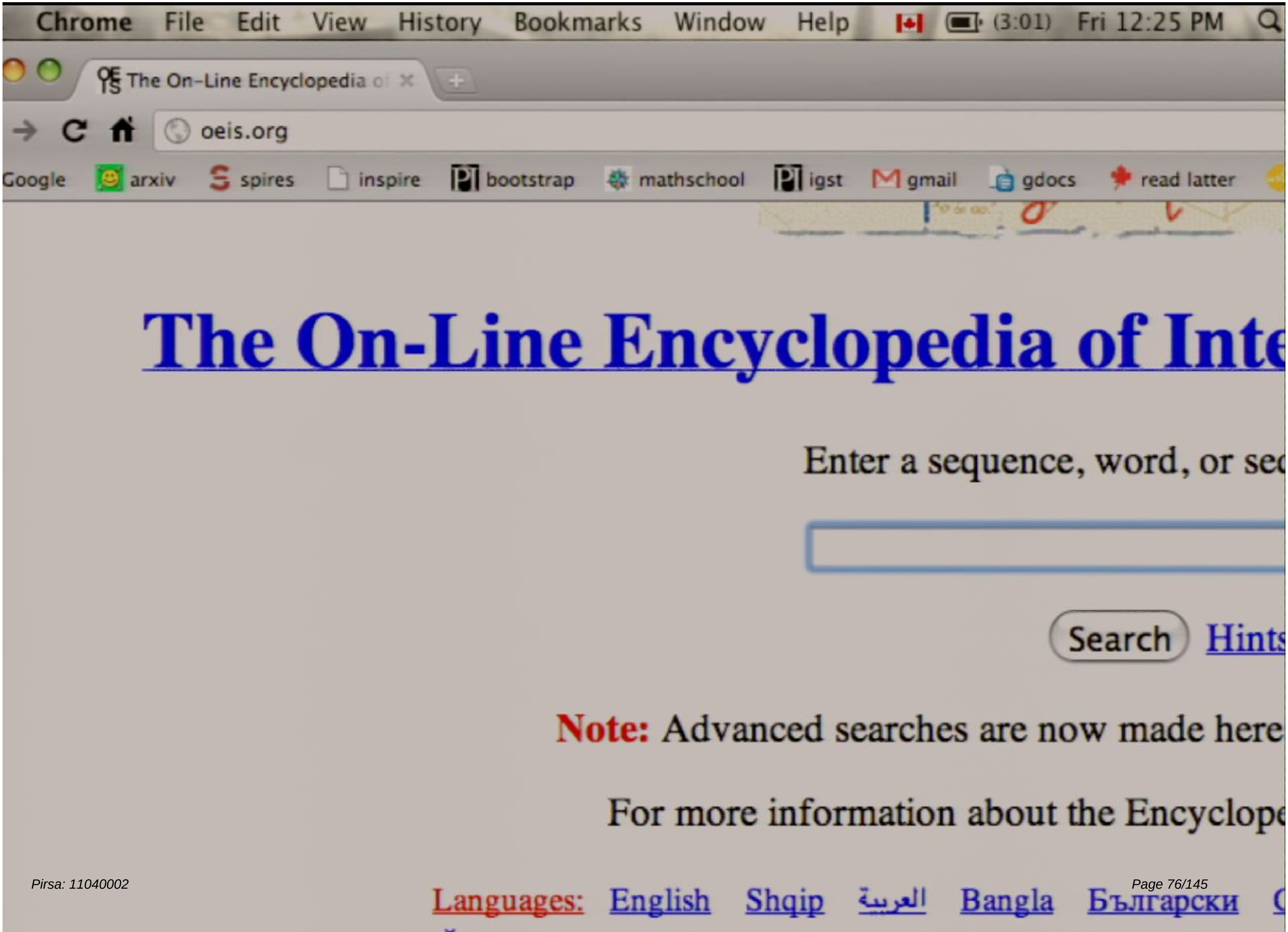
[461]=

{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}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1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, \ 742900, 2674440
1, 2, 5, 14
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1,1,2,5,14
1,1,2,5,14,42

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000108 Catalan numbers: C(n) = binomial(2n,n)/(n+1) = (2n)!/(n!(n+1)!). Also called Segner numbers. +20 1918

(Formerly M1459 N0577)

1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324 (list; graph; listen; history; internal format)

OFFSET 0,3

COMMENTS The solution to Schroeder's first problem. A very large number of combinatorial interpretations are known - see references, esp. Stanley, Enumerative Combinatorics, Volume 2.

Number of ways to insert n pairs of parentheses in a word of n+1 letters. E.g. for n=3 there are 5 ways: ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).

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Untitled-11

```
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
  n > -1] // FunctionExpand
```

[460]=

$$\frac{(-1)^n\, 2^{2n}\, \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

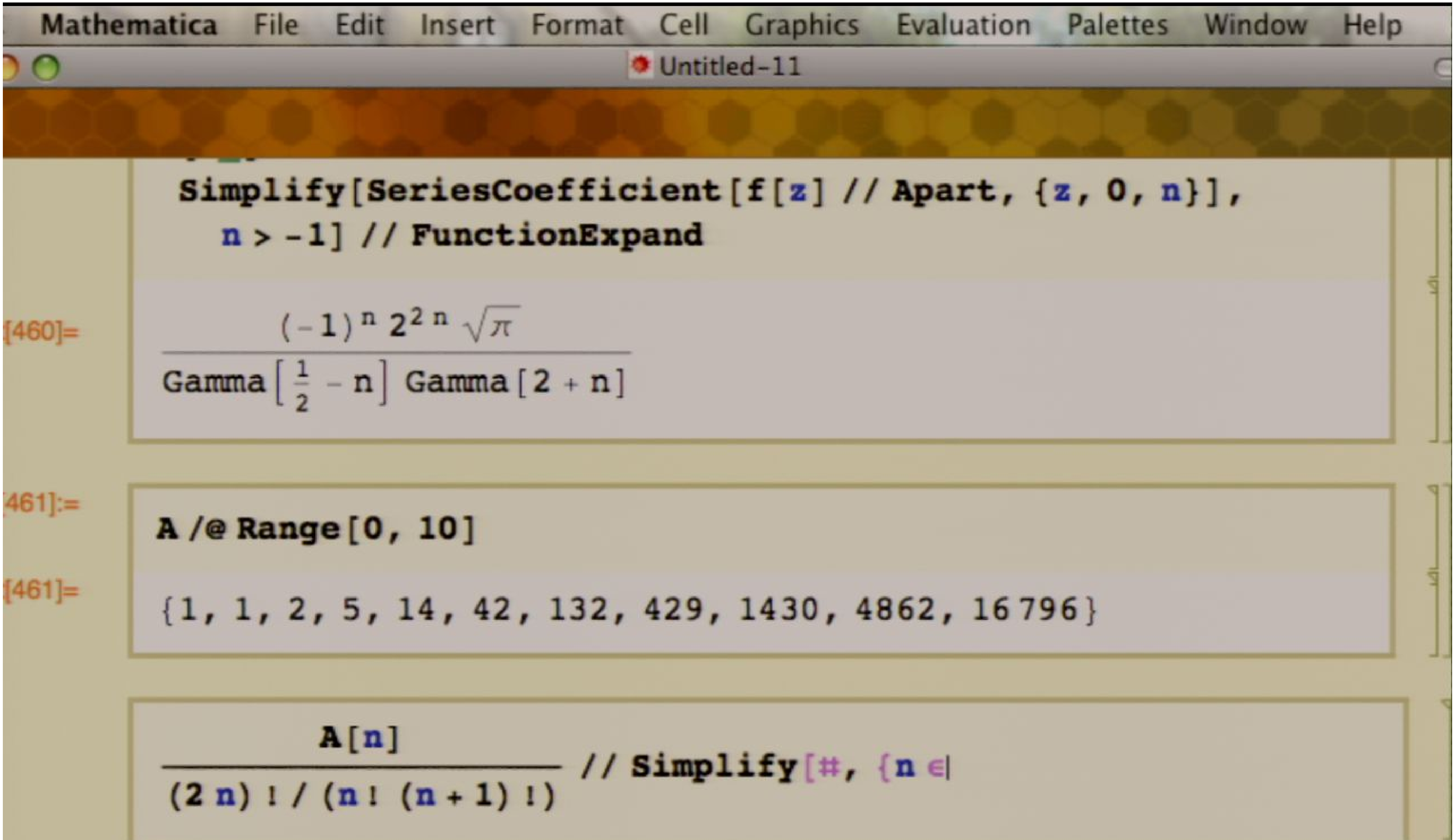
461]:=

```
A /@ Range[0, 10]
```

[461]=

{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}

```
(2 n) ! / (n ! (n + 1) ! )
```



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Untitled-11
Simplify[SeriesCoefficient[f[z] // Apart, {z, 0, n}],
n > -1] // FunctionExpand
[460]=
(-1)^n 2^(2 n) √π / (Gamma[1/2 - n] Gamma[2 + n])
[461]:=
A /@ Range[0, 10]
[461]=
{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}
A[n] / ((2 n)! / (n! (n + 1)!)) // Simplify[#, {n ∈

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Untitled-11

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A /@ Range[0, 10]
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{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16 796}

462]:=

$$\frac{\text{A[n]}}{\text{(2 n) ! / (n ! (n + 1) !)}}$$ // Simplify[#, {n ∈ Integer}] &

Element::bset :
The second argument Integer of Element should be one of: Primes, Integers, Rationals, Algebraics, Reals, Complexes, or Booleans. >>

[462]=

$$\frac{(-4)^n \sqrt{\pi}\, n!\, (1 + n)!}{(2 n)!\, \text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

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```
n > -1] // FunctionExpand
```

[460]=

$$\frac{(-1)^{n}\, 2^{2\,n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2}-n\right]\, \text{Gamma}[2+n]}$$

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A /@ Range[0, 10]
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[461]=

{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16 796}

463]:=

$$\frac{\text{A}[n]}{(2\,n)\,!\,/\,(n\,!\,(n+1)\,!)}$$ // Simplify[#, {n ∈ Integers}] &

[463]=

$$\frac{(-4)^{n} \sqrt{\pi}\, n\,!\, (1+n)\,!}{(2\,n)\,!\, \text{Gamma}\left[\frac{1}{2}-n\right]\, \text{Gamma}[2+n]}$$

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```
n > -1] // FunctionExpand
```

[460]=

$$\frac{(-1)^n\, 2^{2n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

461]:=

```
A /@ Range[0, 10]
```

[461]=

```
{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}
```

464]:=

$$\frac{\text{A}[n]}{(2\,n)!\,/\,(n!\,(n+1)!)}$$ `// FullSimplify[#, {n ∈ Integers}] &`

