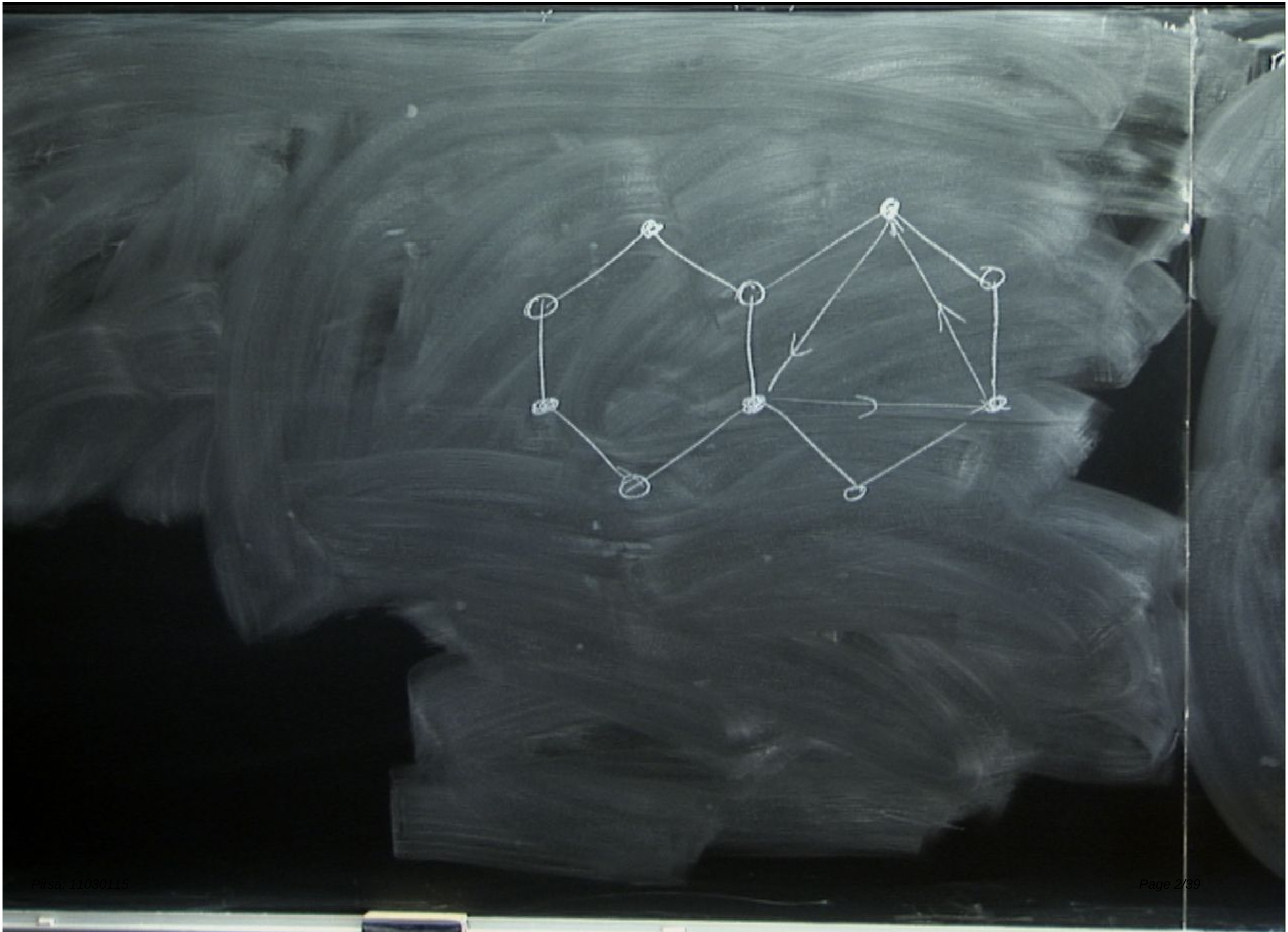
Title: Explorations in Condensed Matter - Lecture 12

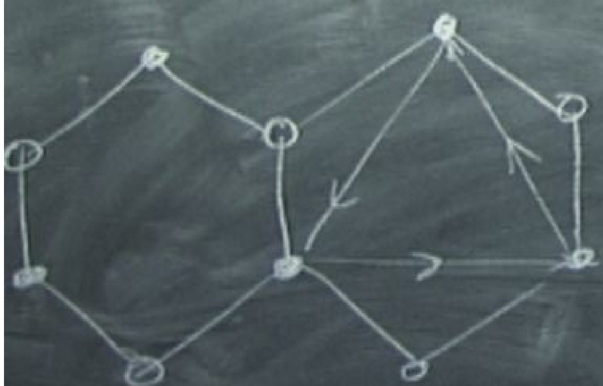
Date: Mar 29, 2011 10:15 AM

URL: http://pirsa.org/11030115

Abstract:

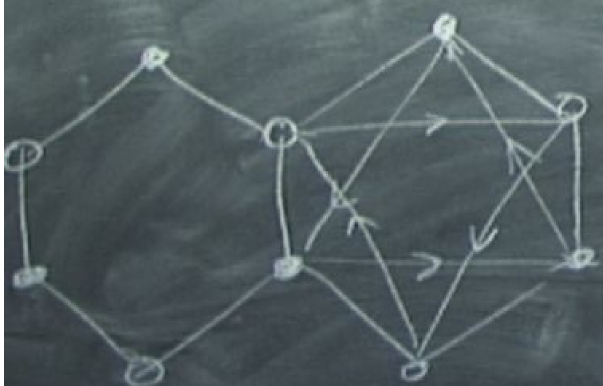


$$H = t \sum_{\langle ij \rangle} c_i^{\dagger} c_j + i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^{\dagger} c_j$$



$$H = t \sum_{\langle ij \rangle} c_i^{+} c_j + i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij} c_i^{+} c_j$$

$$\nu_{ij} = \begin{cases} 1 & i \rightarrow j \\ -1 & j \leftarrow i \end{cases}$$



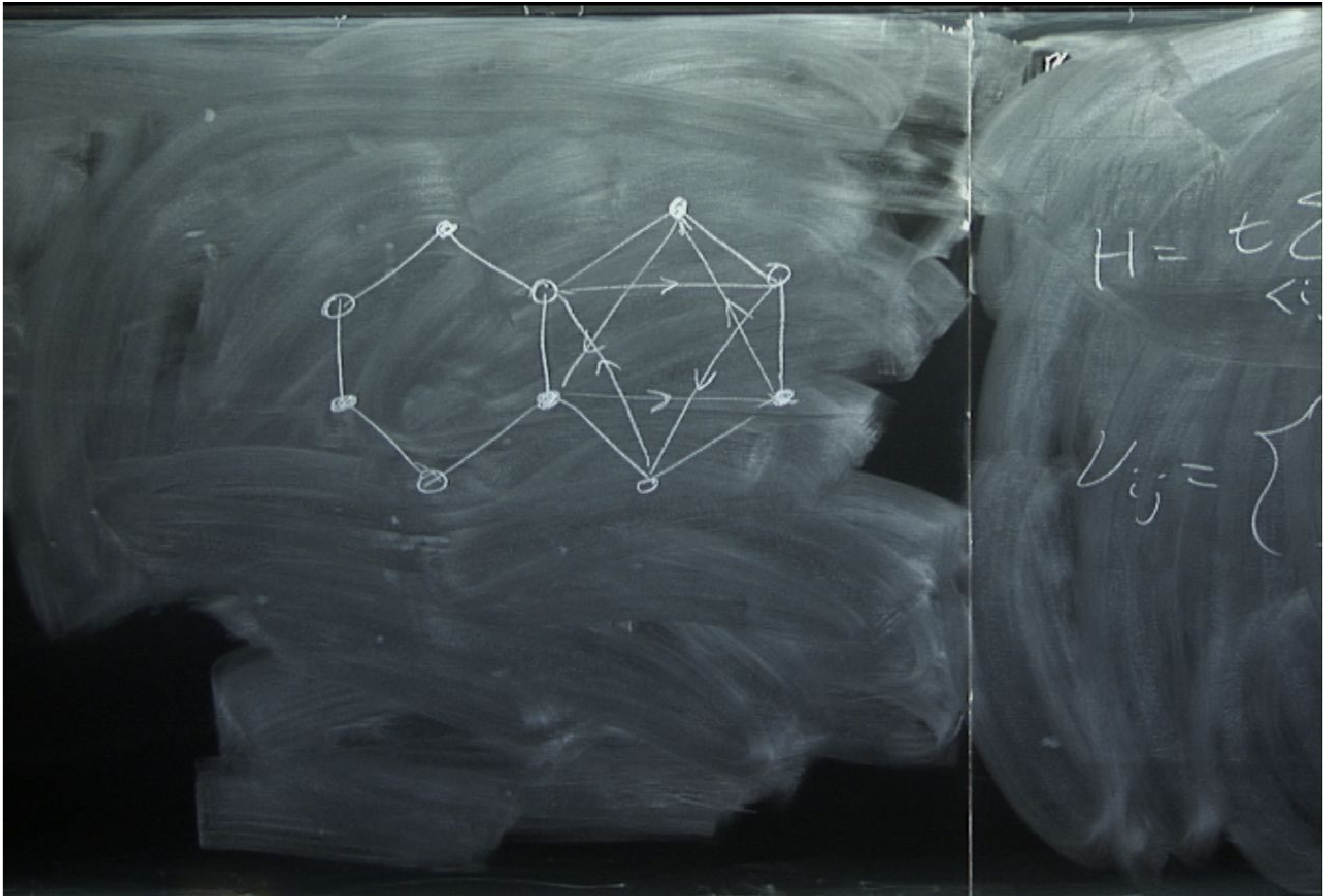
$$H = t\sum_{\langle ij\rangle} c_i^{\dagger} c_j + i\lambda \sum_{\langle\langle ij\rangle\rangle} \nu_{ij}\, c_i^{\dagger} c_j$$

