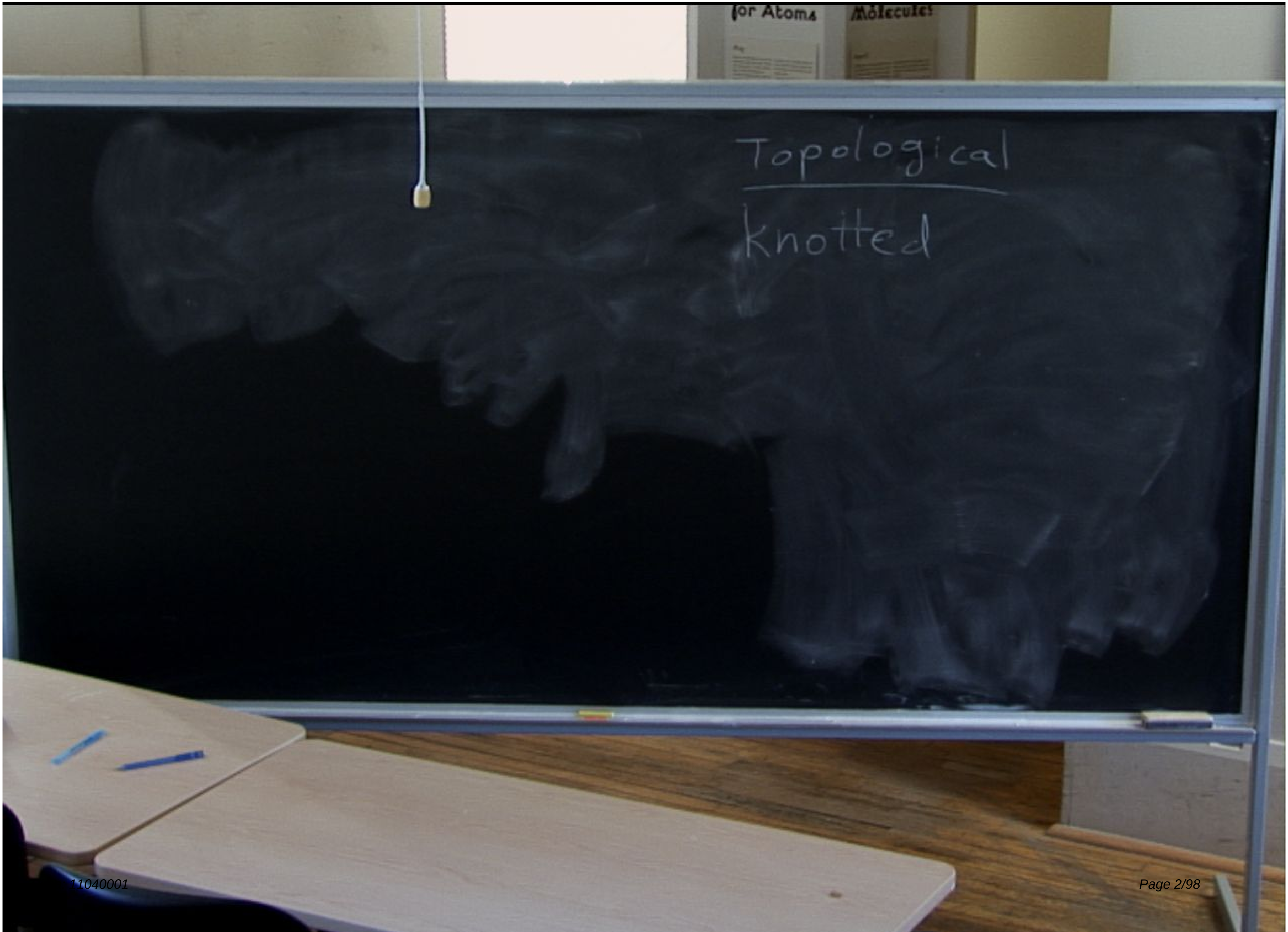
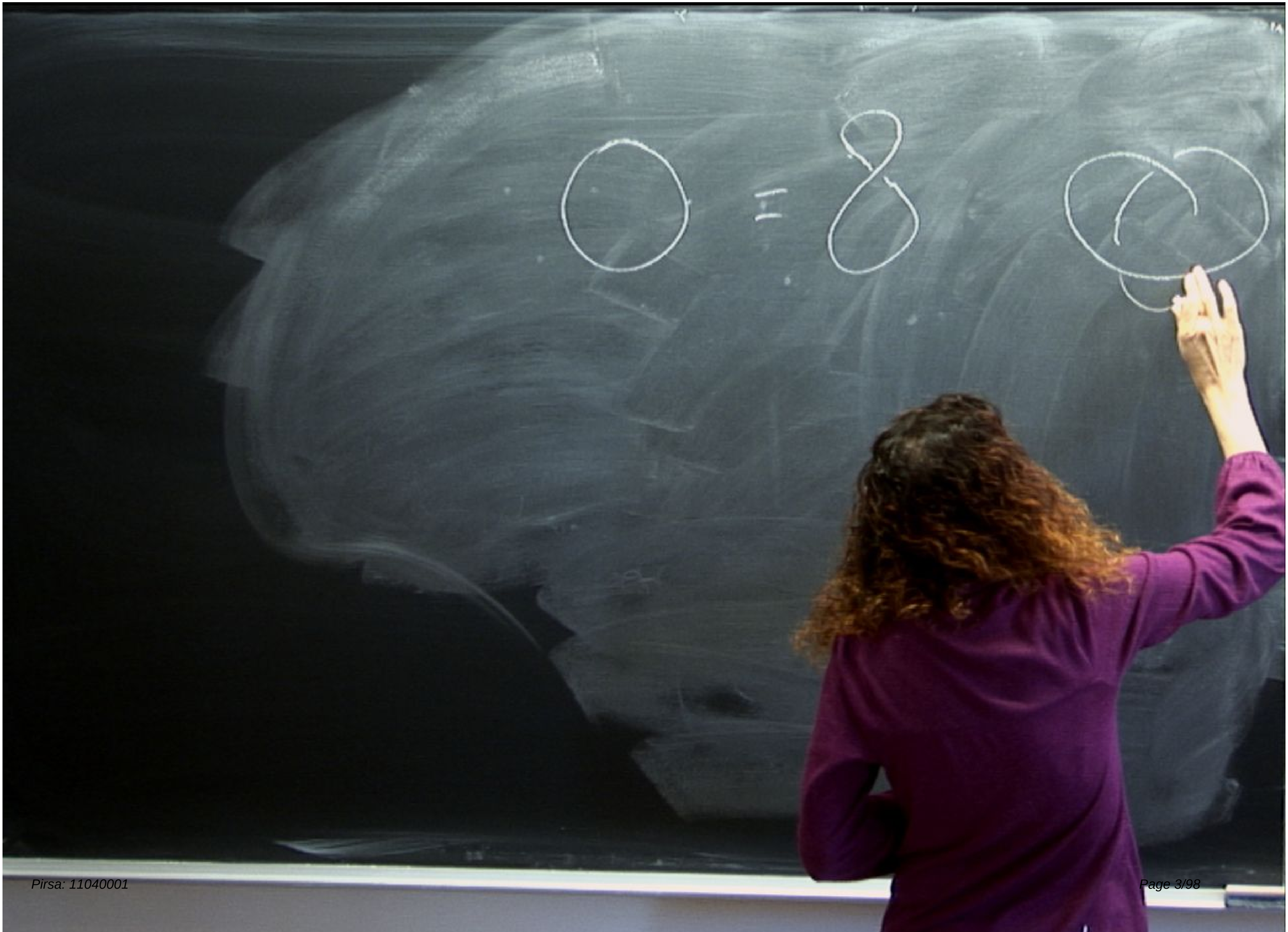
Title: Explorations in Condensed Matter - Lecture 11

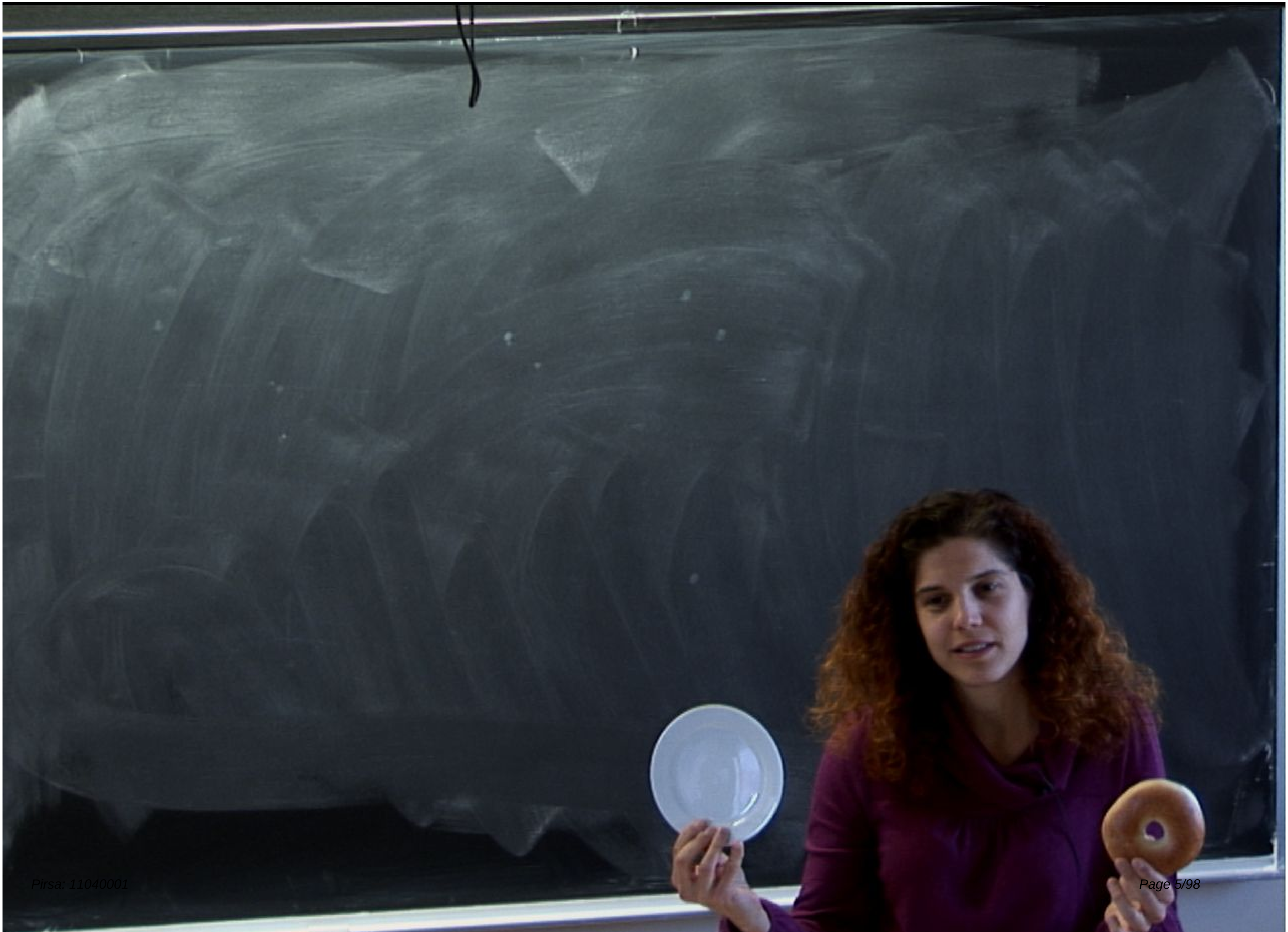
Date: Mar 28, 2011 10:15 AM

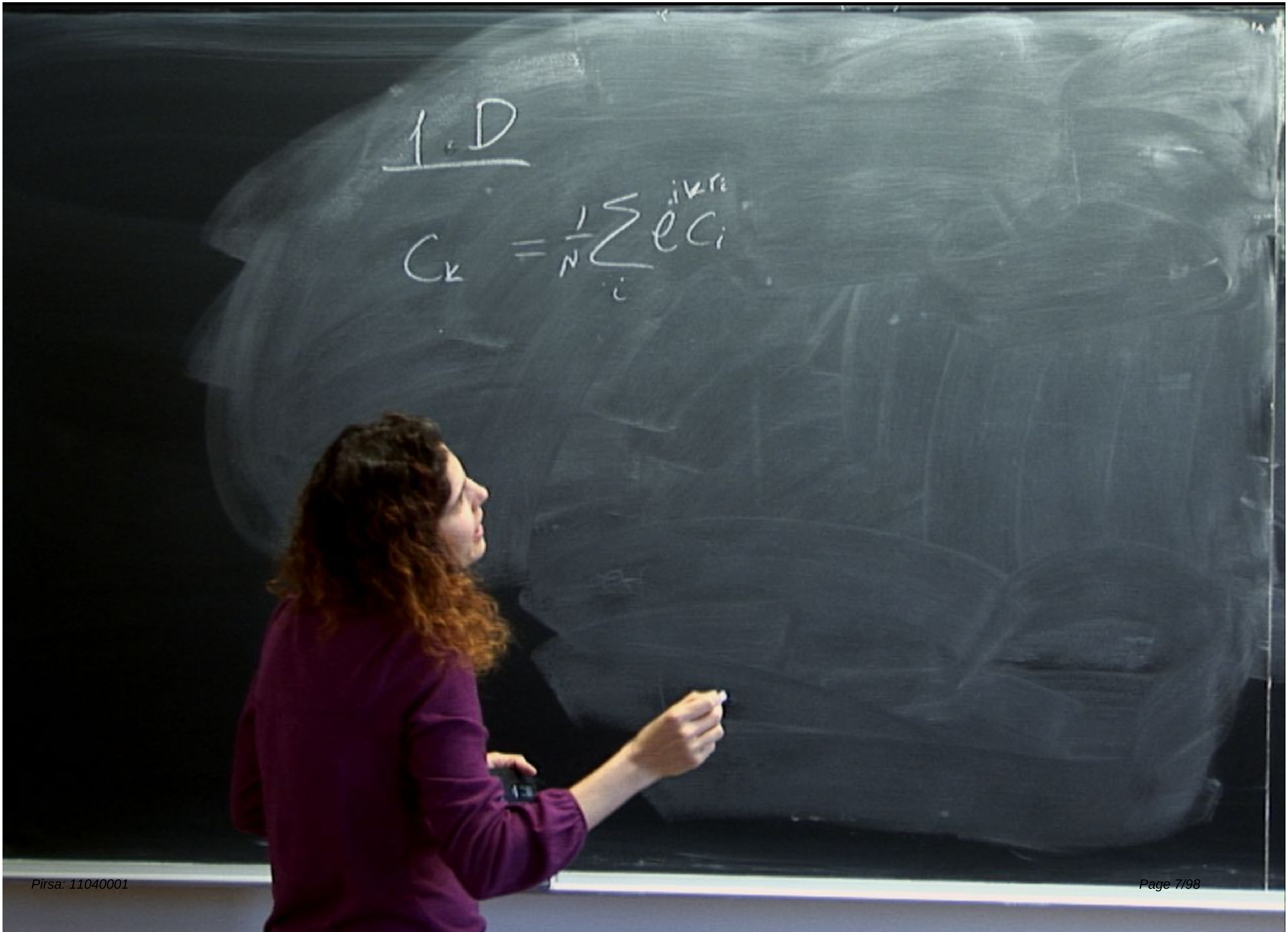
URL: http://pirsa.org/11040001

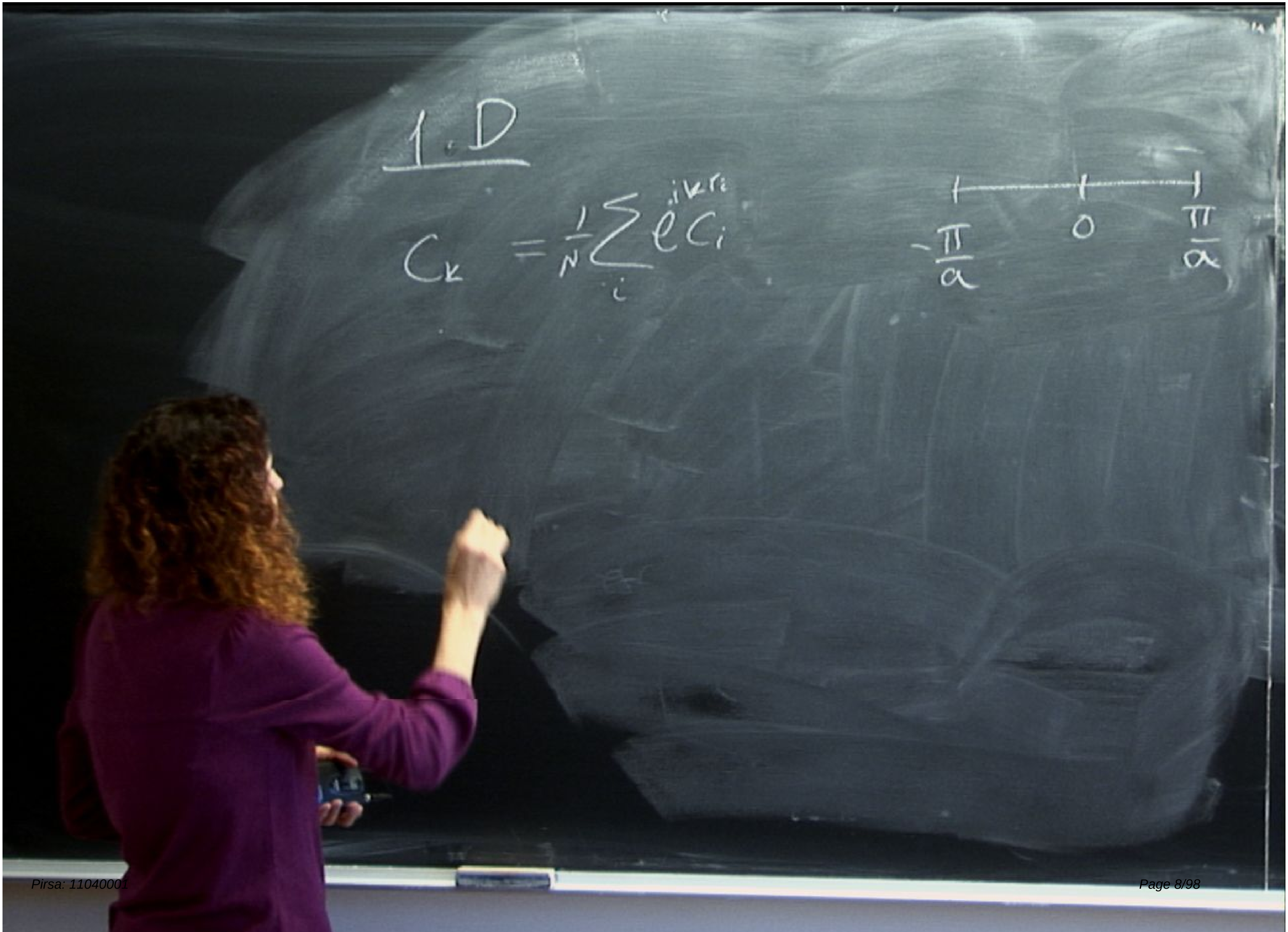
Abstract:

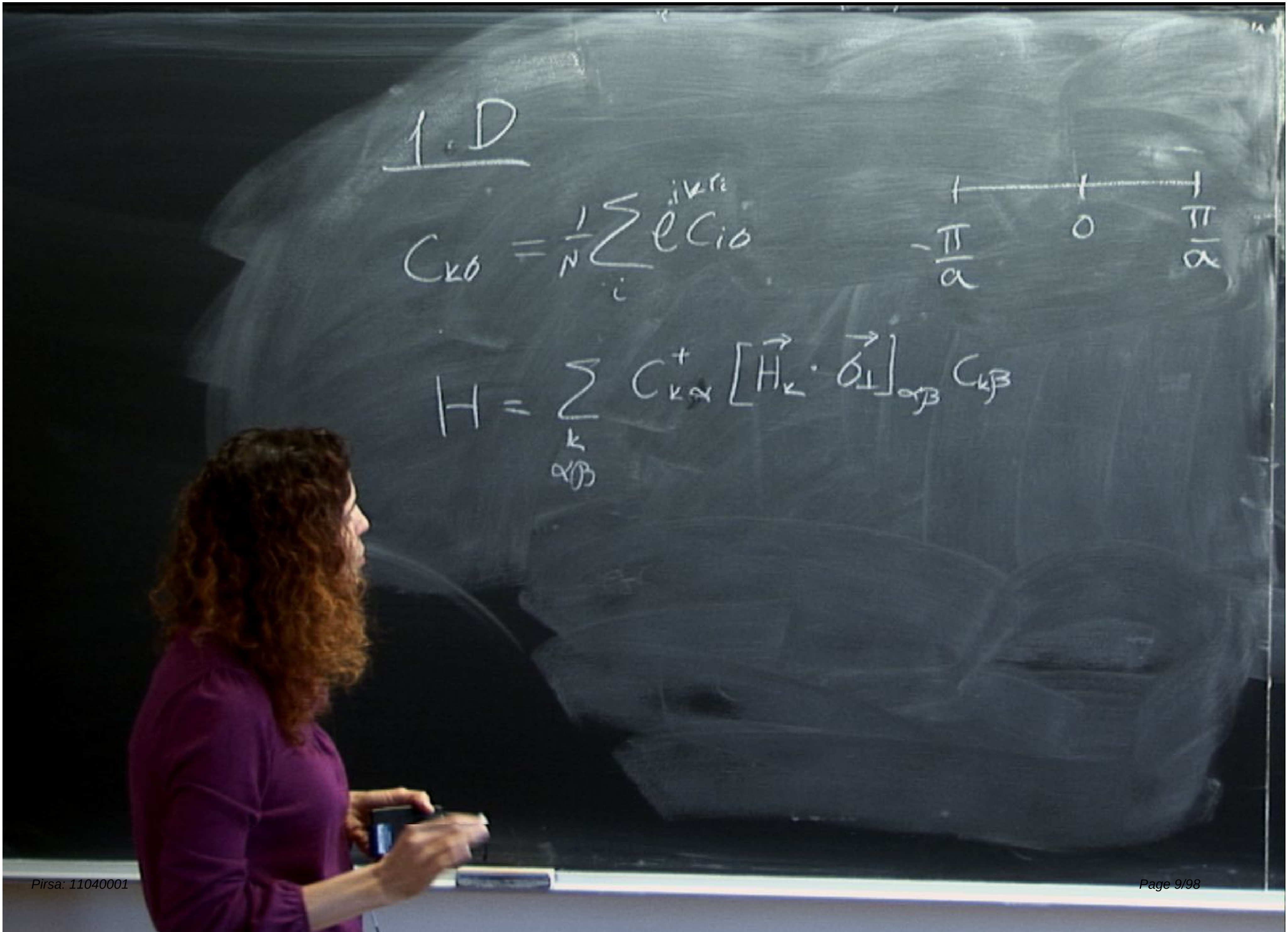

Topological
knotted

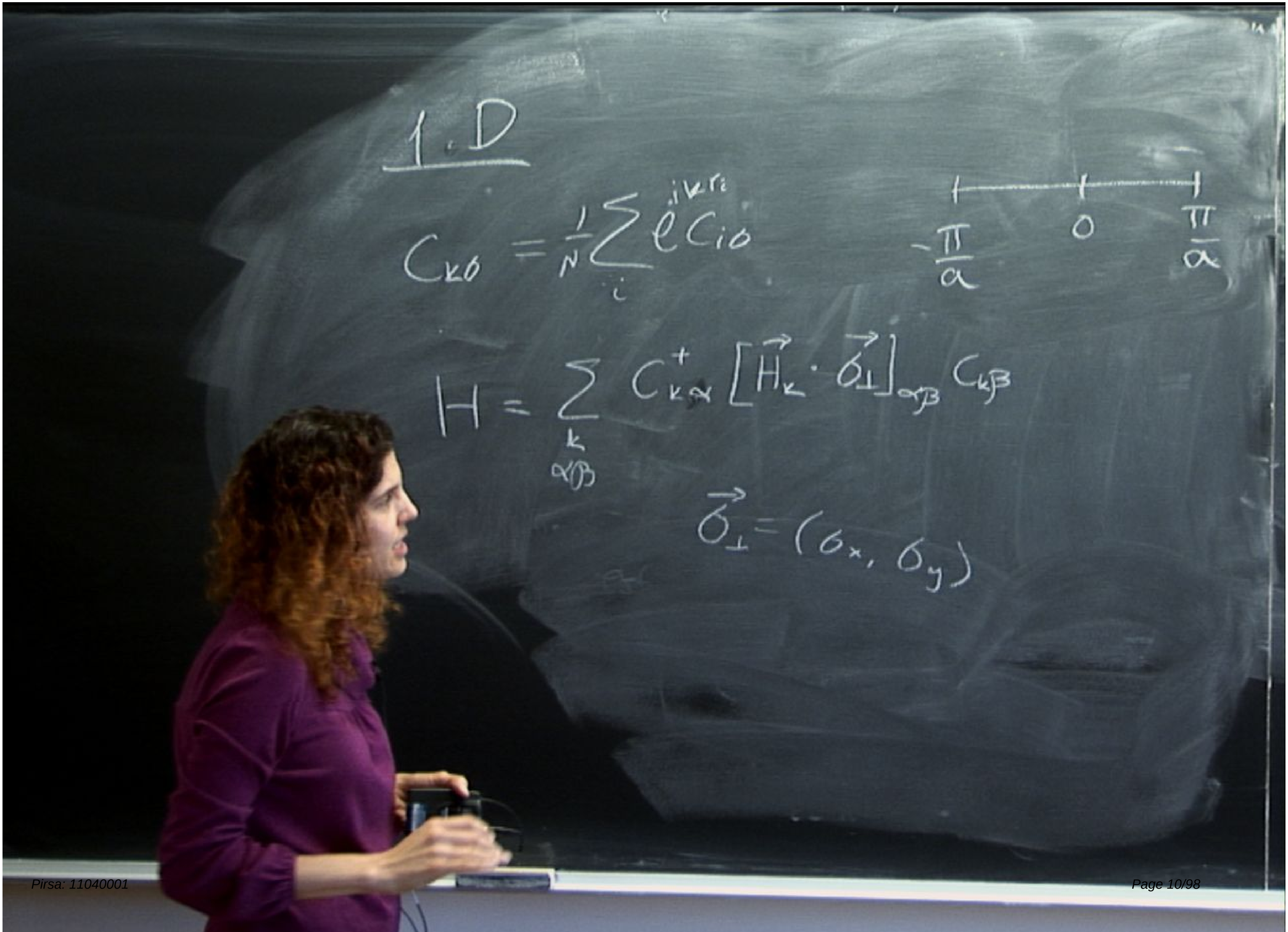


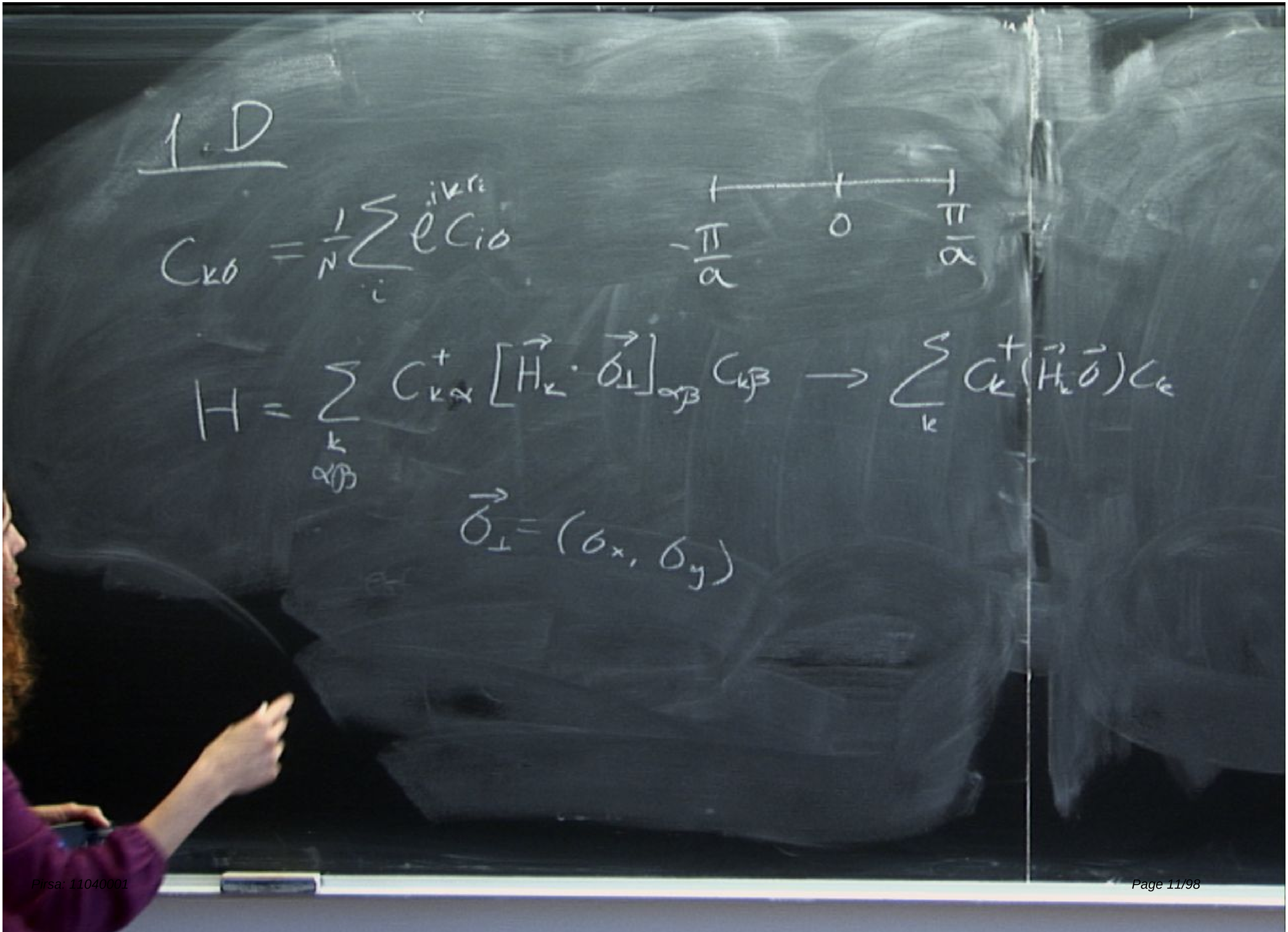



1.D
$C_k = \frac{1}{N}\sum_i e^{ikr_i} c_i$


1.D
$C_k = \frac{1}{N}\sum_i e^{ikr_i} c_i$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$


1.D
$C_{k\sigma} = \frac{1}{N} \sum_i e^{ikr_i} c_{i\sigma}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$H = \sum_{k, \alpha\beta} C^{+}_{k\alpha} [\vec{H}_k \cdot \vec{\sigma}_\perp]_{\alpha\beta} C_{k\beta}$


1.D
$C_{k\sigma} = \frac{1}{N}\sum_i e^{ikr_i} c_{i\sigma}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$H = \sum_{k,\alpha\beta} C^{+}_{k\alpha} [\vec{H}_k \cdot \vec{\sigma}_\perp]_{\alpha\beta} C_{k\beta}$
$\vec{\sigma}_\perp = (\sigma_x, \sigma_y)$

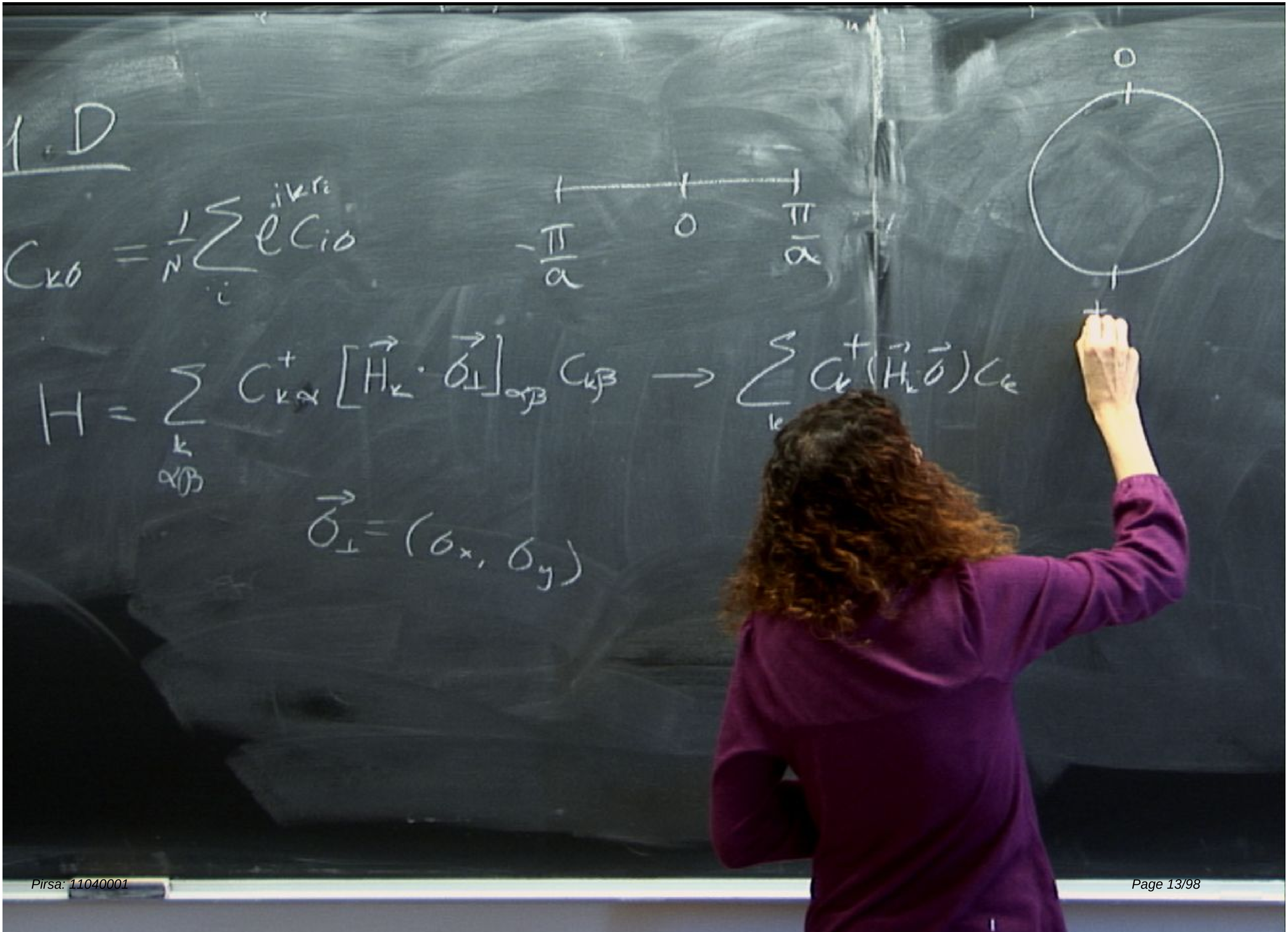

1.D
$C_{k\sigma} = \frac{1}{N}\sum_i e^{ikr_i} C_{i\sigma}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$H = \sum_{k,\alpha\beta} C^{\dagger}_{k\alpha} [\vec{H}_k \cdot \vec{\sigma}_\perp]_{\alpha\beta} C_{k\beta} \longrightarrow \sum_k C^{\dagger}_k (\vec{H}_k \vec{\sigma}) C_k$
$\vec{\sigma}_\perp = (\sigma_x, \sigma_y)$

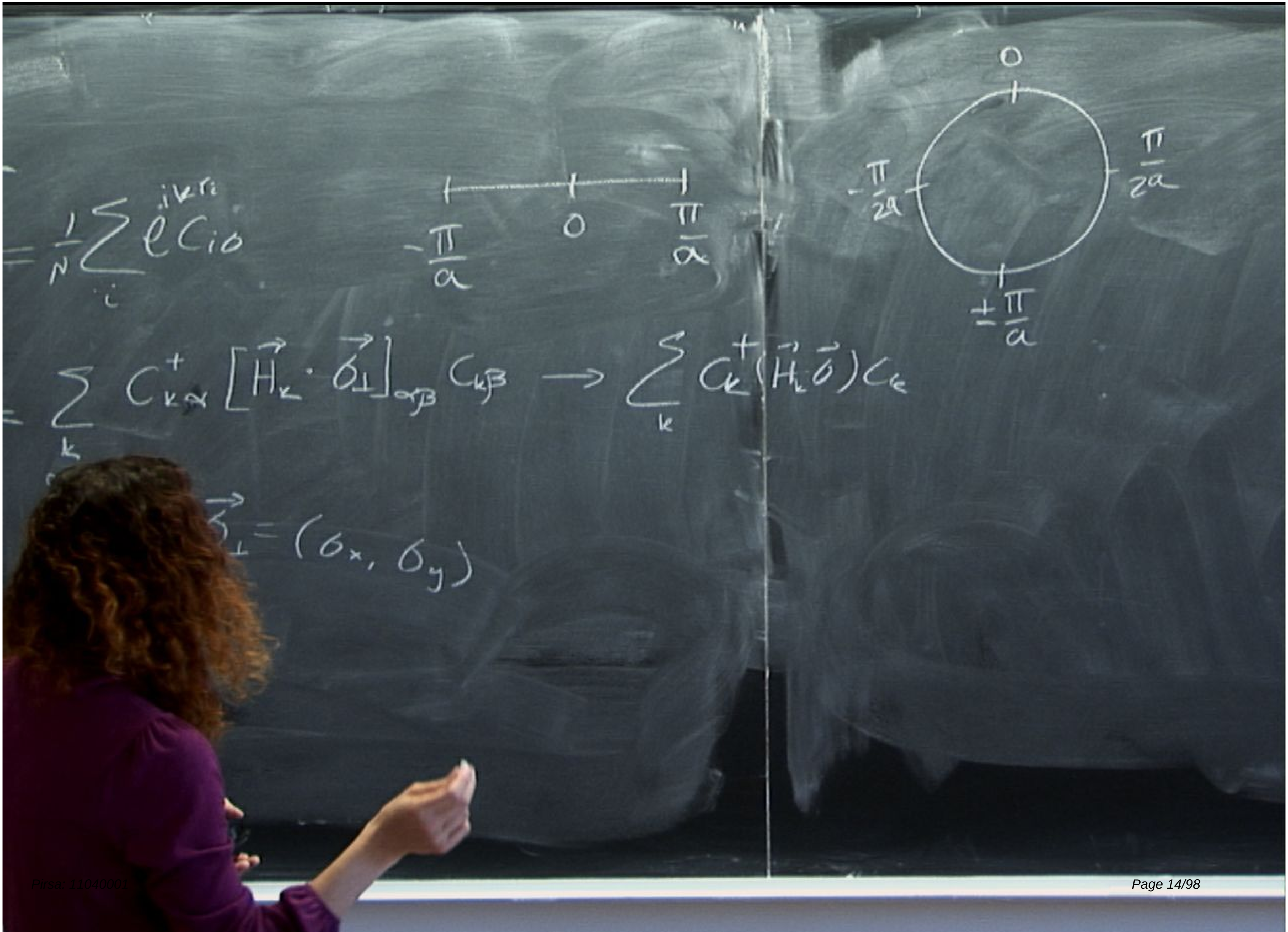
1.D

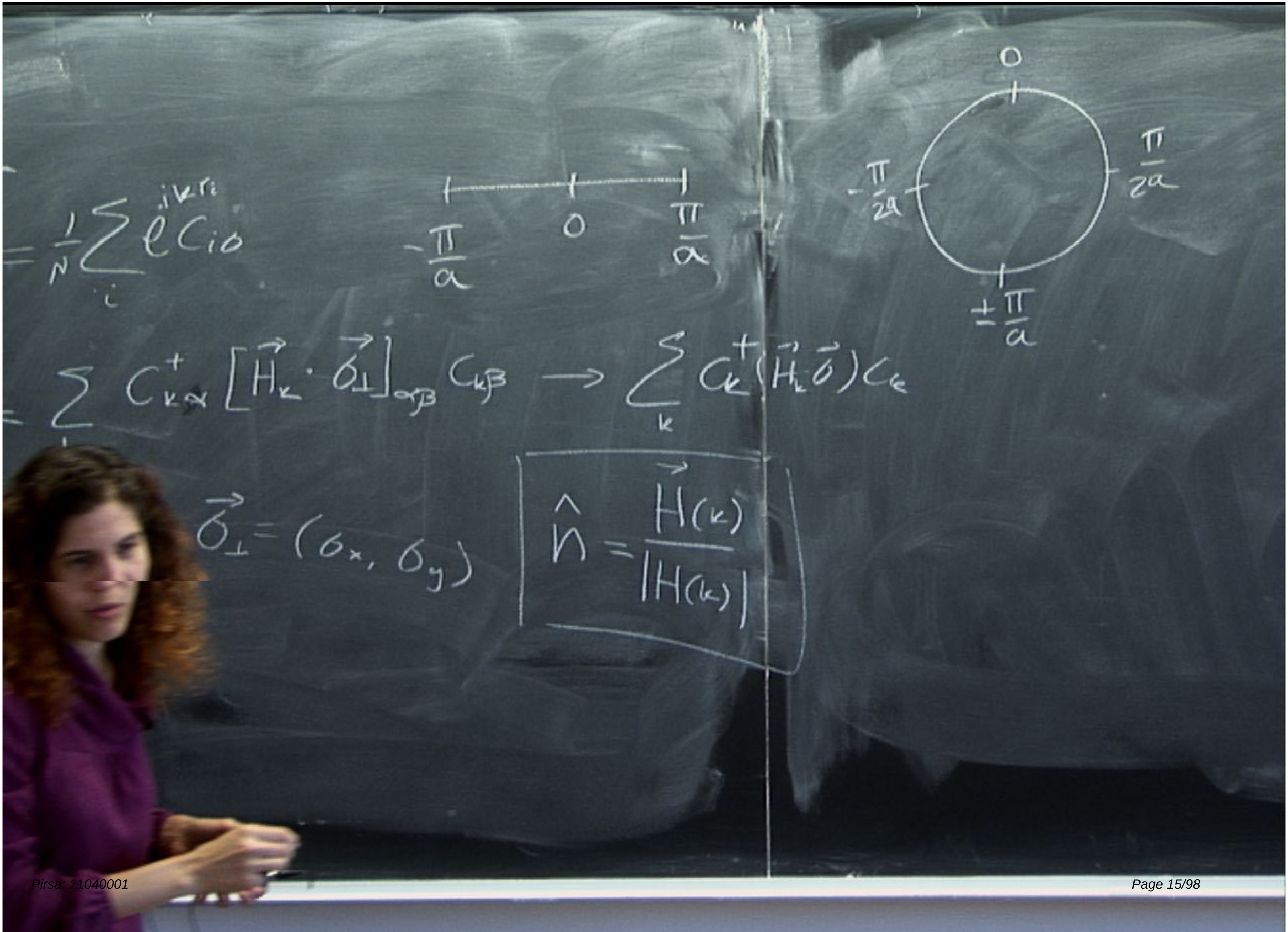
$$c_{k\sigma} = \frac{1}{N}\sum_i e^{ikr_i} c_{i\sigma}$$

