

Title: Higher Spin Theories - Lecture 4

Date: Mar 29, 2011 09:30 AM

URL: <http://pirsa.org/11030098>

Abstract:

$$d_x W + W * W = 0$$

$$d_z W + d_x S + W * S + S * W = 0$$

$$d_z S + S * S = e^{i\theta} B * K d\bar{z} + e^{i\theta} B * \bar{K} dz^2$$

$$d_x B + W * B - B * \pi(W) = 0$$

$$d_z B + S * B - B * \pi(S) = 0$$

$\theta = 0$ type A
 $\theta = \pi/2$ type B

K-P-S-S CONJECTURES

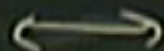
MINIMAL VASILIEV'S
TYPE A/B HS ON AS_n



$O(n)$ FREE/C

K-P-S-S CONJECTURES

MINIMAL VASILIEV'S
TYPE I HS ON A_{S-1}



O(N) FREE / CRITICAL

VECTOR MODELS

(BOSONIC / FERMIONIC)
TYPE A TYPE B

K-P-S-S CONJECTURES

MINIMAL VASILIEV'S

TYPE A/B HS ON AS_n



O(N) FREE / CRITICAL

VECTOR MODELS

(BOSONIC / FERMIONIC)
TYPE A TYPE II

MINIMAL VASILIEV'S
TYPE A/B HS ON AdS_4

$O(N)$ FREE / CRITICAL

VECTOR MODELS

(BOSONIC / FERMIONIC)
TYPE A TYPE B

SCALAR, $\varphi(z, \vec{x})$ $m^2 = -2$

$$\varphi(z, \vec{x}) = z^{\Delta_+} \alpha(\vec{x}) + z^{\Delta_-} \beta(\vec{x}) \quad \begin{matrix} \Delta_- = 1 \\ \Delta_+ = 2 \end{matrix}$$

" Δ_- b.c.": $\alpha(\vec{x}) = 0 \quad \beta(\vec{x}) = \langle \mathcal{O}_{\Delta_-}(\vec{x}) \rangle$

" Δ_+ b.c.": $\beta(\vec{x}) = 0 \quad \alpha(\vec{x}) = \langle \mathcal{O}_{\Delta_+}(\vec{x}) \rangle$

MINIMAL VASILIEV'S
TYPE A/B HS ON AdS_4

$O(N)$ FREE / CRITICAL

VECTOR MODELS
(BOSONIC / FERMIONIC)
TYPE A TYPE B

SCALAR, $\varphi(z, \bar{z})$ $m^2 = -2$

$z \rightarrow 0$ $\gamma(z)$ $z^{\Delta_+} \alpha(\bar{x}) + z^{\Delta_-} \beta(\bar{x})$

$\Delta_- = 1$
 $\Delta_+ = 2$

" Δ

$\alpha(\bar{x}) = 0$ $\beta(\bar{x}) = \langle \mathcal{O}_{\Delta_-}(\bar{x}) \rangle$

$\beta(\bar{x}) = 0$ $\alpha(\bar{x}) = \langle \mathcal{O}_{\Delta_+}(\bar{x}) \rangle$

MINIMAL VASILIEV'S
TYPE A/B HS ON AdS_4

$O(N)$ FREE / CRITICAL

VECTOR MODELS
(BOSONIC / FERMIONIC)
TYPE A TYPE B

SCALAR, $\varphi(z, \vec{x})$ $m^2 = -2$

$$z \rightarrow 0 \quad \varphi(z, \vec{x}) = z^{\Delta_+} \alpha(\vec{x}) + z^{\Delta_-} \beta(\vec{x}) \quad \begin{matrix} \Delta_- = 1 \\ \Delta_+ = 2 \end{matrix}$$

" Δ_- bc": $\alpha(\vec{x}) = 0$ $\beta(\vec{x}) \sim \mathcal{O}_{\Delta_-}(\vec{x})$

" Δ_+ bc": $\beta(\vec{x}) = 0$ $\alpha(\vec{x}) \sim \mathcal{O}_{\Delta_+}(\vec{x})$

K-P-S-S CONJECTURES

$s=0,1,2,\dots$
MINIMAL VASILIEV'S

TYPE A/B HS ON ALS_4



$O(N)$ FREE / CRITICAL

VECTOR MODELS

(BOSONIC / FERMIONIC)
TYPE A TYPE B

SCALAR, $\varphi(z, \bar{z})$ $m^2 = -2$

$$z \rightarrow 0 \quad \varphi(z, \bar{z}) = z^{\Delta_+} \alpha(\bar{x}) + z^{\Delta_-} \beta(\bar{x}) \quad \begin{matrix} \Delta_- = 1 \\ \Delta_+ = 2 \end{matrix}$$

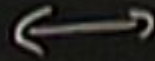
" Δ_- bc": $\alpha(\bar{x}) = 0 \quad \beta(\bar{x}) = \langle \mathcal{O}_{\Delta_-}(\bar{x}) \rangle$

" Δ_+ bc": $\beta(\bar{x}) = 0 \quad \alpha(\bar{x}) = \langle \mathcal{O}_{\Delta_+}(\bar{x}) \rangle$

NON-MINIMAL HS THEORIES

$$S = 0, 1, 2, \dots, \infty$$

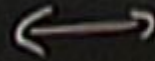
NON-MINIMAL HS THEORIES
 $S = 0, 1, 2, \dots, \infty$



COMPLEX VECTOR MODELS
IN $U(N)$ SINGLET SECTOR

$$S = \int$$

NON-MINIMAL HS THEORIES
 $S = 0, 1, 2, \dots, \infty$



COMPLEX VECTOR MODELS
IN $U(N)$ SINGLET SECTOR

$$S_{\text{eff}} = \int d^4x \, \phi^\dagger \partial_\mu \phi$$

$$J_{\mu\nu} = \phi^\dagger (\partial_\mu \phi) - (\partial_\nu \phi) \phi$$

CHECKS of CONJECTURES

CHECKS of CONJECTURES

$$\langle \mathcal{I}_{S_1} \mathcal{I}_{S_2} \mathcal{I}_{S_3} \rangle_{\text{CF}} \longleftrightarrow$$



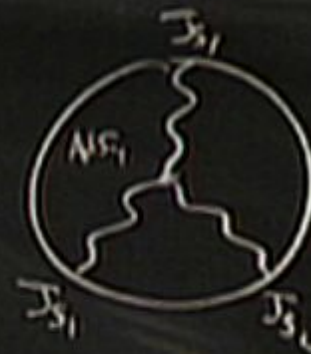
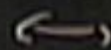
CHECKS of CONJECTURES

$$\langle \mathcal{I}_{S_1} \mathcal{I}_{S_2} \mathcal{I}_{S_3} \rangle_{\text{CFI}}$$



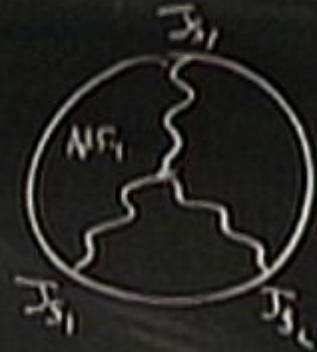
CHECKS of CONJECTURES

$$\langle \mathcal{I}_{S_1} \mathcal{I}_{S_2} \mathcal{I}_{S_3} \rangle_{\text{CF-7}}$$



CHECKS of CONJECTURES

$$\langle T_{S_1} T_{S_2} T_{S_3} \rangle_{\text{CF}} \longleftrightarrow$$



$$\langle O(x_1) O(x_2) O(x_3) \rangle$$

$$\langle J_{S_1} J_{S_2} J_{S_3} \rangle_{CFV}$$

$\longleftrightarrow$



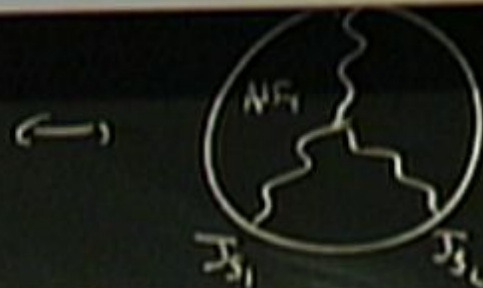
$$\langle O(x_1) O(x_2) O(x_3) \rangle$$

$$\gamma T_{\mu\nu} = 0$$

$$\langle TTT \rangle$$

$$\langle TTT \rangle_B + a_2 \langle TTT \rangle_F$$

$$\langle T_{S_1} T_{S_2} T_{S_3} \rangle_{CFV}$$

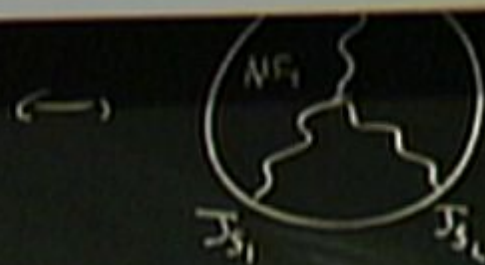


$$\langle O(x_1) O(x_2) O(x_3) \rangle$$

$$\langle \dots \rangle = a_1 \langle TTT \rangle_B + a_2 \langle TTT \rangle_F$$

Osherson-Pitkow

$$\langle T_{S_1} T_{S_2} T_{S_3} \rangle_{CFV}$$



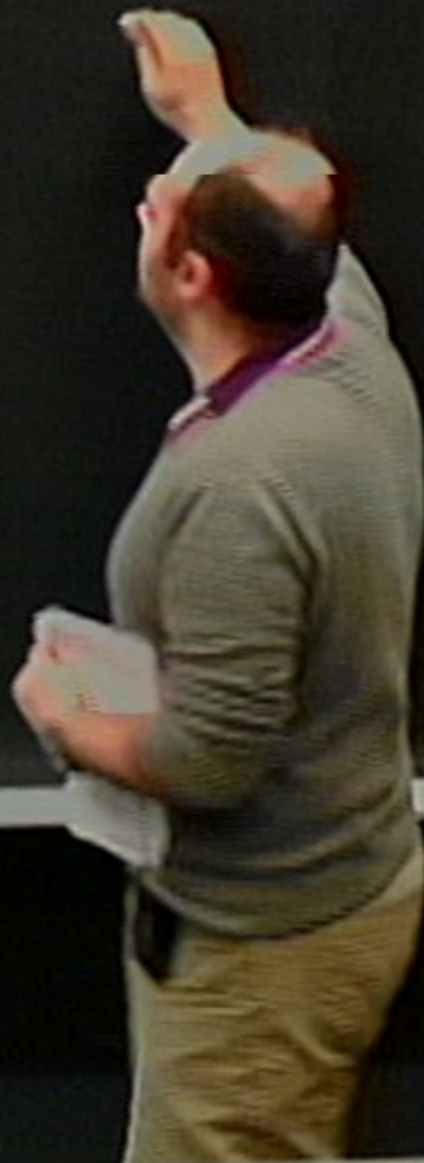
$$\langle O(x_1) O(x_2) O(x_3) \rangle$$

$$\gamma T_{\mu\nu} = 0$$

$$\langle TTT \rangle = a_1 \langle TTT \rangle_B + a_2 \langle TTT \rangle_F + b \langle TTT \rangle_{\text{parity odd}}$$

