Title: Explorations in Condensed Matter - Lecture 13

Date: Mar 30, 2011 10:15 AM

URL: http://pirsa.org/11030116

Abstract:

Topological insulators

experiments

Quantum Spin Hall

The Quantum Spin Hall Effect:
Theory and Experiment

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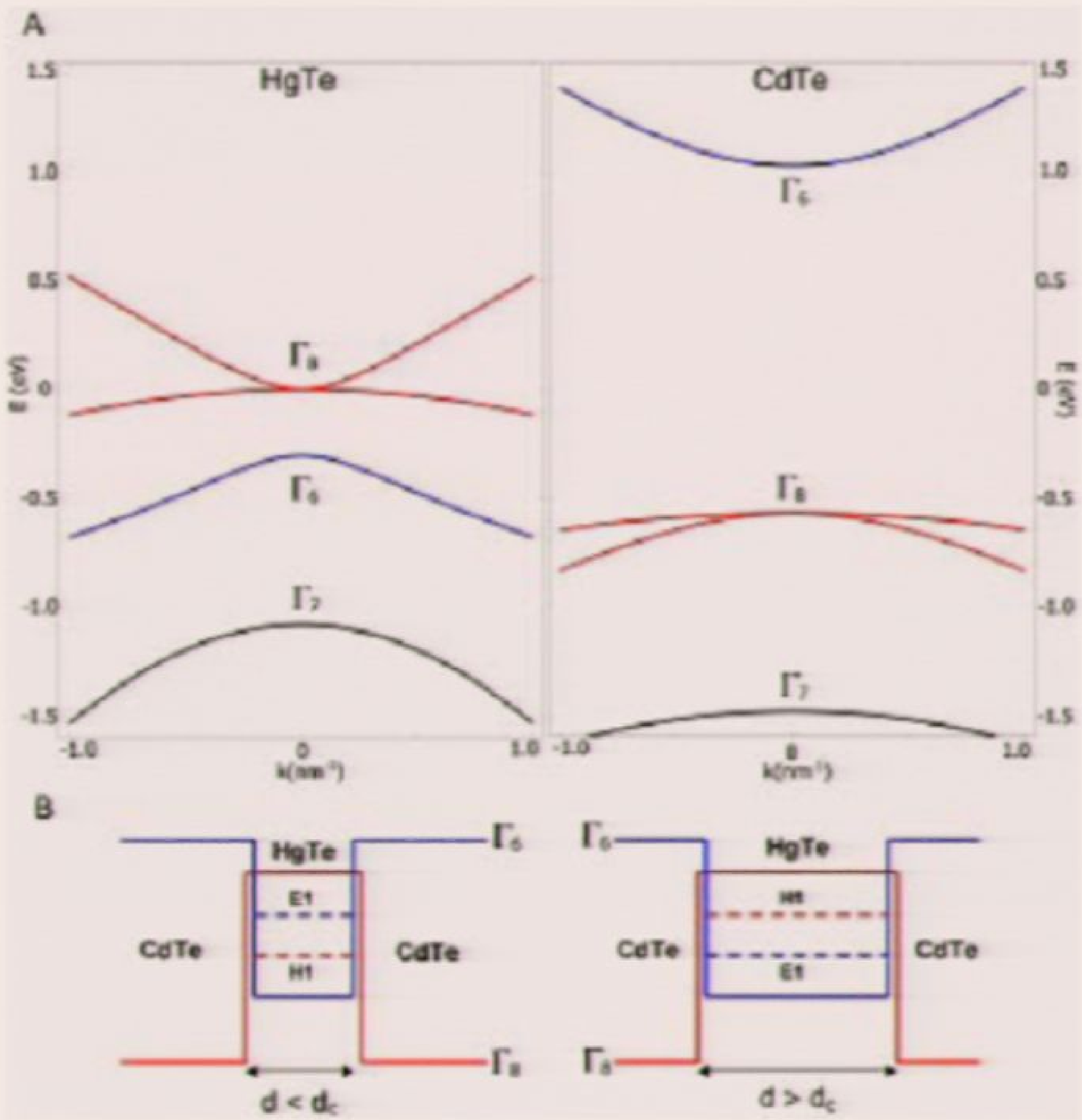
Stanford, CA 94305-4045
[3]Center for Advanced Study, Tsinghua University
Beijing, 100084, China

Quantum Spin Hall



Taylor l ... mp[1],

... eng Zhang[2]

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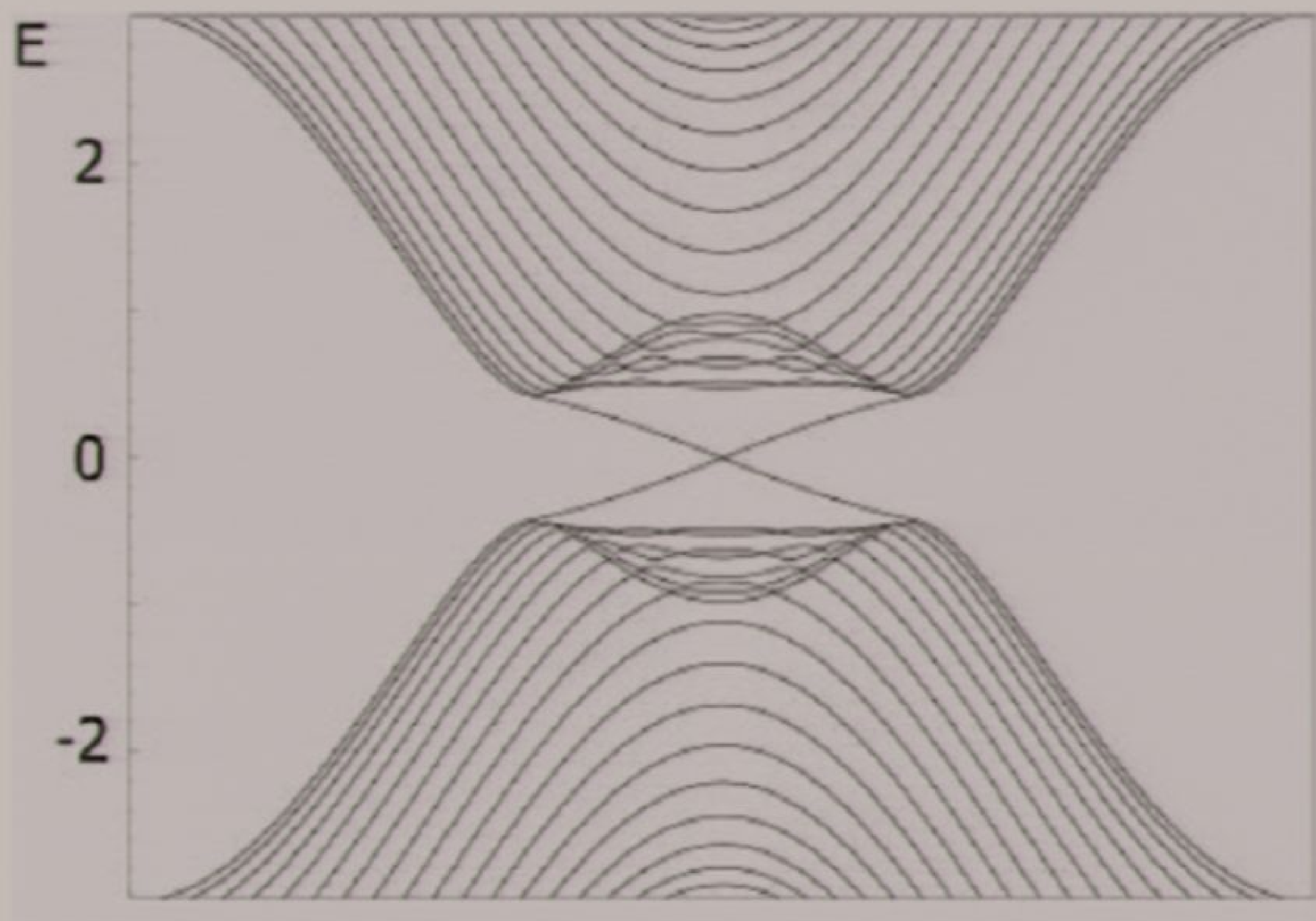
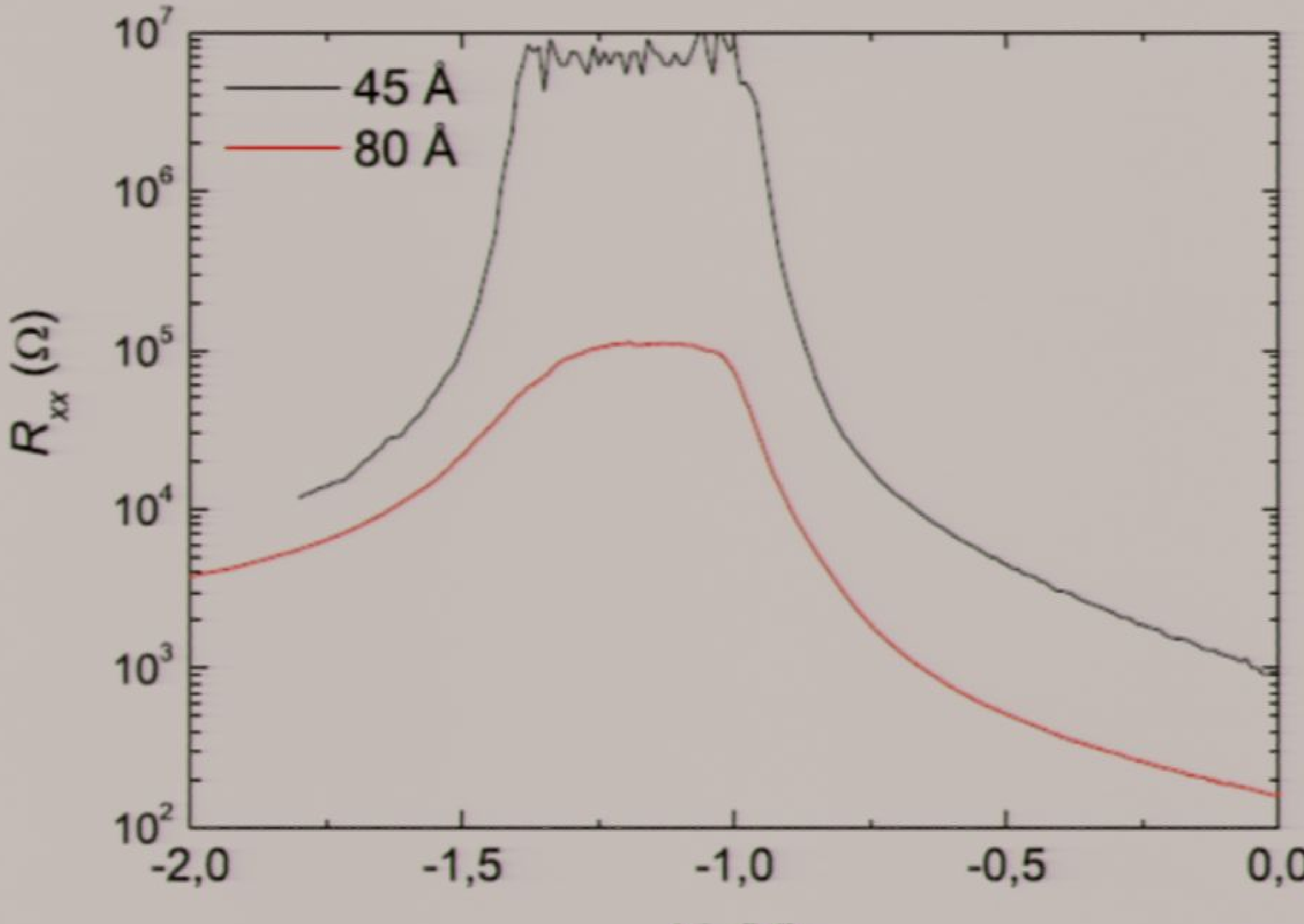
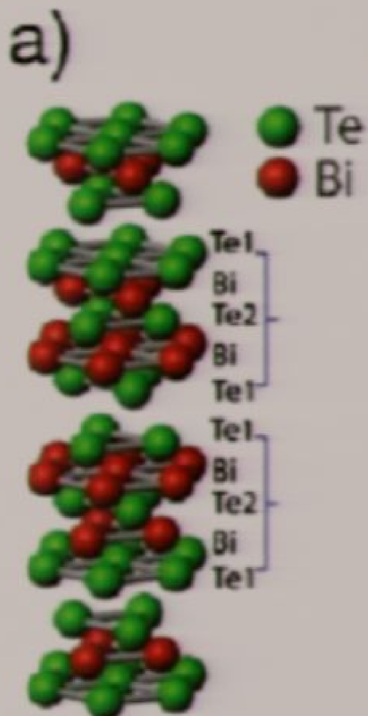
Bulk and edge modes

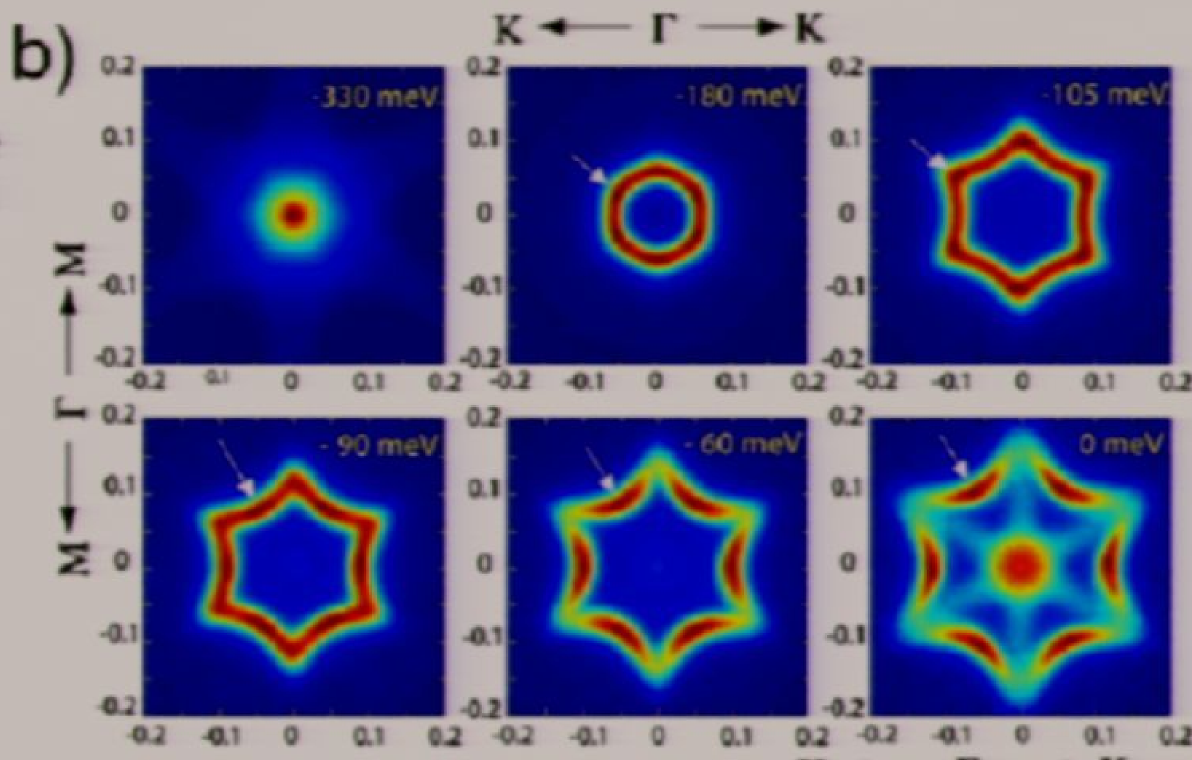


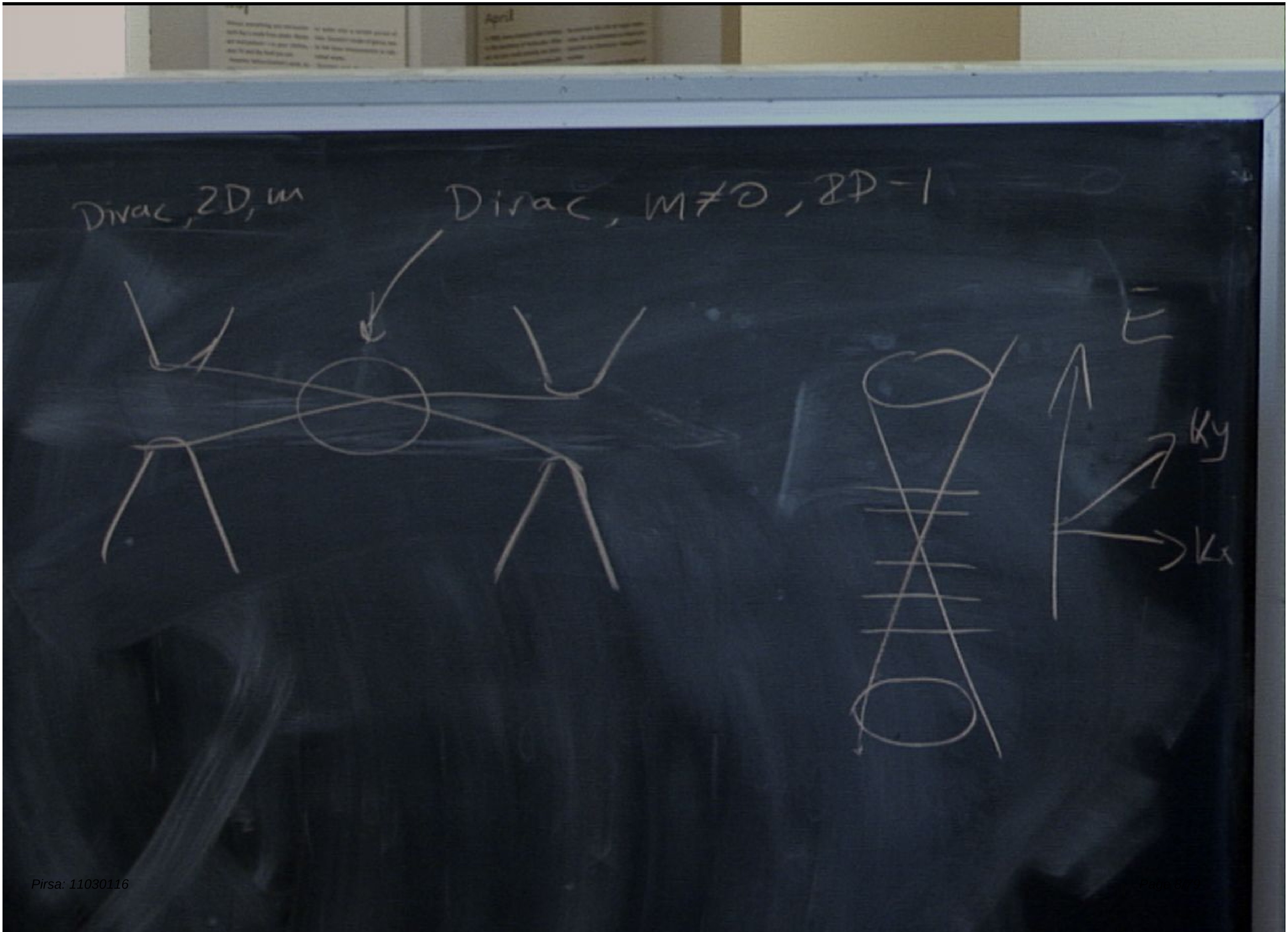
Longitudinal conductivity



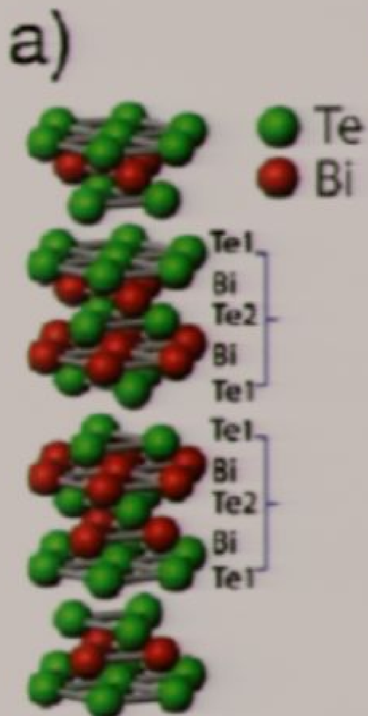
Surface of a 3D Topological insulator
Edge mode dispersion

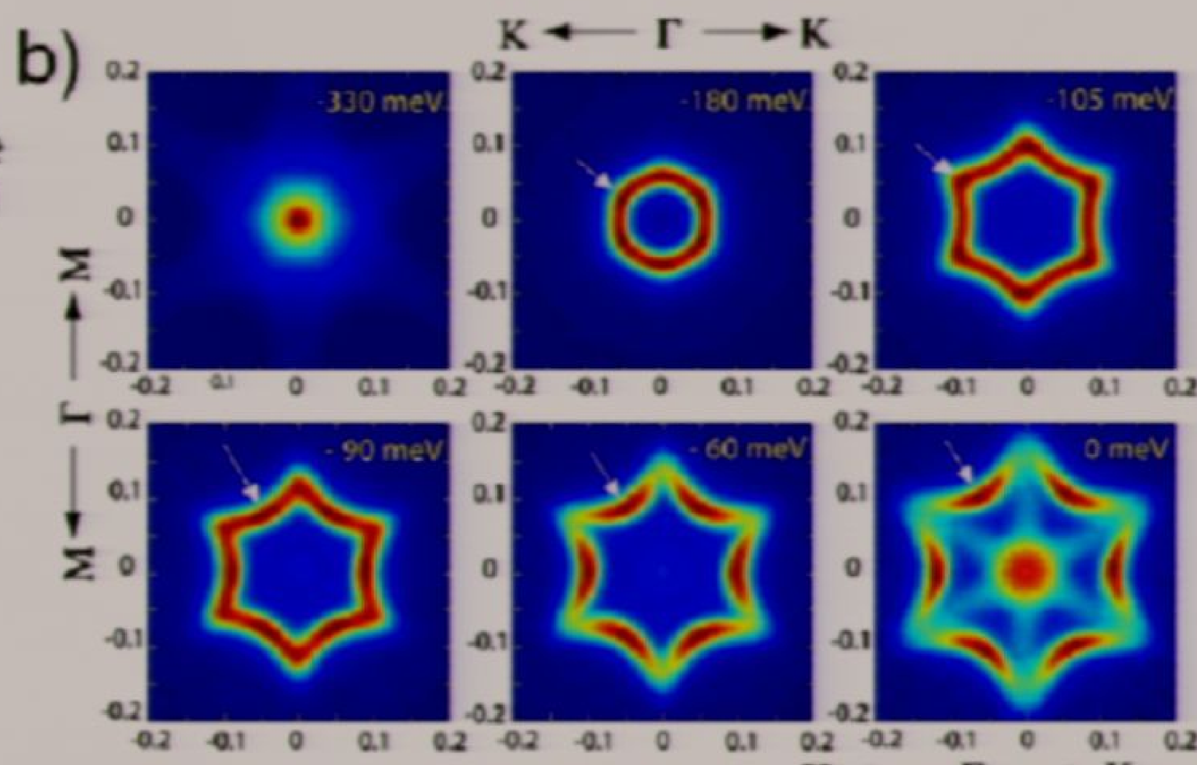


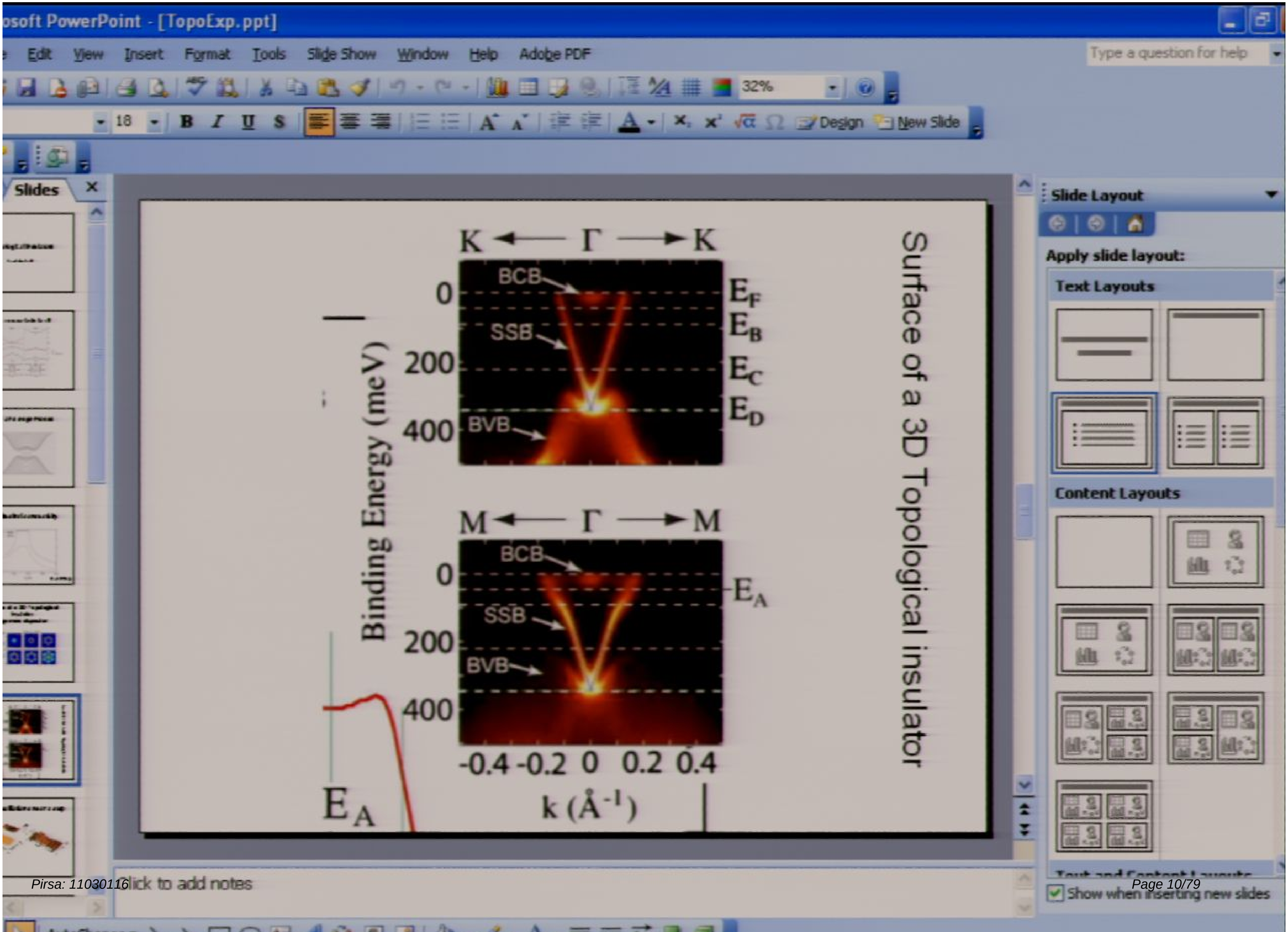



Dirac, 2D, m
Dirac, m≠0,
E
ky
kx

Surface of a 3D Topological insulator
Edge mode dispersion







osoft PowerPoint - [TopoExp.ppt]
Surface of a 3D Topological insulator
K Γ K
M Γ M
BCB
SSB
BVB
Binding Energy (meV)
0
200
400
E_F
E_B
E_C
E_D
E_A
-0.4 -0.2 0 0.2 0.4
k (Å^-1)
Slide Layout
Apply slide layout:
Text Layouts
Content Layouts
Click to add notes
Show when inserting new slides

