Title: Higher Spin Theories - Lecture 2

Date: Mar 25, 2011 09:30 AM

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Abstract:

FRAME FORMULATION

gravity

$$g_{\mu\nu} \longrightarrow \{ e_\mu^a , \omega_\mu^{ab} \}$$

$$e_\mu^a \downarrow P_a \qquad \omega_\mu^{ab} \downarrow M_{ab}$$

FRAME FORMULATION

gravity

$$g_{\mu\nu} \longrightarrow \{ e_\mu^a , \omega_\mu^{ab} \}$$

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(P_a, M_{ab}) $\quad SO(3,2) \quad \Lambda \neq 0$

$$[M_{ab}, M_{cd}] = i(\eta_{ac} M_{bd} + \cdots)$$

$$[P_a, M_{bc}] = i(\eta_{ac} P_b - \eta_{ab} P_c)$$

FRAME FORMULATION

gravity

$$\begin{array}{ccccc} g_{\mu\nu} & \longrightarrow & \{ e_\mu^a, & \omega_\mu^{ab} \} \\ & & \downarrow & \downarrow \\ & & P_a & M_{ab} \end{array}$$

(P_a, M_{ab}) $SO(3,2)$ $\Lambda \neq 0$

$$[M_{ab}, M_{cd}] = i(\eta_{ac} M_{bd} + \dots)$$

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$$[P_a, P_b] = \frac{-i}{R_{AdS}^2} M_{ab}$$

FRAME FORMULATION

gravity

$$g_{\mu\nu} \longrightarrow \left\{ e^a_\mu , \; \omega^{ab}_\mu \right\}$$

$$\begin{array}{cc} e^a_\mu & \omega^{ab}_\mu \\ \downarrow & \downarrow \\ P_a & M_{ab} \end{array}$$

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FRAME FORMULATION

gravity

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$$[P_a, P_b] = \frac{-i}{R^2_{AdS}} M_{ab} \quad R_{AdS} = 1$$

SO(3,2) GAUGE FIELD

W

FRAME FORMULATION

gravity

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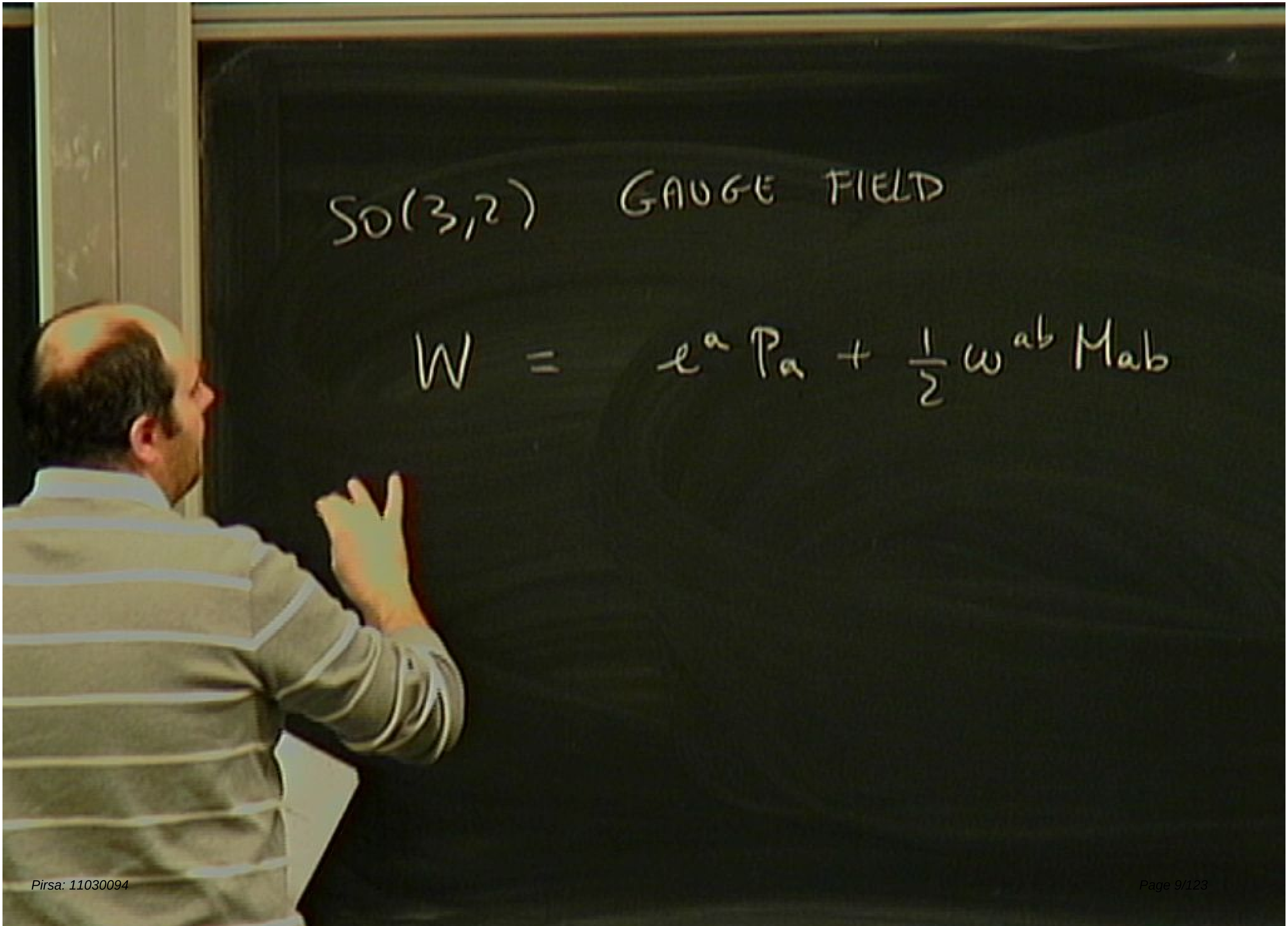
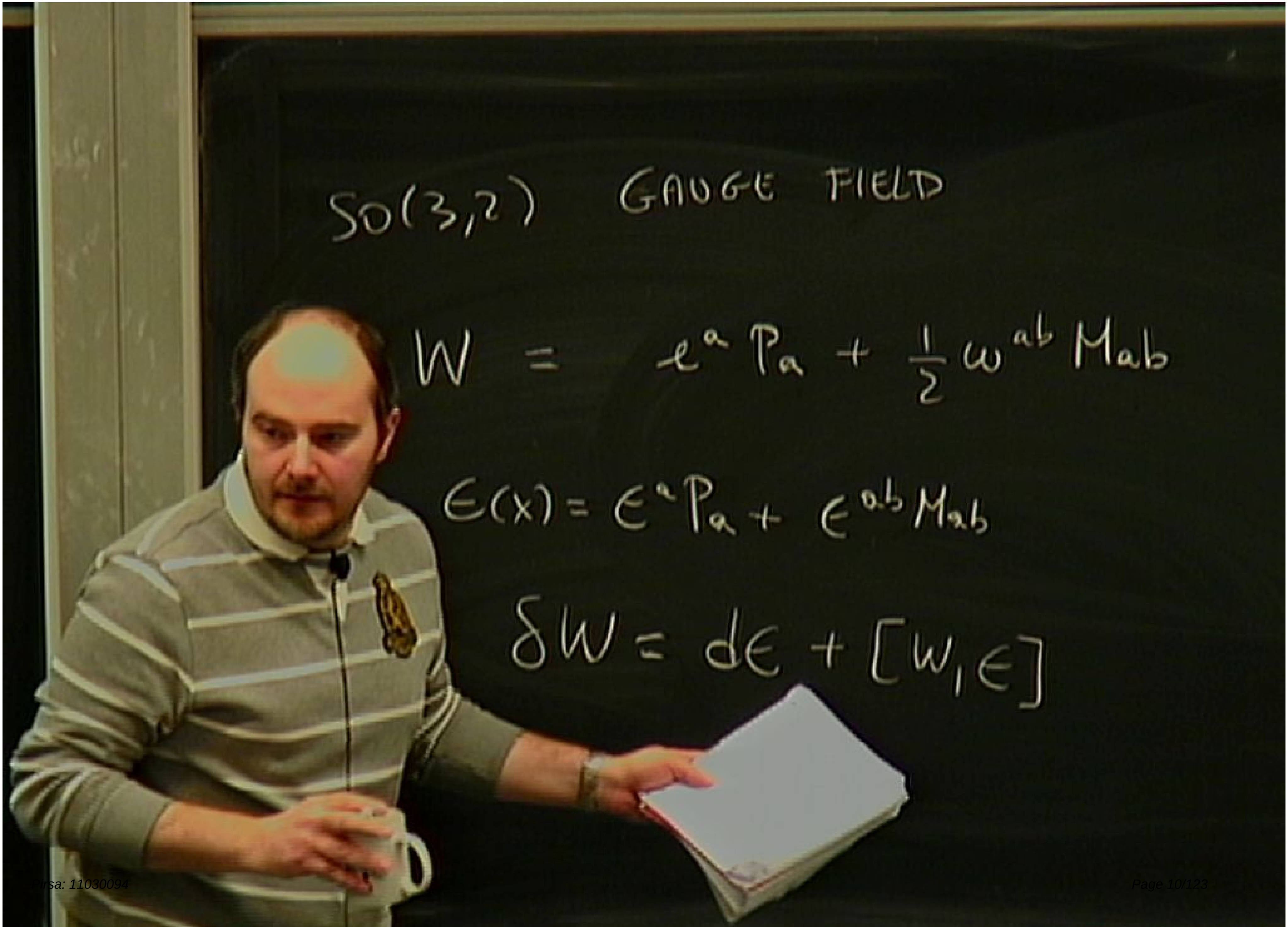
$R^2_{AdS} = 1$


SO(3,2) GAUGE FIELD
$W = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}$


SO(3,2) GAUGE FIELD
$W = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}$
$\epsilon(x) = \epsilon^a P_a + \epsilon^{ab} M_{ab}$
$\delta W = d\epsilon + [W, \epsilon]$

SO(3,2) GAUGE FIELD

$$W = e^a P_a + \frac{1}{2}\omega^{ab} M_{ab}$$

$$\epsilon(x) = \epsilon^a P_a + \epsilon^{ab} M_{ab}$$

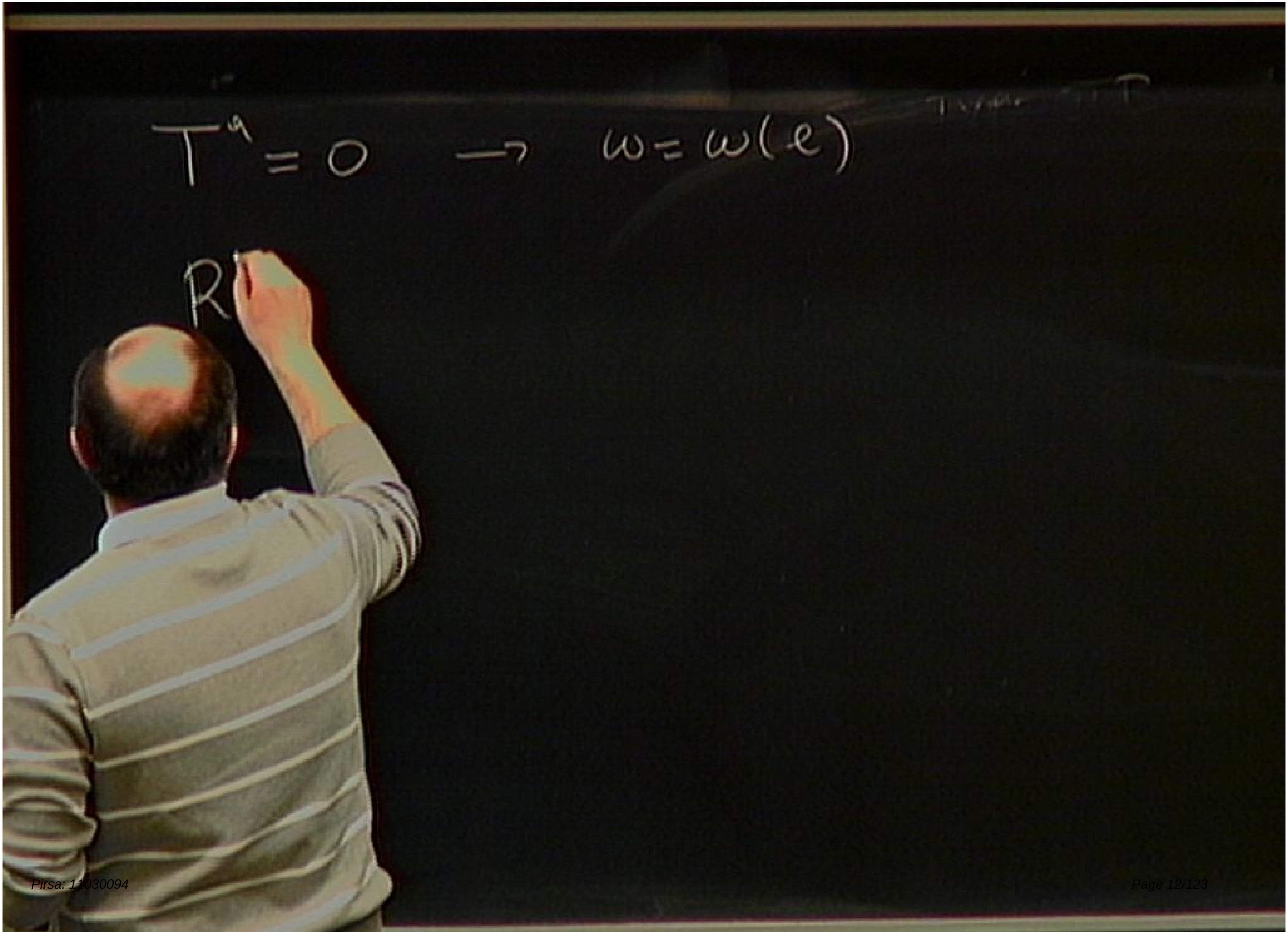
$$\delta W = d\epsilon + [W, \epsilon]$$

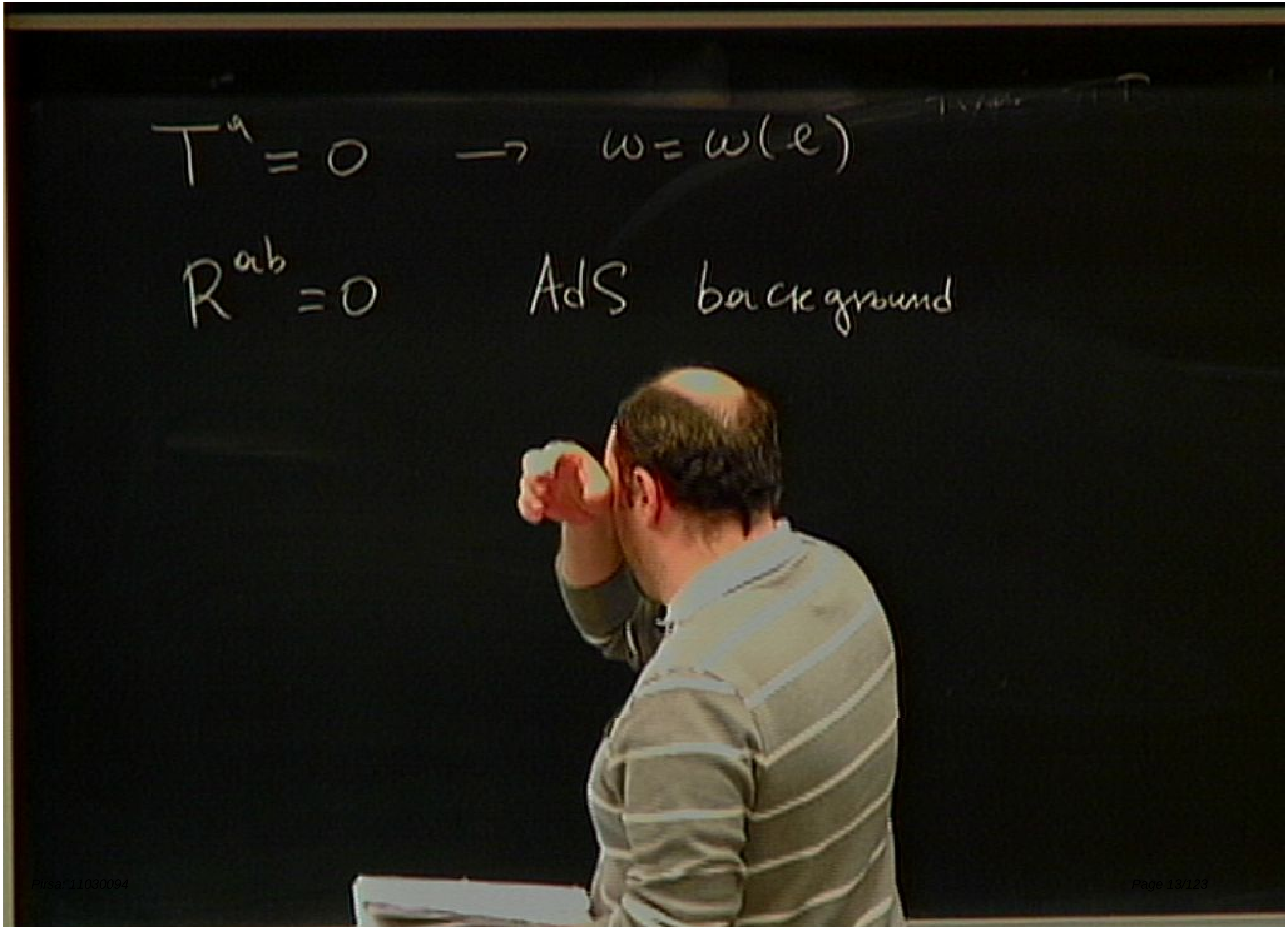
$$R = dW + W \wedge W$$

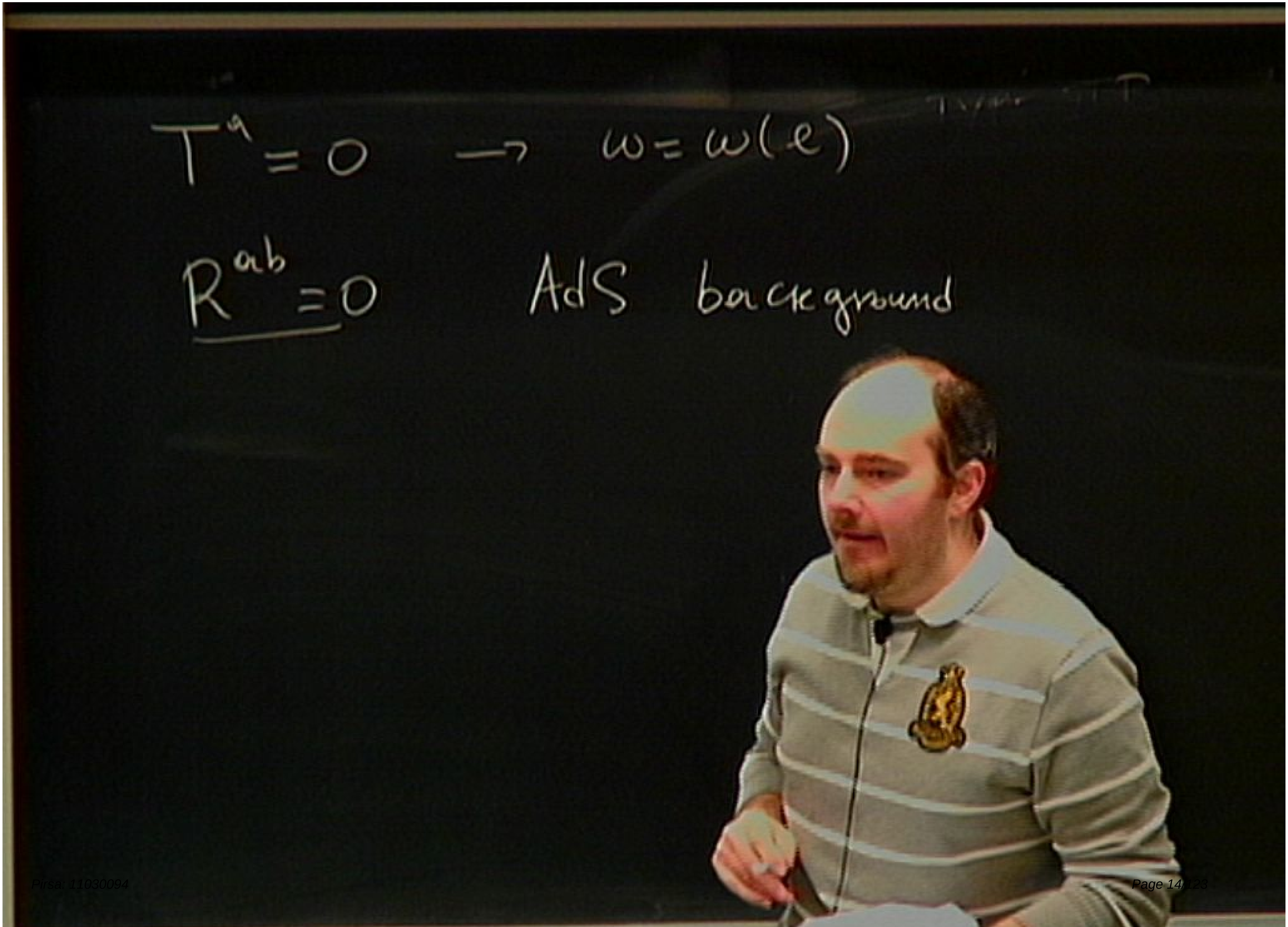
$$\delta R = [R, \epsilon]$$

$$R = \overbrace{(de^a + \omega^a{}_b \wedge e^b)}^{T^a} P_a$$

$$+ \underbrace{(d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} - e^a \wedge e^b)}_{R^{ab}} M_{ab}$$


$T^a = 0 \longrightarrow \omega = \omega(e)$
R


$T^a = 0 \longrightarrow \omega = \omega(e)$
$R^{ab} = 0$ AdS background


$T^a = 0 \longrightarrow \omega = \omega(e)$
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SO(3,2) GAUGE FIELD

$$W = e^a P_a + \frac{1}{2}\omega^{ab} M_{ab}$$

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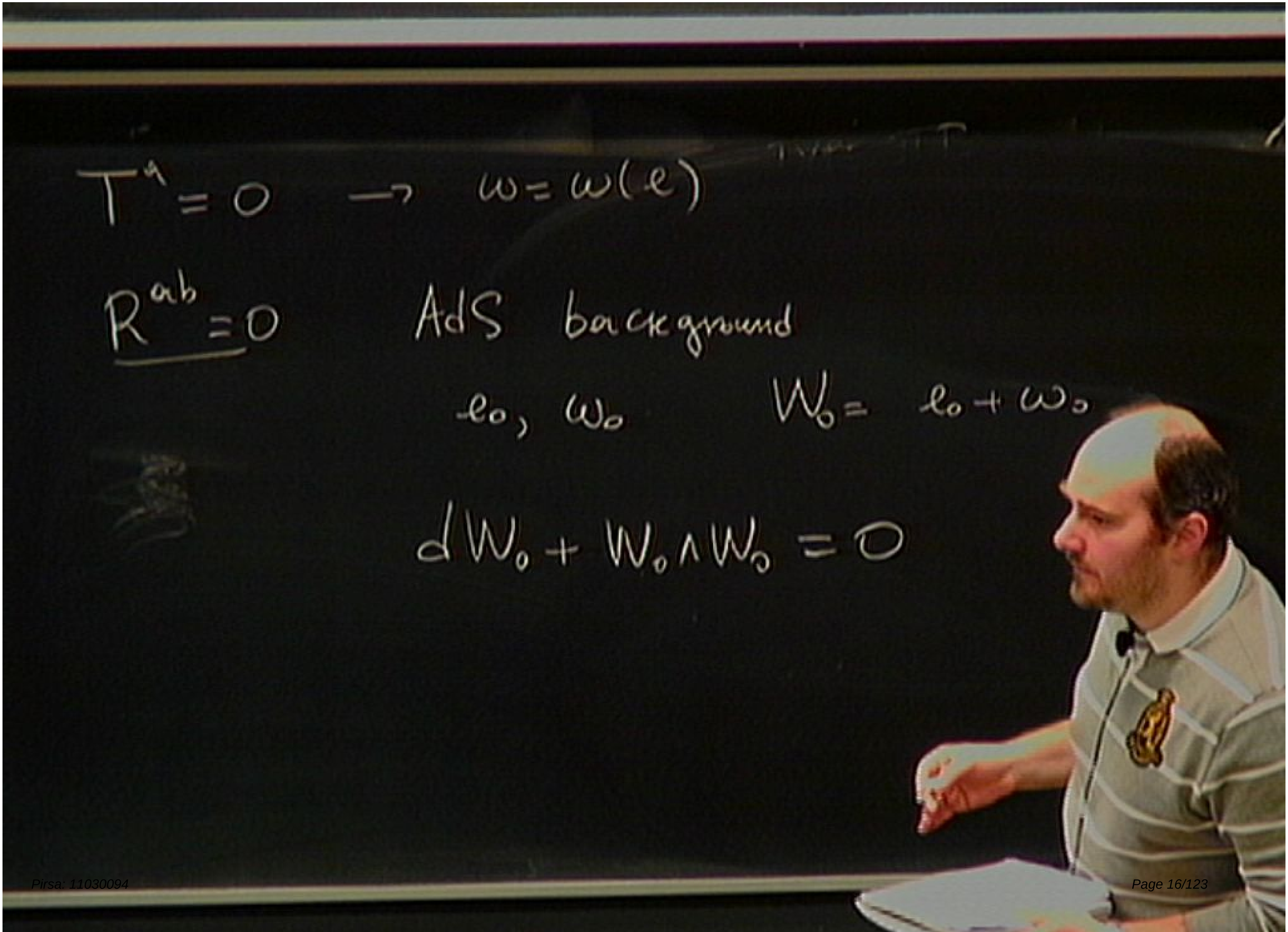
$T^a = 0 \longrightarrow \omega = \omega(e)$
$R^{ab} = 0$
AdS background
e_0, ω_0
$W_0 = e_0 + \omega_0$
$dW_0 + W_0 \wedge W_0 = 0$

