Title: Geodesically Complete Analytic Solutions to a Cyclic Universe

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Abstract: I will present analytic solutions to a class of cosmological models described by a canonical scalar field minimally coupled to gravity and experiencing self interactions through a hyperbolic potential. Using models and methods of solution inspired by 2T-physics, I will show how analytic solutions can be obtained including radiation and spacial curvature. Among the analytic solutions, there are many interesting geodesically complete cyclic solutions, both singular and non-singular ones. Cyclic cosmological models provide an alternative to inflation for solving the horizon and flatness problems as well as generating scale-invariant perturbations. I will argue in favor of the geodesically complete solutions as being more attractive for constructing a more satisfactory model of cosmology. When geodesic completeness is imposed, it restricts models and their parameters to certain a parameter subspace, including some quantization conditions on parameters. I will explain the theoretical origin of our model from the point of view of 2T-gravity as well as from the point of view of the colliding branes scenario. If time permits, I will discuss how to associate solutions of the quantum Wheeler-deWitt equation with the classical analytic solutions, physical aspects of some of the cyclic solutions, and outline future directions.

Geodesically Complete Analytic Solutions for a Cyclic Universe

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March 1st @ Perimeter Institute

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1103.xxxx

Collaborating with Itzhak Bars (USC, Perimeter)
Paul Steinhardt (Princeton)
Neil Turok (Perimeter)

Outline

- Motivation
- Models of cosmology
- Solve the model using 2T-Gravity
- Generic solution
- Geodesically complete solutions
- Summary
- Future Directions

Motivation

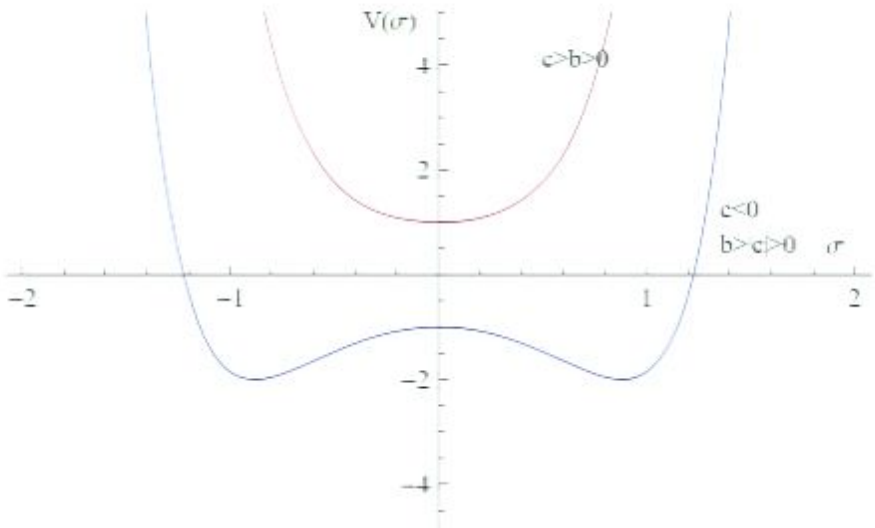
- What happens beyond initial singularity?
  (cyclic cosmology, ekpyrotic model Steinhardt, Turok 2002)
- Can one avoid singularity? (non-singular bounce)
- How to deal with singularity? (WdW equation)

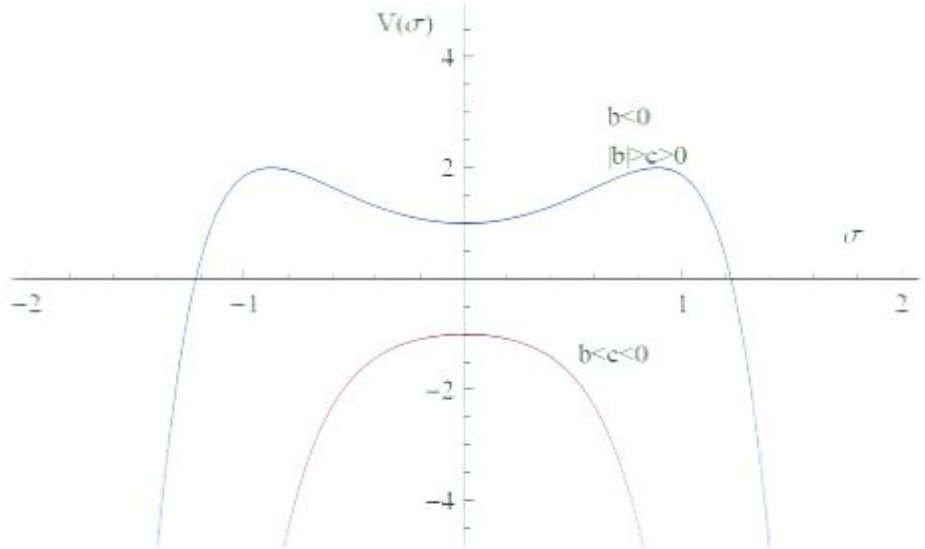
Models of Cosmology

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \frac{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}{2} - V(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\}$$

$K^{-1}=2.43\times10^{18}$GeV

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa}\right)^4\left[b\cosh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right) + c\sinh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right)\right]$$





$$ds^2 = a^2(\tau)\left(-d\tau^2 + \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$$

K>0: closed, K<0: open, K=0 flat

Equations of motion

Assuming homogeneous σ and A_μ

Friedmann Equations

$$\frac{\dot{a}^2}{a^4} = \frac{\kappa^2}{3}\left[\frac{\dot{\sigma}^2}{2a^2} + V(\sigma) + \frac{\rho}{a^4}\right] - \frac{K}{a^2},$$

$$\frac{\ddot{a}}{a^3} - \frac{\dot{a}^2}{a^4} = -\frac{\kappa^2}{3}\left[\frac{\dot{\sigma}^2}{a^2} - V(\sigma) + \frac{\rho}{a^4}\right],$$

where

$$\frac{\rho}{a^4} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + a^{-2}F_{0\rho}F_0{}^{\rho}$$

e.o.m for σ and Maxwell Equations

$$\frac{\ddot{\sigma}}{a^2} + 2\frac{\dot{a}}{a^3}\dot{\sigma} + V'(\sigma) = 0,$$

$$\nabla_\mu F^{\mu\nu} = 0$$

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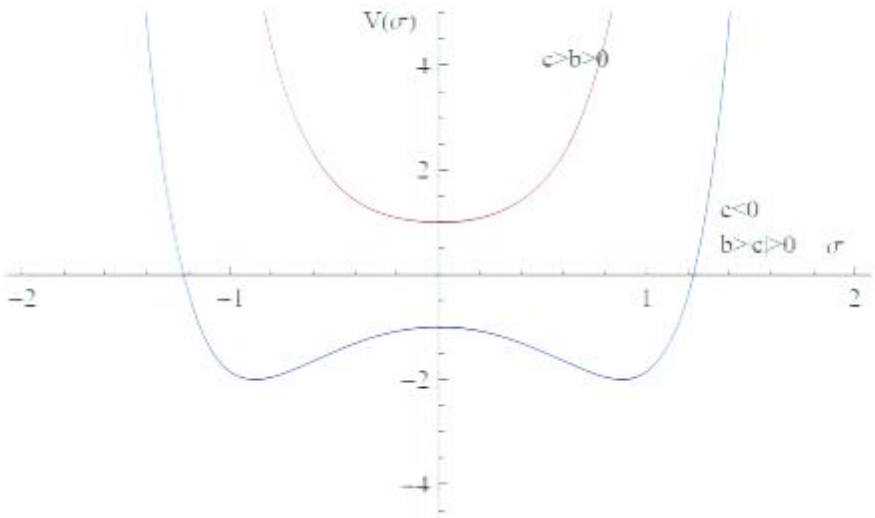

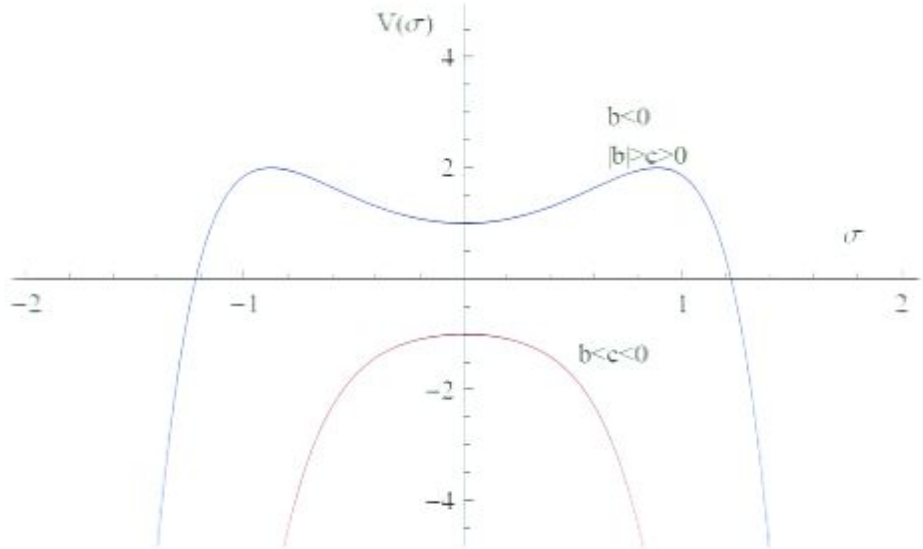

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Conclusions

Among all parameters in the model

$$b, c, \epsilon, \rho, K$$

- For all singular cyclic solutions, there exist a quantization equation to be satisfied.
- For all non singular cyclic solutions, only need to satisfy a inequality equation that restrict the range of the parameters.

Solving the model using 2T-Gravity

- Fundamental Gauge Principle SP(2R)

$$S_{2T} = \int d^6 X \sqrt{G} L\left(x^\mu, w, u\right)$$

I. Bars (1998)

- Gauge fixing (general coordinate symmetry and 2T-Gaugy symmetry) as well as put part of the e.o.m on-shell

$$_{conformal} = \int d^4x \sqrt{-g} \left[\frac{(\partial\phi)^2}{2} - \frac{(\partial s)^2}{2} + \frac{1}{12}\left(\phi^2 - s^2\right) R - \phi^4 f\left(\frac{s}{\phi}\right) - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \right]$$

- No dimensional parameter
- f is an arbitrary function of s/Φ
- Φ has wrong sign kinetic term
- Weyl symmetry (thus ghost free)

$$\phi' = e^{-\lambda(x)}\phi,\ s' = e^{-\lambda(x)} s,\ g'_{\mu\nu} = e^{2\lambda(x)} g_{\mu\nu},\ A'_\mu = A_\mu$$

Einstein Frame (gauge)

If Φ^2-s^2 stay positive $\left(\phi'^2 - s'^2\right) = e^{-2\lambda(x)}\left(\phi^2 - s^2\right)$

$$\frac{\left(\phi_E^2 - s_E^2\right)}{12} = \frac{1}{2k^2}$$

$$\phi_E = \pm\frac{\sqrt{6}}{\kappa}\cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right), s_E = \pm\frac{\sqrt{6}}{\kappa}\sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)$$

$$S_{\text{Einstein}} = \int d^4x\sqrt{-g_E}\left[\frac{R(g_E)}{2\kappa^2} - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}g_E^{\mu\rho}g_E^{\nu\sigma}\right]$$

For every V(σ) in Einstein frame there exist a corresponding f(s/Φ)

$$f\left(\frac{s}{\phi}\right) = c\left(\frac{s}{\phi}\right)^4 + b \longrightarrow V(\sigma) = \left(\frac{\sqrt{6}}{\kappa}\right)^4\left[b\cosh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right) + c\sinh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right)\right]$$

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Gamma Frame (gauge)

$$a_\gamma(\tau) = 1$$

$$R_\gamma = 6K$$

$$S_\gamma = \int d^4x \sqrt{-g_\gamma} \left[\frac{(\partial\phi_\gamma)^2}{2} - \frac{(\partial s_\gamma)^2}{2} + \frac{1}{2}\left(\phi_\gamma^2 - s_\gamma^2\right) K - \phi_\gamma^4 f\left(\frac{s_\gamma}{\phi_\gamma}\right) - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g_\gamma^{\mu\rho} g_\gamma^{\nu\sigma} \right]$$

Specialize to the potential $f\left(\frac{s_\gamma}{\phi_\gamma}\right) = c\left(\frac{s_\gamma}{\phi_\gamma}\right)^4 + b$

$$S_\gamma = \int d^4x \sqrt{-g_\gamma} \left\{ \begin{array}{c} \frac{(\partial\phi_\gamma)^2}{2} + \frac{1}{2} K \phi_\gamma^2 - b\phi_\gamma^4 \\ -\frac{(\partial s_\gamma)^2}{2} - \frac{1}{2} K s_\gamma^2 - c s_\gamma^4 \\ -\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g_\gamma^{\mu\rho} g_\gamma^{\nu\sigma} \end{array} \right\}$$

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The E.O.M for g_{00} should still be satisfied

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When specialized to homogeneous fields

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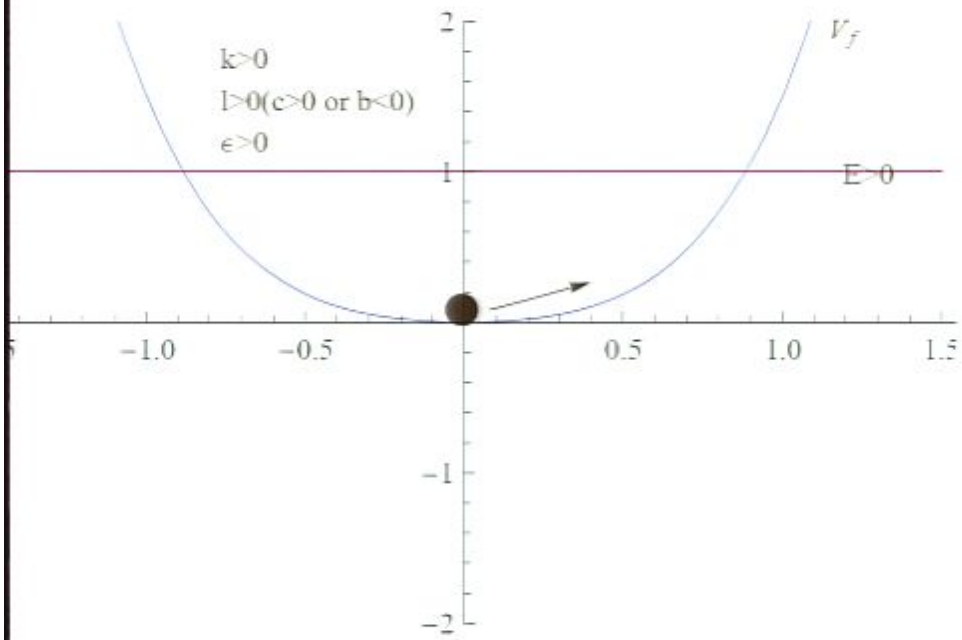

