Title: Effects of black holes to inflation perturbation

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URL: http://pirsa.org/11030122

Abstract: We calculate scalar quantum fluctuations during inflation in the presence of a black hole. The implications to the cosmic microwave background anisotropy are briefly mentioned.

# Effects of Black Holes on Inflaton Perturbation

**Kin-Wang Ng**

**Academia Sinica, Taiwan**
**&**
**Stanford University**

**Perimeter Institute**
**29 March, 2011**

Collaborators: I-Chin Wang, Hing-Tong Cho, Shang-Yung Wang

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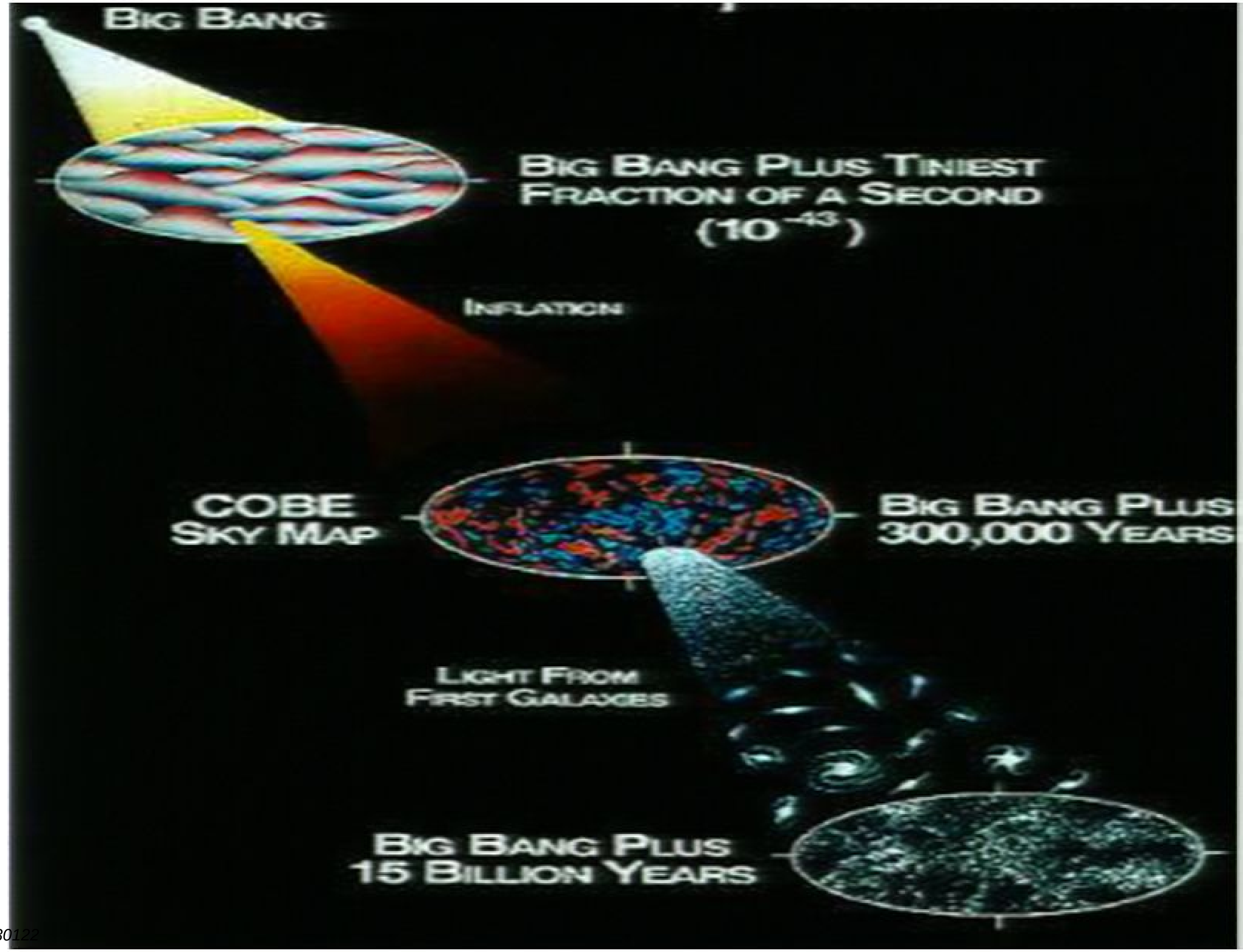
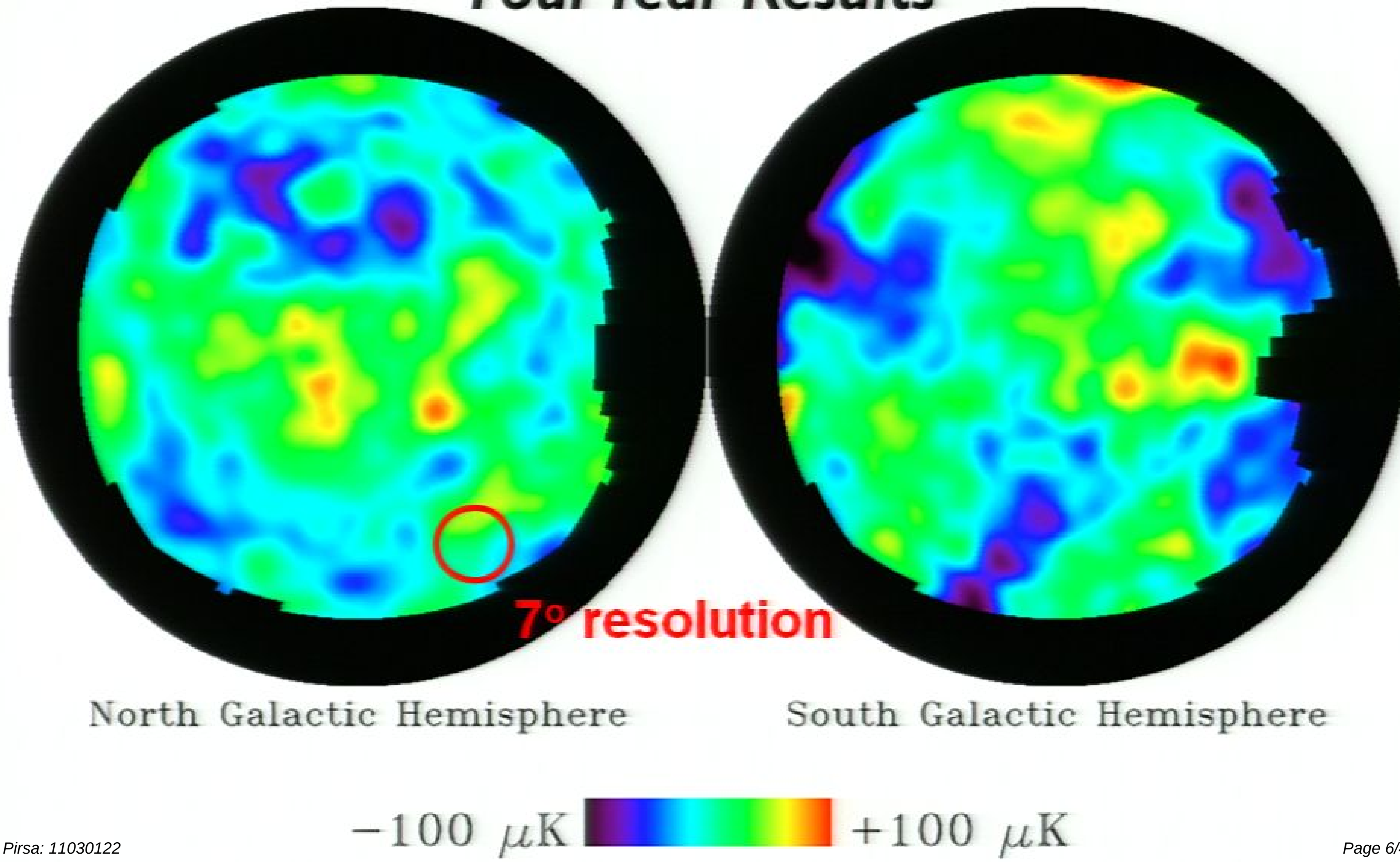
# Motivation –

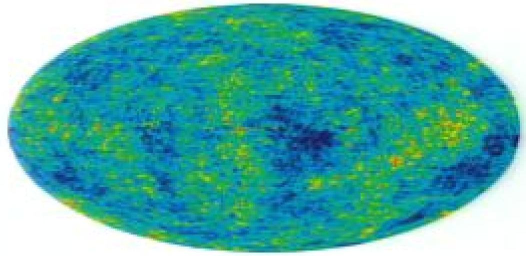
## Large-scale CMB anomalies


Big Bang
Big Bang Plus Tiniest Fraction of a Second (10^-43)
Inflation
COBE Sky Map
Big Bang Plus 300,000 Years
Light From First Galaxies
Big Bang Plus 15 Billion Years

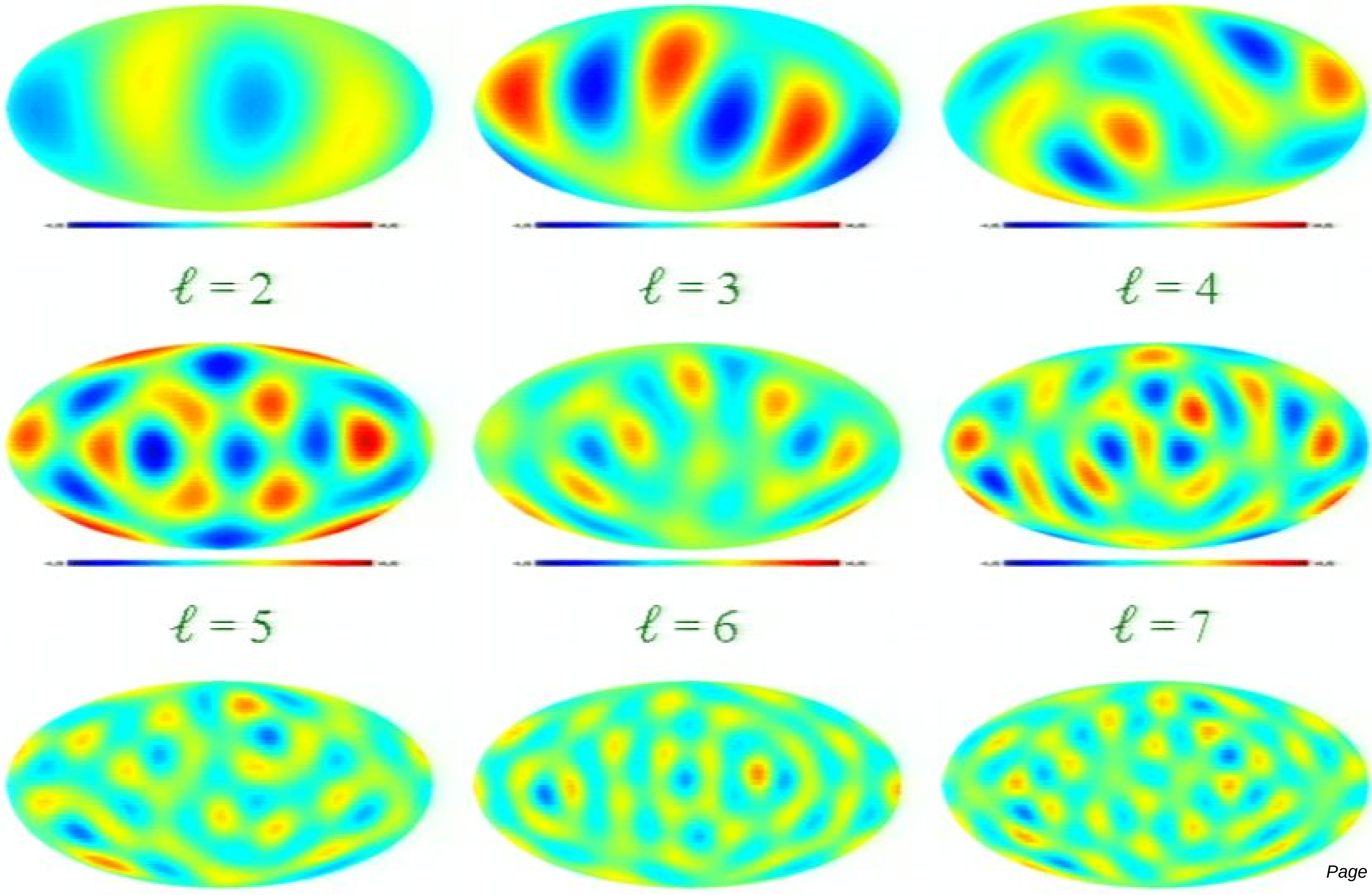
# COBE - DMR *Map* of CMB Anisotropy
## Four Year Results

