Title: Timelike entanglement in the quantum vacuum

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Abstract: For quantum fields with m=0, it is pointed out that timelike separated
fields are quantized as independent subsystems. This allows us to ask the question of whether the field in the future region is entangled with the field in the past region of Minkowski space, in the Minkowski vacuum state. I will show that the answer is "yes," and then explore some consequences, including a thermal effect and a procedure for extracting
the timelike entanglement with two inertial Unruh-DeWitt detectors.

Timelike Entanglement in the Quantum Vacuum

March 3, 2011
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Waterloo, Ontario

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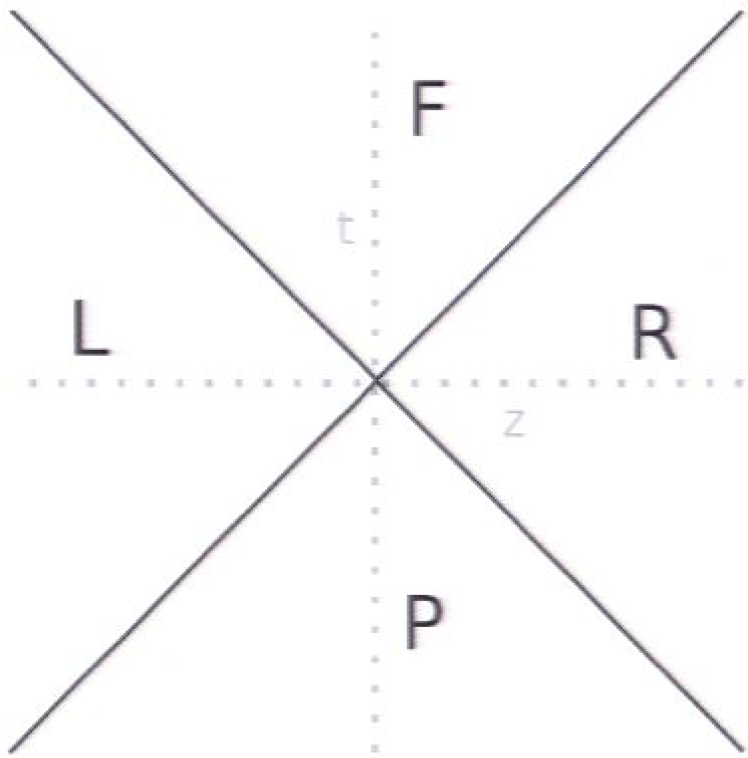
Outline

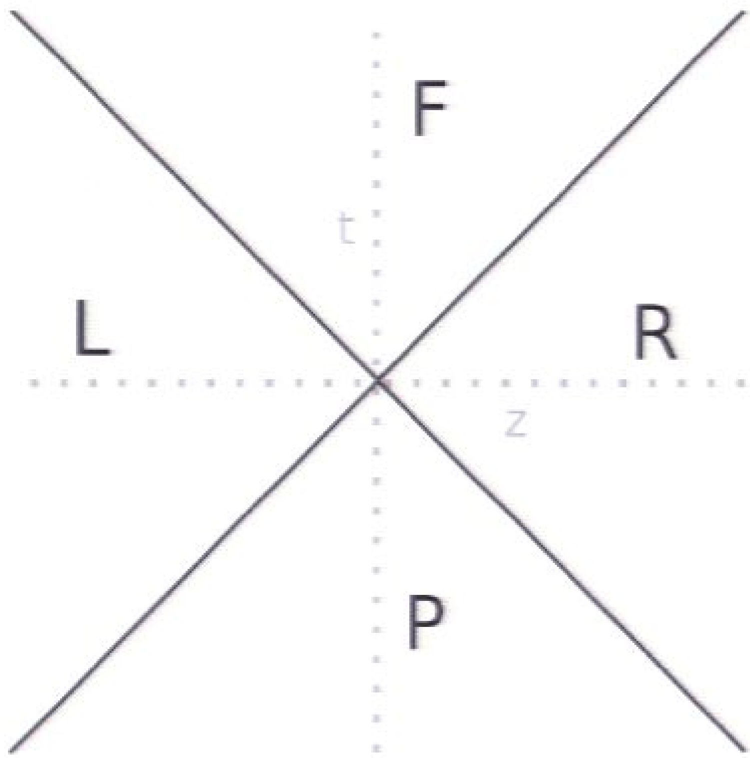
- Rindler space vacuum entanglement (reminder)
- Future/Past nonseparability aka timelike entanglement
- Thermal effects on Unruh-DeWitt detectors
- Extraction of timelike entanglement
- Conclusions / possible applications



All fields satisfy:

$$[\phi(x_L), \phi(x_R)] = 0$$

commuting fields → independent subsystems → possibility of entanglement



R:
$$t = a^{-1} e^{a\epsilon} \sinh(a\tau)$$
$$z = a^{-1} e^{a\epsilon} \cosh(a\tau)$$

L:
$$t = -a^{-1} e^{a\bar{\epsilon}} \sinh(a\bar{\tau})$$
$$z = -a^{-1} e^{a\bar{\epsilon}} \cosh(a\bar{\tau})$$

$$\left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\epsilon^2}\right)_R \phi = 0$$

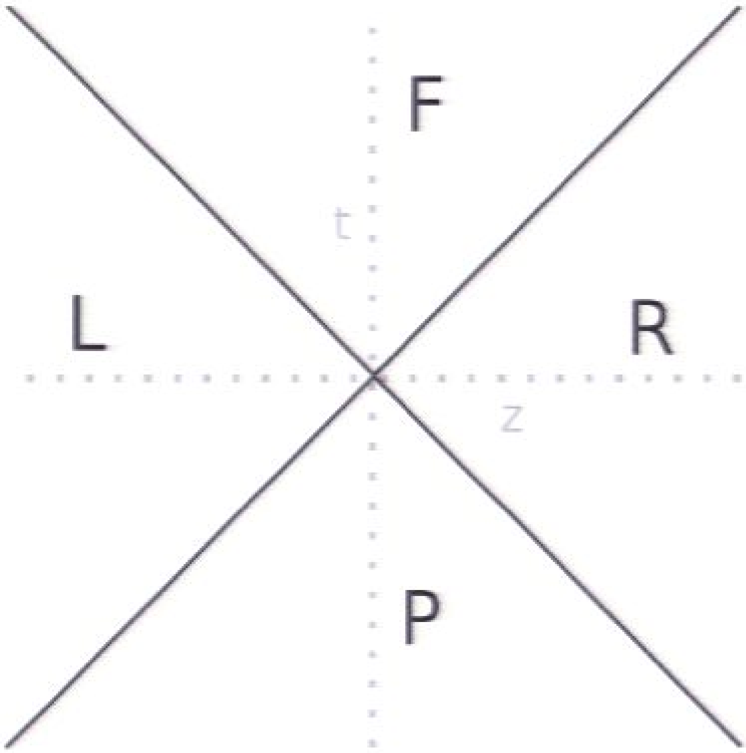
$$g^R_\omega(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}$$

$$\chi = \tau + \epsilon$$

$$\left(\frac{\partial^2}{\partial\bar{\tau}^2} - \frac{\partial^2}{\partial\bar{\epsilon}^2}\right)_L \phi = 0$$

