Title: The quantum adiabatic theorem and eigenpath traversal

Date: Mar 14, 2011 04:00 PM

URL: http://pirsa.org/11030085

Abstract: We review situations under which a standard quantum adiabatic condition fails. We reformulate the problem of adiabatic evolution as the problem of Hamiltonian eigenpath traversal, and give convergence conditions in terms of the length of the eigenpath and the minimum energy gap of the Hamiltonians. We introduce a randomized evolution method that can be used to traverse the eigenpath and prove its convergence and cost. We then describe more efficient methods for the same task and show that their implementation complexity is close to optimal.

1. Motivation
2. Problems with “folk” versions
3. Randomization
4. Fast quantum algorithms for eigenpath traversal
5. The cost path / gap is optimal

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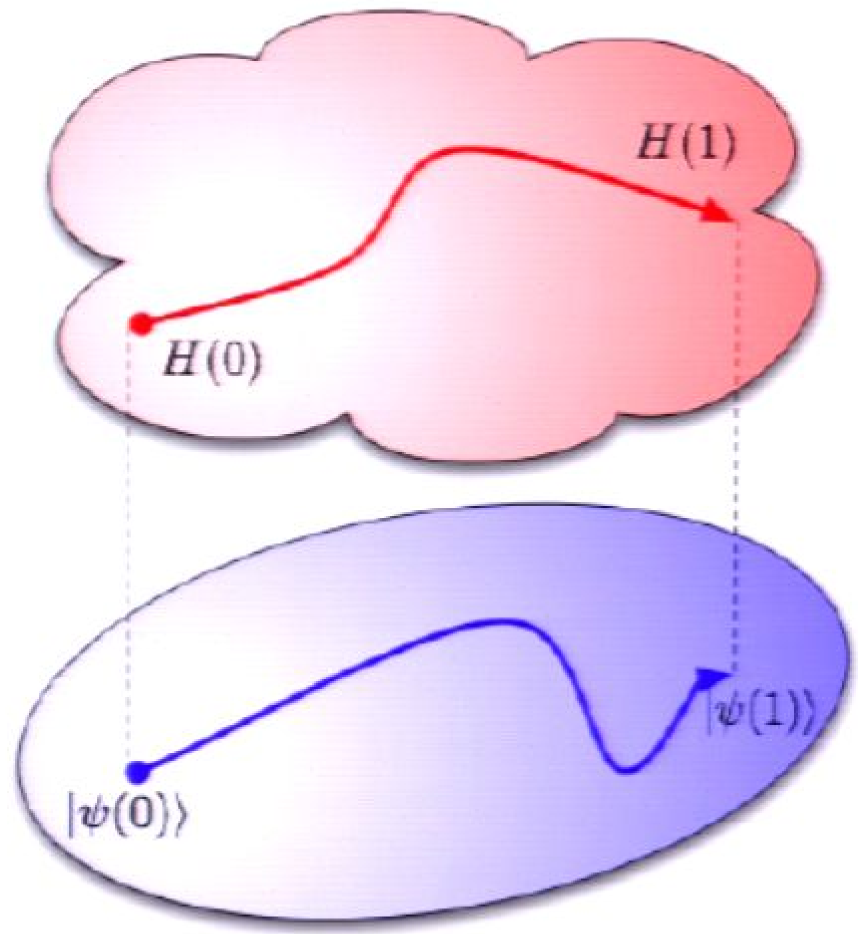
1. If the system is at eigenstate $|\psi(s)\rangle$
2. and the Hamiltonian $H(s)$ is changed very "slowly"
3. the system remains an eigenstate $|\psi(s')\rangle$.



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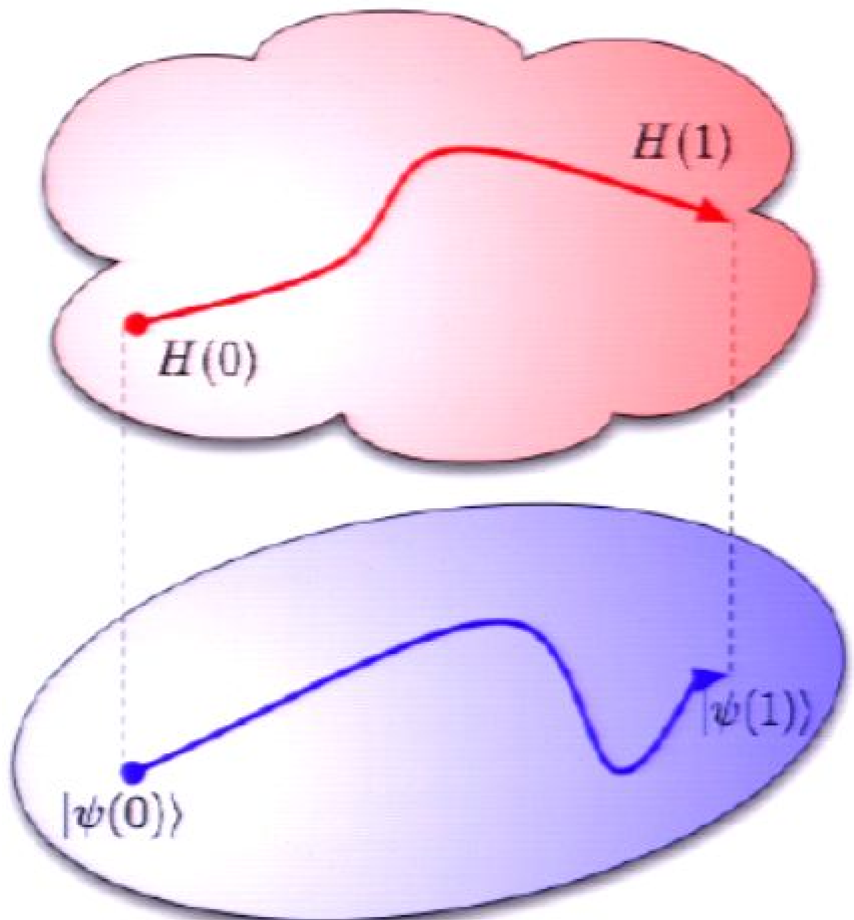
Adiabatic approximation

How slow should the change be? For gap energy Δ,

$$\frac{\|\partial_t H(s(t))\|}{\Delta^2} \ll 1 \quad ???$$



Motivation

- The adiabatic theorem and the adiabatic approximation are widely used in physics and chemistry: quantum Hall, Born-Oppenheimer, STIRAP, geometric phases...

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- Adiabatic quantum computation is universal.
- Key method for quantum simulations and a powerful heuristic for satisfiability problems.
- Quantum speedups of classical Monte-Carlo and other algorithms

  Somma, Boixo, Barnum and Knill and other works.

Summary of results

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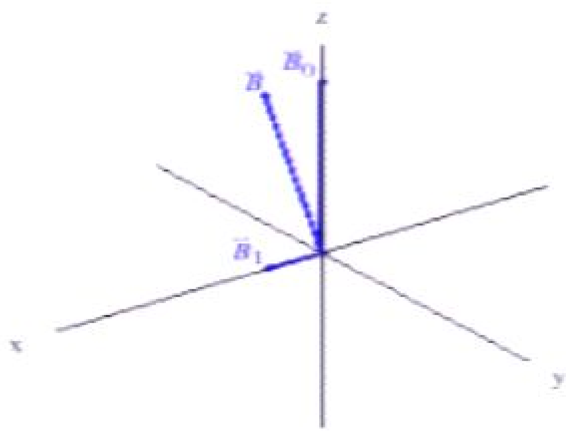
3. Randomized adiabatic approximations are more robust and have better scaling, L^2/Δ. They amount to instantaneous phase decoherence.
4. The cost of (algorithmic) eigenpath traversal is L/Δ. Not even continuity is necessary.
5. The scaling with L/Δ is necessary.