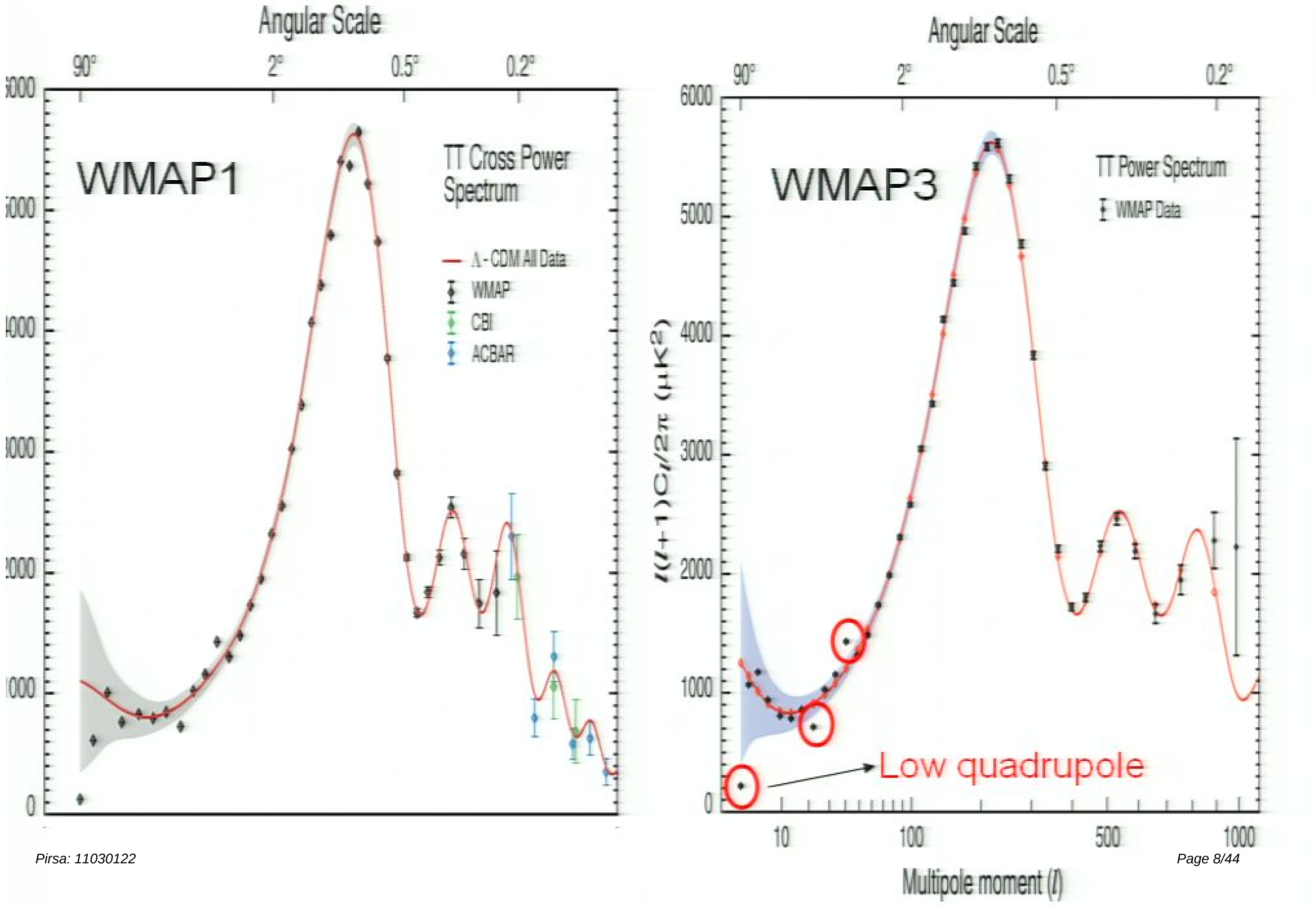


# WMAP3 CMB sky map

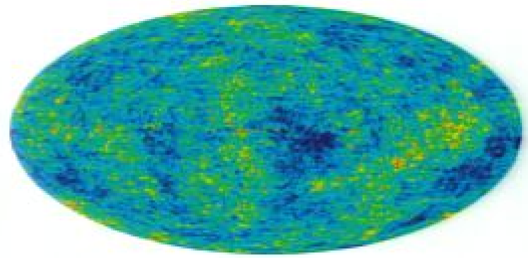


$$\delta T(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$




Angular Scale
90°
2°
0.5°
0.2°
WMAP1
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
WMAP3
TT Power Spectrum
WMAP Data
Low quadrupole
10
100
500
1000
Multipole moment (l)

WMAP3
CMB sky map



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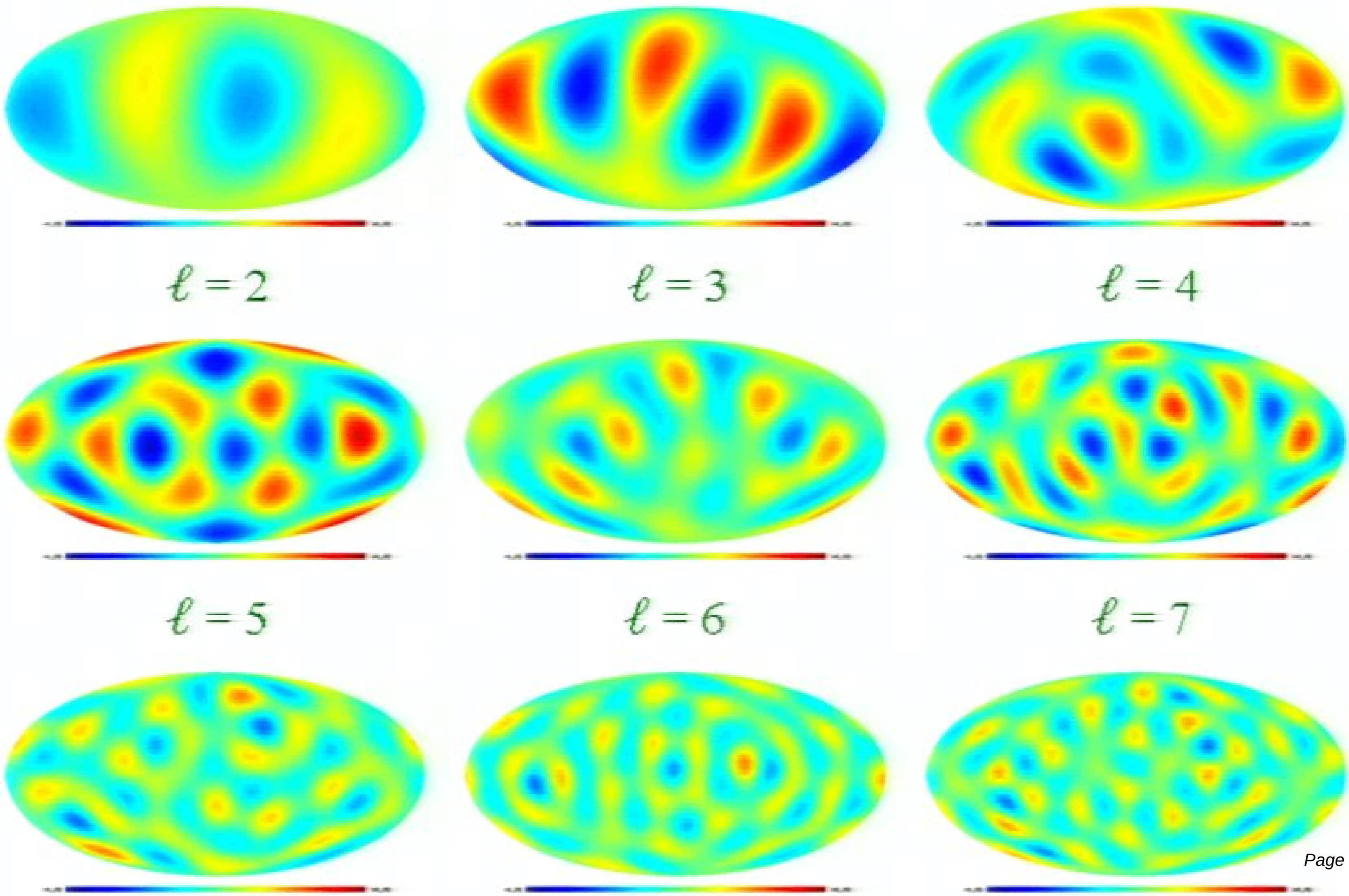