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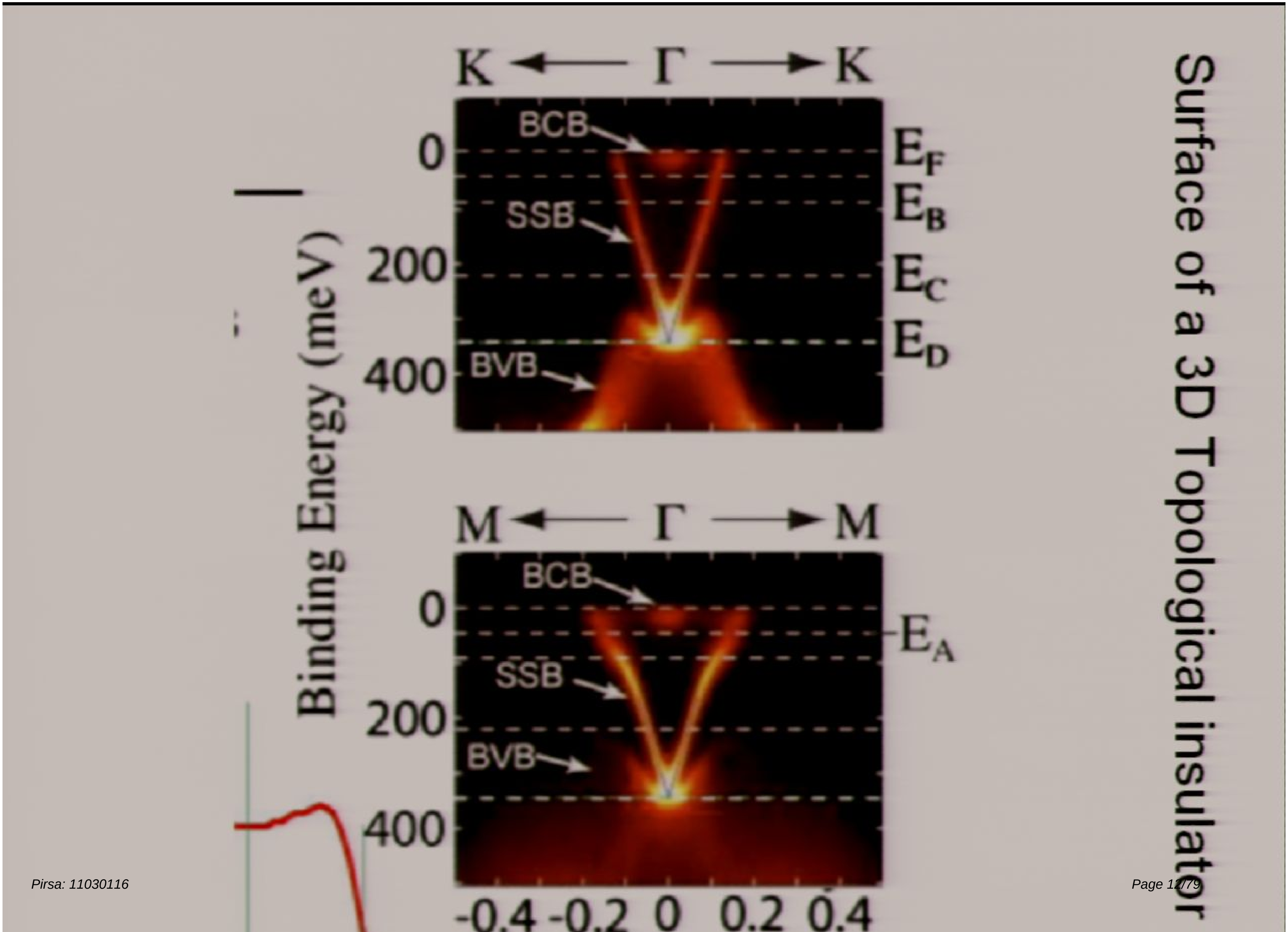
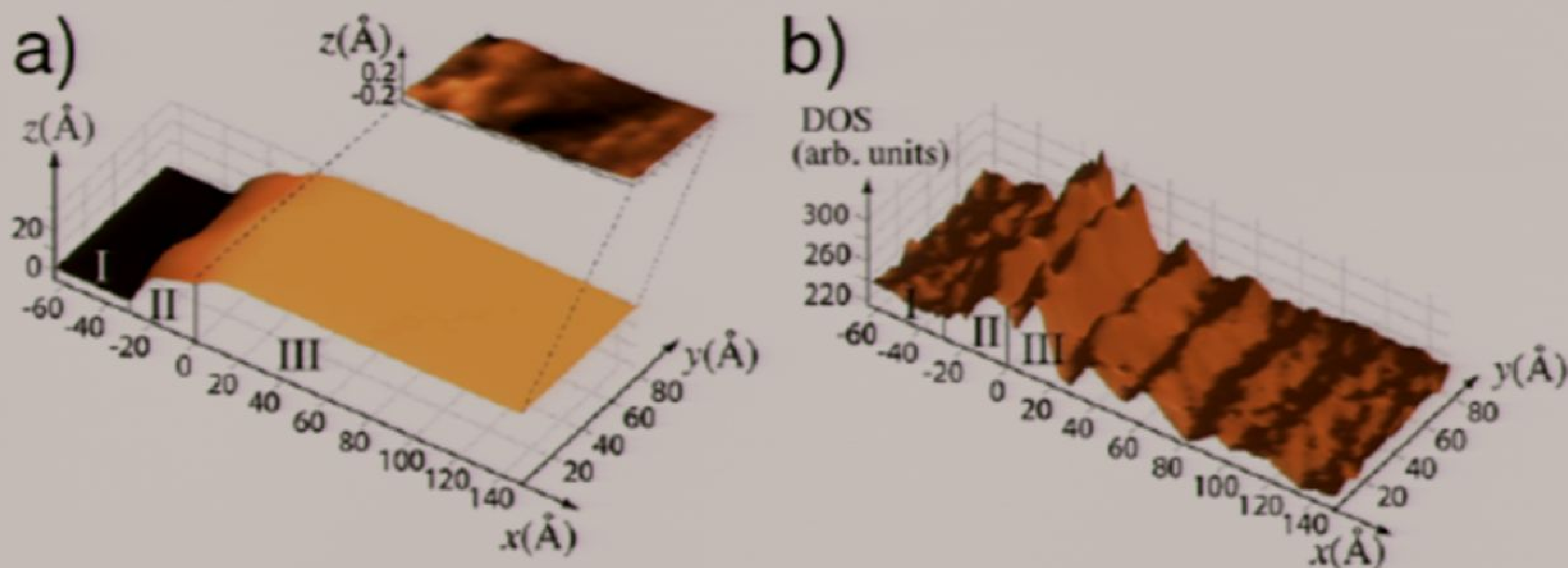
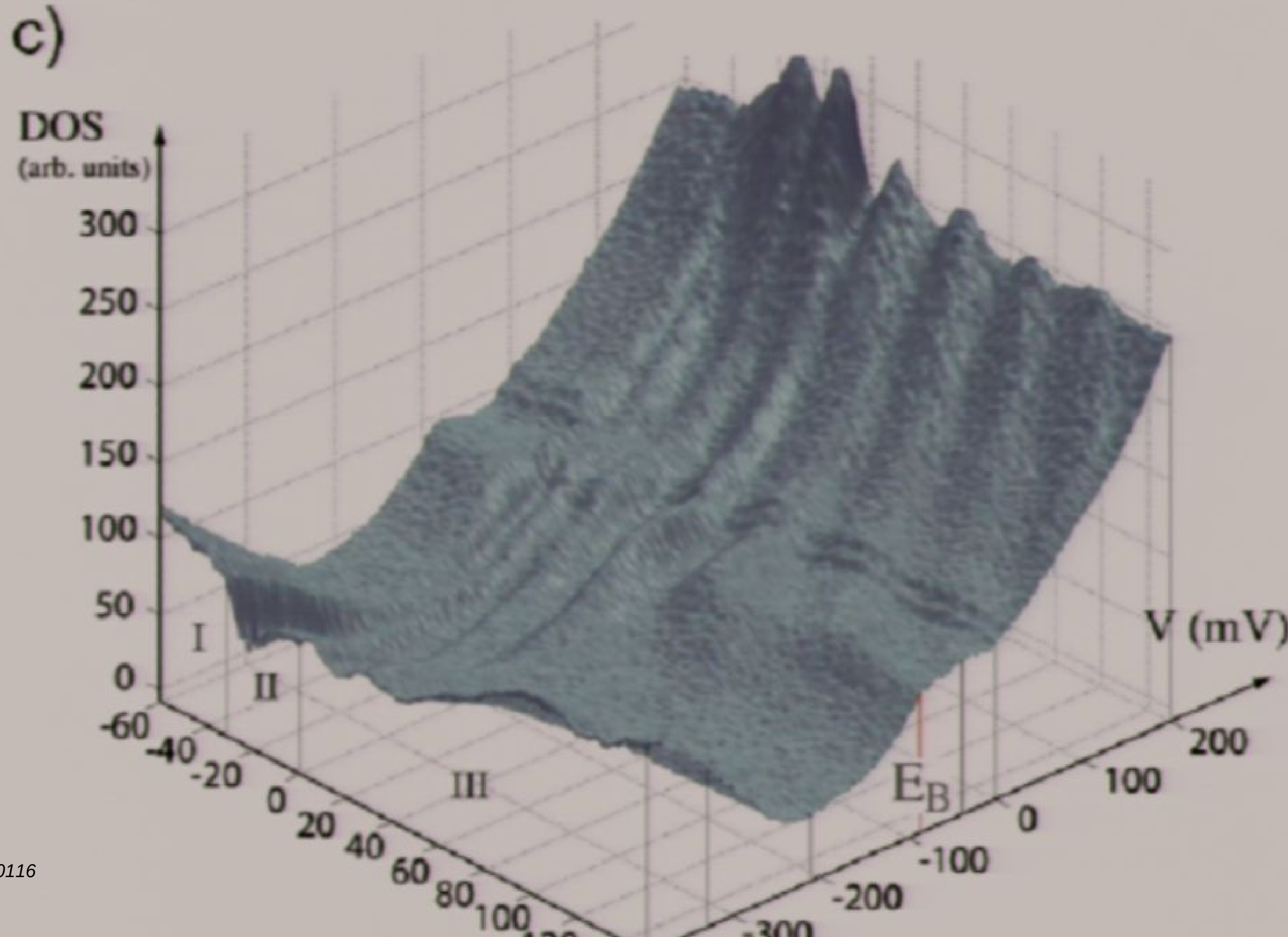

Surface of a 3D Topological insulator
K ← Γ → K
BCB
SSB
BVB
E_F
E_B
E_C
E_D
M ← Γ → M
BCB
SSB
BVB
E_A
Binding Energy (meV)
0
200
400
-0.4 -0.2 0 0.2 0.4

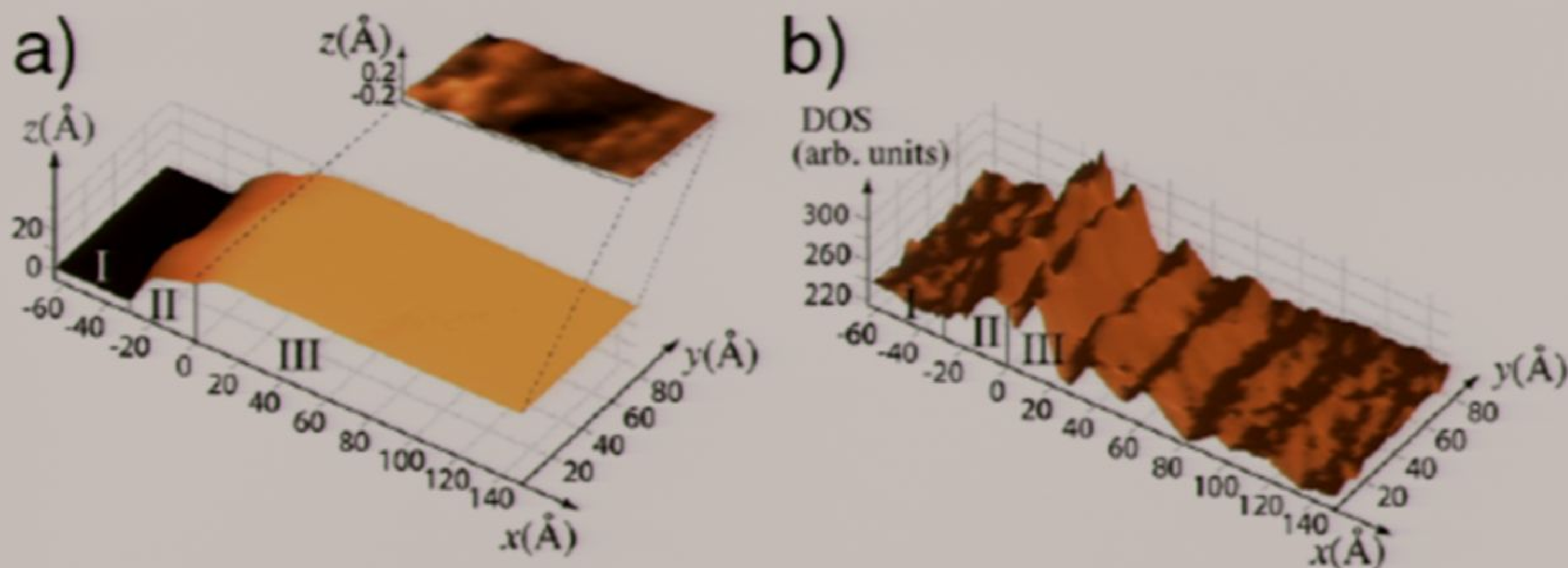
LDOS oscillations near a step



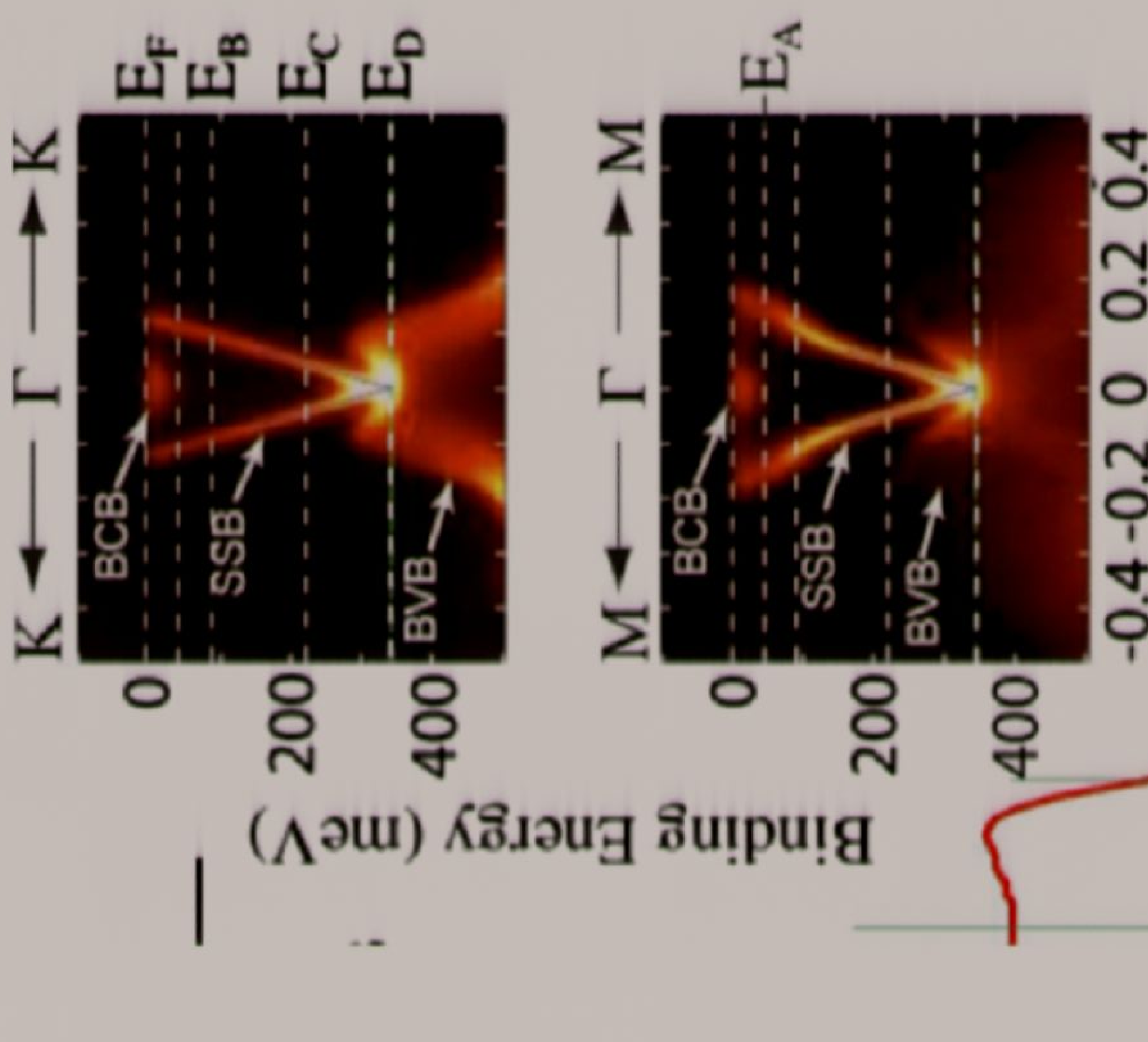
LDOS oscillations near a step



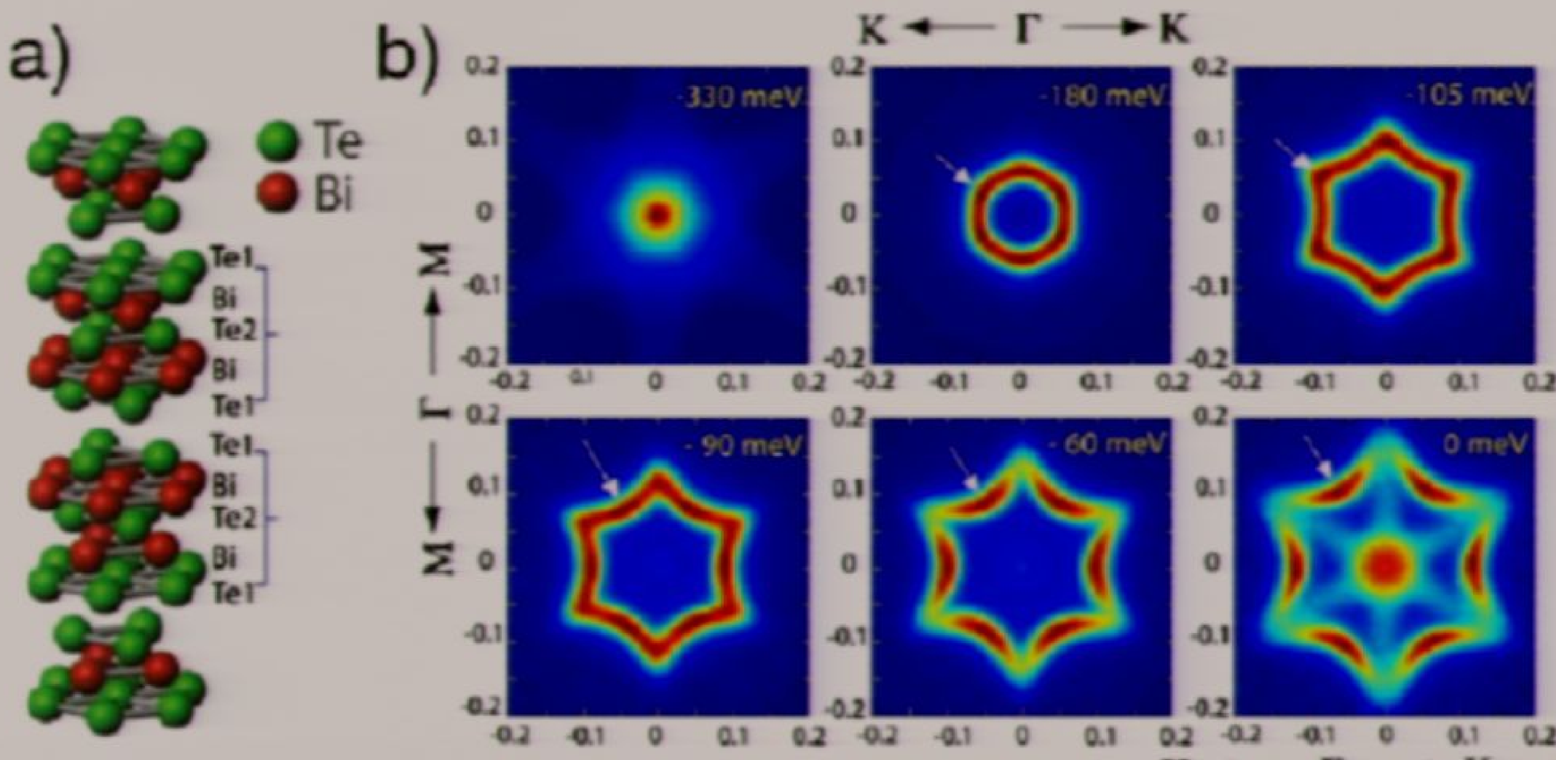
LDOS oscillations near a step



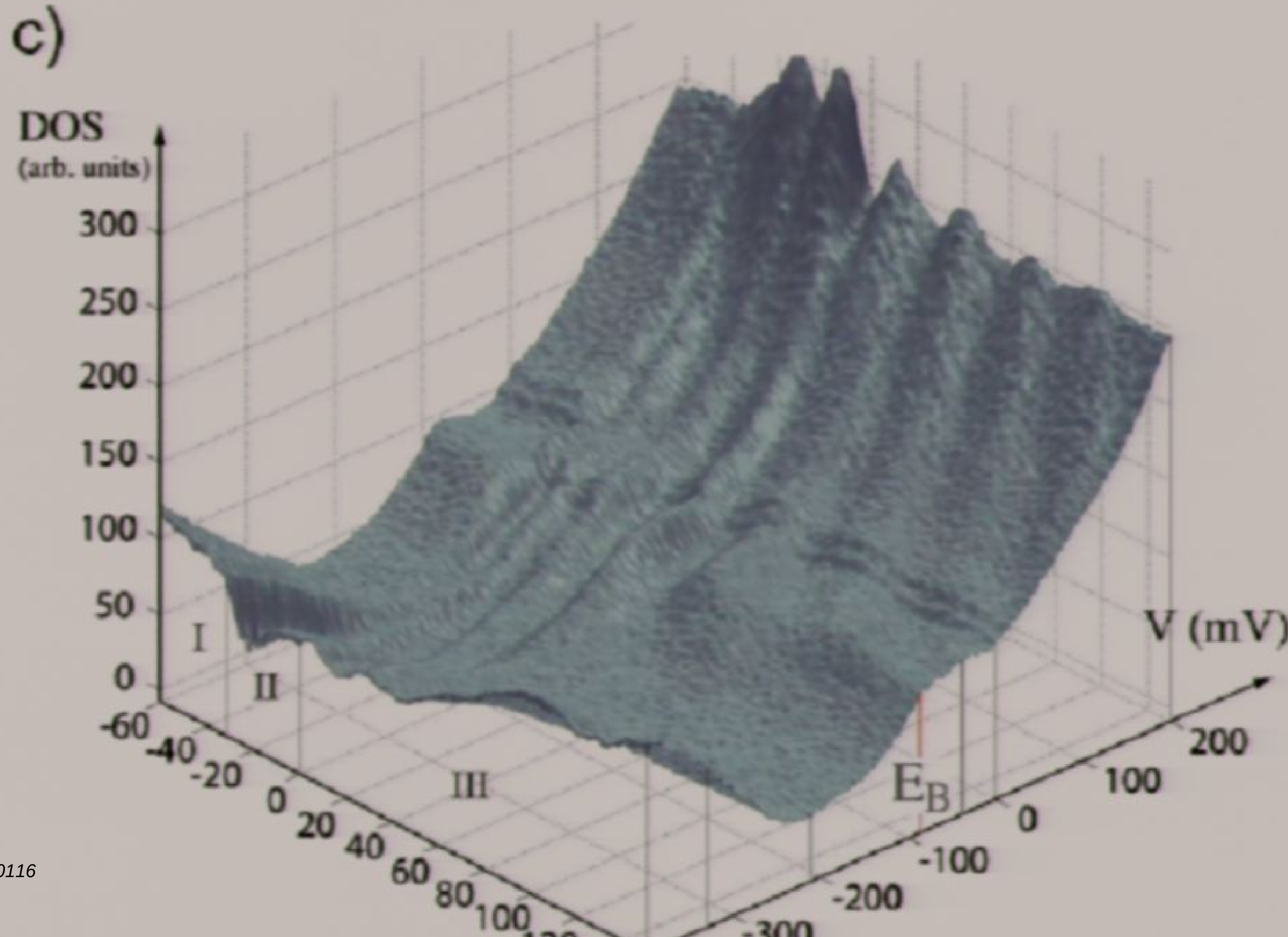
Surface of a 3D Topological insulator



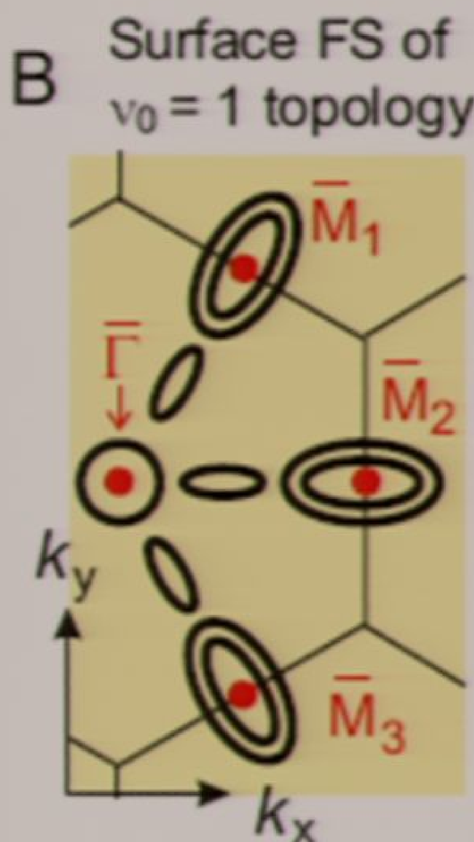
Surface of a 3D Topological insulator
Edge mode dispersion