Oshida - Pithon

$$J_s(\bar{x}, \bar{\varepsilon}) = J_{i_1 \dots i_s}(\bar{x}) \in \mathbb{R}^{n_1 \dots n_s}$$



$$J_s(\vec{x}, \vec{E}) = J_{i_1 \dots i_s}(\vec{x}) \in^{i_1} \dots \in^{i_s}$$

$$\vec{E} \cdot \vec{E} = 0$$

$$J_s(\vec{x}, \vec{e}) = J_{i_1 \dots i_s}(\vec{x}) e^{i_1} \dots e^{i_s}$$

$$\vec{e} \cdot \vec{e} = 0$$

$$(\vec{e} \cdot \vec{e})_{\mu p}$$

$$J_s(\vec{x}, \vec{e}) = J_{i_1 \dots i_s}(\vec{x}) e^{i_1} \dots e^{i_s}$$

$$\vec{e} \cdot \vec{e} = 0$$

$$(\vec{e} \cdot \vec{e})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$\mathcal{I}_s(\vec{x}, \vec{E}) = \mathcal{I}_{i_1 \dots i_s}(\vec{x}) \in^{i_1} \dots \in^{i_s}$$

$$\vec{E} \cdot \vec{E} = 0$$

$$(\vec{E} \cdot \vec{E})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$(\vec{E})_{\alpha\beta}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = (1, \sigma^1, \sigma^2, \sigma^3)_{\alpha\dot{\alpha}}$$

$$\mathcal{J}_s(\vec{x}, \vec{E}) = \mathcal{J}_{i_1 \dots i_s}(\vec{x}) \in^{i_1} \dots \in^{i_s}$$

$$\vec{E} \cdot \vec{E} = 0$$

$$(\vec{E} \cdot \vec{E})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$(\vec{E})_{\alpha\beta}$$

$$(\vec{E})_{\alpha\beta} = \left(\frac{\sigma^0}{2}, \sigma^1, \sigma^2, \sigma^3 \right)_{\alpha\beta}$$

$$P = (\vec{\sigma} \cdot \vec{\sigma}')_{\alpha\beta}$$

$$J_s(\vec{x}, \vec{E}) = J_{i_1 \dots i_s}(\vec{x}) \in^{i_1} \dots \in^{i_s}$$

$$\vec{E} \cdot \vec{E} = 0$$

$$(\vec{E} \cdot \vec{E})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$(\vec{E})_{\alpha\beta}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = \left(\frac{\sigma^0}{1}, \sigma^1, \sigma^2, \sigma^3 \right)_{\alpha\dot{\alpha}}$$

$$(\vec{E})_{\alpha\beta} = (\vec{\sigma} \cdot \vec{\sigma})_{\alpha\beta}$$

$$\mathcal{J}_S(\vec{x}, \vec{E}) = \mathcal{J}_{i_1 \dots i_s}(\vec{x}) \in^{i_1} \dots \in^{i_s}$$

$$\vec{E} \cdot \vec{E} = 0$$

$$(\vec{E} \cdot \vec{E})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$(\vec{E})_{\alpha\beta}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = \left(\begin{matrix} 1 & \sigma^x & \sigma^y & \sigma^z \\ \sigma^x & 1 & \sigma^z & \sigma^y \\ \sigma^y & \sigma^z & 1 & \sigma^x \\ \sigma^z & \sigma^y & \sigma^x & 1 \end{matrix} \right)_{\alpha\dot{\alpha}}$$

$$(\vec{E})_{\alpha\beta} = (\vec{\sigma} \cdot \vec{\sigma})_{\alpha\beta}$$

$$J_s(\bar{x}, \bar{\epsilon}) = J_{i_1 \dots i_s}(\bar{x}) \epsilon^{i_1} \dots \epsilon^{i_s}$$

$$\bar{\epsilon} \cdot \bar{\epsilon} = 0$$

$$(\bar{\epsilon} \cdot \bar{\epsilon})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$J_s(\bar{x}, \lambda) = J_{\alpha_1 \dots \alpha_s}(\bar{x}) \lambda^{\alpha_1} \dots \lambda^{\alpha_s}$$

$$(\bar{\epsilon})_{\alpha\beta}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = (\mathbb{1}, \sigma^1, \sigma^2, \sigma^3)_{\alpha\dot{\alpha}}$$

$$(\bar{\epsilon})_{\alpha\beta} = (\bar{\sigma} \sigma^1)_{\alpha\beta}$$

$$\mathcal{J}_S(\vec{x}, \vec{\epsilon}) = \mathcal{J}_{i_1 \dots i_5}(\vec{x}) \epsilon^{i_1} \dots \epsilon^{i_5}$$

$$\vec{\epsilon} \cdot \vec{\epsilon} = 0$$

$$(\vec{\epsilon} \cdot \vec{\epsilon})_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

$$(\vec{\epsilon})_{\alpha\beta}$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = (\mathbb{1}, \sigma^1, \sigma^2, \sigma^3)_{\alpha\dot{\alpha}}$$

$$\mathcal{J}_S(\vec{x}, \lambda) = \mathcal{J}_{\underbrace{\alpha_1 \dots \alpha_{25}}_{\text{indices}}}(\vec{x}) \lambda^{\alpha_1} \dots \lambda^{\alpha_{25}}$$

$$(\vec{\epsilon})_{\alpha\beta} = (\vec{\sigma} \cdot \vec{\sigma})_{\alpha\beta}$$

$$\phi(z, x) \leftrightarrow \mathcal{O}(x)$$

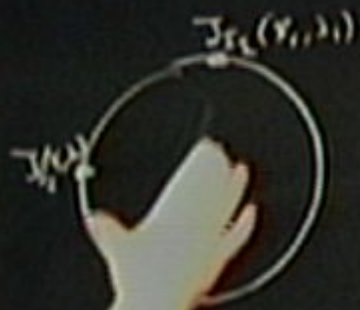
$$\phi(z, \bar{z}) \sim z^{\Delta} \langle \mathcal{O}(\bar{z}) \rangle$$

$$\phi(r, \vec{x}) \leftrightarrow \mathcal{O}(\vec{x})$$

$$\phi(r, \vec{x}) \sim r^\Delta \langle \mathcal{O}(\vec{x}) \rangle$$

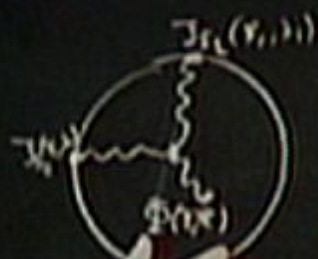
$$\phi(z, \bar{z}) \leftrightarrow \odot(\bar{z})$$

$$\phi(z, \bar{z}) \sim z^{\Delta} \langle \odot(\bar{z}) \rangle$$

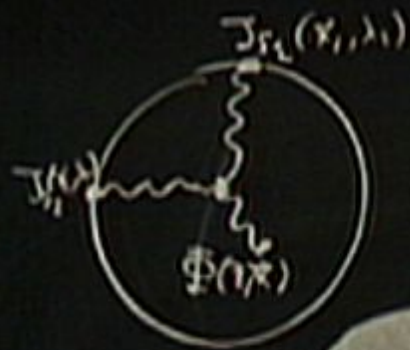


$$\phi(r, x) \leftarrow \odot(x)$$

$$\phi(r, x) \sim r^{\Delta} \langle \odot(x) \rangle$$

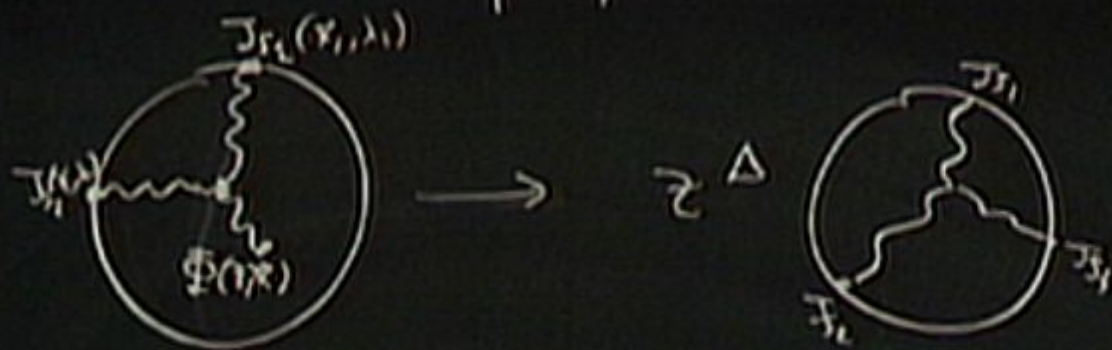


$$\phi(z, \vec{x}) \sim z^\Delta \langle \odot(\vec{x}) \rangle$$



CAUTION
 Do not touch the blackboard
 and the whiteboard with your hands.
 It is dangerous to touch
 the blackboard and the whiteboard.
 Please keep your hands away from them.

$$\phi(z, \vec{x}) \sim z^\Delta \langle \mathcal{O}(\vec{x}) \rangle$$



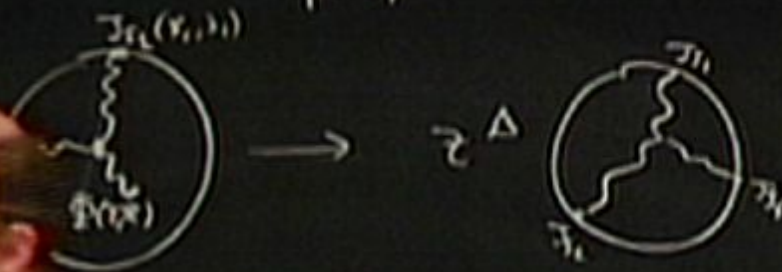
$$\langle \Phi_{(S_1)}(z, \vec{x}) \rangle_{\mathcal{J}_{S_1}, \mathcal{J}_{S_2}} \rightarrow \langle \mathcal{J}_{S_3} \mathcal{J}_{S_2} \mathcal{J}_{S_1} \rangle$$