1. Motivation
2. **Problems with “folk” versions**
3. Randomization
4. Fast quantum algorithms for eigenpath traversal
5. The cost path [illegible] gap is optimal

Adiabatic toy model

NMR or two-level atom



$$H(s) = -\frac{\Delta}{2}\left(\cos(\theta)\sigma_z + \sin(\theta)\left(\cos(\nu s)\sigma_x - \sin(\nu s)\sigma_y\right)\right) .$$

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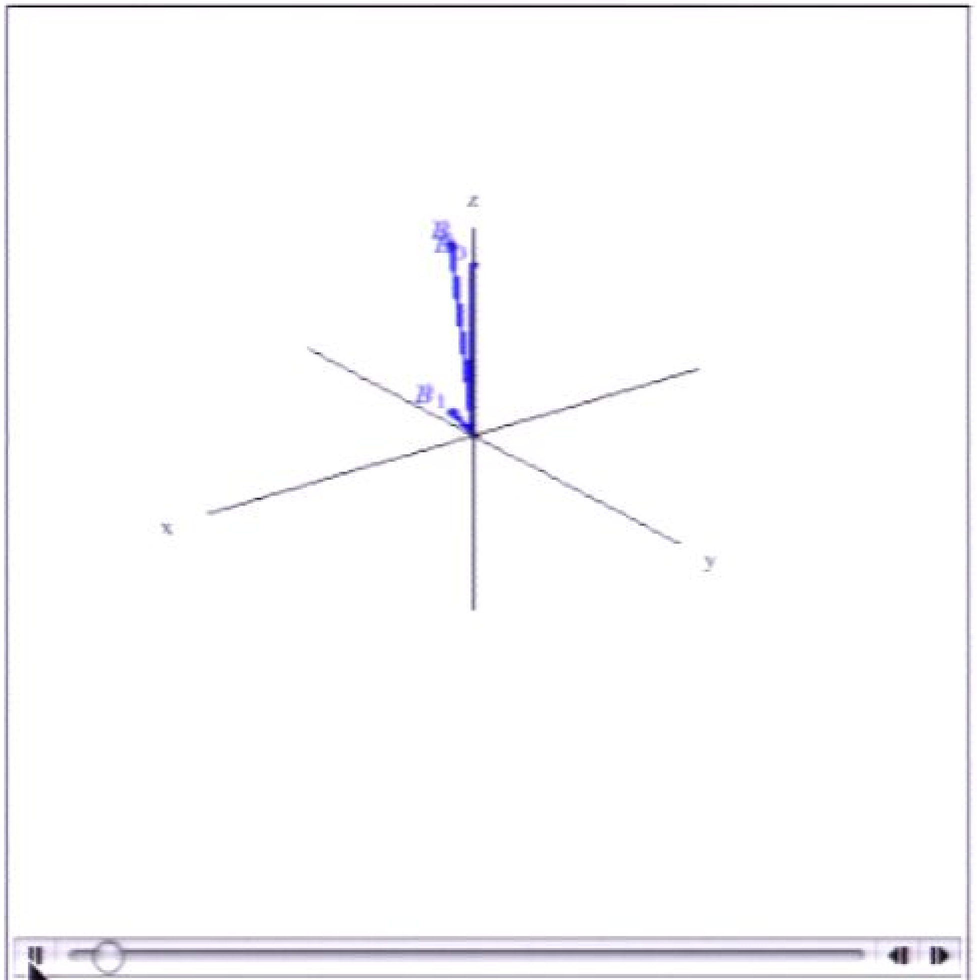

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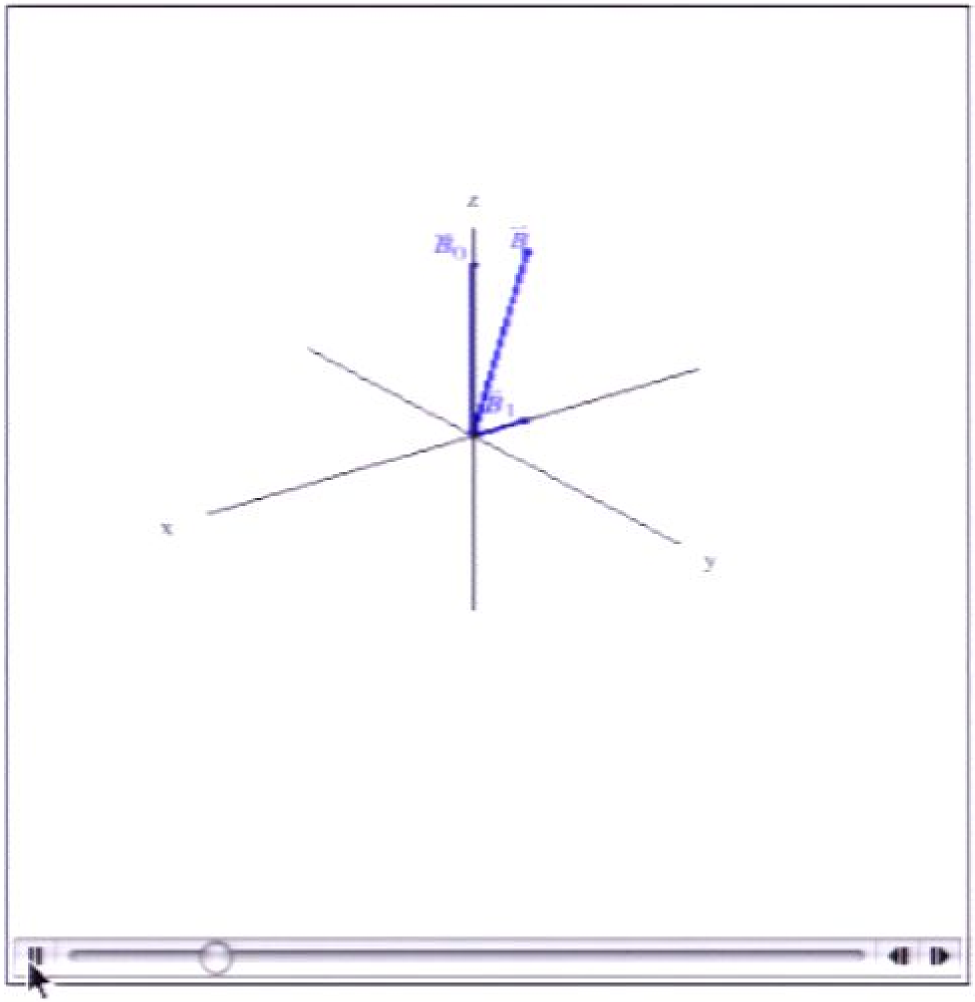

