Title: Non-equilibrium drive near a quantum critical point

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Abstract: We will discuss the effect of non-equilibrium drive near a quantum critical point in itinerant electron systems. Non-equilibrium field theory is formulated in terms of the Keldysh functional integral. The renormalization group approach is used to study the universality class of the non-equilibrium phase transition in the steady state system. The role of the non-equilibrium drive in the quantum-classical crossover will be discussed using the example of the Hertz-Millis theory and the generalization thereof.

Non-equilibrium Drive near a Quantum Critical Point

Yong Baek Kim
University of Toronto

Perimeter Institute, March 28, 2011

Canada Research Chairs NSERC CRSNG CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH

Prelude

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Relation to thermal transitions ?

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Renormalization Group theory of
non-equilibrium transitions

Outline

Equilibrium magnetic quantum phase transitions in itinerant magnets; Hertz-Millis-Moriya theory

Non-equilibrium drive (current, electric field) near an equilibrium magnetic quantum critical point

Open system (coupled to reservoirs); Steady state non-equilibrium state

Development of Renormalization Group scheme

Study of Universality Classes

People involved in the work

So Takei (MPI-Stuttgart ⇒ Maryland)

Aditi Mitra (New York Univ)

Andy Millis (Columbia)

William Witczak-Krempa (Toronto)

A. Mitra, S. Takei, Y. B. Kim, A. J. Millis, Phys. Rev. Lett. 97, 236808 (2006)

S. Takei, W. Witczak-Krempa, Y. B. Kim, Phys. Rev. B 81, 125430 (2010)

Related Works

D. Dalidovich and P. Philips (2004)

Non-linear transport near insulator-superconductor transition

D. E. Feldman (2005)

Effect of voltage bias on ferromagnetic transition in "closed" one-dimensional itinerant electron system

P. M. Hogan and A. G. Green (2006)

Non-linear transport near ferromagnetic quantum critical point

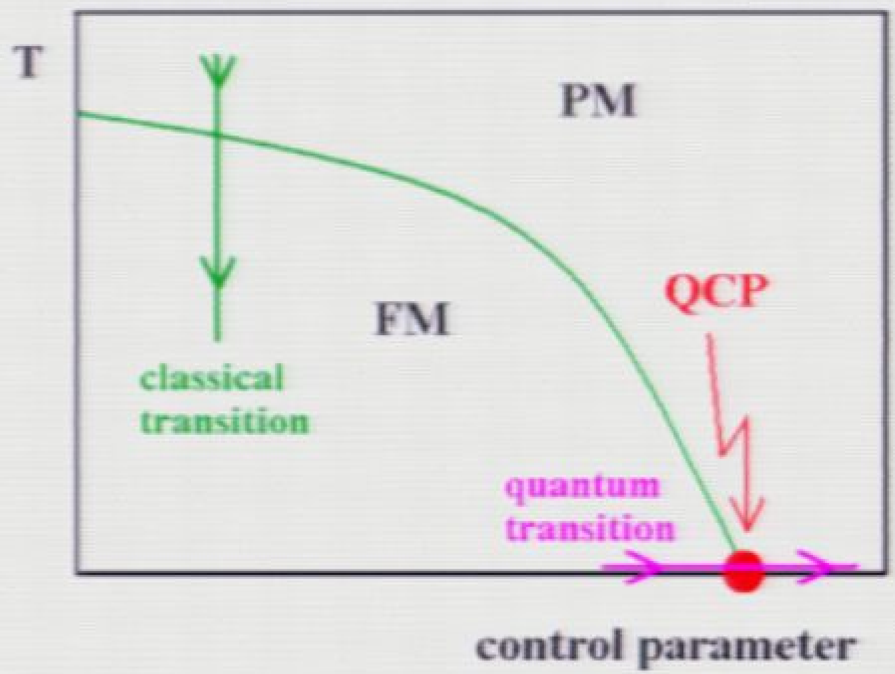
E. G. Dalla Torre, E. Demler, T. Giamarchi, E. Altman (2009)

Effect of non-equilibrium noise on quantum critical state in zero and one-dimensional systems

Equilibrium quantum phase transitions

Phase transitions at T=0 tuned by control parameters (e.g. pressure, magnetic fields, chemical doping)

Change in broken symmetries at a quantum critical point



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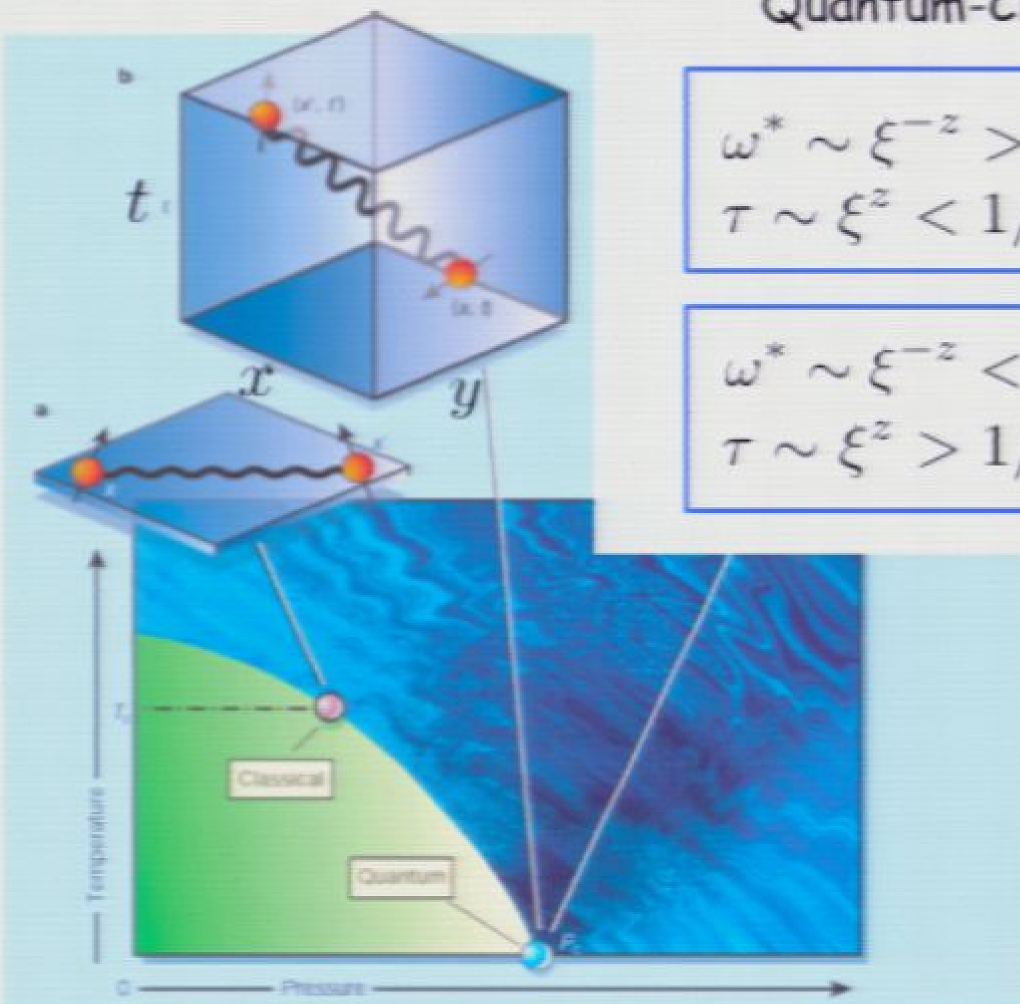