$$g^L_\omega(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}}$$

$$\bar{\chi} = -\bar{\tau} - \bar{\epsilon}$$



$$g^R_\omega(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}$$

$$g^L_\omega(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}}$$

$$\chi = \tau + \epsilon \qquad \bar{\chi} = -\bar{\tau} - \bar{\epsilon}$$

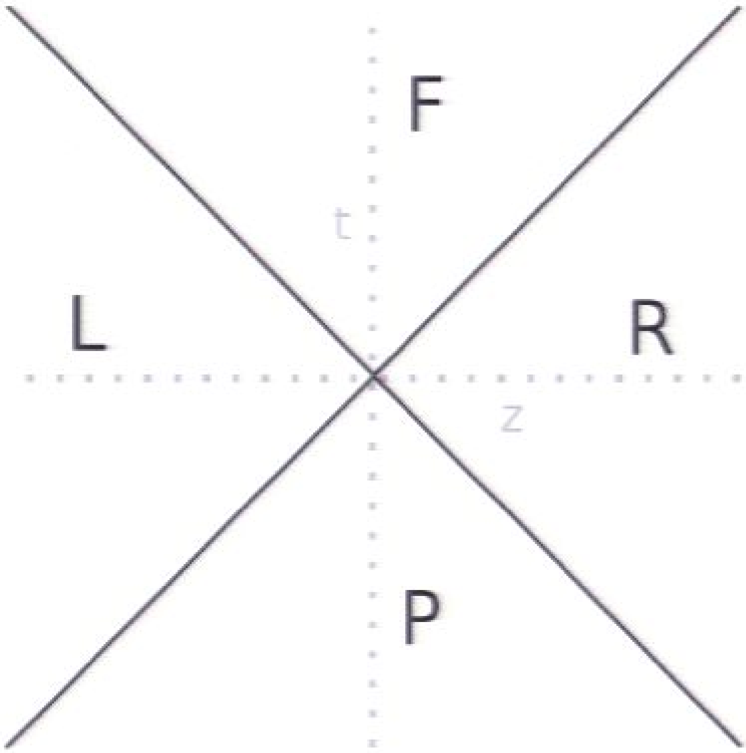
$$u_k(V) = (4\pi k)^{-1/2} e^{-ikV}$$

$$V = t + z$$

$$V = a^{-1} e^{a\chi} \qquad V = -a^{-1} e^{-a\bar{\chi}}$$

$$\theta(V) g^R_\omega(\chi) = \int_0^\infty dk \left(\alpha^R_{\omega k} u_k(V) + \beta^R_{\omega k} u^*_k(V)\right)$$

$$\theta(-V) g^L_\omega(\bar{\chi}) = \int_0^\infty dk \left(\alpha^L_{\omega k} u_k(V) + \beta^L_{\omega k} u^*_k(V)\right)$$



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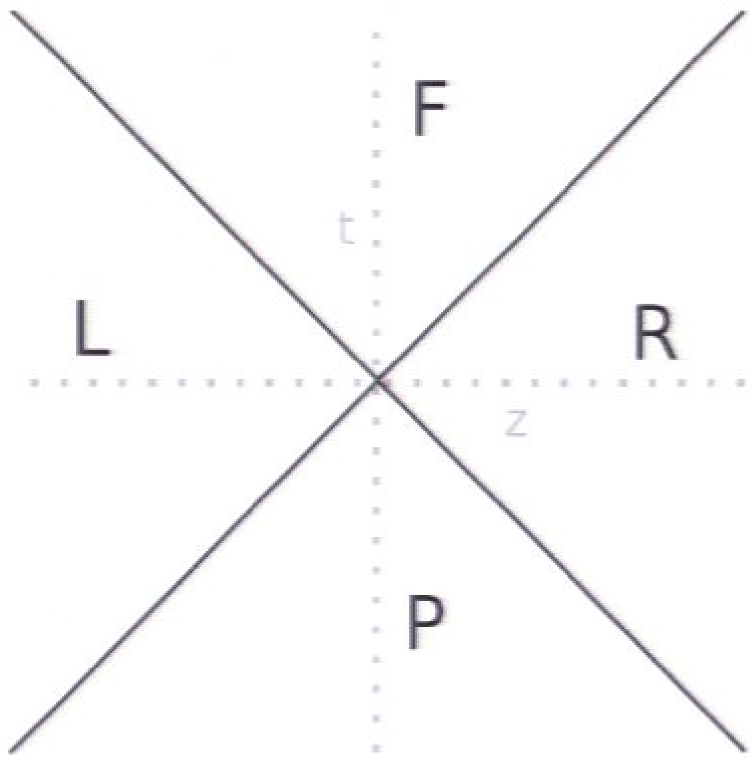

$$g^R_\omega(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}$$

$$g^L_\omega(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}}$$

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$$V = a^{-1} e^{a\chi} \qquad V = -a^{-1} e^{-a\bar{\chi}}$$

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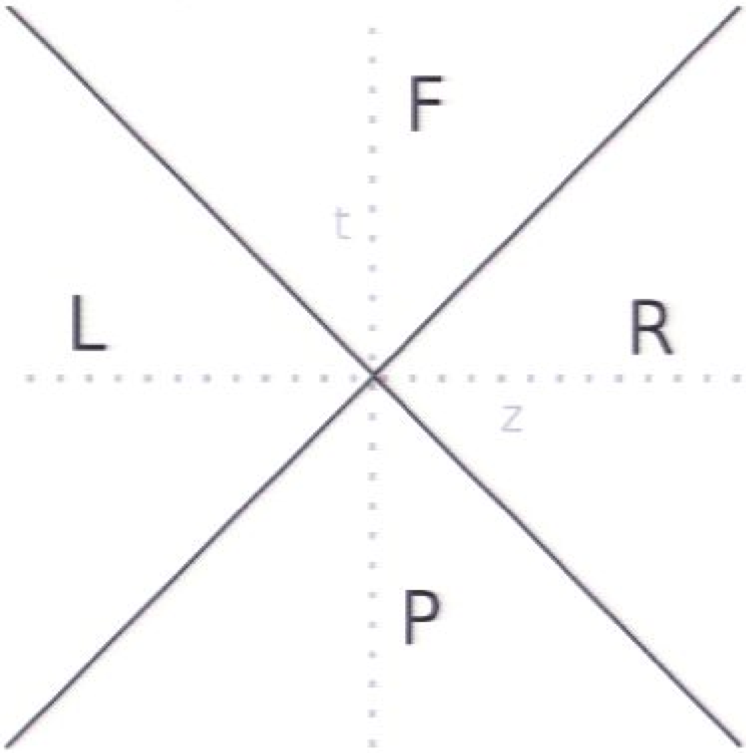


$$\beta^L_{\omega k} = -e^{-\pi\omega/a}\alpha^{R*}_{\omega k}$$

$$\beta^R_{\omega k} = -e^{-\pi\omega/a}\alpha^{L*}_{\omega k}$$

$$G_\omega(V) = \theta(V)g^R_\omega(\nu) + \theta(-V)e^{-\pi\omega/a}g^{L*}_\omega(\bar{\nu})$$

$$\bar{G}_\omega(V) = \theta(-V)g^L_\omega(\bar{\nu}) + \theta(V)e^{-\pi\omega/a}g^{R*}_\omega(\nu)$$

Pure positive frequency functions of Minkowski time --> shares Minkowski vacuum state $|0_M\rangle$.