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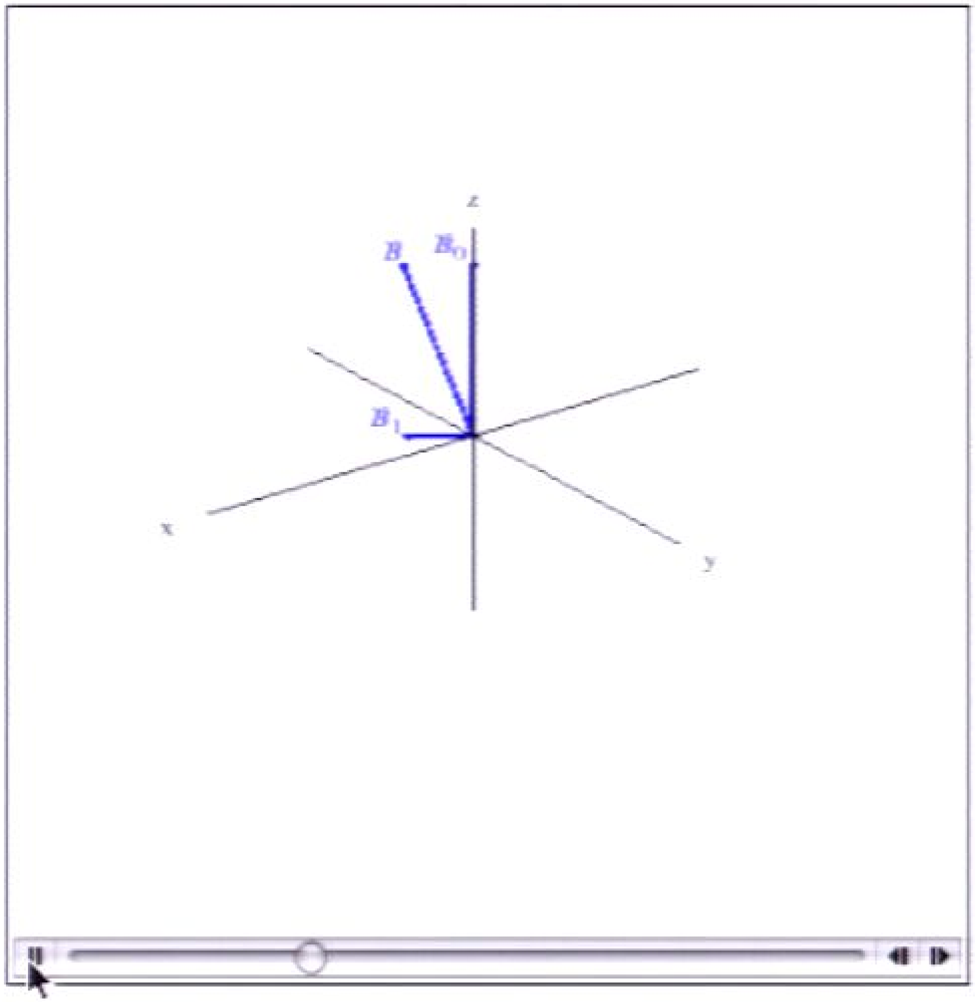

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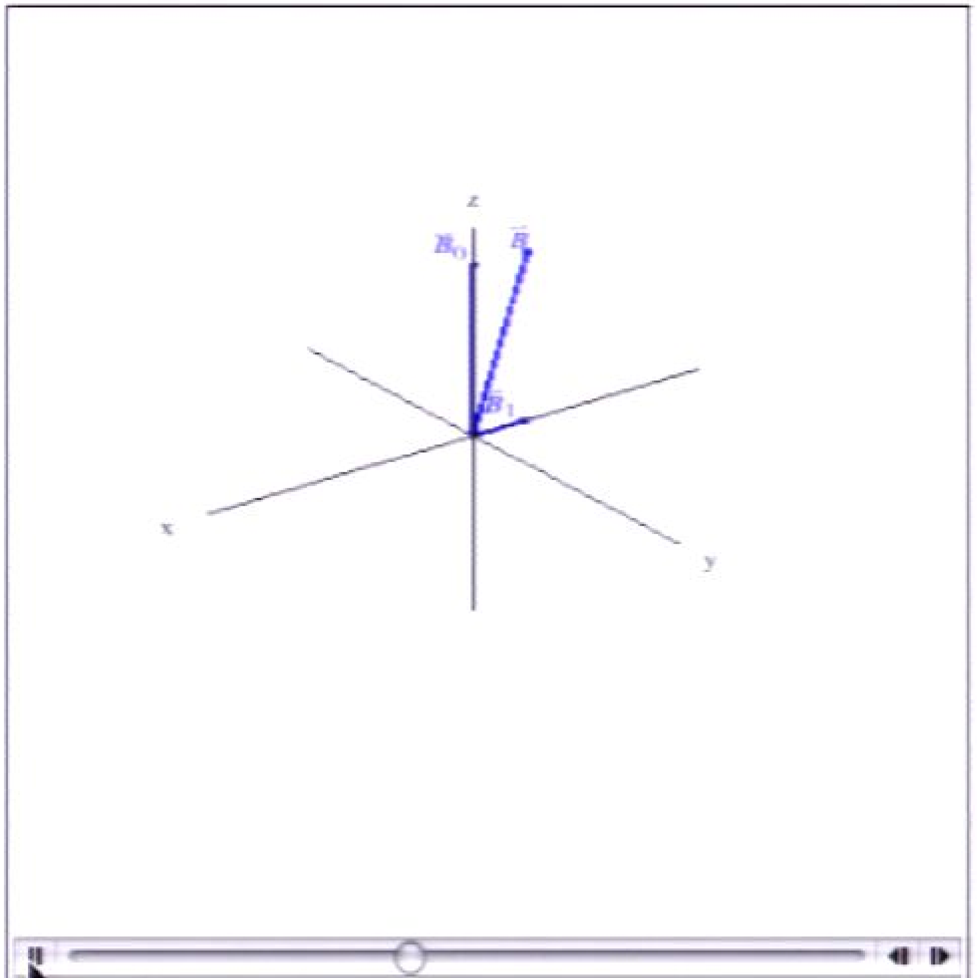

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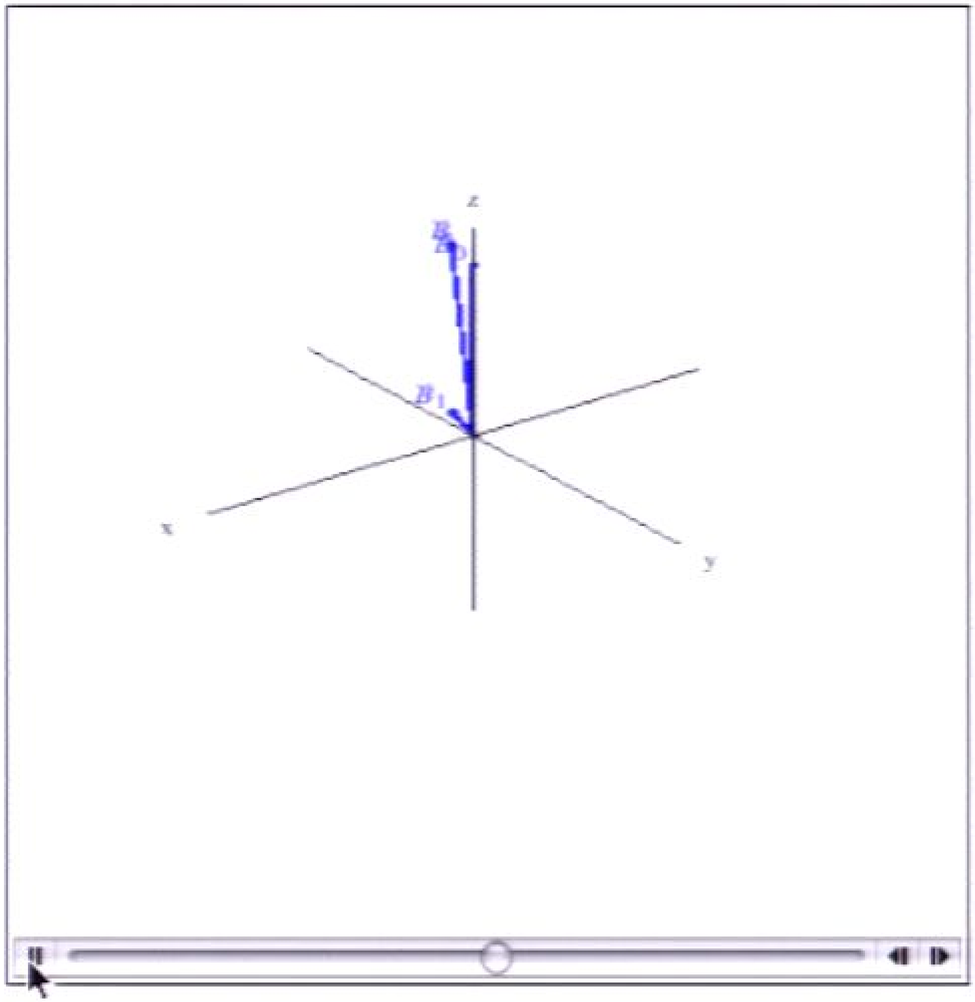