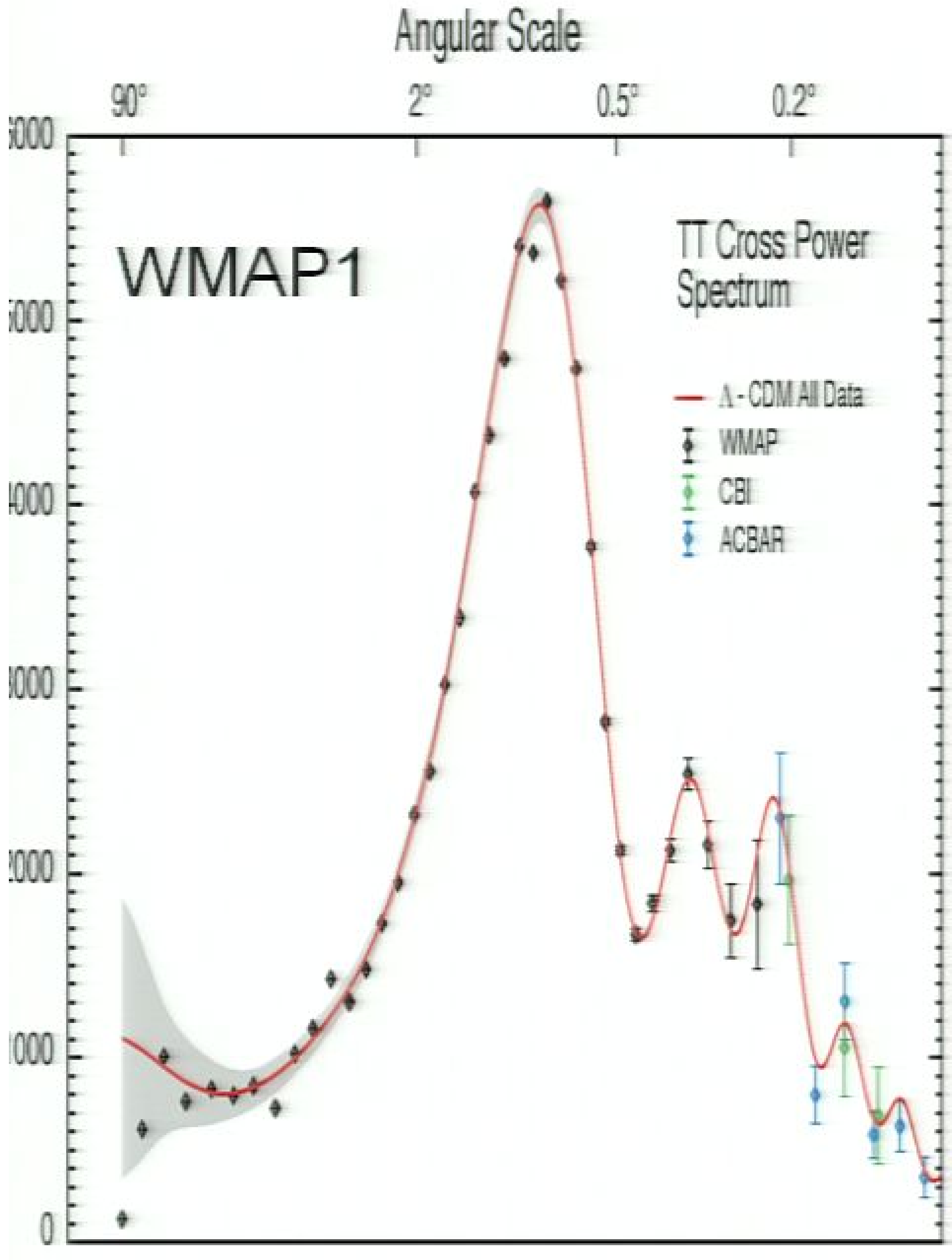
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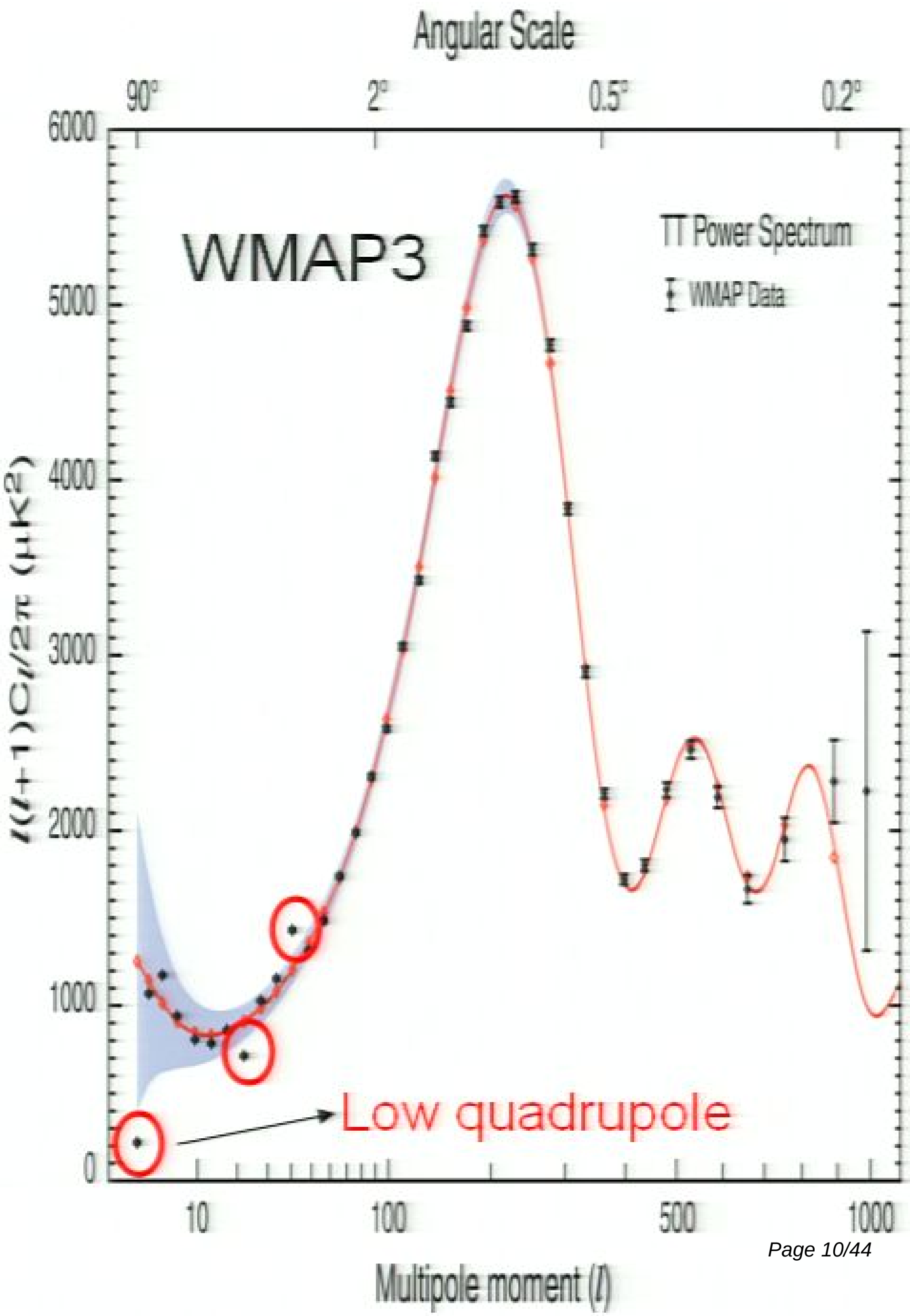
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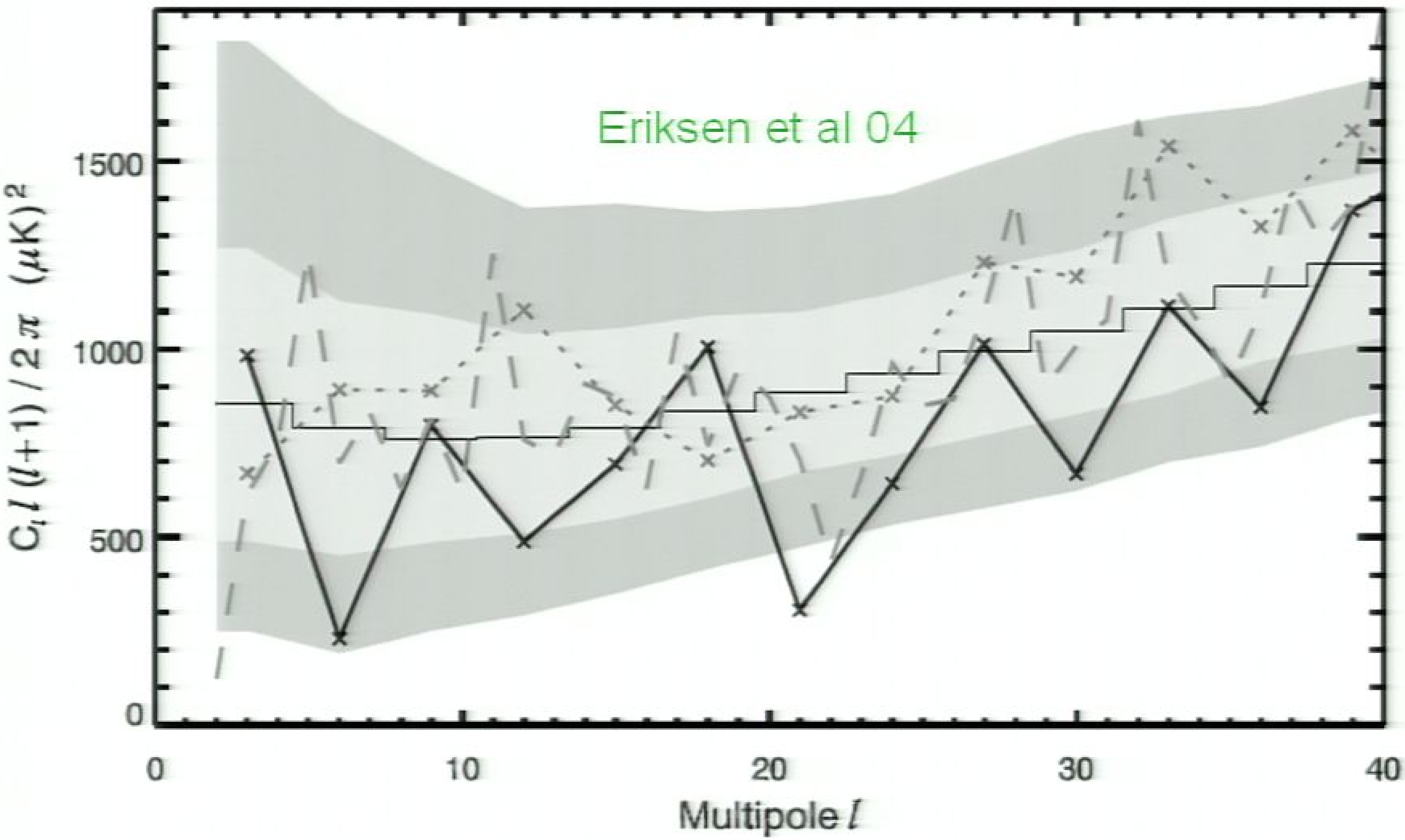
Low quadrupole
Multipole moment ($l$)