Quantum-Classical Crossover

$\omega^* \sim \xi^{-z} > T$ $\tau \sim \xi^z < 1/T$	Quantum Regime

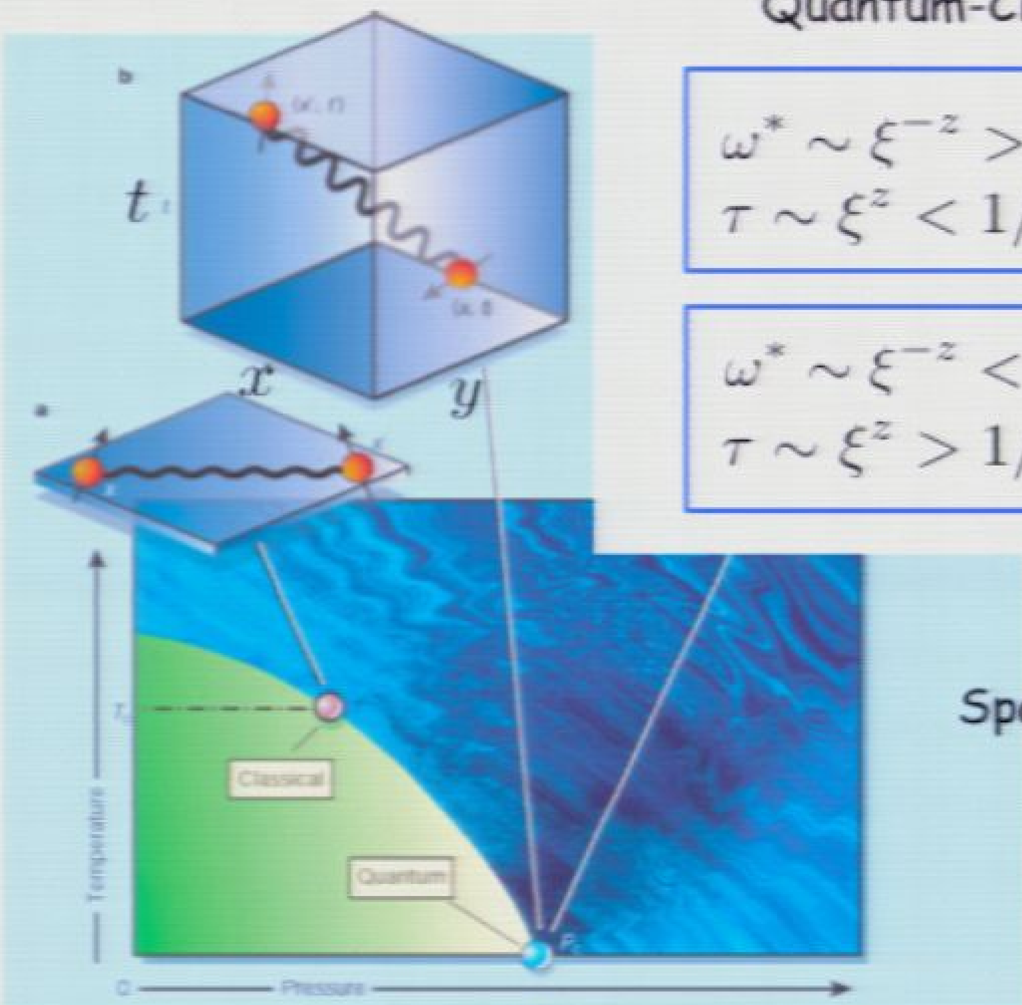
$\omega^* \sim \xi^{-z} < T$ $\tau \sim \xi^z > 1/T$	Classical Regime (quantum critical)

Effective dimensionality

$$\tau \propto \xi^z$$

$$d_{\text{eff}} = d + z > d$$

Equilibrium quantum phase transitions



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Spatial and Temporal fluctuations
are coupled in
quantum phase transitions

Theory of Quantum Criticality in Equilibrium
Itinerant Magnetic Phase Transitions

Effective field theory of the order parameter in **d+z** dimensions (effective space-time dimensions)

$$\omega \propto q^z \qquad \tau \propto \xi^z$$

$$S_{\text{eff}} = \sum_{\mathbf{q},\omega} \chi^{-1}(\mathbf{q},\omega)|\vec{m}(\mathbf{q},\omega)|^2 + u \int d^d x \; dt \; [\vec{m}(\mathbf{x},t)]^4$$

$$\chi^{-1}(\mathbf{q},\omega) = -i\frac{\omega}{\Gamma(q)} + q^2 + \delta \qquad \delta = 1 - UN(0)$$

Stoner parameter

(c.f. $F = \int d^d x \; ([\nabla \vec{m}(\mathbf{x})]^2 + \delta[\vec{m}(\mathbf{x})]^2 + u[\vec{m}(\mathbf{x})]^4)$ Landau free energy)

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$\Gamma(q) \propto q, q^2$ Clean/Dirty Ferromagnet (z=3,4)

$\Gamma(q) \propto \text{constant}$ Open System (z=2)

- Renormalization Group Analysis;
  quartic term is irrelevant ($d_{\mathrm{eff}} > 4$); relevant ($d_{\mathrm{eff}} < 4$)