[464]=

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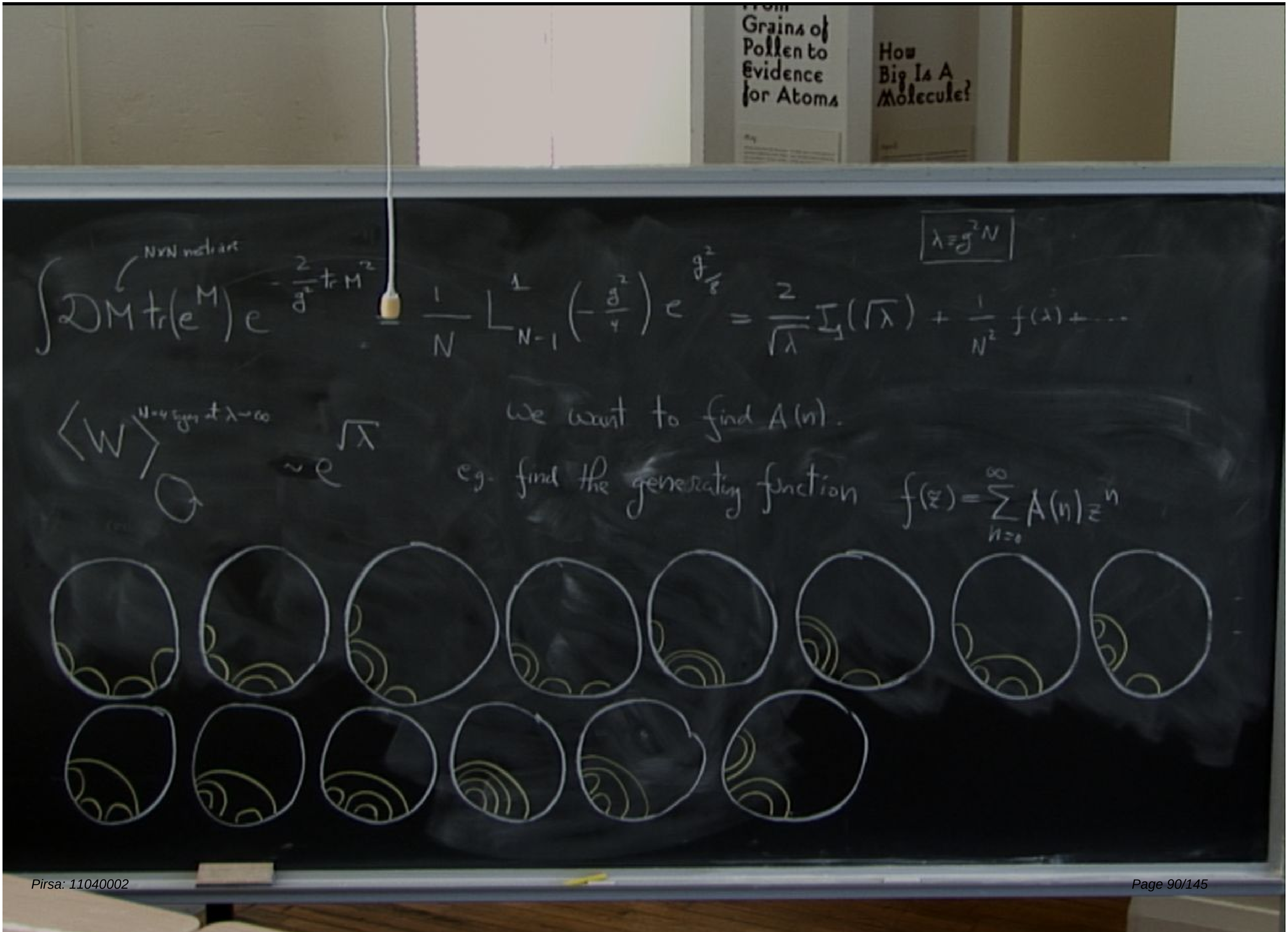
1
```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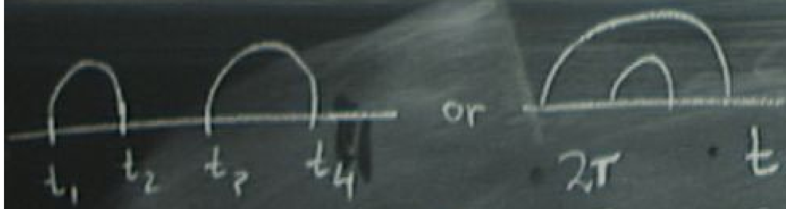


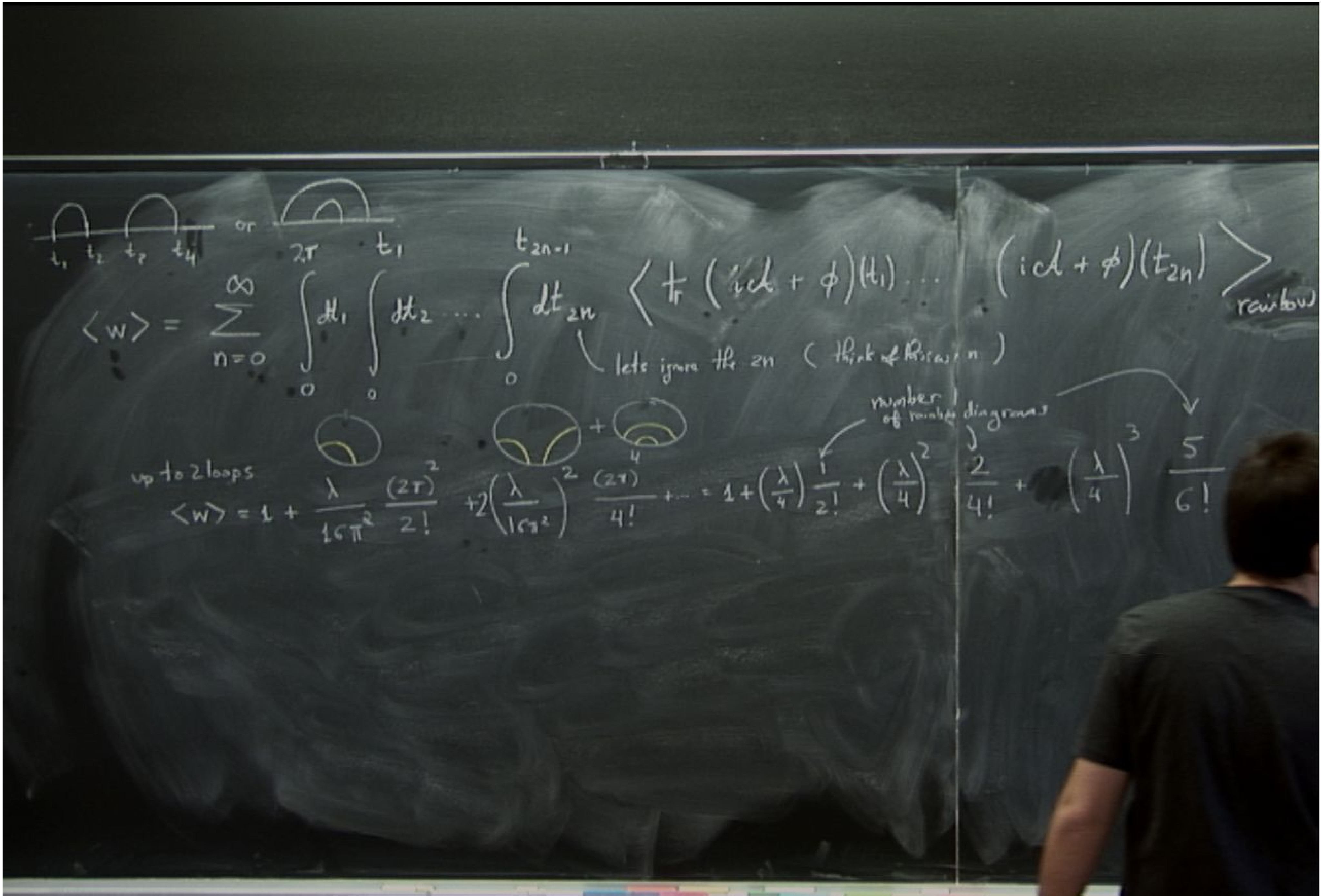
$$\langle W\rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\,(iA + \phi)(t_1) \cdots \right.$$

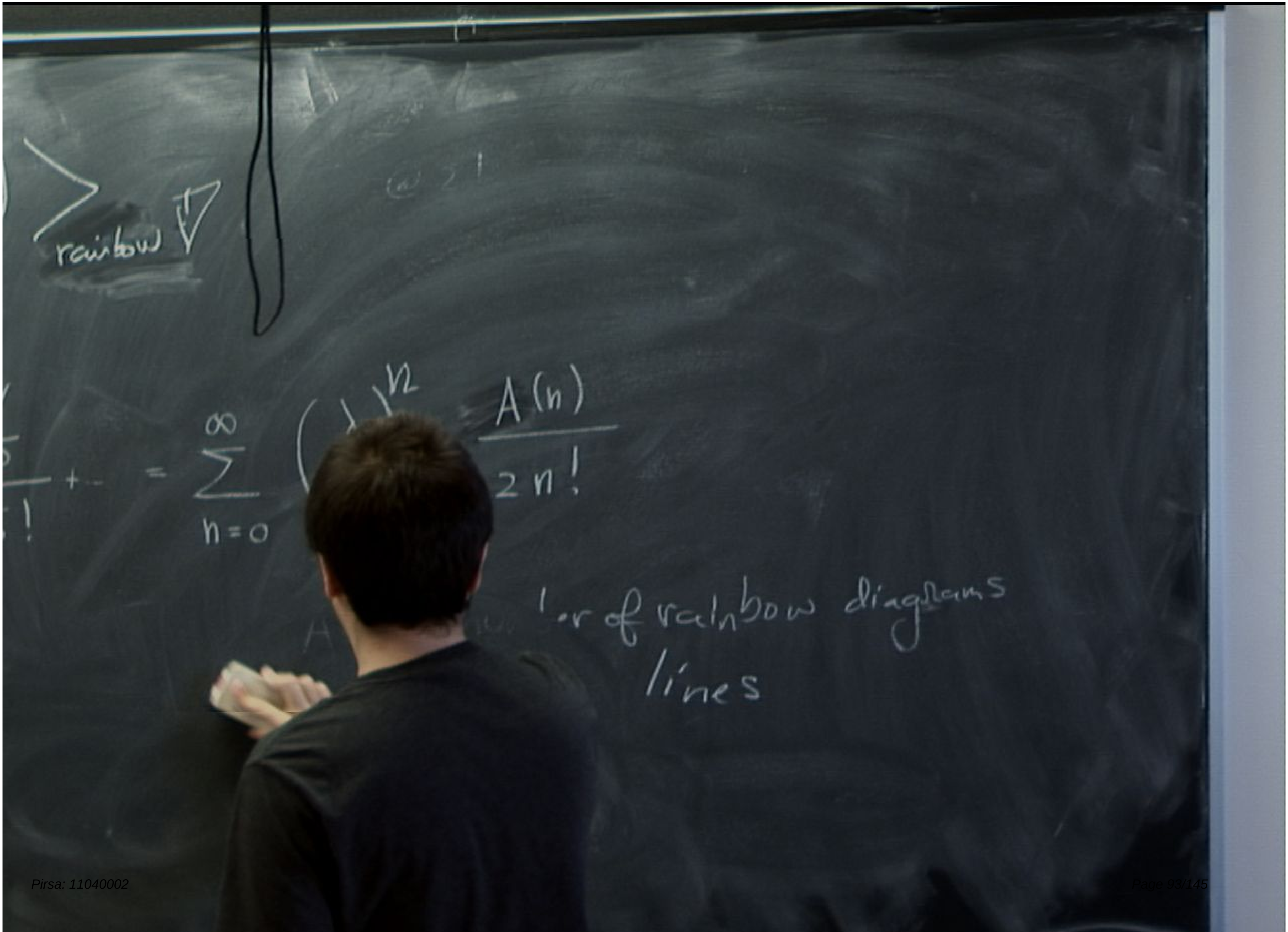
lets ignore the 2n (think of this as

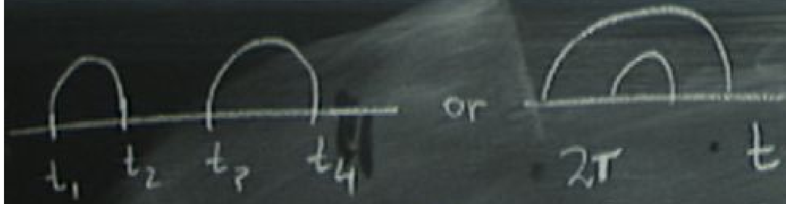
number of rainbow d

up to 2 loops

$$\langle W\rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \dots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$


or
lets ignore the 2n
up to 2 loops
number of rainbow diagrams
rainbow
$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr}(iA + \phi)(t_1) \cdots (iA + \phi)(t_{2n}) \rangle_{\text{rainbow}}$
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2\frac{(2\pi)^4}{4!} + \dots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3\frac{5}{6!}$


rainbow
A(n)
2n!
n=0
of rainbow diagrams
lines

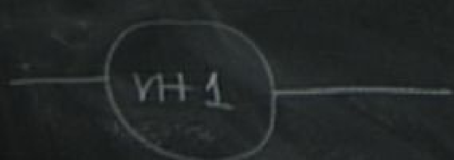


$$\langle W\rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(iA + \phi\right)(t_1) \cdots \right.$$

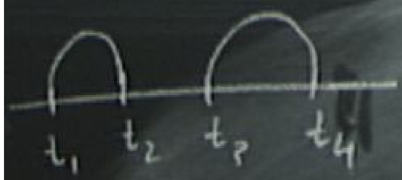
lets ignore the 2n (think of this as

up to 2 loops

$$\langle W\rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2\frac{(2\pi)^4}{4!} + \dots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$



$A(n+1)$



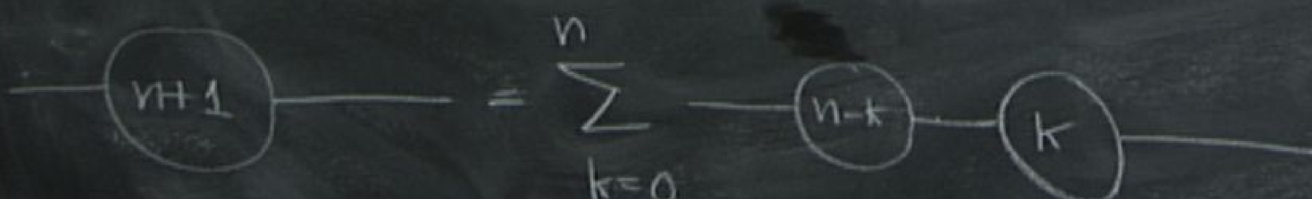
or

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(i\dot{x} + \phi \right)(t_1) \cdots \right.$$

lets ignore the 2n (think of this as ...

up to 2 loops

$$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \dots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$



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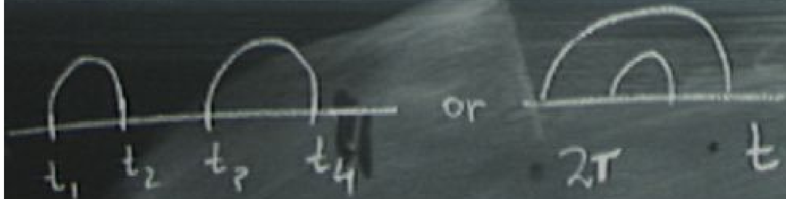

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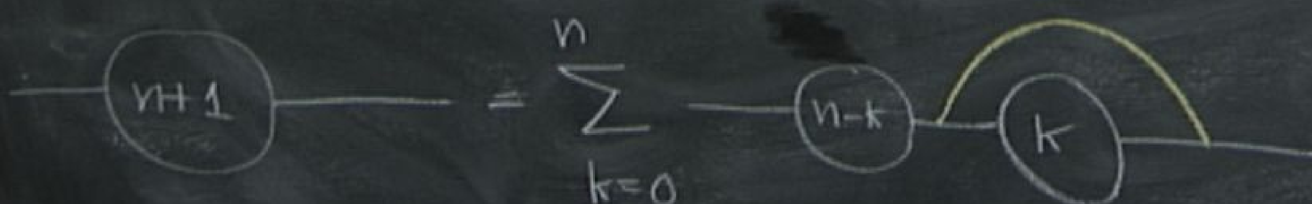

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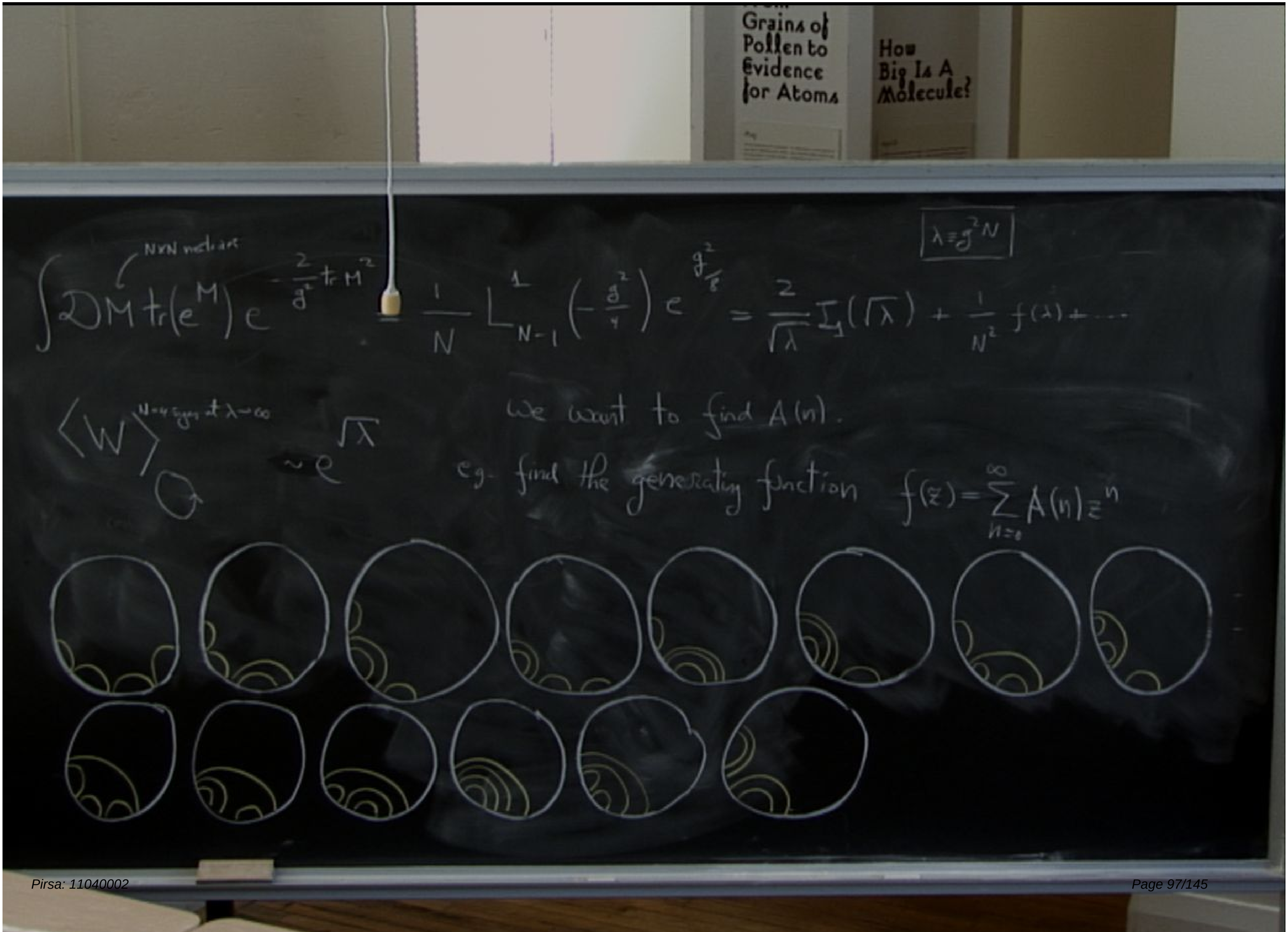
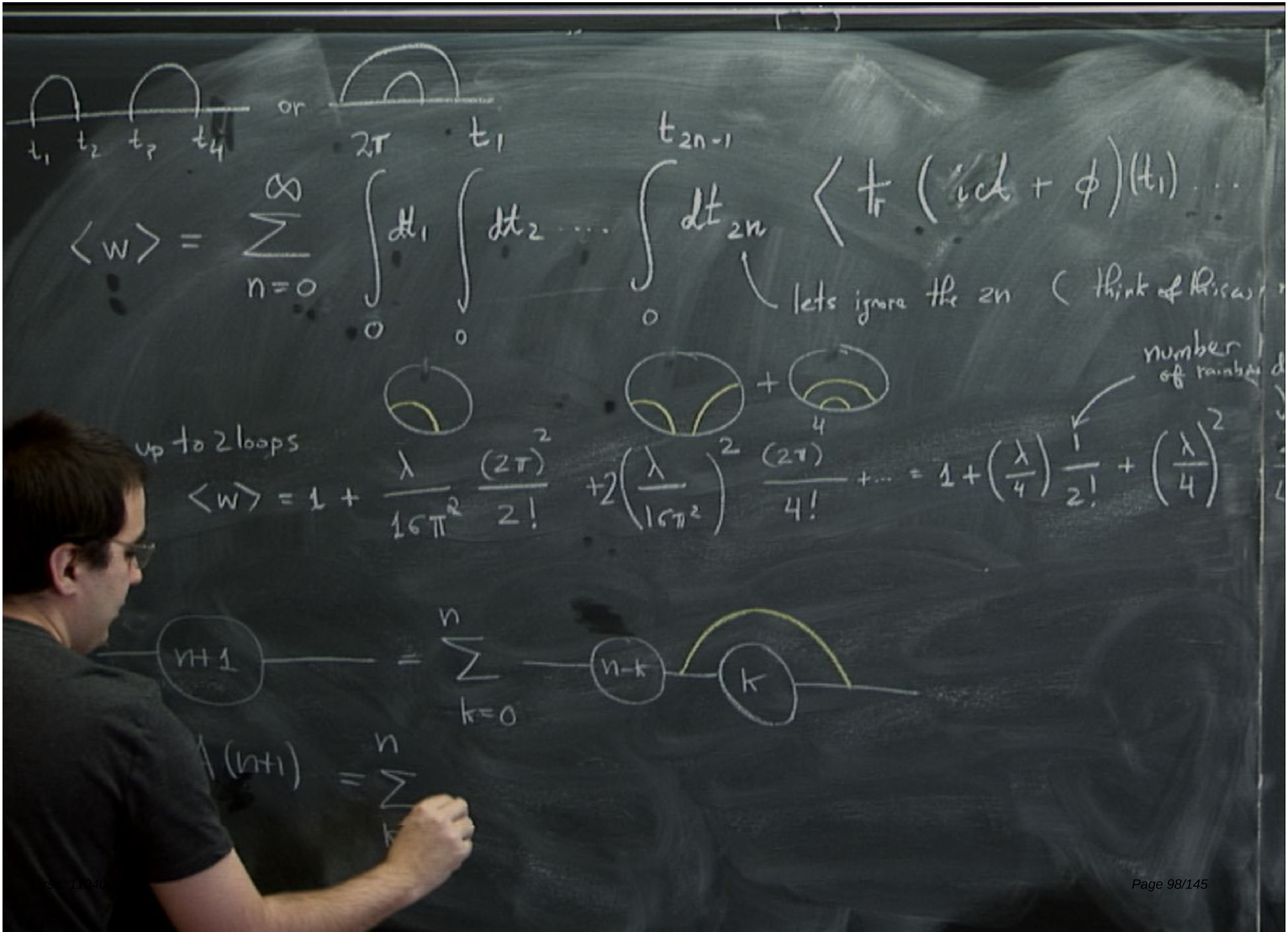
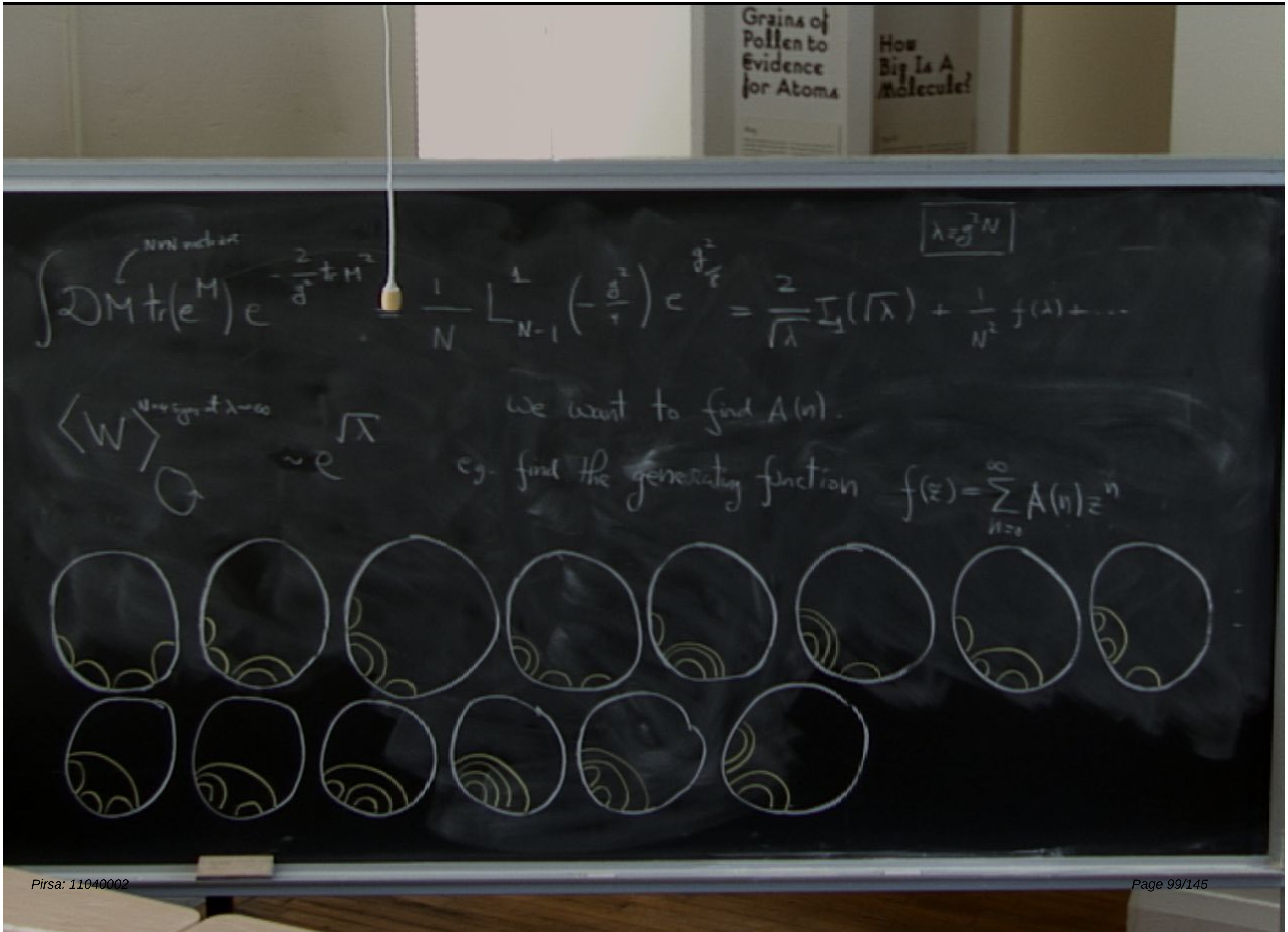

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
λ ≡ g²N
N×N matrices
∫DM tr(e^M) e^(−(2/g²) tr M²) = (1/N) L¹_(N−1)(−g²/4) e^(g²/8) = (2/√λ) I₁(√λ) + (1/N²) f(λ) + ...
⟨W⟩ ~ e^(√λ)
N=4 Sym at λ→∞
We want to find A(n).
e.g. find the generating function f(z) = Σ_(n=0)^∞ A(n) zⁿ



t_1 t_2 t_3 t_4 or
$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \langle \mathrm{tr}\,(iA + \phi)(t_1) \cdots$
lets ignore the 2n (think of this as
number of rainbow
up to 2 loops
$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \ldots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$