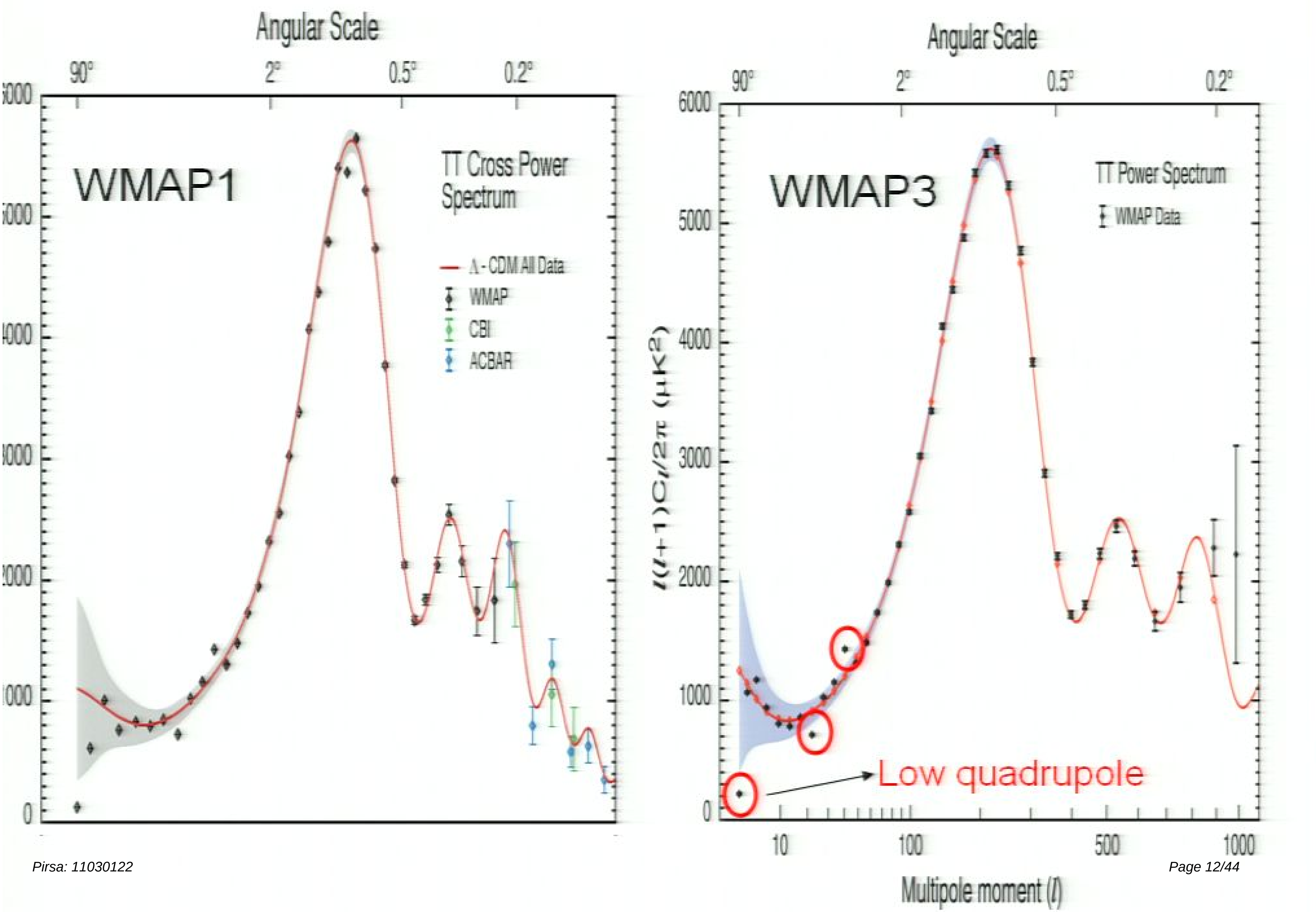
# South-North Power Asymmetry

Eriksen et al 04
Park 04




northern hemisphere
southern hemisphere

North pole (80°,57°)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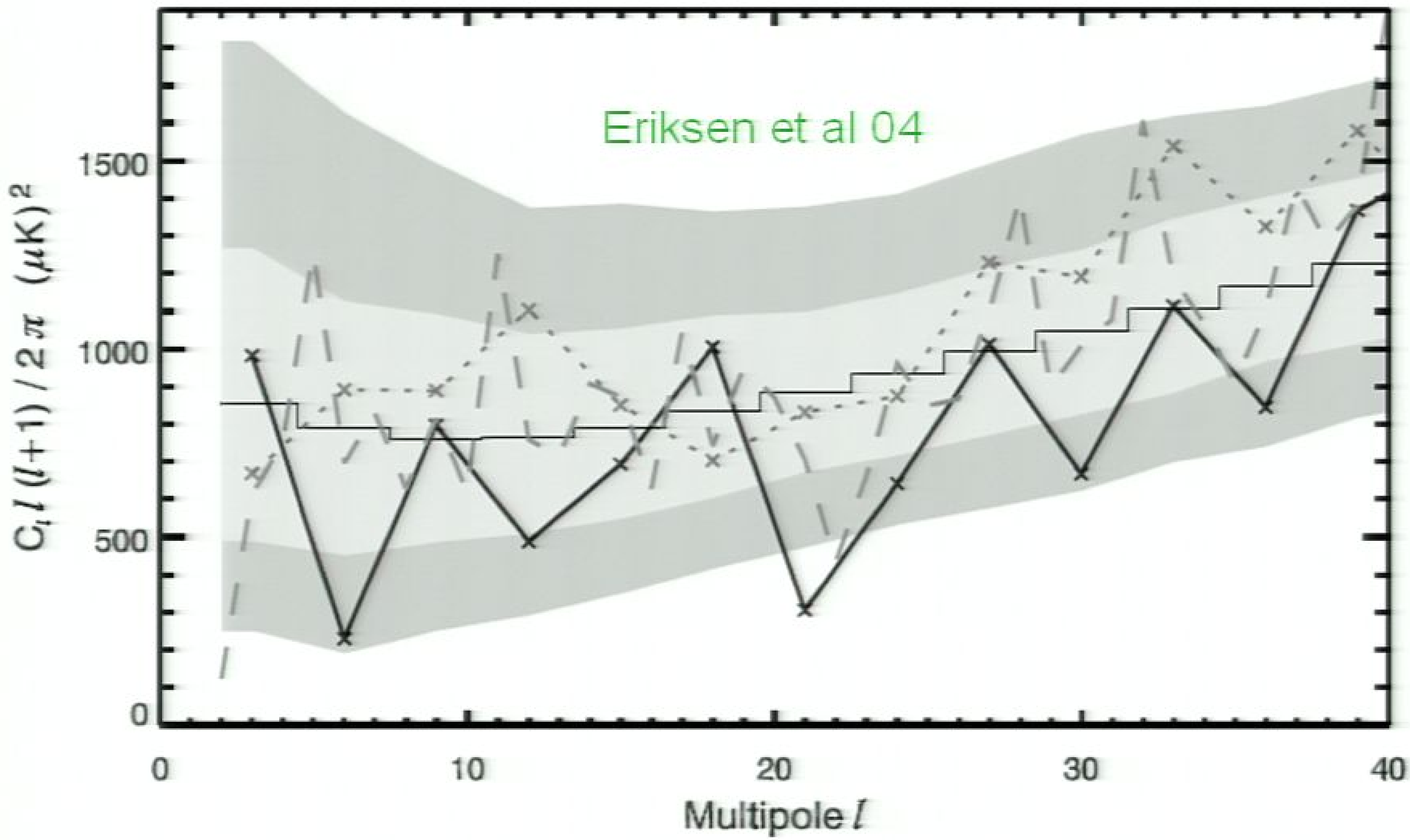


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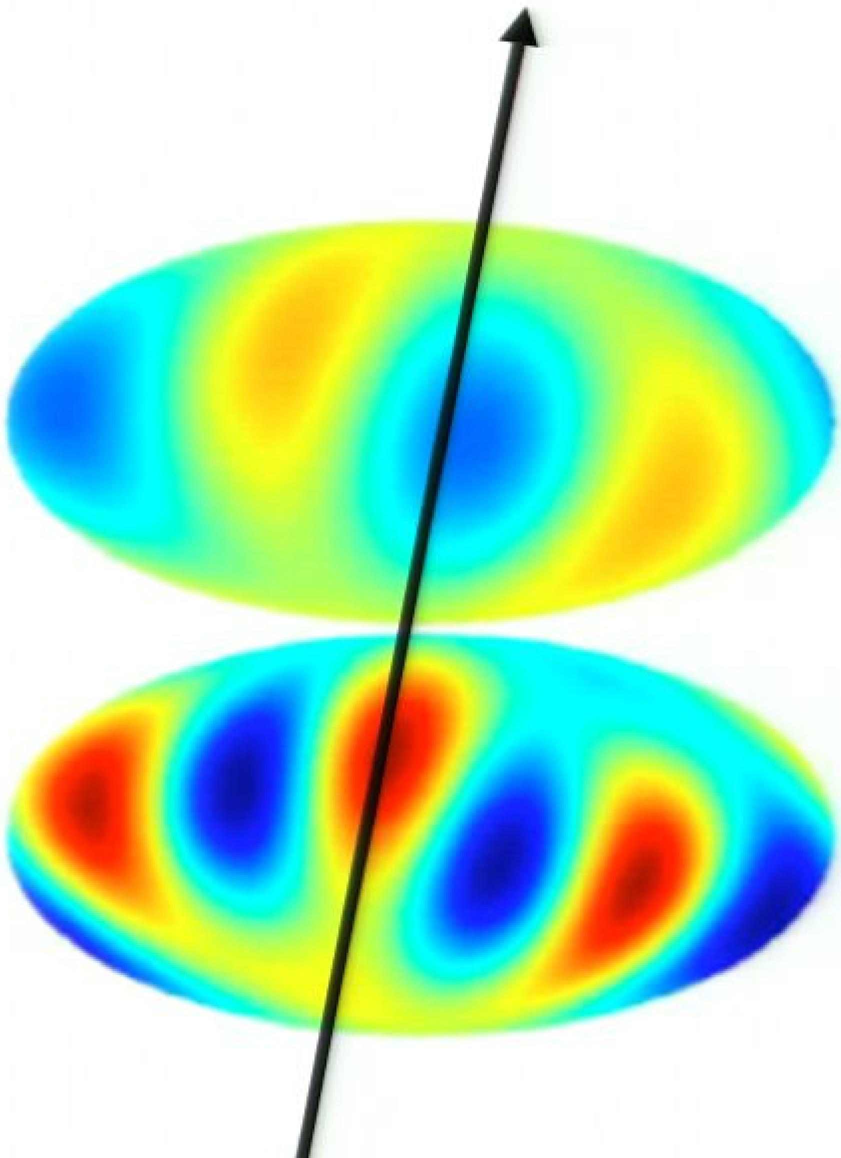
southern hemisphere

North pole (80°,57°)

Land & Magueijo 05

# "Axis of Evil" $(l,b) \sim (-110°, 60°)$



PHYSICAL REVIEW D **69**, 063516 (2004)

# Significance of the largest scale CMB fluctuations in WMAP

Angélica de Oliveira-Costa,[1,*] Max Tegmark,[1] Matias Zaldarriaga,[2] and Andrew Hamilton[3]
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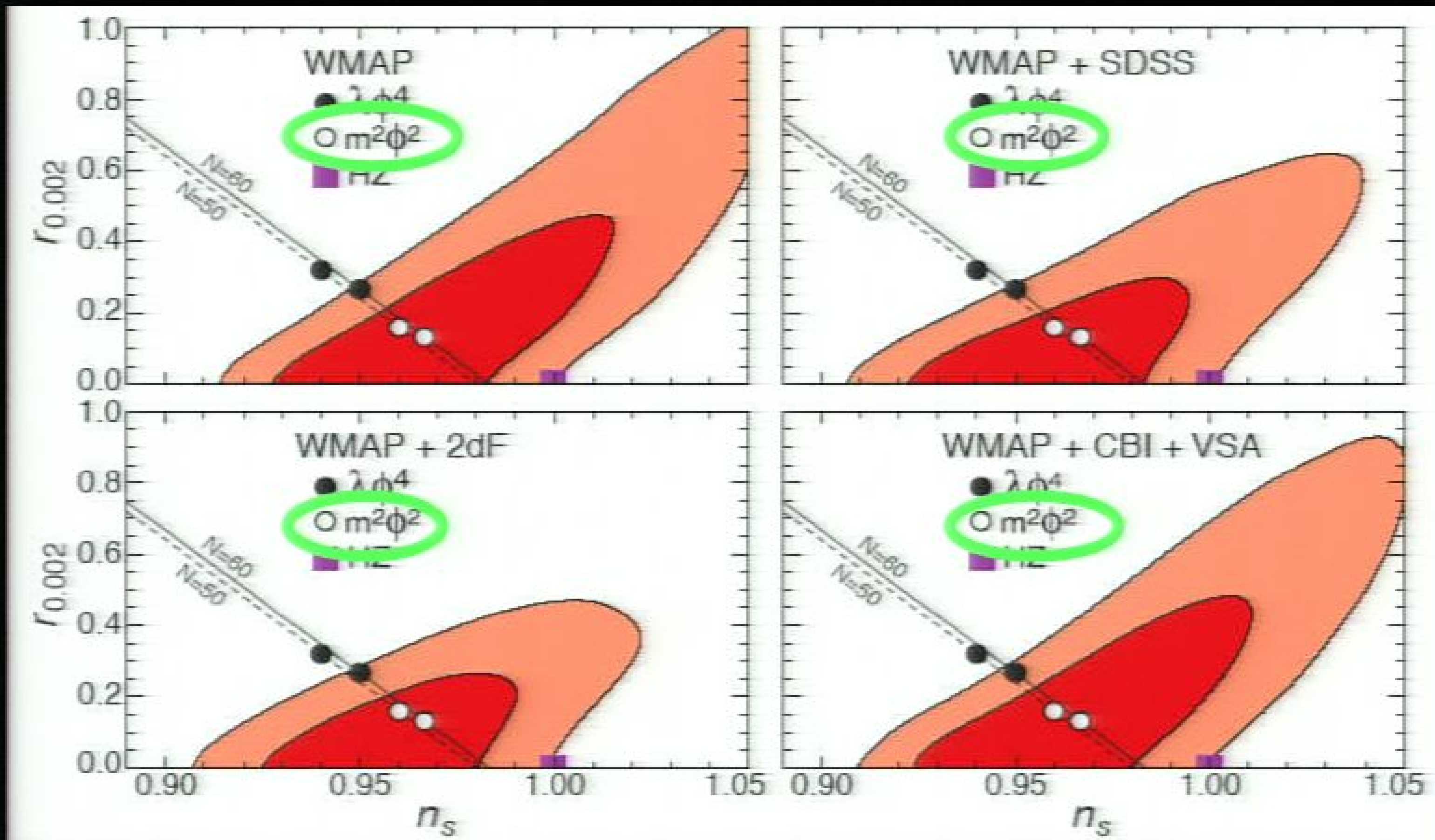
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
*A & Department of Astrophysics & Planetary Sciences, University of Colorado, Boulder, Colorado 80309, USA*
(Received 16 July 2003; published 25 March 2004)

We investigate anomalies reported in the cosmic microwave background maps from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite on very large angular scales and discuss possible interpretations. Three independent anomalies involve the quadrupole and octopole: (1) The cosmic quadrupole on its own is anomalous at the 1-in-20 level by being low (the cut-sky quadrupole measured by the WMAP team is more strikingly low, apparently due to a coincidence in the orientation of our Galaxy of no cosmological significance); (2) the cosmic octopole on its own is anomalous at the 1-in-20 level by being very planar; (3) the alignment between the quadrupole and octopole is anomalous at the 1-in-66 level. Although the *a priori* chance of all three occurring is 1 in 24000, the multitude of alternative anomalies one could have looked for dilutes the significance of such *a posteriori* statistics. The simplest small universe model where the universe has toroidal topology with one small dimension of order one-half the horizon scale, in the direction toward Virgo, could explain the three items above. However, we rule this model out using two topological tests: the *S* statistic and

