Title: Gravitational wave phenomenology for LIGO, LISA and co from effective field theories

Date: Mar 31, 2011 01:00 PM

URL: http://pirsa.org/11030114

Abstract: The effective field theory framework yields a systematic treatment of gravitational bound states such as binary systems. Gravitational waves emitted from compact binaries are one of the prime event candidates at direct detection experiments. Due to the multiple scales involved in the binary problem, an effective field theory treatment yields many advantages in perturbative calculations. My talk will review the setup of the effective field theory framework and report on recent progress in gravitational wave phenomenology.

Outline

1. Motivation

gravitational waves, LIGO, VIRGO, LISA..., binary inspirals with black hole or neutron star constituents, post-Newtonian expansion

2. EFT Setup

scales in binary inspiral, integrating out short-distance physics step-by-step, potential and radiation gravitons, finite size effects, spin

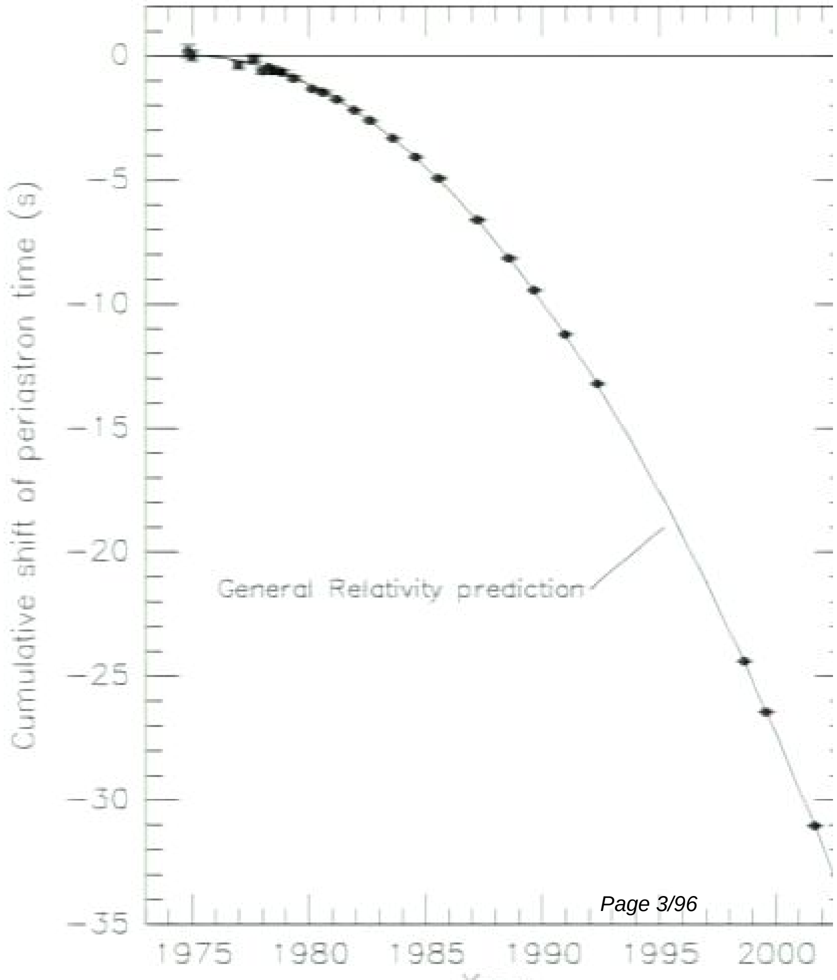
3. Radiation Sector, Radiative Effects & Spin

expansion parameters, calculating the power loss, tail effects, classical RG running of multipoles, PN matching for multipoles, spin effects

4. Outlook & Conclusion

Motivation

- Einstein's GR predicts gravitational waves (GWs)
- so far "only" indirect evidence for GWs
  → Hulse & Taylor, Nobel Prize 1993
- direct detection expected in the next few years

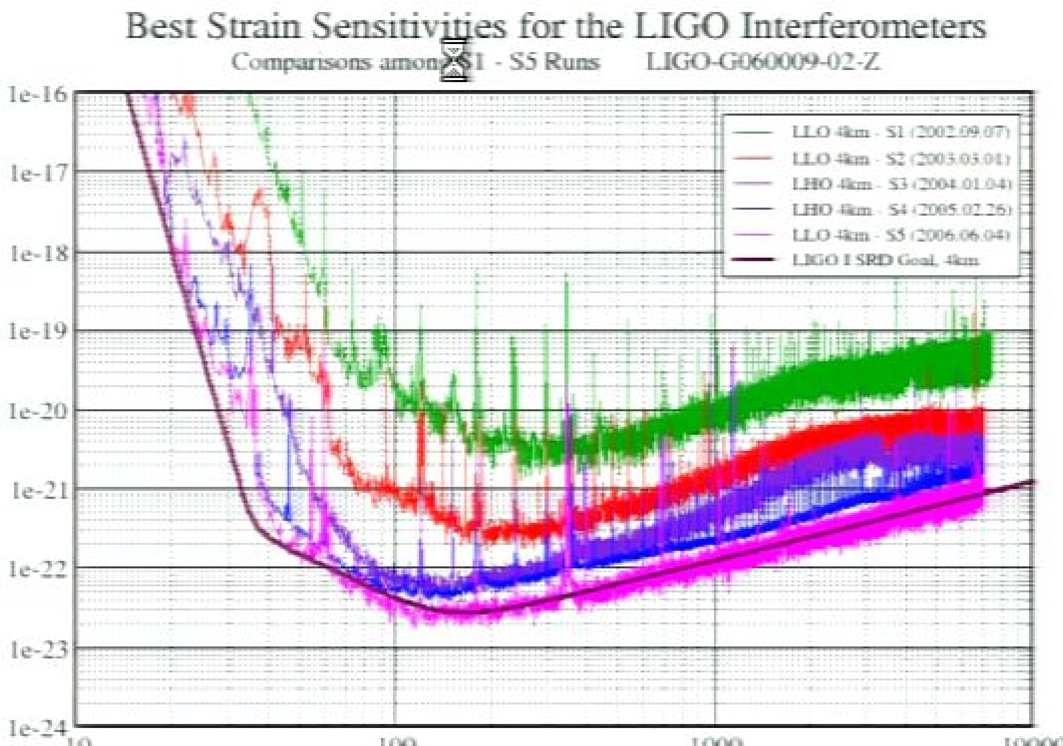


Motivation

- interferometers for GW detection LIGO, VIRGO, TAMA, GEO (now, earth) & LISA (future, space)

Motivation

- ground-based interferometers for GW detection LIGO, VIRGO, TAMA, GEO
- sensitivity $\Delta L/L \lesssim 10^{-21}$ for $\nu \sim 10 - 10^4 Hz$

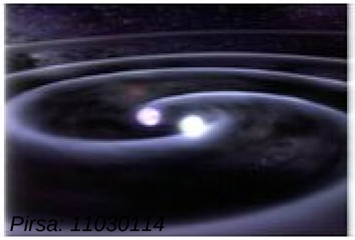


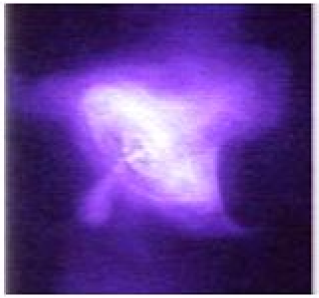
from LIGO website

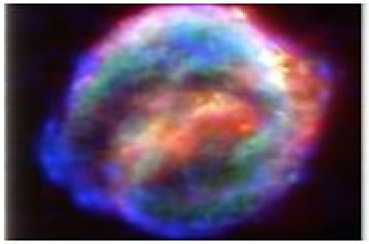
Motivation

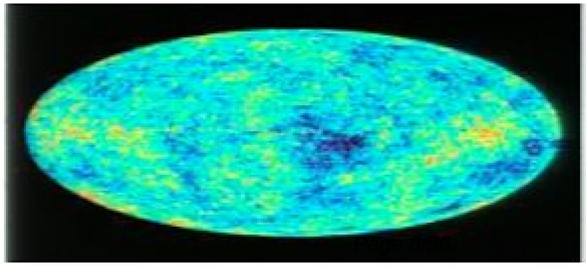
Possible Sources for GWs at LIGO & co:

- coalescence of compact binaries, with black hole (BH) or neutron star (NS) constituents
- pulsars
- supernovae/bursts
- stochastic background









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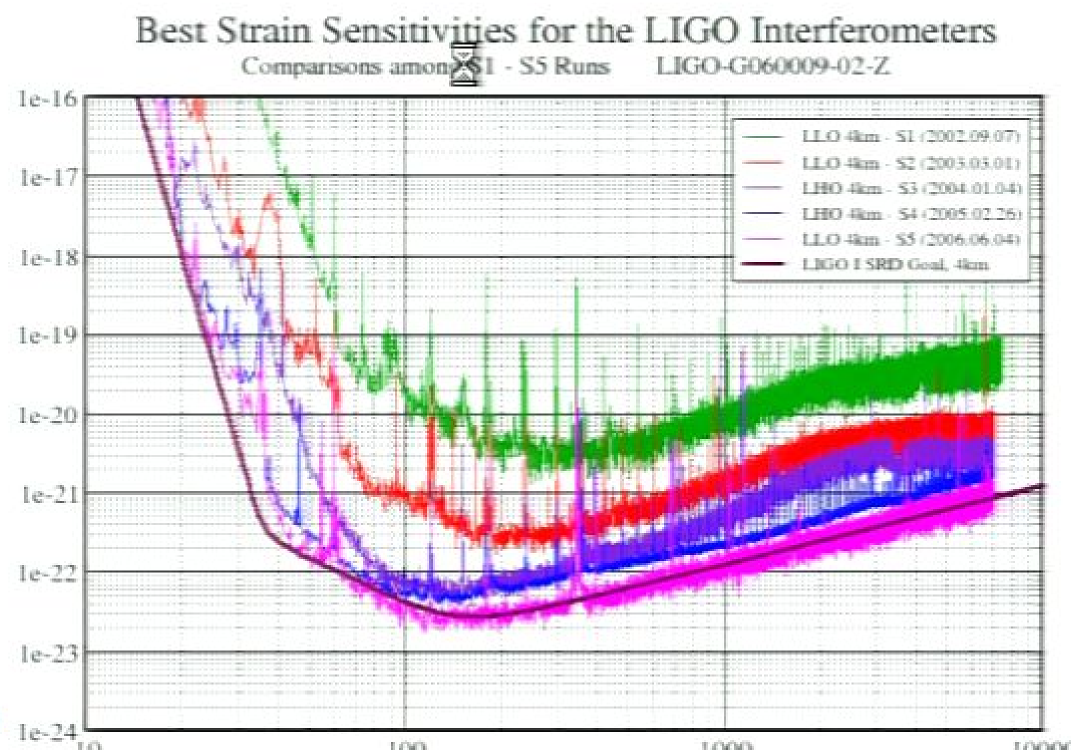

