Title: Upper and lower bounds on the quantum violation of tripartite Bell correlation inequalities

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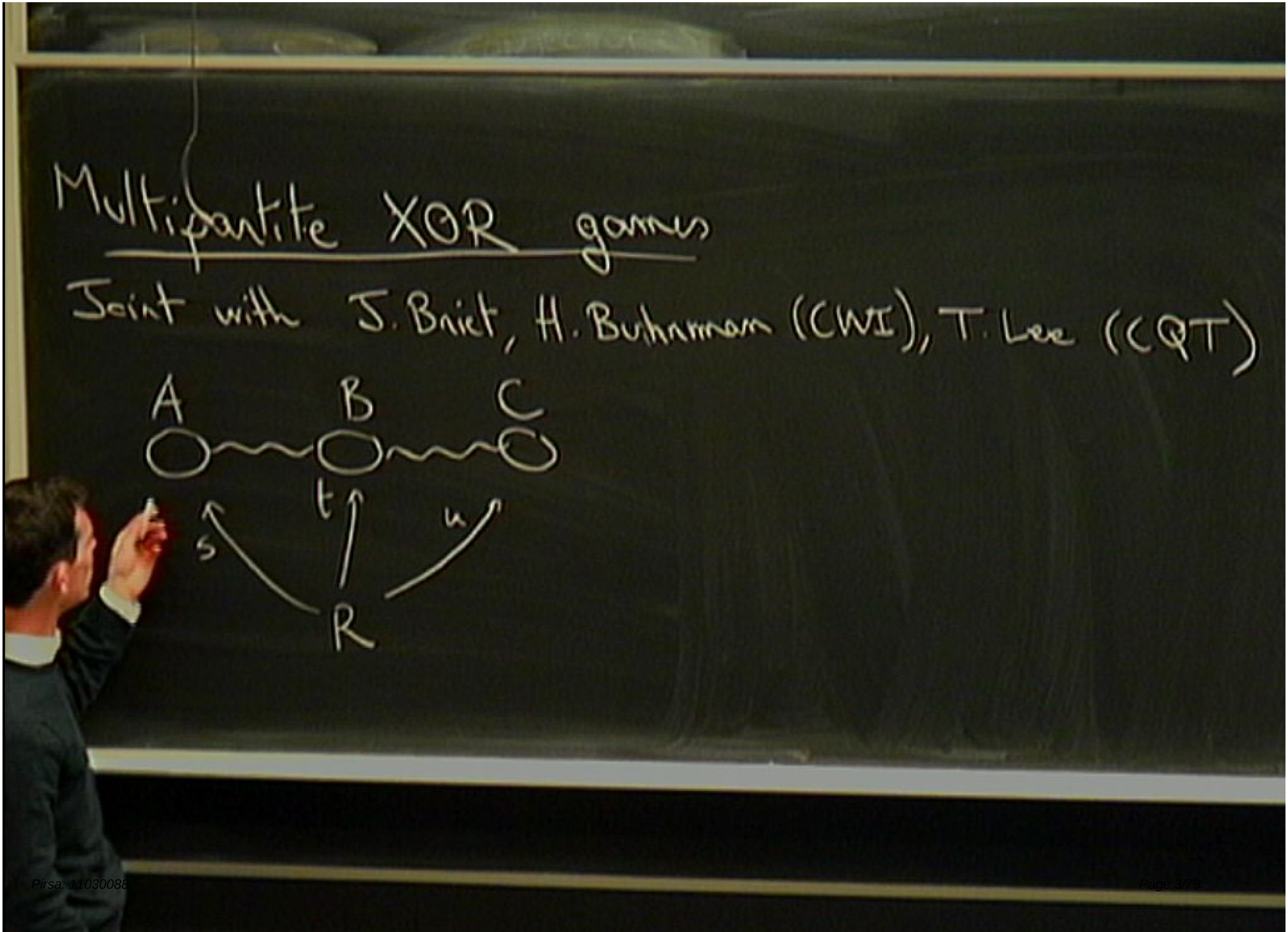
Abstract: Two-party Bell correlation inequalities (that is, inequalities involving only correlations between dichotomic observables at each site, such as the CHSH inequality) are well-understood: Grothendieck's inequality stipulates that the quantum bias can only be a constant factor larger than the classical bias, and the maximally entangled state is always the most nonlocal resource. In part due to the complex nature of multipartite entanglement, tripartite inequalities are much more unwieldy. In a recent breakthrough result, Perez-Garcia et. al. (quant-ph/0702189) showed using tools originating from the study of operator algebras that in this setting the quantum-classical violation could be arbitrarily large. Moreover, they showed that GHZ states could only lead to bounded violations, so that they were not the most non-local states. We extend and simplify their results in a number of ways: - We show that large families of states, including generalizations of GHZ states and stabilizer states, can only lead to bounded violations. - We prove bounds on the maximal quantum-classical violation as a function both of the local dimension (this was already shown in Perez-Garcia et. al., but we give a much simpler proof), and of the number of settings per site. - We provide a simple probabilistic construction of an inequality for which there is an unbounded quantum-classical gap. Our construction is simpler, and has better parameters, than the one in Perez-Garcia et.al. It is essentially optimal in terms of the local dimension of one of the parties, and off by a quadratic factor in terms of the number of settings. In this talk I will survey some of these results, focusing on the tools that have so far been useful for their analysis (and do not involve operator algebras!). Based on joint works with Jop Briet, Harry Buhrman, and Troy Lee. Some of this work is available at arXiv:0911.4007.

Multipartite XOR games

Joint with J. Briet, H. Buhrman (CWI), T. Lee (CQT)


Multipartite XOR games
Joint with J. Briet, H. Buhrman (CWI), T. Lee (CQT)
A
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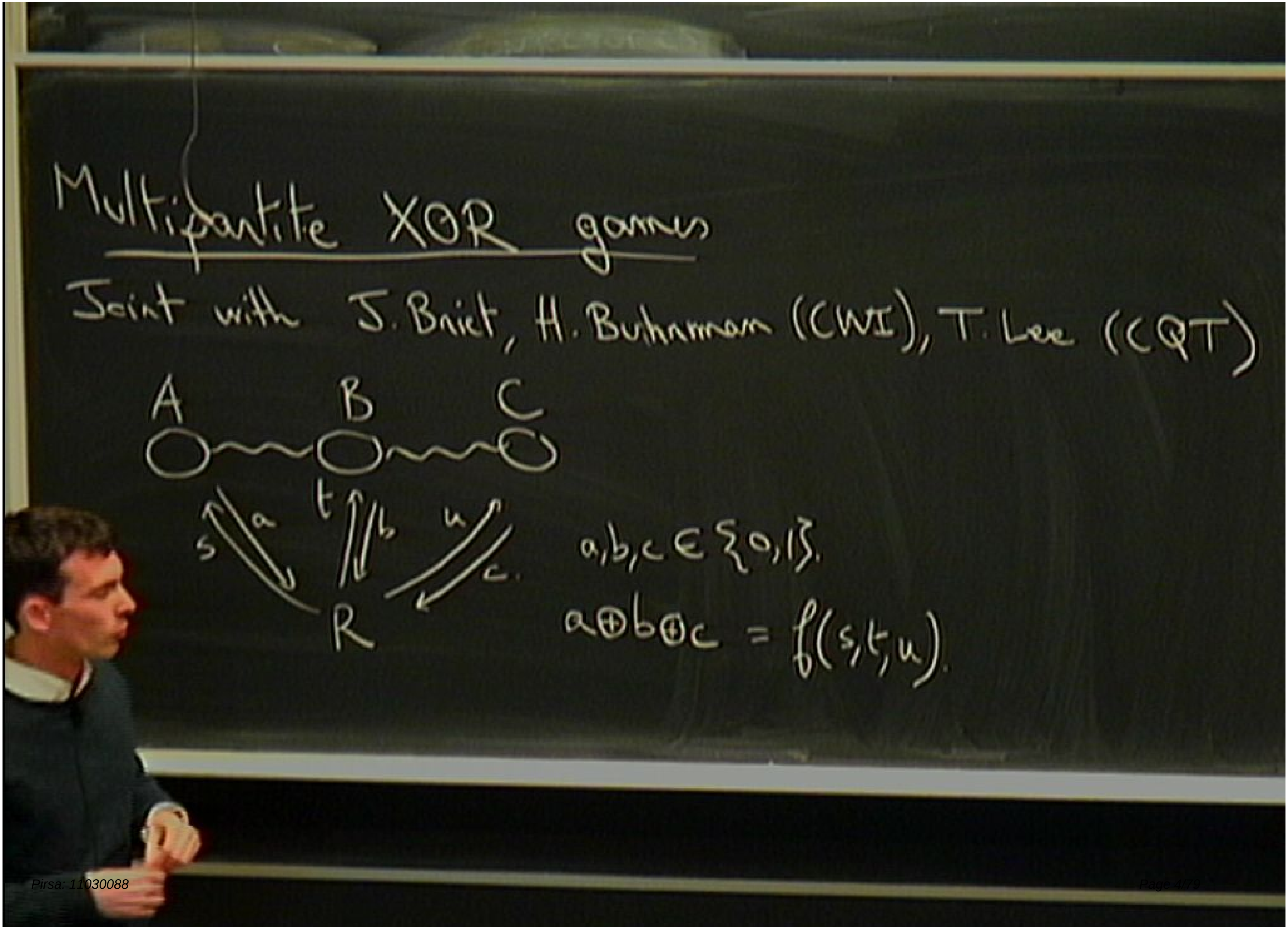

Multipartite XOR games
Joint with J. Briet, H. Buhrman (CWI), T. Lee (CQT)
A
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s
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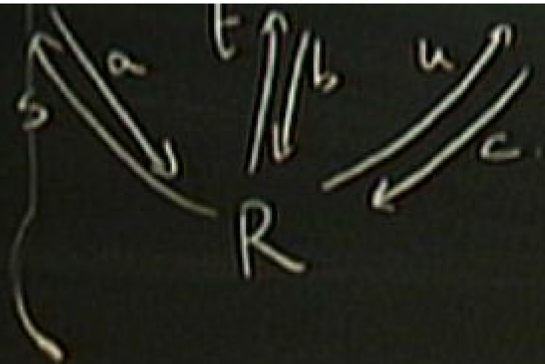
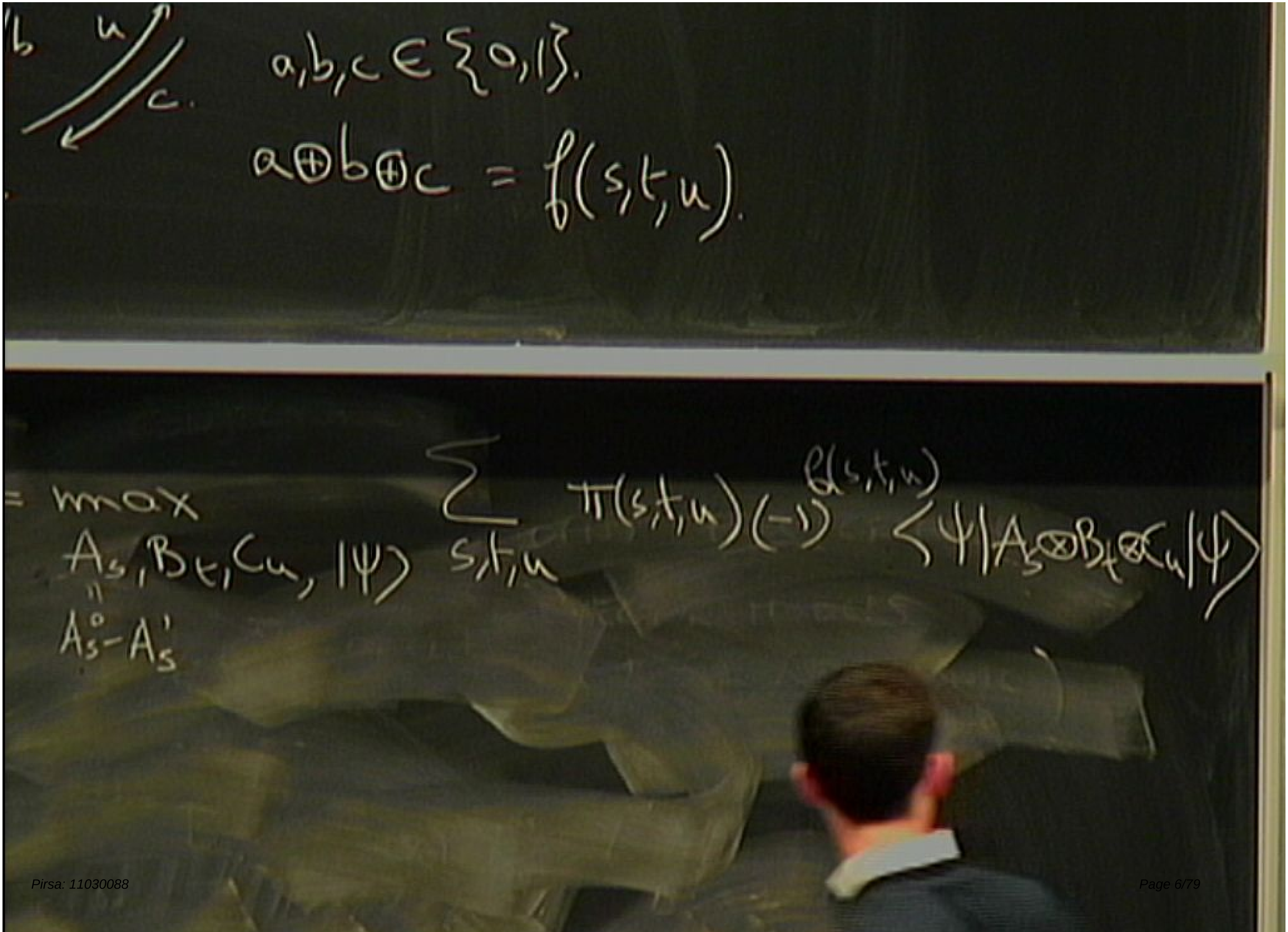
a,b,c ∈ {0,1}.
a⊕b⊕c = f(s,t,u).

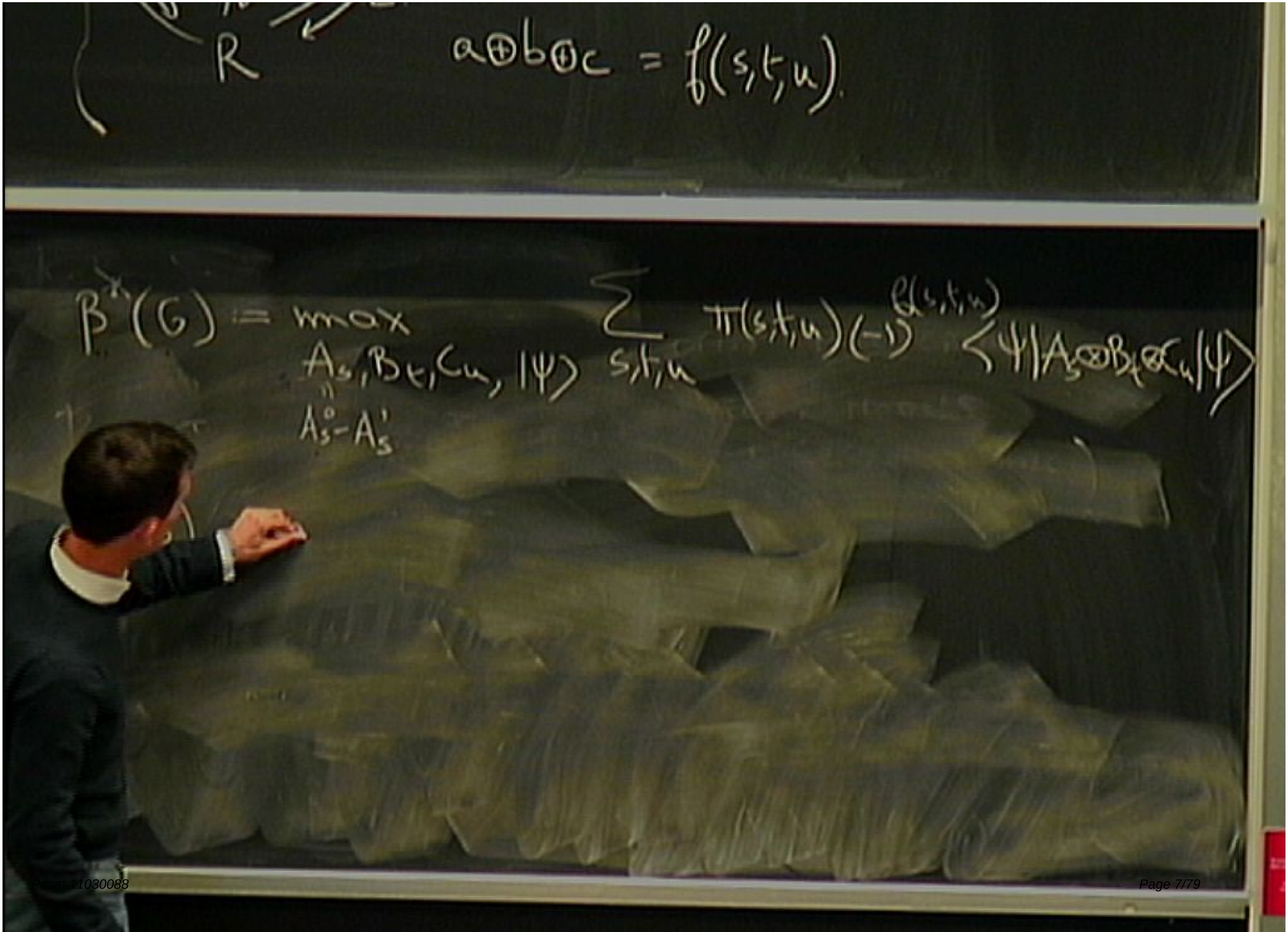


$a, b, c \in \{0,1\}$.

$a \oplus b \oplus c = f(s,t,u)$.