CAUTION

DO NOT TOUCH THE BOARD
OR THE CHALK
IF YOU HAVE TO
PLEASE ASK THE
LECTURER

$$\phi(z, \bar{z}) \leftrightarrow \odot(\bar{z})$$

$$\phi(z, \bar{z}) \sim z^\Delta \langle \odot(\bar{z}) \rangle$$

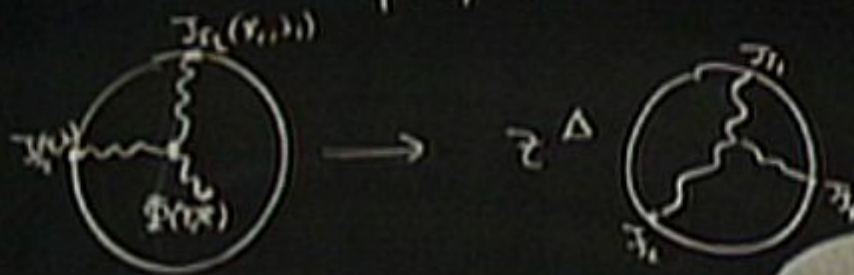


$$\langle \Phi_{S_1}(z, \bar{z}) \rangle_{\mathcal{T}_{S_1}, \mathcal{T}_{S_2}} \rightarrow \langle \mathcal{T}_{S_3} \rangle_{\mathcal{T}_{S_1}, \mathcal{T}_{S_2}} = \langle \mathcal{T}_{S_1} \mathcal{T}_{S_2} \mathcal{T}_{S_3} \rangle$$

$$\phi(z, \bar{z}) \leftrightarrow \odot(\gamma)$$

$$\phi(z, \bar{z}) \sim z^{\Delta} \langle \odot(\gamma) \rangle$$

$$\Phi_{(1)}^{lin}, \Phi_{(2)}^{lin}$$



$\mathcal{D}$

$$\langle \Phi_{(s_1)}(z, \bar{z}) \rangle_{\gamma_{s_1}, \gamma_{s_2}}$$

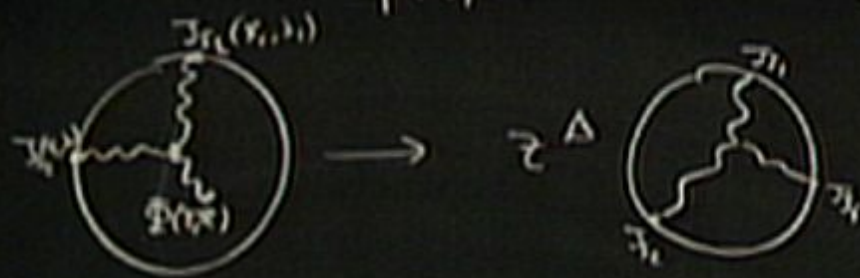
$$\langle \gamma_{s_1} \gamma_{s_2} \gamma_{s_3} \rangle$$

$$\phi(z, \bar{z}) \leftrightarrow \odot(z)$$



$$\phi(z, \bar{z}) \sim z^\Delta \langle \odot(z) \rangle$$

$$\Phi_{(1)}^{lin}, \Phi_{(2)}^{lin}$$



$$\mathcal{D} \Phi^{quad} = \Phi_{(1)} * \Phi_{(1)}$$

$$\langle \Phi_{(1)}(z, \bar{z}) \rangle_{J_{S_1}, J_{S_2}} \rightarrow \langle J_{S_3} J_{S_1} J_{S_2} \rangle = \langle \bar{J}_{S_1} \bar{J}_{S_2} \bar{J}_{S_3} \rangle$$

$$\leftrightarrow \odot(\tilde{x})$$

$$\tilde{x}_1 \text{ (wavy line in a circle)}$$

$$\sim z^\Delta \langle \odot(\tilde{x}) \rangle$$

$$\Phi_{(1)}^{lin}, \Phi_{(2)}^{lin}$$

$$z^\Delta \text{ (circle with wavy lines and labels } \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \text{)}$$

$$" \mathcal{D} \Phi^{lin} = \Phi_{(1)} * \Phi_{(2)} "$$

$$\gamma_{\tilde{x}_1, \tilde{x}_2} \rightarrow \langle \tilde{x}_3 \gamma_{\tilde{x}_1, \tilde{x}_2} \rangle = \langle \tilde{x}_1 \tilde{x}_2 \tilde{x}_3 \rangle$$

$$\leftrightarrow \odot(\tilde{x})$$



$$\sim z^\Delta \langle \odot(\tilde{x}) \rangle$$

$$\Phi_{(1)}^{\text{lin}}, \Phi_{(2)}^{\text{lin}}$$



$$" \mathcal{D} \Phi^{\text{nonlin}} = \Phi_{(1)} * \Phi_{(2)} "$$

$$\gamma_{T_{s1}, T_{s2}} \rightarrow \langle T_{s3} \gamma_{T_{s1}, T_{s2}} \rangle = \langle T_{s1} T_{s2} T_{s3} \rangle$$

HS dof

W

: gauge fields

$W_{\mu}^{(S-1, S-1)}$

$\rightsquigarrow$

HS d.o.f.

W : gauge fields $W_{\mu}^{(s-1, s-1)} \rightsquigarrow \varphi_{a_1 \dots a_s} \rightarrow J_{i_1 \dots i_s}$

B : Weyl curvature $B^{(2s, 0)}, B^{(0, 2s)}$

HS d.o.f.

W : gauge fields $W_{\mu}^{(s-1, s-1)} \rightsquigarrow \varphi_{a_1 \dots a_s} \rightarrow J_{i_1 \dots i_s}$

$\rightarrow B$: Weyl curvature $B^{(2s, 2s)}$

$z \rightarrow 0$ $B \propto \varphi_{i_1 \dots i_s}$
 $(z, t) \rightarrow (z, t)$

HS d.o.f.

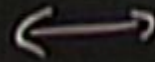
W : gauge fields $W_{\mu}^{(s-1, s-1)} \rightsquigarrow \varphi_{a_1 \dots a_s} \rightarrow J_{i_1 \dots i_s}$

$\rightarrow B$: Weyl curvature $B^{(2s, 2)} , B^{(0, 2s)}$

$z \rightarrow 0$ $B \propto \varphi_{i_1 \dots i_s}$

$a = (z, t) \quad i = 1, 2, \dots$

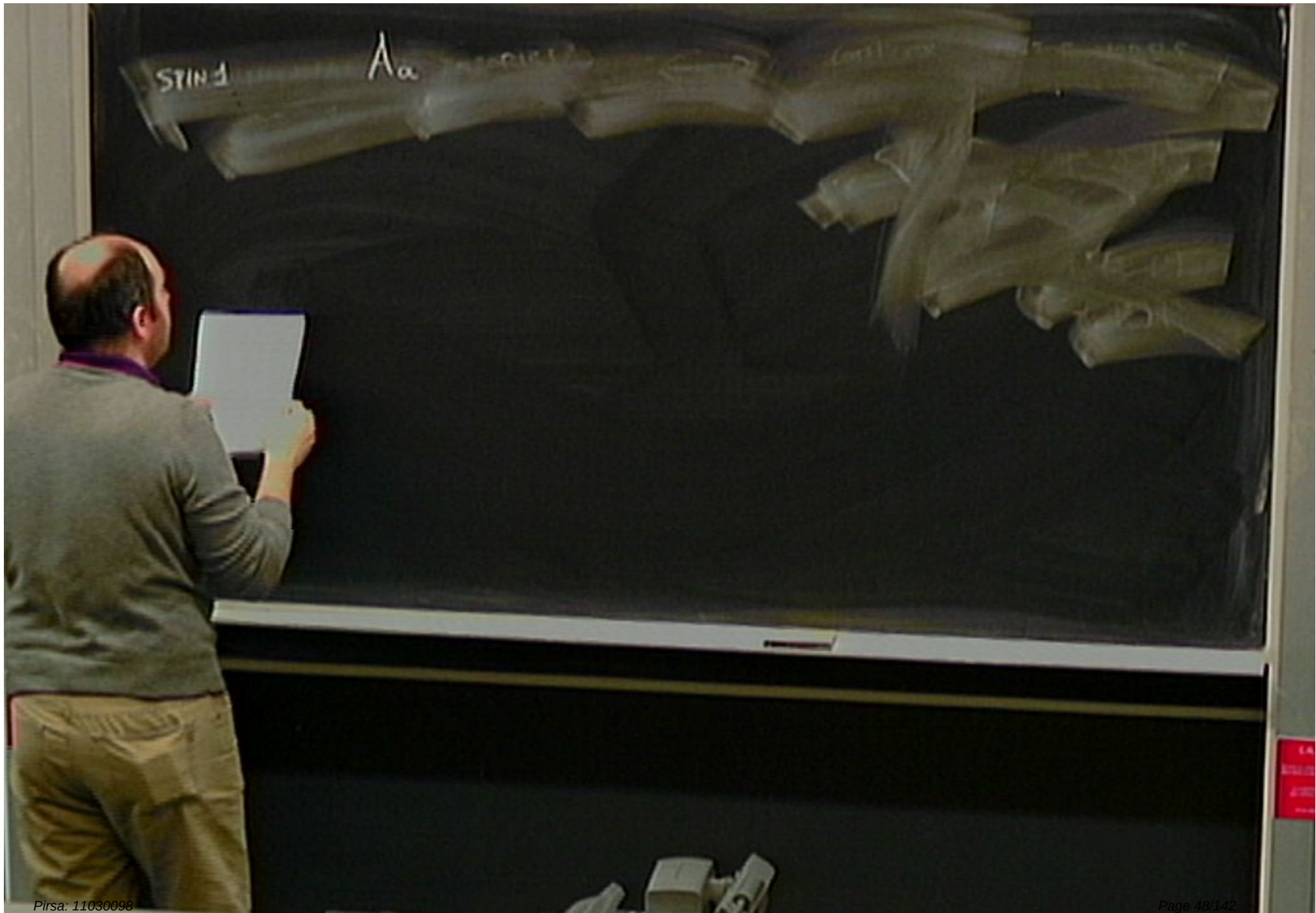
NON-MINIMAL HS THEORIES
 $S = 0, 1, 2, \dots, \infty$



COMPLEX VECTOR MODELS
IN $U(N)$ SINGLET SECTOR

$$S_{\text{fm}} = \int \gamma^\mu \phi_i^* \partial_\mu \phi^i$$

$$J_{\mu\nu} = \phi_i^* (\partial_\mu \phi^\nu - \partial_\nu \phi^\mu)$$



$$A_i(z, \vec{x}) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$A_z(z, \vec{x}) \sim z^3$$