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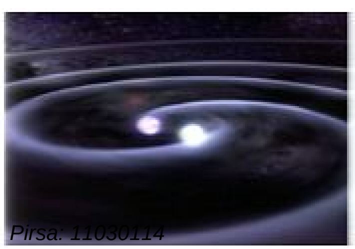
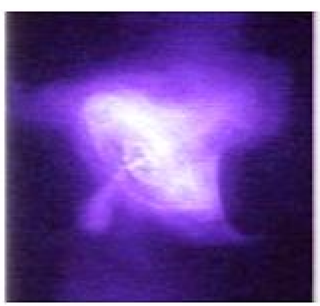
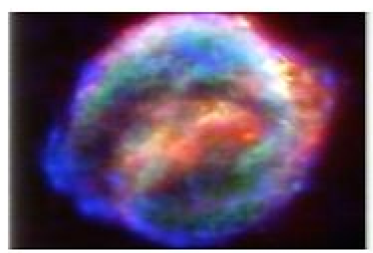
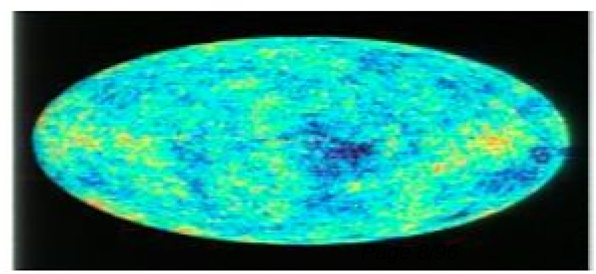

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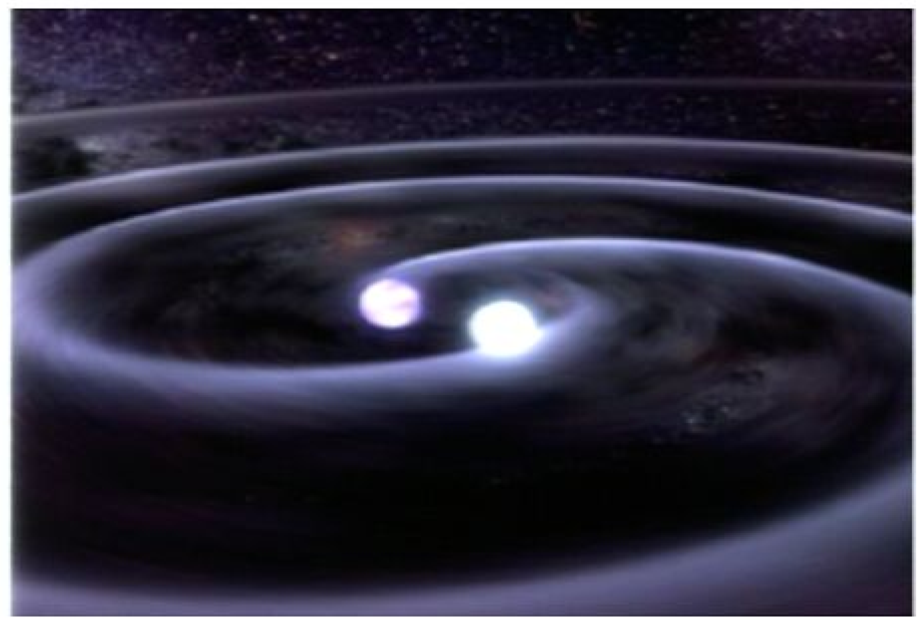


→ this talk: focus on **compact binaries**

Motivation

- upgrade to Advanced LIGO, running in 2015
- Advanced LIGO has roughly 10 times higher sensitivity, which increases the event rate by a factor of ~ 1000
- estimated event rate for binary inspirals:

BH/BH: $\sim 1 - 500$ per year

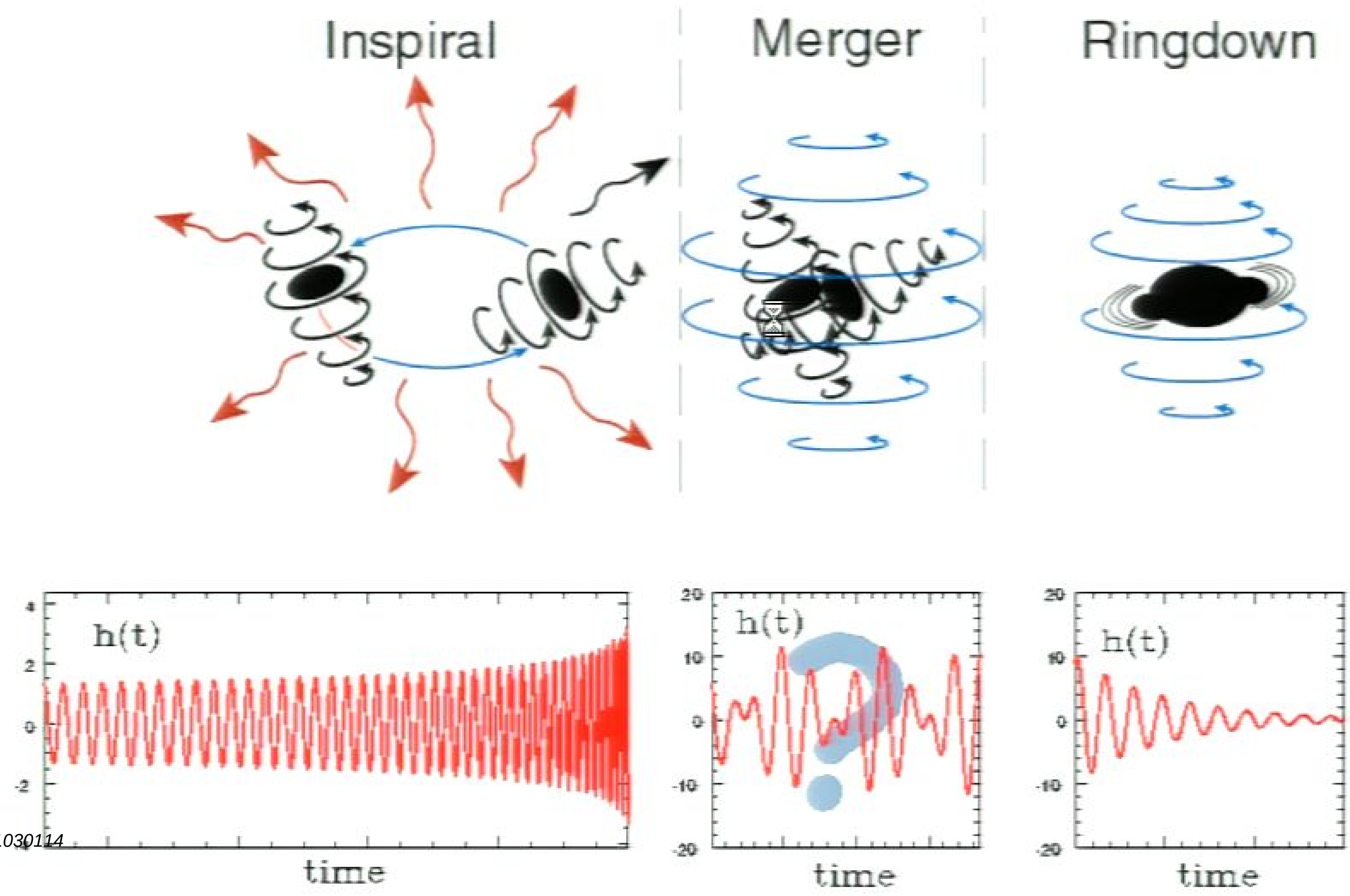
BH/NS: $\sim 1 - 30$ per year

NS/NS: $\sim 10 - 100$ per year

from: M. Maggiore, "Gravitational Waves"

Motivation

Binary Black Hole Mergers



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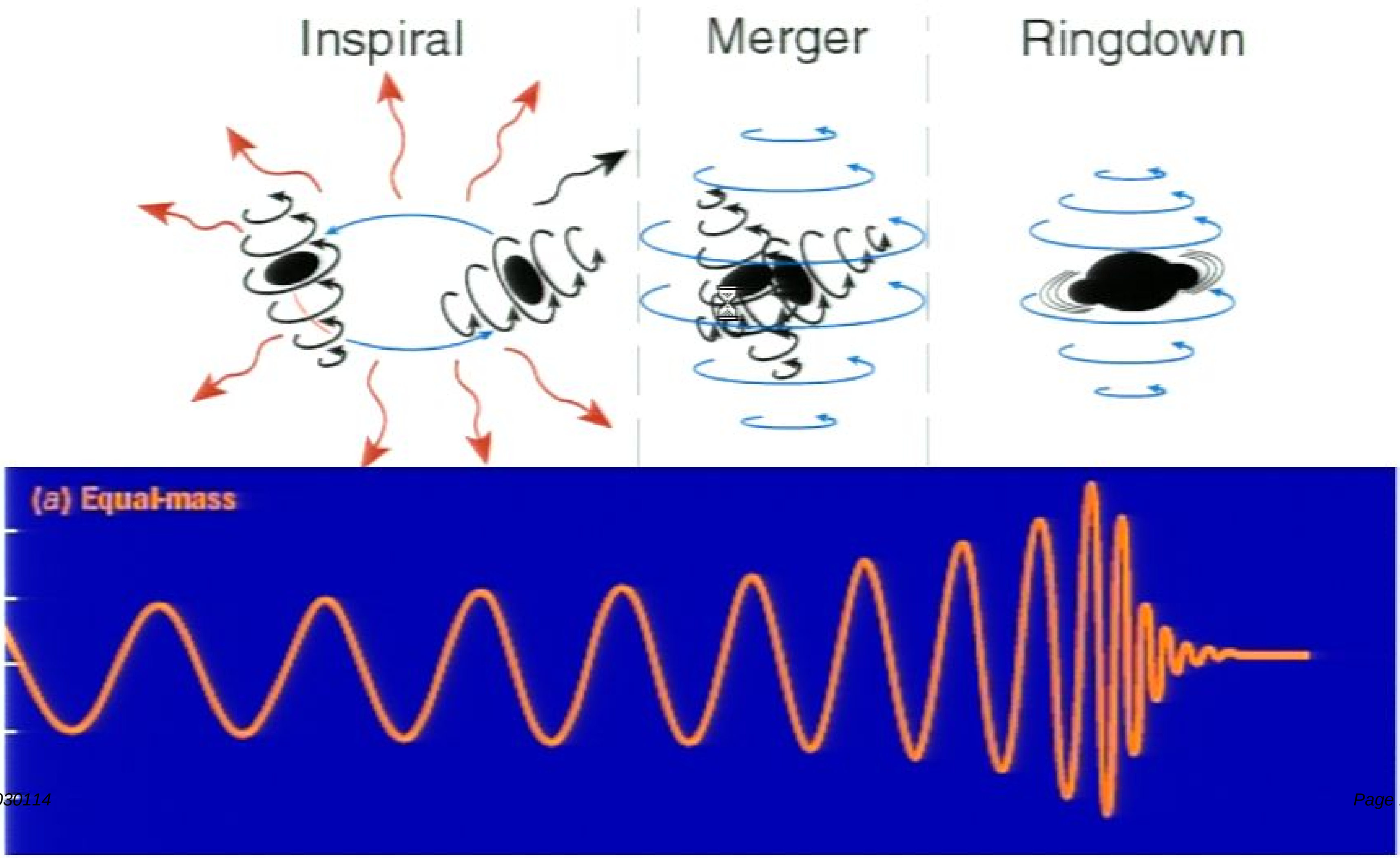