$\beta^{\chi}(G)$


$a, b, c \in \{0,1\}$.
$a \oplus b \oplus c = f(s,t,u)$.
$= \max_{A_s, B_t, C_u, |\psi\rangle} \sum_{s,t,u} \pi(s,t,u)(-1)^{f(s,t,u)} \langle\psi| A_s \otimes B_t \otimes C_u |\psi\rangle$
$A_s = A_s^0 - A_s^1$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$a \oplus b \oplus c = f(s,t,u)$
$\beta^*(G) := \max_{A_s, B_t, C_u, |\psi\rangle} \sum_{s,t,u} \pi(s,t,u)(-1)^{f(s,t,u)} \langle\psi| A_s \otimes B_t \otimes C_u |\psi\rangle$
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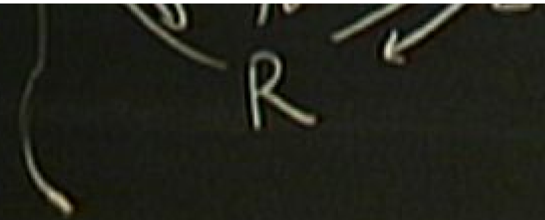

$$a \oplus b \oplus c = f(s,t,u)$$

$$\beta^{*}(G) := \max_{A_s, B_t, C_u, |\psi\rangle} \sum_{s,t,u} \pi(s,t,u)(-1)^{f(s,t,u)} \langle \psi | A_s \otimes B_t \otimes C_u | \psi \rangle$$

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$$\beta_c(G) = \max$$

R

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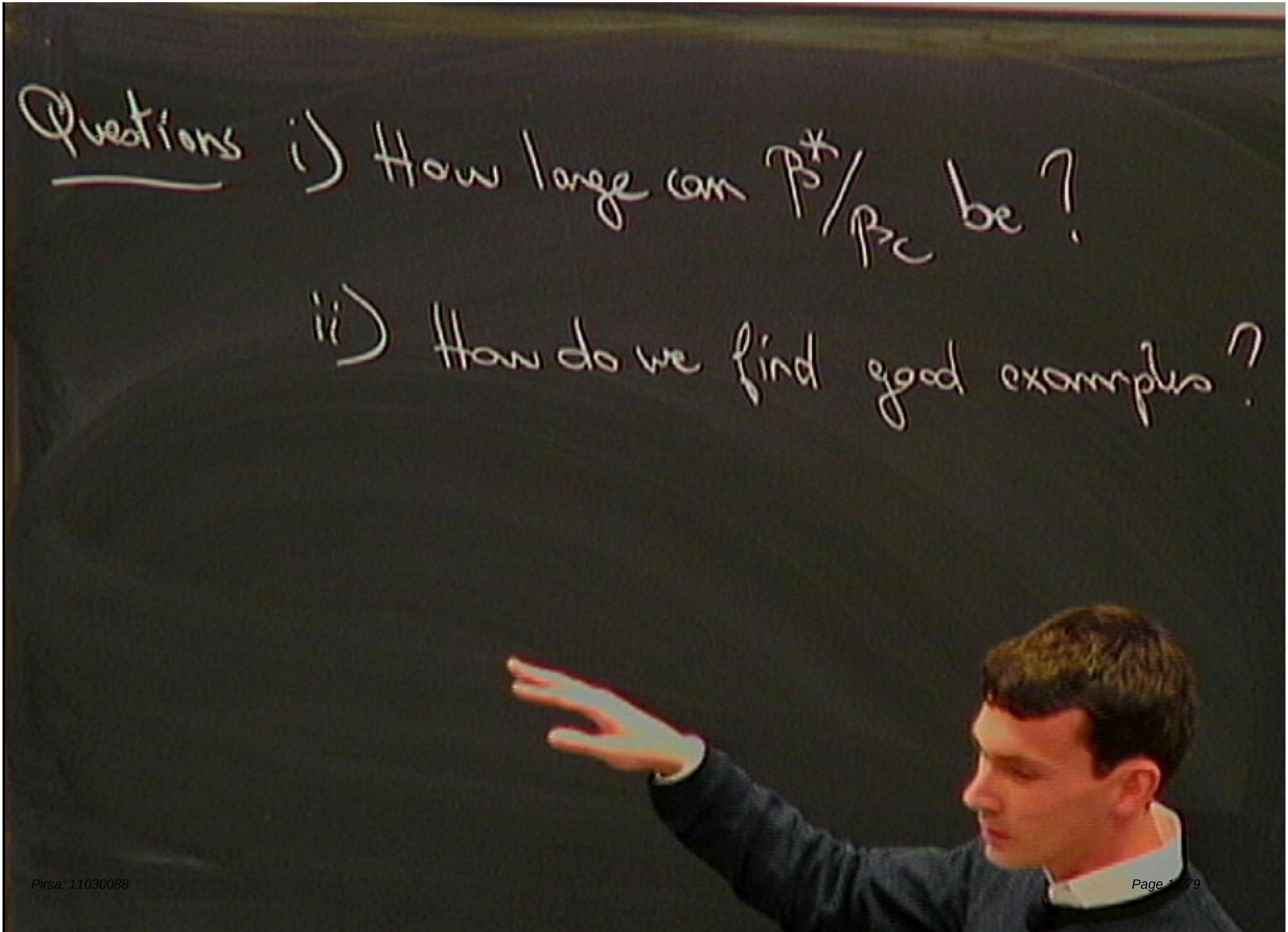
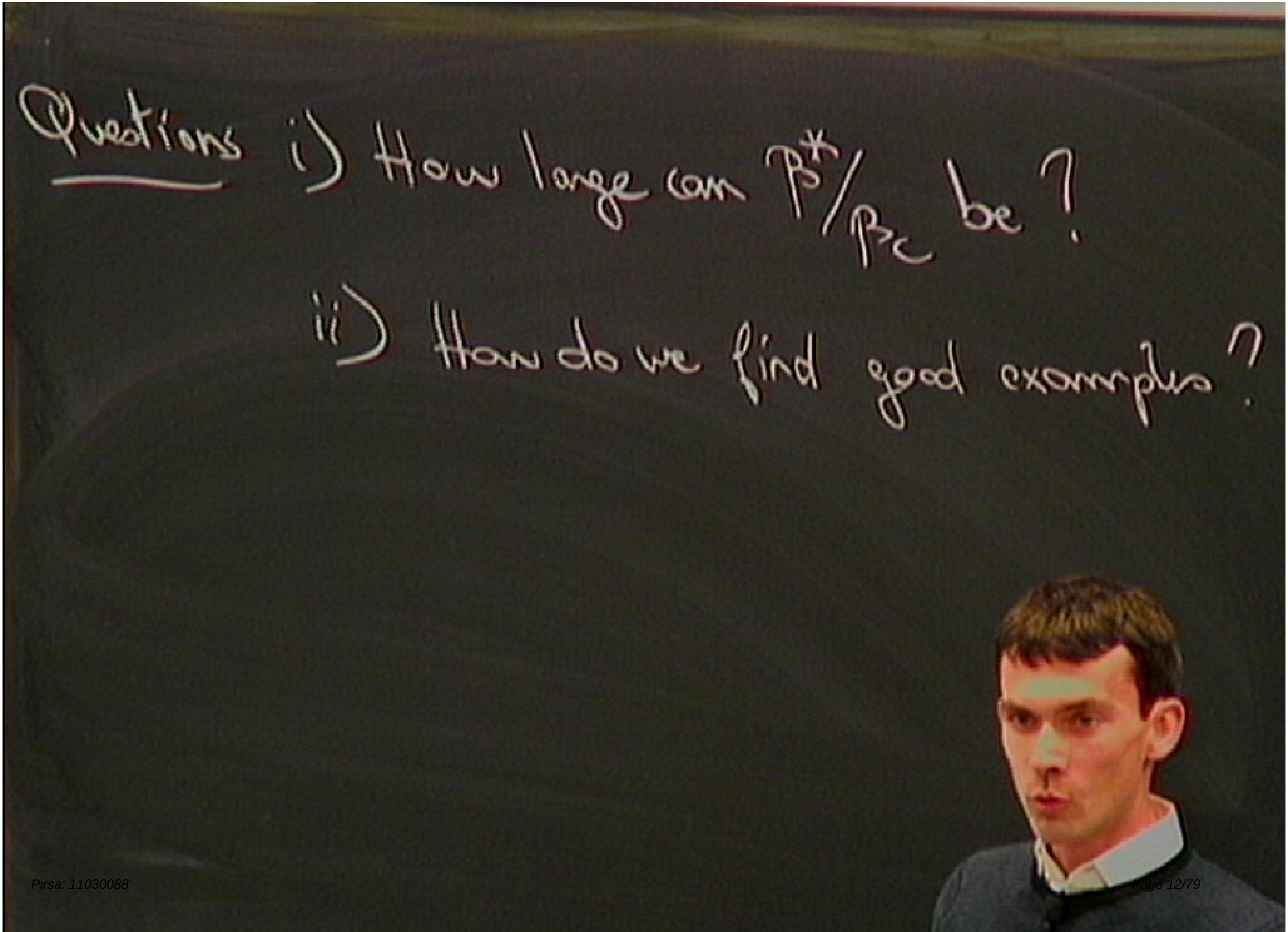
$$\beta_c(G) = \max_{a_s, b_t, c_u \in \{\pm 1\}} \sum_{s,t,u} \pi(s,t,u)(-1)^{f(s,t,u)} a_s b_t c_u$$

<u>Questions</u> i) How large can β^*/β_c be?


Questions
i) How large can β^* / β_c be?
ii) How do we find good examples?


Questions
i) How large can β^*/β_c be ?
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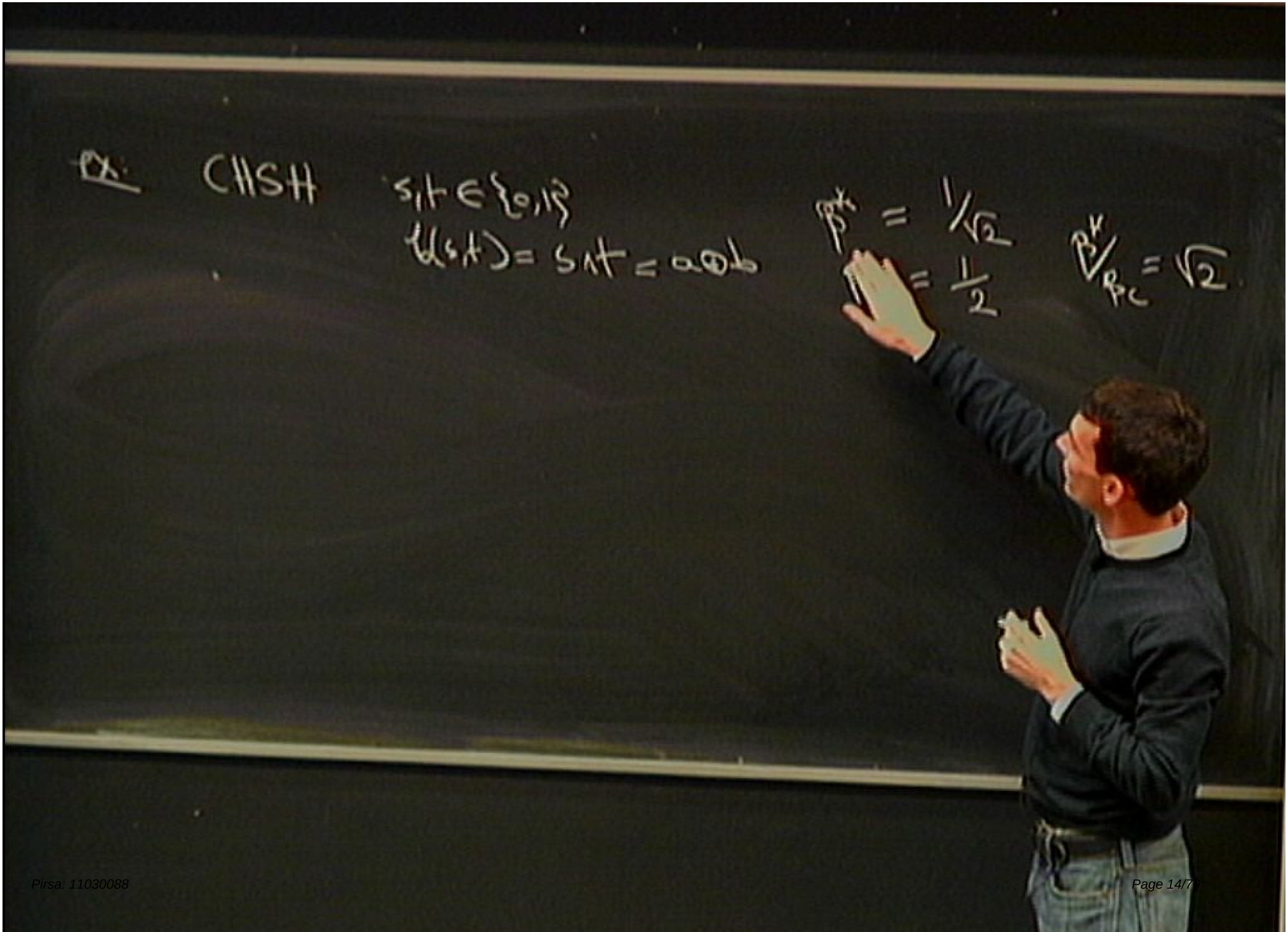
Questions i) How large can β^{m}/β_{c} be?

ii) How do we find good examples?

iii) What are the most nonlocal states?

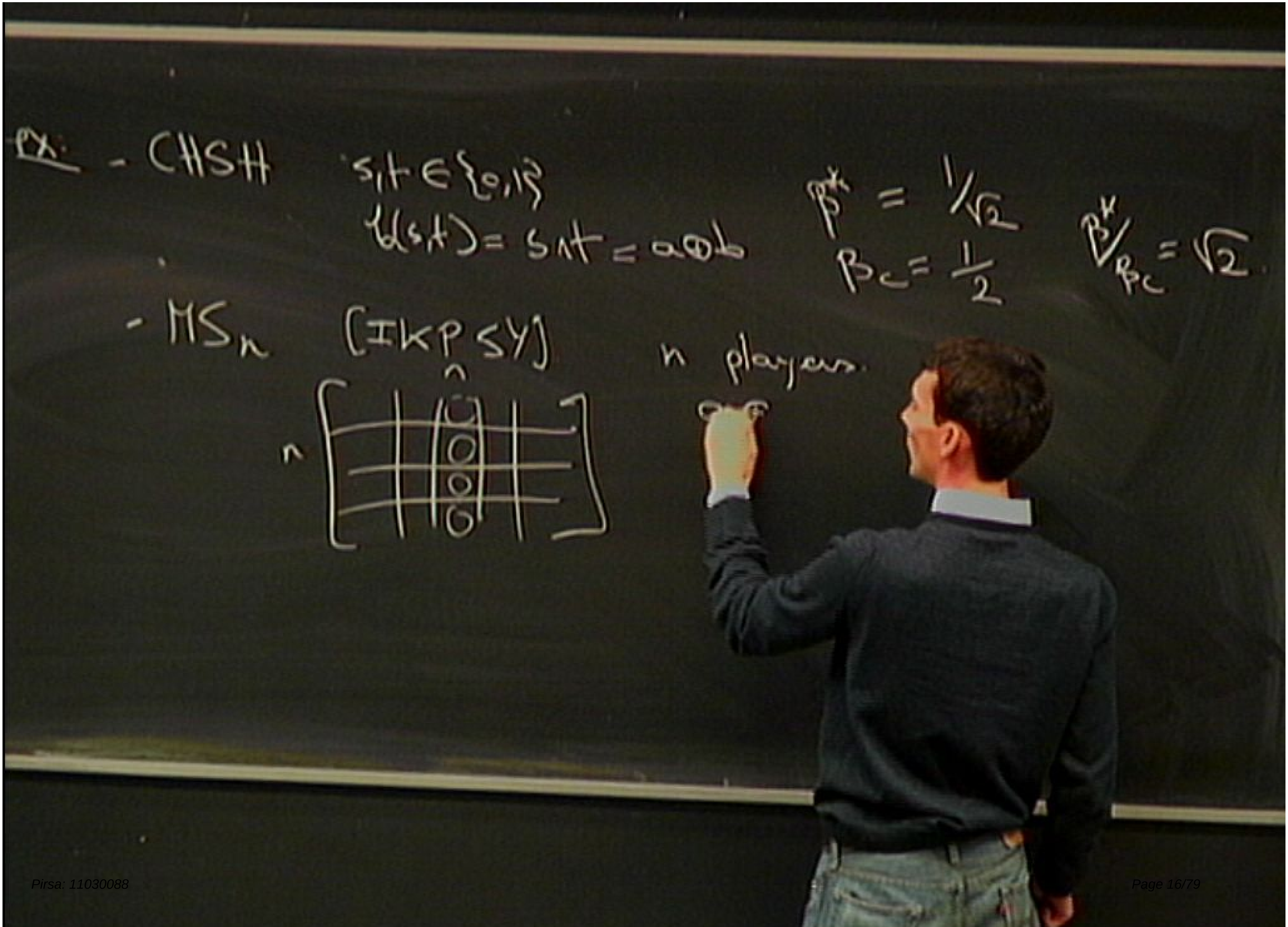

Ex: CHSH
s,t ∈ {0,1}
f(s,t) = s∧t = a⊕b
= 1/√2
= 1/2
= √2

Ex: - CHSH $s,t \in \{0,1\}$

$f(s,t) = s \wedge t = a \oplus b$

$\beta^* = 1/\sqrt{2}$ $\beta^*/\beta_c = \sqrt{2}$

$\beta_c = \frac{1}{2}$

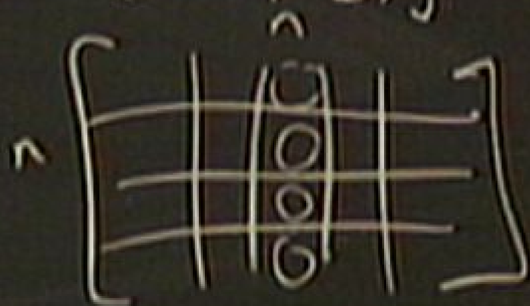

Ex: - CHSH
s,t ∈ {0,1}
V(s,t) = s∧t = a⊕b
β* = 1/√2
βc = 1/2
β*/βc = √2
- MSn [IKPSY]
n players

Ex: - CHSH $s,t \in \{0,1\}$

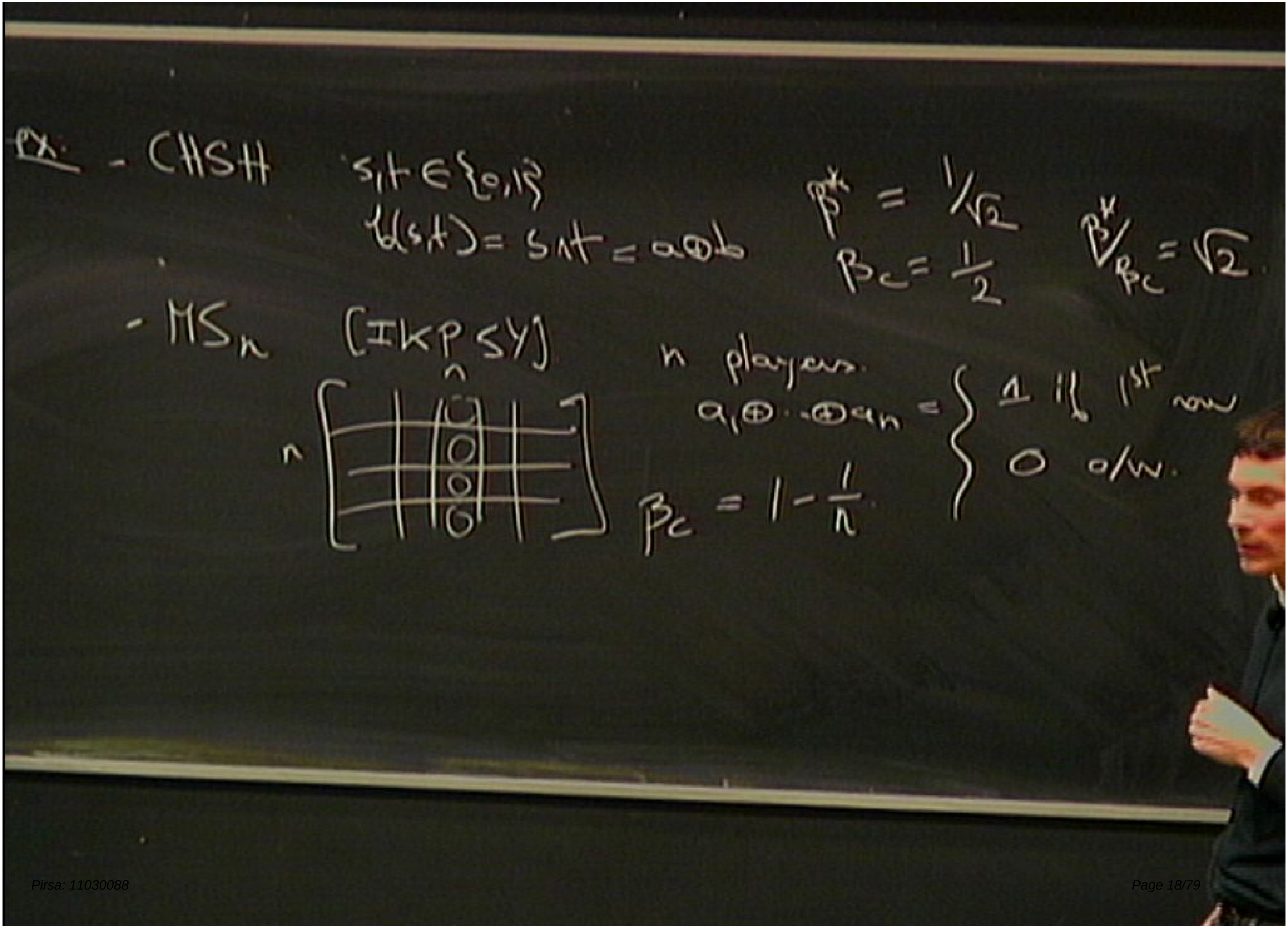
$f(s,t) = s \wedge t = a \oplus b$

$\beta^* = 1/\sqrt{2}$ $\beta_c = \frac{1}{2}$ $\beta^*/\beta_c = \sqrt{2}$

- MS_n [IKPSY] n players



$a_1 \oplus \cdots \oplus a_n = \begin{cases} 1 & \text{iff } 1^{st} \text{ row} \\ 0 & \text{o/w.} \end{cases}$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- MS_n [IKPSY]
n players
a_1⊕ ... ⊕a_n = { 1 iff 1st row; 0 o/w.
β_c = 1 - 1/n