F

$$A_i(z, \bar{z}) \sim z^2 \langle J_i(z) \rangle$$

$$A_z(z, \bar{z}) \sim z^3$$

$F_{ab} : F_{zi}$

$$A_i(z, \vec{x}) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$A_z(z, \vec{x}) \sim z^3$$

$$F_{ab}: \quad F_{zi} \sim z(\partial_z A_i - \partial_i A_z) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$A_i(z, \vec{x})$$

$$A_z(z, \vec{x}) \sim z^3$$

$$F_{ab} : F_{zi} \sim z(\partial_z A_i - \partial_i A_z) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$F_{ij}$$

$$A_z(z, \mathbf{x}) \sim z^3$$

$$F_{ab}: \quad F_{zi} \sim z(\partial_z A_i - \partial_i A_z) \sim z^2 \langle \mathcal{J}_i(\mathbf{x}) \rangle$$

$$F_{ab} \rightarrow \epsilon_{\alpha\beta} F_{\alpha\mu} + \epsilon_{\mu\beta} F_{\alpha\mu}$$

Tab. 1: Tab. 2: $\mathcal{L}(X) = \mathcal{L}(Y) \iff \mathcal{L}(X) \subseteq \mathcal{L}(Y) \text{ and } \mathcal{L}(Y) \subseteq \mathcal{L}(X)$

F_{ij}

$$F_{ab} \rightarrow \epsilon_{ijk} F_{ij} + \epsilon_{ijk} F_{jk}$$

$$B^{(2)} \sim \partial \dots \partial \varphi_{i_1 \dots i_r}$$

$$z=0$$

$$B \propto \varphi_{i_1 \dots i_r}$$

$$a=(z,t) \quad i=1,2,3$$

Tab: $F_{ab} = \partial_a A_b - \partial_b A_a$

F_{ij}

$$F_{ab} \rightarrow \epsilon_{ab} F_{\mu\nu} + \epsilon_{\mu\nu} F_{ab}$$

$$B^{(25,0)}, B^{(0,25)} \sim \partial \dots \partial \varphi_{a_1 \dots a_5}$$

$$B \propto \varphi_{i_1 \dots i_5}$$

$$a = (z, t) \quad i=1,2,3$$

$$A_i(z, \vec{x}) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$A_z(z, \vec{x}) \sim z^3$$

$$F_{ab} : F_{zi} \sim z(\partial_z A_i - \partial_i A_z) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$F_{ij}$$

$$F_{ab} \rightarrow \epsilon_{ijk} F_{ij} + \epsilon_{ijk} F_{ik}$$

$$B^{(rs,0)}, B^{(0,rs)} \sim \frac{\partial \dots \partial}{\partial z^s} \varphi_{a_1 \dots a_s}$$

$$B \propto \varphi_{i_1 \dots i_s}$$

$$a = (z, i) \quad i = 1, 2, 3$$

SPIN 1

A_a

$$A_i(z, \vec{x}) \sim z^2 \langle J_i(\vec{x}) \rangle$$

$$A_z(z, \vec{x}) \sim z^3$$

F_{ab} :

$$F_{zi} \sim z(\partial_z A_i - \partial_i A_z) \sim z^2 \langle J_i(\vec{x}) \rangle$$

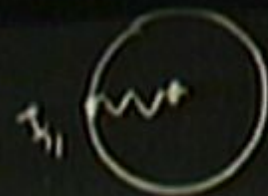
$$F_{ab} \rightarrow \epsilon_{\alpha\beta} F_{\alpha\gamma} + \epsilon_{\alpha\gamma} F_{\beta\gamma}$$

$$B^{(2s,0)}, B^{(0,2s)} \sim \frac{\partial \dots \partial}{\partial z^s} \varphi_{a_1 \dots a_s}$$

$$B \propto \varphi_{i_1 \dots i_s}$$

$$a = (z, \vec{i}) \quad \vec{i} = 1, 2, 3$$

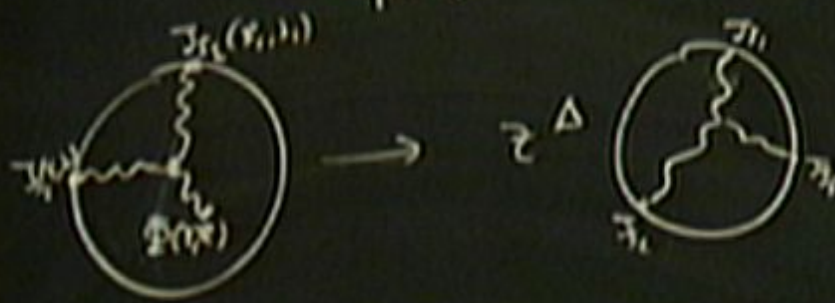
$$\phi(z, \bar{z}) \leftrightarrow \odot(\bar{z})$$



$$\phi(z, \bar{z}) \sim z^\Delta \langle \odot(\bar{z}) \rangle$$

$$\Phi_{(1)}^{l,n}, \Phi_{(2)}^{l,n}$$

$$D \Phi^{l,n} = \Phi_{(1)} * \Phi_{(2)}$$



$$\langle \Phi_{(1)}(z, \bar{z}) \rangle_{T_1, T_2} \rightarrow \langle T_3 \rangle_{T_1, T_2} = \langle T_1 T_2 T_3 \rangle$$

K-P-S-S CONJECTURES

$B_2(\overline{x}, z \rightarrow 1)$

K-P-S-S CONJECTURES

$$B_2(\overline{x}, z \rightarrow 0 \mid y_2, \overline{y}_2 = 0, \overline{z}_2, \overline{z}_2 = 0)$$

K-P-S-S CONJECTURES

$$B_2(\overline{x}, z \rightarrow 0 \mid \gamma_\alpha, \overline{\gamma}_\alpha = 0, \overline{z}_\alpha, \overline{z}_\alpha = 0)$$

$$z^{s+1}$$

K-P-S-S CONJECTURES

$$B_2(\bar{x}, z \rightarrow 0 \mid \gamma_\alpha, \bar{\gamma}_\alpha = 0, \bar{z}_\alpha, \bar{z}_\alpha = 0)$$

