

Title: Higher Spin Theories - Lecture 1

Date: Mar 24, 2011 09:30 AM

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Abstract:

# HIGHER SPINS AND HOLOGRAPHY

Res.

hp-th/9611024 }  
/ 9910096 } Vasiliev (ADS)

/ 0210114 Klebanov-Polyakov

/ 030540 Sergio Sundell

0912.3462 }  
1004.3736 } SG, Yin

# HIGHER SPINS AND HOLOGRAPHY

Ref.

hep-th/9611024 } Vasiliev (ADS)  
• / 9910096 }

• / 0210114 Klebanov-Polyakov

• / 030540 Sergio Sundell

0912.3462 } SG, Yin  
1004.3736 }



# HIGHER SPINS AND HOLOGRAPHY

Ref.

hep-th/9611024 } Vasiliev (ADS)  
/ 9910096 }

/ 021011

/ 0305

0912.2450

1008

FREE N-VECTOR  $\phi^i$   $i = 1, \dots, N$



# HIGHER SPINS AND Holography

Ref.

hep-th/9611029 } Vasiliev (ADS)  
 / 99

/ 02 } Pilyavsky  
 / 03 } Sundell

0912  
 1001

FREE N-VECTOR  $\phi^i$   $i=1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$



# HIGHER SPINS AND HLOGRAPHY

Ref.  
hep-th/9611029 } Vasiliev (ADS)  
/ 9910096

/ 0210110

/ 030541

091

FREE N-VECTOR  $\phi^i$   $i=1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$T_\mu = \phi_i^* \partial_\mu \phi^i - (\partial_\mu \phi_i^*) \phi^i$$

Spin 1



# HIGHER SPINS AND HOLOGRAPHY

Ref.

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09123462 } So, Yin  
 1004.3736 }

FREE N-VECTOR  $\phi^i$   $i = 1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$T_\mu = \phi_i^* \partial_\mu \phi^i - (\partial_\mu \phi_i^*) \phi^i$$

$$\partial^\mu T_\mu = 0$$

$$\partial_\mu \partial^\mu \phi^i = 0$$

Spin 1

$$T_{\mu\nu} = \partial_\mu \phi^i \partial_\nu \phi^i$$



$$T_{\mu\nu} = \partial_{(\mu} \phi^{\mu} \partial_{\nu)} \phi^i - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_{\mu} \partial_{\nu} + g_{\mu\nu} \partial^2) \phi^2$$

$$T_{\mu\nu} = \partial_{(\mu} \phi^{\mu} \partial_{\nu)} \phi^{\nu} - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_{\mu} \partial_{\nu} + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^{\mu} T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$



# HIGHER SPINS AND HOLOGRAPHY

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FREE N-VECTOR  $\phi^i$   $i = 1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$T_\mu = \phi_i^* \partial_\mu \phi^i - (\partial_\mu \phi_i^*) \phi^i$$

Spin 1

$$\partial^\mu T_\mu = 0 \quad \partial_\mu \partial^\mu \phi^i = 0$$

$$\Delta(T_\mu) = d - 2 + 1 = d - 1$$

$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$\dots \mu_\nu = \phi_{\mu}^2 \partial_{\nu} \dots \partial_{\mu} \phi^2 + \dots$$

$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$\mu_\nu = \phi_\nu^\mu \partial_\mu \cdot \partial_{\mu\nu} \phi^\nu + \dots$$

=



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^i \partial_{\nu)} \phi^i - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$T_{\mu_1 \dots \mu_s} = \phi_i^i \partial_{\mu_1} \dots \partial_{\mu_s} \phi^i + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)!} (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^i)$$

$$T_{\mu\nu} = \partial_{(\mu} \phi^{\nu)} \partial_{\nu)} \phi^{\mu} - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_{\mu} \partial_{\nu} + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^{\mu} T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$T_{\mu_1 \dots \mu_s} = \phi^{\nu} \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi^{\nu}) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$J_{\mu_1 \dots \mu_s} = \phi_{\nu}^2 \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_{\nu}^2) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$

$$\partial^\mu J_{\mu\mu_2 \dots \mu_s} = 0$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$\mathcal{J}_{\mu_1 \dots \mu_s} = \phi_{\nu}^2 \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_{\nu}^2) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$

$$\partial^\mu \mathcal{J}_{\mu\mu_2 \dots \mu_s} = 0$$

$$\Delta(\mathcal{J}_n) = d-2+s$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^{\mu} \partial_{\nu)} \phi^{\nu} - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_{\mu} \partial_{\nu} + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^{\mu} T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$J_{\mu_1 \dots \mu_s} = \phi_{\nu}^{\nu} \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_{\nu}^{\nu}) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$

$$\partial^{\mu} J_{\mu\mu_2 \dots \mu_s} = 0$$

$$\Delta(J_n) = d-2+s$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$T_{\mu_1 \dots \mu_s} = \phi_{\nu}^2 \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots \quad \text{Tr}(\phi^3)$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_{\nu}^2) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$

$$\partial^\mu T_{\mu\mu_2 \dots \mu_s} = 0$$

$$\Delta(T_n) = d-2+s$$



# HIGHER SPINS AND HOLOGRAPHY

Ref.

hep-th/9611029 } Vasiliev (ANS)  
 / 9910096

/ 0210114 Klebanov-Polyakov  
 / 030540 Sergio-Sendin

0912.3462 } So, Yin  
 1004.3736

FREE N-vector  $\phi^i \quad i=1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$J_\mu = \phi_i^* \partial_\mu \phi^i - (\partial_\mu \phi_i^*) \phi^i$$

Spin 1

$$\partial^\mu J_\mu = 0 \quad \partial_\mu \partial^\mu \phi^i = 0$$

$$\Delta(J_\mu) = d-2+1 = d-1$$



$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2)\partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$\text{Tr}(\phi^2)$$

$$\mathcal{J}_{\mu_1 \dots \mu_s} = \phi_{\nu}^2 \partial_{\mu_1} \dots \partial_{\mu_s} \phi^{\nu} + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_{\nu}^2) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^{\nu}) - \text{traces}$$