Ex: - CHSH

$s,t \in \{0,1\}$

$V(s,t) = s \wedge t = a \oplus b$

$\beta^* = 1/\sqrt{2}$

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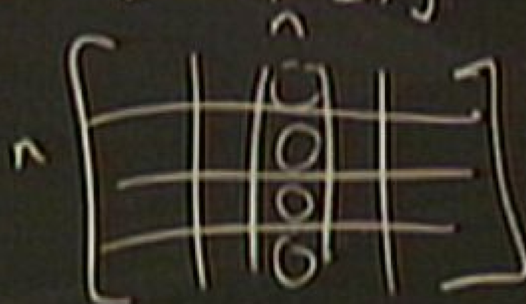

n players

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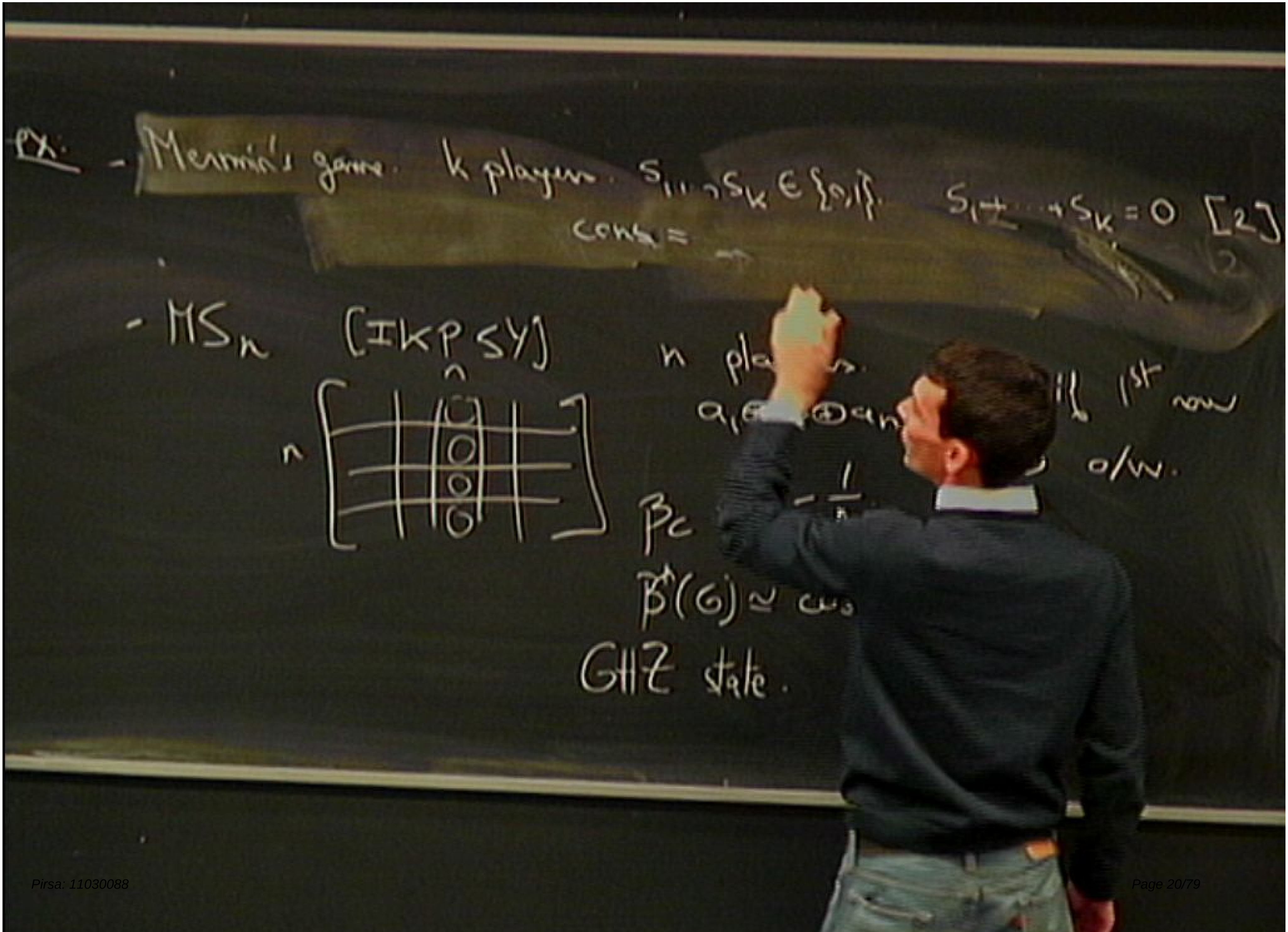
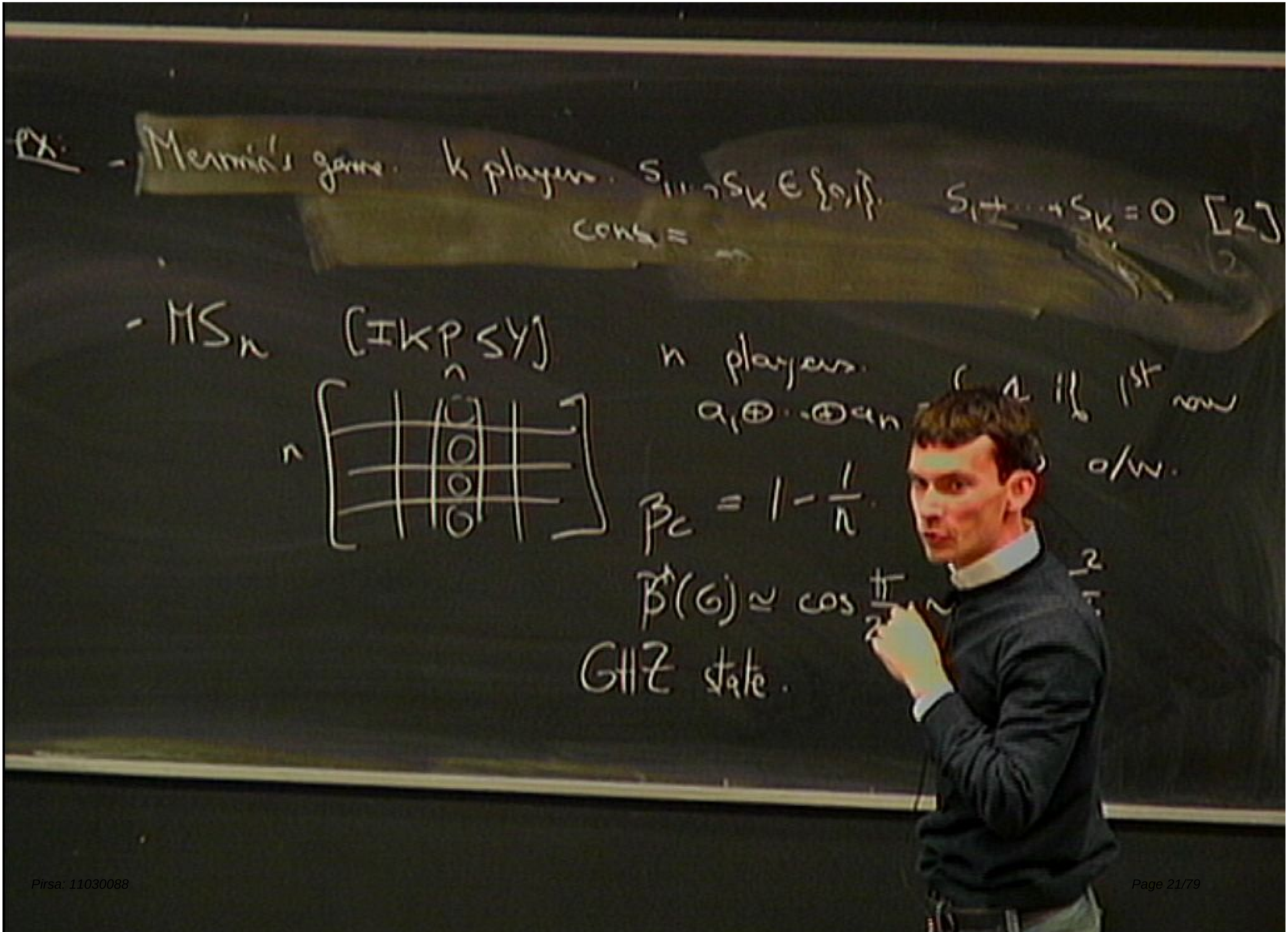
$\beta_c = 1 - \frac{1}{n}$

$\beta^*(G) \simeq \cos \frac{\pi}{2n} \sim 1 - \frac{\pi^2}{8n^2}$



Ex. - Mermin's game. k players. s1,...,sk ∈ {0,1}. s1+...+sk = 0 [2]
cons =
- MSn [IKPSY]
n players.
1st row
o/w.
GHZ state.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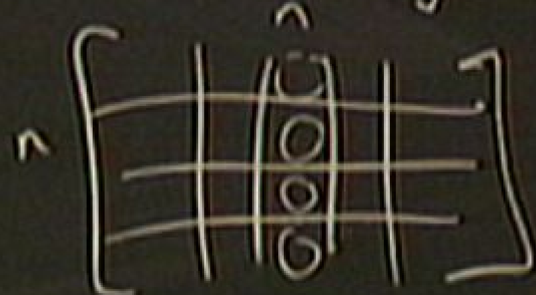
cond $= a_1 \oplus \dots \oplus a_k = \frac{s_1 + \dots + s_k}{2}$

- MS_n [IKPSY] n players.



$$a_1 \oplus \dots \oplus a_n = \begin{cases} 1 & \text{1st row} \\ 0 & \text{o/w.} \end{cases}$$

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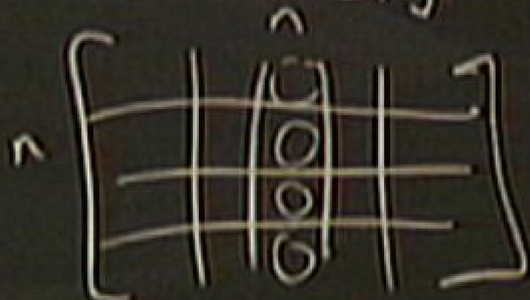

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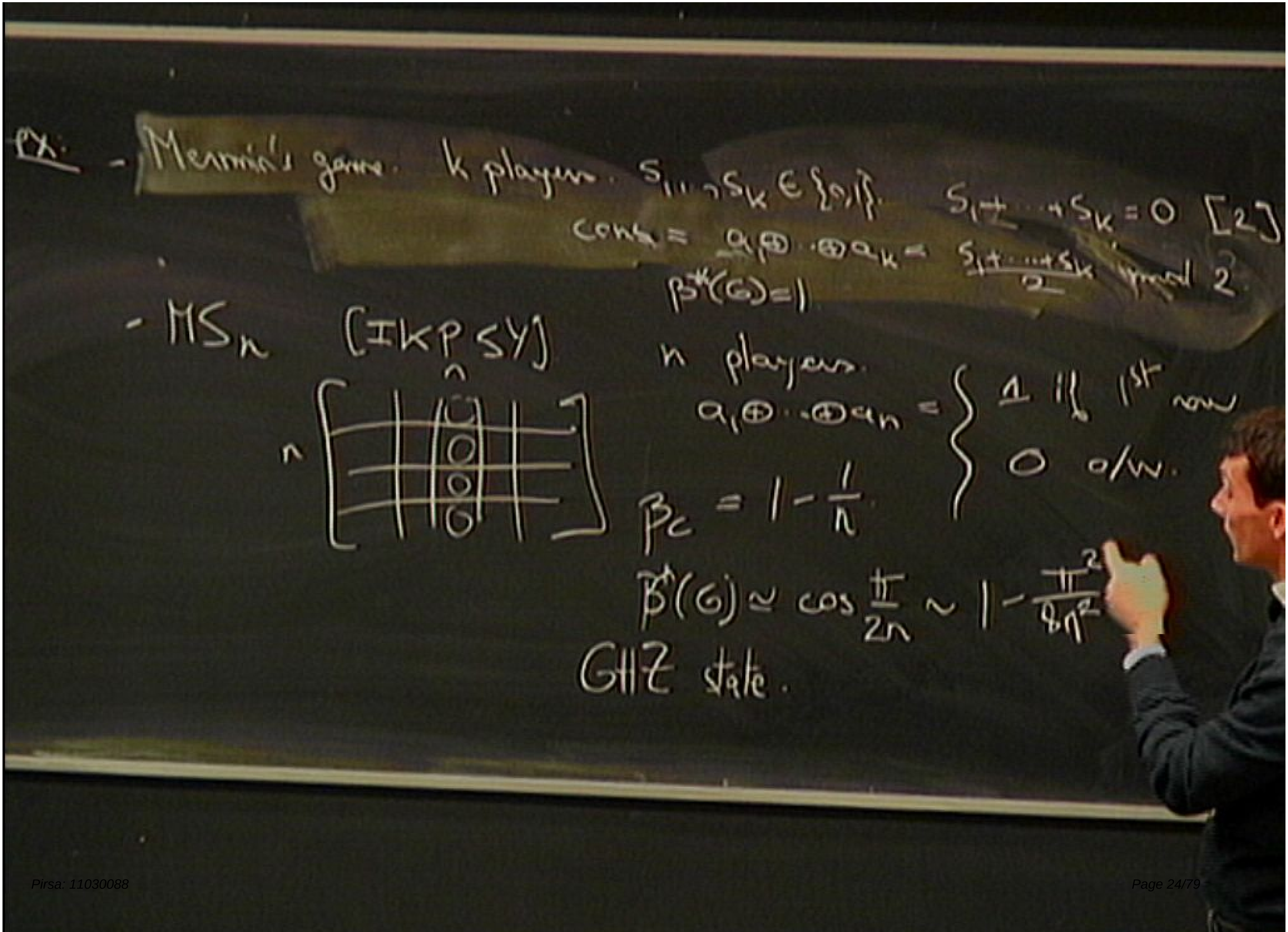
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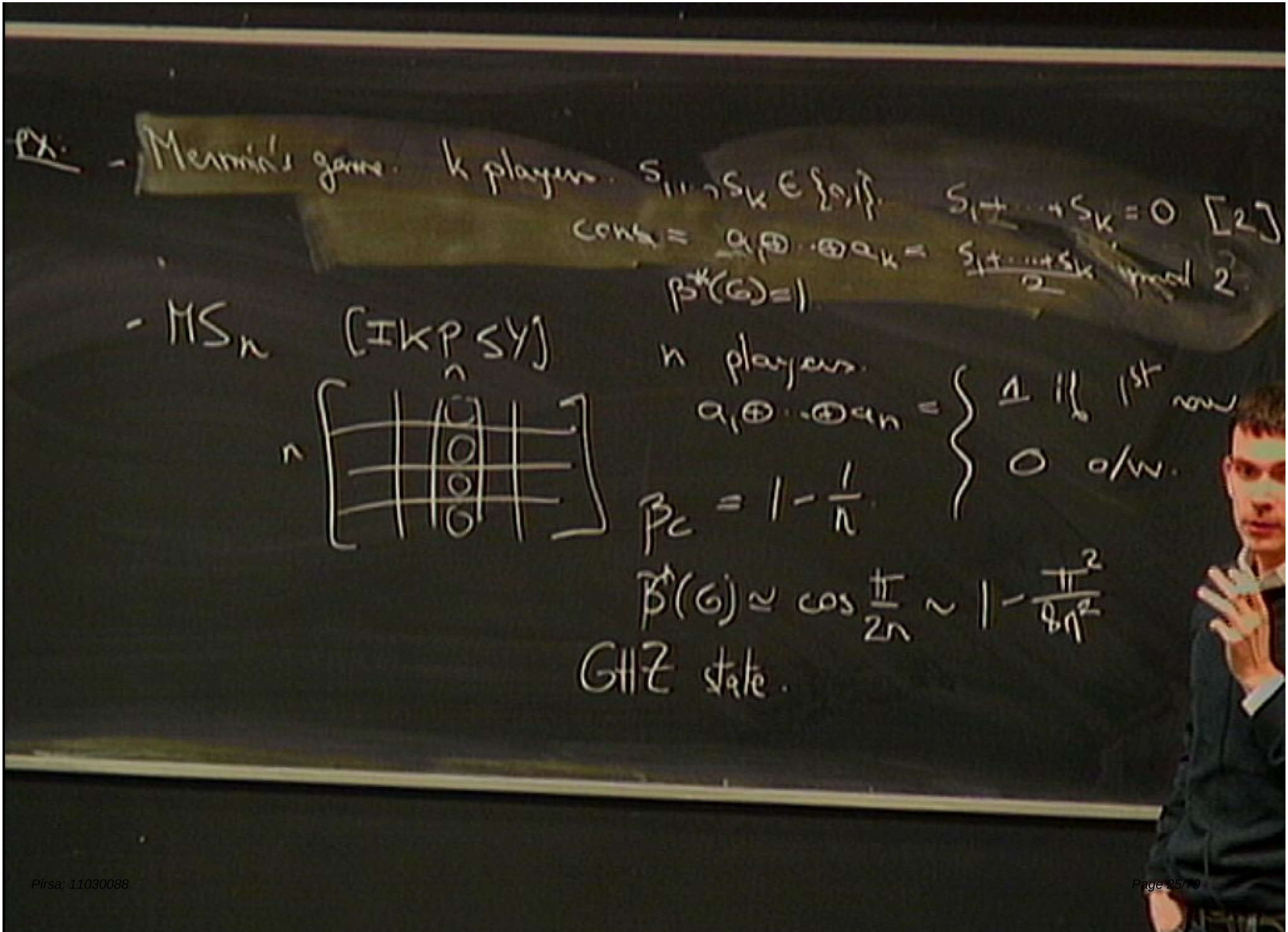
cons = a_1 ⊕ ... ⊕ a_k = (s_1 + ... + s_k)/2 mod 2
β*(G) = 1
- MS_n [IKPSY] n players.
a_1 ⊕ ... ⊕ a_n = { 1 iff 1st row ; 0 o/w.
β_c = 1 - 1/n
β*(G) ≃ cos π/2n ~ 1 - π²/8n²
GHZ state.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cond: $a_1 \oplus \dots \oplus a_k = \frac{s_1 + \dots + s_k}{2} \bmod 2$
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- MS_n [IKPSY]
n players.
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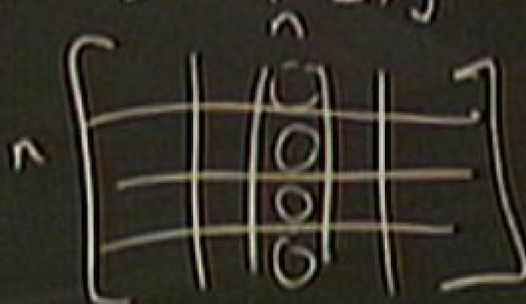
- MS_n [IKPSY] n players.



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$\beta^*(G) \simeq \cos \frac{\pi}{2n} \sim 1 - \frac{\pi^2}{4}$

GHZ state.

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$\text{cond} = a_1 \oplus \dots \oplus a_k = \frac{s_1 + \dots + s_k}{2} \bmod 2$

$\beta^*(G) = 1 \qquad \beta_c(G) = \frac{?}{2^{k/2}}$

- MS_n [IKPSY]

n players.

$n \begin{bmatrix} \ \end{bmatrix}$

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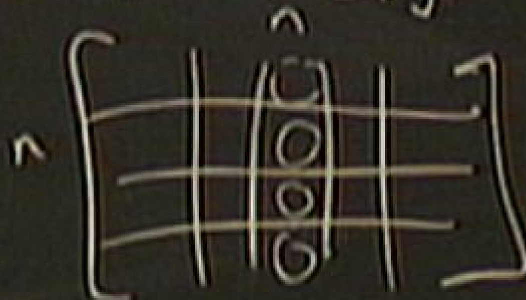
$\beta^*(G) = 1$ $\beta_c(G) = \frac{2}{2^{k/2}}$ $\frac{\beta^*}{\beta} = 2^{k/2-1}$

- MS_n [IKPSY] n players.