n+1 $= \sum_{k=0}^{n}$ n-k k
$A(n+1) = \sum^{n}$


Grains of Pollen to Evidence for Atoms
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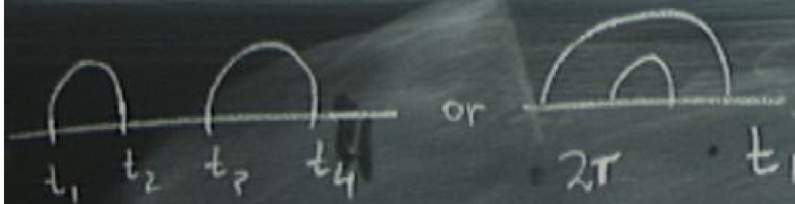
N×N matrix
$\lambda = g^2 N$
$\int \mathcal{D}M \, \mathrm{tr}(e^M) \, e^{-\frac{2}{g^2}\mathrm{tr} M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{g^2/8} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \ldots$
$\langle W \rangle \sim e^{\sqrt{\lambda}}$
We want to find A(n).
e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$



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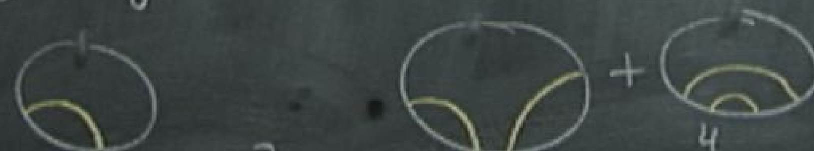
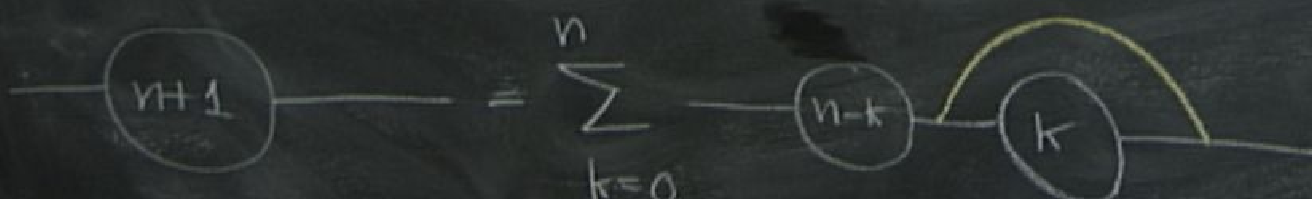


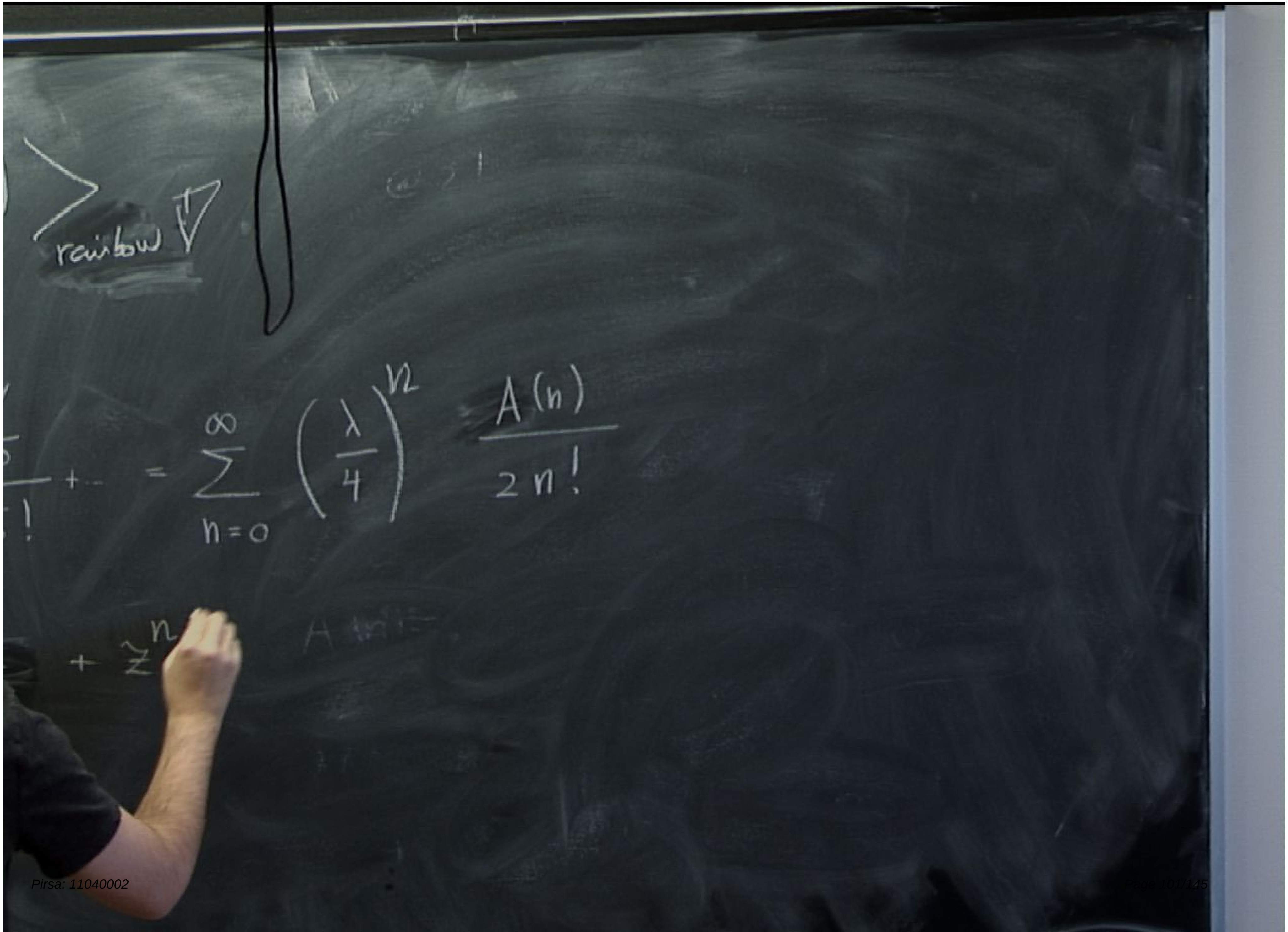
number of rainbow d…

up to 2 loops

$$\langle W \rangle = 1 + \frac{\lambda}{16\pi^2}\frac{(2\pi)^2}{2!} + 2\left(\frac{\lambda}{16\pi^2}\right)^2 \frac{(2\pi)^4}{4!} + \cdots = 1 + \left(\frac{\lambda}{4}\right)\frac{1}{2!} + \left(\frac{\lambda}{4}\right)^2$$



$$A(n+1) = \sum_{k=0}^{n} A(n-k)\, A(k)$$


rainbow
$= \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^{n} \frac{A(n)}{2n!}$
$+ z^{n}$

rainbow

$$+ \cdots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^{n} \frac{A(n)}{2n!}$$

$$+ \; z^{n} \left(\sum_{k=0}^{n} A(k) A(n-k) \right) + \cdots$$

$)(t_1) \ldots (i\partial + \phi)(t_{2n}) \rangle_{\text{rainbow}}$ ∇

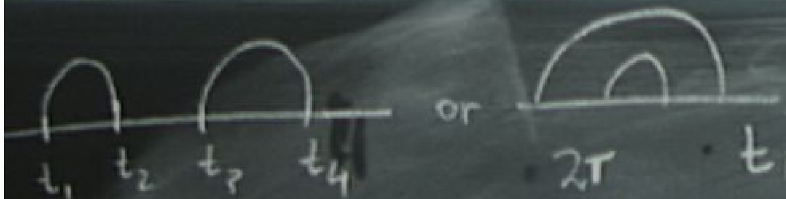
think of this as n)

number of rainbow diagrams

$$\ldots + \left(\frac{\lambda}{4}\right)^2 \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!}$$

$$f^2(z) = \ldots + z^n \left(\sum_{k=0}^{n} A(k)A(n-k) \right) + \ldots$$

$$f(z)$$



$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr} \left(i\dot{x} + \phi \right)(t_1) \cdots \right.$$

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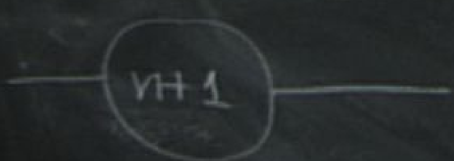
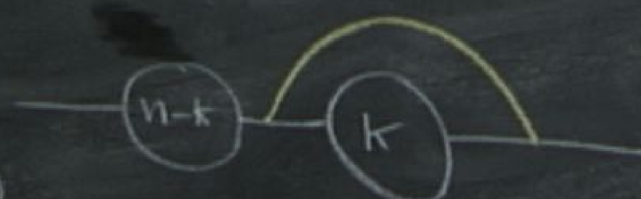
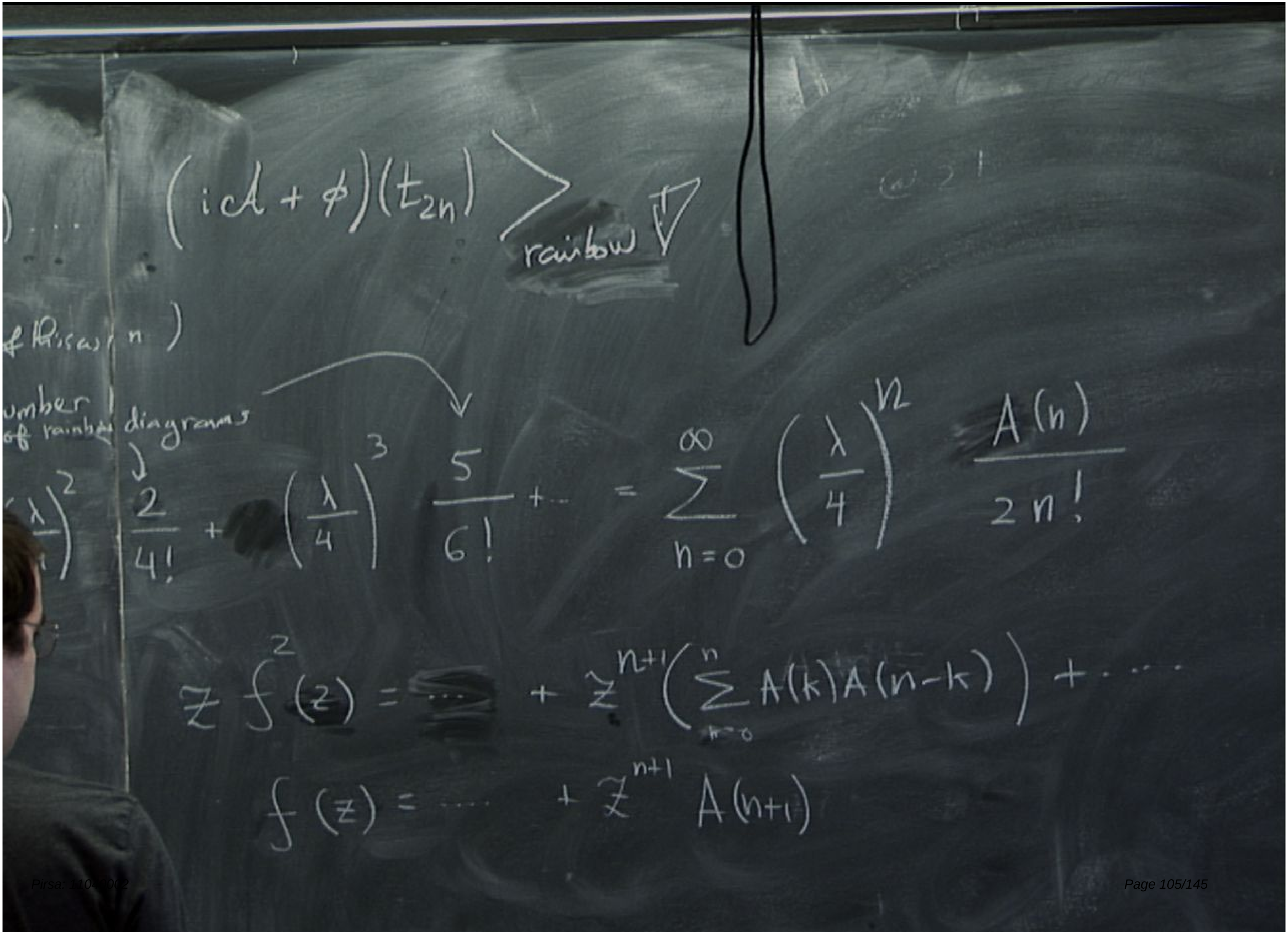


$$= \sum_{k=0}^{n}$$



$$A(n+1) = \sum_{k=0}^{n} A(n-k)\, A(k)$$


(icd + φ)(t2n)
rainbow
number of rainbow diagrams
2/4! + (λ/4)^3 5/6! + ... = Σ_{n=0}^∞ (λ/4)^n A(n)/2n!
z f^2(z) = ... + z^{n+1} (Σ_{k=0}^n A(k)A(n-k)) + ...
f(z) = ... + z^{n+1} A(n+1)

$$)(t_1) \ldots (i\,\partial + \phi)(t_{2n}) \Big\rangle_{\text{rainbow}}$$

think of this as ... n)

number of rainbow diagrams

$$\ldots + \left(\frac{\lambda}{4}\right)^2 \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!}$$

$$z f^2(z) = \ldots + z^{n+1}\left(\sum_{k=0}^{n} A(k)A(n-k)\right) + \ldots$$

$$f(z) = 1 + z + \ldots + z^{n+1} A(n+1)$$

$t_1) \ldots \quad (i\partial\!\!\!/ + \phi)(t_{2n}) \sum_{\text{rainbow}} \nabla$

k of this as n)

number of rainbow diagrams