$$\partial^\mu \mathcal{J}_{\mu\mu_1 \dots \mu_s} = 0$$

$$\Delta(\mathcal{J}_m) = d-2+s$$



$$(\phi_i^* \phi^i)(\phi_j^* \phi^j) \dots$$

$$\mathcal{J}_0 = \phi_i^* \phi^i$$

$$(\phi_i^* \phi^i)(\phi_j^* \phi^j) \dots$$

$$\mathcal{I}_0 = \phi_i^* \phi^i$$

$$\mathbb{R}^d$$

$$\mathcal{I}_{\mu_1, \dots, \mu_s}$$

$$A[S_{d+1}]$$



$$(\phi_i^* \phi^i)(\phi_j^* \phi^j) \dots$$

$$\mathcal{I}_0 = \phi_i^* \phi^i$$

$$\mathbb{R}^d$$

$$\mathcal{I}_{\mu_1 \dots \mu_s}$$

$$A[S_{d+1}]$$

$$\varphi_{H_1 \dots H_s}$$

$$(\phi_i^* \phi^i)(\phi_j^* \phi^j) \dots$$

$$\mathcal{I}_0 = \phi_i^* \phi^i$$

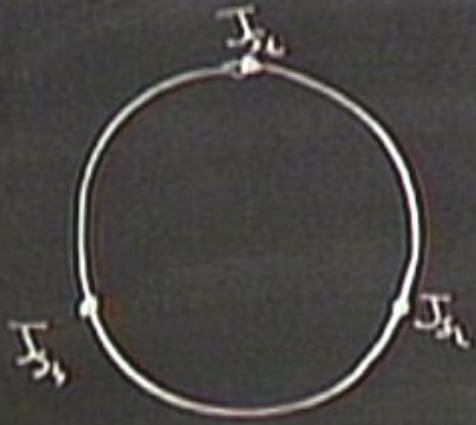
$$\mathbb{R}^d$$

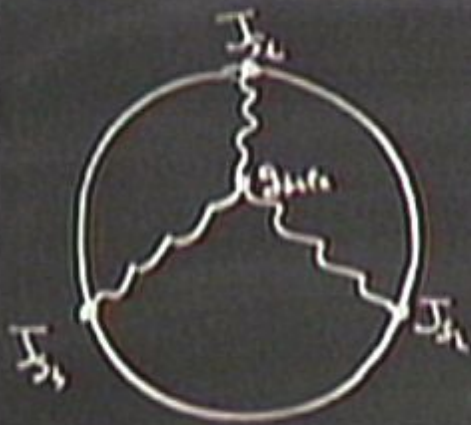
$$\mathcal{I}_{\mu_1 \dots \mu_s}$$

$$A[S_{d+1}]$$

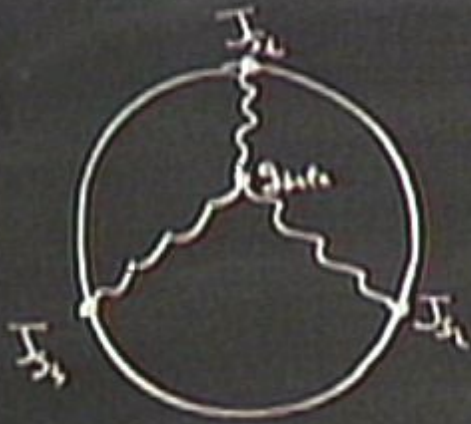
$$\varphi_{H_1 \dots H_s}$$



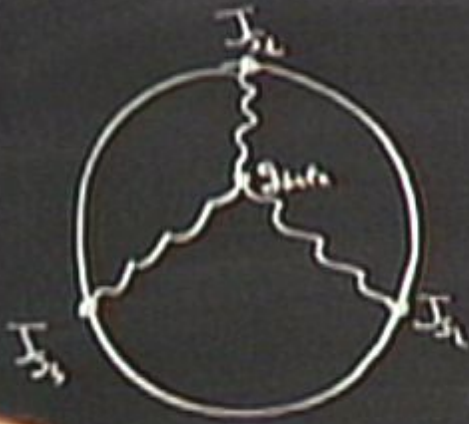








$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

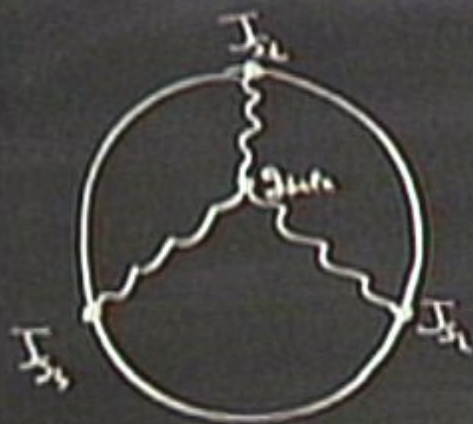


$$J_{s_1} J_{s_2} > \sim O(1)$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu$$

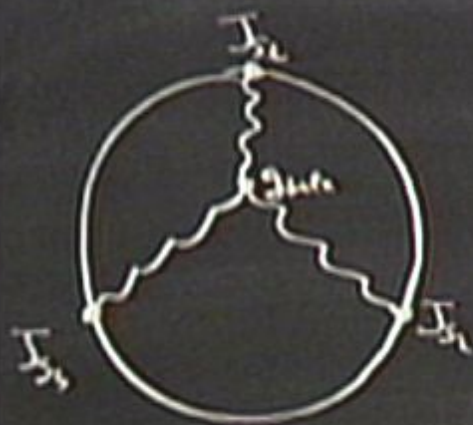
$$\langle \underbrace{\phi_i^a \phi_i^b \phi_j^c \phi_j^d}_{\sim N} \rangle \sim N$$





$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu \quad \langle \underbrace{\phi_i^a \phi_i^a \phi_j^b \phi_j^b}_{\sim N} \rangle \sim N$$



$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{123} \approx \frac{1}{\sqrt{N}}$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu$$

$$\langle \underbrace{\phi_i^\dagger \phi_i(\omega) \phi_j^\dagger \phi_j(\gamma)} \rangle \sim N$$



# HIGHER SPINS AND HOLOGRAPHY

Ref.

hep-th/9611024 } Vasiliev (AdS<sub>4</sub>)  
 / 9910096 }

/ 0210119 KLT  
 / 030540 S

0912.3462 } S  
 1004.3736 }

FREE N-VECTOR  $\phi^i$   $i=1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$T_\mu = \phi_i^* \partial_\mu \phi^i - \frac{1}{2} (\partial_\mu \phi_i^*) \phi^i$$

$$\partial^\mu T_\mu = 0$$

$$\partial_\mu \partial^\mu \phi^i = 0$$

Spin 1

$$(T) \quad d-2+1 = d-1$$



# HIGHER SPINS AND HOLOGRAPHY

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hep-th/9611024 } Vasiliev (AdS)  
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/ 0210114 Klebanov-Polyakov  
 / 030540 Sergio S. Sundell

0912.3462 } So, Yin  
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FREE N-VECTOR  $\phi^i$   $i = 1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

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$$\partial^\mu T_\mu = 0 \quad \partial_\mu \partial^\mu \phi^i = 0$$

$$\Delta(T_\mu) = d - 2 + 1 = d - 1$$

Spin 1



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$

$\Leftrightarrow$

TYPE IIB

$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$



TYPE IIB

$$\frac{R_{\text{AdS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{AdS}}^4}{\ell_{\text{pl}}^4} = N$$