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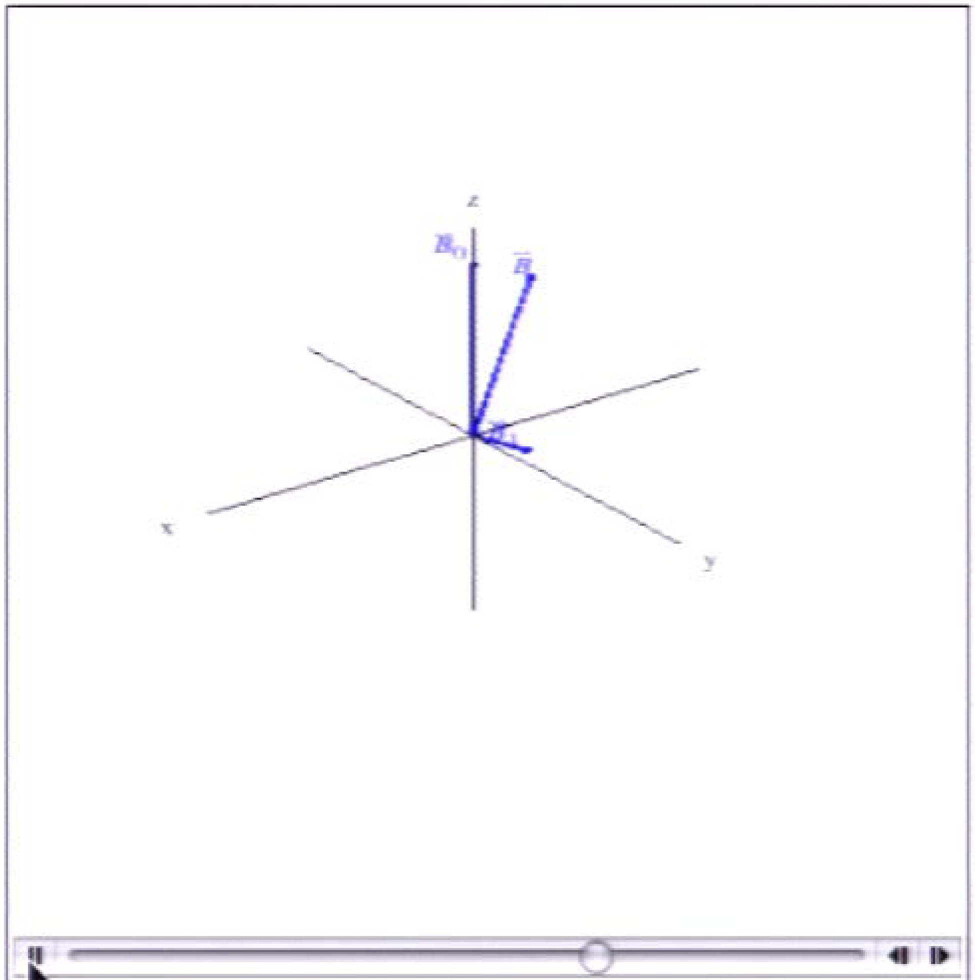

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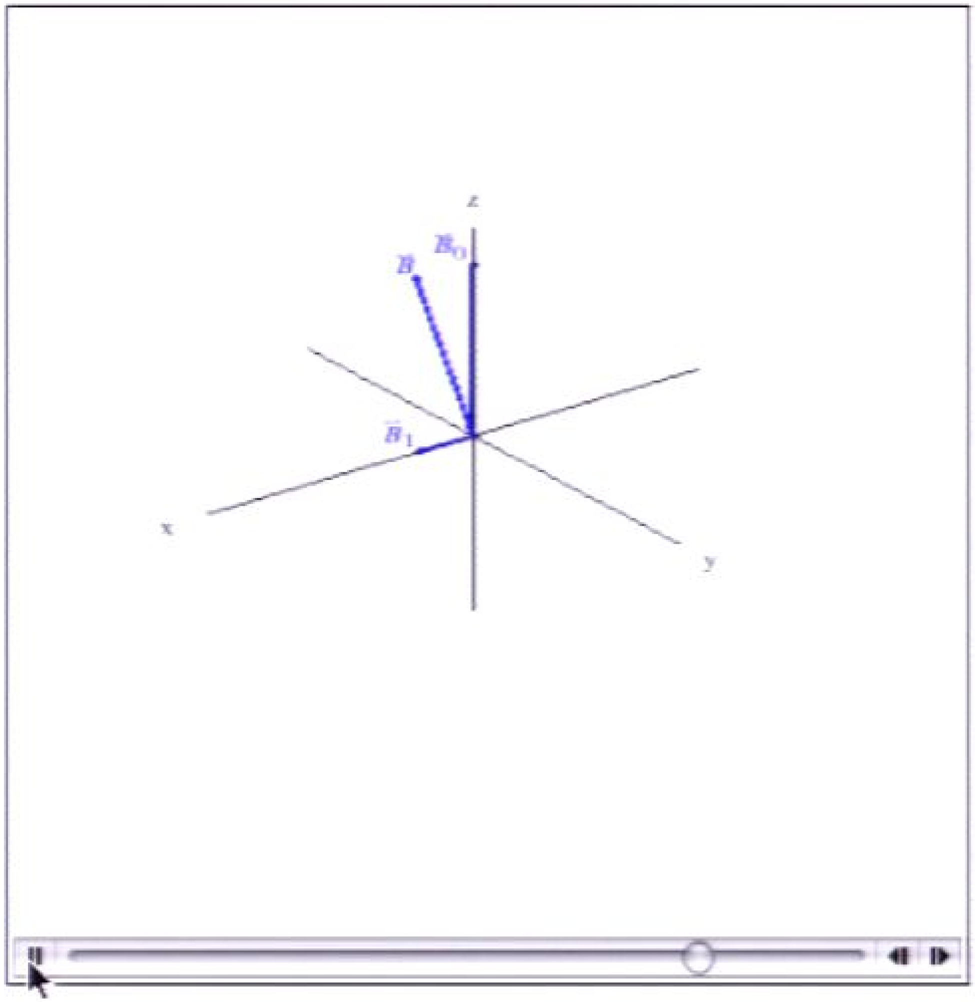