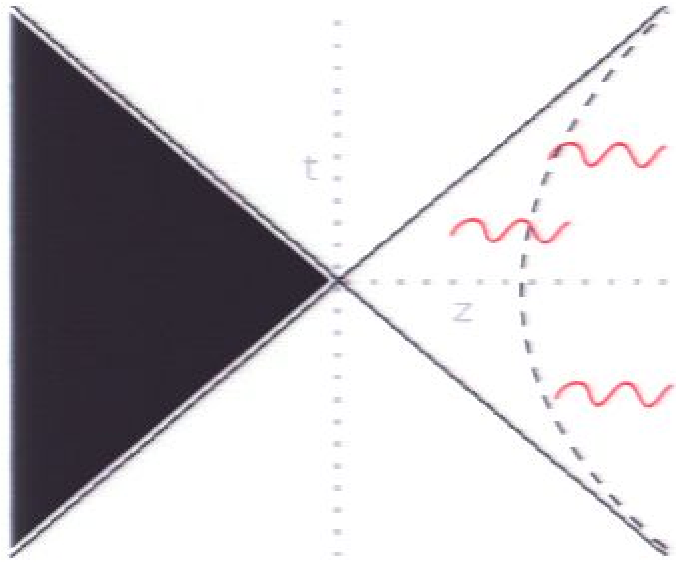
$$\hat{a}_{G\omega}|0_M\rangle = \hat{a}_{\bar{G}\omega}|0_M\rangle = 0$$

$$\hat{a}_{G\omega} = (\hat{a}^R_\omega - e^{-\pi\omega/a}\hat{a}^{L\dagger}_\omega)$$

$$\hat{a}_{\bar{G}\omega} = (\hat{a}^L_\omega - e^{-\pi\omega/a}\hat{a}^{R\dagger}_\omega)$$

$$\longrightarrow (\hat{a}^{R\dagger}_\omega \hat{a}^R_\omega - \hat{a}^{L\dagger}_\omega \hat{a}^L_\omega)|0_M\rangle = 0$$

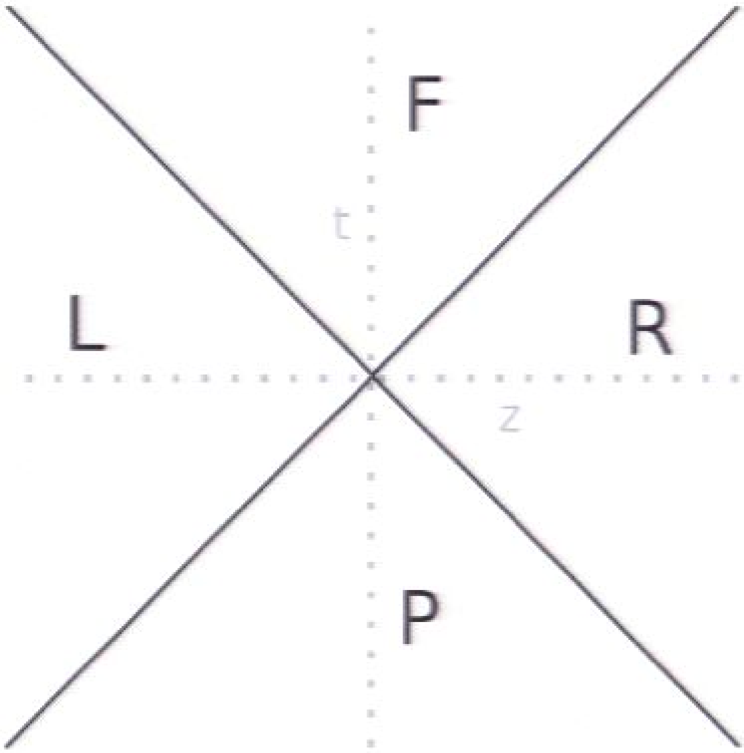
$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!}(\hat{a}^{R\dagger}_{\omega_i}\hat{a}^{L\dagger}_{\omega_i})^{n_i}|0_R\rangle$$



$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle\langle n_i^R| \right]$$

$$T_U = \frac{\hbar a}{2\pi c k_B}$$

One degree corresponds to a = 10^{20} m/s^2.



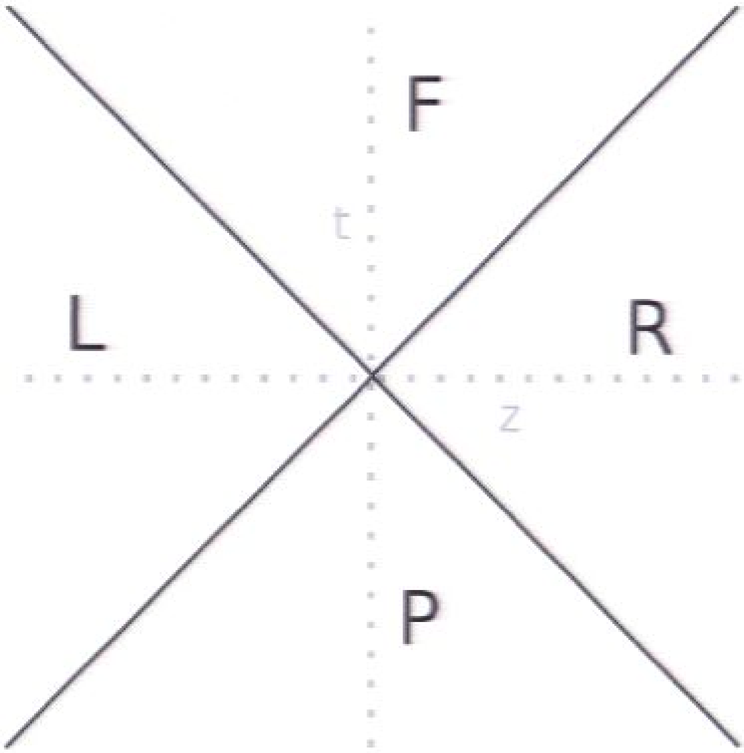
For <u>all</u> fields:

$$[\phi(x_L), \phi(x_R)] = 0$$

For <u>massless</u> fields:

$$[\phi(x_F), \phi(x_P)] = 0$$

commuting fields $\rightarrow$ independent subsystems $\rightarrow$ possibility of entanglement



For all fields:

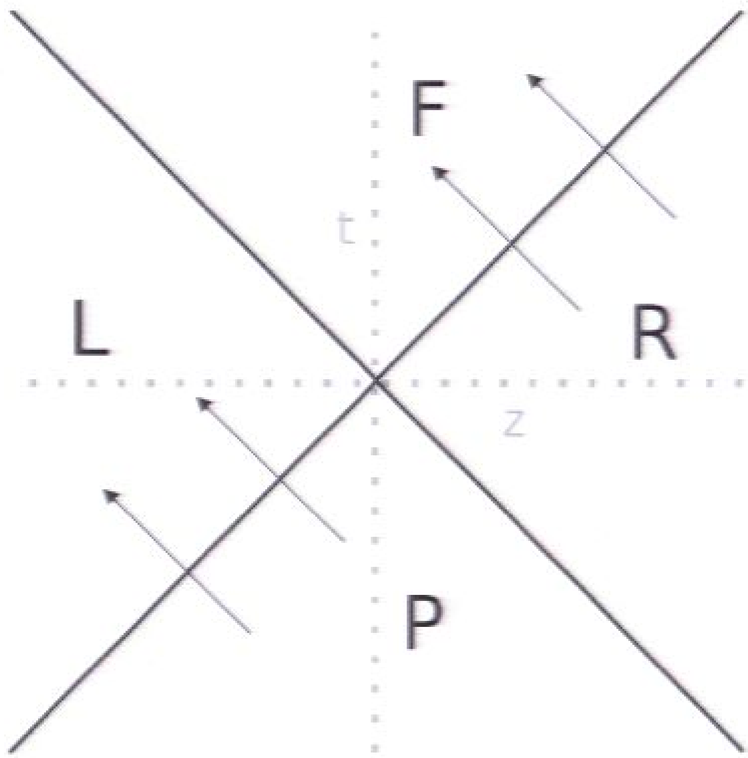
$$[\phi(x_L), \phi(x_R)] = 0$$

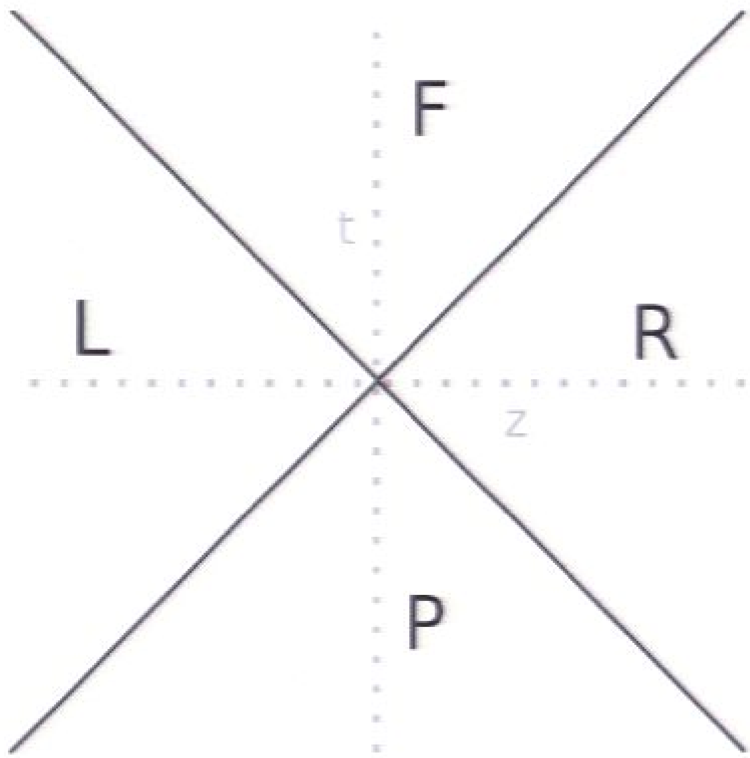
For massless fields:

$$[\phi(x_F), \phi(x_P)] = 0$$

For massive fields: $[\phi(x_F), \phi(x_P)] < m^2$

commuting fields → independent subsystems → possibility of entanglement


F
t
L
R
z
P



F:
$$t = a^{-1} e^{a\eta} \cosh(a\zeta)$$
$$z = a^{-1} e^{a\eta} \sinh(a\zeta)$$

P:
$$t = -a^{-1} e^{a\bar{\eta}} \cosh(a\bar{\zeta})$$
$$z = -a^{-1} e^{a\bar{\eta}} \sinh(a\bar{\zeta})$$

$$\left(\frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \zeta^2}\right)_F \phi = 0$$

$$\left(\frac{\partial^2}{\partial \bar{\eta}^2} - \frac{\partial^2}{\partial \bar{\zeta}^2}\right)_P \phi = 0$$

$$g^F_\omega(\nu) = (4\pi\omega)^{-1/2} e^{-i\omega\nu}$$

$$g^P_\omega(\bar{\nu}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\nu}}$$

$$\nu = \eta + \zeta$$

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$$g_\omega^R(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}$$

$$g_\omega^L(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}}$$

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$$\theta(V) g_\omega^R(\chi) = \int_0^\infty dk \left(\alpha_{\omega k}^R u_k(V) + \beta_{\omega k}^R u_k^*(V)\right)$$

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$$\alpha^F_{\omega k} = \alpha^R_{\omega k} \qquad \beta^F_{\omega k} = \beta^R_{\omega k}$$

$$\alpha^P_{\omega k} = \alpha^L_{\omega k} \qquad \beta^P_{\omega k} = \beta^L_{\omega k}$$

$$\downarrow$$

$$\beta^P_{\omega k} = -e^{-\pi\omega/a}\alpha^{F*}_{\omega k} \qquad \beta^F_{\omega k} = -e^{-\pi\omega/a}\alpha^{P*}_{\omega k}$$

$$\downarrow$$

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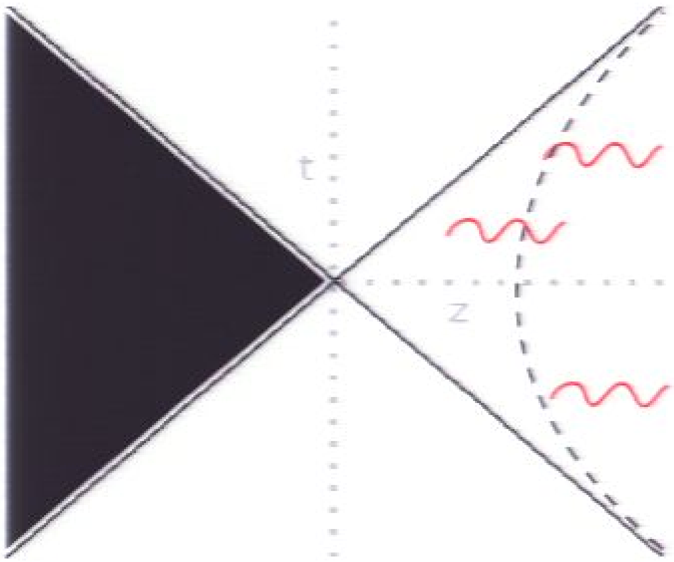
$$\downarrow$$

$$(\hat{a}^{F\dagger}_\omega \hat{a}^F_\omega - \hat{a}^{P\dagger}_\omega \hat{a}^P_\omega)|0_M\rangle = 0$$

$$\downarrow$$

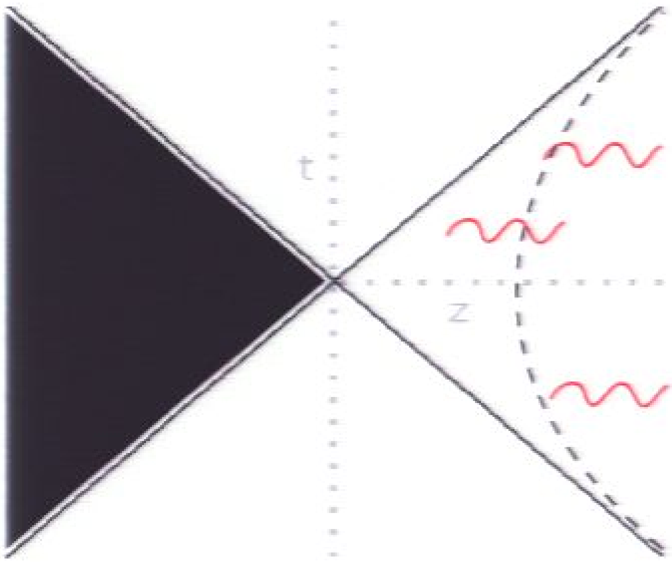
$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!}(\hat{a}^{F\dagger}_{\omega_i}\hat{a}^{P\dagger}_{\omega_i})^{n_i}|0_T\rangle$$



$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle\langle n_i^R| \right]$$

$$T_U = \frac{\hbar a}{2\pi c k_B}$$

One degree corresponds to a = 10^{20} m/s^2.



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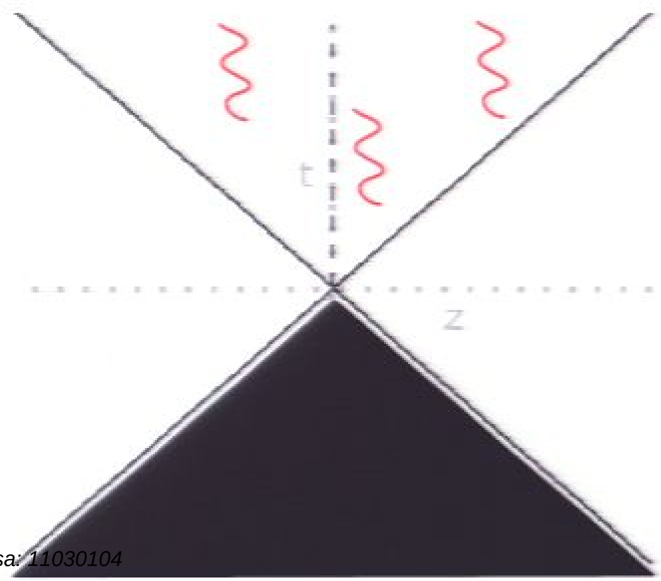

$$\hat{\rho}_F = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^F\rangle\langle n_i^F| \right]$$

$$T = \frac{\hbar a}{2\pi k_B}$$

Detector with trajectory with x=y=z=0 corresponds to: $t = a^{-1} e^{a\eta}$

$$i \frac{\partial}{\partial \eta} \Psi = H_0 \Psi \quad \Longleftrightarrow \quad i \frac{\partial}{\partial t} \Psi = \frac{H_0}{at} \Psi$$