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$

$N \rightarrow \infty$   $\lambda$  fixed



TYPE IIB

$$\frac{R_{\text{AdS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{AdS}}^4}{\ell_{\text{pl}}^4} = N$$

$N=4$  SYM

$$\lambda = g_{YM}^2 N$$

$N$



TYPE IIB

$$\frac{R_{AdS}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{AdS}^4}{\ell_{pl}^4} = N$$

$J \rightarrow \infty$   $\lambda$  fixed

1)  $\lambda \gg 1$

2)  $\lambda \ll 1$



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$



TYPE IIB

$$\frac{R_{\text{AdS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{AdS}}^4}{\ell_{\text{pl}}^4} = N$$

$N \rightarrow \infty$   $\lambda$  fixed

1)  $\lambda \gg 1$

2)  $\lambda < 1$ ,  $\lambda \rightarrow 0$



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$

$N \rightarrow \infty$   $\lambda$  fixed

1)  $\lambda \gg 1$

2)  $\lambda \ll 1$ ,  $\lambda \rightarrow 0$



TYPE IIB

$$\frac{R_{\text{AdS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{AdS}}^4}{\ell_{\text{pl}}^4} = N$$

"TENSIONLESS STRINGS"



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$

$N \rightarrow \infty$   $\lambda$  fixed

1)  $\lambda \gg 1$

2)  $\lambda \ll 1$ ,  $\lambda \rightarrow 0$



TYPE IIB

$$\frac{R_{\text{AdS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{AdS}}^4}{\ell_{\text{pl}}^4} = N$$

"TENSIONLESS STRINGS"

$$J_{\mu_1 \dots \mu_s} = \text{Tr}(\Phi \partial_{\mu_1} \dots \partial_{\mu_s} \Phi)$$



$$J_{\mu_1 \dots \mu_s} = \text{Tr}(\Phi \partial_{\mu_1} \dots \partial_{\mu_s} \Phi)$$

$$\text{Tr}(\Phi \partial \partial \Phi \dots \partial \Phi \dots)$$

$$J_{\mu_1 \dots \mu_s} = \text{Tr}(\Phi \lambda_{\mu_1} \dots \lambda_{\mu_s} \Phi)$$

$$\text{Tr}(\Phi \circ \Phi \dots \Phi \dots)$$



$N=4$  SYM

$$\lambda = g_{\text{YM}}^2 N$$

$N$

$N \rightarrow \infty$   $\lambda$  fixed

1)  $\lambda \gg 1$

2)  $\lambda \ll 1$ ,  $\lambda \rightarrow 0$



TYPE IIB

$$\frac{R_{\text{NS}}^4}{(\alpha')^2} = \lambda$$

$$\frac{R_{\text{NS}}^4}{\ell_{\text{pl}}^4} = N$$

"TENSIONLESS STRINGS"





$$T_{\mu\nu} = \partial_{(\mu} \phi_{\nu)}^2 - \frac{1}{4} \frac{1}{d-1} ((d-2) \partial_\mu \partial_\nu + g_{\mu\nu} \partial^2) \phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 \quad g^{\mu\nu} T_{\mu\nu} = 0 \quad \Delta(T_{\mu\nu}) = d$$

$$\text{Tr}(\phi^3)$$

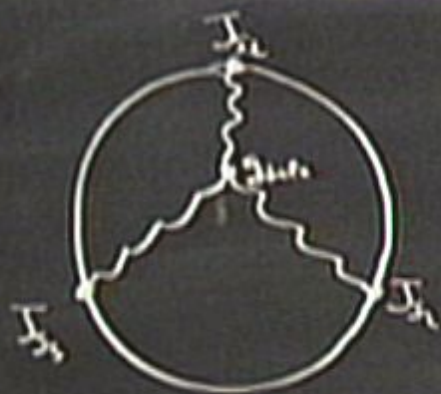
$$= \phi_i^k \partial_{\mu_1} \dots \partial_{\mu_n} \phi^i + \dots$$

$$= \sum_{k=0}^s \frac{(-1)^k}{k! (k + \frac{d-2}{2})! (s-k)! (s-k + \frac{d-2}{2})!} (\partial_{\mu_1} \dots \partial_{\mu_k} \phi_i^k) (\partial_{\mu_{k+1}} \dots \partial_{\mu_s} \phi^i) - \text{traces}$$

$$\mu_1 \dots \mu_s = 0$$

$$\Delta(\mathcal{J}_n) = d-2+s$$



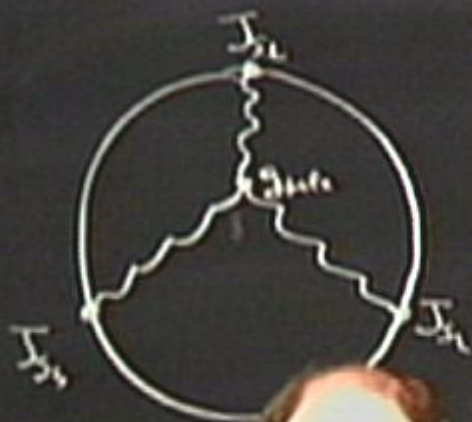


$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{123} \approx \frac{1}{\sqrt{N}}$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu$$

$$\langle \underbrace{\phi_i^a \phi_i^a \phi_j^b \phi_j^b}_{\sim N} \rangle \sim N$$

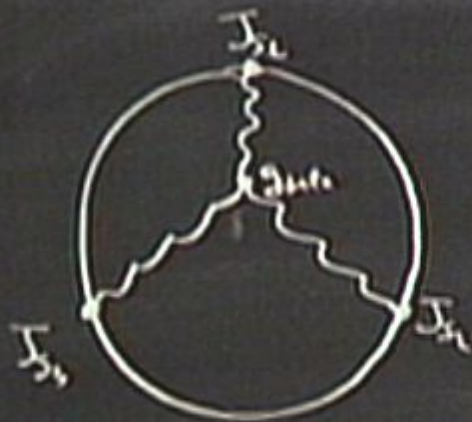


$$\langle J_{s_1}, J_{s_2}, J_{s_3} \rangle$$

$$\langle J_{s_1}, J_{s_2}, J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \sim \frac{1}{\sqrt{N}}$$

$$\langle \underbrace{\phi_i^a \phi_i^b \phi_j^c \phi_j^d}_{\sim N} \rangle \sim N$$





$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \sim \frac{1}{\sqrt{N}}$$

$$J_{s_i} \rightarrow \frac{1}{\sqrt{N}} J_{s_i}$$

$$\langle \underbrace{\phi_i^a \phi_i^a}_{\sim 1} \phi_j^b \phi_j^b \rangle \sim N$$