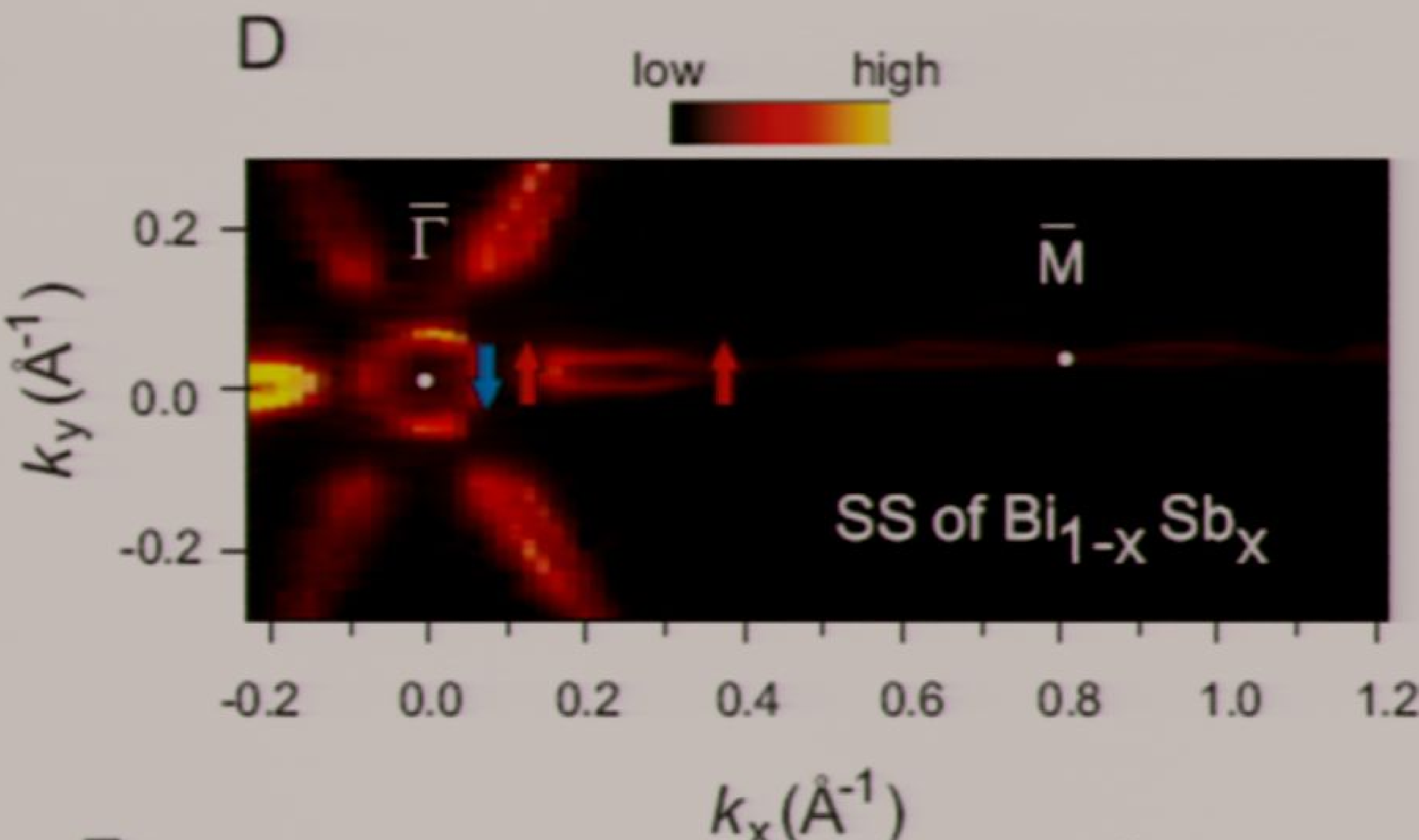


LDOS oscillations near a step

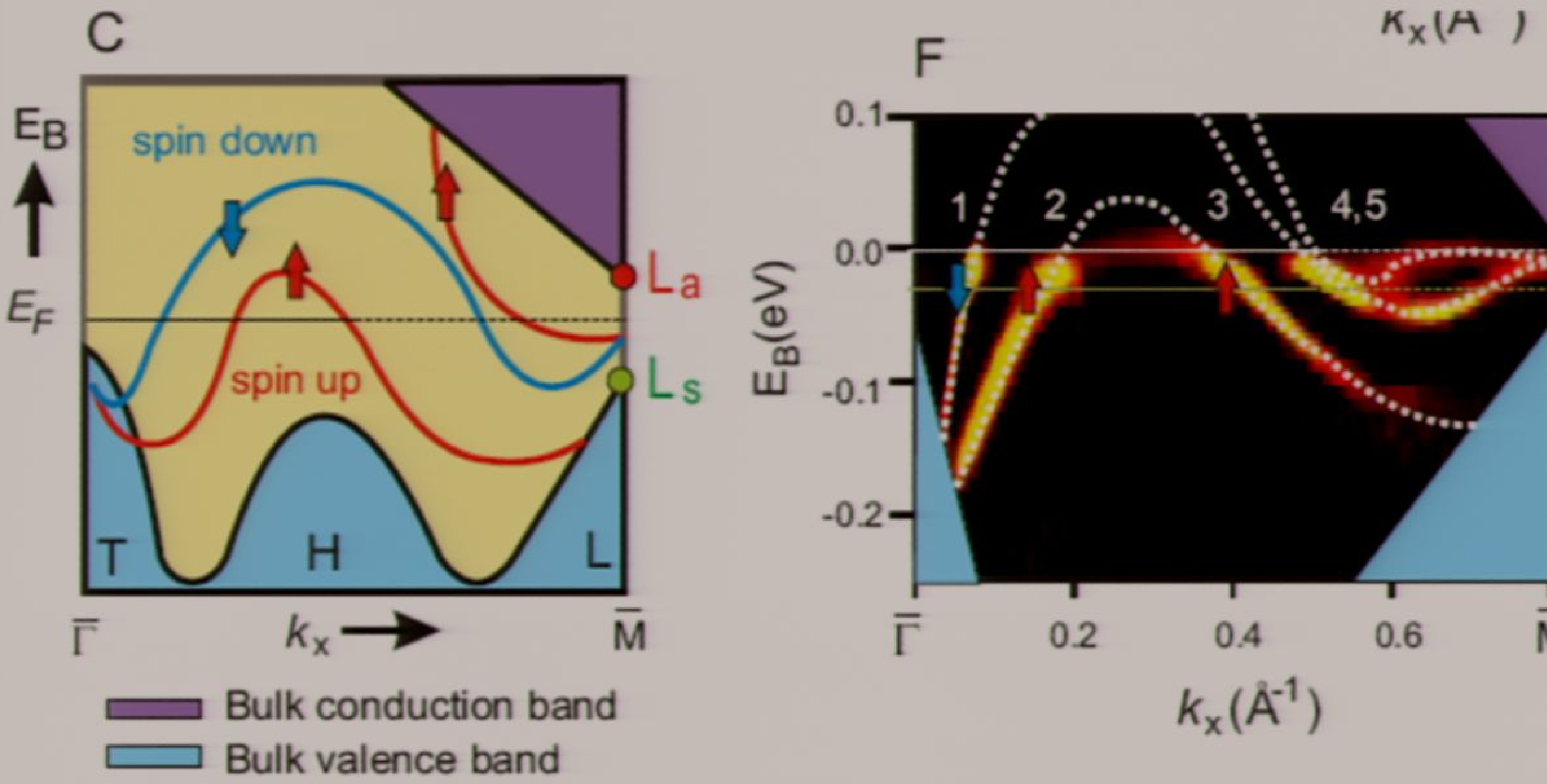


Spin ARPES


B
Surface FS of
ν0 = 1 topology
M1
Γ
M2
M3
ky
kx


D
low
high
Γ
M
SS of Bi1-x Sbx
0.2
0.0
-0.2
ky(Å-1)
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
kx(Å-1)

Edge dispersion



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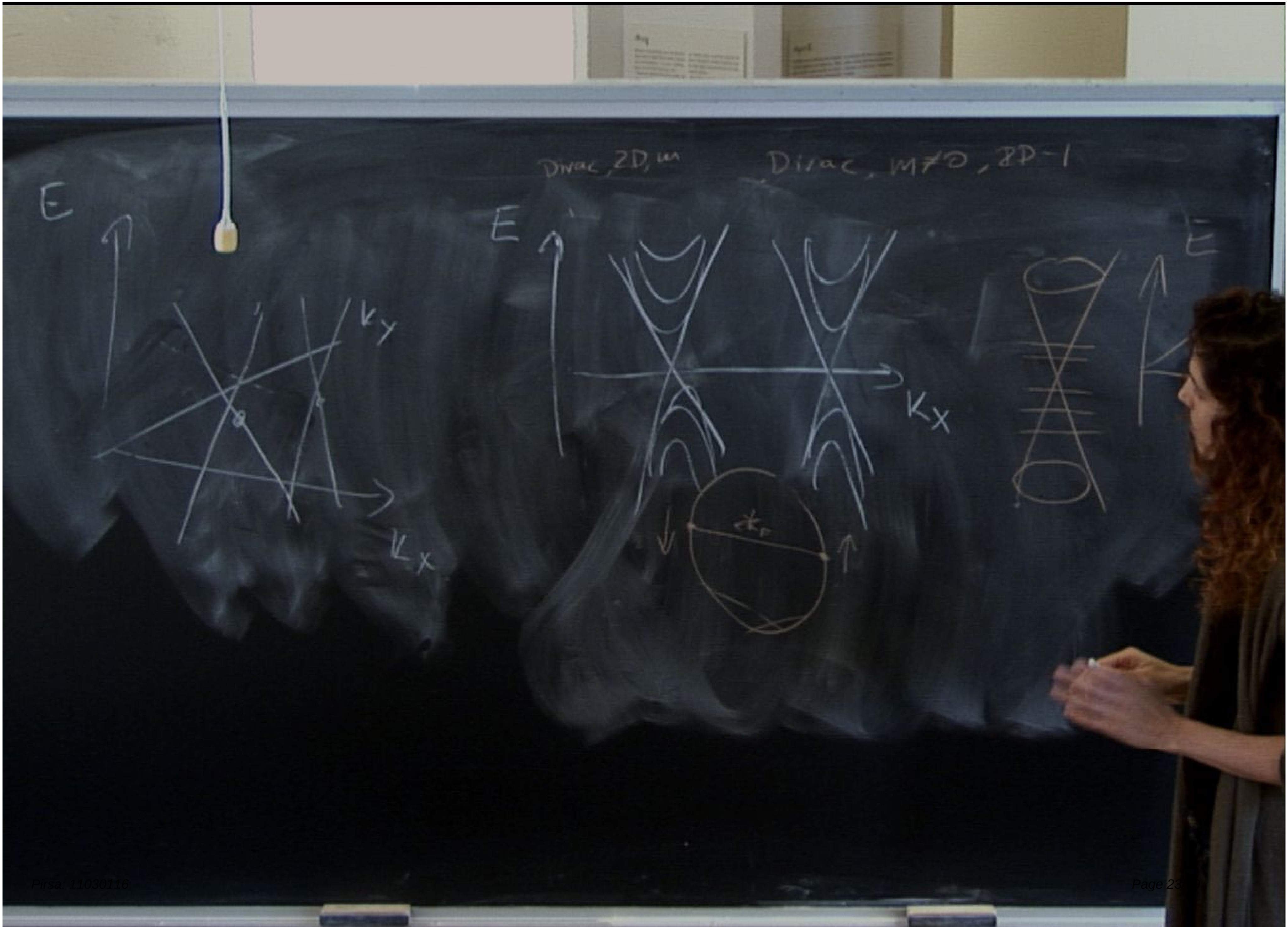
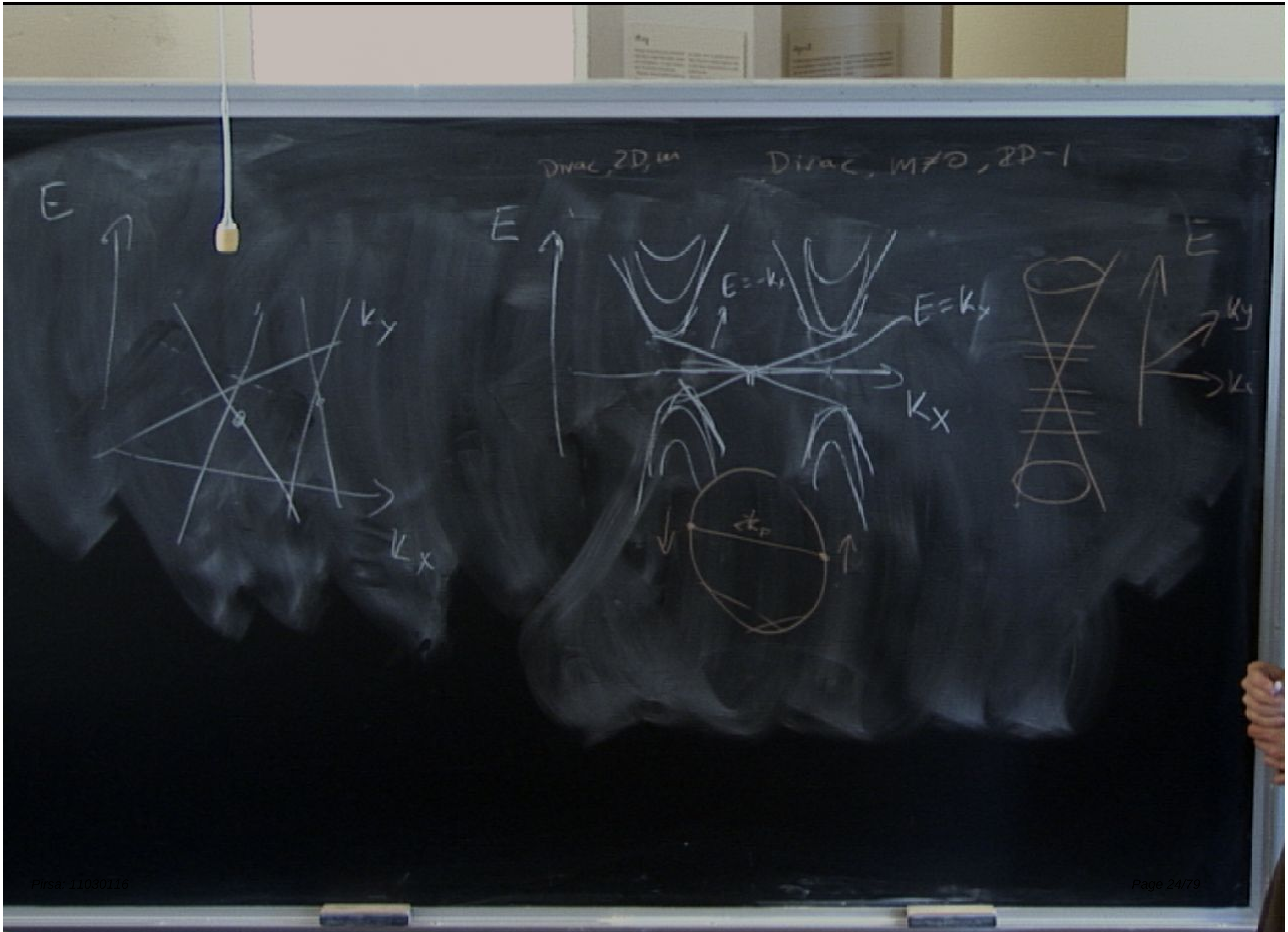
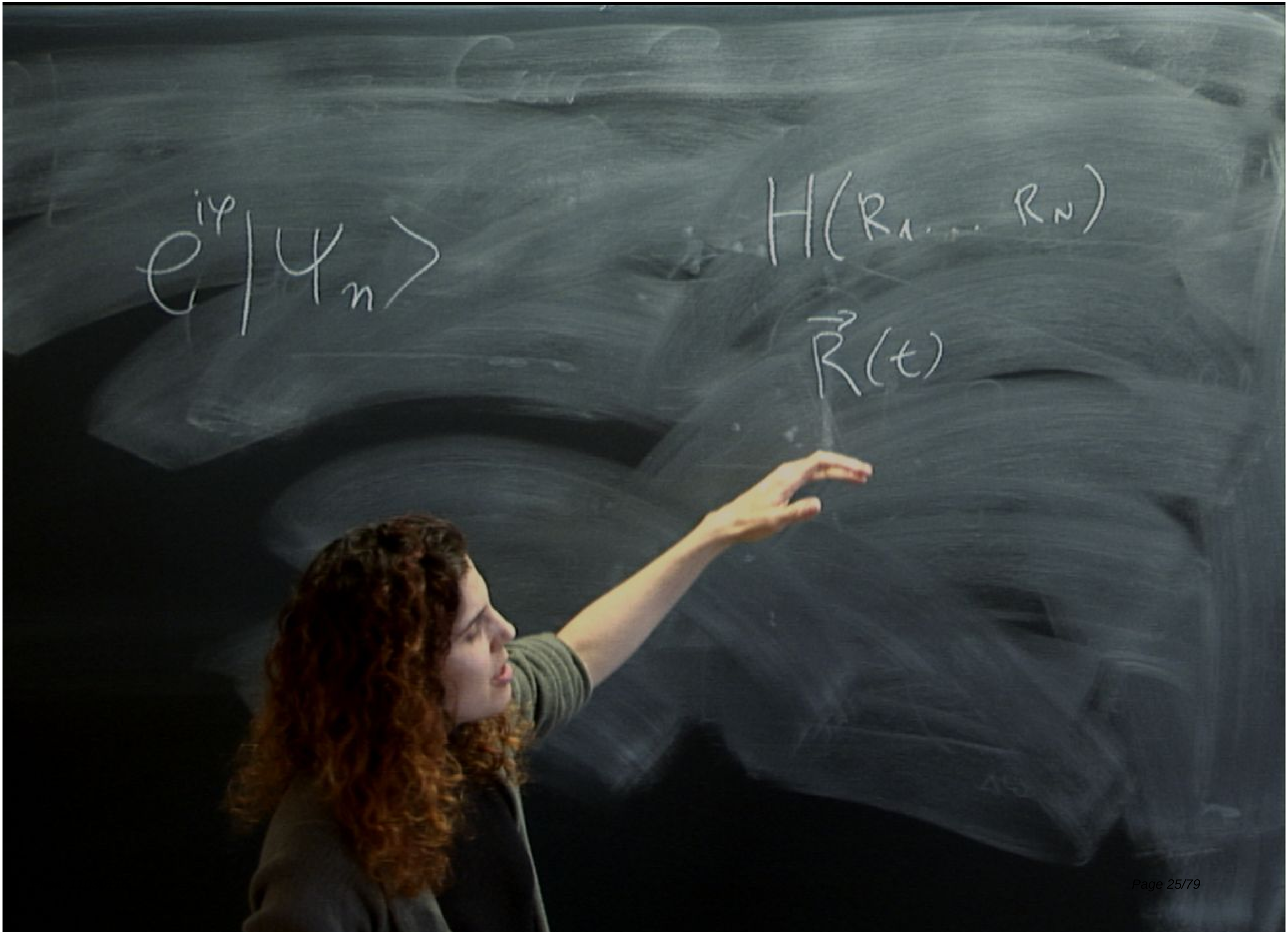
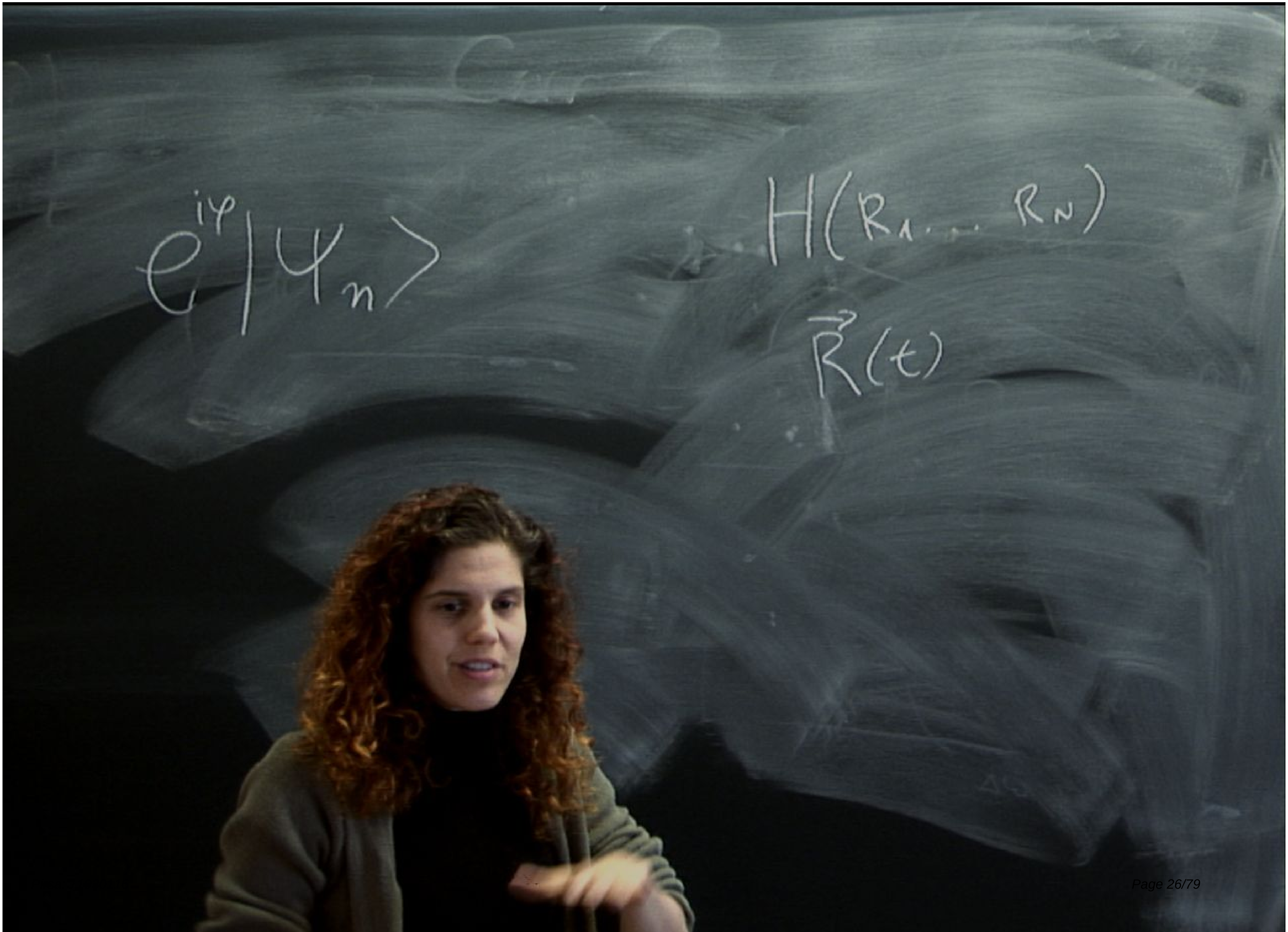
Dirac, 2D, m
Dirac, M≠0, 2D-1
E
ky
kx
E
ky
kx