$$+ \left(\frac{\lambda}{4}\right)^2 \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!}$$

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$$\boxed{z f^2(z) = f(z) - 1}$$

$(id + \phi)(t_{2n}) \sum_{\text{rainbow}} \Gamma$

$n)$

diagrams

$\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \dots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!}$

$A(n)$, Catalan numbers.

$z f^2(z) = 1 + \dots + z^{n+1}\left(\sum_{k=0}^{n} A(k)A(n-k)\right) + \dots$

$f(z) = 1 + z + \dots + z^{n+1} A(n+1)$

$$\boxed{z f^2(z) = f(z) - 1}$$

$\Longrightarrow \quad f(z) = \frac{1 - \sqrt{1-4z}}{2z} = \sum_{n=0}^{\infty} \frac{(2n)!}{n!(n+1)!} z^n$

$$(id + \phi)(t_{2n}) \sum_{\text{rainbow}} \nabla$$

$n\,)$

diagrams

$$\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \dots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!}$$

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1,1,2,5,14 - OEIS

oeis.org/search?q=1%2C1%2C2%2C5%2C14&sort=&language=eng...

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A000108 Catalan numbers: C(n) = binomial(2n,n)/(n+1) = (2n)!/(n!(n+1)!). Also called Segner numbers. +20 1918

(Formerly M1459 N0577)

1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324 (list; graph; listen; history; internal format)

OFFSET 0,3

COMMENTS

The solution to Schroeder's first problem. A very large number of combinatorial interpretations are known - see references, esp. Stanley, Enumerative Combinatorics, Volume 2.

Number of ways to insert n pairs of parentheses in a word of n+1 letters. E.g. for n=3 there are 5 ways: ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).

Consider all the binomial(2n,n) paths on squared paper that (i) start at (0, 0), (ii) end at (2n, 0) and (iii) at each step, either make a (+1,+1) step or a (+1,-1) step. Then the number of such paths which never go never below the x-axis (Dyck paths) is C(n) [Chung-Feller]

a(n) is the number of ordered rooted trees with n nodes, not including the root. See the Conway-Guy reference where these rooted ordered trees are called plane bushes. See also the Bergeron et al. reference, Example 4, p. 167. W. Lang Aug 07 2007.

Shifts one place left when convolved with itself.

For n >= 1 a(n) is also the number of rooted bicolored unicellular maps of genus 0 on n edges. - Ahmed Fares (ahmedfares(AT)my-deja.com), Aug 15 2001

Ways of joining 2n points on a circle to form n nonintersecting chords. (If no such restriction imposed, then ways of forming n chords is given by (2n-1)!!=(2n)!/n!2^n=A001147(n).)

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Arises in Schubert calculus - see Sottile reference.

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Untitled-11