# STRINGS AND HOLOGRAPHY

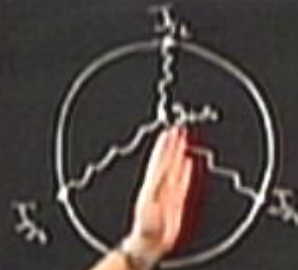
FREE N-VARIABLE  $\phi^i$  ( $i=1, \dots, N$ )

$$S = \int d^d x \partial_\mu \phi^i \partial^\mu \phi^i$$

$$T_{\mu\nu} = \partial_\mu \phi^i \partial_\nu \phi^i - \frac{1}{2} (\partial_\rho \phi^i)^2 \eta_{\mu\nu} \quad \text{sp} = 1$$

$$\partial^\mu T_{\mu\nu} = 0 \quad \partial_\mu \phi^i = 0$$

$$\Delta(T_{\mu\nu}) = d-2+1 = d-1$$



$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle \sim O(1)$$

$$T_{\mu\nu} \sim \frac{1}{\sqrt{d}} T_{\mu\nu}$$

$$\langle T_{\mu\nu} T_{\rho\sigma} T_{\alpha\beta} \rangle \sim \frac{1}{\sqrt{d}} \Rightarrow \partial_\mu$$

$$\langle \phi^i \phi^j \phi^k \phi^l \rangle \sim N$$



# ADS AND Holography

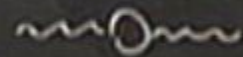
Free N-vector  $\phi^i$  ( $i=1, \dots, N$ )

$$S = \int d^d x \partial_\mu \phi^i \partial^\mu \phi^i$$

$$T_{\mu\nu} = \partial_\mu \phi^i \partial_\nu \phi^i - \frac{1}{2} (\partial_\rho \phi^i)^2 \eta_{\mu\nu}$$

$$\partial^\mu T_{\mu\nu} = 0 \quad \partial_\mu \phi^i = 0$$

$$\Delta(\phi^i) = d - 2 + 1 = d - 1$$

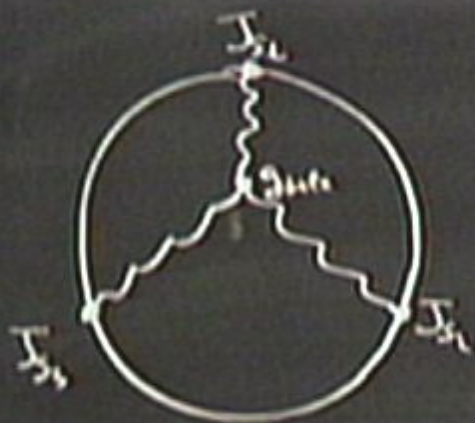


$$\langle T_{21} T_{11} \rangle \sim O(1)$$

$$\langle T_{21} T_{11} T_{11} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow \partial_{\mu\nu}$$

$$T_{\mu\nu} \sim \frac{1}{\sqrt{N}} T_{\mu\nu}$$

$$\langle \phi^i \phi^j \phi^k \phi^l \rangle \sim N$$



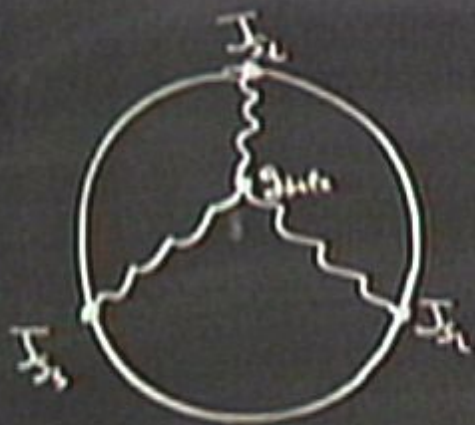
$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \sim \frac{1}{\sqrt{N}}$$

$$J_{s_i} \rightarrow \frac{1}{\sqrt{N}} J_{s_i}$$

$$\langle \underbrace{\phi_i^a \phi_i^a \phi_j^b \phi_j^b}_{(i,j)} \rangle \sim N$$



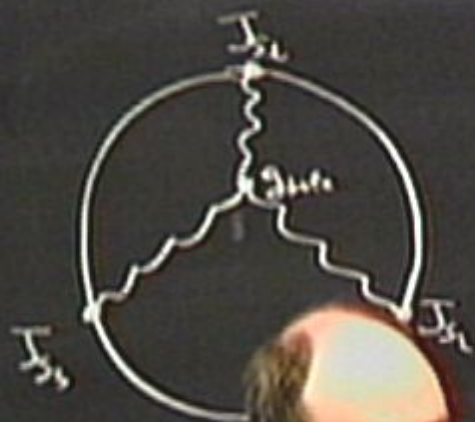


$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{\text{tadpole}} \sim \frac{1}{\sqrt{N}}$$

$$J_{\mu} \rightarrow \frac{1}{\sqrt{N}} J_{\mu}$$

$$\langle \underbrace{\phi_i^{\dagger} \phi^i(\omega) \phi_j^{\dagger} \phi^j(\omega)} \rangle \sim N$$



$$\langle TTT \rangle$$

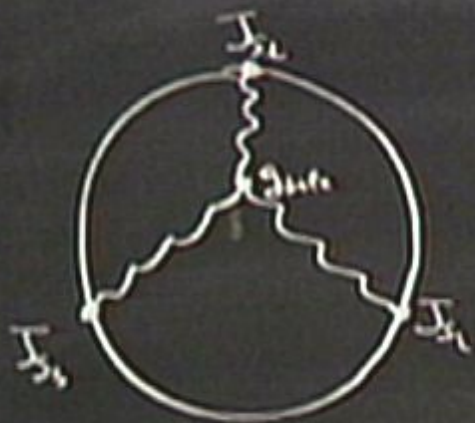
$$\langle J_s \rangle \sim O(1)$$

$$J_s$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \sim \frac{1}{\sqrt{N}}$$

$$\langle \underbrace{\phi_i^a(\omega) \phi_j^a(\omega) \phi_i^b(\gamma) \phi_j^b(\gamma)} \rangle \sim N$$





$$\langle TTT \rangle = a_B \langle TTT \rangle_B + a_F \langle TTT \rangle_F$$

$$\langle J_{s_1} J_{s_2} \rangle \sim O(1)$$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \sim \frac{1}{\sqrt{N}}$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu$$

$$\langle \underbrace{\phi_i^a \phi_i^a \phi_j^b \phi_j^b}_{\sim N} \rangle \sim N$$

Ref.