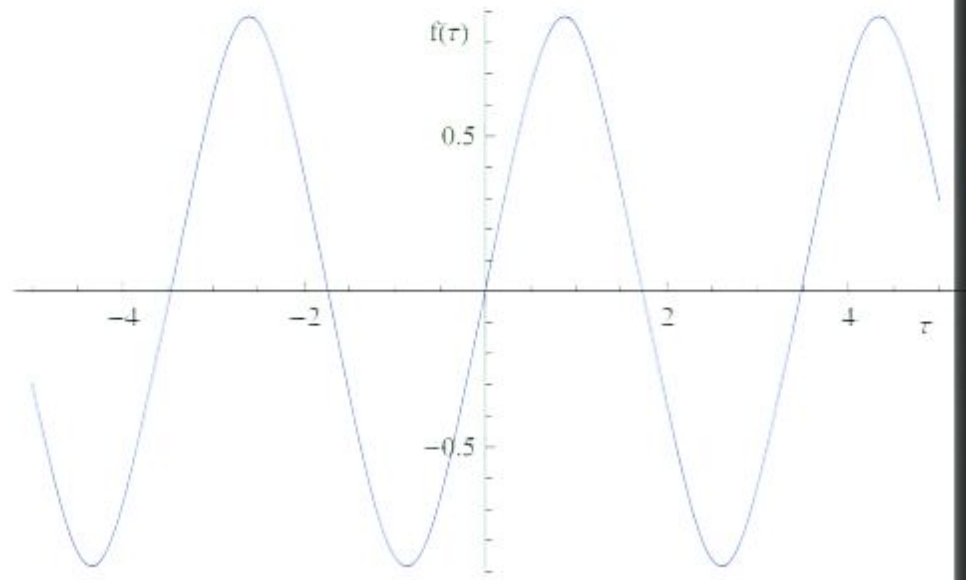

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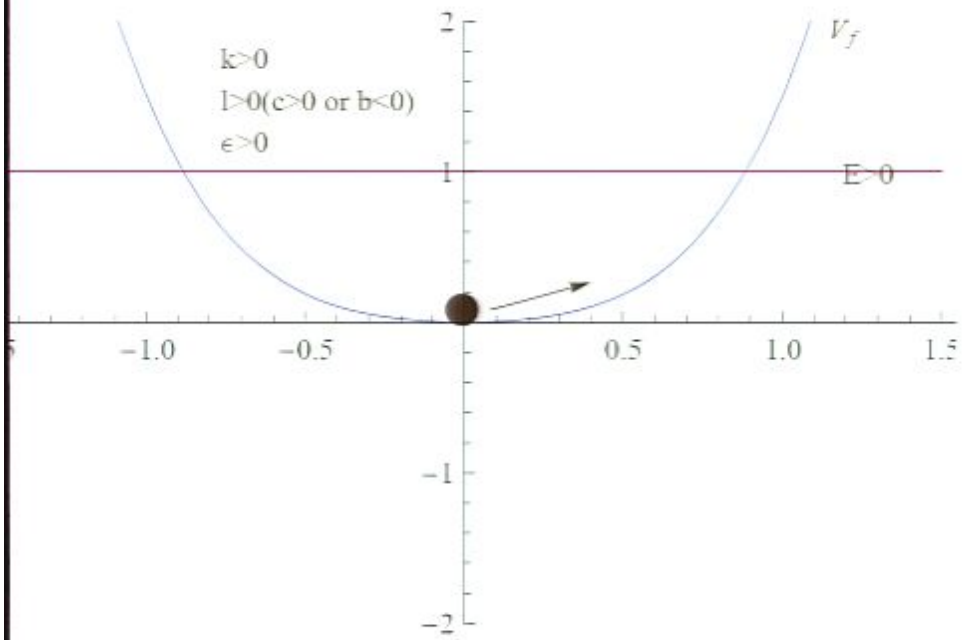

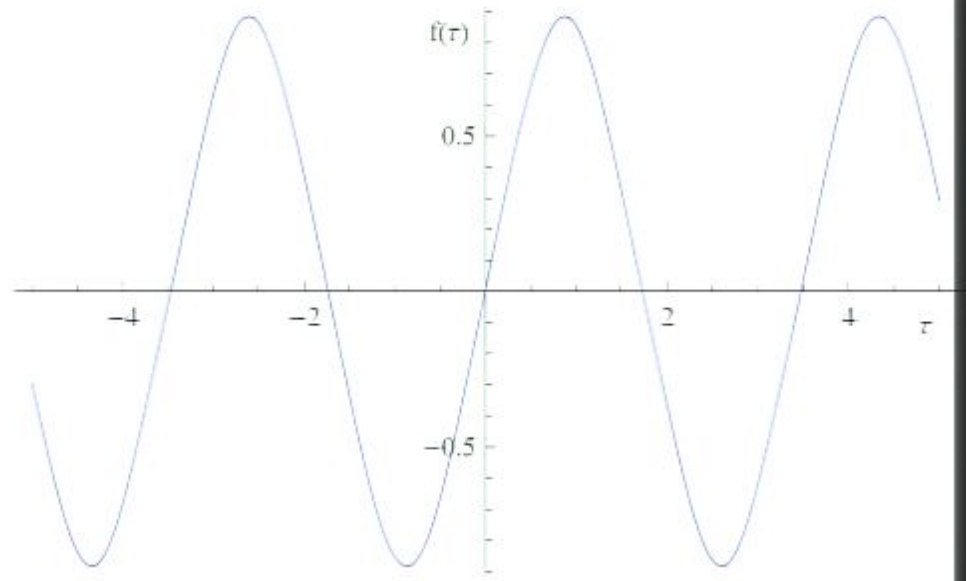

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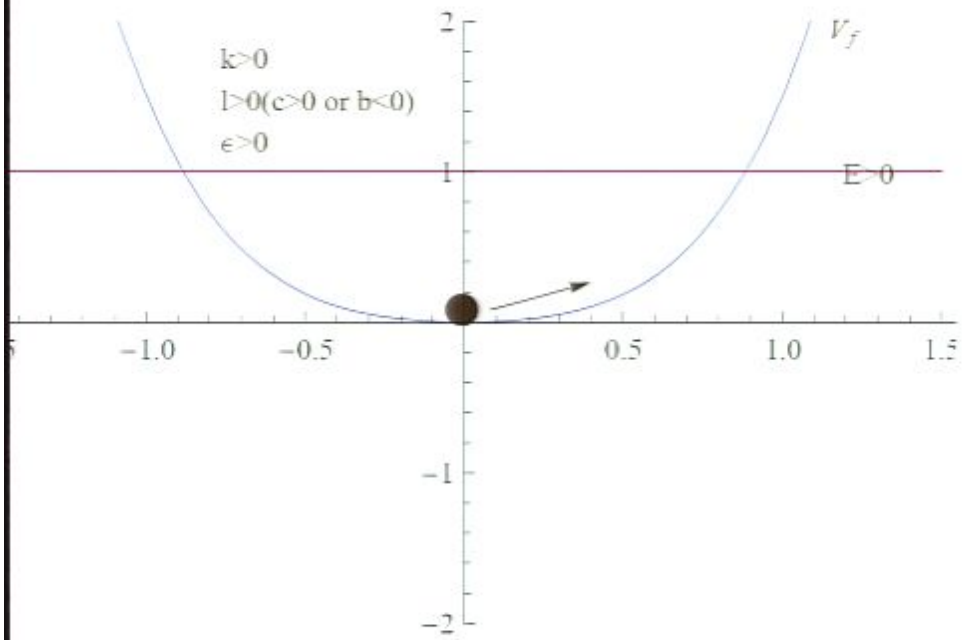

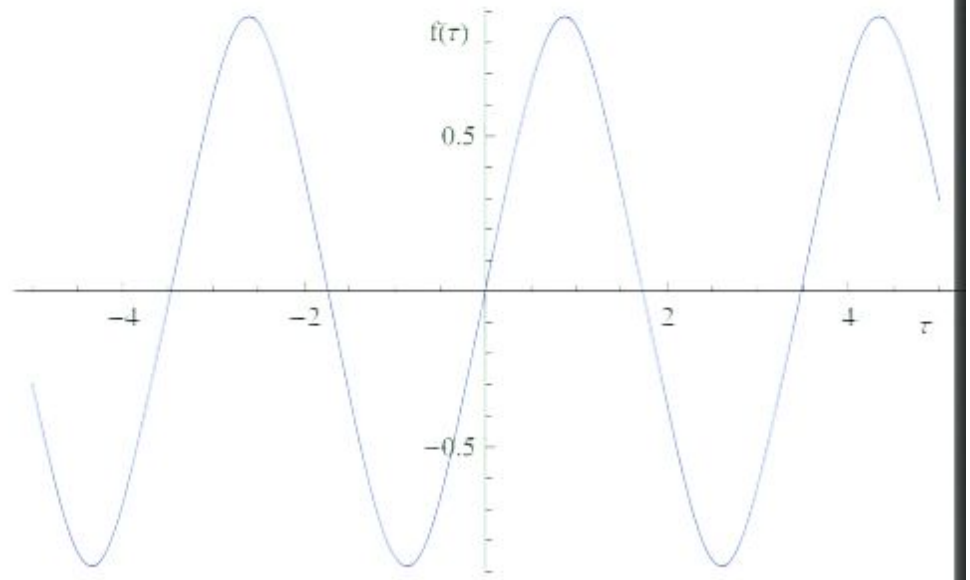

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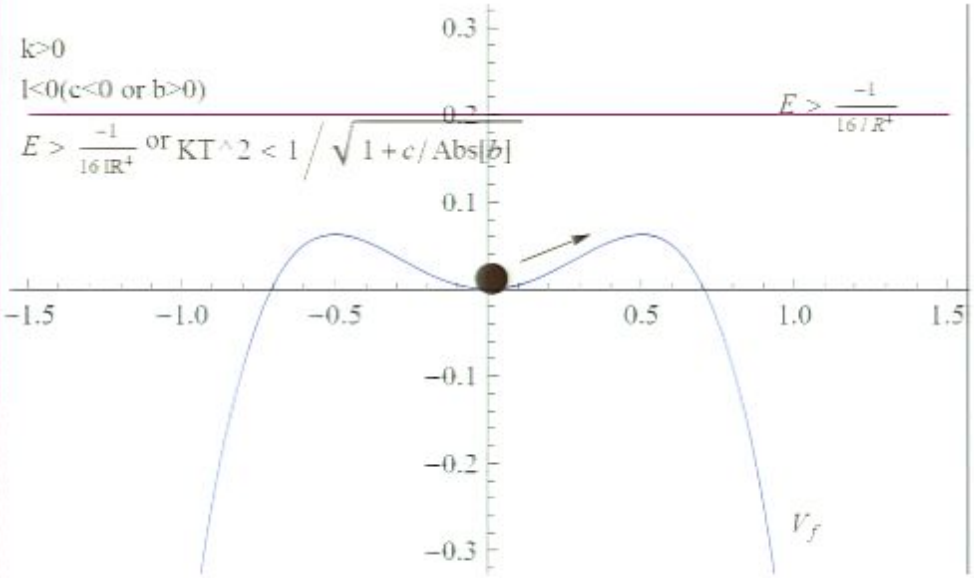
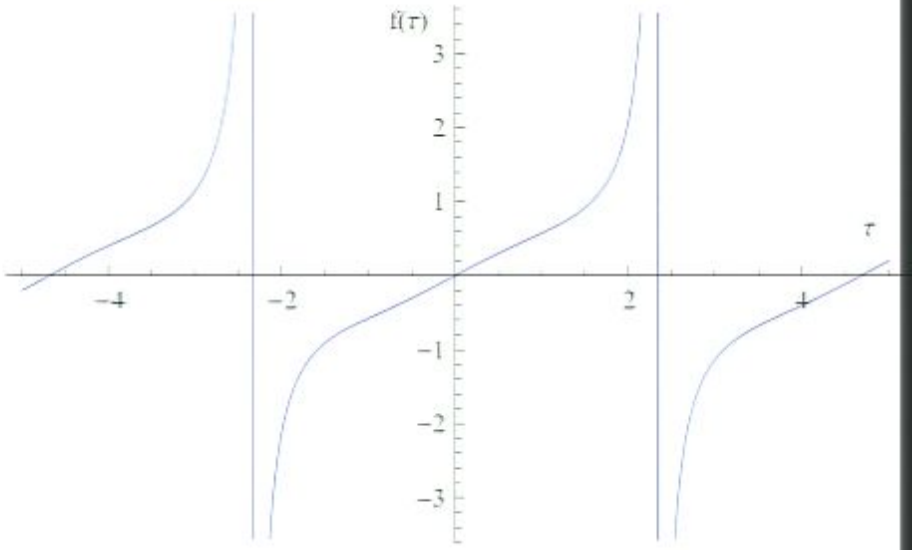


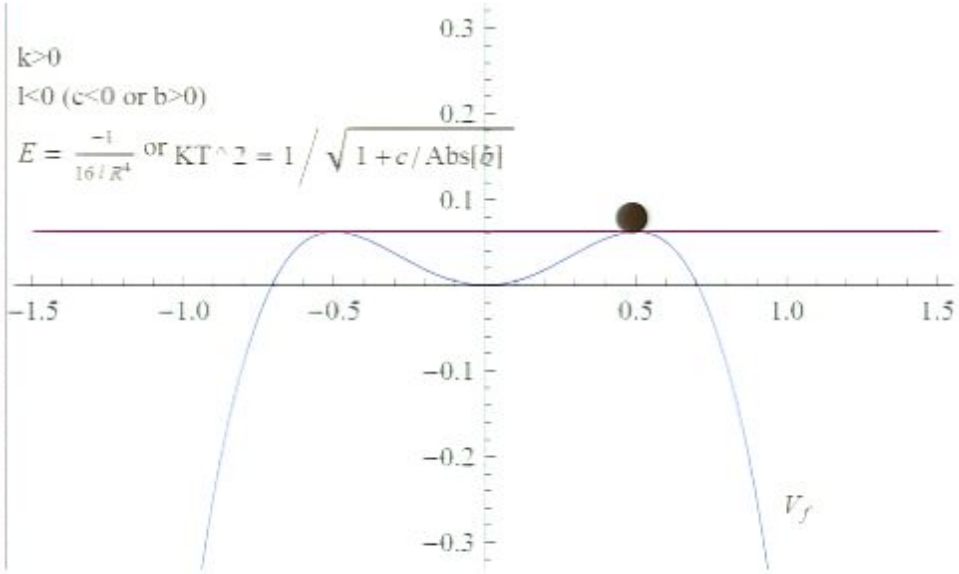


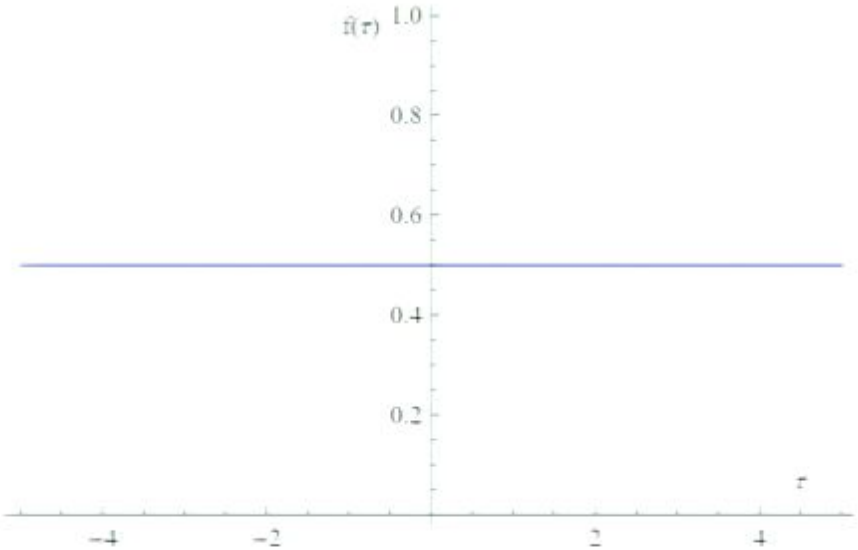
$$f[t\_,\ l\_,\ \epsilon\_,\ K\_] := \left(\frac{-\epsilon}{l}\right)^{\frac{1}{4}} \frac{\mathrm{JacobiSN}\left[\frac{t}{T_+[\epsilon,l]},\ m_+[\epsilon,\ l,\ K]\right]}{1 + \mathrm{JacobiCN}\left[\frac{t}{T_+[\epsilon,l]},\ m_+[\epsilon,\ l,\ K]\right]}$$