Ordinary Unruh-DeWitt interaction term, in conformal time:

$$i \frac{\partial}{\partial \eta} \Psi = \left(H_0 + e^{a\eta} H_I \right) \Psi$$

Resulting detector response function:

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} e^{a(\eta+\eta')} D^+(\eta, \eta')$$

New factor

$$F_{scaled}(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} e^{a(\eta+\eta')} D^{+}(\eta,\eta')$$

$$D^{+}(x,x') = -\left[(t-t'-i\epsilon)^2 - (\vec{x}-\vec{x}')^2\right]^{-1}$$

Scaled inertial detector.

$$D^{+}(x(\eta),x(\eta')) = -\frac{a^2 e^{-a(\eta+\eta')}}{4\sinh^2(\frac{a}{2}(\eta-\eta')-i\epsilon)}$$

Accelerated detector.

$$T = \frac{\hbar a}{2\pi k_B} \qquad D^{+}(x(\tau),x(\tau')) = -\frac{a^2}{4\sinh^2(\frac{a}{2}(\tau-\tau')-i\epsilon)}$$

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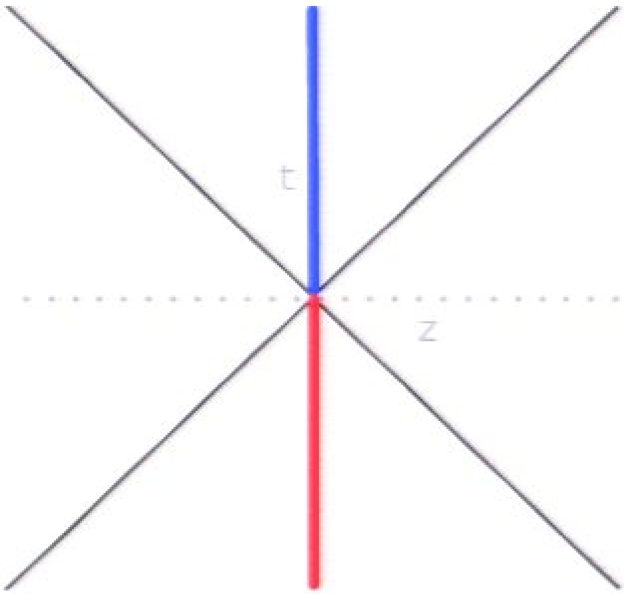


Timelike entanglement can be extracted from the vacuum, by means of two scaled Unruh-DeWitt detectors, one operating in the past, and the other in the future.

Window functions describe the switching of the interaction.

$$\chi_F(\eta) \qquad \chi_P(\bar{\eta})$$

$$H_F = \frac{H_0}{at} + H_I \qquad H_P = \frac{-H_0}{at} + H_I$$

$$H_I = \alpha\chi(\eta)\hat{\phi}(x(\eta))[|0\rangle\langle 1| + |1\rangle\langle 0|]$$

At t=-∞, we have the state |00>, we wish to know whether the state at t=∞ is entangled.

Phys. Rev. A 71, 042104 (2005) Reznik, Retzker, Silman for spacelike

Perturbation theory final state:

$$
\begin{aligned}
|\Psi\rangle = & (1-C)|0_M\rangle|00\rangle \\
& -i\int_{-\infty}^{\infty} d\eta\,\chi_F(\eta)e^{a\eta}e^{-iE\eta}\hat{\phi}(\eta)|0_M\rangle|01\rangle \\
& -i\int_{-\infty}^{\infty} d\bar{\eta}\,\chi_P(\bar{\eta})e^{a\bar{\eta}}e^{iE\bar{\eta}}\hat{\phi}(\bar{\eta})|0_M\rangle|10\rangle \\
& -\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\,\chi_F(\eta)\chi_P(\bar{\eta})e^{a(\eta+\bar{\eta})}e^{-iE(\eta-\bar{\eta})}\hat{\phi}(\eta)\hat{\phi}(\bar{\eta})|0_M\rangle|11\rangle
\end{aligned}
$$

$$
= (1-C)|0_M\rangle|00\rangle - i|A_F\rangle|01\rangle - i|A_P\rangle|10\rangle - |X\rangle|11\rangle
$$

Reduced two-detector density operator at t=∞.

$$\rho = \begin{pmatrix} N & 0 & 0 & -\langle X|0_M\rangle \\ 0 & \langle A_F|A_F\rangle & -\langle A_P|A_F\rangle & 0 \\ 0 & -\langle A_F|A_P\rangle & \langle A_P|A_P\rangle & 0 \\ -\langle 0_M|X\rangle & 0 & 0 & \langle X|X\rangle \end{pmatrix}$$

The necessary and sufficient condition for non-separability is that the negativity be positive. To lowest order, the negativity is:

$$\mathcal{N}(\rho) = |\langle 0_M|X\rangle| - \sqrt{\langle A_F|A_F\rangle\langle A_P|A_P\rangle}$$

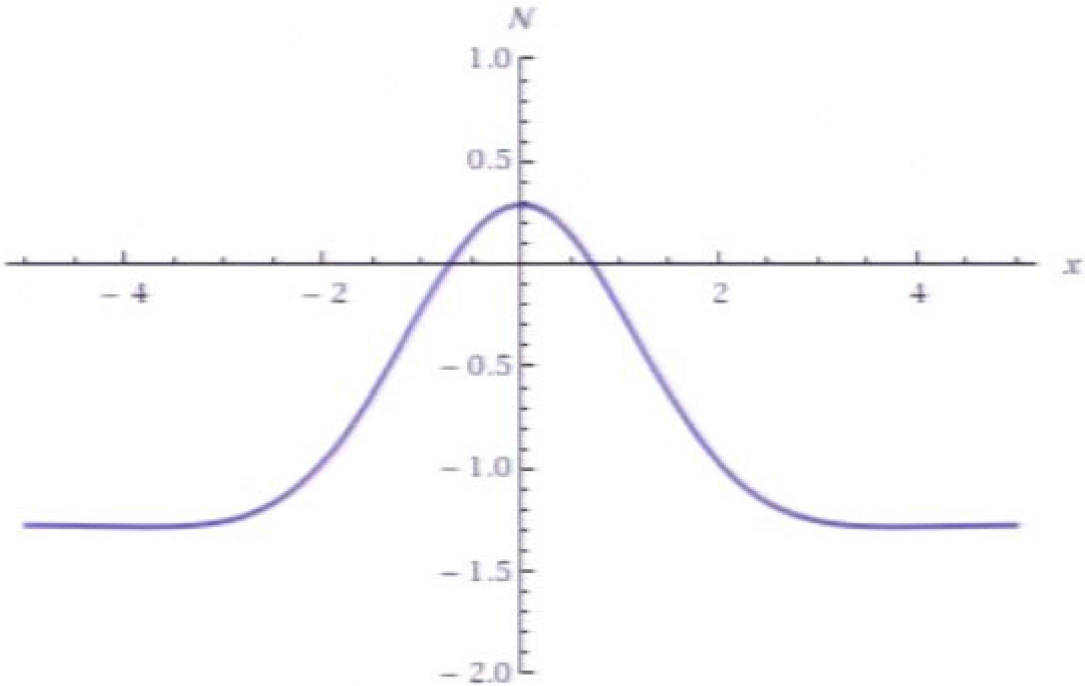
This amounts to the following condition for entanglement:

$$\left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_F(\eta)\chi_P(\bar{\eta})e^{a(\eta+\bar{\eta})}e^{-iE(\eta-\bar{\eta})}\langle 0_M|\hat{\phi}(\eta)\hat{\phi}(\bar{\eta})|0_M\rangle\right|$$

$$> \left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, \chi_F(\eta)\chi_F(\eta')e^{a(\eta+\eta')}e^{-iE(\eta-\eta')}\langle 0_M|\hat{\phi}(\eta)\hat{\phi}(\eta')|0_M\rangle\right|$$