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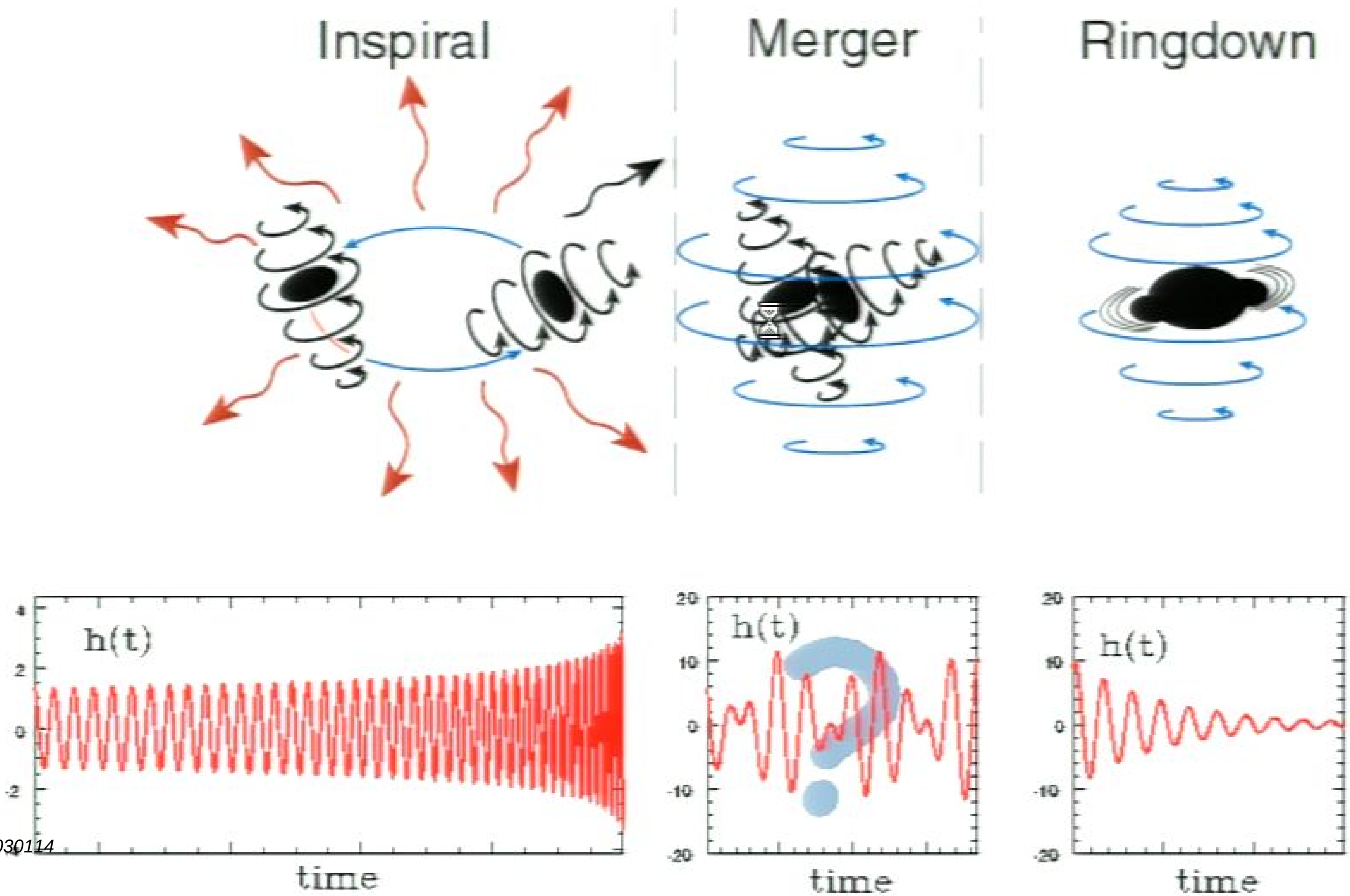

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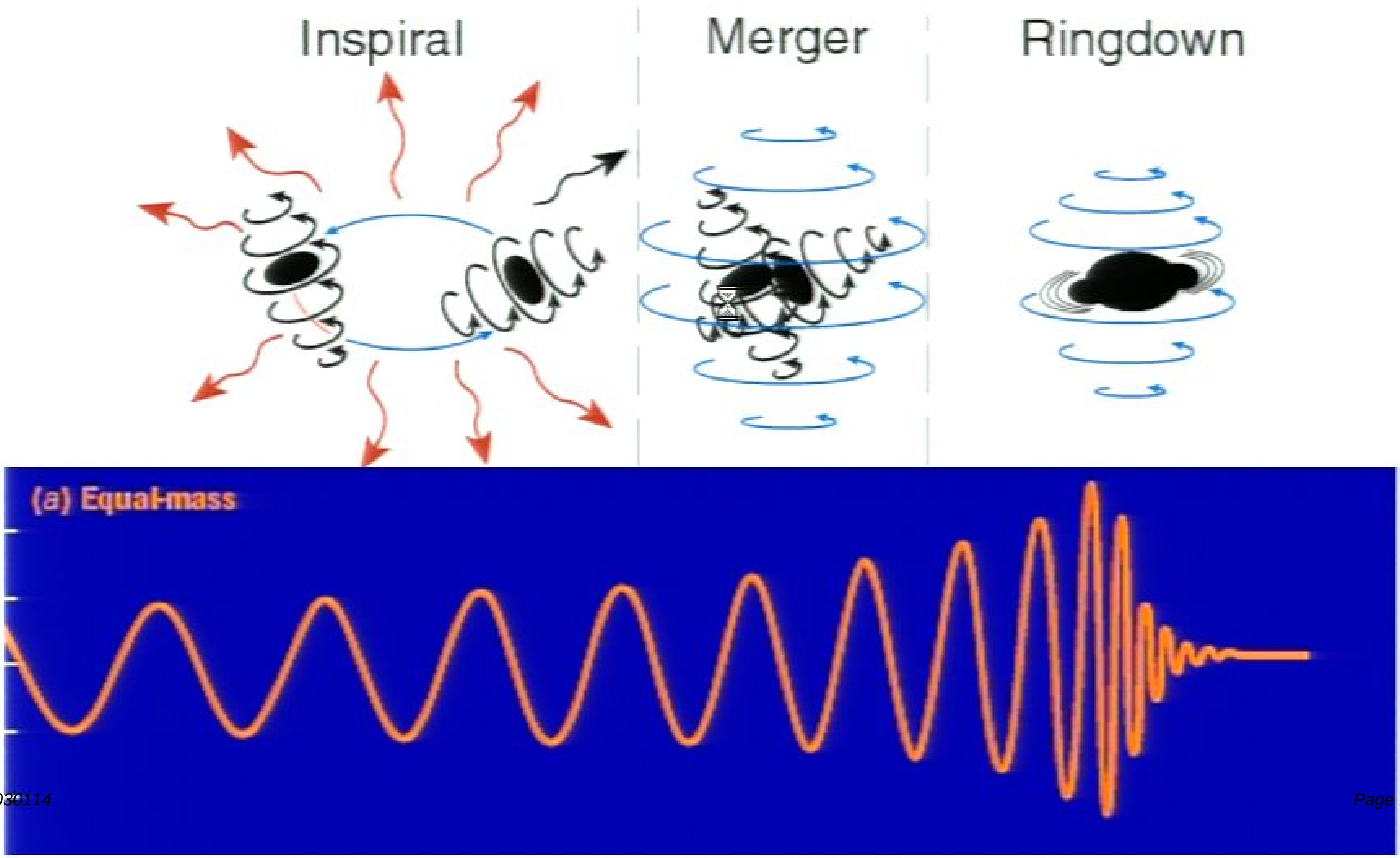

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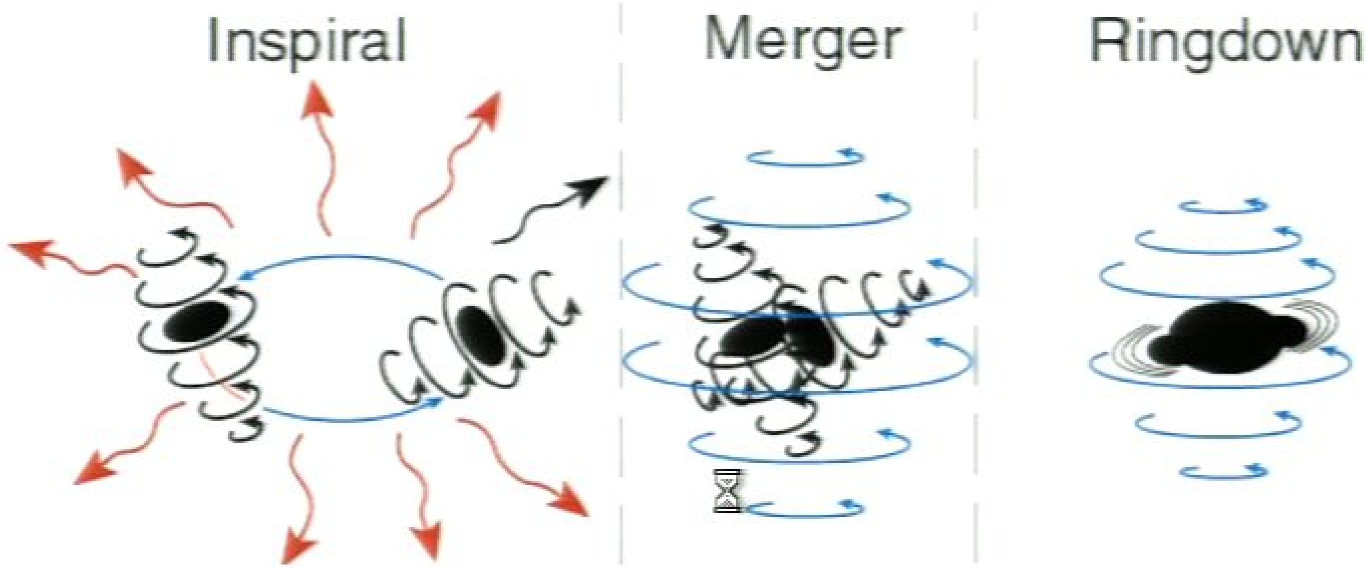
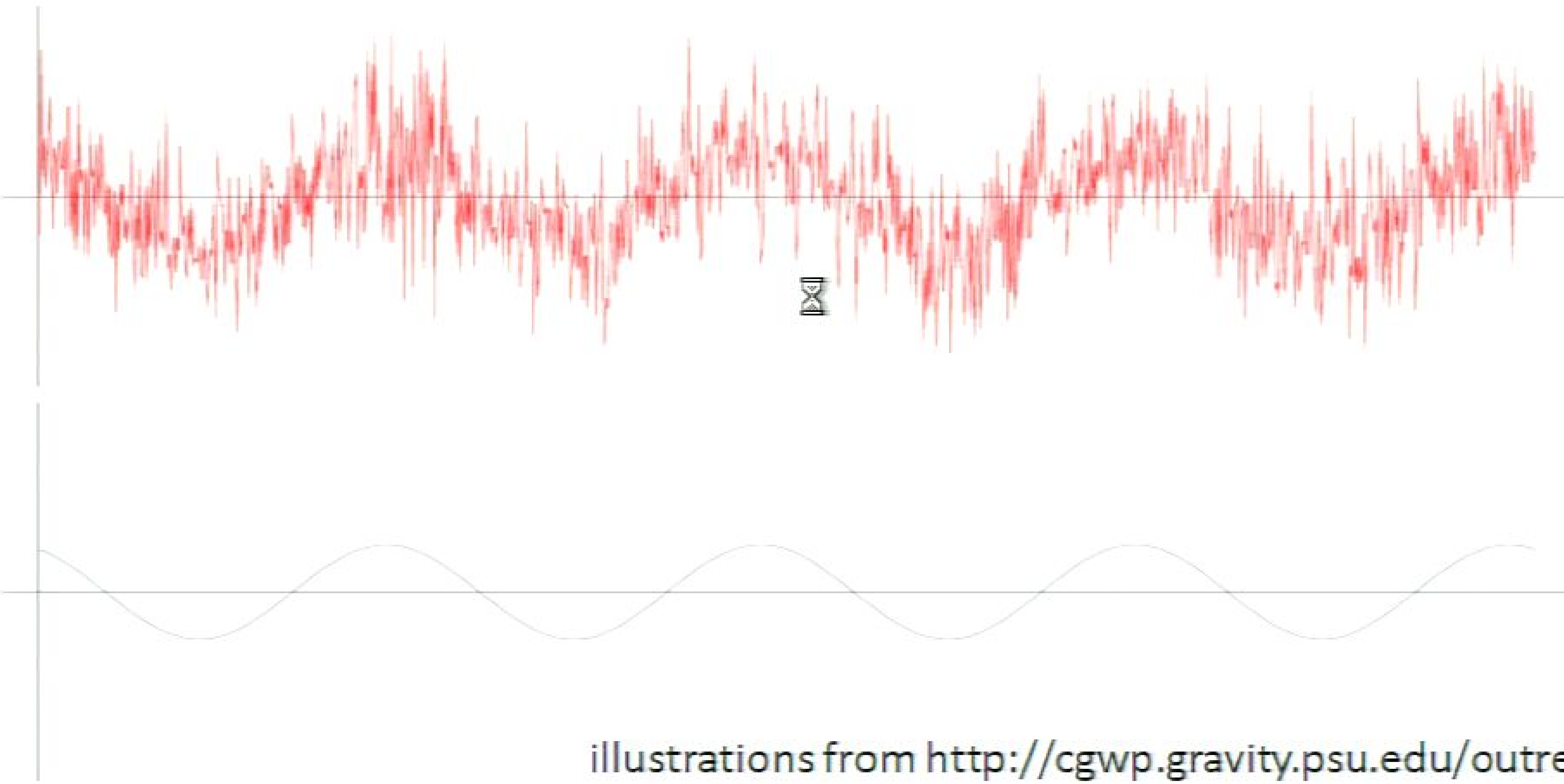