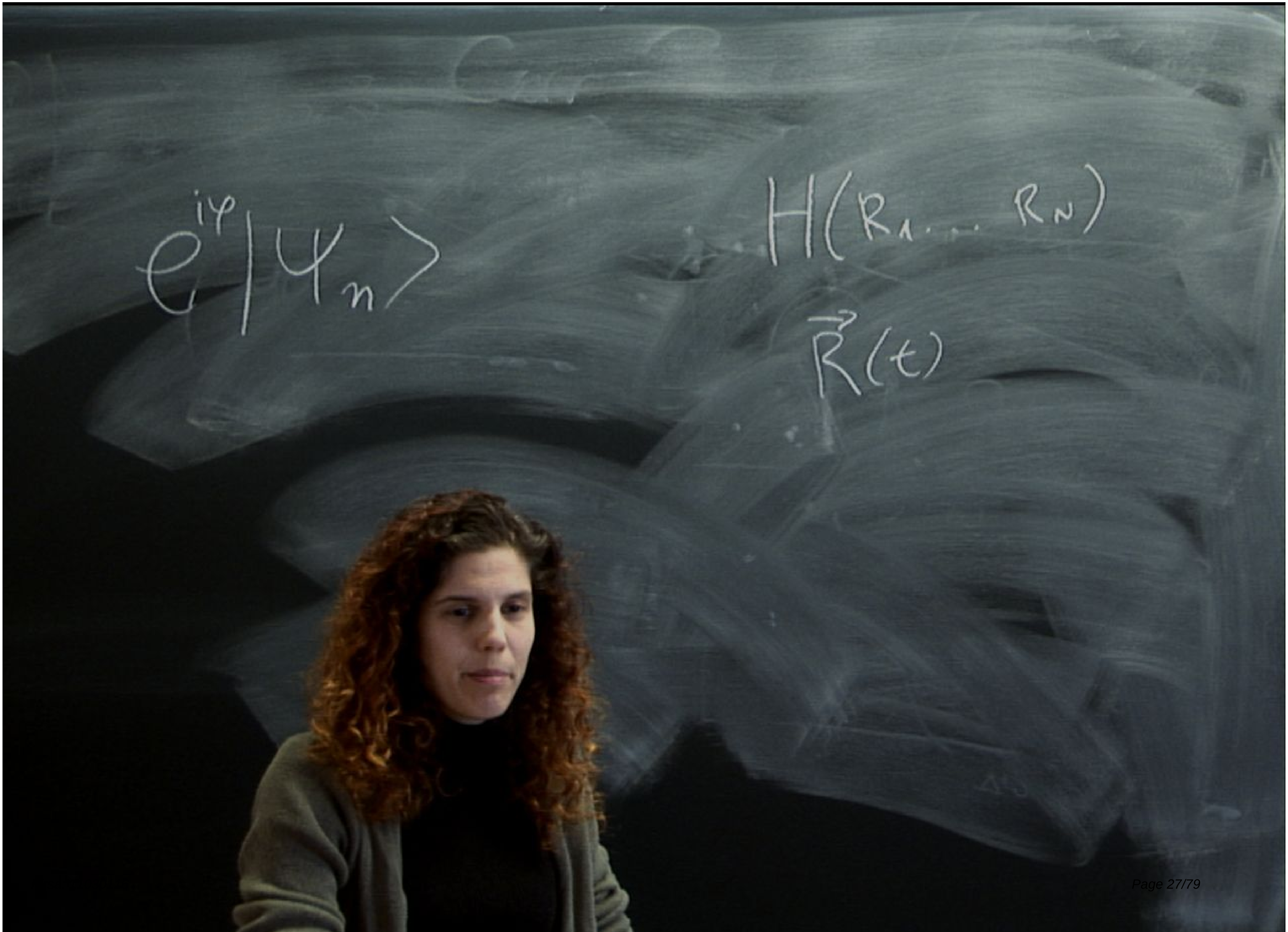

Dirac, 2D, m
Dirac, m≠0, 2D-1
E
k_y
k_x
E
k_x
E
k_F

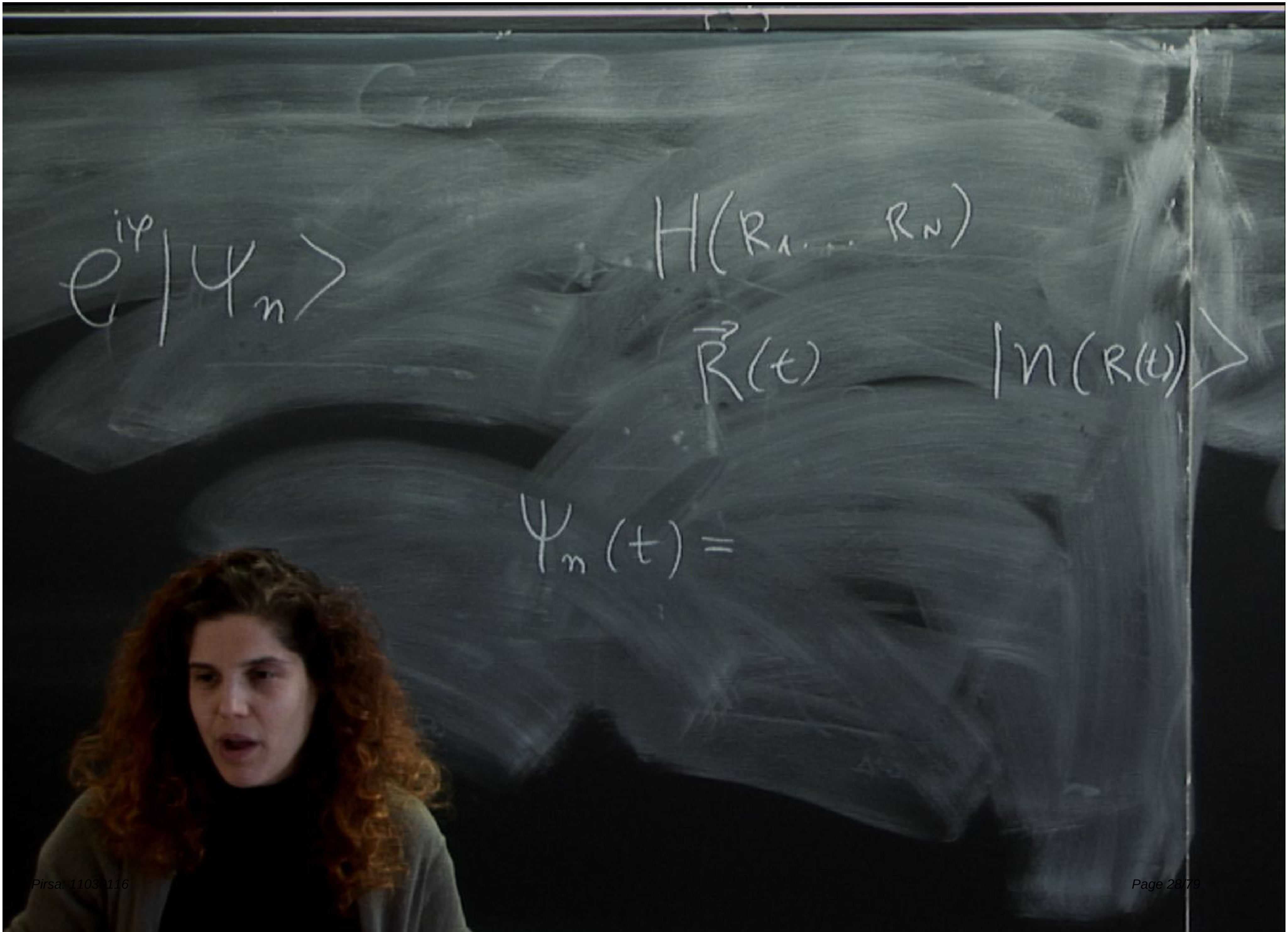

Dirac, 2D, m
Dirac, M≠0, 2D-1
E
ky
kx
E
E=-kx
E=ky
Kx
kF
E
ky
kx



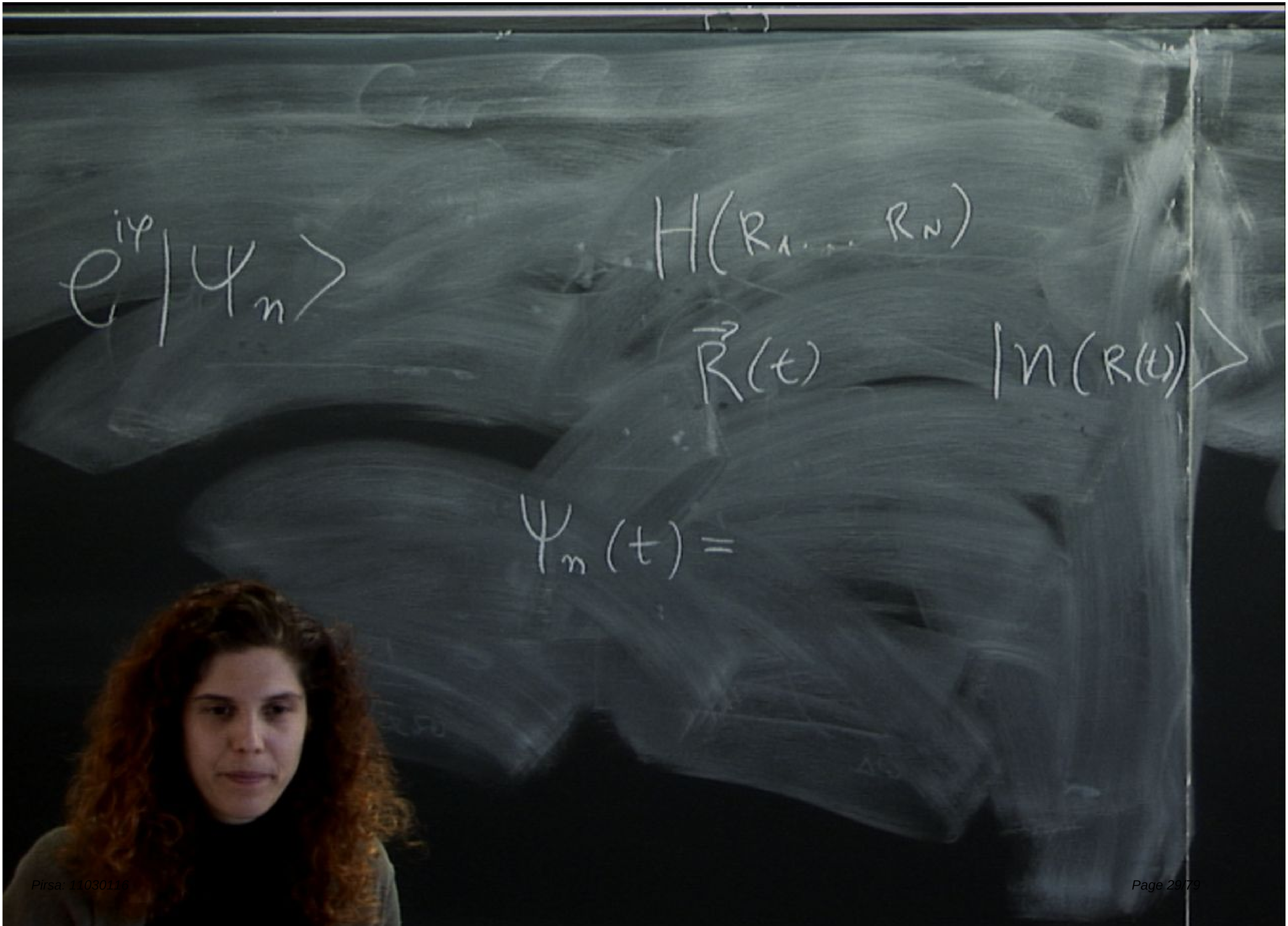
$e^{i\varphi}|\psi_n\rangle$
$H(R_1, \ldots R_N)$
$\vec{R}(t)$

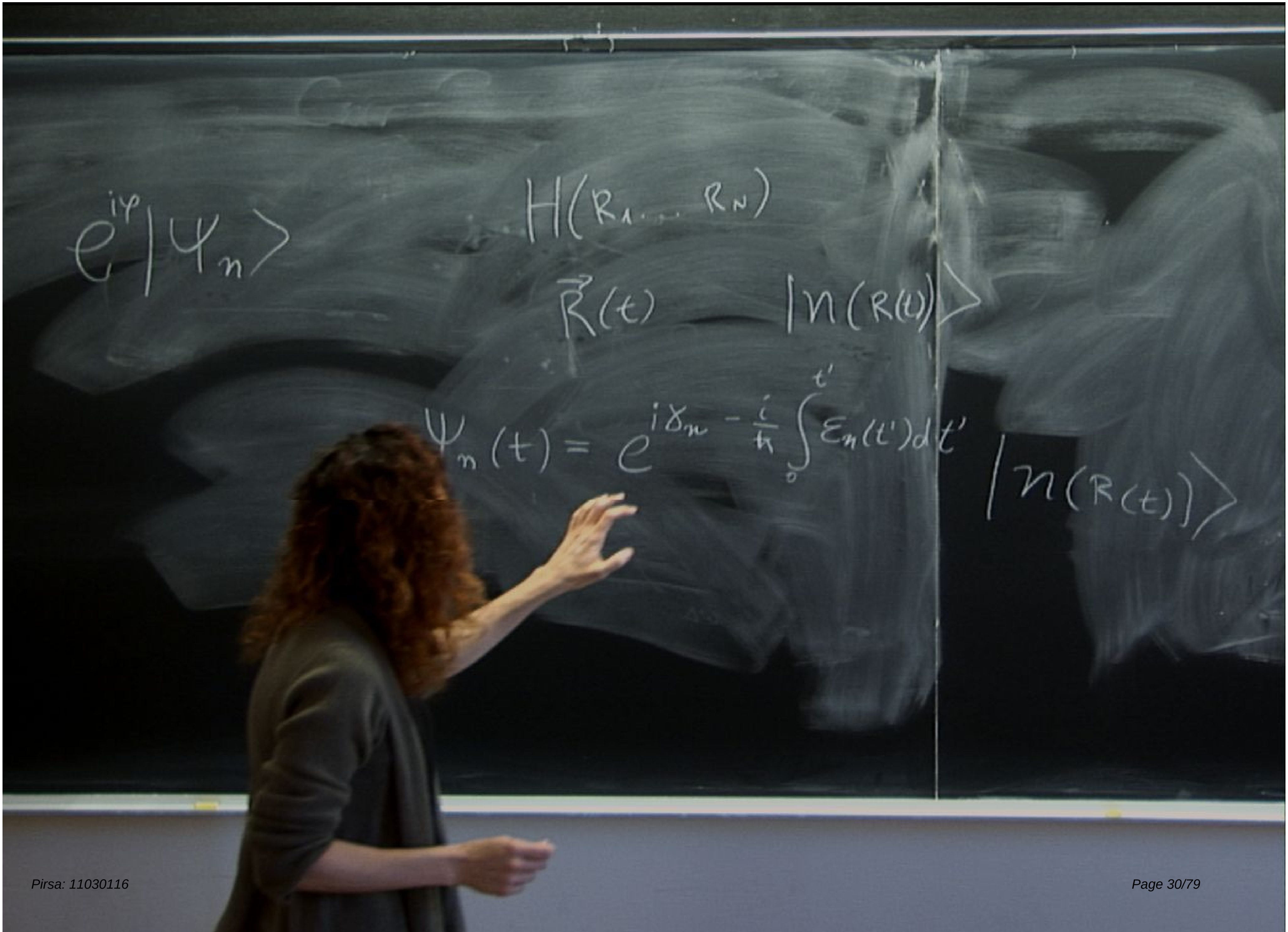

$e^{i\varphi}|\psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$

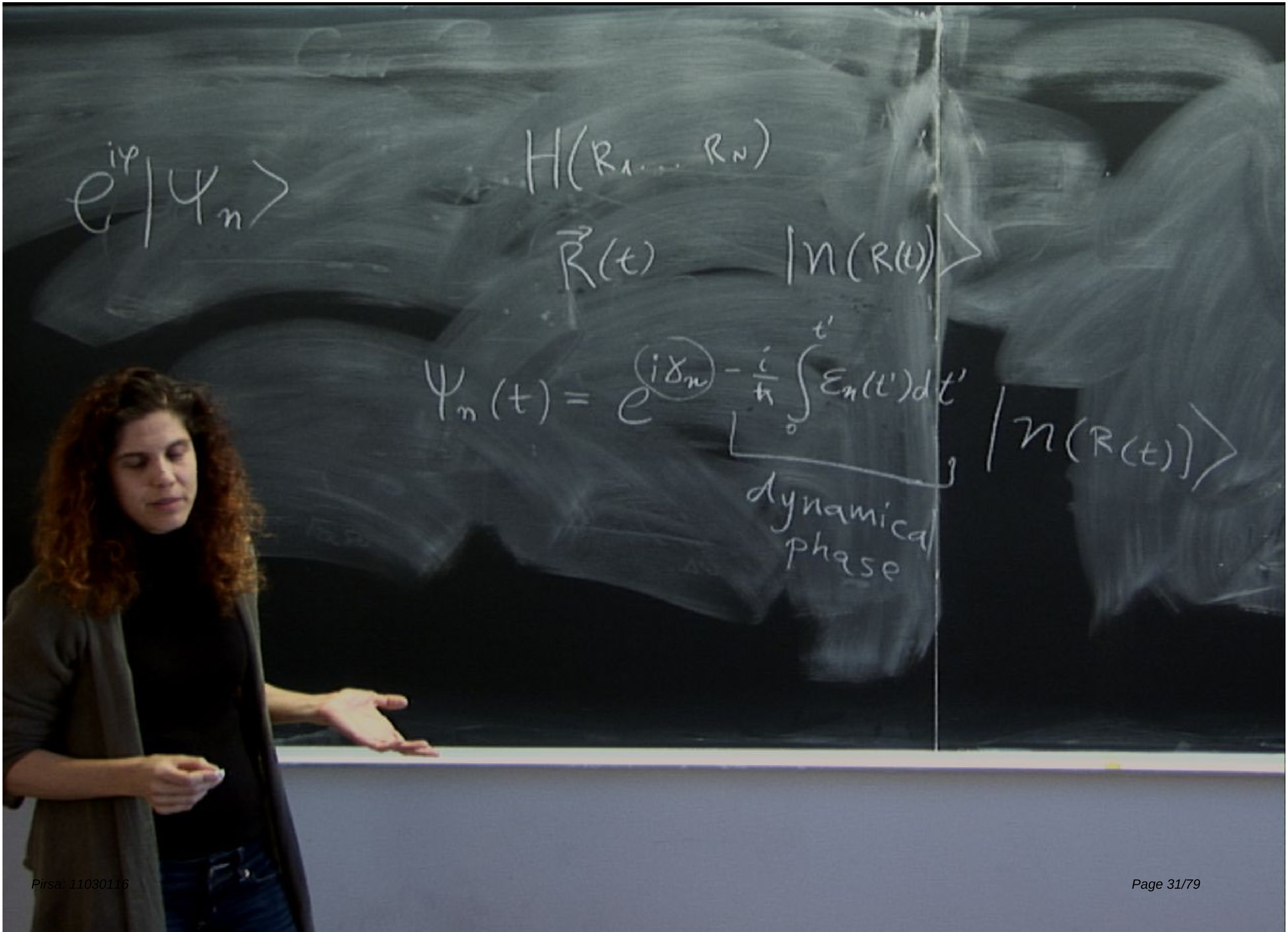

$e^{i\varphi}|\psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$

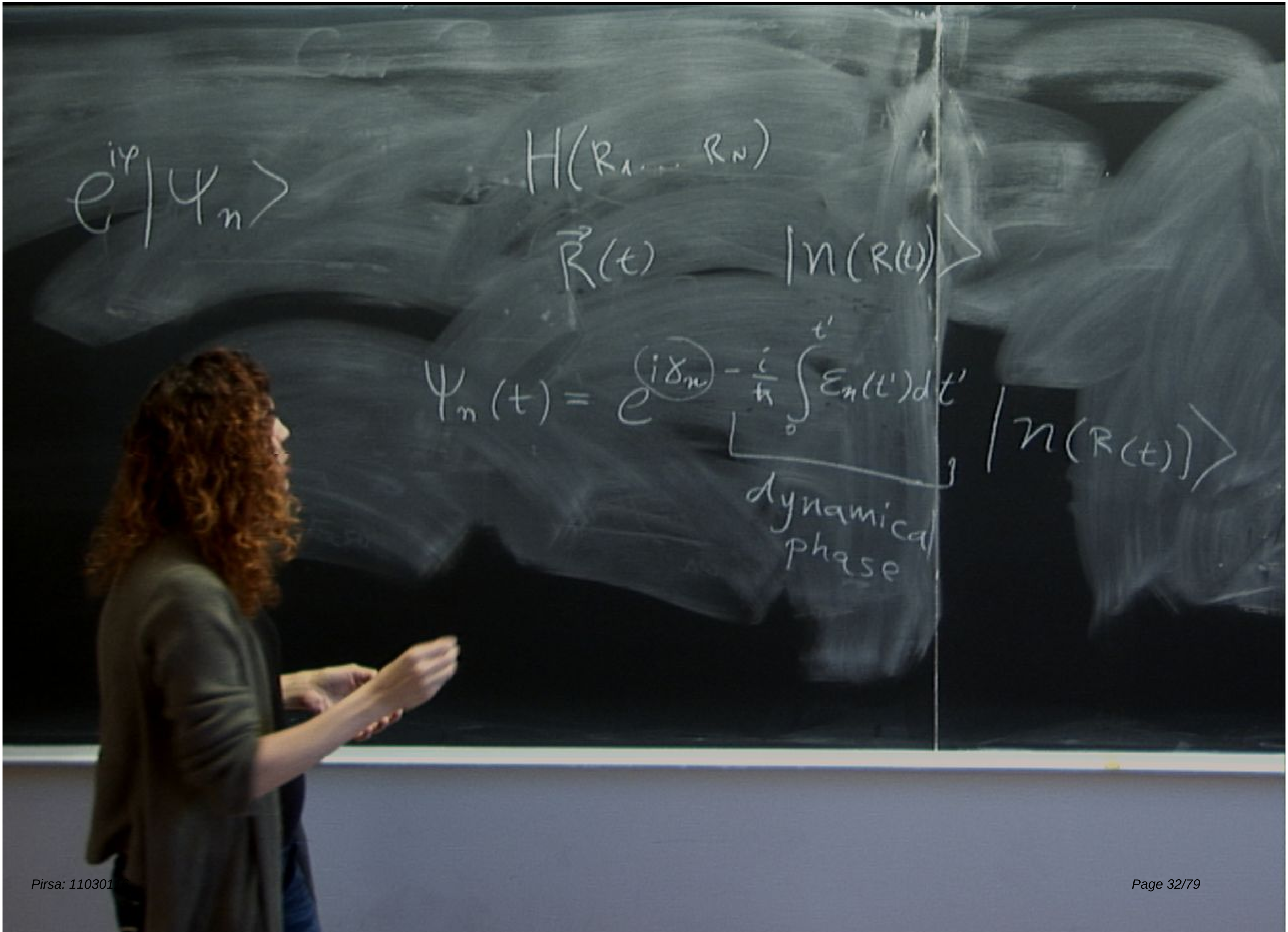


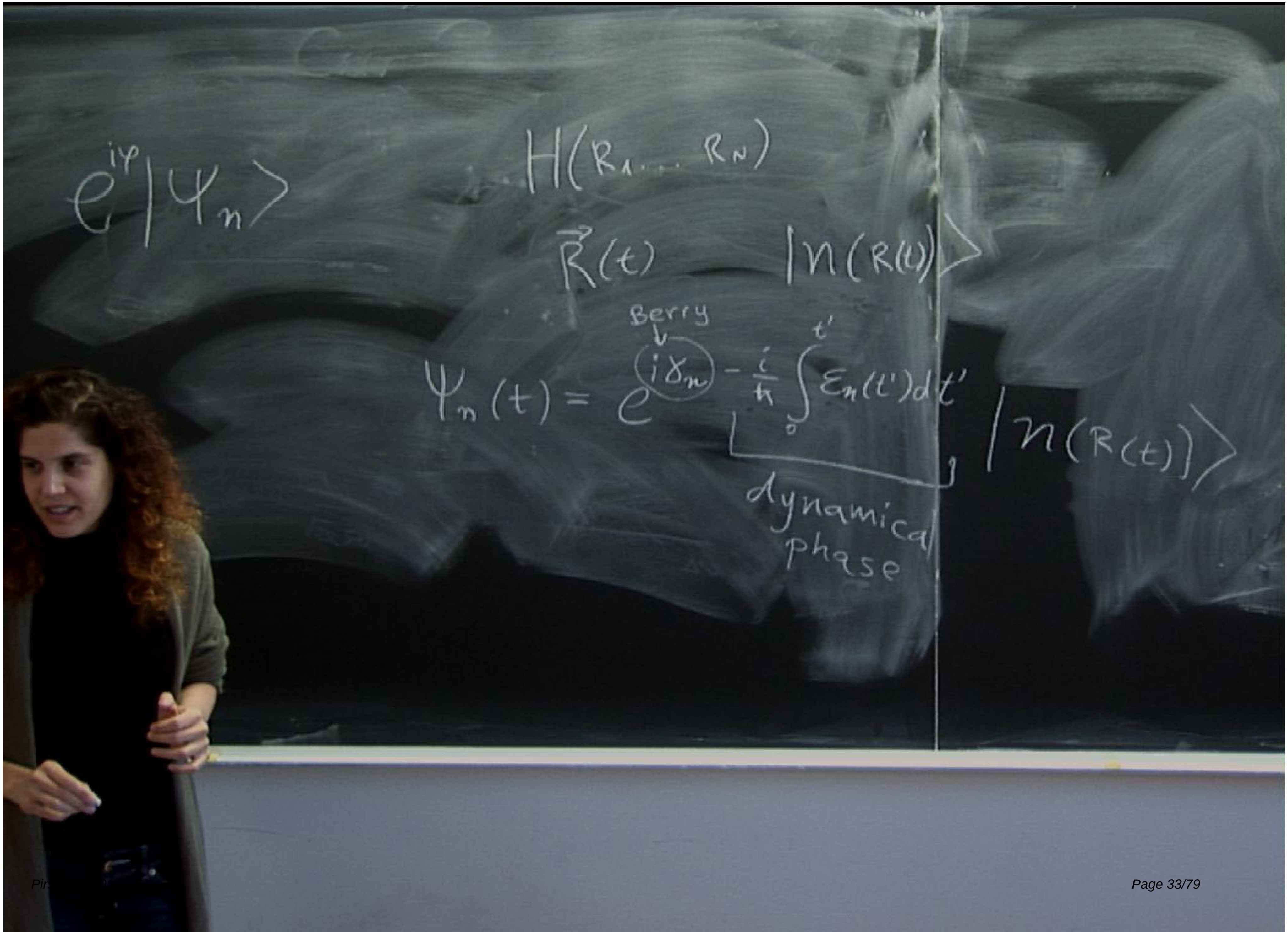
$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
$\Psi_n(t) =$


