Title: Explorations in Condensed Matter - Lecture 14

Date: Mar 31, 2011 10:15 AM

URL: http://pirsa.org/11030117

Abstract:

BCS :

$$H^{BCS} = \sum_{k\sigma} (\varepsilon_k - \mu)\, c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{k\sigma\sigma'} \underbrace{\Delta_{k\sigma\sigma'}\, \overbrace{c_{k\sigma}}^{A}\, \overbrace{c_{-k\sigma'}}^{A}} + h.c.$$

$k \to -k$

$\sigma \leftrightarrow \sigma'$

$$c_{k\sigma} c_{-k\sigma'} \to c_{-k\sigma'} c_{k\sigma} = - c_{k\sigma} c_{-k\sigma'}$$

$$\left.\Delta_{k\sigma\sigma'} = \chi_{\sigma\sigma'} \Delta_k \right\} A$$

$\uparrow\uparrow$

$\downarrow\downarrow$

$\uparrow\downarrow + \downarrow\uparrow$

χ symm

Δ anti

p-wave

χ anti

Δ_k symm

s-wave, d-wave ...

h.c $\xrightarrow{\text{spinless}}$ $H = \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \begin{pmatrix} \xi_k & \Delta_k \\ \Delta_k^* & -\xi_k \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$

$$\hat{\psi}_k = \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

$$u_k c_k^\dagger + v_k c_{-k}$$

$$\xi_k = \varepsilon_k - \mu$$

$$E_k = \pm\sqrt{\xi_k^2 + |\Delta_k|^2}$$

$$(\xi_k - E)\, u_k = -\Delta_k v_k$$

$$\Delta_k^* u_k = (\xi_k + E)\, v_k$$

-wave ...

spinless $\longrightarrow$ $H = \begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \begin{pmatrix} \xi_k & \Delta_k \\ \Delta_k^* & -\xi_k \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$

$$\xi_k = \varepsilon_k - \mu$$

$$E_k = \pm\sqrt{\xi_k^2 + |\Delta_k|^2}$$

$$(\xi_k - E)\, u_k = -\Delta_k v_k$$

$$\Delta_k^* u_k^* = (\xi_k + E)\, v_k^*$$

$$\boxed{\frac{u_k}{u_k^*} = \frac{v_k}{v_k^*} \cdot \frac{-\Delta_k}{(\xi_k + E)} \, \frac{\Delta_k}{(\xi_k - E)}}$$

$$\hat{\psi}_k = \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

$$\boxed{\psi_k^\dagger = u_k^* c_k^\dagger + v_k^* c_{-k}}$$

$$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$$

$$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$$

$$\Psi_{gs} = \prod_k \left(u_k + v_k c_k^\dagger c_{-k}^\dagger\right)|0\rangle$$

BCS

$$\left(\frac{u_k}{u_k^*}\right) = e^{2i\alpha} = \frac{-\Delta_k^2}{\xi_k^2 - E^2} = \frac{\Delta_k^2}{|\Delta_k|^2} = e^{2i\varphi_\Delta}$$

$$u_k = u_0 e^{i\alpha}$$

$$\begin{pmatrix} |u| e^{i\varphi} \\ |v| \end{pmatrix}$$

BCS :

$$\left(\frac{u_k}{u_k^*}\right) = e^{2i\alpha} = \frac{-\Delta_k^2}{\xi_k^2 - E^2} = \frac{\Delta_k^2}{|\Delta_k|^2} = e^{2i\varphi_\Delta}$$

$$u_k = u_0 e^{i\alpha}$$

$$\boxed{\Delta_k = k_x + i k_y}$$

$$\sigma_2 H \sigma_2 = -H^*$$

$$H\psi = E\psi$$

$$H\sigma_2\psi^* = -(\sigma_2 H^* \sigma_2)\sigma_2\psi^* =$$

$$= -\sigma_2 H^* \psi^* = -\sigma_2 (H\psi)^* =$$

$$= -E\sigma_2\psi^*$$

spinless $\longrightarrow$

$$H\Psi_0 = 0$$

$$H\sigma_2\Psi_0^* = 0$$

$\nearrow$ $\Psi_0 = \sigma_2\Psi_0^*$ 1 zero mode

$\searrow$ $\underline{\Psi_1} = \Psi_0 \,,\ \underline{\Psi_2 = \sigma_2\Psi_0^*}$ $\Psi_1 \neq \Psi_2$ 2 zero modes