$T^a = 0 \longrightarrow \omega = \omega(e)$

$\underline{R^{ab} = 0}$ AdS background

$e_0, \omega_0 \qquad W_0 = e_0 + \omega_0$

$$\underline{dW_0 + W_0 \wedge W_0 = 0} \qquad \text{Ad}\underline{S}$$

2-COMPONENT SPINORS

$$x^\mu \longrightarrow x^{\alpha\dot\alpha} = x^\mu (\sigma^\mu)^{\alpha\dot\alpha}$$

$\alpha = 1,2$ $\dot\alpha = 1,2$

χ^α , $\bar\chi^{\dot\alpha}$

$$\chi^\alpha \psi_\alpha = \epsilon^{\alpha\beta} \chi_\beta \psi_\alpha \quad \text{etc}$$

$\varphi_{\mu_1 \dots}$

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$$\chi^\alpha \,,\ \bar\chi^{\dot\alpha} \qquad \chi^\alpha \psi_\alpha = \epsilon^{\alpha\beta}\chi_\beta \psi_\alpha \quad \text{etc}$$

$$\varphi_{(\mu_1\cdots\mu_s)} \longrightarrow \varphi_{\alpha_1\cdots\alpha_s\dot\alpha_1\cdots\dot\alpha_s}$$

2-COMPONENT SPINORS

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$$\varphi_{(\mu_1 \cdots \mu_s)} \longrightarrow \varphi_{\alpha_1 \cdots \alpha_s \dot{\alpha}_1 \cdots \dot{\alpha}_s}$$

$$F_{[\mu\nu]} \longrightarrow F_{\alpha\beta} \, , \, F_{\dot{\alpha}\dot{\beta}}$$

$$C_{\mu\nu\rho\sigma}$$

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Weyl $\quad C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta} \ , \ C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$$e^a \to e^{\alpha\dot\alpha}$$

$$\omega^{ab} \to \omega^{\alpha\beta} \ , \ \omega^{\dot\alpha\dot\beta}$$

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Weyl $C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta} \,,\, C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

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$$F_{[\mu\nu]} \longrightarrow F_{\alpha\beta} \,,\, F_{\dot\alpha\dot\beta} \qquad \omega^{ab} \to \omega^{\alpha\beta}, \omega^{\dot\alpha\dot\beta}$$

Weyl $C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta} \,,\, C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

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Weyl $C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta}\,, C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$$e^a \to e^{\alpha\dot\alpha} \qquad P_{\alpha\dot\alpha}$$

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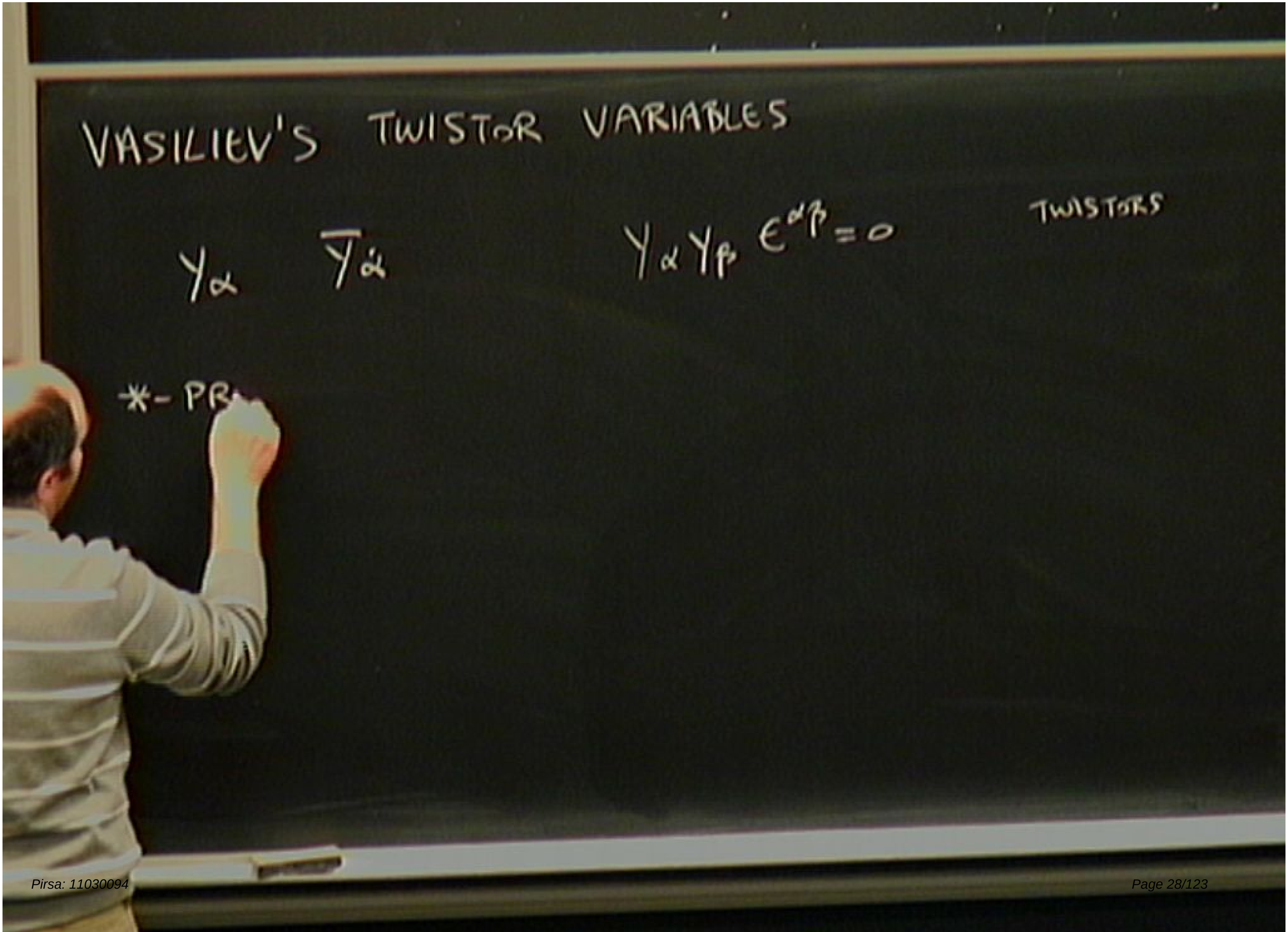
Weyl $\quad C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta}\ ,\ C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$$\text{dir}\,e_\mu^{\ a} \rightarrow \text{dir}\,e_\mu^{\ \alpha\dot\alpha} \qquad P_{\alpha\dot\alpha}$$

$$\omega^{ab} \rightarrow \omega^{\alpha\beta}, \omega^{\dot\alpha\dot\beta} \qquad M_{\alpha\beta}, M_{\dot\alpha\dot\beta}$$


VASILIEV'S TWISTOR VARIABLES
Y_α $\bar{Y}_{\dot\alpha}$
$Y_\alpha Y_\beta \epsilon^{\alpha\beta} = 0$
TWISTORS
*- PR

VASILIEV'S TWISTOR VARIABLES

TWISTORS

$y_\alpha \quad \bar{y}_{\dot\alpha}$

$y_\alpha y_\beta \epsilon^{\alpha\beta} = 0$

<u>*- PRODUCT</u>

VASILIEV'S TWISTOR VARIABLES

$y_\alpha \quad \bar{y}_{\dot\alpha}$ $\qquad y_\alpha y_\beta \epsilon^{\alpha\beta} = 0$ TWISTORS

*- PRODUCT

$$\left[\begin{array}{l} y^\alpha * y^\beta = y^\alpha y^\beta + \epsilon^{\alpha\beta} \\ \bar{y}^{\dot\alpha} * \bar{y}^{\dot\beta} = \bar{y}^{\dot\alpha} \bar{y}^{\dot\beta} + \epsilon^{\dot\alpha\dot\beta} \\ y^\alpha * \bar{y}^{\dot\alpha} = y^\alpha \bar{y}^{\dot\alpha} \end{array}\right.$$

$$[y_\alpha, y_\beta]_* = 2\epsilon_{\alpha\beta}$$

$$[y_\alpha, \bar{y}_{\dot\beta}]_* = 0$$

VASILIEV'S TWISTOR VARIABLES

$y_\alpha \quad \bar{y}_{\dot\alpha}$

$y_\alpha y_\beta \epsilon^{\alpha\beta} = 0$

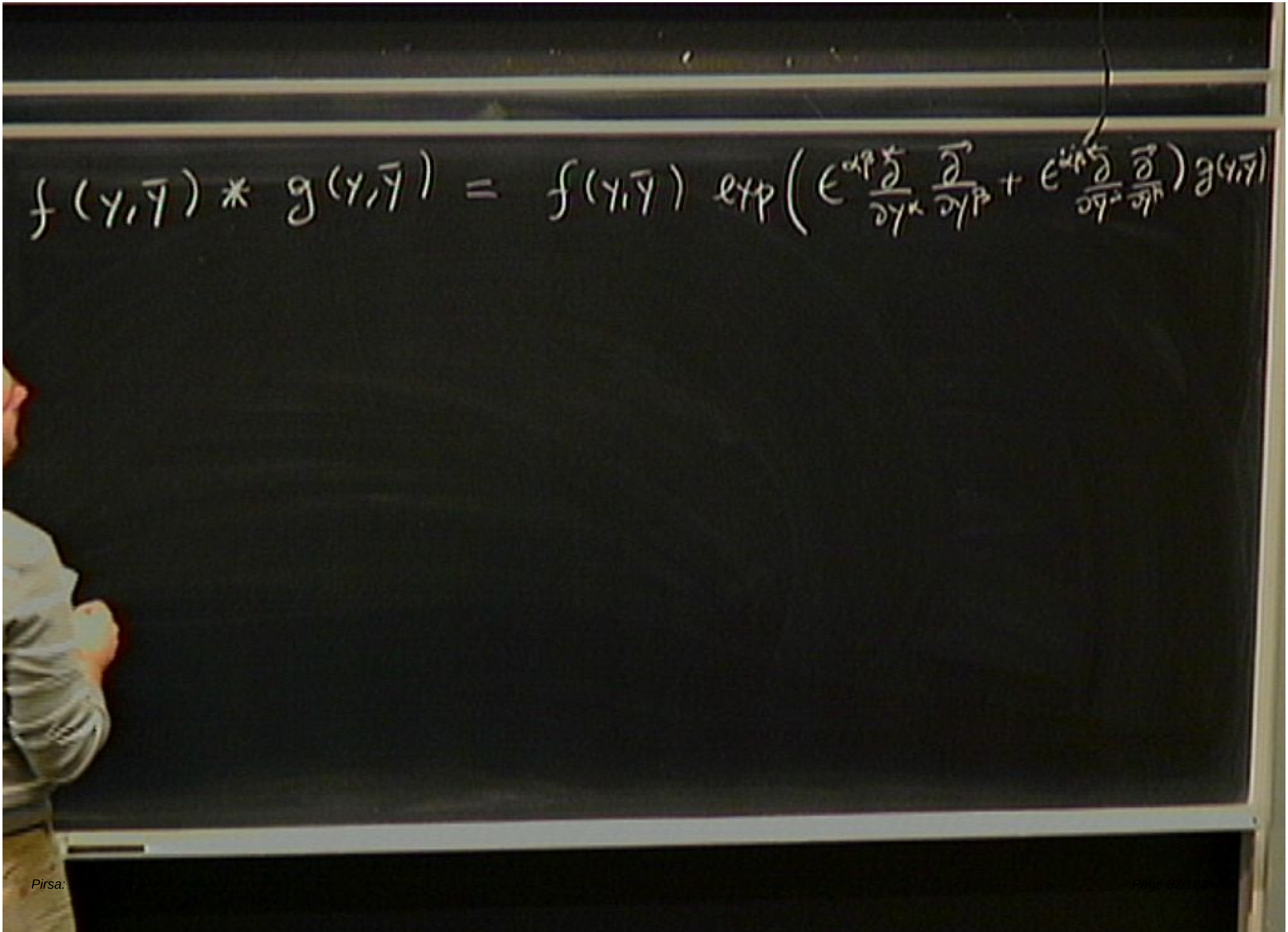
TWISTORS

*- PRODUCT

$$\begin{cases} y^\alpha * y^\beta = y^\alpha y^\beta + \epsilon^{\alpha\beta} \\ \bar{y}^{\dot\alpha} * \bar{y}^{\dot\beta} = \bar{y}^{\dot\alpha} \bar{y}^{\dot\beta} + \epsilon^{\dot\alpha\dot\beta} \\ y^\alpha * \bar{y}^{\dot\alpha} = y^\alpha \bar{y}^{\dot\alpha} \end{cases}$$

$$[y_\alpha, y_\beta]_* = 2\epsilon_{\alpha\beta}$$

$$[y_\alpha, \bar{y}_{\dot\beta}]_* = 0$$


$f(y,\bar{y}) * g(y,\bar{y}) = f(y,\bar{y})\, \exp\left(\epsilon^{\alpha\beta} \frac{\overleftarrow{\partial}}{\partial y^\alpha} \frac{\overrightarrow{\partial}}{\partial y^\beta} + \epsilon^{\dot\alpha\dot\beta} \frac{\overleftarrow{\partial}}{\partial \bar{y}^{\dot\alpha}} \frac{\overrightarrow{\partial}}{\partial \bar{y}^{\dot\beta}} \right) g(y,\bar{y})$

$$f(y,\bar{y}) * g(y,\bar{y}) = f(y,\bar{y}) \exp\left(\epsilon^{\alpha\beta} \frac{\overleftarrow{\partial}}{\partial y^{\alpha}} \frac{\overrightarrow{\partial}}{\partial y^{\beta}} + \epsilon^{\dot{\alpha}\dot{\beta}} \frac{\overleftarrow{\partial}}{\partial \bar{y}^{\dot{\alpha}}} \frac{\overrightarrow{\partial}}{\partial \bar{y}^{\dot{\beta}}}\right) g(y,\bar{y})$$

$$f(y,\bar{y}) * g(y,\bar{y}) = \int d^2u \, d^2\bar{u} \, d^2v \, d^2\bar{v} \, e^{u^{\alpha} v_{\alpha} + \bar{u}^{\dot{\alpha}} \bar{v}_{\dot{\alpha}}} f(y+u, \bar{y}+\bar{u}) \, g(y+v, \bar{y}+\bar{v})$$

$$f(y,\bar{y}) * g(y,\bar{y}) = f(y,\bar{y}) \exp\left(\epsilon^{\alpha\beta} \frac{\overleftarrow{\partial}}{\partial y^\alpha} \frac{\overrightarrow{\partial}}{\partial y^\beta} + \epsilon^{\dot\alpha\dot\beta} \frac{\overleftarrow{\partial}}{\partial \bar{y}^{\dot\alpha}} \frac{\overrightarrow{\partial}}{\partial \bar{y}^{\dot\beta}} \right) g(y,\bar{y})$$

$$f(y,\bar{y}) * g(y,\bar{y}) = \int d^2u\, d^2\bar{u}\, d^2v\, d^2\bar{v}\; e^{u^\alpha v_\alpha + \bar{u}^{\dot\alpha} \bar{v}_{\dot\alpha}}\, f(y+u, \bar{y}+\bar{u})\, g(y+v, \bar{y}+\bar{v})$$

$$\int d^2u\, e^{u^\alpha (v_\alpha - v'_\alpha)} = \delta(v_\alpha - v'_\alpha)$$

$$f(y,\bar{y}) * g(y,\bar{y}) = f(y,\bar{y})\, \exp\left(\epsilon^{\alpha\beta} \frac{\overleftarrow{\partial}}{\partial y^\alpha} \frac{\overrightarrow{\partial}}{\partial y^\beta} + \epsilon^{\dot\alpha\dot\beta} \frac{\overleftarrow{\partial}}{\partial \bar{y}^{\dot\alpha}} \frac{\overrightarrow{\partial}}{\partial \bar{y}^{\dot\beta}} \right) g(y,\bar{y})$$

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$SO(3,2) \qquad [\ ,\ ] \to [\ ,\ ]_*$

$$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar{y}_{\dot\alpha}$$

$$M_{\alpha\beta} = \frac{1}{2} y_\alpha y_\beta \qquad M_{\dot\alpha\dot\beta} = \frac{1}{2} \bar{y}_{\dot\alpha} \bar{y}_{\dot\beta}$$

$SO(3,2)$ $\qquad [\ ,\ ] \to [\ ,\ ]_*$

$$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar{y}_{\dot\alpha}$$

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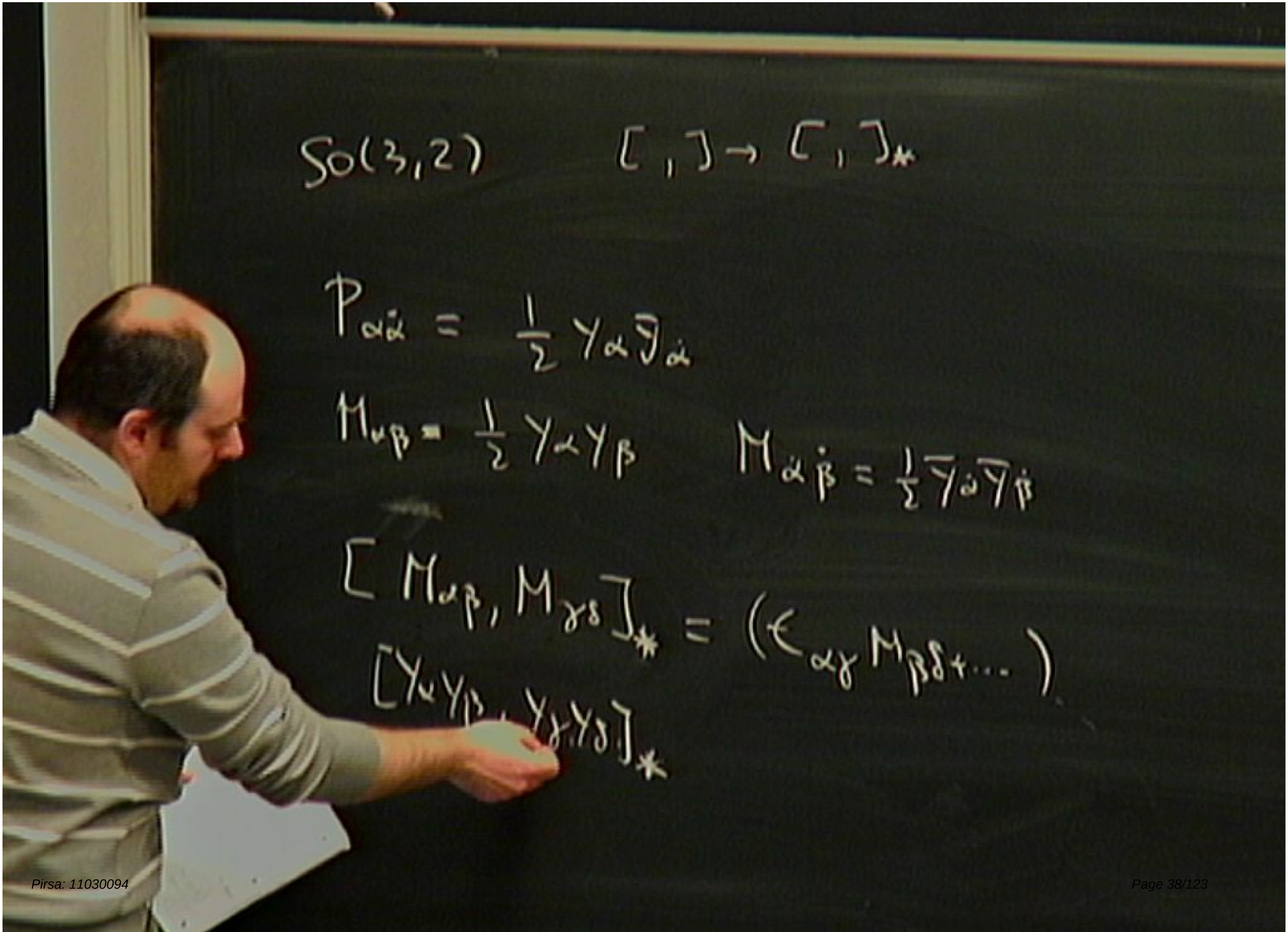
$$[M_{\alpha\beta}, M_{\gamma\delta}]_* = (\epsilon_{\alpha\gamma} M_{\beta\delta} + \dots)$$


$SO(3,2)$ $[\ ,\ ] \to [\ ,\ ]_*$
$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar{y}_{\dot\alpha}$
$M_{\alpha\beta} = \frac{1}{2} y_\alpha y_\beta$ $M_{\dot\alpha\dot\beta} = \frac{1}{2} \bar{y}_{\dot\alpha} \bar{y}_{\dot\beta}$
$[M_{\alpha\beta}, M_{\gamma\delta}]_* = (\epsilon_{\alpha\gamma} M_{\beta\delta} + \dots)$
$[y_\alpha y_\beta, y_\gamma y_\delta]_*$

$SO(3,2) \qquad [\ ,\ ] \to [\ ,\ ]_*$

$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar{y}_{\dot\alpha}$

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$SO(3,2)$ $\quad [\,,\,] \to [\,,\,]_*$

$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar{y}_{\dot\alpha}$

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$[M_{\alpha\beta}, M_{\gamma\delta}]_* = (\epsilon_{\alpha\gamma} M_{\beta\delta} + \ldots)$