$$\nu_{ij} = \begin{cases} 1 & i \rightarrow j \\ -1 & j \leftarrow i \end{cases}$$

$$H = t \sum_{\langle ij \rangle} c_i^{+} c_j + i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij} c_i^{+} c_j$$

$$\nu_{ij} = \begin{cases} 1 & i \longrightarrow j \\ -1 & j \longleftarrow i \end{cases}$$

$$t_{ij} \rightarrow t\, e^{i\int_i^j A\, dr}$$


H = t
ν_{ij} =

sign

$$H = t \sum_{\langle ij \rangle} c_i^{\dagger} c_j + i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^{\dagger} c_j$$

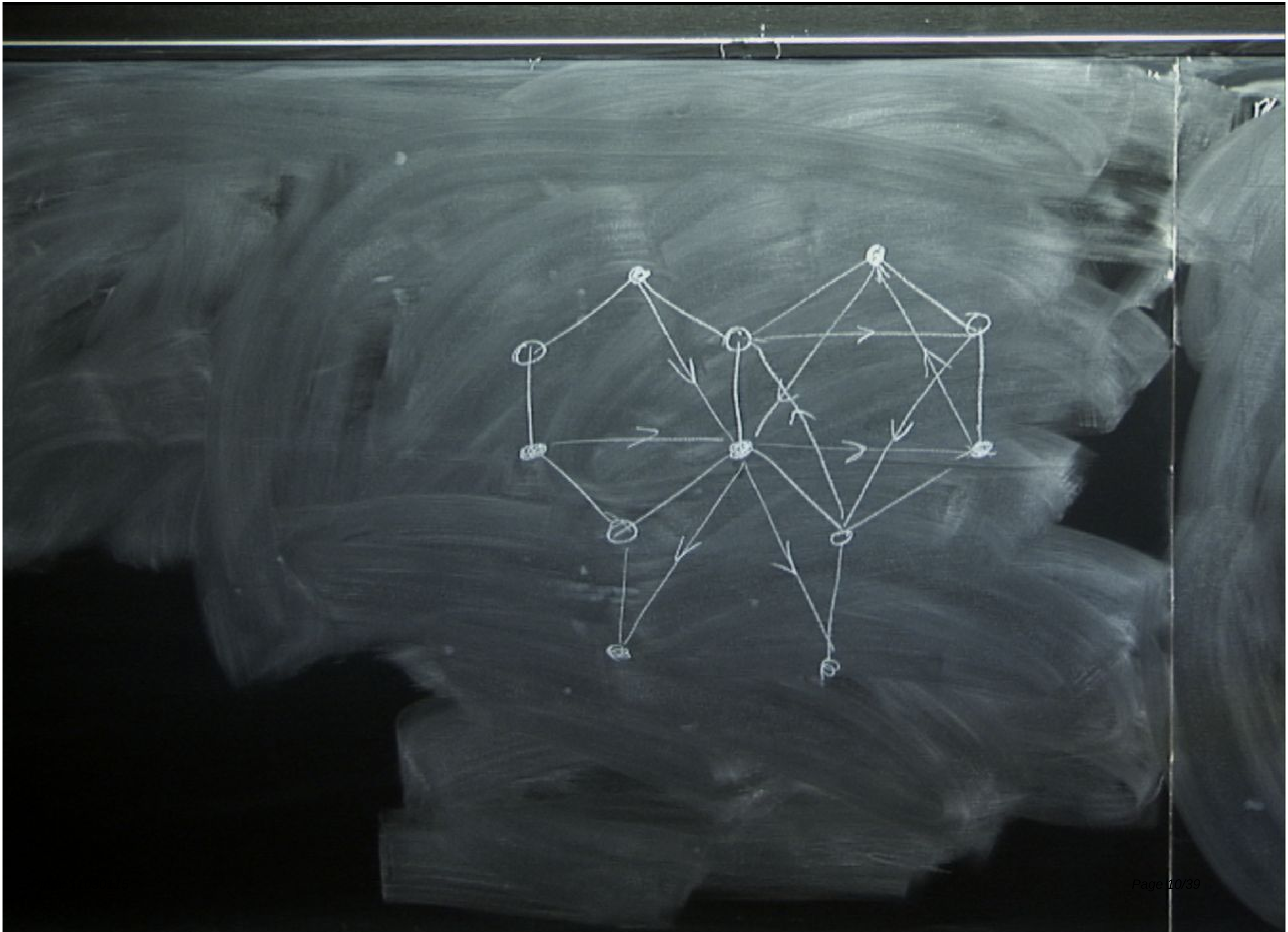
$$\sum \psi_k^{\dagger} H_k \psi_k \qquad H_k = \begin{pmatrix} & \gamma_k \\ \gamma_k^{*} & \end{pmatrix}$$

$$-t\left(e^{i\frac{k_y}{\sqrt{3}}} + 2\cos\frac{k_x}{2}\, e^{-i\frac{k_y}{2\sqrt{3}}}\right)$$

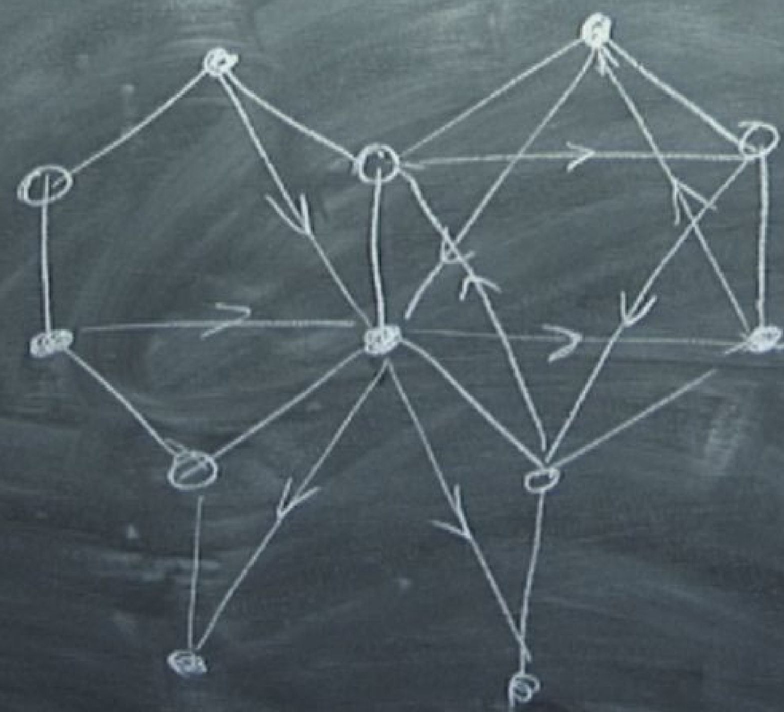
$$H = t\sum_{\langle ij\rangle} c_i^\dagger c_j + i\lambda \sum_{\langle\langle ij\rangle\rangle} \overset{\text{sign}}{\nu_{ij}}\, c_i^\dagger c_j$$

$$= \sum_k \Psi_k^\dagger H_k \Psi_k \qquad H_k = \begin{pmatrix} M_k & \gamma_k \\ \gamma_k^* & -M_k \end{pmatrix}$$