```
n > -1] // FunctionExpand
```

[460]=

$$\frac{(-1)^n 2^{2n} \sqrt{\pi}}{\text{Gamma}\left[\frac{1}{2} - n\right] \text{Gamma}[2 + n]}$$

461]:=

```
A /@ Range[0, 10]
```

[461]=

{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16 796}

464]:=

$$\frac{A[n]}{(2n)!/(n!\,(n+1)!)}$$ // FullSimplify[#, {n ∈ Integers}] &

[464]=

1

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Untitled-11

[461]=

```
{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}
```

[464]:=

$$\frac{\text{A[n]}}{\text{(2 n)! / (n! (n + 1)!)}}$$ `// FullSimplify[#, {n ∈ Integers}] &`

[464]=

```
1
```

```
(2 n)! / (n! (n + 1)!)
```

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Untitled-11

Out[461]=

```
{1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}
```

In[464]:=

```
A[n] / ((2 n)! / (n! (n + 1)!)) // FullSimplify[#, {n ∈ Integers}] &
```

Out[464]=

```
1
```

```
(2 n)! / (n! (n + 1)!)
```

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Untitled-11

[461]= {1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796}

[464]:= $\frac{\texttt{A[n]}}{(2\,n)!\,/\,(n!\,(n+1)!)}$ // FullSimplify[#, {n ∈ Integers}] &

[464]= 1

[465]:= Sum$\left[\frac{\left(\frac{\lambda}{4}\right)^{n}}{(n!\,(n+1)!)}, \{n, 0, \infty\}\right]$

[465]= $\frac{2\,\text{BesselI}\left[1, \sqrt{\lambda}\right]}{\sqrt{\lambda}}$

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Untitled-11

[464]=

1

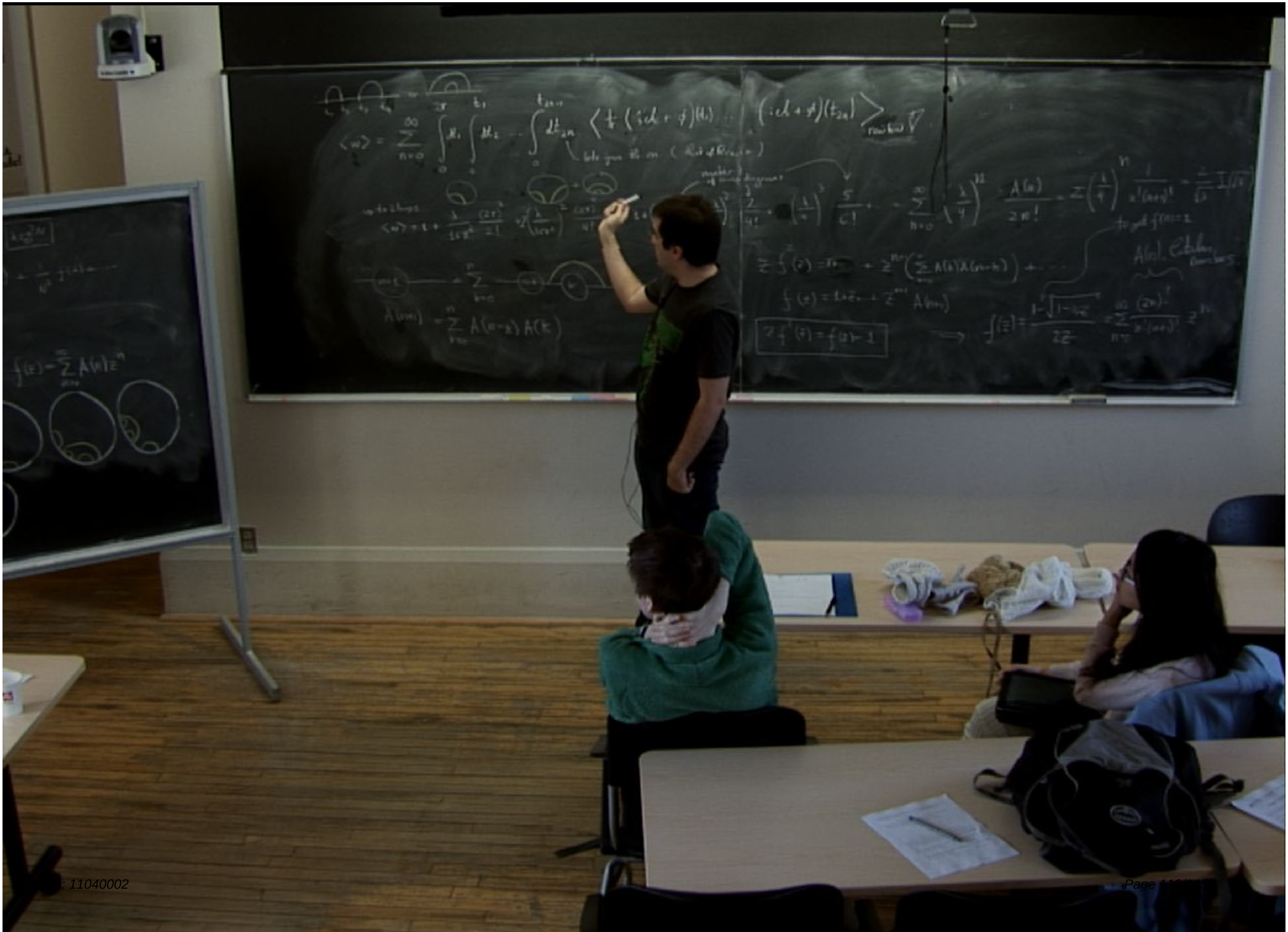
[465]:=

$$\text{Sum}\left[\frac{\left(\frac{\lambda}{4}\right)^{n}}{(n!\,(n+1)!)},\ \{n,\ 0,\ \infty\}\right]$$

[465]=

$$\frac{2\,\text{BesselI}\left[1,\ \sqrt{\lambda}\right]}{\sqrt{\lambda}}$$

[redacted]



$\lambda \equiv g^2 N$

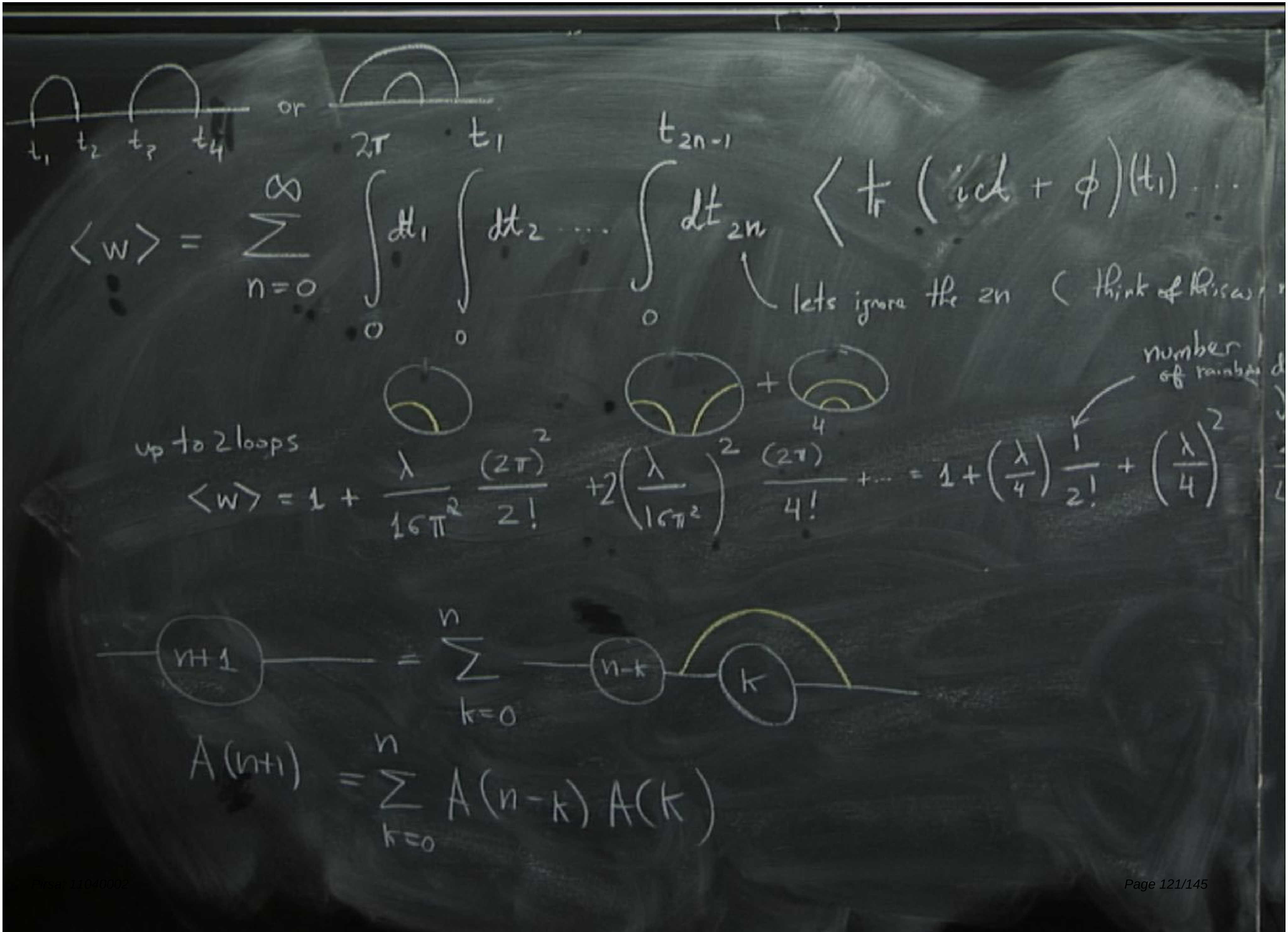
(N×N matrices)

$$\int \mathcal{D}M \, \mathrm{tr}(e^M)\, e^{-\frac{2}{g^2}\mathrm{tr} M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{\frac{g^2}{8}} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \dots$$

$$\langle W \rangle_{\bigcirc}^{N=4 \text{ SYM at } \lambda \to \infty} \sim e^{\sqrt{\lambda}}$$

We want to find A(n).

e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$



t1 t2 t3 t4 or
2π t1 t2n-1