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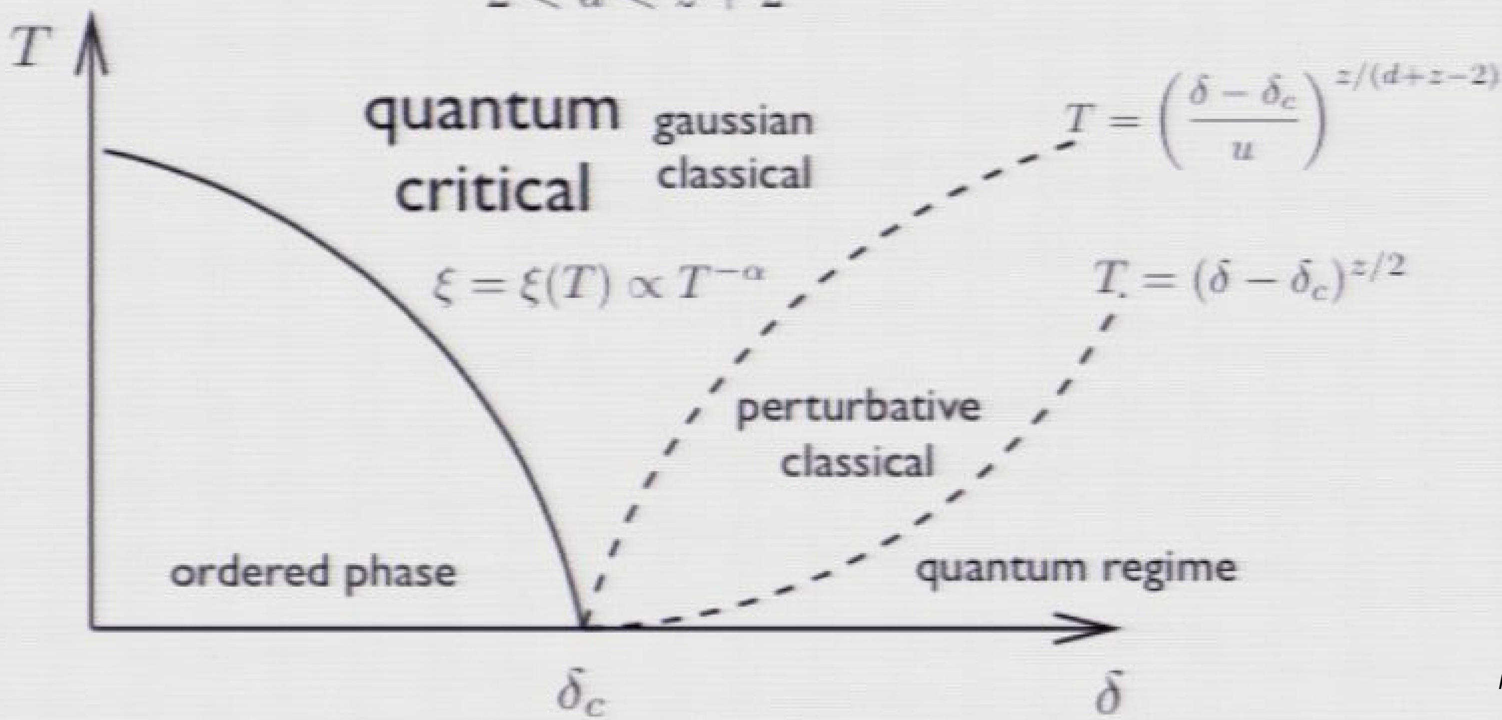
- T is a relevant perturbation $\frac{dT(b)}{d\ln b} = zT(b)$

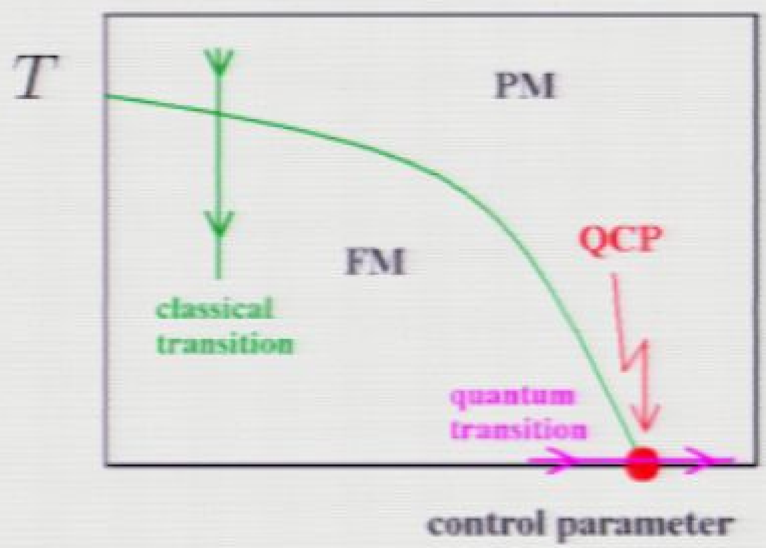
  Thermal fluctuations decouple the dynamics and statics - classical transitions in d-dimensions

$2 < d < z+2$



General Consideration

The fields which drive the system out of equilibrium typically increase its energy and destroy phase coherence; this may be analogous to temperature ?
similarity between non-equilibrium transitions and thermal transitions ?



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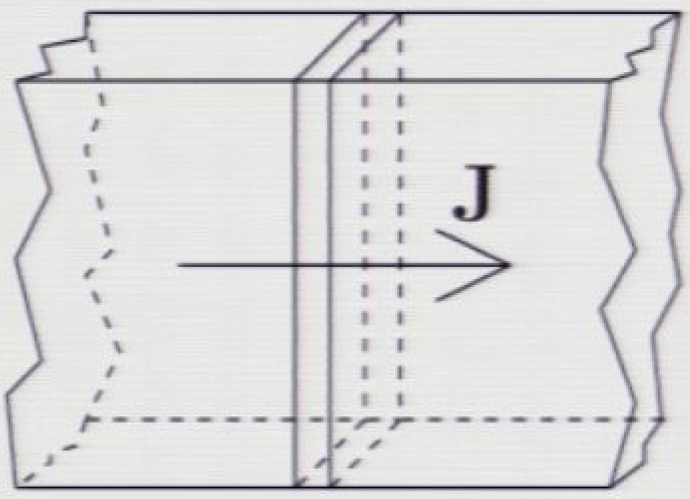
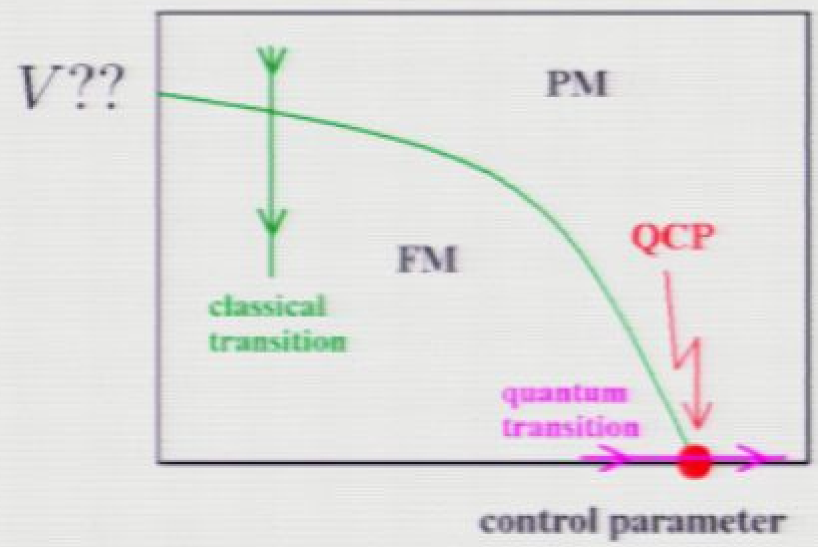
$$V = \mu_L - \mu_R$$





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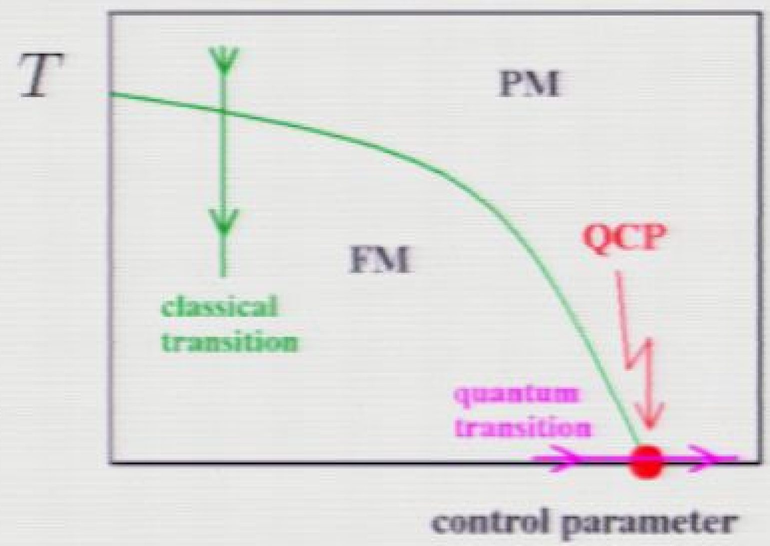