$$T_+[\epsilon\_,\ l\_] := (64\ (-\epsilon\ l))^{\frac{-1}{4}}$$

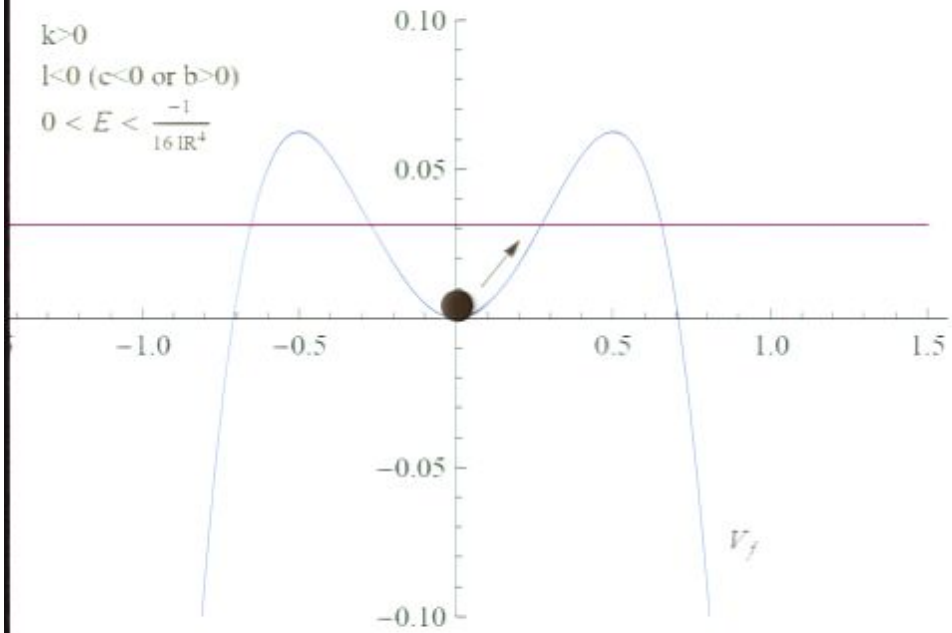
$$m_+[\epsilon\_,\ l\_,\ K\_] := \frac{1}{2} + \frac{K}{8\ (-\epsilon\ l)^{\frac{1}{2}}}$$

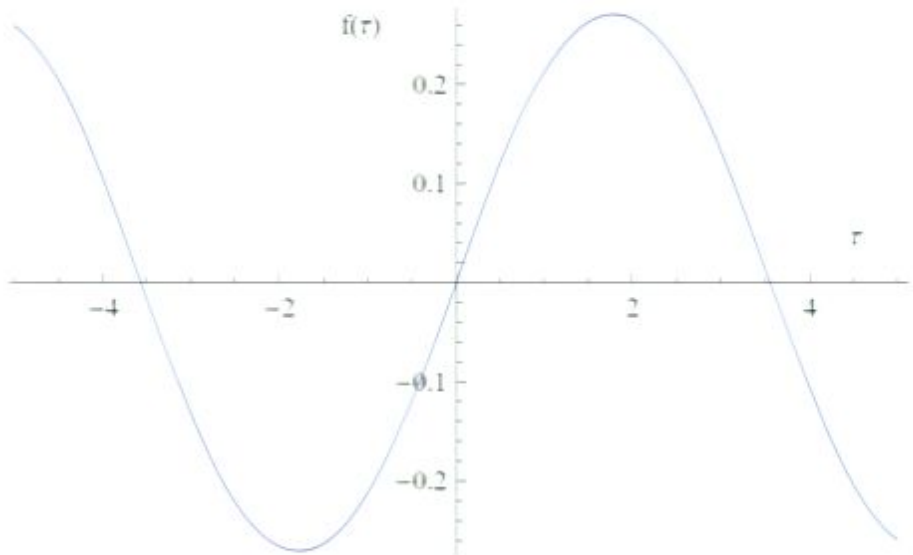
$$\mathrm{Pe}[\epsilon\_,\ l\_,\ K\_] := 4\ T_+[\epsilon,\ l]\ \mathrm{EllipticK}[m_+[\epsilon,\ l,\ K]]$$





$$f[t\_, I\_, K\_] := \left(\frac{-K}{4\,I}\right)^{\frac{1}{2}}$$



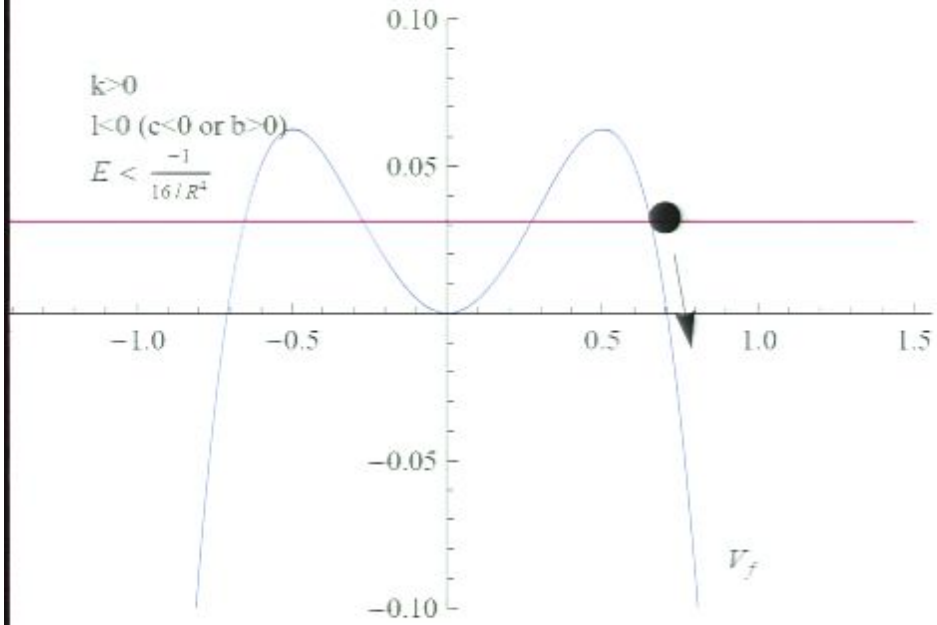


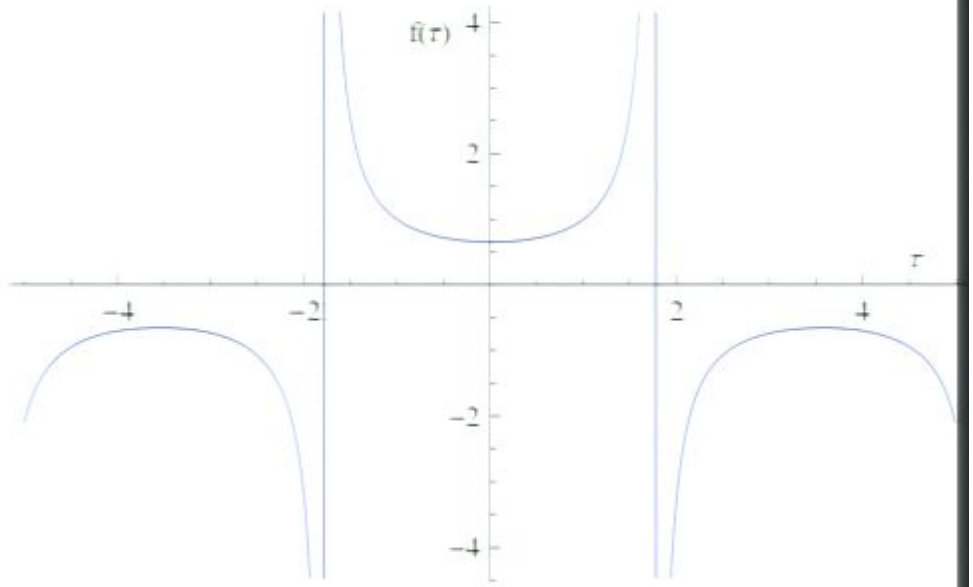
$$f[t\_, \epsilon\_, l\_, K\_] := \sqrt{\frac{K}{-2\,l} - \frac{K + \sqrt{K^2 + 16\,l\,\epsilon}}{-4\,l}}\ \text{JacobiSN}\left[\frac{t}{T_i[\epsilon, l, K]}, m_i[\epsilon, l, K]\right]$$

$$T_i[\epsilon\_, l\_, K\_] := \frac{1}{\sqrt{\frac{K + \sqrt{K^2 + 16\,l\,\epsilon}}{2}}}$$

$$m_i[\epsilon\_, l\_, K\_] := \frac{2\,K}{K + \sqrt{K^2 + 16\,l\,\epsilon}} - 1$$

$$pe[\epsilon\_, l\_, K\_] := 4\,T_i[\epsilon, l, K]\ \text{EllipticK}[m_i[\epsilon, l, K]]$$



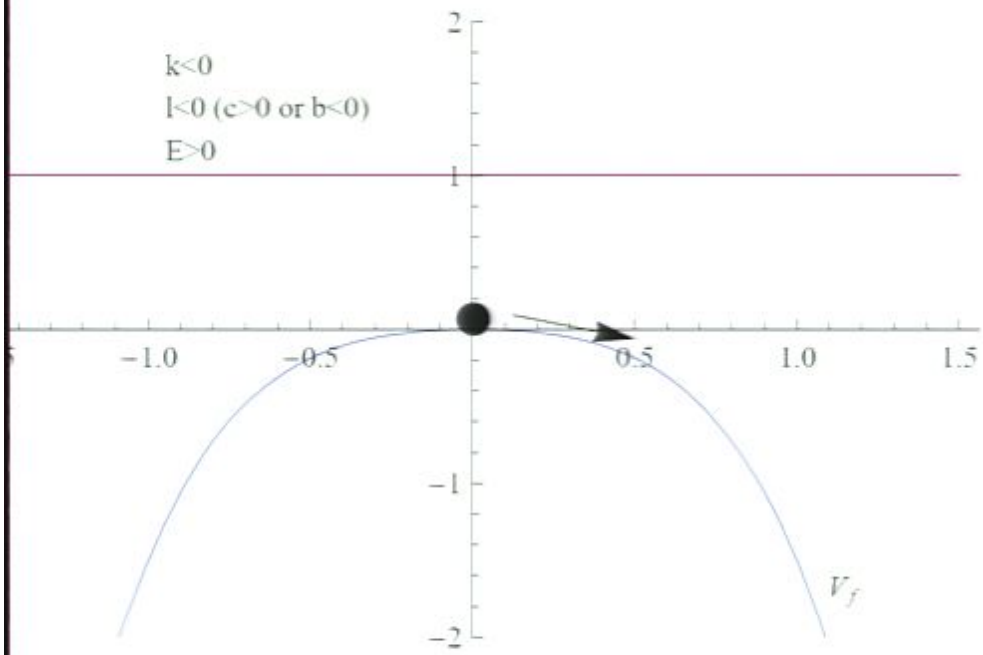


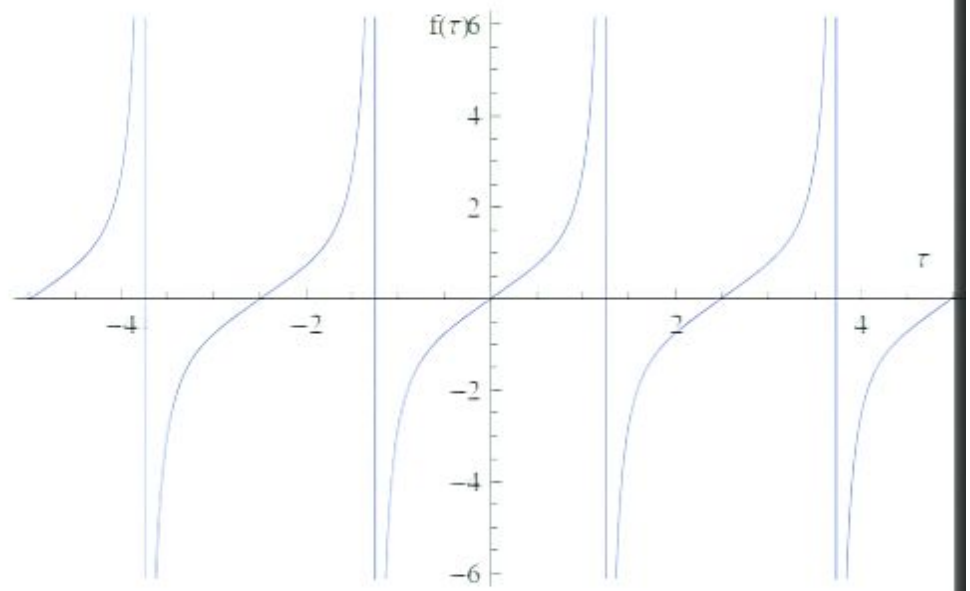
$$f[t\_, \epsilon\_, l\_, K\_] := \frac{\left(\frac{\frac{-1}{R^2} - \left(\frac{1}{R^4} + 16\, l\, \epsilon\right)^{\frac{1}{2}}}{4\, l}\right)^{\frac{1}{2}}}{\text{JacobiCN}\left[\frac{t}{T_0[\epsilon, l, K]}, m_0[\epsilon, l, K]\right]}$$

$$T_0[\epsilon\_, l\_, K\_] := \left(K^2 + 16\, l\, \epsilon\right)^{\frac{-1}{4}}$$

$$m_0[\epsilon\_, l\_, K\_] := \frac{1}{2} - \frac{1}{2\left(1 + 16\, \frac{l\, \epsilon}{K^2}\right)^{\frac{1}{2}}}$$

$$\text{Pe}[\epsilon\_, l\_, K] := 4\, T_0[\epsilon, l, K]\, \text{EllipticK}[m_0[\epsilon, l, K]]$$