# WMAP3 and chaotic inflation

r : tenor/scalar



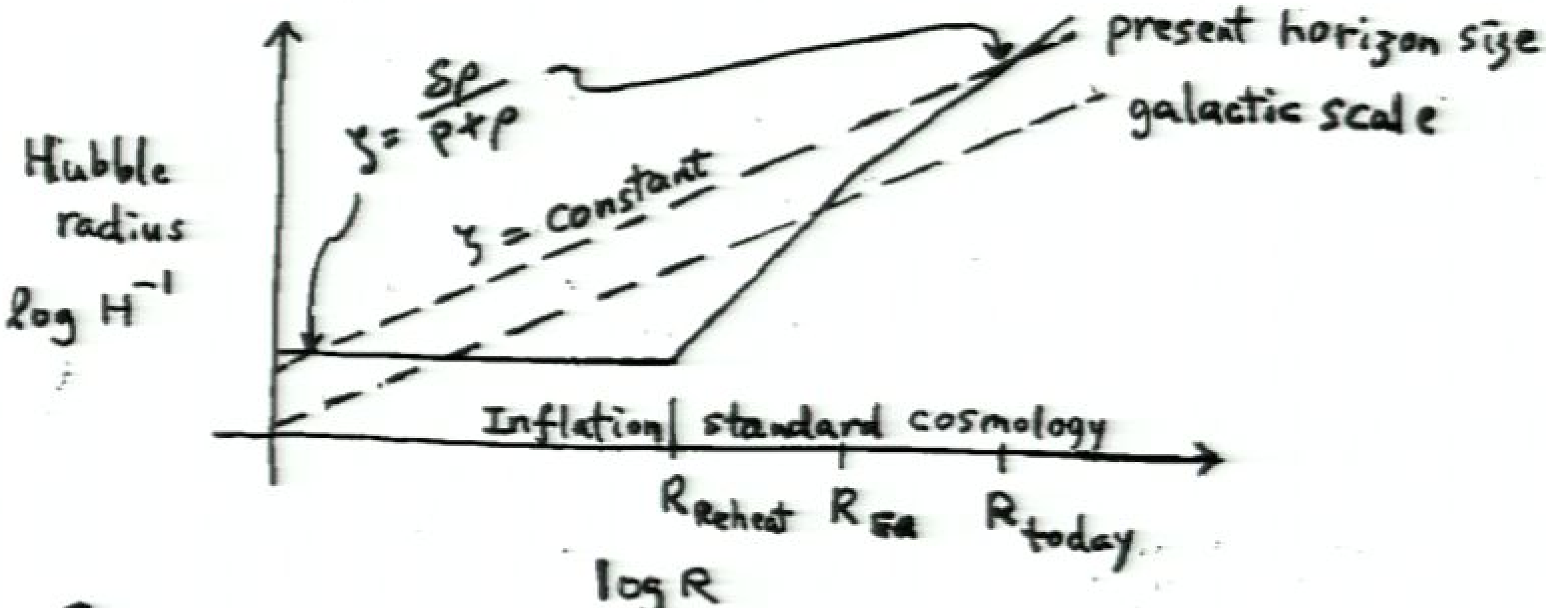

Spectral index $n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$

Spergel et al (2006)

$m \sim 10^{13}$ GeV

# Inflation and Primordial Density Fluctuations

## Evolution of cosmological density perturbation




Evolution of gauge-invariant $\zeta$ — Bardeen, Steinhardt & Turner 1983

- For super-horizon modes $\zeta$ = constant
- At horizon crossing $(\sim H^{-1})$ $\zeta = \frac{\delta\rho}{p+\rho}$

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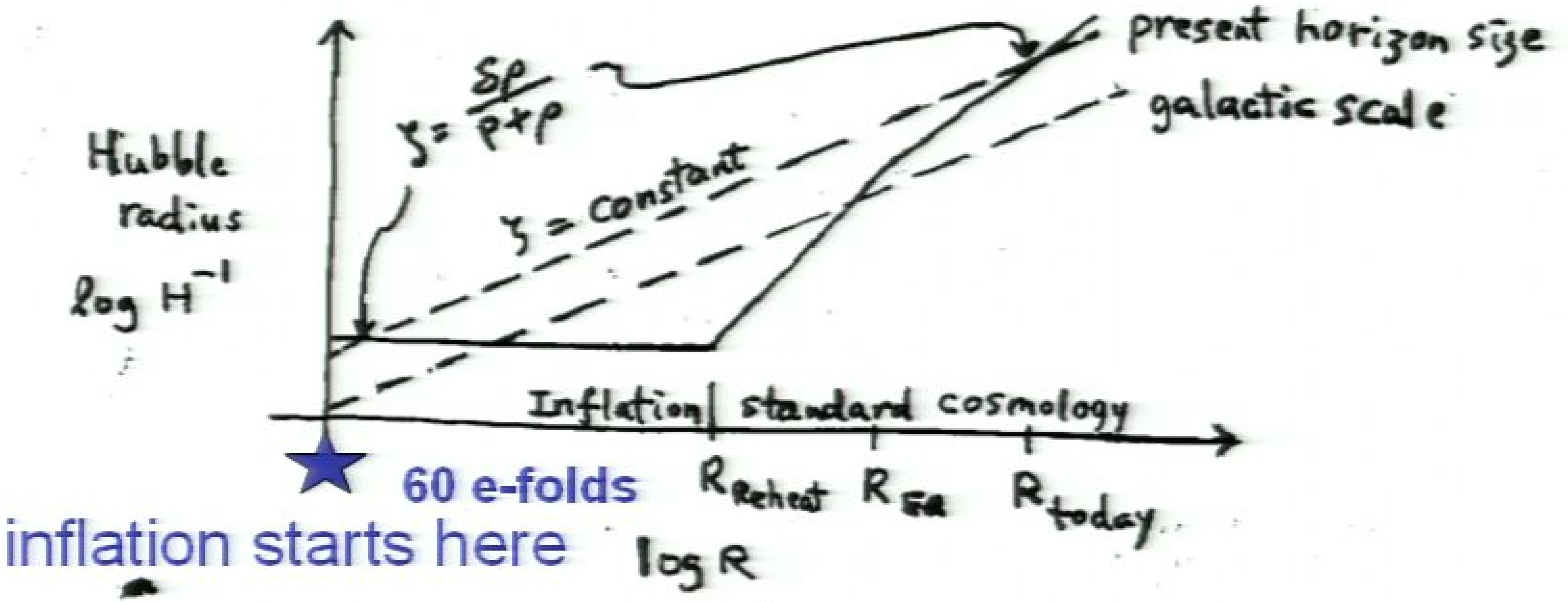


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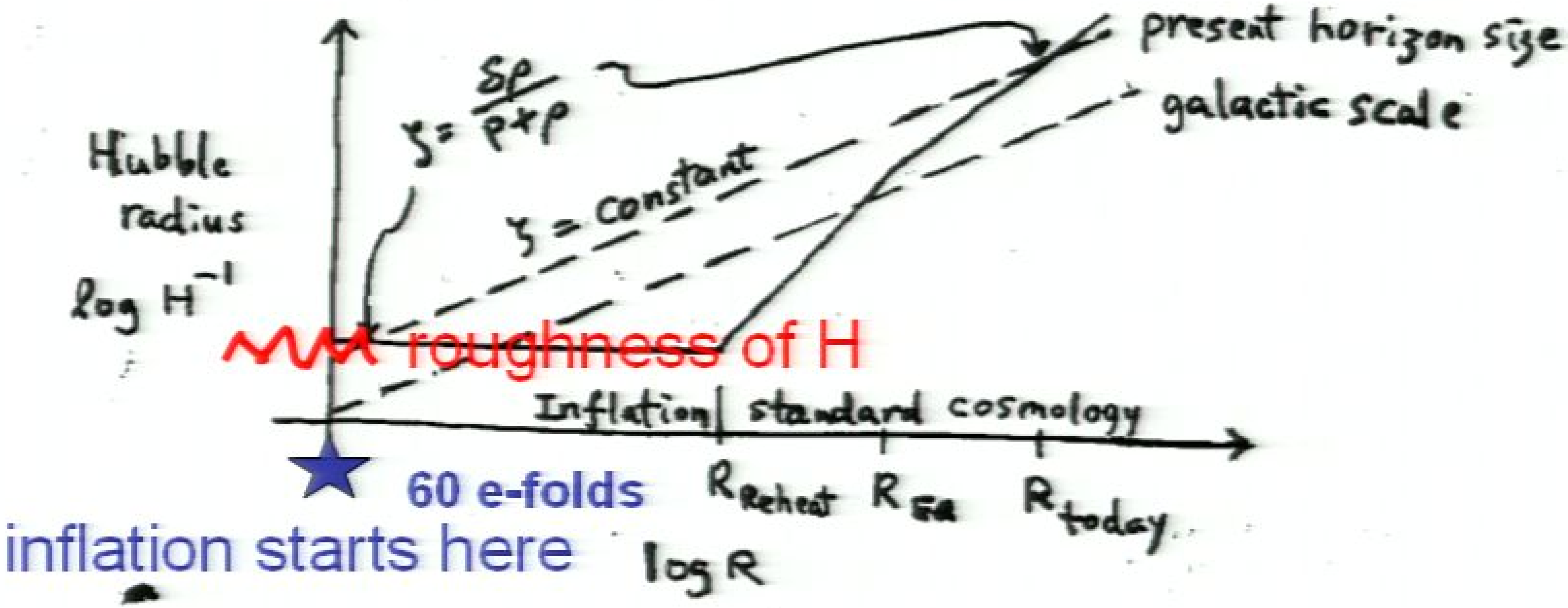


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# A Challenge to Standard Slow-roll inflation!?

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

**Slow-roll kinematics**

- Slow-roll conditions violated after horizon crossing (Leach et al)
- General slow-roll condition (Steward)
  |n-1|~|dn/dlnk|
- Multi-field (Vernizzi, Tent, Rigopoulos, Yokoyama et al)
- etc

**Quantum fluctuations**

- Chaotic inflation – classical fluctuations driven by a white noise (Starobinsky) or by a colored noise (Liguori, Matarrese et al.) coming from high-k inflaton
- Driven by a colored noise from interacting quantum environment (Wu et al)
- Others

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# Summary

- Hints from WMAP data on beyond standard slow-roll inflation !?
- A fine tuning – physics just at 60 e-foldings
- Maybe there is a window to see the first few e-foldings of inflation !?
- Many models give a suppressed CMB low multipoles
- Or we are all fooled by probability – it is indeed a Gaussian quantum process
- Nongaussianity is an important check

Following is an effort to go from homogenous to directional effects

(also refer to vector inflation and etc.)

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(also refer to vector inflation and etc.)