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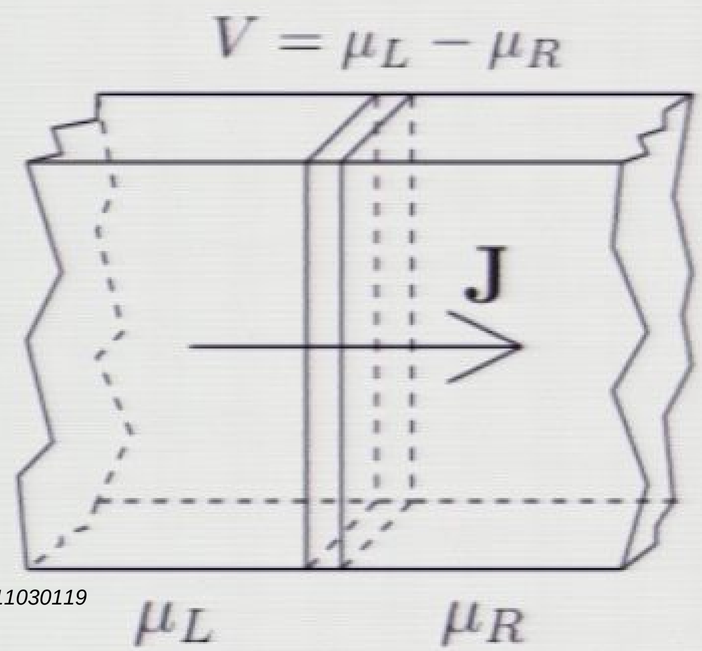
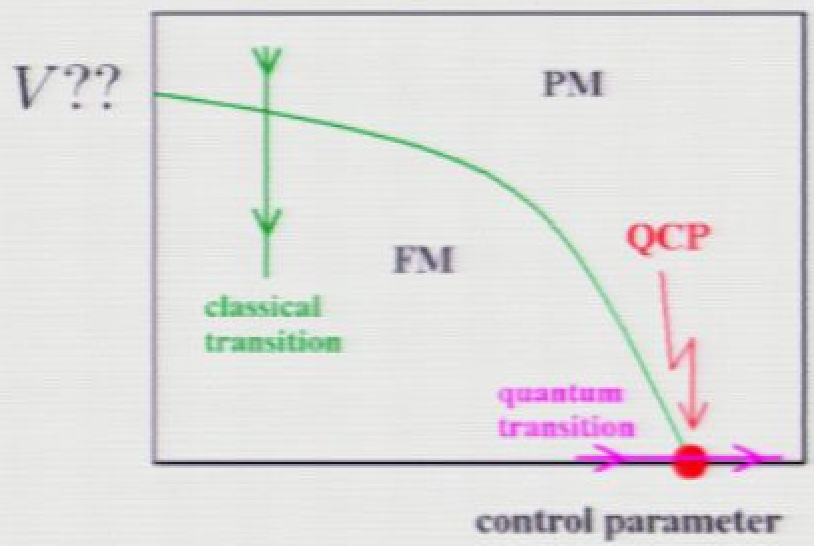
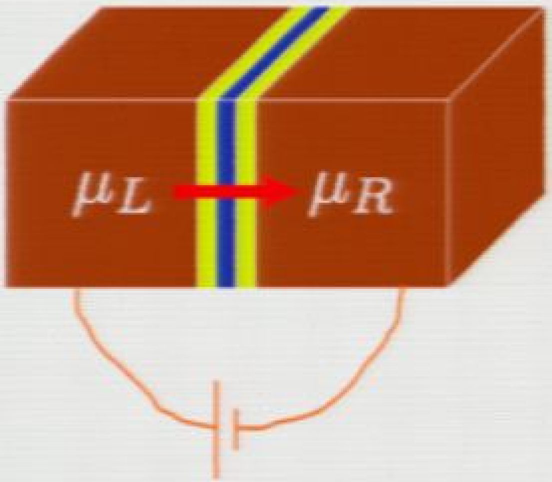




Open systems coupled to reservoirs - non-conserved order parameter



$$V = \mu_L - \mu_R$$

$$H = H_{layer} + H_{mix} + H_{leads}$$

$$H_{layer} = \sum_{i,\delta,\sigma} t_\delta c^\dagger_{i+\delta,\sigma} c_{i,\sigma} + H_{int}$$

$$H_{mix} = \sum_{i,k,\sigma,b=L,R} \left(t_{k,b} c^\dagger_{i,\sigma} a_{i,k,\sigma,b} + h.c. \right)$$

2D Itinerant electron system coupled to two 3D leads

Let us consider the ISING limit (longitudinal magnetization)

Goals

Difficulty in the formalism: Hertz-Millis-Moriya theory is based on a quantum generalization of the Landau-Ginzberg free energy.

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Find a way to express nonequilibrium problems
in a Feynman path integral form;
sum over histories on the Keldysh time contour

Determine the nature of quantum phase transition;
determine dynamic and static universality classes;
generalize renormalization group scheme
to nonequilibrium systems

Need for Keldysh Formalism

General consideration

$$\langle \hat{O}(t) \rangle \equiv \mathrm{Tr}\{\hat{O}\hat{\rho}(t)\}$$

$$\hat{\rho}(t) = \hat{\mathcal{U}}_{t,-\infty}\hat{\rho}(-\infty)[\hat{\mathcal{U}}_{t,-\infty}]^{\dagger} = \hat{\mathcal{U}}_{t,-\infty}\hat{\rho}(-\infty)\hat{\mathcal{U}}_{-\infty,t}$$

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In equilibrium, we can avoid this ...

$$\langle \mathrm{GS}|\hat{O}|\mathrm{GS}\rangle = \langle 0|\hat{\mathcal{U}}_{-\infty,t}\hat{O}\hat{\mathcal{U}}_{t,-\infty}|0\rangle \qquad |\mathrm{GS}\rangle = \hat{\mathcal{U}}_{t,-\infty}|0\rangle$$

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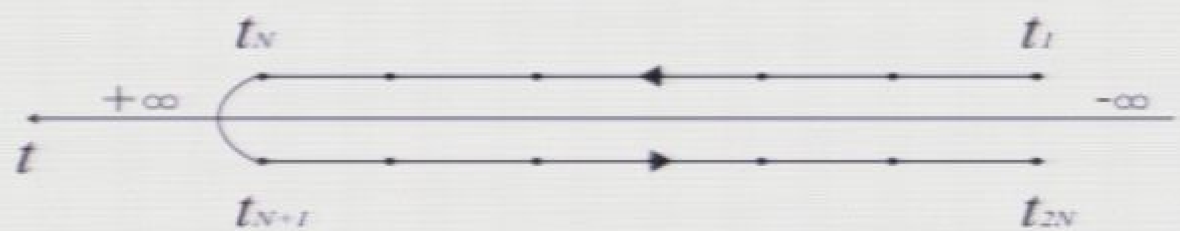