- inspiral can be calculated perturbatively in an expansion in $v^2 \sim GM/r$, up to $\sim 10^4$ cycles observable in LIGO band
- merger from numerical computations
- ringdown from quasi normal modes analytically

Motivation

Detection of GW signal via matched filtering



illustrations from http://cgwp.gravity.psu.edu/outreach/

- phase needs to match during all cycles observed

Motivation

- need to know phase to precision $\Delta\phi/\phi \lesssim 10^{-3}$
- from $P = -\frac{dE}{dt}$ obtain phase $\frac{d\phi}{d\omega^2} \sim \frac{dE/d\omega}{P}$

  $\rightarrow$ need to know $E(\omega)$ and $P(\omega)$ to $\mathcal{O}(v^6)$ beyond leading "Newtonian" order in the post-Newtonian expansion in $v^2 \sim GM/r$, i.e. at "3PN" Cutler et al., astro-ph/9208005
- amplitude of higher precision important, e.g. for improving LISA's angular resolution or for comparison of analytic and numerical results

Motivation

State of the art & goals for current project

- neglecting spin: 3.5PN phase & 3PN amplitude
  Blanchet et al., gr-qc/0105099, gr-qc/0406012; arXiv:0802.1249
- including spin: 2.5PN phase & 2PN amplitude
  Buonanno et al., gr-qc/0605140; arXiv:0810.5336 [gr-qc]
- spin for BHs in binaries commonly close to maximal
  -> match precision of waveforms with spin
- in collaboration with Rafael Porto, Ira Rothstein and Michele Vallisneri

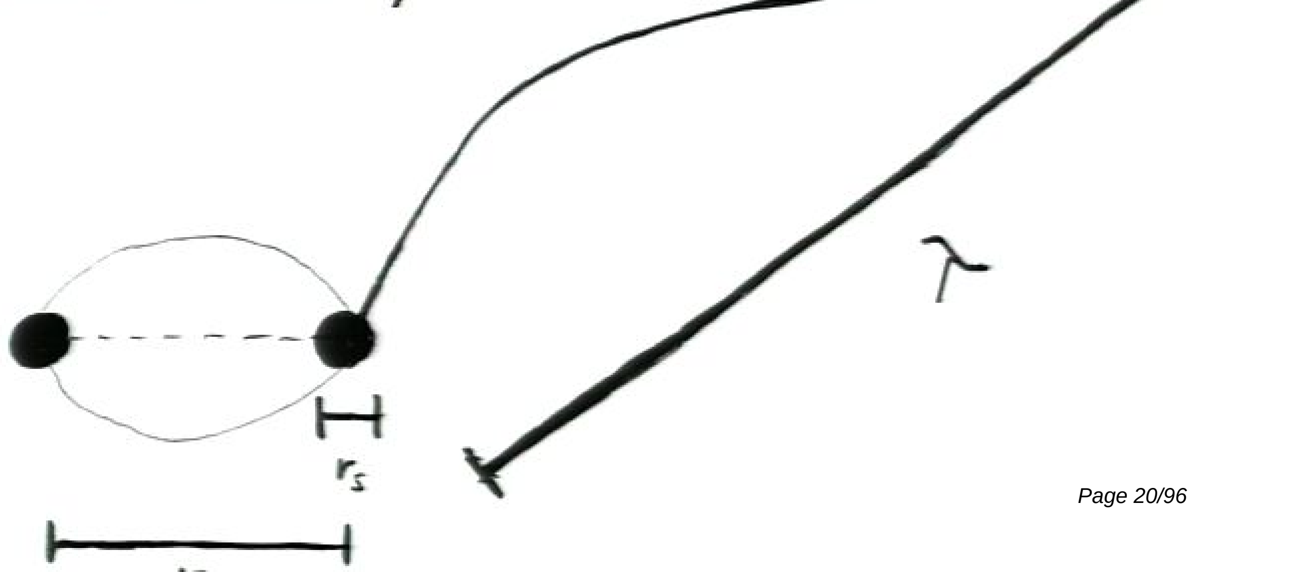
Motivation

What can we learn from binary inspiral signals?

- stringent test of strong gravitational dynamics
- masses & spins of BHs and NSs
- astrophysical abundances of such binary systems
- structure of NSs/BHs, e.g. NS equation of state (?
- can use binary inspirals as "standard sirens" for measurements of dark energy and structure formation e.g. Cutler & Holz, arxiv:0906.3752 [astro-ph.CO]

EFT Setup

- build effective theories based on hierarchy of scales in binary $r_s \ll r \ll \lambda$
- scales correlated $r_s \sim v^2 r, \quad \lambda \sim r/v$ by single expansion parameter v which is orbital velocity



EFT Setup

full theory: extended objects coupled to GR

matching ———————————————— $\frac{1}{\mu} = r$

relativistic point particles coupled to GR

↓

NR 2-body problem

matching ———————————————— $\frac{1}{\mu} = r$

composite multipole object coupled to GWs

↓

———————————————— $\frac{1}{\mu} = \frac{r}{v}$

EFT Setup

Features of effective field theory treatment:

- systematically accounts for physics at each scale
- disentangles physical effects at different scales and thus makes calculations more tractable
- definite power counting
- divergences well understood, standard regularization & renormalization program
- renormalization group methods

→ alternative to traditional PN methods

(Blanchet, Buonanno, Damour, Schäfer, Will, ...)

EFT Setup

- action: point particle coupled to gravity

$$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{-g} R$$

$$S_{GF} = m_{Pl}^2 \int d^4x \sqrt{-g}\, \Gamma^\mu \Gamma^\nu g_{\mu\nu}$$ harmonic gaug

$$S_{pp} = -\sum_{N=1}^{2} m_N \int d\tau_N + \cdots$$

finite size effects

$$d\tau_N = \sqrt{g_{\mu\nu}(x_N) dx_N^\mu dx_N^\nu}$$

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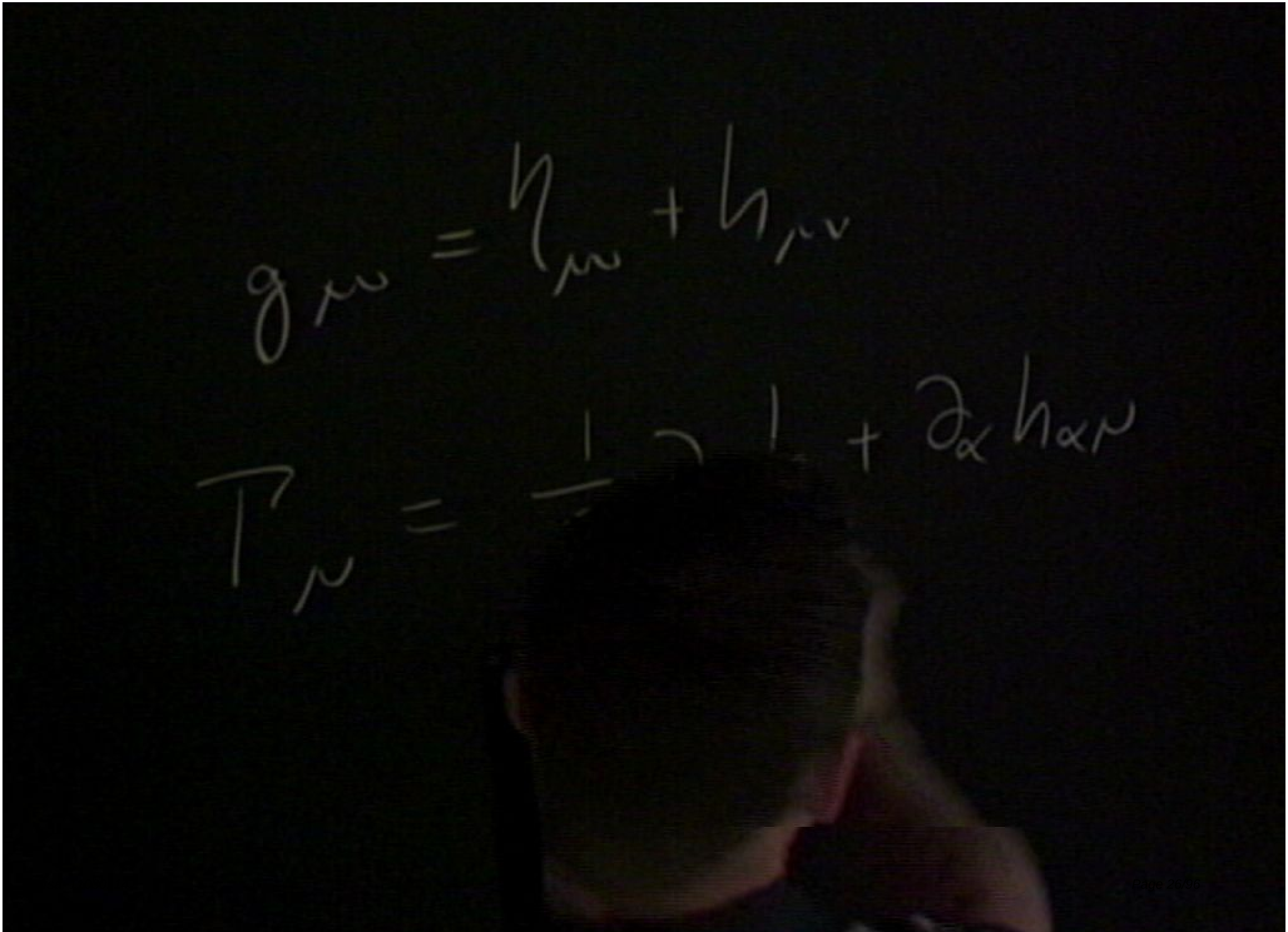
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$\Gamma_{\nu} =$
$+ \partial_{\alpha} h_{\alpha\nu}$