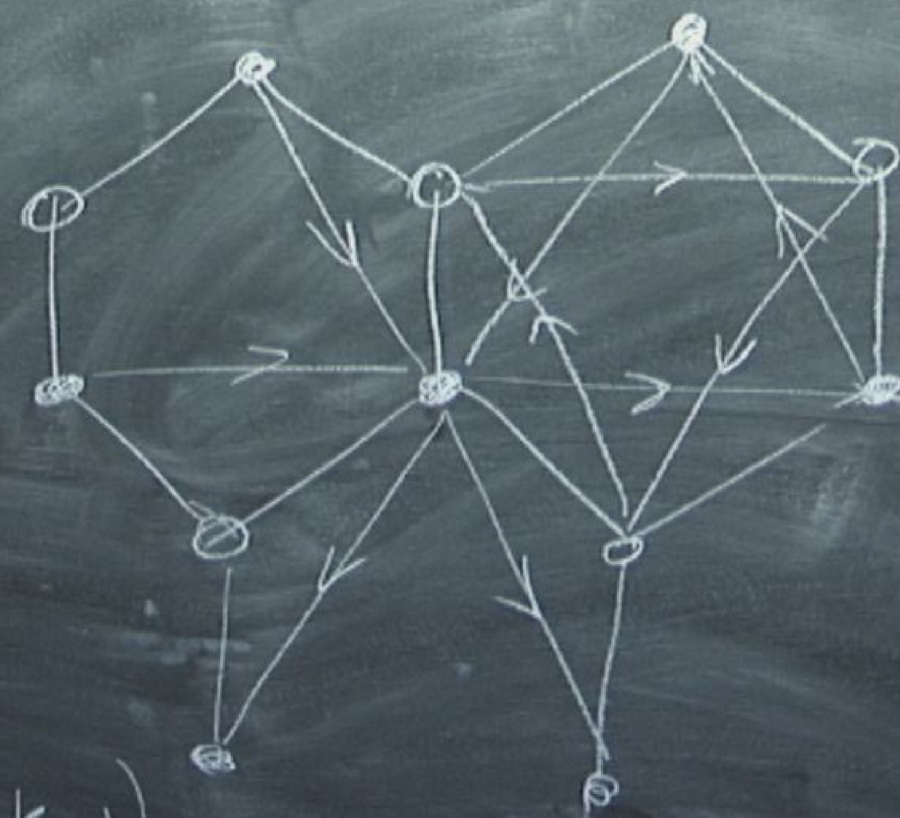
$$\gamma_k = t\left(e^{i\frac{k_y}{\sqrt{3}}} + 2\cos\frac{k_x}{2}\, e^{-i\frac{k_y}{2\sqrt{3}}}\right)$$

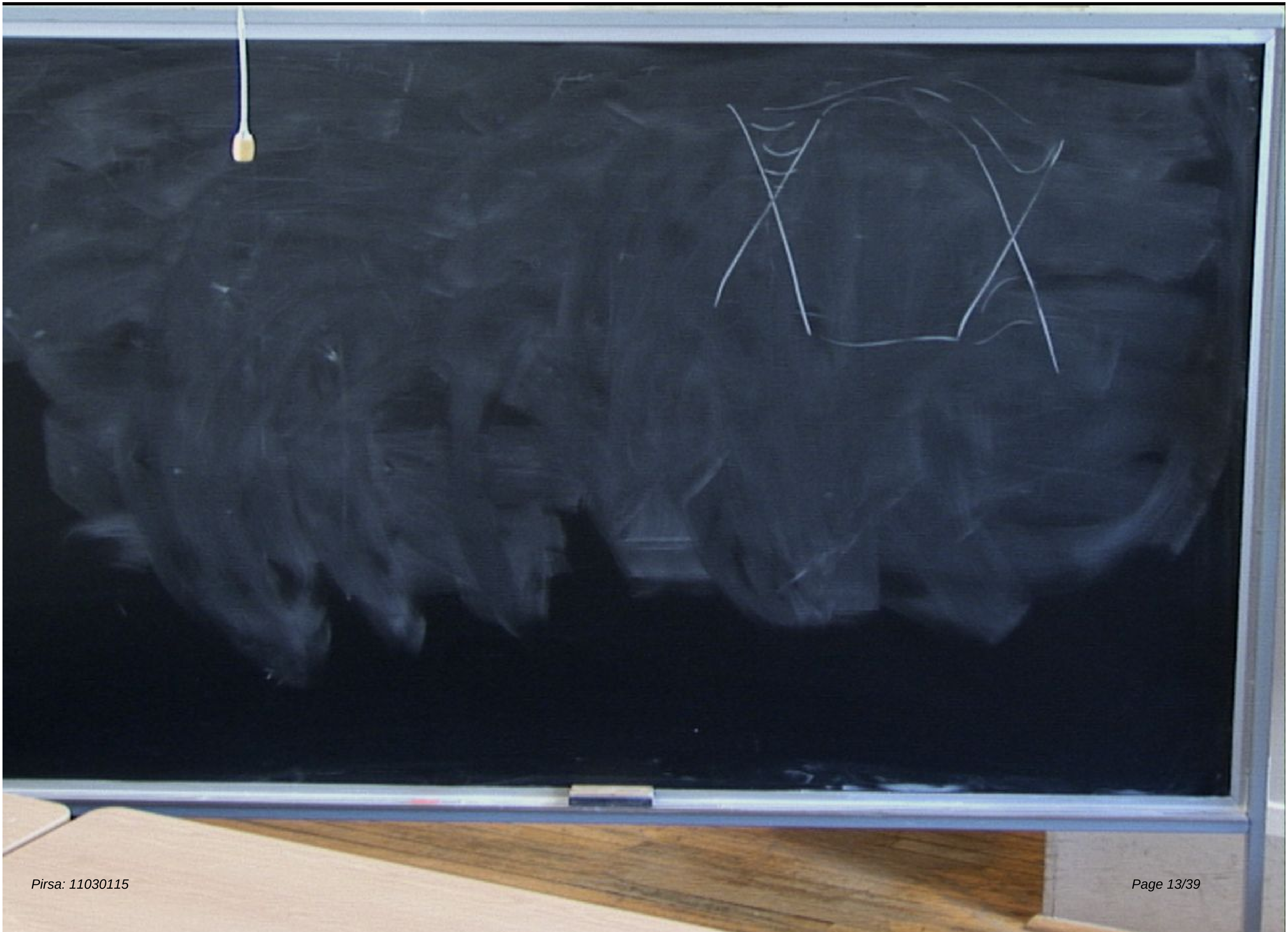


$i\lambda(2\sin k_x +$



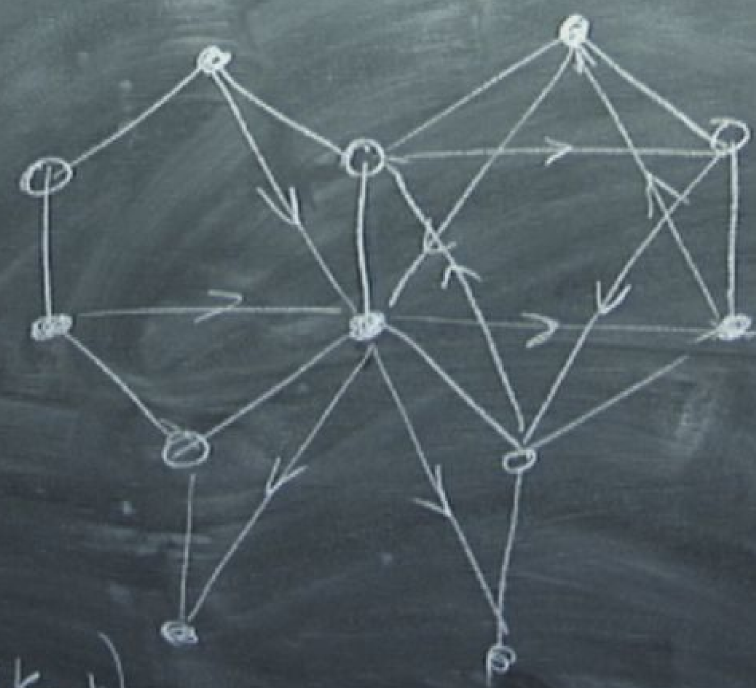
$$M_k = \lambda\left(2\sin k_x + 4\cos\left(\frac{\sqrt{3}k_y}{2}\right)\sin\left(\frac{k_x}{2}\right)\right)$$