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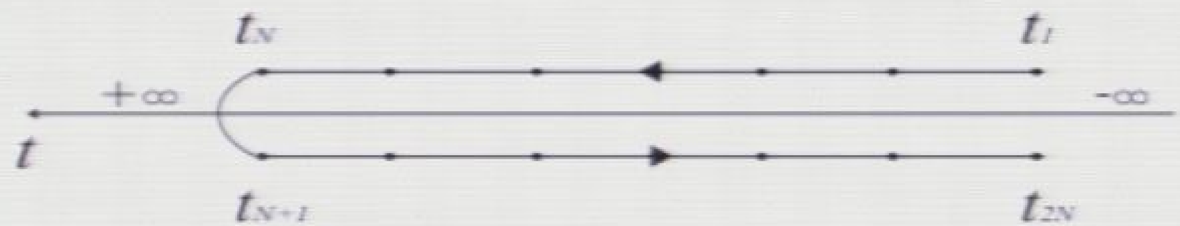

Keldysh generating functional

$$Z_K[\eta] = \mathrm{Tr}\{\hat{\mathcal{U}}_C\hat{\rho}(-\infty)\} \qquad \hat{H}_O(t) \equiv \hat{H}(t) \pm \hat{O}\eta(t)/2$$

$$\langle \hat{O}(t) \rangle = \left. \frac{\delta \ln Z_K[\eta]}{\delta \eta(t)} \right|_{\eta=0}$$

Keldysh Path Integral Formalism

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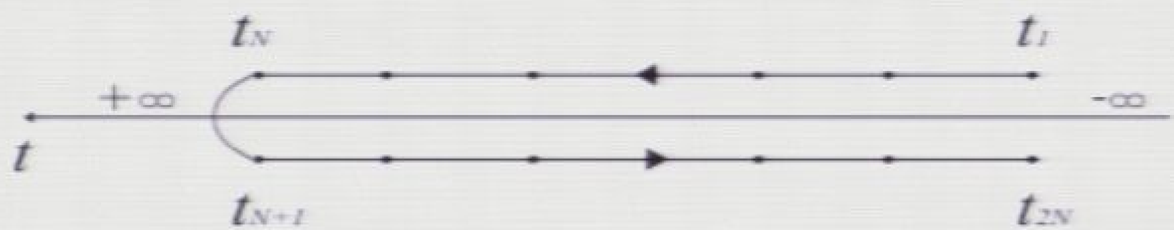
Keldysh approach; let the quantum system evolve forward in time then rewind its evolution back; then no knowledge of the distant future is necessary



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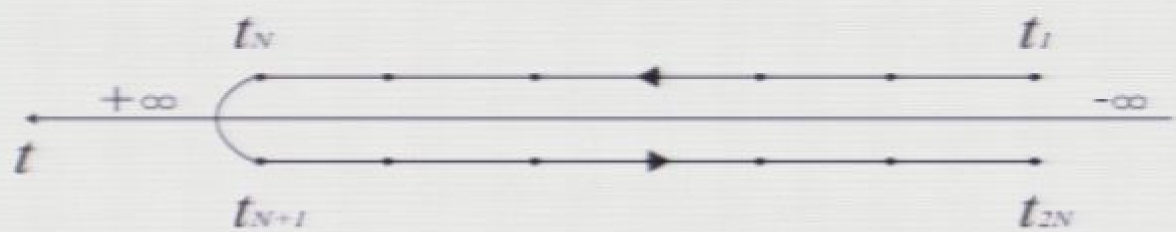

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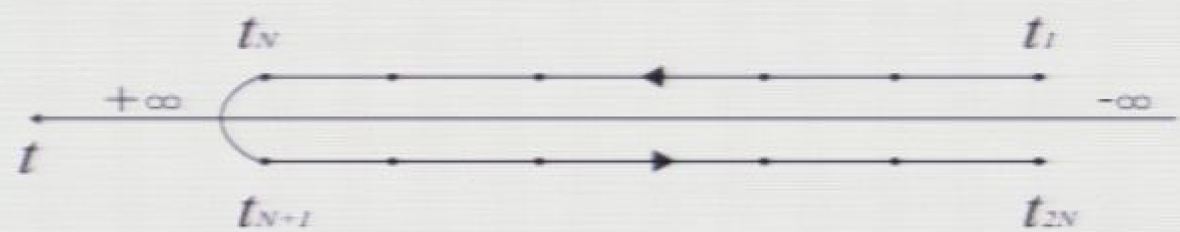

Keldysh path integral in terms of
fields on the forward and backward paths

$$Z_K \sim \int \prod_{i=1}^{6} \mathcal{D}\{\phi_i\} \langle\phi_6|\hat{\mathcal{U}}_{-\delta_t}|\phi_5\rangle\langle\phi_5|\hat{\mathcal{U}}_{-\delta_t}|\phi_4\rangle\langle\phi_4|\hat{1}|\phi_3\rangle\langle\phi_3|\hat{\mathcal{U}}_{+\delta_t}|\phi_2\rangle\langle\phi_2|\hat{\mathcal{U}}_{+\delta_t}|\phi_1\rangle\langle\phi_1|\hat{\rho}_0|\phi_6\rangle$$

N=3 case

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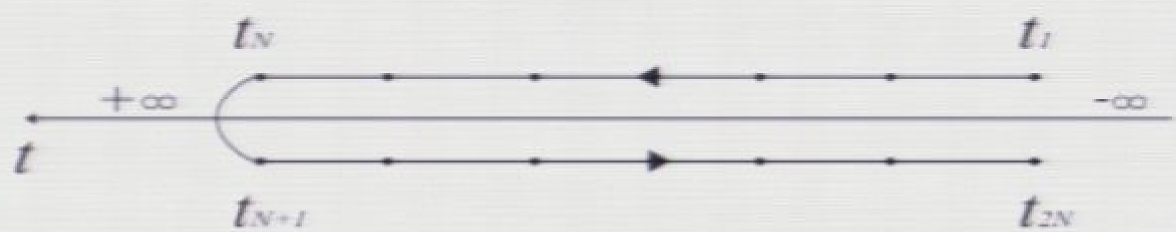

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$$Z_K \sim \sum_{\text{all conf. of } \phi_+,\phi_-} e^{iS_K[\{\phi_+,\phi_-\}]}$$

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Keldysh path integral in terms of order parameter fields on the forward and backward paths

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Effective Field Theory

Integrate out electronic degree of freedom and obtain an effective field theory of order parameter

$$S_K^{\text{eff}} = S^{(2)} + S^{(4)} + \cdots \qquad m_{cl} = \frac{1}{2}(m_- + m_+) \quad m_q = \frac{1}{2}(m_- - m_+)$$

$$S^{(2)} = -i \int dt\, dt' \int d^d r\, d^d r' \left(m_{cl}(t,r),\ m_q(t,r)\right) \begin{pmatrix} 0 & \left[\chi^{-1}\right]^A \\ \left[\chi^{-1}\right]^R & \left[\chi^{-1}\right]^K \end{pmatrix} \begin{pmatrix} m_{cl}(t',r') \\ m_q(t',r') \end{pmatrix}$$