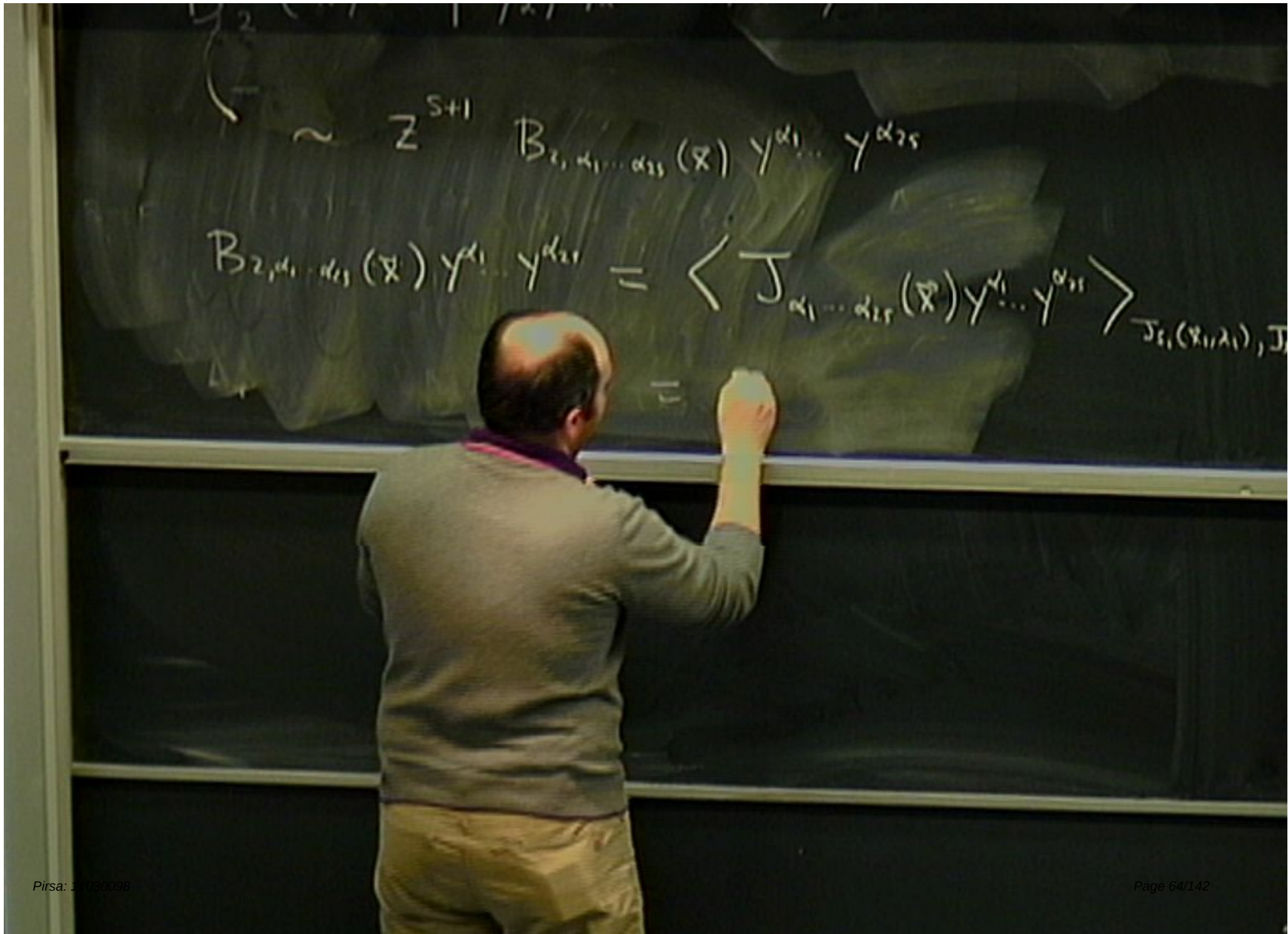
$$\sim z^{s+1}$$

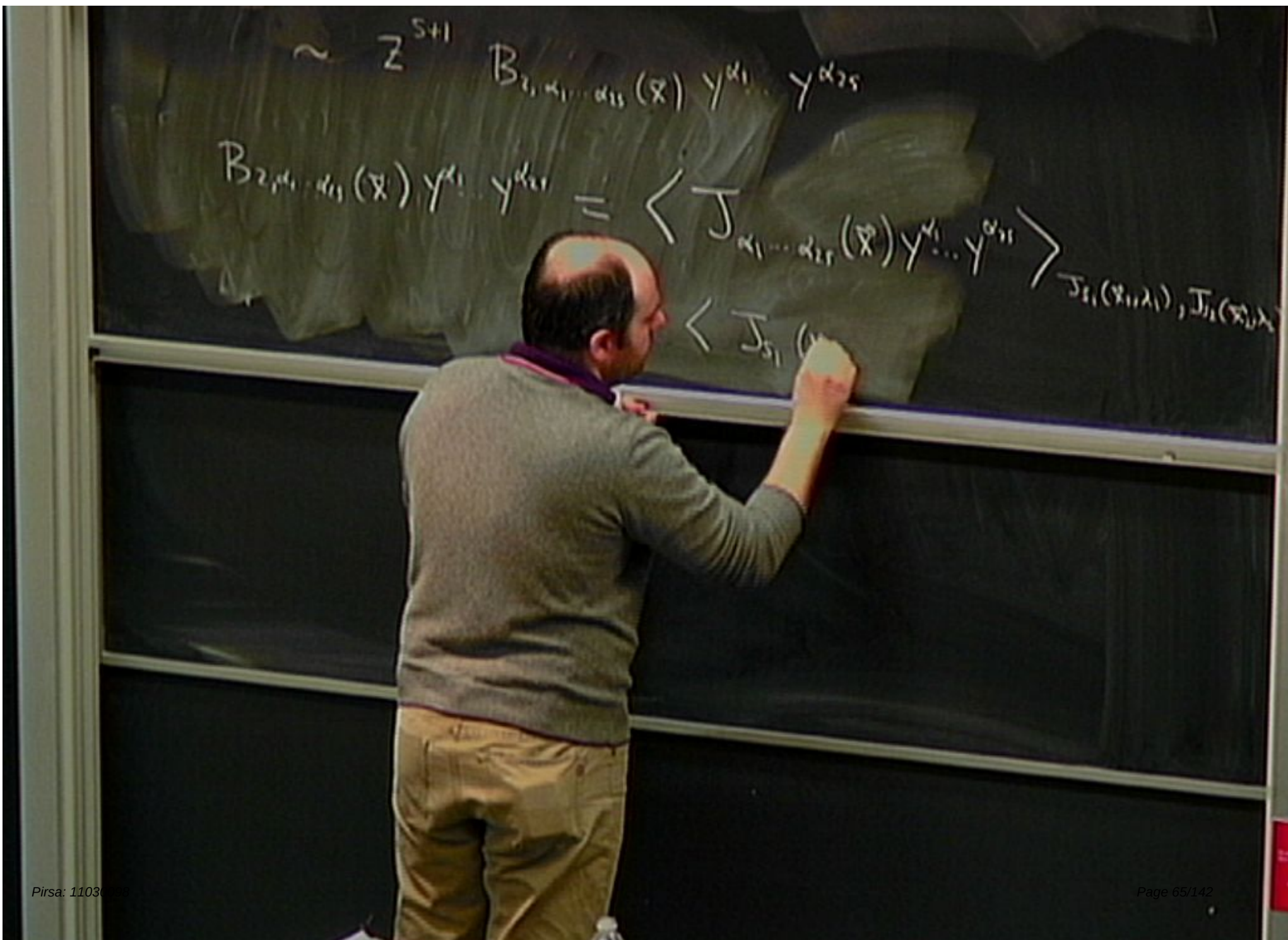
$$\alpha_{11}(\bar{x})$$

K-P-S-S CONJECTURES

$$B_2(\bar{x}, z \rightarrow 0 \mid \gamma_\alpha, \bar{\gamma}_\alpha = 0, \bar{z}_\alpha, \bar{z}_\alpha = 0)$$

$$\sim z^{s+1} B_{z, \alpha_1 \dots \alpha_{2s}}(\bar{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2s}}$$





$$\mathcal{R}_2(\vec{x}, z \rightarrow 0 \mid \gamma_\lambda, \bar{\gamma}_\lambda = 0, \bar{z}_\lambda, \bar{z}_\lambda = 0)$$

$$\sim z^{S+1} B_{\alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}}$$

$$B_{\alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} = \langle J_{\alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} \rangle_{J_{S_1}(\vec{x}_1, \lambda_1), J_{S_2}(\vec{x}_2, \lambda_2)} \\ = \langle J_{S_1}(\vec{x}_1, \lambda_1) J_{S_2}(\vec{x}_2, \lambda_2) J_S(\vec{x}, \lambda_3 = \gamma) \rangle$$

$$\mathcal{R}_2(\vec{x}, z \rightarrow 0 \mid \gamma_\alpha, \bar{\gamma}_\alpha = 0, \bar{z}_\alpha, \bar{z}_\alpha = 0)$$

$$\sim z^{S+1} B_{z, \alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}}$$

$$B_{z, \alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} = \langle \mathcal{J}_{\alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} \rangle_{\mathcal{J}_{\mathcal{L}_1}(\vec{x}_1, \lambda_1), \mathcal{J}_{\mathcal{L}_2}(\vec{x}_2, \lambda_2)}$$

$$= \langle \dots \mathcal{J}_{\mathcal{L}_2}(\vec{x}_2, \lambda_2) \mathcal{J}_S(\vec{x}, \lambda_3 = \gamma) \rangle$$

$$\mathcal{P}_2(\vec{x}, z \rightarrow 0 \mid \gamma_\alpha, \bar{\gamma}_\alpha = 0, z_\alpha, \bar{z}_\alpha = 0)$$

$$\sim z^{S+1} B_{2, \alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}}$$

$$B_{2, \alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} = \langle \mathcal{J}_{\alpha_1 \dots \alpha_{2S}}(\vec{x}) \gamma^{\alpha_1} \dots \gamma^{\alpha_{2S}} \rangle_{\mathcal{J}_1(\vec{x}_1, \lambda_1), \mathcal{J}_2(\vec{x}_2, \lambda_2), \mathcal{J}_S(\vec{x}, \lambda_S)}$$

$$= \langle \mathcal{J}_{S_1}(\vec{x}_1, \lambda_1) \mathcal{J}_{S_2}(\vec{x}_2, \lambda_2) \mathcal{J}_S(\vec{x}, \lambda_S) \rangle$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x) = g^{-1}(x|Y, z) * d_x g(x|Y, z)$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|Y, z) = g^{-1}(x|Y, z) * d_x g(x|Y, z)$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|Y, z) = g^{-1}(x|Y, z) * d_x g(x|Y, z)$$

$$S = g^{-1} * d_z g + g^{-1} * S' * g$$

$$B = g^{-1} * B' * \pi(g)$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|Y, z) = g^{-1}(x|Y, z) * d(x|Y, z)$$

$$S = g^{-1} * d_2 g +$$

$$B = g^{-1} * B' * \pi(\gamma)$$

$$d_x B' = d_Y S'$$

$$d_x W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|y,z) = g^{-1}(x) * d_x g(x|y,z)$$

$$S = g^{-1} * d_z$$

$$B = g^{-1} * \dots$$

$$d_x B' = d_y S' = 0$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$g(x|Y, z) = g^{-1}(x|Y, z) * d_x g(x|Y, z)$$

$$= g^{-1} * d_z g + g^{-1} * S' * g$$

$$g^{-1} * B' * \pi(g)$$

$$\begin{cases} d_x B' = d_x S' = 0 \\ d_z S' + S' * S' = e^{i\theta} B' * k dz + c.c. \\ d_z B' + S' * B' - B' * \pi(S') = 0 \end{cases}$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|Y, z) = g^{-1}(\dots) * d_x g(x|Y, z)$$

$$S = g^{-1} * S' * g$$

$$B = g^{-1}$$

$$\begin{cases} d_x B' = d_x S' = 0 \\ d_z S' + S' * S' = e^{i\theta} B' * k dz + \dots \\ d_z B' + S' * B' - B' * \pi(S') = 0 \end{cases}$$

$$d_x W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|y, z) = \dots * d_x g(x|y, z)$$

$$S = \dots * S' * g$$

$$B =$$

$$\begin{cases} d_x B' = d_y S' = 0 \\ d_z S' + S' * S' = e^{i\theta} B' * k dz + c.c. \\ d_z B' + S' * B' - B' * \pi(S') = 0 \end{cases} (r, z)$$

$$d_Y W + W * W = 0$$

$$W = W_0^{NS} + W_1 + W_2 + \dots$$

$$W(x|Y, z) = g^{-1}(x|Y, z) * d_x g(x|Y, z)$$

$$S = g^{-1} * d_z g + g^{-1} * S' * g$$

$$B = g^{-1} * B' * \pi(g)$$

$$\left\{ \begin{array}{l} d_x B' = d_x S' = 0 \\ d_z S' + S' * S' = e^{i\theta} B' * k dz + c.c. \\ d_z B' + S' * B' - B' * \pi(S') = 0 \end{array} \right. (Y, z)$$

$$\mathcal{B}^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) \star \mathcal{B}'(Y, z) \star \pi(g(x|Y, z))$$

$$\mathcal{B}^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) \star \mathcal{B}'(Y, z) \star \pi(g(x|Y, z))$$

$$g^{-1} \star d_Y g = W_0(x|Y) + W_1(x|Y, z) + \dots$$

$$g(x|Y) = L(x|Y) + g_{\text{lin}}(x|Y, z) + \dots$$

$$\mathcal{B}^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) \star \mathcal{B}'(Y, z) \star \pi(g(x|Y, z))$$

$$g^{-1} \star d_Y g = W_0(x|Y) + W_1(x|Y, z) + \dots$$

$$g(x|Y, z) = L(x|Y) + g_{\text{lin}}(x|Y, z) + \dots$$

$$B^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) * B'(Y, z) * \pi(g(x|Y, z))$$

$$g^{-1} * d_Y g = W_0(x|Y) + W_1(x|Y, z) + \dots$$

$$g(x|Y, z) = L(x|Y) + g_{\text{lin}}(x|Y, z) + \dots$$

$$B_2^{\text{phys}} = L^{-1}(x|Y) * B'_2(Y, z) * L(x|Y) + \dots$$

$$B^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) * B'(Y, z) * \pi(g(x|Y, z))$$

$$g^{-1} * d_Y g = W_0(x|Y) + W_1(x|Y, z) + \dots$$

$$g(x|Y, z) = \underbrace{L(x|Y)}_{\uparrow} + g_{\text{lin}}(x|Y, z) + \dots$$

$$B_2^{\text{phys}} = L^{-1}(x|Y) * B'_2(Y, z) * L(x|Y) + \dots$$

$$S = g^{-1} * dz g + g^{-1} * S' * g$$

$$B = g^{-1} * B' * \pi(g)$$

$$\begin{aligned} dx B' &= dx S' = 0 \\ dz S' + S' * S' &= e^{i\theta} B' * k dz + c.c. \\ dz B' + S' * B' - B' * \pi(S') &= 0 \end{aligned}$$