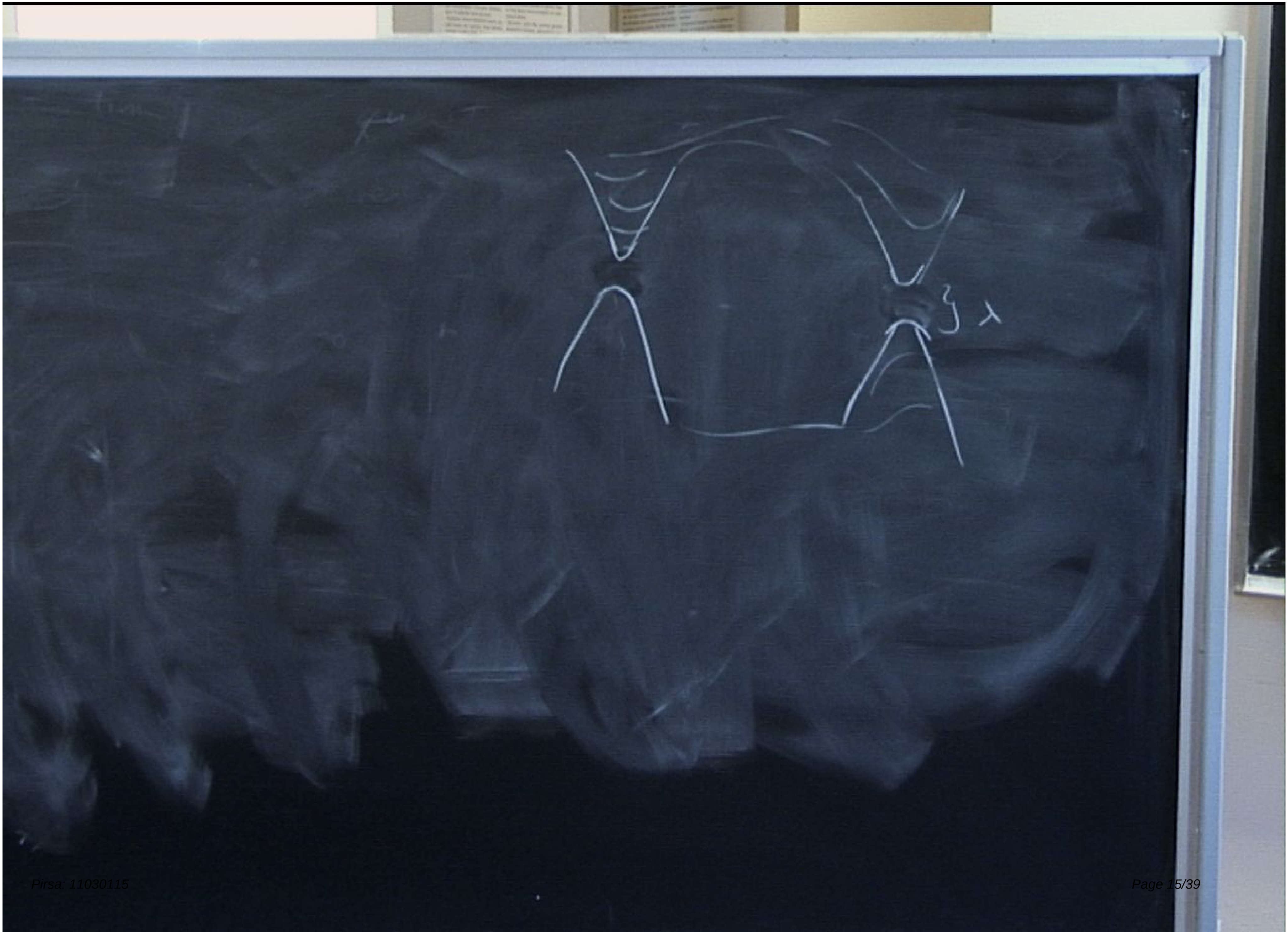


$$K = \left(\frac{4\pi}{3}, 0\right)$$

$$K' = \left(-\frac{4\pi}{3}, 0\right)$$



$$M_k = \lambda\left(2\sin k_x + 4\cos\left(\frac{\sqrt{3}k_y}{2}\right)\sin\left(\frac{k_x}{2}\right)\right)$$


λ

Linearized Haldane model

$$H_k \to \begin{pmatrix} & k_x+ik_y & & \\ k_x-ik_y & & & \\ & & & -k_x+ik_y \\ & & -k_x-ik_y & \end{pmatrix}$$