Effective Field Theory

Integrate out electronic degree of freedom and obtain an effective field theory of order parameter

$$S_K^{\text{eff}} = S^{(2)} + S^{(4)} + \cdots \qquad m_{cl} = \frac{1}{2}(m_- + m_+) \quad m_q = \frac{1}{2}(m_- - m_+)$$

$$S^{(2)} = -i\int dt\, dt' \int d^dr\, d^dr' \left(m_{cl}(t,r),\ m_q(t,r)\right) \begin{pmatrix} 0 & [\chi^{-1}]^A \\ [\chi^{-1}]^R & [\chi^{-1}]^K \end{pmatrix} \begin{pmatrix} m_{cl}(t',r') \\ m_q(t',r') \end{pmatrix}$$

$$[\chi^{-1}]^R(\mathbf{q},\Omega) = [\chi^R(\mathbf{q},\Omega)]^{-1} = \delta - i\frac{\Omega}{\gamma} + \xi_0^2 q^2$$

$$[\chi^{-1}]^K(\mathbf{q},\Omega) = -2i\sum_{ab} \coth\frac{\Omega + \mu_a - \mu_b}{2T} \frac{(\Omega + \mu_a - \mu_b)}{\gamma^{ab}} \rightarrow -2i\sum_{ab} \frac{|\Omega + \mu_a - \mu_b|}{\gamma^{ab}}$$

$$\delta = 1 - UN(\mu_L, \mu_R, \Gamma_L, \Gamma_R)$$

γ_{ab}, γ some function of Γ_L, Γ_R

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$[\chi^{-1}]^K$ acts as a "mass" for quantum fluctuations

Effective Field Theory

$[\chi^{-1}]^K$ acts as a "mass" for quantum fluctuations

At T=0 and at equilibrium (V=0)

$[\chi^{-1}]^K(\omega \to 0) = 0$ strong quantum fluctuations

At finite T and at equilibrium (V=0)

$[\chi^{-1}]^K(\omega \to 0) = \frac{T}{\gamma'} \neq 0$ quantum fluctuations suppressed by T

At T=0 and in non-equilibrium (finite V)

$[\chi^{-1}]^K(\omega \to 0) = \frac{V}{\gamma'} \neq 0$ quantum fluctuations suppressed by V

Quartic Interactions

$$S^{(4)} = -i\int (d\{k\})[u_1 m_q m_{cl}^3 + u_2 m_q^2 m_{cl}^2 + u_3 m_q^3 m_{cl} + u_4 m_q^4]$$

u_i are interaction functions depending on the momenta and frequencies of all the fields

Renormalization Group

Integrate out fluctuations with $\Lambda/b < q < \Lambda$

Rescaling $q \to q'/b, (\Omega, T, V) \to (\Omega', T', V')/b^z, m_{cl,q} \to m_{cl,q} b^{1+(d+z)/2}$

Renormalization Group

$$\frac{dT(b)}{d\ln b} = zT(b)$$

$$\frac{dV(b)}{d\ln b} = zV(b)$$

$$\frac{d\delta}{d\ln b} = 2\delta + C_1 u_1(b)$$

$$\frac{du_i}{d\ln b} = [4 - (d+z)]u_i + \mathcal{O}(u_i u_j)$$

Voltage is a relevant perturbation

Renormalization Group

$$\frac{dT(b)}{d\ln b} = zT(b) \qquad \frac{dV(b)}{d\ln b} = zV(b) \qquad \frac{d\delta}{d\ln b} = 2\delta + 3u_1(b)g$$

$$\frac{du_1}{d\ln b} = \epsilon u_1 - 18u_1^2(f^{KR} + f^{KA}) + 12u_1\bar{u}_2 f^{RA}$$

$$\frac{d\bar{u}_2}{d\ln b} = \epsilon\bar{u}_2 - 2\bar{u}_2\left[15u_1(f^{KR} + f^{KA}) - 2\bar{u}_2 f^{RA}\right] + 18u_1\left[u_1 f^{KK} - 2u_3 f^{RA}\right]$$

$$\frac{d(u_1-u_3)}{d\ln b} = \epsilon(u_1 - u_3) - 12u_1\left[6u_4 f^{RA} - u_2 f^{KK}\right] - 8\bar{u}_2^2(f^{KR} + f^{KA})$$

$$-(u_1 - u_3)\left[18u_1(f^{KR} + f^{KA}) - 12\bar{u}_2 f^{RA}\right]$$

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$\bar{u}_{2,4} = iu_{2,4}$ $\qquad \epsilon = 4 - d - z$ $\qquad f, g$ depend on $T, V, \delta(T, V)$

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Solutions of the Scaling Equations

The quartic interaction is irrelevant/relevant for d+z>4 or <4

d=z=2 case is marginal; details of the crossover complicated

At the end of scaling $\delta(b^*) \sim 1$

$$\delta(b^*) = [b^*]^2 r \sim 1 \qquad b^* \sim r^{-1/2} \qquad r = \delta - \delta_c$$

$$V(b^*) = [b^*]^z V \sim r^{-z/2} V$$

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$V > r^{z/2}$ classical regime

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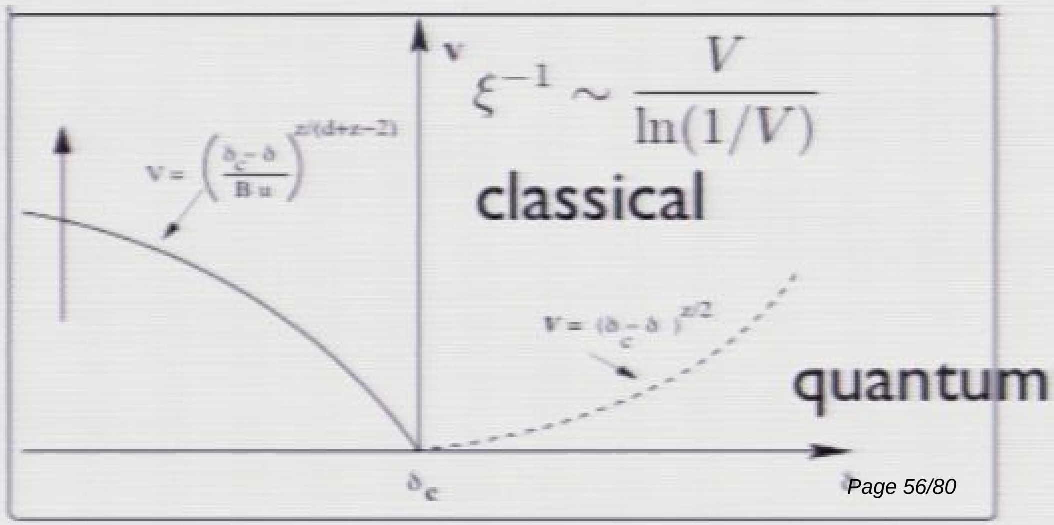