(1,2)

$$B' = B_1 + b_2 + \dots$$

$$S' = S_1' + S_2' + \dots$$

$$d_z B_1' = 0$$

B

$$g^{-1} * d_z g + g^{-1} * S' * g$$

$$g^{-1} * B' * \pi(g)$$

$$d_x B' = d_x S' = 0$$

$$d_z S' + S_1' * S' = e^{i\theta} B_1' * k dz + \dots$$

$$(d_z B' + S_1' * B' - B_1' * \pi(S')) = 0$$

(1, 2)

$$B' = B'_1 + B'_2 + \dots$$

$$S' = S'_1 + S'_2 + \dots$$

$$d_z B'_1 = 0 \rightarrow B'_1(Y)$$

$$d_z S'_1 = e^{iH} d_z^2 + \dots$$

$$B' = B_1' + B_2' + \dots$$

$$S' = S_1' + S_2' + \dots$$

$$d_z B_1' = 0 \rightarrow B_1'(Y)$$

$$d_z S_1' = e^{i\theta} B_1' * \kappa dz^2 + \dots$$

$$B' = B'_1 + B'_2 + \dots$$

$$S' = S'_1 + S'_2 + \dots$$

$$d_{\mathbb{R}} B'_1 = 0 \rightarrow B'_1(Y) =$$

$$d_{\mathbb{R}} S'_1 = e^{i\theta} B'_1 \wedge e^{\mathbb{R}} d\mathbb{R} \dots$$

$$B'_1(Y) = L(x|Y) \star \tilde{B}'^{(n)}_1(x|Y) \star$$

$$B = g^{-1} \star B' \star \pi(g)$$

$$\left(\begin{array}{c} d_{\mathbb{R}} B' \\ (Y, Z) \end{array} \right)$$

$$e d\mathbb{R} d\mathbb{R} \dots$$

$$= 0$$

$$\partial_z^s \gamma_{i_1 \dots i_s}$$

$$B_1^1(Y) = L(X|Y) * B_1^{\text{phys}}(X|Y) \# \pi(L^{-1})$$

$$L(X|Y)$$

$$B^{\text{phys}}(x|Y, z) = g^{-1}(x, Y, z) * B'(Y, z) * \pi(g(x|Y, z))$$

$$g^{-1} * d_Y g = W_0(x|Y) + W_1(x|Y, z) + \dots$$

$$g(x|Y, z) = \underbrace{L(x|Y)}_{\uparrow} + \underbrace{g_{\text{lin}}(x|Y, z)}_{\dots}$$

$$B_2^{\text{phys}} = L^{-1}(x|Y) * B'_2(Y, z) * L(x, Y) + \dots$$

$$B' = B'_1 + B'_2 + \dots$$

$$S' = S'_1 + S'_2 + \dots$$

$$d_t B'_1 = 0 \rightarrow B'_1(Y) =$$

$$d_t S'_1 = e^{i\theta} B'_1 * k dt^2 \dots$$

$$B'_1(Y) = L(X|Y) * B_1^{phn}(X|Y) * \pi(L^{-1})$$

$$L(X|Y) = P_{L,Y} * \left(- \int_Y^{x_0} W_0''(x'|Y) dx' \right)$$

$$B = g^{-1} * B' * \pi(s)$$

$$d_t B' + S'_1 * B' - B'_1 * \pi(s') = 0$$

$$\rightarrow B_1^1(Y) = L(x|Y) * B_1^{\text{phys}}(x|Y) \pi(L^{-1})$$

$$L(x|Y) = P_{\text{LTP}} * \left(- \int_x^{x_0} W_0^{\mu}(x'|Y) dx' \right)$$

$$\rightarrow B_1^1(Y)$$

$$L(x_0|Y)$$

$$\rightarrow B_1'(Y) = L(x|Y) * B_1^{\text{phys}}(x|Y) * \pi(L^{-1})$$

$$L(x|Y) = P_{L,Y} P_{*} \left(- \int_x^{x_0} W_0^{\mu}(x'|Y) dx'_\mu \right)$$

$$L(x_0|Y) = 1$$

$$B_1'(Y) =$$

$$\vec{x}_1, \lambda_1, \vec{x}_2, \lambda_2$$

$$g' * B' * \pi(g)$$

$$)=0$$

$$\rightarrow B_1'(Y) = L(x|Y) * B_1^{\text{phys}}(x|Y) * \pi(L^{-1})$$

$$L(x|Y) = P_{L+P*} \left(- \int_x^{x_0} W_0^\mu(x'|Y) dx'_\mu \right)$$

$$L(Y) = 1$$

$$B_1'(x_0|Y) = B_1^{\text{phys}}(x_0|Y; \overline{x}_1, \lambda_1, \overline{x}_2, \lambda_2)$$

$$g' * B' * \pi(g)$$

$$s' * B' + s' * B' - B' * \pi(s') = 0$$

$$\rightarrow B'_1(Y) = L(X|Y) * B_1^{\text{phys}}(X|Y) * \pi(L^{-1})$$

$$L(X|Y) = P_{\text{LTP}} * \left(- \int_X^{x_0} W_0^\mu(X'|Y) dX'_\mu \right)$$

$$L(X_0|Y) = 1$$

$$B'_1(Y) = B_1^{\text{phys}}(X_0|Y; \vec{x}_i, \lambda_i, \vec{x}_e, \lambda_e)$$

$$\begin{cases} dz S' + S' * S' = e^{i\theta} B' * k dz_{\text{c.c.}} \\ dz B' + S' * B' - B' * \pi(S') = 0 \end{cases}$$

$$(\gamma, z)$$

$$B' = B'_1 + B'_2 + \dots$$

$$S' = S'_1 + S'_2 + \dots$$

$$d_z B'_1 = 0 \rightarrow B'_1(Y)$$

$$d_z S'_1 = e^{i\theta} B'_1 \wedge k dz_2 \dots$$

$$\rightarrow B'_1(Y) = L(X|Y)$$

$$L(X|Y) = P_{\star} P_{\star}^* \left(- \int_Y^{x_0} W'_0(x'|Y) dx'_1 \right)$$

$$L(X_0|Y) = 1$$

$$B'_1(Y) = \mathcal{B}'_1^{phys}(X_0|Y, \nabla, \Delta, \nabla, \Delta)$$



$$S = g \star d_z g + g' \star S \star g$$

$$B = g^{-1} \star B' \star \pi(g)$$

$$\begin{cases} d_z S'_1 + S'_1 \star S'_1 = e^{i\theta} B'_1 \wedge k dz_2 \dots \\ d_z B'_1 + S'_1 \star B'_1 - B'_1 \star \pi(S'_1) = 0 \end{cases}$$

(Y, Z)

$$B_1(\gamma, \bar{\gamma}; x, \lambda)$$



$$B_1(\gamma, \bar{\gamma}; \bar{x}, \lambda) = e^{-\gamma(\sigma^2 + 2\bar{\gamma}_1 \bar{\sigma} - \sigma^2)}$$

$$B_1(\gamma, \bar{\gamma}; \bar{x}, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2\frac{\bar{x}_1 \bar{\sigma} - \sigma^2}{\gamma_1^2 + 1})\bar{\gamma}}}{\gamma_1^2 + 1} \left\{ \exp\left[-2\gamma \frac{\bar{x}_1 \bar{\sigma} \sigma^2 - 1}{\gamma_1^2 + 1} \lambda_1 + \dots\right] \right\}$$

$$B' = B'_1 + B'_2 + \dots$$

$$S' = S'_1 + S'_2 + \dots$$

$$d_z B'_1 = 0 \rightarrow B'_1(Y)$$

$$d_t S'_1 = e^{i\theta} B'_1 \wedge k dz_2 \dots$$

$$\rightarrow B'_1(Y) = L(x|Y) \star \tilde{B}'_1(x|Y) \wedge \pi(L)$$

$$L(x|Y) = P_{\star} \wedge P_{\star} \left(- \int_x^{x_0} W'_0(x'|Y) dx'_1 \right)$$

$$L(x_0|Y) = 1$$

$$x_0 = (\vec{0}, 1)$$



$$B'_1(Y) = \mathcal{B}'_1^{Phys}(x_0|Y; \gamma_i, \lambda_i, \bar{\gamma}_i, \lambda_i)$$

$$C = g \star d_z g + g' \wedge S' \star g$$

$$B = g' \star B' \star \pi(g)$$

$$d_z S' + S' \wedge S' = e^{i\theta} B' \wedge k dz_2 \wedge \dots$$

$$d_z B' + S' \wedge B' - B' \wedge \pi(S') = 0$$

(Y, Z)

$$B_1(\gamma, \bar{\gamma}; \bar{x}_1, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2\frac{\bar{x}_1 \bar{\sigma} - \sigma^2}{x_1^2 + 1})\bar{\gamma}}}{x_1^2 + 1} \left\{ e^{4\rho} \left[-2\gamma \frac{\bar{x}_1 \bar{\sigma} - \sigma^2}{x_1^2 + 1} \lambda_1 + \dots \right] \right\}$$

$$B_1'(\gamma, \bar{\gamma}; \bar{x}_1, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2\frac{\bar{x}_1 \bar{\sigma} - \sigma^2}{\gamma_1^2 + 1})\bar{\gamma}}}{\gamma_1^2 + 1} \left\{ e^{4\rho \left[-2\gamma \frac{\bar{x}_1 \bar{\sigma} - \sigma^2}{\gamma_1^2 + 1} \lambda \right] + \dots} \right\}$$

$\lambda \rightarrow \mu$

$$\tilde{B}_1(\gamma, \bar{\gamma}; \bar{x}_{1+\mu}) = \int d^2\lambda e^{\lambda^* \mu_\lambda} B_1'(\gamma, \bar{\gamma}, \bar{x}_1)$$

$$B'_1(\gamma, \bar{\gamma}; \bar{x}_1, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2\frac{\bar{x}_1 \cdot \bar{\sigma} - \sigma^2}{x_1^2 + 1})\bar{\gamma}}}{x_1^2 + 1} \left\{ e^{4\rho} \left[-2\gamma \frac{\bar{x}_1 \cdot \bar{\sigma} \sigma^2}{x_1^2 + 1} \lambda \right] + \dots \right\}$$

$\lambda \rightarrow \mu$

$$\tilde{B}_1(\gamma, \bar{\gamma}; \bar{x}_1, \mu) = \int d^2\lambda e^{\lambda^* \mu \lambda} B'_1(\gamma, \bar{\gamma}; \bar{x}_1, \lambda_1)$$

$$B_1'(\gamma, \bar{\gamma}; \bar{x}_1, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2\frac{\bar{\gamma}_1 \bar{\sigma} \sigma^2 - \sigma^2}{x_1^2 + 1})\bar{\gamma}}}{x_1^2 + 1} \left\{ e^{4\rho \left[-2\gamma \frac{\bar{\gamma}_1 \bar{\sigma} \sigma^2 - 1}{x_1^2 + 1} \lambda \right]} + \dots \right\}$$