hep-th/9611024 } Vasiliev (ADS)  
 + / 9910096

4 / 0210114 Klebanov-Polyakov  
 4 / 030540 Sergio Susskind

0912.3462 } So, Yin  
 1004.3736

FREE N-VECTOR  $\phi^i$   $i=1 \dots N$

$$S = \int d^d x \partial_\mu \phi^i \partial^\mu \phi^i$$

$$\psi^\dagger \gamma^\mu \partial_\mu \psi$$

$$T_\mu = \phi^i \partial_\mu \phi^i - \phi^i \partial_i \phi^\mu$$

$$\partial^\mu T_\mu = 0$$

$$\Delta(T_\mu) = d-2$$



Ref.

hep-th/9611024 } Vasiliev (ANS)

• / 9910096

• / 0210119 Klebanov-Papadopoulos

• / 030540 Sergio Susskind

0912.3462

1004.3736 } So, Yin

FREE N-VECTOR  $\phi^i$   $i=1, \dots, N$

$$S = \int d^d x \partial_\mu \phi_i^* \partial^\mu \phi^i$$

$$\psi^i \gamma^\mu \partial_\mu \psi^i$$

$$T_\mu = \phi_i^* \partial_\mu \phi^i - \phi (\partial_\mu \phi_i^*) \phi^i$$

Spin 1

$$\partial^\mu T_\mu = 0 \quad \partial_\mu \partial^\mu \phi^i = 0$$

$$\Delta(T_\mu) = d-2+1 = d-1$$

Ref.

hep-th/9611024 } Vasiliev (ANS)

• / 9910096

• / 0210114 Klebanov-Papadopoulos

• / 030540 Sergio Susskind

0912.3462

1004.3736 } So, Yin

FREE N-VECTOR  $\phi^i$   $i = 1, \dots, N$

$$S = \int d^d x \partial_\mu \phi^i \partial^\mu \phi^i$$

$$\psi^i \gamma^\mu \partial_\mu \psi^i$$

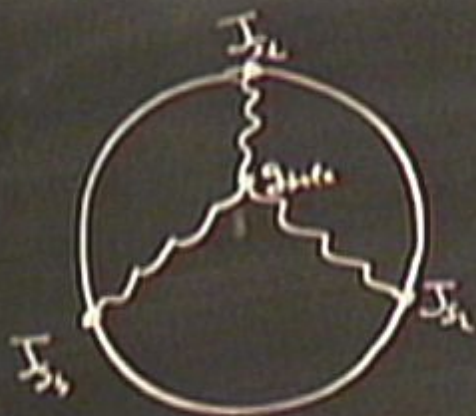
$$T_\mu = \phi^i \partial_\mu \phi^i - \frac{1}{2} (\partial_\mu \phi^i)^2$$

Spin 1

$$\partial^\mu T_\mu = 0 \quad \partial_\mu \partial^\mu \phi^i = 0$$

$$\Delta(T_\mu) = d - 2 + 1 = d - 1$$





$$\langle TTT \rangle = a_B \langle TTT \rangle_B + a_F \langle TTT \rangle_F$$

$$\langle TTT \rangle_{CH} \rightarrow a_B = a_F$$

$$\langle J_{S_1} J_{S_2} \rangle \sim O(1) \quad \langle J_{S_1} J_{S_L} J_{S_1} \rangle \sim \frac{1}{\sqrt{N}} \Rightarrow g_{bulk} \approx \frac{1}{\sqrt{N}}$$

$$J_\mu \rightarrow \frac{1}{\sqrt{N}} J_\mu \quad \langle \underbrace{\phi_i^\dagger \phi_i^\dagger(\omega) \phi_j^\dagger \phi_j^\dagger(\gamma)} \rangle \sim N$$

# HIGHER SPIN GAUGE FIELDS

$$T_{\mu\nu} = \frac{1}{2} (\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \eta_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi)$$

$$T_{\mu\nu} = \frac{1}{2} (\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \eta_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi)$$



# HIGHER SPIN GABGE FIELDS

Massive HS

# HIGHER SPIN GAUGE FIELDS

Massive HS

Fierz-Pauli (139)

Flat space

$$(\square - m^2)\varphi_{\mu_1 \dots \mu_s} = 0$$



# HIGHER SPIN GAUGE FIELDS

Massive HS

Fierz-Pauli (139)

Flat space

$$(\square - m^2) \varphi_{\mu_1 \dots \mu_s} = 0$$

$$\partial^\mu \varphi_{\mu \mu_2 \dots \mu_s} = 0 \quad \varphi^\mu_{\mu \mu_2 \dots \mu_s} = 0$$

(25.11)

Fronsdal ('78)

$$\varphi_{\mu_1 \dots \mu_s}$$

DOUBLE TRACELESS

$$\varphi^{\mu}{}_{\mu}{}^{\nu}{}_{\nu} \dots = 0$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}$$

$$\delta A_{\mu} = \partial_{\mu} \epsilon$$

$$\delta h_{\mu\nu} = \partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}$$

$$\square \varphi_{\mu_1 \dots \mu_s} - s \partial_{(\mu_1} \partial^{\mu} \varphi_{\mu_2 \dots \mu_s)}_{\mu} + \frac{s(s-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s)}^{\mu} =$$



Fronsdal ('78)

$$\varphi_{\mu_1 \dots \mu_s}$$

DOUBLE TRACELESS

$$\varphi^{\mu}{}_{\mu}{}^{\nu}{}_{\nu} \dots = 0$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)} \\ \epsilon^{\mu}{}_{\mu \mu_2 \dots \mu_s} = 0$$

$$\delta A_{\mu} = \partial_{\mu} \epsilon$$

$$\delta h_{\mu\nu} = \partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}$$

$$G_{\mu_1 \dots \mu_s} = \square \varphi_{\mu_1 \dots \mu_s} - s \partial_{(\mu_1} \partial^{\mu} \varphi_{\mu_2 \dots \mu_s)}{}_{\mu} + \frac{s(s-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s)}{}^{\mu}{}_{\mu} =$$

$$\delta G_{\mu_1 \dots \mu_s} = \frac{s(s-1)(s-2)}{2} \partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \epsilon^{\mu}{}_{\mu \mu_4 \dots \mu_s}$$

Fronsdal ('78)

$$\varphi_{\mu_1 \dots \mu_s}$$

DOUBLE TRACELESS

$$\varphi^{\mu}{}_{\mu}{}^{\nu}{}_{\nu} \dots = 0$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}$$

$$\epsilon^{\mu}{}_{\mu \mu_2 \dots \mu_s} = 0$$

$$\delta A_{\mu} = \partial_{\mu} \epsilon$$

$$\delta h_{\mu\nu} = \partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}$$

$$G_{\mu_1 \dots \mu_s} = \square \varphi_{\mu_1 \dots \mu_s} - s \partial_{(\mu_1} \partial^{\mu} \varphi_{\mu_2 \dots \mu_s)}_{\mu} + \frac{s(s-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s)}^{\mu}{}_{\mu} =$$

$$\delta G_{\mu_1 \dots \mu_s} = \frac{s(s-1)(s-2)}{2} \partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \epsilon^{\mu}{}_{\mu \mu_4 \dots \mu_s}$$