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EFT Setup

Finite size effects (spinless)

- need to include in S_{pp} all operators allowed by symmetries, RPI & general covariance

$$S_{pp} = -m\int d\tau + c_1\int d\tau R + c_2\int d\tau R_{\mu\nu}v^\mu v^\nu + c_E\int d\tau E_{\mu\nu}E^{\mu\nu} + c_B\int d\tau B_{\mu\nu}B^{\mu\nu}$$

- since vacuum eoms $R_{\mu\nu} = 0$

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- only terms with Riemann tensors can yield finite size effects
- can show that first enter at $\mathcal{O}(v^{10})$ or 5PN
- for NS possible enhancement of finite size effects since $r_{NS}/r_s \sim$ a few and effect $\sim r_{NS}^5$

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Spin Porto, gr-qc/0511061, arXiv:0710.5150 [hep-th]

- introduced as additional degrees of freedom on the worldline
- complications arise since rotations comprise only 3 out of 6 dofs of Lorentz transformations → need to impose constraints to project out spin
- spin formalism gives effectively new vertices of gravitational field coupling to worldline
- finite size effects for spinning objects arise

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EFT Setup

- expanding the action around flat space, i.e. plugging in $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we could derive Feynman rules and start calculating
- BUT: that is not efficient for treating the non-relativisitic 2-body problem! Need a definite power counting in small expansion parameter v
- similar setup to NRQED/NRQCD where modes are divided into potential gravitons and radiation gravitons, and radiation modes are multipole expanded → **NRGR**

EFT Setup

Potential modes $H_{\mu\nu}$

- yield binding dynamics of binary
- 4-momenta $p^{\mu} \sim (v/r, 1/r)$
- cannot be on-shell, so integrate out

Radiation modes $\bar{h}_{\mu\nu}$

- GWs which propagate out to detector, on-shell
- 4-momenta $k^{\mu} \sim (v/r, v/r)$
- treat as background field

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + H_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} + H_{\mu\nu}$$

EFT Setup

- for complete NRGR power counting rules, see Goldberger & Rothstein, hep-th/0409156
- integrate out potential modes

$$e^{iS_{eff}[x_N,\bar{h}]} = \int \mathcal{D}H_{\mu\nu}\, e^{iS[x_N,\bar{h}+H]}$$

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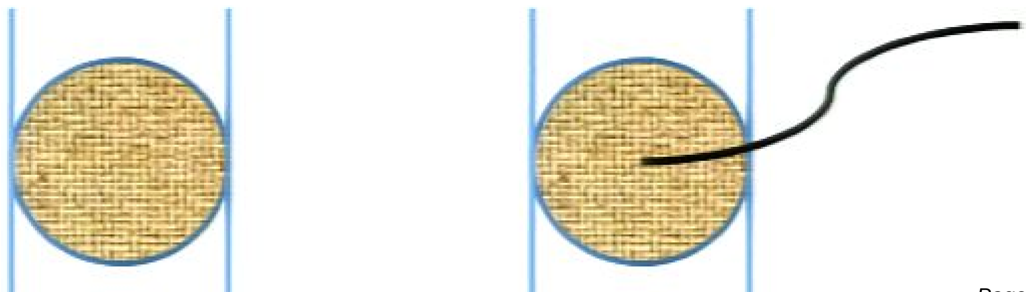

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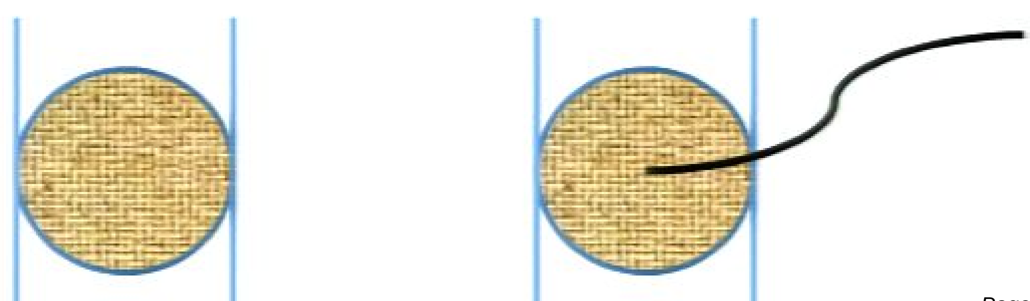

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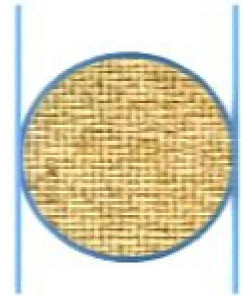
$$S_0[x_N] = \int dt L[x_N]$$



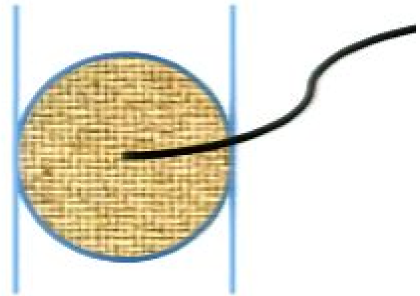
- calculate all graphs without external radiation gravitons to obtain conservative dynamics of binary system $\rightarrow E(\omega)$
- 1PN: Goldberger&Rothstein, hep-th/0409156
- 2PN: Gilmore&AR, arXiv:0810.1328
- 3PN: work in progress

EFT Setup

$$iS_{eff}[x_N, \bar{h}] = iS_0[x_N] + iS_1[x_N, \bar{h}] + \cdots$$

$$S_1[x_N, \bar{h}] = -\frac{1}{2m_{pl}} \int d^4x\, T^{\mu\nu}(x) \bar{h}_{\mu\nu}(x)$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$



- power counting forces us to Taylor expand $\bar{h}_{\mu\nu}$ in action S_1 around a single point
- yields a single worldline EFT with action S_1 in form of a multipole expansion
- form fixed by gauge and reparam. invariance

Radiation Sector

$$S = -m\int d\tau - \frac{1}{2}\int dx^{\mu} L_{ab}\,\omega_{\mu}^{ab}(\tau)$$
$$+\frac{1}{2}\sum_{n=0}^{\infty}\int d\tau c_n^{(I)} I^{aba_1\ldots a_n}(\tau)\nabla_{a_1}\cdots\nabla_{a_n}E_{ab}(x)$$
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- describe an arbitrary source of gravitational radiation in the long wavelength limit
- single worldline EFT endowed with multipoles

Radiation Sector

- two distinct expansions:

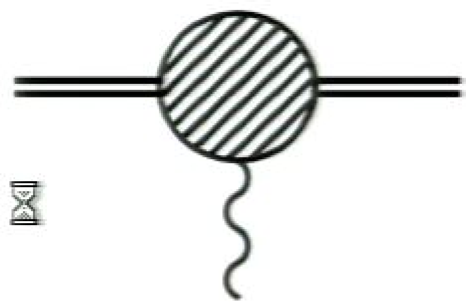
  1. multipole expansion in $a/\lambda \ll 1$
  2. post-Minkowskian exp. in $\eta = Gm/\lambda \ll 1$

- in PN regime $a/\lambda \sim v$ and $\eta \sim v^3$
- multipole moments are Wilson coefficients which encode short distance physics
- given a description of the short distance physics we can perform a matching calculation to determine these Wilson coefficients

Radiation Sector

Calculating observables

- start from single graviton emission amplitude

$$i\mathcal{A}_h(\mathbf{k}) =$$ 

- graviton emission rate $d\Gamma_h(\mathbf{k}) = \frac{1}{T}\frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}|\mathcal{A}_h(\mathbf{k})|$
- 4-momentum flux $\dot{P}^\mu\Big|_{h=\pm 2} = \int k^\mu d\Gamma_h(\mathbf{k})$
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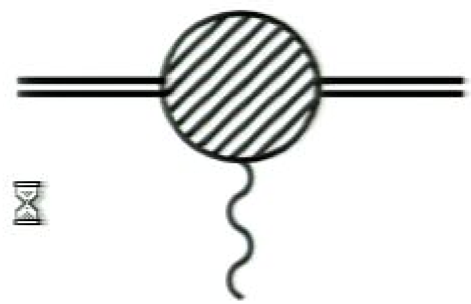
$$i\mathcal{A}_h(\mathbf{k}) =$$ 

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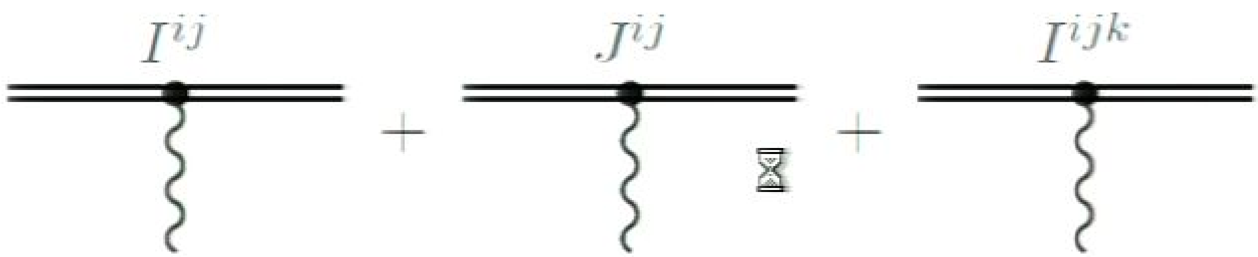
- at order η^0 the amplitudes are trivial and give rise to standard results

$\mathcal{A}_h(\mathbf{k}) =$  $+ \cdots$

$$= \frac{i}{4m_{Pl}}\epsilon^*_{ij}(\mathbf{k}, h)\left[\mathbf{k}^2 I^{ij}(k) + \frac{4}{3}|\mathbf{k}|\,\mathbf{k}^l\epsilon^{ikl}J^{jk}(k) - \frac{i}{3}\mathbf{k}^2\mathbf{k}^l I^{ijl}(k) + \cdots\right.$$

$$\dot{P}^0 = \frac{G_N}{\pi T}\int_0^\infty dk\left[\frac{k^6}{5}\left|I^{ij}(k)\right|^2 + \frac{16}{45}k^6\left|J^{ij}(k)\right|^2 + \frac{k^8}{189}\left|I^{ijk}(k)\right|^2 + \cdots\right.$$

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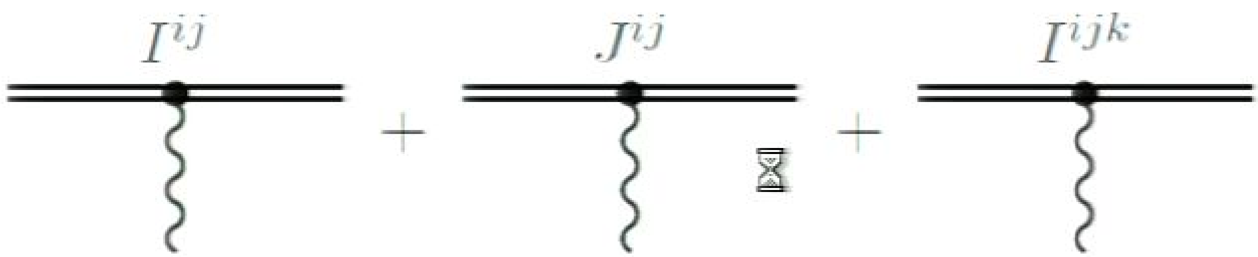