# A black hole in inflation Cho, Ng, Wang 09

Schwarzschild-de Sitter

**M - black hole mass**
**H - Hubble parameter**

$$ds^2 = -\left(1 - \frac{2GM}{r} - H^2 r^2\right) dt^2 + \left(1 - \frac{2GM}{r} - H^2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2,$$

## Static -------> Planar

$$ds^2 = -f(r,\tau)d\tau^2 + h(r,\tau)(dr^2 + r^2 d\Omega^2),$$

$$f(r,\tau) = a^2(\tau)\left[1 - \frac{GM}{2a(\tau)r}\right]^2 \left[1 + \frac{GM}{2a(\tau)r}\right]^{-2}, \quad h(r,\tau) = a^2(\tau)\left[1 + \frac{GM}{2a(\tau)r}\right]^4$$

## Inflaton fluctuations

Klein-Gordon equation $\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)=0.$

$$\hat{\phi}(x)=\int_0^\infty dk\sum_{lm}\left[\hat{a}_{klm}\varphi_{klm}(x)+\hat{a}^\dagger_{klm}\varphi^*_{klm}(x)\right],$$

$$[\hat{a}_{klm},\hat{a}_{k'l'm'}]=[\hat{a}^\dagger_{klm},\hat{a}^\dagger_{k'l'm'}]=0,$$
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$$\varphi_{klm}(x)=\varphi_{kl}(r,\tau)Y_{lm}(\theta,\phi).$$

$d\tau=a^{-1}(\tau)dt$ is the conformal time

$$-\frac{1}{\sqrt{fh}}\partial_\tau\left(\sqrt{\frac{h^3}{f}}\partial_\tau\varphi_{kl}(r,\tau)\right)+\frac{1}{r^2\sqrt{fh}}\partial_r\left(r^2\sqrt{fh}\partial_r\varphi_{kl}(r,\tau)\right)-\frac{l(l+1)}{r^2}\varphi_{kl}(r,\tau)=0$$

$$\varphi_l=\varphi_l^{(0)}+\varphi_l^{(1)}+\varphi_l^{(2)}+\cdots$$

$\epsilon\equiv GMH$ Expansion parameter

$$\partial_\tau^2\varphi_{kl}^{(0)}(r,\tau)-\frac{2}{\tau}\partial_\tau\varphi_{kl}^{(0)}(r,\tau)-\partial_r^2\varphi_{kl}^{(0)}(r,\tau)-\frac{2}{r}\partial_r\varphi_{kl}^{(0)}(r,\tau)+\frac{l(l+1)}{r^2}\varphi_{kl}^{(0)}(r,\tau)=0.$$

$$\partial_\tau^2\varphi_{kl}^{(1)}(r,\tau)-\frac{2}{\tau}\partial_\tau\varphi_{kl}^{(1)}(r,\tau)-\partial_r^2\varphi_{kl}^{(1)}(r,\tau)-\frac{2}{r}\partial_r\varphi_{kl}^{(1)}(r,\tau)+\frac{l(l+1)}{r^2}\varphi_{kl}^{(1)}(r,\tau)=J_1$$

where the source term $J_1(r,\tau)=\frac{4\epsilon\tau}{r}\left[\partial_\tau^2\varphi_{kl}^{(0)}(r,\tau)-\frac{1}{\tau}\partial_\tau\varphi_{kl}^{(0)}(r,\tau)\right]$

# Solutions

Zero order $\varphi^{(0)}_{kl}(r,\tau) = k^2 j_l(kr)\varphi^{(0)}_{kl}(\tau),$ $\varphi^{(0)}_{kl}(\tau) = -\frac{H\tau}{k\sqrt{\pi k}}\left(1-\frac{i}{k\tau}\right)e^{-ik\tau}$

First order $\varphi^{(1)}_{kl}(r,\tau) = \int_0^\infty dr' r'^2 \int_{\tau_i}^0 d\tau' G(r,\tau;r',\tau') J_1(r',\tau')$

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# Power spectrum

$$P_{kl}(r,\tau) \;=\; \frac{k}{4\pi j_l^2(kr)}\left|\varphi_{kl}^{(0)}(r,\tau)+\varphi_{kl}^{(1)}(r,\tau)+\cdots\right|^2 = P_{kl}^{(0)}(r,\tau)\left[1+\epsilon\Delta_{kl}^{(1)}(r,\tau)+\cdots\right]$$

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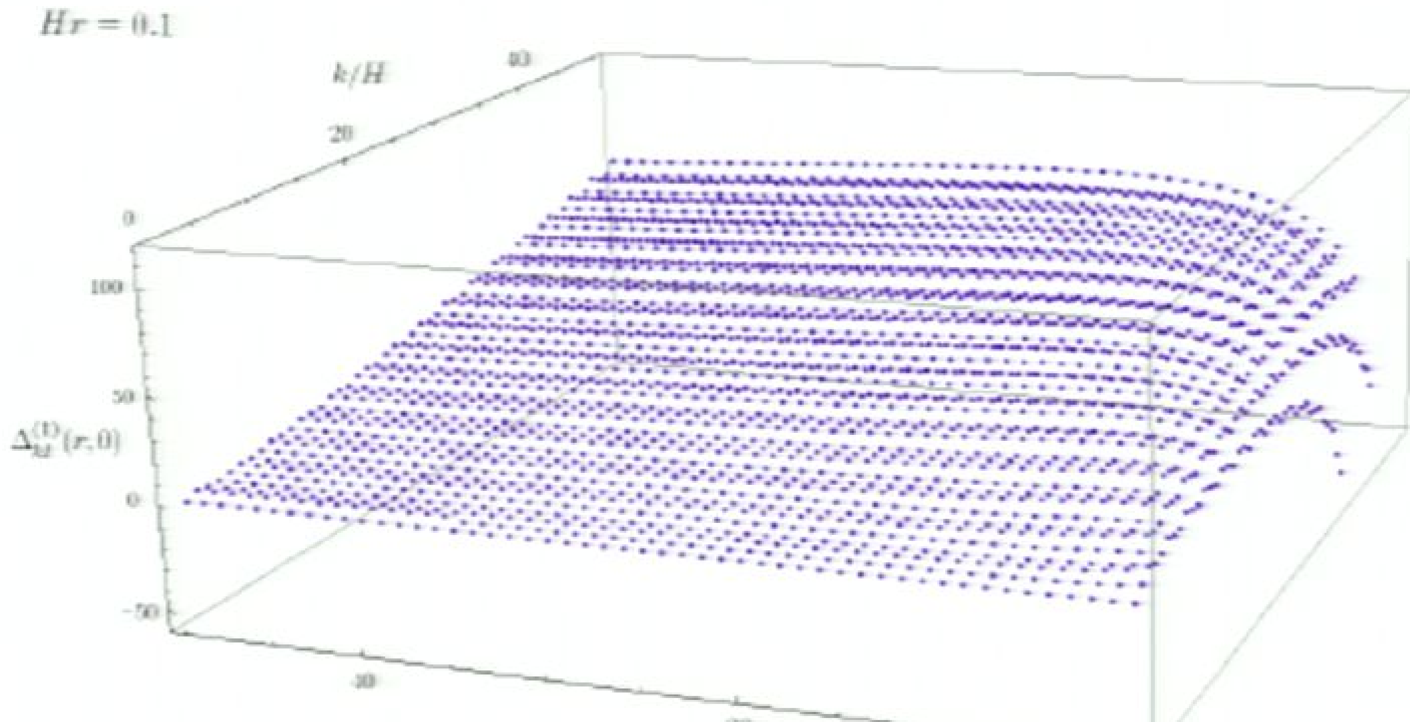
End of inflation $\tau \to 0$ $\quad P_{kl}^{(1)} \longrightarrow \frac{H^2}{4\pi^2}\epsilon\Delta_{kl}, \qquad \epsilon \equiv GMH$



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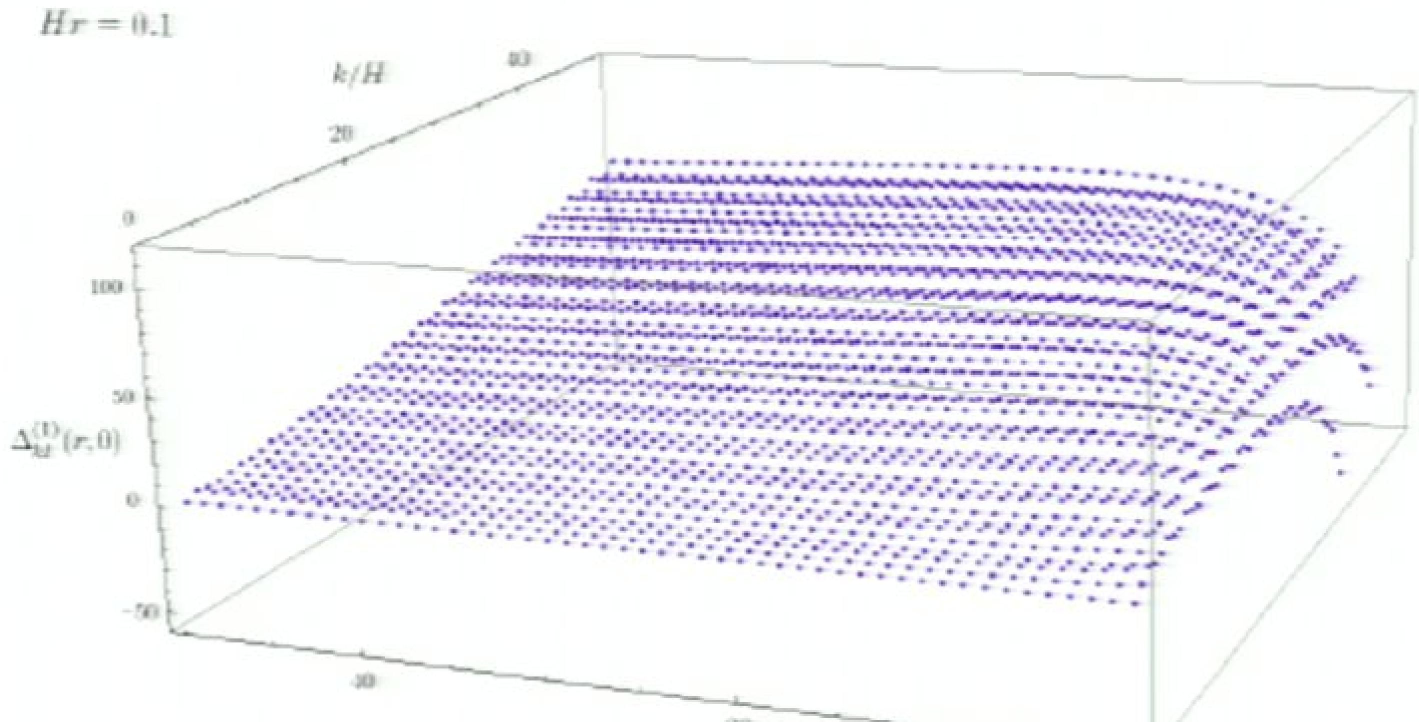
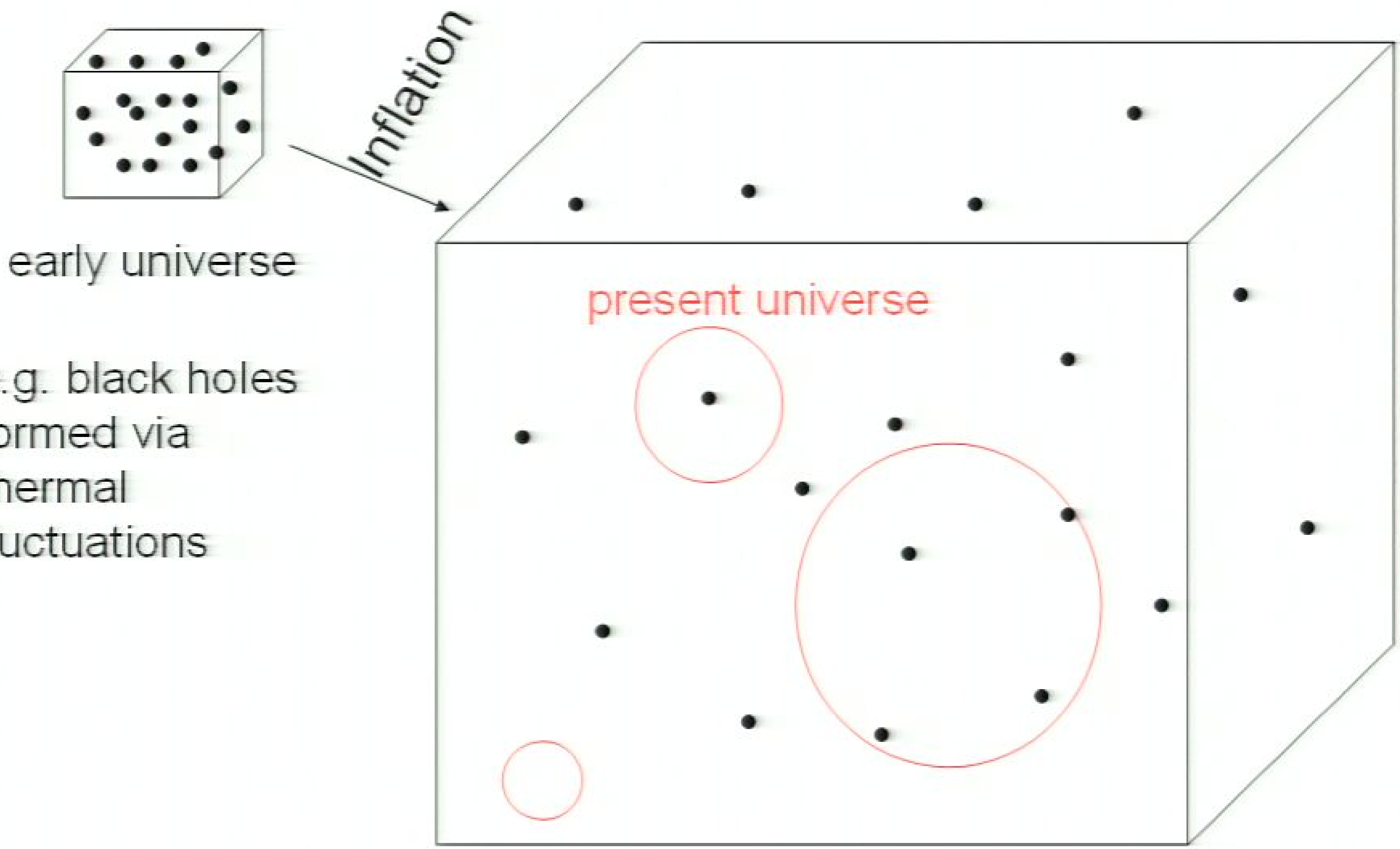