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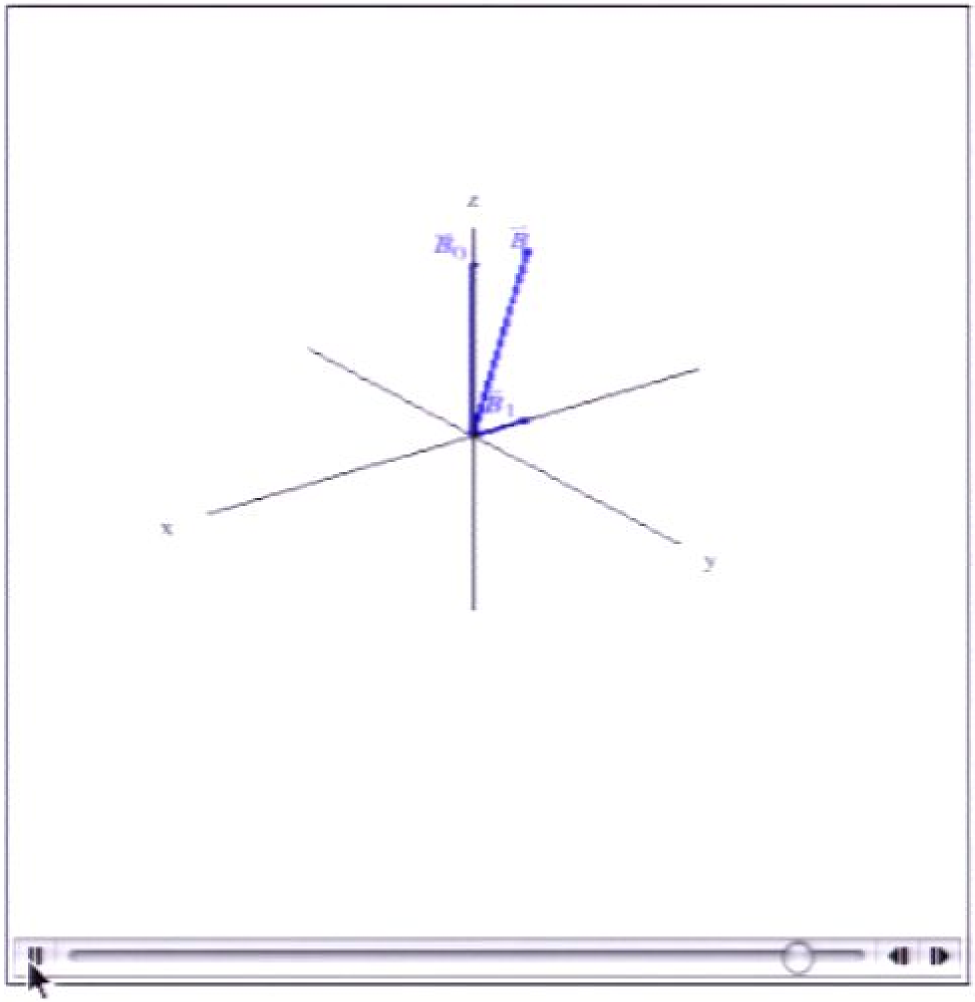

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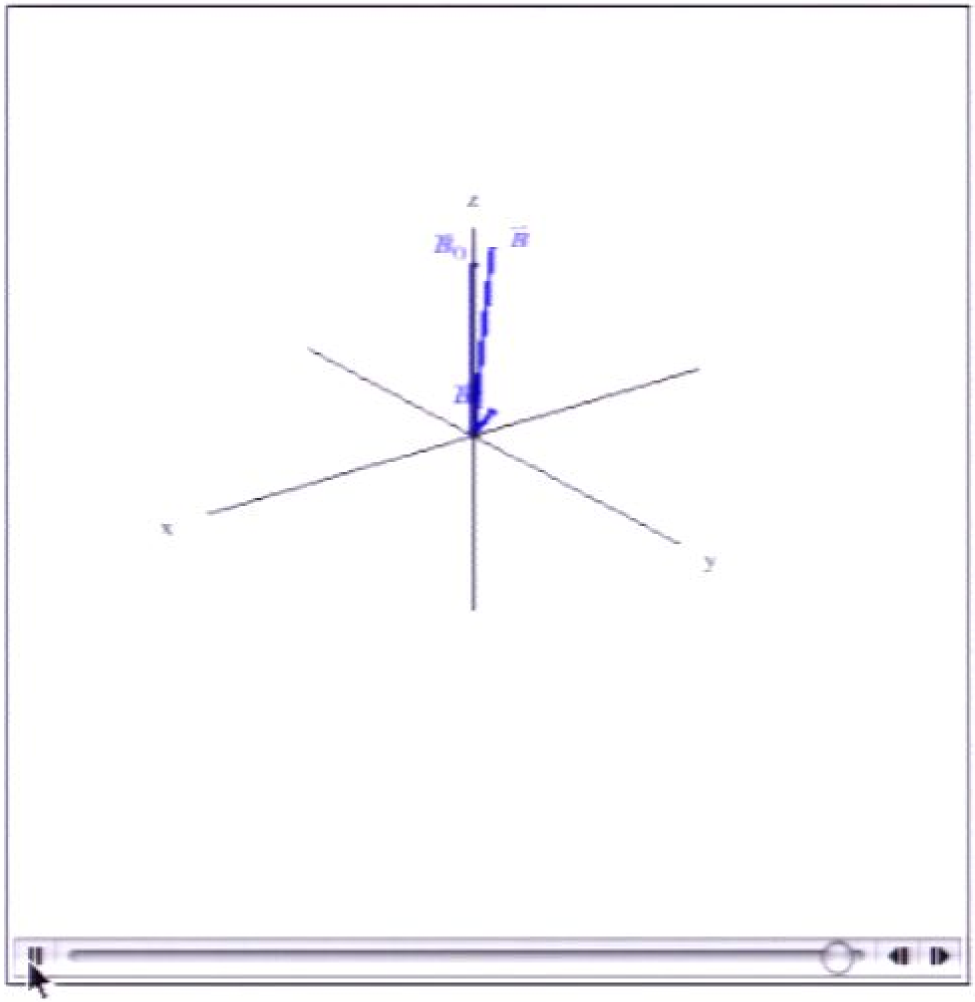

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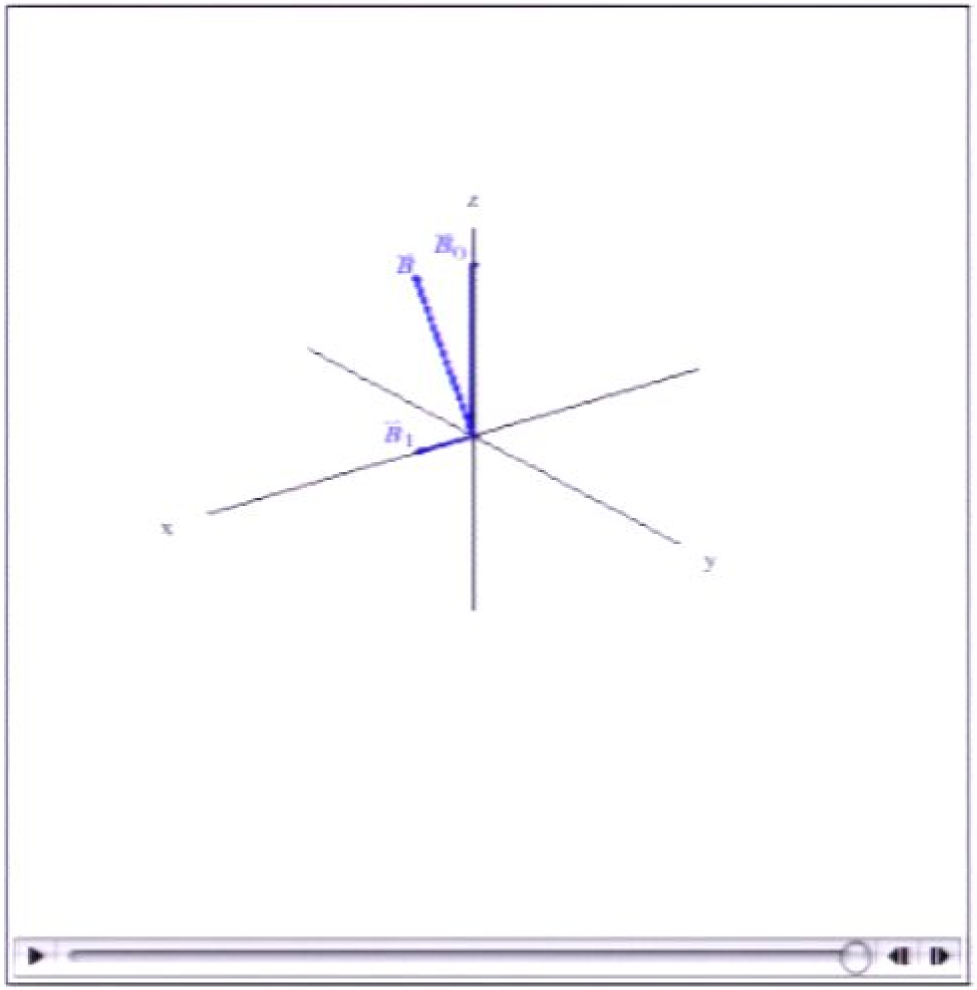