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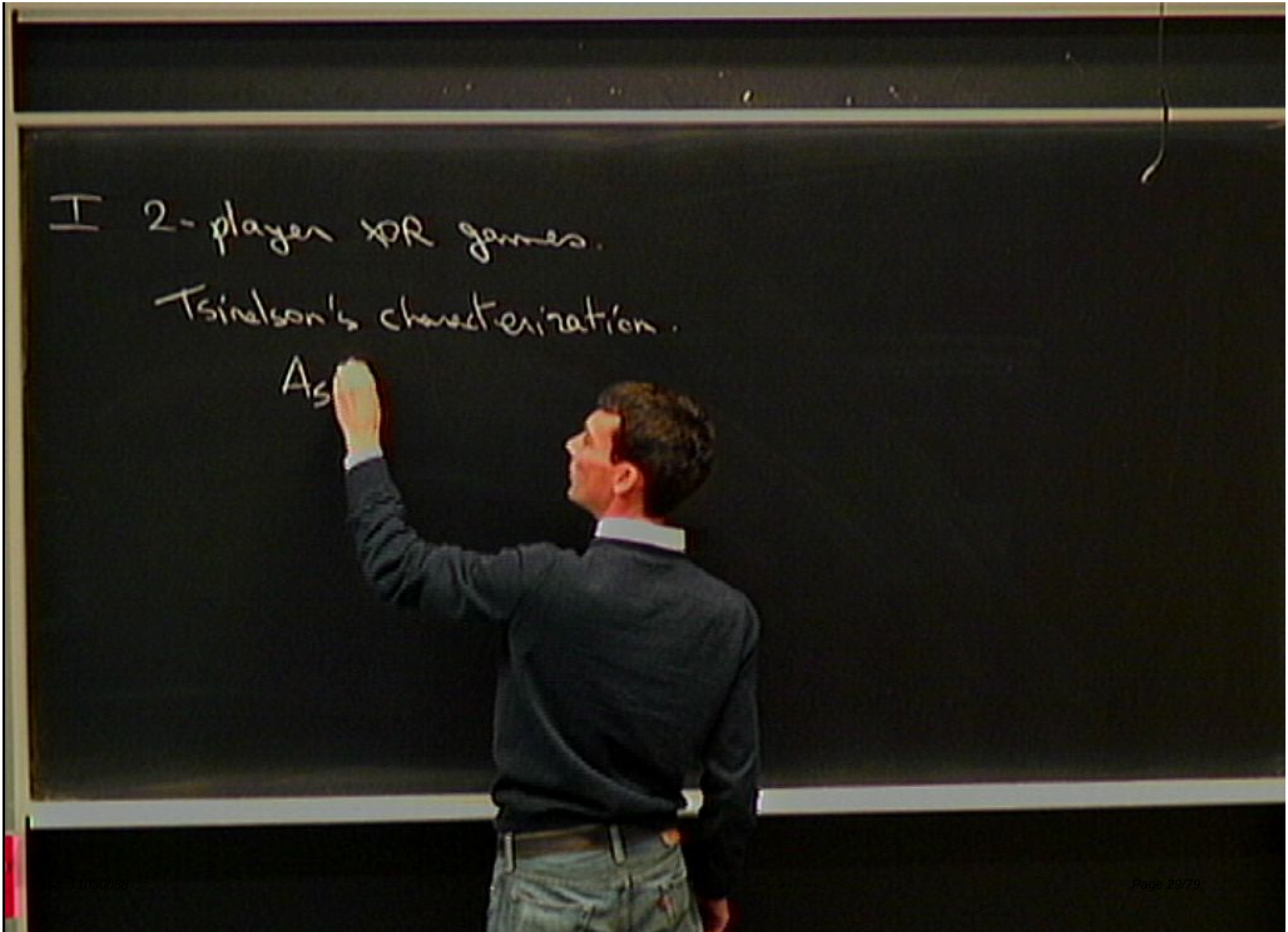
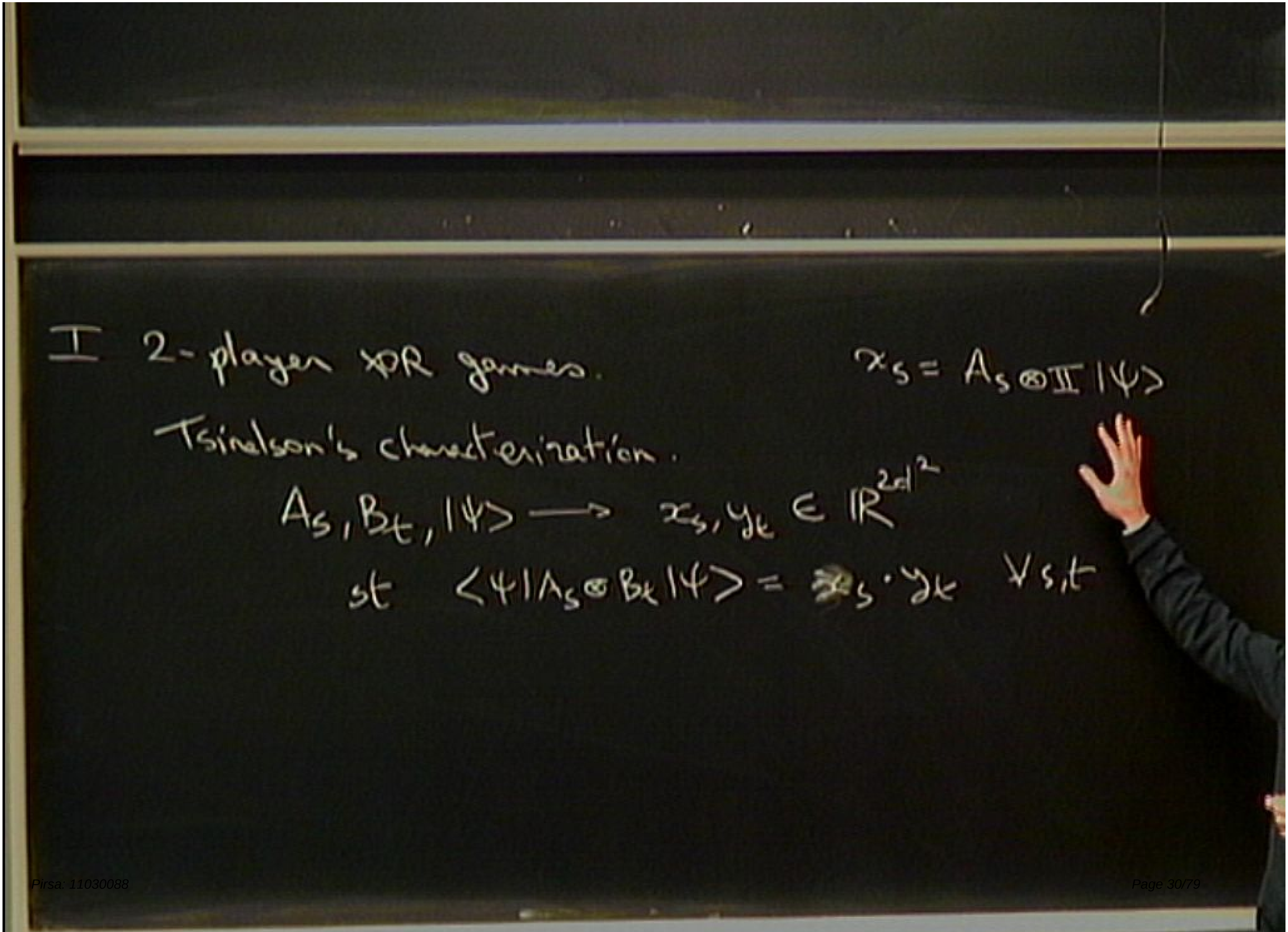

I 2-player XOR games.
Tsirelson's characterization.
As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$x_s = A_s \otimes \mathbb{I}|\psi\rangle$
Tsirelson's characterization.
$A_s, B_t, |\psi\rangle \longrightarrow x_s, y_t \in \mathbb{R}^{2d^2}$
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$$\beta^*(G) = \max_{\substack{x_s, y_t \in \mathbb{R}^{2n} \\ \|x_s\| = \|y_t\| = 1}} \sum_{s,t} \pi(s,t)(-1)^{f(s,t)} \, x_s \cdot y_t$$

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[... W04]

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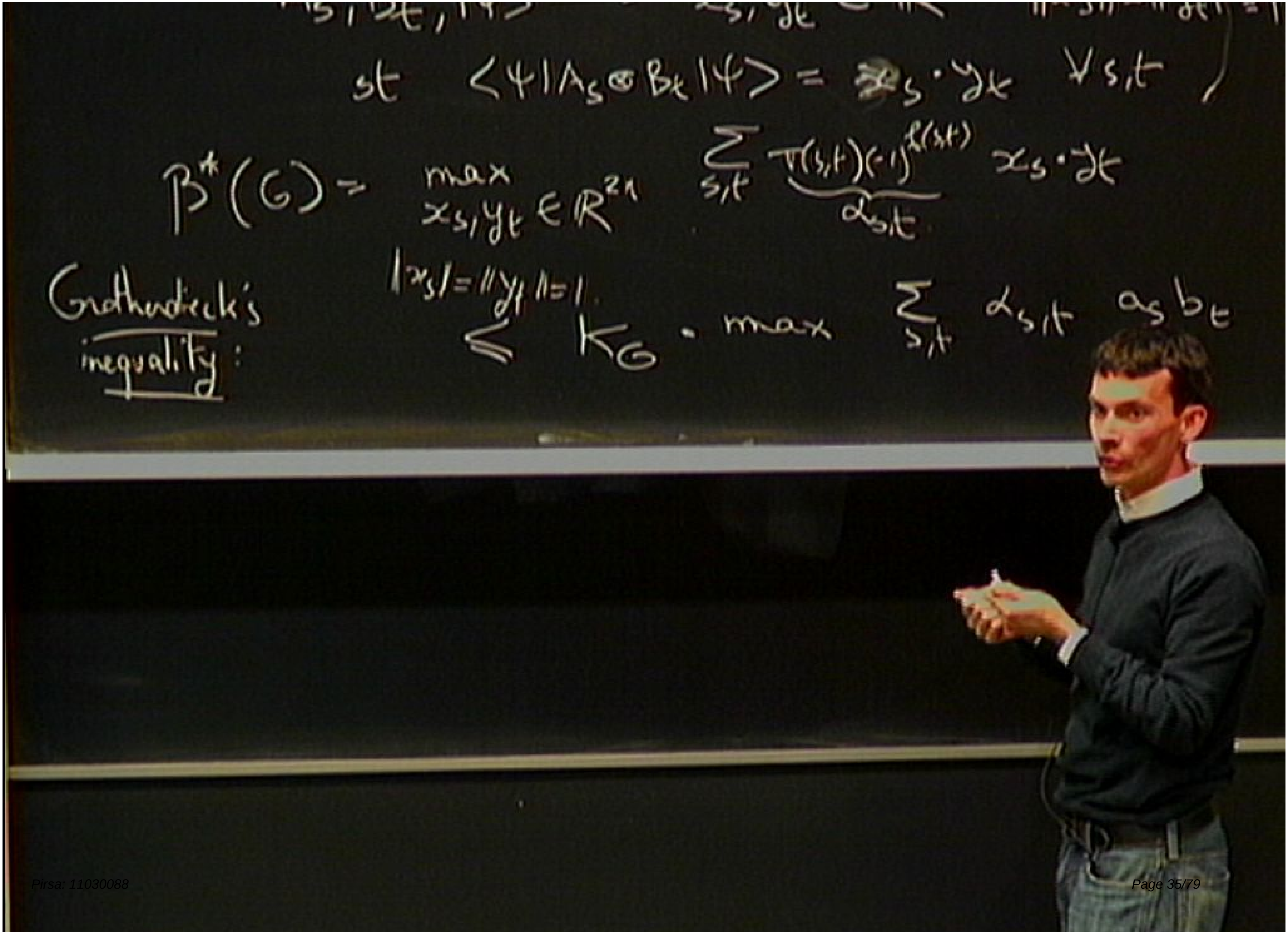
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$\beta^*(G) = \max_{x_s, y_t \in \mathbb{R}^{2n}} \sum_{s,t} \underbrace{\pi(s,t)(-1)^{f(s,t)}}_{\alpha_{s,t}} x_s \cdot y_t$
$\|x_s\| = \|y_t\| = 1$
Grothendieck's inequality:
$\leq K_G \cdot \max \sum_{s,t} \alpha_{s,t} a_s b_t$

[CHTW04]

I 2-player XOR games.

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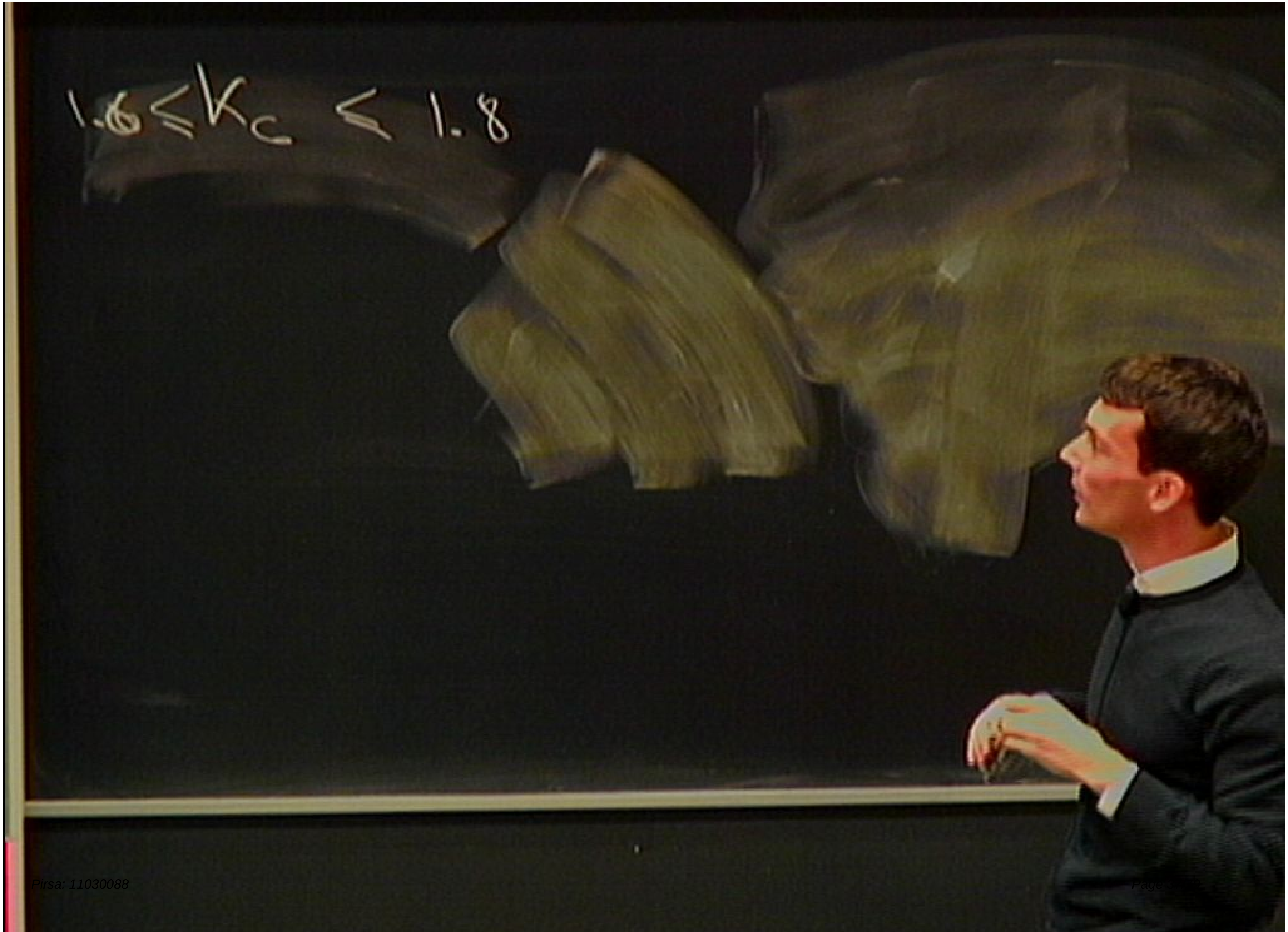
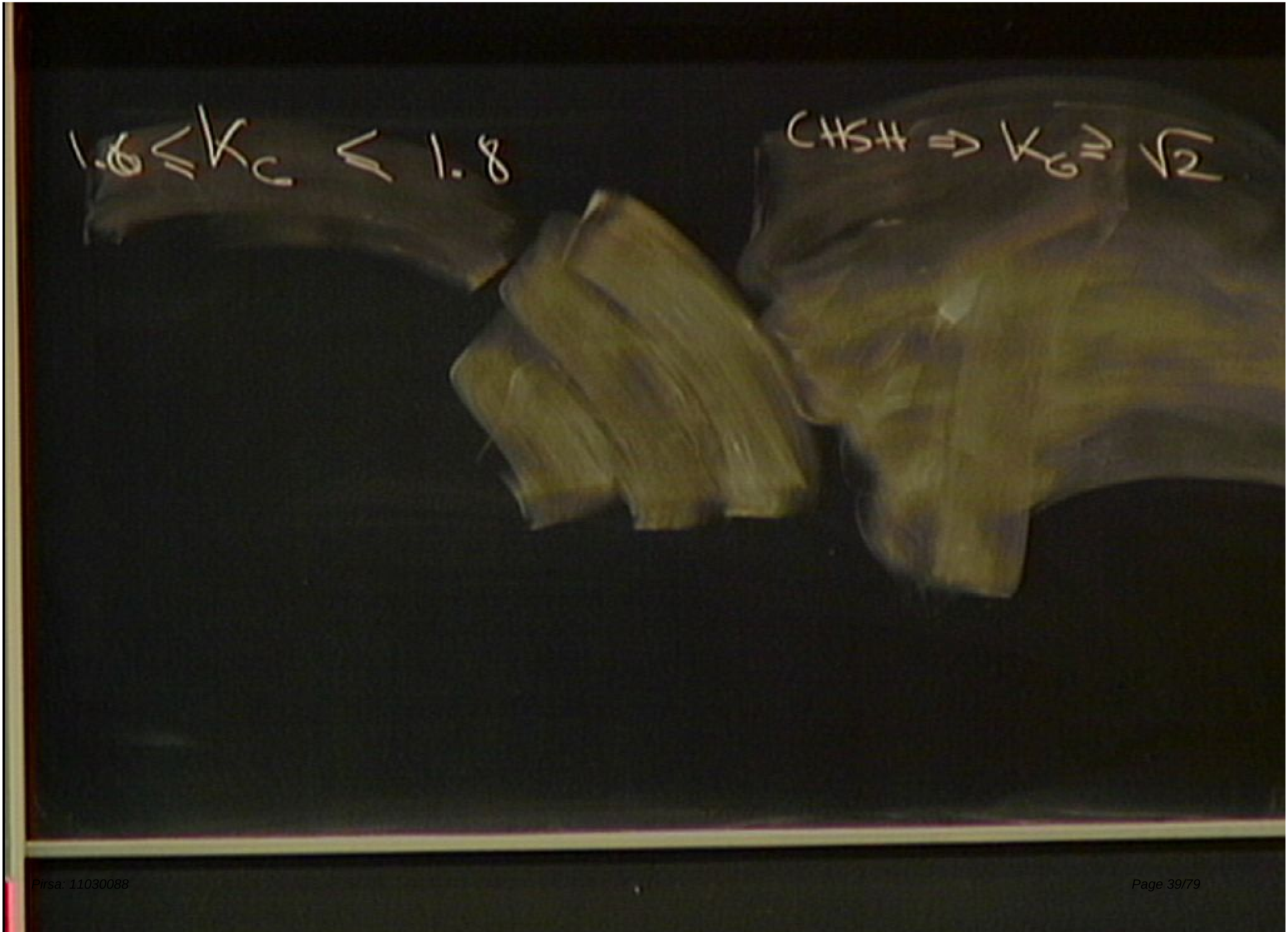
Grothendieck's inequality:

$$\leq K_G \cdot \max_{a_s, b_t \in \{\pm 1\}} \sum_{s,t} \alpha_{s,t} a_s b_t$$


1.6 ≤ Kc ≤ 1.8

$1.6 \leq K_C \leq 1.8$

CHSH $\Rightarrow K_G \geq \sqrt{2}$


1.6 ≤ K_C ≤ 1.8
CHSH ⇒ K_G ≥ √2

II 3-player XOR games

PG W PV J.

- GHZ can only give bounded $\beta^*/\beta_c \leq 2^{\frac{3}{2}(n-1)} K_G$

- If $|\psi\rangle$ has local dimension d then $\beta^*/\beta \leq$

III 3-player XOR games

PG W PV J.

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II 3-player XOR games

PG W PV / J.

- GHZ can only give bounded $\beta^*/\beta_c \leq 2^{\frac{3}{2}(n-1)} K_G$

- If $|\psi\rangle$ has local dimension d then $\beta^*/\beta \leq O(\sqrt{d})$

- $\forall d \; \exists G$ st $\beta^*/\beta \geq \Omega(\sqrt{d})$ and $|\psi\rangle$ has dim $d \times D \times D$.