$[y_\alpha y_\beta, y_\gamma y_\delta]_*$

$SO(3,2) \qquad [\ ,\ ] \to [\ ,\ ]_*$

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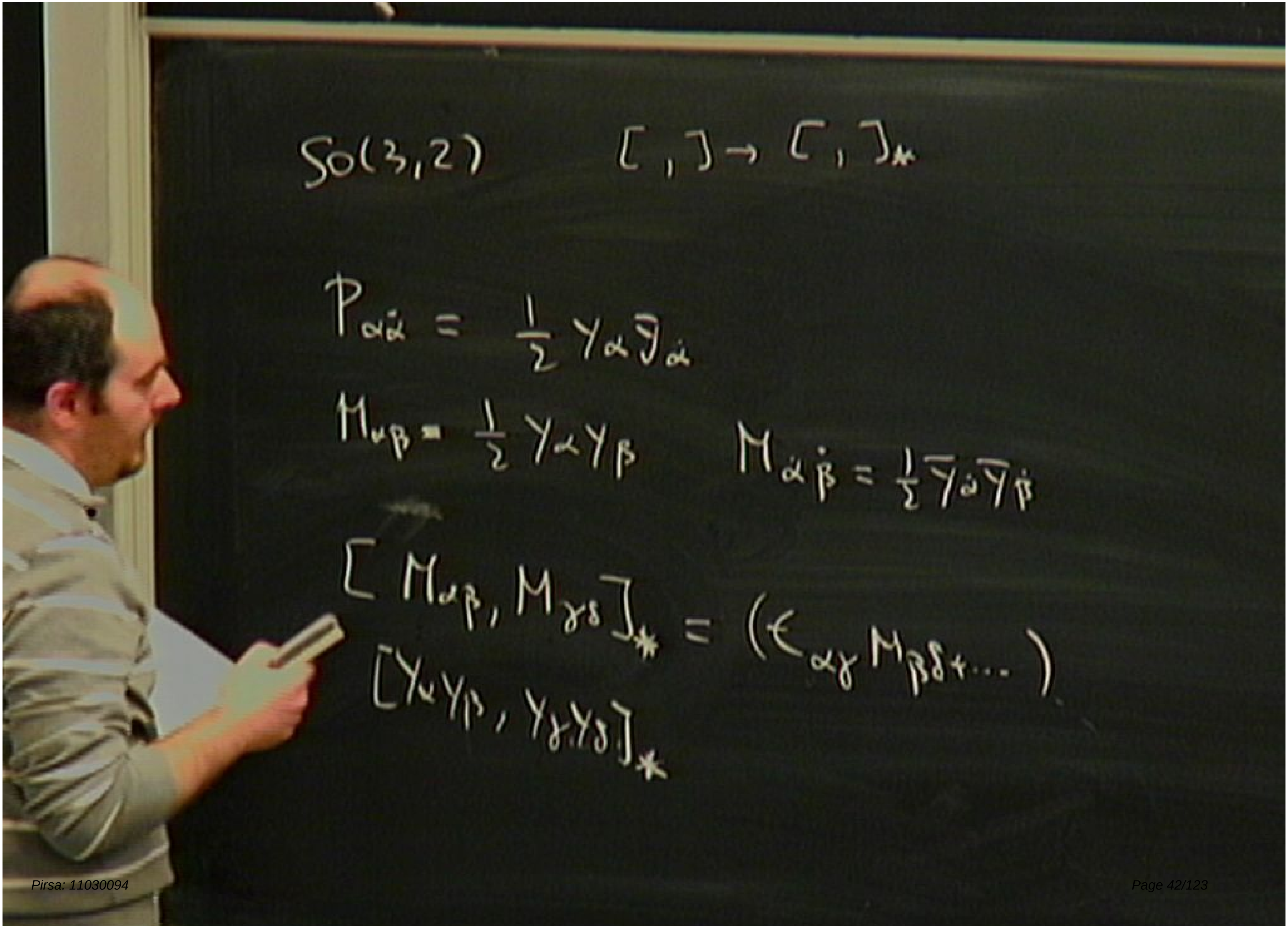
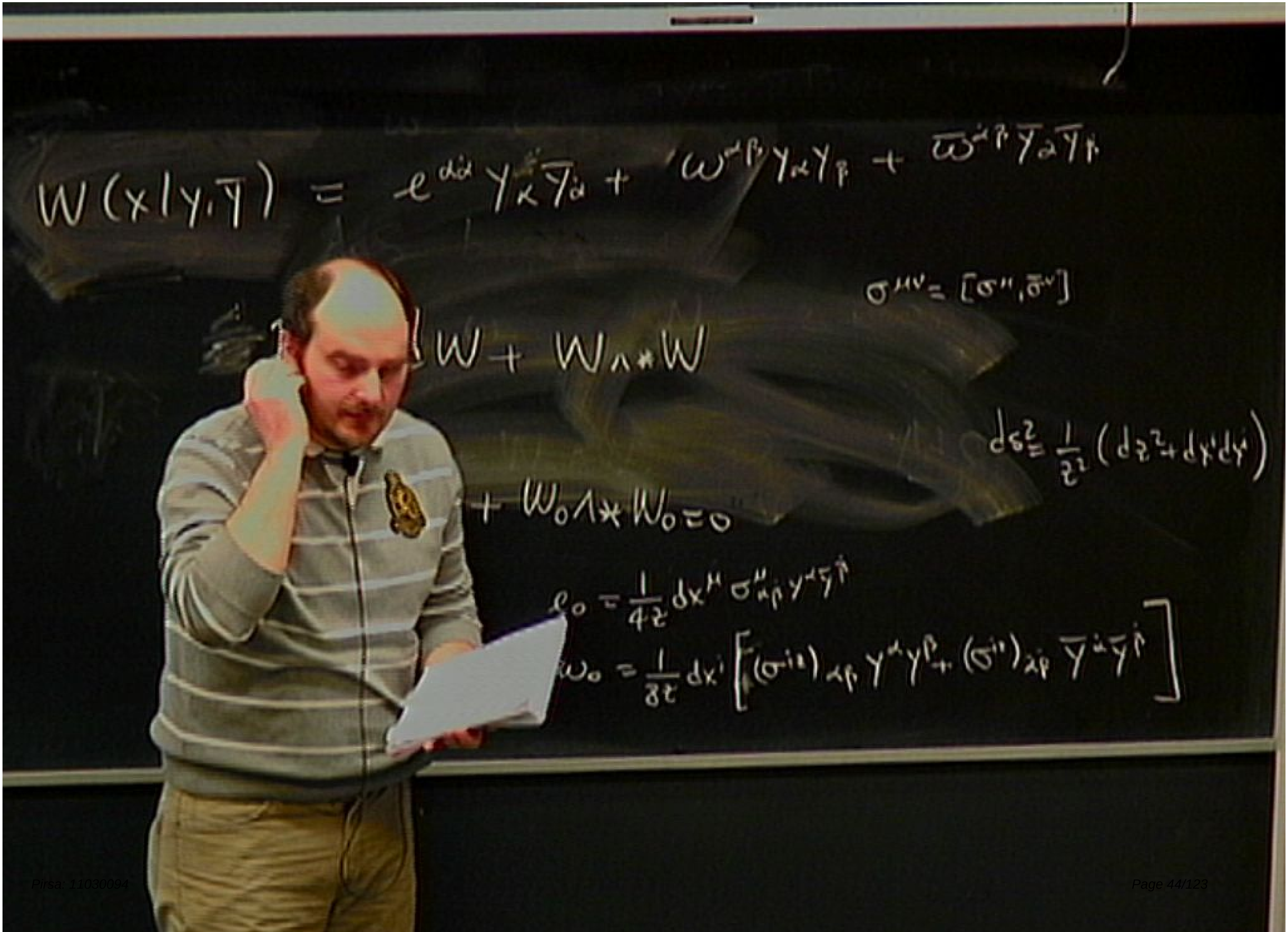
$$[M_{\alpha\beta}, M_{\gamma\delta}]_* = (\epsilon_{\alpha\gamma} M_{\beta\delta} + \dots)$$

$$[y_\alpha y_\beta, y_\gamma y_\delta]_*$$


$SO(3,2)$ $[\ ,\ ] \to [\ ,\ ]_*$
$P_{\alpha\dot\alpha} = \frac{1}{2} y_\alpha \bar\partial_{\dot\alpha}$
$M_{\alpha\beta} = \frac{1}{2} y_\alpha y_\beta$ $M_{\dot\alpha\dot\beta} = \frac{1}{2} \bar y_{\dot\alpha} \bar y_{\dot\beta}$
$[M_{\alpha\beta}, M_{\gamma\delta}]_* = (\epsilon_{\alpha\gamma} M_{\beta\delta} + \dots)$
$[y_\alpha y_\beta, y_\gamma y_\delta]_*$

$$W(x|y,\bar{y}) = e^{\alpha\dot{\alpha}}\, y_{\alpha}\bar{y}_{\dot{\alpha}} + \omega^{\alpha\beta} y_{\alpha} y_{\beta} + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}$$


$W(x|y,\bar{y}) = e^{\alpha\dot{\alpha}} y_\alpha \bar{y}_{\dot{\alpha}} + \omega^{\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}$
$\sigma^{\mu\nu} = [\sigma^\mu, \bar{\sigma}^\nu]$
$W + W \wedge * W$
$ds^2 = \frac{1}{z^2}(dz^2 + dx^i dx^i)$
$+ W_0 \wedge * W_0 = 0$
$e_0 = \frac{1}{4z} dx^\mu \sigma^\mu_{\alpha\dot{\beta}} y^\alpha \bar{y}^{\dot{\beta}}$
$\omega_0 = \frac{1}{8z} dx^i \left[(\sigma^{iz})_{\alpha\beta} y^\alpha y^\beta + (\bar{\sigma}^{iz})_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}} \right]$

$$W(x|y,\bar{y}) = e^{\alpha\dot{\alpha}} y_\alpha \bar{y}_{\dot{\alpha}} + \omega^{\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}$$

$$\sigma^{\mu\nu} = [\sigma^\mu, \bar{\sigma}^\nu]$$

$$R = dW + W \wedge * W$$

$$ds^2 = \frac{1}{z^2}\left(dz^2 + dx^i dx^i\right)$$

AdS $\quad dW_0 + W_0 \wedge * W_0 = 0$

$$W_0 = e_0 + \omega_0$$

$$e_0 = \frac{1}{4z} dx^\mu \sigma^\mu_{\alpha\dot{\beta}} y^\alpha \bar{y}^{\dot{\beta}}$$

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$$W(x|y,\bar{y}) = e^{\alpha\dot{\alpha}} y_\alpha \bar{y}_{\dot{\alpha}} + \omega^{\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}$$

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$$W_0 = e_0 + \omega_0 \qquad e_0 = \frac{1}{4z} dx^\mu \sigma^\mu_{\alpha\dot{\beta}} y^\alpha \bar{y}^{\dot{\beta}}$$

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$$\sigma^{\mu\nu} = [\sigma^\mu, \bar{\sigma}^\nu]$$

$$R = dW + W \wedge * W$$

$$\sigma^\mu = (1, \vec{\sigma}) \qquad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

$$ds^2 = \frac{1}{z^2}\left(dz^2 + dx^i dx^i\right)$$

AdS $\quad dW_0 + W_0 \wedge * W_0 = 0$

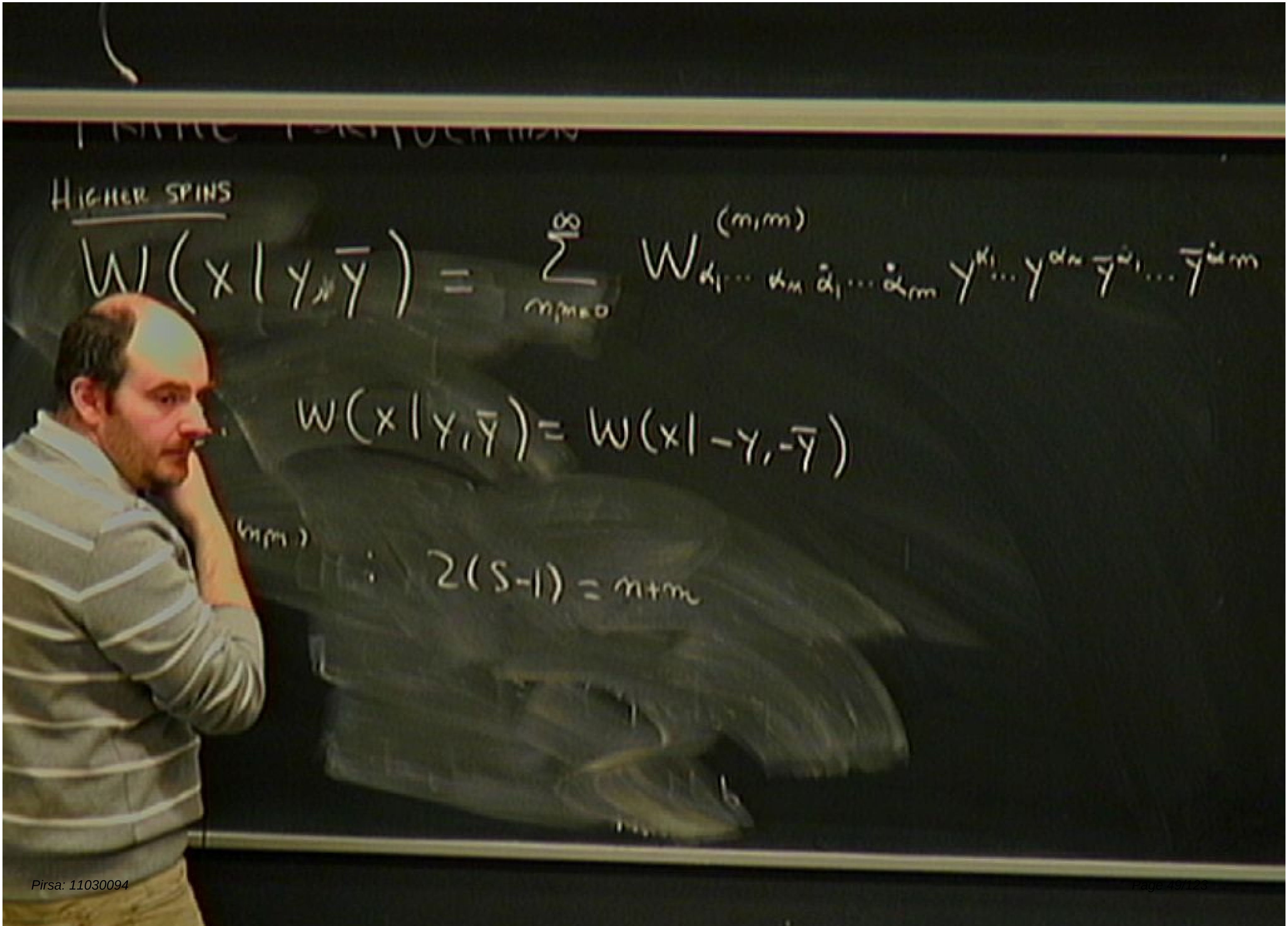
$$W_0 = e_0 + \omega_0 \qquad e_0 = \frac{1}{4z} dx^\mu \sigma^\mu_{\alpha\dot{\beta}} y^\alpha \bar{y}^{\dot{\beta}}$$

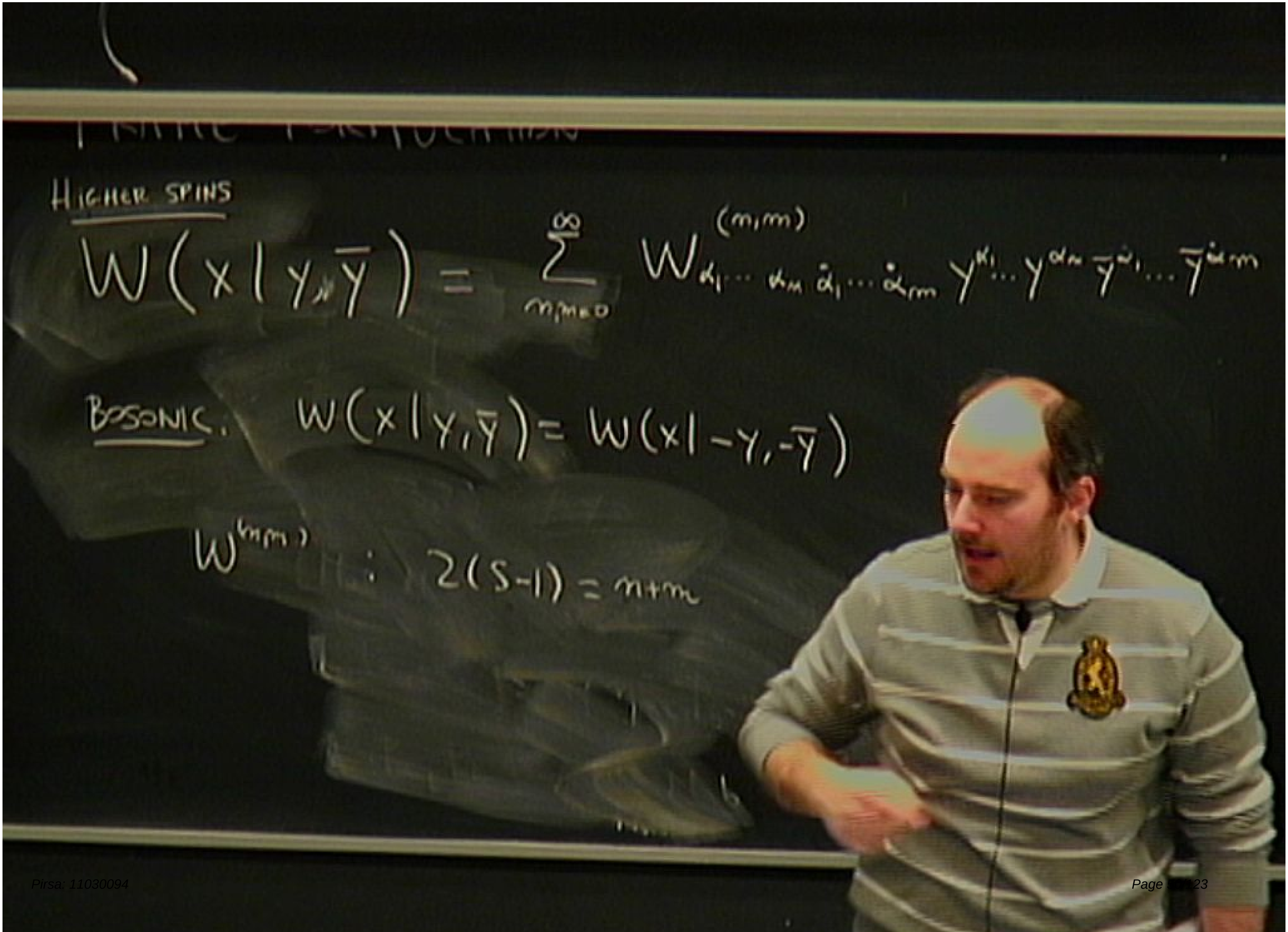
$$\omega_0 = \frac{1}{8z} dx^i \left[(\sigma^{iz})_{\alpha\beta} y^\alpha y^\beta + (\bar{\sigma}^{iz})_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}} \right]$$

HIGHER SPINS

$$W(x\,|\,y,\bar{y}) = \sum_{m,n=0}^{\infty} W^{(n,m)}_{\alpha_1\cdots\alpha_n\,\dot{\alpha}_1\cdots\dot{\alpha}_m}\, y^{\alpha_1}\cdots y^{\alpha_n}\,\bar{y}^{\dot{\alpha}_1}\cdots\bar{y}^{\dot{\alpha}_m}$$

BOSONIC: $W(x\,|\,y,\bar{y}) = W(x\,|\,-y,-\bar{y})$


HIGHER SPINS
$W(x|y,\bar{y}) = \sum_{n,m=0}^{\infty} W^{(n,m)}_{\alpha_1\cdots\alpha_n\dot{\alpha}_1\cdots\dot{\alpha}_m} y^{\alpha_1}\cdots y^{\alpha_n}\bar{y}^{\dot{\alpha}_1}\cdots\bar{y}^{\dot{\alpha}_m}$
$W(x|y,\bar{y}) = W(x|-y,-\bar{y})$
$2(s-1) = n+m$


HIGHER SPINS
$W(x|y,\bar{y}) = \sum_{n,m=0}^{\infty} W^{(n,m)}_{\alpha_1\cdots\alpha_n\dot{\alpha}_1\cdots\dot{\alpha}_m} y^{\alpha_1}\cdots y^{\alpha_n}\bar{y}^{\dot{\alpha}_1}\cdots\bar{y}^{\dot{\alpha}_m}$
BOSONIC:
$W(x|y,\bar{y}) = W(x|-y,-\bar{y})$
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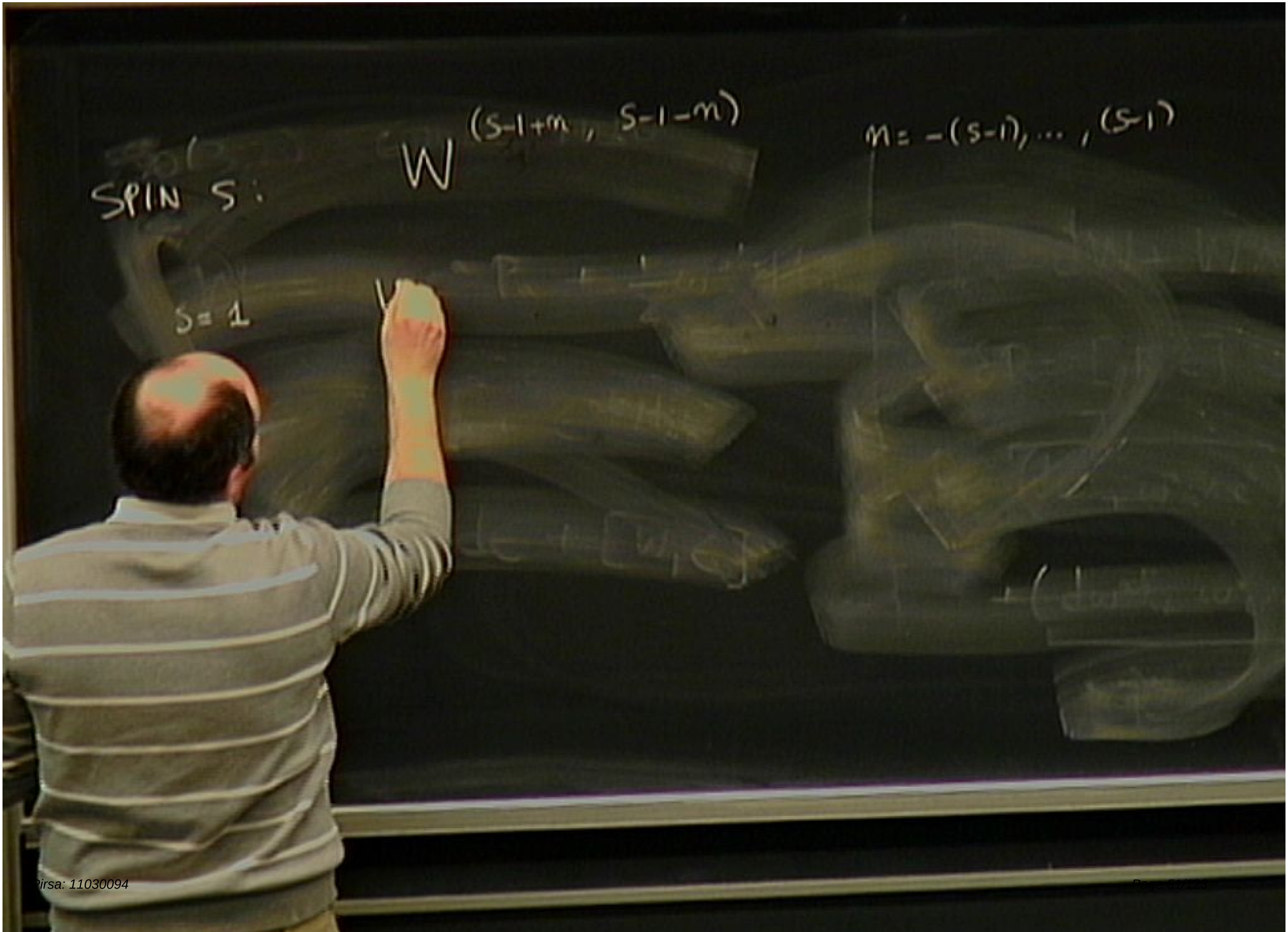

SPIN S: $W^{(s-1+m,\ s-1-m)}$
$m = -(s-1), \dots, (s-1)$
$s = 1$

SPIN S: $W^{(S-1+m,\ S-1-m)}$ $\quad m = -(S-1), \dots, (S-1)$

$S=1$ $\quad W^{(0,0)} \to A_\mu$

$S=2$ $\quad \underset{e^{\alpha\dot\alpha}}{W^{(1,1)}} \quad \underset{\omega^{\alpha\beta}}{W^{(2,0)}} \quad \underset{\omega^{\dot\alpha\dot\beta}}{W^{(0,2)}}$

$S=3$ $\quad W^{(2,2)} \quad \underbrace{W^{(3,1)} \quad W^{(4,0)} \quad W^{(0,4)}}_{\text{"}\omega\text{"}} \quad W^{(1,3)}$

$W^{(2,2)} \to e^{\alpha_1\alpha_2\dot\alpha_1\dot\alpha_2}$

SPIN S: $W^{(S-1+n,\ S-1-n)}$ $\qquad n = -(S-1), \dots, (S-1)$

$S=1$: $\quad W^{(0,0)} \to A_\mu$

$S=2$: $\quad \underset{e^{\alpha\dot\alpha}}{W^{(1,1)}} \quad \underset{\omega^{\alpha\beta}}{W^{(2,0)}} \quad \underset{\omega^{\dot\alpha\dot\beta}}{W^{(0,2)}}$

$S=3$: $\quad \underset{\displaystyle\downarrow\ e^{\alpha_1\alpha_2\dot\alpha_1\dot\alpha_2}}{W^{(2,2)}} \quad \underbrace{W^{(3,1)} \quad W^{(4,0)} \quad W^{(0,4)} \quad W^{(1,3)}}_{\text{“}\omega\text{”}}$

$W^{(s-1,\,s-1)}$ $\quad e^{\alpha_1 \cdots \alpha_{s-1}\,\dot{\alpha}_1 \cdots \dot{\alpha}_{s-1}}$ $\quad e_{\mu}^{\overbrace{a_1 \cdots a_s}^{\text{symm. traceless}}}$

$W^{(s-1,\,s-1)}$ $\quad e^{\alpha_1 \cdots \alpha_{s-1}\ \dot{\alpha}_1 \cdots \dot{\alpha}_{s-1}}$ $\quad e_\mu^{\overbrace{a_1 \cdots a_s}^{\text{symm. traceless}}}$

$\Downarrow$

$e_{(\mu\, a_1 \cdots a_s)} \rightarrow \varphi_{\mu a_1 \cdots a_s}$

DOUBLE TRACELESS

SPIN s: $W^{(s-1+n,\ s-1-n)}$ $\quad n = -(s-1), \dots, (s-1)$

$s=1$ $\quad W^{(0,0)} \to A_\mu$

$s=2$ $\quad W^{(1,1)} \quad W^{(2,0)} \quad W^{(0,2)}$

$e^{\alpha\dot{\alpha}} \quad \omega^{\alpha\beta} \quad \omega^{\dot{\alpha}\beta}$

$s=3$ $\quad W^{(2,2)} \quad \underbrace{W^{(3,1)} \quad W^{(4,0)} \quad W^{(0,4)} \quad W^{(1,3)}}_{\text{"}\omega\text{"}}$

$W^{(2,2)} \to e^{\alpha_1\alpha_2\dot{\alpha}_1\dot{\alpha}_2}$

HIGHER SPINS

$$\text{dyr} \, W_\mu(x \mid y, \bar{y}) = \sum_{n,m=0}^{\infty} W^{(n,m)}_{\alpha_1 \cdots \alpha_n \dot{\alpha}_1 \cdots \dot{\alpha}_m} y^{\alpha_1} \cdots y^{\alpha_n} \bar{y}^{\dot{\alpha}_1} \cdots \bar{y}^{\dot{\alpha}_m}$$

BOSONIC: $W(x \mid y, \bar{y}) = W(x \mid -y, -\bar{y})$

$W^{(n,m)}$: $2(s-1) = n+m$

$W^{(s-1,s-1)}$ $\qquad e^{\alpha_1 \dots \alpha_{s-1}\ \dot\alpha_1 \dots \dot\alpha_{s-1}}$ $\qquad e_\mu^{\overbrace{a_1 \dots a_s}^{\text{symm. traceless}}}$

$\Downarrow$

$e_{(\mu\, a_1 \dots a_s)} \rightarrow \varphi_{\mu a_1 \dots a_s}$

DOUBLE TRACELESS

HS ALGEBRA

$$[\,P_1(y,\bar y),\ P_2(y,\bar y)\,]_*$$

$W^{(s-1,s-1)}$ $\qquad e^{\alpha_1\cdots\alpha_{s-1}\ \dot\alpha_1\cdots\dot\alpha_{s-1}}$ $\qquad e_\mu^{\overbrace{a_1\cdots a_s}^{\text{symm. traceless}}}$