Take the window functions to be the following:

$$\chi_P = e^{-\bar{\eta}^2} \qquad \chi_F = e^{-(\eta - x)^2}$$



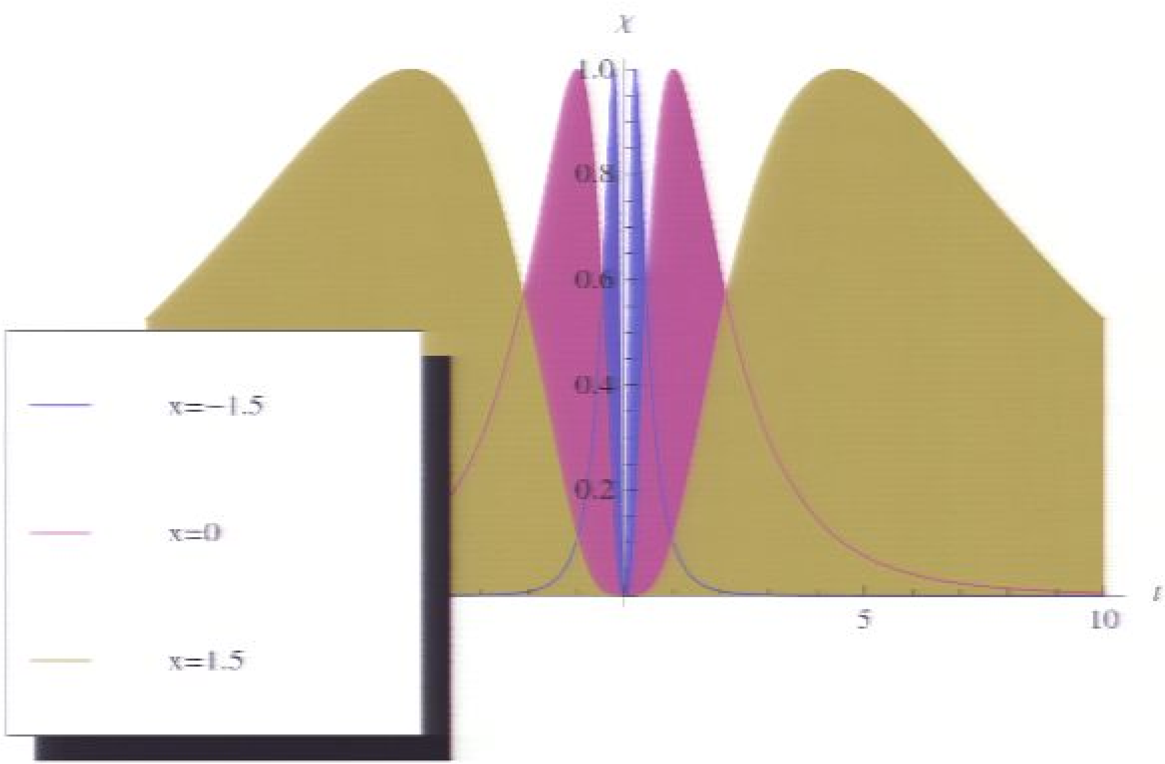
For E = 1, a=2, this is a plot of the following:

$$\left| \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_F(\eta)\chi_P(\bar{\eta}) e^{a(\eta+\bar{\eta})} e^{-iE(\eta-\bar{\eta})} \langle 0_M | \hat{\phi}(\eta)\hat{\phi}(\bar{\eta}) | 0_M \rangle \right|$$

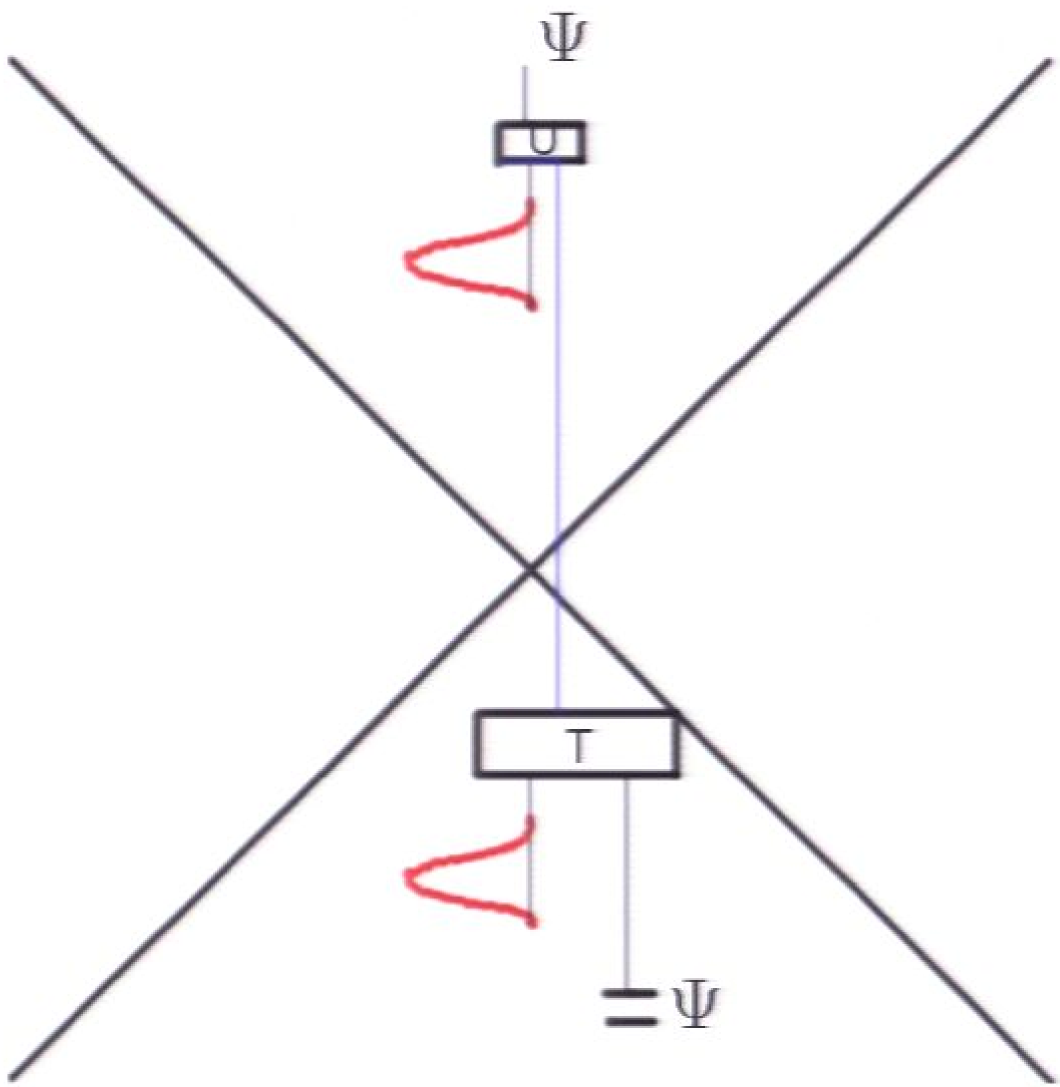
$$- \left| \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, \chi_F(\eta)\chi_F(\eta') e^{a(\eta+\eta')} e^{-iE(\eta-\eta')} \langle 0_M | \hat{\phi}(\eta)\hat{\phi}(\eta') | 0_M \rangle \right|$$

For symmetrical gaussian window functions in conformal time, the entanglement remains constant.

$$\chi_P = e^{-(\bar{\eta}-x)^2} \qquad \chi_F = e^{-(\eta-x)^2}$$



Pairs of symmetrical window functions plotted in Minkowski time t.