⟨W⟩ = Σ_{n=0}^{∞} ∫_0 dt1 ∫_0 dt2 … ∫_0 dt2n ⟨ tr (iA + φ)(t1) …
lets ignore the 2n (think of this as
number of rainbow d
up to 2 loops
⟨W⟩ = 1 + λ/16π² (2π)²/2! + 2(λ/16π²)² (2π)⁴/4! + … = 1 + (λ/4) 1/2! + (λ/4)²
n+1 = Σ_{k=0}^{n} n-k k
A(n+1) = Σ_{k=0}^{n} A(n-k) A(k)

$$(i\,d + \phi)(t_{2n}) \sum_{\text{rainbow}} \nabla$$

$\ldots, n)$

... diagrams

$$\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \ldots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!} = \Sigma \left(\frac{\lambda}{4}\right)^n \frac{1}{n!(n+1)!} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

to get $f(0)=1$

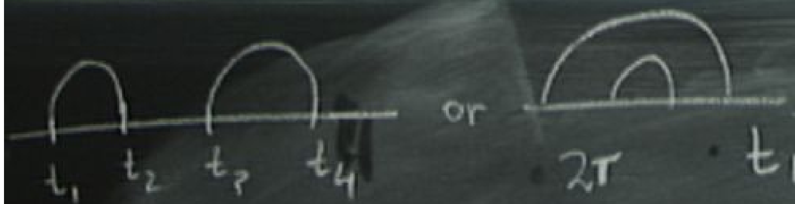
$A(n)$, Catalan numbers.

$$z f^2(z) = \ldots + z^{n+1}\left(\sum_{k=0}^{n} A(k)A(n-k)\right) + \ldots$$

$$f(z) = 1 + z + \ldots + z^{n+1} A(n+1)$$

$$\boxed{z f^2(z) = f(z) - 1}$$

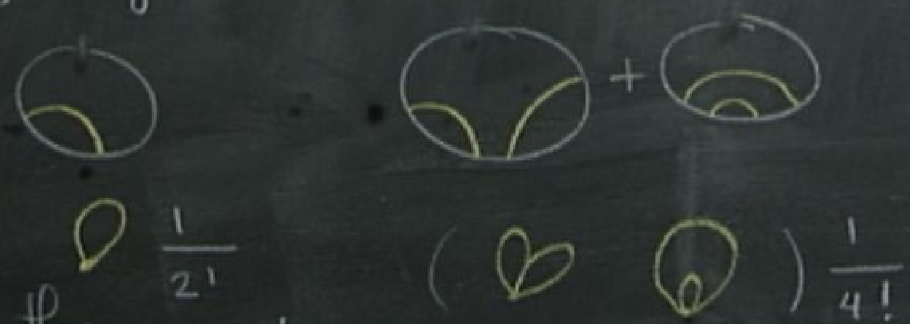
$$\longrightarrow \quad f(z) = \frac{1-\sqrt{1-4z}}{2z} = \sum_{n=0}^{\infty} \frac{(2n)!}{n!(n+1)!} z^n$$



$$\langle W\rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(iA + \phi\right)(t_1)\cdots\right.$$

lets ignore the 2n (think of this as …

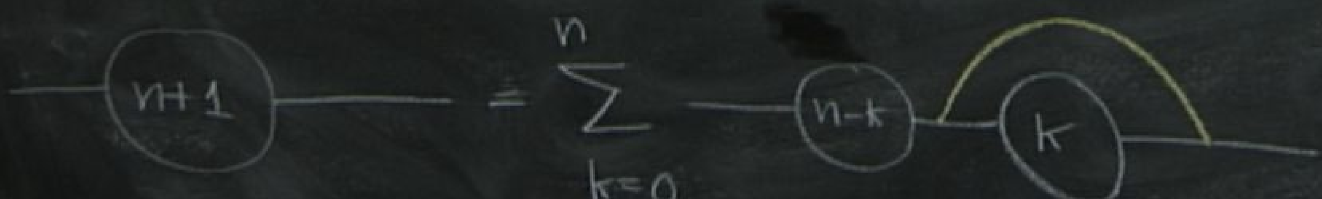
number of rainbow d…



$\frac{1}{2!}$ the prop is const

$\frac{1}{4!}$

$+\left(\frac{\lambda}{4}\right)^2$



$$= \sum_{k=0}^{n}$$

$$A(n+1) = \sum_{k=0}^{n} A(n-k)\,A(k)$$

$t_1 \quad t_2 \quad t_3 \quad t_4$ or 

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \operatorname{tr}\left(iA + \phi \right)(t_1) \cdots \right.$$

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$\left(\quad \right) \frac{1}{4!}$

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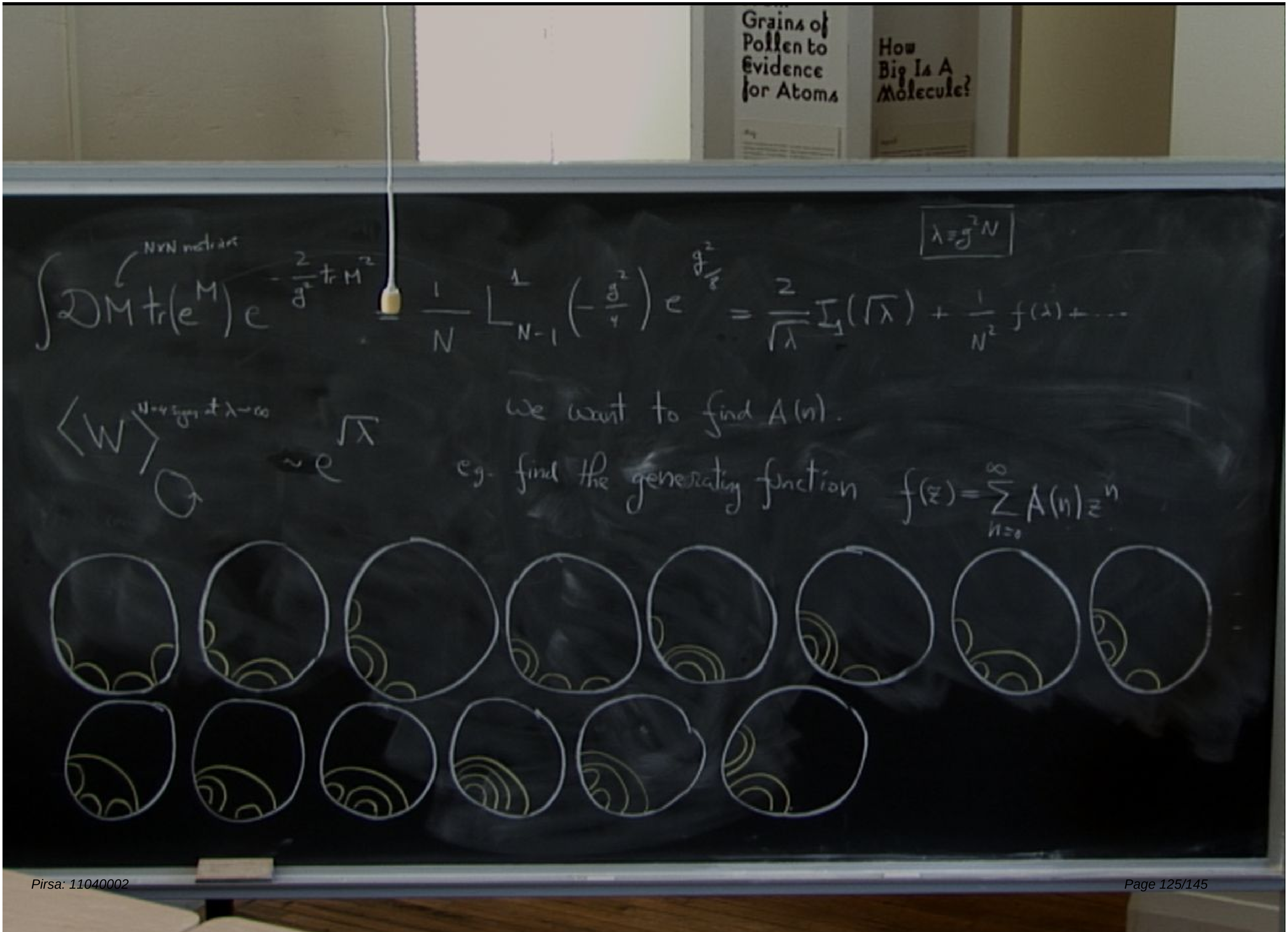
$$\langle e^M \rangle = \left\langle 1 + M + \frac{M^2}{2!} + \frac{M^3}{3!} + \frac{M^4}{4!} + \cdots \right\rangle$$

$$= 1 + 0 + \frac{1}{2!}\,\text{O} + 0 + \frac{1}{4!}\,\text{O} + \text{O} + \cdots$$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\lambda = g^2 N$
N×N matrices
$\int \mathcal{D}M \, \mathrm{tr}(e^M) \, e^{-\frac{2}{g^2}\mathrm{tr}\, M^2} = \frac{1}{N} L^1_{N-1}\left(-\frac{g^2}{4}\right) e^{g^2/8} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{1}{N^2} f(\lambda) + \dots$