$H =$

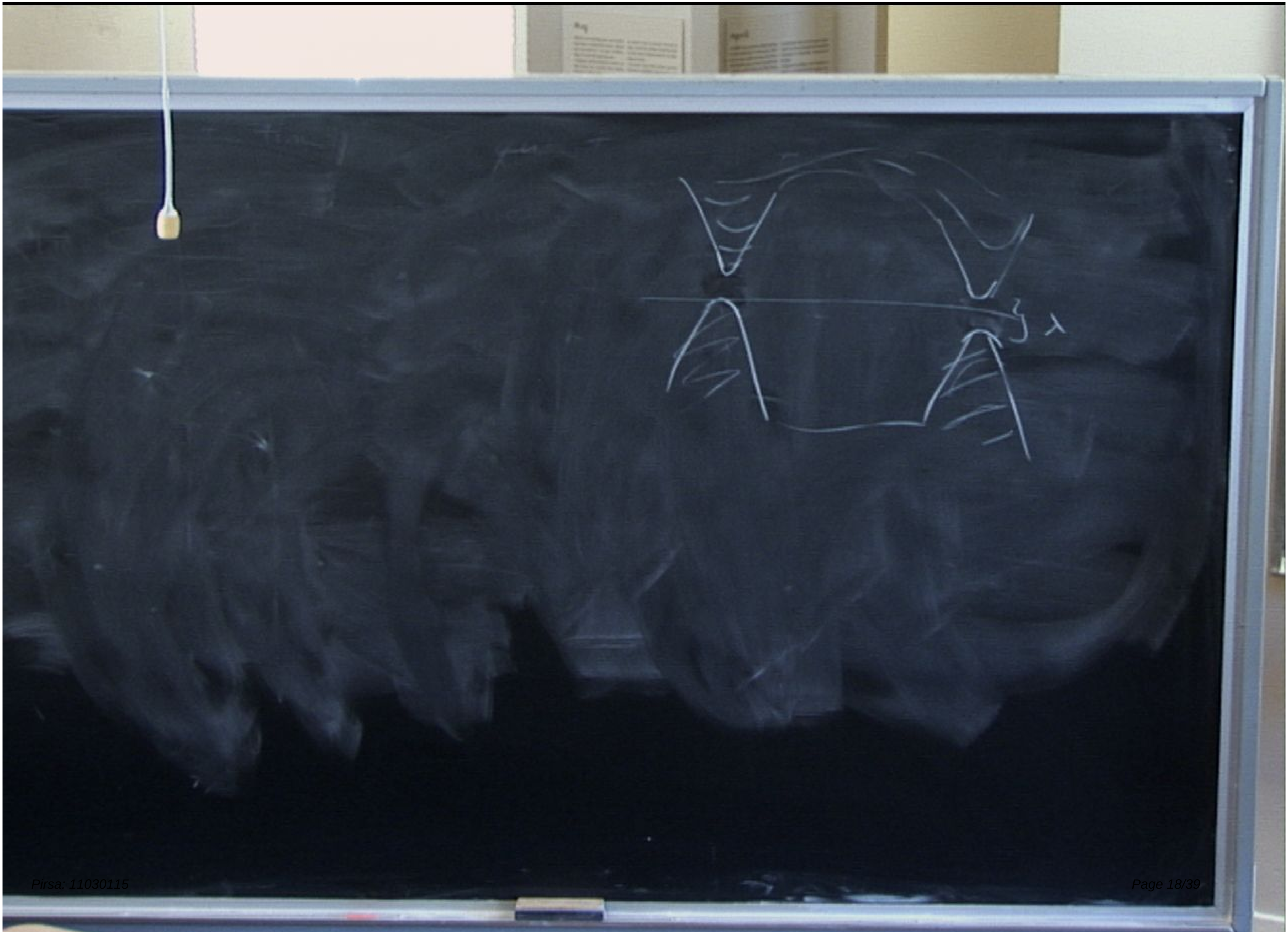
$\sin\left(\frac{k_x}{2}\right)\Big)$

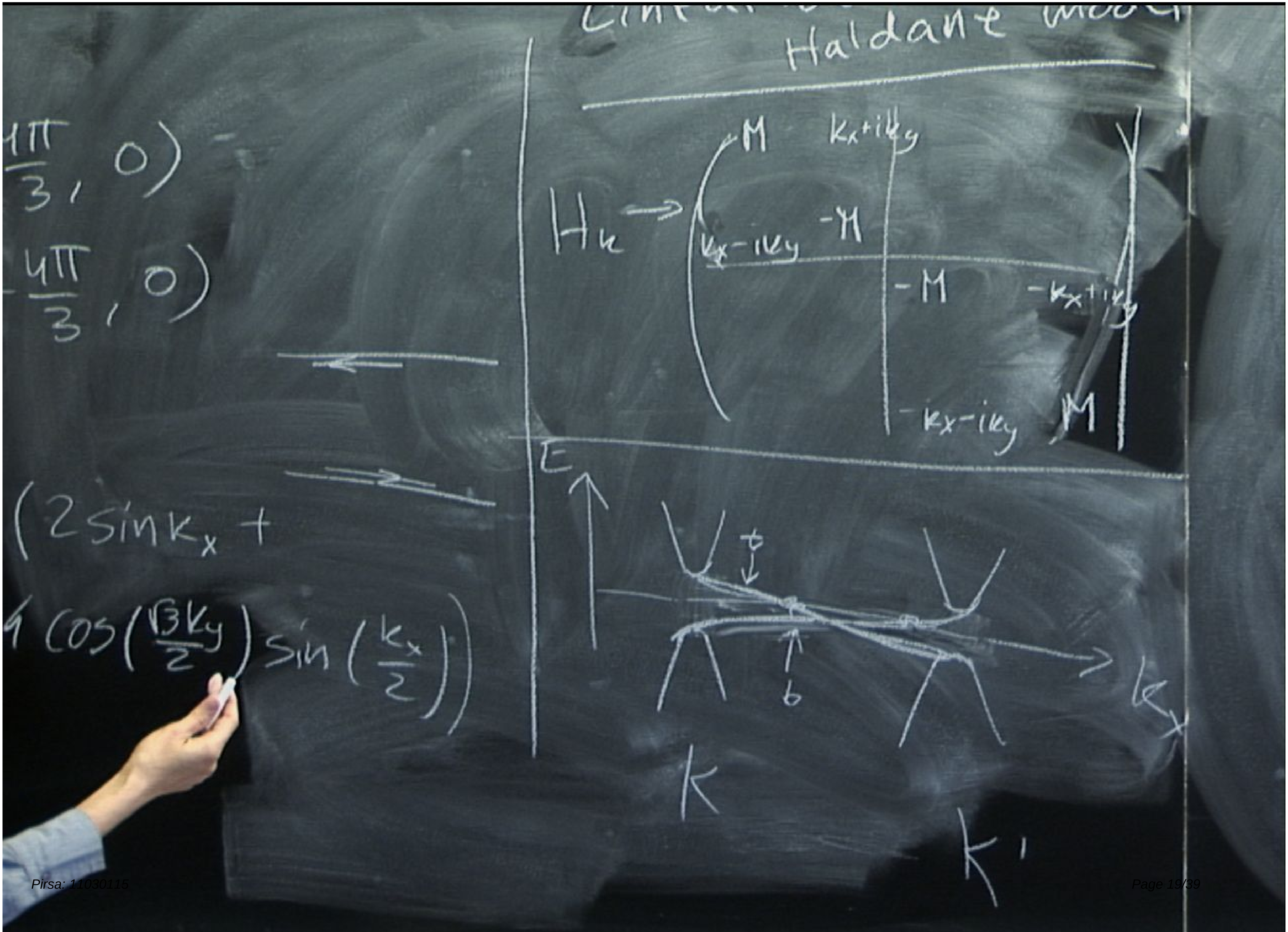
Linearized Haldane model

$$H_k \longrightarrow \begin{pmatrix} M & k_x + i k_y & & \\ k_x - i k_y & -M & & \\ & & -M & -k_x + i k_y \\ & & -k_x - i k_y & M \end{pmatrix}$$

$H =$

$\sin\left(\frac{k_x}{2}\right)\Big)$


λ


Haldane model
(4π/3, 0)
(-4π/3, 0)
(2 sin kx +
4 cos(√3 ky/2) sin(kx/2))
Hk →
M kx+iky
kx−iky −M
−M −kx+iky
−kx−iky M
E
t
b
kx
K
K'