$\#\, q \quad 2^{d^2} \times 2^{D^2} \times 2^{D^2}$

III 3-player XOR games

PG W PV J

- GHZ can only give bounded $\beta^*/\beta_c \le 2^{\frac{3}{2}(n-1)} K_G$

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II 3-player XOR games

PG W PV J

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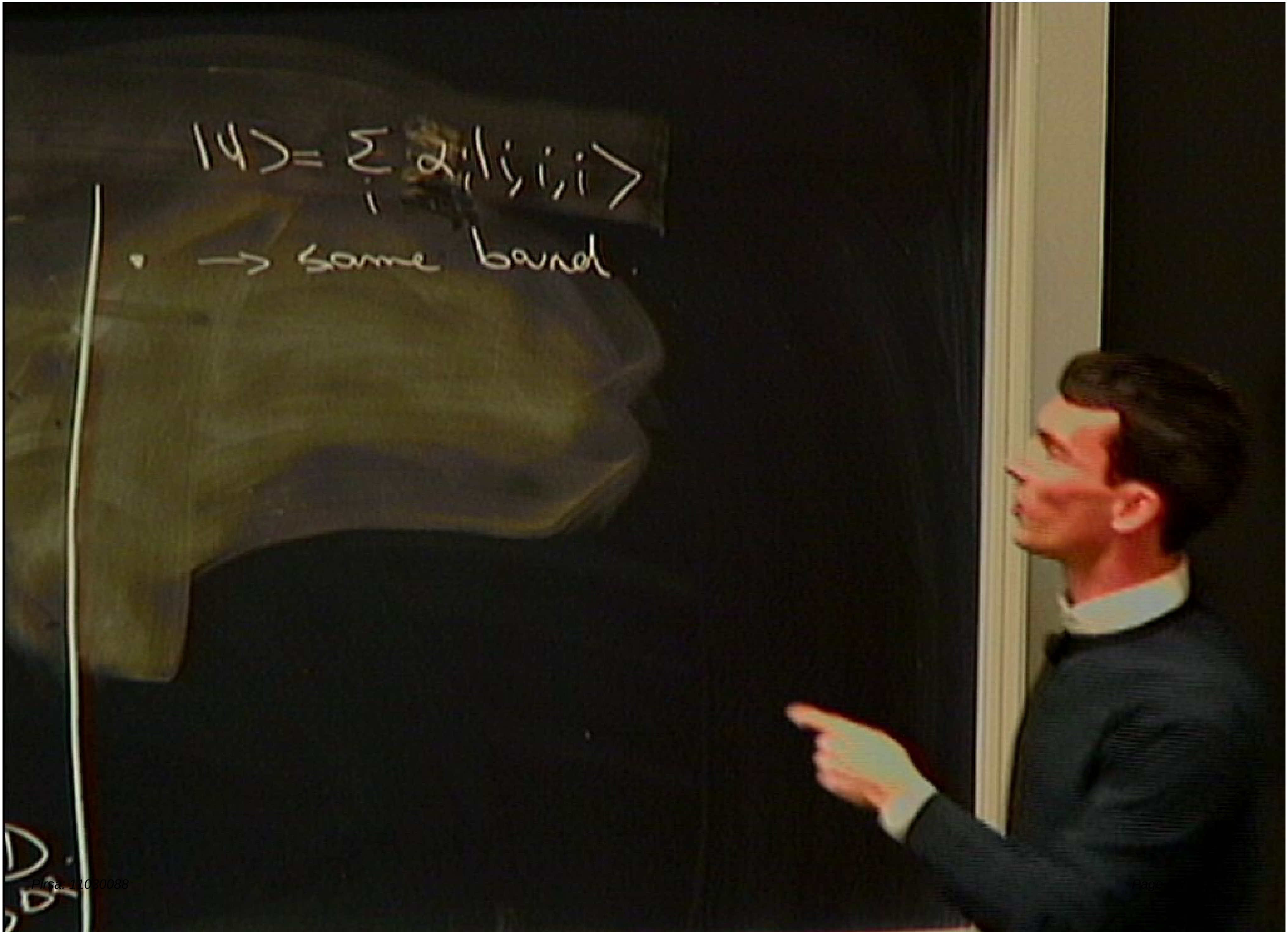
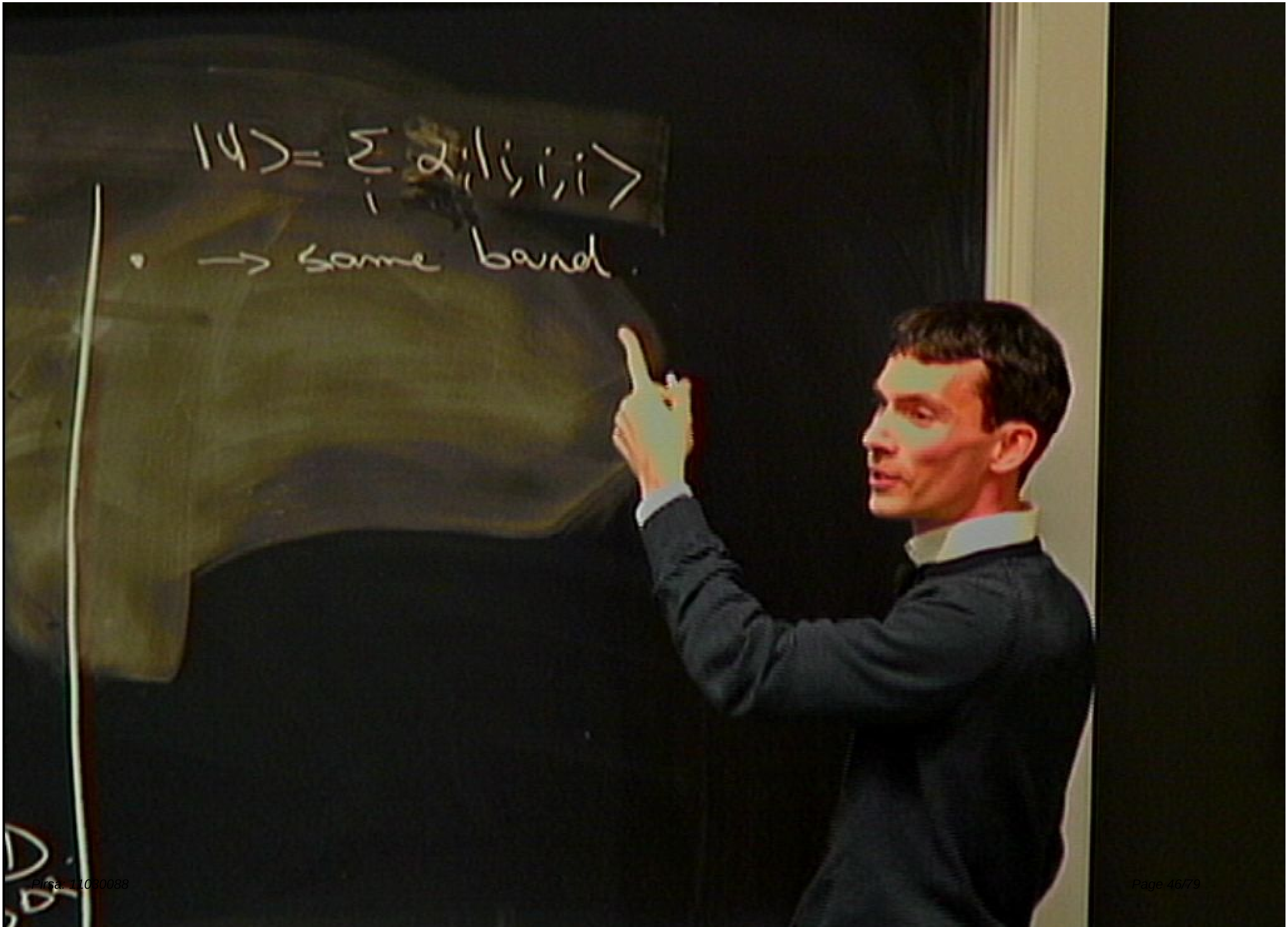
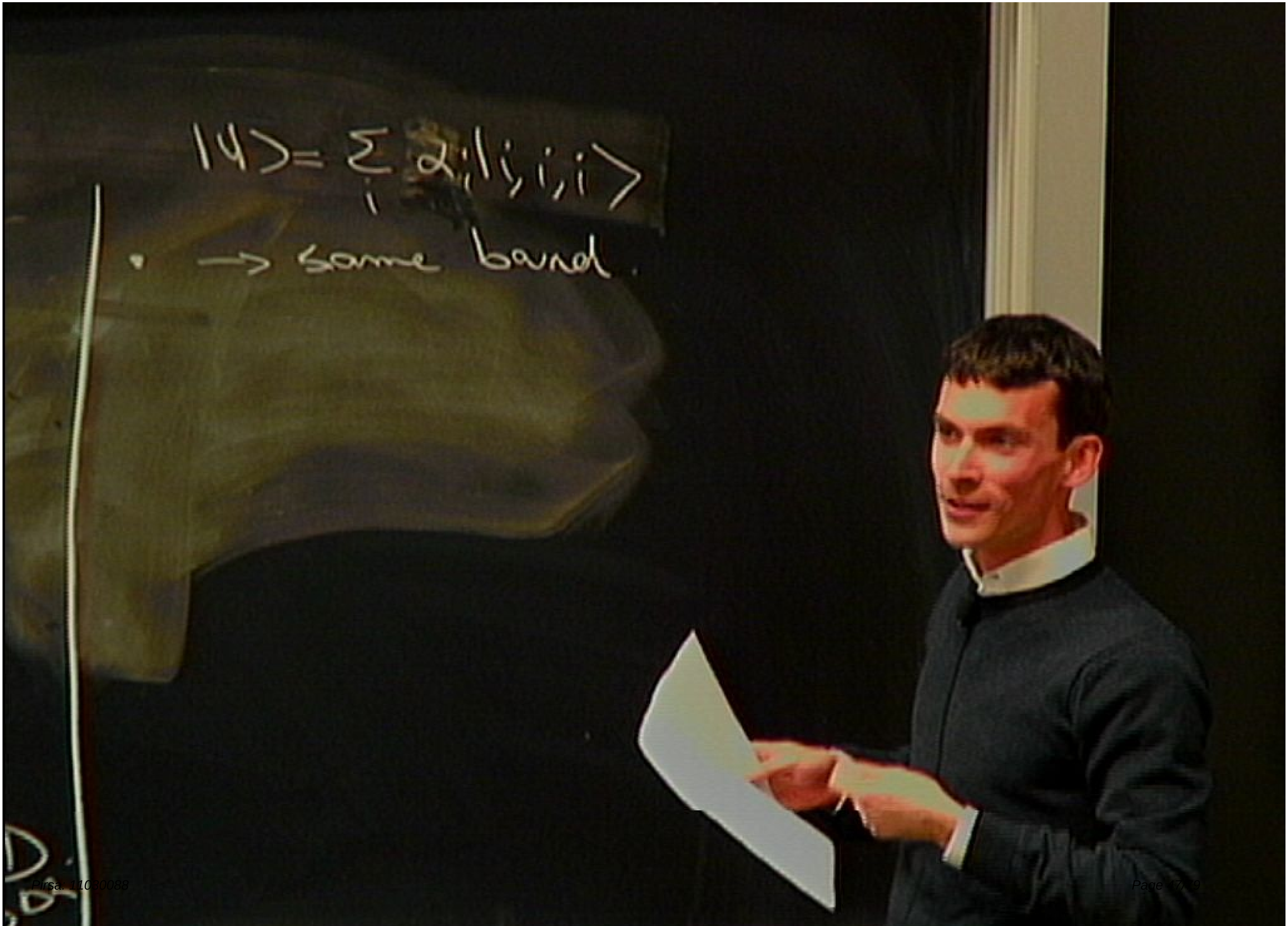
$\#\ q \quad 2^{d^2} \times 2^{D^2} \times 2^{D^2}$


|ψ⟩ = Σ_i α_i |i, i, i⟩
• → same band.


$|\psi\rangle = \sum_i \alpha_i |i,i,i\rangle$
• → same bond


|ψ⟩= Σ α_i|i,i,i⟩
→ same band

$|\psi\rangle = \sum_i \alpha_i |i,i,i\rangle$

- $\rightarrow$ same band.
- if $\# q = n$ then $\beta^*/\beta_c \leq o(\sqrt{n})$
- $\forall d \; \exists G$ s.t. $\beta^*/\beta = \Omega(\sqrt{d})$.

$|\psi\rangle$ has dim $d \times d \times d$.

$\# q$ $\quad d^2 \times d^2 \times d^2$.

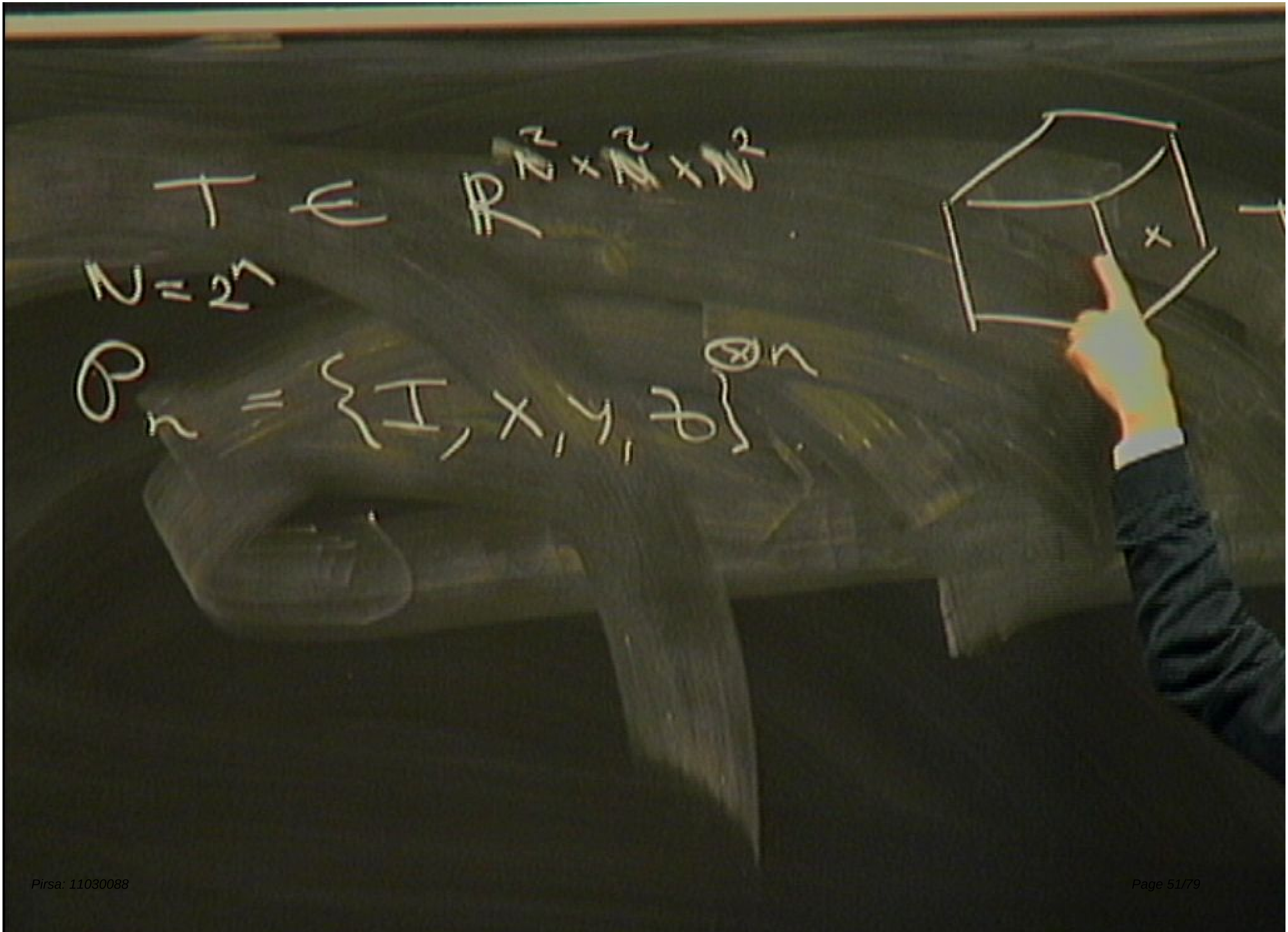
$$|\psi\rangle = \sum_i \alpha_i |i,i,i\rangle$$

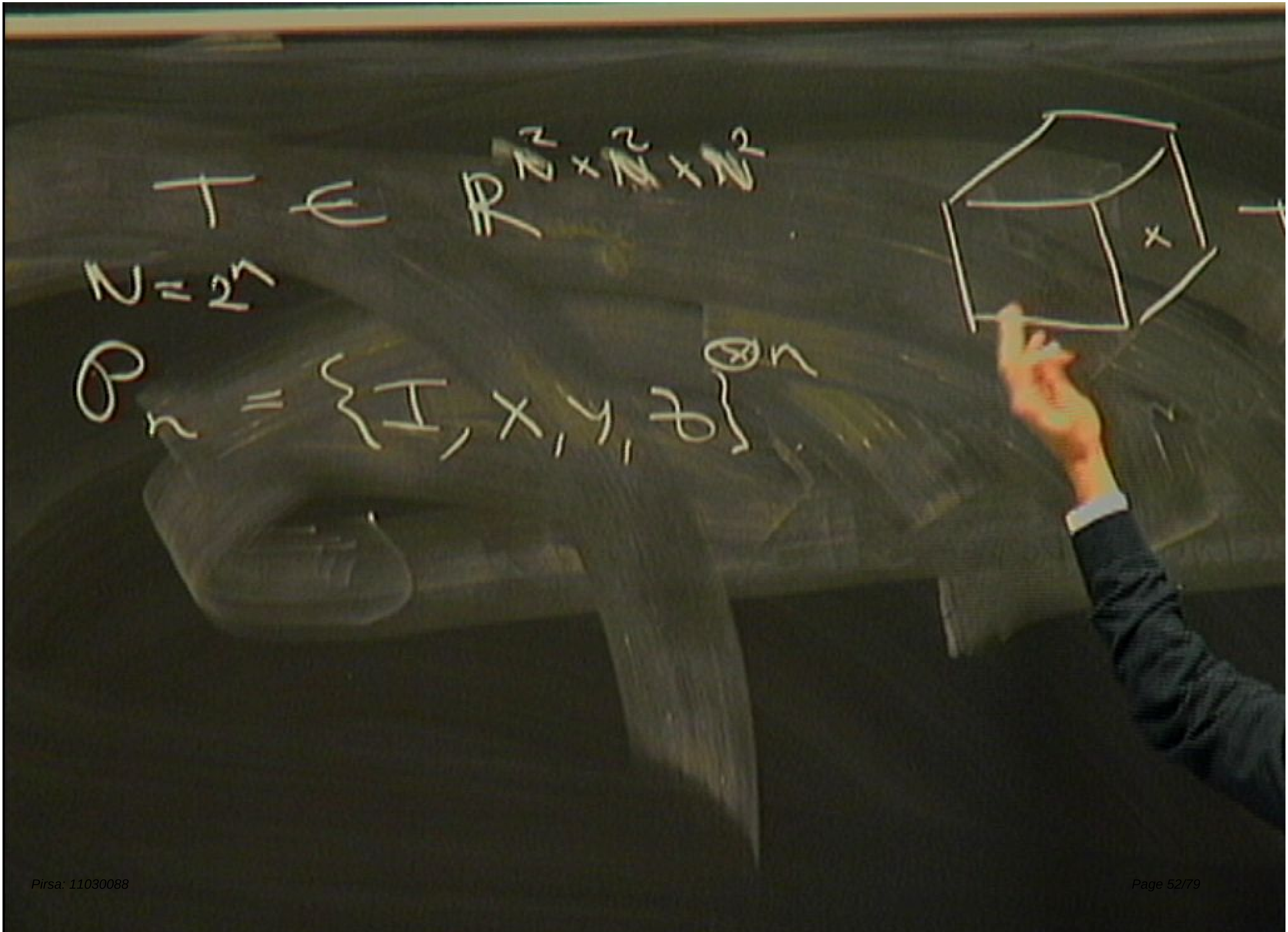
- $\rightarrow$ same band.
- if $\#q = n$ then $\beta^* / \beta_c \leq O(\sqrt{n})$
- $\forall d \ \exists G$ st $\beta^* / \beta = \Omega(\sqrt{d})$.

$|\psi\rangle$ has dim $d \times d \times d$.

$\#q \quad d^2 \times d^2 \times d^2$.

$$T \in \mathbb{R}^{n^2 \times n^2 \times n^2}$$


T ∈ R^(N^2 × N^2 × N^2).
N = 2^n
P_n = {I, X, Y, Z}^⊗n.