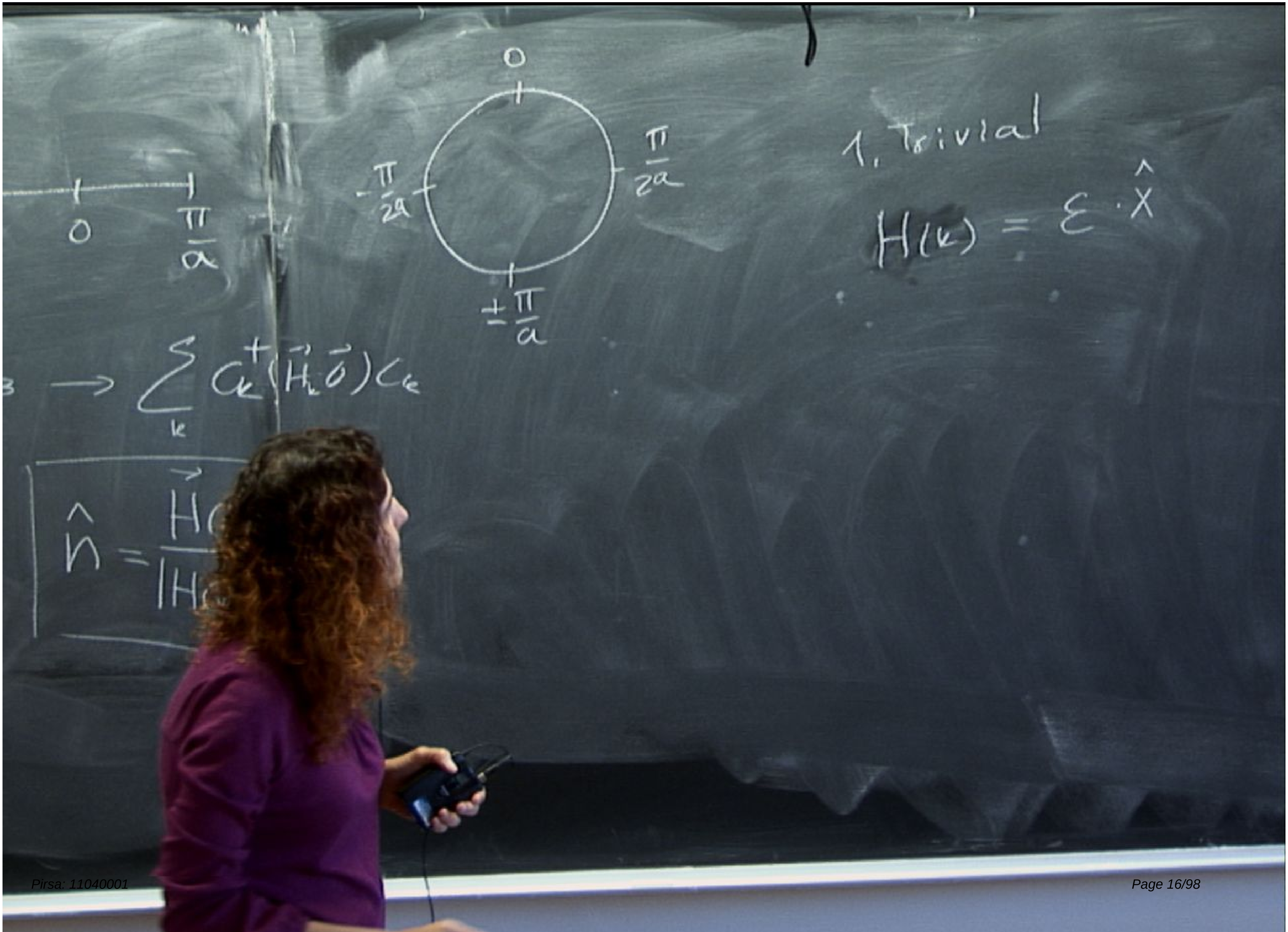
$$H = \sum_{\substack{k \\ \alpha\beta}} c^\dagger_{k\alpha}\left[\vec{H}_k\cdot\vec{\sigma}_\perp\right]_{\alpha\beta} c_{k\beta} \longrightarrow \sum_k c_k^\dagger(\vec{H}_k\cdot\vec{\sigma})c_k$$

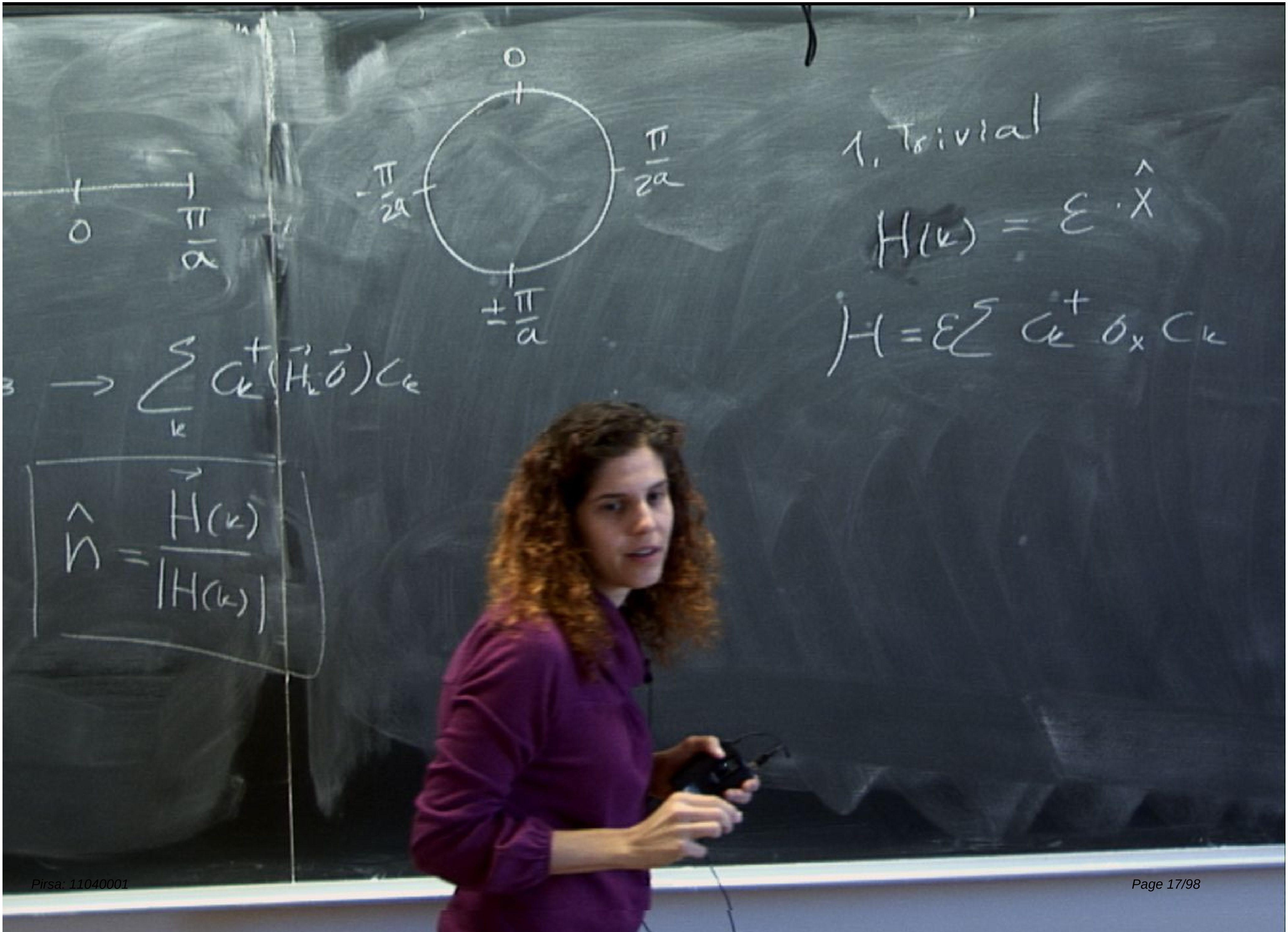
$$\vec{\sigma}_\perp = (\sigma_x, \sigma_y)$$

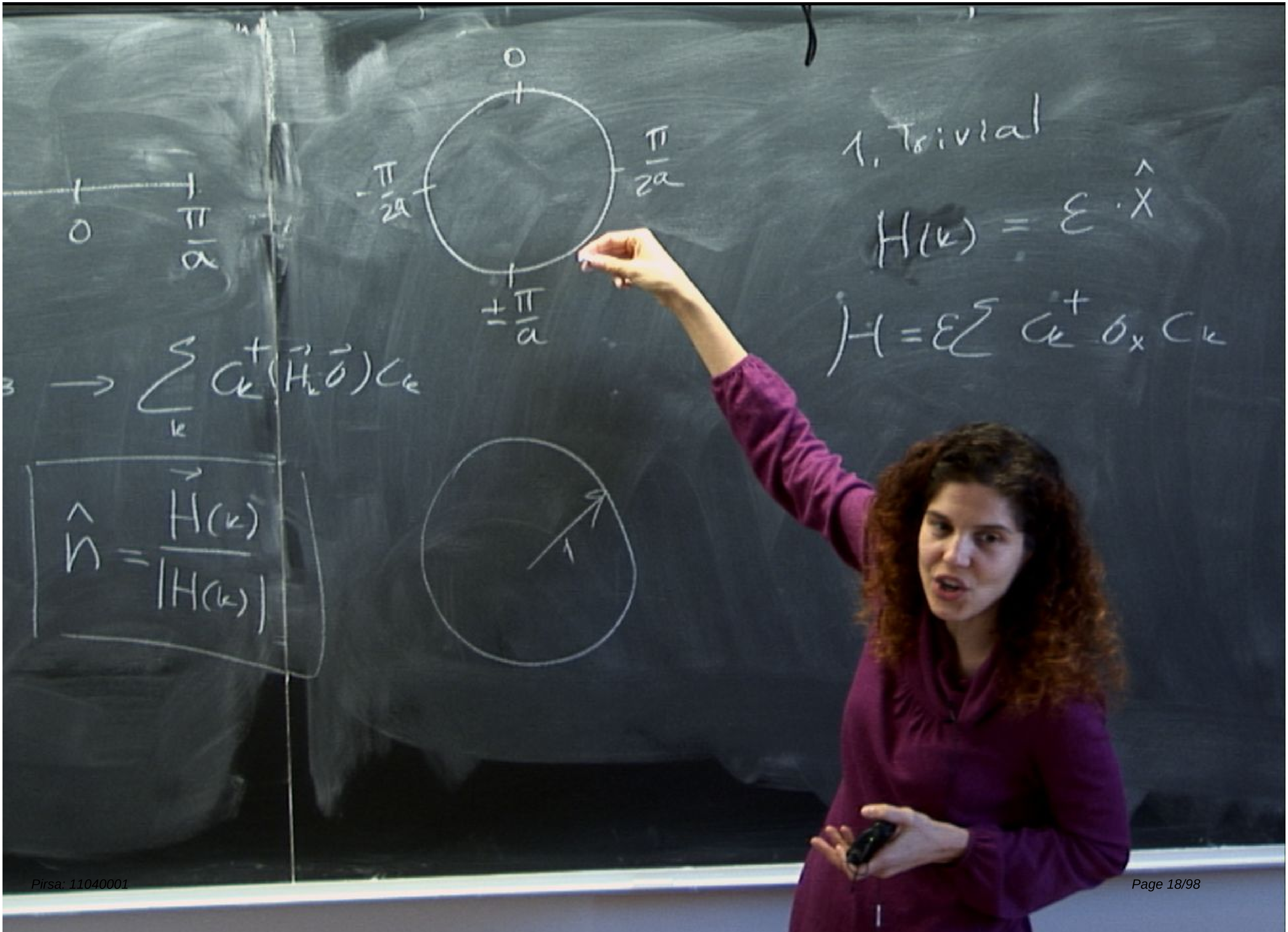

1.D
$C_{k\sigma} = \frac{1}{N}\sum_i e^{ikr_i} C_{i\sigma}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$H = \sum_{k,\alpha\beta} C^+_{k\alpha} [\vec{H}_k \cdot \vec{\sigma}_\perp]_{\alpha\beta} C_{k\beta} \longrightarrow \sum_k C^+_k (\vec{H}_k \cdot \vec{\sigma}) C_k$
$\vec{\sigma}_\perp = (\sigma_x, \sigma_y)$
0

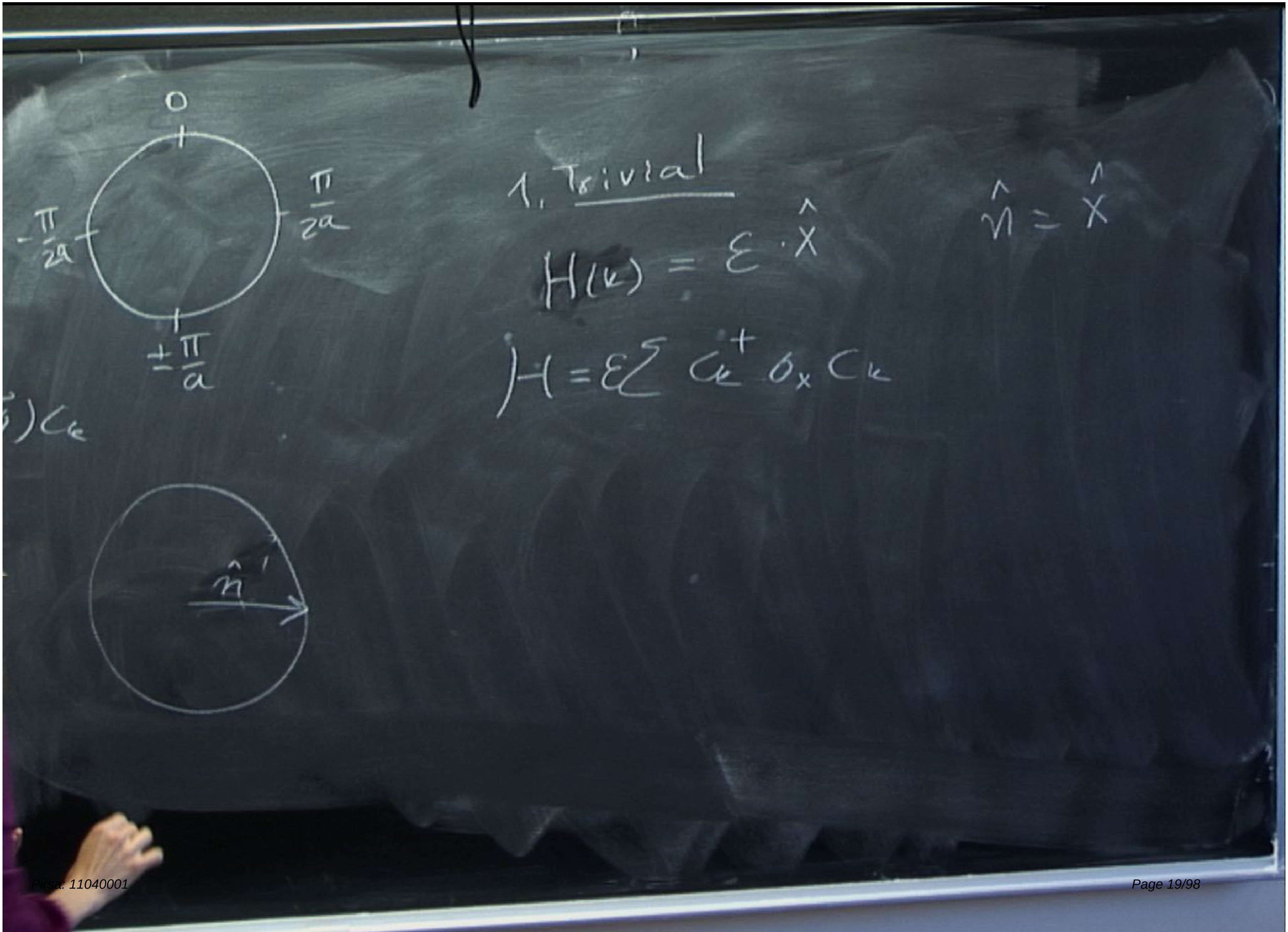

0
π/a
−π/a
π/2a
−π/2a
±π/a

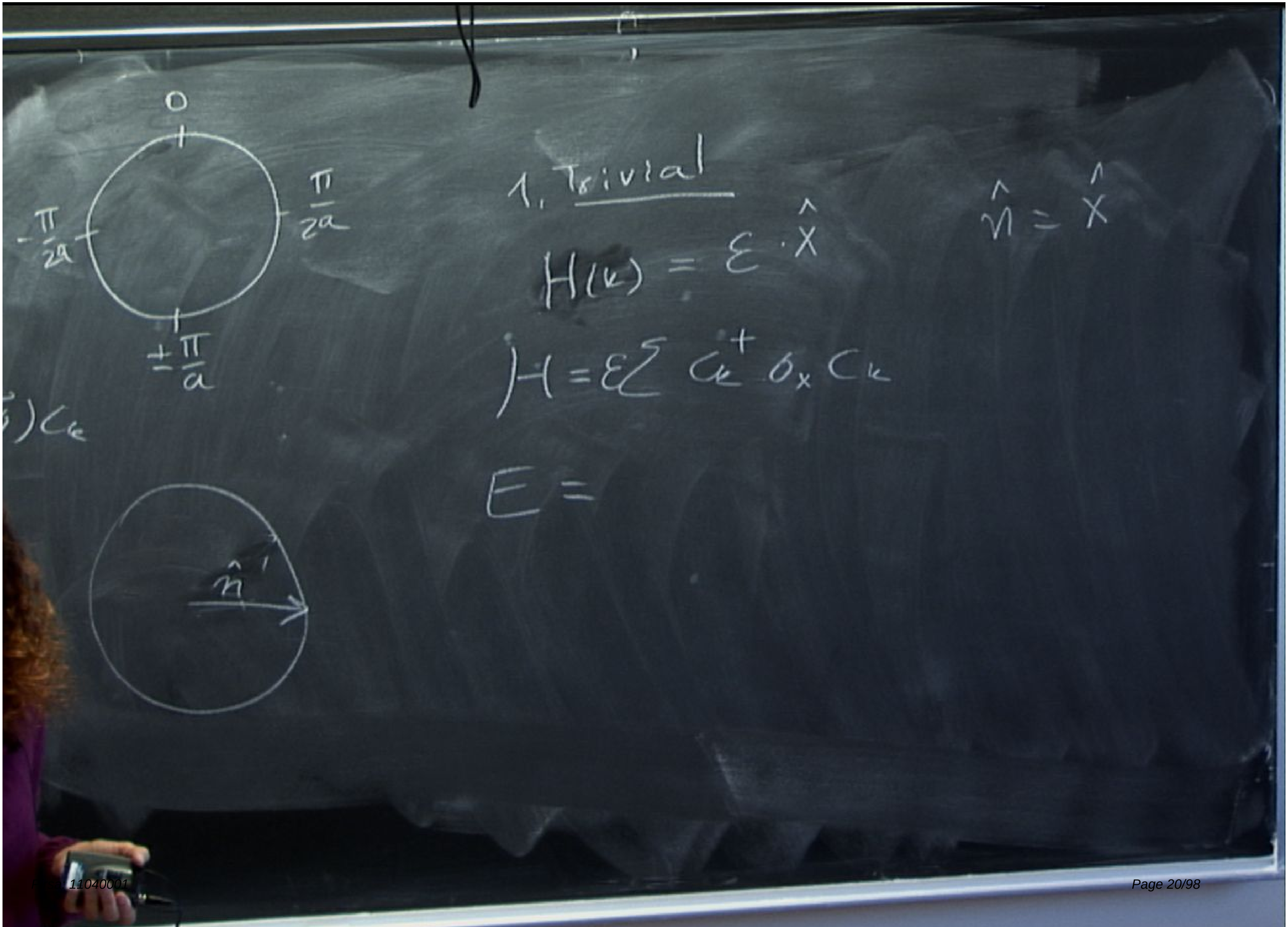

= 1/N Σ_i e^{ik r_i} C_iσ
−π/a 0 π/a
0
−π/2a π/2a
±π/a
= Σ C†_kα [H_k · σ⊥]_αβ C_kβ → Σ_k C†_k (H_k · σ) C_k
σ⊥ = (σx, σy)
n̂ = H(k) / |H(k)|

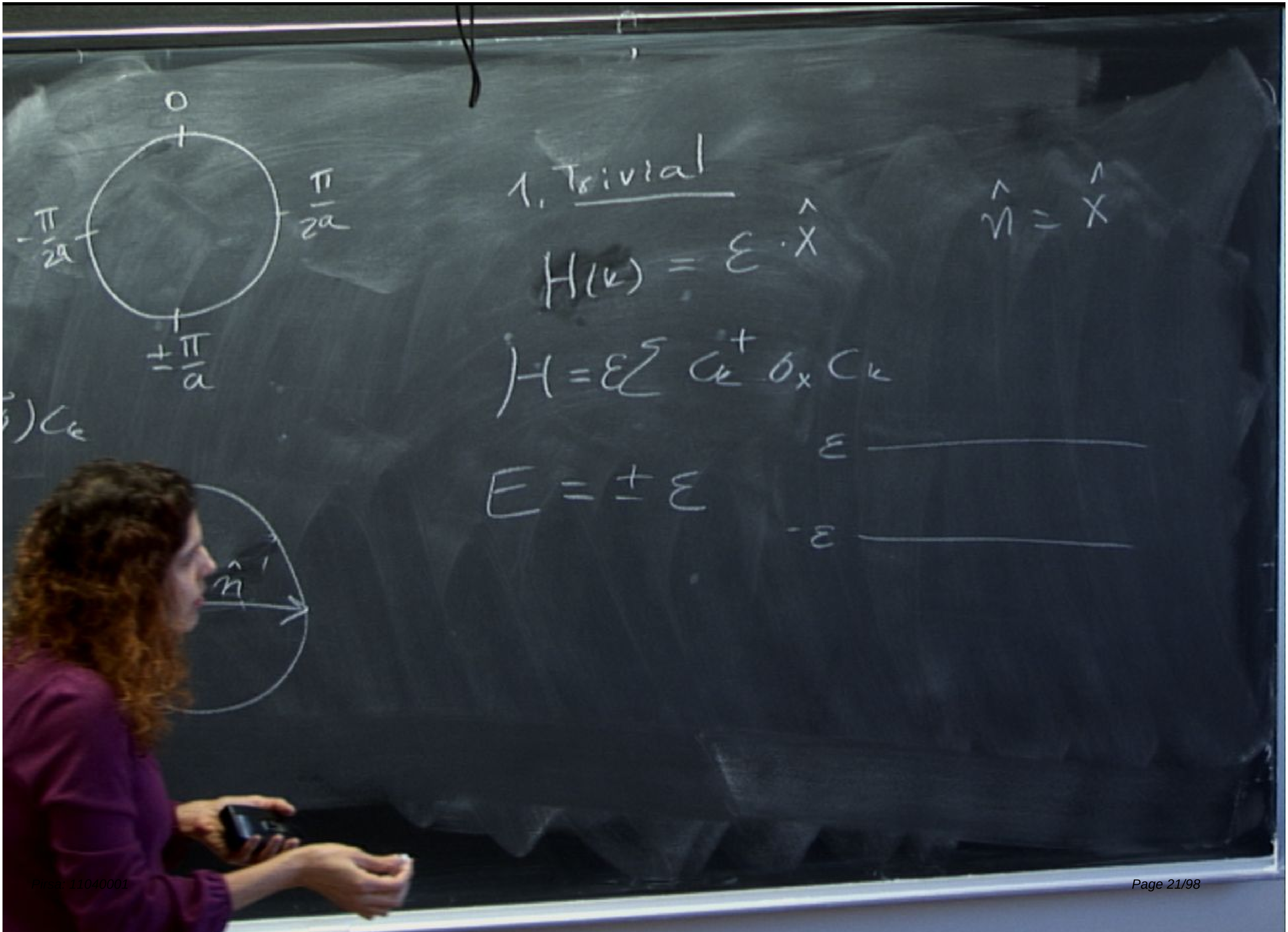

0
π/a
0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂

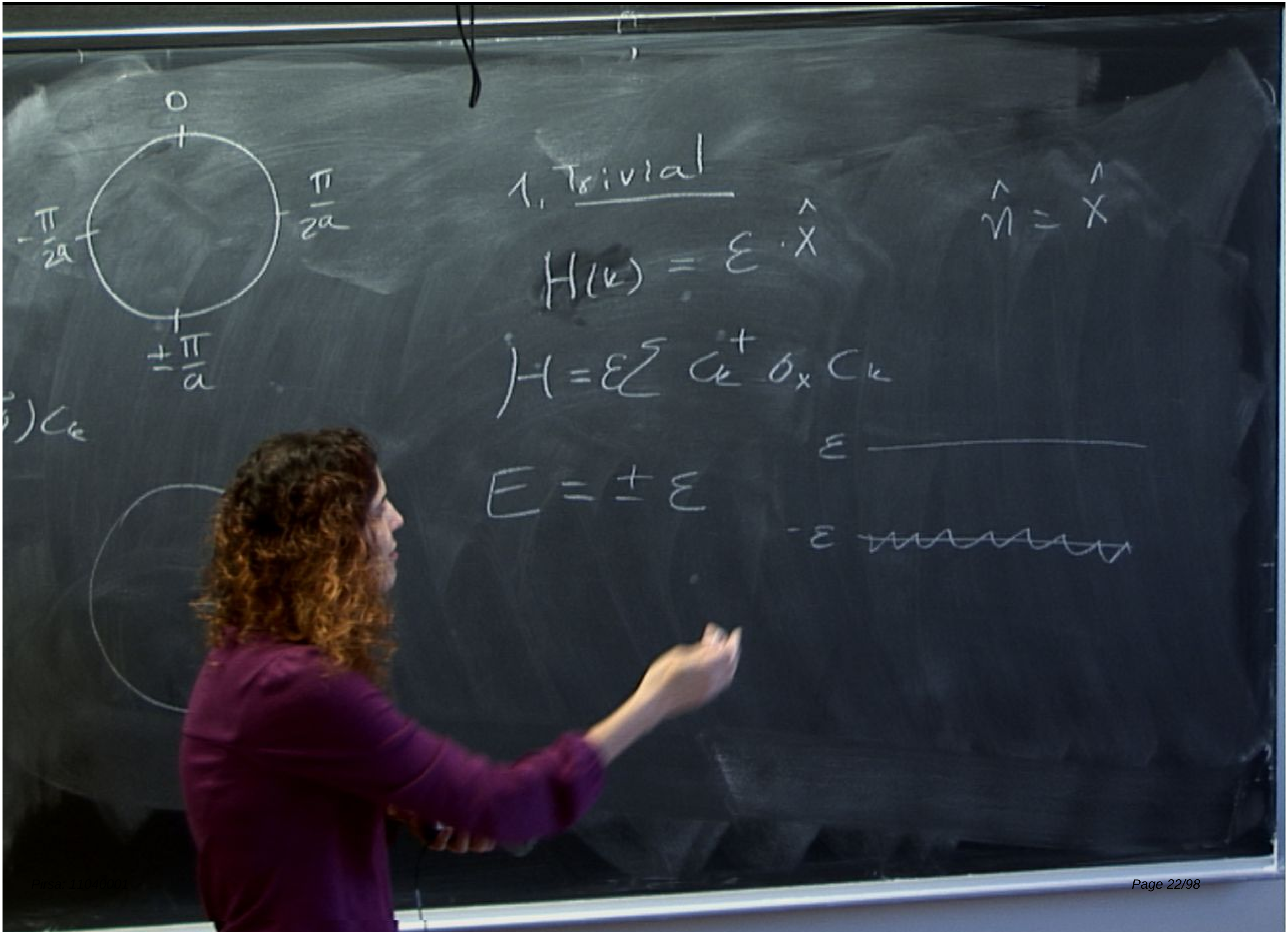

1. Trivial
H(k) = ε · X̂
H = ε Σ c_k† σ_x c_k
Σ_k c_k† (H⃗_k · σ⃗) c_k
n̂ = H⃗(k) / |H(k)|
0
π/a
π/2a
-π/2a
±π/a

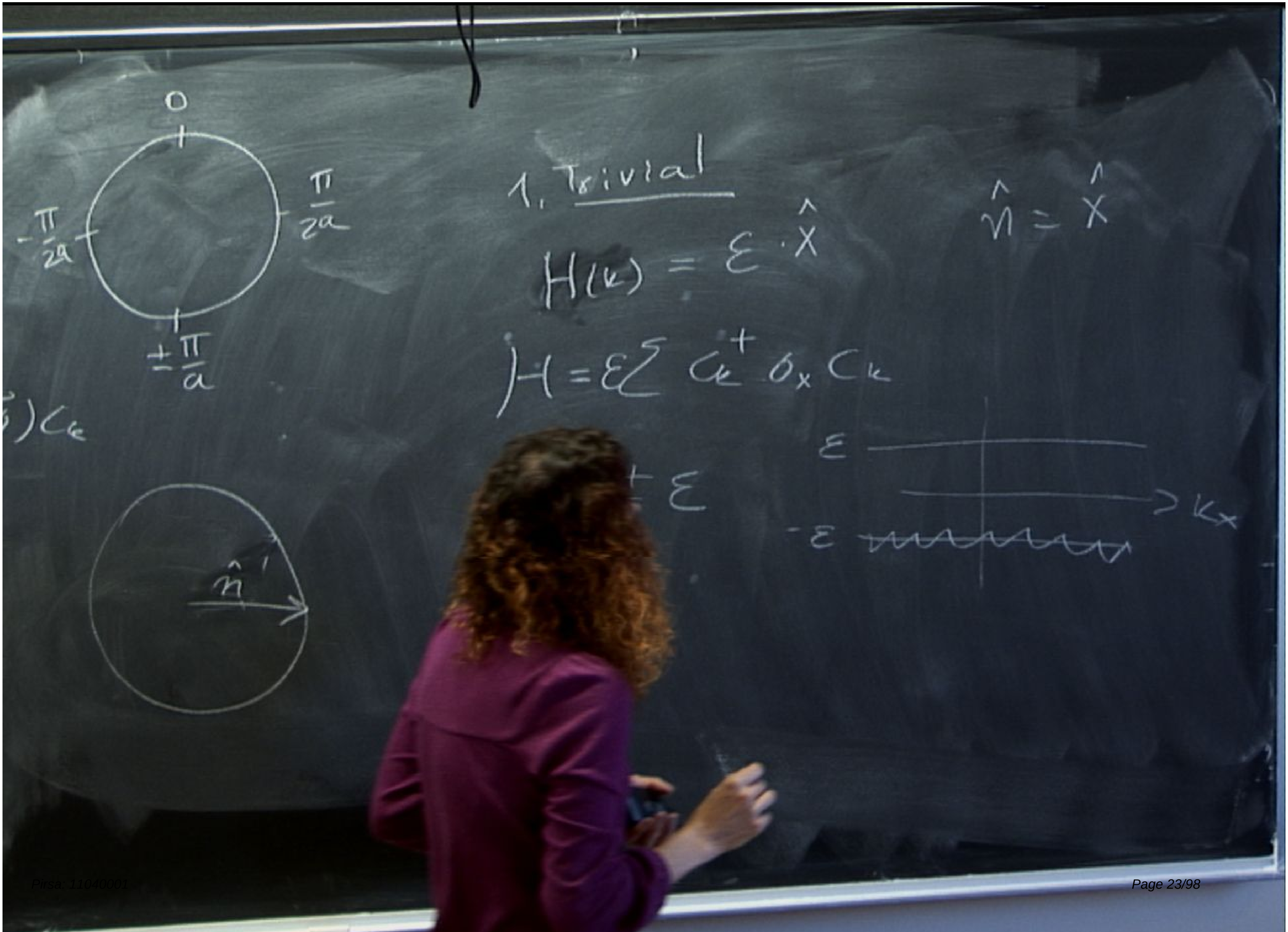

1. Trivial
H(k) = ε · X̂
H = ε Σ c_k^+ σ_x c_k
Σ_k c_k^+ (H_k · σ) c_k
n̂ = H(k) / |H(k)|
0
π/a
-π/2a
π/2a
±π/a
1

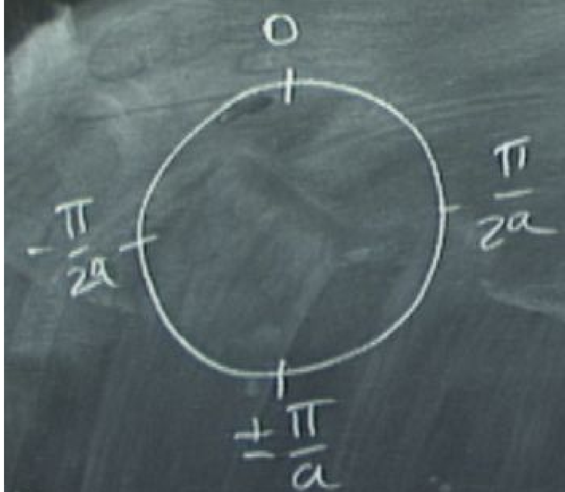

0
π/2a
-π/2a
±π/a
)c_k
n̂
1. Trivial
H(k) = ε · X̂
H = ε Σ c_k^† σ_x c_k
n̂ = X̂


0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂
H = ε Σ c_k^† σ_x c_k
E =
n̂ = X̂
n̂


0
π/2a
-π/2a
±π/a
n̂
1. Trivial
H(k) = ε · X̂
H = ε Σ c_k^+ σ_x c_k
E = ± ε
ε
-ε
n̂ = X̂


0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂
n̂ = X̂
H = ε Σ c_k^+ σ_x c_k
E = ± ε
ε
-ε


0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂
n̂ = X̂
H = ε Σ c_k^+ σ_x c_k
±ε
ε
-ε
k_x
n̂



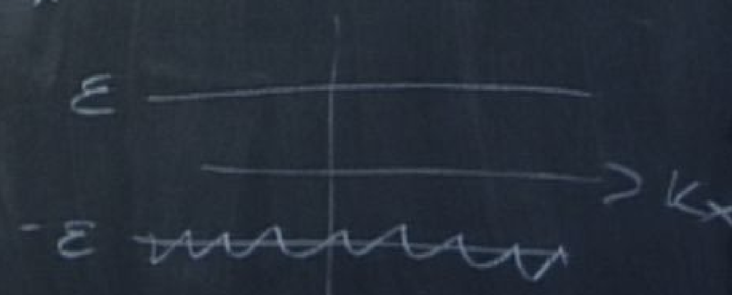
1. Trivial

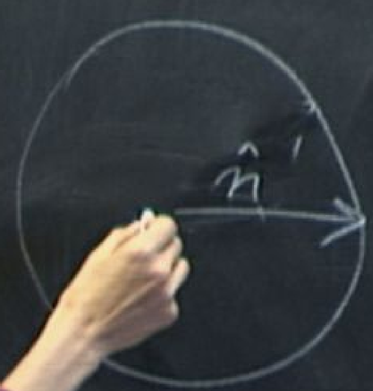
$$H(k) = \varepsilon \cdot \hat{X}$$

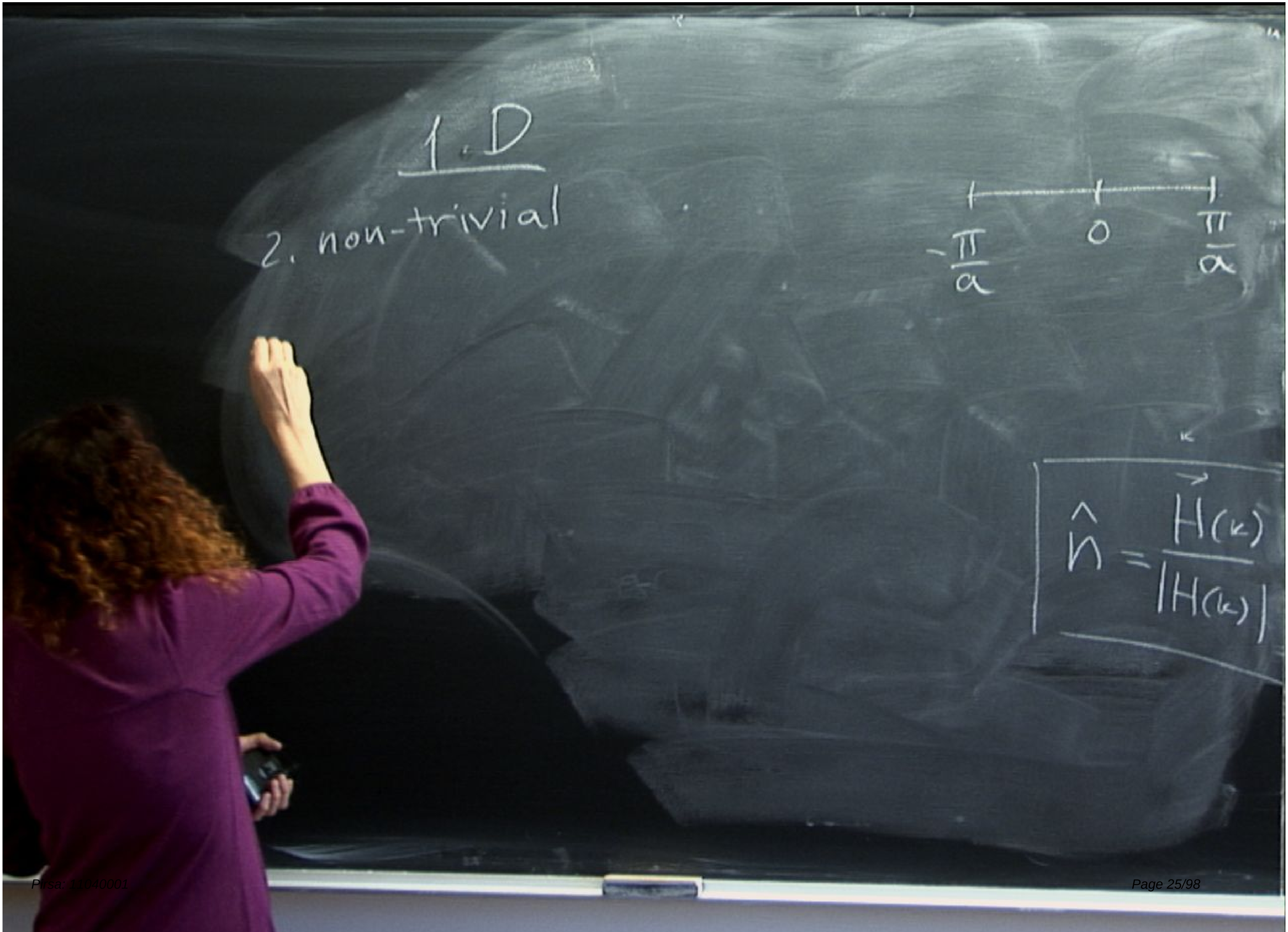
$$\hat{n} = \hat{X}$$

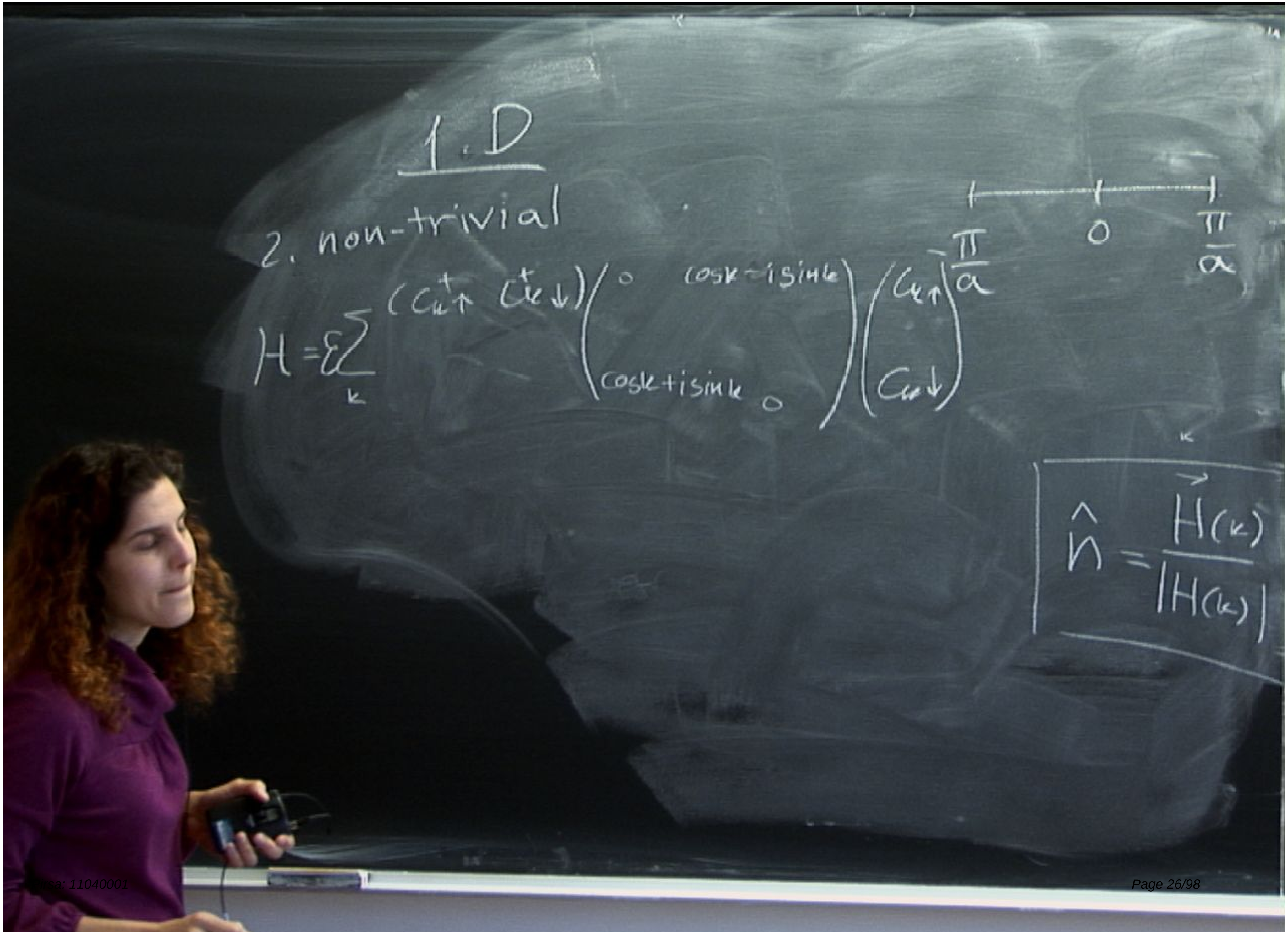
$$H = \varepsilon \sum c_k^{\dagger} \sigma_x c_k$$

$$E = \pm \varepsilon$$






1. D
2. non-trivial
-π/a
0
π/a
k
n̂ = H⃗(k) / |H(k)|


1. D
2. non-trivial
$H = \varepsilon \sum_k (c^\dagger_{k\uparrow}\ c^\dagger_{k\downarrow}) \begin{pmatrix} 0 & \cos k - i\sin k \\ \cos k + i\sin k & 0 \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \end{pmatrix}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$\hat{n} = \frac{\vec{H}(k)}{|H(k)|}$