$\lambda \rightarrow \mu$

$$\begin{aligned} \tilde{B}_1(\gamma, \bar{\gamma}; \bar{\gamma}_1, \mu) &= \int d^2\lambda e^{\lambda^* \mu} B_1'(\gamma, \bar{\gamma}; \bar{x}_1, \lambda_1) \\ &= \delta(\gamma - \chi_1) e^{\bar{\gamma} \bar{\chi}_1} + \delta(\bar{\gamma} - \bar{\chi}_1) e^{\gamma \chi_1} \end{aligned}$$

$$B_1(\gamma, \bar{\gamma}; \bar{x}_1, \lambda) = \frac{e^{-\gamma(\sigma^2 + 2 \frac{\bar{x}_1 \cdot \bar{\sigma} - \sigma^2}{x_1^2 + 1})\bar{\gamma}}}{x_1^2 + 1} \left\{ \exp\left[-2\gamma \frac{\bar{x}_1 \cdot \bar{\sigma} - \sigma^2}{x_1^2 + 1} \lambda\right] + \dots \right\}$$

$$\begin{aligned} \tilde{B}_1(\gamma, \bar{\gamma}; \bar{x}_1) &= \int d^2\lambda e^{\lambda_1^* \mu_1} B_1(\gamma, \bar{\gamma}; \bar{x}_1, \lambda_1) \\ &= \delta(\gamma - \chi_1) e^{\bar{\gamma} \bar{\chi}_1} + \delta(\bar{\gamma} - \bar{\chi}_1) e^{\gamma \chi_1} \\ \chi_1 &= \sigma^2 (\bar{x}_1 \cdot \bar{\sigma} + \sigma^2) \mu_1 \end{aligned}$$

$$B'_2(y, z) = \int$$

$$B'_2(Y, Z) = \int d^4U d^4V B(Y, U, X_1, X_2, M, M_2)$$

$$B'_2(Y, Z) = \int d^4U d^4V \left[B'_1(U, \bar{x}_1, \bar{x}_2, \mu_1, \mu_2) \cdot B'_1(V, \bar{x}_1, \bar{x}_2, \mu_1, \mu_2) \cdot F(Y, Z; U, V) \right]$$

$$B'_2(Y, Z) = \int d^4U d^4V \mathcal{B}'_1(U, \vec{x}_1, \vec{x}_2, \mu_1, \mu_2) \cdot \mathcal{B}'_1(V, \vec{x}_1, \vec{x}_2, \mu_1, \mu_2) \cdot \mathcal{F}(Y, Z; U, V)$$

$U = (u^\mu, u^\nu)$

$$B'_2(Y, Z) = \int d^4U d^4V \mathcal{B}'_1(U, \bar{X}_1, \bar{X}_2, \mu_1, \mu_2) \cdot \mathcal{B}'_1(V, \bar{X}_1, \bar{X}_2, \mu_1, \mu_2) \cdot \mathcal{F}(Y, Z; U, V)$$

$U = (u^\mu, \bar{u}^{\dot{\mu}})$



$$B'_2(Y, Z) = \int d^4U d^4V \cdot B'_1(U, \bar{x}_1, \bar{x}_2, \mu_1, \mu_2) \cdot B'_1(V, \bar{x}_1, \bar{x}_2, \mu_1, \mu_2) \cdot F(Y, Z; U, V)$$

$$B_2^{\text{phys}}(\bar{x}, z | Y, Z) = L^{-1} * B'_2(Y, Z) * \pi(L)$$

$$z^{-s-1} \beta_2^{1/2} \left(x \rightarrow 0 \mid y, \bar{y} = z, \bar{z} = 0 \right)$$

$$(- \int W_0''(x'|y) dx'_x)$$

$$z^{-s-1} \mathcal{B}_2^{\text{phys}} \left(\vec{x}_0, z \rightarrow 0 \mid \gamma, \bar{\gamma} = z_\alpha = \bar{z}_\alpha = 0 \right)$$

$$= \int d^4 U d^4 V e^{u\bar{v} - \bar{u}\bar{v}} \mathcal{B}_1(u, \bar{u}) \mathcal{B}_1(\bar{v}, \bar{v})$$

$$z^{-s-1} \mathcal{B}_2^{\text{phys}}(\bar{x}_0, z \rightarrow 0 \mid \gamma, \bar{\gamma} = z_1 = \bar{z}_2 = 0)$$

$$= \int d^4 U d^4 V \cdot e^{uv - \bar{u}\bar{v}} \mathcal{B}_1(u, \bar{u}) \mathcal{B}_1(v, \bar{v}) \cdot (\gamma \sigma^\mu \bar{u}) \left[\frac{e^{z\gamma(u-v)}}{(\gamma \sigma^\mu \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{z\gamma(u+v)}}{(\gamma \sigma^\mu \alpha_-)(\bar{u} \alpha_-)} \right]$$

$$z^{-s-1} \mathcal{B}_2^{H_1} \left(x_0, z \rightarrow 0 \mid \gamma, \bar{\gamma} = z_1 = \bar{z}_1 = 0 \right)$$

$$= \int d^4 U d^4 V e^{uv - \bar{u}\bar{v}} \mathcal{B}_1(u, \bar{u}) \mathcal{B}_1(v, \bar{v})$$

$$\cdot (\gamma \sigma^\mu \bar{u}) \left[\frac{e^{z\gamma(u-v)}}{(\gamma \sigma^\mu \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{z\gamma(u+v)}}{(\gamma \sigma^\mu \alpha_-)(\bar{u} \alpha_-)} \right]$$

$$\bar{u} = \sigma^\mu u \pm (\bar{u} \pm \sigma^\mu v)$$

$$| \rightarrow B_1'(Y) = L(X|Y) \star B_1^{\text{phys}}(X|Y)$$

$$B_2^{\text{phys}} \left(\vec{X}=0, z \rightarrow 0 \mid \gamma, \bar{\gamma} = z_\pm = \bar{z}_\pm = 0 \right)$$

$$d^4U d^4V e^{u\bar{v} - \bar{u}v} B_1(u, \bar{u}) B_1(v, \bar{v})$$

$$\cdot (\gamma \sigma^\pm \bar{u}) \left[\frac{e^{z\gamma(u-\bar{v})}}{(\gamma \sigma^\pm \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{z\gamma(u+\bar{v})}}{(\gamma \sigma^\pm \alpha_-)(\bar{u} \alpha_-)} \right]$$

$\gamma^\mu (\sigma^\pm \alpha_\pm)_\mu$
 $\langle \gamma \sigma^\pm \alpha_\pm \rangle$

$$\alpha_\pm = (\bar{u} \pm \sigma^\pm v)$$

$$z^{-s-1} B_2^{\text{th}}(\bar{x}_0, z \rightarrow 0 \mid \gamma, \bar{\gamma} = z_1 = \bar{z}_2 = 0)$$

$$= \int d^4 U d^4 V \cdot e^{uv - \bar{u}\bar{v}} B_1(u, \bar{u}) B_1(v, \bar{v}) \cdot (\gamma \sigma^\mu \bar{u}) \left[\frac{e^{z\gamma(u-v)}}{(\gamma \sigma^\mu \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{z\gamma(u+v)}}{(\gamma \sigma^\mu \alpha_-)(\bar{u} \alpha_-)} \right]$$

$$d_\pm = \pm (\bar{u} \pm \sigma^\mu \alpha_\mu)$$

$$= \int d^4 u d^4 v e^{i p \cdot u - i p \cdot v} B_1(u, \bar{u}) B_1(v, \bar{v})$$

$$\cdot (\gamma \sigma^{\mu} \bar{u}) \left[\frac{e^{i \gamma (u-v)}}{(\gamma \sigma^{\mu} \bar{u})(\bar{u} \alpha_{+})} - \frac{e^{i \gamma (u+v)}}{(\gamma \sigma^{\mu} \bar{u})(\bar{u} \alpha_{-})} \right]$$

$$d_{\pm} = \bar{u} - \sigma^{\mu} u \pm (\bar{u} + \sigma^{\mu} v)$$

$$\gamma \sigma^{\mu} \bar{u}, \bar{u} \alpha_{\pm}$$

$$\gamma(\sigma^{\mu} \bar{u})$$

$$\langle \gamma \sigma^{\mu} \bar{u} \rangle$$

$$\gamma \sigma^{\mu} \bar{u}, \bar{u} \alpha_{\pm}$$

$$\gamma$$

$$(r, z)$$

$$r + i z - \beta^{\dagger} \pi(r) = 0$$

$$\alpha_{\pm} = \bar{v} - \sigma^z u \pm (\bar{u} \pm \sigma^z v)$$

$$\gamma \sigma^z \alpha_{+}, \quad \bar{u} \alpha_{+}$$

$$|\gamma \sigma^z \alpha_{+}| = \epsilon_1$$

$$|\bar{u} \alpha_{+}| = \epsilon_2$$

$$(\gamma \sigma^z \alpha_{+})(\bar{u})$$

$$\gamma^{\mu}(\sigma^z \alpha_{+})_{\mu}$$

$$\langle \gamma \sigma^z \alpha_{+} \rangle$$

$$\gamma, \bar{\gamma} = z_2 = \bar{z}_2 = 0)$$

$$-u\bar{v} B_1(u, \bar{u}) B_1(v, \bar{v})$$

$$\gamma \sigma^z \bar{u} \left[\frac{e^{2\gamma(u-v)}}{(\gamma \sigma^z \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{2\gamma(u+v)}}{(\gamma \sigma^z \alpha_-)(\bar{u} \alpha_-)} \right]$$

$$\bar{u} + \sigma^z v)$$

$$\text{Residue} = \pm \gamma \sigma^z \bar{u}$$

$$= \int d^4U d^4V e^{uv - \bar{u}\bar{v}} B_1(u, \bar{u}) B_1(v, \bar{v}) \cdot (\gamma \sigma^{\mu} \bar{u}) \left[\frac{e^{z\gamma(u-v)}}{(\gamma \sigma^{\mu} \alpha_+)(\bar{u} \alpha_+)} - \frac{e^{z\gamma(u+v)}}{(\gamma \sigma^{\mu} \alpha_-)(\bar{u} \alpha_-)} \right]$$

$$\alpha_{\pm} = \bar{v} - \sigma^{\mu} u \pm (\bar{u} \pm \sigma^{\mu} \bar{v})$$

$$\text{Residue} = i \gamma \sigma^{\mu} \bar{u}$$

$$\gamma \sigma^{\mu} \alpha_+$$

$$|\gamma \sigma^{\mu} \alpha_+|$$

$$|\bar{u} \alpha_+|$$

$$B = \frac{1}{2}$$

$$(L + B' + S' + B' - B' \pi(r)) = 0$$

(r, z)