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$$\Psi_0 = \sigma_2\Psi_0^*$$

$$\begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix} & -i\\ i & \end{pmatrix}\begin{pmatrix}u^*\\v^*\end{pmatrix} = \begin{pmatrix}-iv^*\\ iu^*\end{pmatrix}$$

$$u = -iv^*$$
$$|u| = |v|$$
$$\underline{\underline{u = v^*}}$$

$$\begin{pmatrix}1\cdot e^{i\varphi}\\ 1\end{pmatrix} = e^{\frac{i\varphi}{2}}\begin{pmatrix}e^{i\varphi/2}\\ e^{-i\varphi/2}\end{pmatrix}\sigma$$

$$\Psi_a = \Psi_1 + \Psi_2 = \Psi + \sigma_2\Psi^*$$
$$\Psi_b = i(\Psi_1 - \Psi_2) = i(\Psi - \sigma_2\Psi^*)$$

Majorana modes

$$\hat{\psi}_k = g_k c_k + g_k^* c_{-k}^\dagger$$

$$\gamma = \int dr \left[g(r)\, c_r + g^*(r)\, c_r^\dagger \right]$$

$$\boxed{\gamma^\dagger = \gamma} \qquad \{\gamma_i, \gamma_j\}$$

$$\{c_i^\dagger, c_j\} = \delta_{ij}$$

$$\{c_i, c_j\} = 0 = \{c_i^\dagger, c_j^\dagger\}$$

fermions

a modes

$+ g_k^* C_{-k}^+$

$g(r)\, C_r + g^*(r)\, C_r^+ \Big]$

$\boxed{\gamma'}$

$$\{\gamma_i, \gamma_j\} = \int dr\, dr' \Big[\underbrace{\{c_r^+, c_{r'}\}}_{\delta_{rr'}} g_i(r)\, g_j^*(r') + h.c \Big] = \delta_{ij}$$