Haldane model

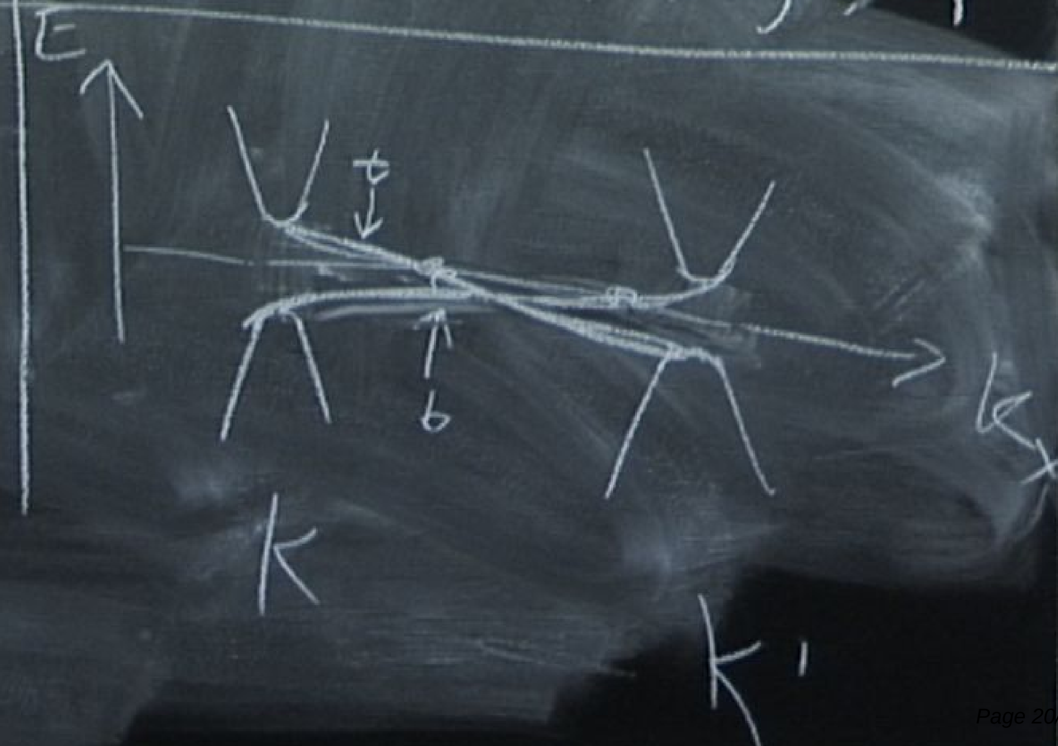
$$\left(\frac{4\pi}{3}, 0\right)$$

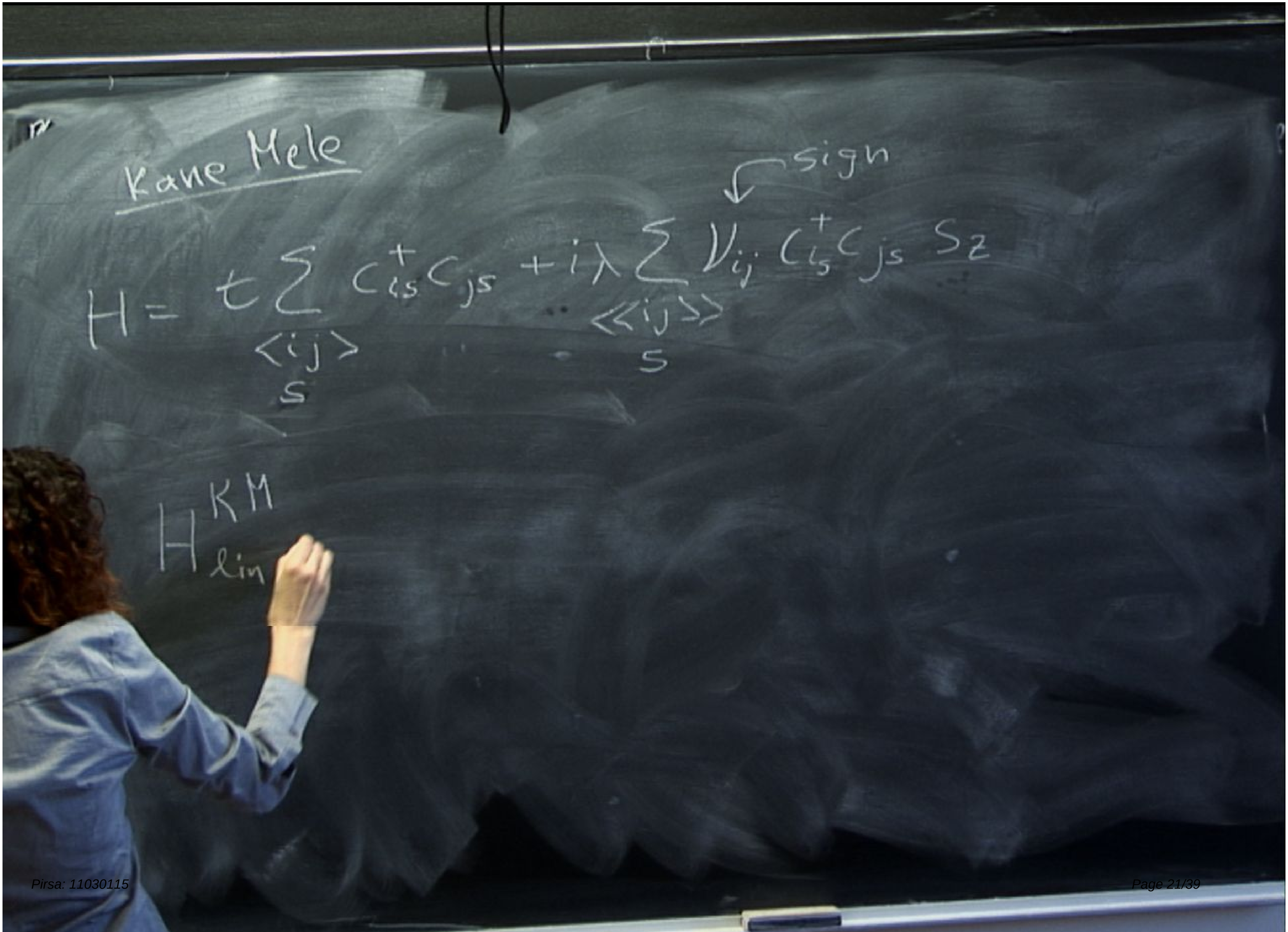
$$\left(-\frac{4\pi}{3}, 0\right)$$

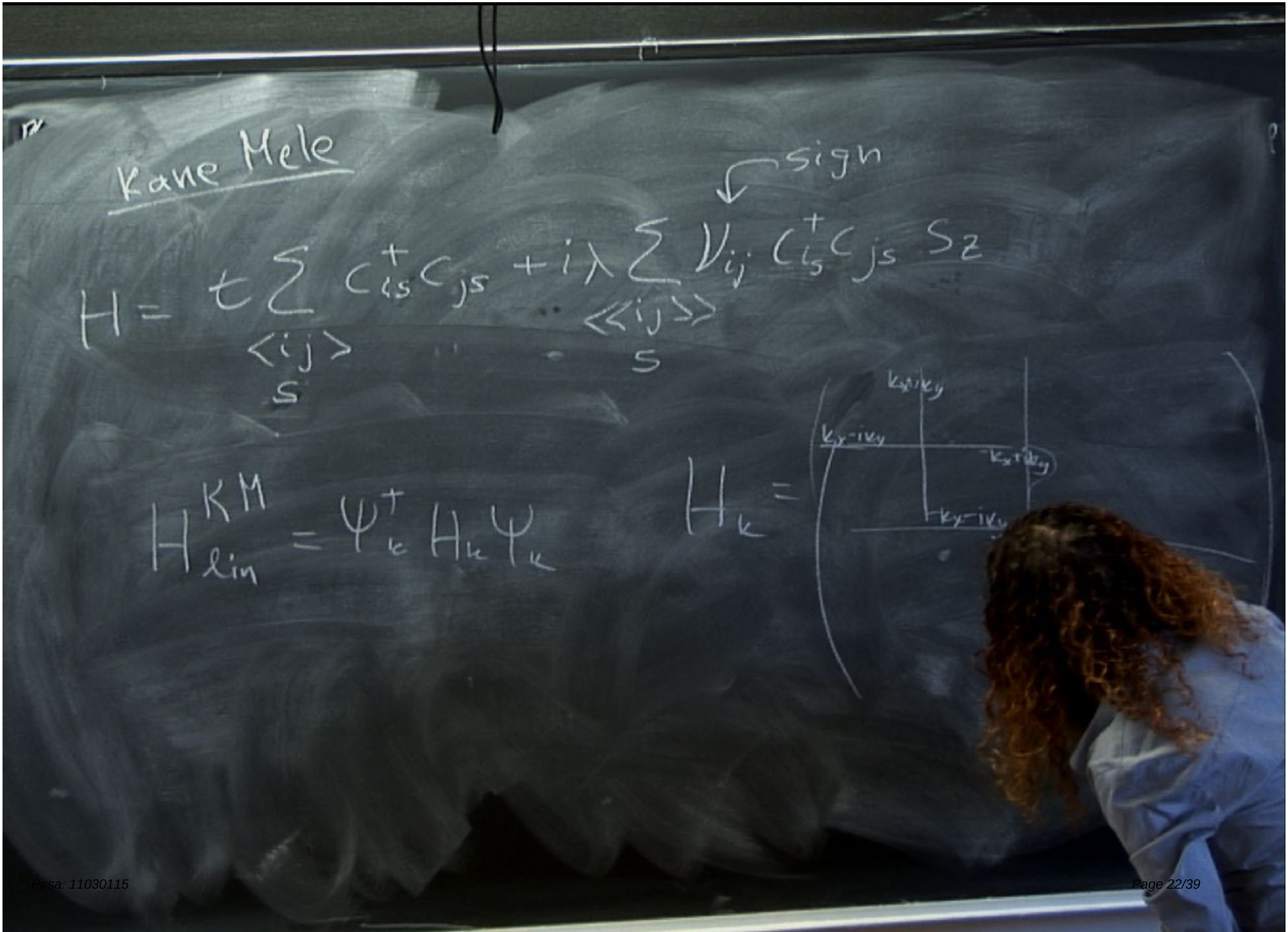
W

$$\left(2\sin k_x + 4\cos\left(\frac{\sqrt{3}k_y}{2}\right)\sin\left(\frac{k_x}{2}\right)\right)$$

$$H_k \longrightarrow \begin{pmatrix} M & k_x+ik_y & & \\ k_x-ik_y & -M & & \\ & & -M & -k_x+ik_y \\ & & -k_x-ik_y & M \end{pmatrix}$$




Kane Mele
sign
$H = t \sum_{\substack{\langle ij \rangle \\ s}} c^{\dagger}_{is} c_{js} + i\lambda \sum_{\substack{\langle\langle ij \rangle\rangle \\ s}} \nu_{ij} c^{\dagger}_{is} c_{js} s_z$
H^{KM}_{lin}


Kane Mele
sign
$H = t \sum_{\substack{\langle ij \rangle \\ s}} c^{\dagger}_{is} c_{js} + i\lambda \sum_{\substack{\langle\langle ij \rangle\rangle \\ s}} \nu_{ij} c^{\dagger}_{is} c_{js} s_z$
$H^{KM}_{lin} = \Psi^{\dagger}_k H_k \Psi_k$
$H_k =$

$$i\lambda \sum_{\substack{\langle\langle ij\rangle\rangle \\ s}} \nu_{ij}\, c^{\dagger}_{is} c_{js}\, s_z$$

$$H_k = \begin{pmatrix} & k_x+ik_y & & & & \\ k_x-ik_y & & -k_x+ik_y & & & \\ & -k_x-ik_y & & & & \\ & & & & k_x+ik_y & \\ & & & k_x-ik_y & & -k_x+ik_y \\ & & & & -k_x-ik_y & \end{pmatrix}$$

$$i\lambda \sum_{\substack{\langle\langle ij \rangle\rangle \\ s}} \nu_{ij}\, c^{\dagger}_{is} c_{js}\, s_z$$

$$
\begin{array}{c}
\begin{array}{cccc} \overbrace{\qquad\qquad}^{K\uparrow} & \overbrace{\qquad\qquad}^{K'\uparrow} & \overbrace{\qquad\qquad}^{K\downarrow} & \overbrace{\qquad\qquad}^{K'\downarrow} \end{array} \\
H_k = \left(\begin{array}{cc|cc|cc|cc}
m & k_x+ik_y & & & & & & \\
k_x-ik_y & -m & & & & & & \\
\hline
 & & -m & -k_x+ik_y & & & & \\
 & & -k_x-ik_y & m & & & & \\
\hline
 & & & & -m & k_x+ik_y & & \\
 & & & & k_x-ik_y & -m & & \\
\hline
 & & & & & & m & -k_x+ik_y \\
 & & & & & & -k_x-ik_y & -m
\end{array}\right)
\end{array}
$$

$$H = t \sum_{\substack{\langle ij \rangle \\ s}} c^{\dagger}_{is} c_{js} + i\lambda \sum_{\substack{\langle\langle ij \rangle\rangle \\ s}} \nu_{ij} c^{\dagger}_{is} c_{js}$$

$$H^{KM}_{lin} = \Psi^{\dagger}_k H_k \Psi_k$$

$$H_k = \begin{pmatrix} \overbrace{\begin{matrix} m & k_x+ik_y \\ k_x-ik_y & -m \end{matrix}}^{K\uparrow} & \\ & -m \end{pmatrix}$$

$\sigma \to$ sublattice

$s \to$ spin

$\tau \to$ valley

$$H_k = \tau k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z$$

$$H = t\sum_{\langle ij\rangle s} c^{\dagger}_{is} c_{js} + i\lambda \sum_{\langle\langle ij\rangle\rangle s} \nu_{ij}\, c^{\dagger}_{is} c_{js}$$

$$H^{KM}_{lin} = \Psi^{\dagger}_{k} H_k \Psi_k \qquad H_k = \begin{pmatrix} m & k_x + i k_y & & \\ k_x - i k_y & -m & & \\ & & -m & \\ & & & \end{pmatrix}$$

$K\uparrow$

$\sigma \rightarrow$ sublattice

$s \rightarrow$ spin

$\tau \rightarrow$ valley

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$

Jackiw & Rebbi PRD 73

Jackiw & Rebbi PRD 75

$$H = -i\vec{\nabla}\cdot\sigma_\perp + m\sigma_z$$

$$\psi_{edge} \propto e^{ik_x x - my}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{pmatrix} m & -i(\partial_x - i\partial_y) \\ -i(\partial_x - i\partial_y) & -m \end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}e^{ikx-my} = E\begin{pmatrix}1\\1\end{pmatrix}e^{ikx-my}$$