1. D

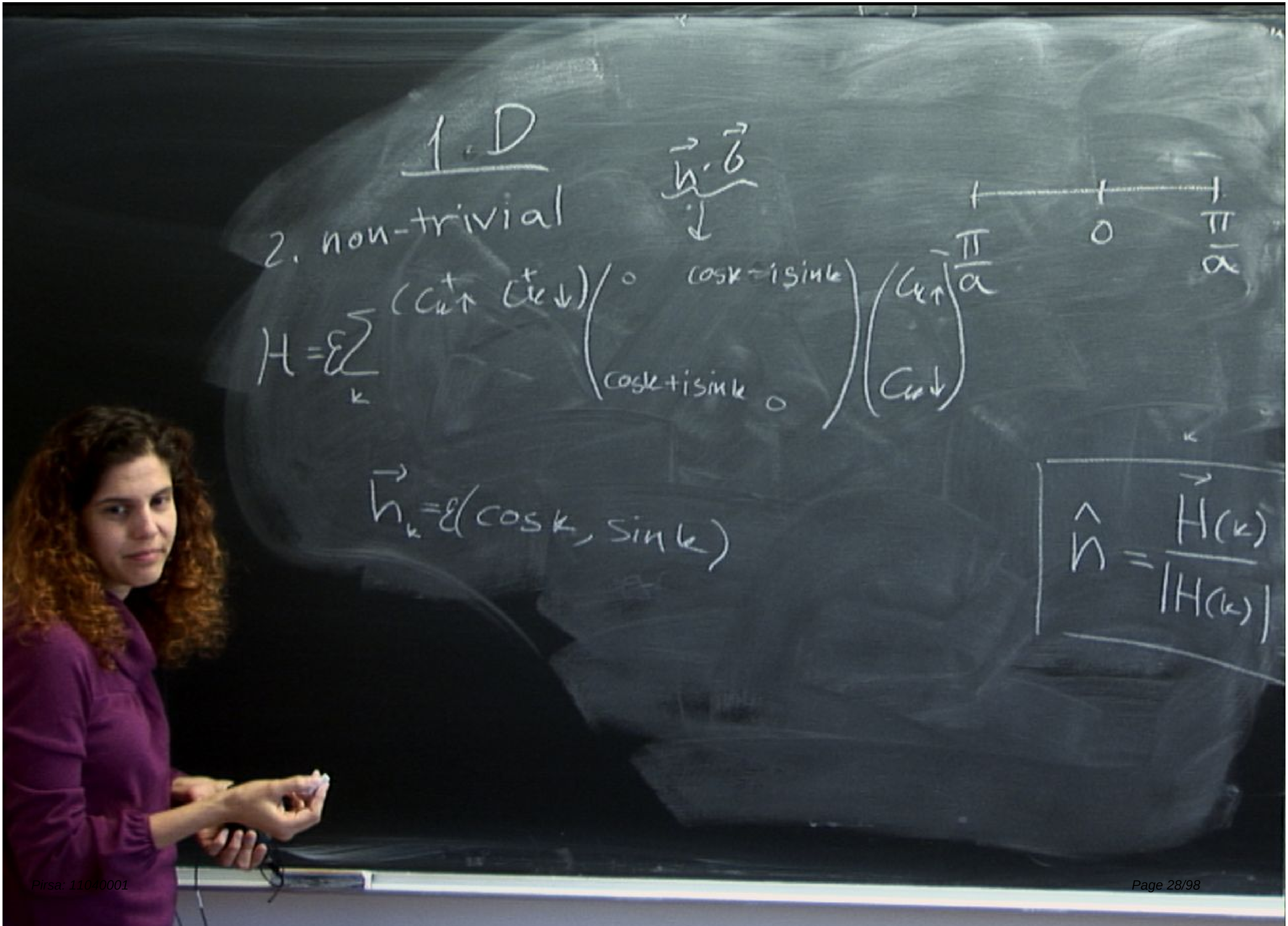
$\vec{h}\cdot\vec{\sigma}$

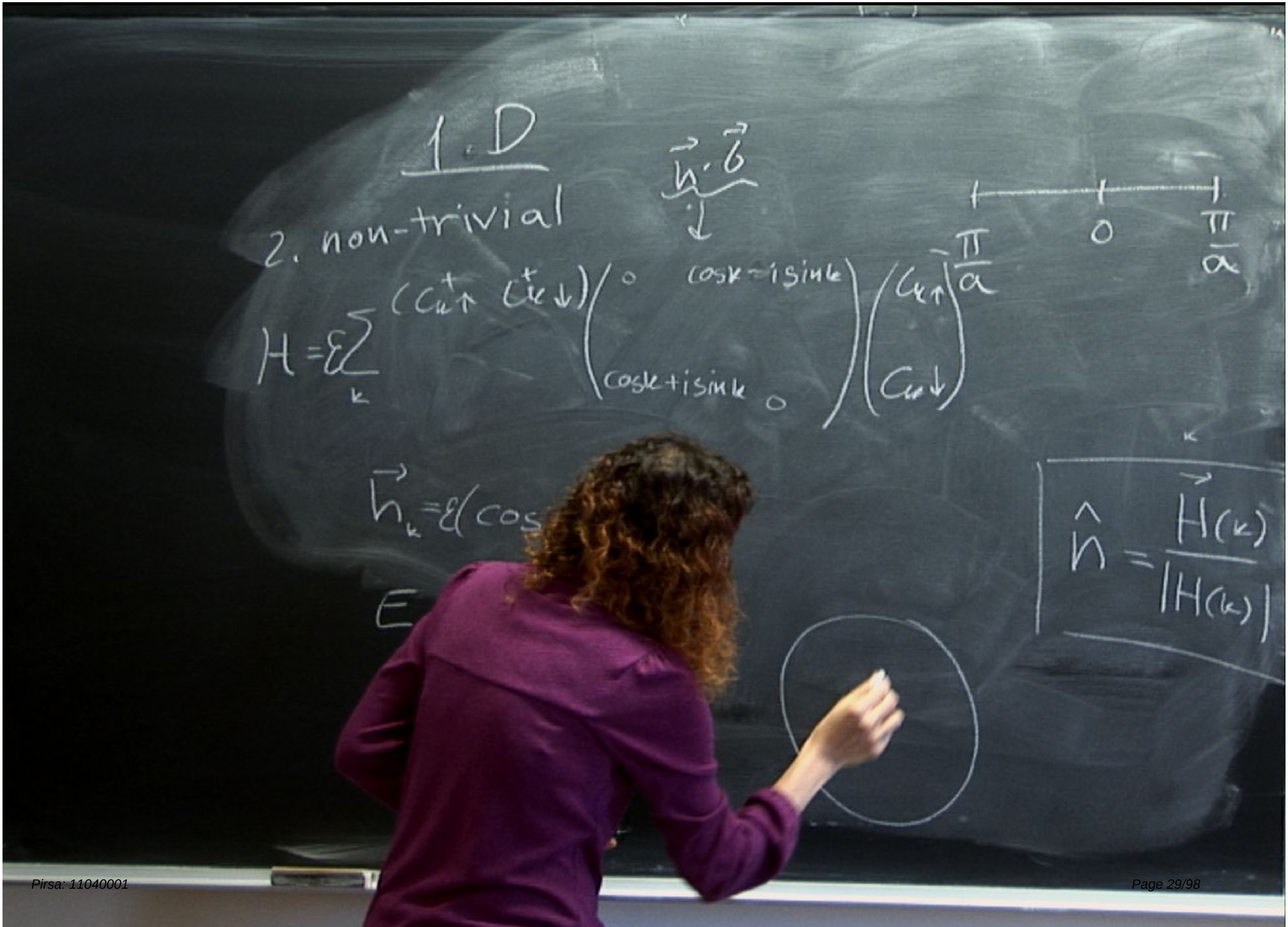
2. non-trivial

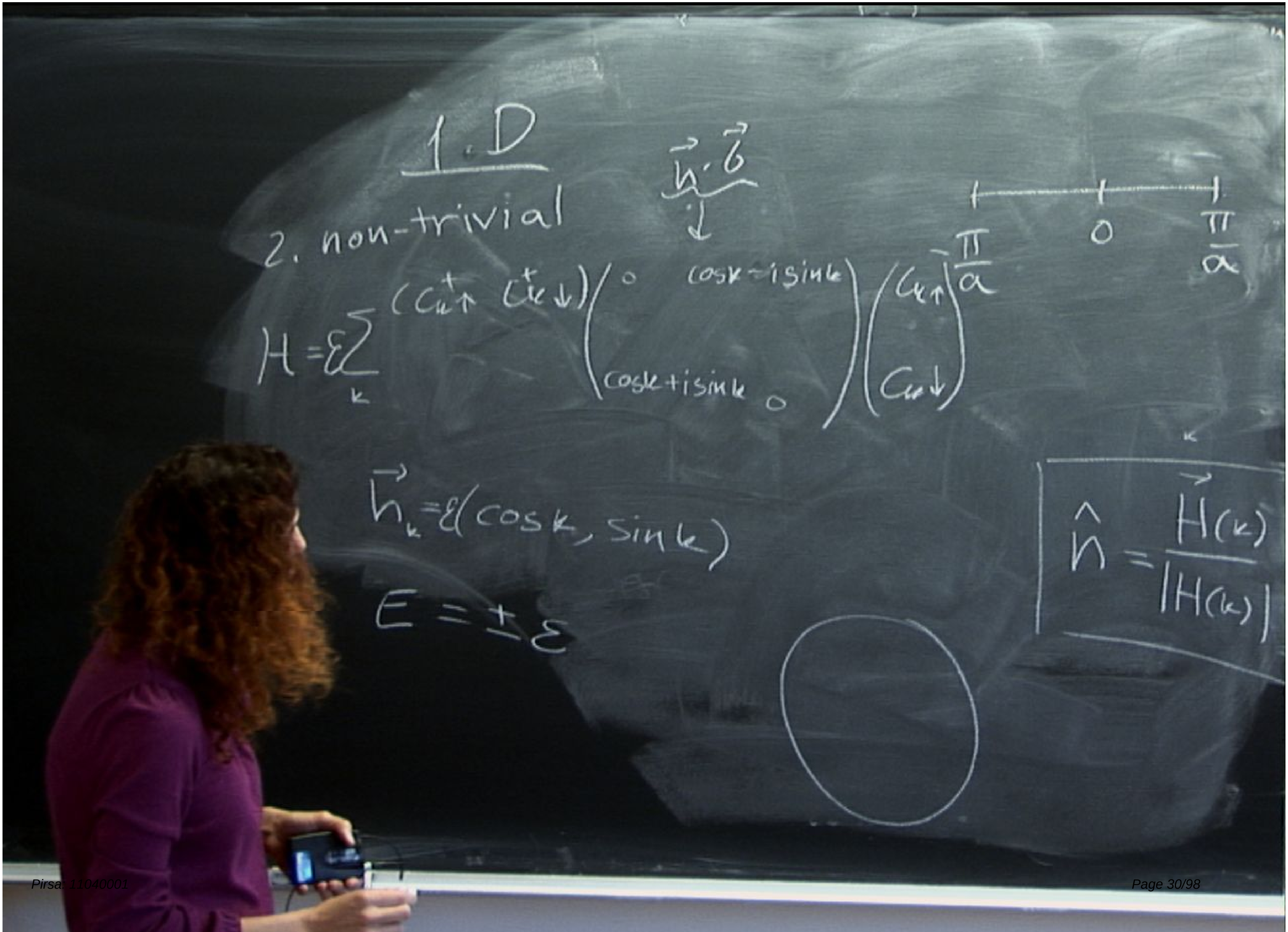
$$H = \varepsilon \sum_k \left(c^\dagger_{k\uparrow} \; c^\dagger_{k\downarrow} \right) \begin{pmatrix} 0 & \cos k - i \sin k \\ \cos k + i \sin k & 0 \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \end{pmatrix}$$

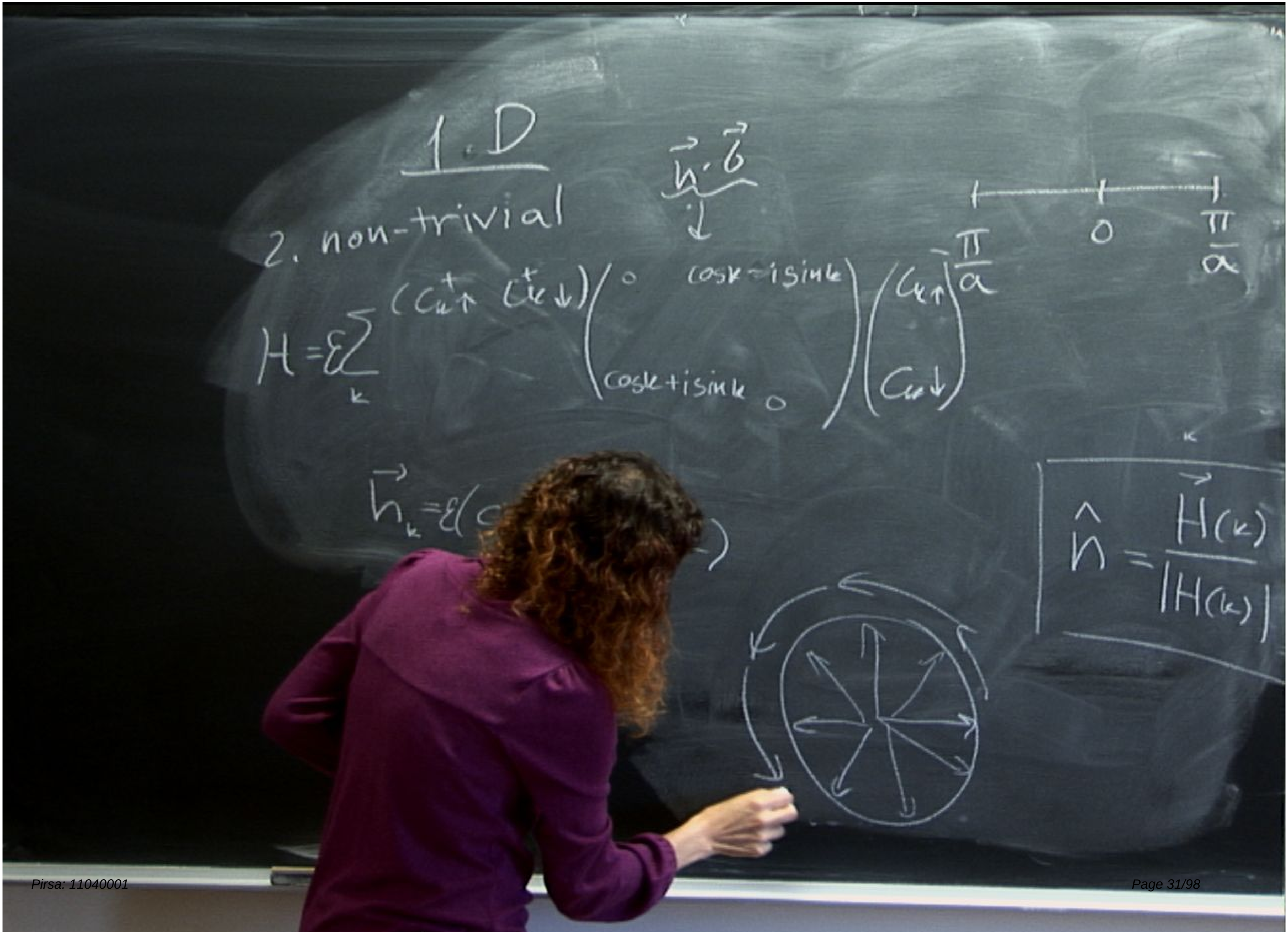
$-\frac{\pi}{a}$

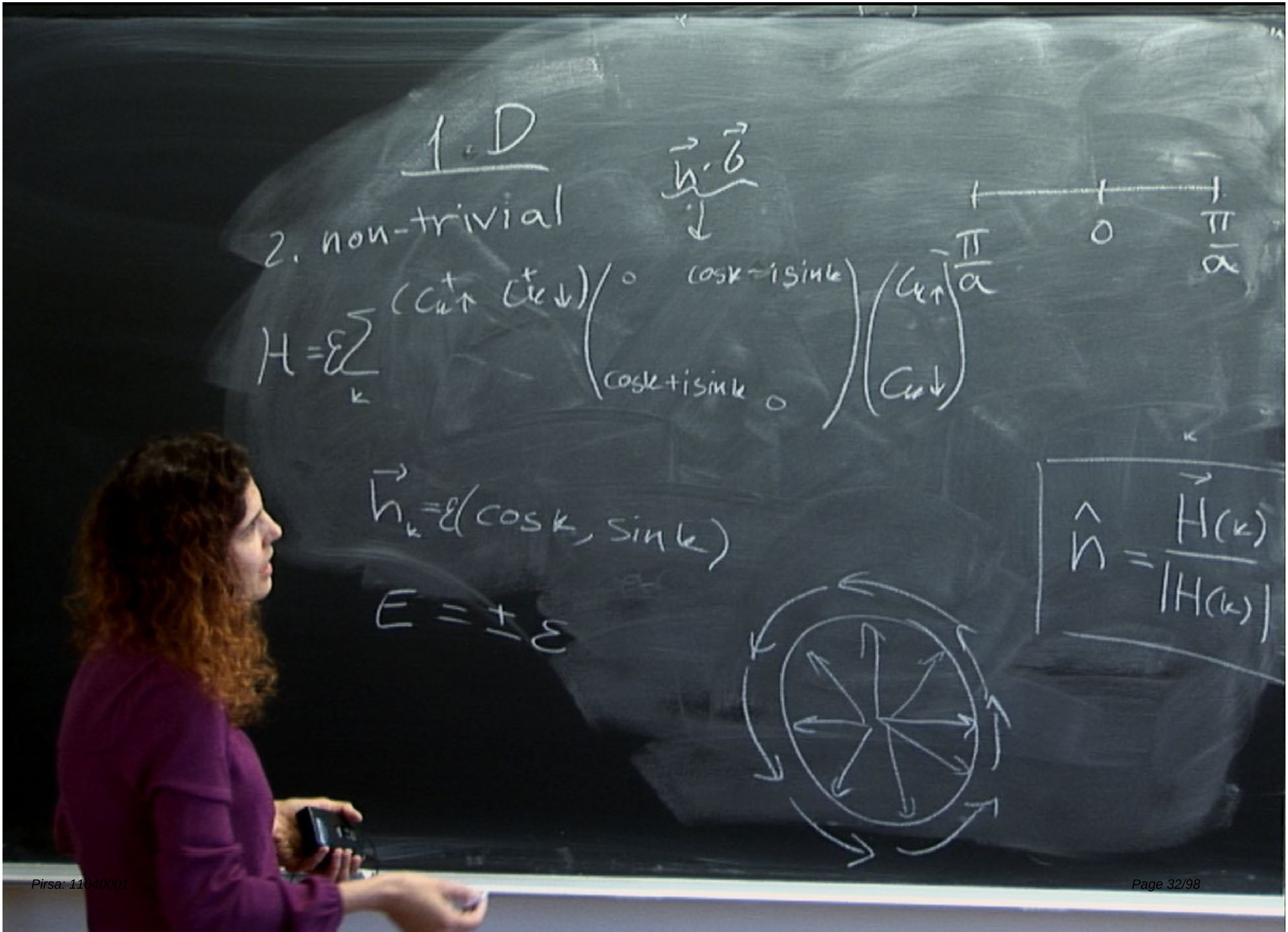
$\vec{h}_k =$

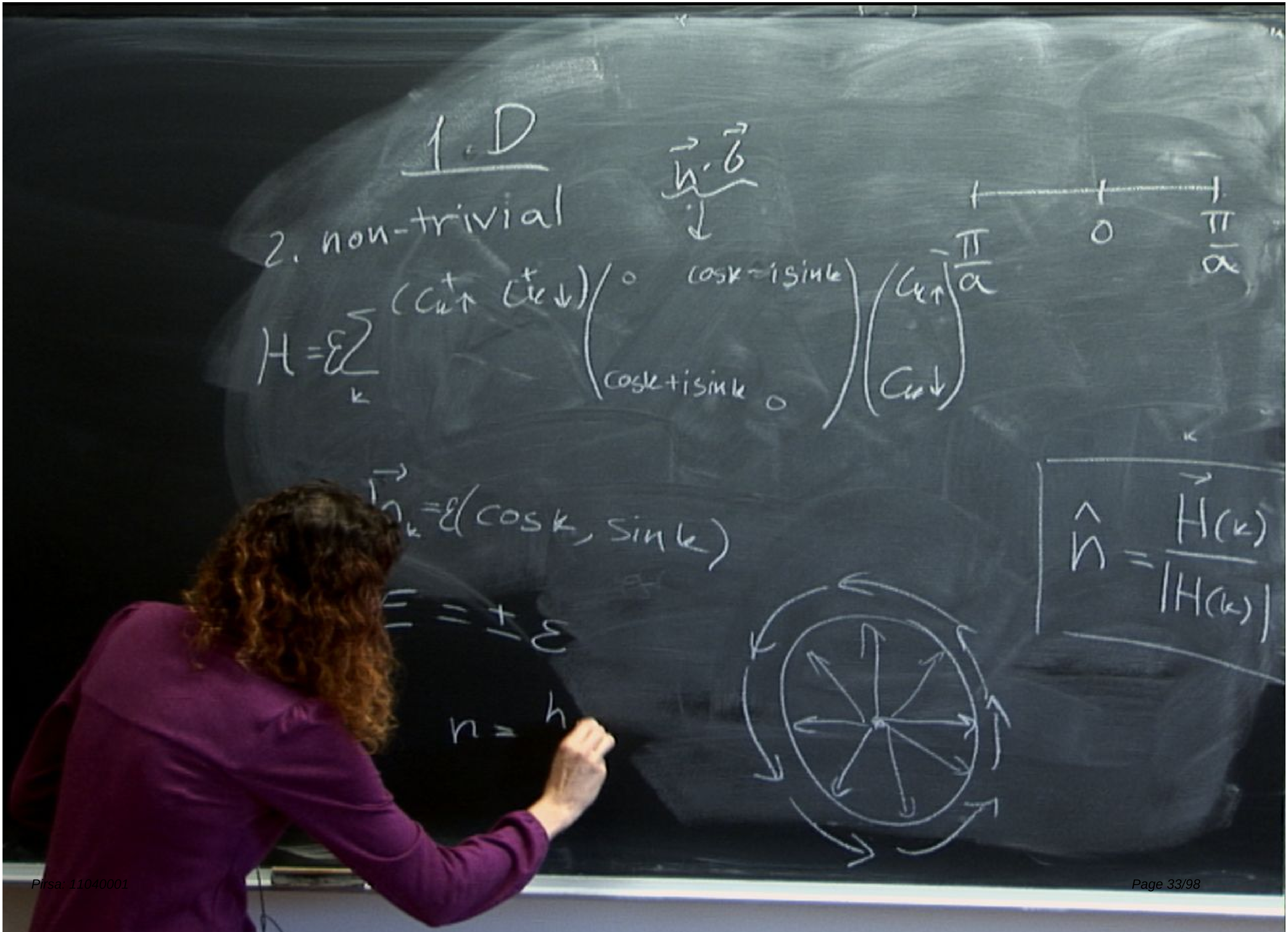

1. D
2. non-trivial
$\vec{h}\cdot\vec{\sigma}$
$H = \varepsilon \sum_k (c^\dagger_{k\uparrow}\ c^\dagger_{k\downarrow}) \begin{pmatrix} 0 & \cos k - i \sin k \\ \cos k + i \sin k & 0 \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \end{pmatrix}$
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$\vec{h}_k = \varepsilon(\cos k, \sin k)$
$\hat{n} = \frac{\vec{H}(k)}{|H(k)|}$

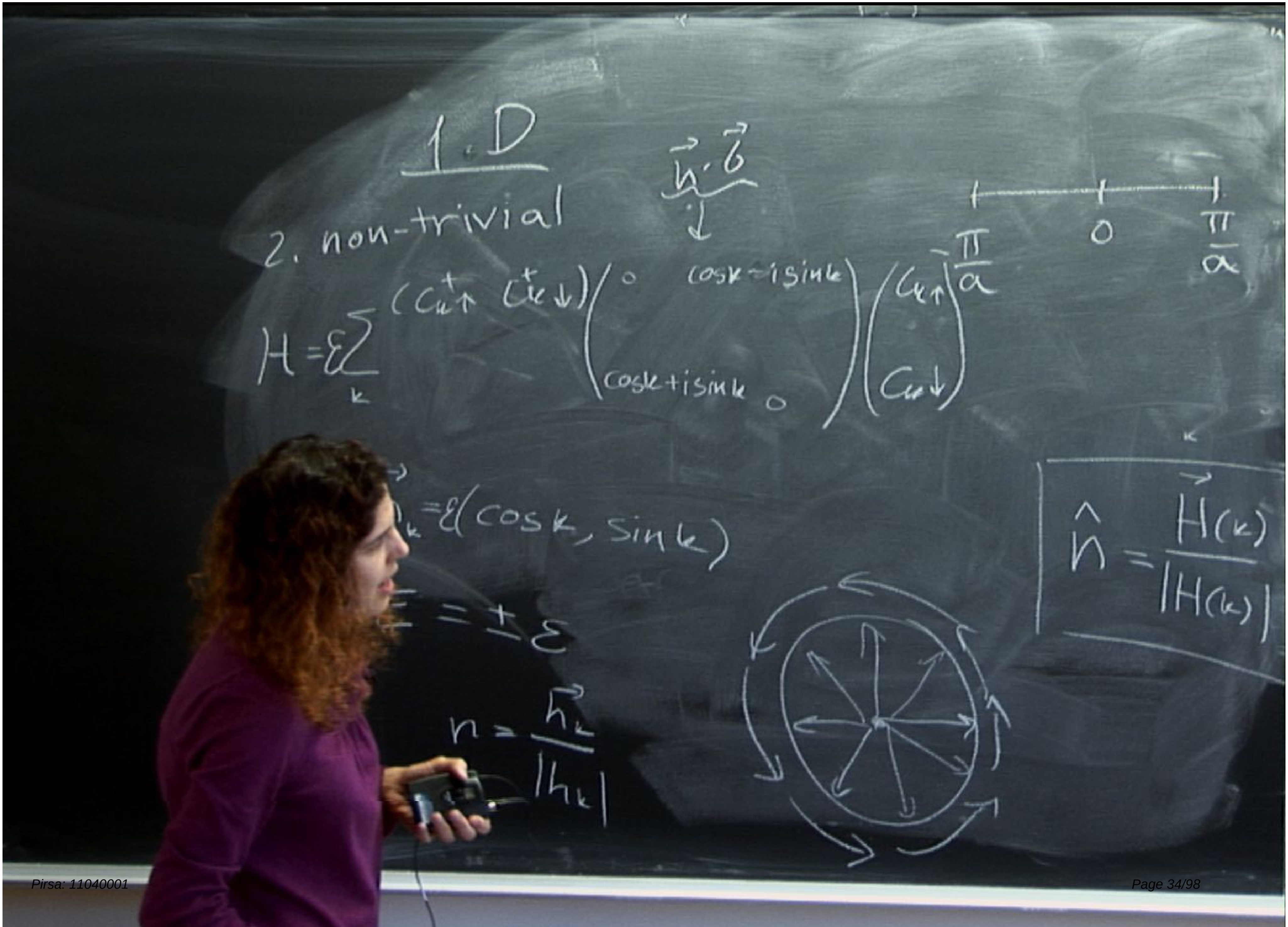

1.D
2. non-trivial
n·σ
H = ε Σ (c†k↑ c†k↓) (0 cos k − i sin k ; cos k + i sin k 0) (ck↑ ; ck↓)
−π/a 0 π/a
h_k = ε(cos
E
n̂ = H(k) / |H(k)|

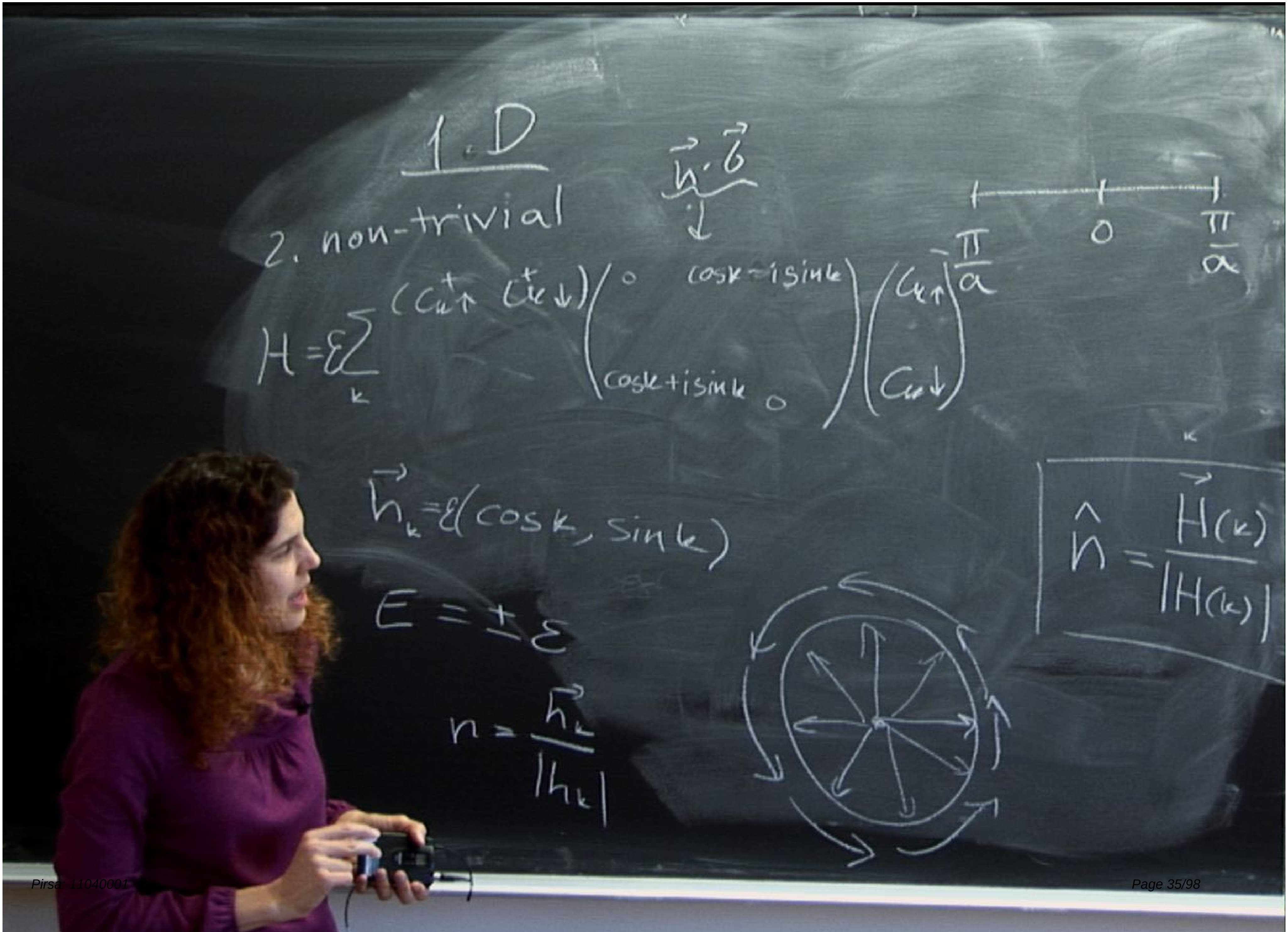

1. D
h · σ
2. non-trivial
H = ε Σ_k (c†_k↑ c†_k↓) (0 cosk − isink ; cosk + isink 0) (c_k↑ ; c_k↓)
−π/a 0 π/a
h_k = ε(cos k, sin k)
E = ±ε
n̂ = H(k) / |H(k)|

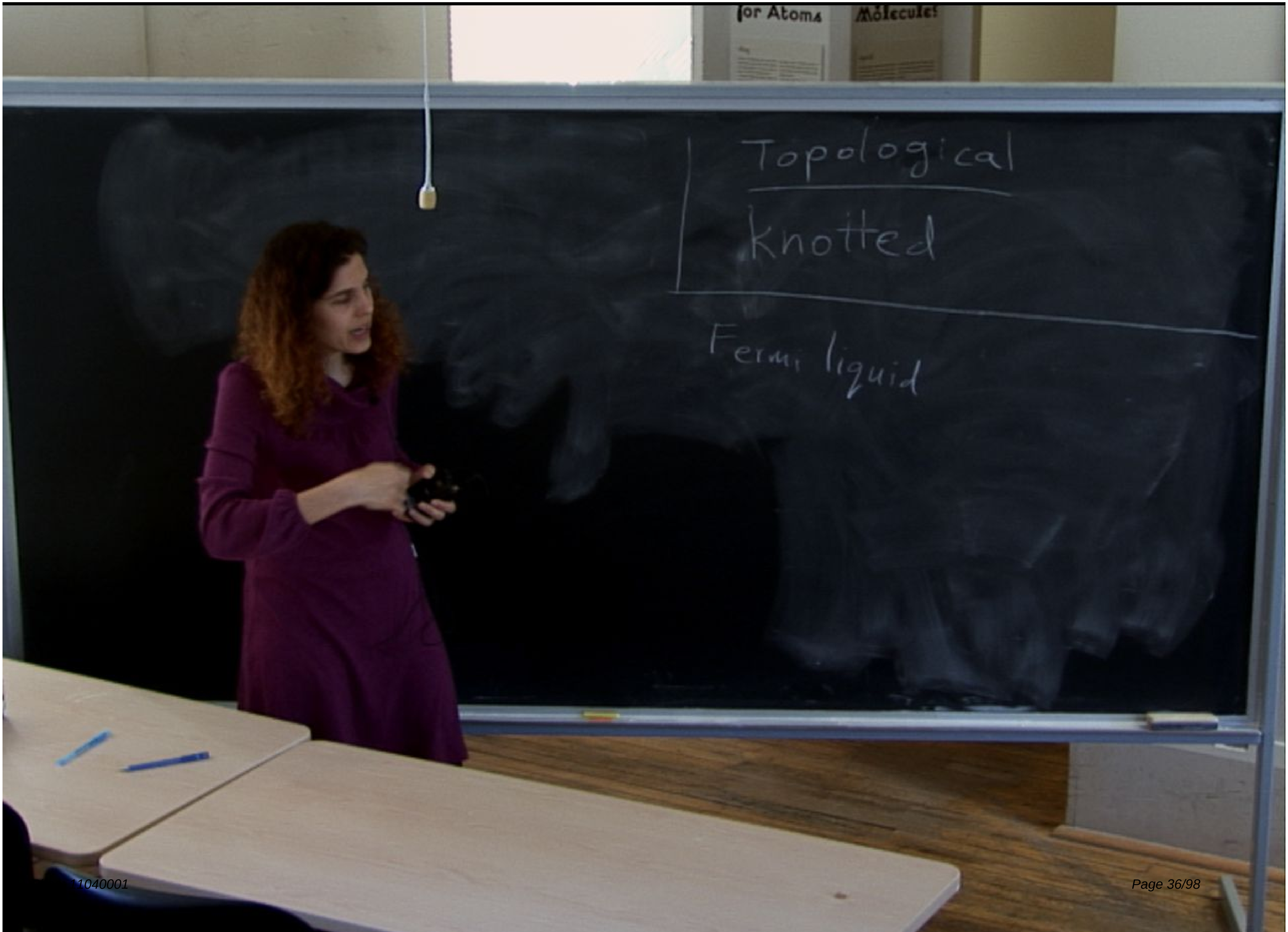

1.D
2. non-trivial
H = ε Σ_k (c†_k↑ c†_k↓) (0 cosk − isink ; cosk + isink 0) (c_k↑ ; c_k↓)
h_k = ε(c
-π/a 0 π/a
n̂ = H(k) / |H(k)|

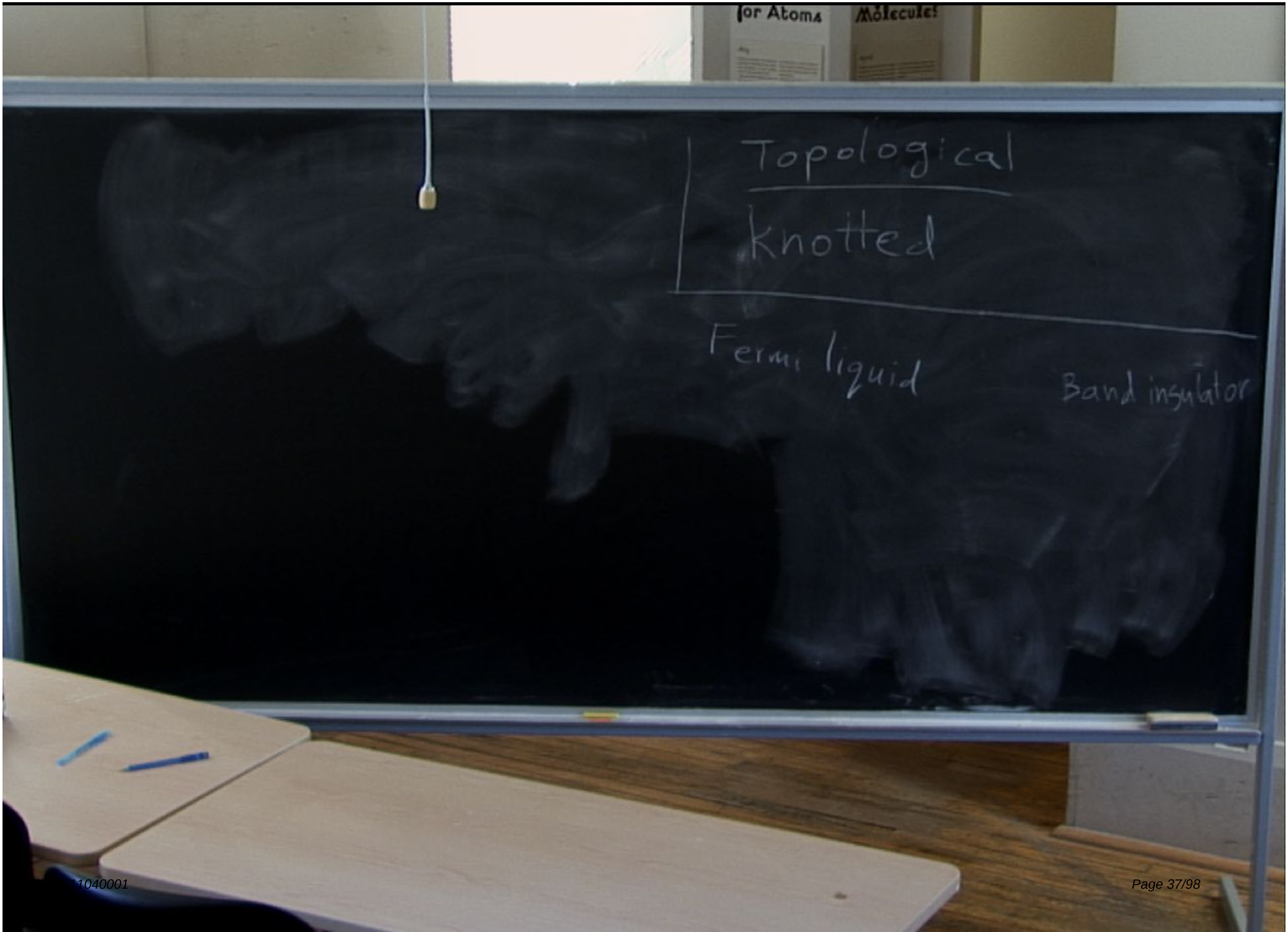

1.D
2. non-trivial
h·σ
H = ε Σ_k (c†_{k↑} c†_{k↓}) (0 cosk − isink ; cosk + isink 0) (c_{k↑} ; c_{k↓})
−π/a 0 π/a
h_k = ε(cos k, sin k)
E = ± ε
n̂ = H(k) / |H(k)|