Fronsdal ('78)

$\varphi_{\mu_1 \dots \mu_s}$

DOUBLE TRACELESS

$$\varphi^{\mu}{}_{\mu}{}^{\nu}{}_{\nu} \dots = 0$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)} \\ \epsilon^{\mu}{}_{\mu \mu_2 \dots \mu_s} = 0$$

$$\delta A_{\mu} = \partial_{\mu} \phi$$

$$\delta h_{\mu\nu} = \partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}$$

$$G_{\mu_1 \dots \mu_s} = \Box \varphi_{\mu_1 \dots \mu_s} - s \partial_{(\mu_1} \partial^{\mu} \varphi_{\mu_2 \dots \mu_s)} + \frac{s(s-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s)} = 0$$

$$\delta G_{\mu_1 \dots \mu_s} = \frac{s(s-1)(s-2)}{2} \partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \epsilon^{\mu}{}_{\mu \mu_4 \dots \mu_s}$$

$$S = \int \left( \frac{1}{2} \dot{\varphi}^2 \right)$$



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} S(S-1) \varphi_{\mu_1}^{\mu_2 \dots \mu_s} G_{\nu}^{\nu \mu_2 \dots \mu_s}(\varphi) \right)$$

Fronsdal ('78)

4d

$\varphi_{\mu_1 \dots \mu_s}$

DOUBLE TRACELESS

$$\varphi^{\mu}{}_{\mu}{}^{\nu}{}_{\nu}{}_{\mu_3 \dots \mu_s} = 0$$

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}$$

$$\epsilon^{\mu}{}_{\mu \mu_3 \dots \mu_s} = 0$$

$$\delta A_{\mu} = \partial_{\mu} \epsilon$$

$$\delta h_{\mu\nu} = \partial_{\mu} \epsilon_{\nu} + \partial_{\nu} \epsilon_{\mu}$$

$$\square_{\mu_1 \dots \mu_s} \varphi_{\mu_1 \dots \mu_s} - s \partial_{(\mu_1} \delta^{\mu} (\varphi_{\mu_2 \dots \mu_s) \mu} + \frac{s(s-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s) \mu}{}^{\mu} = 0$$

$$\varphi_{\mu_1 \dots \mu_s} = \frac{s(s-1)(s-2)}{2} \partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \epsilon^{\mu}{}_{\mu \mu_4 \dots \mu_s}$$



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_{s-1}}(\varphi) \right)$$

$\pm S$

---

Weinberg (1964)



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi^{\mu_1 \mu_2 \dots \mu_s} G_{\mu_1 \mu_2 \dots \mu_s}^{\nu}(\varphi) \right)$$

$\pm S$

berg (164)





$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_s} G_{\nu}^{\nu \mu_3 \dots \mu_s}(\varphi) \right)$$

$\pm S$

Weinberg (1964)



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi^{\mu_1 \mu_2 \dots \mu_s} G_{\nu \mu_2 \dots \mu_s}^{\nu}(\varphi) \right)$$

$\pm S$

ing (164)

$q \rightarrow 0$

$$A(p_1 \dots p_n; q) =$$





$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_s}(\varphi) \right)$$

$\pm S$

Weinberg (1964)

$q \rightarrow 0$

$$A(p_1 \dots p_N; q) =$$



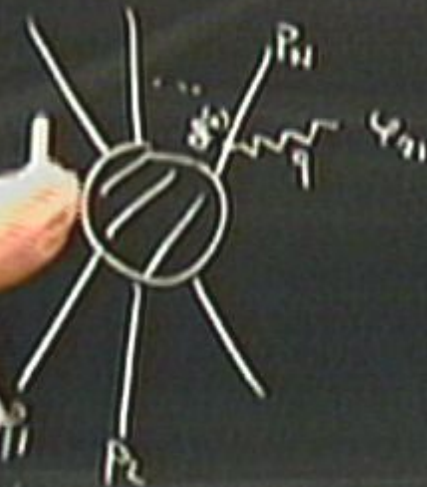
$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_{s-1}}(\varphi) \right)$$

$\pm S$

Weinberg (1964)

$q \rightarrow 0$

$$A(p_1 \dots p_N; q) =$$





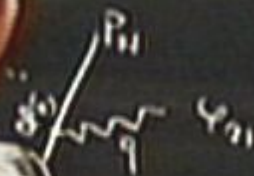
$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_s} G_{\nu}^{\nu \mu_2 \dots \mu_s}(\varphi) \right)$$

$\pm S$

Wein (164)

$q \rightarrow 0$

$$A(p_1 \dots p_n; q) = \sum_{\substack{\delta^{(1)} \\ p_{\mu_1}^i \dots p_{\mu_s}^i \in \mu_{(q)}^{\mu_1 \dots \mu_s}}} \frac{1}{2^{p^1 \cdot q}} A(p_1 \dots p_n)$$



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_s} G_{\nu}^{\nu \mu_2 \dots \mu_s}(\varphi) \right)$$

$\pm S$

Weimb. (64)

$q \rightarrow 0$

$$A(p_1 \dots p_n, q) = \sum_{\substack{p_1^i \dots p_n^i \in \mu_1^{\mu_s}(q)}} \frac{d^{(i)}}{2^{p^1 \cdot q}} A(p_1 \dots p_n)$$



$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_{s-1}}(\varphi) \right)$$

$\pm S$

Weinberg (1964)

$q \rightarrow 0$



$$A(p_1 \dots p_n; q) = \sum_{\substack{\text{diagrams} \\ p_1^i \dots p_n^i \in H_{s-1}^{\mu_1 \dots \mu_{s-1}}(q)}} \frac{1}{2^{p_1 \cdot q}} A(p_1 \dots p_n)$$

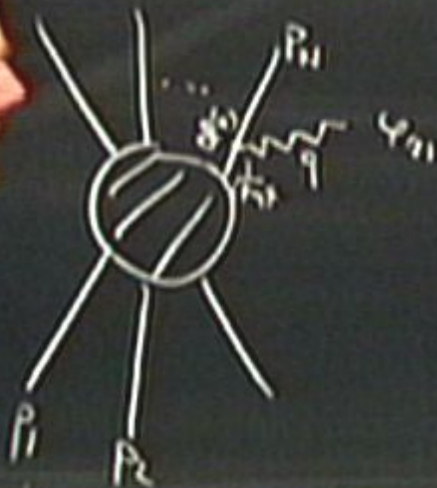
$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_{s-1}}(\varphi) \right)$$