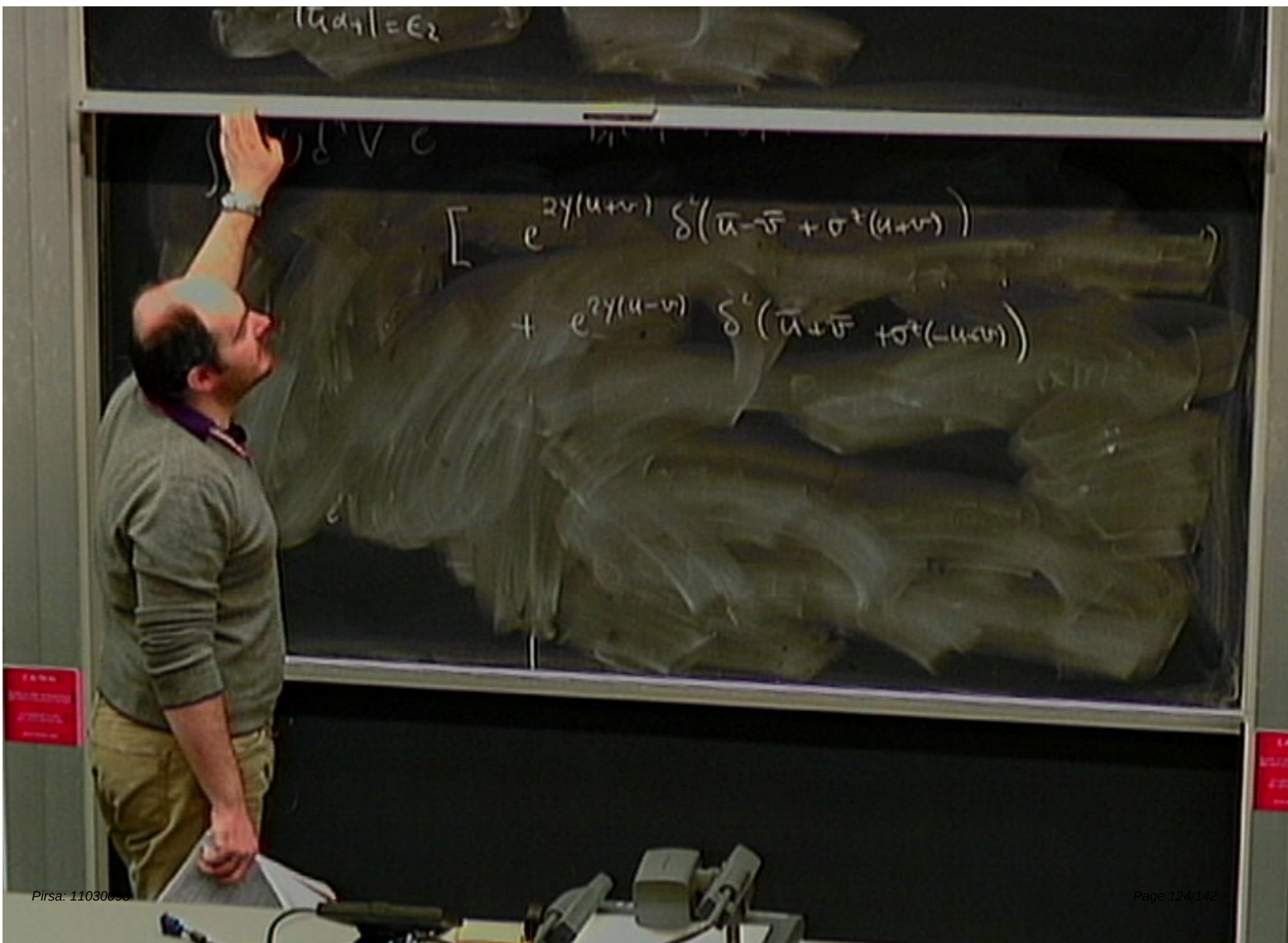
$$\int d^4U d^4V e^{iuv - u\bar{v}} b'_1(u, \bar{u}) B'_1(v, \bar{v})$$

$$\int e^{2\gamma(u)}$$

$$\int d^4u d^4v e^{i\gamma(u-v)} B_1'(u, \bar{u}) B_1'(v, \bar{v})$$

$$e^{2\gamma(u+v)} \delta^2(\bar{u} - \bar{v} + \sigma^2(u+v))$$

$$+ e^{2\gamma(u-v)} \delta^2(\bar{u} + \bar{v} + \sigma^2(-u-v))$$



$$\int d^4u d^4v e^{uv - \bar{u}\bar{v}} B'_1(u, \bar{u}) B'_1(v, \bar{v})$$

$$\left[e^{2\gamma(u+v)} \delta^2(\bar{u} - \bar{v} + \sigma^2(u+v)) \right. \\ \left. + e^{2\gamma(u-v)} \delta^2(\bar{u} + \bar{v} + \sigma^2(-u+v)) \right]$$

$$\int d^4u d^4v e^{uv - \bar{u}\bar{v}} b_1^1(u, \bar{u}) B_1^1(v, \bar{v})$$

$$\left[-2\gamma(u+v) \delta^2(\bar{u} - \bar{v} + \sigma^2(u+v)) \right]$$

$$+ e^{uv} \delta^2(\bar{u} + \bar{v} + \sigma^2(-u-v))$$

$$\int d^4U d^4V e^{uv - \bar{u}\bar{v}} b_1'(u, \bar{u}) B_1'(v, \bar{v})$$

$$\left[e^{2\gamma(u+v)} \delta^2(\bar{u} - \bar{v} + \sigma^z(u+v)) \right.$$

$$\left. + e^{2\gamma(u-v)} \delta^2(\bar{u} + \bar{v} + \sigma^z(-u-v)) \right]$$

$$\int d^4u d^4v e^{uv - \bar{u}\bar{v}} B_1'(u, \bar{u}) B_1'(v, \bar{v})$$

$$\left[e^{2\gamma(u+v)} \delta^2(\bar{u} - \bar{v} + \sigma^2(u+v)) \right.$$

$$\left. + e^{2\gamma(u-v)} \delta^2(\bar{u} + \bar{v} + \sigma^2(-u-v)) \right]$$

1. do U.V. integral
2. do F.T. back

1. do U, V integral

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{X}_3$ $\vec{X}_1 \rightarrow \vec{X}_3$ $\vec{X}_2$

1. do U, V thing

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{X}_3$ $\vec{X}_1 \rightarrow \vec{X}_{13}$ $\vec{X}_2 \rightarrow \vec{X}_{23}$

1

α λ_1 X_{12} X_{23}

$|X_{12}| |X_{23}| |X_{31}|$

back $\mu_{12} \rightarrow \lambda_{12}$

$$\vec{x}_3 \quad \vec{x}_1 \rightarrow \vec{x}_{13} \quad \vec{x}_2 \rightarrow \vec{x}_{23}$$

$$\omega \left(\lambda_1 \frac{x_{12} x_{23} x_{31}}{2 x_{12}^2 x_{31}^2} \lambda_1 + \text{cyclic} \right)$$

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$

$\cosh \left(\lambda_1 \frac{x_{12} x_{23}}{2} \lambda_1 + \text{cyclic} \right)$

$|x_{12}| |x_{23}| |x_{31}|$

$\cosh \left(\lambda_1 x_{13} \lambda_3 \right)$

$$\chi_1 = \sigma^2 (\vec{x}_1 \cdot \vec{\sigma} + \sigma^2) \mu_1$$

1. do $\vec{v}_1, \vec{v}_2$ thing

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$|x_{12}| |x_{23}| |x_{31}|$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \lambda_2 \right)$$

$$\chi_1 = \sigma^1 (\vec{x}_1 \cdot \vec{\sigma} + \sigma^2) \mu_1$$

F.T. back $\mu_{12} \rightarrow \lambda_{12}$
 $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right) \cosh \left(\lambda_3 \frac{x_{31}}{x_{31}^2} \lambda_1 \right)$$

$$\chi_1 = \sigma^1 (\vec{x}_1 \cdot \vec{\sigma} + \sigma^2) \mu_1$$

F.T. back $\mu_{12} \rightarrow \lambda_{12}$
 $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$\left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

2. do f.i. back $\lambda_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

$$\frac{1}{|x_{12}| |x_{23}| |x_{31}|} \cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_1 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

1

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$|x_{12}| |x_{23}| |x_{31}|$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

$$x_{ij}(\vec{x}, \vec{t})_{\lambda_i}$$

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_{23}$, $\gamma \rightarrow \lambda_3$

"TYPE A"

1

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31}}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$|x_{12}| |x_{23}| |x_{31}|$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

$$x_{ij} = (\vec{x} \cdot \vec{e}_i) \wedge \vec{e}_j$$

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$
 3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

"TYPE A"

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31}}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

$\cosh \rightarrow \sinh$

$$x_{ij} = (\vec{x} \cdot \vec{e}_i)_{\perp j}$$

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$
 3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

"TYPE A"

1

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31}}{2} + \text{cyclic} \right)$$

$$|x_{12}| |x_{23}| |x_{31}|$$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_n} \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_n} \right) \cosh \left(\lambda_3 \frac{x_{31}}{x_n} \right)$$

"TYPE B": $\cosh \rightarrow \sinh$

$$\lambda_i = (\vec{x} \cdot \vec{e})_{21}$$

1. do U, V integral

2. do F.T. back $\mu_{12} \rightarrow \lambda_{12}$

3. $\vec{0} \rightarrow \vec{x}_3$ $\vec{x}_1 \rightarrow \vec{x}_3$ $\vec{x}_2 \rightarrow \vec{x}_3$, $\gamma \rightarrow \lambda_3$

"Type A"

1

$$\cosh \left(\lambda_1 \frac{x_{12} x_{23} x_{31} \lambda_1}{2 x_{12}^2 x_{31}^2} + \text{cyclic} \right)$$

$|x_{12}| |x_{23}| |x_{31}|$

$$\cosh \left(\lambda_1 \frac{x_{12}}{x_{12}^2} \lambda_2 \right) \cosh \left(\lambda_1 \frac{x_{13}}{x_{13}^2} \lambda_3 \right) \cosh \left(\lambda_2 \frac{x_{23}}{x_{23}^2} \lambda_3 \right)$$

"Type B": $\cosh \rightarrow \sinh$

$$x_{ij} = (\vec{x} \cdot \vec{e}_i) \wedge \vec{e}_j$$