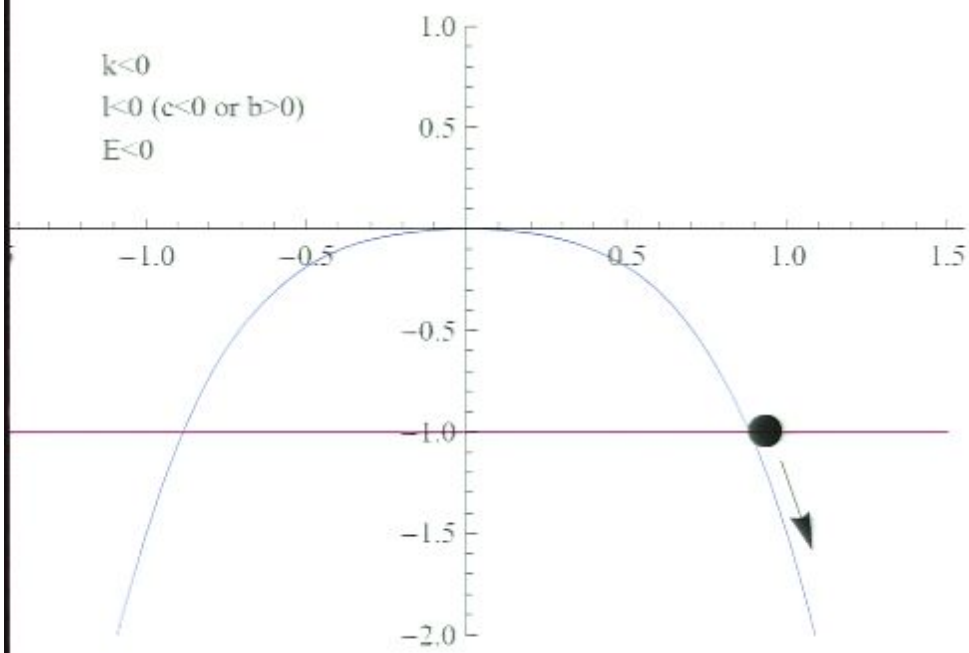


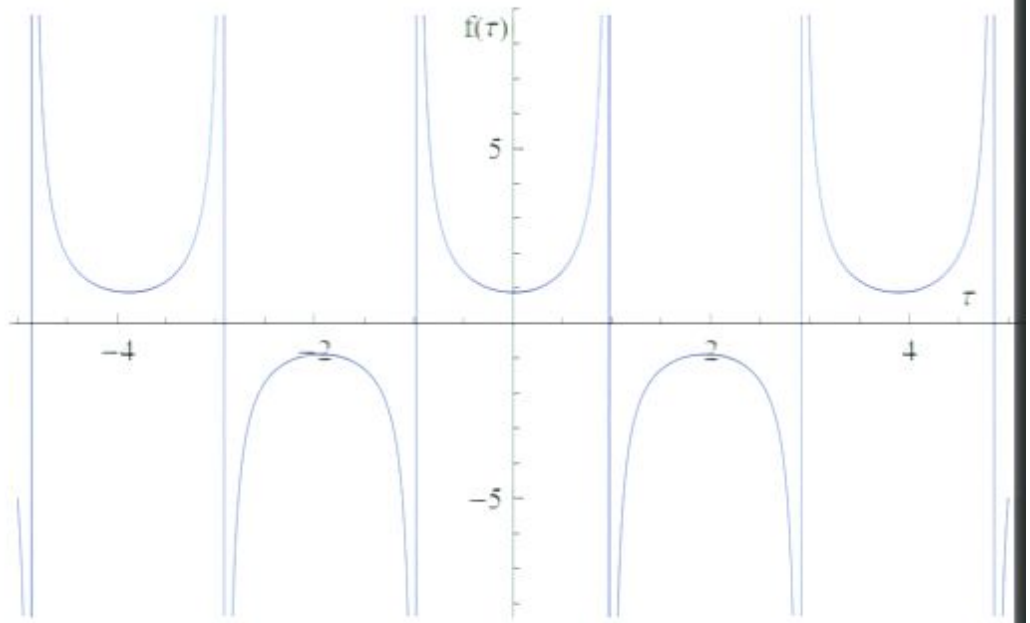
$$\mathtt{f[t\_,\ \epsilon\_,\ l\_,\ K\_]} := \left(\frac{-\epsilon}{l}\right)^{\frac{1}{4}} \frac{\mathtt{JacobiSN}\left[\frac{t}{T_+[\epsilon, l]},\ \mathtt{m}[\epsilon,\ l,\ K]\right]}{1 + \mathtt{JacobiCN}\left[\frac{t}{T_+[\epsilon, l]},\ \mathtt{m}[\epsilon,\ l,\ K]\right]}$$

$$T_+[\epsilon\_,\ l\_] := (-64\ \epsilon\ l)^{\frac{-1}{4}}$$

$$\mathtt{m}[\epsilon\_,\ l\_,\ K\_] := \frac{1}{2} + K\,(T_+[\epsilon,\ l])^2$$

$$\mathtt{P}[\epsilon\_,\ l\_,\ K\_] := 4\,\mathtt{T}[\epsilon,\ l]\ \mathtt{EllipticK}[\mathtt{m}[\epsilon,\ l,\ K]]$$

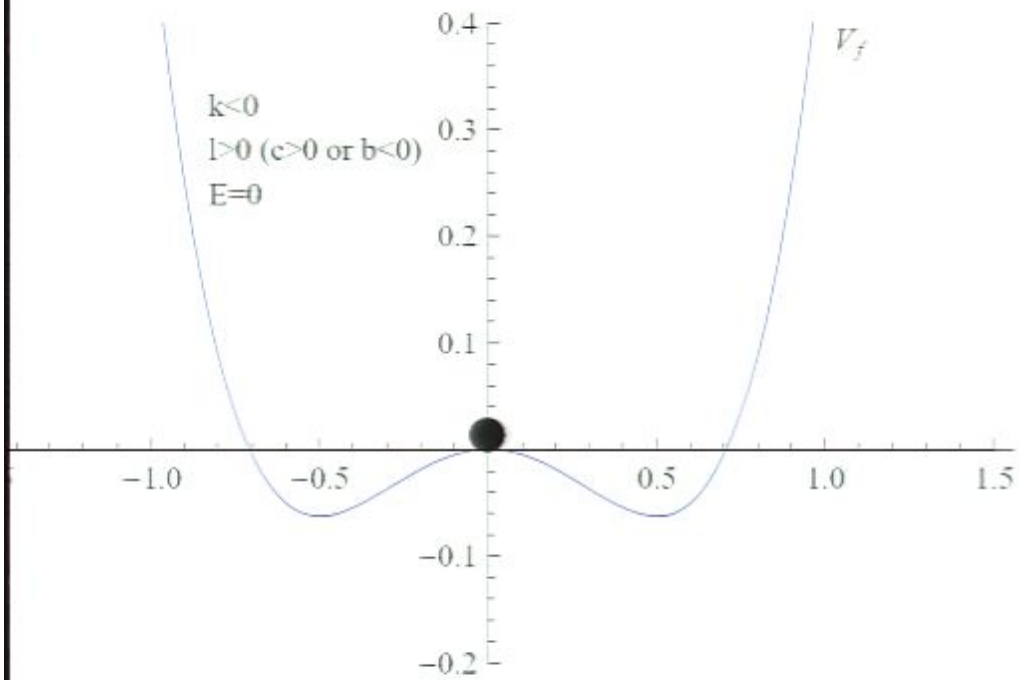


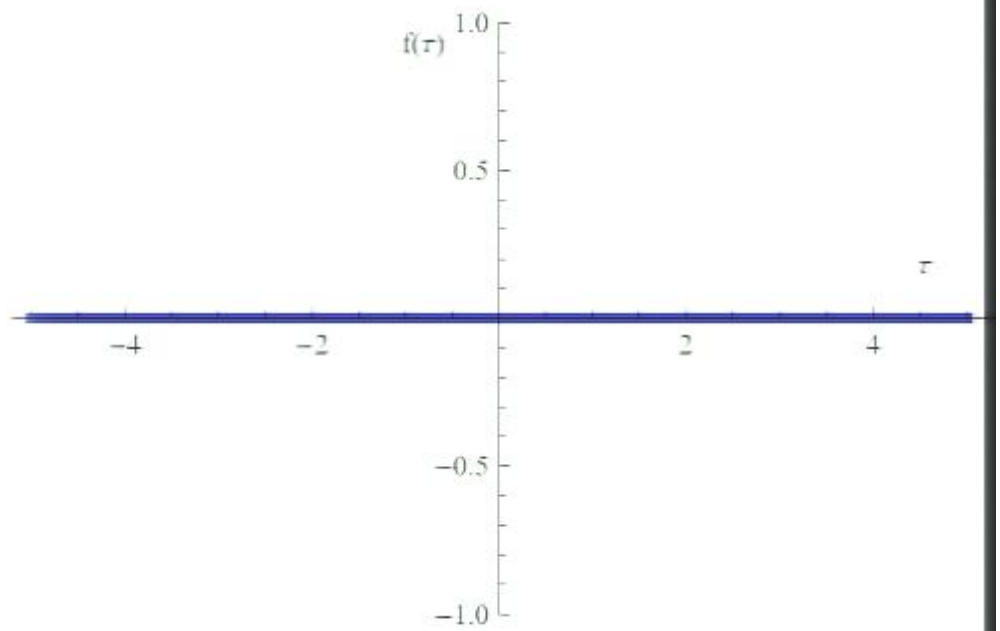


$$f[t\_, \epsilon\_, l\_, K\_] := \left(\frac{1-\left(1+\frac{16}{K^2}\,\epsilon\, l\right)^{\frac{1}{2}}}{-\frac{4\,l}{K}}\right)^{\frac{1}{2}} \frac{1}{\text{JacobiCN}\left[\frac{t}{T_o[\epsilon, l, K]}, m_o[\epsilon, l, K]\right]}$$

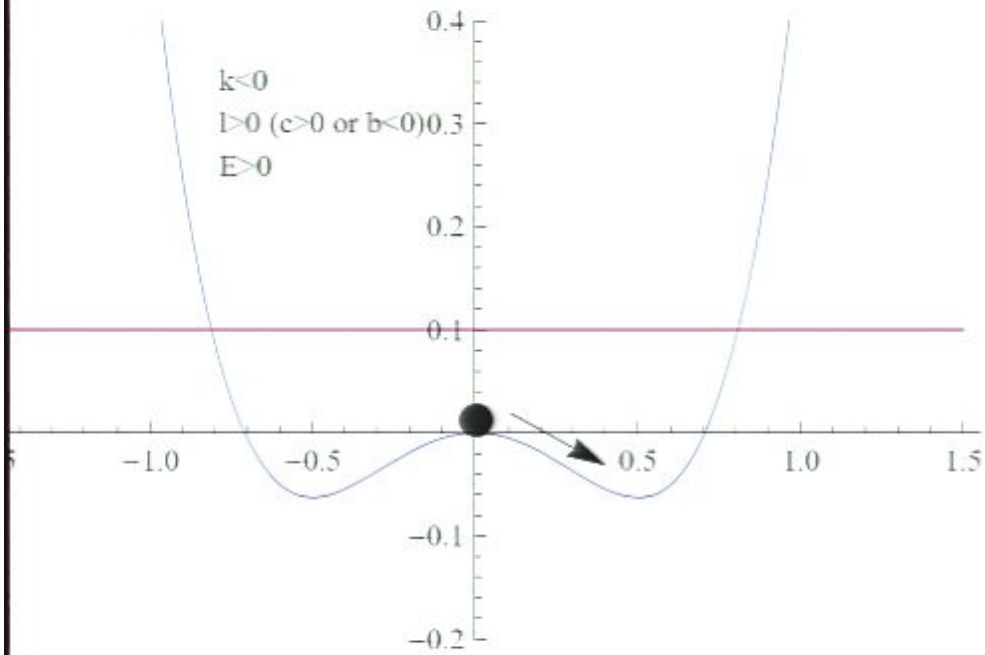
$$T_o[\epsilon\_, l\_, K\_] := \left(K^2 + 16\,\epsilon\, l\right)^{\frac{-1}{4}}$$

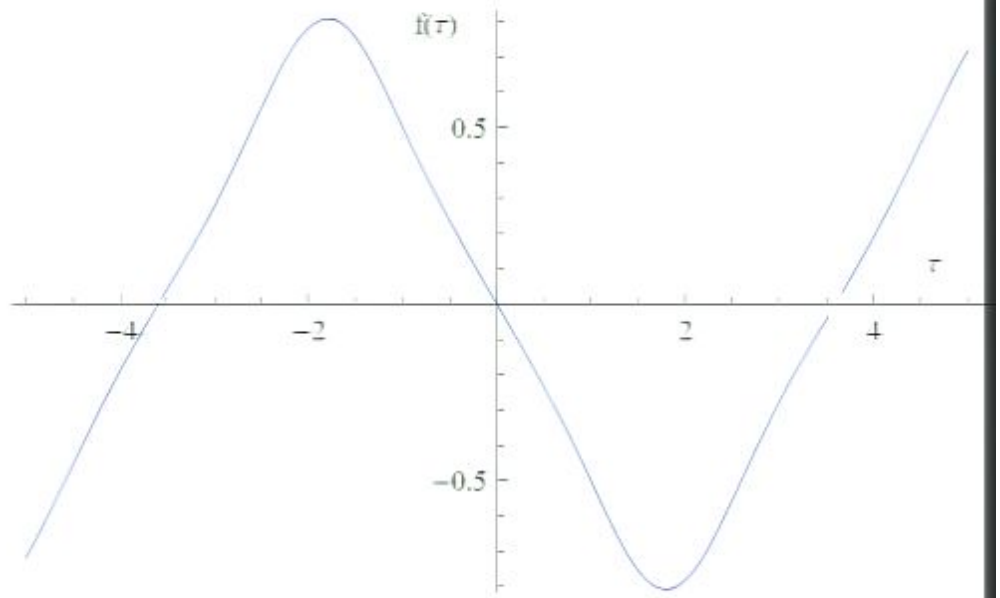
$$m_o[\epsilon\_, l\_, K\_] := \frac{-K + (T_o[\epsilon, l, K])^{-2}}{2\,(T_o[\epsilon, l, K])^{-2}}$$