$\pm S$

Weinberg (1964)

$q \rightarrow 0$

$$A(p_1 \dots p_N; q) = \sum_{i=1}^N \frac{\delta^{(i)} \frac{p_{\mu_1}^i \dots p_{\mu_s}^i \in \mu_1 \dots \mu_s(q)}{2p^i \cdot q}}{2p^i \cdot q} A(p_1 \dots p_N)$$



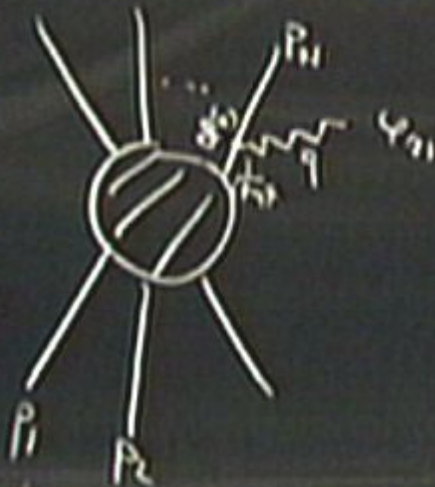


$$S = \int \left( \frac{1}{2} \varphi^{\mu_1 \dots \mu_s} G_{\mu_1 \dots \mu_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{\mu}^{\mu_1 \mu_2 \dots \mu_{s-1}} G_{\nu}^{\nu \mu_2 \dots \mu_{s-1}}(\varphi) \right)$$

$\pm S$

Weinberg (1964)

$q \rightarrow 0$



$$A(p_1 \dots p_N, q) = \sum_{i=1}^N \frac{\delta^{(i)} p_{\mu_1}^i \dots p_{\mu_s}^i \in \mathcal{H}_{\mu_1 \dots \mu_s}^{(q)}}{2p^i \cdot q} A(p_1 \dots p_N)$$

$\pm 5$

Weinberg ('64)

$q \rightarrow 0$



$$A(p_1 \dots p_N; q) = \sum_{i \in 1}^N g^{(i)} \frac{p_{\mu_1}^i \dots p_{\mu_N}^i \in^{\mu_1 \dots \mu_N}(q)}{2p \cdot q} A(p_1 \dots p_N)$$

$$\boxed{\sum_{i \in 1}^N g^{(i)} p_{\mu_1}^i \dots p_{\mu_{N-1}}^i = 0} \quad \in^{\mu_1 \dots \mu_N}(q) = q^{\mu_1} \eta^{\mu_2 \dots \mu_N}$$

c)  $\lambda \ll 1$ ,  $\lambda \rightarrow 0$

"TENSIONLESS STRINGS"



$$S=1$$

$$\sum_{i=1}^N g^{(i)} = 0$$

CHARGE  
CONS.

$$\Delta(T) = 1$$

$$S=2$$

$$\sum_{i=1}^N g^{(i)} p_{\mu}^{(i)} = 0$$

• S=1

$$\sum_{i=1}^N g^{(i)} = 0$$

CHARGE  
CONS.

$$\Delta(T_{\mu\nu}) = 0$$

• S=2

$$\sum_{i=1}^N g^{(i)} p_{\mu}^{(i)} = 0$$

$$g^{(i)} = g, \quad \sum_{i=1}^N p_{\mu}^{(i)} = 0$$

• S=2

$$g^{(i)} = 0$$



Q M1... MS-1

$$S=1$$

$$\sum_{i=1}^N g^{(i)} = 0$$

CHARGE  
CONS.

$$\Delta(T) = 1$$

$$S=2$$

$$\sum_{i=1}^N g^{(i)} p_M^{(i)} = 0$$

$$g^{(i)} = g, \quad \sum_{i=1}^N p_M^{(i)} = 0$$

$$Q^{M_1 \dots M_{S-1}}$$

$$S=2$$

$$g^{(i)} = 0$$



# HIGHER SPIN GAUGE FIELDS

HIS - GRAVITON COUPLING

# HIGHER SPIN GAUGE FIELDS

HS - GRAVITON COUPLING

Aragone - Deser ( '77 )



# HIGHER SPIN GAUGE FIELDS

HIS - GRAVITON COUPLING

Aragone - Deser ( '77 )

# HIGHER SPIN GAUGE FIELDS

HIS - GRAVITON COUPLING

Aragone - Deser (1977)

$$\partial \rightarrow \partial + \Gamma$$



# HIGHER SPIN GAUGE FIELDS

HS - GRAVITON COUPLING

Aragone - Deser (1977)

$$\partial \rightarrow \partial + \Gamma$$

$$\delta S = \int R_{\dots} \epsilon_{\dots} \neq 0$$



# HIGHER SPIN GAUGE FIELDS

HIS - GRAVITON COUPLING

Ameghan - Deser (1979)

$\partial + \mathcal{P}$

$$S = \int R \dots \epsilon \dots \neq 0$$

FLAT SPACE

$\langle p \rangle - h_{\mu\nu}$



# HIGHER SPIN GAUGE FIELDS

HIS - GRAVITON COUPLING

Aragone - Deser (1979)

$$\partial \rightarrow \partial + \Gamma$$

$$\delta S = \int R_{\dots} \epsilon_{\dots} \neq 0$$

1) FLAT SPACE  ~~$\phi_{\mu\nu}$~~

2) NO HS GAUGE SYMM IN ARBITRARY BACKGROUND

AdS

$$S \rightarrow S(\partial \rightarrow \partial + \Gamma) + \frac{1}{R_{\text{AdS}}^2} \varphi \varphi$$



AdS

$$S \rightarrow S(\partial \rightarrow \partial + \Gamma) + \frac{1}{R_{\text{AdS}}^2} \varphi \varphi$$

$$\delta \varphi_{m_1 \dots m_5} = D_{m_1} \epsilon_{m_2 \dots m_5}$$

AdS (dS)

$$S \rightarrow S(\partial \rightarrow \partial + \Gamma) + \frac{1}{R_{\text{AdS}}^2} \varphi \varphi$$

$$\delta \varphi_{n_1 \dots n_5} = D_{n_1} \epsilon_{n_2 \dots n_5}$$



AdS (dS)

$$S \rightarrow S(\partial \rightarrow \partial + \Gamma) + \frac{1}{R_{\text{AdS}}^2} \varphi \varphi$$

$$\delta \varphi_{n_1 \dots n_5} = D_{n_1} \epsilon_{n_2 \dots n_5}$$

Fradicim-Vosiliev ('87)



Fradicim-Vosiliev ('87)

Sint

Fradicim-Vosiliev ('87)

$$L_{int} \sim \frac{1}{\Lambda^{\frac{p+q+r}{2}}} \partial^p \varphi \partial^q \varphi \partial^r \varphi$$