$\sigma \rightarrow$ s

$S \rightarrow$ s

$\tau \rightarrow$

Jackiw & Rebbi   PRD 75

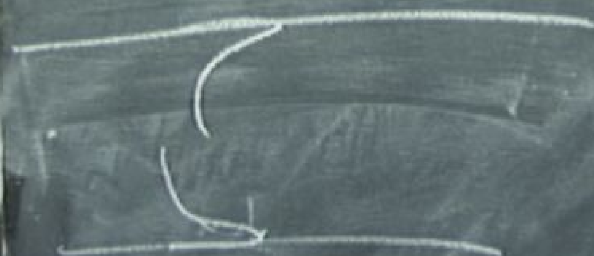
$$H = -i\vec{\nabla}\cdot\sigma_{\perp} + m\sigma_z$$

$$\psi_{edge} \propto e^{ik_x x - my}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\cancel{m} + k - \cancel{m} = E$$

$$\boxed{E = k}$$

$$\begin{pmatrix} m & -i(\partial_x - i\partial_y) \\ -i(\partial_x - i\partial_y) & -m \end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}e^{ikx+my} = E$$



$$H^{KM}_{lin} = \Psi^\dagger_k H_k \Psi_k$$

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$

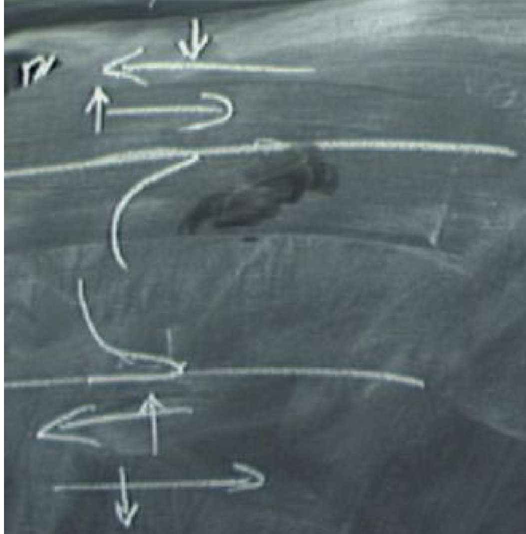
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{ikx - my}$$

$$\begin{pmatrix}1\\1\end{pmatrix} e^{ikx-my}$$

$$H^{KM}_{lin} = \Psi^\dagger_k H_k \Psi_k$$

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$



$$H_{SO} = i\lambda \sum_{\langle\langle ij\rangle\rangle} \nu_{ij}\, c_i^\dagger c_j$$

$$H^{KM}_{lin} = \Psi^\dagger_k H_k \Psi_k$$

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$

$$H_{so} = i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^\dagger c_j$$

$$\vec{S}\cdot(\vec{\nabla} V \times \vec{p})$$

$$H^{KM}_{lin} = \Psi_k^\dagger H_k \Psi_k$$

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m\, \tau_z s_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$

$$H_{SO} = i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^{\dagger} c_j$$

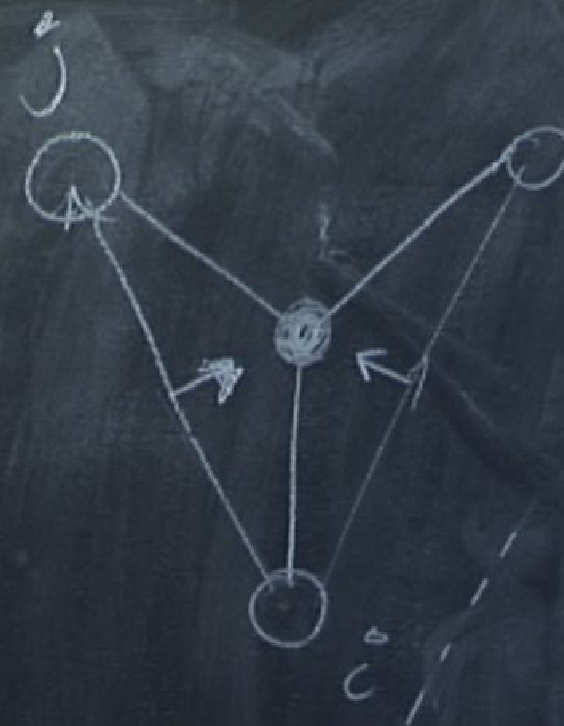
$$\vec{S}_z \cdot (\vec{\nabla} V \times \vec{p})$$

$$H^{KM}_{lin} = \Psi^{\dagger}_k H_k \Psi_k$$

$$\boxed{H_k = \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z S_z \sigma_z}$$

$$E_k = \pm\sqrt{k_x^2 + k_y^2 + m^2}$$

$$H_{so} = i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^\dagger c_j$$

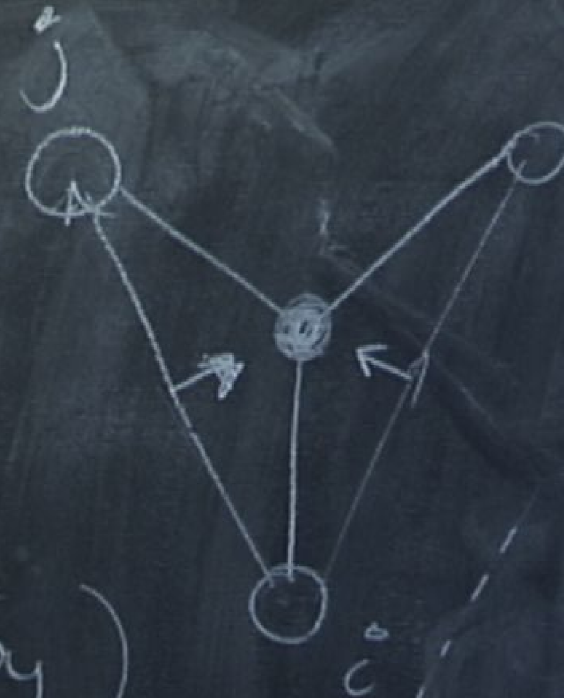


$$\vec{S}_z \cdot (\vec{\nabla} V \times \vec{p})$$

$$\Psi_k^\dagger H_k \Psi_k$$

$$\boxed{= \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z S_z \sigma_z}$$

$$H_{SO} = i\lambda \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^{\dagger} c_j$$



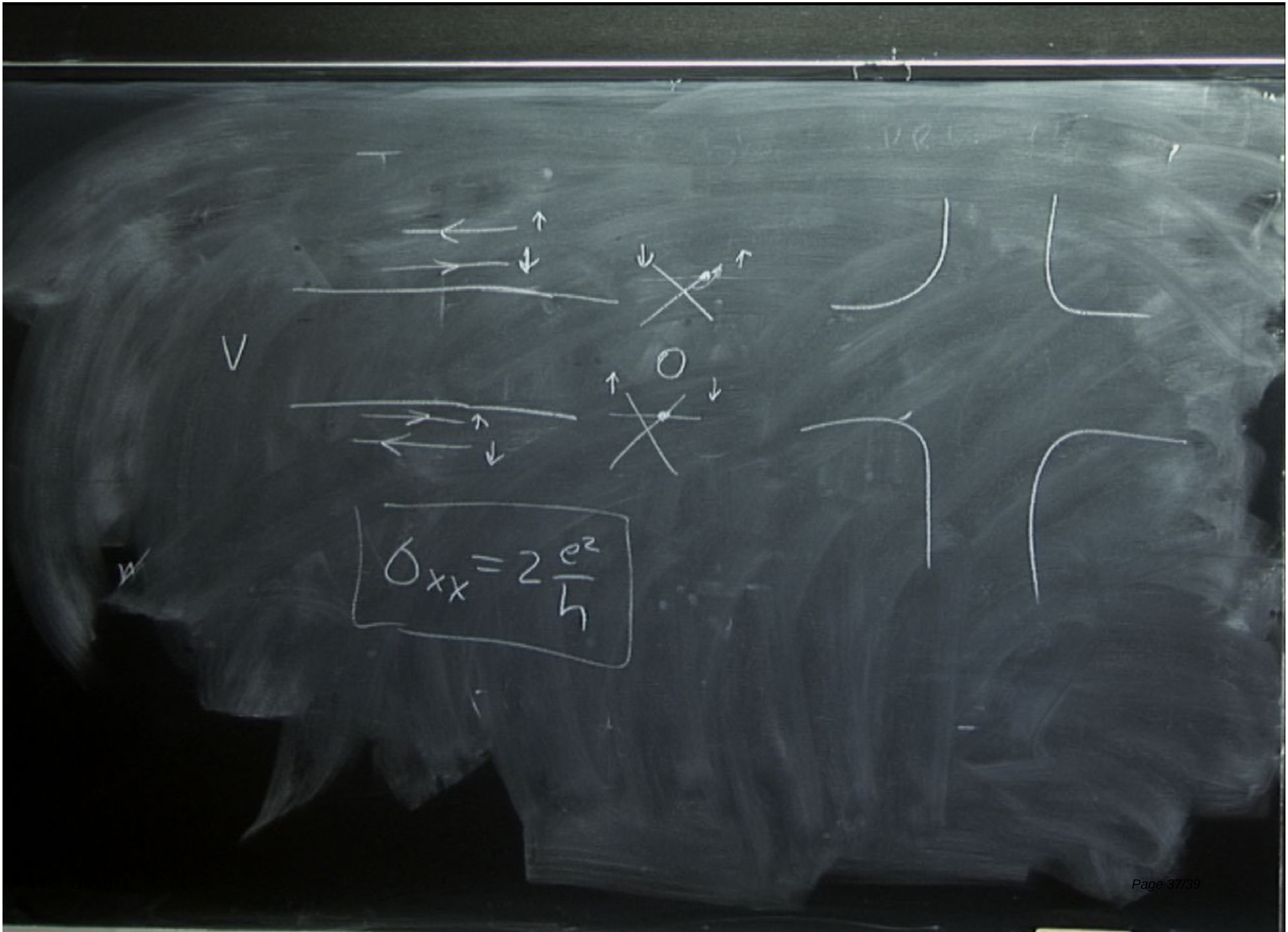
$$\vec{S}_z \cdot (\vec{\nabla} V \times \vec{p})$$