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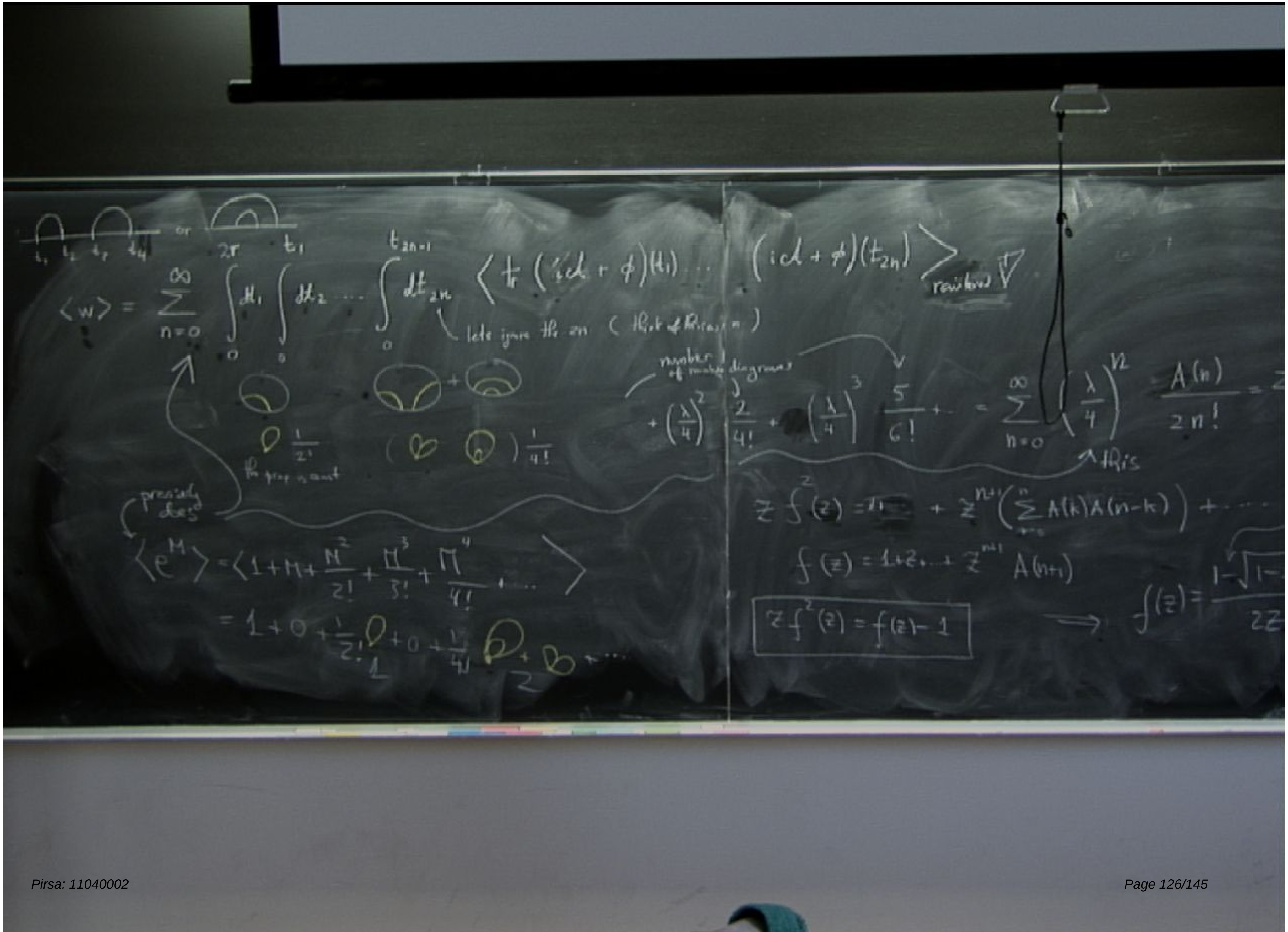
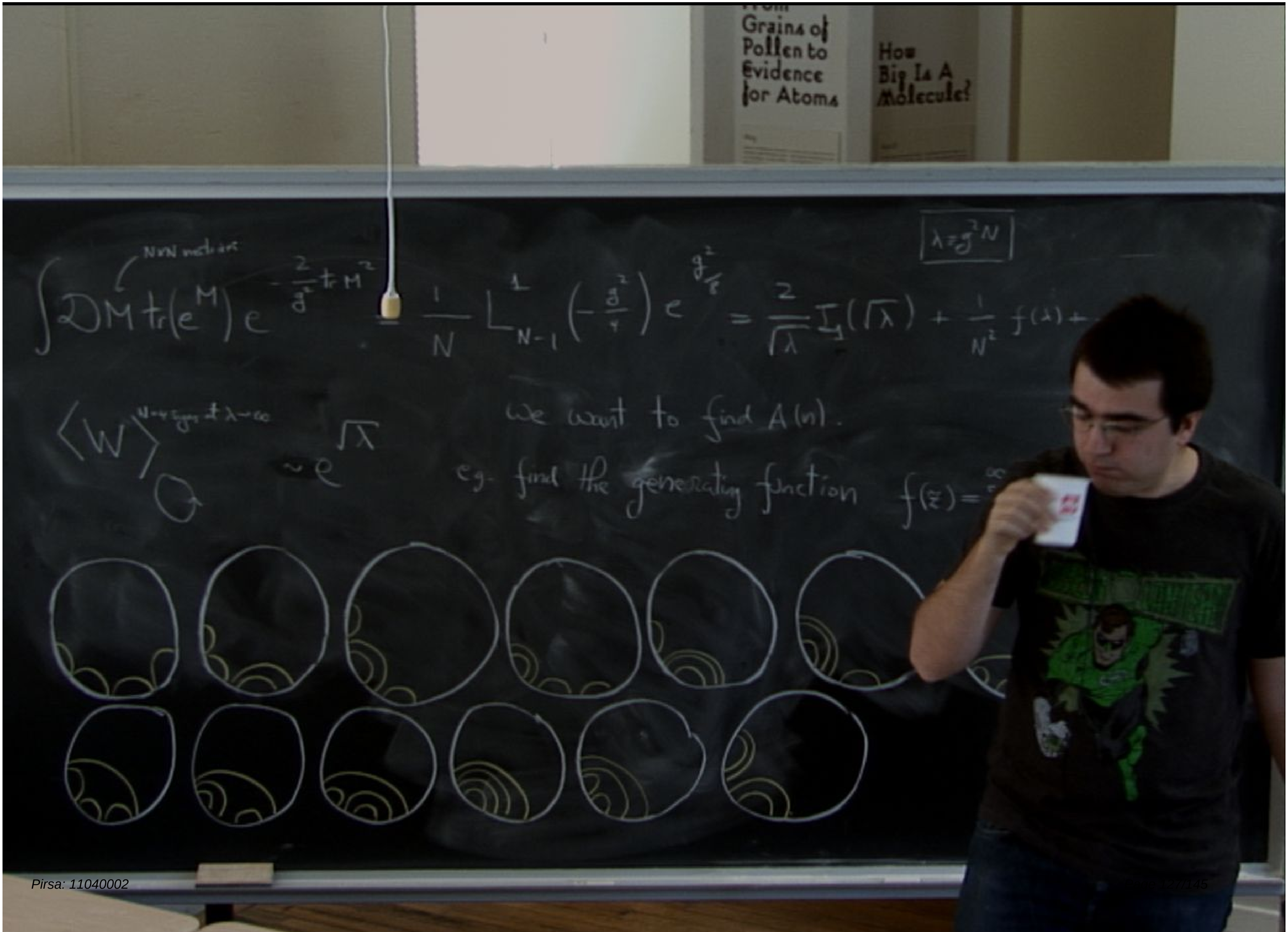
We want to find A(n).
e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$



lets ignore the 2n
number of rainbow diagrams
rainbow
this
previously does
Z f²(z) = f(z) − 1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$t_1 \quad t_2 \quad t_3 \quad t_4$ or $2\pi \quad t_1$

$$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \mathrm{tr}\left(i\dot{x} + \phi \right)(t_1) \cdots \right.$$

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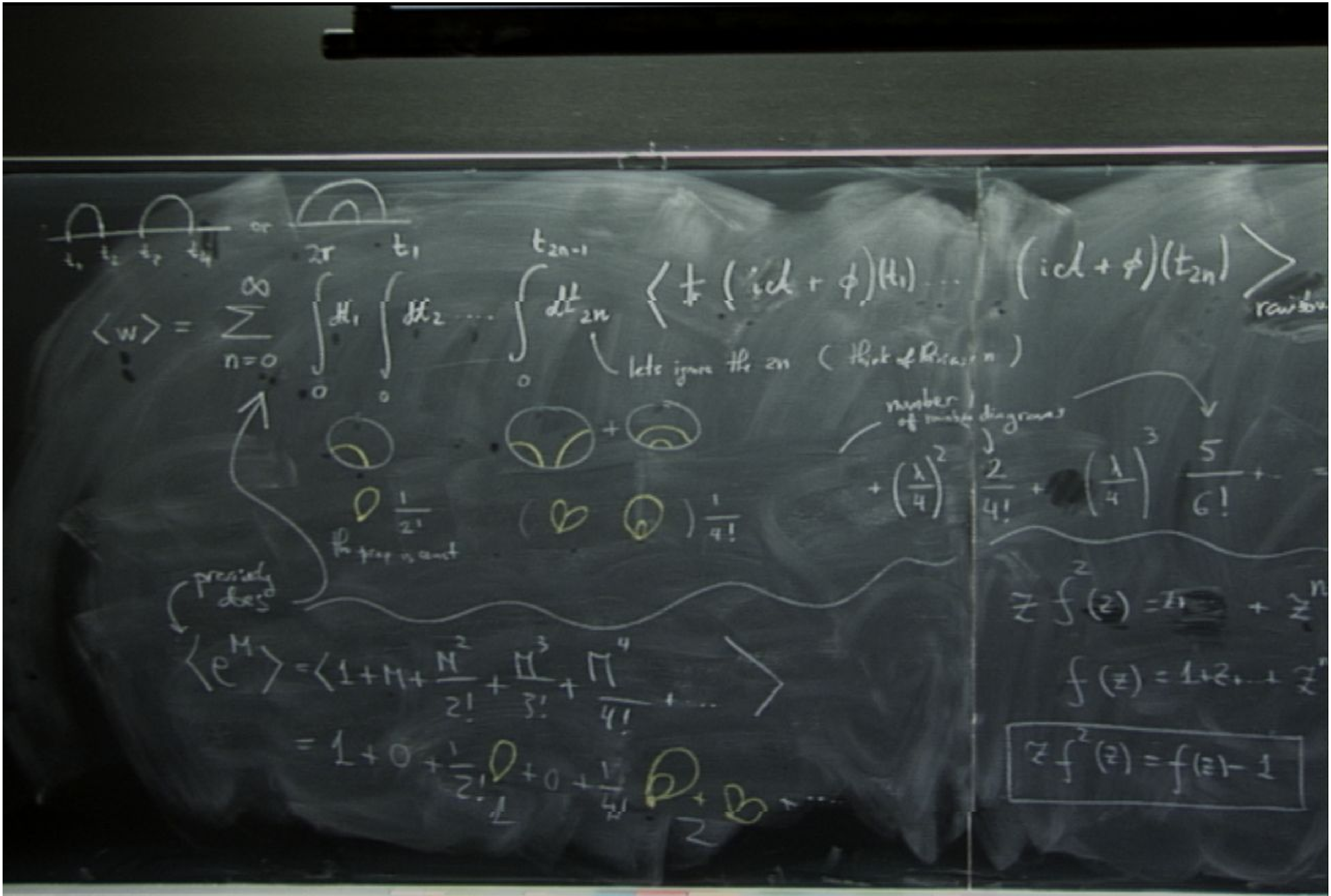
number of rainbo

$+ \left(\frac{\lambda}{4}\right)^2$

precisely does

$$\langle e^M \rangle = \left\langle 1 + M + \frac{M^2}{2!} + \frac{M^3}{3!} + \frac{M^4}{4!} + \cdots \right\rangle$$

$$= 1 + 0 + \frac{1}{2!} \cdot 1 + 0 + \frac{1}{4!} \cdots + \cdots + \cdots$$



or
$\langle W \rangle = \sum_{n=0}^{\infty} \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{2n-1}} dt_{2n} \left\langle \frac{1}{N}\, (i\dot{x} + \phi)(t_1) \cdots (i\dot{x} + \phi)(t_{2n}) \right\rangle$
rainbow
lets ignore the 2n
the prop is const
number of rainbow diagrams
$+ \left(\frac{\lambda}{4}\right)^2 \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \cdots$
precisely does
$\langle e^M \rangle = \left\langle 1 + M + \frac{M^2}{2!} + \frac{M^3}{3!} + \frac{M^4}{4!} + \cdots \right\rangle$
$= 1 + 0 + \frac{1}{2!} + 0 + \frac{1}{4!} + \cdots$
$f(z) = 1 + z + \cdots + z^n$
$z f^2(z) = f(z) - 1$

$$(i d + \phi)(t_{2n}) \quad \sum_{\text{rainbow}}$$

$$\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \dots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!} = \sum \left(\frac{\lambda}{4}\right)^n \frac{1}{n!(n+1)!} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

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$w, n)$

diagrams

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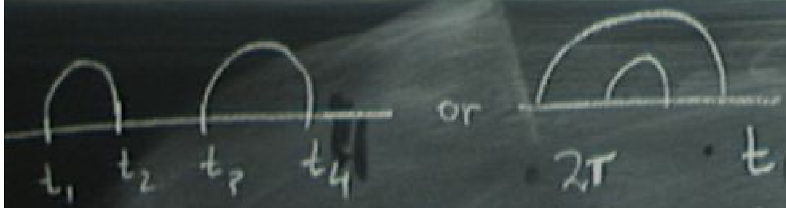
^ this

1) large λ.



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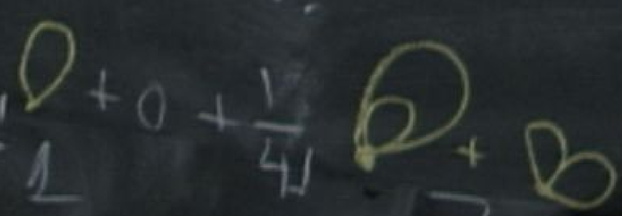
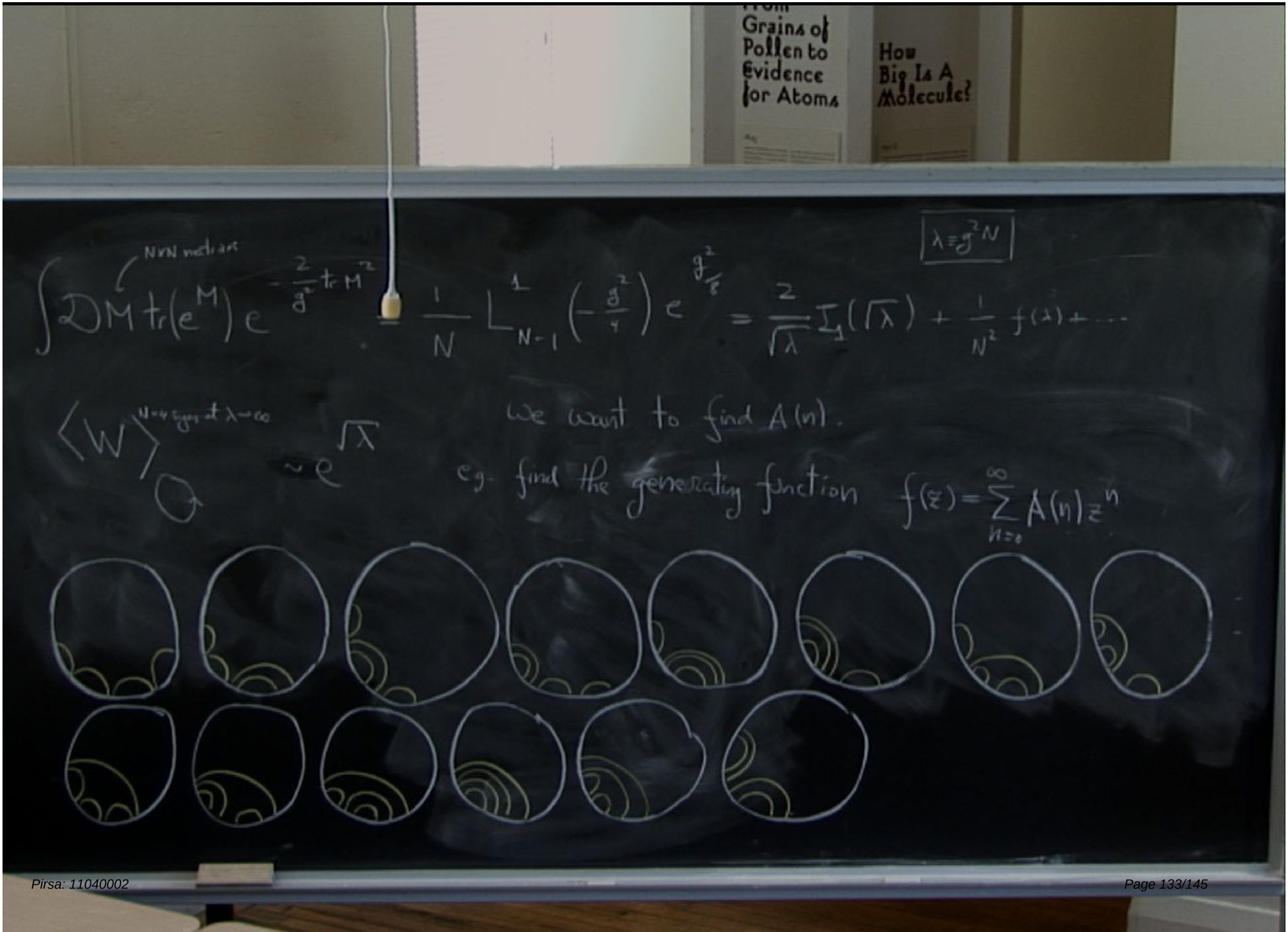
$= 1 + 0 + \frac{1}{2!}$  $+ 0 + \frac{1}{4!}$ (+) $+ \cdots$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We want to find A(n).