AdS (dS)

$$S \rightarrow S(\partial \rightarrow \partial + \Gamma) + \frac{1}{R_{\text{AdS}}^2} \varphi \varphi$$

$$\delta \varphi_{M_1 \dots M_n} = D_{M_1} \epsilon_{M_2 \dots M_n}$$

Fradicim-Vosiliev ('87)

$$L_{int} \sim \frac{1}{\Lambda^{\frac{p+q+r}{2}}} \partial^p \varphi \partial^q \varphi \partial^r \varphi$$



Fradicim-Vosiliev ('87)

$$L_{int} \sim \frac{1}{\Lambda^{\frac{p+q+r}{2}}} \partial^p \varphi_{(s_1)} \partial^q \varphi_{(s_2)} \partial^r \varphi_{(s_3)}$$

$\Lambda \rightarrow 0$  SINGULAR

Fradkin-Vasiliev ('87)

$$\mathcal{L}_{int} \sim \frac{1}{\Lambda^{\frac{p+q+r}{2}}} \partial^p \varphi_{(s_1)} \partial^q \varphi_{(s_2)} \partial^r \varphi_{(s_3)}$$

$\Lambda \rightarrow 0$  SINGULAR

Vasiliev ('80-'90)



Fradkin-Vasiliev ('87)

$$L_{\text{int}} \sim \frac{1}{\Lambda^{\frac{p+1}{2}}} \partial^p \varphi_{(s)} \partial^q \varphi_{(s)} \partial^r \varphi_{(s)}$$

$\Lambda \rightarrow 0$  SINGULAR

Vasiliev ('80-'90)

CONSISTENT E.O.M. FOR ALL SPINS IN  $\text{AdS}_4$

Fradkin-Vasiliev ('87)

$$L_{\text{int}} \sim \frac{1}{\Lambda^{\frac{p+1}{2}}} \partial^p \varphi_{(s)} \partial^q \varphi_{(s)} \partial^r \varphi_{(s)}$$

$\Lambda \rightarrow 0$  SINGULAR

Vasiliev ('80-'90)

[CONSISTENT E.O.M. FOR ALL SPINS IN  $\text{AdS}_4$   
FULLY NON-LINEAR]



Fradkin-Vasiliev ('87)

$$\mathcal{L}_{\text{int}} \sim \frac{1}{\Lambda^{\frac{p+1}{2}}} \partial^p \varphi_{(s)} \partial^q \varphi_{(s)} \partial^r \varphi_{(s)}$$

$\Lambda \rightarrow 0$  SINGULAR

Vasiliev ('80-'90)

[CONSISTENT E.O.M. FOR ALL SPINS IN  $\text{AdS}_4$   
FULLY NON-LINEAR]



Fradkin-Vasiliev ('87)

$$\mathcal{L}_{\text{int}} \sim \frac{1}{\Lambda^{\frac{p+1}{2}}} \partial^p \varphi_{(s_1)} \partial^q \varphi_{(s_2)} \partial^r \varphi_{(s_3)}$$

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ew ('80-'90)

CONSISTENT E.O.M.

FOR ALL SPINS

IN  $AdS_4$

NON-LINEAR

- BOSONIC  
- SUSY



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$V_8$

80-190)

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# FRAME-LIKE FORMULATION

$$g_{\mu\nu} \rightarrow e_{\mu}^a, \omega_{\mu}^{ab}$$

$$\delta e_{\mu}^a = \partial_{\mu} \epsilon^a + \epsilon^a_b e_{\mu}^b$$



## FRAME-LIKE FORMULATION

$$g_{\mu\nu} \rightarrow (e_\mu^a, \omega_\mu^{ab})$$

$$\delta e_\mu^a = \partial_\mu \epsilon^a + \epsilon^a_b \omega_\mu^b$$

$$T^a = 0 = de^a + \omega^a_b \wedge e^b$$

$$g_{\mu a} = e(\mu, a)$$

CURVATURE

$$R_L^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb}$$

$\Lambda / \Lambda \sim 10^{-16}$   $\Leftrightarrow$  TYPE IIB

$$S_{\text{EH}} = \frac{1}{2\kappa^2} \int e^a \wedge e^b \wedge R_{ab}{}^{cd} \epsilon_{abcd}$$

$SO(3,2)$  CONNECTION

$$W = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}$$

$$dW + W \wedge W = T^a P_a + \frac{1}{2} R^{ab} M_{ab}$$



$$R^{ab} = R_L^{ab} + \Lambda e^a \wedge e^b$$

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$$S_{HM} = \frac{1}{2} \int R^{ab} \wedge R^{cd} \epsilon_{abcd}$$



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$$S_{MM} = \frac{1}{4k\Lambda} \int R^{ab} \wedge R^{cd} \epsilon_{abcd}$$

$$1) \frac{1}{2k_1} \int R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$



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$$\frac{1}{2k\Lambda} \int R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$

$$\frac{\Lambda}{4k\Lambda} \int e^a \wedge e^b \wedge e^c \wedge e^d \sim \int \sqrt{g} \Lambda$$

$$1) \frac{1}{2k_1} \int R_L \Lambda e^{\epsilon} \Lambda e^{\epsilon} \epsilon_{abcd}$$

$$2) \frac{\Lambda}{4k_1} \int e^{\epsilon} \Lambda e^{\epsilon} \Lambda e^{\epsilon} \Lambda e^{\epsilon} \sim \int \sqrt{g} \Lambda$$

$$3) \int R_L \Lambda$$



$$2) \frac{\Lambda}{4k_1} \int \epsilon^{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \sim \int \sqrt{g} \Lambda$$

$$3) \frac{1}{\Lambda} \int P^{ab} R_L^{cd} \epsilon_{abcd}$$

$$S_{\text{HH}} = \frac{1}{4k\Lambda} \int R^{ab} \wedge R^{cd} \epsilon_{abcd}$$

$$1) \quad \frac{1}{2k_1} \int R_L^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$

$$2) \quad \frac{\Lambda}{4k_2} \int e^a \wedge e^b \wedge e^c \wedge e^d \sim \int \sqrt{g} \Lambda$$



$$S_{\text{HH}} = \frac{1}{4\kappa\Lambda} \int R^{ab} \wedge R^{cd} \epsilon_{abcd}$$

$$1) \quad \frac{1}{2\kappa} \int R_L^{ab} \wedge e^c \wedge e^d \epsilon_{abcd}$$

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$$2) \frac{1}{4k_1} \int \epsilon^{\lambda\epsilon^b\lambda\epsilon^c\lambda\epsilon^d} \sim \int \sqrt{g} \Lambda$$

$$3) \frac{1}{\Lambda} \int_M R_L^{ab} \wedge R_L^{cd} \epsilon_{abcd} \sim \chi(M)$$