k<0
l>0 (c>0 or b<0)
E=0
V_f
0.4
0.3
0.2
0.1
−0.1
−0.2
−1.0
−0.5
0.5
1.0
1.5


f(τ)
τ
1.0
0.5
−0.5
−1.0
−4
−2
2
4

```
f[t_, l_] := 0
```



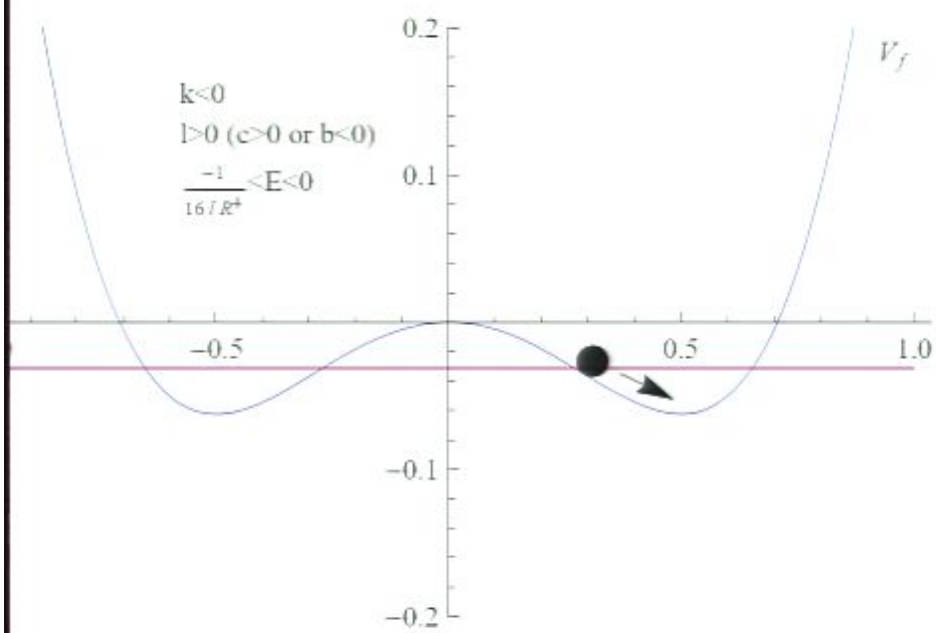


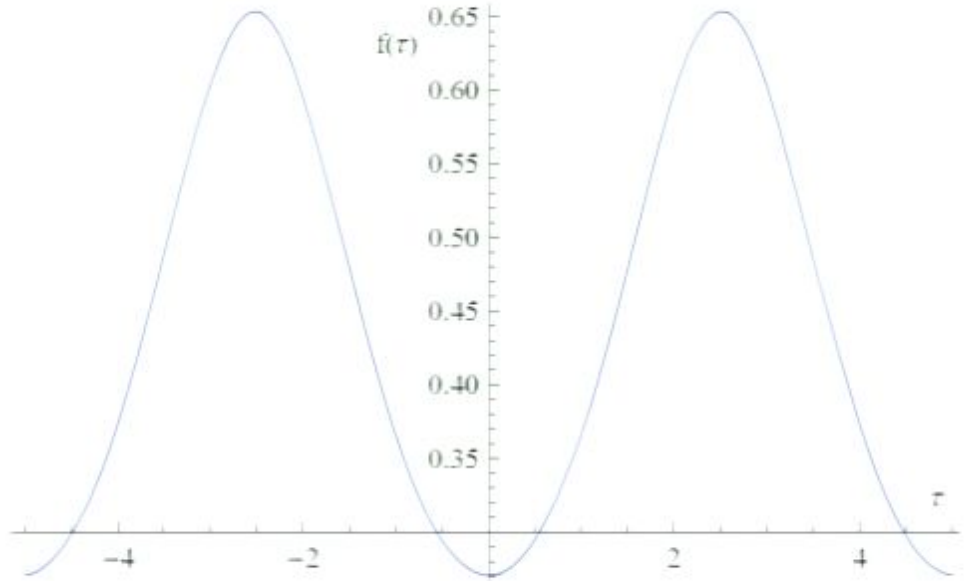
$$f[t\_, \epsilon\_, l\_, K\_] := \left(\frac{-1-\left(K^2+16\,l\,\epsilon\right)^{\frac{1}{2}}}{-4\,l}\right)^{\frac{1}{2}} \text{JacobiDN}\left[\frac{t+\frac{T_i[\epsilon,l,K]}{\sqrt{m_i[\epsilon,l,K]}}\ \text{EllipticK}\left[\frac{1}{m_i[\epsilon,l,K]}\right]}{T_i[\epsilon,l,K]},\ m_i[\epsilon,l,K]\right]$$

$$T_i[\epsilon\_, l\_, K\_] := \frac{1}{\left(\frac{-K+\left(K^2+16\,l\,\epsilon\right)^{\frac{1}{2}}}{2}\right)^{\frac{1}{2}}}$$

$$m_i[\epsilon\_, l\_, K\_] := 2 + K\,(T_i[\epsilon, l, K])^2$$

$$P[\epsilon\_, l\_, K\_] := 4\,\frac{T_i[\epsilon, l, K]}{\sqrt{m_i[\epsilon, l, K]}}\ \text{EllipticK}\left[\frac{1}{m_i[\epsilon, l, K]}\right]$$





$$\mathbf{f}[t\_,\ \epsilon\_,\ l\_,\ K\_] := \left(\frac{-1 + \left(K^2 + 16\, l\, \epsilon\right)^{\frac{1}{2}}}{-4\, l}\right)^{\frac{1}{2}} \mathbf{JacobiDN}\left[\frac{t}{\mathbf{T}\_[\epsilon,\ l,\ K]},\ \mathbf{m}\_[\epsilon,\ l,\ K]\right]$$

$$\mathbf{T}\_[\epsilon\_,\ l\_,\ K\_] := \left(\frac{-K - \left(K^2 + 16\, l\, \epsilon\right)^{\frac{1}{2}}}{2}\right)^{\frac{-1}{2}}$$

$$\mathbf{m}\_[\epsilon\_,\ l\_,\ K\_] := 2 + K\,(\mathbf{T}\_[\epsilon,\ l,\ K])^2$$

$$\mathbf{P}[\epsilon\_,\ l\_,\ K\_] := 2\,\mathbf{T}\_[\epsilon,\ l,\ K]\,\sqrt{1 - \frac{-\mathbf{m}\_[\epsilon,\ l,\ K]}{1 - \mathbf{m}\_[\epsilon,\ l,\ K]}}\ \mathbf{EllipticK}\left[\frac{-\mathbf{m}\_[\epsilon,\ l,\ K]}{1 - \mathbf{m}\_[\epsilon,\ l,\ K]}\right]$$

The Bars-Chen Solution in Einstein Frame

Using gauge invariant quantity $\frac{s_E}{\phi_E} = \frac{s_\gamma}{\phi_\gamma}$

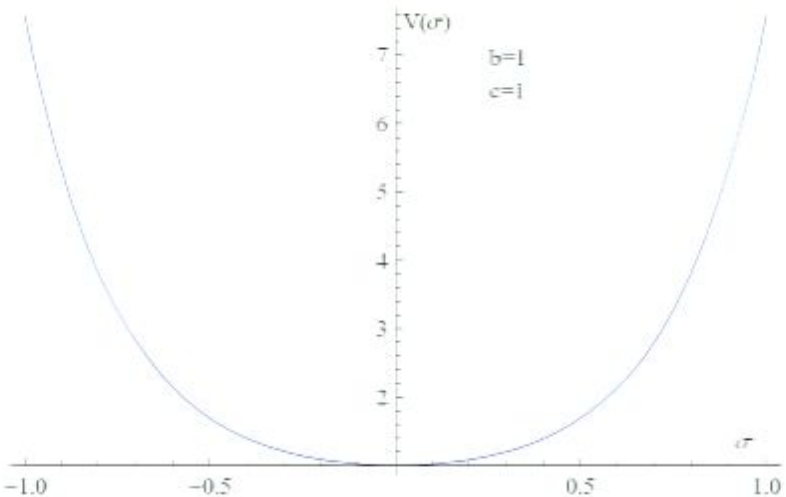
$$\frac{\frac{\sqrt{6}}{\kappa}\sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)}{\frac{\sqrt{6}}{\kappa}\cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)} = \tanh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right) = \frac{s_\gamma}{\phi_\gamma} \longrightarrow \sigma = \frac{\sqrt{6}}{\kappa}\frac{1}{2}\ln\frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma}$$

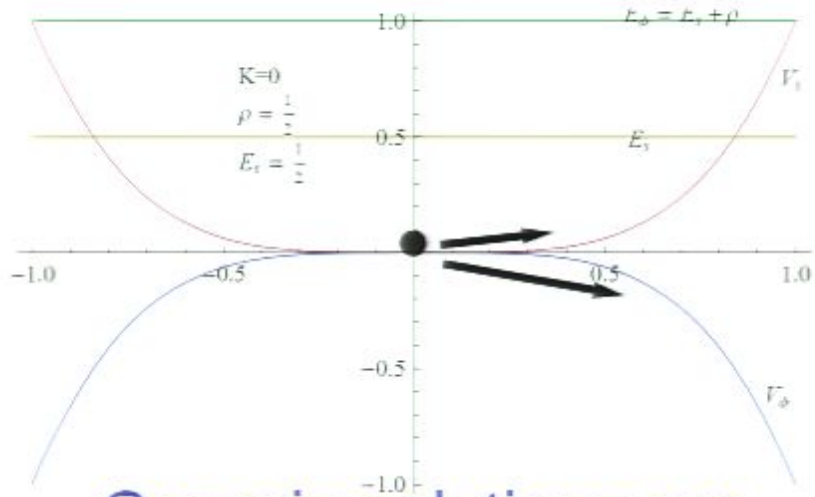
$$a_E = e^{\lambda(x)} a_\gamma$$

$$\phi_E^2 - s_E^2 = \frac{6}{\kappa^2} = e^{-2\lambda}\left(\phi_\gamma^2 - s_\gamma^2\right) \longrightarrow a_E = e^{\lambda} = \frac{\kappa}{\sqrt{6}}\left(\phi_\gamma^2 - s_\gamma^2\right)^{\frac{1}{2}}$$

Generic solutions

Take flat universe as an example:





- Generic solutions are geodesically incomplete

- Can tune the parameters to make a geodesically compete solution if the following quantization rules are satisfied

$$b = \frac{4\,c\,\epsilon}{n^4\,(c+\epsilon)^{22}}$$

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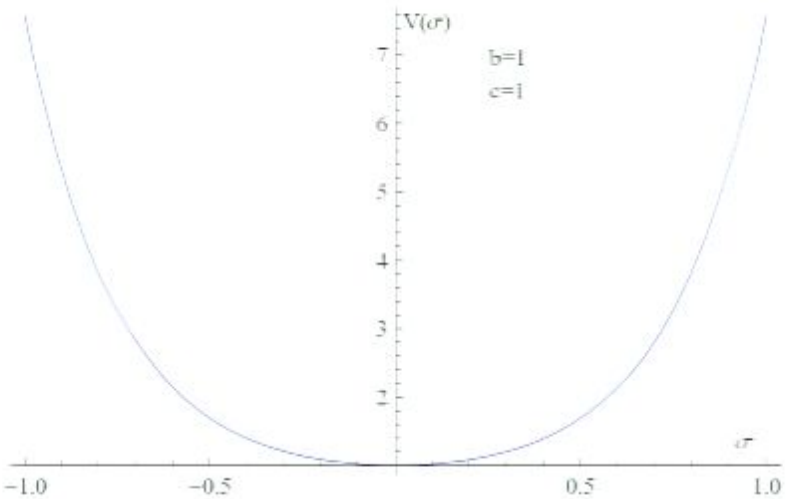

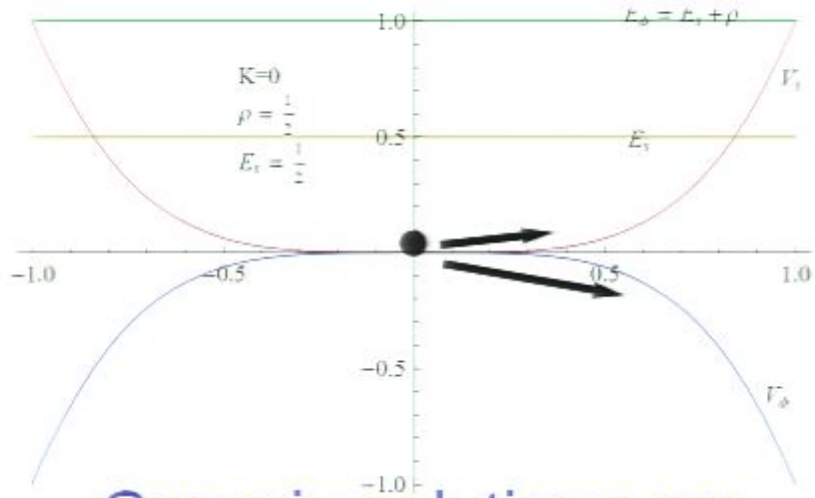

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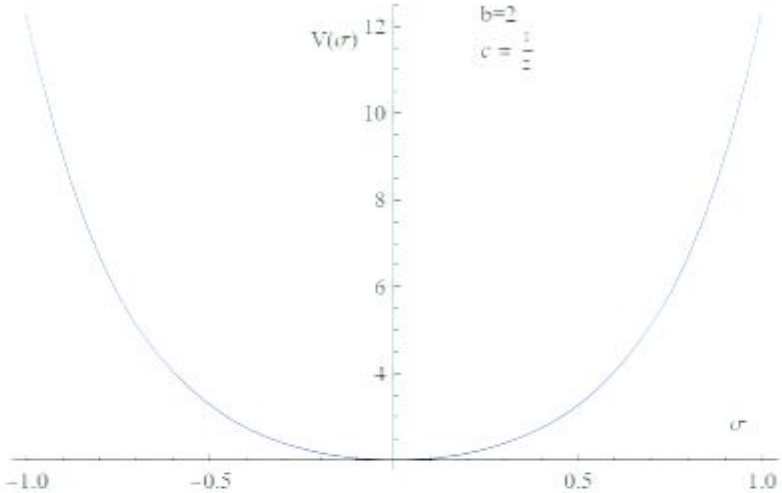
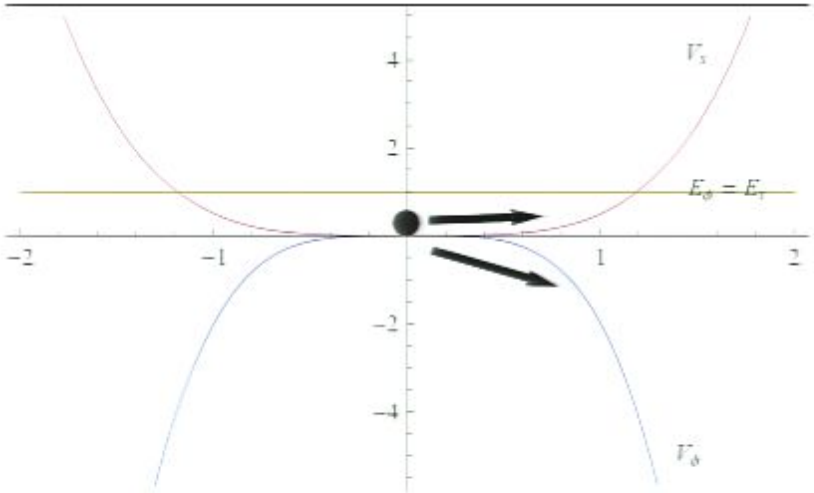
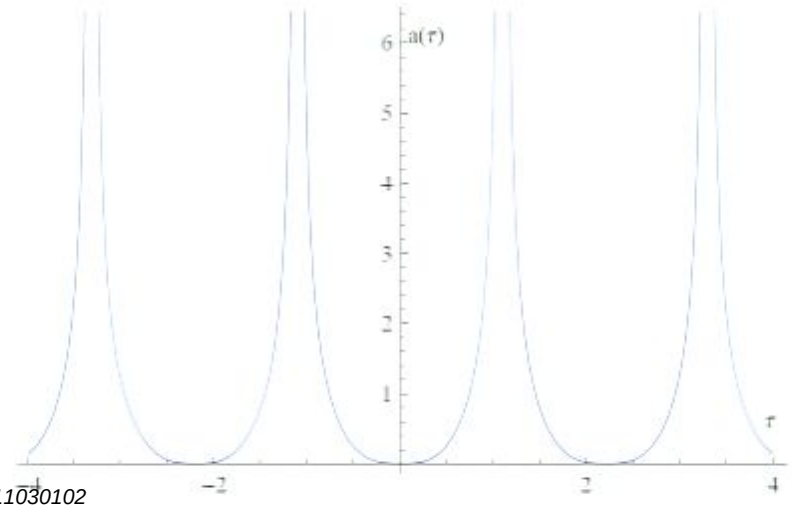
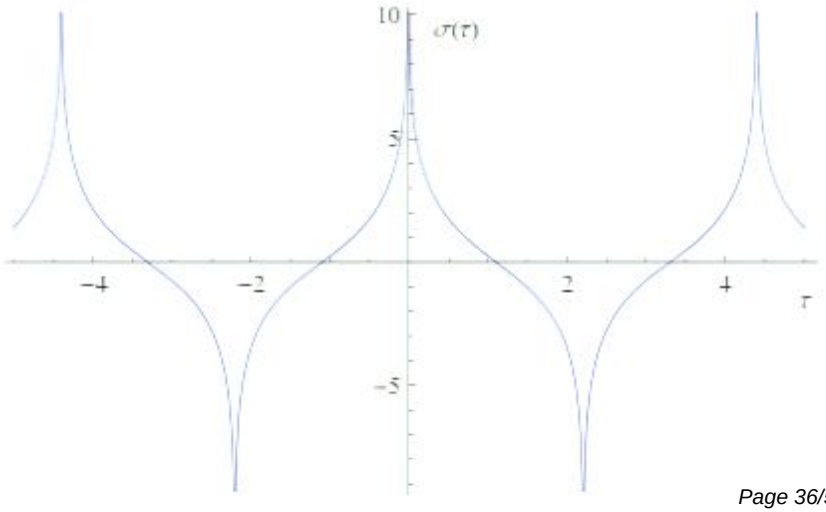
Tuned solution