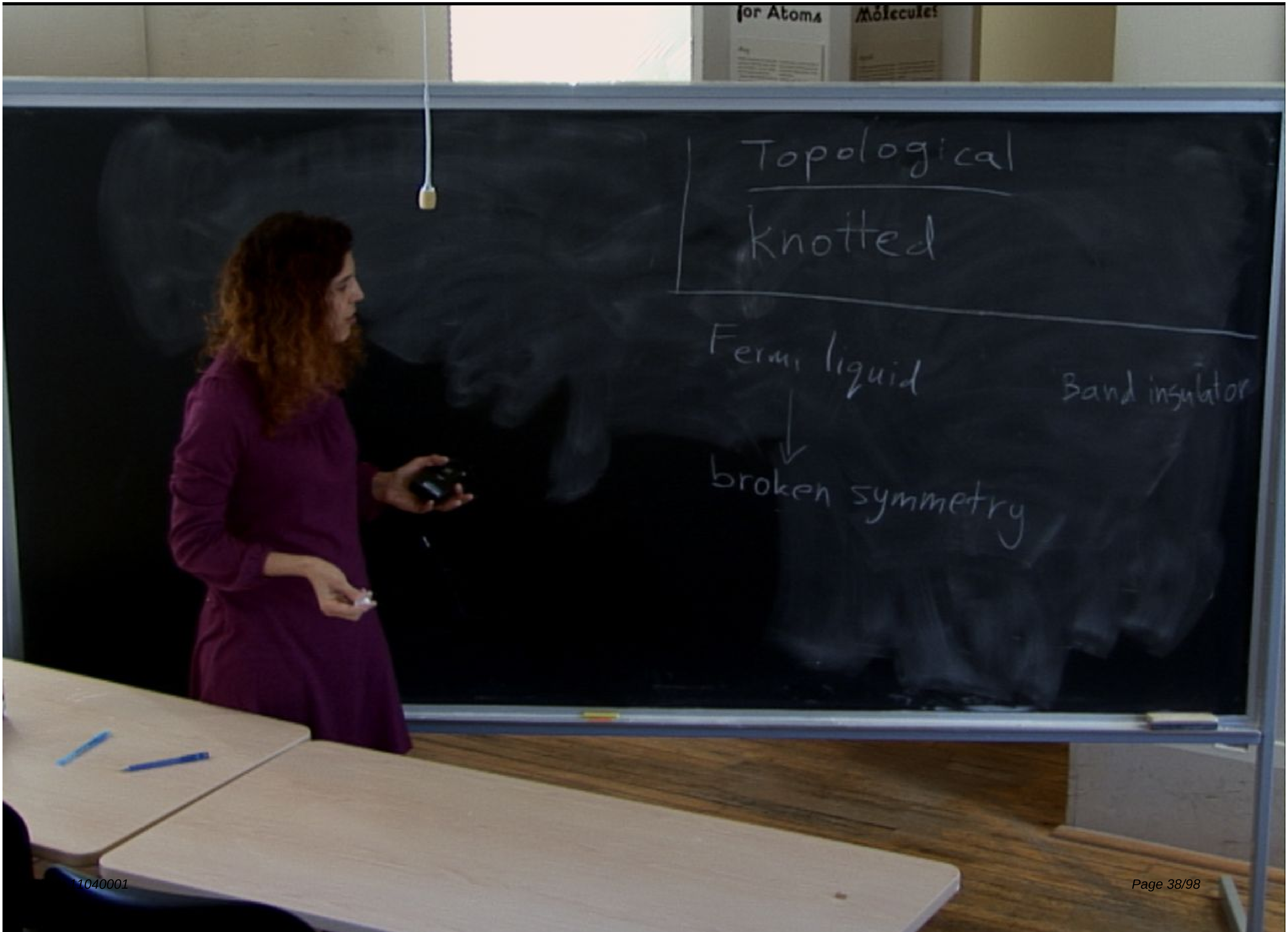

1.D
2. non-trivial
n·σ
−π/a
0
π/a
H = ε Σ_k (c†_k↑ c†_k↓) (0, cosk − isink; cosk + isink, 0) (c_k↑; c_k↓)
n_k = ε(cos k, sin k)
E = ± ε
n = h
n̂ = H(k)/|H(k)|

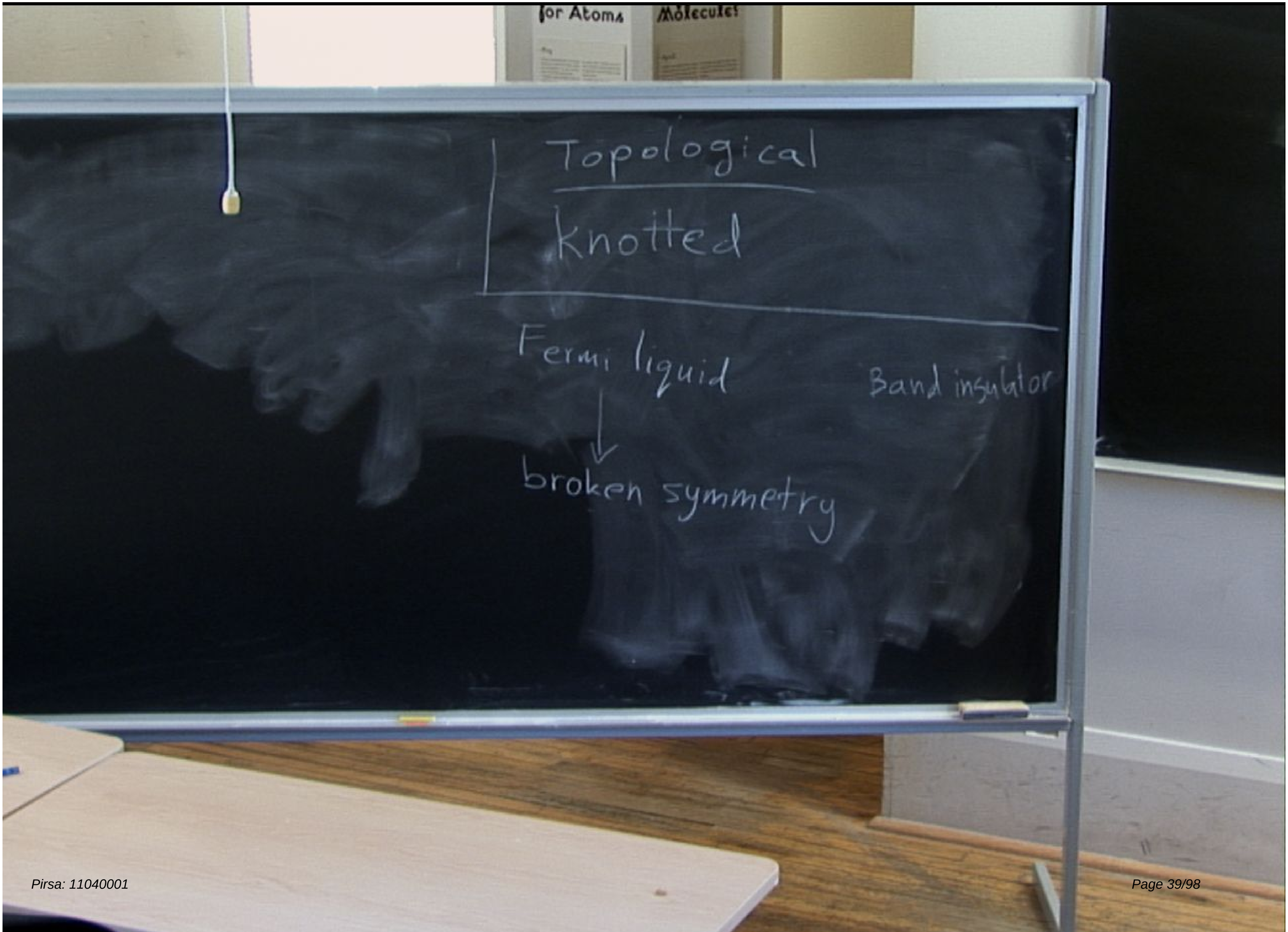

1.D
$\vec{h} \cdot \vec{\sigma}$
2. non-trivial
$-\frac{\pi}{a}$ 0 $\frac{\pi}{a}$
$H = \varepsilon \sum_k (c_{k\uparrow}^{+} \; c_{k\downarrow}^{+}) \begin{pmatrix} 0 & \cos k - i \sin k \\ \cos k + i \sin k & 0 \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \end{pmatrix}$
$= \varepsilon(\cos k, \sin k)$
$= \pm \varepsilon$
$n = \frac{\vec{h}_k}{|h_k|}$
$\hat{n} = \frac{\vec{H}(k)}{|H(k)|}$

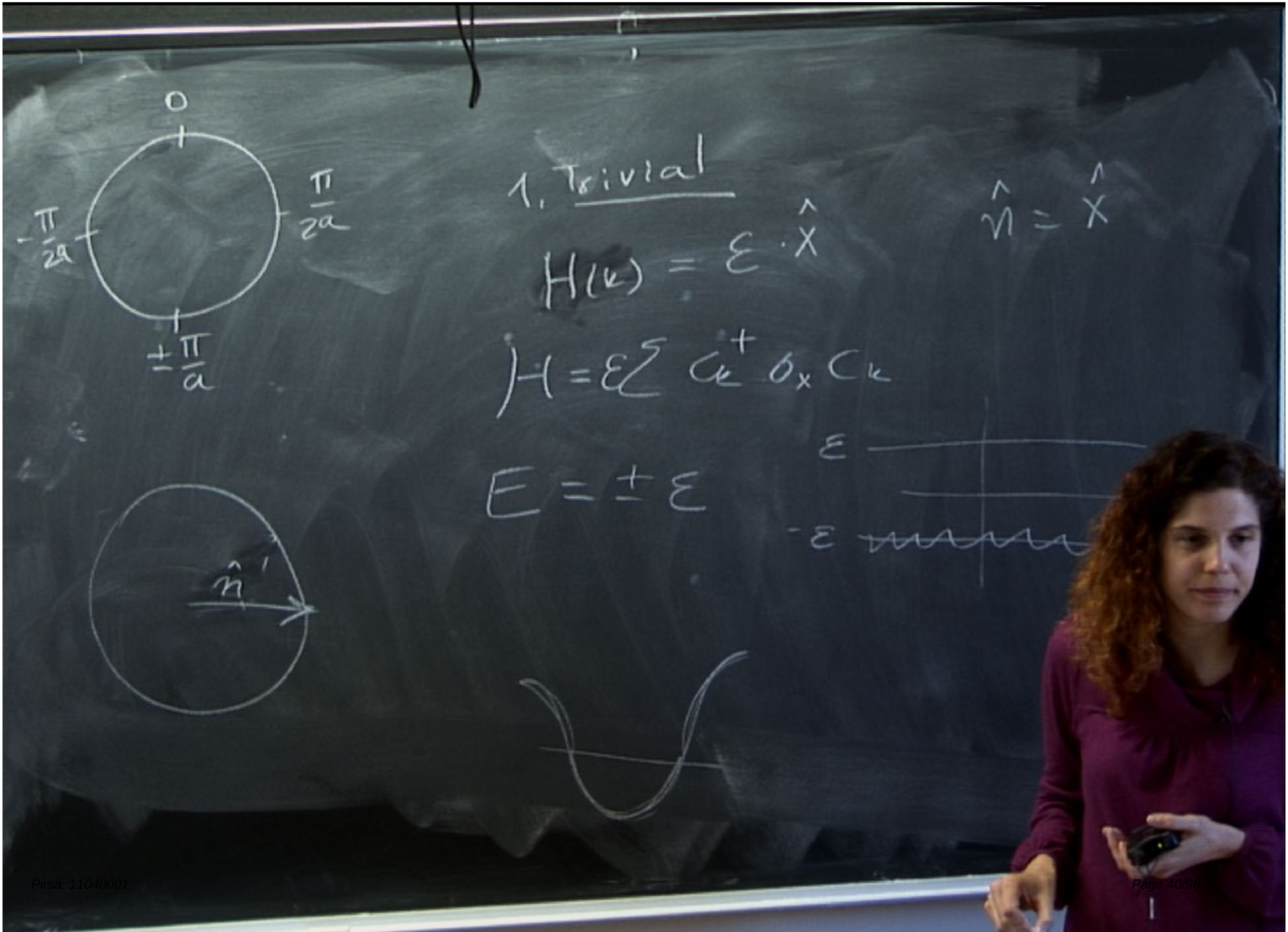

1. D
2. non-trivial
n · σ
H = ε Σ_k (c†_k↑ c†_k↓) (0 cos k − i sin k ; cos k + i sin k 0) (c_k↑ ; c_k↓)
−π/a 0 π/a
h_k = ε(cos k, sin k)
E = ± ε
n = h_k / |h_k|
n̂ = H(k) / |H(k)|

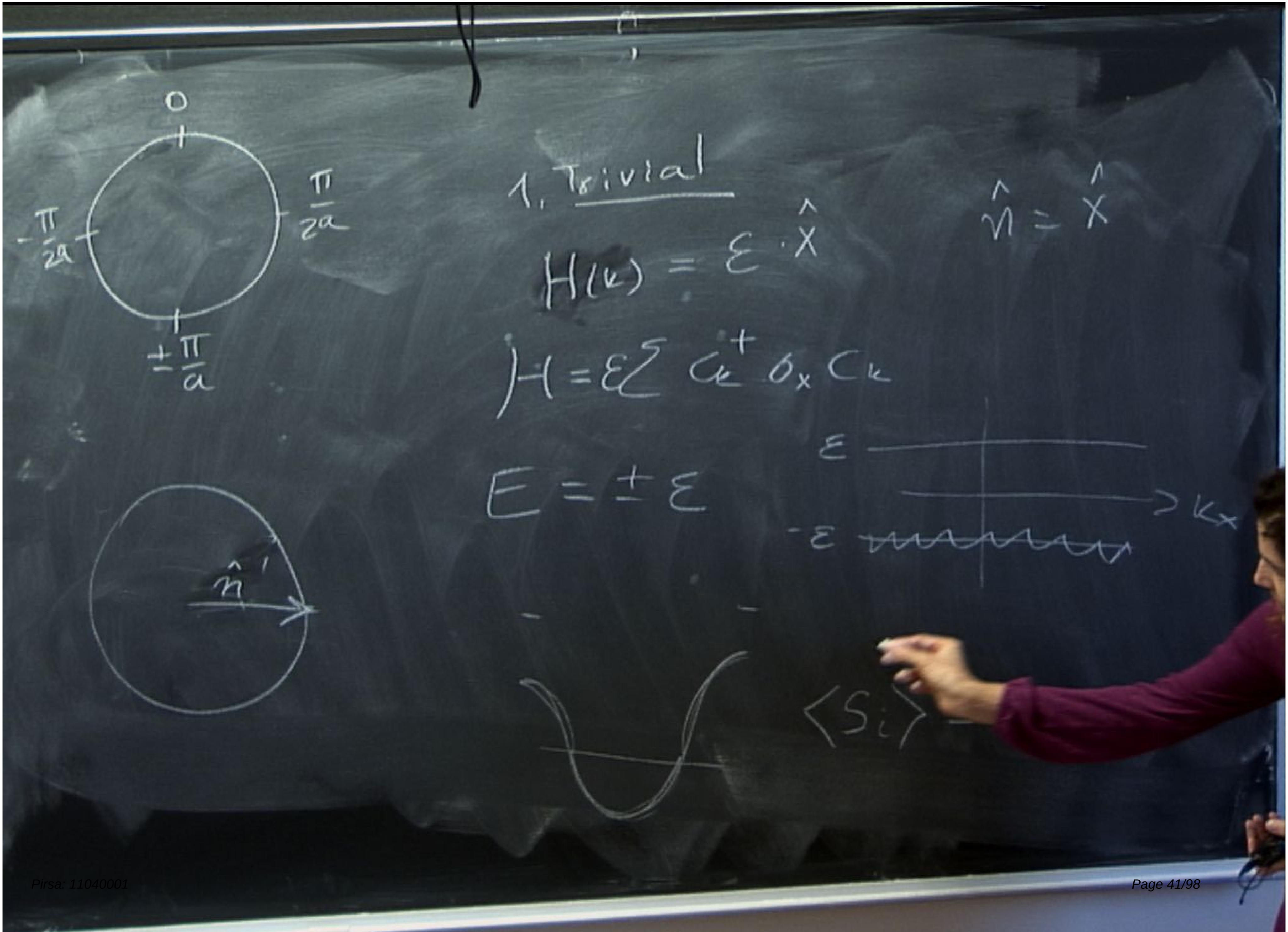

for Atoms
Molecules
Topological
knotted
Fermi liquid

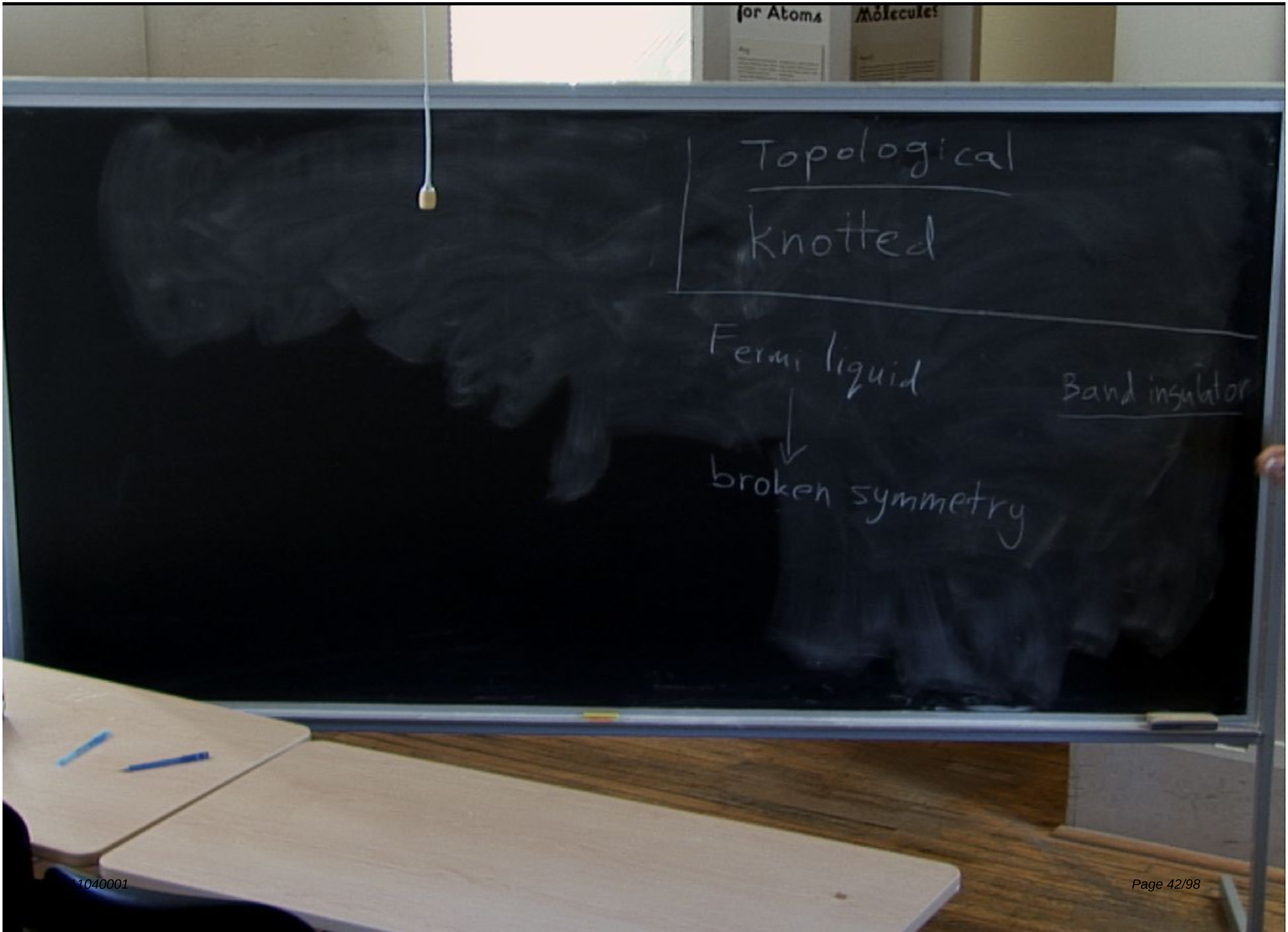

for Atoms
Molecules
Topological
knotted
Fermi liquid
Band insulator


for Atoms
Molecules
Topological
Knotted
Fermi liquid
Band insulator
broken symmetry

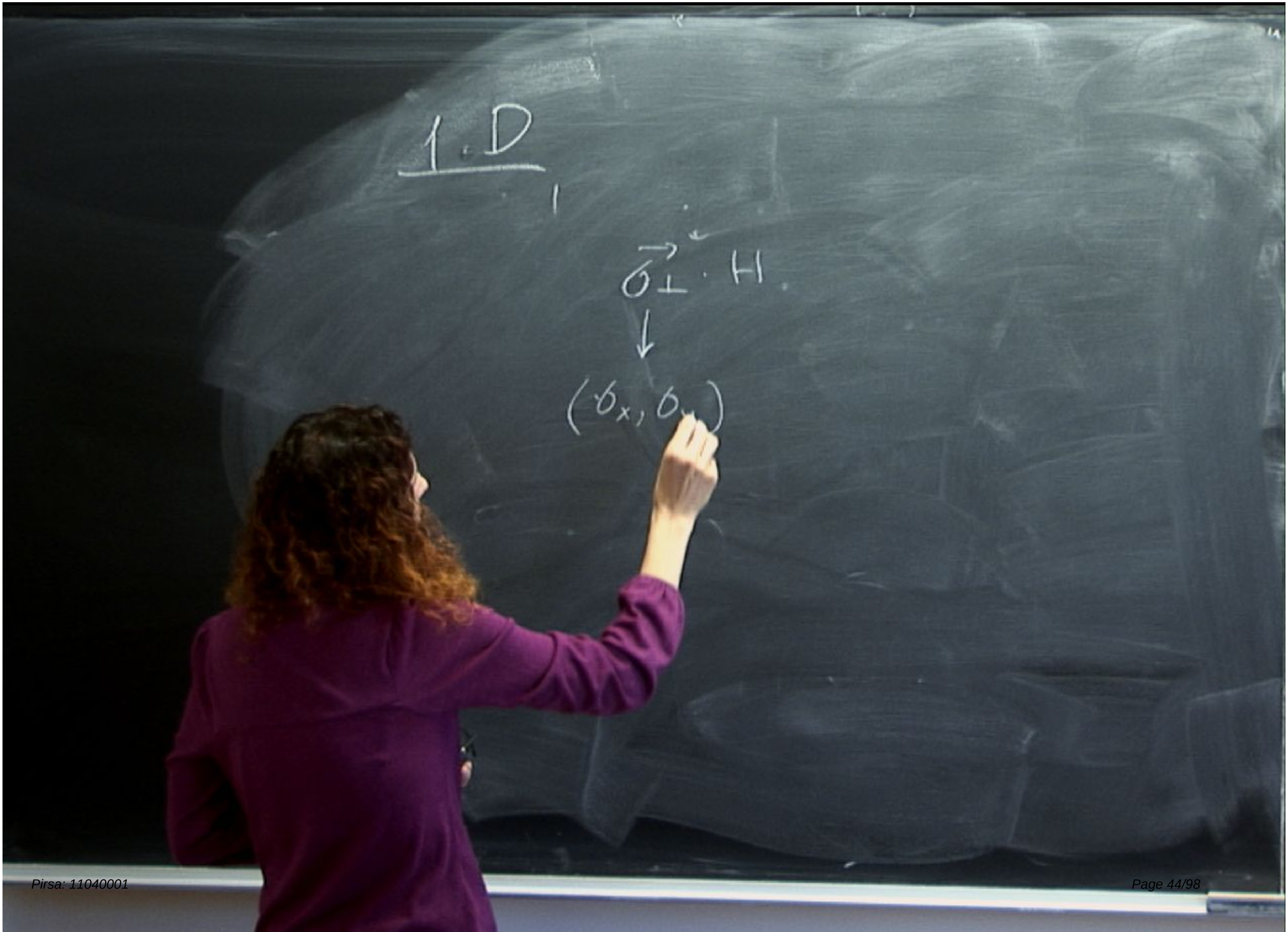

for Atoms
Molecules
Topological
knotted
Fermi liquid
broken symmetry
Band insulator

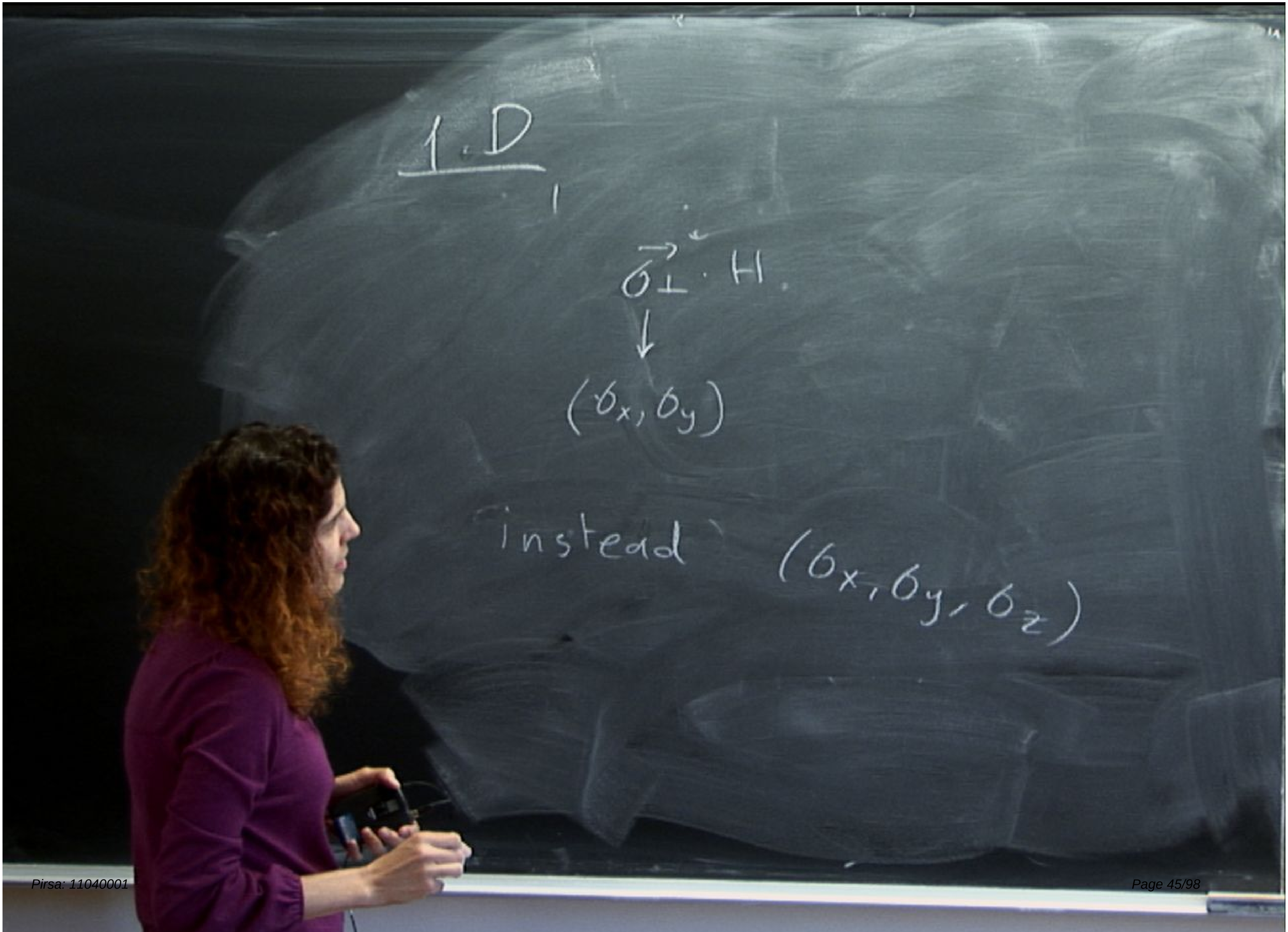

0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂
n̂ = X̂
H = ε Σ c_k^† σ_x c_k
E = ± ε
ε
-ε
n̂

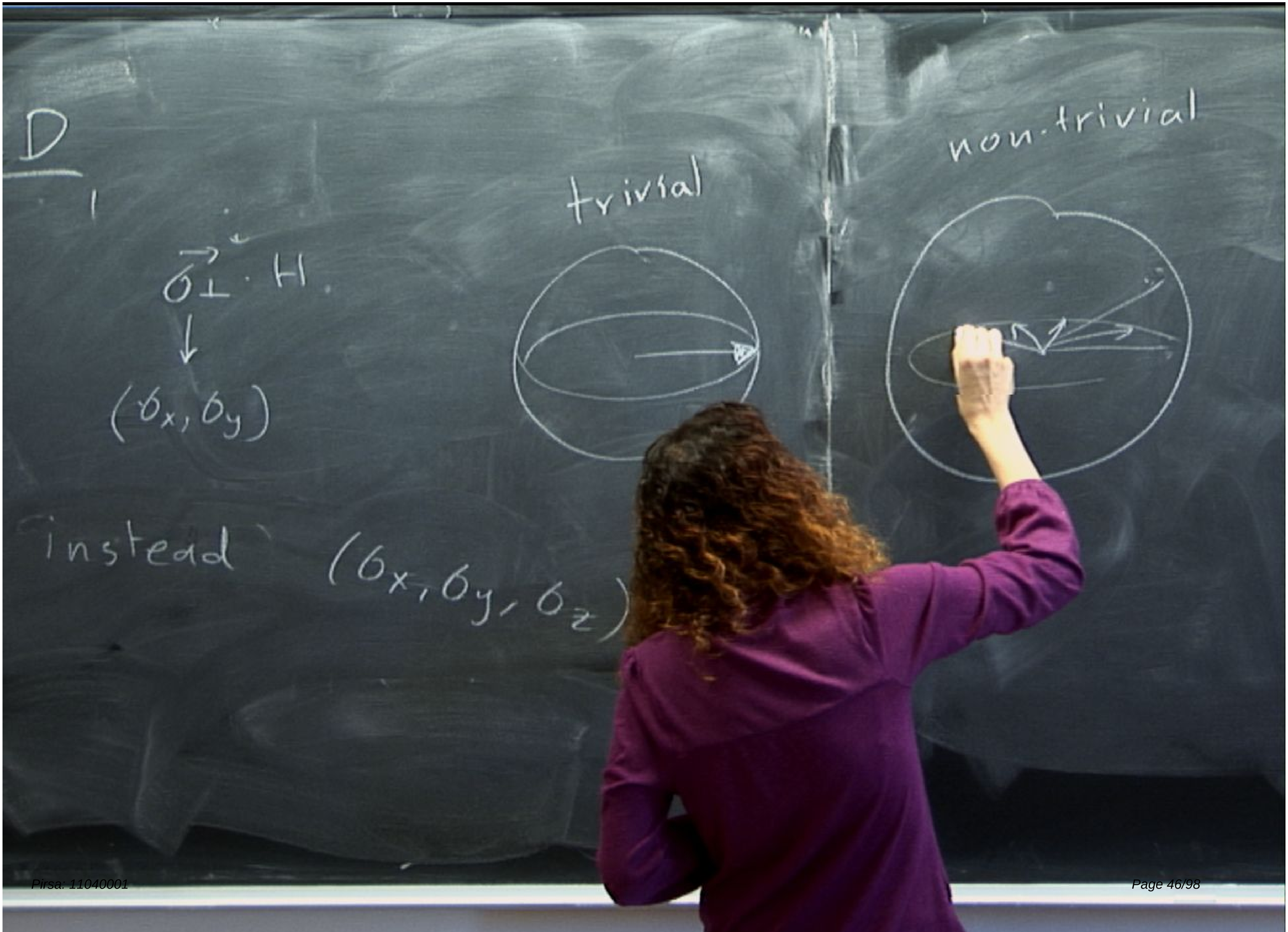

0
$\frac{\pi}{2a}$
$-\frac{\pi}{2a}$
$\pm\frac{\pi}{a}$
1. Trivial
$H(k) = \varepsilon \cdot \hat{x}$
$\hat{n} = \hat{x}$
$H = \varepsilon \sum c_k^+ \sigma_x c_k$
$E = \pm \varepsilon$
ε
k_x
$-\varepsilon$
$\hat{n}$
$\langle S_i \rangle$

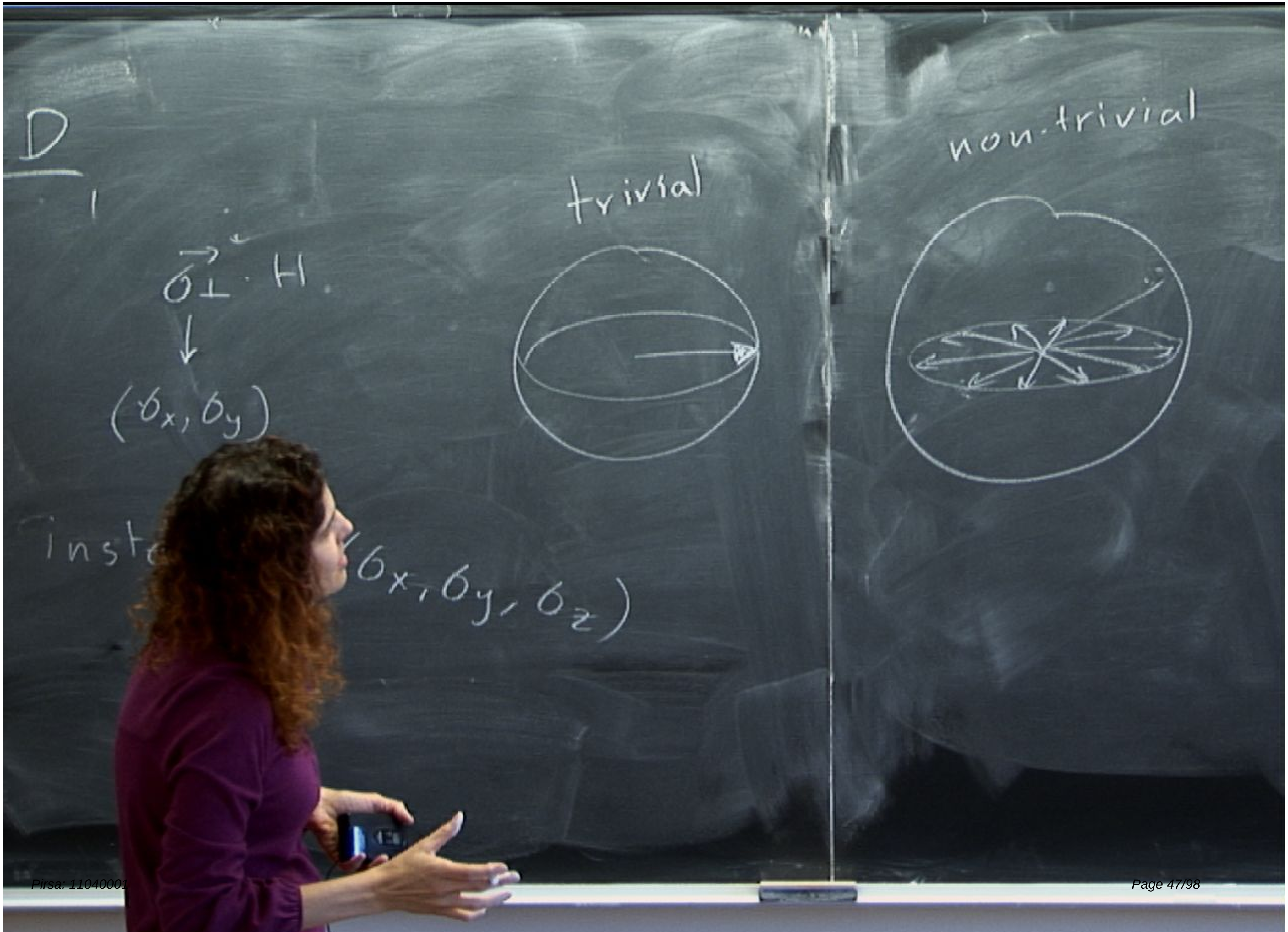

for Atoms
Molecules
Topological
knotted
Fermi liquid
broken symmetry
Band insulator

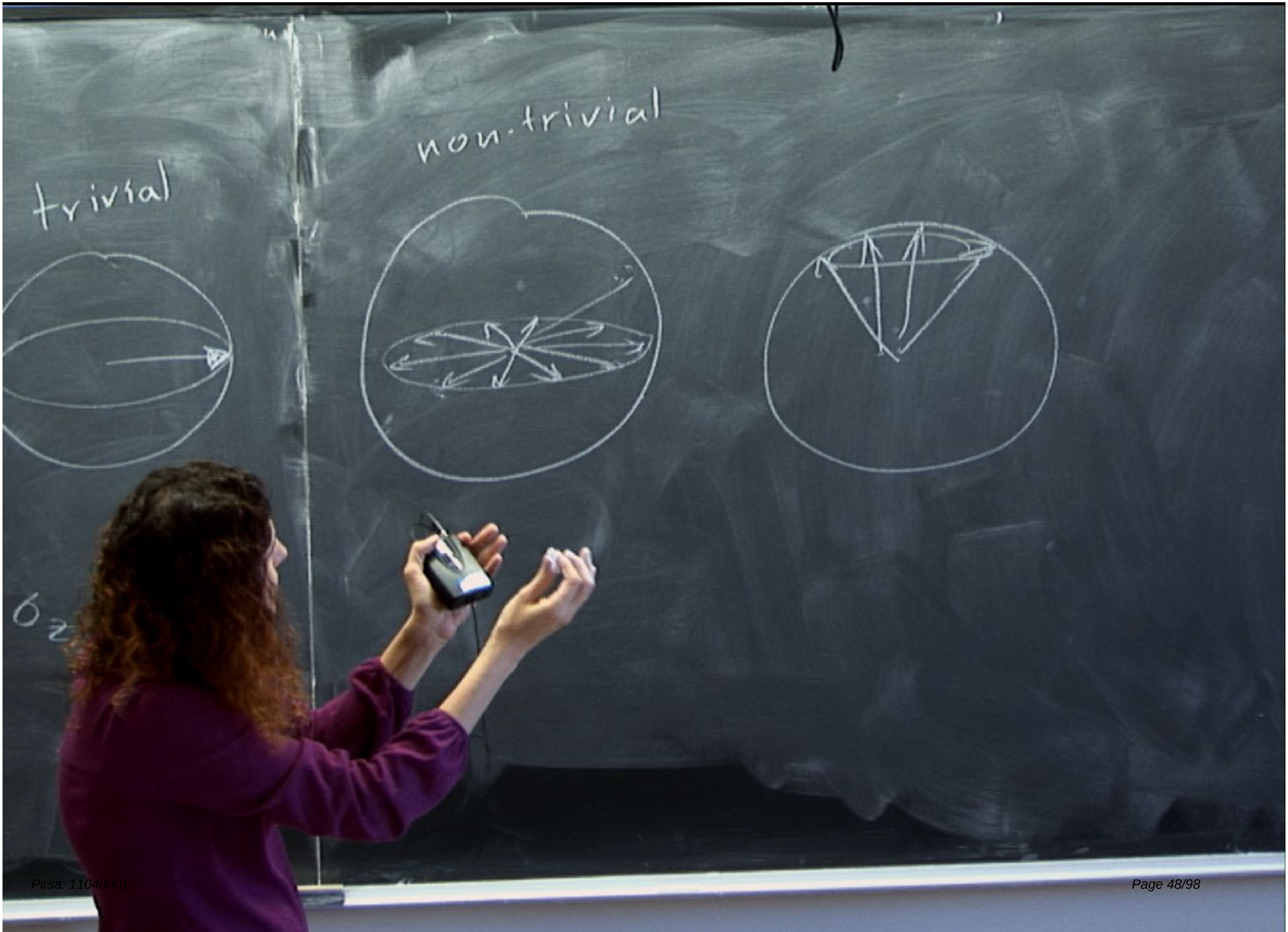

0
π/2a
-π/2a
±π/a
1. Trivial
H(k) = ε · X̂
n̂ = X̂
H = ε Σ c_k^+ σ_x c_k
E = ± ε
ε
-ε
k_x
⟨S_i⟩ = M


1.D
σ⊥ · H

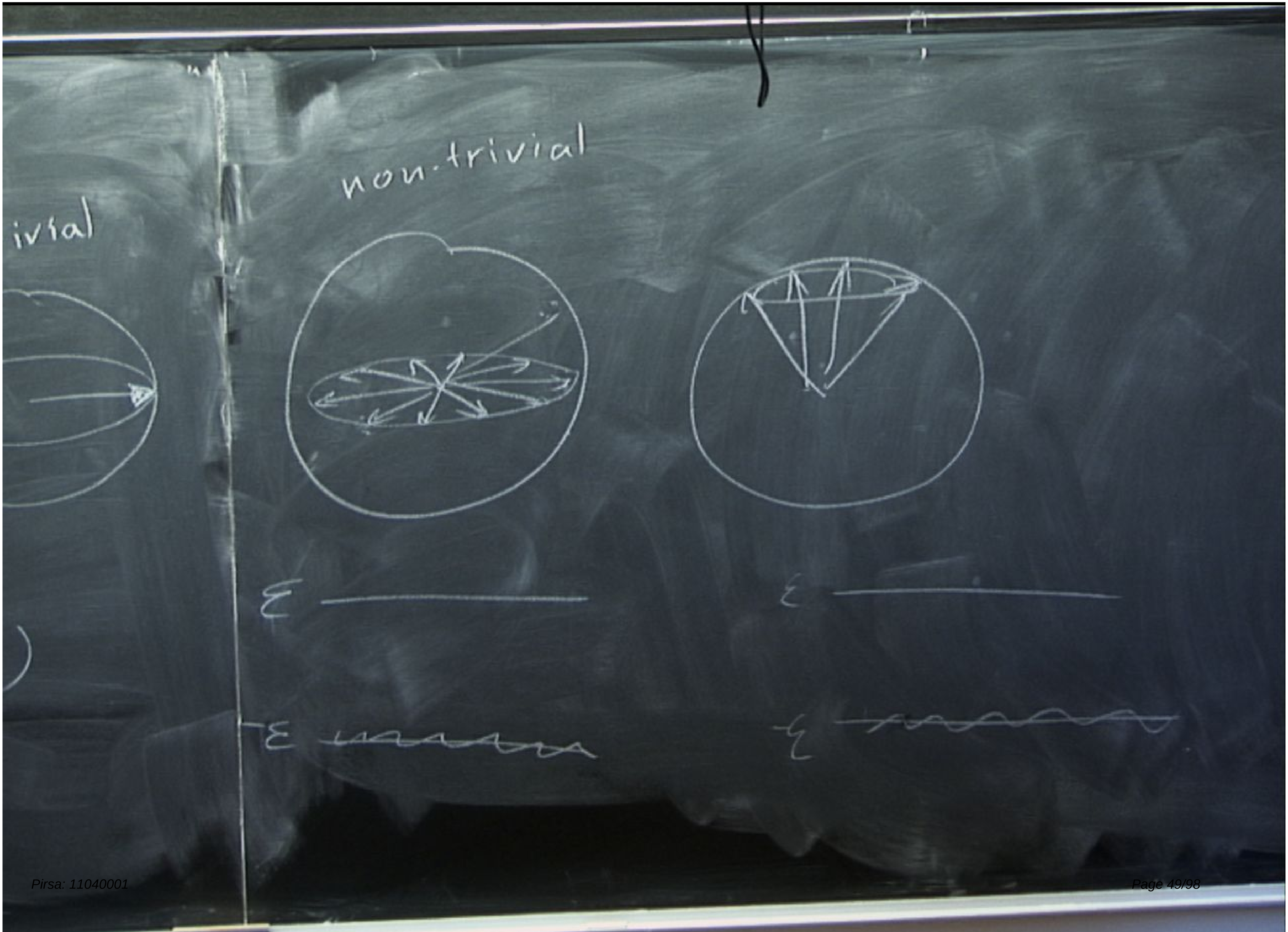

1.D
σ⊥ · H
(σx, σy)
instead (σx, σy, σz)