# Possible effects to CMB anisotropy




e.g. black holes
formed via
thermal
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Carroll, Tseng, & Wise 08 preferred point, line, or plane

# Speculations

- Is it possible not to fine tune inflation duration to 60 efolds?
- Then there must be something happening during slow-roll inflation
- Formation rate must not be far below the expansion rate of inflation

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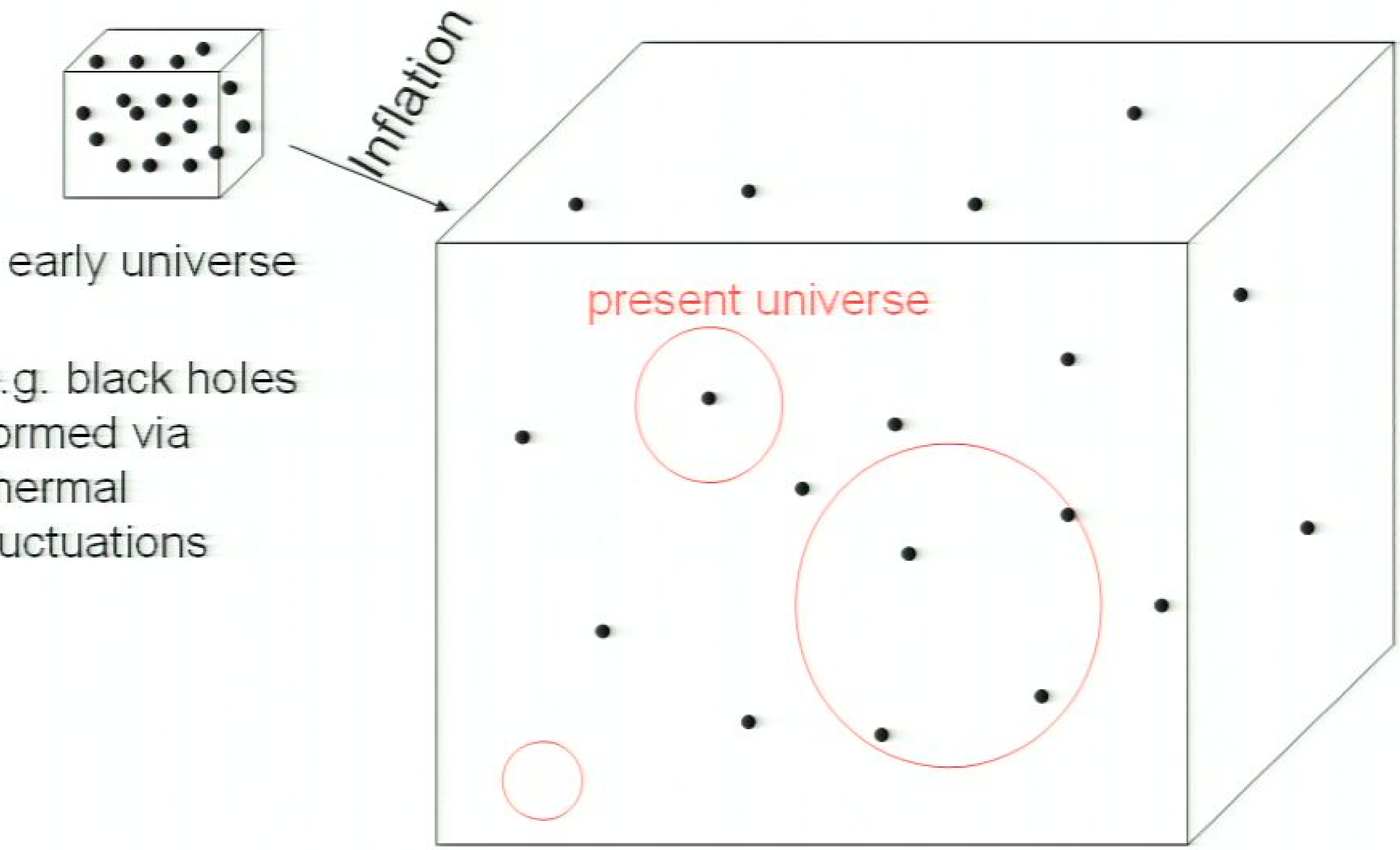


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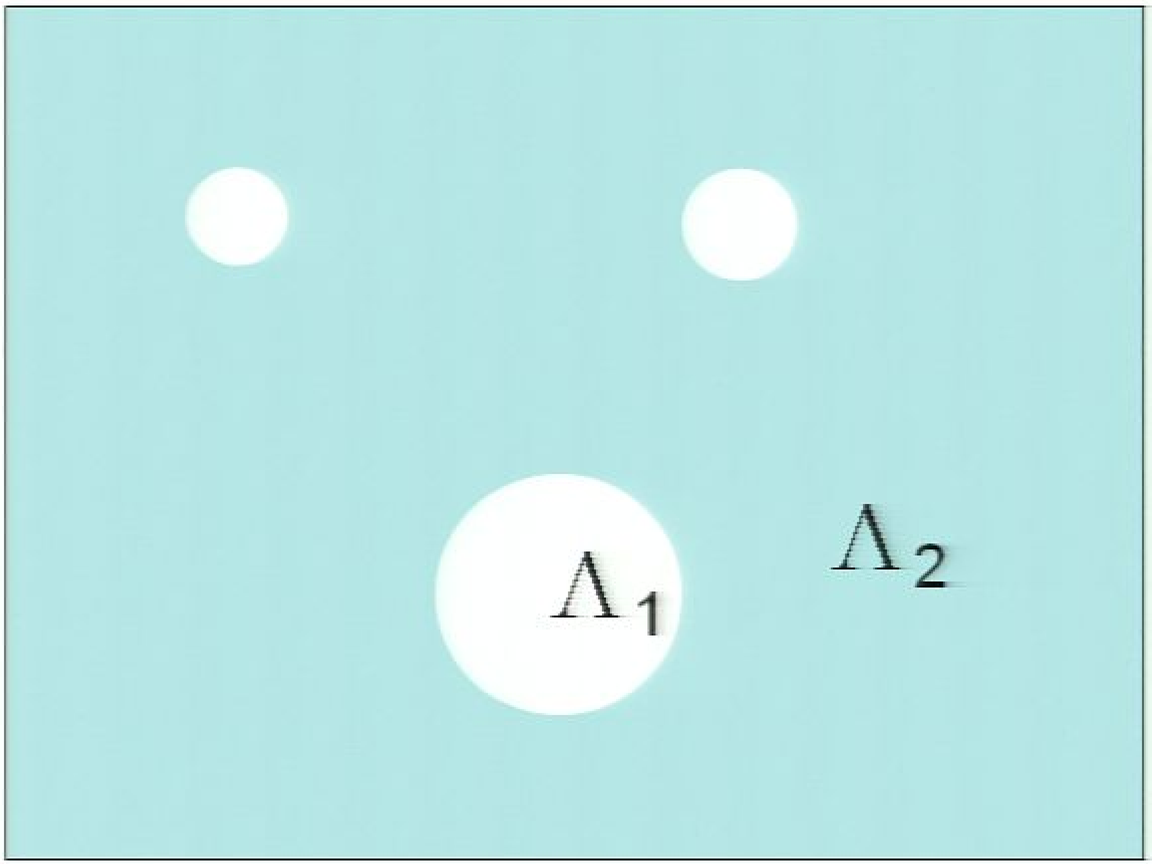
# (I) Inhomogeneity due to quantum fluctuations in inflation

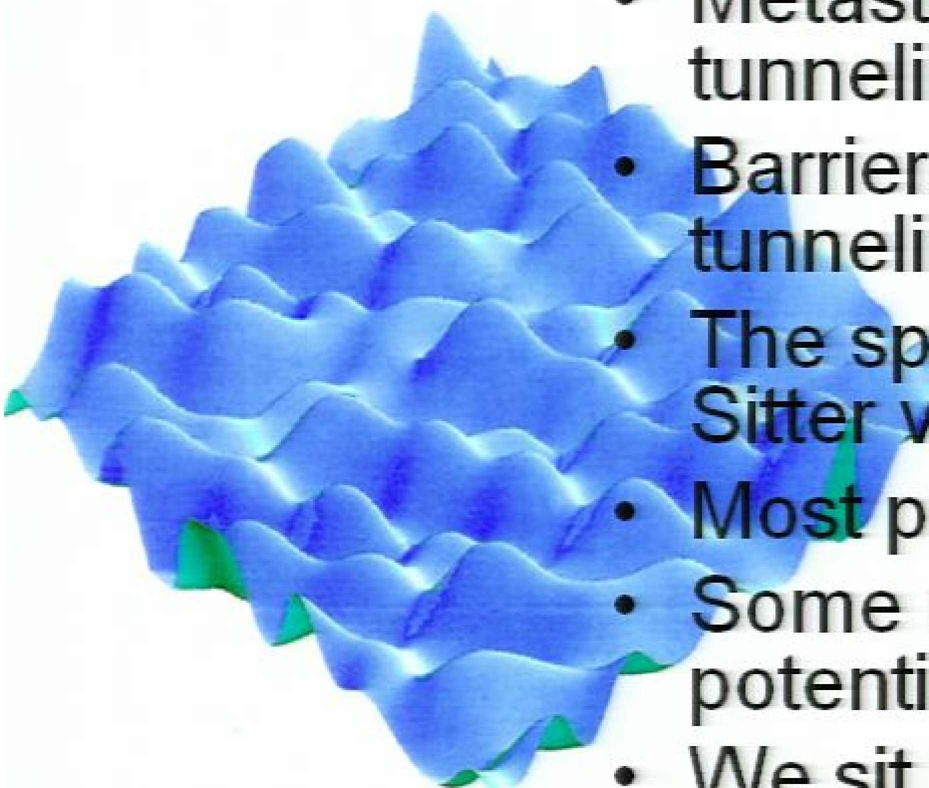
Nambu 94,….

slow-roll inflation in a de Sitter vauum $\Lambda_2$




Will these spherical inhomogeneity $\Lambda_1$ collapse into black holes?