$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
$\Psi_n(t) =$

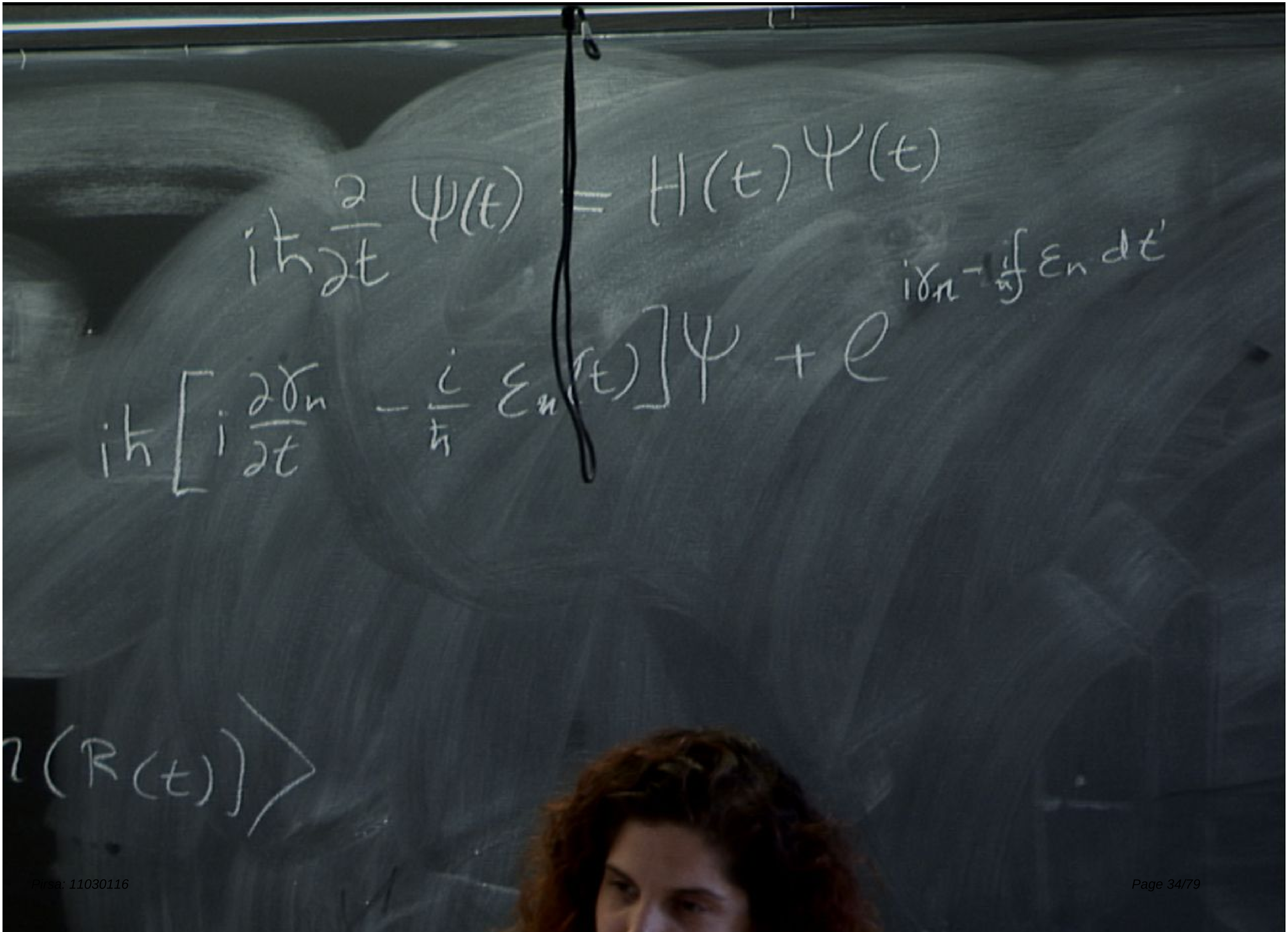

$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
$\Psi_n(t) = e^{i\gamma_n - \frac{i}{\hbar}\int_0^{t'} \varepsilon_n(t')dt'} |n(R(t))\rangle$

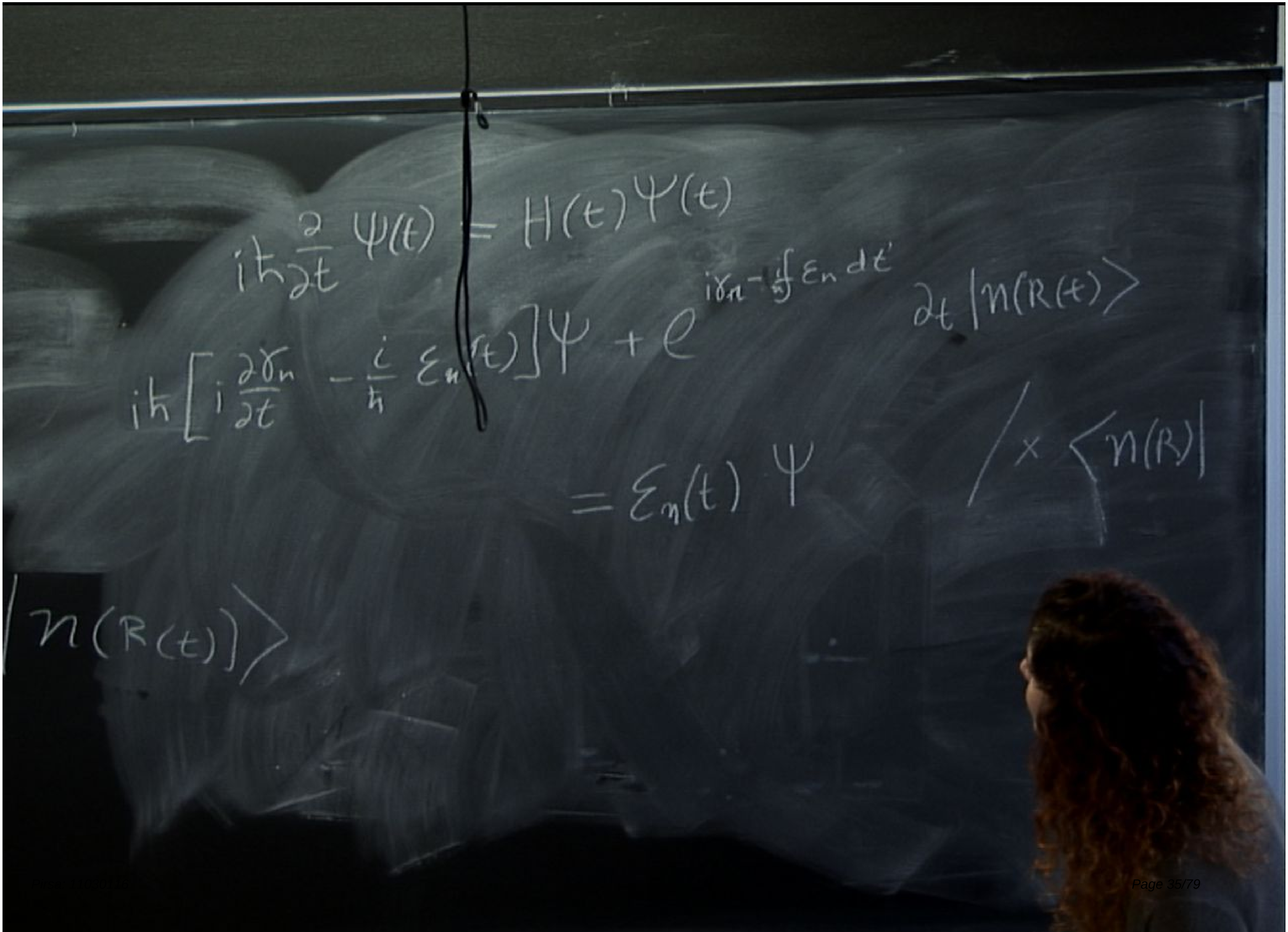

$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
$\Psi_n(t) = e^{i\gamma_n - \frac{i}{\hbar}\int_0^{t'} \varepsilon_n(t')dt'} |n(R(t))\rangle$
dynamical phase


$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \dots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
$\Psi_n(t) = e^{i\gamma_n - \frac{i}{\hbar}\int_0^{t'} \varepsilon_n(t')dt'} |n(R(t))\rangle$
dynamical phase



$e^{i\varphi}|\Psi_n\rangle$
$H(R_1 \ldots R_N)$
$\vec{R}(t)$
$|n(R(t))\rangle$
Berry
$\Psi_n(t) = e^{i\gamma_n - \frac{i}{\hbar}\int_0^{t'} \varepsilon_n(t')dt'}$
$|n(R(t))\rangle$
dynamical phase


$i\hbar \frac{\partial}{\partial t} \psi(t) = H(t)\Psi(t)$
$i\hbar \left[i \frac{\partial \gamma_n}{\partial t} - \frac{i}{\hbar} \varepsilon_n(t) \right] \Psi + e^{i\gamma_n - \frac{i}{\hbar}\int \varepsilon_n dt'}$
$n(R(t))\rangle$



$i\hbar \frac{\partial}{\partial t} \psi(t) = H(t)\Psi(t)$
$e^{i\gamma_n - \frac{i}{\hbar}\int \varepsilon_n dt'}$
$\partial_t |n(R(t))\rangle$
$i\hbar\left[i\frac{\partial \gamma_n}{\partial t} - \frac{i}{\hbar}\varepsilon_n(t)\right]\Psi + e$
$/ \times \langle n(R)|$
$= \varepsilon_n(t)\ \Psi$
$|n(R(t))\rangle$

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = H(t) \Psi(t)$$

$$i\hbar \left[i \frac{\partial \gamma_n}{\partial t} - \frac{i}{\hbar} \mathcal{E}_n(t) \right] \Psi + e^{i\gamma_n - \frac{i}{\hbar}\int \mathcal{E}_n dt'}$$

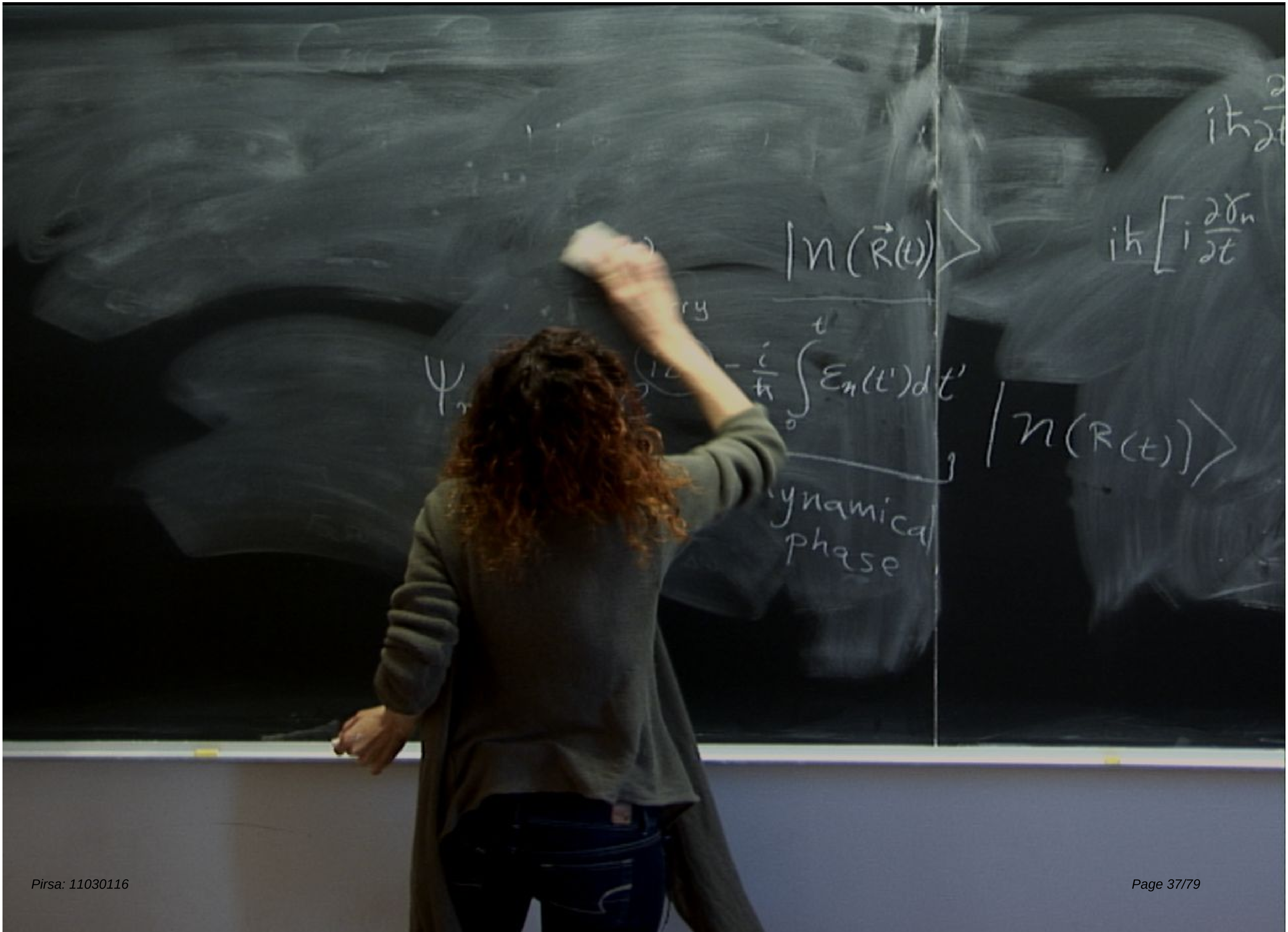
$$\partial_t | n(R(t)) \rangle \, i\hbar$$

$$= \mathcal{E}_n(t) \, \Psi$$

$$/ \times \langle n(R) |$$

$$n(R(t)) \rangle$$

$$i \frac{\partial \gamma_n}{\partial t} = \langle n(R(t)) | \partial_t | n(R(t)) \rangle$$


dynamical
phase

$$\gamma_n = i \int_0^t dt' \underbrace{\langle n(R) | \partial_{t'} | n(R) \rangle}_{\frac{\partial \vec{R}}{\partial t'} \cdot \langle n(R) | \vec{\nabla}_R | n(R) \rangle} =$$

$$= \int_C$$

$$\gamma_n = i \int_0^t dt' \, \underbrace{\langle n(R) | \partial_{t'} | n(R) \rangle}_{\frac{\partial \vec{R}}{\partial t'} \cdot \langle n(R) | \vec{\nabla}_R | n(R) \rangle} =$$

$$= \int_C d\vec{R} \cdot \underbrace{\langle n(R) | \vec{\nabla}_R | n(R) \rangle}$$

$$\gamma_n = i \int_0^t dt' \, \underbrace{\langle n(R) | \partial_{t'} | n(R) \rangle}_{\frac{\partial \vec{R}}{\partial t'} \cdot \langle n(R) | \vec{\nabla}_R | n(R) \rangle} =$$

$$= \int_C d\vec{R} \cdot \underbrace{\langle n(R) | \vec{\nabla}_R | n(R) \rangle}_{\vec{A}}$$

$\vec{A} \equiv$ Berry Connection

$$|n(R)\rangle \longrightarrow e^{i\xi(R)} |n(R)\rangle$$

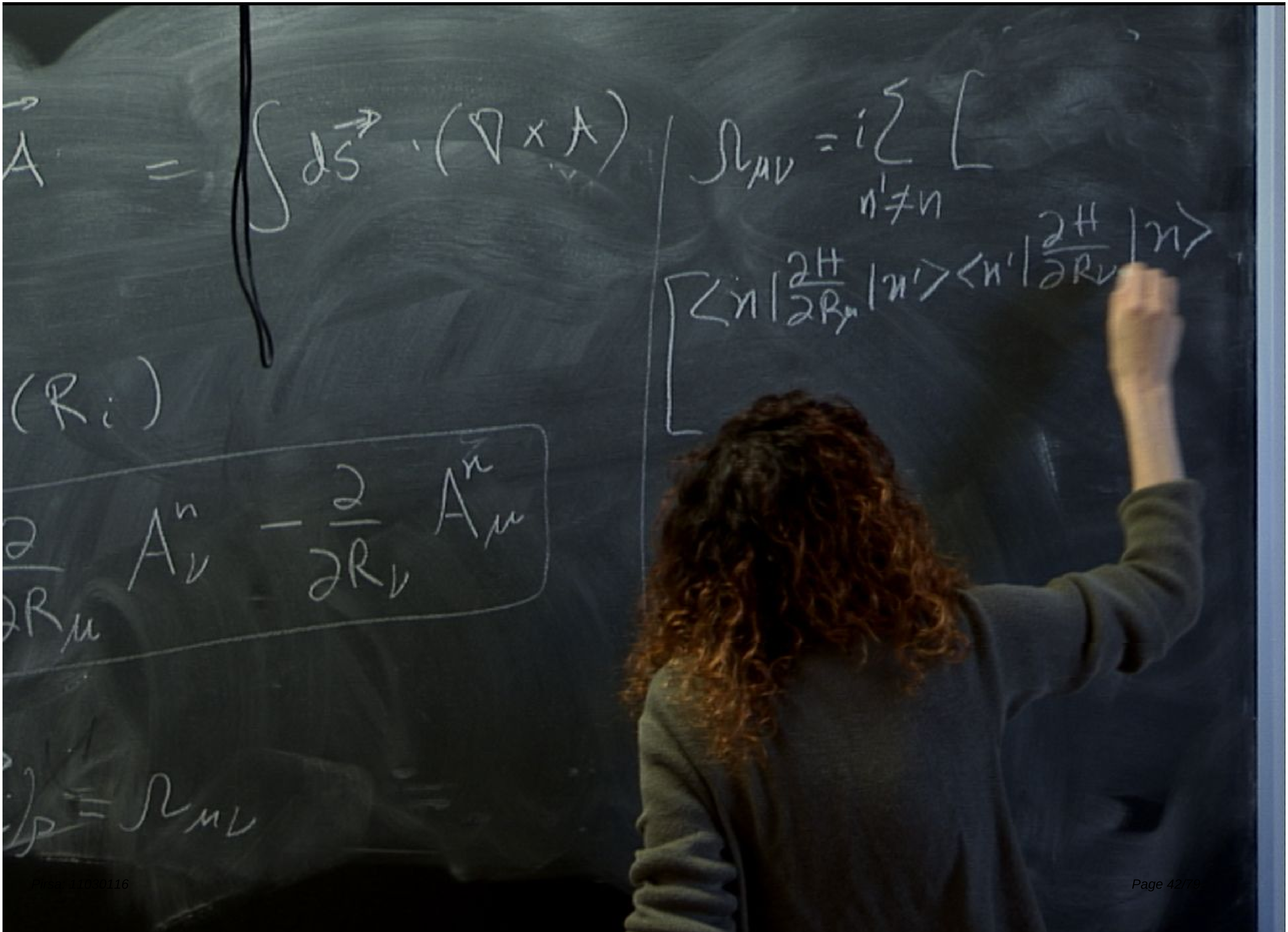
$$A \longrightarrow A + \vec{\nabla}\xi$$

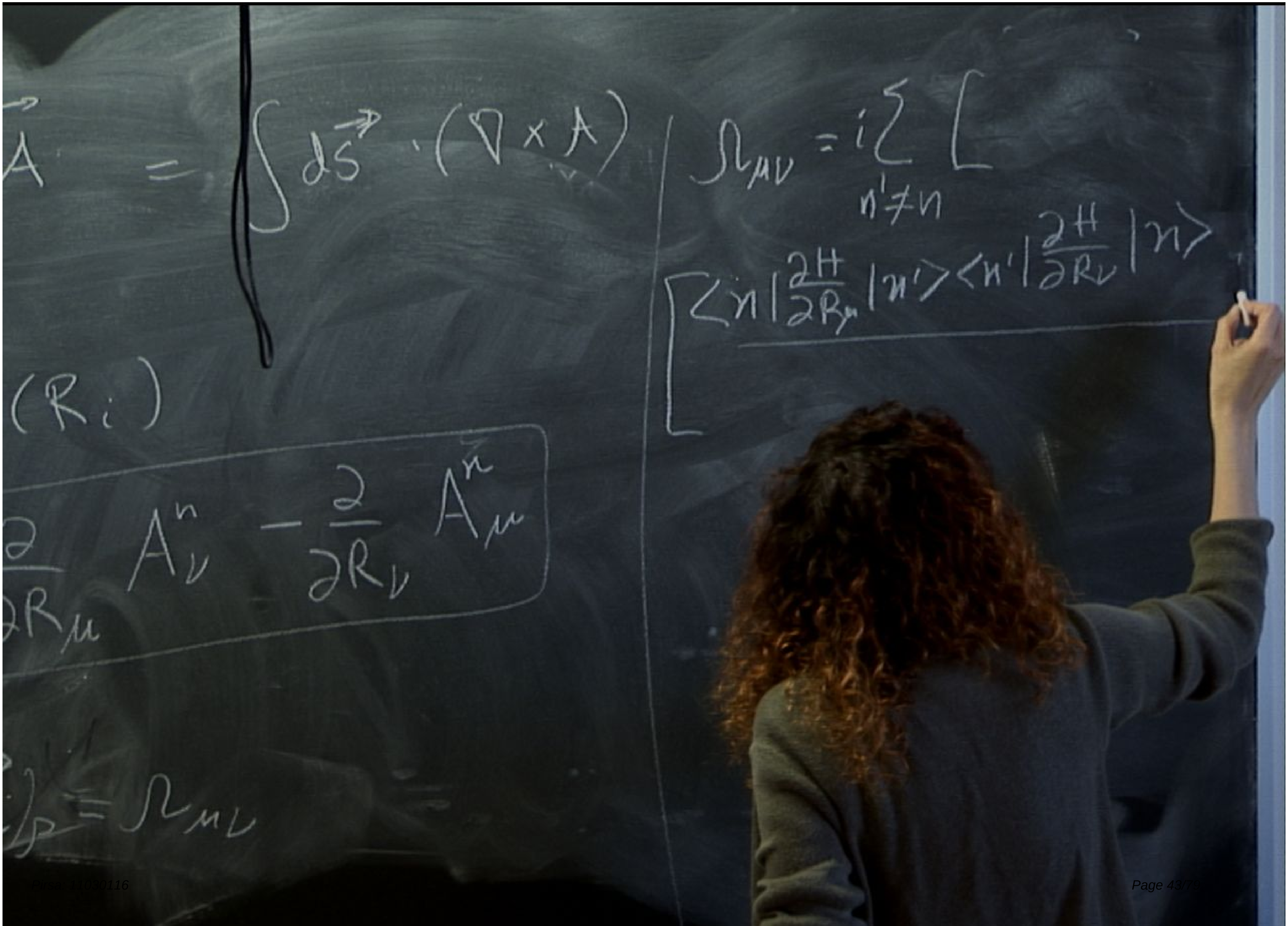
$$\gamma_n = \oint d\vec{R}\cdot\vec{A} = \int d\vec{S}\cdot(\nabla\times A)$$