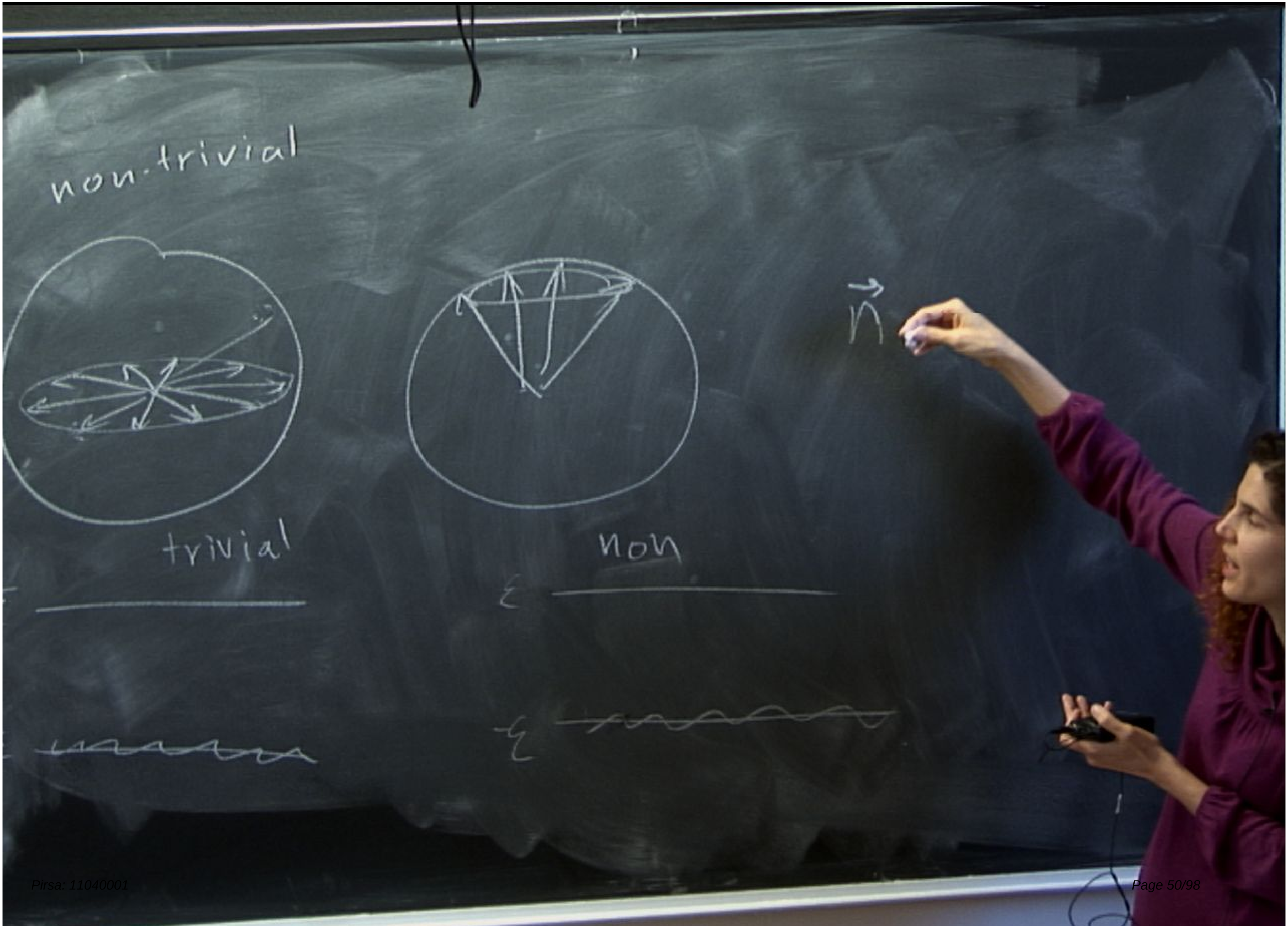

D
$\vec{\sigma}_\perp \cdot H$
(σ_x, σ_y)
trivial
non-trivial
instead
$(\sigma_x, \sigma_y, \sigma_z)$

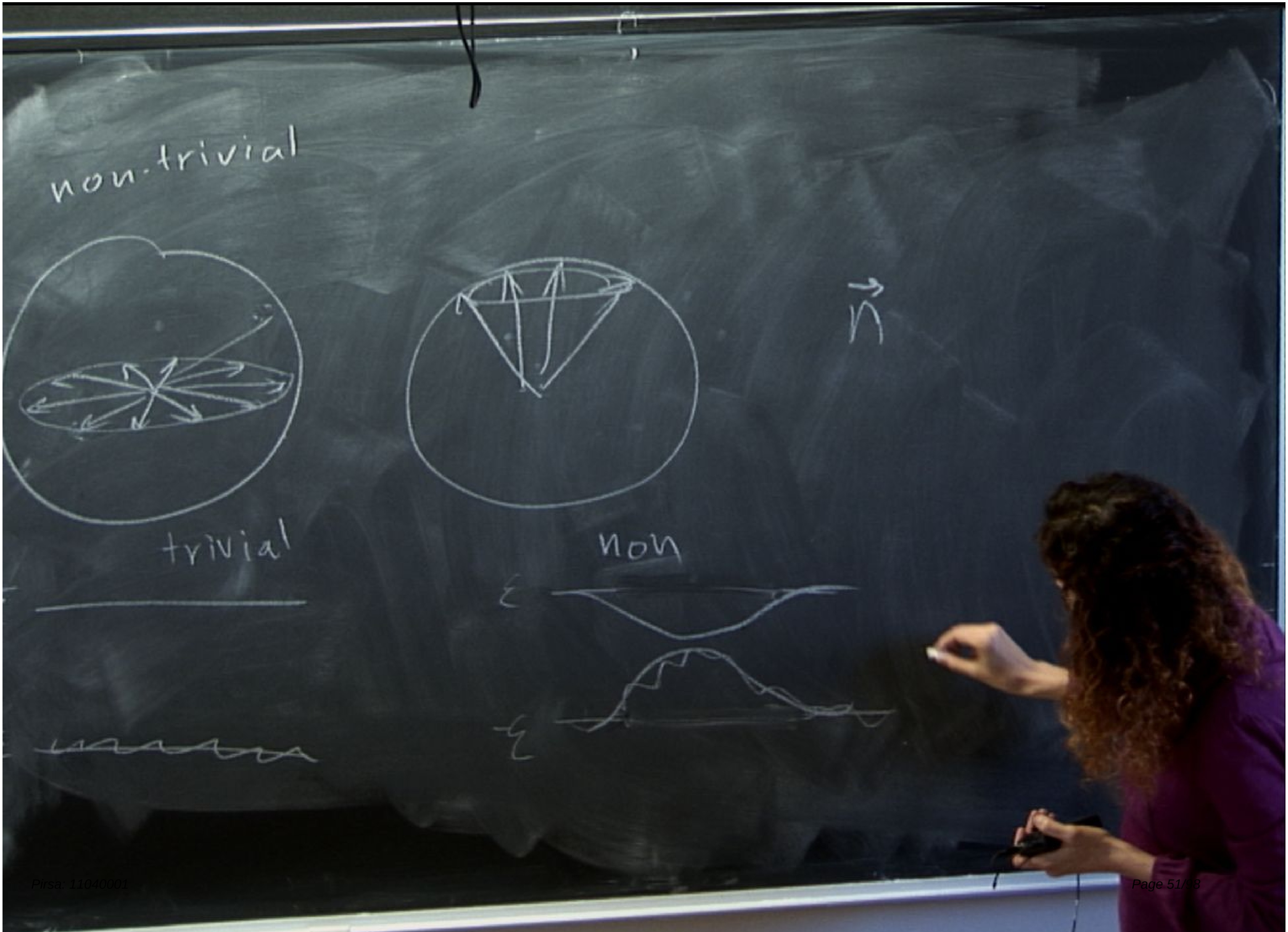

D
σ⊥ · H
(σx, σy)
trivial
non-trivial
(σx, σy, σz)

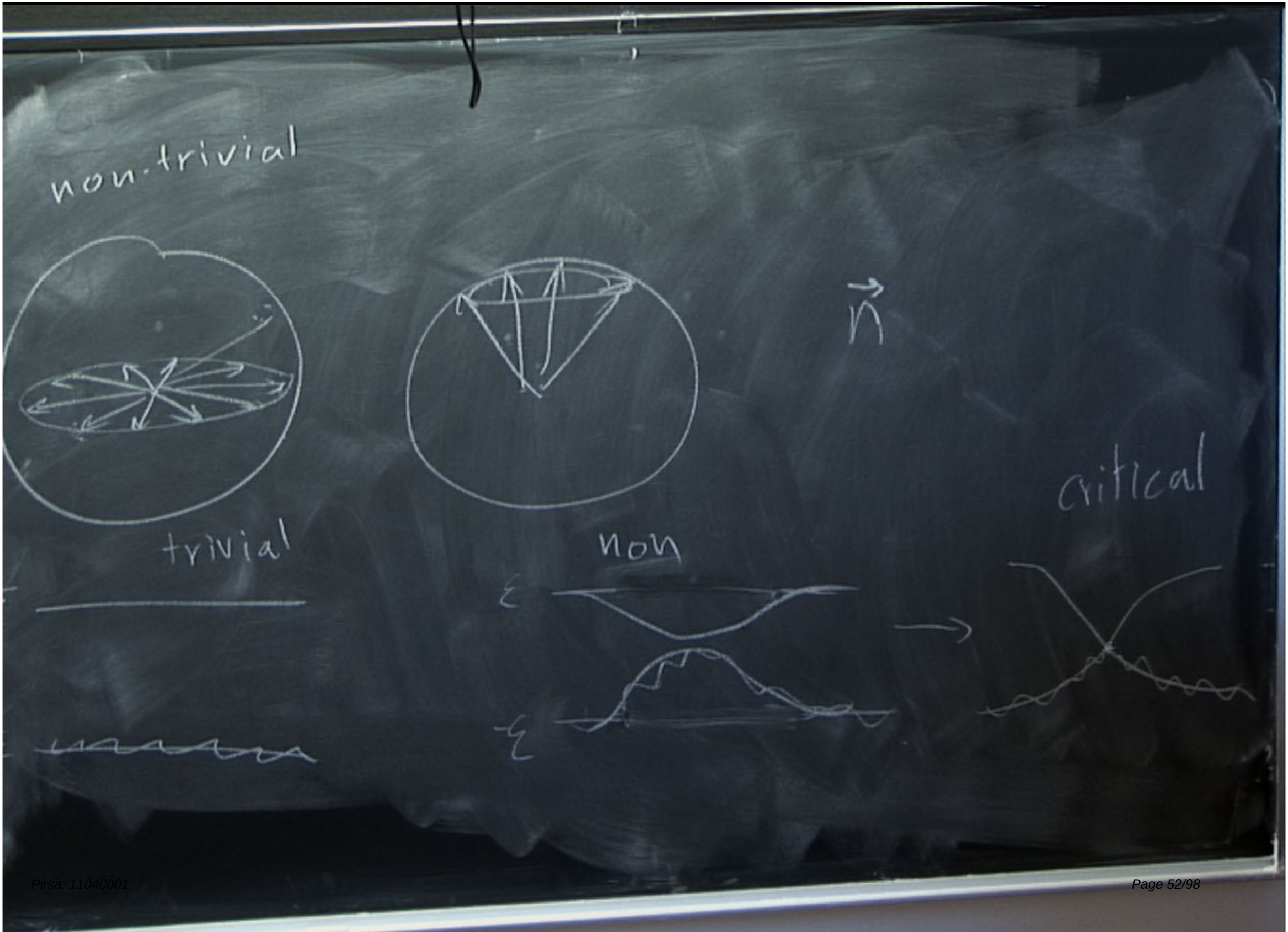


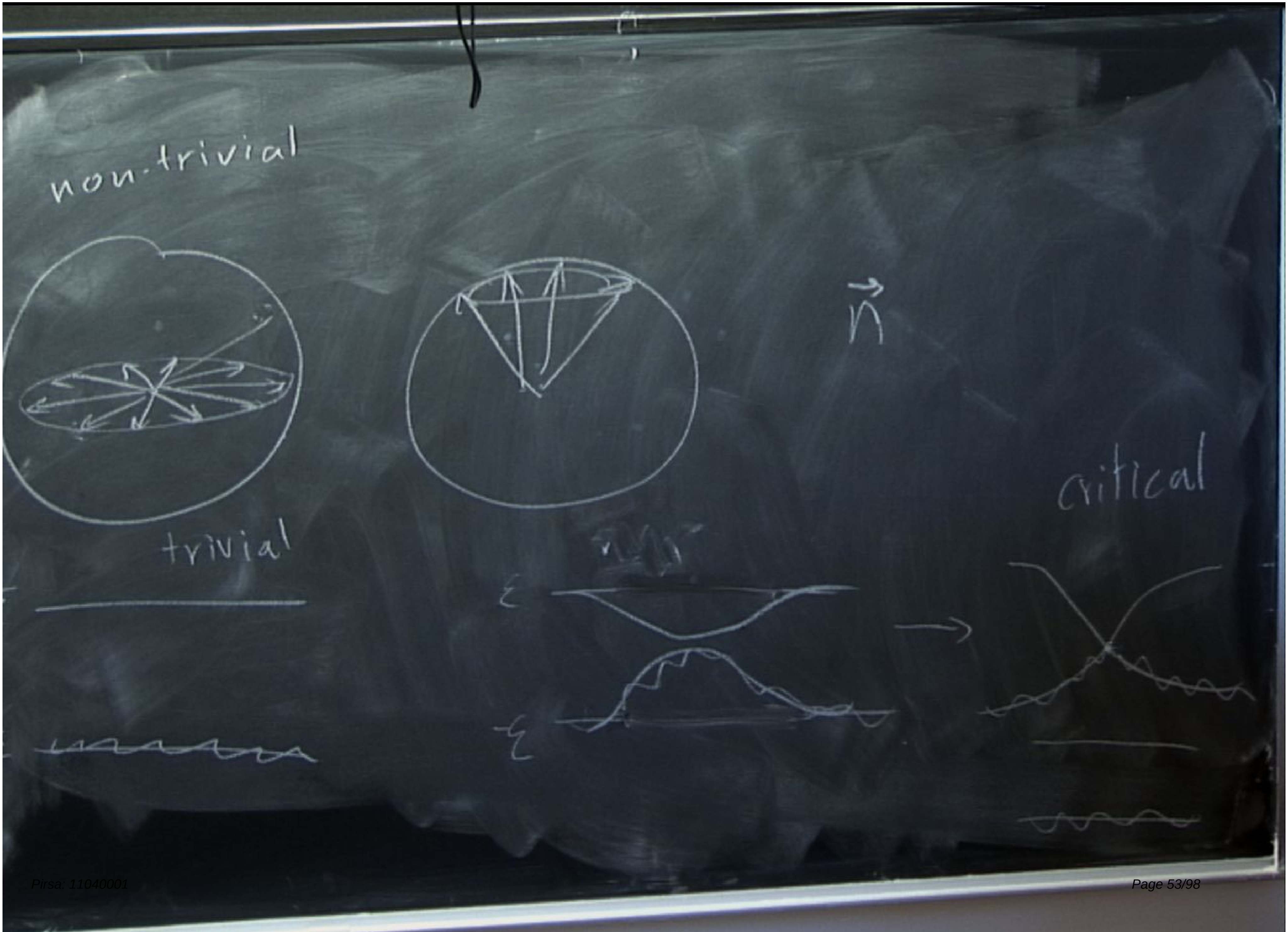
trivial
non-trivial

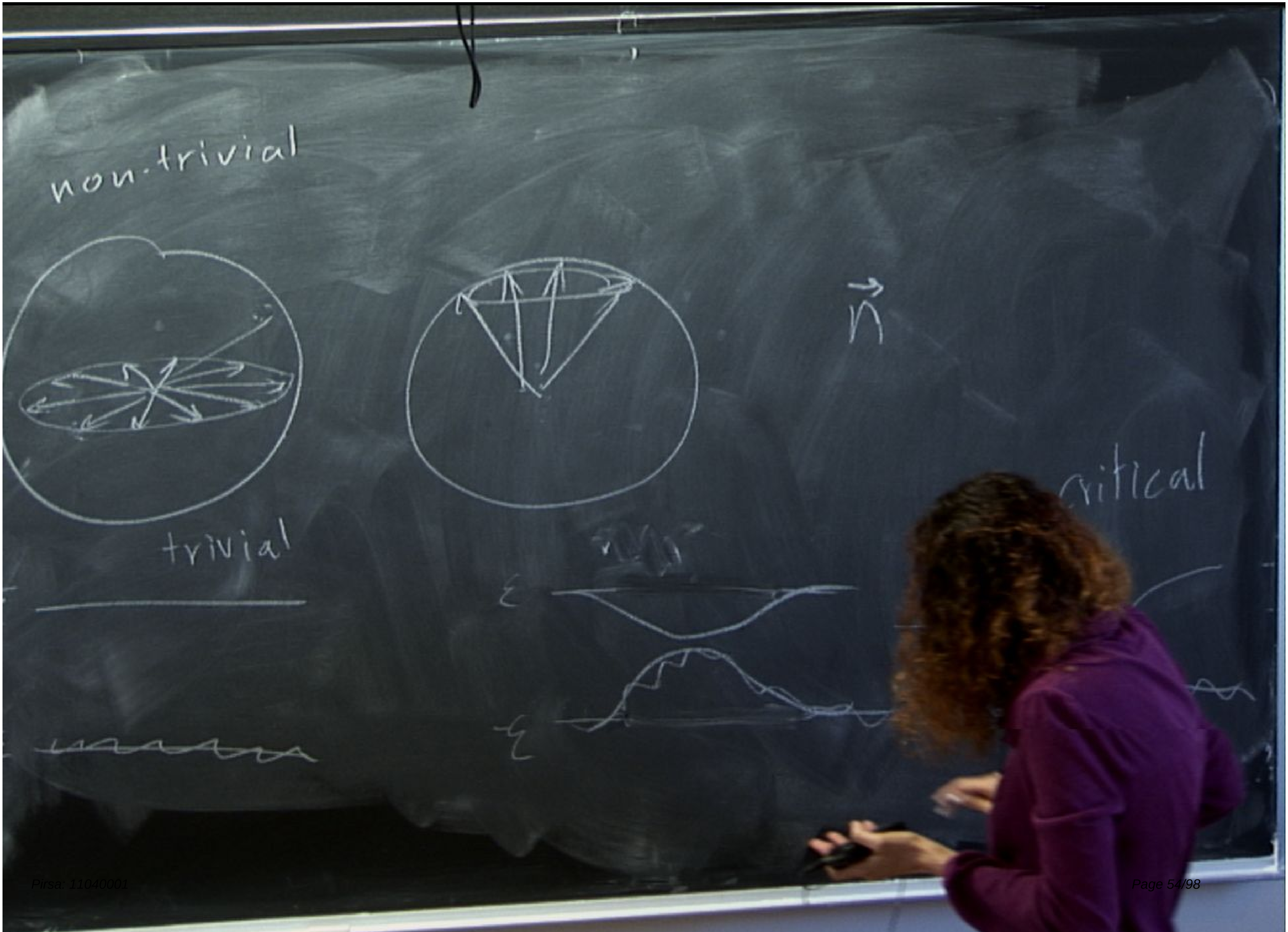

ivial
non-trivial

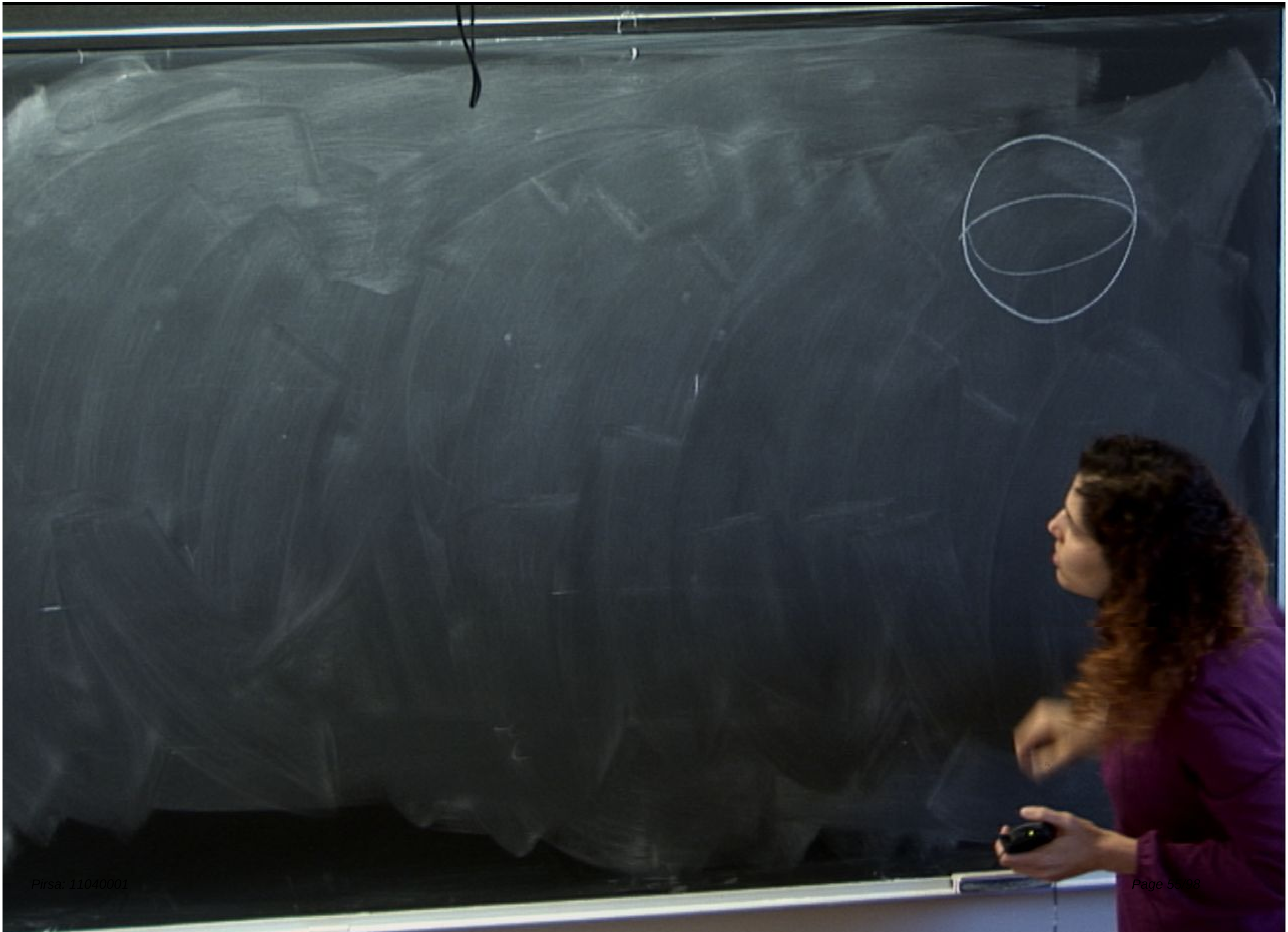

non-trivial
trivial
non
ε
ε
$\vec{n}$

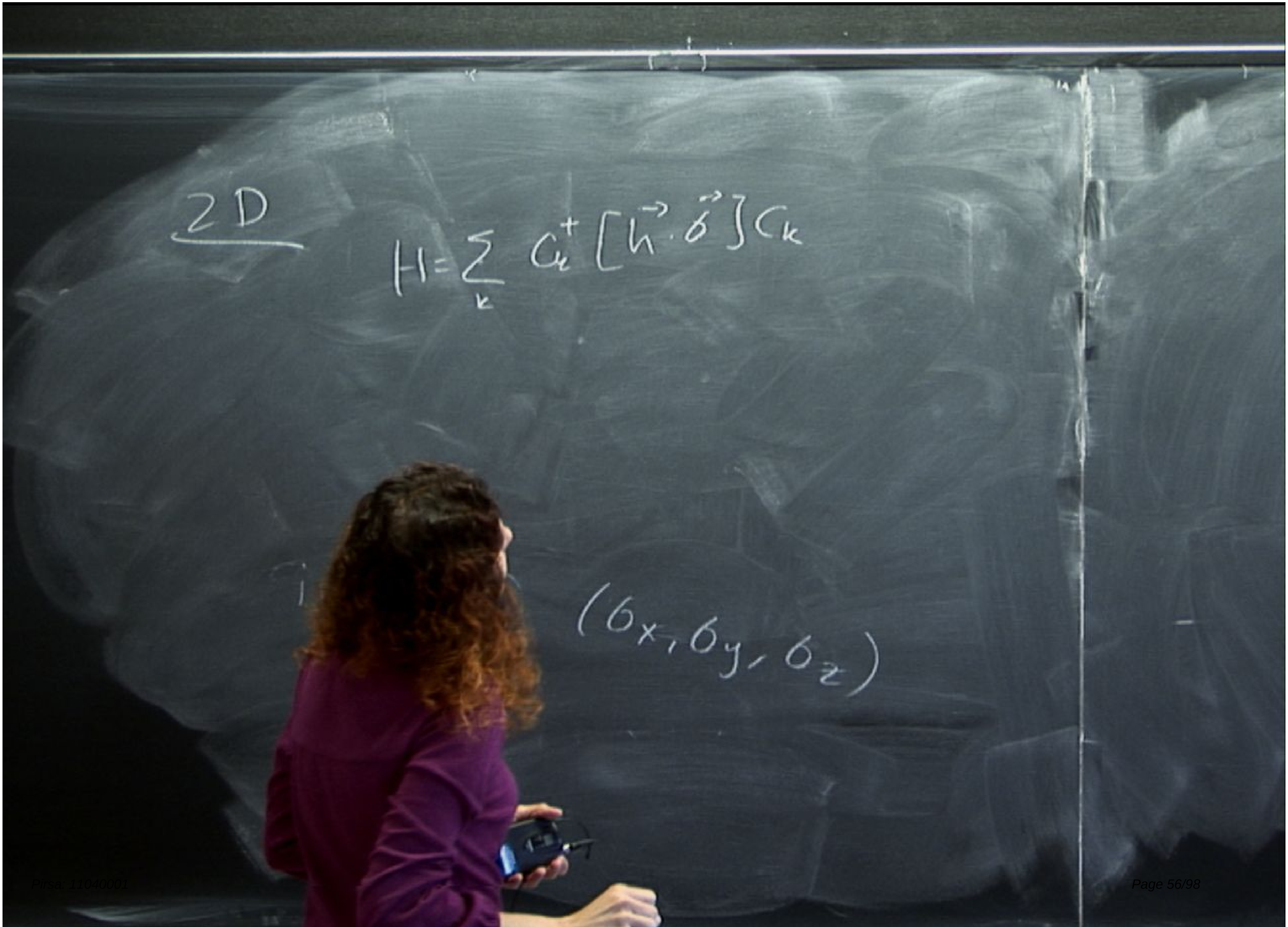

non-trivial
trivial
non
$\vec{n}$
ε
ε

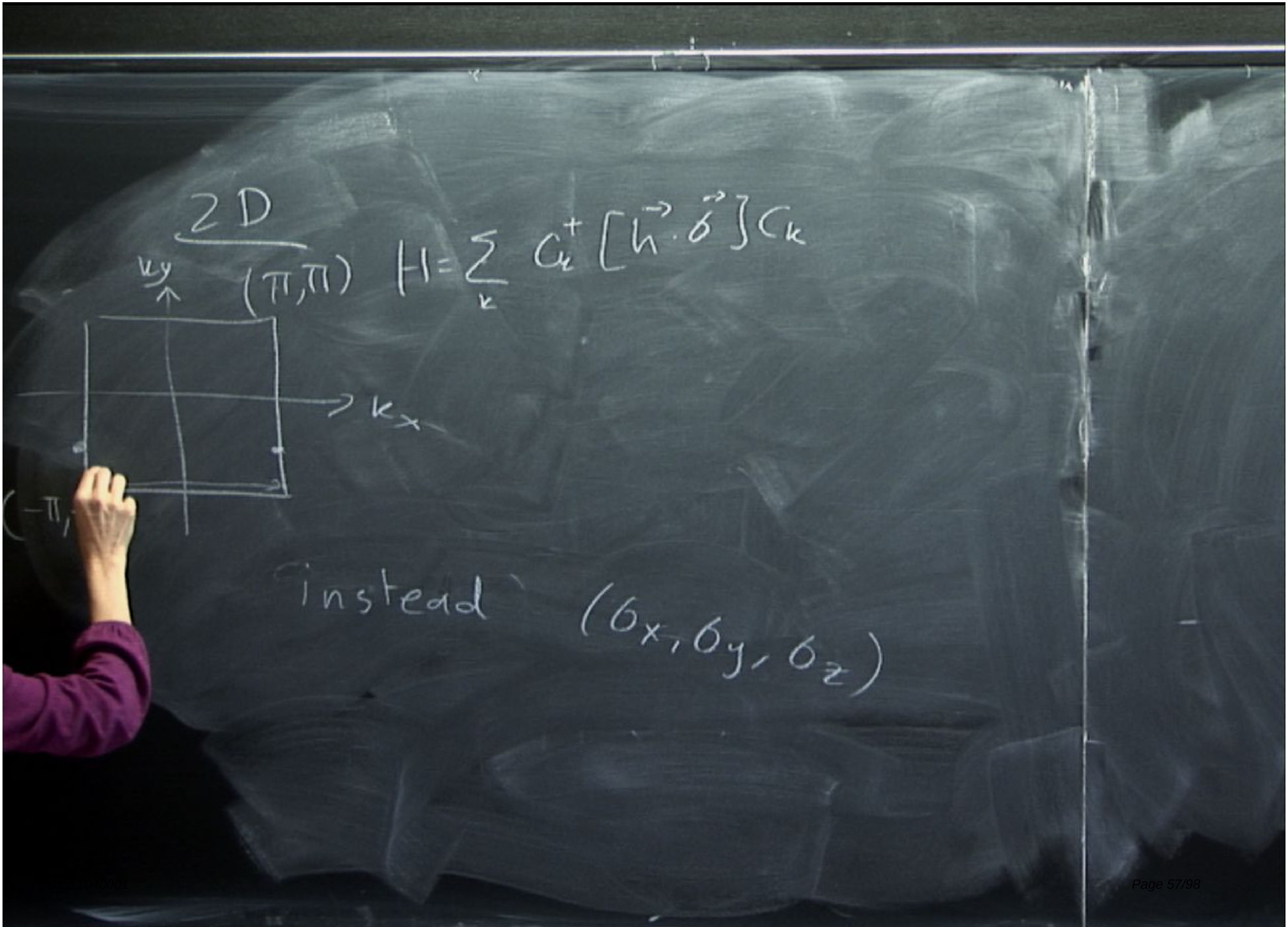

non-trivial
$\vec{n}$
critical
trivial
non
ε
ε

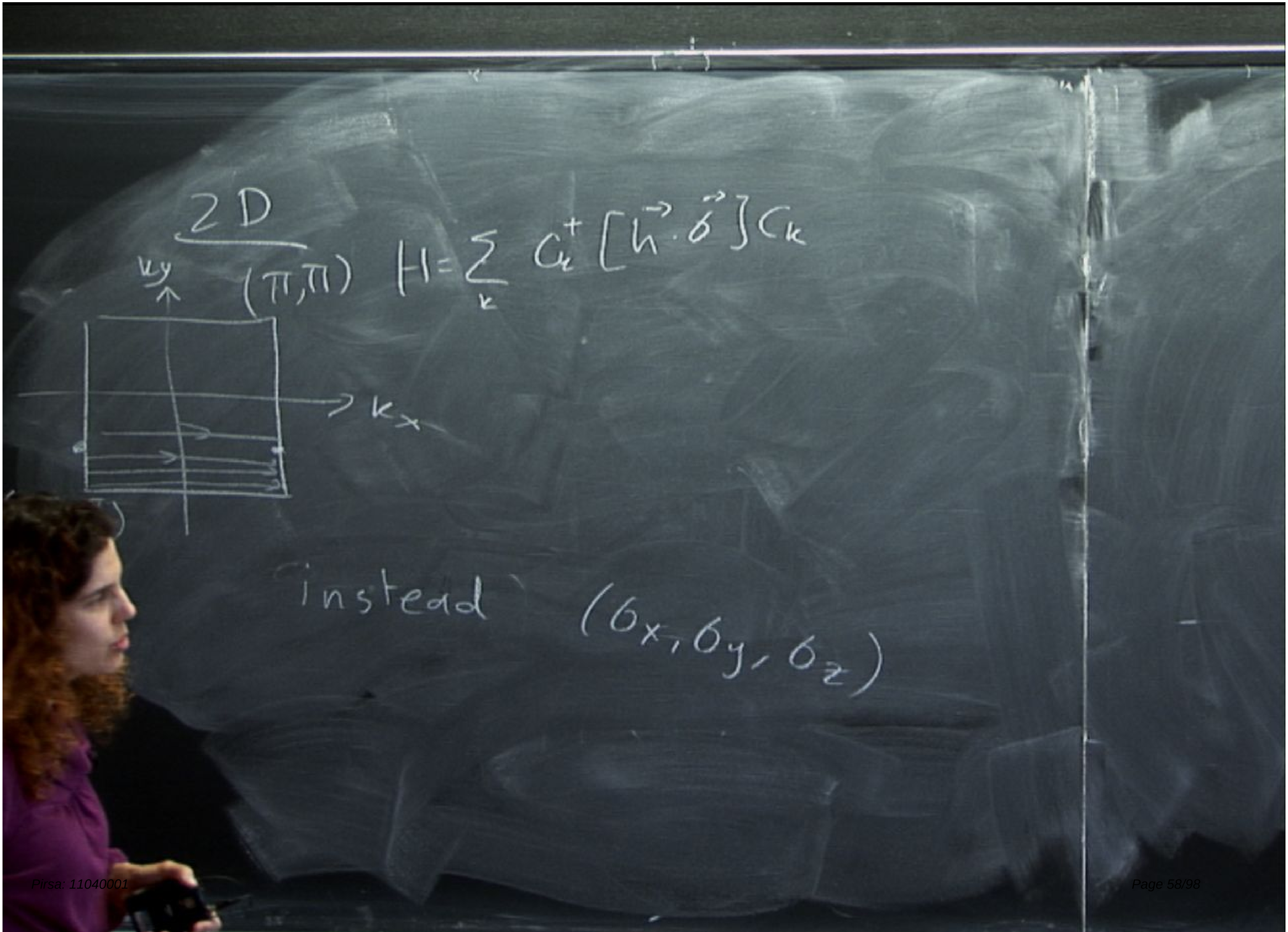

non-trivial
trivial
$\vec{n}$
critical
ε
$-\varepsilon$

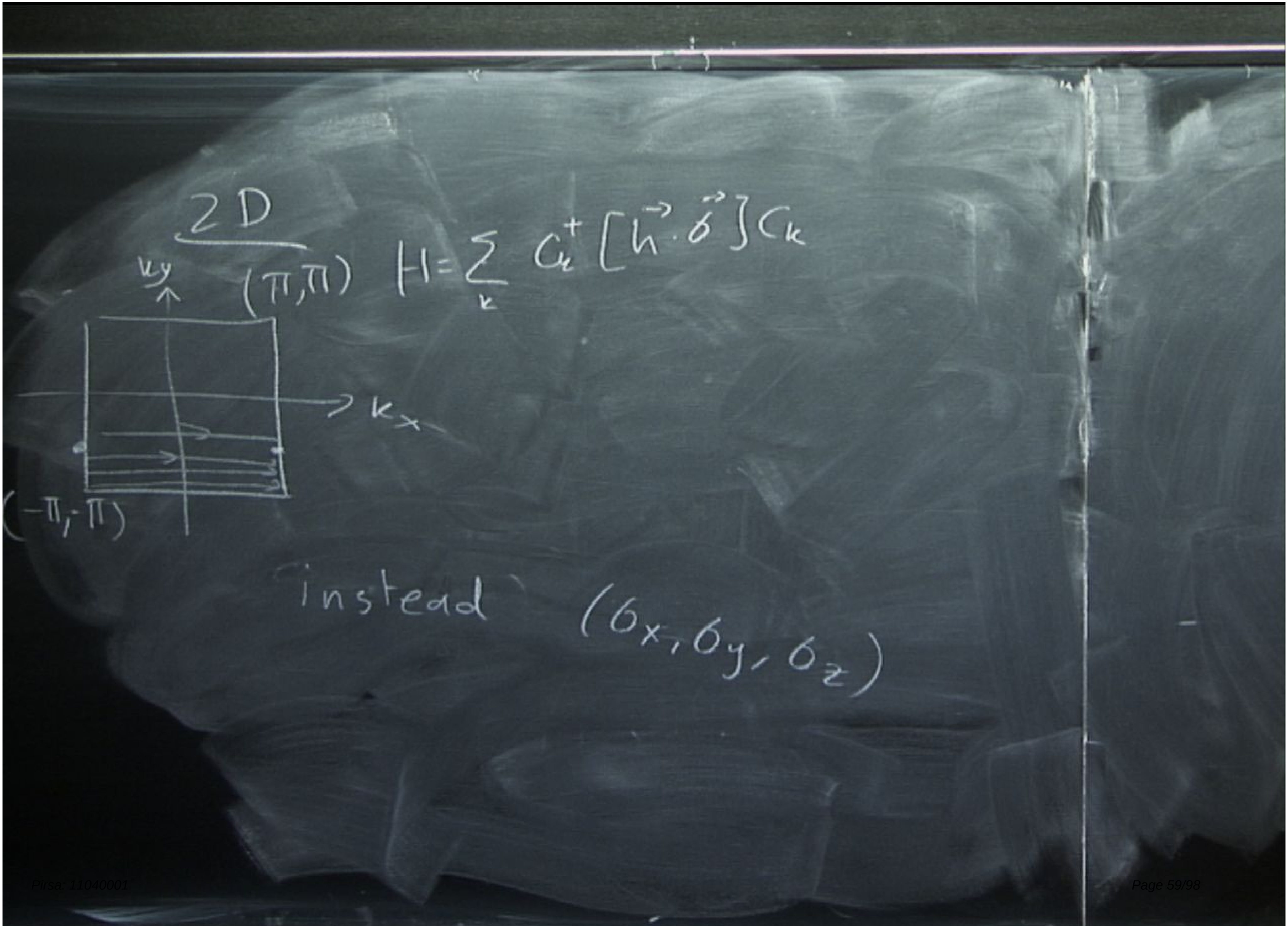

non-trivial
trivial
critical

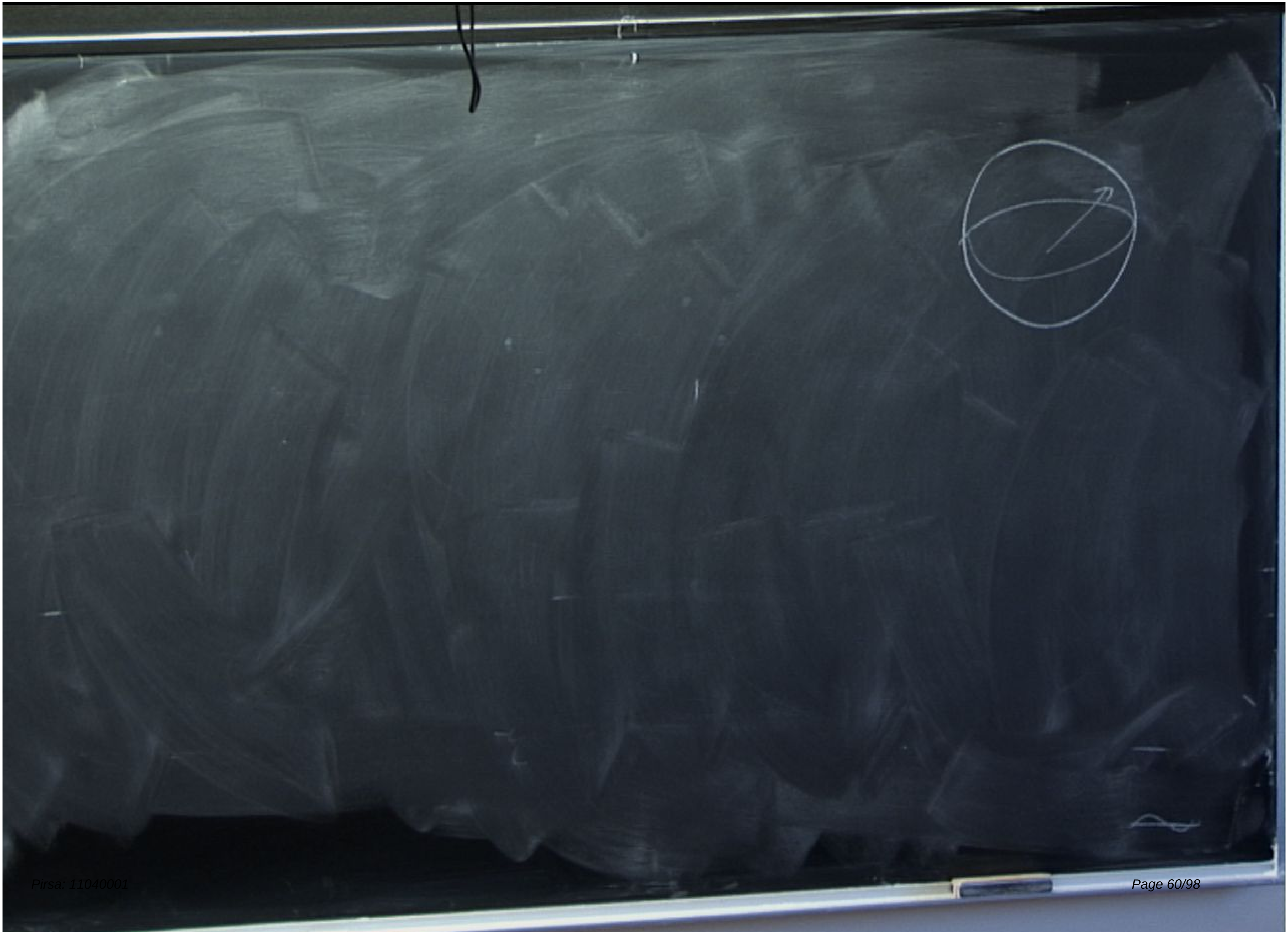


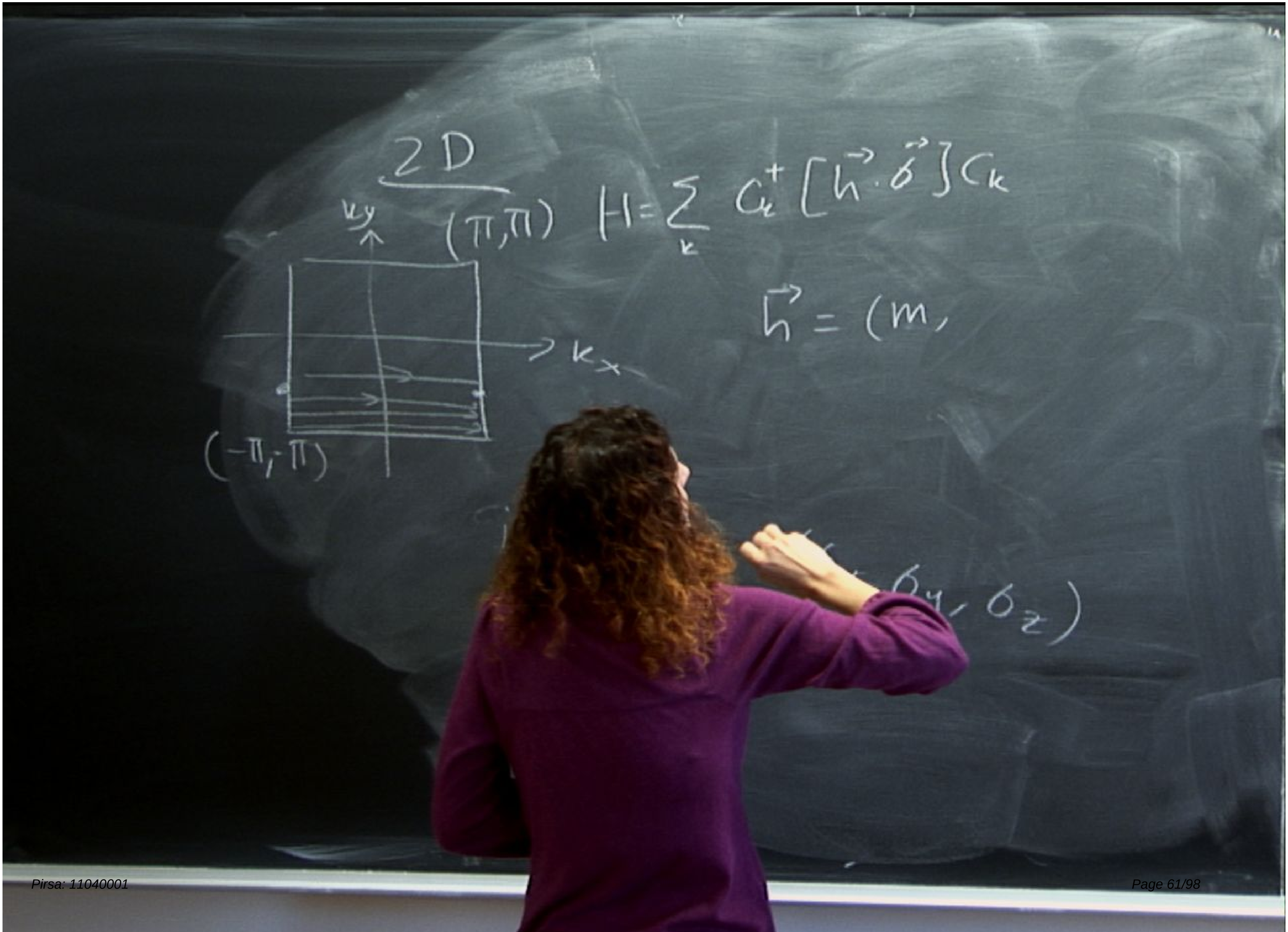

2D
$H = \sum_k c_k^\dagger [\vec{h} \cdot \vec{\sigma}] c_k$
$(\sigma_x, \sigma_y, \sigma_z)$