Ψ
U
T
= Ψ

arxiv:1003.0720 “Entanglement between the future and past in the quantum vacuum” (in press, PRL)

arxiv:1101.2565 “Extraction of timelike entanglement from the quantum vacuum”

Detector with trajectory with x=y=z=0 corresponds to: $t = a^{-1} e^{a\eta}$

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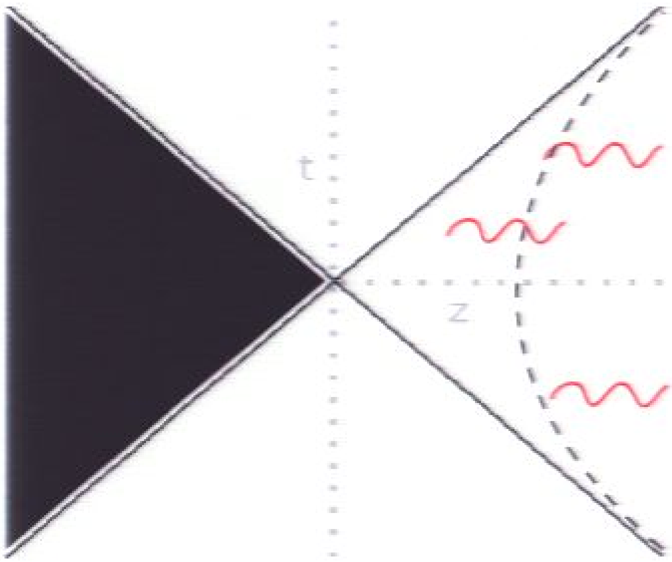
↑ New factor



$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle\langle n_i^R| \right]$$

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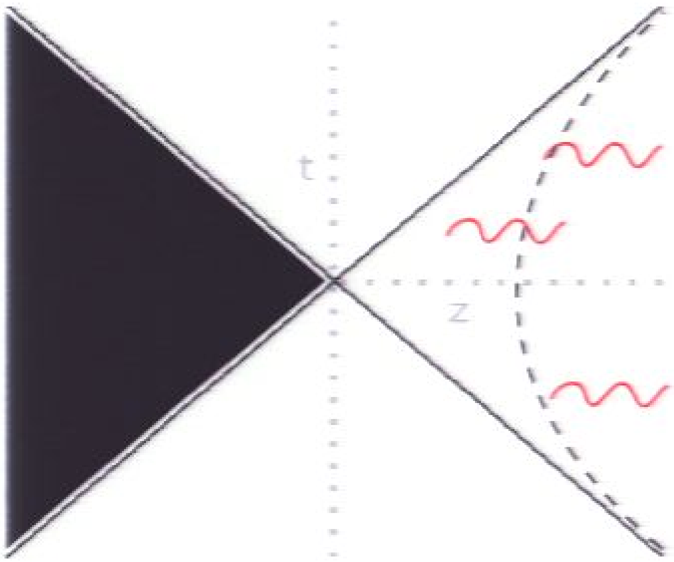

$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle\langle n_i^R| \right]$$

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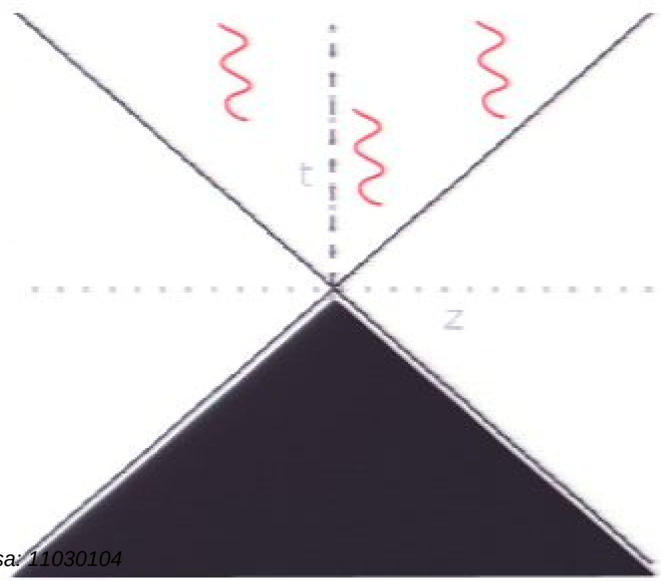

$$\hat{\rho}_F = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^F\rangle\langle n_i^F| \right]$$

$$T = \frac{\hbar a}{2\pi k_B}$$

$$\alpha^F_{\omega k} = \alpha^R_{\omega k} \qquad \beta^F_{\omega k} = \beta^R_{\omega k}$$

$$\alpha^P_{\omega k} = \alpha^L_{\omega k} \qquad \beta^P_{\omega k} = \beta^L_{\omega k}$$

$$\downarrow$$

$$\beta^P_{\omega k} = -e^{-\pi\omega/a}\alpha^{F*}_{\omega k} \qquad \beta^F_{\omega k} = -e^{-\pi\omega/a}\alpha^{P*}_{\omega k}$$

$$\downarrow$$

$$G_\omega(V) = \theta(V) g^F_\omega(\nu) + \theta(-V) e^{-\pi\omega/a} g^{P*}_\omega(\bar{\nu})$$

$$\bar{G}_\omega(V) = \theta(-V) g^P_\omega(\bar{\nu}) + \theta(V) e^{-\pi\omega/a} g^{F*}_\omega(\nu)$$

$$\downarrow$$

$$(\hat{a}^{F\dagger}_\omega \hat{a}^F_\omega - \hat{a}^{P\dagger}_\omega \hat{a}^P_\omega)|0_M\rangle = 0$$

$$\downarrow$$

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!} (\hat{a}^{F\dagger}_{\omega_i} \hat{a}^{P\dagger}_{\omega_i})^{n_i} |0_T\rangle$$

$$g_\omega^R(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}$$

$$g_\omega^L(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}}$$

$$\chi = \tau + \epsilon \qquad \bar{\chi} = -\bar{\tau} - \bar{\epsilon}$$

$$u_k(V) = (4\pi k)^{-1/2} e^{-ikV}$$

$$V = t + z$$

$$V = a^{-1} e^{a\chi} \qquad V = -a^{-1} e^{-a\bar{\chi}}$$

$$\theta(V) g_\omega^R(\chi) = \int_0^\infty dk \left(\alpha_{\omega k}^R u_k(V) + \beta_{\omega k}^R u_k^*(V)\right)$$

$$\theta(-V) g_\omega^L(\bar{\chi}) = \int_0^\infty dk \left(\alpha_{\omega k}^L u_k(V) + \beta_{\omega k}^L u_k^*(V)\right)$$

$$g_\omega^F(\nu) = (4\pi\omega)^{-1/2} e^{-i\omega\nu}$$

$$g_\omega^P(\bar{\nu}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\nu}}$$