$\Updownarrow$

$e_{(\mu\, a_1\cdots a_s)} \longrightarrow \varphi_{\mu a_1\cdots a_s}$

DOUBLE TRACELESS

HS ALGEBRA

$$[\,P_1(y,\bar y),\ P_2(y,\bar y)\,]_*$$

$W^{(s-1,s-1)}$ $\qquad e^{\alpha_1\ldots\alpha_{s-1}\,\dot\alpha_1\ldots\dot\alpha_{s-1}}$

symm. traceless

$e_\mu^{\;\underbrace{a_1\ldots a_s}}$

$\Updownarrow$

$e_{(\mu a_1\ldots a_s)} \to \varphi_{\mu a_1\ldots a_s}$

DOUBLE TRACELESS

HS ALGEBRA

$$[\,P_1(y,\bar y),\ P_2(y,\bar y)\,]_*$$

$$\delta W = d\epsilon(x|y,\bar y) + [W,\epsilon(x|y,\bar y)]_*$$

$$\delta W = d\epsilon + [W_0,\epsilon]_* = D_0\epsilon$$

$$\ldots W_0 + W_0\wedge_* W_0 = 0$$

$$W_0 = e_0 + \omega_0 \qquad e_0 = \frac{1}{4z}dx^\mu\,\sigma^\mu_{\alpha\dot\beta}\,y^\alpha\bar y^{\dot\beta}$$

$$\omega_0 = \frac{1}{8z}dx^i\left[(\sigma^{ij})_{\alpha\beta}\,y^\alpha y^\beta + (\bar\sigma^{ij})_{\dot\alpha\dot\beta}\,\bar y^{\dot\alpha}\bar y^{\dot\beta}\right]$$

symm. traceless

$$W^{(s-1,s-1)} \quad e^{\alpha_1\cdots\alpha_{s-1}\,\dot\alpha_1\cdots\dot\alpha_{s-1}} \qquad e_\mu^{\overbrace{a_1\cdots a_s}}$$

$$e_{(\mu\, a_1\cdots a_s)} \rightarrow \varphi_{\mu a_1\cdots a_s}$$

DOUBLE TRACELESS

HS ALGEBRA

$$[\, P_1(y,\bar y),\ P_2(y,\bar y)\,]_*$$

$$\delta W = d\epsilon(x|y,\bar y) + [W, \epsilon(x|y,\bar y)]_*$$

$$\delta W = d\epsilon + [W_0, \epsilon]_* = D_0\epsilon$$

$$\delta\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)}$$

$$W_0 = e_0 + \omega_0 \qquad e_0 = \frac{1}{4i} dx^\mu \sigma^\mu_{\alpha\beta} y^\alpha \bar y^\beta$$

$$\omega_0 = \frac{1}{8i} dx^i \left[(\sigma^{ik})_{\alpha\beta} y^\alpha y^\beta + (\bar\sigma^{ik})_{\dot\alpha\dot\beta} \bar y^{\dot\alpha} \bar y^{\dot\beta} \right]$$

$$W^{(s-1,\,s-1)} \qquad e^{\alpha_1\cdots\alpha_{s-1}\,\dot\alpha_1\cdots\dot\alpha_{s-1}} \qquad e_\mu^{\overbrace{a_1\cdots a_s}^{\text{symm. traceless}}}$$

$$\Updownarrow$$

$$e_{(\mu\, a_1\cdots a_s)} \longrightarrow \varphi_{\mu a_1\cdots a_s}$$

DOUBLE TRACELESS

HS ALGEBRA

$$[\,P_1(y,\bar y),\ P_2(y,\bar y)\,]_*$$

$$\delta W = d\epsilon(x|y,\bar y) + [W,\ \epsilon(x|y,\bar y)]_*$$

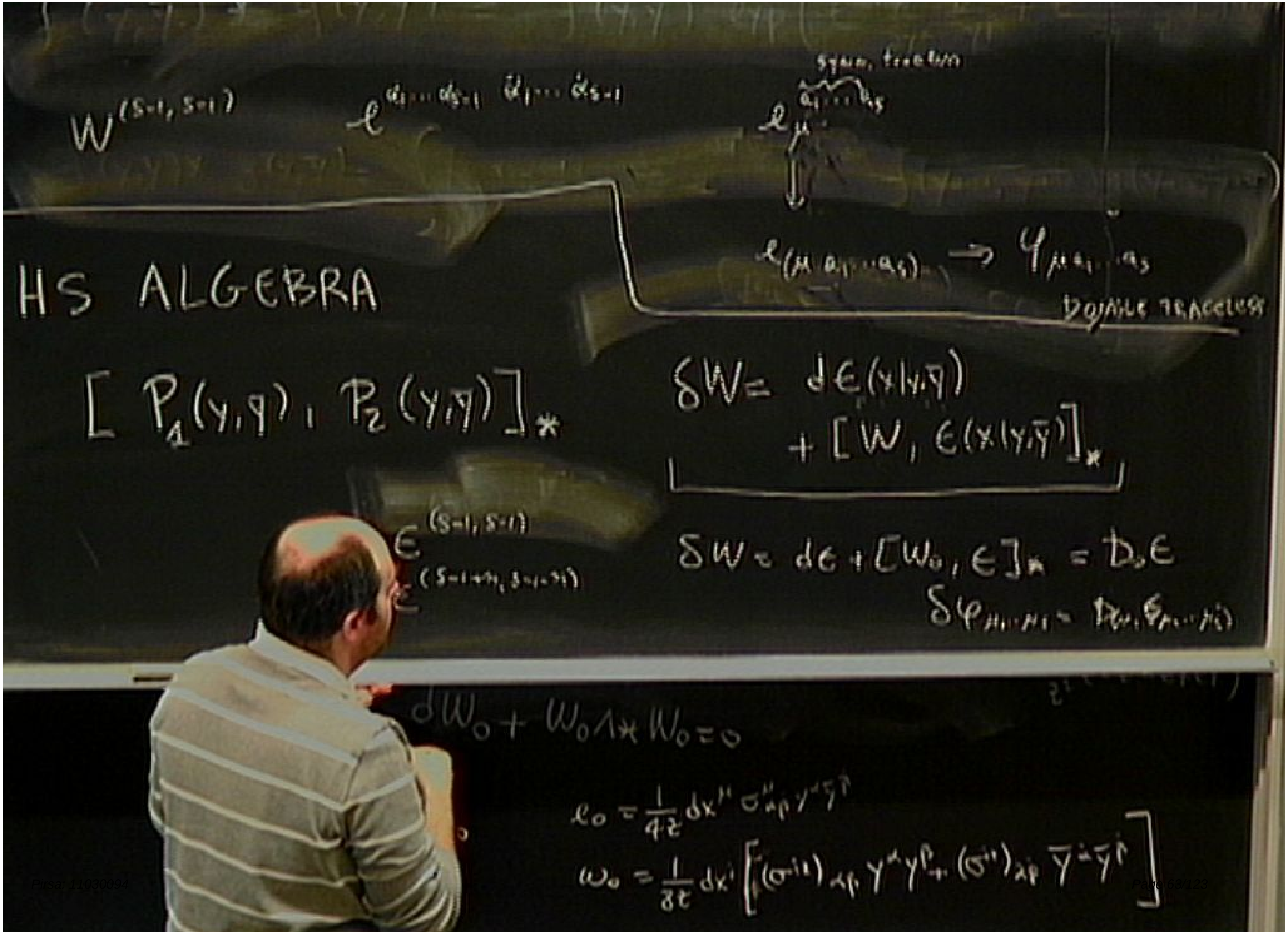
$$\delta W = d\epsilon + [W_0, \epsilon]_* = D_0\epsilon$$

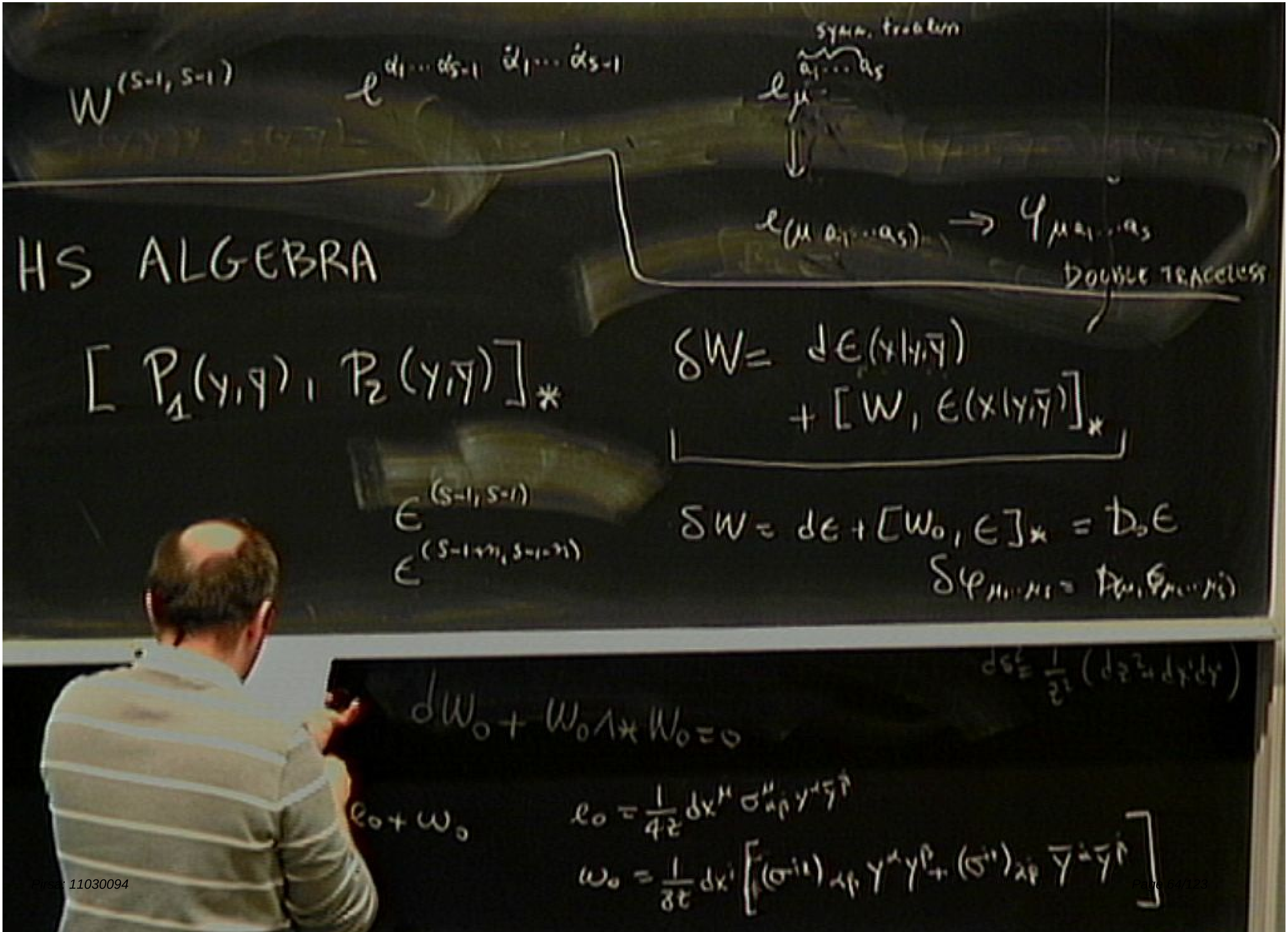
$$\delta\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

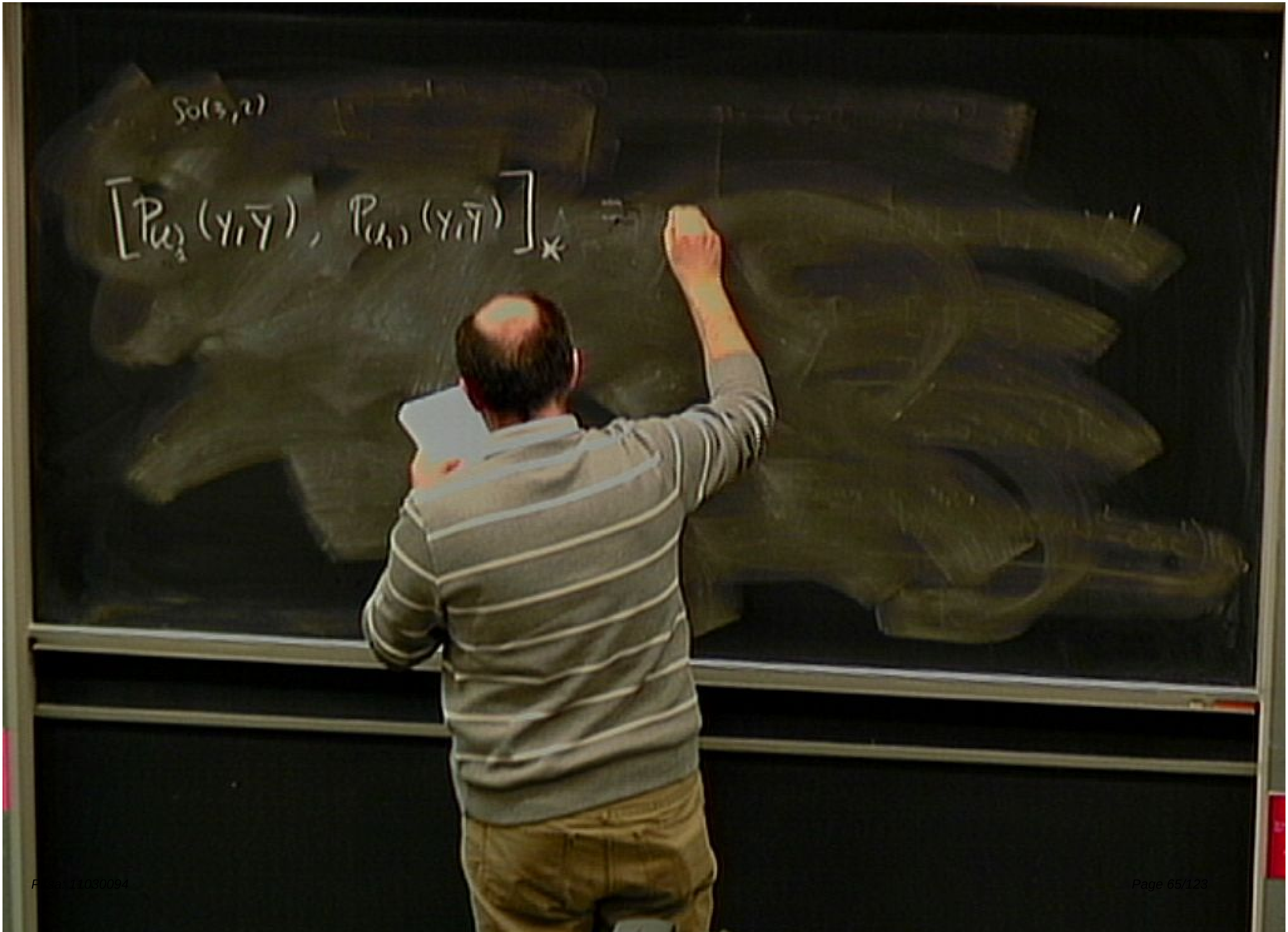
$$\cdots W_0 + W_0 \wedge_* W_0 = 0$$

$$W_0 = e_0 + \omega_0 \qquad e_0 = \frac{1}{4z}\, dx^\mu\, \sigma^\mu_{\alpha\dot\beta}\, y^\alpha \bar y^{\dot\beta}$$

$$\omega_0 = \frac{1}{8z}\, dx^i \left[(\sigma^{ij})_{\alpha\beta}\, y^\alpha y^\beta + (\bar\sigma^{ij})_{\dot\alpha\dot\beta}\, \bar y^{\dot\alpha} \bar y^{\dot\beta} \right]$$


HS ALGEBRA
symm. traceless
DOUBLE TRACELESS
dW₀ + W₀∧*W₀ = 0


sym. traceless
HS ALGEBRA
DOUBLE TRACELESS


SO(3,2)

$SO(3,2)$

$$\left[P_{(\ell_2)}(y,\bar{y}),\ P_{(\ell_1)}(y,\bar{y})\right]_* = \sum_{\ell=|\ell_1-\ell_2|+2}^{\ell_1+\ell_2-2} P_{(\ell)}(y,\bar{y})$$

$SO(3,2)$

$$\left[P_{(l_1)}(Y,\bar{Y}),\ P_{(l_2)}(Y,\bar{Y})\right]_* = \sum_{l=|l_1-l_2|+2}^{l_1+l_2-2} P_{(l)}(Y,\bar{Y})$$

$$\left[P_{(4)},\ P_{(4)}\right]_* = P_{(2)} + P_{(6)}$$

$s=2$ $\qquad$ $s=4$

$SO(3,2)$

$$\left[P_{(\ell_1)}(Y,\bar{Y}),\ P_{(\ell_2)}(Y,\bar{Y}) \right]_* = \sum_{\ell=|\ell_1-\ell_2|+2}^{\ell_1+\ell_2-2} P_{(\ell)}(Y,\bar{Y})$$

$$\left[P_{(4)},\ P_{(4)} \right]_* = \underset{s=2}{P_{(2)}} + \underset{s=4}{P_{(6)}}$$

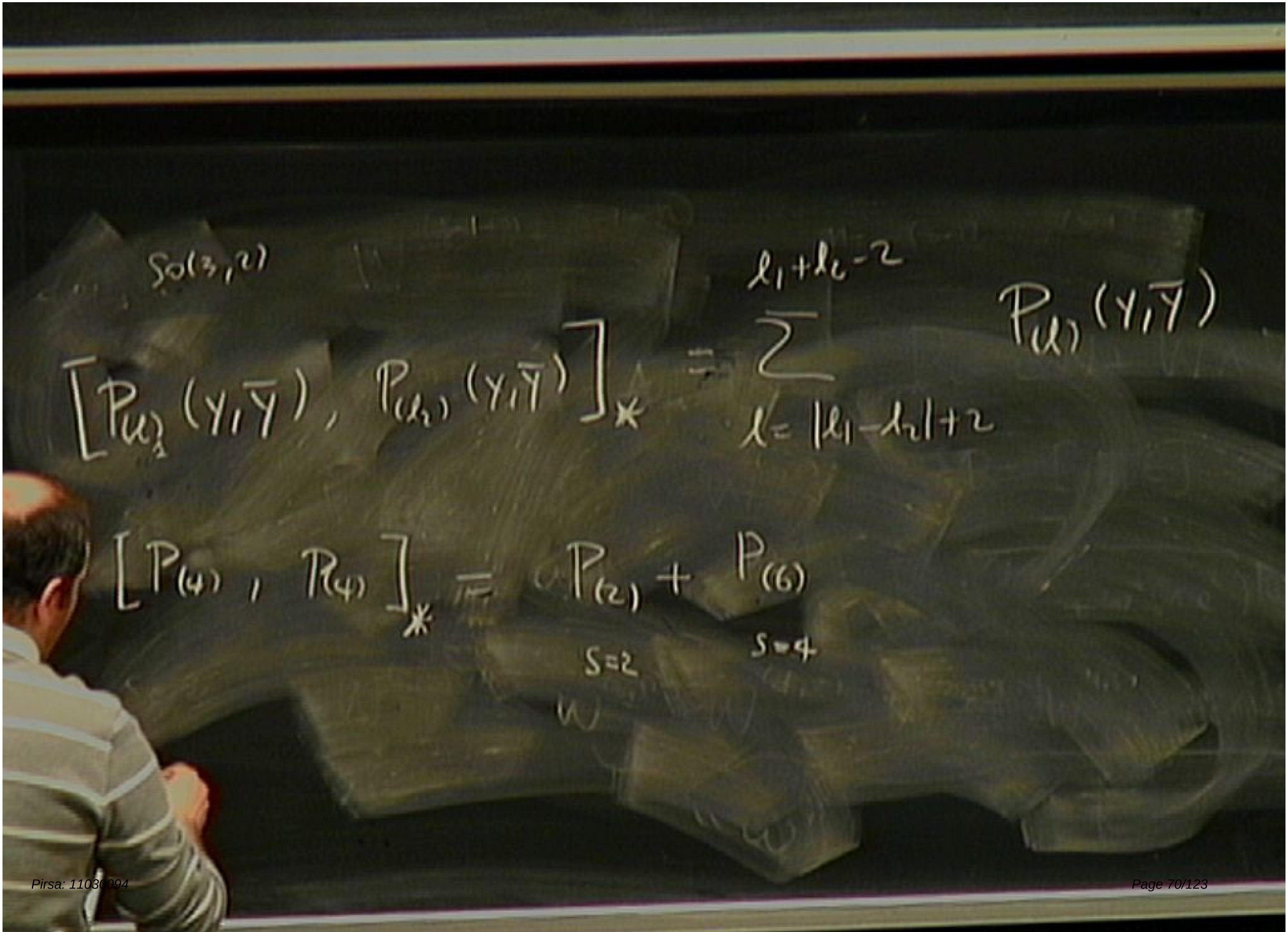
$so(3,2)$

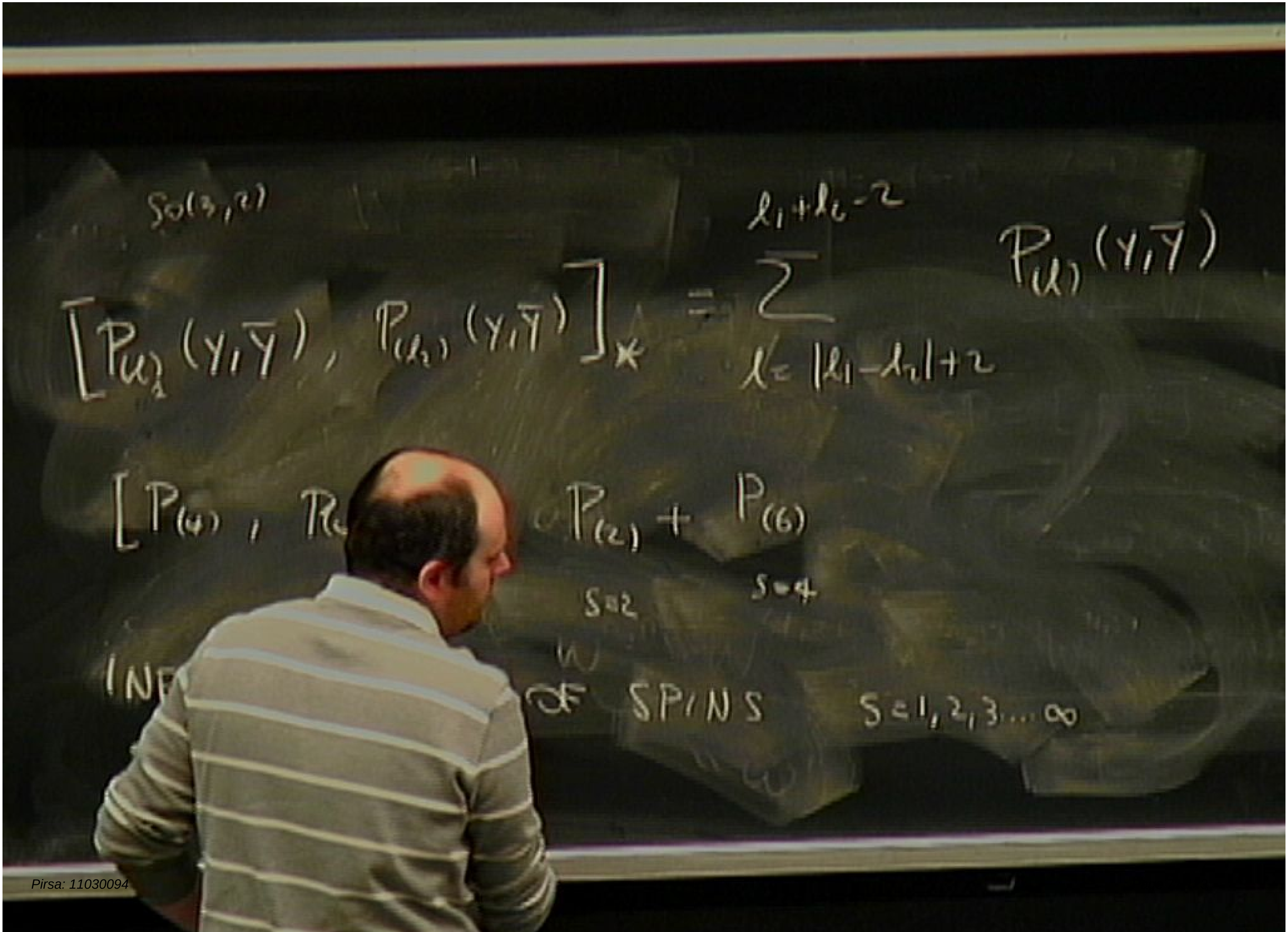
$$\left[P_{(l_1)}(Y,\bar{Y}),\ P_{(l_2)}(Y,\bar{Y})\right]_* = \sum_{l=|l_1-l_2|+2}^{l_1+l_2-2} P_{(l)}(Y,\bar{Y})$$

$$\left[P_{(4)},\ P_{(4)}\right]_* = P_{(2)} + P_{(6)}$$

$s=2$ $\quad$ $s=4$

ω


SO(3,2)
$$\left[P_{(\ell_1)}(Y,\bar{Y}),\ P_{(\ell_2)}(Y,\bar{Y})\right]_* = \sum_{\ell=|\ell_1-\ell_2|+2}^{\ell_1+\ell_2-2} P_{(\ell)}(Y,\bar{Y})$$
$$\left[P_{(4)},\ P_{(4)}\right]_* = P_{(2)} + P_{(6)}$$
s=2
s=4


so(3,2)
[P(l1)(Y,Ȳ), P(l2)(Y,Ȳ)]* = Σ l=|l1−l2|+2 to l1+l2−2 P(l)(Y,Ȳ)
[P(4), P… P(2) + P(6)
s=2
s=4
INF… OF SPINS s=1,2,3…∞