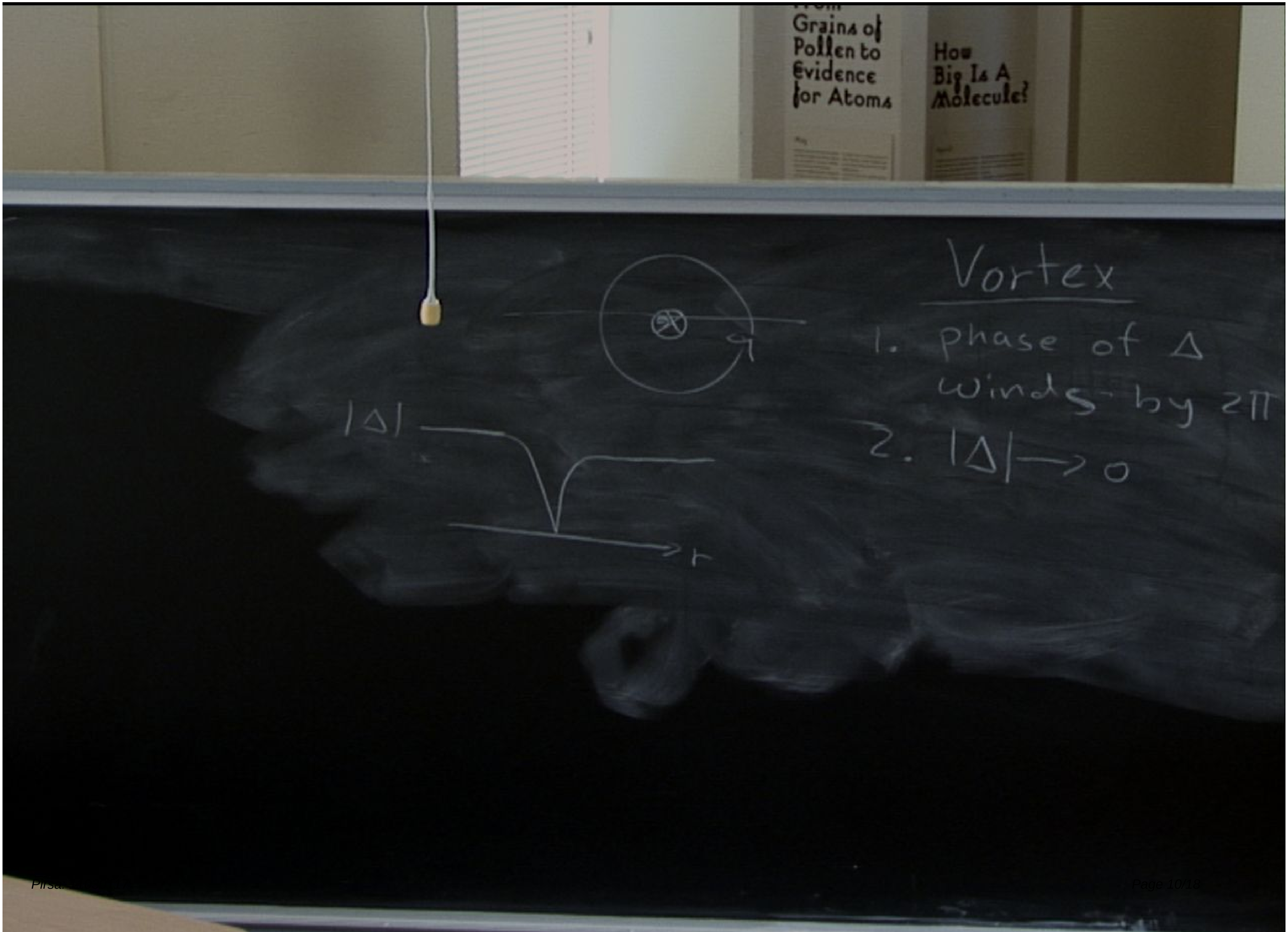
$$= \{\gamma_i, \gamma_j^+\}$$

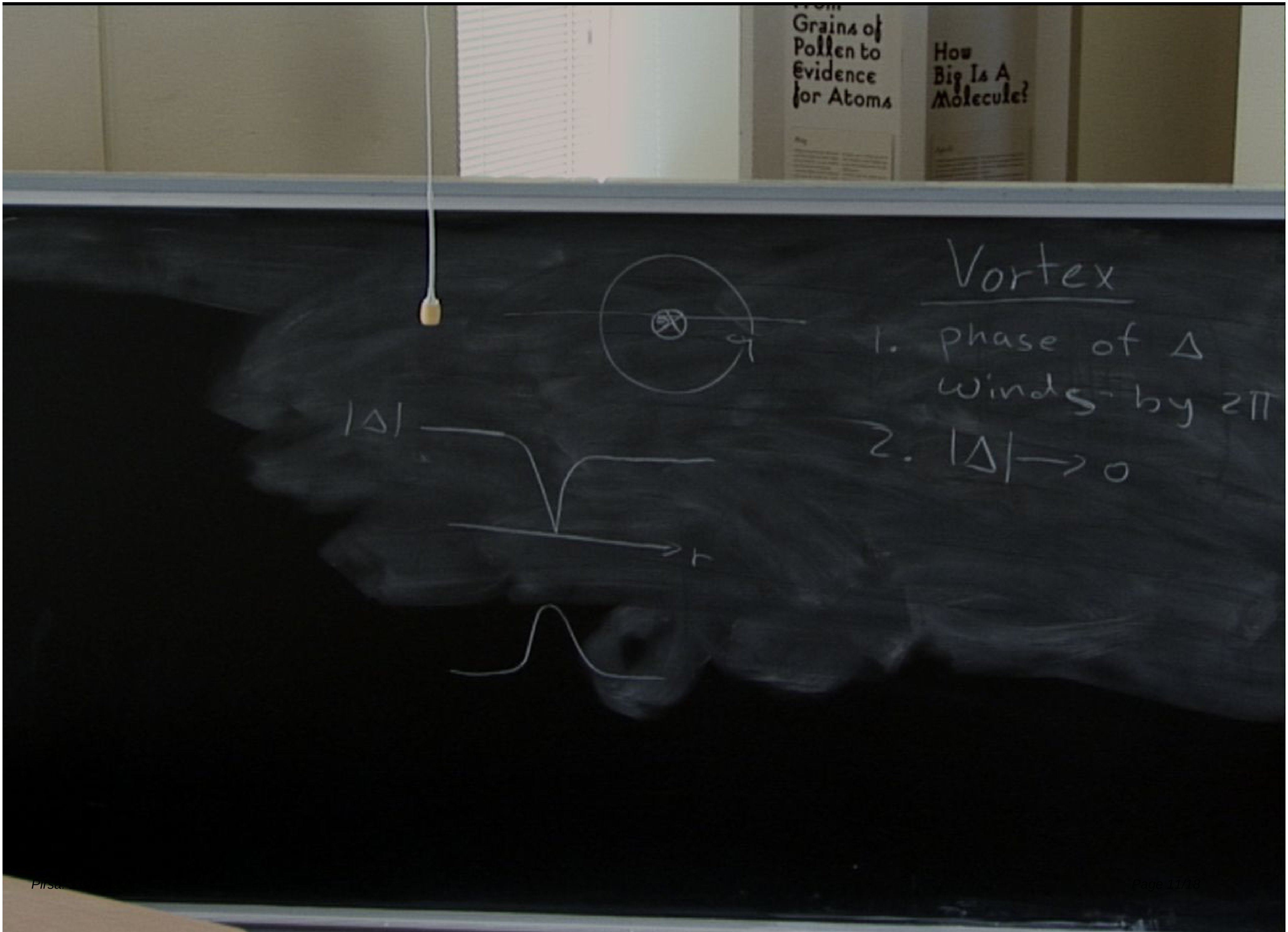
ij

fermions

$= \{c_i^+, c_j^+\}\Big\}$