Geodesically Complete solutions

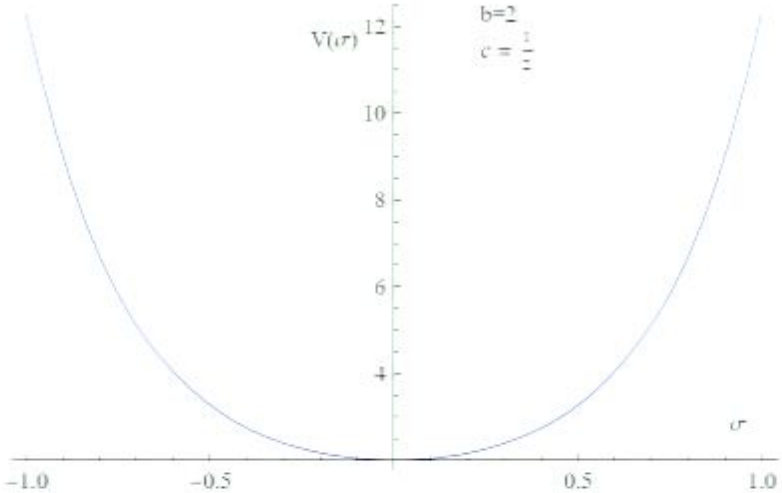
1. Initial condition must be tuned so both s and Φ start from zero
2. The amplitude of Φ should always greater or equal to the amplitude of s
3. The frequency of s should be integer times the frequency of Φ

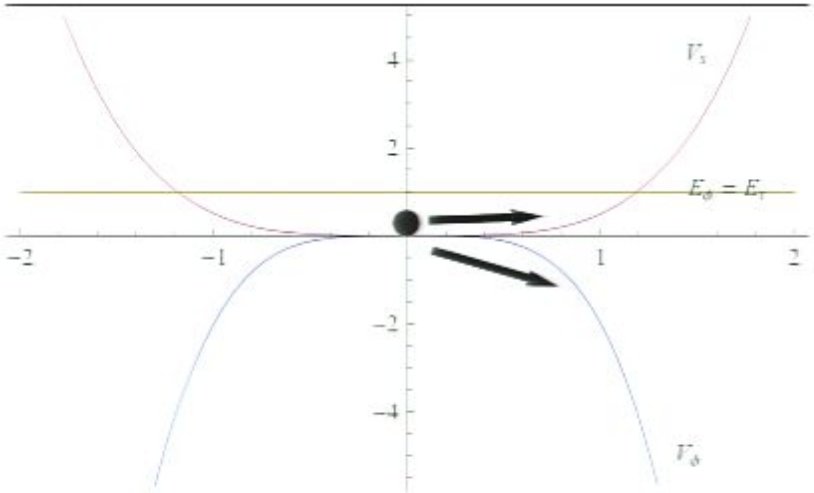
Another example in flat universe: if c>0 b<0

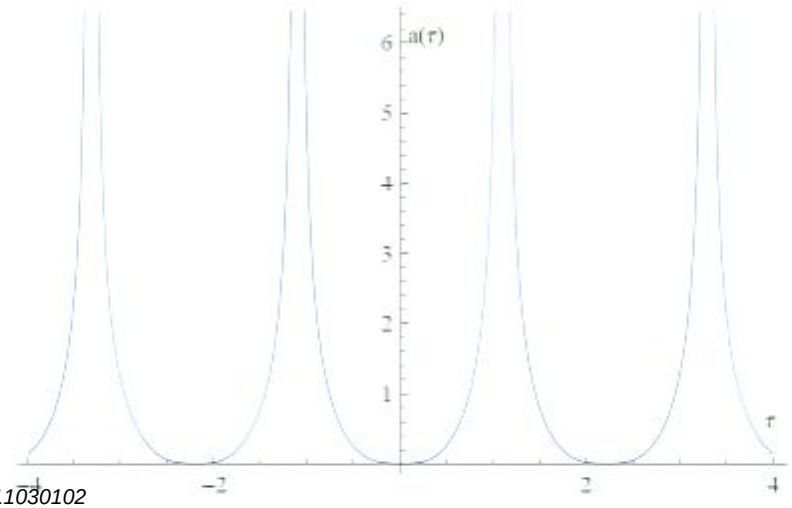
$$b = \frac{-c\,(\epsilon)}{(n+1)^4\,(\epsilon+\rho)}$$

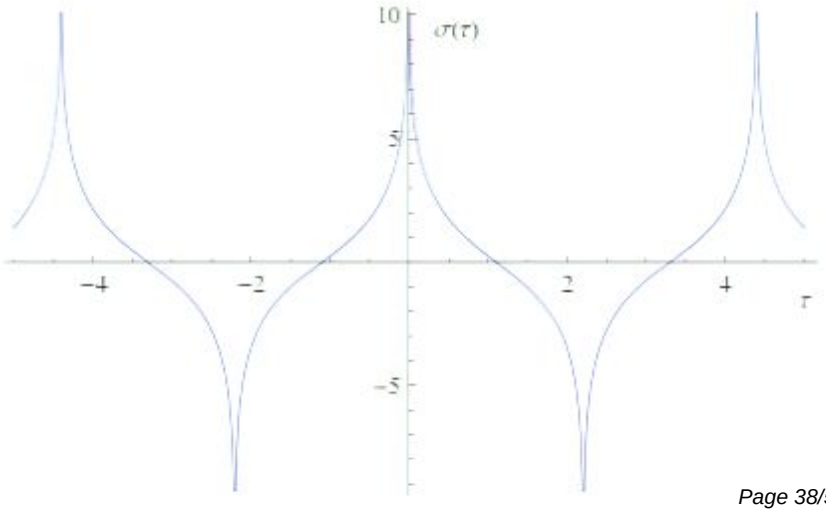
Choose $b=-2^{-4}$, $c=1$, $\rho=0$

Tuned solution









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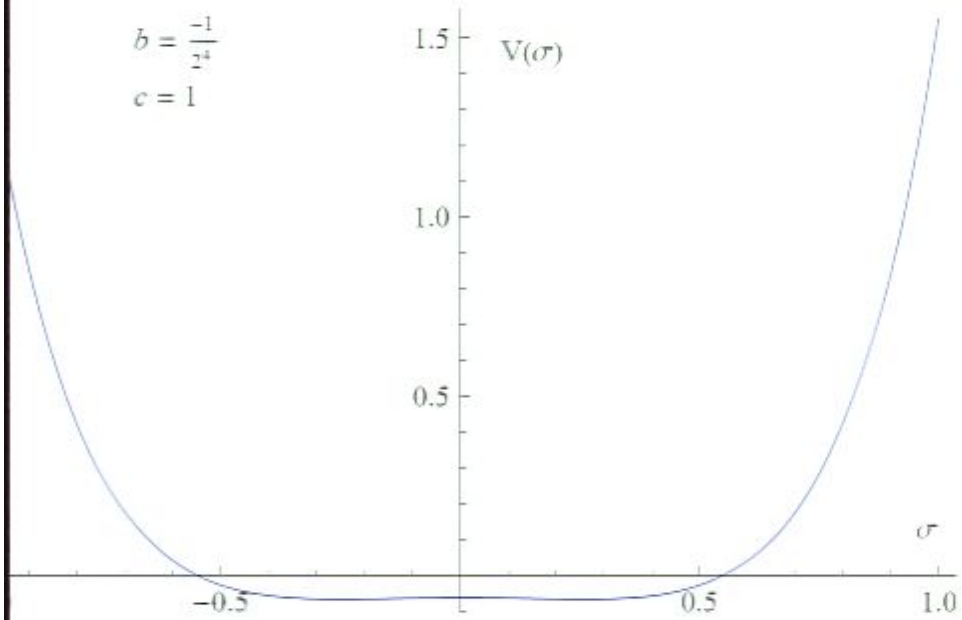
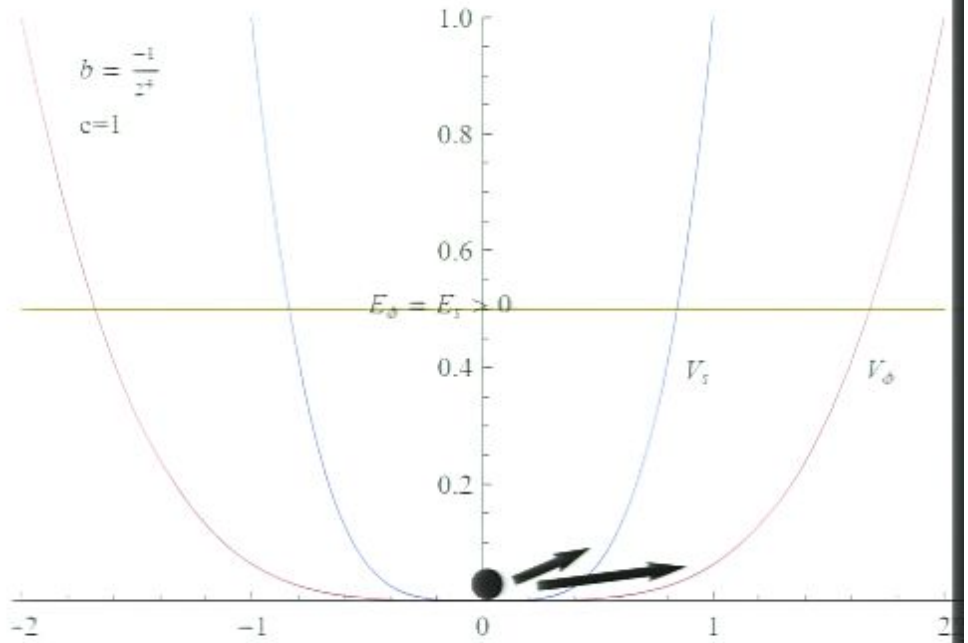
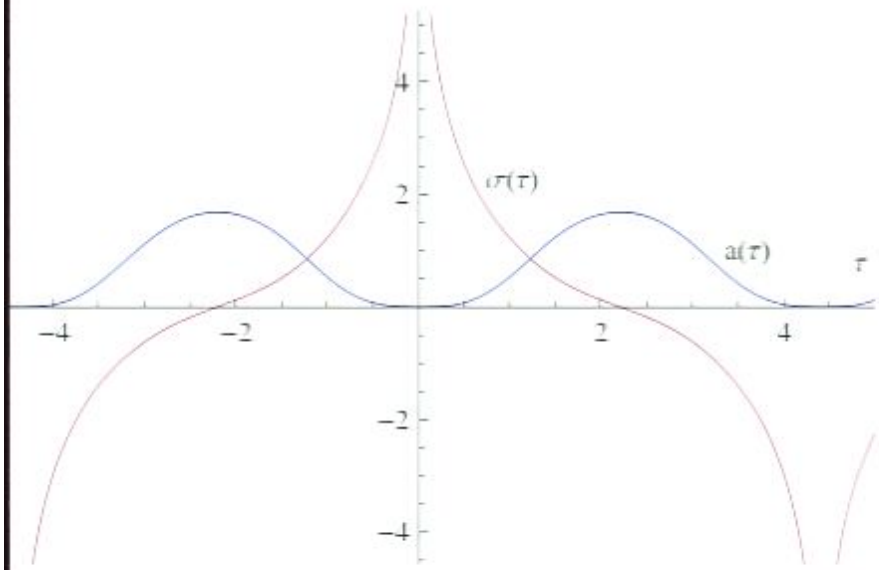




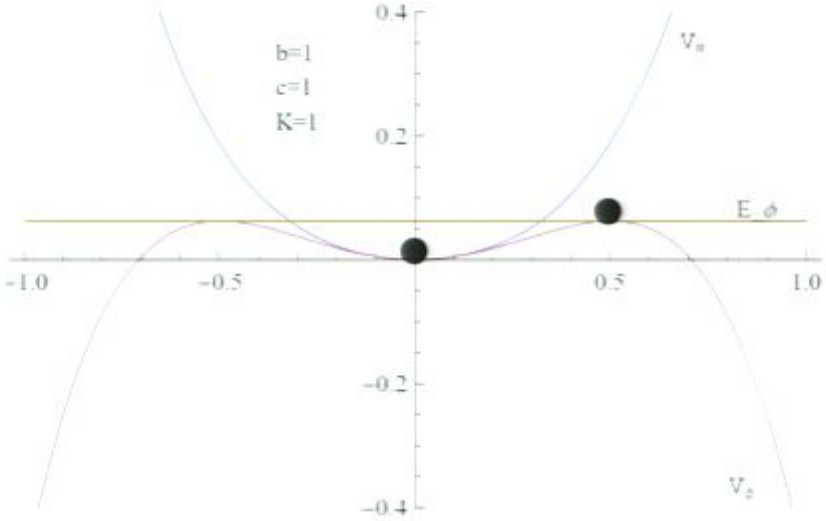


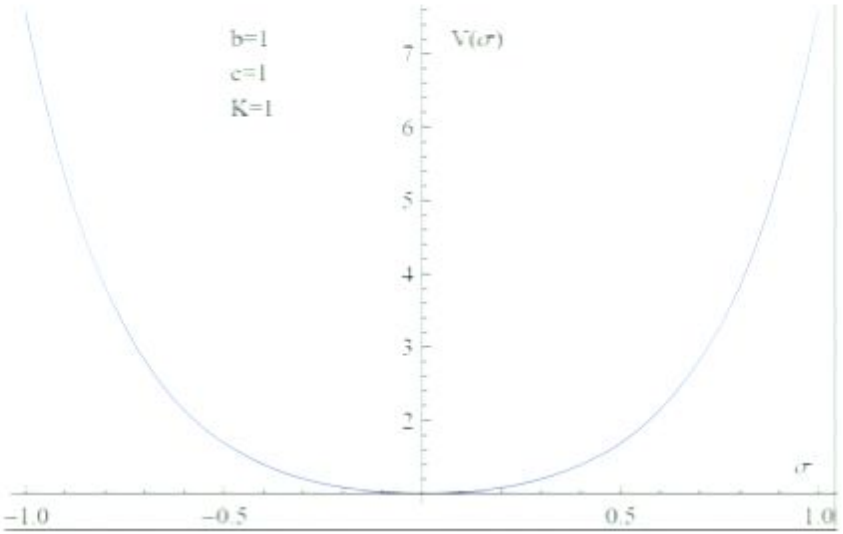
- The cyclic solution requires fine tuning Of initial condition and almost impossible To be found without the s and Φ method.