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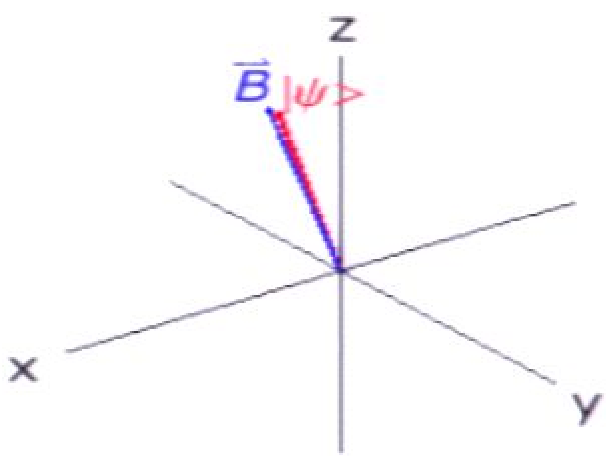
Off-resonant radiation

Low frequency field

$\nu \ll \Delta$



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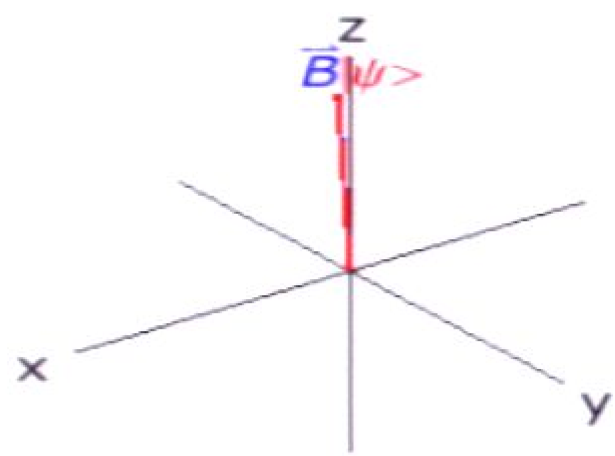

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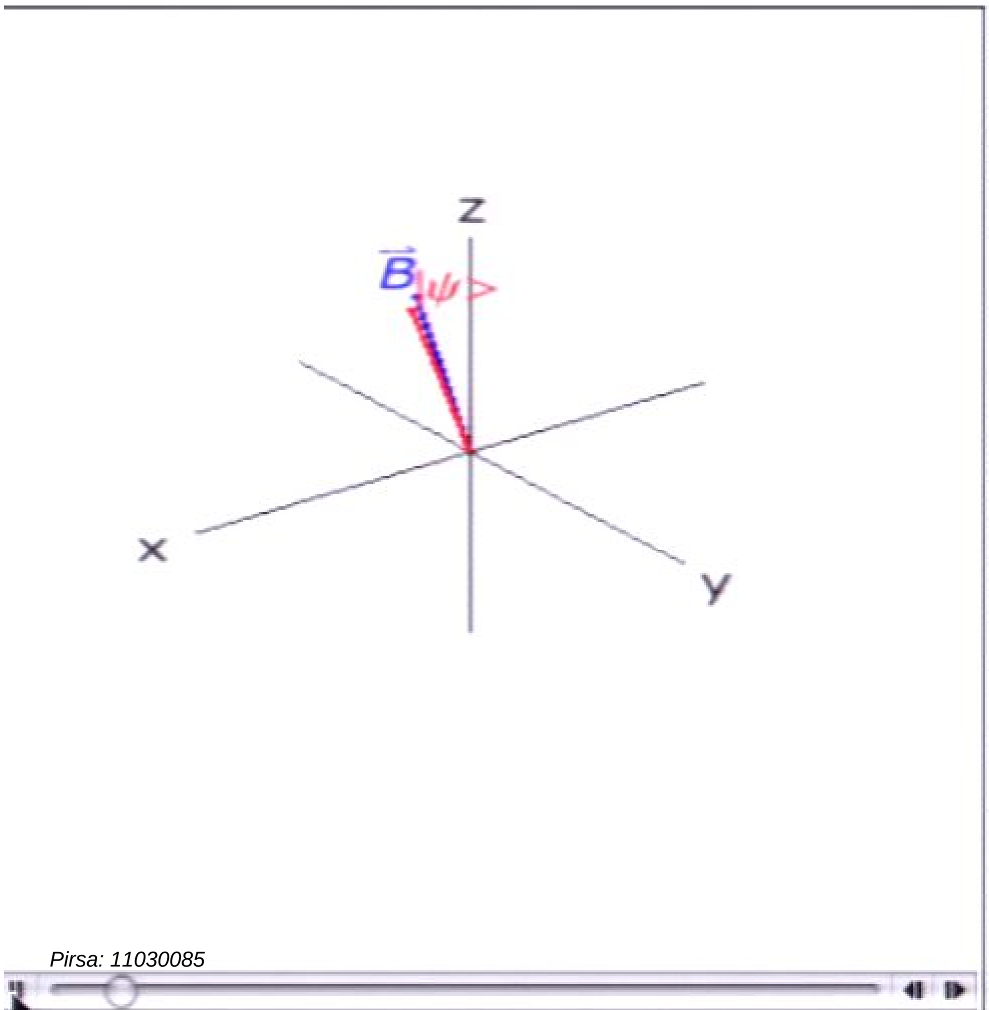

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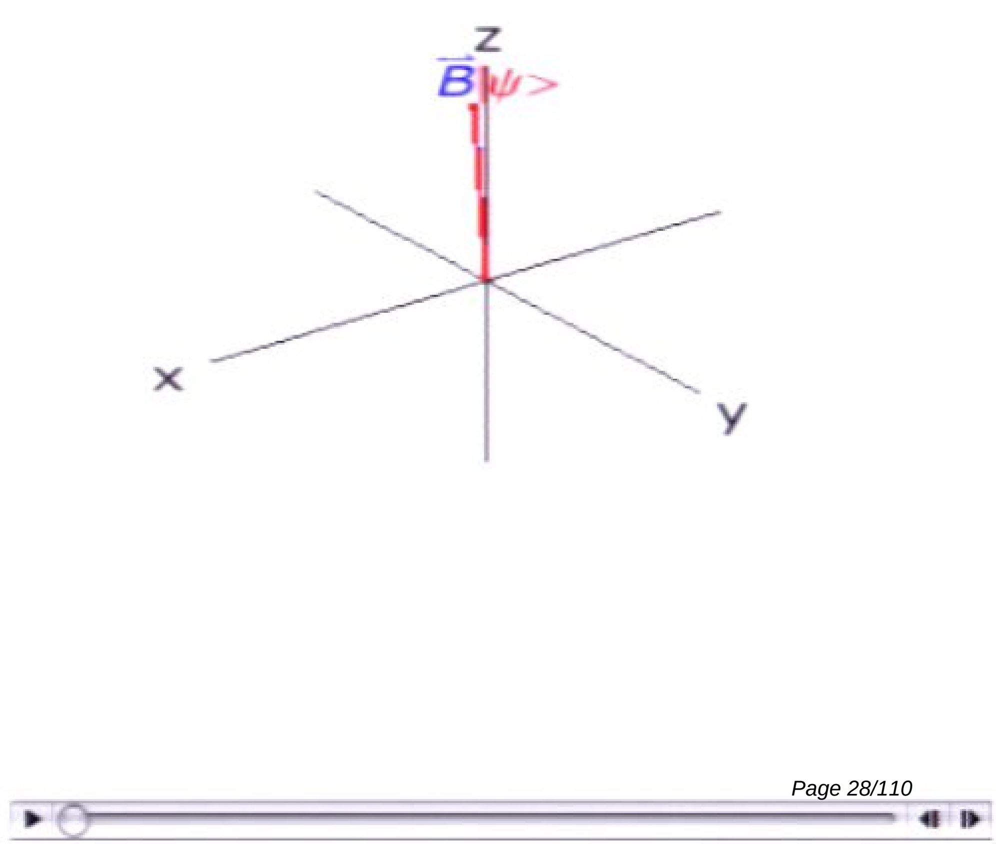