spin


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Vortex
1. phase of Δ winds by 2π
2. |Δ| → 0
|Δ|
r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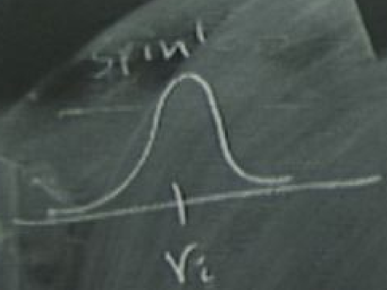
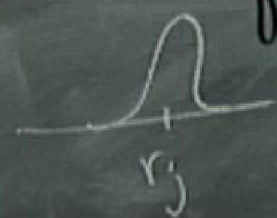
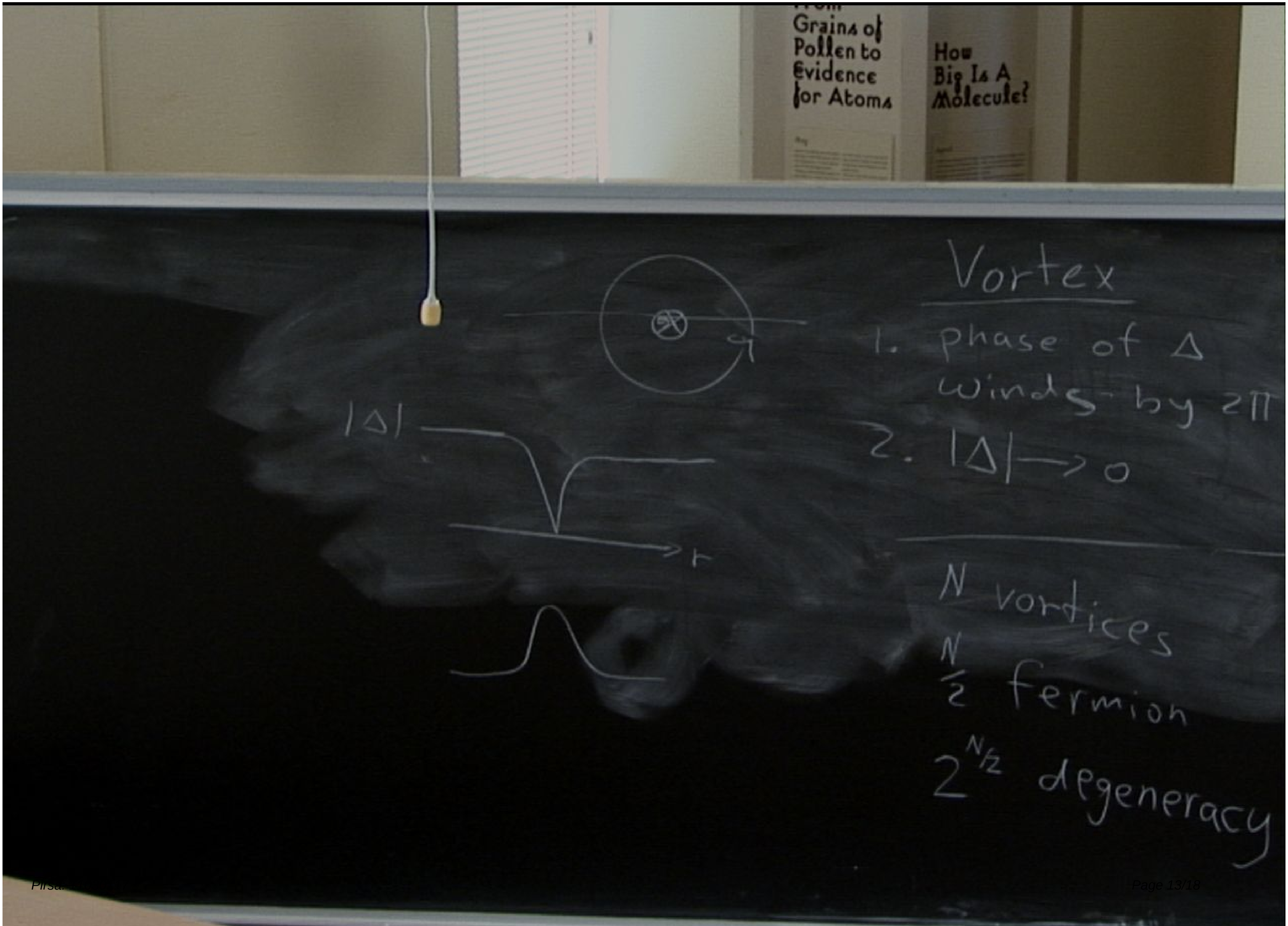