$so(3,2)$

$$\left[P_{(l_1)}(y,\bar{y}),\ P_{(l_2)}(y,\bar{y})\right]_* = \sum_{l=|l_1-l_2|+2}^{l_1+l_2-2} P_{(l)}(y,\bar{y}) \ \ldots$$

$$\left[P_{(4)},\ P_{(4)}\right]_* = \underbrace{P_{(2)}}_{s=2} + \underbrace{P_{(6)}}_{s=4}$$

INFINITE TOWER OF SPINS $s=1,2,3\ldots\infty$

MINIMAL HS ALG. $s=2,4,\ldots\infty$

$SO(3,2)$

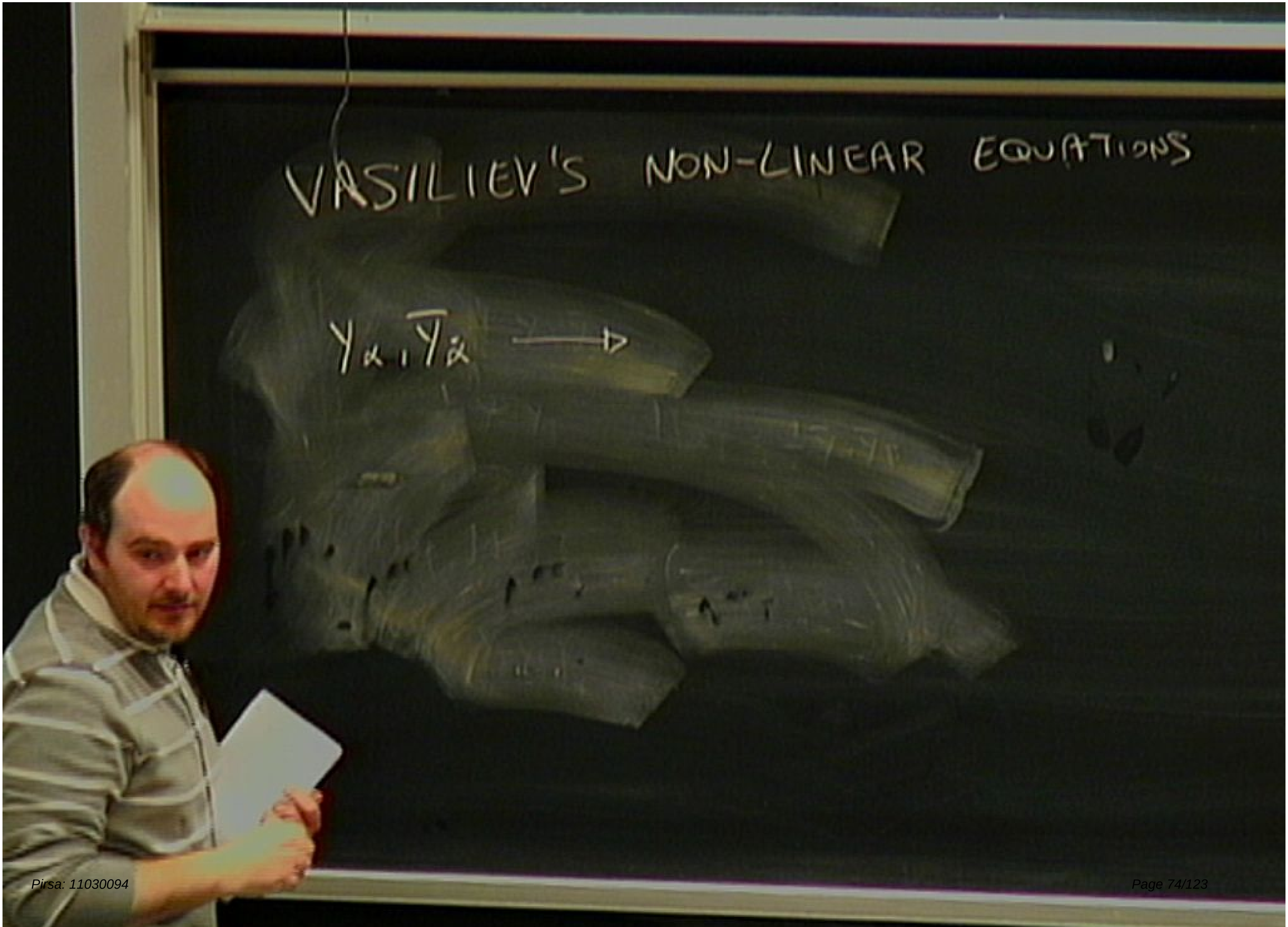
$$\left[P_{(\ell_1)}(y,\bar{y}),\ P_{(\ell_2)}(y,\bar{y})\right]_* = \sum_{\ell=|\ell_1-\ell_2|+2}^{\ell_1+\ell_2-2} P_{(\ell)}(y,\bar{y})$$

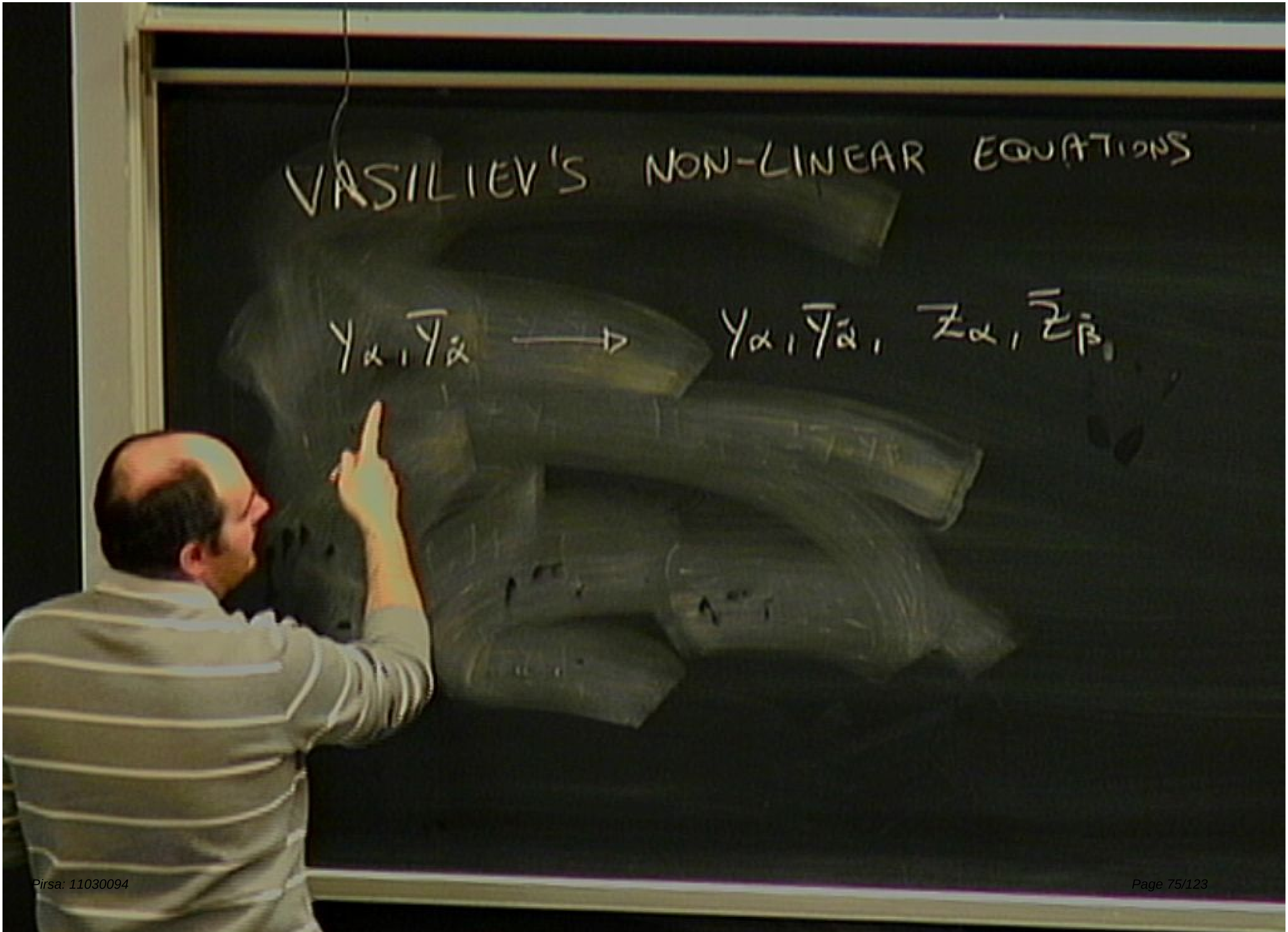
$$\left[P_{(4)},\ P_{(4)}\right]_* = \underset{S=2}{P_{(2)}} + \underset{S=4}{P_{(6)}}$$

ω

INFINITE TOWER OF SPINS $S=1,2,3\ldots\infty$

MINIMAL HS ALG. $S=2,4,\ldots\infty$


VASILIEV'S NON-LINEAR EQUATIONS
$Y_\alpha , \bar{Y}_{\dot{\alpha}} \longrightarrow$


VASILIEV'S NON-LINEAR EQUATIONS
$y_\alpha, \bar{y}_{\dot\alpha} \longrightarrow y_\alpha, \bar{y}_{\dot\alpha}, z_\alpha, \bar{z}_{\dot\beta}$

VASILIEV'S NON-LINEAR EQUATIONS

$$Y_\alpha, \bar{Y}_{\dot\alpha} \longrightarrow Y_\alpha, \bar{Y}_{\dot\alpha}, Z_\alpha, \bar{Z}_{\dot\beta}$$

$$Z_\alpha * Z_\beta = Z_\alpha Z_\beta - \epsilon_{\alpha\beta}$$

$$Y_\alpha * Z_\beta = Z Y_\alpha Z_\beta - \epsilon_{\alpha\beta}$$

VASILIEV'S NON-LINEAR EQUATIONS

$$y_\alpha, \bar{y}_{\dot\alpha} \longrightarrow y_\alpha, \bar{y}_{\dot\alpha}, z_\alpha, \bar{z}_{\dot\beta}$$

$$z_\alpha * z_\beta = z_\alpha z_\beta - \epsilon_{\alpha\beta}$$

$$y_\alpha * z_\beta = y_\alpha z_\beta - \epsilon_{\alpha\beta}$$

VASILIEV'S NON-LINEAR EQUATIONS

$$y_\alpha, \bar{y}_{\dot\alpha} \longrightarrow y_\alpha, \bar{y}_{\dot\alpha}, z_\alpha, \bar{z}_{\dot\beta}$$

$$z_\alpha * z_\beta = z_\alpha z_\beta - \epsilon_{\alpha\beta}$$

$$y_\alpha * z_\beta = y_\alpha z_\beta - \epsilon_{\alpha\beta}$$

$$y^{\dot\alpha} * \bar{y}^{\dot\beta} = \bar{y}^{\dot\alpha}\bar{y}^{\dot\beta} + \epsilon^{\dot\alpha\dot\beta}$$

$$y^{\alpha} * \bar{y}^{\dot\alpha} = y^{\alpha}\bar{y}^{\dot\alpha}$$

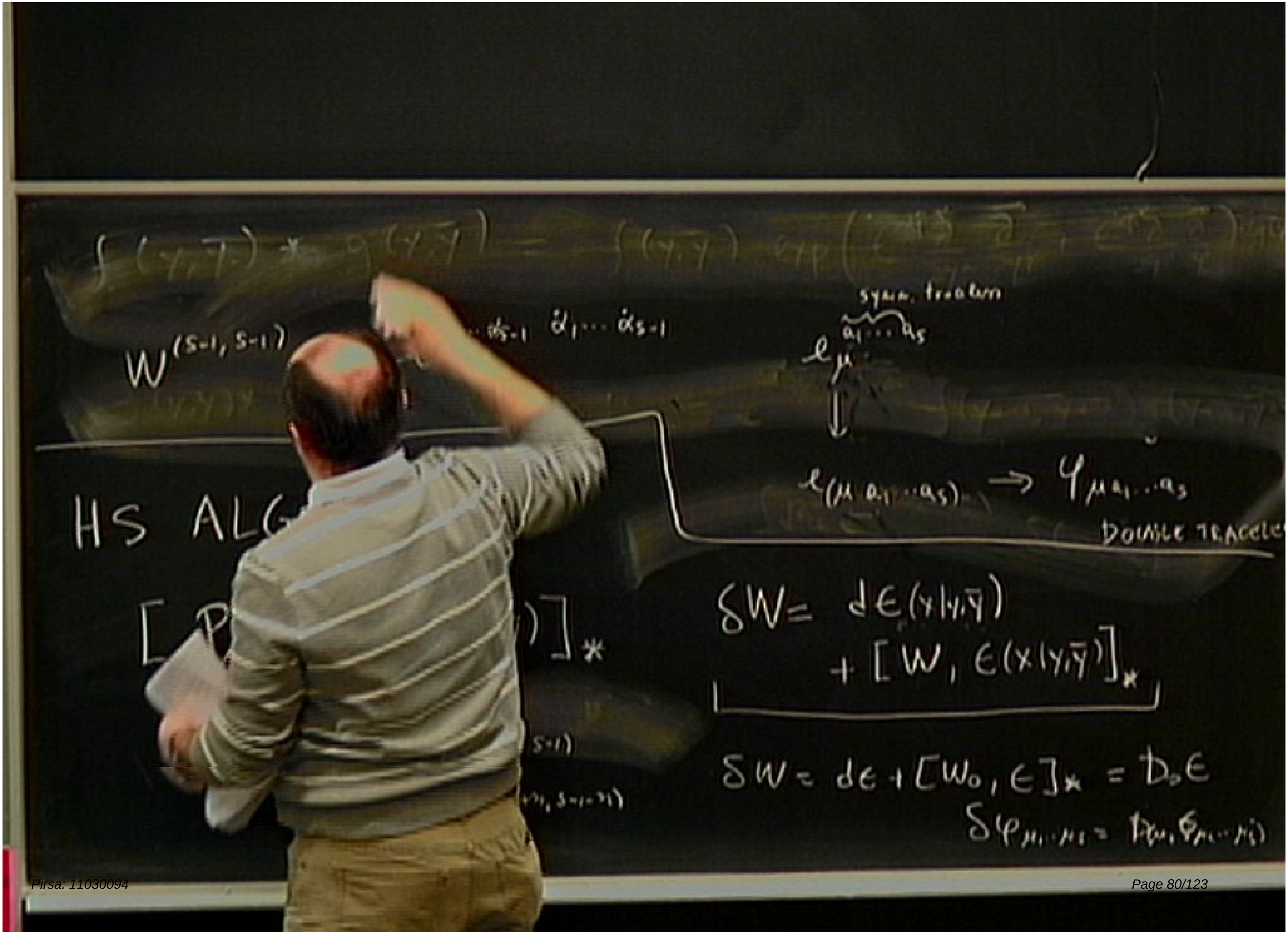
$$[y_\alpha, \bar{y}_{\dot\beta}]_* = 0$$

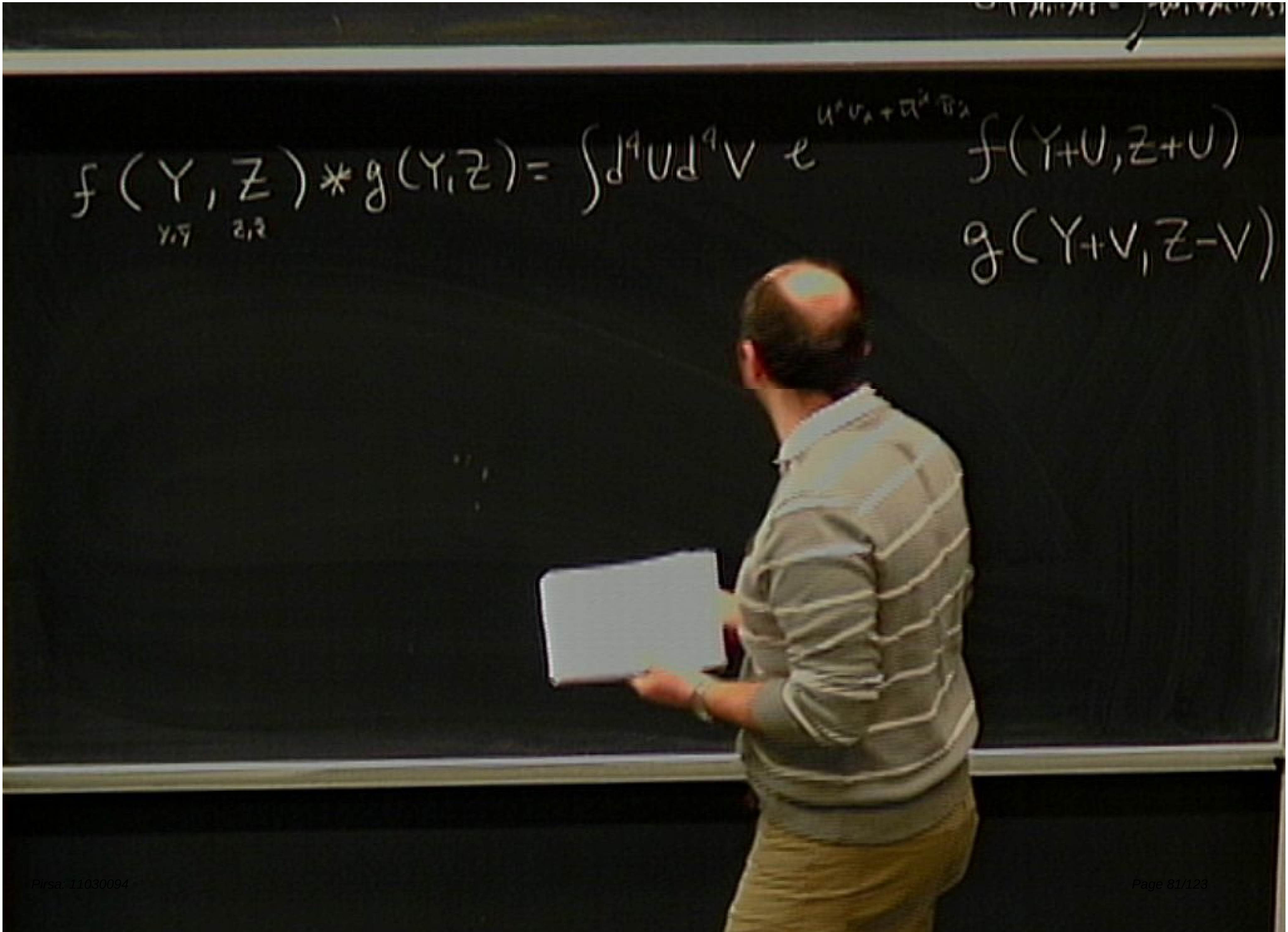
$$[P_{(l_2)}(y,\bar{y}), P_{(l_1)}(y,\bar{y})]_* \qquad l = |l_1 - l_2| + 2$$

$$\underset{*}{[P_{(4)}, P_{(4)}]} = \underset{s=2}{P_{(2)}} + \underset{s=4}{P_{(6)}}$$

INFINITE TOWER OF SPINS $s = 1, 2, 3 \dots \infty$

MINIMAL HS ALG. $s = 2, 4, \dots \infty$


W^(S-1, S-1)
symm. traceless
HS ALG
e_(μ a1...as) → φ_μ a1...as
DOUBLE TRACELESS
δW = dε(x|y,ȳ) + [W, ε(x|y,ȳ)]*
δW = dε + [W0, ε]* = D0ε


$f(\underset{Y,\bar{Y}}{Y}, \underset{Z,\bar{Z}}{Z}) * g(Y,Z) = \int d^4U d^4V \, e^{U^a V_a + \bar{U}^{\dot{a}} \bar{V}_{\dot{a}}} f(Y+U, Z+U) \, g(Y+V, Z-V)$

$$f(\underset{y,\bar{y}}{Y},\underset{z,\bar{z}}{Z}) * g(Y,Z) = \int d^4U\, d^4V\, e^{u^\alpha v_\alpha + \bar{u}^{\dot\alpha}\bar{v}_{\dot\alpha}}\, f(Y+U, Z+U)\, g(Y+V, Z-V)$$

$$k = e^{z^\alpha y_\alpha} \qquad \bar{k} = e^{\bar{z}^{\dot\alpha}\bar{y}_{\dot\alpha}}$$

$$k * k = 1 \qquad \bar{k} * \bar{k} = 1$$

$$k * f(y,\bar{y},z,\bar{z}) * k = f(-y,\bar{y},-z,\bar{z})$$

$$\underset{y,\bar y}{f(Y},\underset{z,\bar z}{Z)} * g(Y,Z) = \int d^4U\, d^4V\, e^{u^\alpha v_\alpha + \bar u^{\dot\alpha} \bar v_{\dot\alpha}}\, f(Y+U, Z+U)\, g(Y+V, Z-V)$$

$$\kappa = e^{z^\alpha y_\alpha} \qquad \bar\kappa = e^{\bar z^{\dot\alpha} \bar y_{\dot\alpha}}$$

$$\kappa * \kappa = 1 \qquad \bar\kappa * \bar\kappa = 1$$

$$\kappa * f(y,\bar y, z, \bar z) * \kappa = f(-y, \bar y, -z, \bar z)$$

$$W_\mu(x|y,\bar{y},z,\bar{z})$$

$$S(x|y,\bar{y},z,\bar{z}) = S_\alpha dz^\alpha + S_{\dot{\alpha}} d\bar{z}^{\dot{\alpha}}$$

$$B(x|y,\bar{y},z,\bar{z})$$

$$W_\mu(x|y,\bar{y},z,\bar{z})$$

$$S(x|y,\bar{y},z,\bar{z}) = S_\alpha dz^\alpha + S_{\dot\alpha} d\bar{z}^{\dot\alpha}$$