T ∈ ℝ^{N²×N²×N²}
N = 2^n
P_n = {I, X, Y, Z}^{⊗n}
X

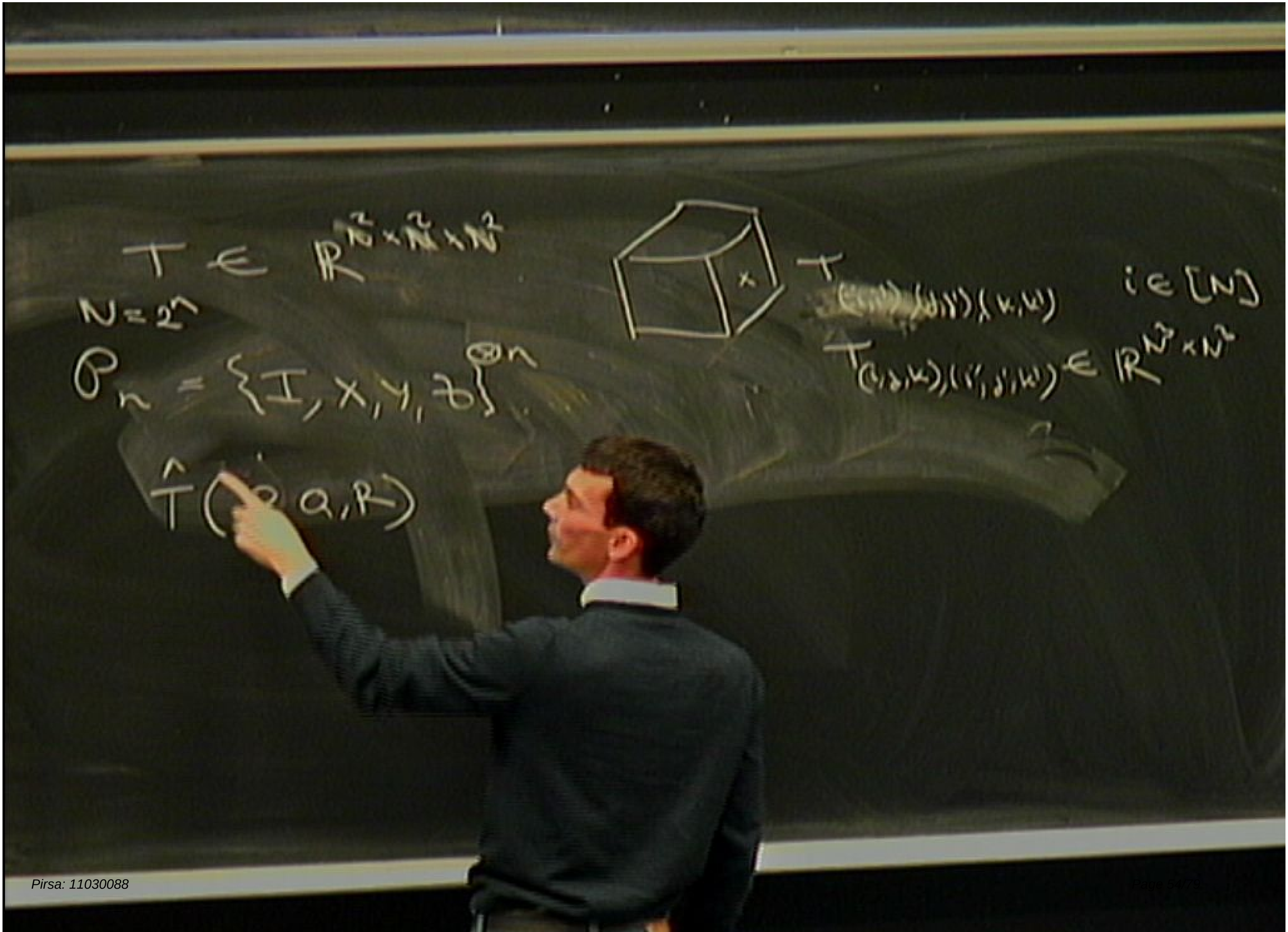
$T \in \mathbb{R}^{N^2 \times N^2 \times N^2}$

$N = 2^n$

$\mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$

$T_{(i,i')(j,j')(k,k')} \quad i \in [N]$

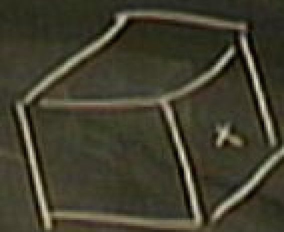
$T_{(i,j,k)(i',j',k')} \in \mathbb{R}^{N^3 \times N^3}$


T ∈ R^{N^2×N^2×N^2}
N=2^n
P_n = {I,X,Y,Z}^{⊗n}
T̂(P,Q,R)
T_{(i,i'),(j,j'),(k,k')}
i ∈ [N]
T_{(i,j,k),(i',j',k')} ∈ R^{N^3×N^3}
Pirsa: 11030088

$T \in \mathbb{R}^{N^2 \times N^2 \times N^2}$

$N = 2^n$

$\mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$



$T_{(i,i')(j,j')(k,k')}$ $\quad i \in [N]$

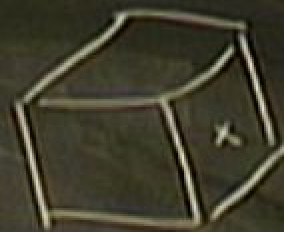
$T_{(i,j,k)(i',j',k')} \in \mathbb{R}^{N^3 \times N^3}$

$$\hat{T}(P, Q, R) = \langle T, P \otimes Q \otimes R \rangle = \sum_{\substack{ijk \\ i'j'k'}} T_{(ijk)(i'j'k')} P_{ii'} Q_{jj'} R_{kk'}$$

$T \in \mathbb{R}^{N^2 \times N^2 \times N^2}$

$N = 2^n$

$\mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$



$T_{(i,i')(j,j')(k,k')} \quad i \in [N]$

$T_{(i,j,k)(i',j',k')} \in \mathbb{R}^{N^3 \times N^3}$

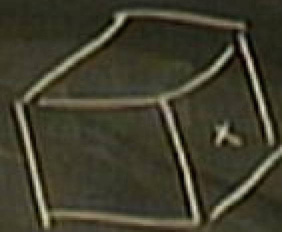
$$\hat{T}(P,Q,R) = \langle T, P \otimes Q \otimes R \rangle = \sum_{\substack{i,j,k \\ i',j',k'}} T_{(ijk)(i'j'k')} P_{ii'} Q_{jj'} R_{kk'}$$

$$T = \frac{1}{N^3} \sum_{P,Q,R} \hat{T}(P,Q,R) \cdot P \otimes Q \otimes R$$

$T \in \mathbb{R}^{N^2 \times N^2 \times N^2}$

$N = 2^n$

$\mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$



$T_{(i,i')(j,j')(k,k')}$ $\quad i \in [N]$

$T_{(i,j,k)(i',j',k')} \in \mathbb{R}^{N^3 \times N^3}$

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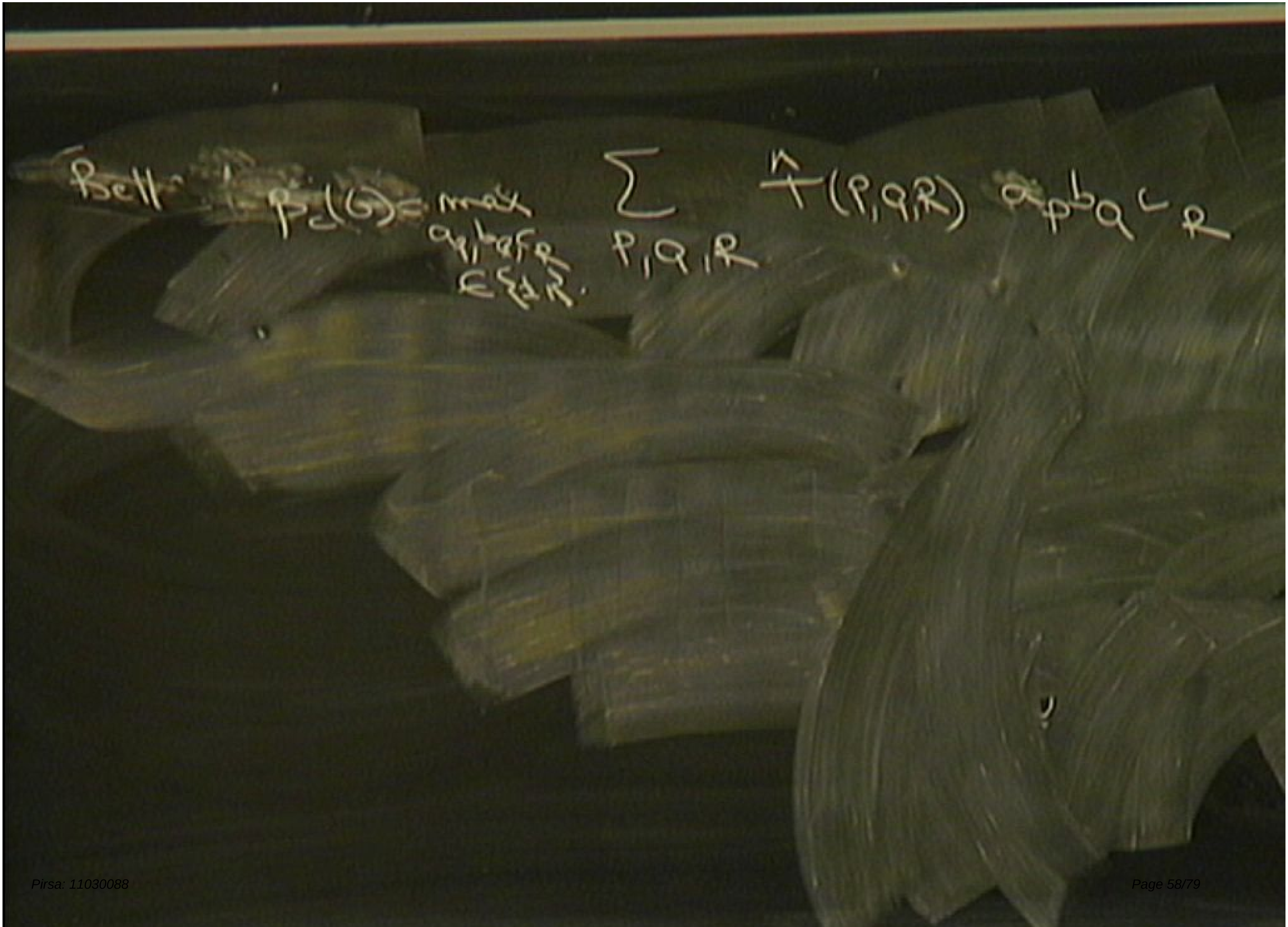
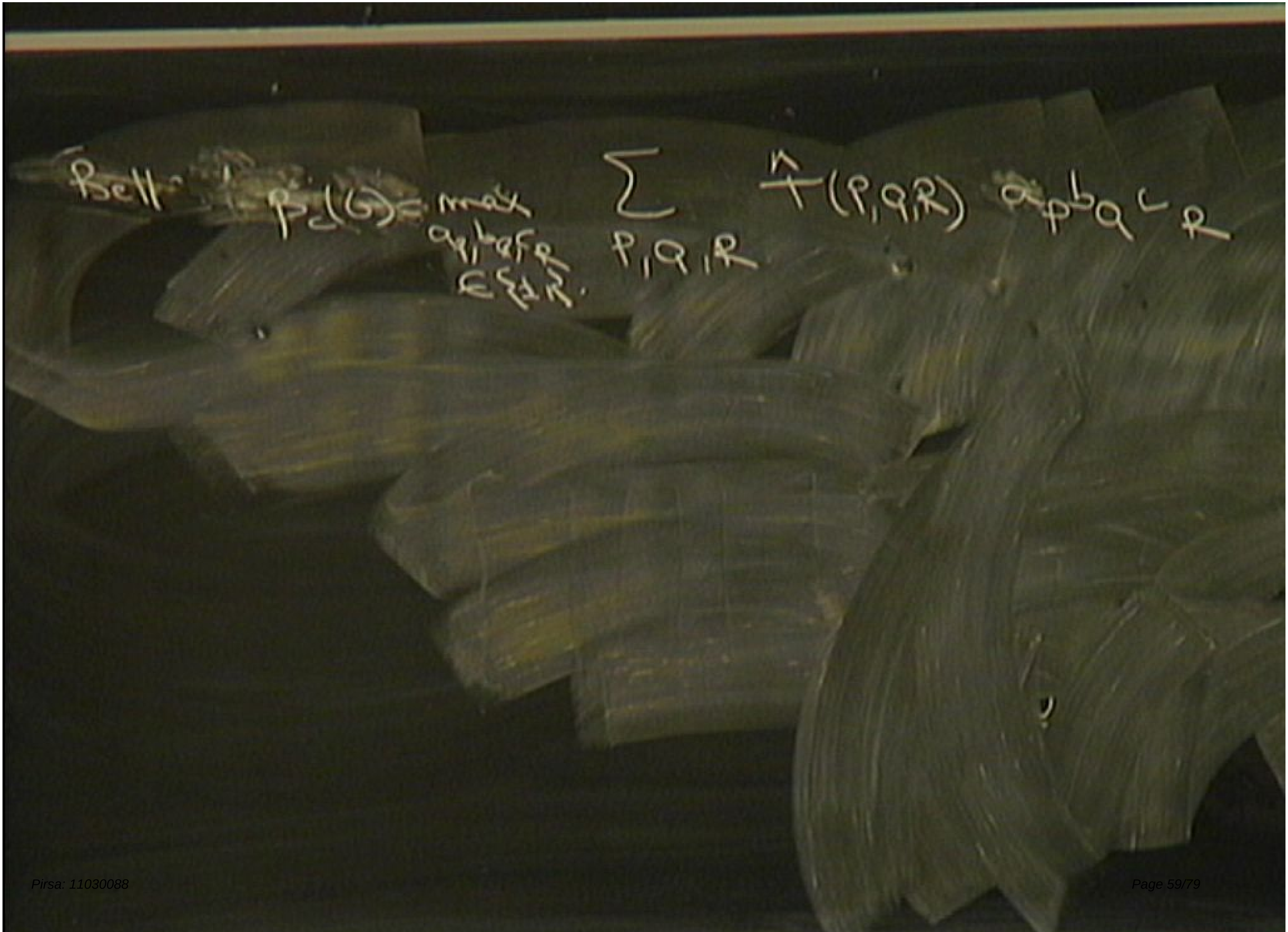

Bell
β_c(G) = max
a_p, b_q, c_R ∈ {±1}
Σ
P, Q, R
f̂(P, Q, R)
R


β_Bell = β_C(G) = max_{a_Q, b_Q, c_R ∈ {±1}} Σ_{P,Q,R} f̂(P,Q,R) a_P b_Q c_R

$$\beta_{Bell} = \beta_c(G) = \max_{a_q, b_q, c_R \in \{\pm 1\}} \sum_{P,Q,R} \hat{f}(P,Q,R)\, a_P b_Q c_R$$

$$\beta^*(G) \geq \sum_{P,Q,R} \hat{f}(P,Q,R)$$

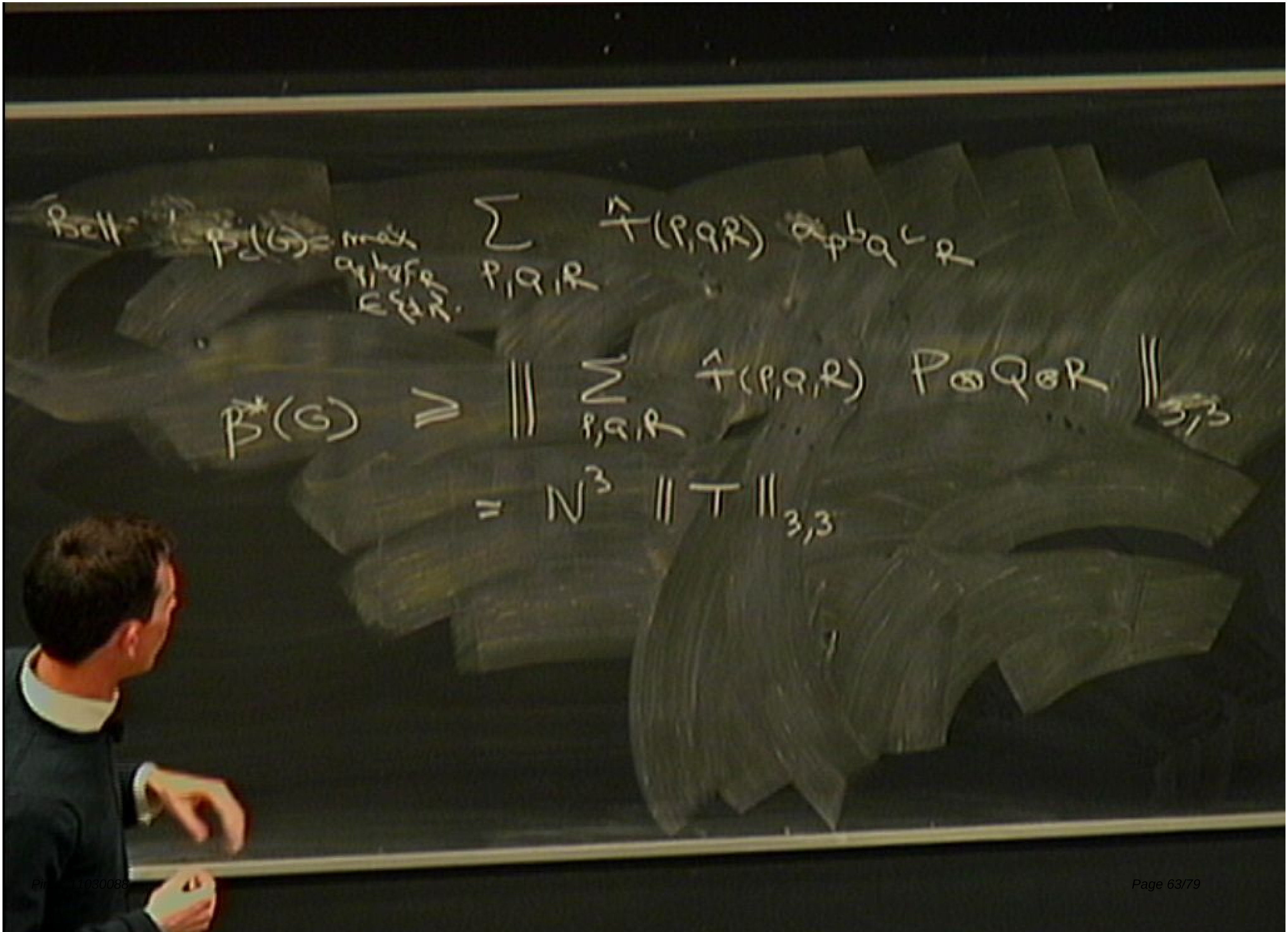
$$\beta_{Bell} = \beta_c(G) = \max_{\substack{a_P, b_Q, c_R \\ \in \{\pm 1\}}} \sum_{P,Q,R} \hat{f}(P,Q,R)\, a_P b_Q c_R$$