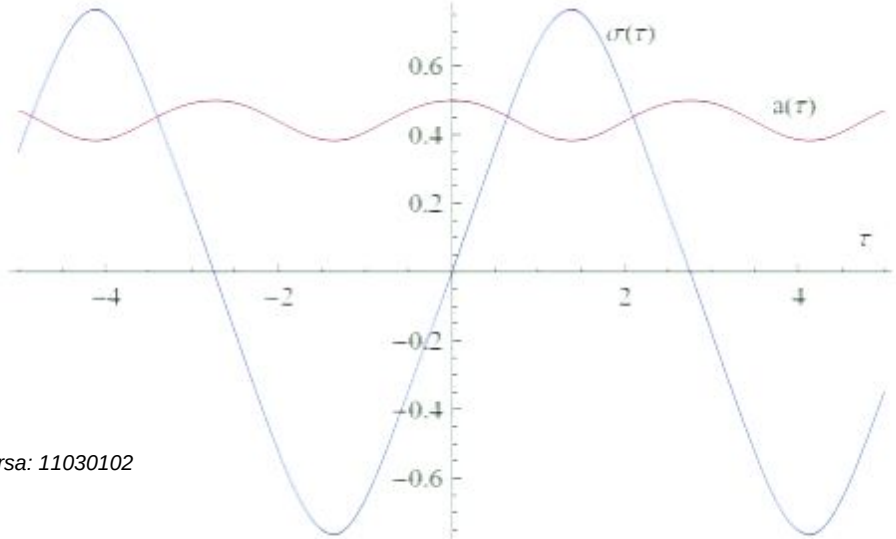
- Turnaround at a finite scale factor
- w>>1 when V<0, ekpyrotic model?

- Radiation will spoiled the smooth transition of a from crunch to bang

Non-singular bounce in closed universe



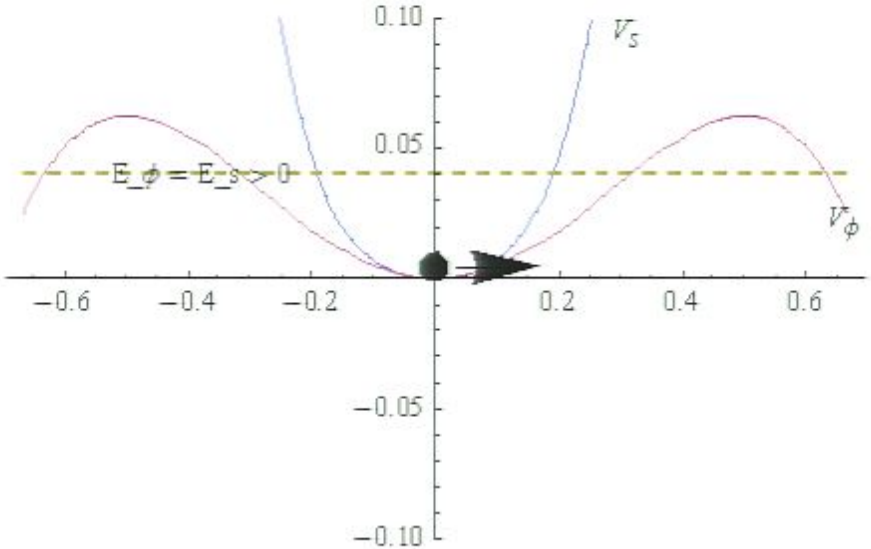


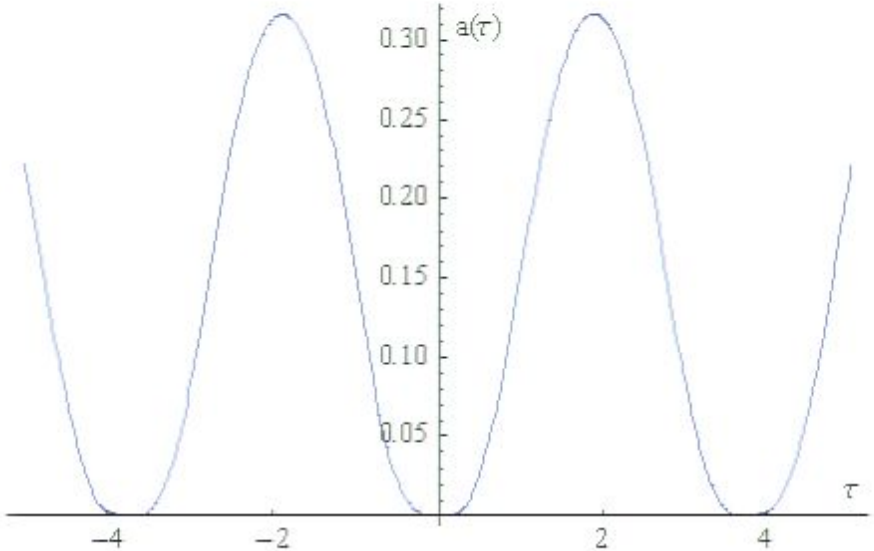


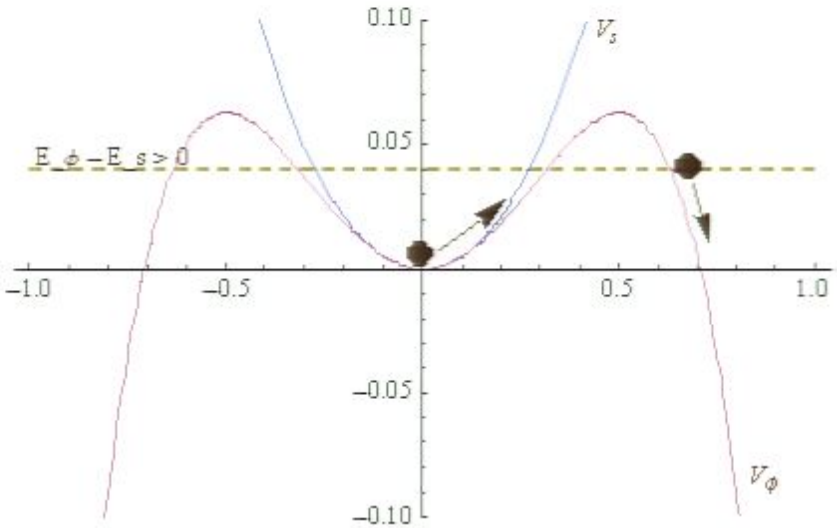
$$E_\phi = \frac{K}{16\,b}$$

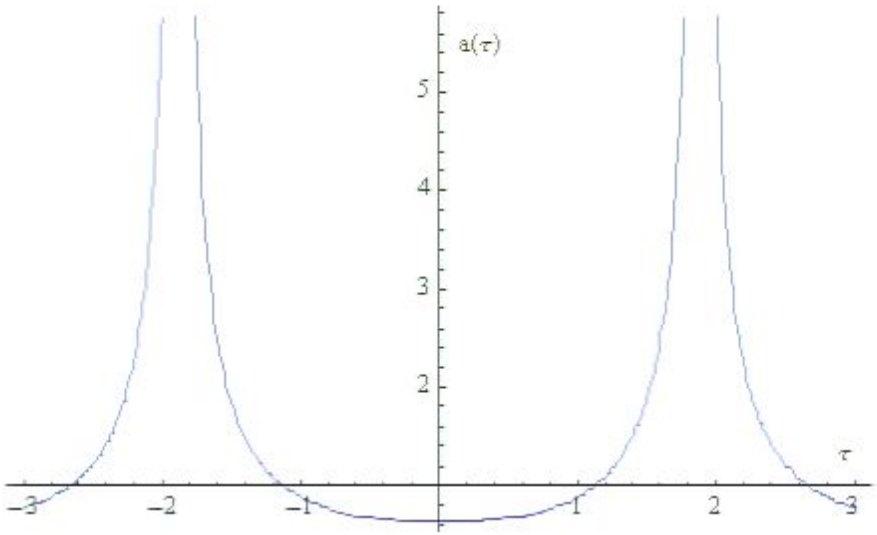
$$0 < E_s < \frac{K}{16\,b}$$

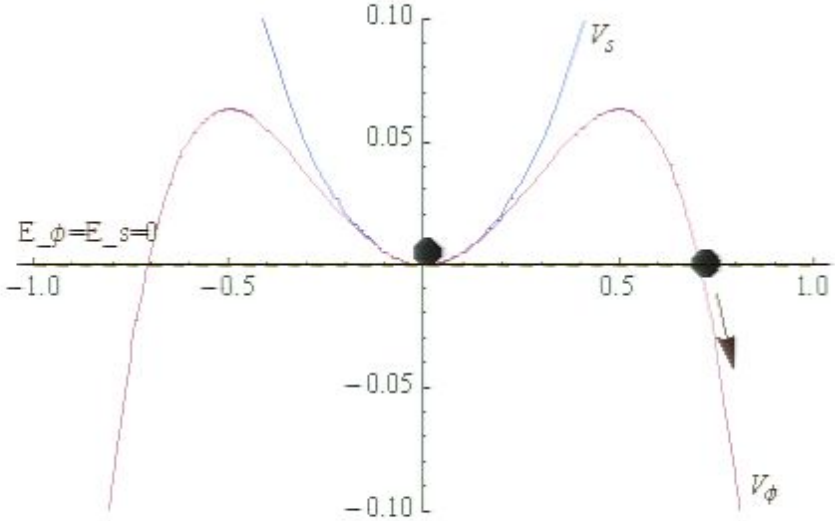
Many other examples

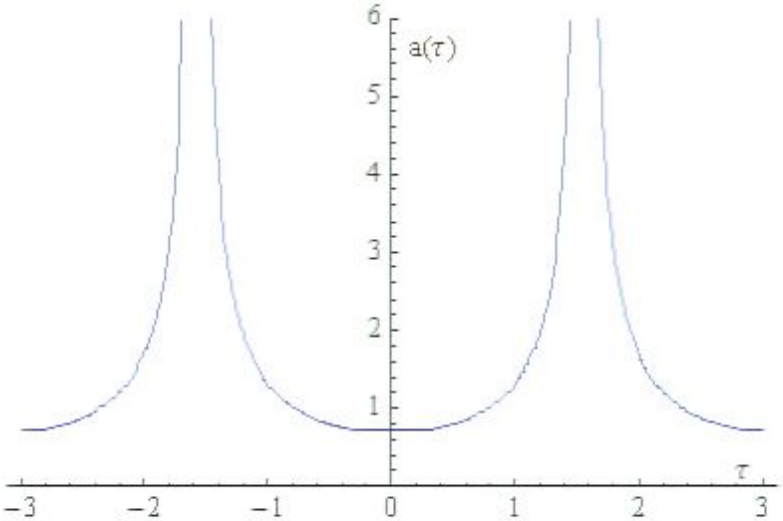





0.10
0.05
−0.05
−0.10
−1.0
−0.5
0.5
1.0
V_s
V_ϕ
E_ϕ − E_s > 0


a(τ)
τ
5
4
3
2
−3
−2
−1
1
2
3


0.10
0.05
−0.05
−0.10
−1.0
−0.5
0.5
1.0
V_s
V_ϕ
E_ϕ=E_s=0


a(τ)
τ
6
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−1
0
1
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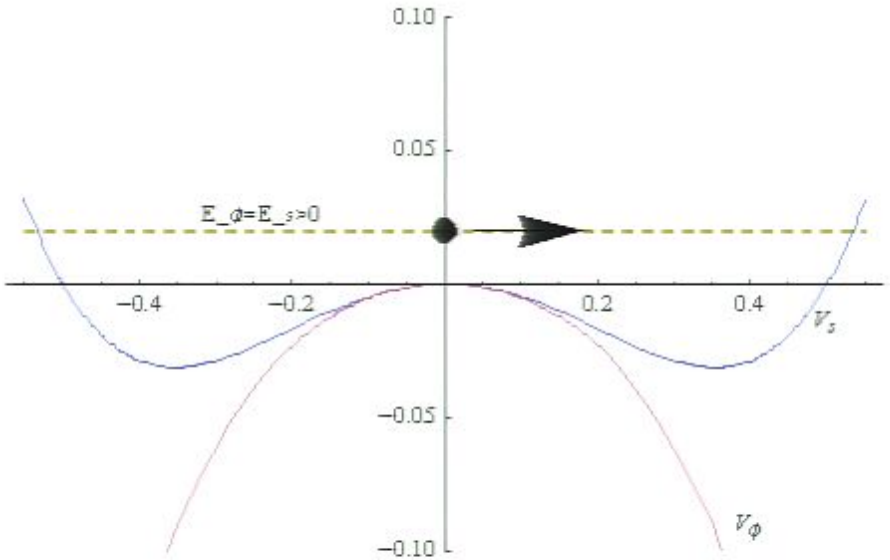
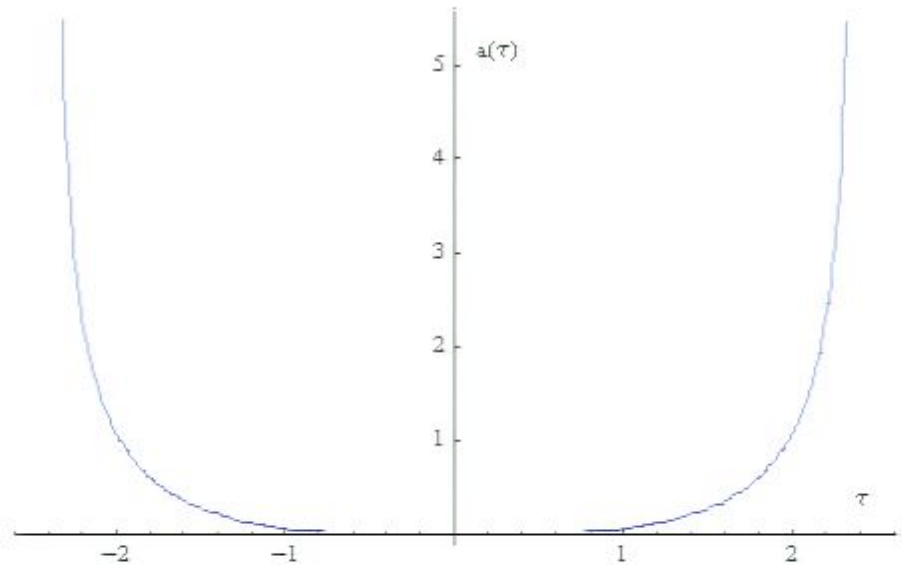
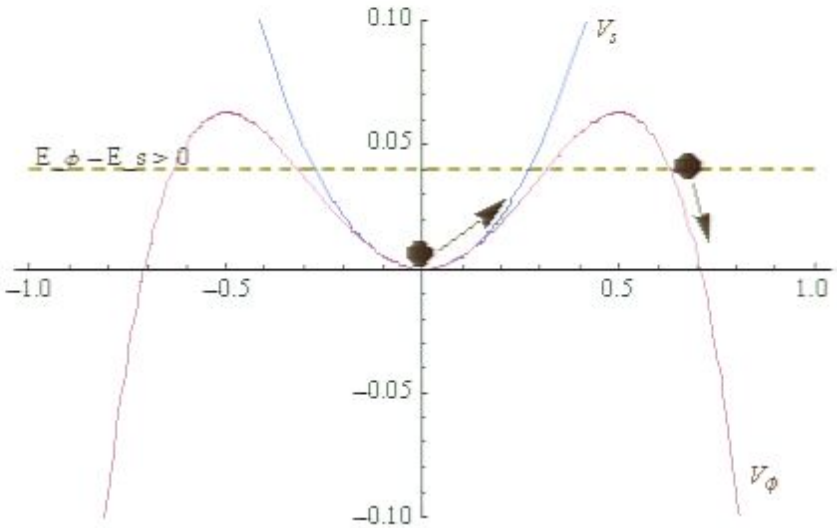




$$\frac{P_\phi[K,\ b,\ \rho,\ E]}{P_s[K,\ c,\ E]} = \frac{n}{2}, \quad n = 1,\ 2,\ 3\ \ldots$$