$$B(x|y,\bar{y},z,\bar{z})$$

SCALAR

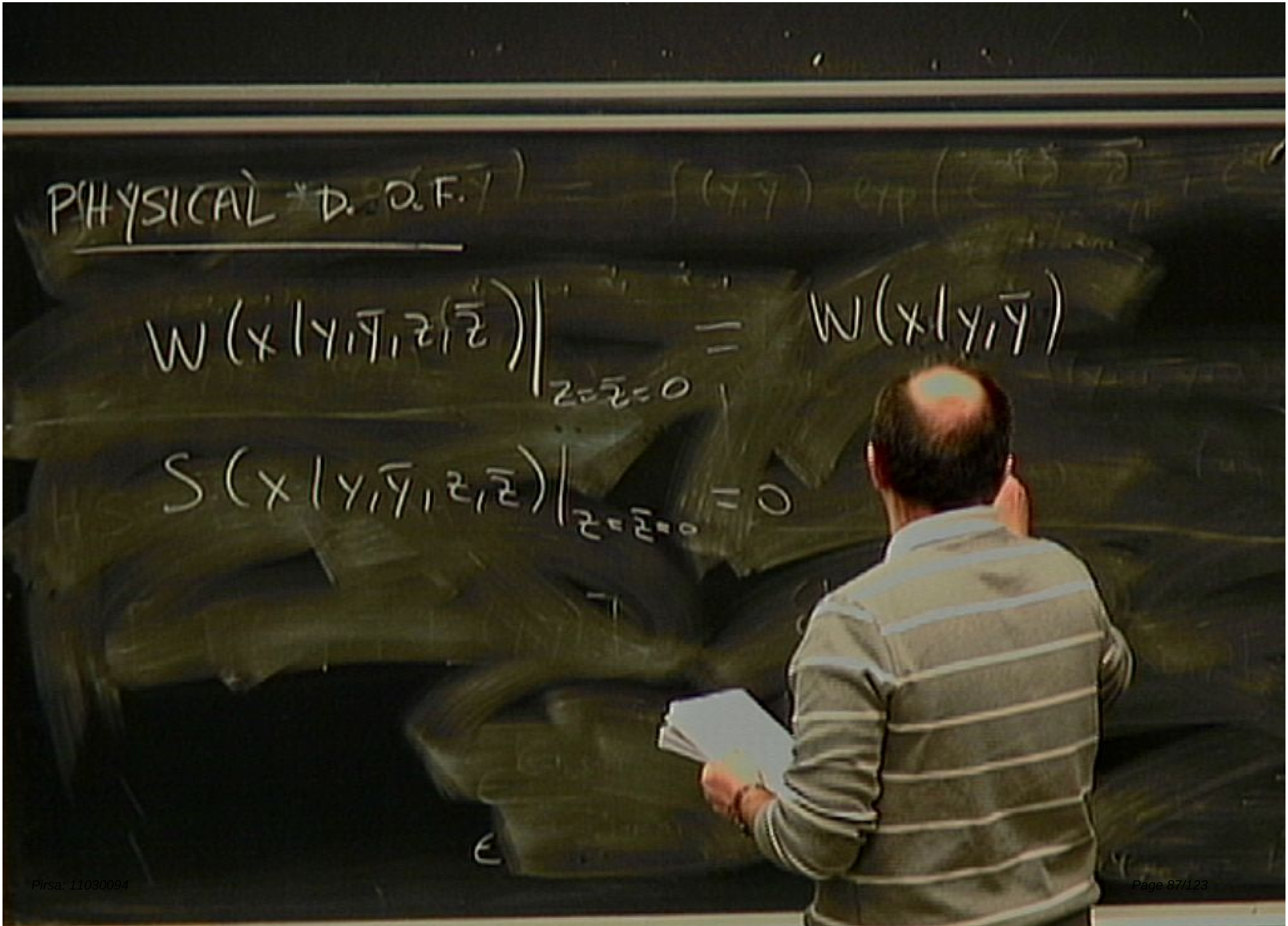
$$W(-y,-z) = W(y,z)$$

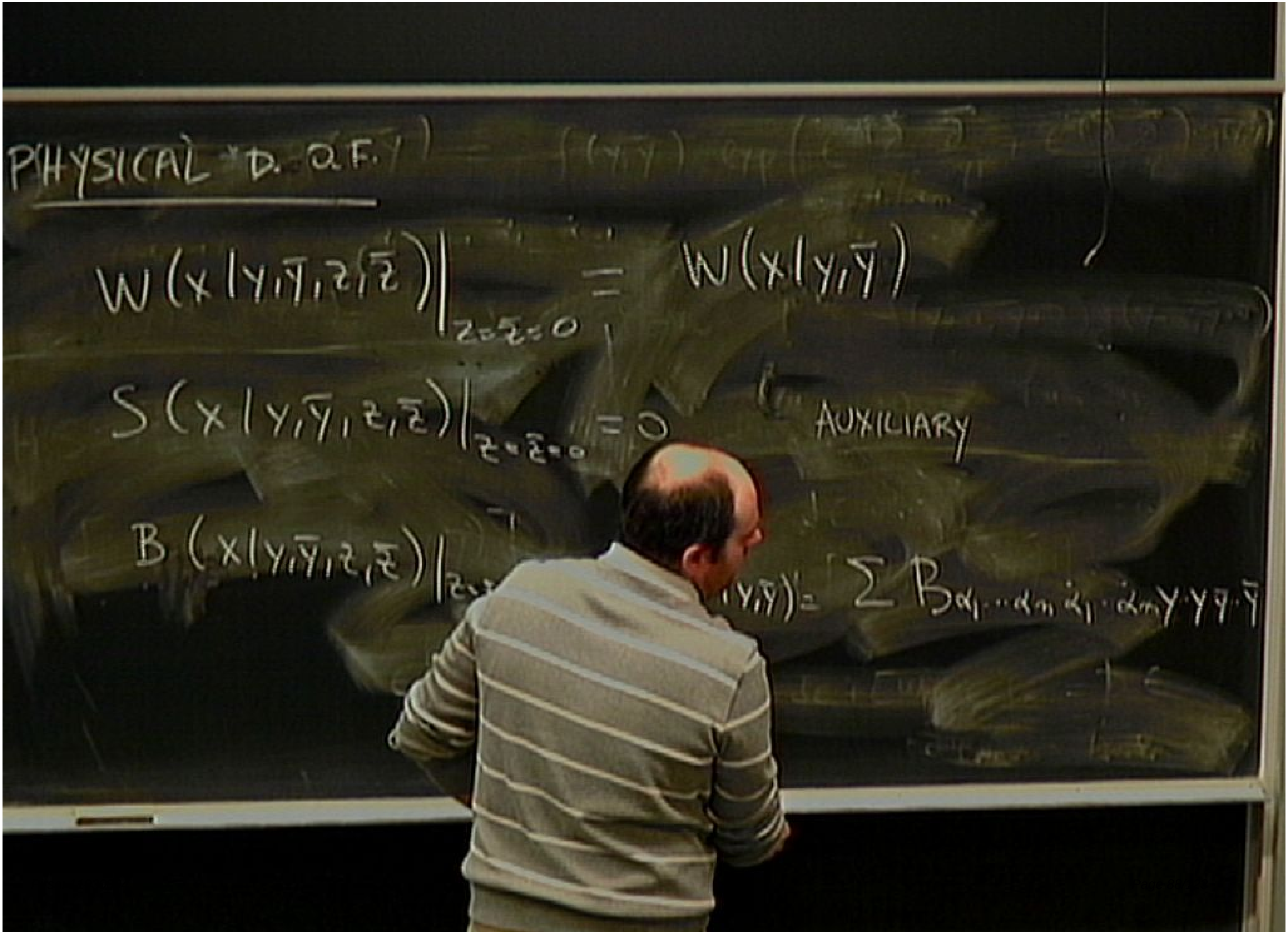
$$S_\alpha(-y,-z) = -S_\alpha(y,z)$$

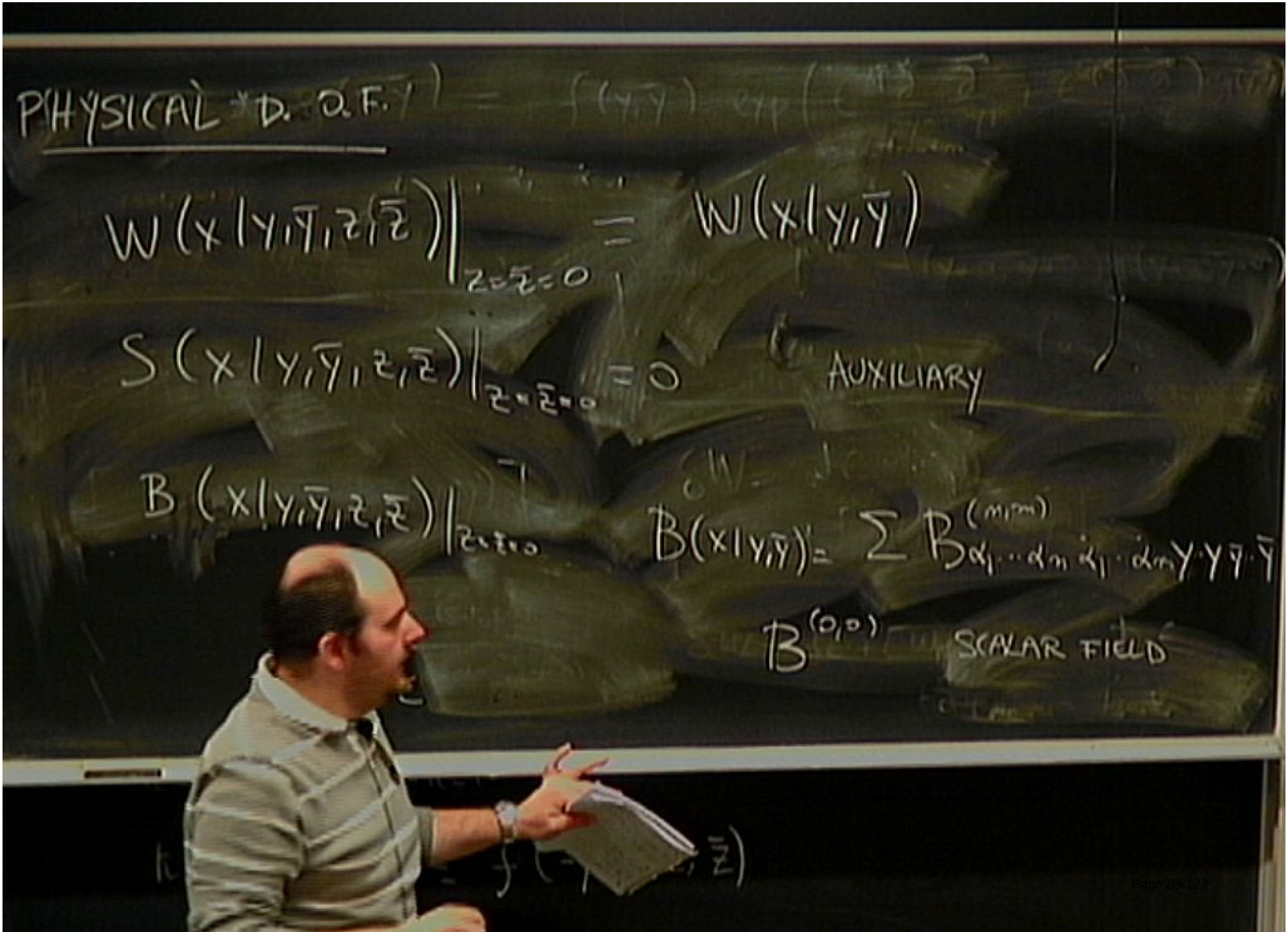
PHYSICAL D.O.F.

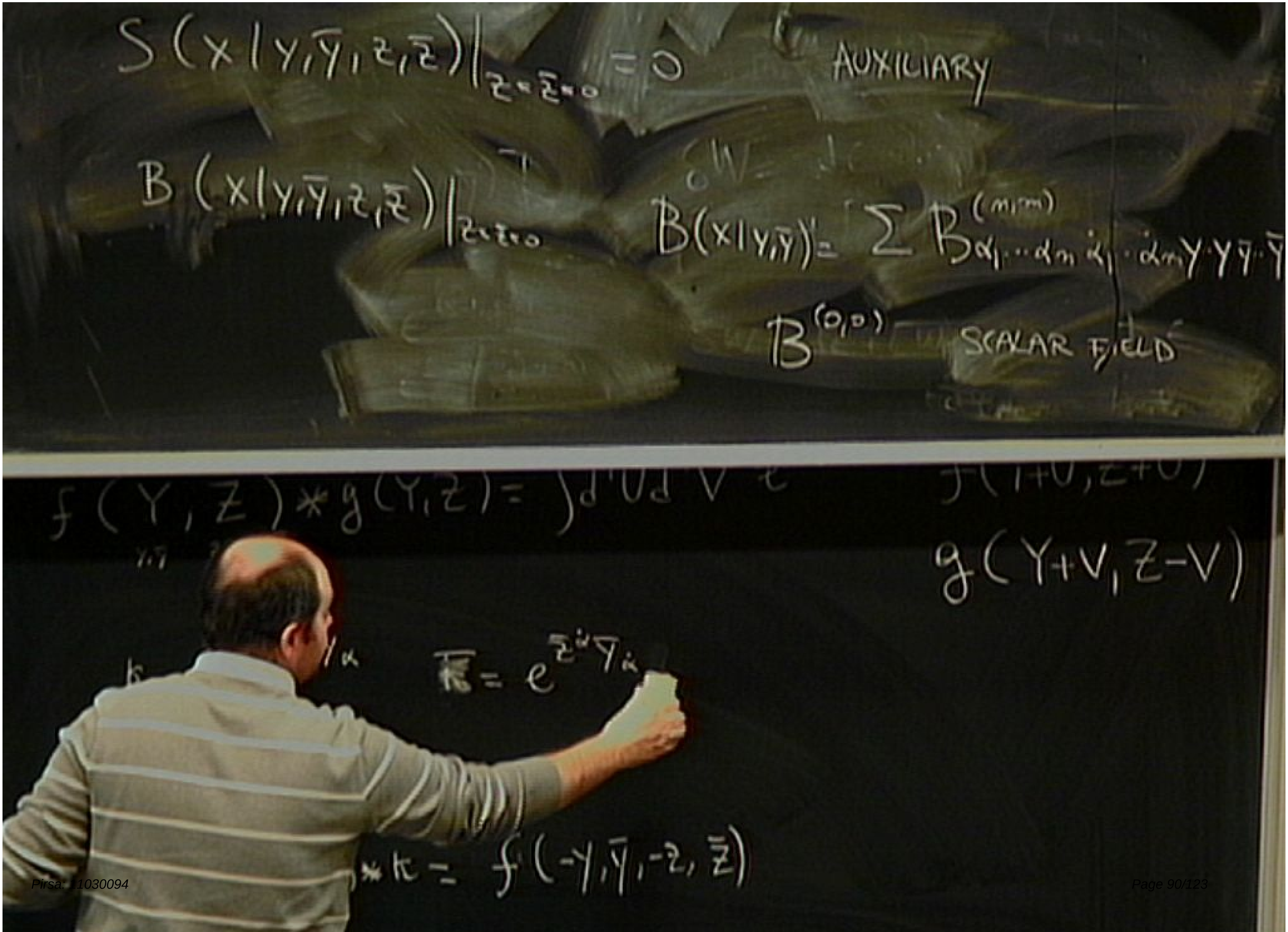
$$W(x|y,\bar{y},z,\bar{z})\big|_{z=\bar{z}=0} = W(x|y,\bar{y})$$

$S($


PHYSICAL D.O.F.
$W(x|y,\bar{y},z,\bar{z})\big|_{z=\bar{z}=0} = W(x|y,\bar{y})$
$S(x|y,\bar{y},z,\bar{z})\big|_{z=\bar{z}=0} = 0$


PHYSICAL D.O.F.
$W(x|y,\bar{y},z,\bar{z})\big|_{z=\bar{z}=0} = W(x|y,\bar{y})$
$S(x|y,\bar{y},z,\bar{z})\big|_{z=\bar{z}=0} = 0$
AUXILIARY
$B(x|y,\bar{y},z,\bar{z})\big|$
$(y,\bar{y}) = \sum B_{\alpha_1\ldots\alpha_n\dot{\alpha}_1\ldots\dot{\alpha}_m} y\cdots y\bar{y}\cdots\bar{y}$


PHYSICAL D.O.F.
$W(x|y,\bar{y},z,\bar{z})|_{z=\bar{z}=0} = W(x|y,\bar{y})$
$S(x|y,\bar{y},z,\bar{z})|_{z=\bar{z}=0} = 0$
AUXILIARY
$B(x|y,\bar{y},z,\bar{z})|_{z=\bar{z}=0}$
$B(x|y,\bar{y}) = \sum B^{(m,m)}_{\alpha_1\ldots\alpha_m\dot{\alpha}_1\ldots\dot{\alpha}_m} y\cdots y\bar{y}\cdots\bar{y}$
$B^{(0,0)}$
SCALAR FIELD


$S(x|y,\bar{y},z,\bar{z})|_{z=\bar{z}=0}=0$
AUXILIARY
$B(x|y,\bar{y},z,\bar{z})|_{z=\bar{z}=0}$
$B(x|y,\bar{y})=\sum B^{(n,m)}_{\alpha_1\cdots\alpha_n\dot{\alpha}_1\cdots\dot{\alpha}_m}y\cdots y\bar{y}\cdots\bar{y}$
$B^{(0,0)}$
SCALAR FIELD
$f(Y,Z)*g(Y,Z)=\int d^4U\,d^4V\,e$
$g(Y+V,Z-V)$
$\bar{k}=e^{\bar{z}^{\dot{\alpha}}\bar{y}_{\dot{\alpha}}}$
$*k=f(-y,\bar{y},-z,\bar{z})$

PHYSICAL D. O. F.

$$W(x|y,\bar{y},z,\bar{z})\Big|_{z=\bar{z}=0} = W(x|y,\bar{y})$$

$$S(x|y,\bar{y},z,\bar{z})\Big|_{z=\bar{z}=0} = 0 \quad \text{AUXILIARY}$$

$$B(x|y,\bar{y},z,\bar{z})\Big|_{z=\bar{z}=0}$$

$$B(x|y,\bar{y}) = \sum B^{(m,m)}_{\alpha_1\cdots\alpha_m\dot{\alpha}_1\cdots\dot{\alpha}_m} y\cdots y\bar{y}\cdots\bar{y}$$

$$B^{(0,0)} \quad \text{SCALAR FIELD}$$

$$f(Y,Z) * g(Y,Z) = \int d^4U\, d^4V\, e \cdots$$

$$g(Y+V, Z-V)$$

VASILIEV'S TWISTOR VARIABLES

$B^{(2s,0)}$ $\qquad$ $B^{(0,2s)}$

$B_{\alpha_1 \cdots \alpha_{2s}}$ $\qquad$ $\bar{B}_{\dot{\alpha}_1 \cdots \dot{\alpha}_{2s}}$ $\qquad$ Weyl CURVATURE

$s=1$ $\qquad$ $F_{\alpha\beta}$ $\quad$ $F_{\dot{\alpha}\dot{\beta}}$

$s=2$ $\qquad$ $C_{\alpha\beta\gamma\delta}$ $\quad$ $C_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}$

VASILIEV'S TWISTOR VARIABLES

$B^{(2s,0)}$ $\quad B^{(0,2s)}$

$B_{\alpha_1\cdots\alpha_{2s}}$, $\bar{B}_{\dot\alpha_1\cdots\dot\alpha_{2s}}$ Weyl CURVATURE

$s=1$ $\quad F_{\alpha\beta}$ $\quad F_{\dot\alpha\dot\beta}$

$s=2$ $\quad C_{\alpha\beta\gamma\delta}$, $C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$B^{(2s+n,n)}$ $\quad B^{(n,2s+n)}$

$n=0,\ldots,\infty$

$B^{(2s,0)}$ $\quad B^{(0,2s)}$

$B_{\alpha_1 \ldots \alpha_{2s}}$, $\bar{B}_{\dot\alpha_1 \ldots \dot\alpha_{2s}}$ Weyl CURVATURE

$s=1$ $\quad F_{\alpha\beta}$ $\quad F_{\dot\alpha\dot\beta}$

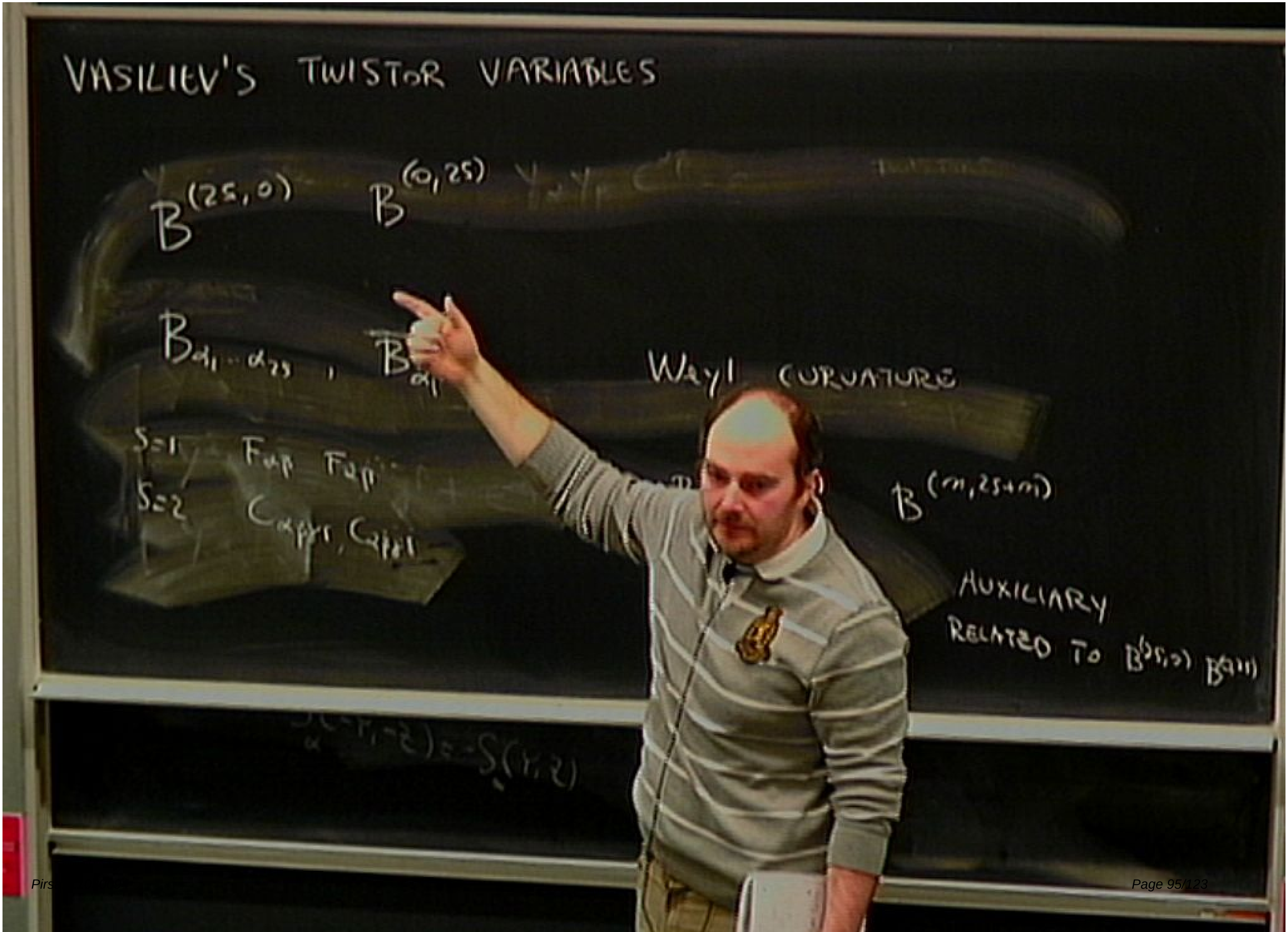
$s=2$ $\quad C_{\alpha\beta\gamma\delta}$, $C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$B^{(2s+m,m)}$ $\quad B^{(m,2s+m)}$

$m=0,\ldots,\infty$

AUXILIARY

RELATED TO $B^{(2s,0)}$ $B^{(0,2s)}$


VASILIEV'S TWISTOR VARIABLES
B(2s,0) B(0,2s)
Weyl CURVATURE
s=1
s=2
AUXILIARY
RELATED TO

VASILIEV'S TWISTOR VARIABLES

$B^{(2s,0)}$ $B^{(0,2s)}$

$B_{\alpha_1 \dots \alpha_{2s}}$, $\bar{B}_{\dot\alpha_1 \dots \dot\alpha_{2s}}$ WEYL CURVATURE

$s=1$ $F_{\alpha\beta}$ $F_{\dot\alpha\dot\beta}$

$s=2$ $C_{\alpha\beta\gamma\delta}$, $C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$B^{(2s+n,n)}$ $B^{(n,2s+n)}$

$n=0,\dots,\infty$

AUXILIARY

RELATED TO $B^{(2s,0)}$ $B^{(0,1)}$

$S_\alpha(-y,-z) = -S_\alpha(y,z)$

$$W_\mu(x|y,\bar{y},z,\bar{z})$$

$$S(x|y,\bar{y},z,\bar{z}) = S_\alpha dz^\alpha + S_{\dot{\alpha}} d\bar{z}^{\dot{\alpha}}$$

$$B(x|y,\bar{y},z,\bar{z})$$

SCALAR

$$W(-Y,-Z)=W(Y,Z)$$

$$S_\alpha(-Y,-Z)=-S_\alpha(Y,Z)$$

VASILIEV'S NON-LINEAR EQUATIONS

GAUGE SYMMETRIES

$\epsilon(x \mid y, \bar{y}, z, \bar{z})$

$$\delta W = d_x \epsilon + [W, \epsilon]_*$$

$$\delta W = d_z S + [S, \epsilon]_*$$

δB

VASILIEV'S NON-LINEAR EQUATIONS

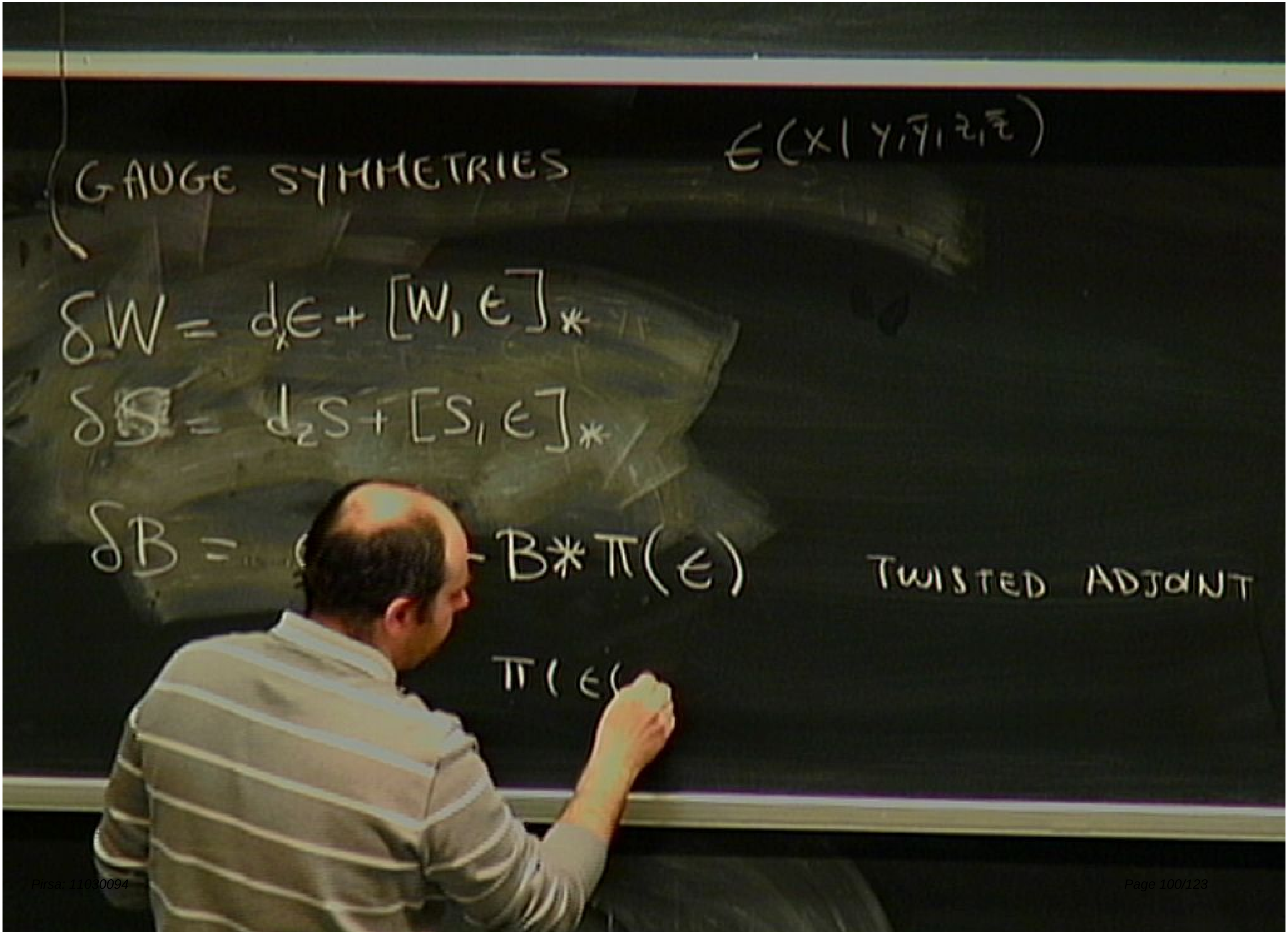
GAUGE SYMMETRIES

$$\epsilon(x \mid y, \bar{y}, z, \bar{z})$$

$$\delta W = d_x \epsilon + [W, \epsilon]_*$$

$$\delta S = d_z S + [S, \epsilon]_*$$

$$\delta B = \epsilon * B - B * \pi(\epsilon)$$


GAUGE SYMMETRIES
$\epsilon(x \mid y, \bar{y}, z, \bar{z})$
$\delta W = d_x \epsilon + [W, \epsilon]_*$
$\delta S = d_z S + [S, \epsilon]_*$
$\delta B =$
$B * \pi(\epsilon)$
TWISTED ADJOINT
$\pi(\epsilon($