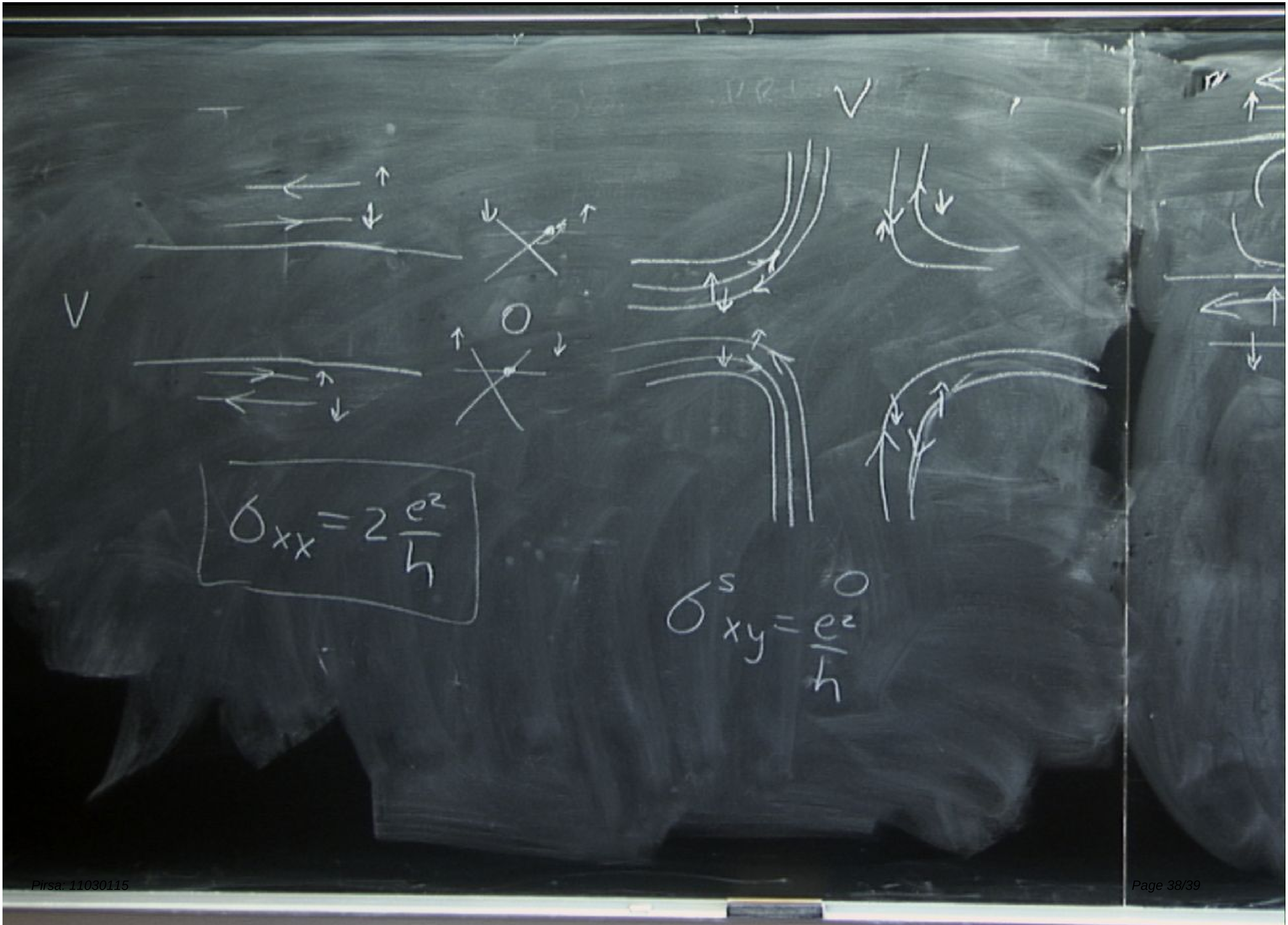
Rashba

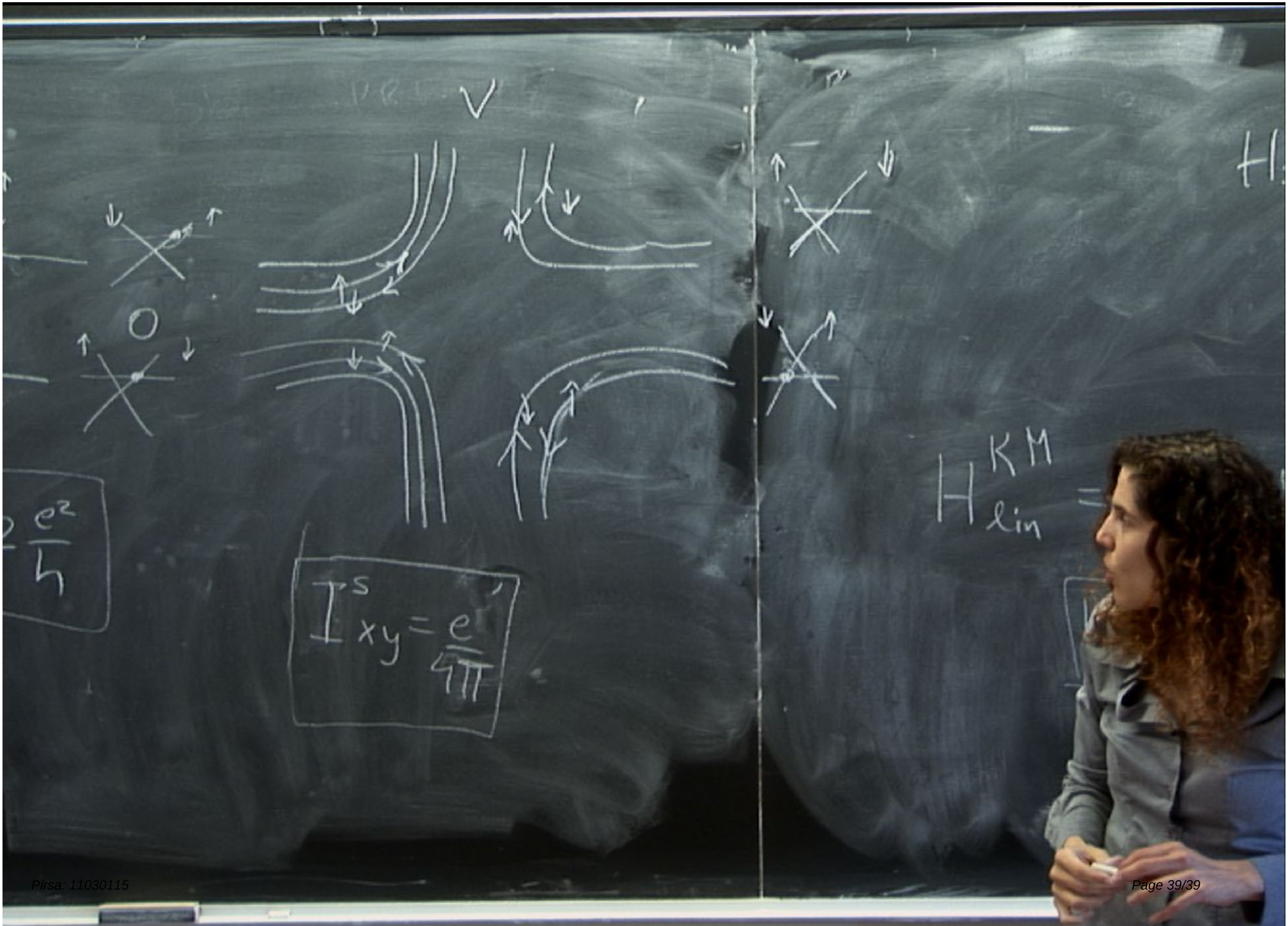
$$\Lambda(\tau_3 \sigma_y s_x - \sigma_x s_y)$$

$$\Psi_k^{\dagger} H_k \Psi_k$$

$$= \tau_z k_x \sigma_x + k_y \sigma_y + m \tau_z s_z \sigma_z$$


V
0
$\sigma_{xx} = 2\frac{e^2}{h}$


V
V
0
$\sigma_{xx} = 2\frac{e^2}{h}$
$\sigma^s_{xy} = \frac{e^2}{h}$
0


V
O
$\frac{e^2}{h}$
$I^{s}_{xy} = \frac{e}{4\pi}$
$H^{KM}_{lin} =$