$$\nu = \eta + \zeta \qquad \bar{\nu} = -\bar{\eta} - \bar{\zeta}$$

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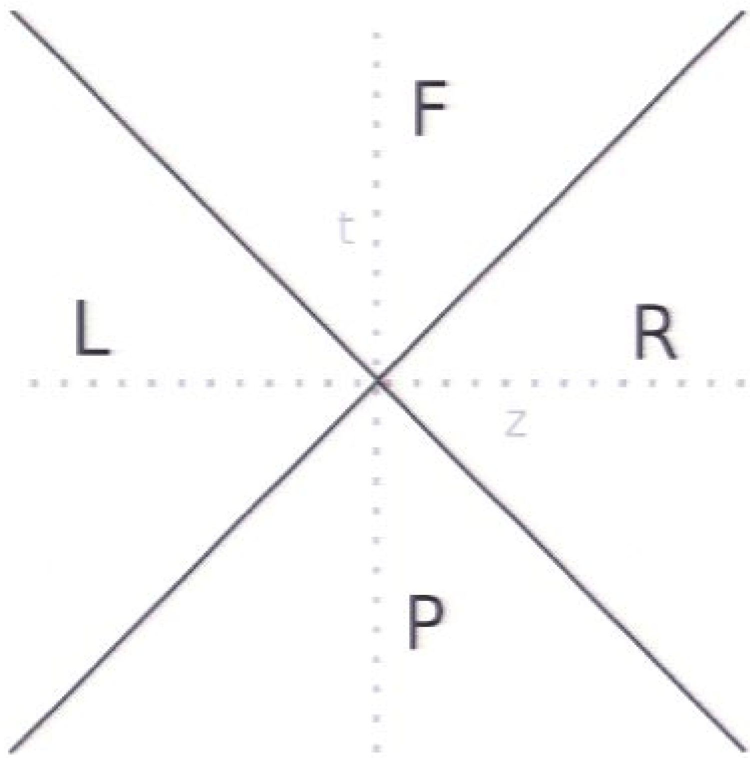

F:
$$t = a^{-1} e^{a\eta} \cosh(a\zeta)$$
$$z = a^{-1} e^{a\eta} \sinh(a\zeta)$$

P:
$$t = -a^{-1} e^{a\bar{\eta}} \cosh(a\bar{\zeta})$$
$$z = -a^{-1} e^{a\bar{\eta}} \sinh(a\bar{\zeta})$$

$$\left(\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\zeta^2}\right)_F \phi = 0$$

$$\left(\frac{\partial^2}{\partial\bar{\eta}^2} - \frac{\partial^2}{\partial\bar{\zeta}^2}\right)_P \phi = 0$$

$$g^F_\omega(\nu) = (4\pi\omega)^{-1/2} e^{-i\omega\nu}$$

$$g^P_\omega(\bar{\nu}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\nu}}$$

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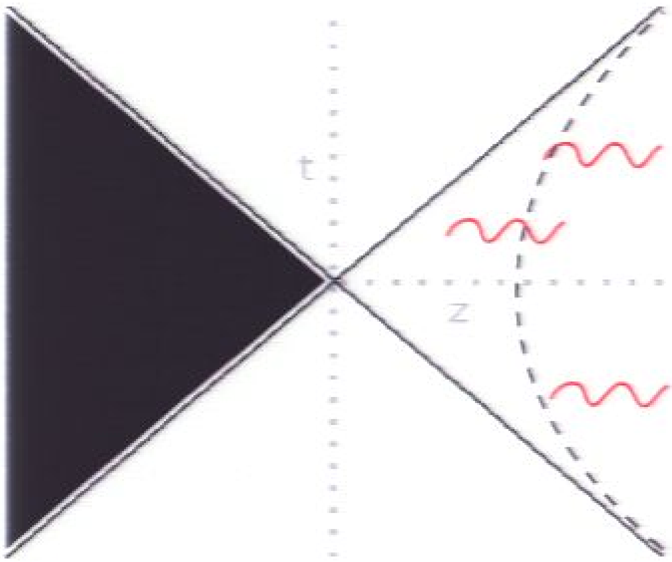

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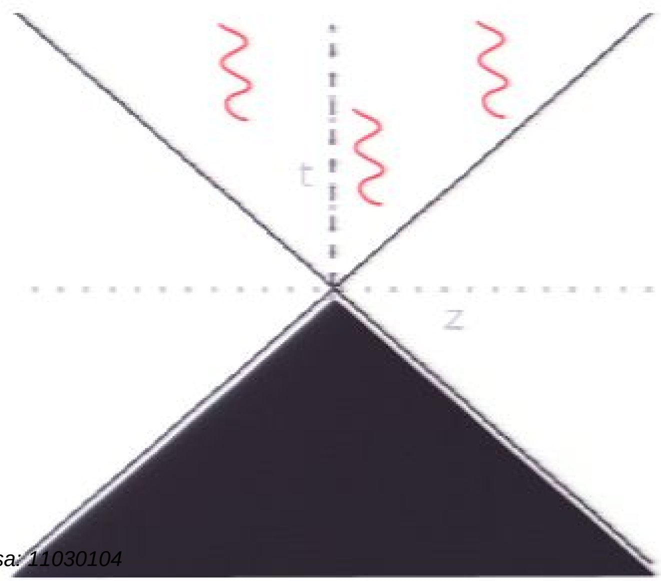

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$$T = \frac{\hbar a}{2\pi k_B}$$

Detector with trajectory with x=y=z=0 corresponds to: $t = a^{-1} e^{a\eta}$

$$i\frac{\partial}{\partial\eta}\Psi = H_0\Psi \quad \Longleftrightarrow \quad i\frac{\partial}{\partial t}\Psi = \frac{H_0}{at}\Psi$$

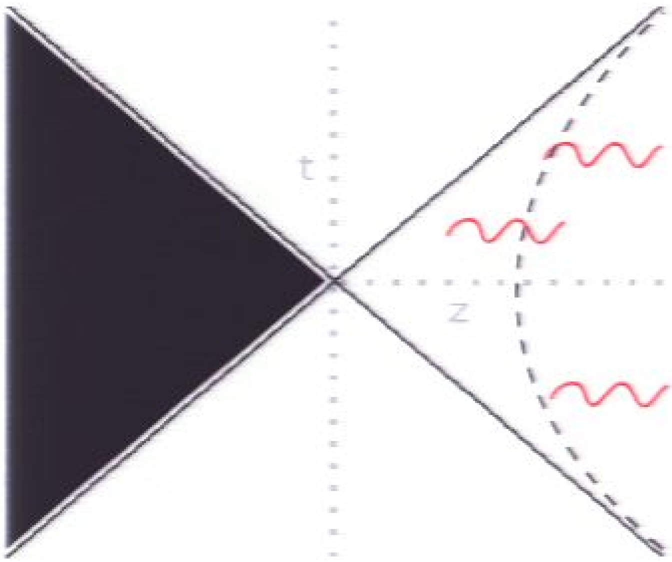
Ordinary Unruh-DeWitt interaction term, in conformal time:

$$i\frac{\partial}{\partial\eta}\Psi = \left(H_0 + e^{a\eta} H_I\right)\Psi$$

Resulting detector response function:

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} e^{a(\eta+\eta')} D^+(\eta, \eta')$$

↑ New factor



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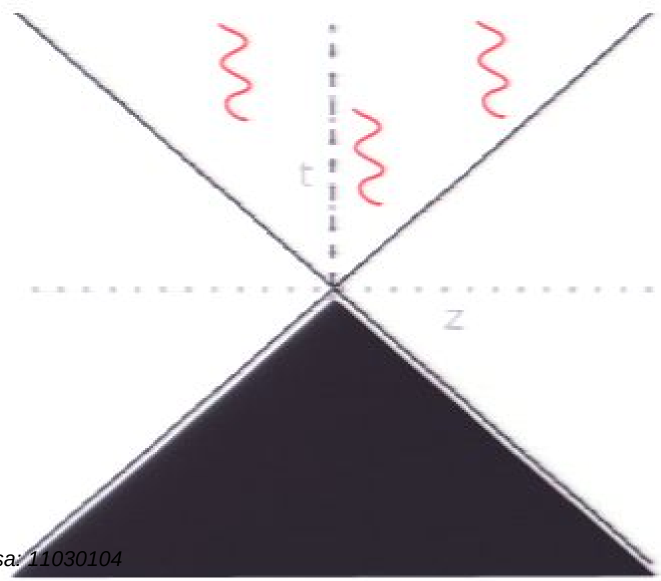

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