0.10
0.05
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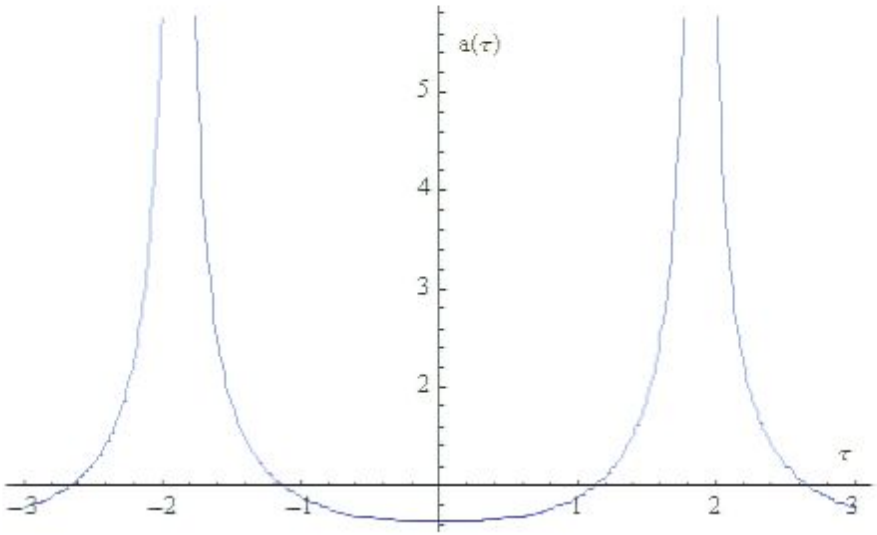
−0.10
−1.0
−0.5
0.5
1.0
V_s
V_ϕ
E_ϕ − E_s > 0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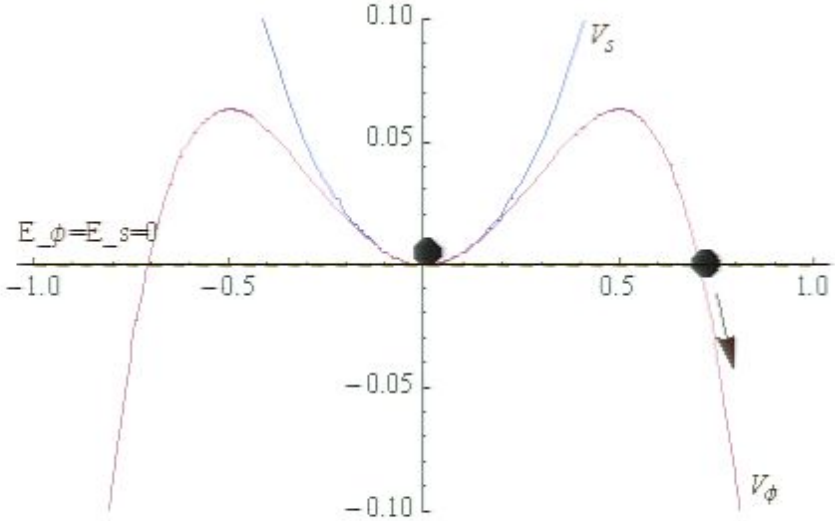
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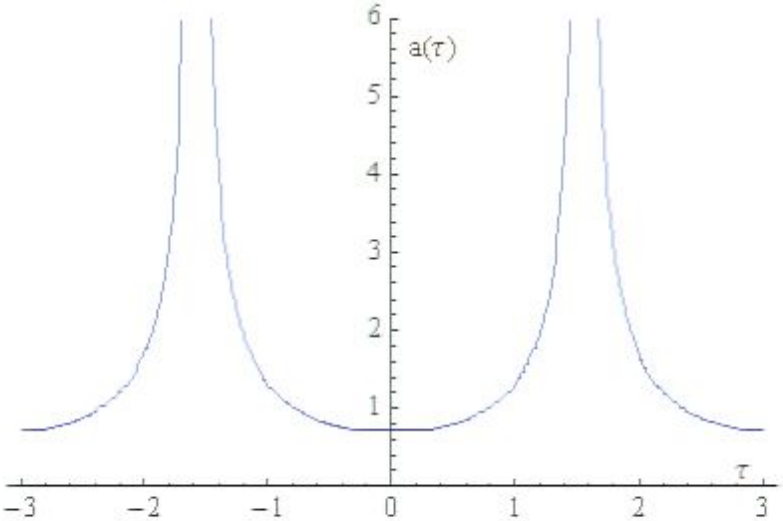
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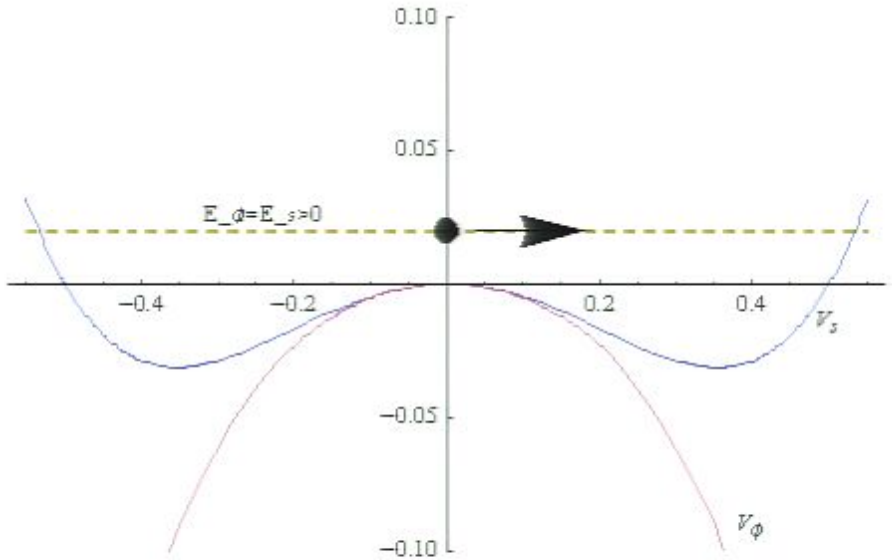

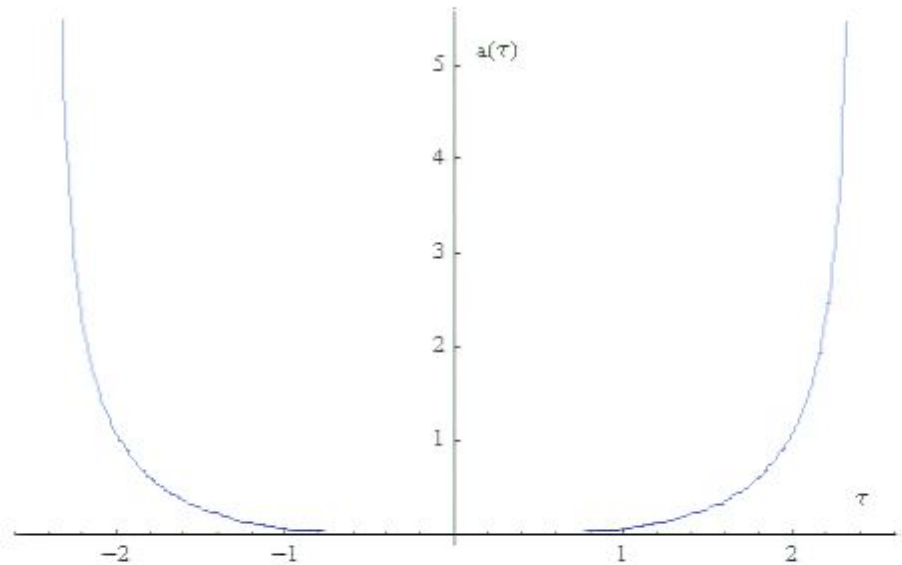

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Summary

•We constructed all cosmological solution for

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \frac{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}{2} - V(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\}$$

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa}\right)^4\left[b\cosh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right) + c\sinh^4\left(\frac{\sigma}{\sqrt{6}}\right)\right]$$

• We have derived the quantization conditions and the inequality equations for singular and non-singular cyclic solutions by using simple mechanics picture in gamma frame.

Future Directions

These cyclic solution may not be stable under perturbation. Study Wheeler-deWitt equation with anisotropy to study the quantum behavior of the universe close to singularity

Construct viable cosmological model that solve cosmological problem as well as generate scale invariant curvature perturbation

If 2T-Physics is the fundamental theory, how to interpret the solutions frame that are not geodesically complete in Einstein, namely, what is the meaning of negative gravitational Constant?

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Thank You!

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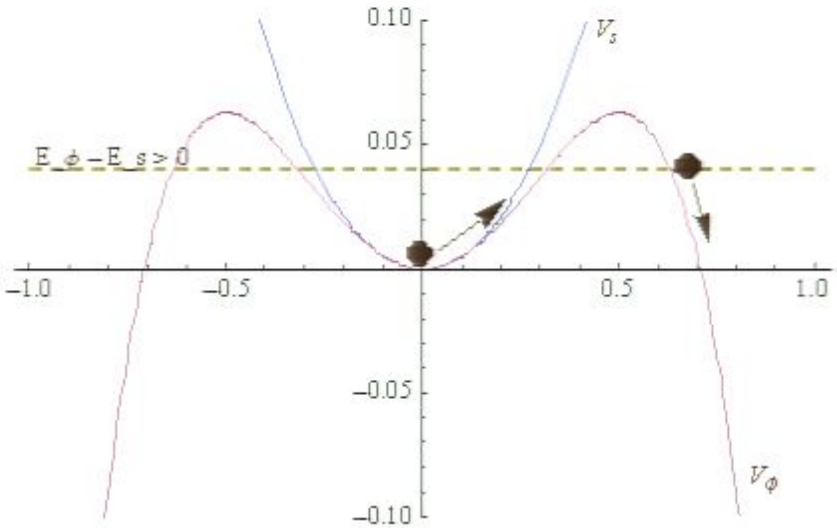
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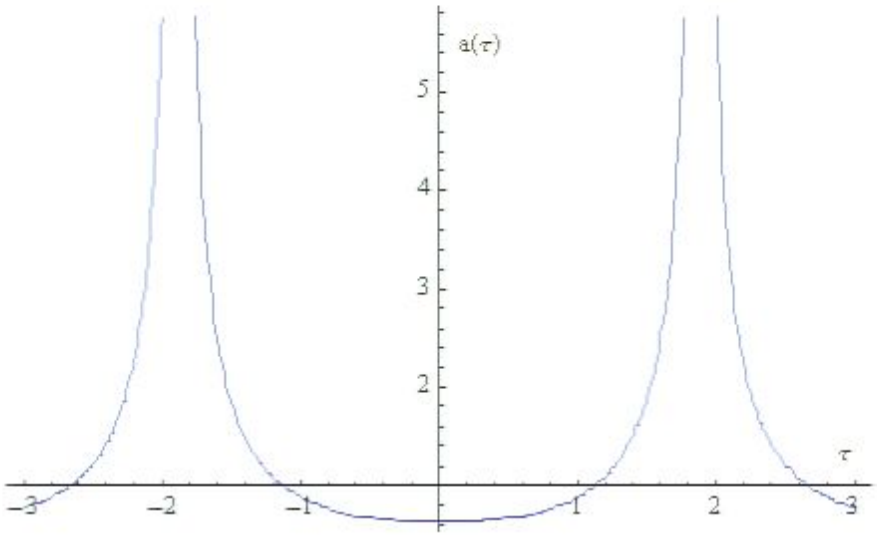
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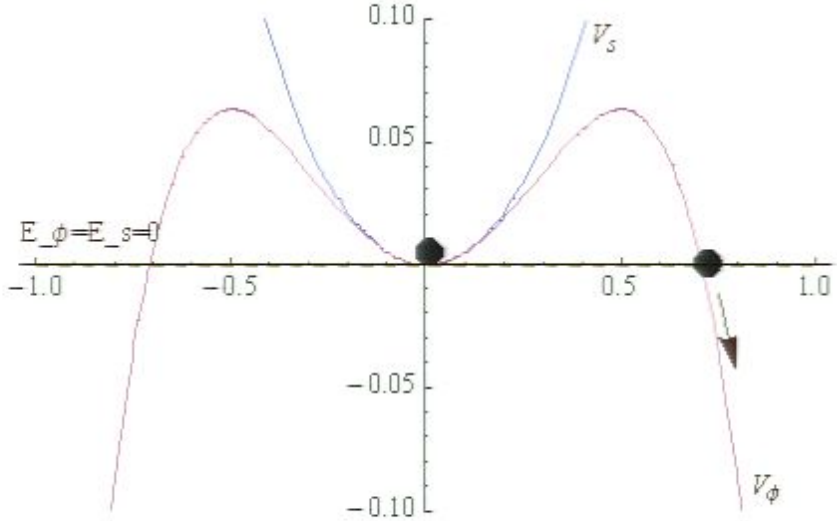
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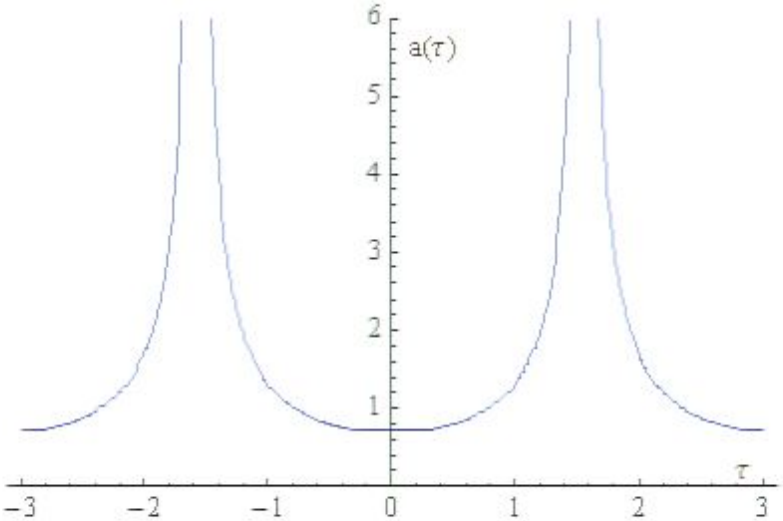
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