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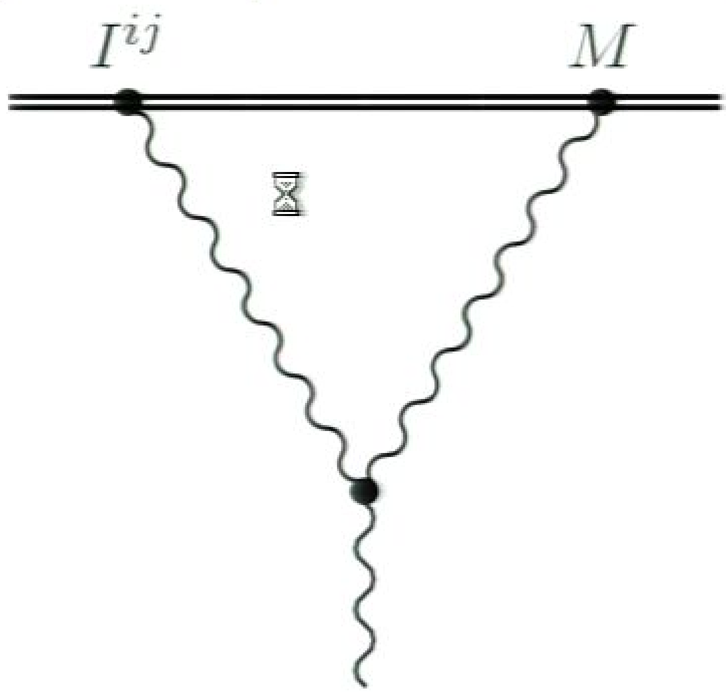
Radiative Corrections

- long distance EFT contains more than just single graviton emission diagrams
- nonlinear interactions of multipoles
- consider interaction of a single radiating moment with the mass monopole -> **tail effects**
- physically tail effects arise from GWs propagating in Schwarzschild background due to binary

Radiative Corrections

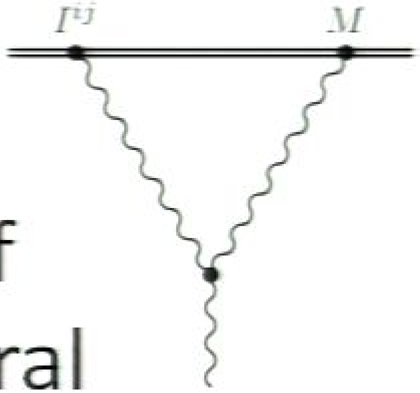
Leading Tail Effect at order η

- simplest case: quadrupole + monopole



- effect of 1/r potential on GW propagation

Radiative Corrections



Leading Tail Effect at order η

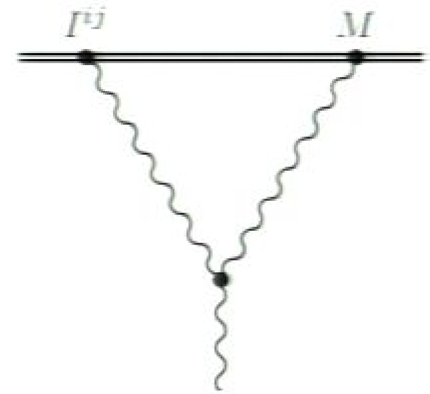
- amplitude includes an integration of the form of a 1-loop Feynman integral
- integral part linear combination of integrals

$$\left(\frac{1}{\mathbf{k}^2}\right)^n \int \frac{d^{d-1}\mathbf{q}}{(2\pi)^{d-1}} \left(\frac{1}{\mathbf{q}^2}\right)^{1-n} \frac{1}{\mathbf{k}^2 - (\mathbf{k}+\mathbf{q})^2 + i\epsilon}$$

- For n=0, long distance behavior $(\mathbf{q} \to 0)$ is

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{\mathbf{q}^2} \frac{1}{\mathbf{k}\cdot\mathbf{q}}$$ → logarithmic IR divergence

Radiative Corrections



Leading Tail Effect at order η

- amplitude reads

$$i\mathcal{A}_{\eta^1} = i\mathcal{A}_{\eta^0} \times iGM|\mathbf{k}|\,\frac{\Gamma[(1-d)/2]}{(4\pi)^{(d-3)/2}}\,\frac{d^4 - 8d^3 + 23d^2 - 28d + 24}{d^3 - 6d^2 + 8d}\left(-\frac{\mathbf{k}^2 + i\epsilon}{\mu^2}\right)^{(d-4)/}$$

$$= i\mathcal{A}_{\eta^0} \times iGM|\mathbf{k}|\left[\frac{2}{d-4} + \log\frac{-\mathbf{k}^2 - i\epsilon}{\pi\mu^2} + \gamma_E - \frac{11}{6} + \mathcal{O}(d-4)\right]$$

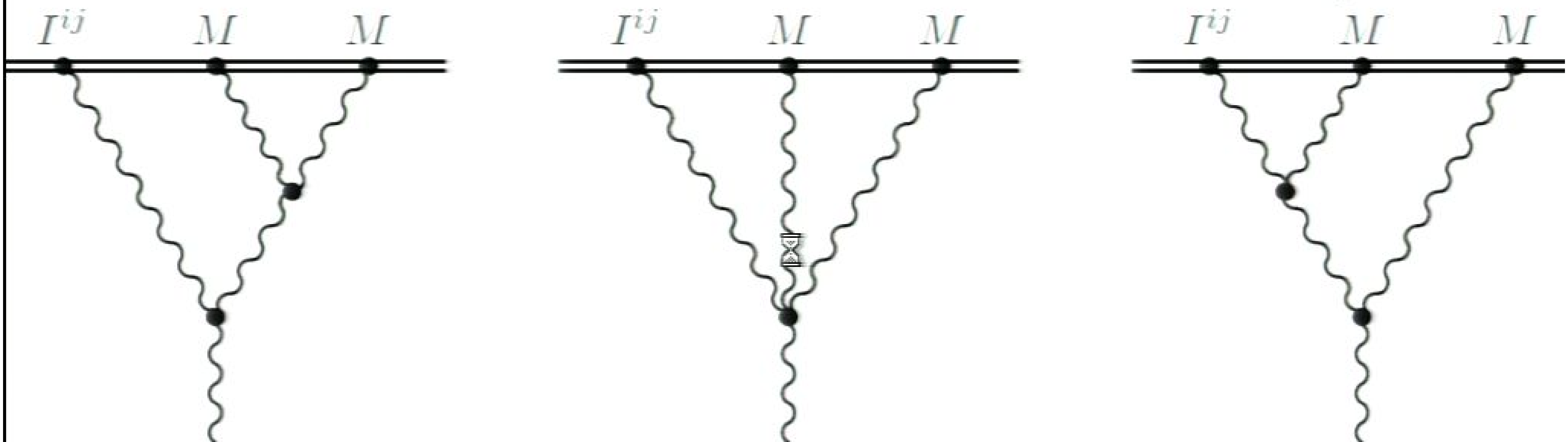
- IR divergence from long-ranged 1/r potential
- leading tail effect enters observables from

$$\left|\frac{\mathcal{A}}{\mathcal{A}_{\eta^0}}\right|^2 = 1 + 2\mathrm{Re}\frac{\mathcal{A}_{\eta^1}}{\mathcal{A}_{\eta^0}} + \mathcal{O}(\eta^2) = 1 + 2\pi G_N m|\mathbf{k}| + \mathcal{O}(\eta^2)$$

- universal factor, independent of rad. multipole

Radiative Corrections

Tail-of-Tail & Tail-Squared Effects at order η^2



(a) (b) (c)

- calculations challenging, with integrals corresponding to 2-loop Feynman integrals
- IR and UV divergences

Radiative Corrections

Treating the Divergences

- IR divergences cancel in any observable, and to all orders they exponentiate to a phase

$$\frac{|\mathcal{A}|^2}{|\mathcal{A}_{\eta^0}|^2} = 1 + 2\pi GM|\mathbf{k}|$$

$$+ (GM|\mathbf{k}|)^2 \left[-\frac{214}{105}\left(\frac{1}{\epsilon_{UV}} + \log\frac{\mathbf{k}^2}{\pi\mu^2} + \gamma_E\right) + \frac{4}{3}\pi^2 + \frac{63491}{4410C}\right.$$

- UV divergence and arbitrary scale μ **must** be canceled by renormalization of quadrupole moment $\quad i\mathcal{A}_{\eta^0} = \frac{i}{4m_{Pl}}\epsilon^*_{ij}(\mathbf{k},h)\mathbf{k}^2 I^{ij}(k)$

Radiative Corrections

Renormalization and the RG

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- from requirement of μ independence of physics

$$\mu \frac{d}{d\mu} I^{ij}(|\mathbf{k}|, \mu) = -\frac{214}{105} (GM|\mathbf{k}|)^2 \, I^{ij}(|\mathbf{k}|, \mu)$$

$$I^{ij}(|\mathbf{k}|, \mu) = \exp \left[-\frac{214}{105} (GM|\mathbf{k}|)^2 \log \frac{\mu}{\mu_0} \right] I^{ij}(|\mathbf{k}|, \mu_0)$$

- typically choose $\mu = |\mathbf{k}|$ and $\mu_0 \sim 1/a$

Radiative Corrections

matching

full theory: extended objects coupled to GR

relativistic point particles coupled to GR

$\frac{1}{\mu} = r_s$

matching

NR 2-body problem

composite multipole object coupled to GWs

$\frac{1}{\mu} = r$

$\frac{1}{\mu} = \frac{r}{v}$

Radiative Corrections

Renormalization and the RG

- EFTs are useful tool to resum large logs via RG, where large logs means a correction of the form

$$(1 + \alpha \log \mu/\mu_0) \sim (1 + \mathcal{O}(1))$$

- many examples where this is essential, e.g. QCD corrections to weak decays
- unfortunately in gravitational wave physics the logarithms cannot become large

$$(1 + \eta^2 \log a/\lambda) \quad \text{where} \quad \eta \sim r_s/\lambda$$

Radiative Corrections

Renormalization and the RG

- nevertheless RG yields interesting information about the dynamics since it constrains the pattern of logs in amplitude squared

$$\left.\frac{\mathcal{A}(\omega)}{\mathcal{A}_{\eta^0}(\omega,\mu_0)}\right|^2_{\text{leading log}} = 1 - \frac{428}{105}(G_N m\omega)^2 \ln\frac{\omega}{\mu_0} + \frac{91592}{11025}(G_N m\omega)^4 \ln^2 \frac{\omega}{\mu_0}$$
$$- \frac{39201376}{3472875}(G_N m\omega)^6 \ln^3 \frac{\omega}{\mu_0} + \cdots .$$

- independent of short distance physics
- in PN case, predicts log^(n) term at (3n)PN