e.g. find the generating function $f(z) = \sum_{n=0}^{\infty} A(n) z^n$

$(i d + \phi)(t_{2n}) >_{\text{rainbow}} \nabla$

... n)

... diagrams

$$\frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \dots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!} = \sum \left(\frac{\lambda}{4}\right)^n \frac{1}{n!(n+1)!} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

this

1) large λ.

$$\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \sim \lambda^{\#} e^{+\sqrt{\lambda}} \quad !!$$

$(i d + \phi)(t_{2n}) \sum_{\text{rainbow}} \nabla$

$w, n)$

diagrams

$\frac{\lambda}{4} \frac{2}{4!} + \left(\frac{\lambda}{4}\right)^3 \frac{5}{6!} + \dots = \sum_{n=0}^{\infty} \left(\frac{\lambda}{4}\right)^n \frac{A(n)}{2n!} = \sum \left(\frac{\lambda}{4}\right)^n \frac{1}{n!(n+1)!} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$

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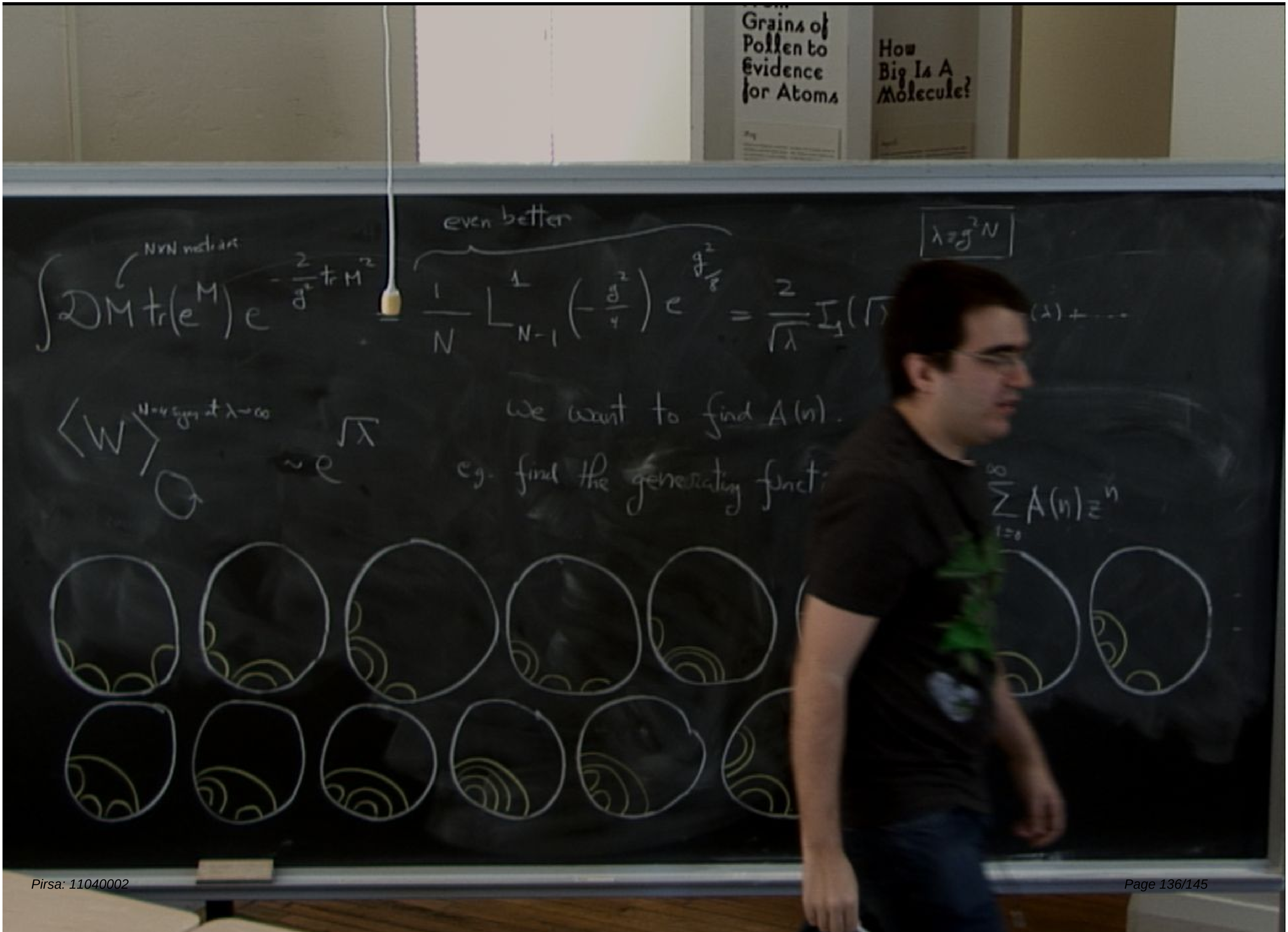
What is expected from

$\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \sim \lambda^{\#} e^{+\sqrt{\lambda}}$!!



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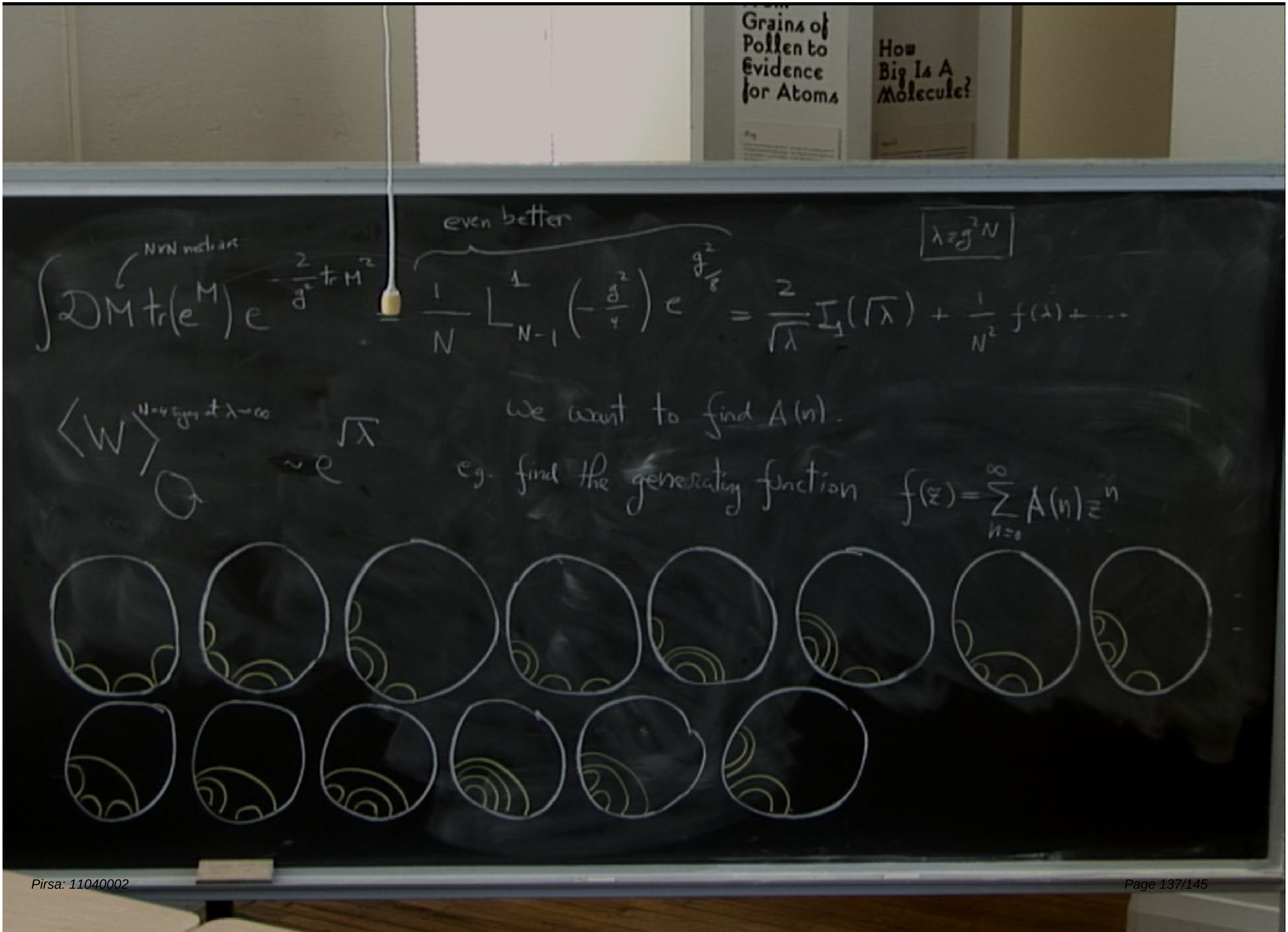
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N=4 Sym at $\lambda \to \infty$
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$$(i\,d + \phi)(t_{2n}) \quad \sum_{\text{rainbow}} \nabla^{\dagger}$$

n)

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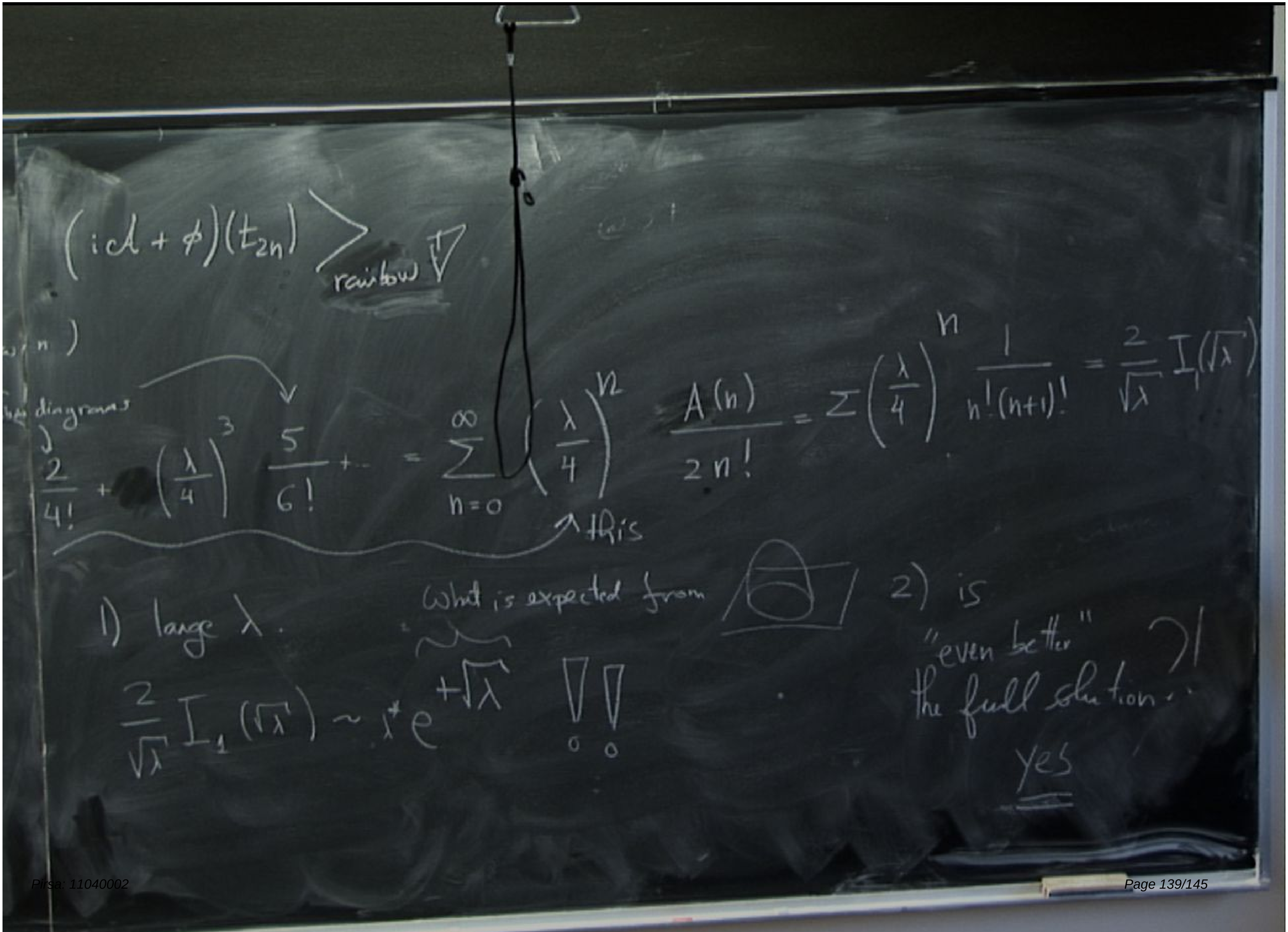
2) is "even better" the full solution?!

yes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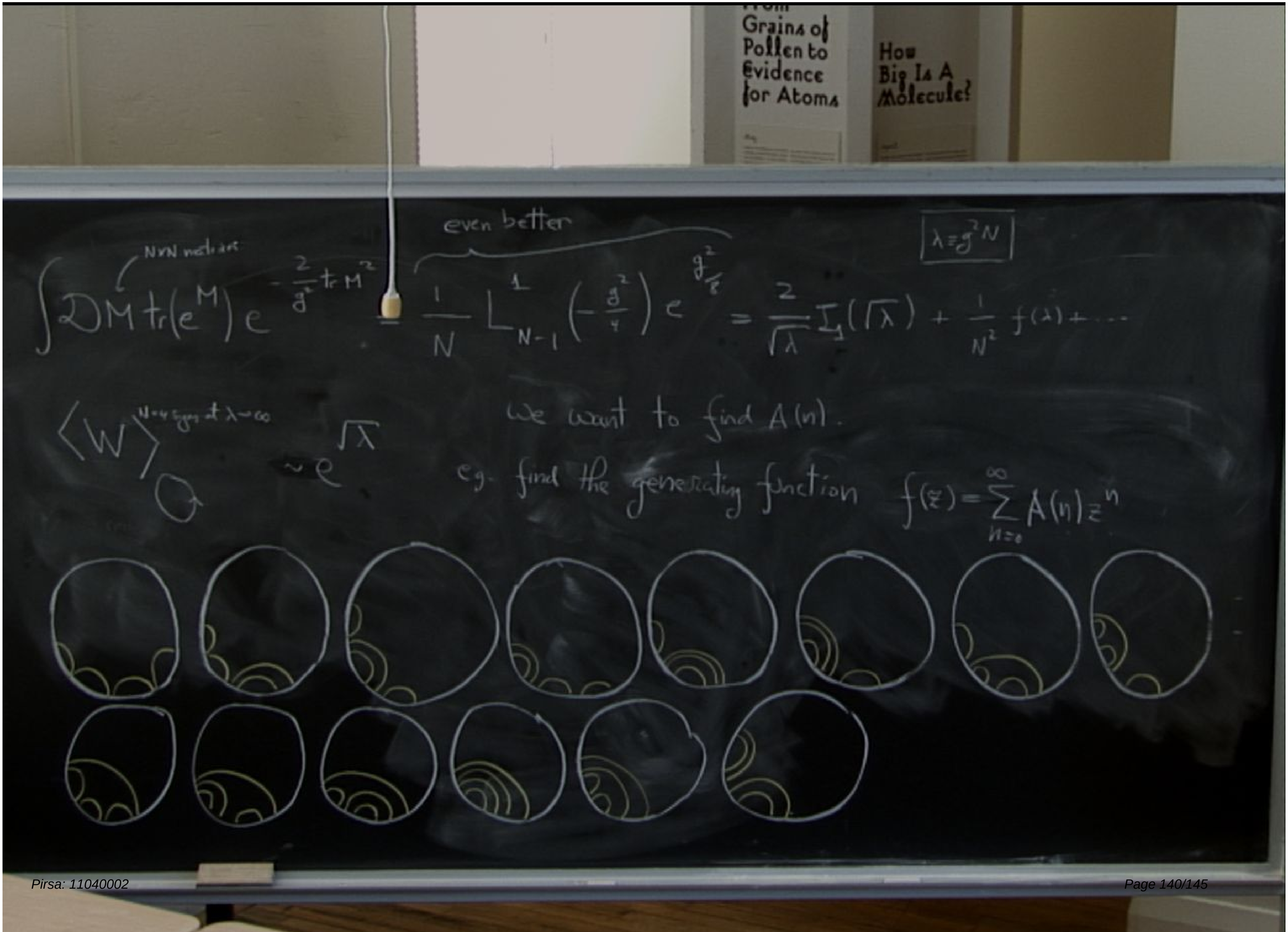
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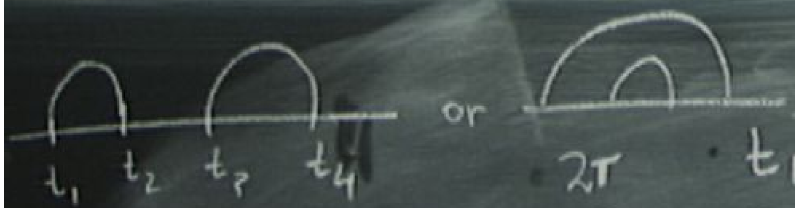

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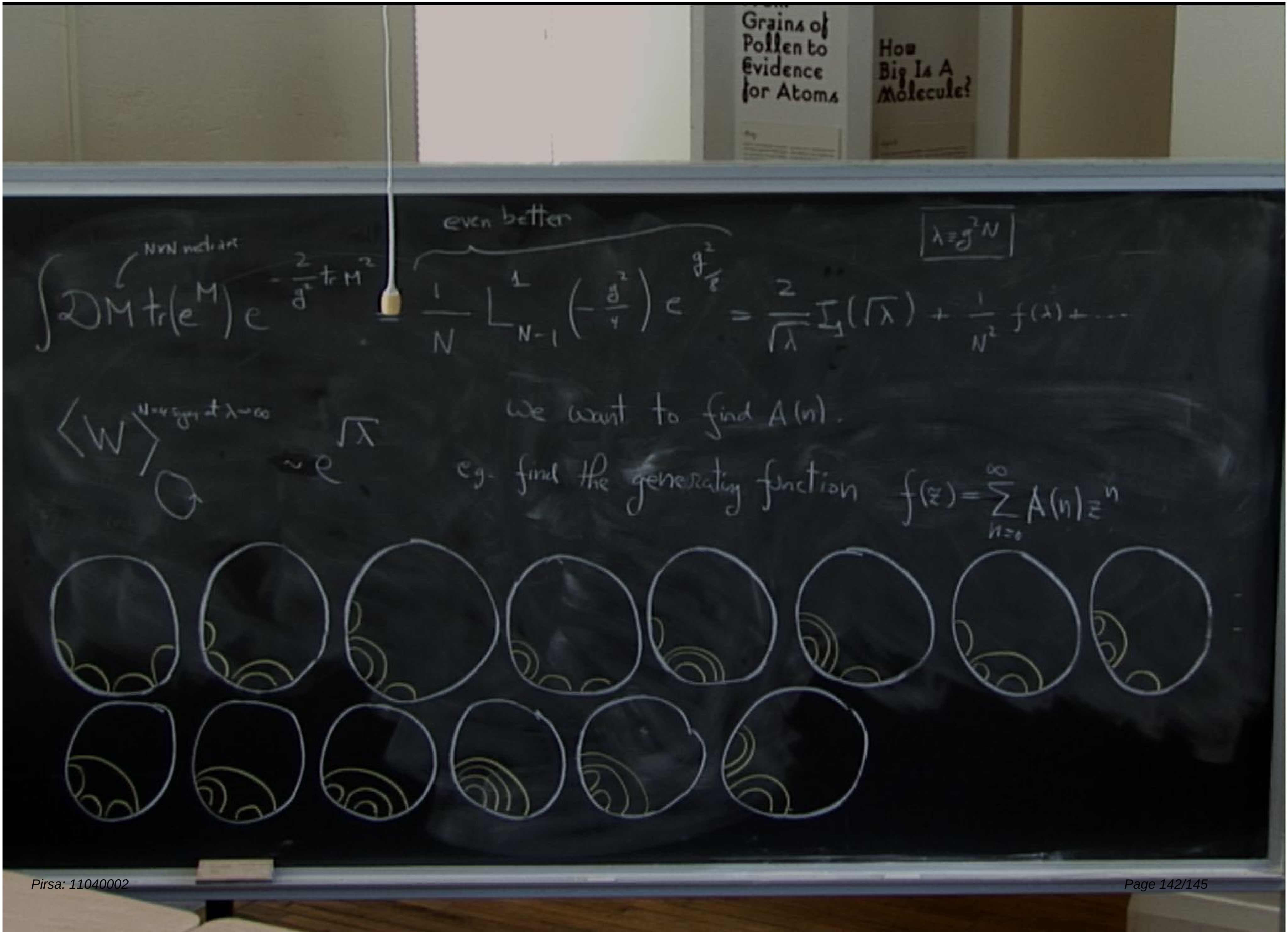
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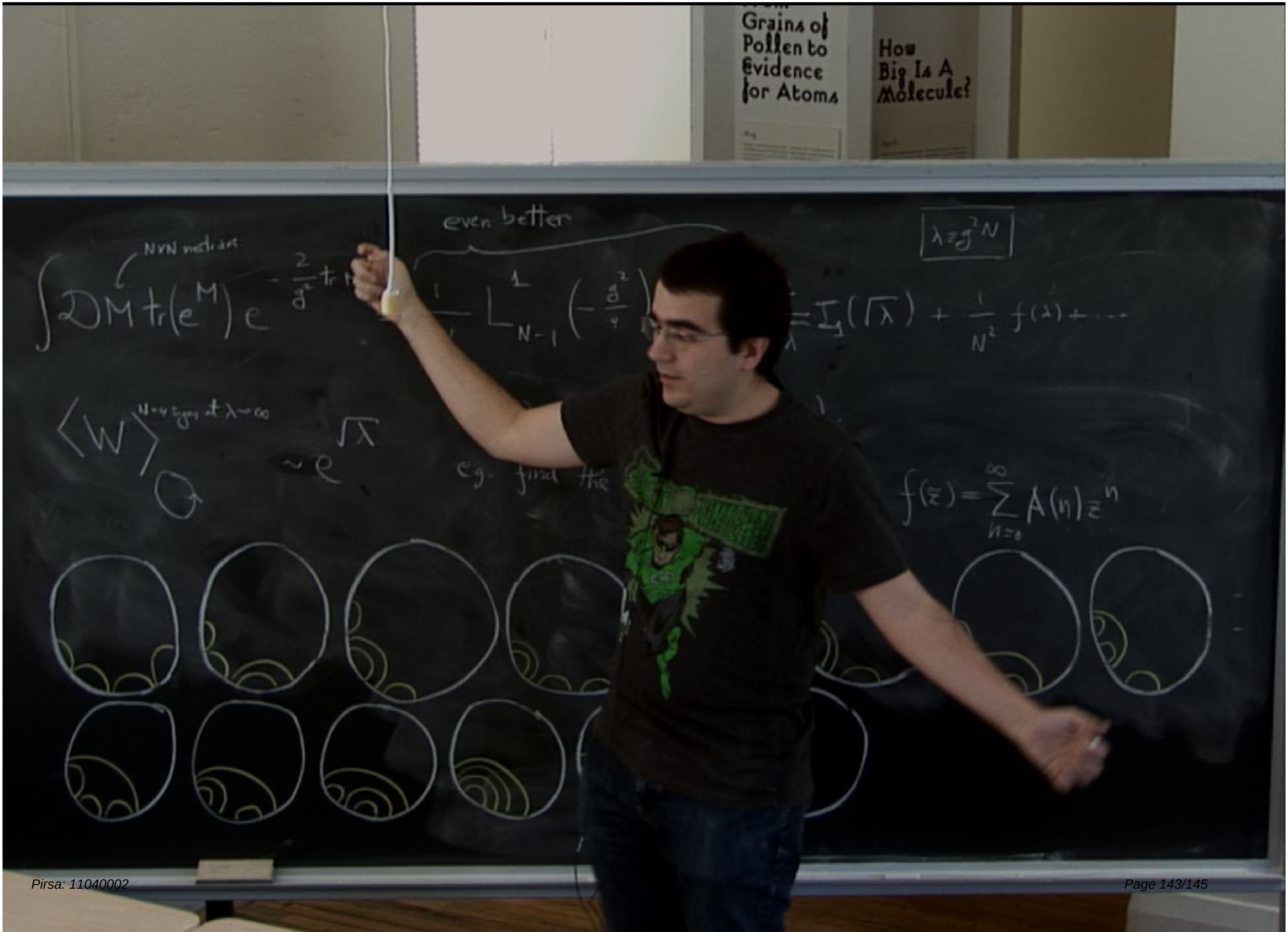
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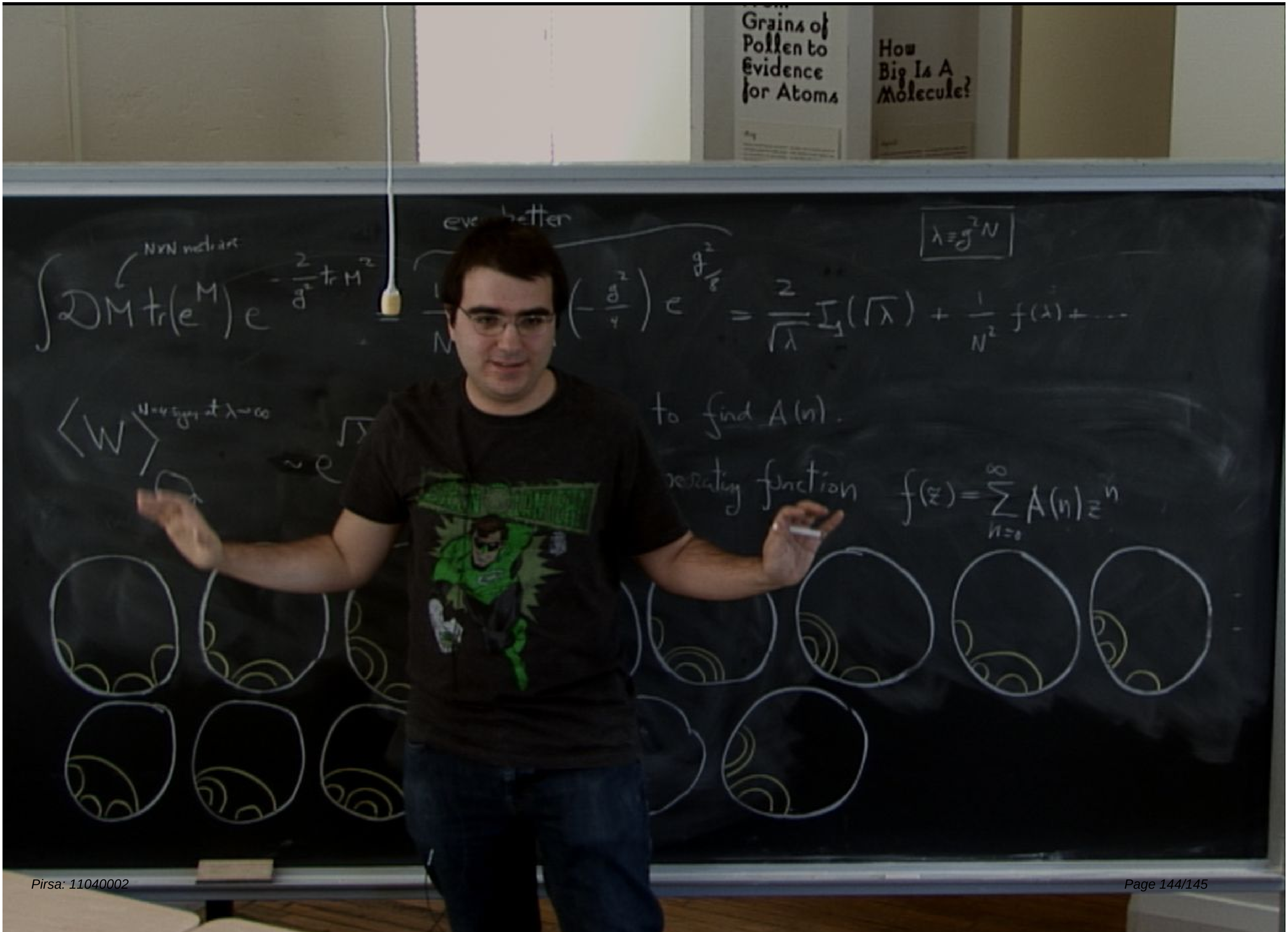
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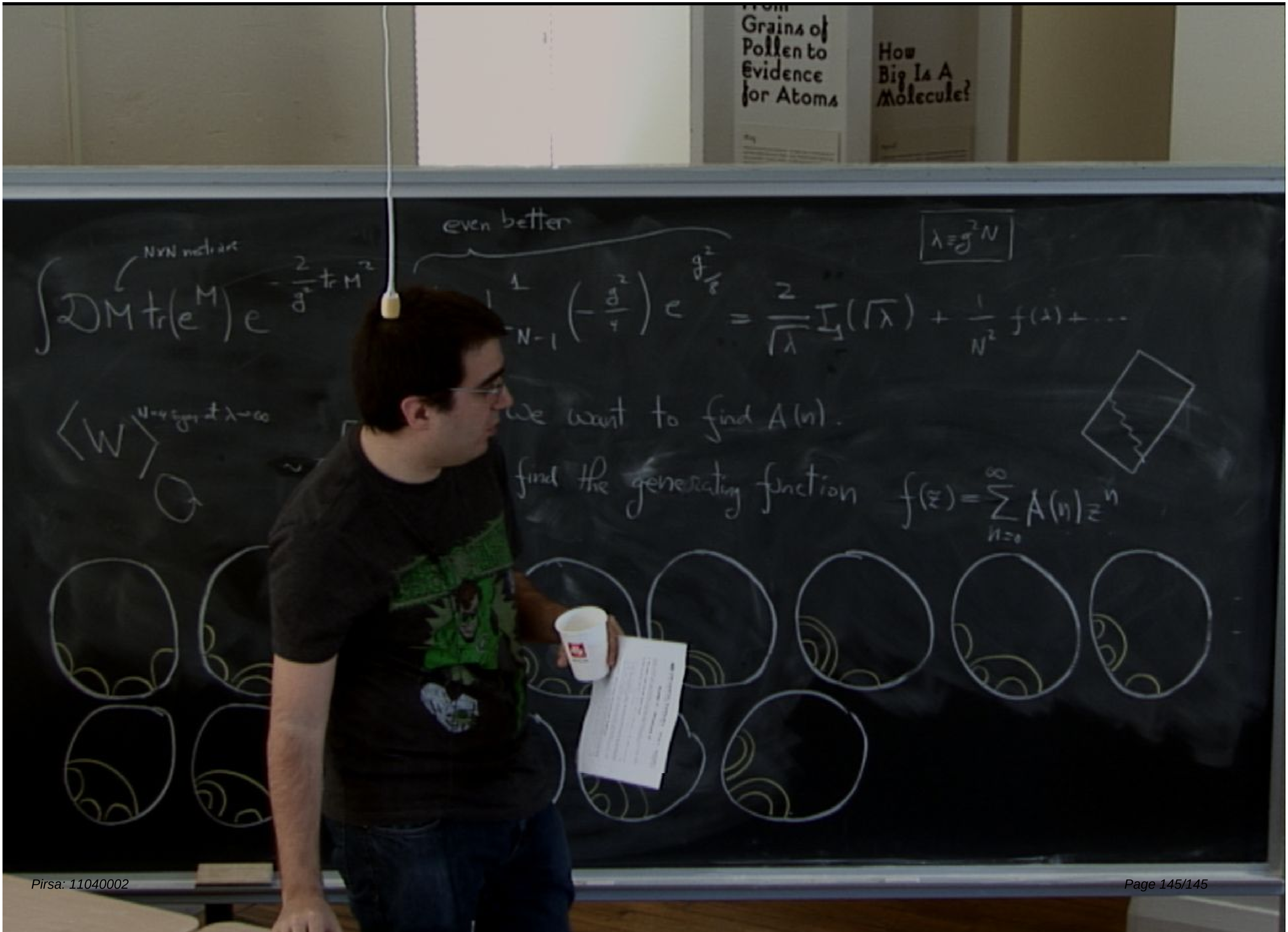
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