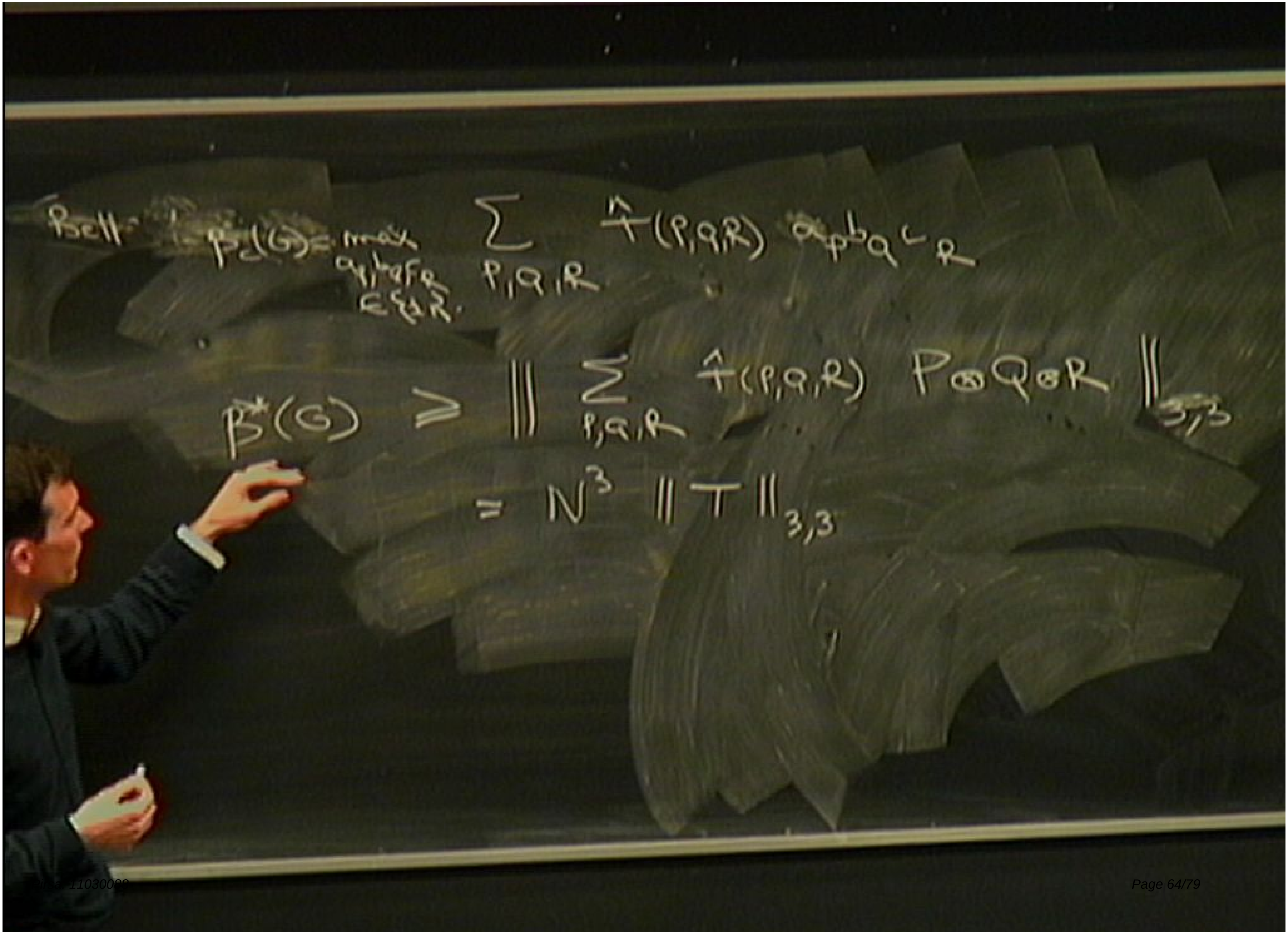
$$\beta^*(G) \geq \sum_{P,Q,R} \hat{f}(P,Q,R)\; P \otimes Q \otimes R \,\Big\|_{3/3}$$

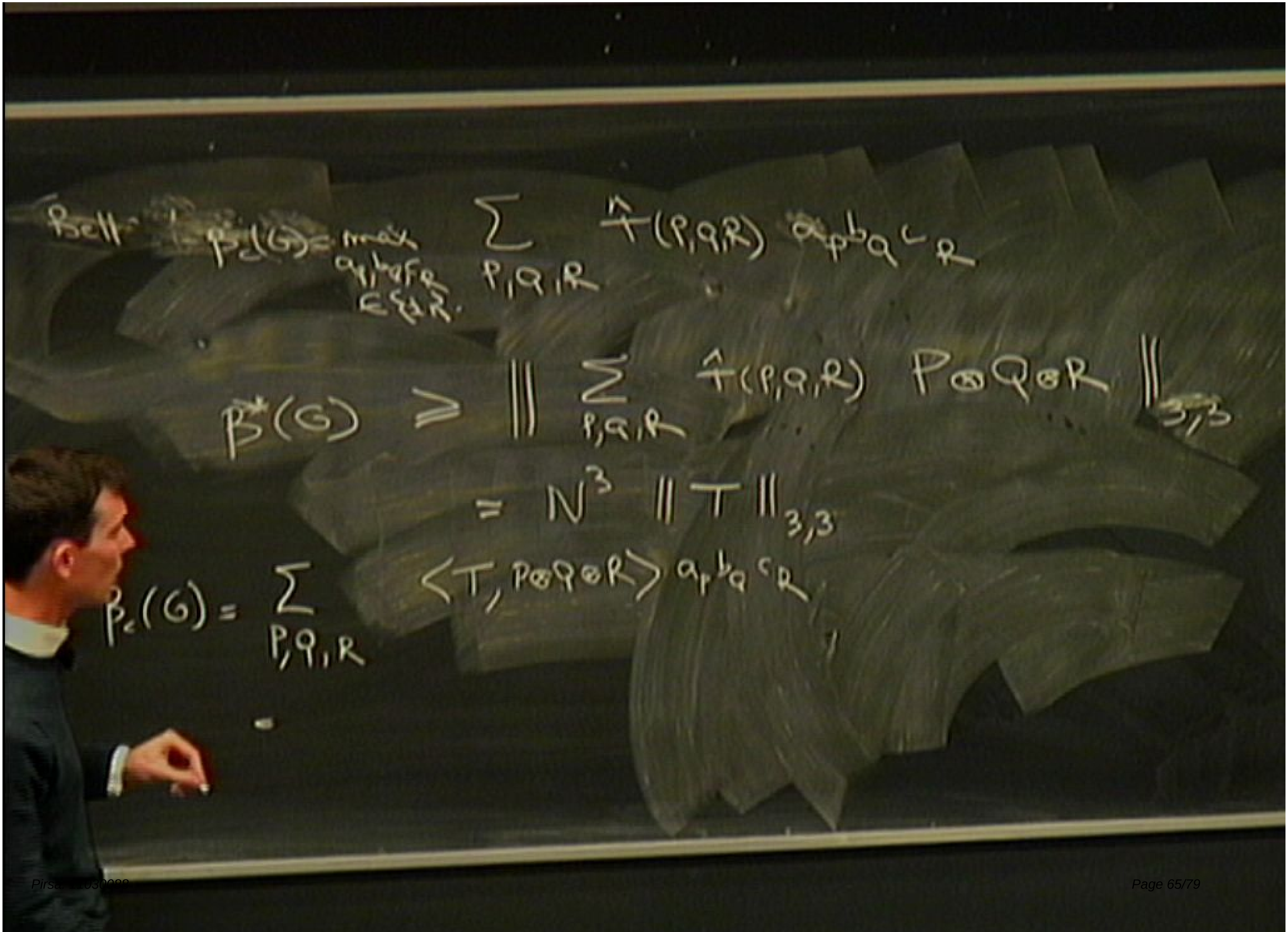
$$\beta_{Bell} = \beta_c(G) = \max_{a_P, b_Q, c_R \in \{\pm 1\}} \sum_{P,Q,R} \hat{f}(P,Q,R)\, a_P b_Q c_R$$

$$\beta^*(G) \geq \Big\| \sum_{P,Q,R} \hat{f}(P,Q,R)\, P \otimes Q \otimes R \Big\|_{3,3}$$

$$= N^3 \, \| T \|_{3,3}$$


Bell: $\beta_c(G) = \max_{a_P, b_Q, c_R \in \{\pm 1\}} \sum_{P,Q,R} \hat{f}(P,Q,R)\, a_P b_Q c_R$
$\beta^*(G) \geq \left\| \sum_{P,Q,R} \hat{f}(P,Q,R)\, P \otimes Q \otimes R \right\|_{3,3}$
$= N^3 \|T\|_{3,3}$


Bell: β_c(G) = max_{a_p, b_Q, c_R ∈ {±1}} Σ_{P,Q,R} f̂(P,Q,R) a_P b_Q c_R
β*(G) ≥ ‖ Σ_{P,Q,R} f̂(P,Q,R) P⊗Q⊗R ‖_{3,3}
= N^3 ‖T‖_{3,3}


Bell
β_c(G) = max Σ f̂(P,Q,R) a_P b_Q c_R
a_P, b_Q, c_R ∈ {±1}   P,Q,R
β*(G) ≥ ‖ Σ f̂(P,Q,R) P⊗Q⊗R ‖_3,3
P,Q,R
= N^3 ‖T‖_3,3
β_c(G) = Σ ⟨T, P⊗Q⊗R⟩ a_P b_Q c_R
P,Q,R

$$\beta_{Bell} = \beta_c(G) = \max_{\substack{a_P, b_Q, c_R \\ \in \{\pm 1\}}} \sum_{P,Q,R} \hat{f}(P,Q,R)\, a_P b_Q c_R$$

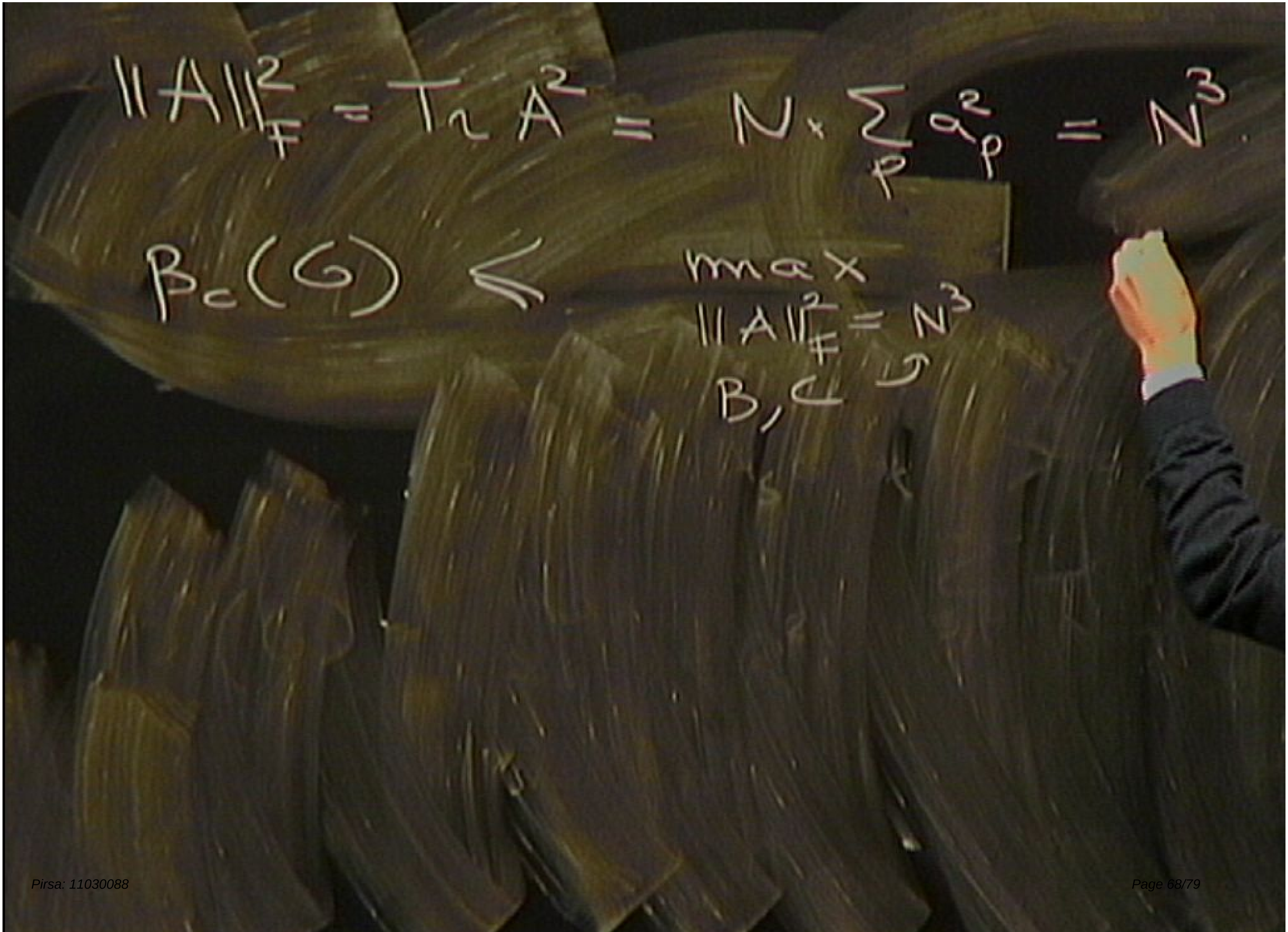
$$\beta^*(G) \geq \left\| \sum_{P,Q,R} \hat{f}(P,Q,R)\, P \otimes Q \otimes R \right\|_{3,3}$$

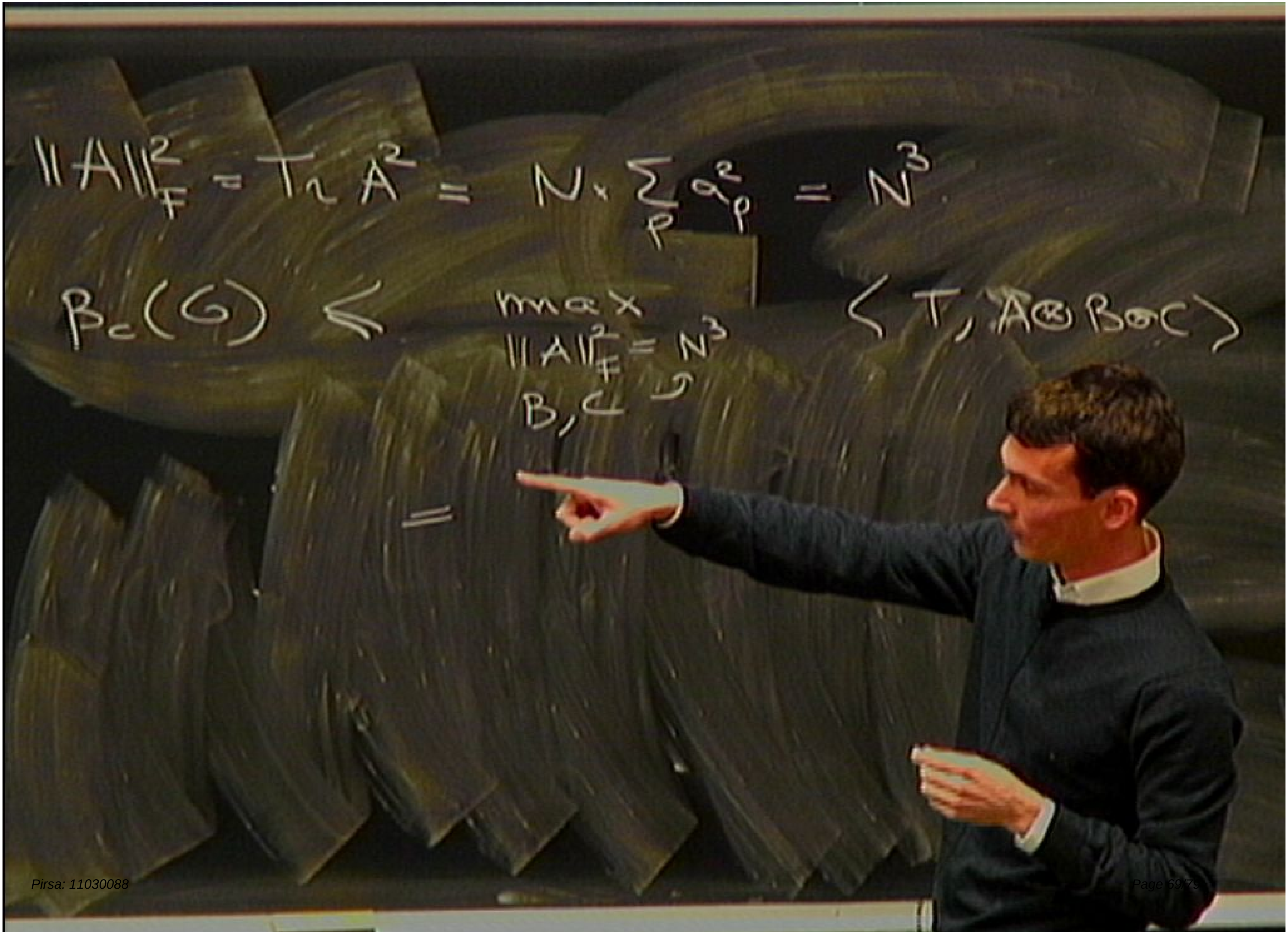
$$= N^3 \, \| T \|_{3,3}$$

$$\beta_c(G) = \sum_{P,Q,R} \langle T, P \otimes Q \otimes R \rangle\, a_P b_Q c_R$$

$$= \Big\langle T, \underbrace{\Big(\sum_P a_P P\Big)}_{A} \otimes \underbrace{\Big(\sum_Q b_Q Q\Big)}_{B} \otimes \underbrace{\Big(\sum_R c_R R\Big)}_{C} \Big\rangle$$

$$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \times \sum_p a_p^2 = N^3$$


$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3$
$\beta_c(G) \leq \max_{\|A\|_F^2 = N^3,\ B, C, D}$


||A||²_F = Tr A² = N × Σ_p a_p² = N³
β_c(G) ≤ max ⟨T, A⊗B⊗C⟩
||A||²_F = N³
B, C
=

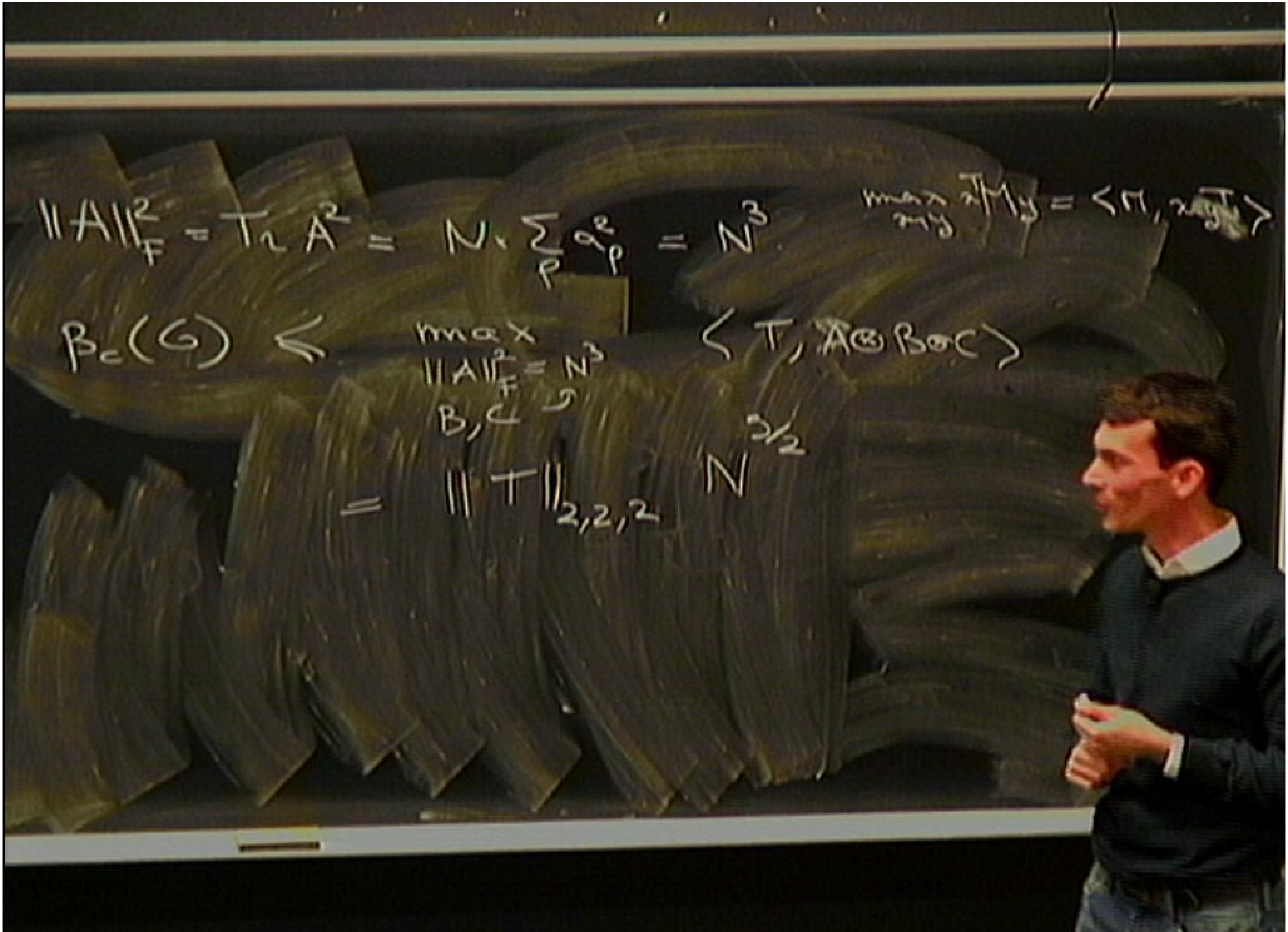
$$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3$$

$$\beta_c(G) \leq \max_{\substack{\|A\|_F^2 = N^3 \\ B, C \circlearrowleft}} \langle T, A \otimes B \otimes C \rangle = \|T\|_{2,2,2}\, N^{3/2}$$