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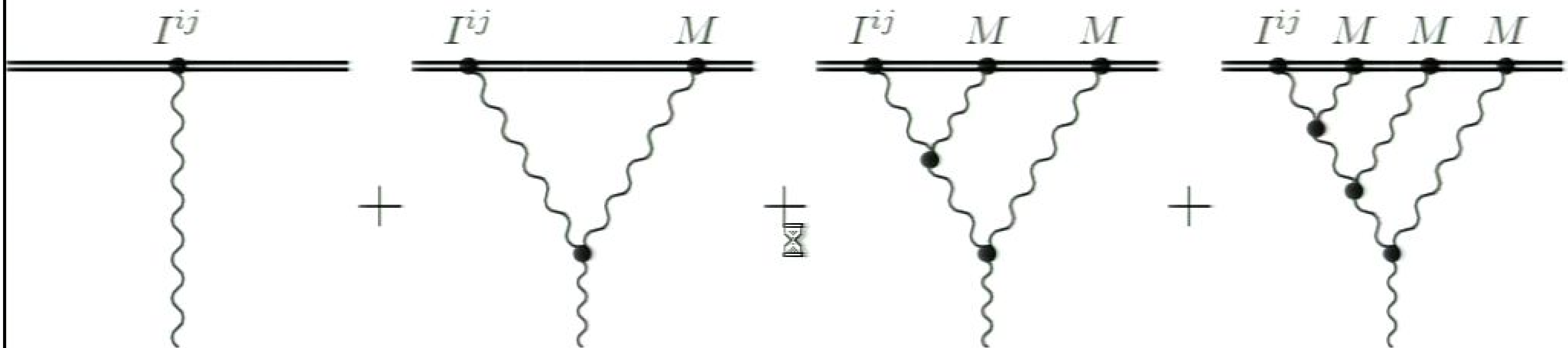
Radiative Corrections

Resumming the leading IR tail effect



- summing ladder diagrams corresponds to solving wave equation with Coulomb potential yields a Sommerfeld factor $\dfrac{4\pi GM|\mathbf{k}|}{1-\exp(-4\pi GM|\mathbf{k}|)}$

Khriplovich et al.(1997), Damour et al.(2007), Asada et al.(1997)

- factorization of IR & UV resummations

Matching

$$S = -m\int d\tau - \frac{1}{2}\int dx^\mu L_{ab}\,\omega_\mu^{ab}(\tau)$$

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Radiative Corrections

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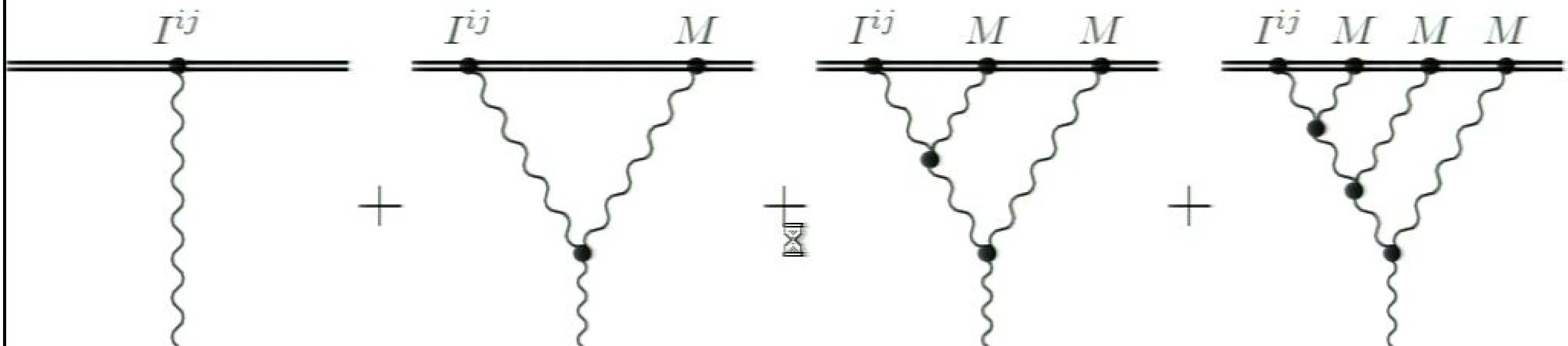

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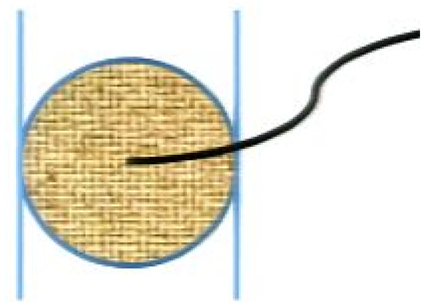

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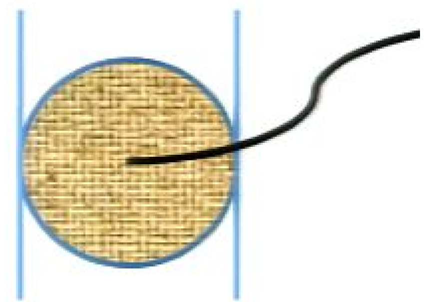

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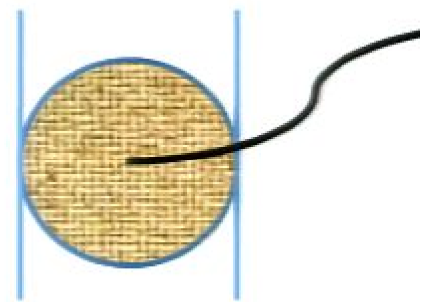

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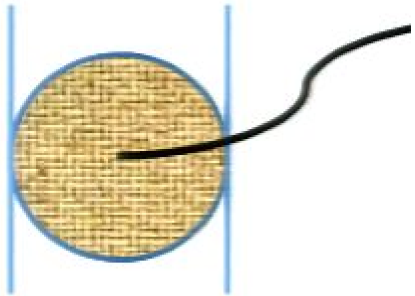

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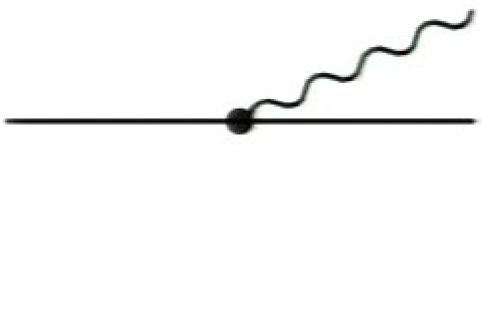
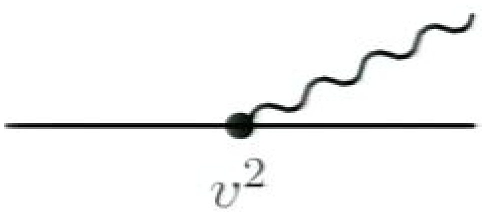
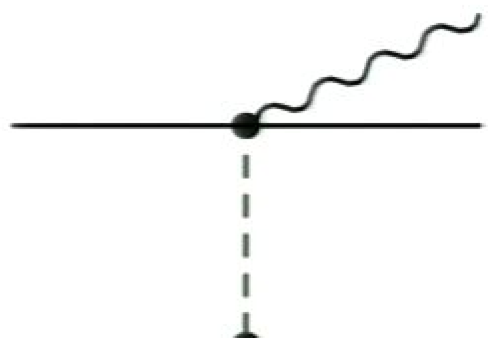
Matching for PN

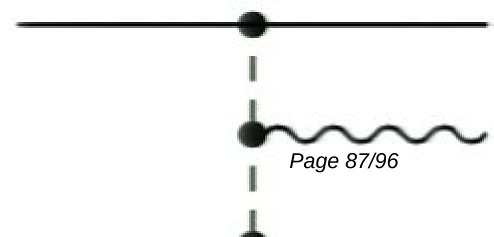
- $T^{\mu\nu}(x^0, \mathbf{k}) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} T^{\mu\nu}(x^0, \mathbf{x})$ from Feynman diagrams, e.g.



$$\longrightarrow \quad T^{00}(x^0, \mathbf{k}) = \sum_a m_a e^{-i\mathbf{k}\cdot\mathbf{x}_a}$$







Matching for PN

- simple Feynman diagrams yielding

$$I^{ij} = \sum_a m_a \left(1 + \frac{3}{2}\mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right)\left[\mathbf{x}_a^i \mathbf{x}_a^j\right]^{TF}$$

$$+ \frac{11}{42}\sum_a m_a \frac{d^2}{dt^2}\left(\mathbf{x}_a^2 \left[\mathbf{x}_a^i \mathbf{x}_a^j\right]^{TF}\right)$$

$$- \frac{4}{3}\sum_a m_a \frac{d}{dt}\left(\mathbf{x_a} \cdot \mathbf{v}_a \left[\mathbf{x}_a^i \mathbf{x}_a^j\right]^{TF}\right)$$

$$J^{ij} = \frac{1}{2}\sum_a m_a \left((\mathbf{x}_a \times \mathbf{v}_a)^i \mathbf{x}_a^j + (\mathbf{x}_a \times \mathbf{v}_a)^j \mathbf{x}_a^i\right)$$

$$I^{ijk} = \sum_a m_a \left[\mathbf{x}_a^i \mathbf{x}_a^j \mathbf{x}_a^k\right]^{TF}$$

- for 1PN circ. orbit $P = P_{LO}\left[1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) \right.$

Matching for PN

Reality check – comparing with known results

$$P = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right) \pi x^{5/2}$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right] x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}$$

from Blanchet, Living Rev. Relativity, 9, (2006), 4

Matching for PN - spin

Towards the 3PN phase with full spin-dependence

- conservative dynamics obtained to 3PN
  Porto et al., arXiv:0802.0720, arXiv:0804.0260, arXiv:1005.5730
- computed matching of all multipole moments needed for 3PN energy flux/phase
  R. Porto, AR, I. Rothstein arXiv:1007.1312 [gr-qc]
- work in progress: computing observables and bring them into a form which is useful for LIGO/LISA

Matching for PN - spin

Multipole moments for 3PN phase

- need to account for contributions both linear and quadratic in spin
- at which order do spin effects enter multipoles?

$$K_\ell^{\mu\nu} \equiv \int d^3x T^{\mu\nu} \mathbf{x}^{i_1} \ldots \mathbf{x}^{i_\ell}$$

	$\mathcal{O}(\not{\mathbf{S}})$	$\mathcal{O}(\mathbf{S}_A)$	$\mathcal{O}(\mathbf{S}_A^2)$
K_ℓ^{00}	mr^ℓ	$mr^\ell v^3$	$mr^\ell v^4$
K_ℓ^{0i}	$mr^\ell v$	$mr^\ell v^2$	$mr^\ell v^5$
K_ℓ^{ij}	$mr^\ell v^2$	$mr^\ell v^3$	$mr^\ell v^6$

$$\dot{P}^0 = \frac{G_N}{\ldots T} \int^\infty dk \left[\frac{k^6}{\ldots} |I^{ij}(k)|^2 + \frac{16}{\ldots} k^6 |J^{ij}(k)|^2 + \frac{k^8}{\ldots} |I^{ijk}(k)|^2 + \cdots \right.$$