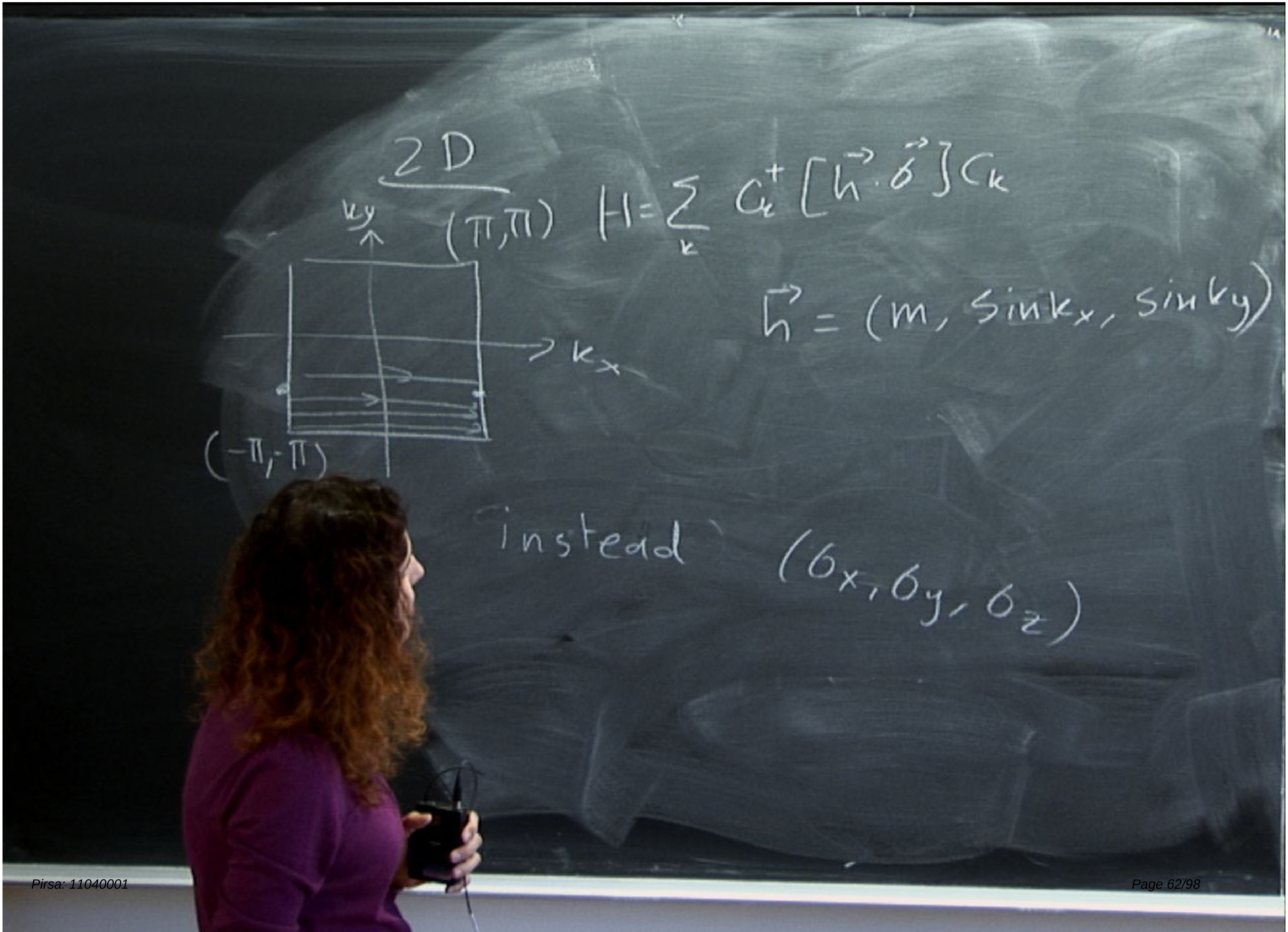

2D
(π,π)
k_y
k_x
$(-\pi,$
$H=\sum_k c_k^+ [\vec{h}\cdot\vec{\sigma}] c_k$
instead
$(\sigma_x, \sigma_y, \sigma_z)$

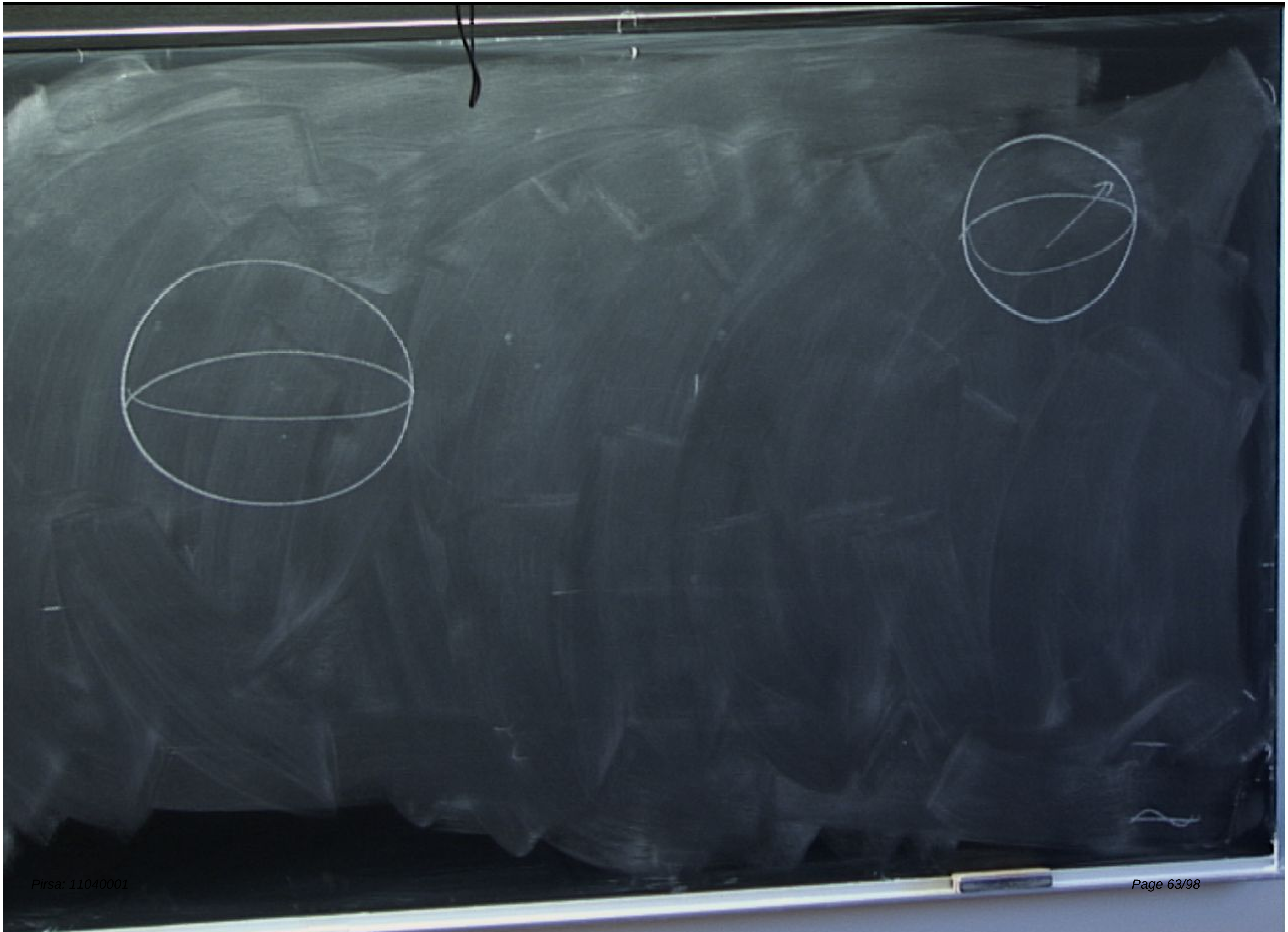

2D
(π,π)
H = Σ_k c_k^+ [h⃗ · σ⃗] c_k
k_y
k_x
instead
(σx, σy, σz)

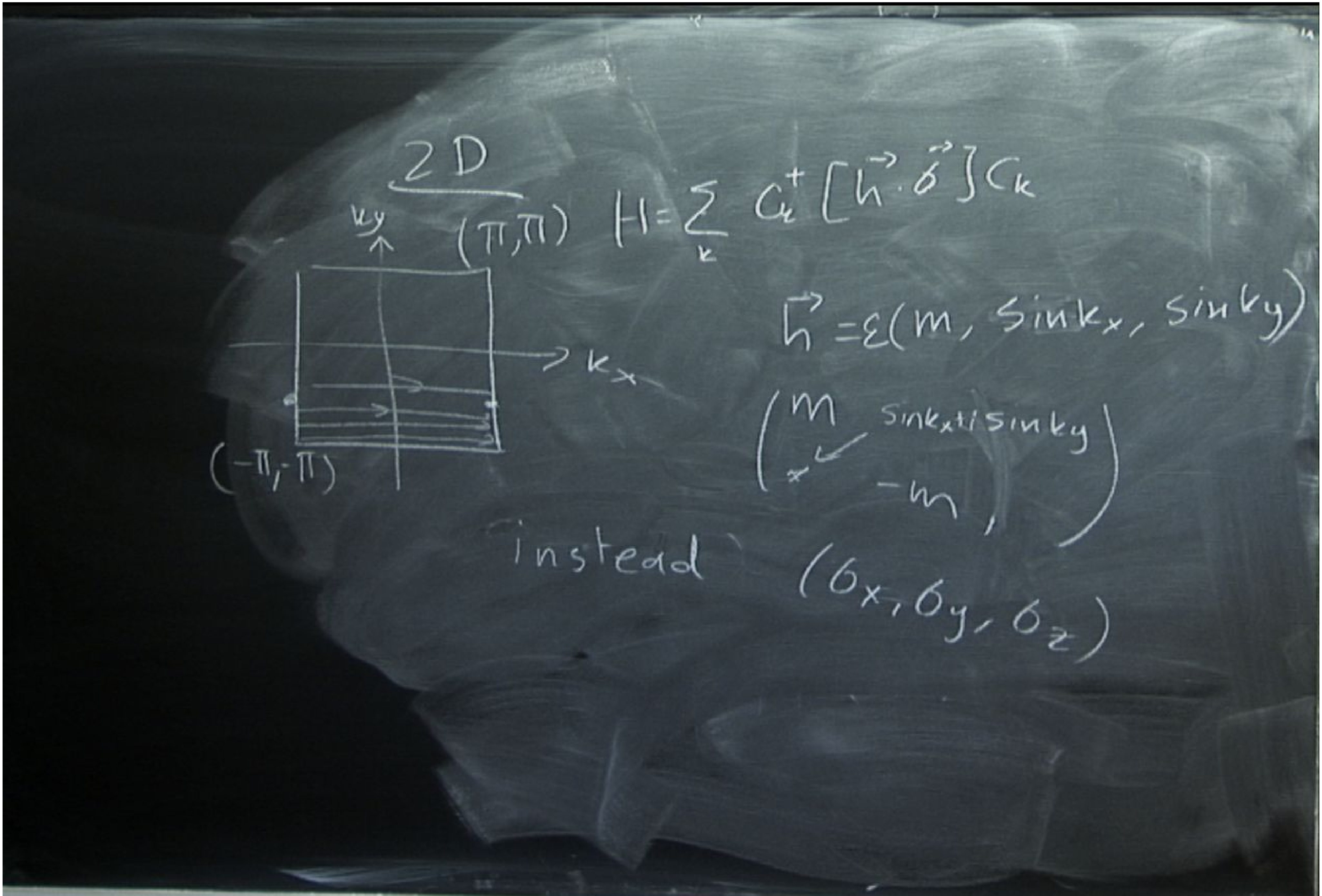

2D
H = Σ_k c_k^† [h⃗ · σ⃗] c_k
(π, π)
k_y
k_x
(−π, −π)
instead
(σ_x, σ_y, σ_z)

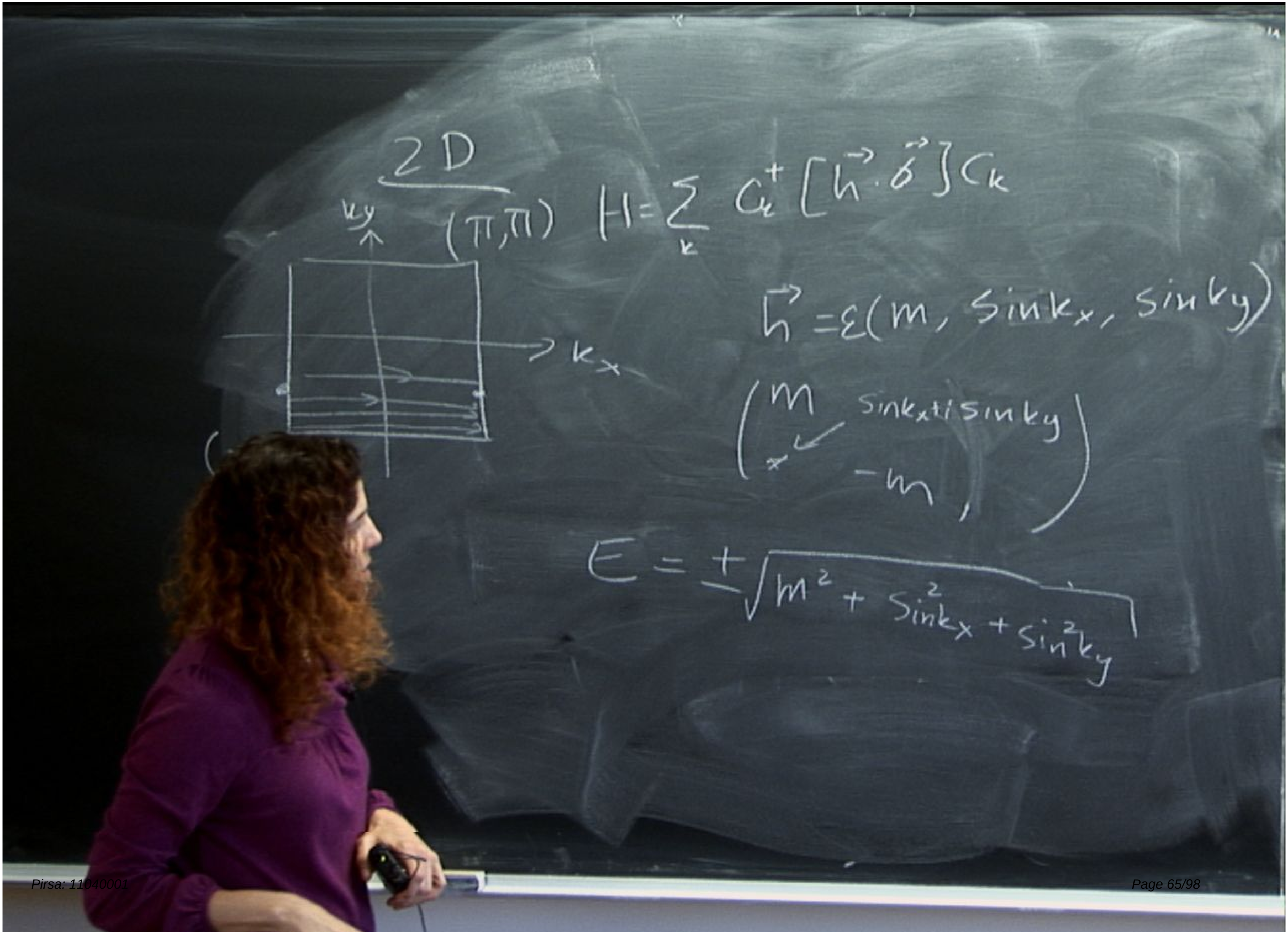


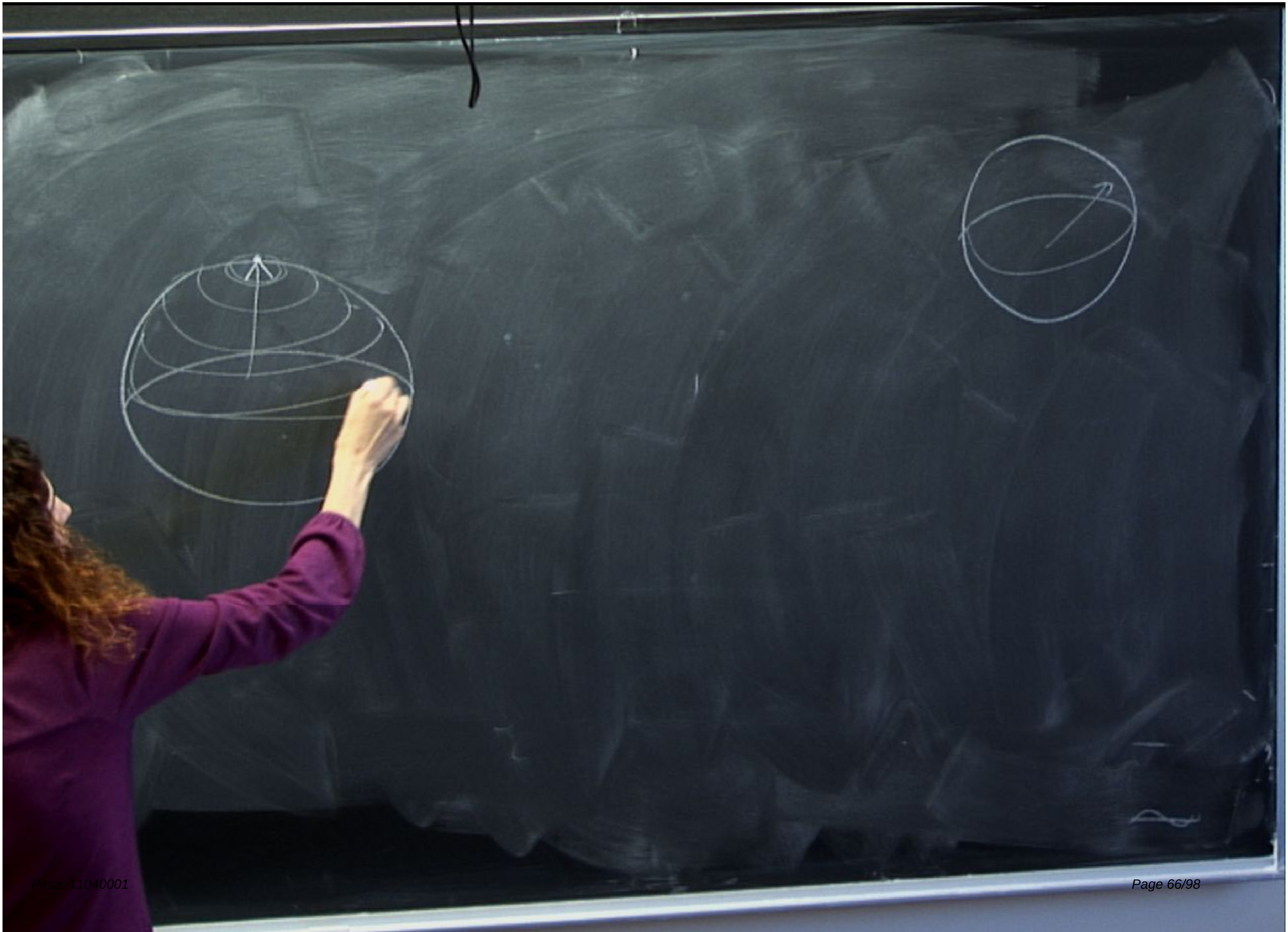

2D
k_y
(π,π)
$H = \sum_k c_k^\dagger [\vec{h}\cdot\vec{\sigma}] c_k$
$\vec{h} = (m,$
k_x
$(-\pi,-\pi)$
$\sigma_y, \sigma_z)$

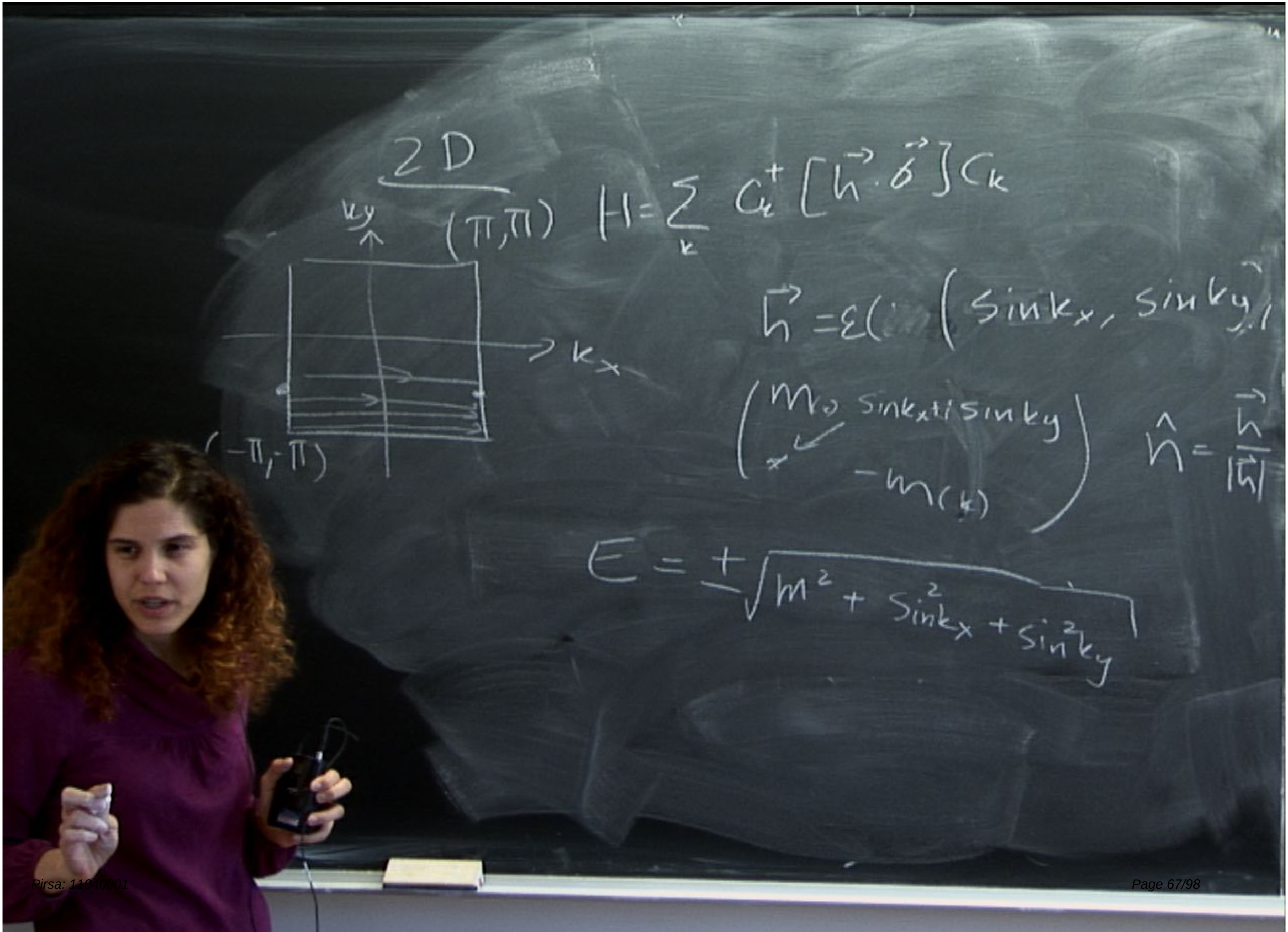

2D
(π,π)
(-π,-π)
ky
kx
H = Σk ck† [h⃗·σ⃗] ck
h⃗ = (m, sin kx, sin ky)
instead
(σx, σy, σz)

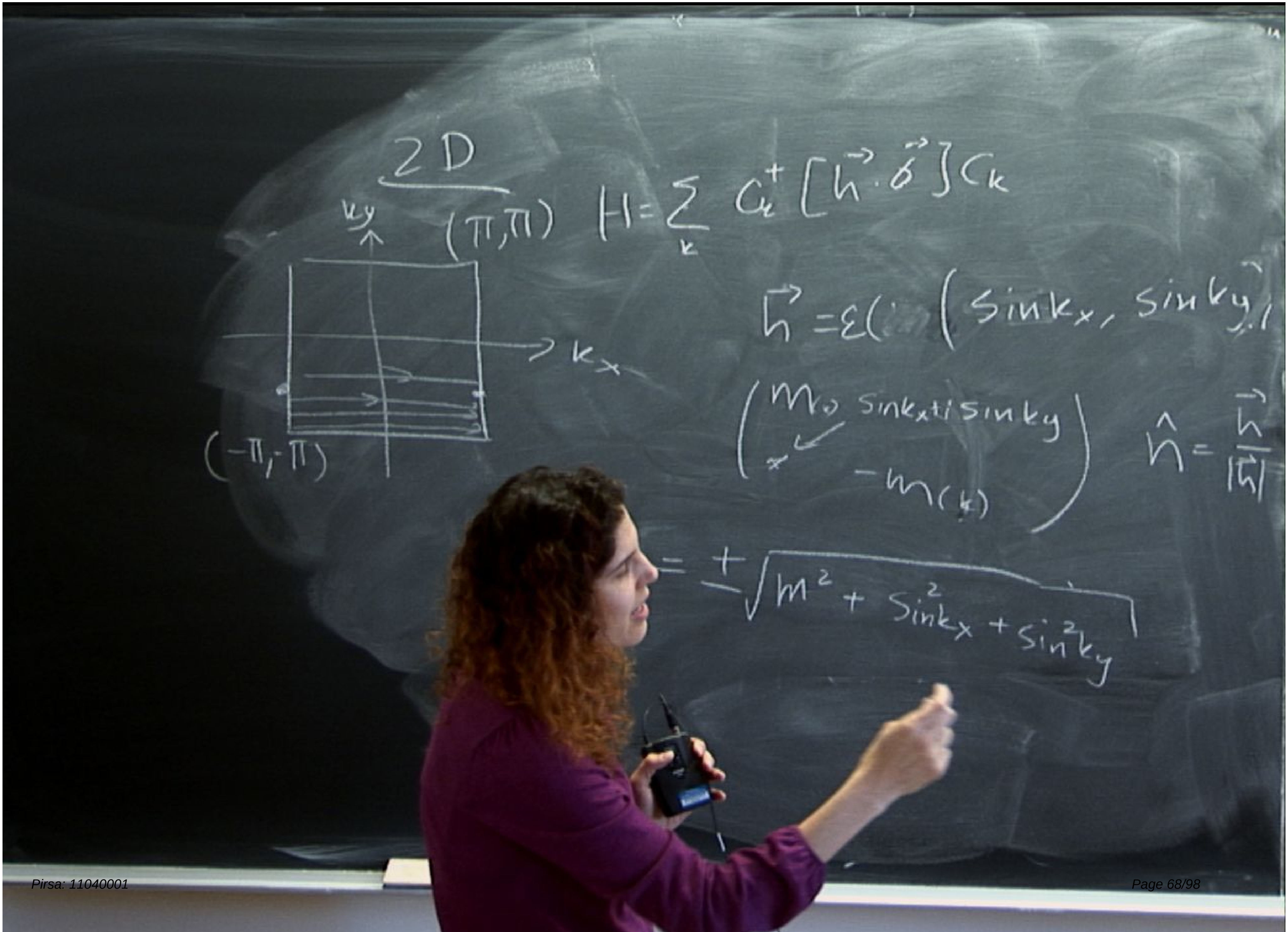



2D
(π,π) $H=\sum_k c_k^\dagger[\vec{h}\cdot\vec{\sigma}]c_k$
k_y
k_x
$(-\pi,-\pi)$
$\vec{h}=\varepsilon(m, \sin k_x, \sin k_y)$
$\begin{pmatrix} m & \sin k_x + i \sin k_y \\ * & -m \end{pmatrix}$
instead
$(\sigma_x, \sigma_y, \sigma_z)$

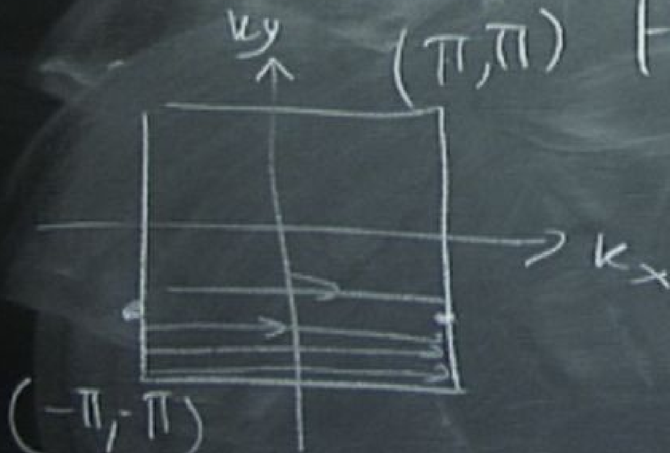

2D
k_y
(π,π)
k_x
$H=\sum_k c_k^\dagger [\vec{h}\cdot\vec{\sigma}] c_k$
$\vec{h}=\varepsilon(m, \sin k_x, \sin k_y)$
$\begin{pmatrix} m & \sin k_x + i \sin k_y \\ * & -m \end{pmatrix}$
$E=\pm\sqrt{m^2+\sin^2 k_x+\sin^2 k_y}$




2D
ky
(π,π)
kx
(-π,-π)
H = Σ_k c_k^† [h⃗ · σ⃗] c_k
h⃗ = ε((sin kx, sin ky,
m(k) sin kx + i sin ky
-m(k)
ĥ = h⃗ / |h⃗|
E = ± √(m² + sin² kx + sin² ky)


2D
(π,π)
H = Σ_k c_k^† [h · σ] c_k
k_y
k_x
(-π,-π)
h = ε((sin k_x, sin k_y,
ĥ = h / |h|
sin k_x + i sin k_y
-m(k)
= ± √(m² + sin² k_x + sin² k_y)

2D



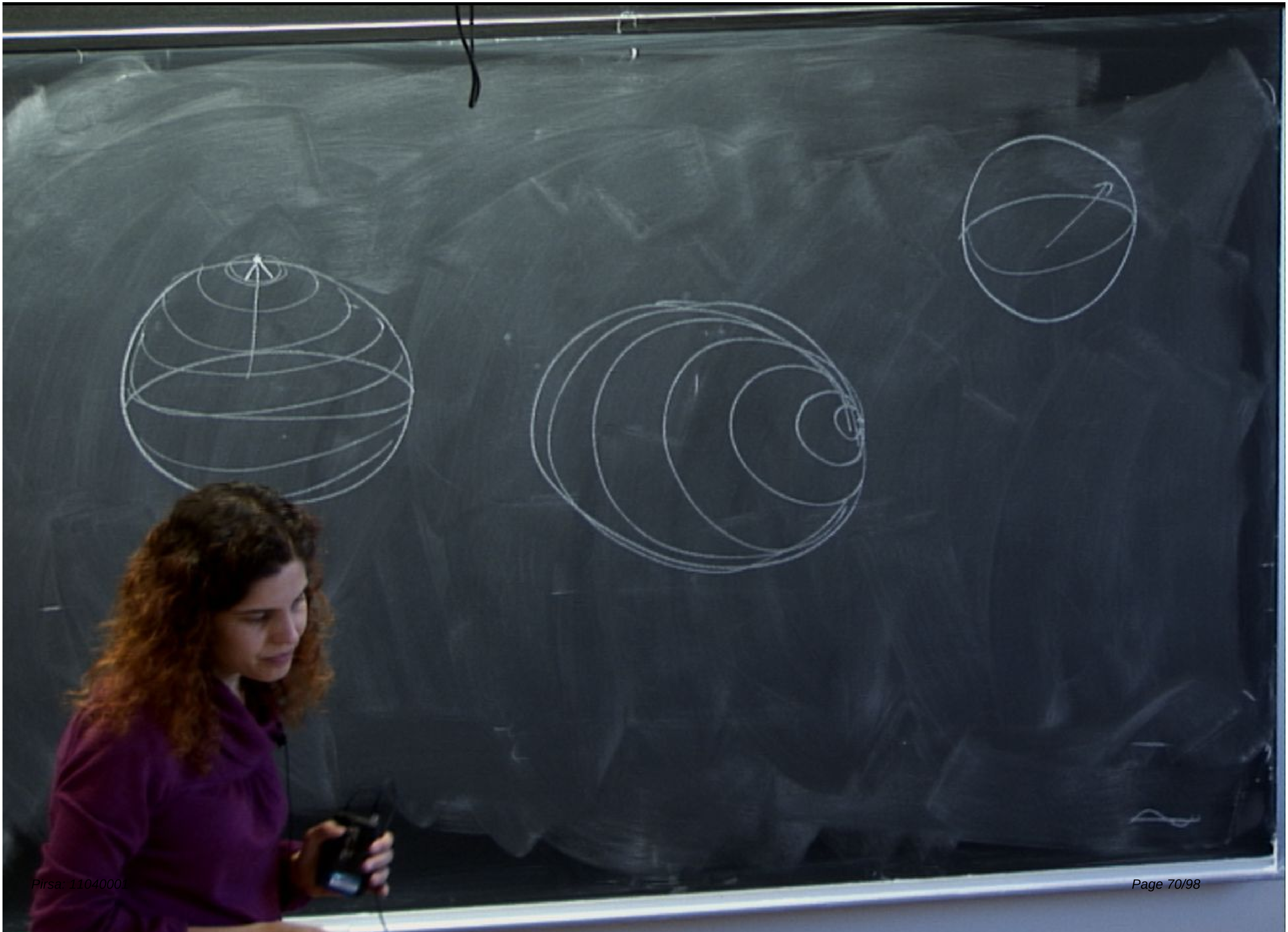
$$H=\sum_k c_k^\dagger\,[\vec{h}\cdot\vec{\sigma}]\,c_k$$