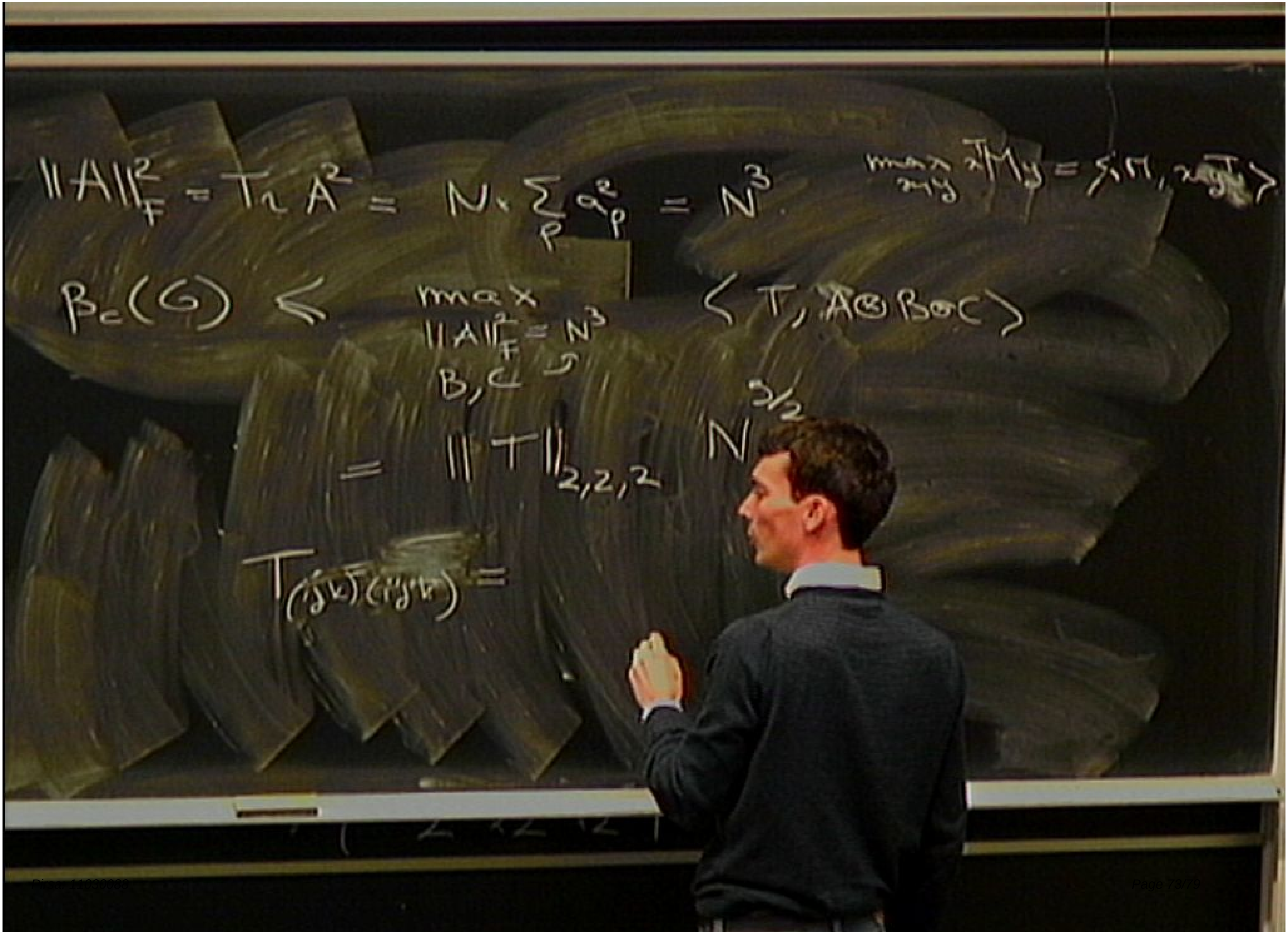
$$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3 \qquad \max_{x,y} x^T M y = \langle M, x y^T \rangle$$

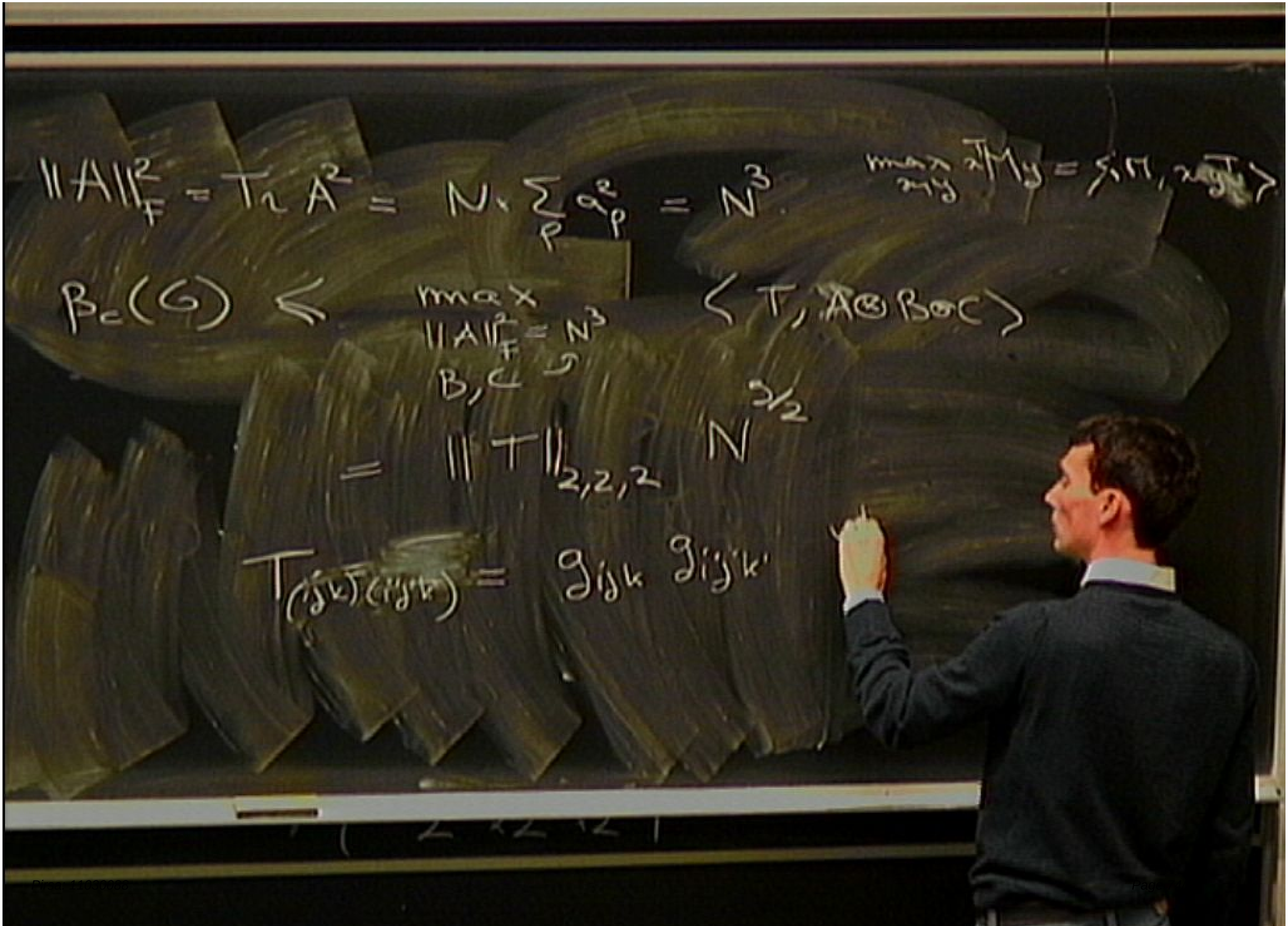
$$\beta_c(G) \leq \max_{\substack{\|A\|_F^2 = N^3 \\ B, C\ \text{[illegible]}}} \langle T, A \otimes B \otimes C \rangle$$

$$= \|T\|_{2,2,2}\, N^{3/2}$$



$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3$
$\max_{x,y} x^T M y = \langle M, x y^T \rangle$
$\beta_c(G) \leq \max_{\|A\|_F^2 = N^3,\ B, C} \langle T, A \otimes B \otimes C \rangle$
$= \|T\|_{2,2,2}\, N^{3/2}$


$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3$
$\max_{x,y} x^T M y = \langle M, xy^T \rangle$
$\beta_c(G) \leq \max_{\|A\|_F^2 = N^3,\ B, C} \langle T, A \otimes B \otimes C \rangle$
$= \|T\|_{2,2,2}\, N^{3/2}$
$T_{(ijk)(i'j'k')} =$



||A||_F^2 = Tr A^2 = N · Σ_p a_p^2 = N^3
max_{x,y} x^T M y = ⟨M, x y^T⟩
β_c(G) ≤ max_{||A||_F^2 = N^3, B, C} ⟨T, A⊗B⊗C⟩
= ||T||_{2,2,2} N^{3/2}
T_{(ijk)(i'j'k')} = g_{ijk} g_{i'j'k'}

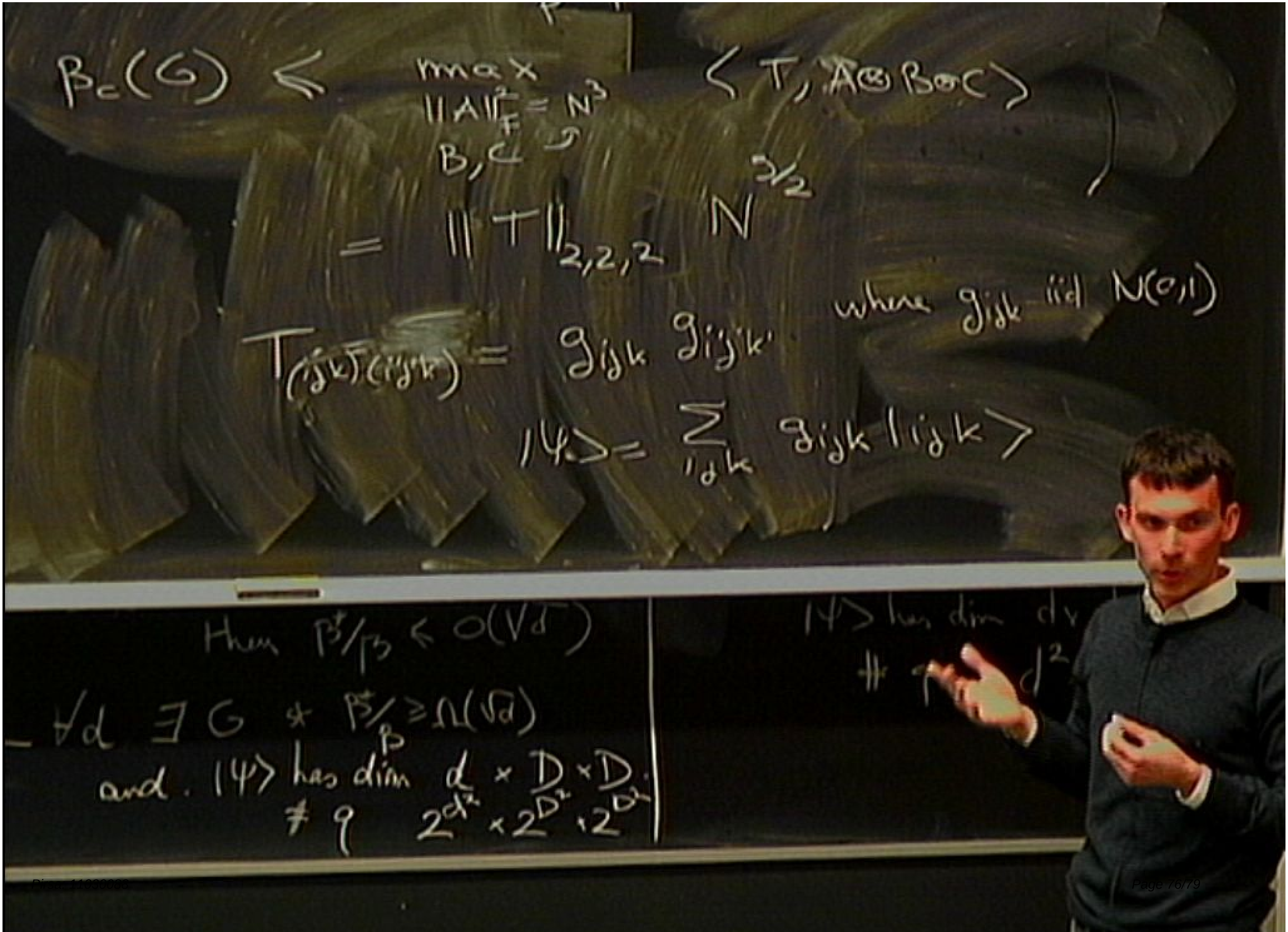
$$\|A\|_F^2 = \mathrm{Tr}\, A^2 = N \cdot \sum_p a_p^2 = N^3$$

$$\max_{x,y} x^T M y = \langle M, x y^T \rangle$$

$$\beta_c(G) \leq \max_{\substack{\|A\|_F^2 = N^3 \\ B, C \,\circlearrowleft}} \langle T, A \otimes B \otimes C \rangle$$

$$= \|T\|_{2,2,2} \; N^{3/2}$$

$$T_{(ijk)(i'j'k')} = g_{ijk}\, g_{i'j'k'} \quad \text{where } g_{ijk} \text{ iid } N(0,1)$$



βc(G) ≤ max ⟨T, A⊗B⊗C⟩
‖A‖F² = N³
B, C ↺
= ‖T‖2,2,2 N^3/2
T(ijk)(i'j'k') = gijk gi'j'k' where gijk iid N(0,1)
|ψ⟩ = Σijk gijk |ijk⟩
then β*/β ≤ O(√d)
∀d ∃ G s.t. β*/β ≥ Ω(√d)
and |ψ⟩ has dim d × D × D
q 2^d² × 2^D² × 2^D²
|ψ⟩ has dim d ×
q d²

$$|\psi\rangle = \sum_{ijk} \alpha_{ijk} |ijk\rangle$$

3-player XOR games

PG W PV / J

- GHZ can only give bounded $\beta^*/\beta_c \le 2^{\frac{1}{2}(n-1)} K_G$
- If $|\psi\rangle$ has local dimension d then $\beta^*/\beta \le O(\sqrt{d})$
- $\forall d\ \exists G$ s.t. $\beta^*/\beta \ge \Omega(\sqrt{d})$ and $|\psi\rangle$ has dim $d \times D \times D$, # q $2^{d^2} \times 2^{D^2} \times 2^{D^2}$

$$|\psi\rangle = \sum_i \alpha_i |i,i,i\rangle$$

- → same bound
- if # q = n then $\beta^*/\beta_c \le O(\sqrt{n})$
- $\forall d\ \exists G$ s.t. $\beta^*/\beta = \Omega(\sqrt{d})$, $|\psi\rangle$ has dim $d \times d \times d$, # q $d^2 \times d^2 \times d^2$.

$$|\psi\rangle = \sum_{ijk} \alpha_{ijk} |ijk\rangle$$

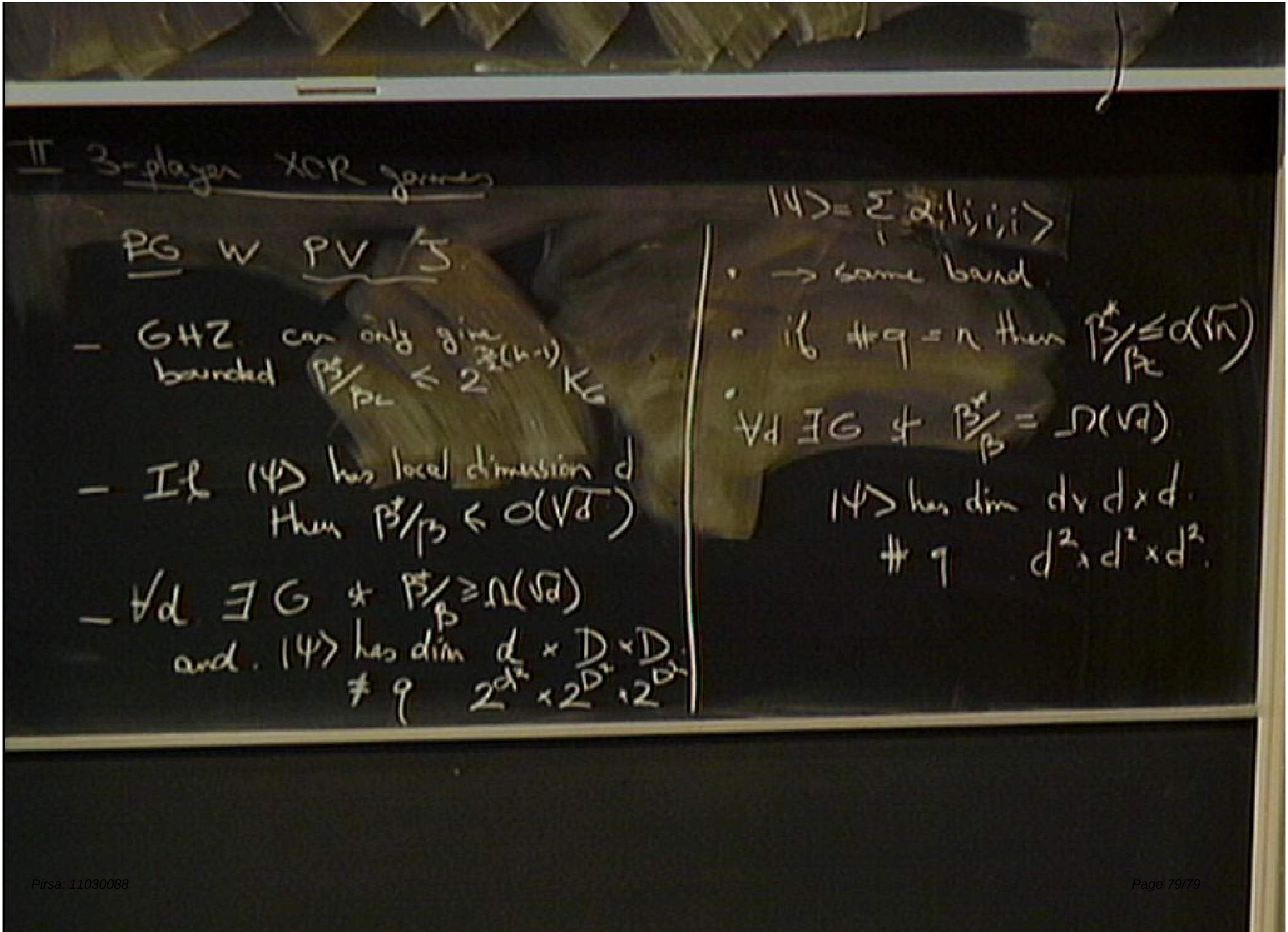
3-player XOR games

BG W PV / J

- GHZ can only give bounded $\beta^*/\beta_c \le 2^{\frac{3}{2}(n-1)} K_G$
- If $|\psi\rangle$ has local dimension d then $\beta^*/\beta \le O(\sqrt{d})$
- $\forall d\ \exists G$ st $\beta^*/\beta \ge \Omega(\sqrt{d})$ and $|\psi\rangle$ has dim $d \times D \times D$
  # q $2^{d^2} \times 2^{D^2} \times 2^{D^2}$

$$|\psi\rangle = \sum_i \alpha_i |i,i,i\rangle$$

- → same bound
- if # q = n then $\beta^*/\beta_c \le O(\sqrt{n})$
- $\forall d\ \exists G$ st $\beta^*/\beta = \Omega(\sqrt{d})$
  $|\psi\rangle$ has dim $d \times d \times d$
  # q $d^2 \times d^2 \times d^2$


II 3-player XOR games
PG W PV / J
– GHZ can only give bounded β*/βc ≤ 2
– If |ψ⟩ has local dimension d then β*/β ≤ O(√d)
– ∀d ∃G st β*/β ≥ Ω(√d) and |ψ⟩ has dim d × D × D
q 2^{d²} × 2^{D²} × 2^{D²}
|ψ⟩ = Σ_i α_i |i,i,i⟩
• → same bound
• if #q = n then β*/βc ≤ O(√n)
• ∀d ∃G st β*/β = Ω(√d)
|ψ⟩ has dim d × d × d
q d² × d² × d²