Matching for PN - spin

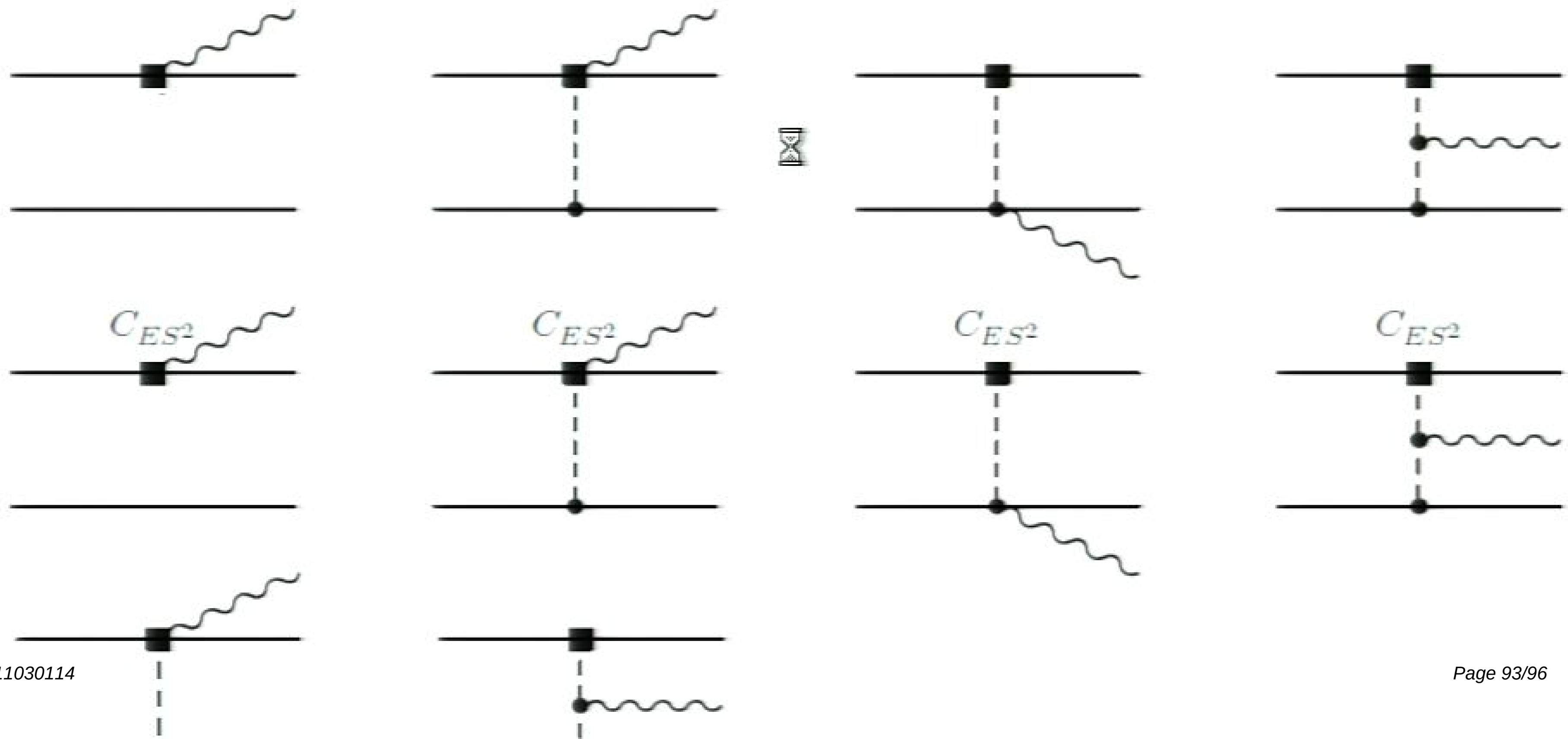
Multipole moments for 3PN phase

- mass quadrupole with $\mathcal{O}(\mathbf{S}_A)$ and $\mathcal{O}(\mathbf{S}_A^2)$ components to NLO & leading $\mathcal{O}(\mathbf{S}_A\mathbf{S}_B)$ terms
- current quadrupole with leading $\mathcal{O}(\mathbf{S}_A^2)$ and up to NLO $\mathcal{O}(\mathbf{S}_A)$ contributions
- mass octupole with leading $\mathcal{O}(\mathbf{S}_A)$ and $\mathcal{O}(\mathbf{S}_A^2)$
- current octupole with leading $\mathcal{O}(\mathbf{S}_A)$
- the spin-independent 1PN corrections to the mass and current quadrupole moments

Matching for PN - spin

Multipole moments for 3PN phase

- straightforward from simple Feynman diagrams



Matching for PN - spin

$$
{}^{ij}_{\mathbf{S}_A\mathbf{S}_A^2,\mathbf{S}_A\mathbf{S}_B} = \sum_A \Bigg[\frac{8}{3}(\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j - \frac{4}{3}(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{v}_A^j - \frac{4}{3}(\mathbf{x}_A \times \dot{\mathbf{S}}_A)^i \mathbf{x}_A^j \qquad (79)
$$

$$
\frac{4}{3}\frac{d}{dt}\left\{\mathbf{v}_A \cdot \mathbf{x}_A (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j\right\} + \frac{1}{7}\frac{d^2}{dt^2}\Bigg\{\frac{1}{3}\mathbf{x}_A \cdot \mathbf{v}_A (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j
$$

$$
4\mathbf{x}_A^2(\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j + \mathbf{x}_A^2(\mathbf{S}_A \times \mathbf{x}_A)^i \mathbf{v}_A^j - \frac{5}{6}(\mathbf{v}_A \times \mathbf{S}_A)\cdot \mathbf{x}_A\, \mathbf{x}_A^i \mathbf{x}_A^j\Bigg\}\Bigg]_{\mathrm{STF}}
$$

$$
\sum_{A,B} \frac{2Gm_B}{r^3}\Big[(\mathbf{v}_B \times \mathbf{S}_A)\cdot \mathbf{r}(\mathbf{x}_B^i \mathbf{x}_B^j - 2\mathbf{x}_A^i \mathbf{x}_A^j) + (\mathbf{v}_A \times \mathbf{S}_A)\cdot \mathbf{r}(\mathbf{x}_A^i \mathbf{x}_A^j + \mathbf{x}_B^i \mathbf{x}_B^j)
$$

$$
2r^2\left\{(\mathbf{v}_B \times \mathbf{S}_A)^i(\mathbf{x}_B^j - \mathbf{x}_A^j) + (\mathbf{r} \times \mathbf{S}_A)^i\left(\mathbf{v}_B^j - \mathbf{v}_A^j - \frac{\mathbf{v}_B \cdot \mathbf{r}}{r^2}(\mathbf{x}_A^j + \mathbf{x}_B^j)\right)\right\}\Big]_{\mathrm{STF}}
$$

$$
\frac{2}{3}\sum_{A,B}\frac{d}{dt}\Bigg[\frac{Gm_B}{r^3}\Big\{r^2\Big((\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}_A^j - 3(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j + 3(\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}_B^j - (\mathbf{x} \times \mathbf{S}_A)^i \mathbf{x}_B^j\Big)
$$

$$
2\mathbf{r}\cdot \mathbf{x}_B\, (\mathbf{r} \times \mathbf{S}_A)^i(\mathbf{x}_A^j + \mathbf{x}_B^j) + (\mathbf{x}_A \times \mathbf{S}_A)\cdot \mathbf{x}_B(\mathbf{x}_A^i \mathbf{x}_A^j - 2\mathbf{x}_B^i \mathbf{x}_B^j)\Big\}\Bigg]_{\mathrm{STF}}
$$

$$
\sum_A \frac{C_{ES^2}^{(A)}}{m_A}\Bigg[\mathbf{S}_A^i \mathbf{S}_A^j\left(-1 + \frac{13}{42}\mathbf{v}_A^2 + \frac{17}{21}\mathbf{a}_A \cdot \mathbf{x}_A\right) + \mathbf{S}_A^2\left(-\frac{11}{21}\mathbf{v}_A^i \mathbf{v}_A^j + \frac{10}{21}\mathbf{a}_A^i \mathbf{x}_A^j\right)
$$

$$
\frac{8}{21}\mathbf{x}_A^i \mathbf{S}_A^j \mathbf{a}_A \cdot \mathbf{S}_A + \frac{4}{7}\mathbf{v}_A^i \mathbf{S}_A^j \mathbf{S}_A \cdot \mathbf{v}_A - \frac{22}{21}\mathbf{a}_A^i \mathbf{S}_A^j \mathbf{S}_A \cdot \mathbf{x}_A\Bigg]_{\mathrm{STF}}
$$

$$
\sum_{A,B}\frac{G}{2r^3}\Bigg[\frac{C_{ES^2}^{(B)} m_A}{m_B}\left(\mathbf{S}_B^2 + 9(\mathbf{S}_B \cdot \mathbf{n})^2\right)\mathbf{x}_B^i \mathbf{x}_B^j + 6\frac{C_{ES^2}^{(B)} m_A}{m_B} r^2 \mathbf{S}_B^i \mathbf{S}_B^j
$$

$$
\left(\frac{C_{ES^2}^{(B)} m_A}{m_B}\left(3(\mathbf{S}_B \cdot \mathbf{n})^2 - \mathbf{S}_B^2\right) + 12\mathbf{S}_A \cdot \mathbf{n}\mathbf{S}_B \cdot \mathbf{n} - 4\mathbf{S}_A \cdot \mathbf{S}_B\right)\mathbf{x}_A^i \mathbf{x}_A^j
$$

$$
4\frac{C_{ES^2}^{(B)} m_A}{m_B}\mathbf{S}_B^2 \mathbf{x}_A^i \mathbf{x}_B^j + 4\left(3\frac{C_{ES^2}^{(B)} m_A}{m_B}\mathbf{S}_B \cdot \mathbf{r} + 2\mathbf{S}_A \cdot \mathbf{r}\right)\mathbf{S}_B^i \mathbf{x}_B^j\Bigg]_{\mathrm{STF}}
$$

$$
\begin{aligned}
J^{ij}_{\mathbf{S}_A\mathbf{S}_A^2} = \sum_A \Bigg[&\frac{C_{ES^2}^{(A)}}{m_A}(\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{S}_A^j + \mathbf{S}_A^i \mathbf{x}_A^j\left(\frac{3}{2} + \frac{2}{7}\mathbf{v}_A^2 - \frac{5}{7}\mathbf{a}_A \cdot \mathbf{x}_A\right) - \frac{3}{7}\mathbf{v}_A^i \mathbf{S}_A^j \mathbf{v}_A \cdot \mathbf{x}_A \\
&+\frac{11}{28}\mathbf{S}_A^i \mathbf{a}_A^j \mathbf{x}_A^2 + \frac{2}{7}\mathbf{S}_A \cdot \mathbf{x}_A \mathbf{a}_A^i \mathbf{x}_A^j + \frac{1}{7}\mathbf{x}_A^i \mathbf{x}_A^j \mathbf{a}_A \cdot \mathbf{S}_A - \frac{3}{7}\mathbf{S}_A \cdot \mathbf{v}_A \mathbf{v}_A^i \mathbf{x}_A^j + \frac{11}{14}\mathbf{S}_A \cdot \mathbf{x}_A \mathbf{v}_A^i \mathbf{v}_A^j \\
&+\sum_{A,B}\frac{Gm_B}{2r^3}\Big[3\mathbf{S}_A \cdot \mathbf{x}_B(\mathbf{x}_B^i \mathbf{x}_B^j - \mathbf{x}_A^i \mathbf{x}_A^j) + \mathbf{S}_A \cdot \mathbf{x}_A(2\mathbf{x}_A^i \mathbf{x}_A^j + \mathbf{x}_A^i \mathbf{x}_B^j - 3\mathbf{x}_B^i \mathbf{x}_B^j) \\
&+\mathbf{S}_A^i \mathbf{x}_A^j(\mathbf{x}_A \cdot \mathbf{r} - 6r^2)\Big]_{\mathrm{STF}}
\end{aligned}
$$

$$
J^{ijk}_{\mathbf{S}_A} = 2\sum_A \left[\mathbf{x}_A^i \mathbf{x}_A^j \mathbf{S}_A^k\right]_{\mathrm{STF}}
$$

Outlook

- catching up slowly for spinless binary systems
- going beyond 3PN for spinless systems?
- full spin-dependence in phase to 3PN
- amplitude with full spin-dependence to 2.5PN
- eventually reach 3.5PN precision for binary systems with spin

Conclusions

- EFT framework is a systematic and transparent way to organize classical calculations of GW observables for binary systems
- not limited to non-rel. PN expansion, can also expand in m/M ...
- method not limited to gravitational applications
- catching up with traditional post-Newtonian calculations
- new resummation from renormalization group