GAUGE SYMMETRIES

$$\delta W = d_1 \epsilon + [W, \epsilon]_*$$

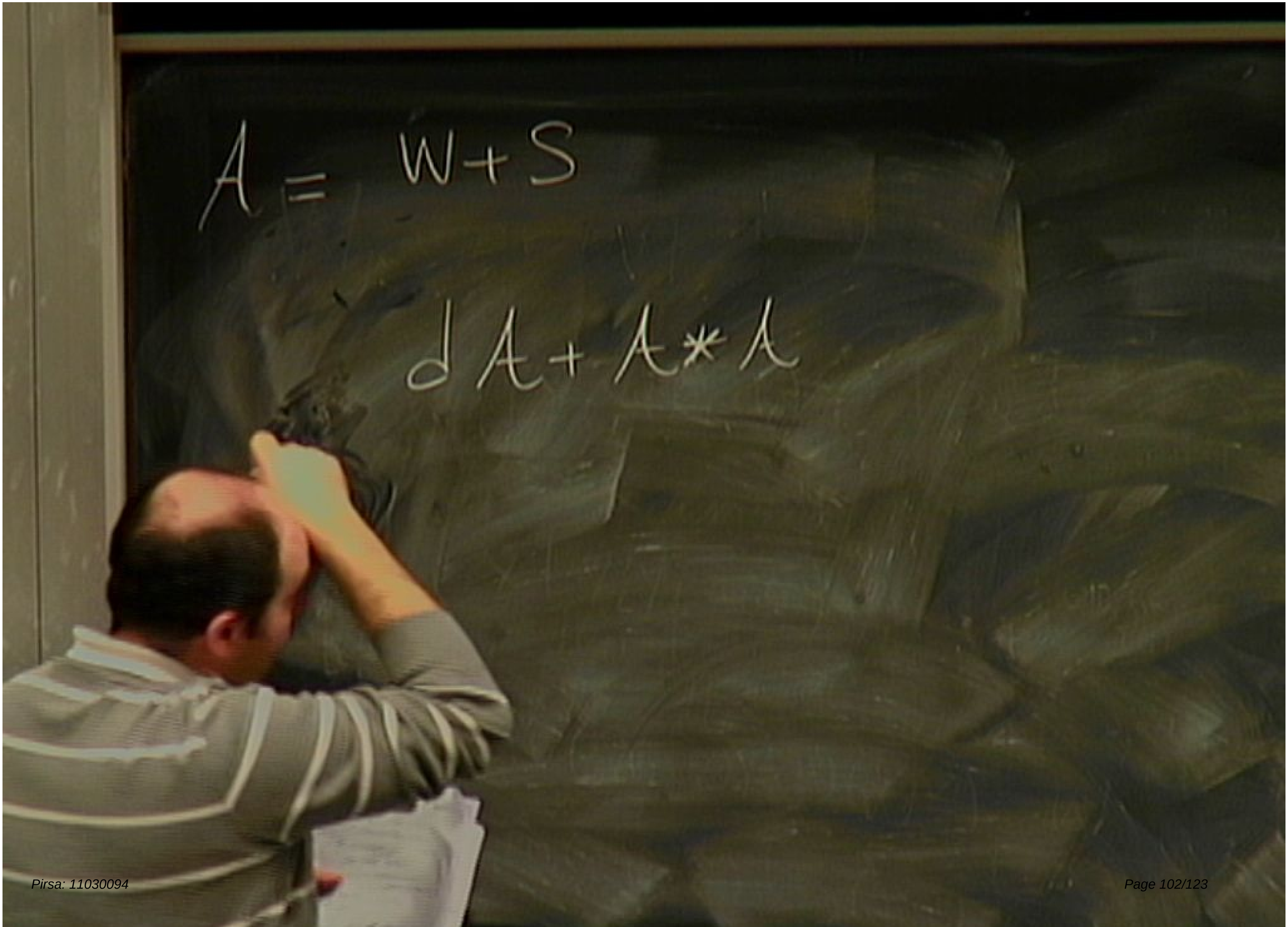
$$\delta S = d_2 S + [S, \epsilon]_*$$

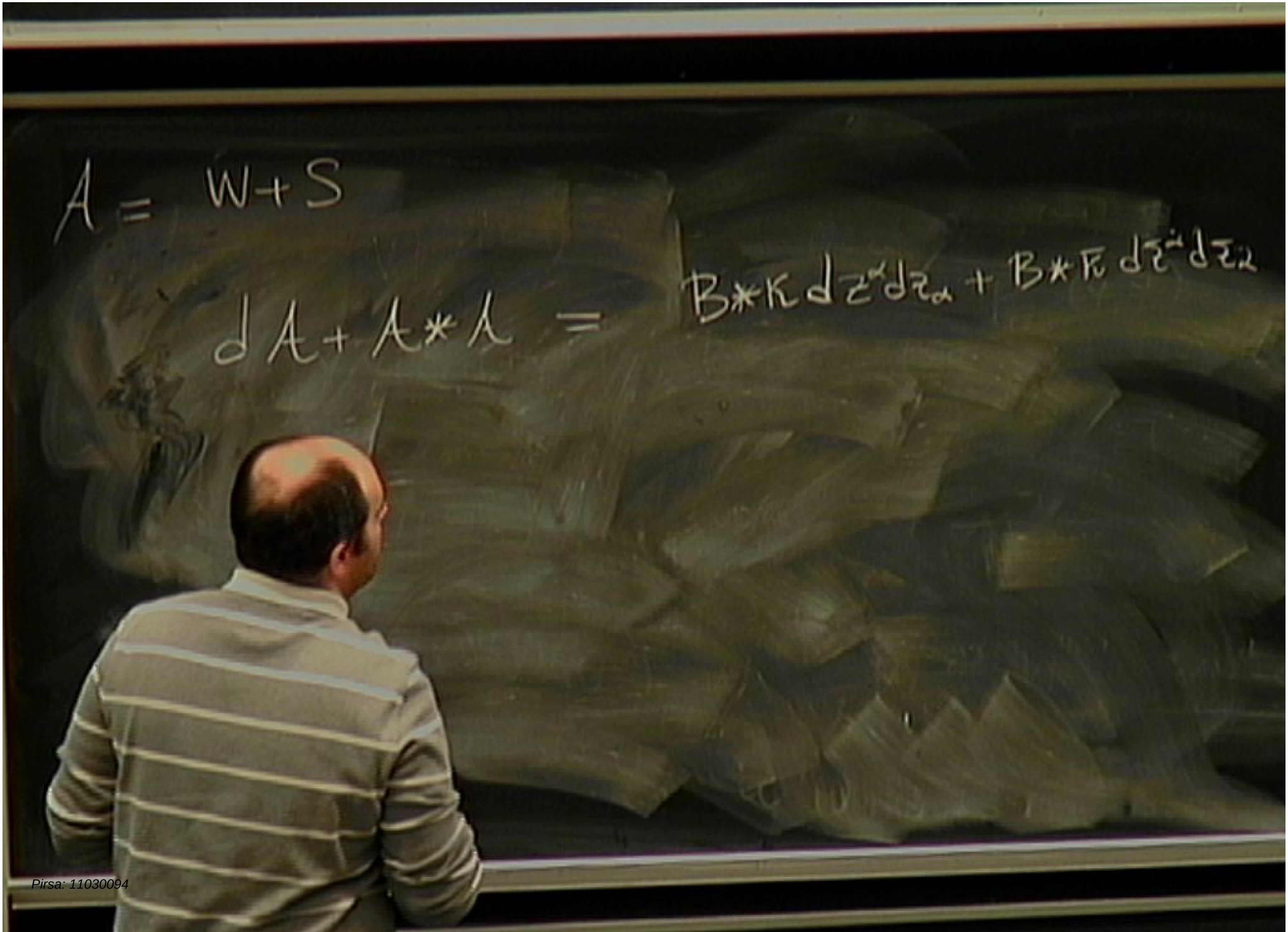
$$\delta B = \epsilon * B - B * \pi(\epsilon)$$

TWISTED ADJOINT

$$\pi(\epsilon(y,\bar{y},z,\bar{z})) = \epsilon(-y,\bar{y},-z,\bar{z}) = k * \epsilon * k$$

$$2(S-1) = n+m$$


A = W+S
dA + A*A


$A = W + S$
$dA + A * A = B * K\, dz^\alpha dz_\alpha + B * \bar{K}\, d\bar{z}^\alpha d\bar{z}_\alpha$

$$A = W + S$$

$$\begin{cases} dA + A * A = B * K\, dz^{\alpha} dz_{\alpha} + B * \bar{K}\, d\bar{z}^{\dot\alpha} d\bar{z}_{\dot\alpha} \\ dB + A * B - B * \pi(A) = 0 \end{cases}$$

$m=0,\ldots,\infty$

AUXILIARY

RELATED TO $B^{(25,1)}$ $B^{(9,1)}$

$A = W + S$

$$\left[\begin{array}{l} dA + A * A = B * K \, dz^{\alpha} dz_{\alpha} + B * \bar{K} \, d\bar{z}^{\dot\alpha} d\bar{z}_{\dot\alpha} \\ dB + A * B - B * \pi(A) = 0 \end{array}\right]$$

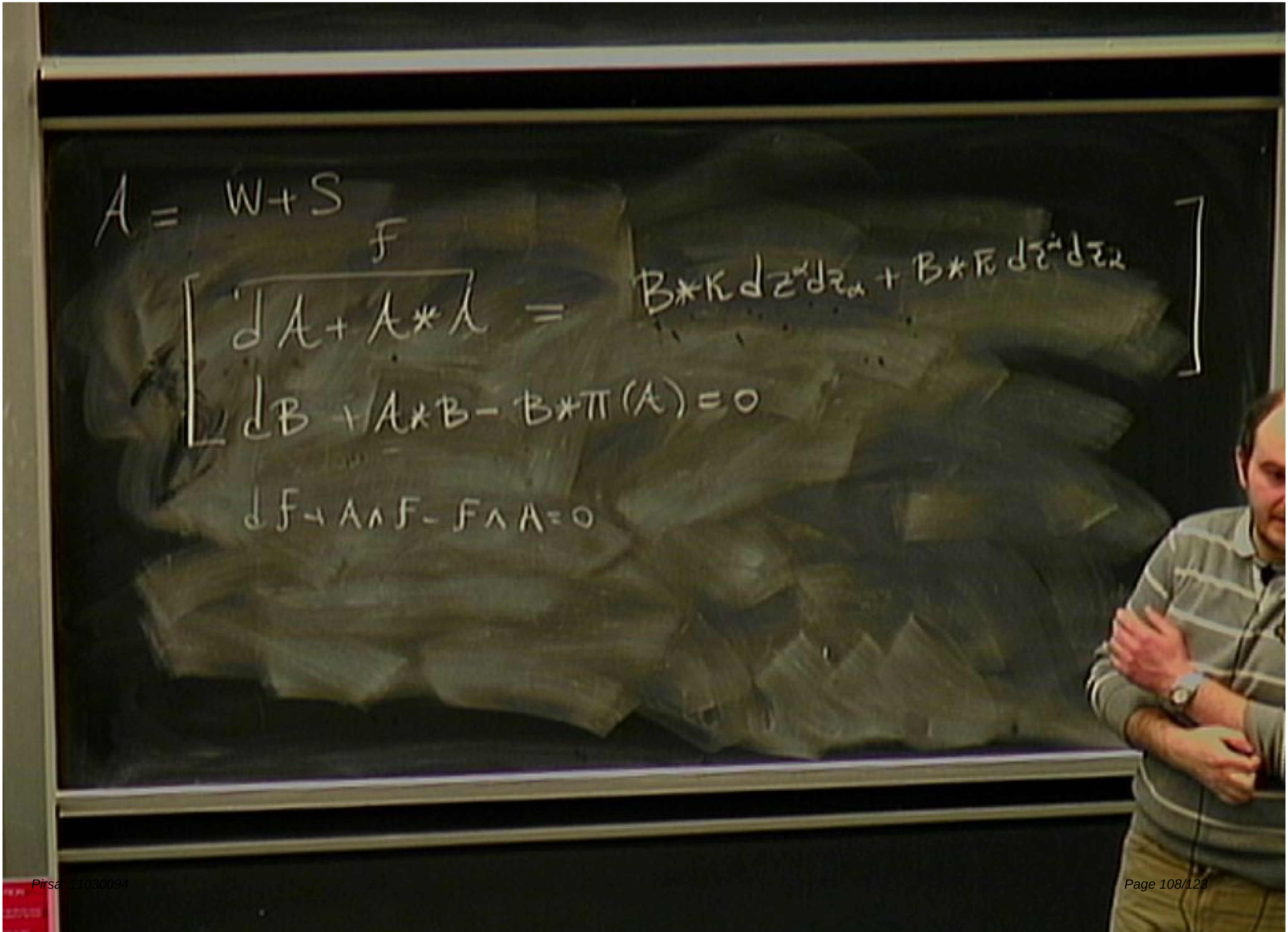
$$A = W + S$$

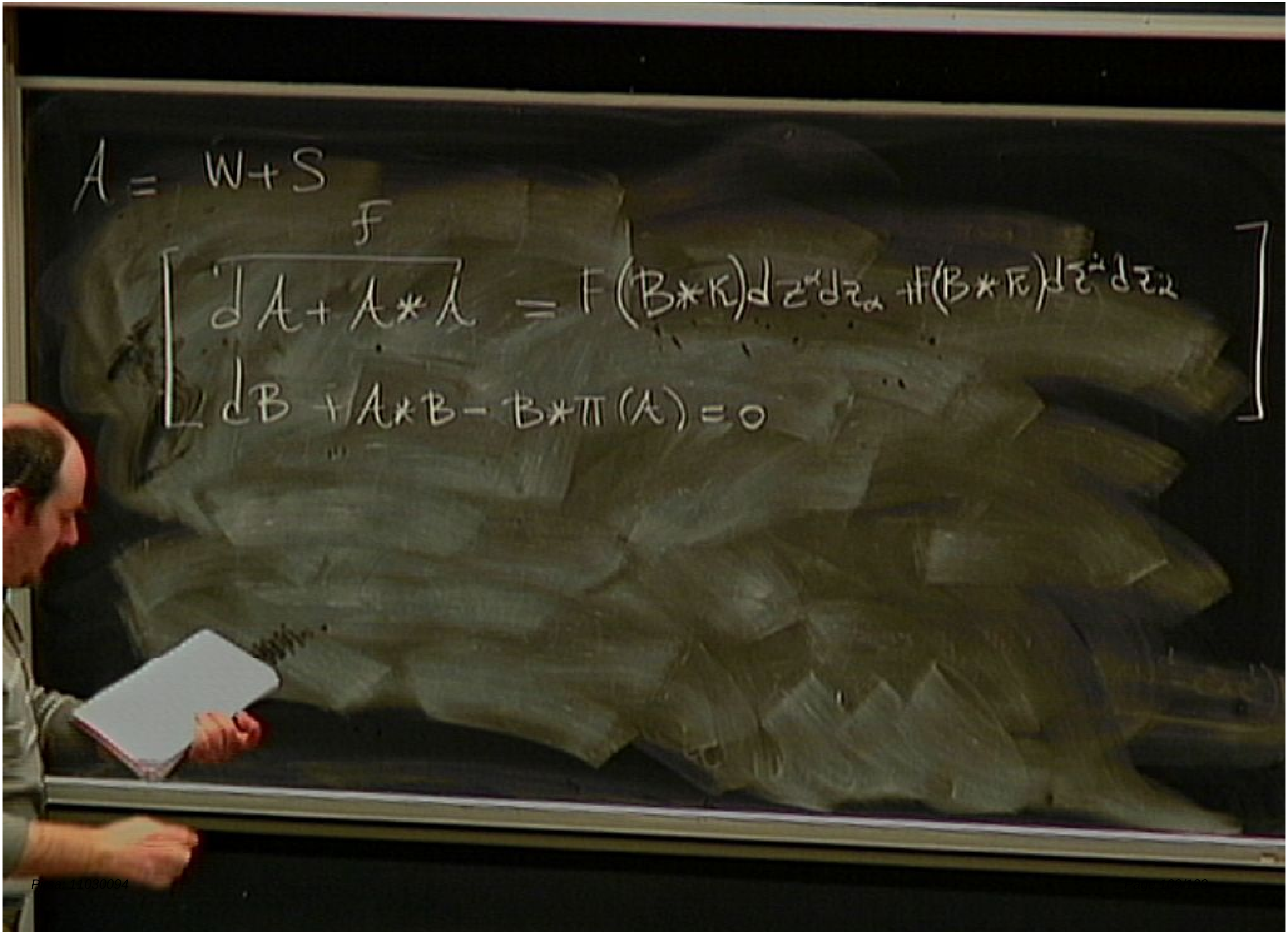
$$\left[\begin{array}{l} dA + A * A = B * K \, dz^{\alpha} dz_{\alpha} + B * \bar{K} \, d\bar{z}^{\alpha} d\bar{z}_{\alpha} \\ dB + A * B - B * \pi(A) = 0 \end{array}\right]$$

$$\mathcal{A} = W + S$$

$$\left[\begin{array}{l} \overbrace{d\mathcal{A} + \mathcal{A} * \mathcal{A}}^{F} = B * K\, dz^{\alpha} dz_{\alpha} + B * \bar{K}\, d\bar{z}^{\alpha} d\bar{z}_{\alpha} \\ dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0 \end{array}\right]$$

$$dF + A \wedge F - F \wedge A = 0$$


$A = W + S$
F
$dA + A * A = B * K \, dz^{\alpha} dz_{\alpha} + B * \bar{K} \, d\bar{z}^{\alpha} d\bar{z}_{\alpha}$
$dB + A * B - B * \pi(A) = 0$
$dF + A \wedge F - F \wedge A = 0$


A = W+S
F
dA + A*A = F(B*K)dz^α dz_α + F(B*K̄)dz̄^α dz̄_α
dB + A*B − B*π(A) = 0

$F_{\mu\nu} \longrightarrow F_{\alpha\beta}, F_{\dot\alpha\dot\beta}$

$C_{\mu\nu\rho\sigma} \longrightarrow C_{\alpha\beta\gamma\delta}, C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

$\omega^{ab} \to \omega^{\alpha\beta}, \omega^{\dot\alpha\dot\beta}$ $\quad M_{\alpha\beta}, M_{\dot\alpha\dot\beta}$

$\epsilon(x|y,\bar y, z, \bar z)$

GAUGE SYMMETRIES

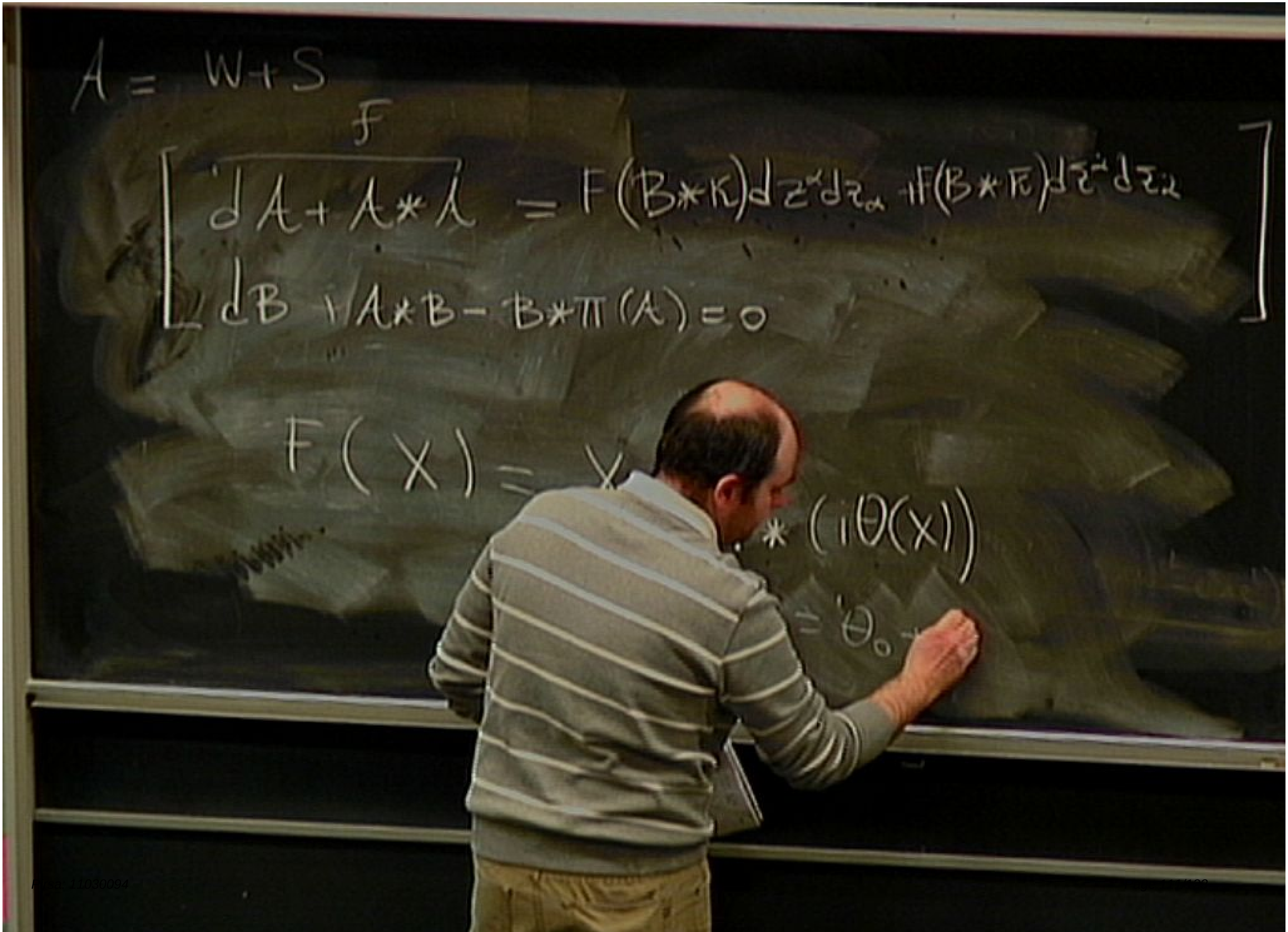
$$\delta W = d_x \epsilon + [W, \epsilon]_*$$