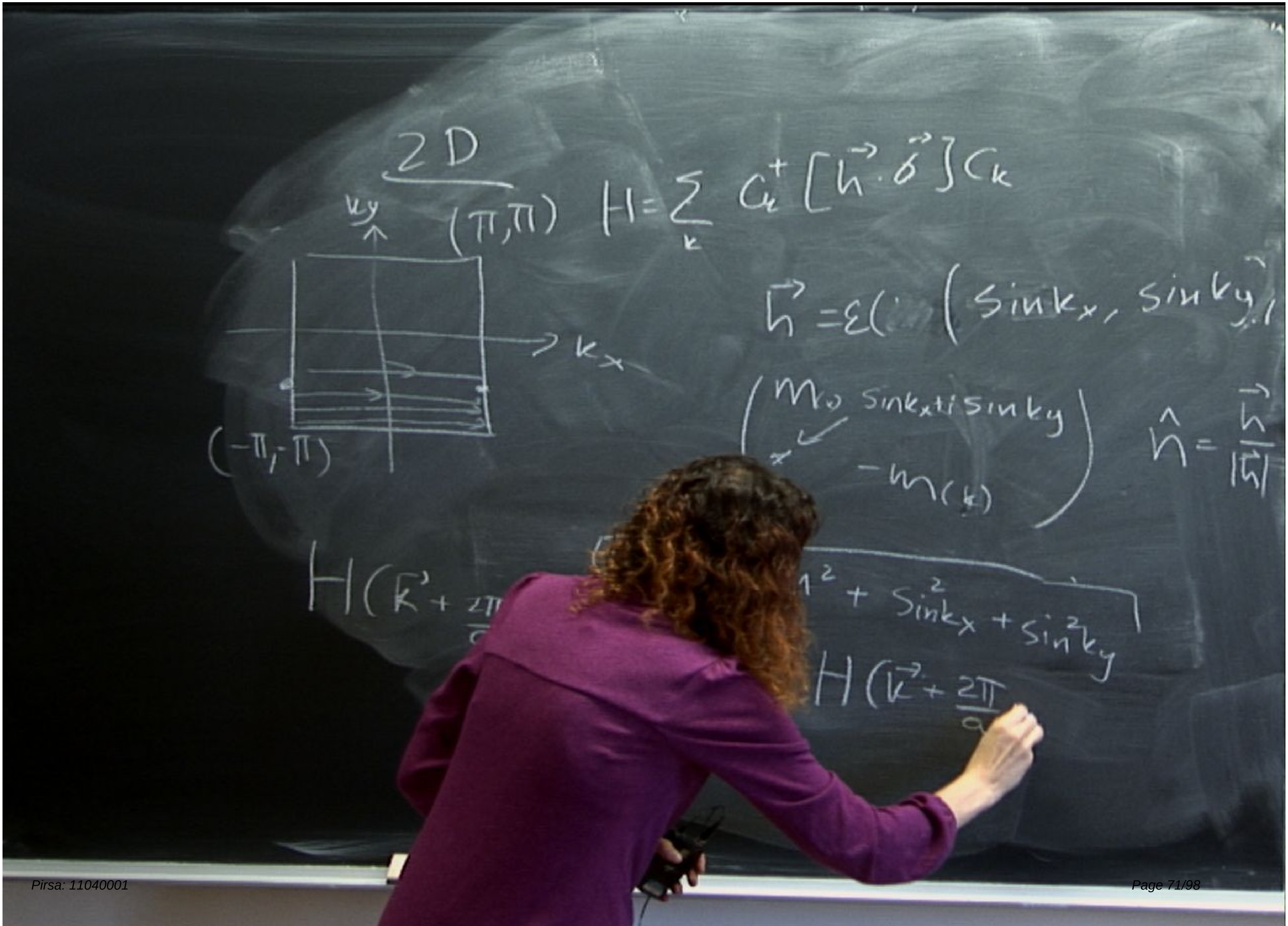
$$\vec{h}=\varepsilon(\ \ (\sin k_x,\ \sin k_y,$$

$$\begin{pmatrix} m(k) & \sin k_x + i\sin k_y \\ * & -m(k) \end{pmatrix}$$

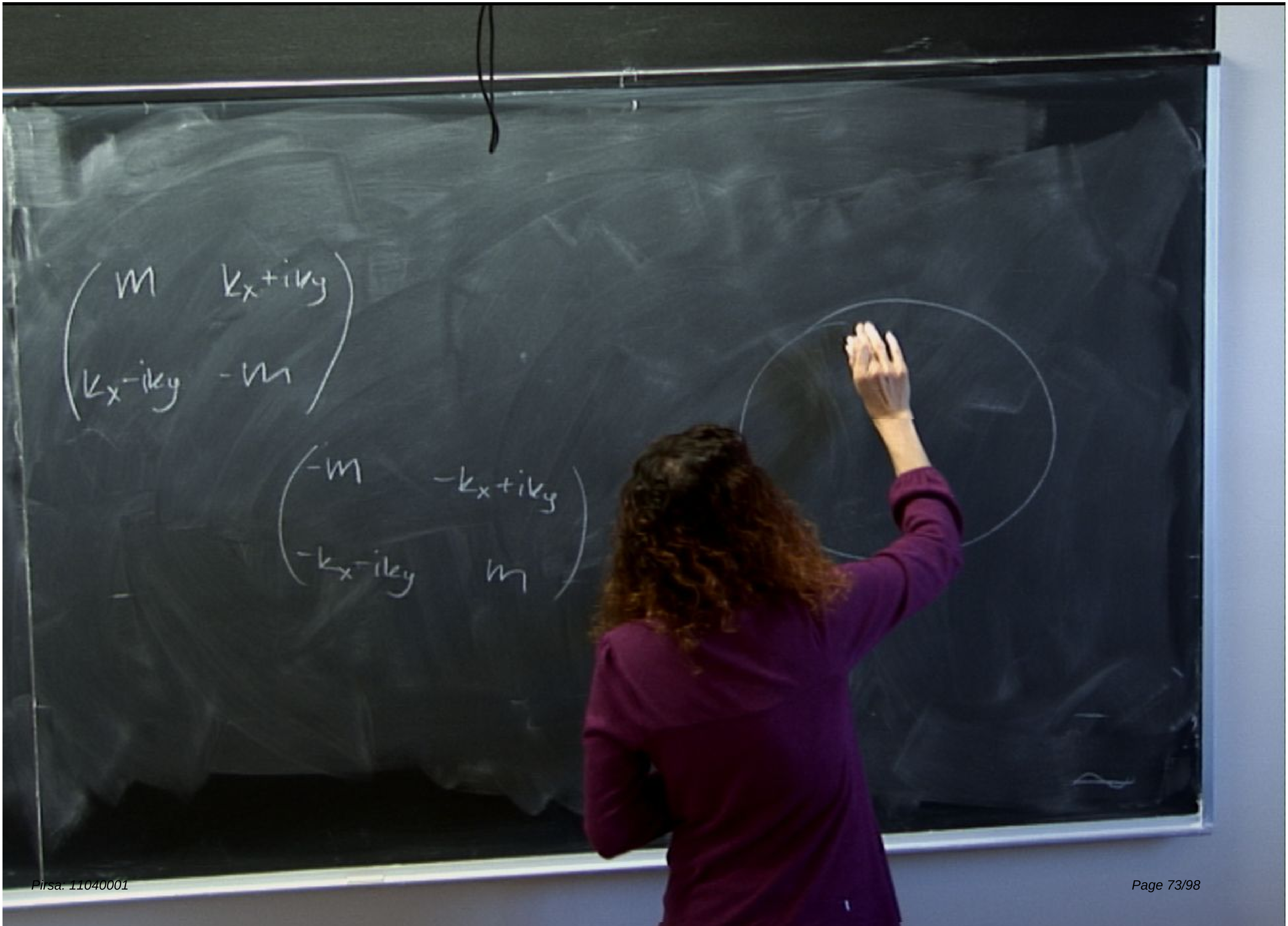
$$\hat{n}=\frac{\vec{h}}{|\vec{h}|}$$

$$E=\pm\sqrt{m^2+\sin^2 k_x+\sin^2 k_y}$$




2D
H = Σ_k c_k^+ [h⃗·σ⃗] c_k
k_y
(π,π)
k_x
(−π,−π)
h⃗ = ε((sin k_x, sin k_y,
m(k) sin k_x + i sin k_y
−m(k)
ĥ = h⃗ / |h⃗|
H(k⃗ + 2π/a
+ sin² k_x + sin² k_y
H(k⃗ + 2π/a

$$\begin{pmatrix} m & k_x + ik_y \\ k_x - ik_y & -m \end{pmatrix}$$

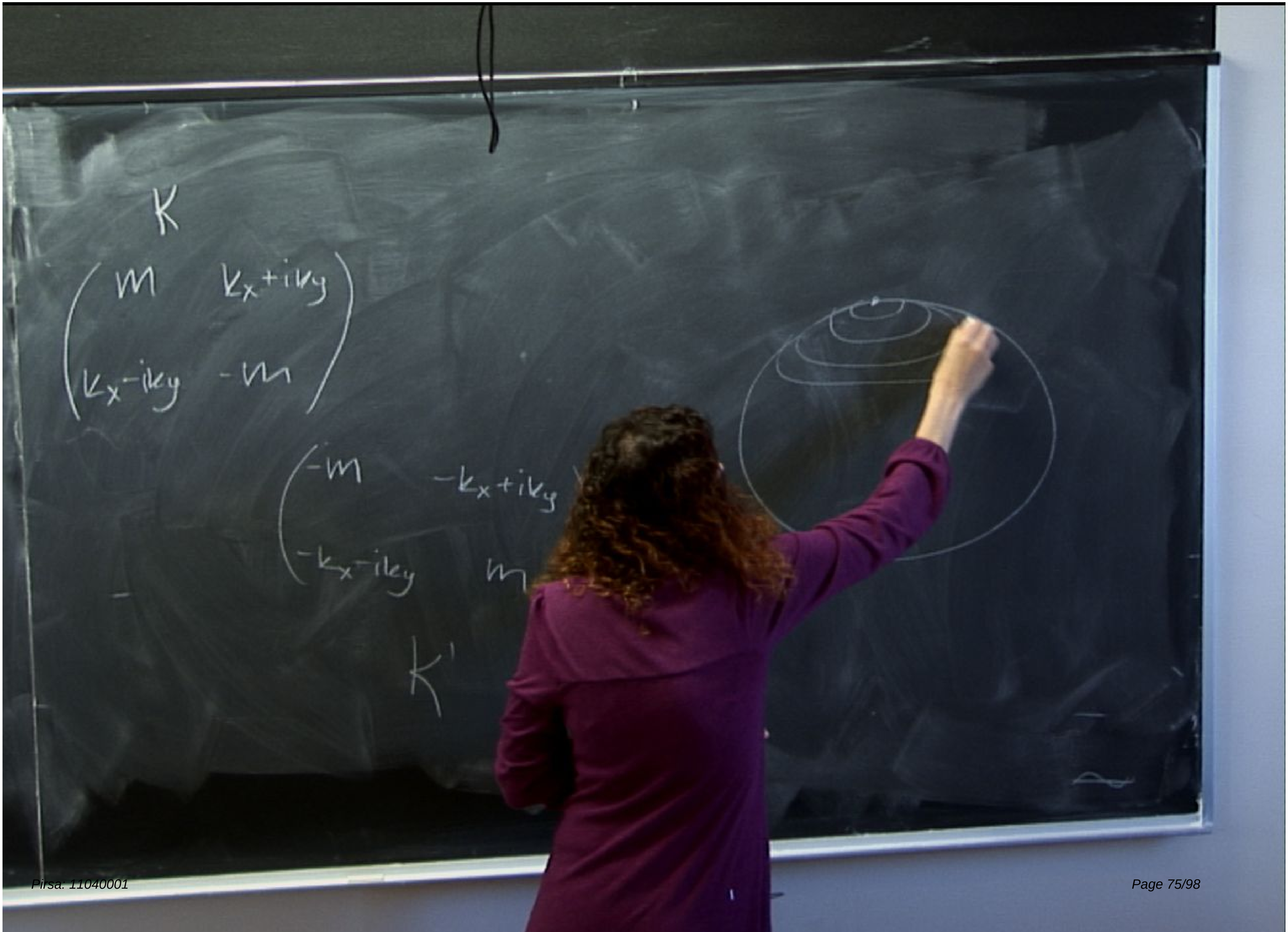

m kx+iky
kx-iky -m
-m -kx+iky
-kx-iky m

K

$$\begin{pmatrix} m & k_x + ik_y \\ k_x - ik_y & -m \end{pmatrix}$$

$$\begin{pmatrix} -m & -k_x + ik_y \\ -k_x - ik_y & m \end{pmatrix}$$

K'

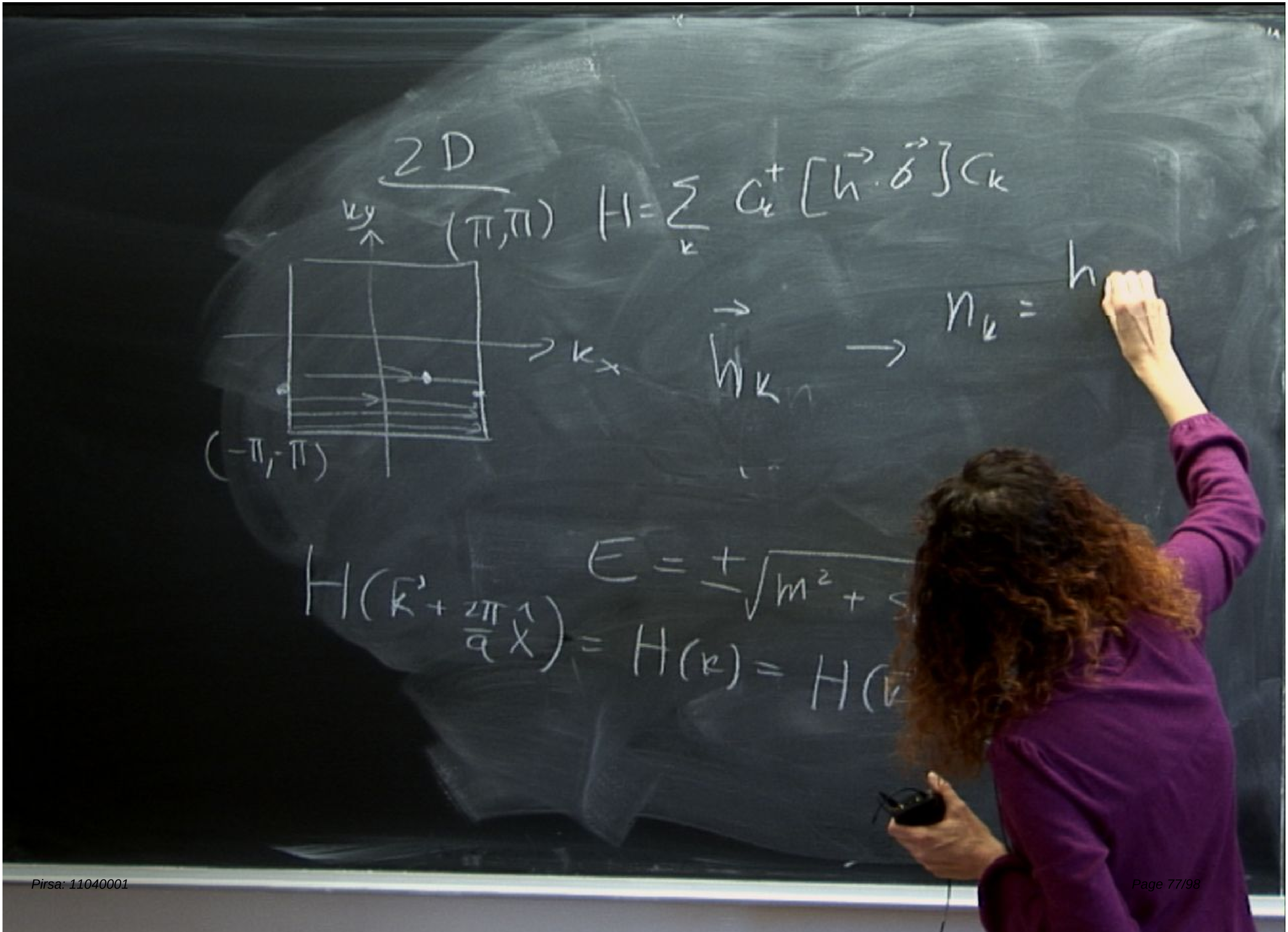

K
m   kx+iky
kx-iky   -m
-m   -kx+iky
-kx-iky   m
K'

K

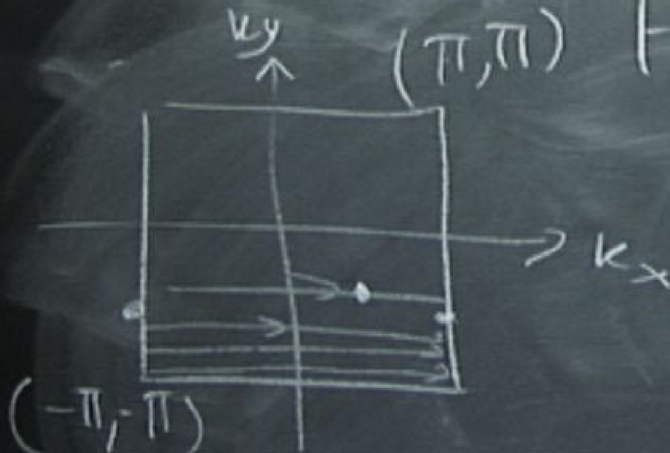
$$\begin{pmatrix} m & k_x + ik_y \\ k_x - ik_y & -m \end{pmatrix}$$

$$\begin{pmatrix} -m & -k_x + ik_y \\ -k_x - ik_y & m \end{pmatrix}$$

K'


2D
(π,π)
H = Σk ck† [h⃗ · σ⃗] ck
ky
kx
(−π,−π)
h⃗k → nk = h
E = ± √(m² +
H(k⃗ + 2π/q x̂) = H(k) = H(k

2D

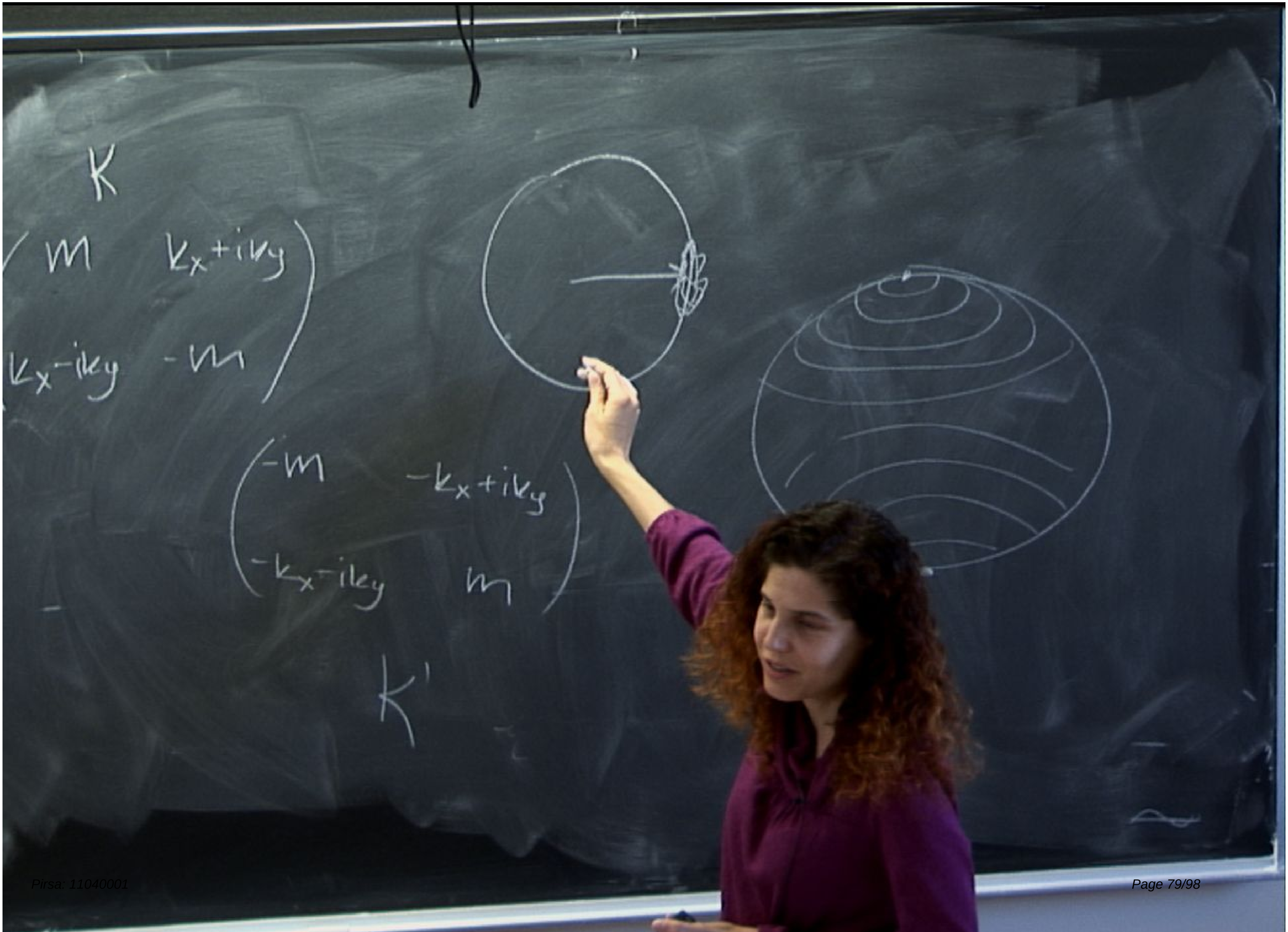


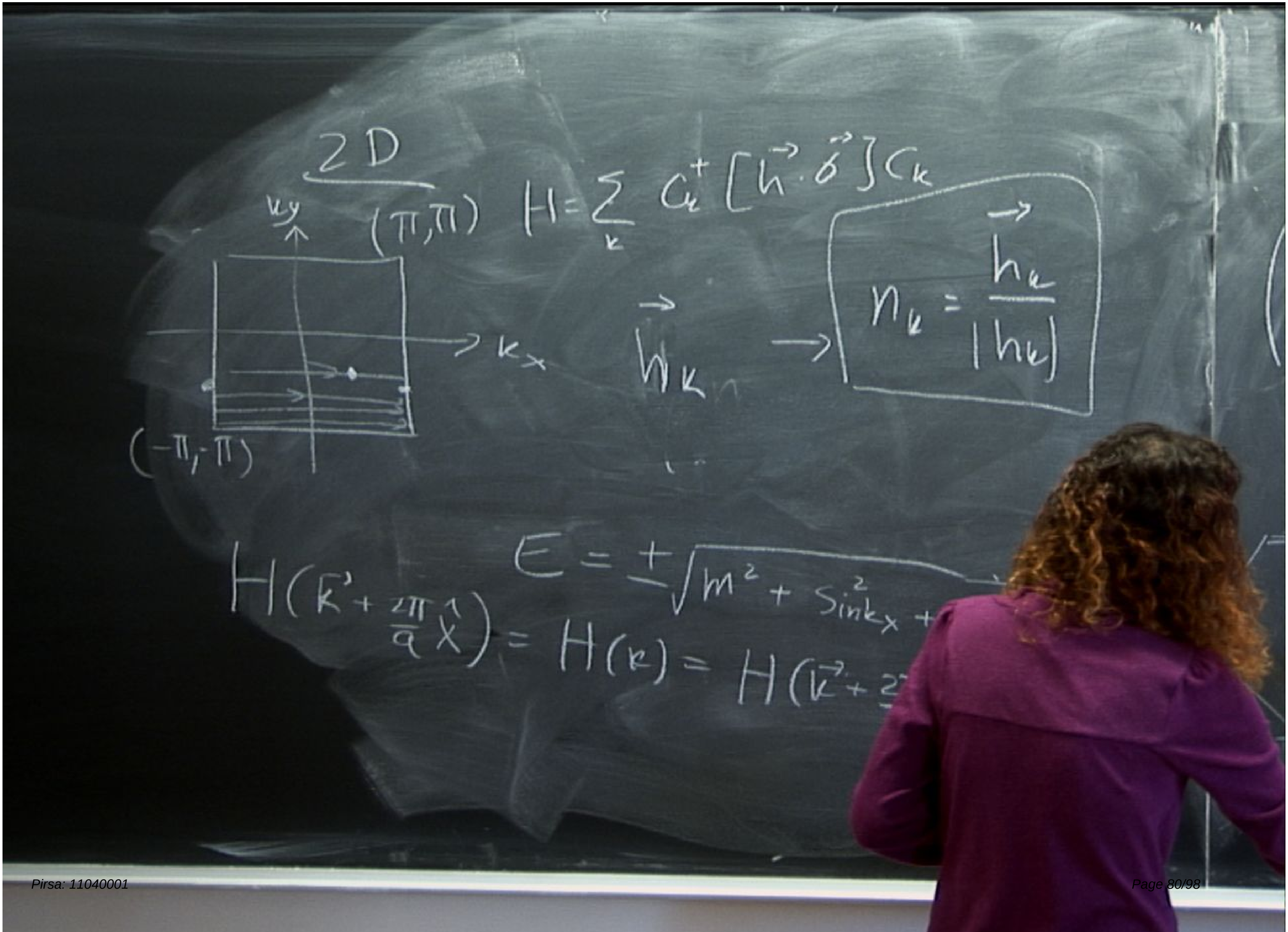
$$H = \sum_k c_k^\dagger [\vec{h}\cdot\vec{\sigma}] c_k$$

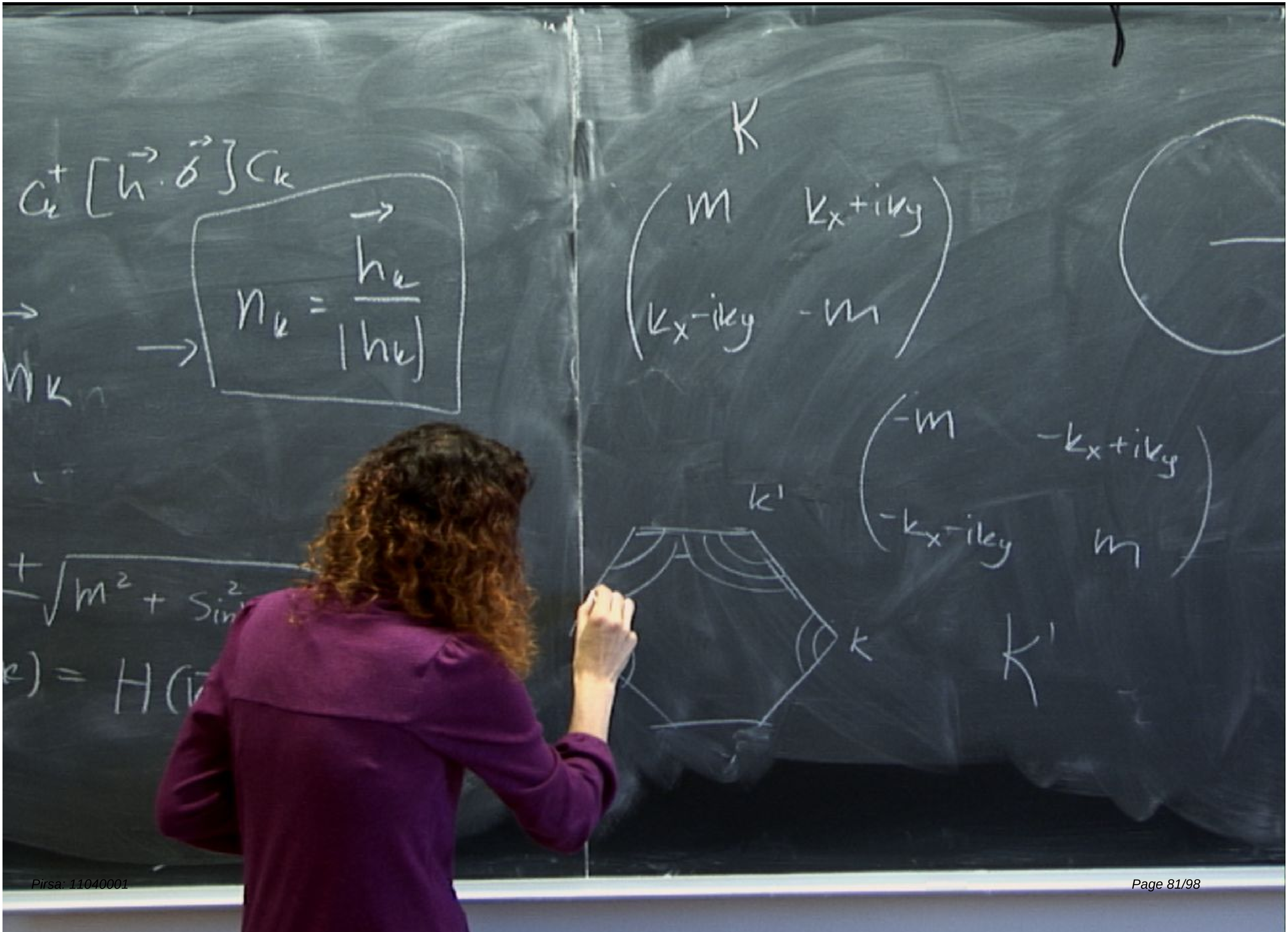
$$\vec{h}_k \longrightarrow n_k = \frac{\vec{h}_k}{|h_k|}$$

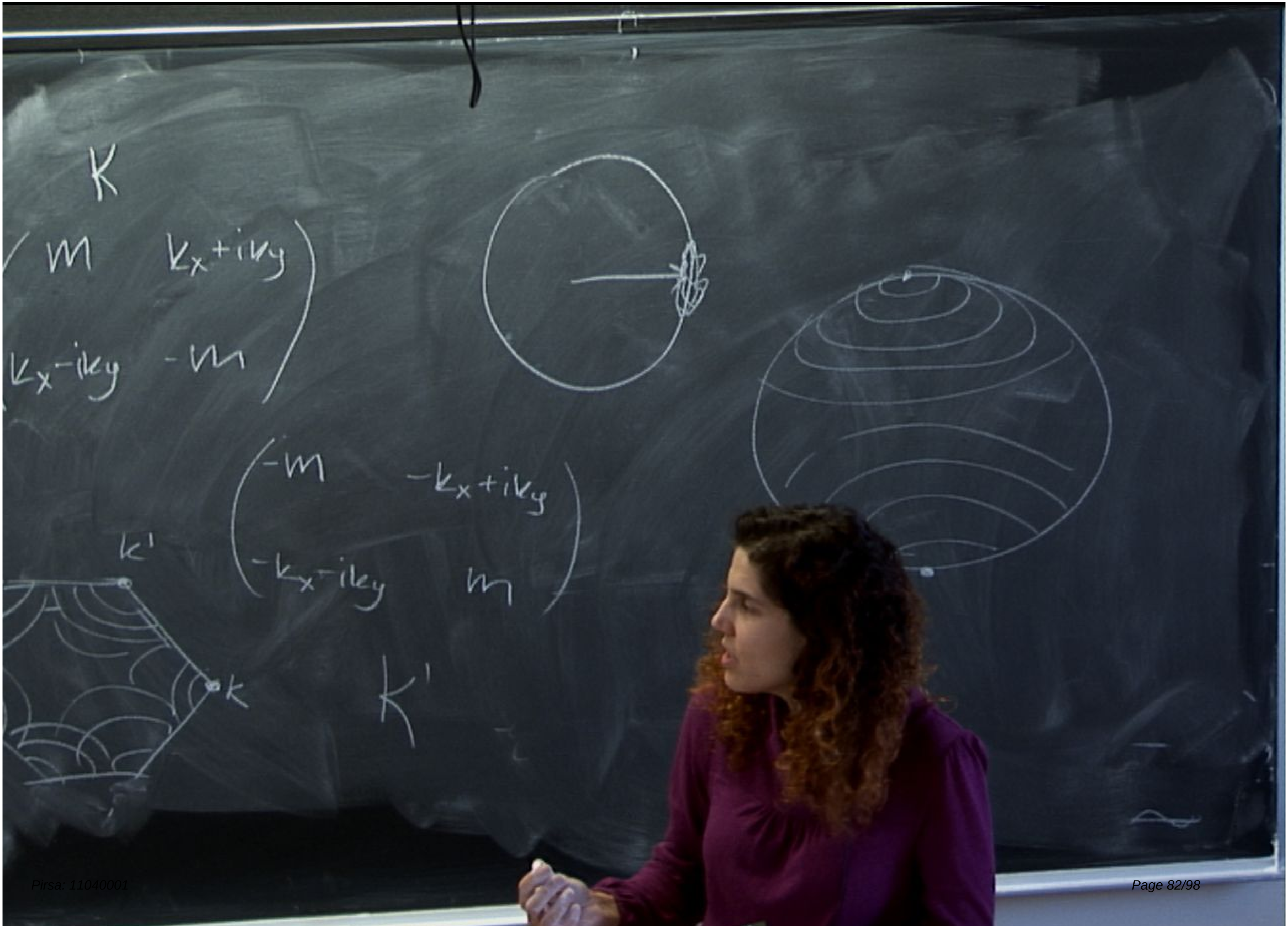
$$E = \pm\sqrt{m^2 + \sin^2 k_x + \sin^2 k_y}$$

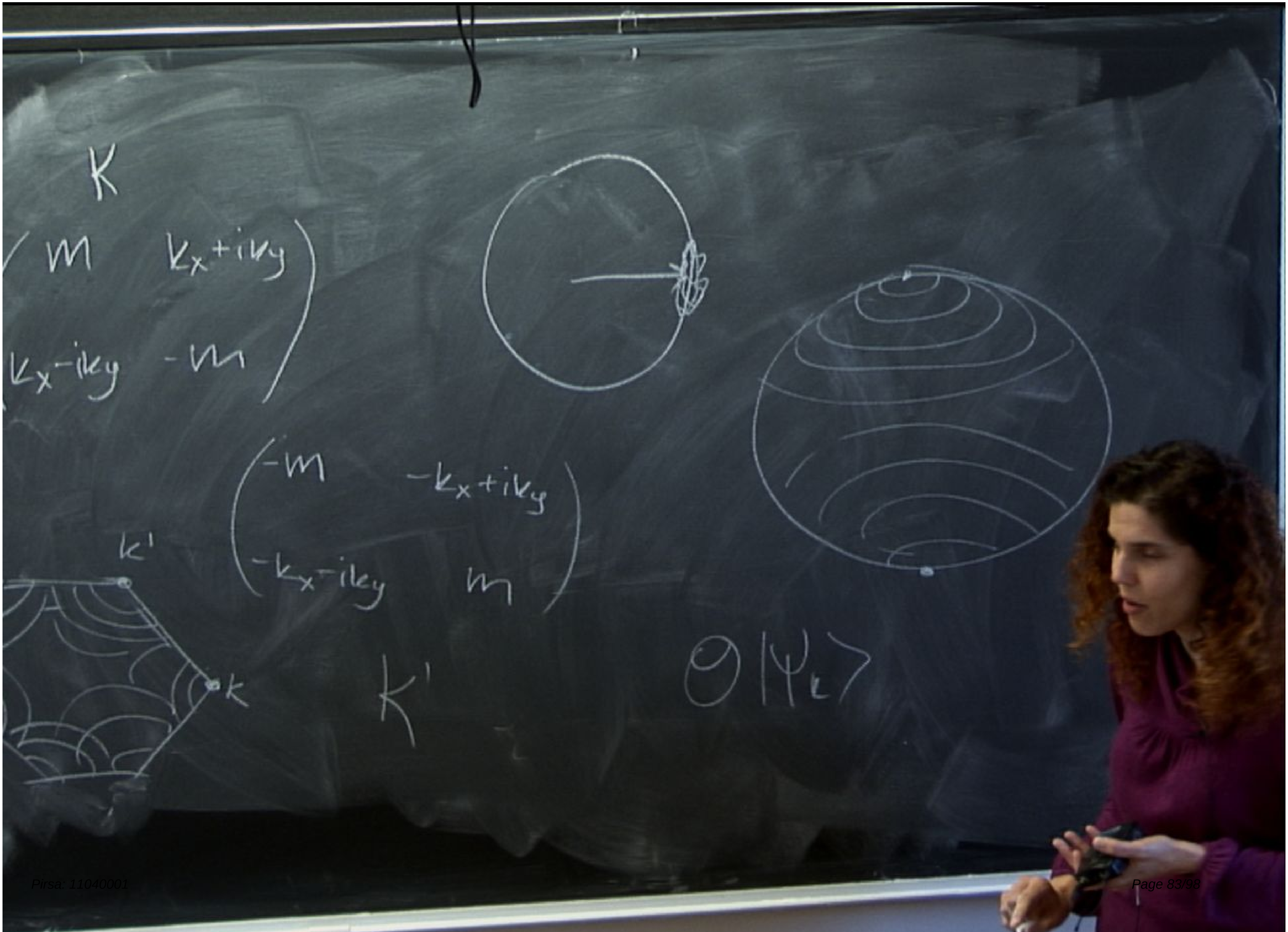
$$H(\vec{k} + \frac{2\pi}{a}\hat{x}) = H(k) = H(\vec{k} + \frac{2\pi}{a}\hat{y})$$

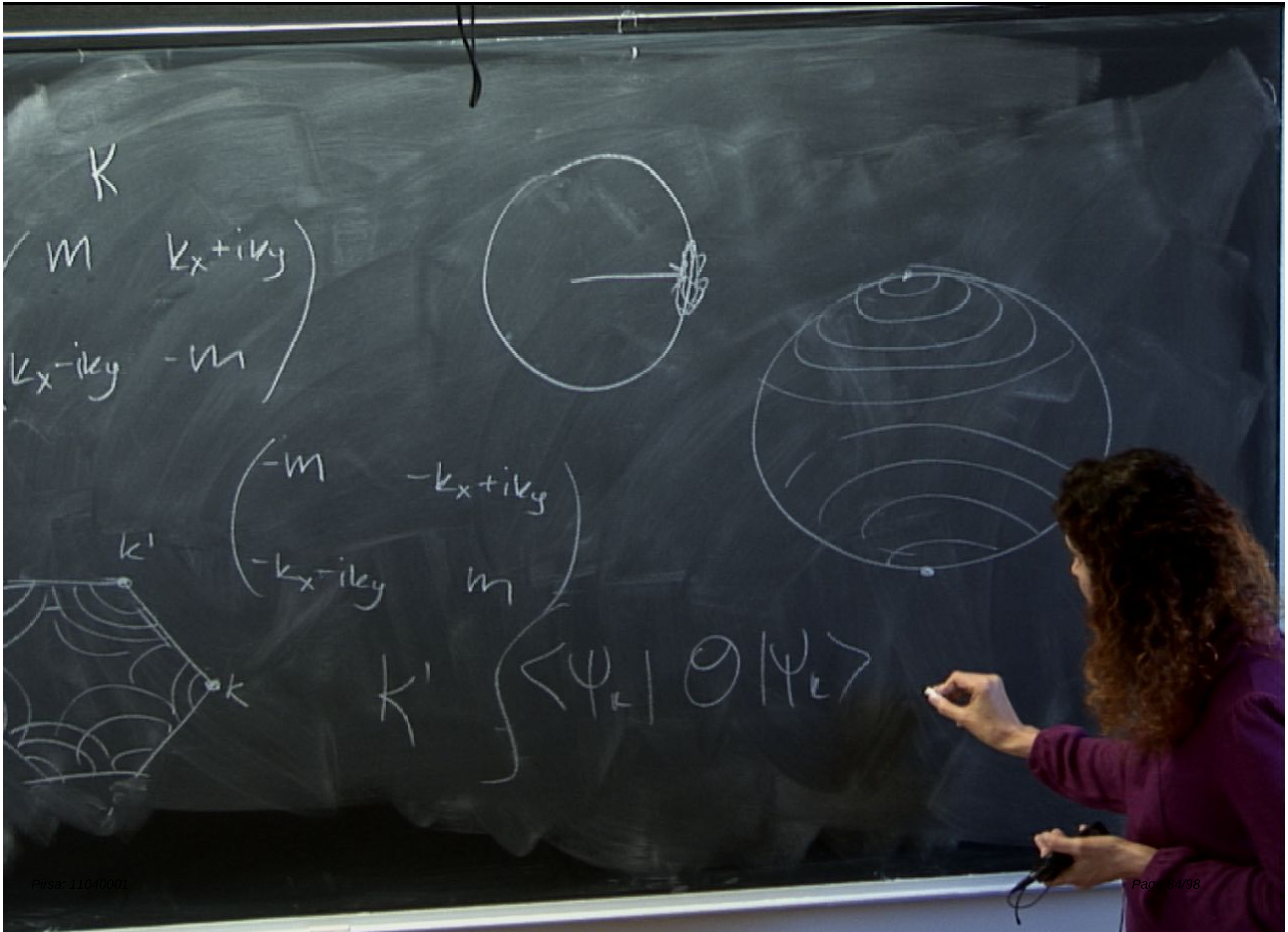

K
m   kx+iky
kx-iky   -m
-m   -kx+iky
-kx-iky   m
K'


2D
(π,π)
H = Σ_k c_k^+ [h⃗·σ⃗] c_k
k_y
k_x
(−π,−π)
h⃗_k → n_k = h⃗_k / |h_k|
E = ± √(m² + sin²k_x +
H(k⃗ + 2π/a x̂) = H(k) = H(k⃗ + 2


K
m   kx+iky
kx−iky   −m
−m   −kx+iky
−kx−iky   m
K'
K
K'


K
m   kx+iky
kx−iky   −m
−m   −kx+iky
−kx−iky   m
k'
k
K'


K
m   kx+iky
kx-iky   -m
-m   -kx+iky
-kx-iky   m
k'
k
K'
Θ|ψk>


K
m   kx+iky
kx-iky   -m
-m   -kx+iky
-kx-iky   m
k'
k
K'   <Ψk| O |Ψk>

K

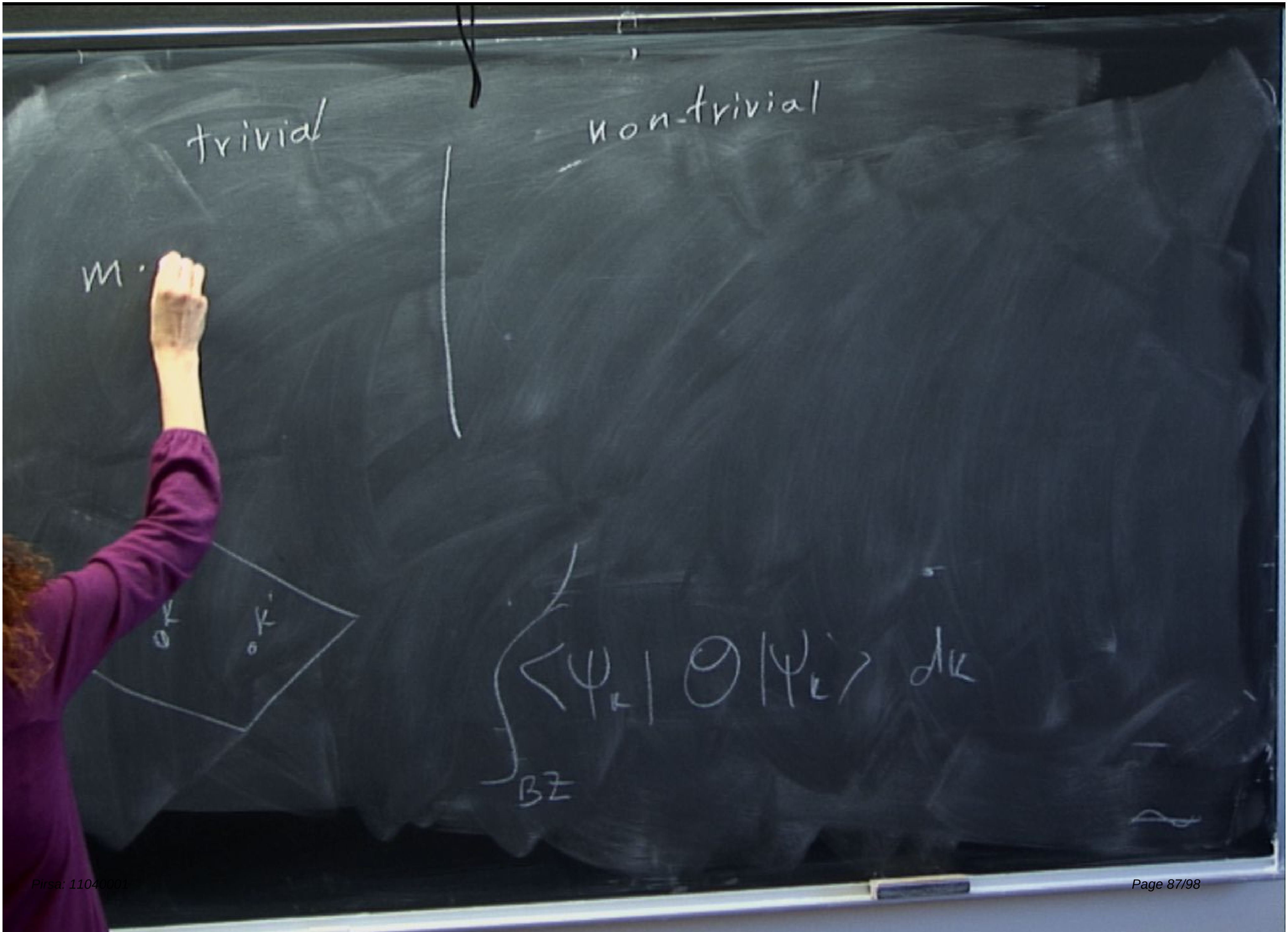
$$\begin{pmatrix} m & k_x+ik_y \\ k_x-ik_y & -m \end{pmatrix}$$