# (II) String Landscape

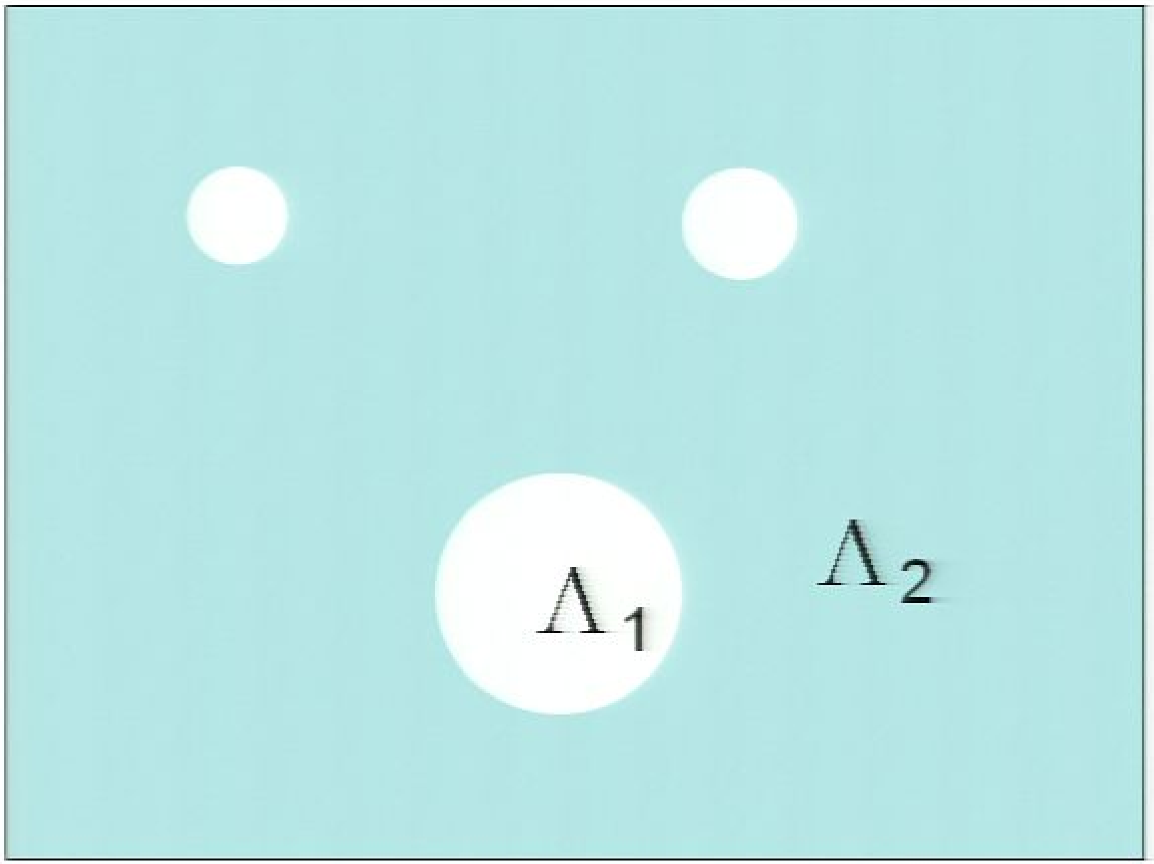


- $10^{500}$ de Sitter vacua
- Metastable, bubble nucleation via tunneling
- Barriers of string scale, slow tunneling rate
- The spacetime is a hierachy of de Sitter vacuum bubbles
- Most part in eternal inflation
- Some regions tunnel down to flat potential for slow-roll infaltion
- We sit in a vacuum with a small cosmological constant today

# Efficient and rapid tunneling

Tye, Shiu,...

slow-roll inflation in a de Sitter vauum $\Lambda_2$




Will these bubbles $\Lambda_1$ collapse into black holes?

# Motion of the bubble wall

$$ds_{\pm}^2 = -F_{\pm}(r)dt^2 + F_{\pm}{}^{-1}(r)dr^2 + r^2 d\Omega^2 \ , \qquad (18)$$

where

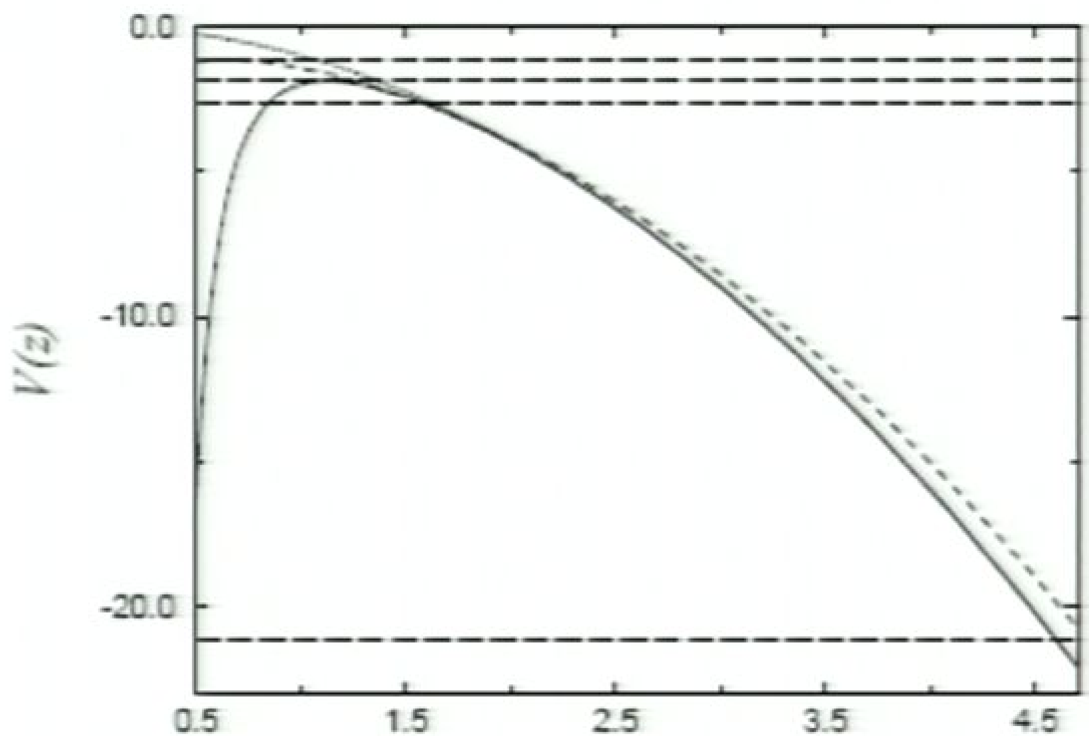
$\kappa \equiv 4\pi G\sigma$

surface tension

$$F_{\pm}(r) = \begin{cases} 1 - \chi_1{}^2 r^2 & \text{interior spacetime,} \\ 1 - 2MG/r - \chi_2{}^2 r^2 & \text{exterior spacetime.} \end{cases} \qquad (19)$$

Here $M$ is an as yet undetermined parameter, $\chi_1{}^2 = \Lambda_1/3$, $\chi_2{}^2 = \Lambda_2/3$, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The cor-

bubble radius




Ng, Wang 11 and references therein

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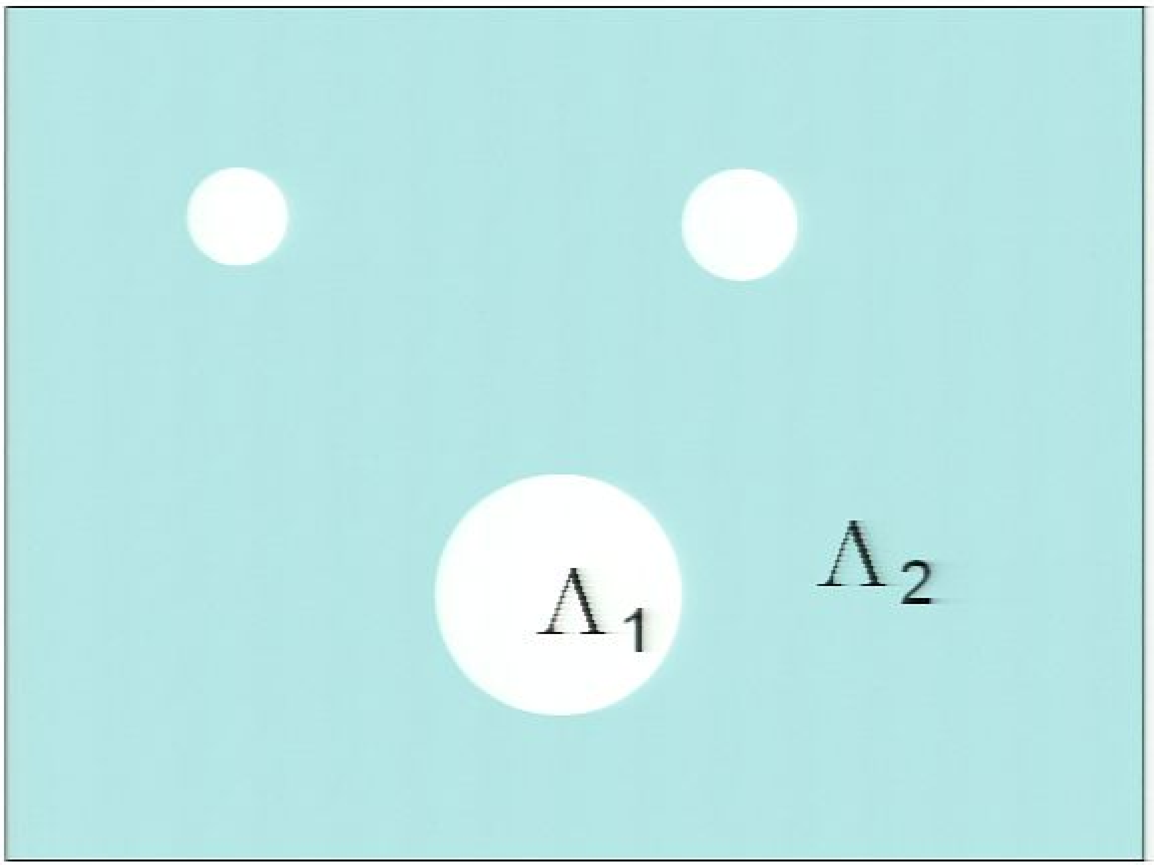


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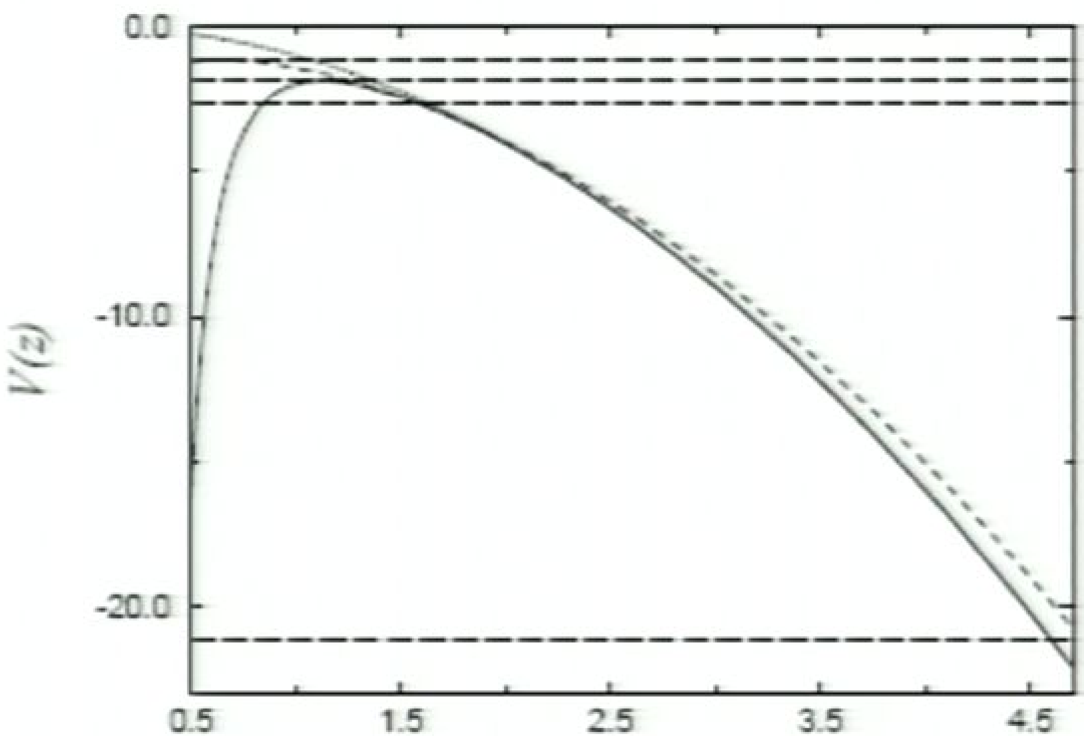


Ng, Wang 11 and references therein

Work is in progress towards understanding the large-scale CMB anomalies

thank you