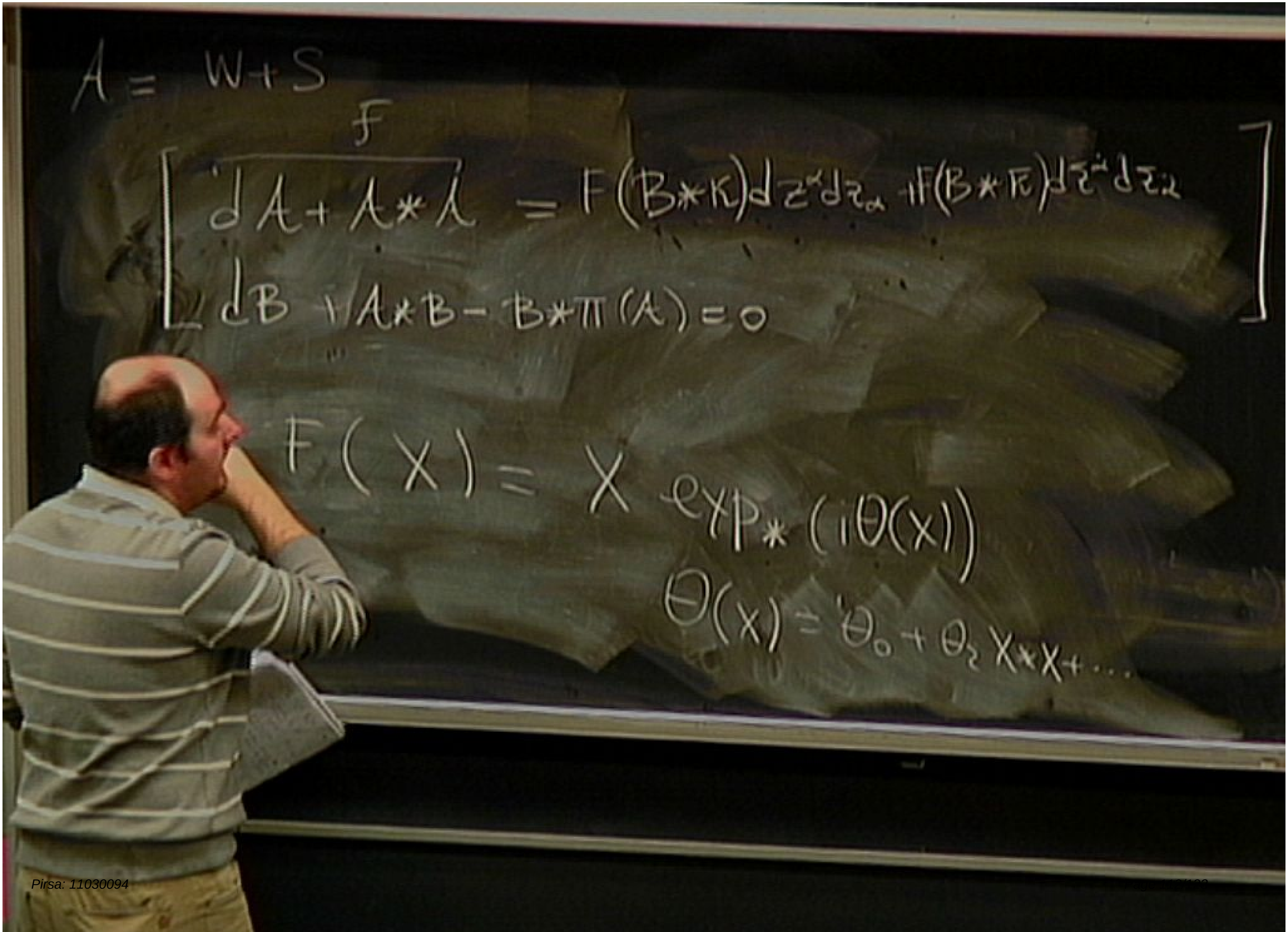
$$\delta S = d_z \epsilon + [S, \epsilon]_*$$

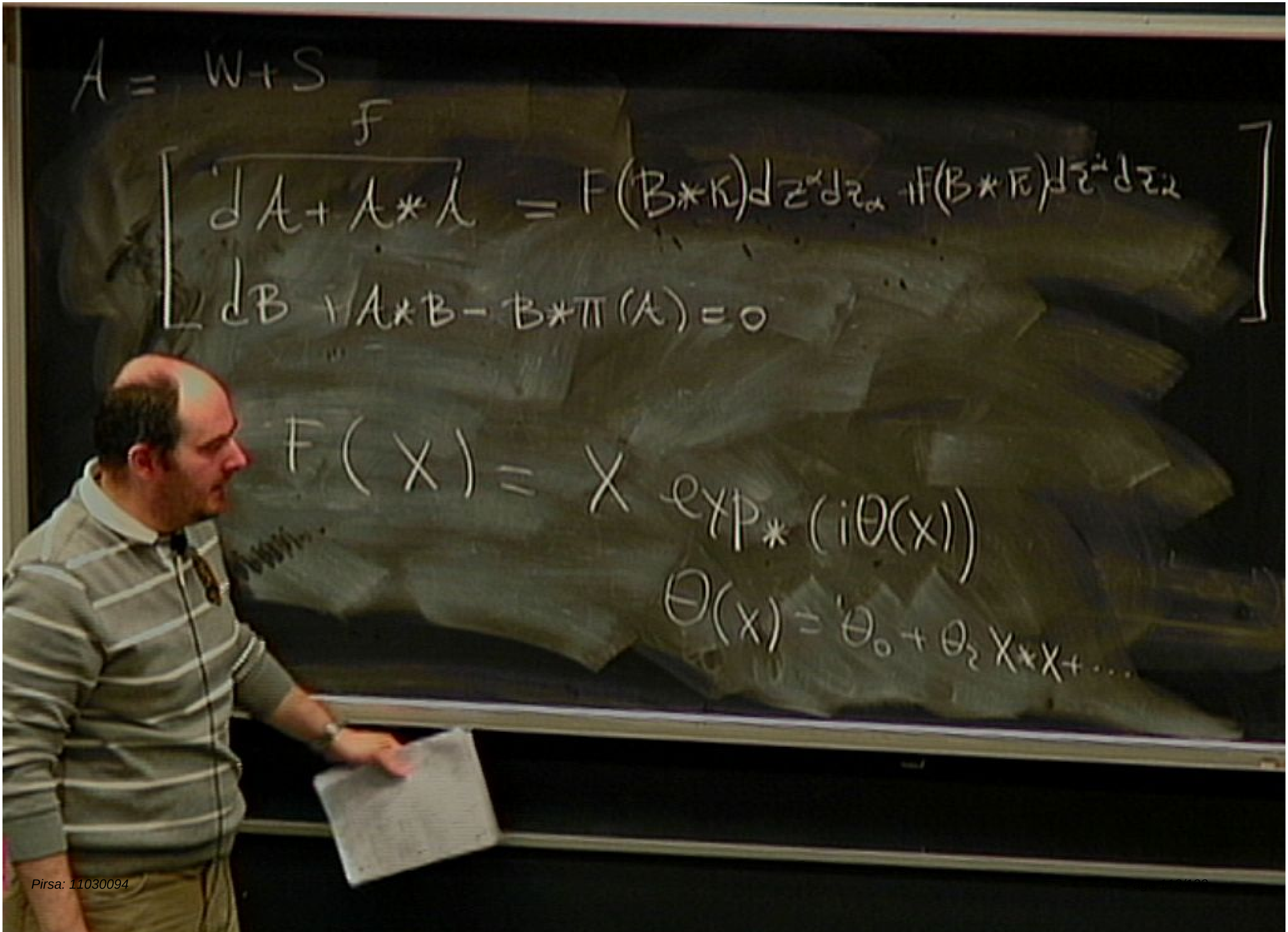
$$\delta B = \epsilon * B - B * \pi(\epsilon)$$

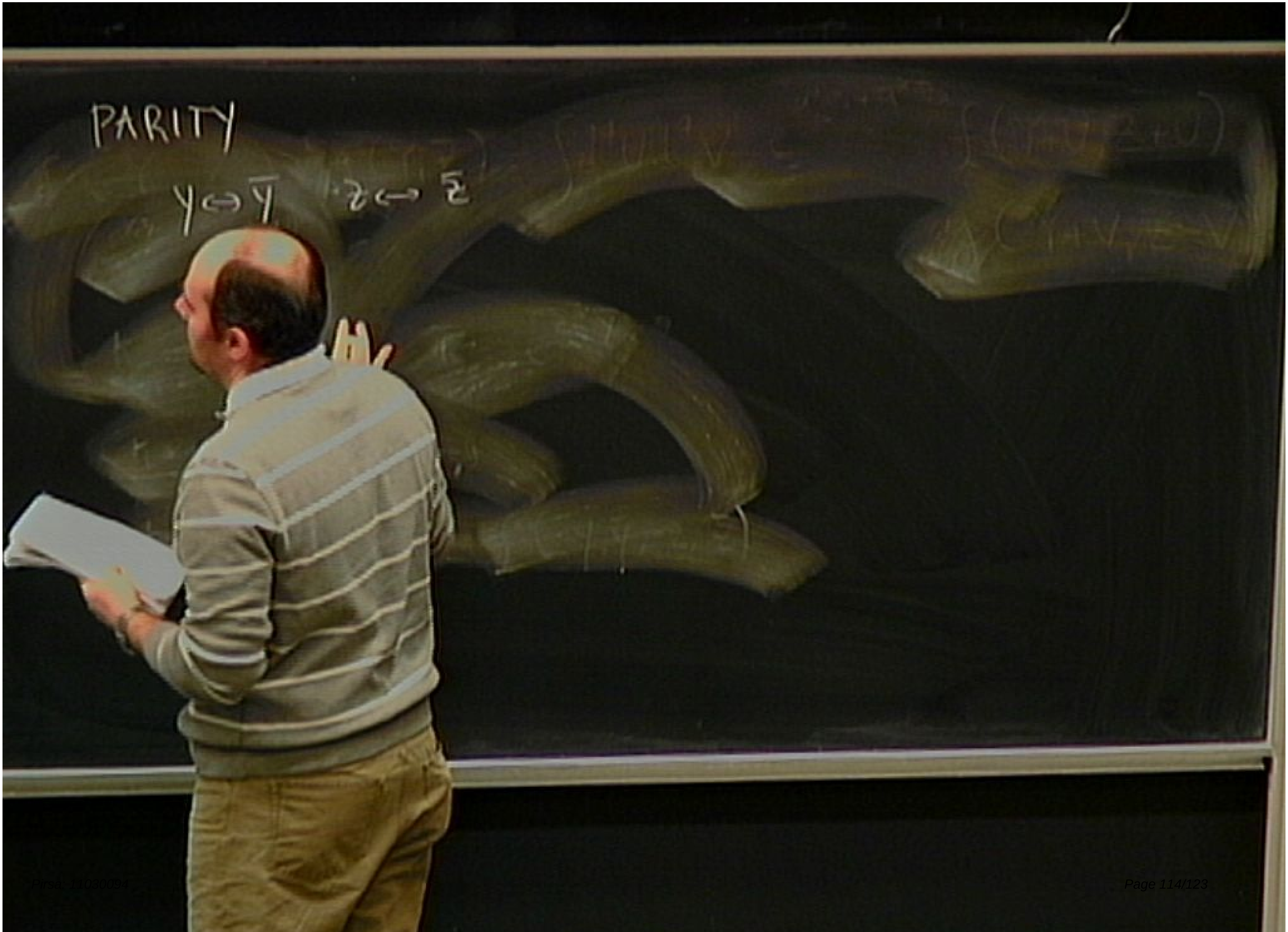
TWISTED ADJOINT

$$\pi(\epsilon(y,\bar y, z, \bar z)) = \epsilon(-y, \bar y, -z, \bar z) = k * \epsilon * k$$


A = W+S
F
dA + A*A = F(B*K)dz^α dz_α + F(B*K)dz^α dz_α
dB + A*B − B*π(A) = 0
F(X) = X
* (iθ(X))
= θ_0 +


A = W+S
F
dA + A*A = F(B*K)dz^α dz_α + F(B*K̄)dz̄^α̇ dz̄_α̇
dB + A*B − B*π(A) = 0
F(X) = X exp_*(iθ(X))
θ(X) = θ_0 + θ_2 X*X + ...


A = W + S
F
dA + A*A = F(B*K) dz^α dz_α + F(B*K̄) dz̄^α dz̄_α
dB + A*B − B*π(A) = 0
F(X) = X exp_*(iΘ(X))
Θ(X) = θ_0 + θ_2 X*X + ...

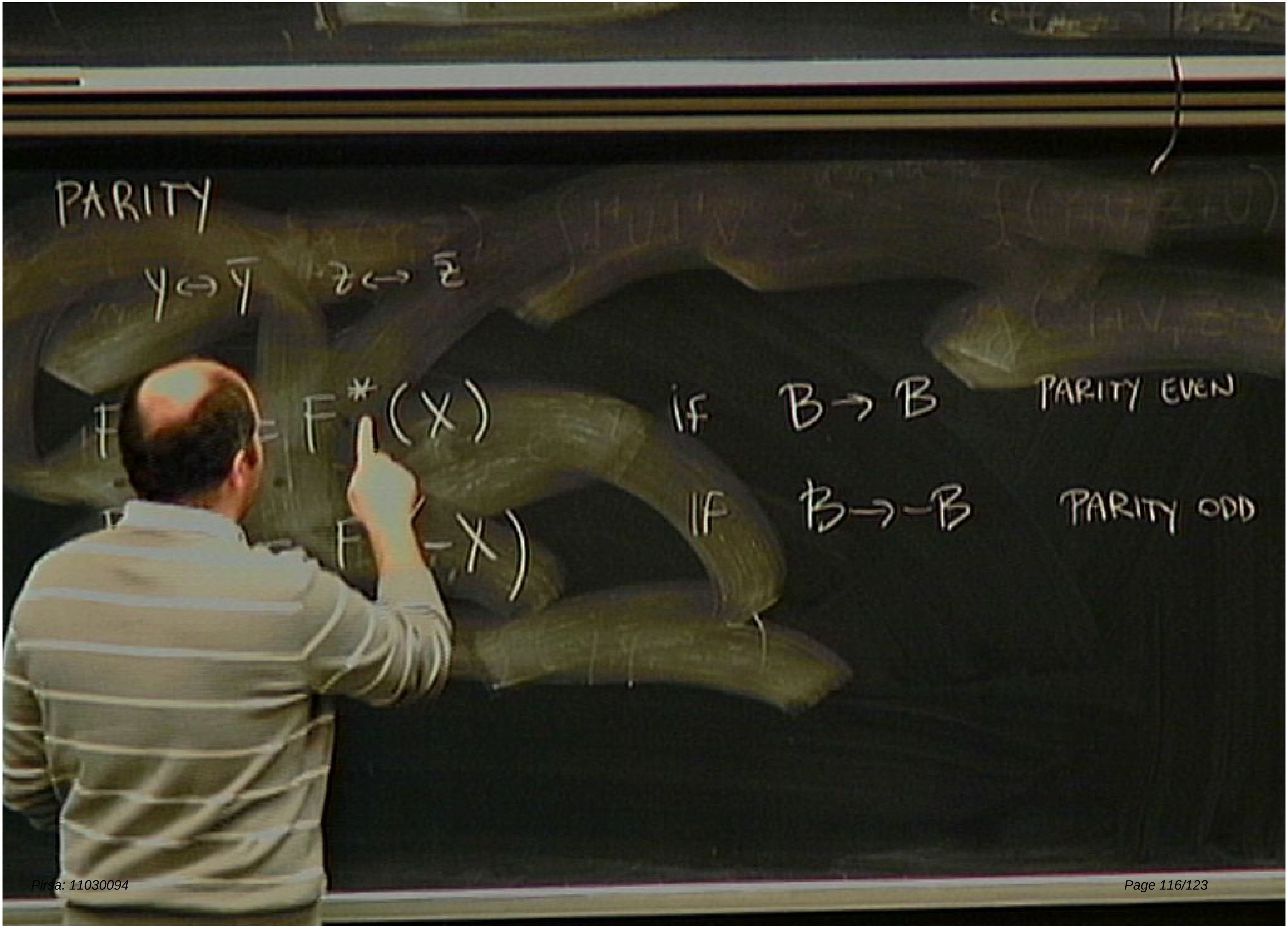

PARITY
$y \leftrightarrow \bar{y}$ $z \leftrightarrow \bar{z}$

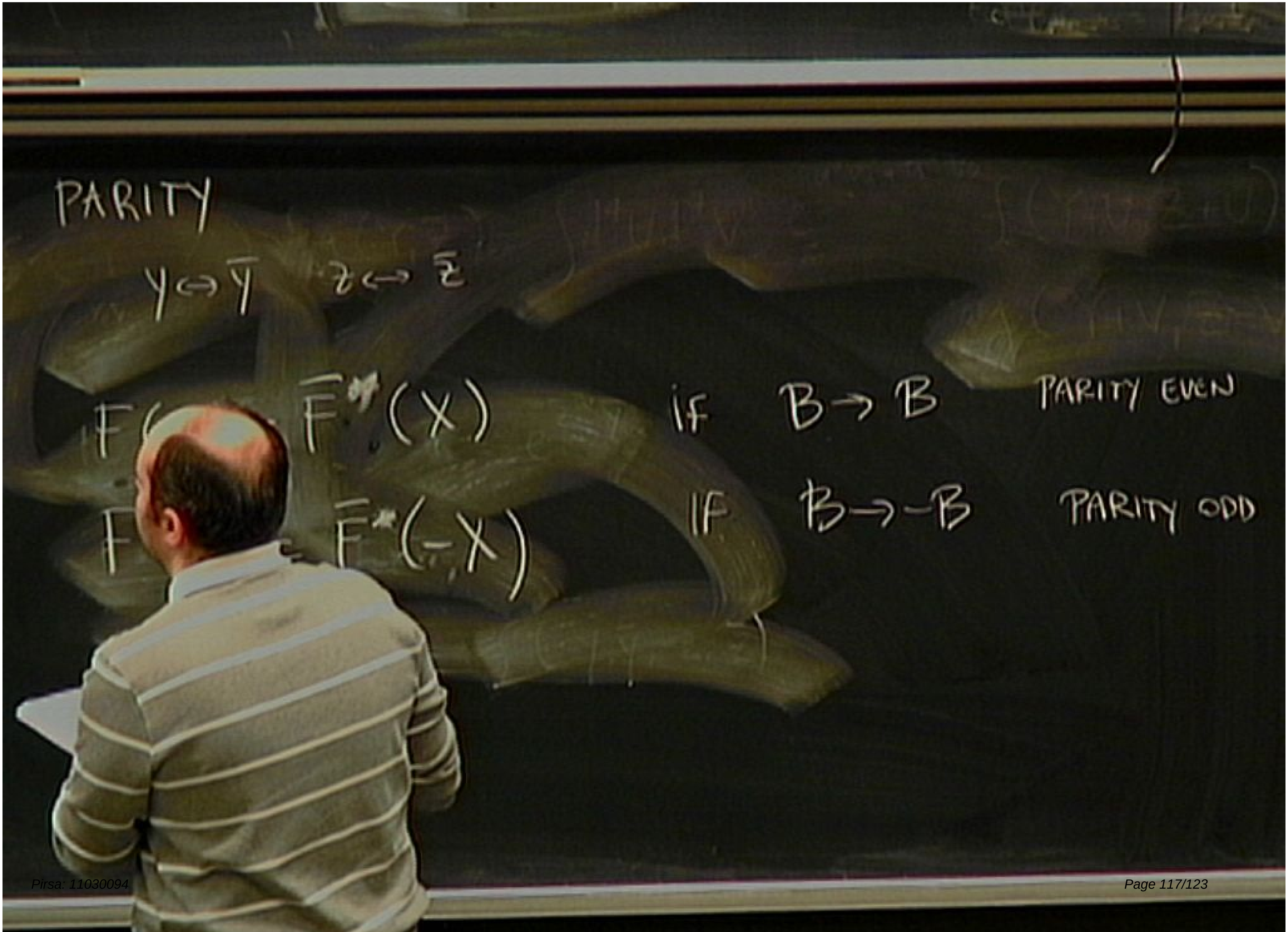
$$A = W + S$$

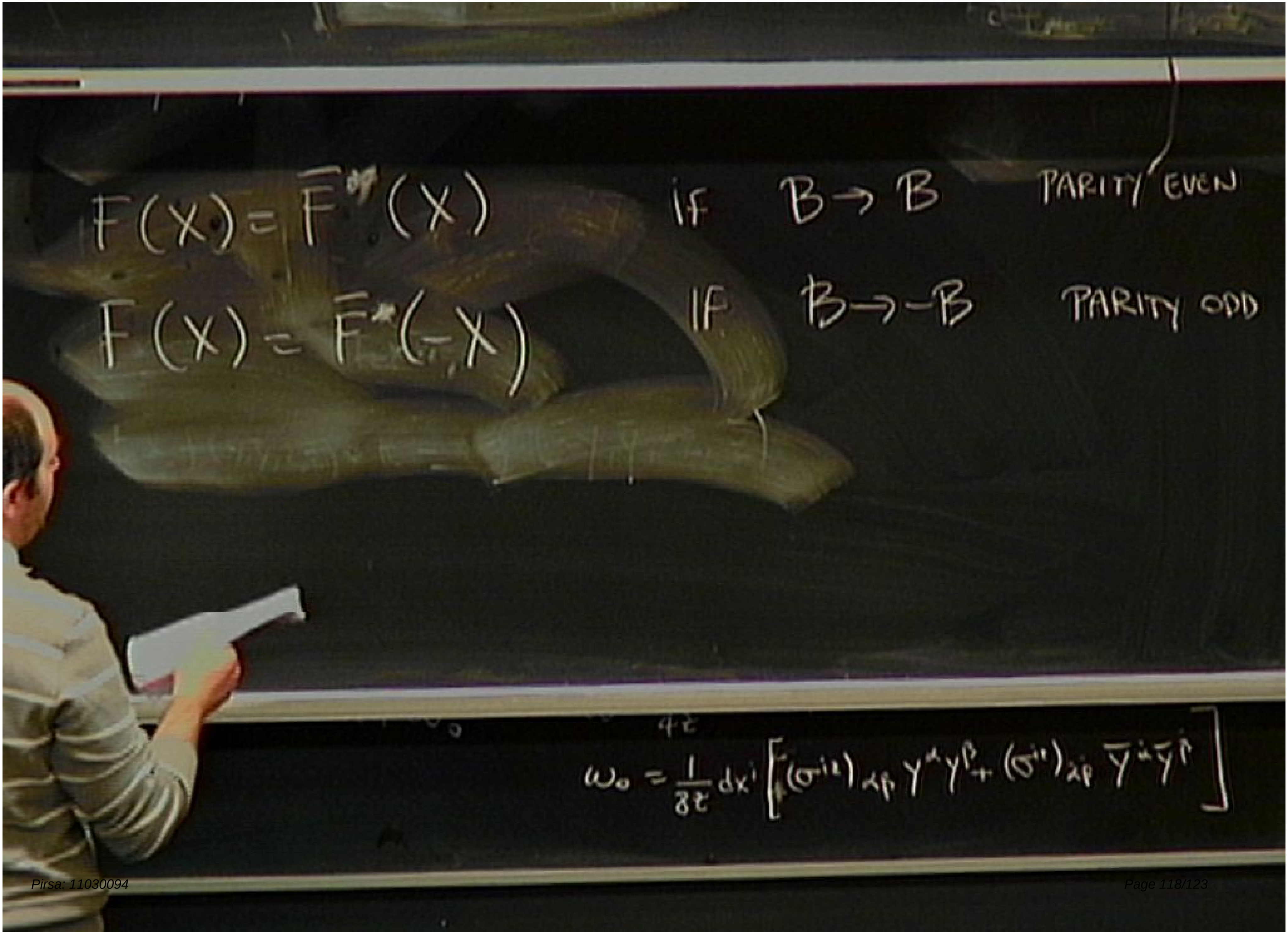
$$\left[\begin{array}{l} \overbrace{dA + A * A}^{F} = F(B * K)\, dz^{\alpha} dz_{\alpha} + \bar{F}(B * \bar{K})\, d\bar{z}^{\alpha} d\bar{z}_{\alpha} \\ dB + A * B - B * \pi(A) = 0 \end{array}\right]$$

$$F(X) = X \exp_{*}(i\theta(X))$$

$$\theta(X) = \theta_0 + \theta_2 X * X + \ldots$$


PARITY
y ↔ ȳ z ↔ z̄
if B → B PARITY EVEN
IF B → -B PARITY ODD


PARITY
y ↔ ȳ
z ↔ z̄
F*(X)
if B → B
PARITY EVEN
F*(-X)
IF B → -B
PARITY ODD


F(X) = F*(X)   IF B → B   PARITY EVEN
F(X) = F*(−X)   IF B → −B   PARITY ODD
ω₀ = 1/8π dx^i [(σ^{i2})_{αβ} y^α y^β + (σ̄^{i0})_{α̇β̇} ȳ^α̇ ȳ^β̇]

$y \leftrightarrow \bar{y}$ $\quad z \leftrightarrow \bar{z}$

1) $F(x) = F^*(x)$ if $B \to B$ PARITY EVEN

2) $F(x) = F^*(-x)$ IF $B \to -B$ PARITY ODD

1) $\theta(x) = 0$ $\quad F(x) = x$

2) $\theta(x) = \pi/2$ $\quad F(x) = ix$

$$A = W + S$$

$$\left[\begin{array}{l} \overbrace{dA + A * A}^{F} = i\,(B * K)\,dz^{\alpha} dz_{\alpha} - i\,(B * \bar{K})\,d\bar{z}^{\alpha} d\bar{z}_{\alpha} \\ dB + A * B - B * \pi(A) = 0 \end{array}\right]$$

$$F(X) = X \exp_*(i\theta(x))$$

$$\theta(x) = \theta_0 + \theta_2\, X * X + \dots$$

$y \leftrightarrow \bar{y} \quad z \leftrightarrow \bar{z}$

1) $F(x) = F^*(x)$ IF $B \to B$ PARITY EVEN

2) $F(x) = F^*(-x)$ IF $B \to -B$ PARITY ODD

1) $\theta(x) = 0 \quad F(x) = X$ "TYPE A"

2) $\theta(x) = \pi/2 \quad F(x) = iX$ "TYPE B"

$y \leftrightarrow \bar{y} \quad z \leftrightarrow \bar{z}$

1) $F(x) = \bar{F}^*(x)$ if $B \to B$ PARITY EVEN

2) $F(x) = \bar{F}^*(-x)$ IF $B \to -B$ PARITY ODD

1) $\theta(x) = 0 \quad F(x) = X$ "TYPE A" $\to$ SCALAR VECTOR M...

2) $\theta(x) = \pi/2 \quad F(x) = iX$ "TYPE B" $\to$ FERMIONIC VECTOR MODELS

PARITY

$y \leftrightarrow \bar{y} \quad z \leftrightarrow \bar{z}$

1) $F(x) = \bar{F}^{*}(x)$ if $B \to B$ PARITY EVEN

2) $F(x) = \bar{F}^{*}(-x)$ IF $B \to -B$ PARITY ODD

1) $\theta(x) = 0 \quad F(x) = x$ "TYPE A" → SCALAR VECTOR MODELS

2) $\theta(x) = \pi/2 \quad F(x) = ix$ "TYPE B" → FERMIONIC VECTOR MODELS