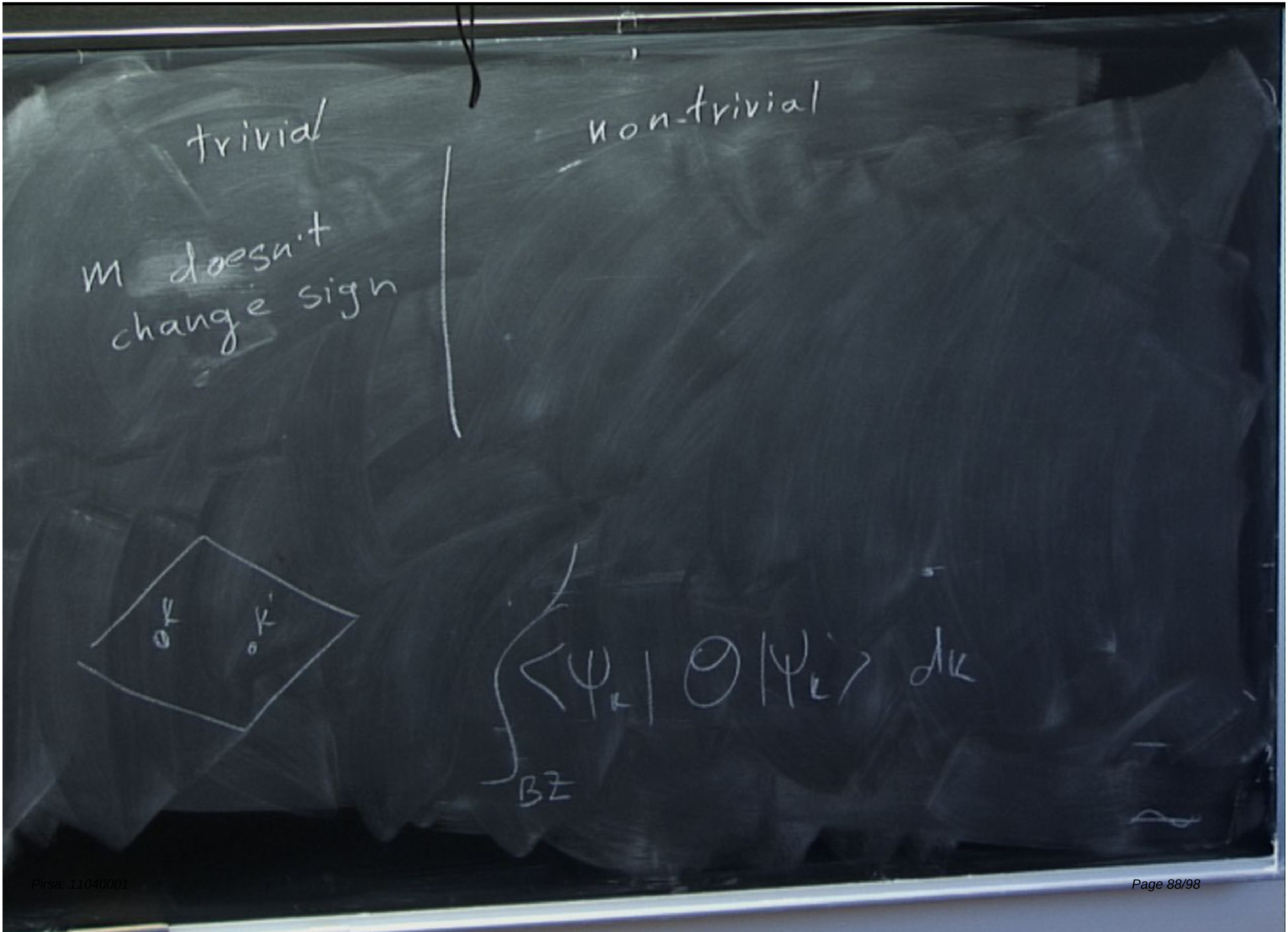
$$\begin{pmatrix} -m & -k_x+ik_y \\ -k_x-ik_y & m \end{pmatrix}$$

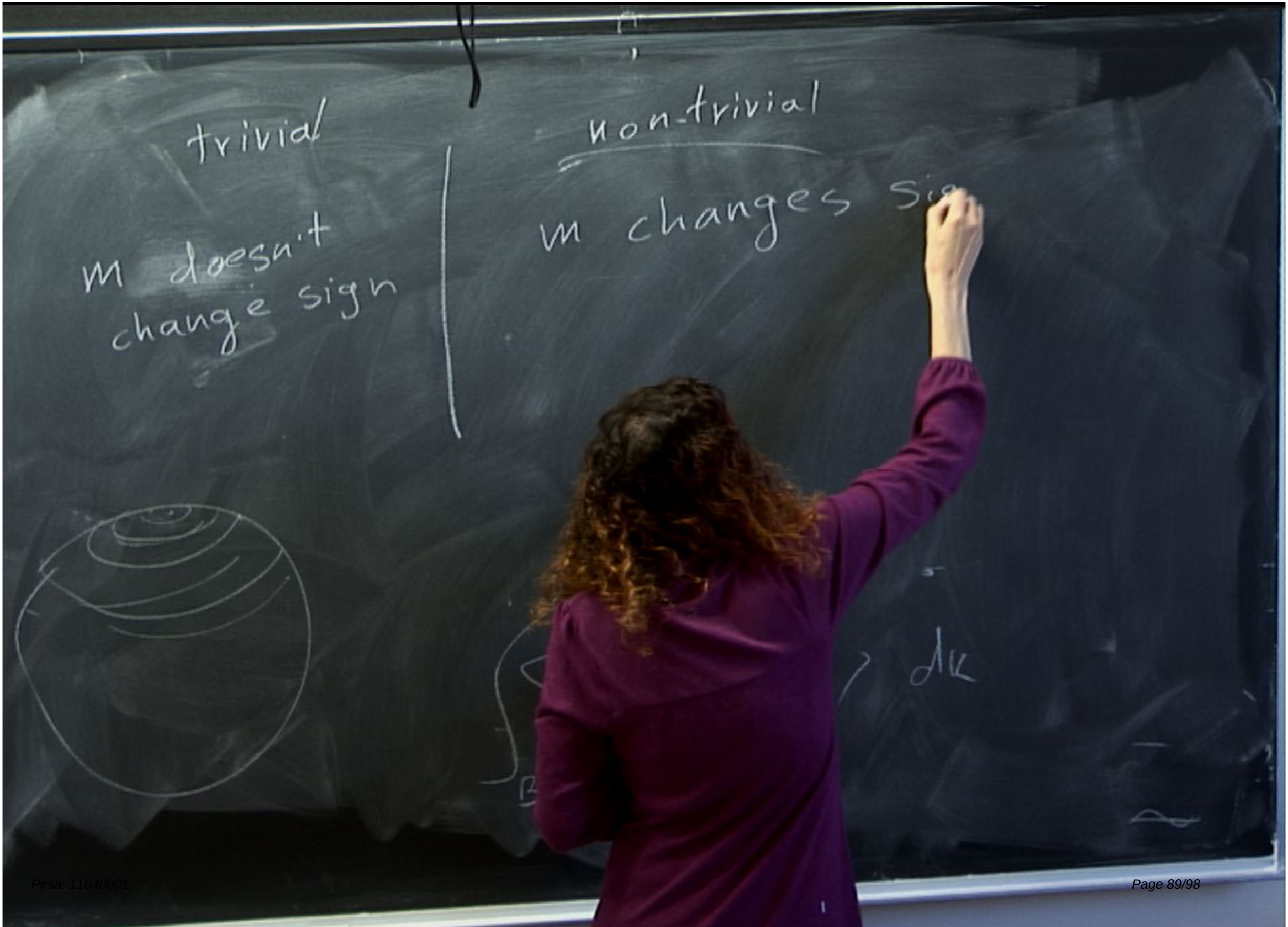
K'

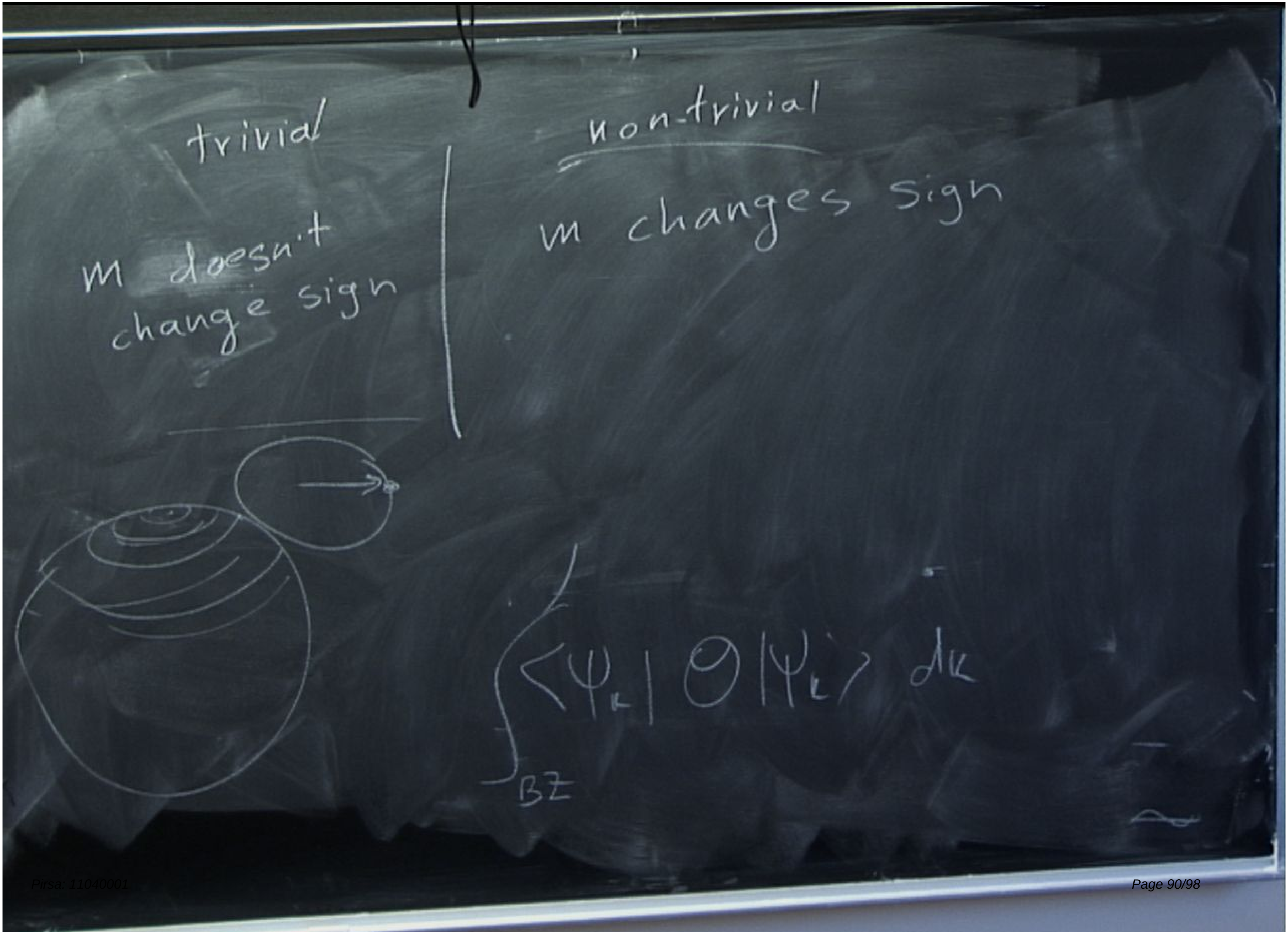
$$\int_{BZ} \langle \psi_k | \Theta | \psi_k \rangle \, dk$$

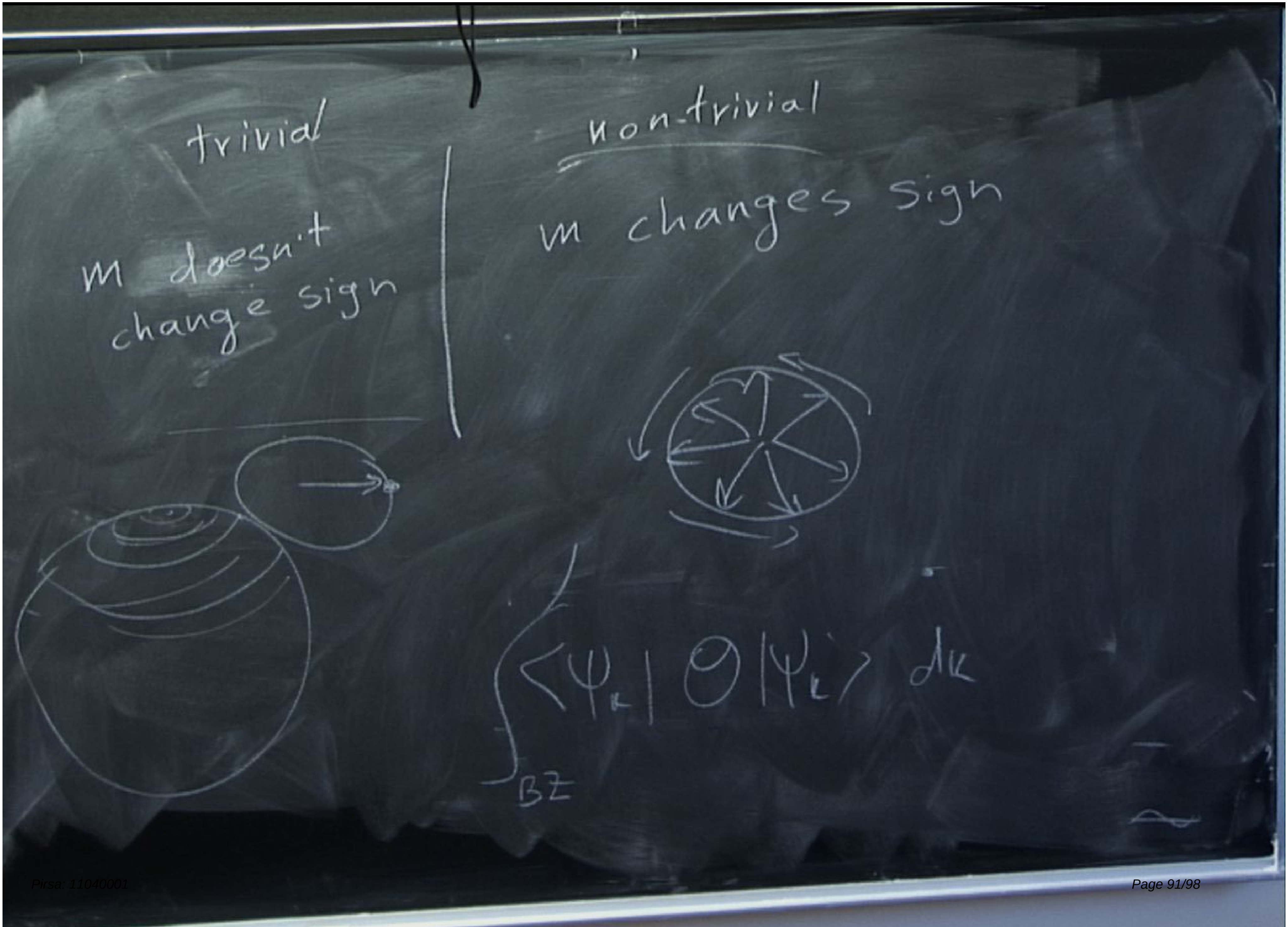

K
K'
$\int_{BZ} \langle \psi_k | O | \psi_{k'} \rangle$

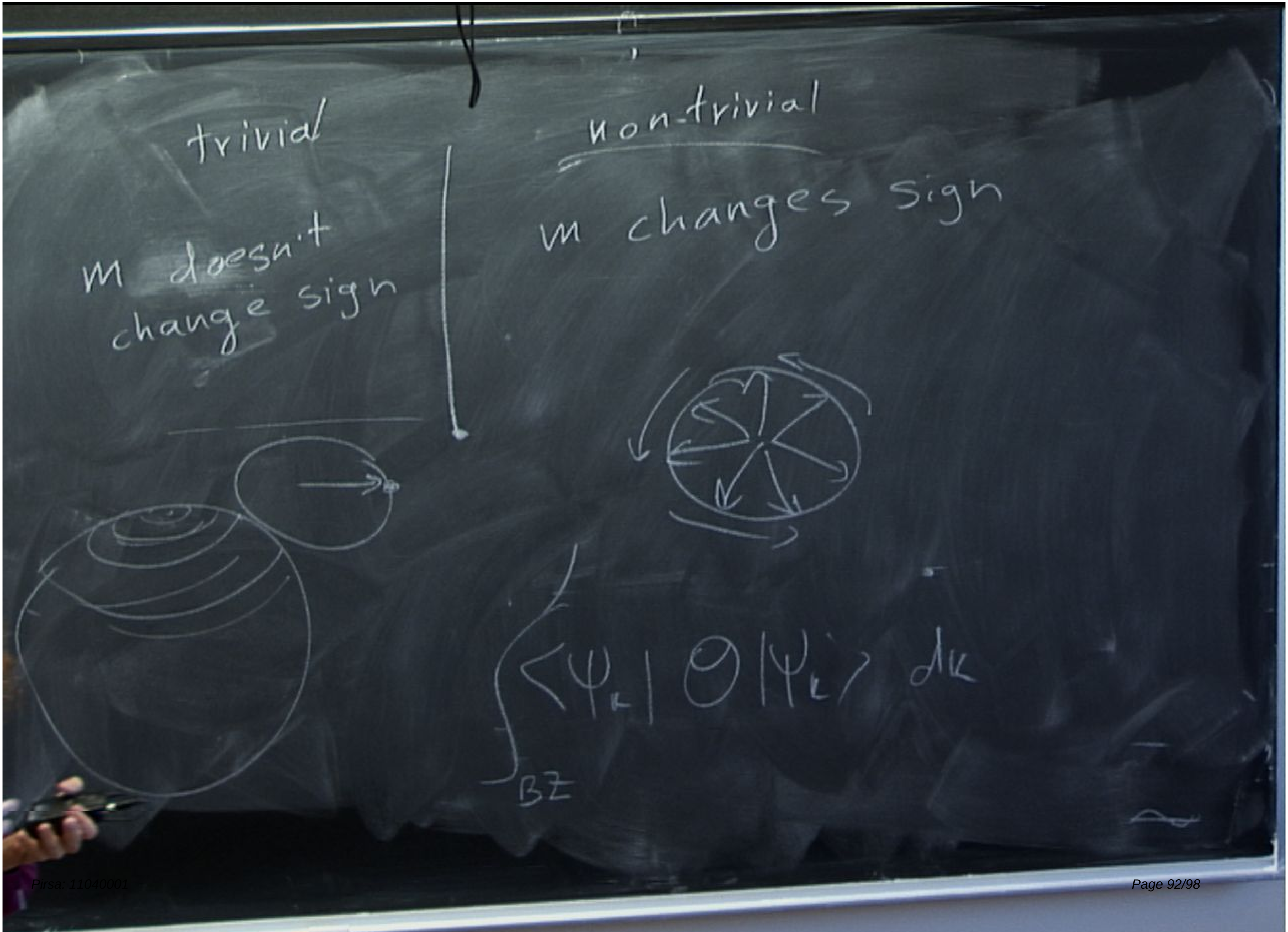

trivial
non-trivial
m ·
k
k'
$\int_{BZ} \langle \psi_k | \mathcal{O} | \psi_{k'} \rangle \, dk$

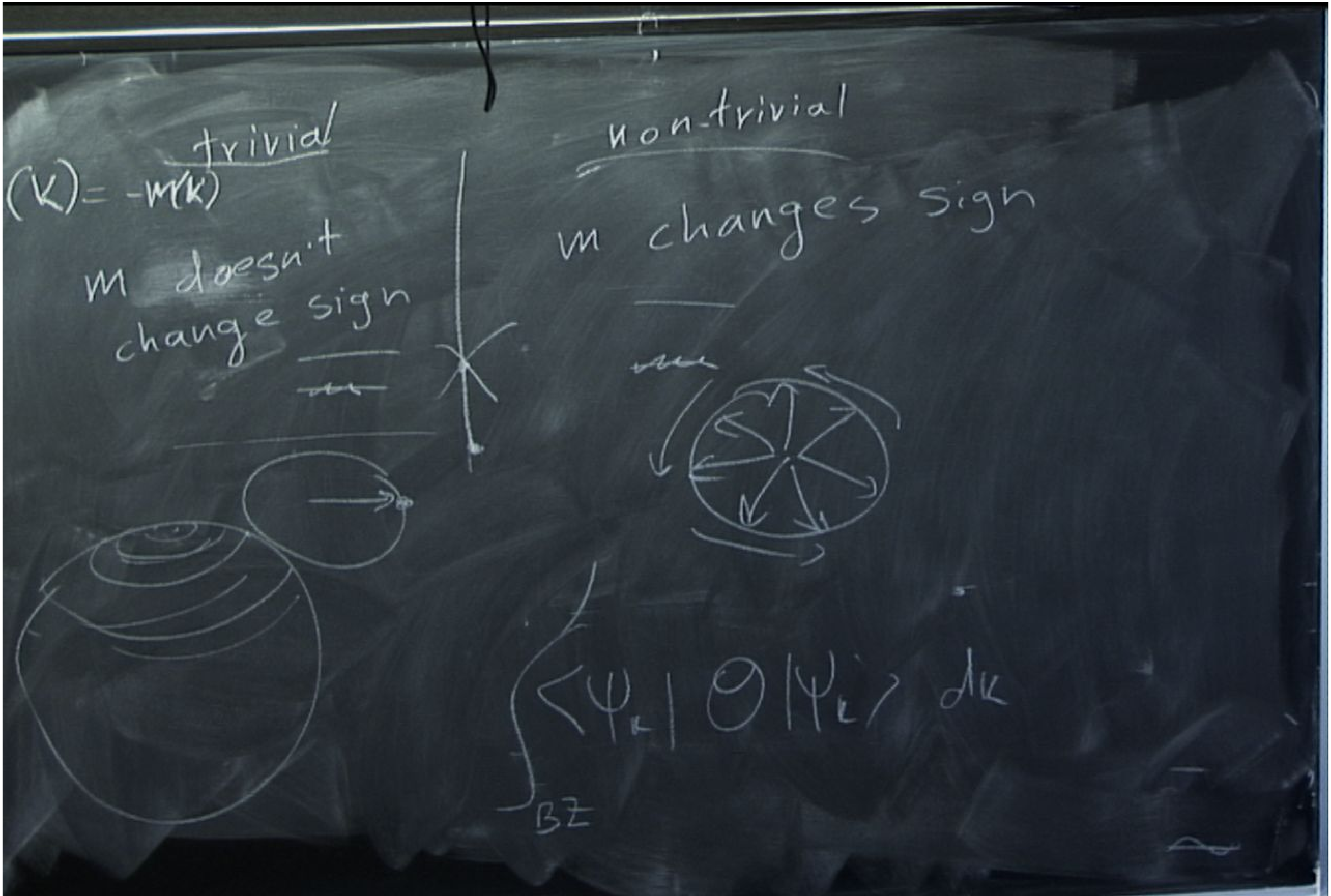

trivial
non-trivial
m doesn't change sign
K
K'
$\int_{BZ} \langle \psi_k | O | \psi_{k'} \rangle \, dk$

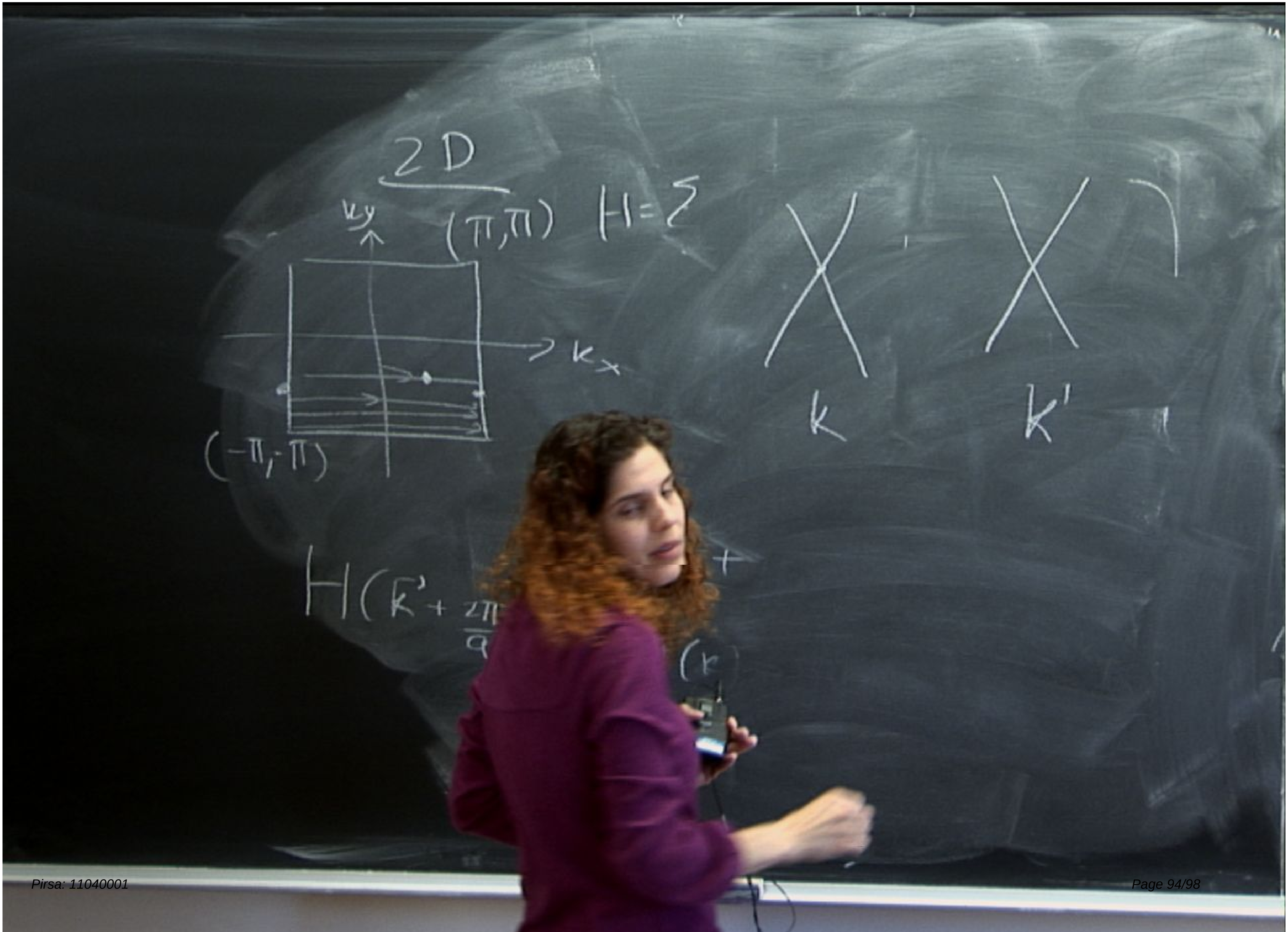

trivial
non-trivial
m doesn't change sign
m changes si
dk

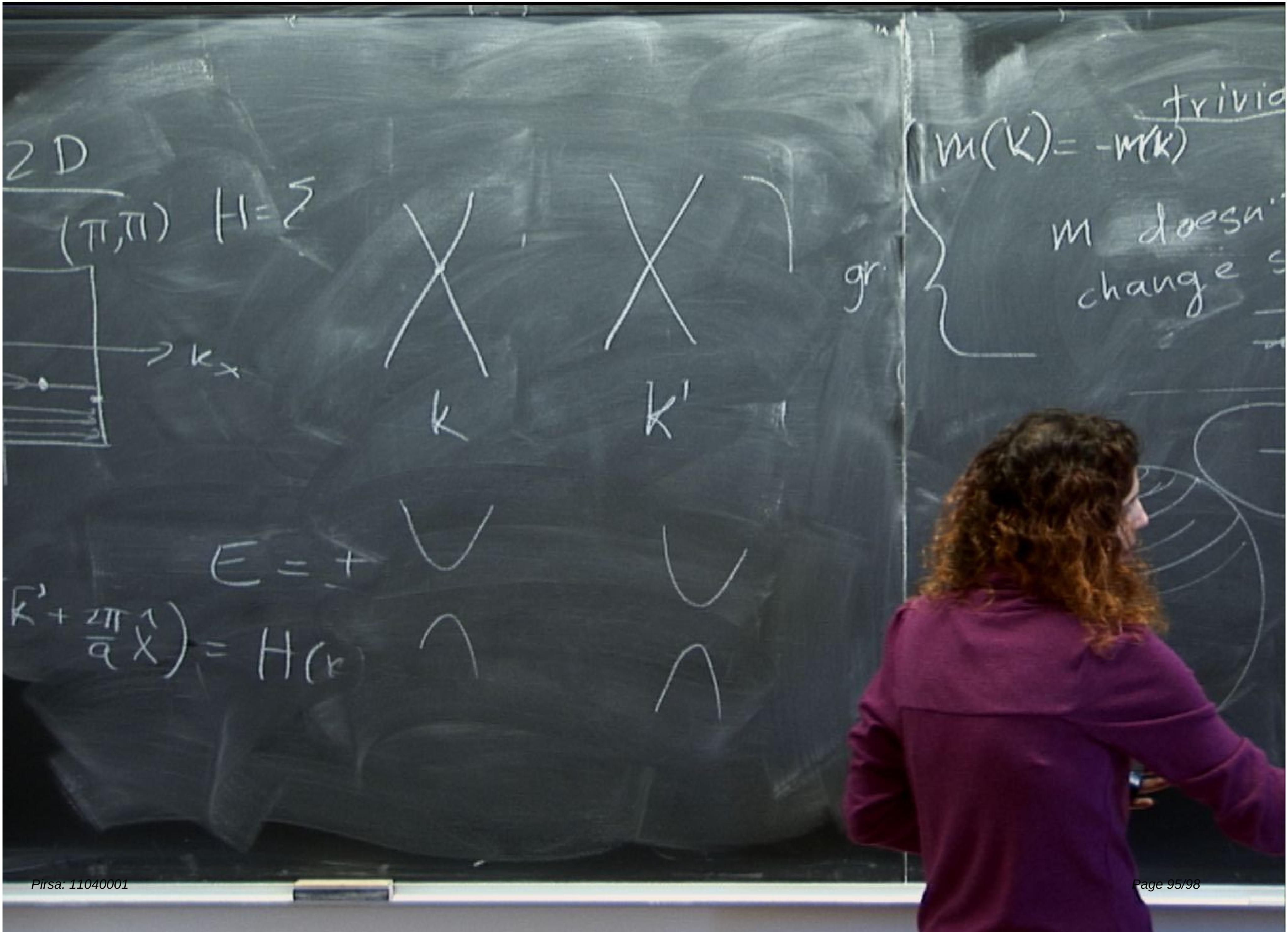

trivial
m doesn't change sign
non-trivial
m changes sign
∫BZ ⟨ψk| O |ψk⟩ dk

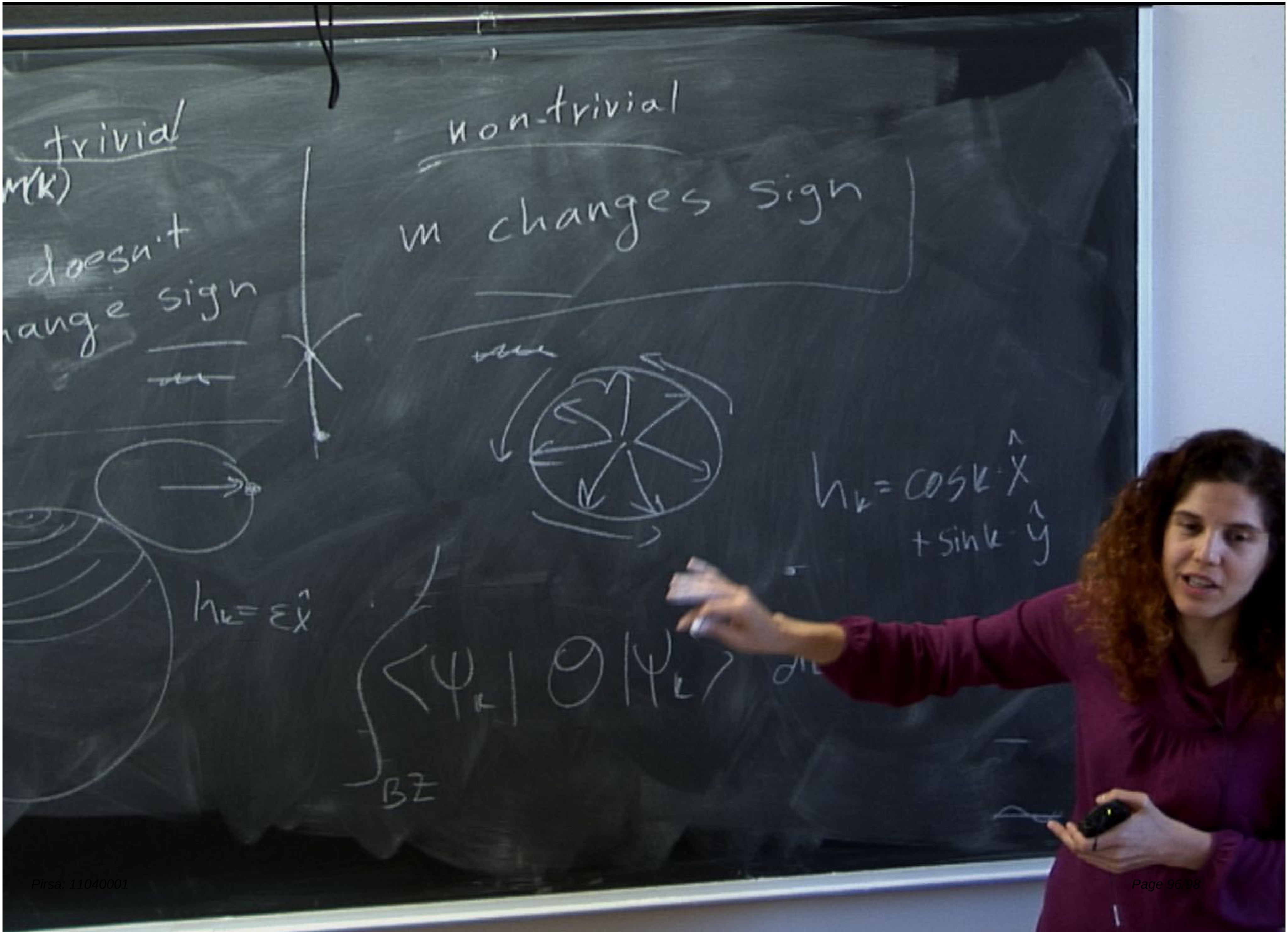

trivial
m doesn't change sign
non-trivial
m changes sign
$\int_{BZ} \langle \psi_k | O | \psi_k \rangle \, dk$

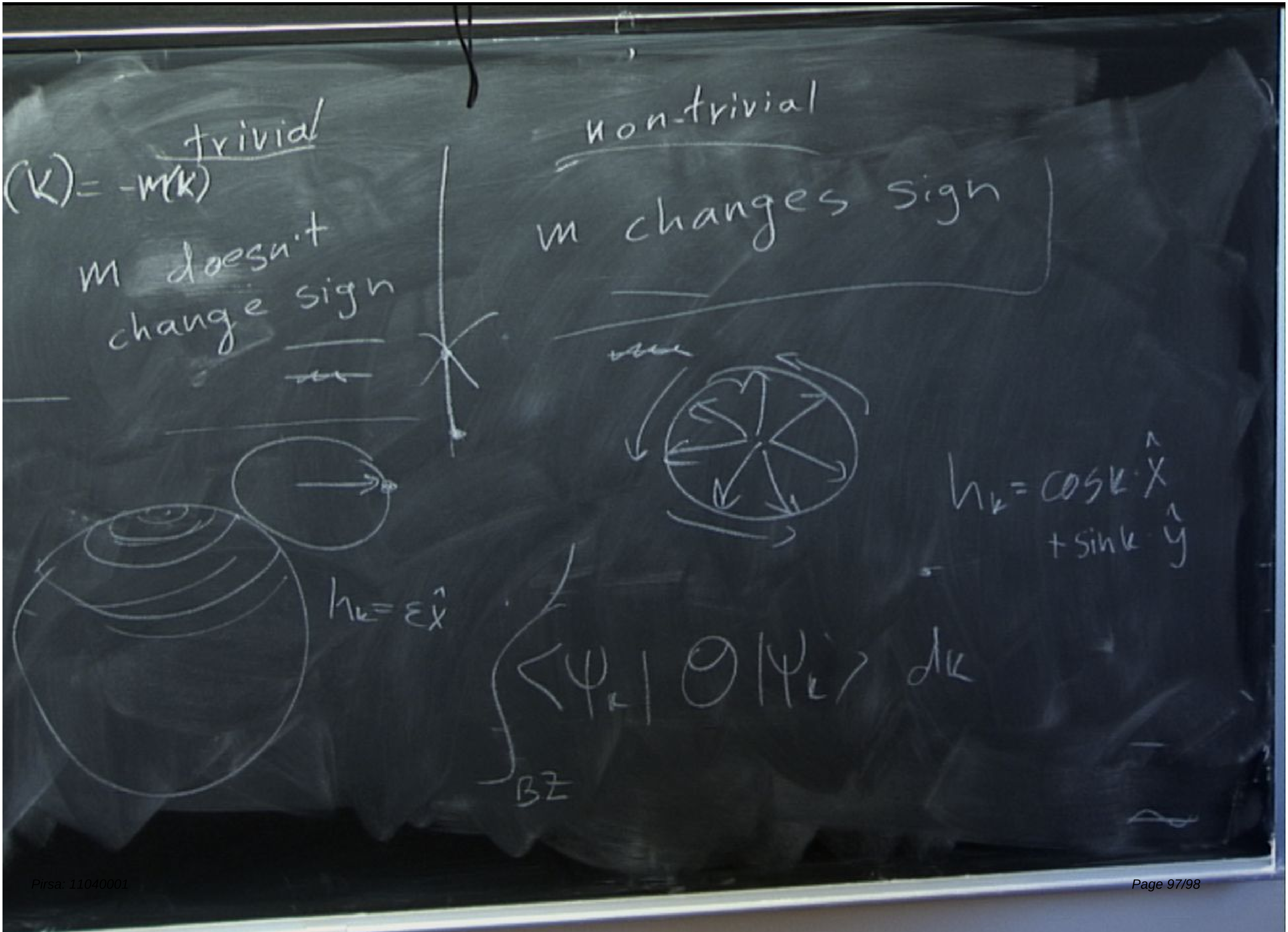

trivial
m doesn't change sign
non-trivial
m changes sign
$\int_{BZ} \langle \psi_k | O | \psi_k \rangle \, dk$

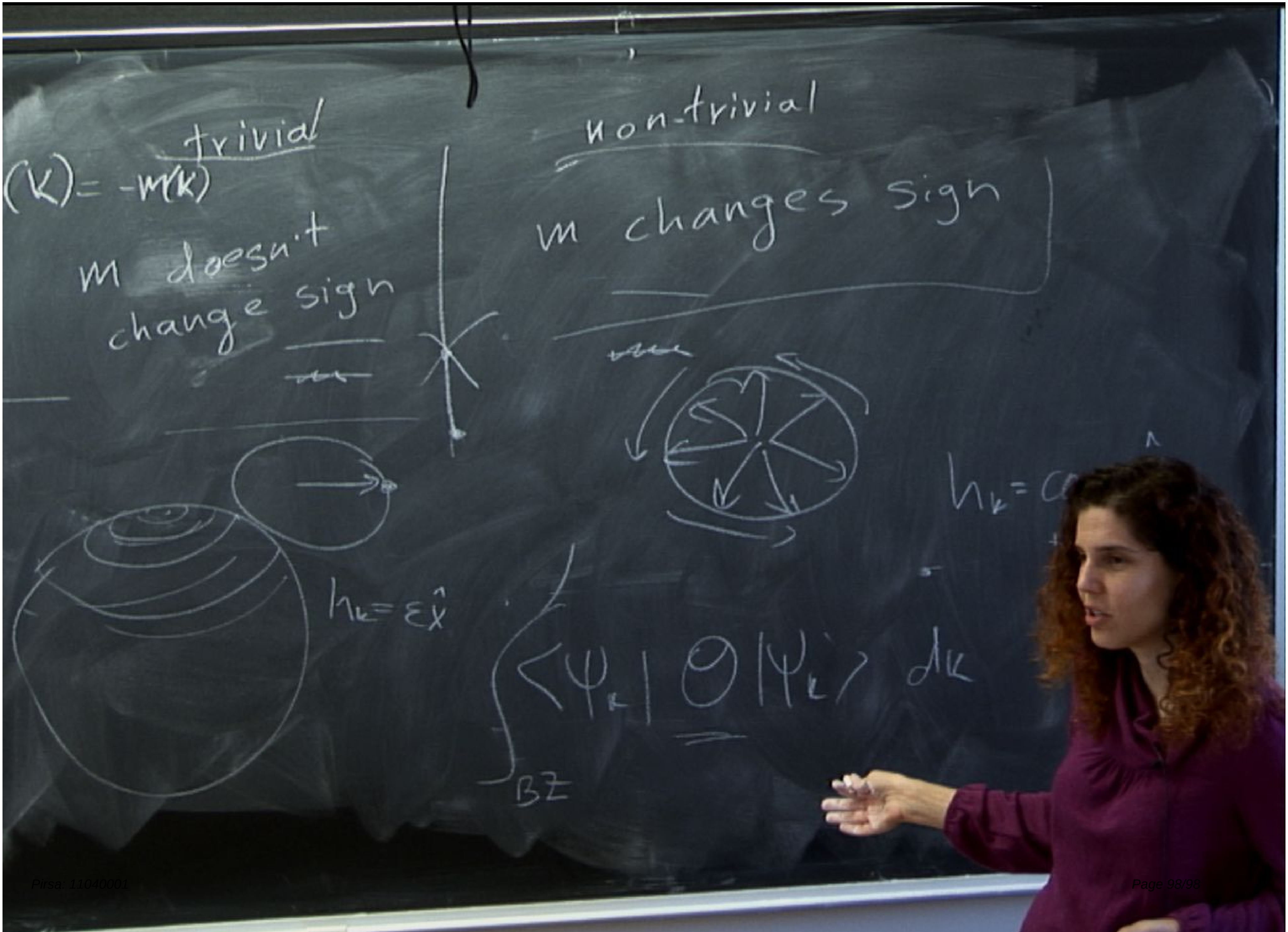

trivial
(k) = -m(k)
m doesn't change sign
non-trivial
m changes sign
∫ ⟨ψk | O | ψk⟩ dk
BZ


2D
ky
(π,π)
H = Σ
kx
(−π,−π)
k
k'
H(k' + 2π/q


2D
(π,π) H = Σ
k_x
k
k'
E = ±
k' + 2π/a x̂) = H(k
gr
m(k) = -m(k)
trivia
m doesn'
change


trivial
m(k)
doesn't
change sign
non-trivial
m changes sign
h_k = cos k · x̂
+ sin k · ŷ
h_k = ε x̂
⟨Ψ_k | O | Ψ_k⟩
BZ


trivial
(k) = -m(k)
m doesn't change sign
non-trivial
m changes sign
h_k = ε x̂
h_k = cos k · x̂ + sin k · ŷ
∫_BZ ⟨ψ_k | O | ψ_k⟩ dk


trivial
(k) = -m(k)
m doesn't change sign
non-trivial
m changes sign
h_k = εx̂
BZ