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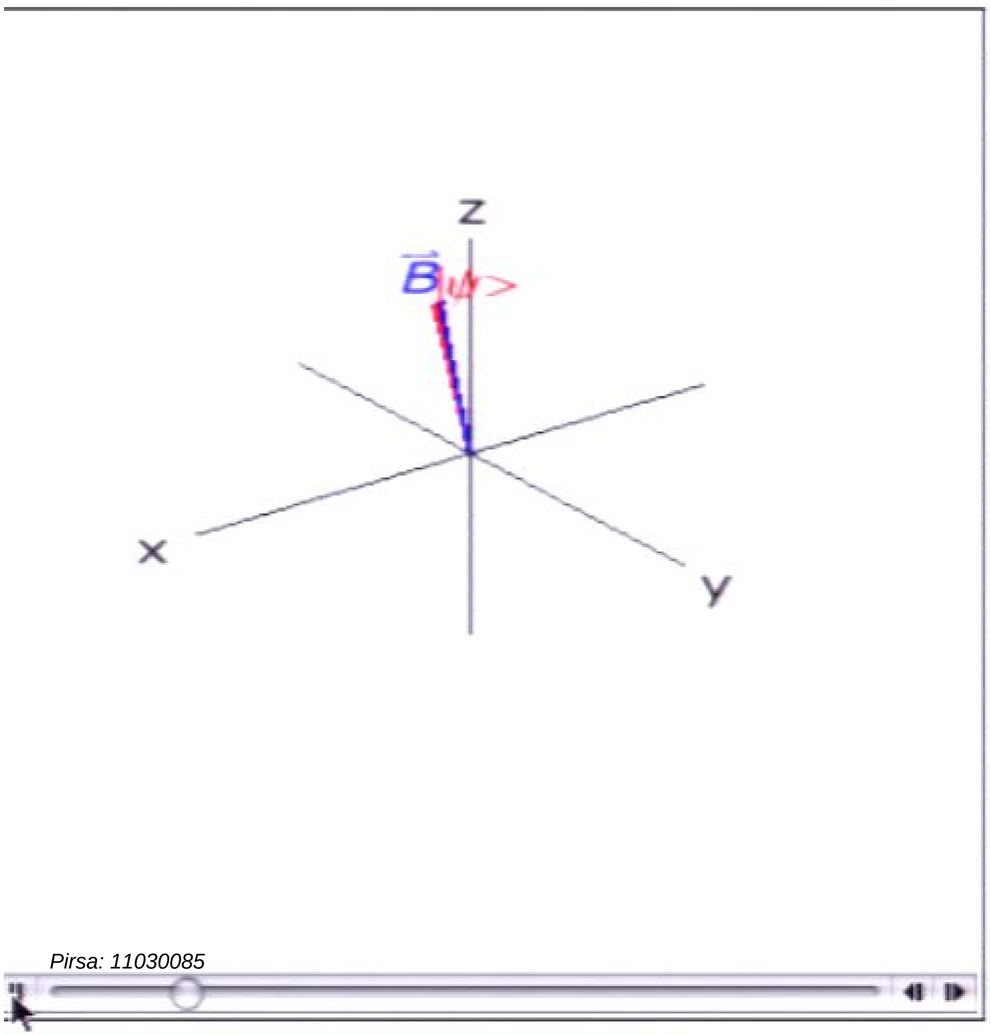

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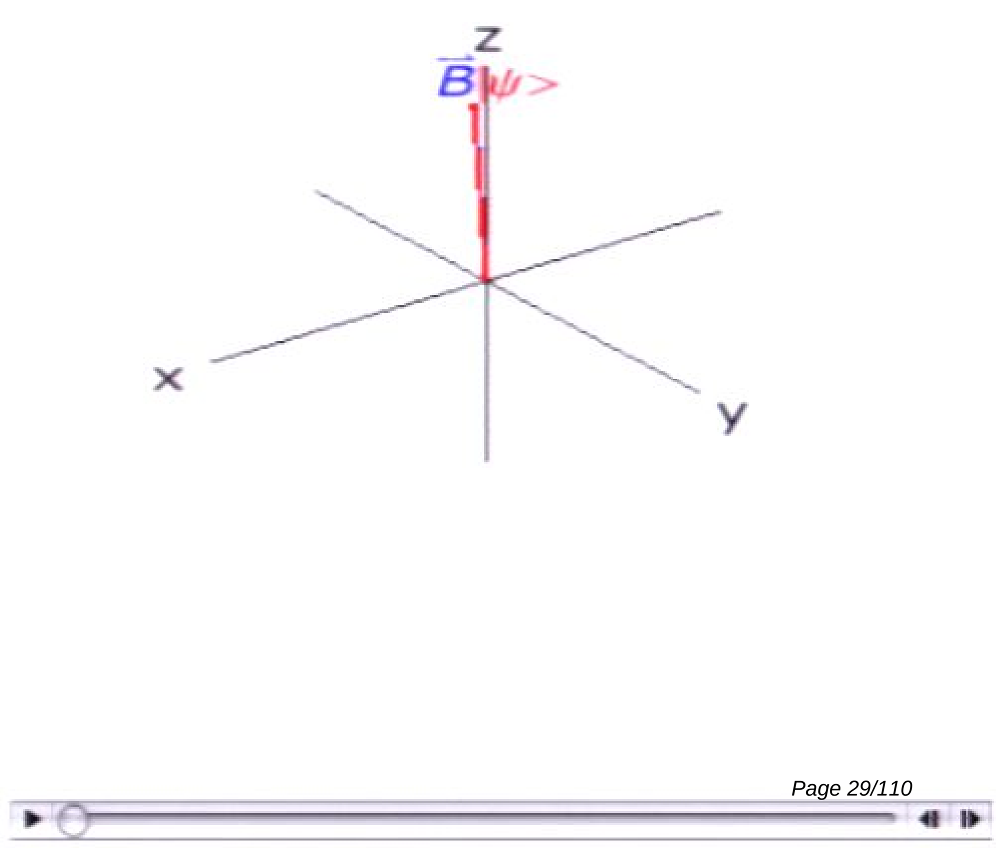