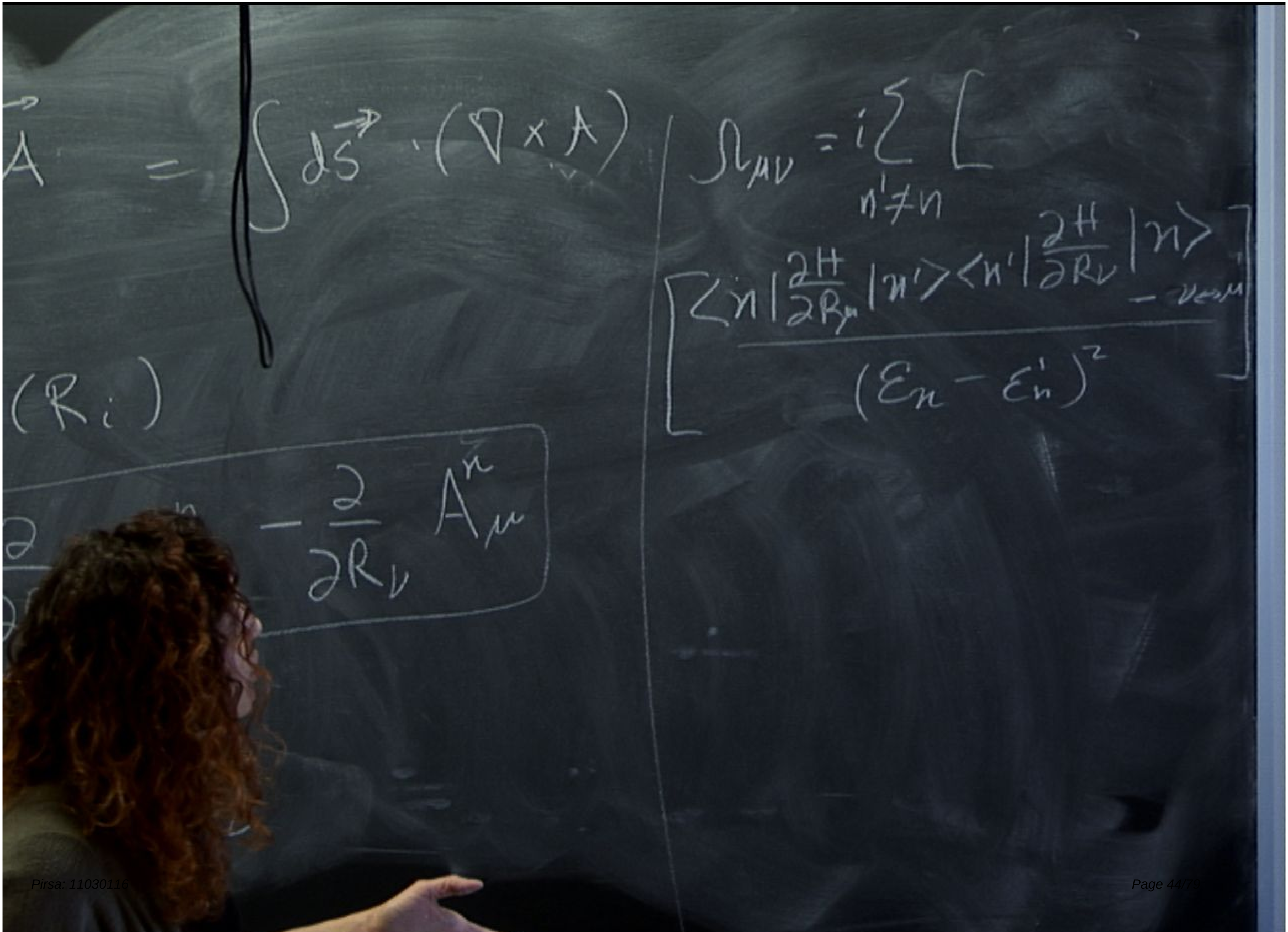
$$\xi(R_f) - \xi(R_i)$$

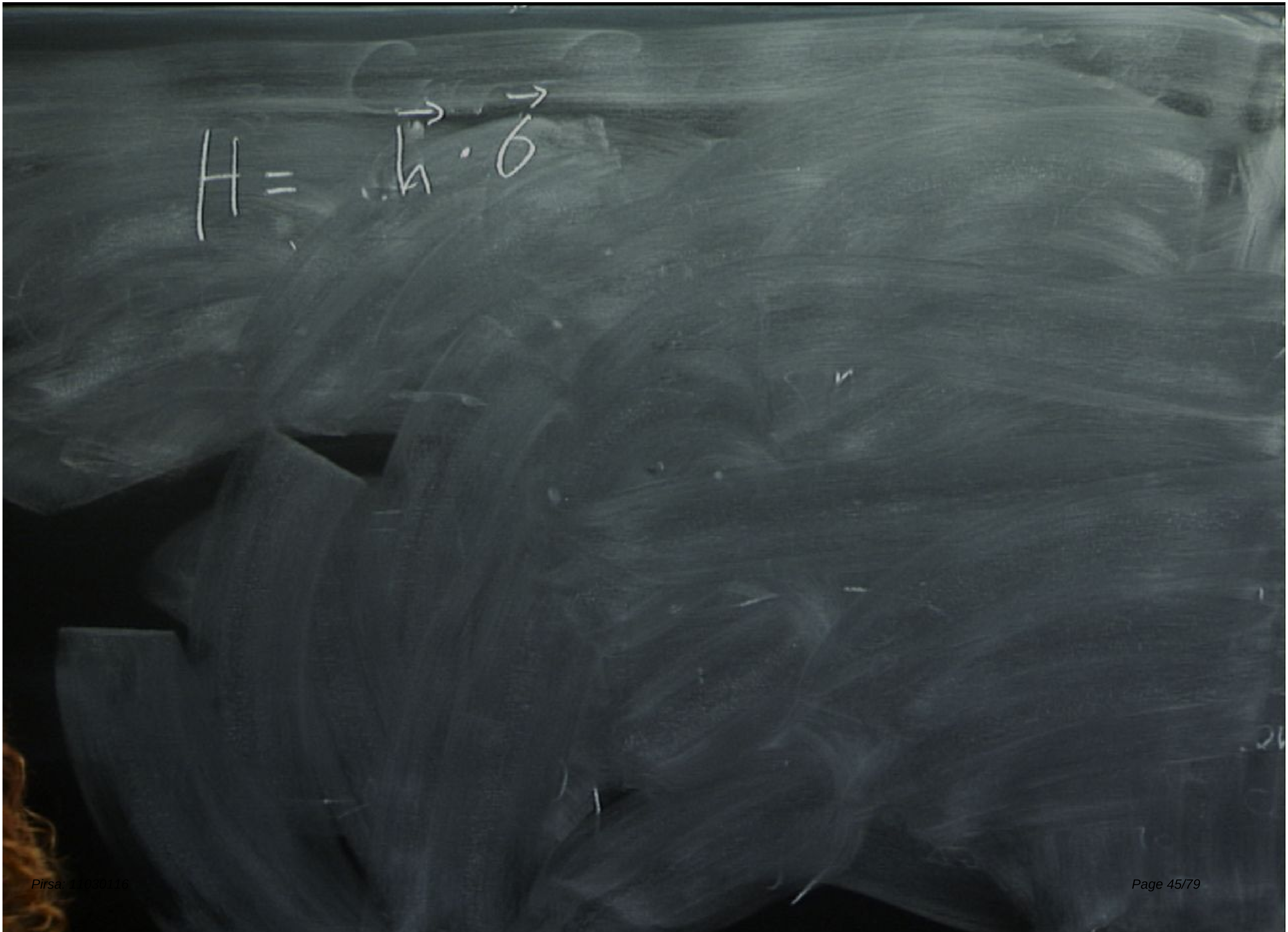
$$\Omega_{\mu\nu} = \frac{\partial}{\partial R_\mu} A^n_\nu - \frac{\partial}{\partial R_\nu} A^n_\mu$$

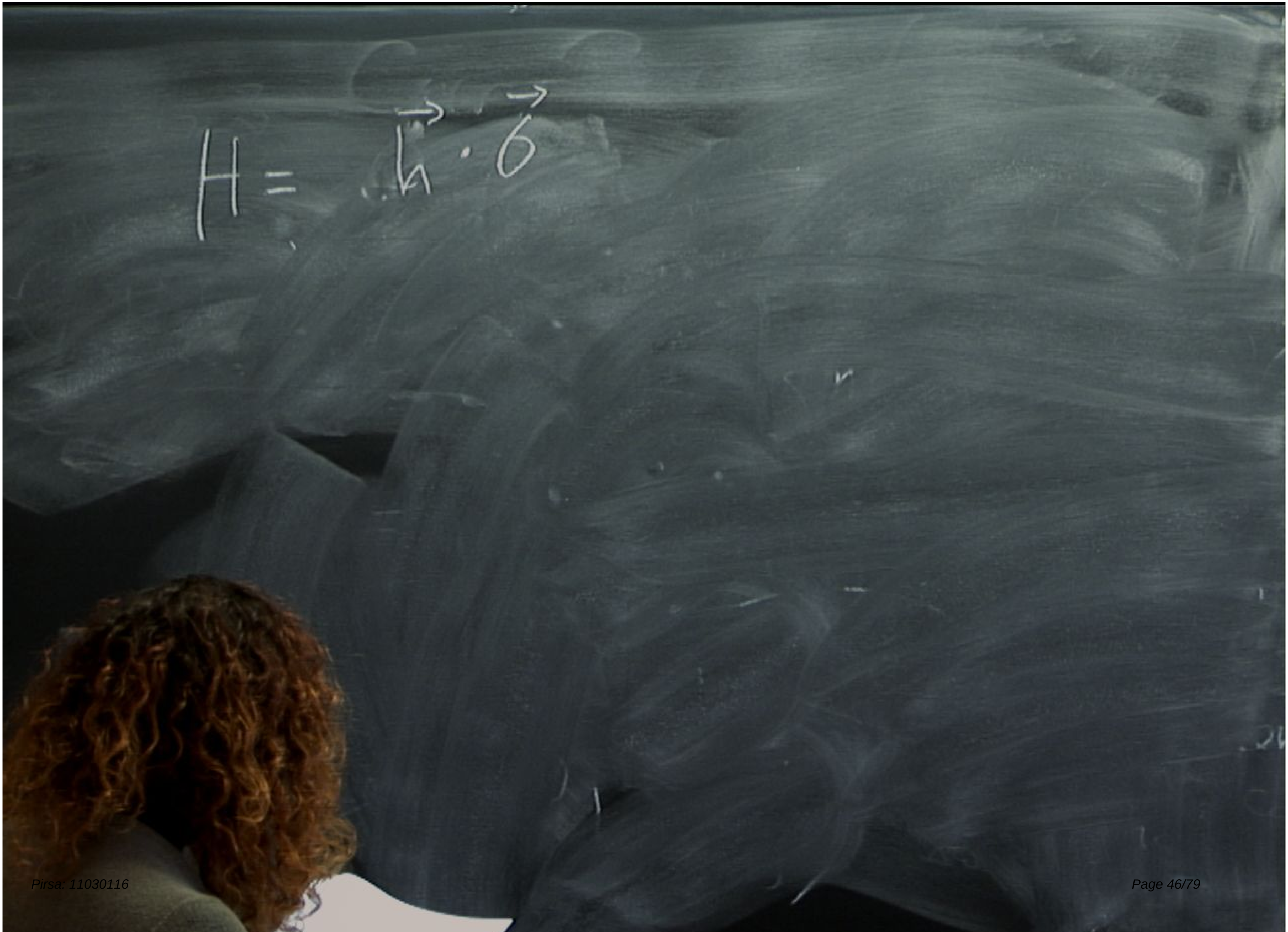
$$\varepsilon_{\mu\nu\rho}\vec{\Omega}$$




$= \int d\vec{S} \cdot (\nabla \times A)$
$\Omega_{\mu\nu} = i \sum_{n' \neq n} [$
$\langle n | \frac{\partial H}{\partial R_\mu} | n' \rangle \langle n' | \frac{\partial H}{\partial R_\nu} | n \rangle$
(R_i)
$\frac{\partial}{\partial R_\mu} A^n_\nu - \frac{\partial}{\partial R_\nu} A^n_\mu$
$= \Omega_{\mu\nu}$




$H = \vec{h} \cdot \vec{\sigma}$


$H = \vec{h} \cdot \vec{\sigma}$

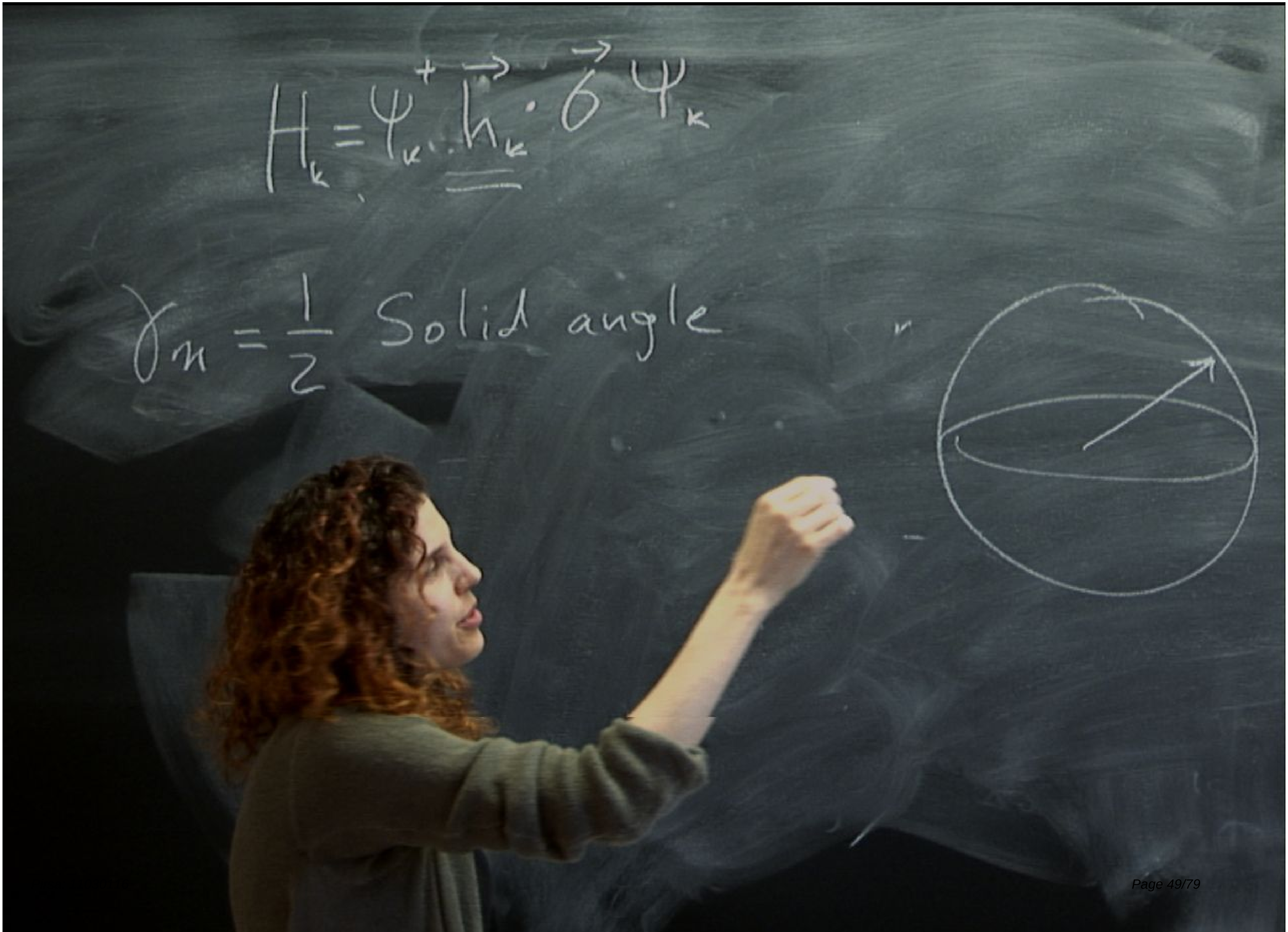
$$H_k = \Psi_k^\dagger \vec{h}_k \cdot \vec{\sigma} \Psi_k$$

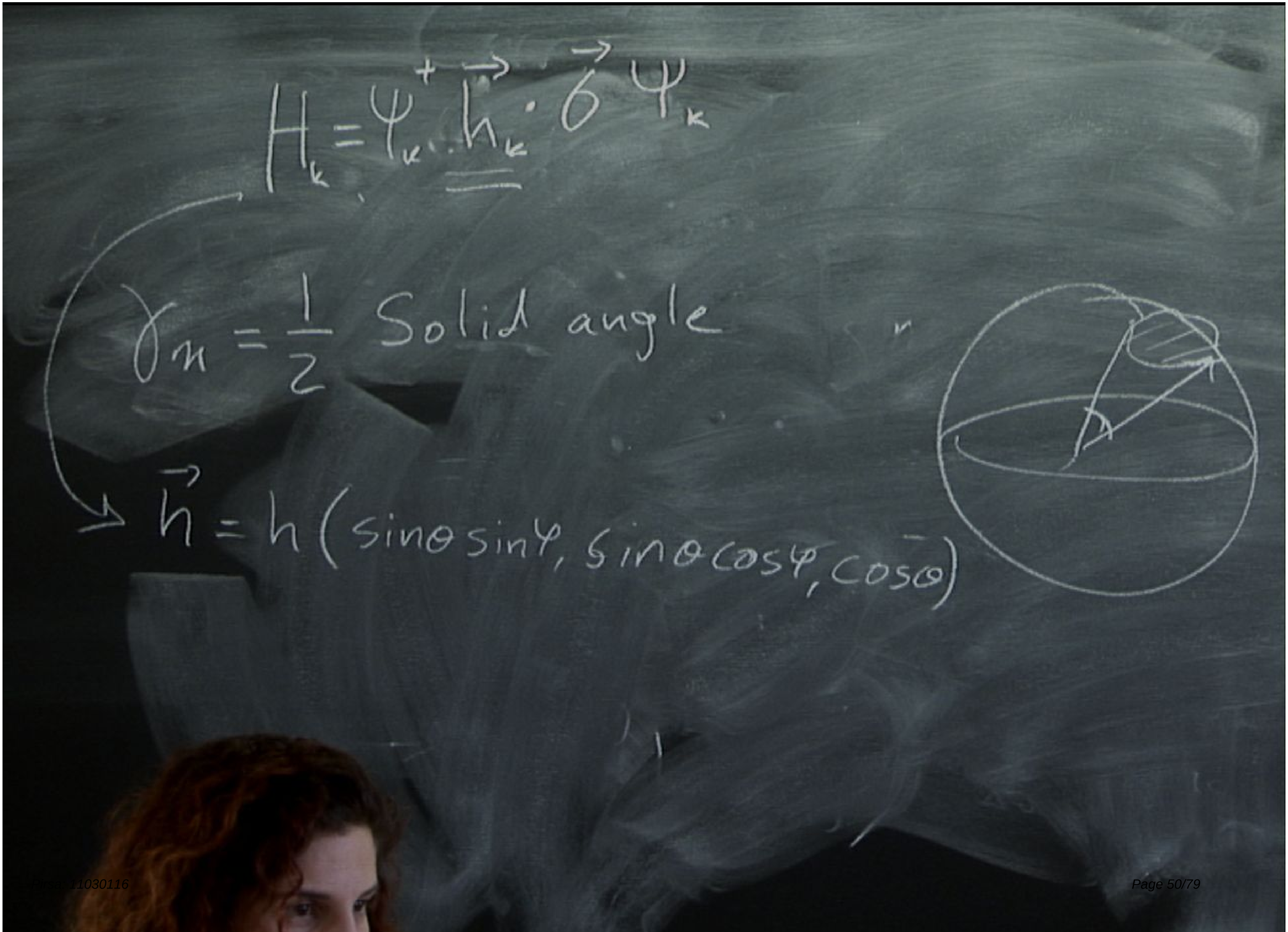
$q)| \nabla_q | u_n(q)\rangle$

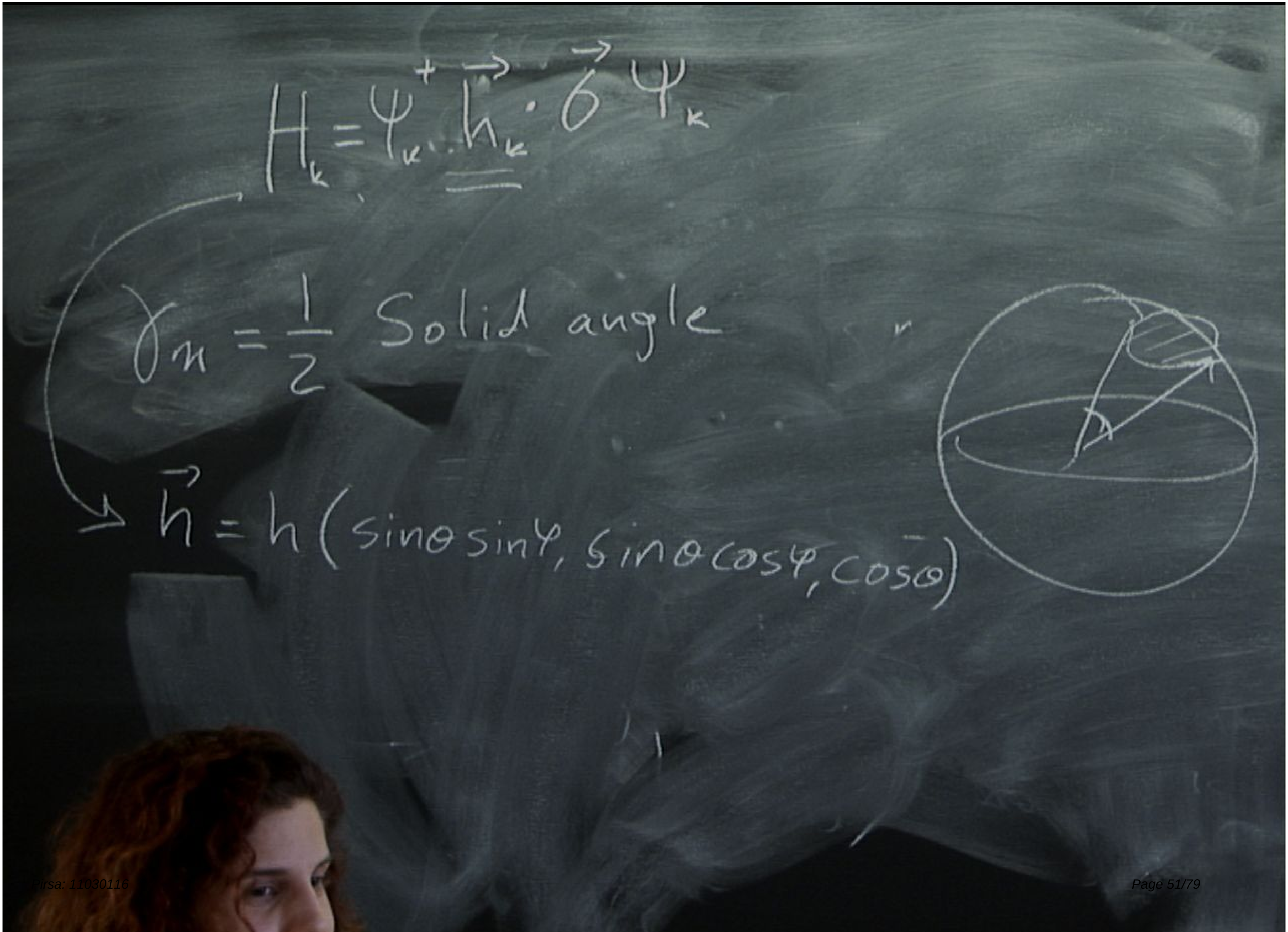
$\Omega_q = \vec{\nabla}_q \times \langle n(q) | \nabla_q | n(q)\rangle$

$$\Omega_{\mu\nu} = i \sum_{n' \neq n} \left[\frac{\langle n | \frac{\partial H}{\partial R_\mu} | n' \rangle \langle n' | \frac{\partial H}{\partial R_\nu} | n \rangle - \nu \leftrightarrow \mu}{(\varepsilon_n - \varepsilon_{n'})^2} \right]$$

$H | n' \rangle \to \varepsilon_{n'} | H$

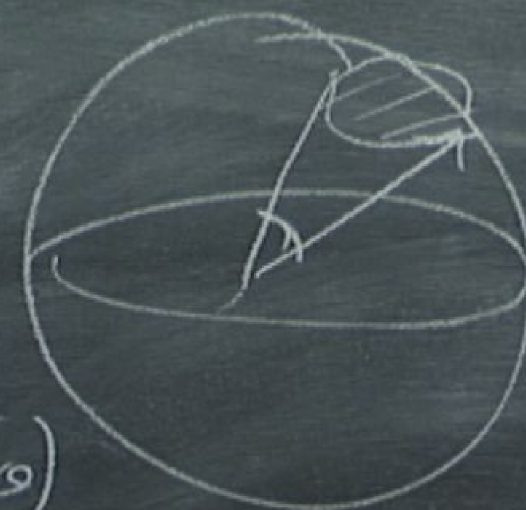

$H_k = \Psi_k^{+} \vec{h}_k \cdot \vec{\sigma} \Psi_k$
$\gamma_n = \frac{1}{2}$ Solid angle


H_k = Ψ_k^+ h_k · σ Ψ_k
γ_n = 1/2 Solid angle
h = h(sinθ sinφ, sinθ cosφ, cosθ)