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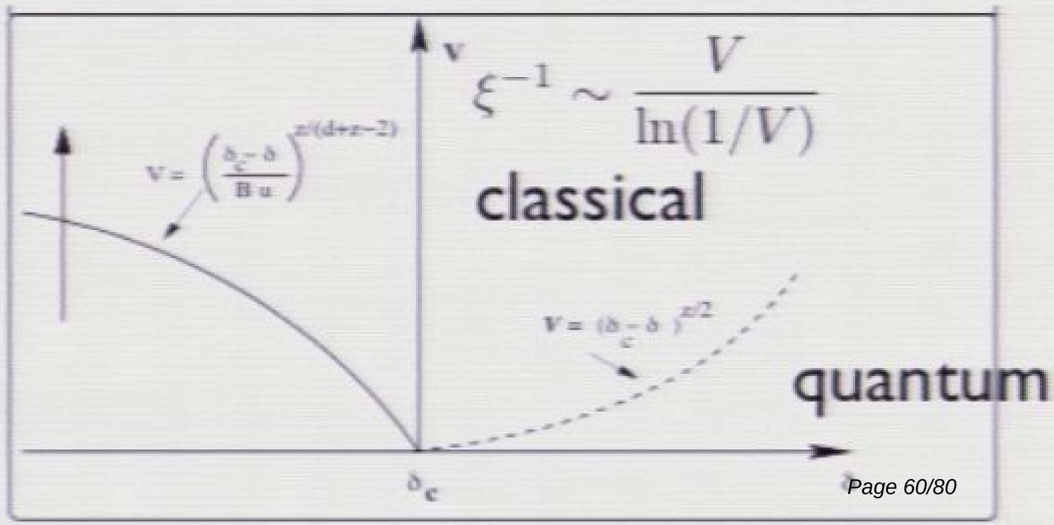

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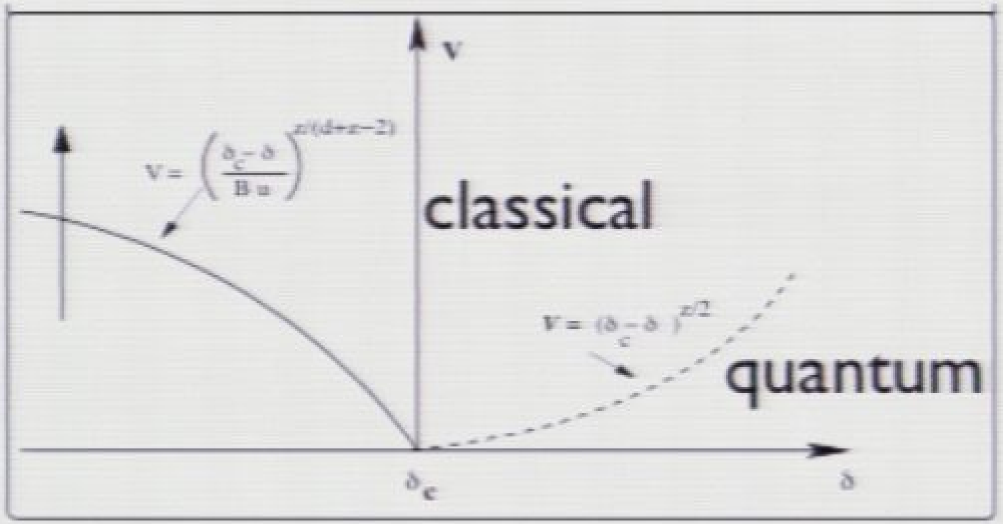
Phase Diagram at d=2



Dynamical critical exponent z=2

V is a relevant perturbation

Generalized Fluctuation-Dissipation Theorem

At T=0 and in the classical regime, V > r

$$\chi^K(\Omega) = \frac{2T_{\rm eff}}{\Omega}[\chi^R(\Omega) - \chi^A(\Omega)] \qquad T_{\rm eff} \sim \frac{\Gamma_L \Gamma_R}{\Gamma} V$$

(c.f. $\chi^K(\Omega) = \coth\left(\frac{\Omega}{2T}\right)[\chi^R(\Omega) - \chi^A(\Omega)]$ in equilibrium at finite T)

fluctuation dissipation

Magnetization Dynamics

Introduce new H-S field

$$-i[\chi^{-1}]^K |m_q|^2 \rightarrow -i\frac{|\xi|^2}{[\chi^{-1}]^K} + i\xi m_q^* + i\xi^* m_q$$

Variation of the Keldysh action w.r.t. m_q

Langevin Equation $$-\frac{1}{\gamma_r}\frac{\partial m_{cl}}{\partial t} = (\delta_r - \xi_0^2\nabla^2 + v_{1,r} m_{cl}^2) m_{cl} + \xi$$

The noise is determined by the Keldysh response function

$$-i\langle \xi(\mathbf{q},\Omega)\xi(\mathbf{q}',\Omega')\rangle = \frac{1}{2}[\chi^{-1}]^K(\mathbf{q},\Omega)\delta(\mathbf{q}+\mathbf{q}')\delta(\Omega+\Omega')$$

Magnetization Dynamics

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In the classical regime, V > r $[\chi^{-1}]^K \propto V$

$\xi(t)$ becomes gaussian white noise

$$\langle\xi(x,t)\xi(x',t')\rangle = \delta(x-x')\delta(t-t')\frac{2V}{\gamma_{rLR}}$$

This is the same as the Model A dynamics;
the voltage driven transition is in the same universality class as the usual thermal Ising transition;
voltage acts like temperature

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The physics of the disordered and quantum-classical crossover regimes is weakly dependent on the spin symmetry

Differences appear in the 'renormalized classical' regime corresponding to adding a weak non-equilibrium drive to an ordered state

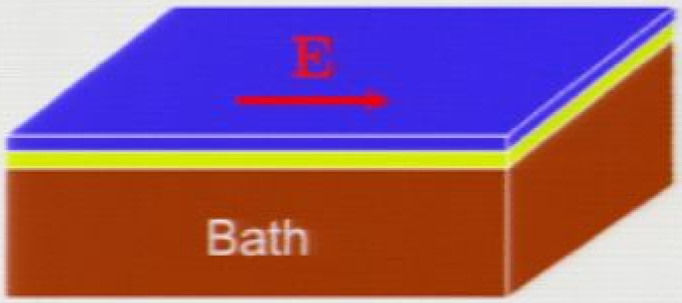
The Langevin Equation in the ordered phase (near QCP)

$$\frac{a_{xx}}{\Gamma}\frac{\partial \vec{m}}{\partial t} + \hat{z} \times \frac{a_{xy}\Delta}{\Gamma^2}\frac{\partial \vec{m}}{\partial t} - \left(b_{xx} - \frac{b_{xy}\Delta V}{\Gamma^2}\hat{z}\times\right)\xi_0^2\nabla^2\vec{m} = \vec{\xi} \quad (V/\Gamma, \Delta/\Gamma \ll 1)$$

spin precession as in the Landau-Lifshitz-Gilbert Eq.

spin-torque effect when $\Gamma_L(\varepsilon_1)\Gamma_R(\varepsilon_2) - \Gamma_L(\varepsilon_2)\Gamma_R(\varepsilon_1) \neq 0$

Symmetry Breaking ?



$$T_{\text{eff}} = eEl_{sc}$$

$$l_{sc} = v_F \tau_{sc}$$

$$v_D \sim T_{\text{eff}}/k_F$$

$$T_{\text{eff}} \tau_{sc} \ll 1$$

$$\Pi^R(q,\Omega) = \delta + Aq^2 - i\tau_{sc}\left(\Omega - \vec{v}_D \cdot \vec{q}\right)$$