$$\Big[\underbrace{\{c^{+}_{r}, c_{r'}\}}_{\delta_{rr'}}\, g_i(r)\, g_j^{*}(r') + h.c\Big] = \delta_{ij} \int |g|^2\, dr = \delta_{ij}$$

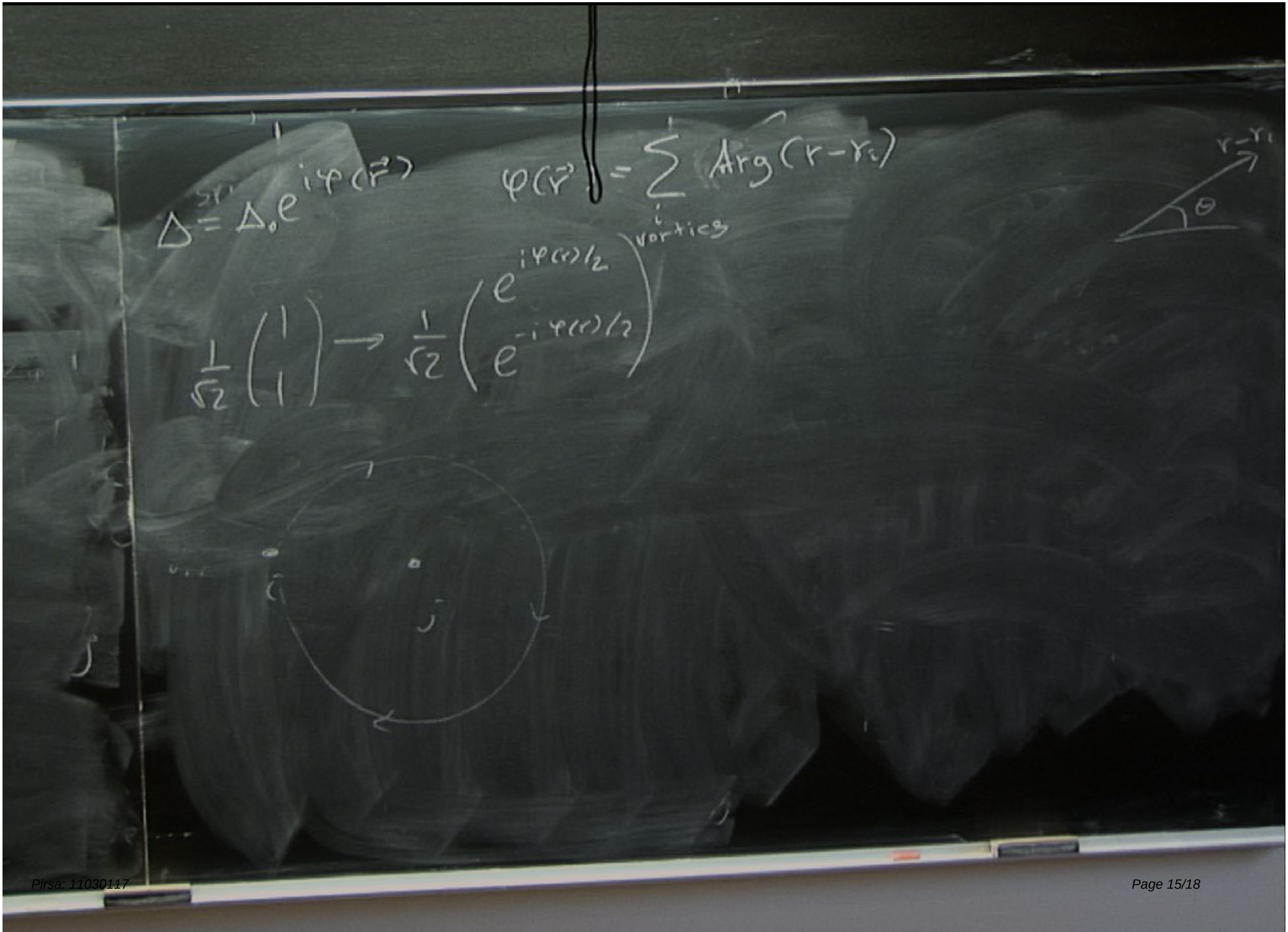

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Vortex
1. phase of Δ winds by 2π
2. |Δ| → 0
|Δ|
r
N vortices
N/2 fermion
2^{N/2} degeneracy

Majorana modes

$$|gs_1\rangle = f_1(r_1 \ldots r_N)$$

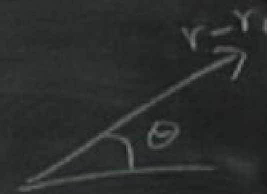
$$\vdots$$

$$|gs_{2^{N/2}}\rangle = f_2(r_1 \ldots r_N)$$


$\Delta = \Delta_0 e^{i\varphi(\vec{r})}$
$\varphi(\vec{r}) = \sum_{i} \mathrm{Arg}(r - r_i)$
vortices
$r - r_i$
θ
$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\varphi(r)/2}\\e^{-i\varphi(r)/2}\end{pmatrix}$
i
j

$$\Delta = \Delta_0 e^{i\varphi(\vec{r})}$$

$$\varphi(\vec{r}) = \sum_{\substack{i \\ \text{vortices}}} \mathrm{Arg}(r - r_i)$$



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\varphi(r)/2} \\ e^{-i\varphi(r)/2} \end{pmatrix}$$

$$\hat{\gamma}_i \to -\hat{\gamma}_i \qquad \gamma_k \Rightarrow \gamma_k$$

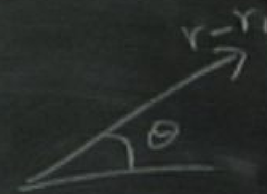
$$\hat{\gamma}_j \to -\hat{\gamma}_j \qquad k \neq i, j$$

$$\underbrace{u^+}_{\gamma_i \gamma_j} \hat{\gamma}_i u = -\gamma_i$$

$$\gamma_i \gamma_j \gamma_i \gamma_j \gamma_i = -\underbrace{\gamma_i \gamma_i}_{1} \underbrace{\gamma_j \gamma_j}_{1} \gamma_i$$

$$\Delta = \Delta_0 e^{i\varphi(\vec{r})}$$

$$\varphi(\vec{r}) = \sum_{i \,\text{vortices}} \text{Arg}(r - r_i)$$



$$u = \gamma_i \gamma_j$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\varphi(r)/2}\\e^{-i\varphi(r)/2}\end{pmatrix}$$

$$\hat{\gamma}_i \rightarrow -\hat{\gamma}_i$$

$$\hat{\gamma}_j \rightarrow -\hat{\gamma}_j$$

$$\gamma_k \Rightarrow \gamma_k \quad k \neq i,j$$

$$|\Psi_{gs}\rangle \longrightarrow \gamma_i \gamma_j |\Psi_{gs}\rangle$$

$$u^\dagger \hat{\gamma}_i u = -\gamma_i$$

$$\gamma_i \gamma_j \gamma_i \gamma_j \gamma_i = -\underbrace{\gamma_i \gamma_i}_{1} \underbrace{\gamma_j \gamma_j}_{1} \gamma_i = -\gamma_i$$

Majorana modes

$$|gs_1\rangle = f_1(r_1 \ldots r_N)$$

$$|gs_2\rangle = f_2(r_1 \ldots r_N)$$