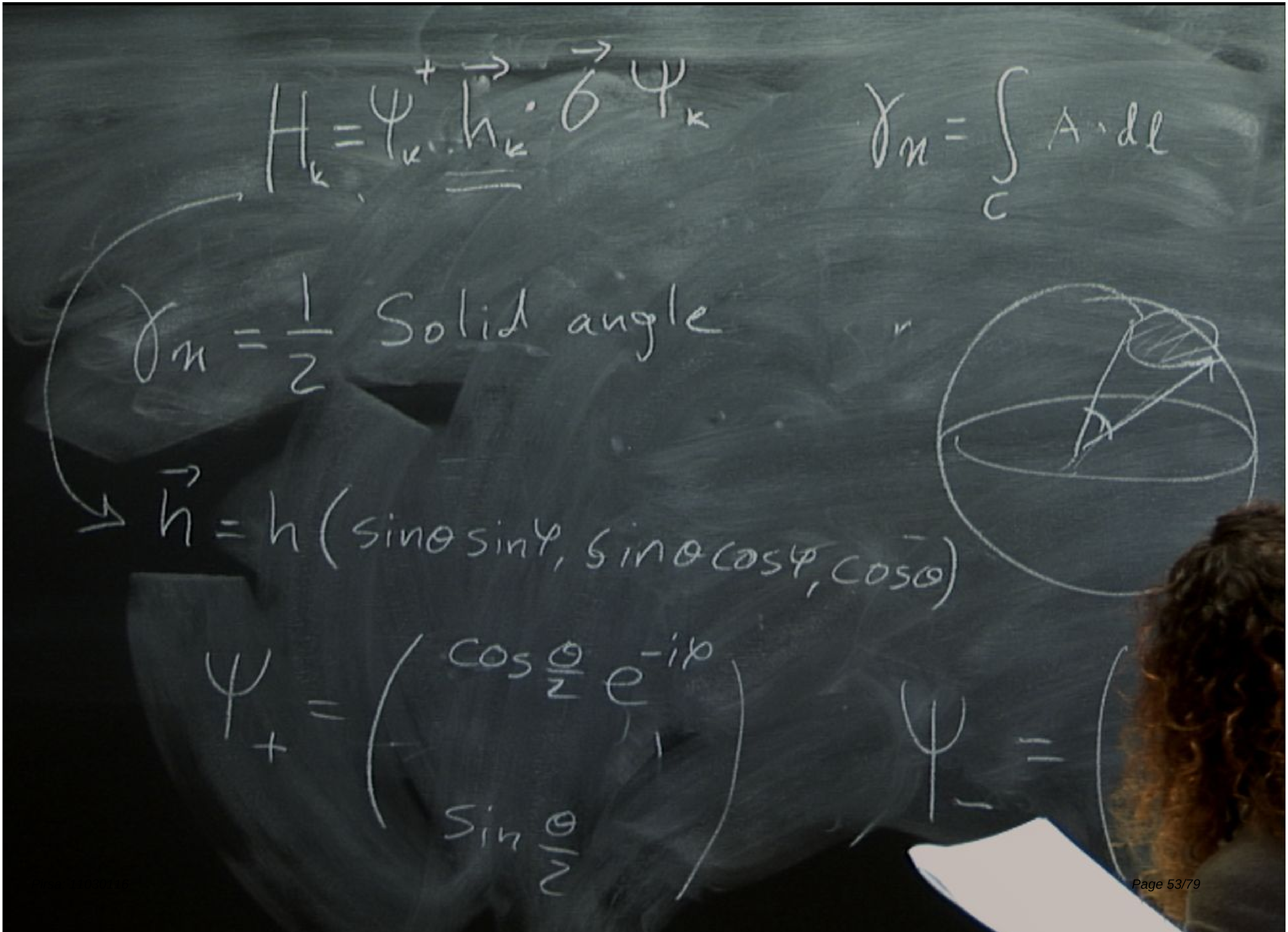

H_k = Ψ_k^+ h_k · σ Ψ_k
γ_n = 1/2 Solid angle
h = h(sinθ sinφ, sinθ cosφ, cosθ)

$$H_k = \Psi_k^{\dagger}\, \vec{h}_k \cdot \vec{\sigma}\, \Psi_k \qquad \gamma_n = \int_C A \cdot dl$$

$$\gamma_n = \frac{1}{2} \text{ Solid angle}$$



$$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta \cos\varphi, \cos\theta)$$

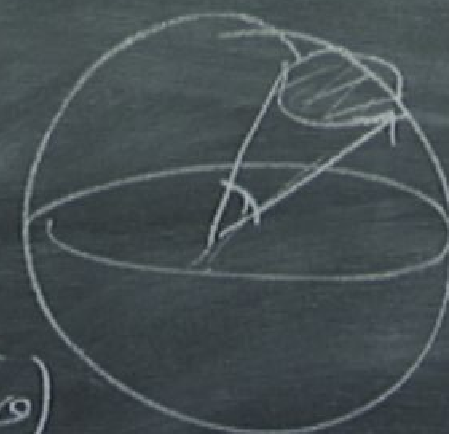

$H_k = \Psi_k^\dagger \vec{h}_k \cdot \vec{\sigma} \Psi_k$
$\gamma_n = \int_C A \cdot dl$
$\gamma_n = \frac{1}{2}$ Solid angle
$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta \cos\varphi, \cos\theta)$
$\Psi_+ = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix}$
$\Psi_- =$

$$H_k = \Psi_k^{\dagger}\, \underline{\underline{\vec{h}_k}} \cdot \vec{\sigma}\, \Psi_k$$

$$\gamma_n = \int_C A \cdot dl$$

$$\gamma_k = \oint_C$$

$$\gamma_n = \frac{1}{2} \text{ Solid angle}$$



$$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta\cos\varphi, \cos\theta)$$

$$\Psi_+ = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$$\gamma_n = \oint_C dq \cdot \langle u_n(q) | \nabla_q | u_n(q) \rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$$\gamma_n = i \int_C \Omega_{\theta\varphi} \, d\theta \, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$\ldots n\frac{\theta}{2} e^{-i\varphi}$

$\ldots s\frac{\theta}{2}$

$$H_k = \Psi_k^{+} \, \underline{\underline{\vec{h}_k}} \cdot \vec{\sigma} \, \Psi_k$$

$$\gamma_n = \int_C A \cdot dl$$

$$\gamma_k = \oint_C$$

$$= \int_B$$

$$\gamma_n = \frac{1}{2} \text{ Solid angle}$$

$$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta\cos\varphi, \cos\theta)$$

$$\Psi_+ = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$$= \oint_C dq \cdot \langle u_n(q) | \nabla_q | u_n(q) \rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$$A_\theta = i \langle \Psi_+ | \partial_\theta | \Psi_+ \rangle =$$

$$= -\sin\frac{\theta}{2} \cdot \frac{1}{2} \cos\frac{\theta}{2}\, i$$

$$+ \cos\frac{\theta}{2} \cdot \frac{1}{2} \cdot \sin\frac{\theta}{2} = 0$$

$$\gamma_n = i \int_C \Omega_{\theta\varphi}\, d\theta\, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$= \oint_C dq \cdot \langle u_n(q)| \nabla_q |u_n(q)\rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$i\varphi$

$$\gamma_n = i \int_C \Omega_{\theta\varphi} \, d\theta \, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$A_\theta = i\langle \psi_+ | \partial_\theta | \psi_+ \rangle =$$

$$= -\sin\frac{\theta}{2}\cdot\frac{1}{2}\cos\frac{\theta}{2}\, i$$

$$+ \cos\frac{\theta}{2}\cdot\frac{1}{2}\cdot\sin\frac{\theta}{2} = 0$$

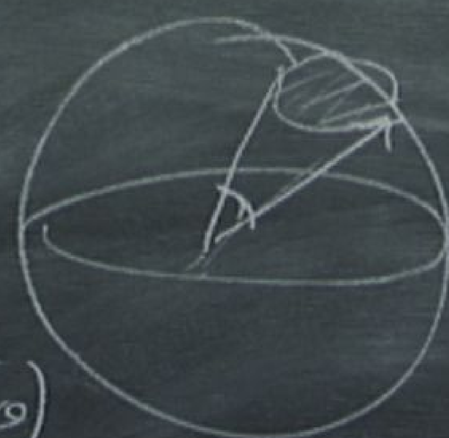
$$A_\varphi = i\langle \psi_+ | \partial_\varphi | \psi_+ \rangle =$$

$$=$$

$$H_k = \Psi_k^{\dagger}\, \vec{h}_k \cdot \vec{\sigma}\, \Psi_k \qquad \gamma_n = \int_C A \cdot dl$$

$$\gamma_k = \oint_C \ldots = \int_B \ldots$$

$$\gamma_n = \frac{1}{2}\ \text{Solid angle}$$



$$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta\cos\varphi, \cos\theta)$$

$$\Psi_+ = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$$= \oint_C dq \cdot \langle u_n(q)| \nabla_q |u_n(q)\rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$i\varphi$)

$$\gamma_n = i\int_C \Omega_{\theta\varphi}\, d\theta\, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$A_\theta = i\langle \psi_+ | \partial_\theta | \psi_+ \rangle =$$

$$= -\sin\frac{\theta}{2} \cdot \frac{1}{2}\cos\frac{\theta}{2}\, i + \cos\frac{\theta}{2} \cdot \frac{1}{2} \cdot \sin\frac{\theta}{2} = 0$$

$$A_\varphi = i\langle \psi_+ | \partial_\varphi | \psi_+ \rangle =$$

$$= i(-i)\cos^2\left(\frac{\theta}{2}\right)$$

$$= \oint_C dq \cdot \langle u_n(q)| \nabla_q |u_n(q)\rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$$\gamma_n = i \int_C \Omega_{\theta\varphi} \, d\theta \, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$A_\theta = i \langle \Psi_+ | \partial_\theta | \Psi_+ \rangle =$$

$$= -\sin\frac{\theta}{2} \cdot \frac{1}{2} \cos\frac{\theta}{2} \, i$$

$$+ \cos\frac{\theta}{2} \cdot \frac{1}{2} \cdot \sin\frac{\theta}{2} = 0$$

$$A_\varphi = i \langle \Psi_+ | \partial_\varphi | \Psi_+ \rangle =$$

$$= i(-i) \cos^2\left(\frac{\theta}{2}\right) =$$

$$= \frac{1}{2} \sin\theta$$

$$= \oint_C dq \cdot \langle u_n(q)| \nabla_q |u_n(q)\rangle$$

$$= \int_{BZ} dq \cdot \Omega_q$$

$$\gamma_n = i\int_C \Omega_{\theta\varphi}\, d\theta\, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$A_\theta = i\langle \psi_+ | \partial_\theta | \psi_+ \rangle =$$

$$= -\sin\frac{\theta}{2}\cdot\frac{1}{2}\cos\frac{\theta}{2}\, i$$

$$+ \cos\frac{\theta}{2}\cdot\frac{1}{2}\cdot\sin\frac{\theta}{2} = 0$$

$$A_\varphi = i\langle \psi_+ | \partial_\varphi | \psi_+ \rangle =$$

$$= i(-i)\cos^2\left(\frac{\theta}{2}\right) =$$

$$= \frac{1}{2} + \frac{1}{2}\sin\theta$$

$$\Omega_{\theta\varphi} = 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}\cdot\frac{1}{2} = \frac{1}{2}\sin\theta$$

$$A_\theta = i\langle\psi_+|\partial_\theta|\psi_+\rangle = -\sin\frac{\theta}{2}\cdot\frac{1}{2}\cos\frac{\theta}{2}\, i + \cos\frac{\theta}{2}\cdot\frac{1}{2}\sin\frac{\theta}{2} = 0$$

$$A_\varphi = i\langle\psi_+|\partial_\varphi|\psi_+\rangle = i(-i)\cos^2\left(\frac{\theta}{2}\right) = \cos^2\left(\frac{\theta}{2}\right)$$

$$\gamma_n = i\int_C \Omega_{\theta\varphi}\, d\theta\, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$\Omega_{\theta\varphi} = 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}\cdot\frac{1}{2} =$$

$$= -\frac{1}{2}\sin\theta$$

$$-\frac{1}{2}\int d\theta\, d\varphi \cdot \sin\theta$$

$$e^{-i\varphi}$$

$$\gamma_n = i\int_C \Omega_{\theta\varphi}\, d\theta\, d\varphi$$

$$\Omega_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta$$

$$A_\theta = i\langle\Psi_+|\partial_\theta|\Psi_+\rangle =$$

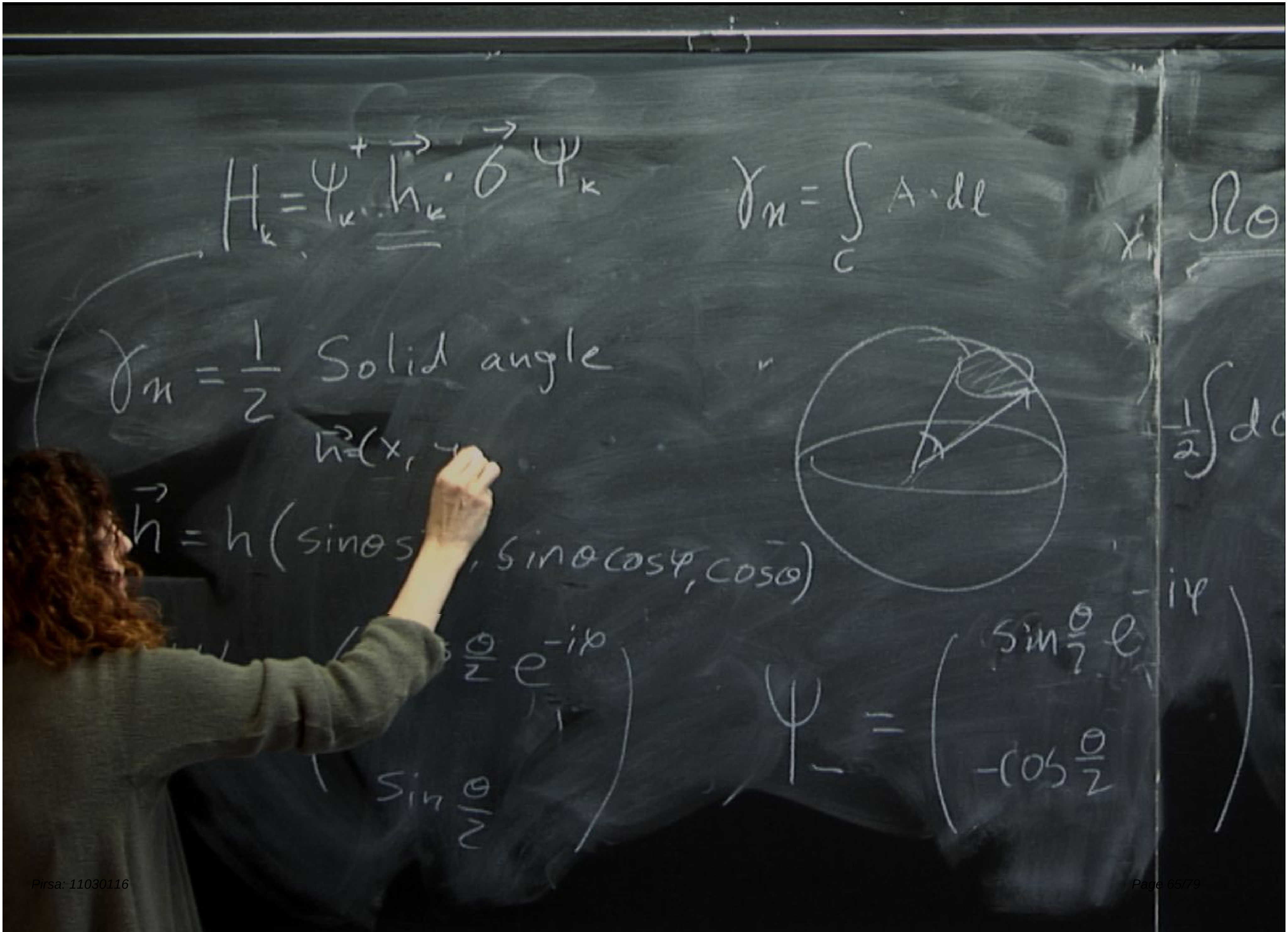
$$= -\sin\frac{\theta}{2}\cdot\frac{1}{2}\cos\frac{\theta}{2}\, i$$

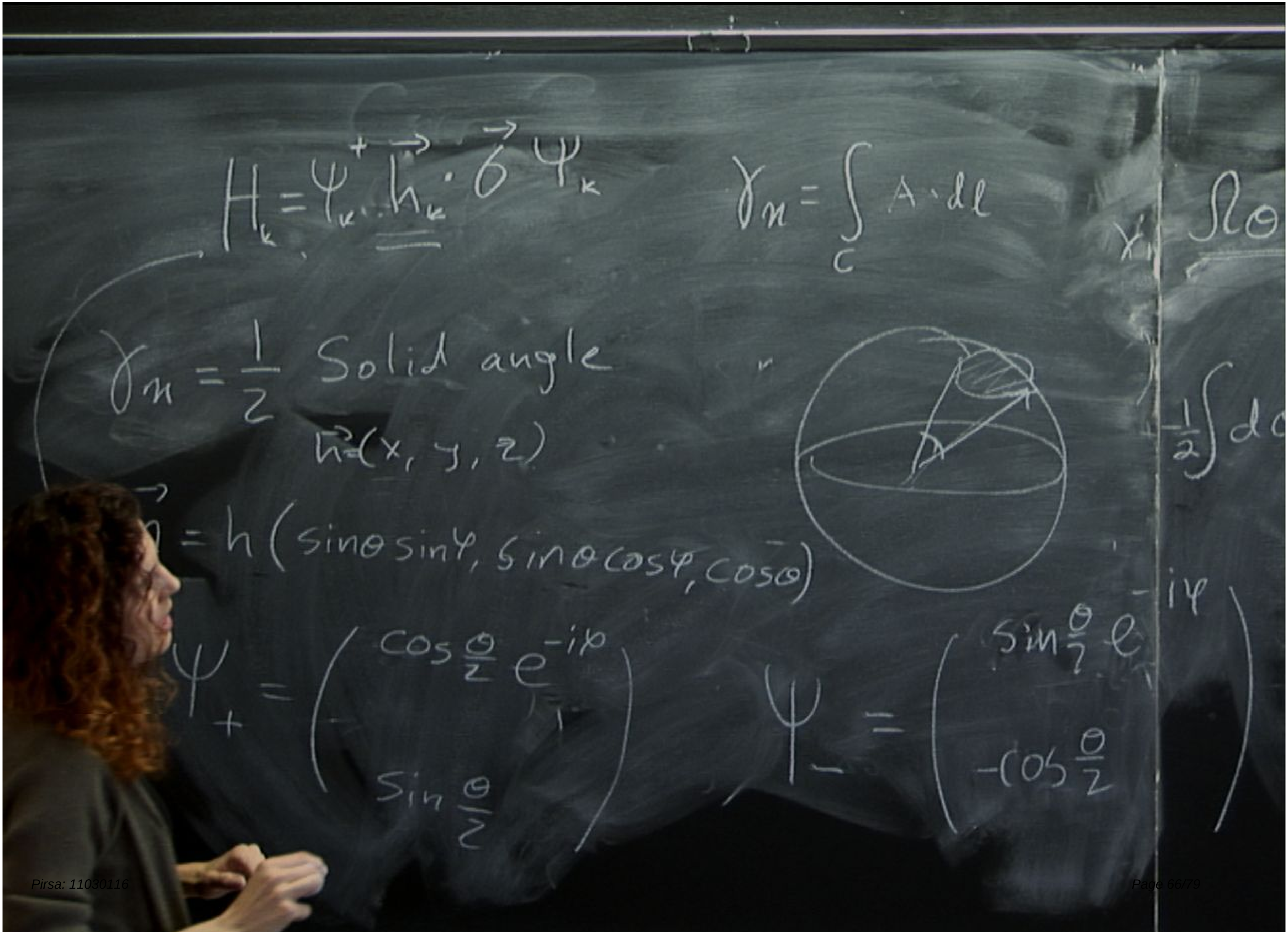
$$+ \cos\frac{\theta}{2}\cdot\frac{1}{2}\cdot\sin\frac{\theta}{2} = 0$$

$$A_\varphi = i\langle\Psi_+|\partial_\varphi|\Psi_+\rangle =$$

$$= i(-i)\cos^2\left(\frac{\theta}{2}\right) =$$

$$= \cos^2\left(\frac{\theta}{2}\right)$$


$H_k = \Psi_k^{+} \vec{h}_k \cdot \vec{\sigma} \Psi_k$
$\gamma_n = \int_C A \cdot dl$
$\gamma_n = \frac{1}{2}$ Solid angle
$\vec{h} = (x, y$
$\vec{h} = h(\sin\theta\cos\varphi, \sin\theta\cos\varphi, \cos\theta)$
$\Psi_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$
$\sin\frac{\theta}{2}$
$-\frac{1}{2}\int d$


H_k = Ψ_k^+ h_k · σ Ψ_k
γ_n = ∫_C A · dl
γ_n = 1/2 Solid angle
n = (x, y, z)
= h (sinθ sinφ, sinθ cosφ, cosθ)
Ψ_+ = (cos θ/2 e^{-iφ} , sin θ/2)
Ψ_- = (sin θ/2 e^{-iφ} , -cos θ/2)

$$H_k = \Psi_k^{\dagger} \vec{h}_k \cdot \vec{\sigma} \Psi_k$$

$$\gamma_n = \int_C A \cdot dl$$

$$\gamma_n = \frac{1}{2} \text{ Solid angle}$$

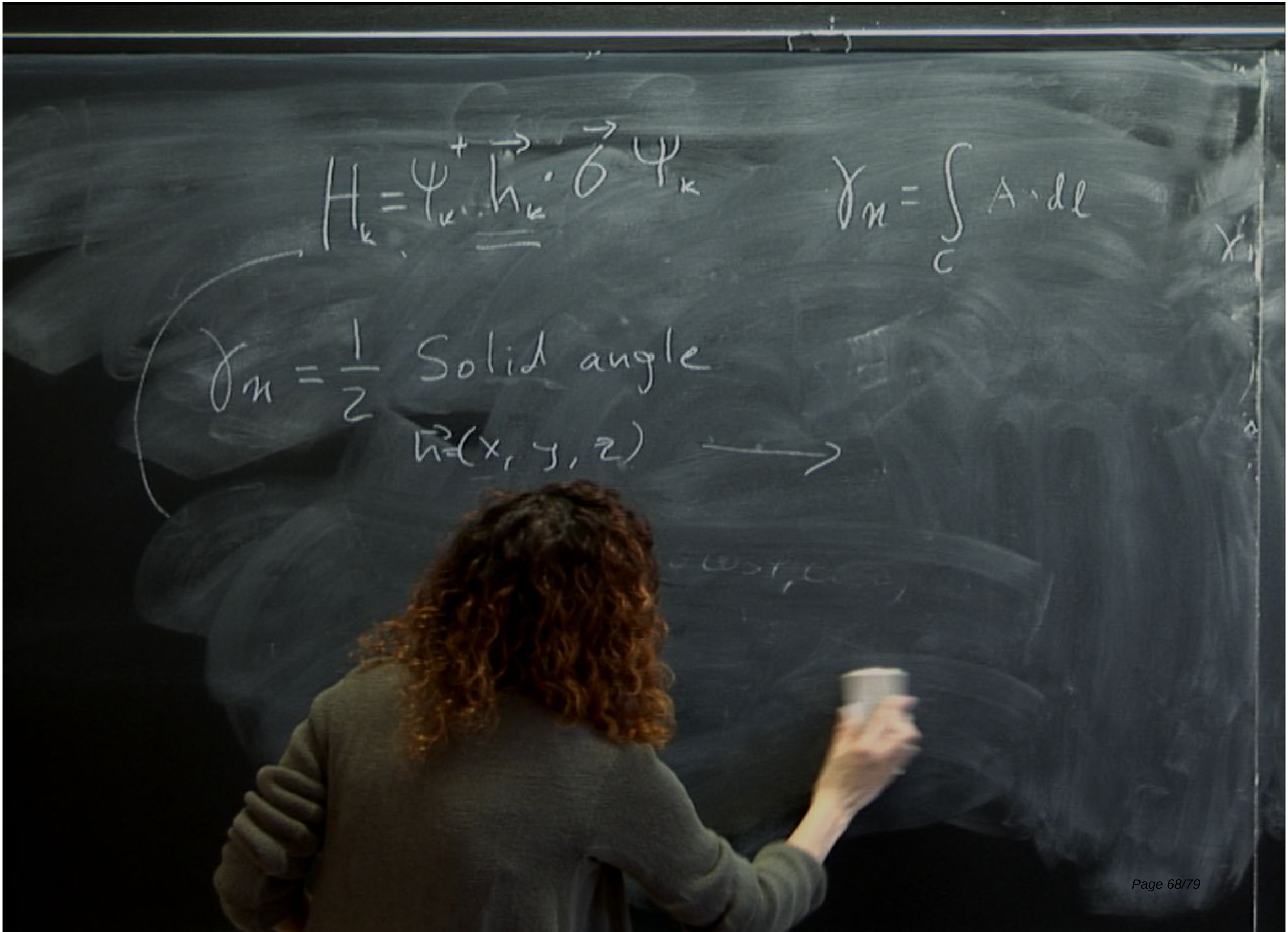
$$\vec{h} = (x, y, z)$$

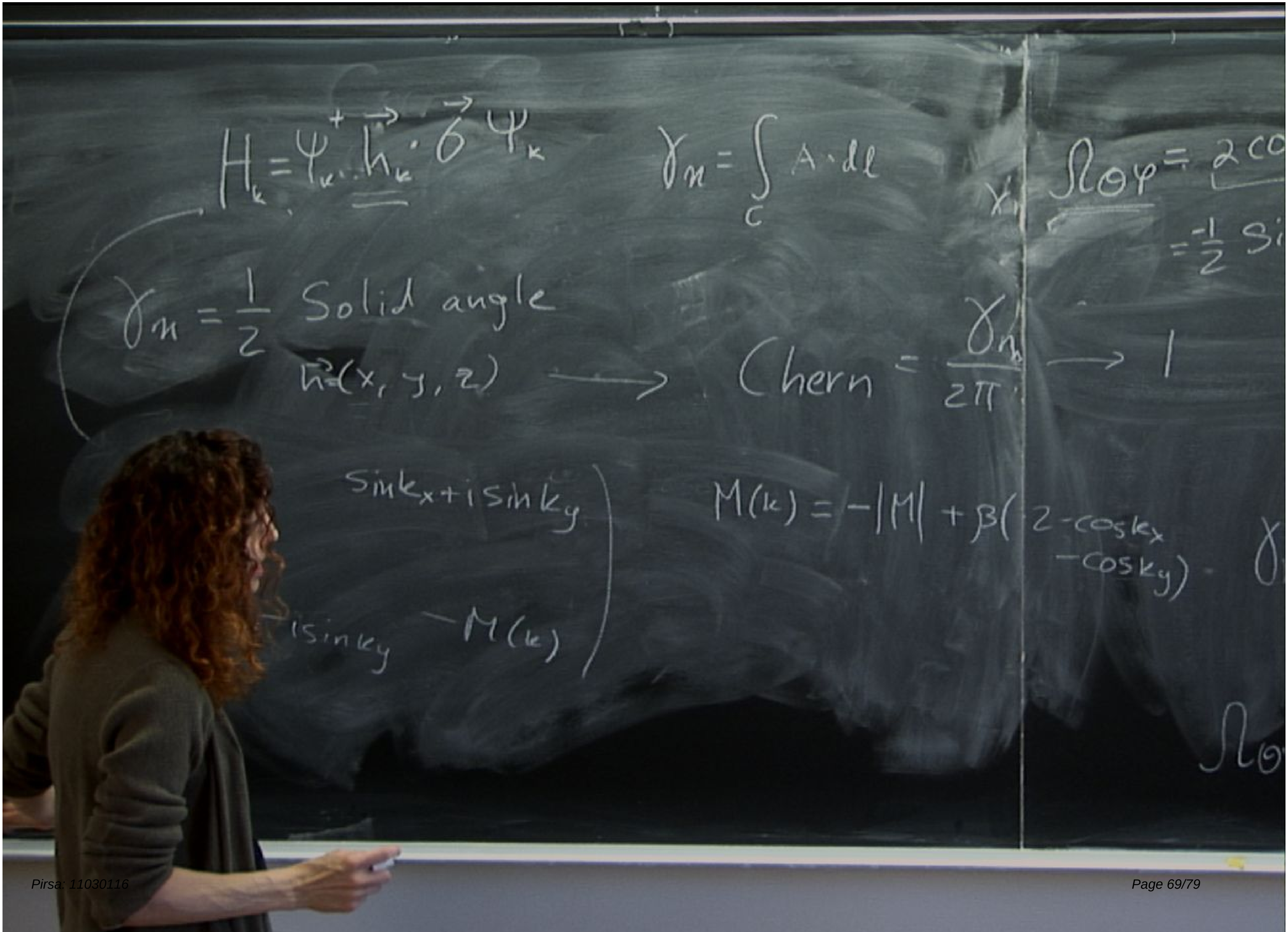
$$\vec{h} = h(\sin\theta \sin\varphi, \sin\theta \cos\varphi, \cos\theta)$$