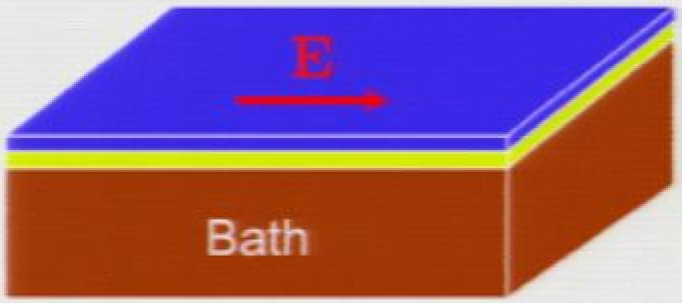
$$f_{bk} = \theta(-\omega) + \frac{1}{2}[\text{sgn}(\omega) + \text{sgn}(\mathbf{E}\cdot\mathbf{v_k})]e^{-\frac{|\omega|}{|e\mathbf{E}\cdot\mathbf{v_k}\tau|}}$$

$$\Pi^K_{2d}(\Omega) = -2i\tau_{sc} \left[|\Omega| + T_{eff}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}|cos\phi|e^{-\frac{|\Omega|}{(T_{eff}|cos\phi|)}}\right]$$

$$\begin{aligned} \frac{dT_{eff}}{dl} &= 2T_{eff} \\ \frac{d\delta}{dl} &= 2\delta + C_1 u_1 \\ \frac{dv_D}{dl} &= v_D \\ \frac{du_i}{dl} &= [2-d]\, u_i + \mathcal{O}(u_k u_l) \end{aligned}$$

A. Mitra, A. J. Millis (2008)

Symmetry Breaking ?



$$v_D k_F \sim T_{\text{eff}} \ll E_F$$

$$\tau_{sc}\left(\frac{\partial}{\partial t} - \vec{v}_D \cdot \vec{\nabla}\right) m_{cl}(x,t) \; = - \left[r - A\nabla^2 + \frac{2\pi^2 m_{cl}^2}{9ln\sqrt{1/T^0_{eff}}} \right] m_{cl} + \xi$$

$$\langle \xi(x,t)\xi(x',t')\rangle = \left(\frac{2\tau_{sc}T_{eff}}{\pi}\right)\delta(t-t')\delta(x-x')$$

Beyond Effective Temperature

- Two 2D electron gases
  - Hubbard interaction (spin channel): $H_{\text{int}} \sim -US_z^2$
  - Near FL-FM quantum critical point

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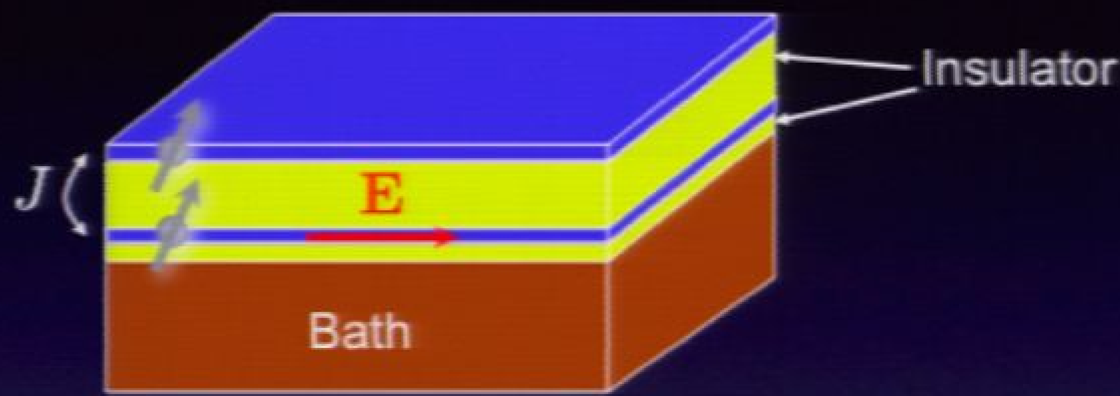
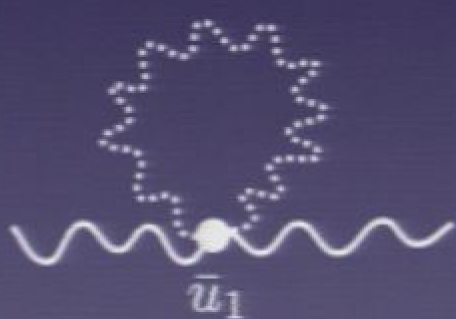


- Two 2D electron gases
  - Hubbard interaction (spin channel): $H_{\text{int}} \sim -U S_z^2$
  - Near FL-FM quantum critical point
- Interlayer interaction
  - FM spin exchange *J*
  - No particle tunneling
- In-plane time-independent electric field for bottom layer
- Reservoir (free Fermionic bath): gives steady-state

RG analysis

- Solutions to RG flow:

$$
\begin{aligned}
T_{\text{eff}}(b) &= T_{\text{eff}} b^3 \\
v_d(b) &= v_d b^2 \\
\tau(b) &= \tau b^{-1}, \\
\bar{u}_i(b) &= \bar{u}_i b^{-1} \\
\Delta(b) &= b^2[\Delta + 3\bar{u}_1 \int_0^{\ln b} dx e^{-3x} f(T_{\text{eff}} e^{3x}, \tau e^{-x})]
\end{aligned}
$$



$$= f(T_{\text{eff}}(b), \tau(b)) = \frac{i}{2\pi} \int \frac{d\Omega}{2\pi} \chi_K(1, \Omega)$$

Summary

studied the steady-state nonequilibrium magnetic quantum critical phenomena in open systems

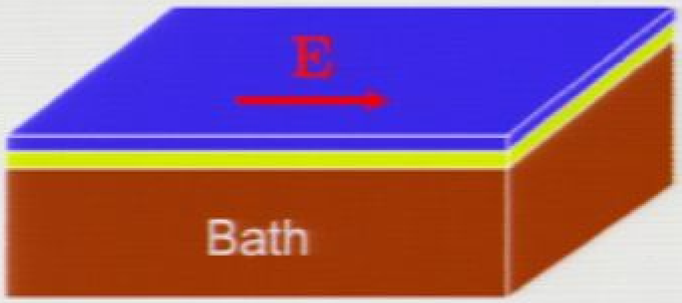
generalized renormalization group scheme to nonequilibrium systems

voltage playing a role of effective temperature

sub-leading scaling may differ from equilibrium analogs for "tensorial" order parameters

identification of universality classes

Symmetry Breaking ?



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The noise is given by $\langle \xi_i(q,t)\xi_j(q',t')\rangle = 2E\delta_{ij}\delta(q+q')\delta(t-t')$

$$E = \frac{V a_{xx}\Gamma_L\Gamma_R}{\Gamma^3} \text{ for } T = 0, V \neq 0$$

The solution of the Langevin Equation

$$\langle m_+(q,t)m_-(q't')\rangle = \frac{\Gamma E\delta(q+q')e^{-D_{\text{eff}}q^2\xi_0^2|t-t'|-i\omega_{\text{eff}}(t-t')}}{(a_{xx}b_{xx}-\frac{a_{xy}b_{xy}\Delta^2 V}{\Gamma^3})q^2\xi_0^2}$$

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