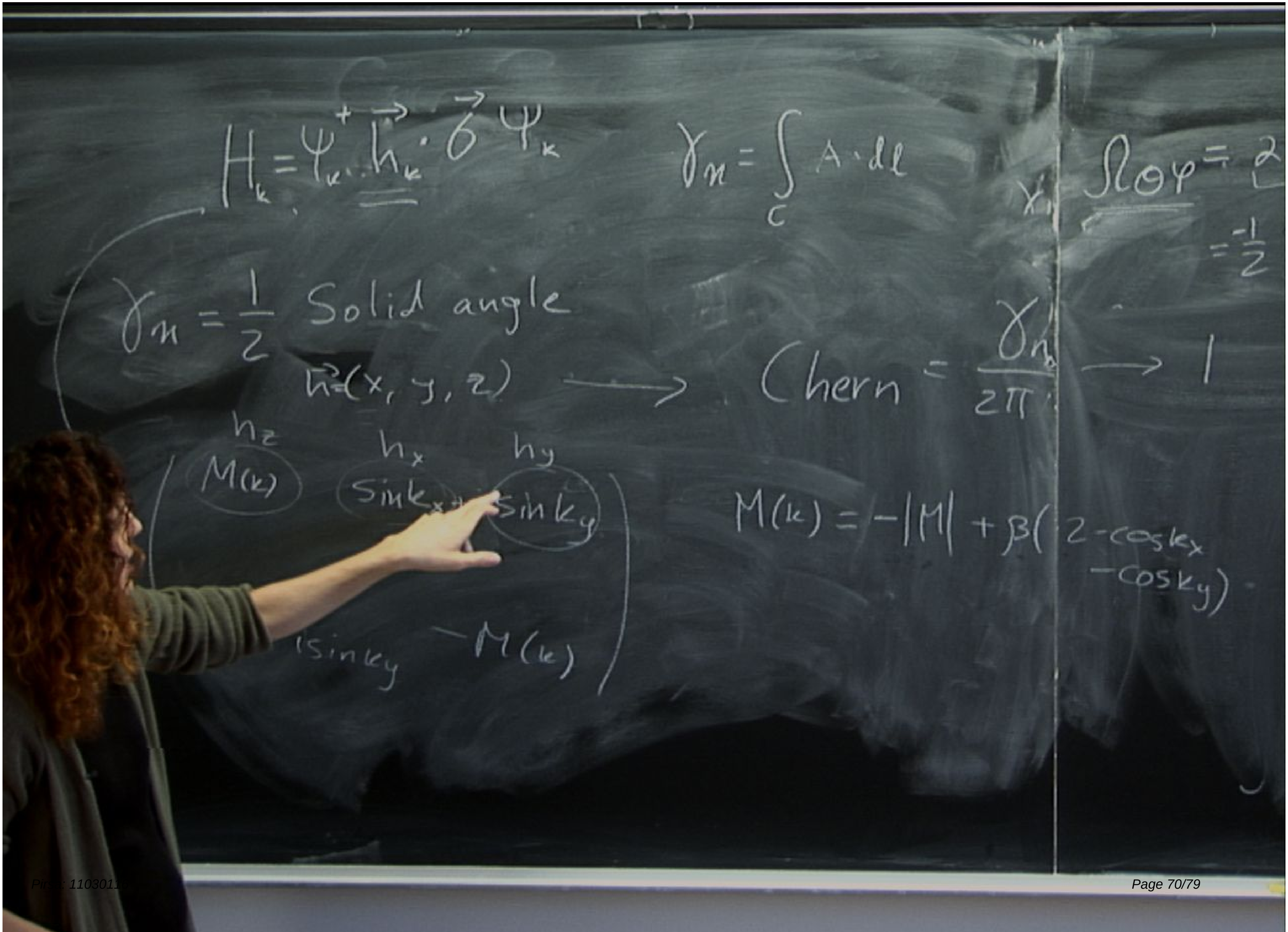
$$\Psi_+ = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ \sin\frac{\theta}{2} \end{pmatrix} \qquad \Psi_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$$\Omega\theta$$

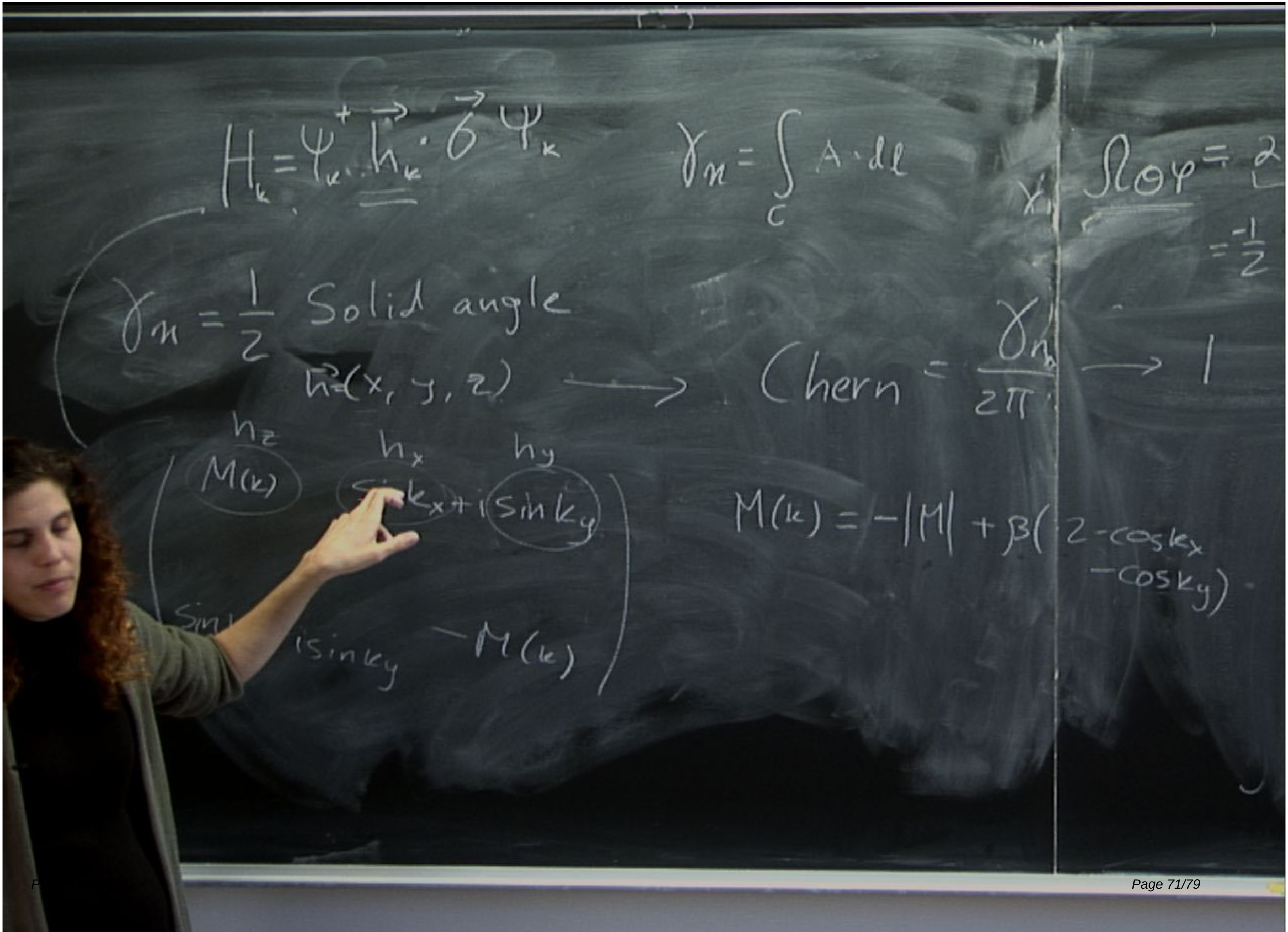
$$-\frac{1}{2}\int d$$


$H_k = \psi_k^\dagger \vec{h}_k \cdot \vec{\sigma} \psi_k$
$\gamma_n = \int_C A \cdot dl$
$\gamma_n = \frac{1}{2}$ Solid angle
$\vec{h} = (x, y, z) \longrightarrow$


$H_k = \Psi_k^\dagger \vec{h}_k \cdot \vec{\sigma} \Psi_k$
$\gamma_n = \int_C A \cdot dl$
$\gamma_n = \frac{1}{2}$ Solid angle
$\vec{n}(x, y, z)$
Chern $= \frac{\gamma_n}{2\pi} \to 1$
$\sin k_x + i \sin k_y$
$i \sin k_y$
$-M(k)$
$M(k) = -|M| + \beta(2 - \cos k_x - \cos k_y)$
$= \frac{-1}{2}$



$H_k = \Psi_k^{+} \vec{h}_k \cdot \vec{\sigma} \Psi_k$
$\gamma_n = \int_C A \cdot dl$
$\gamma_n = \frac{1}{2}$ Solid angle
$\vec{h}(x, y, z)$
$Chern = \frac{\gamma_n}{2\pi} \to 1$
h_z h_x h_y
$M(k)$ $\sin k_x$ $\sin k_y$
$i \sin k_y$ $-M(k)$
$M(k) = -|M| + \beta(2 - \cos k_x - \cos k_y)$
$= \frac{-1}{2}$


H_k = Ψ_k^+ h_k · σ Ψ_k
γ_n = ∫_C A · dl
γ_n = 1/2 Solid angle
n(x, y, z)
Chern = γ_n / 2π
h_z
h_x
h_y
M(k)
sin k_x + i sin k_y
i sin k_y
−M(k)
M(k) = −|M| + β(2 − cos k_x − cos k_y)
= −1/2

$$H_k = \Psi_k^{\dagger}\, \vec{h}_k \cdot \vec{\sigma}\, \Psi_k \qquad \gamma_n = \int_C A \cdot dl$$

$\Omega\theta\varphi = 2$

$= -\frac{1}{2}$

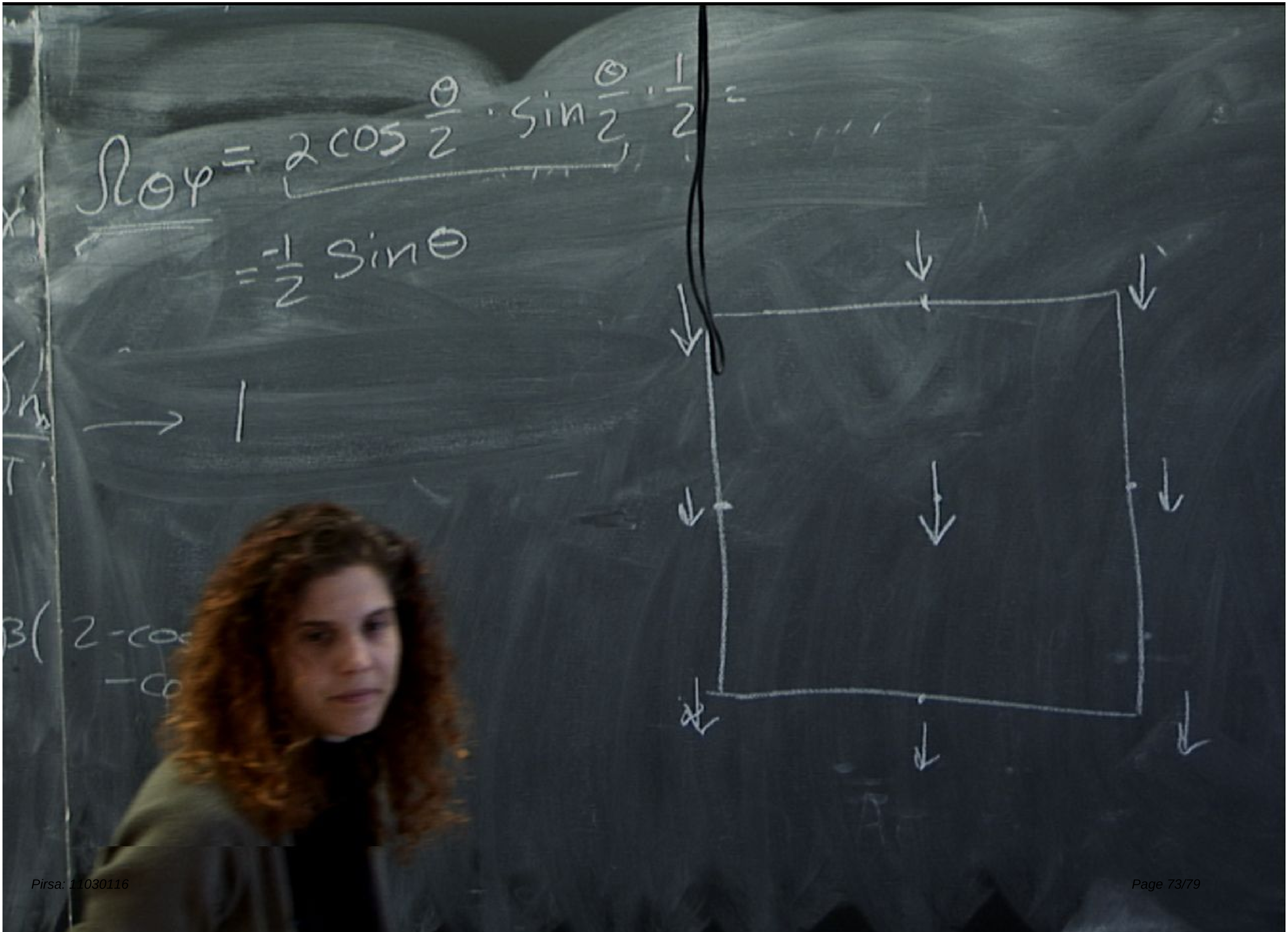
$\gamma_n = \frac{1}{2}$ Solid angle

$\vec{h}(x, y, z) \longrightarrow$ Chern $= \frac{\gamma_n}{2\pi} \longrightarrow 1$

$h_z \quad h_x \quad h_y$

$$\begin{pmatrix} M(k) & \sin k_x + i \sin k_y \\ \sin k_x - i \sin k_y & -M(k) \end{pmatrix}$$

$$M(k) = -|M| + \beta(2 - \cos k_x - \cos k_y)$$

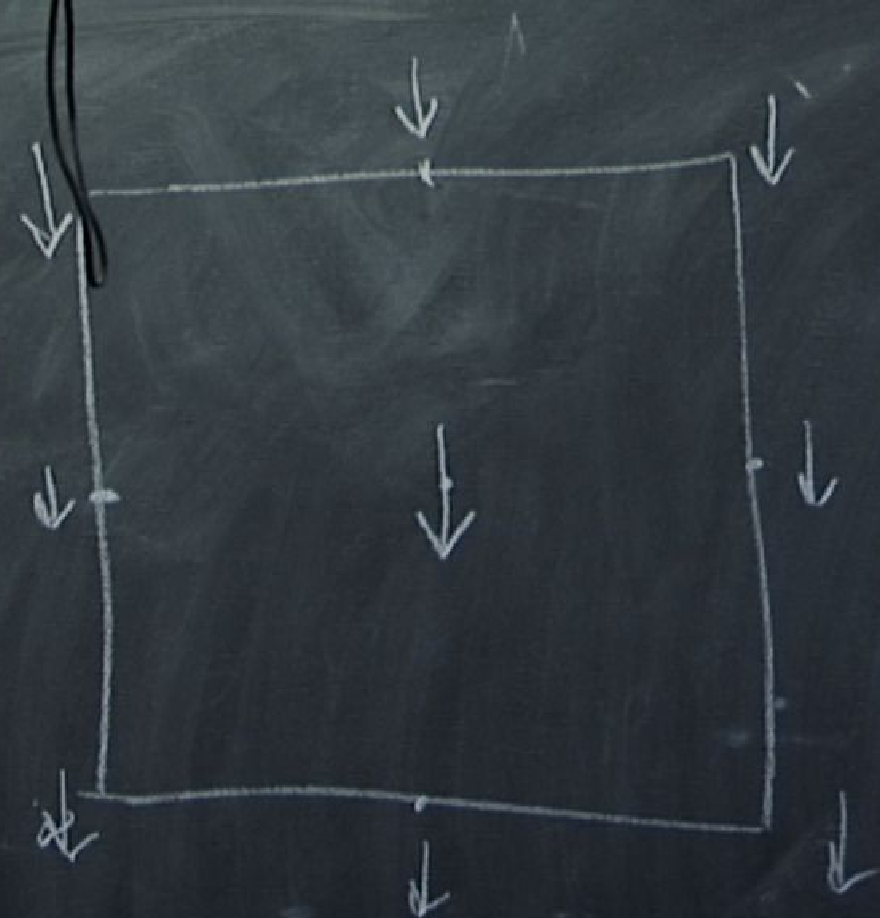

Ωθφ = 2 cos θ/2 · sin θ/2 · 1/2 =
= -1/2 sinθ
→ 1

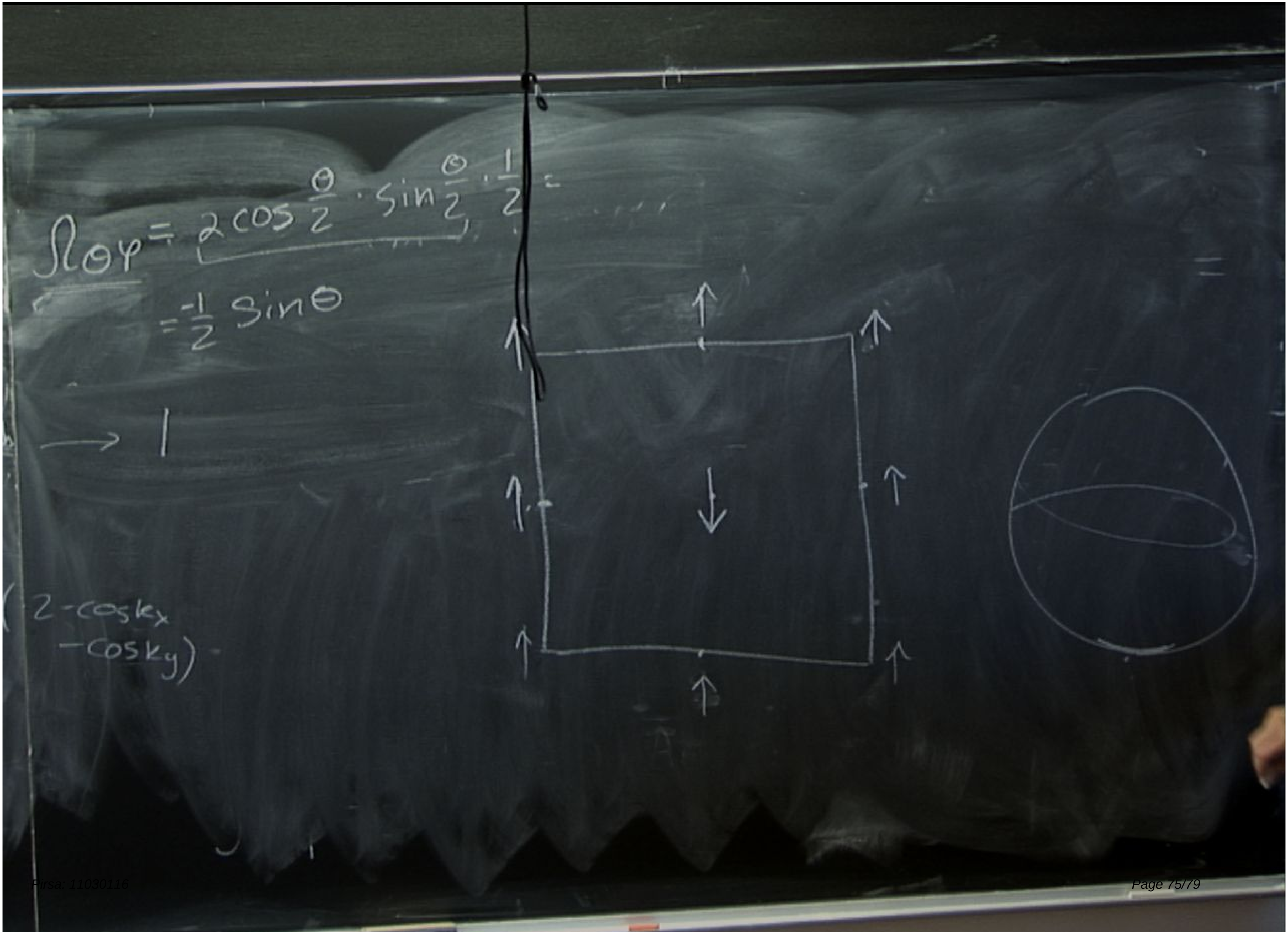
$$\Omega_{\theta\varphi} = 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}\cdot\frac{1}{2} = \dots$$

$$= -\frac{1}{2}\sin\theta$$

$\rightarrow 1$

$3(2 - \cos k_x - \cos k_y)$

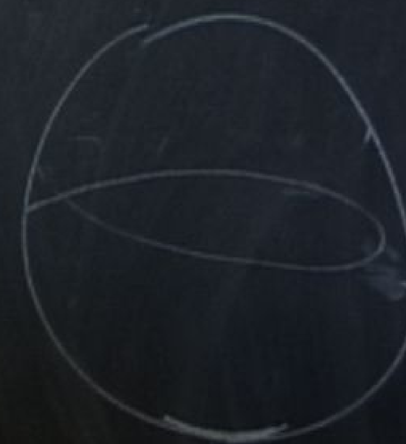



Ωθφ = 2 cos θ/2 · sin θ/2 · 1/2 =
= -1/2 sin θ
→ 1
(2 - cos kx
- cos ky)

$$\Omega_{\theta\varphi} = 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}\cdot\frac{1}{2} = \ldots$$

$$= -\frac{1}{2}\sin\theta$$

$$\longrightarrow 1$$



$$(2 - \cos k_x - \cos k_y)$$