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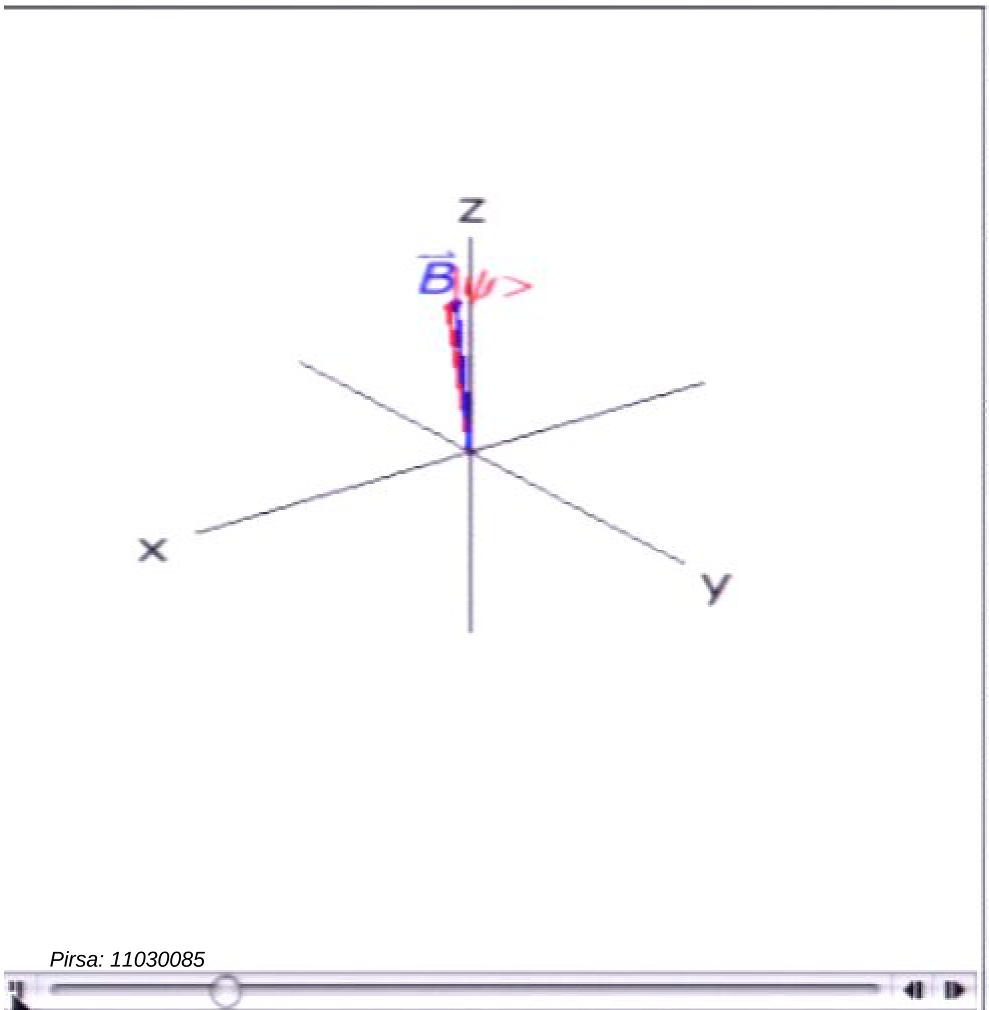

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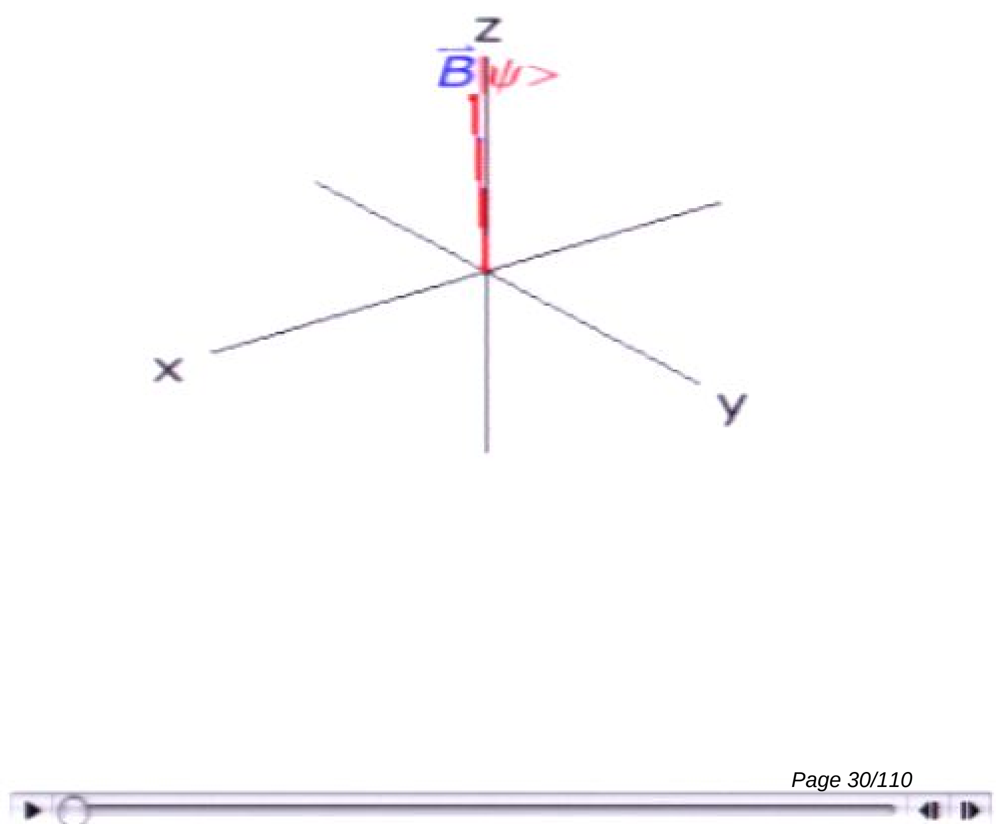

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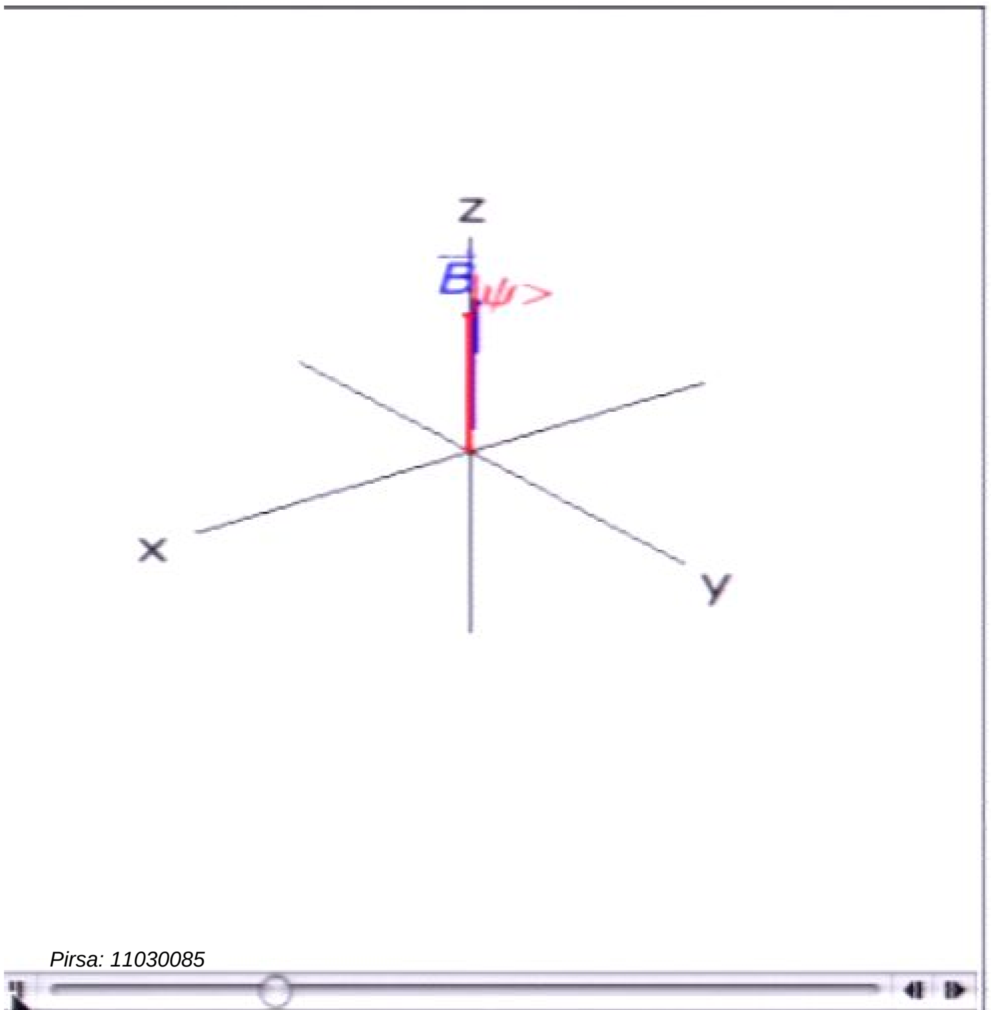

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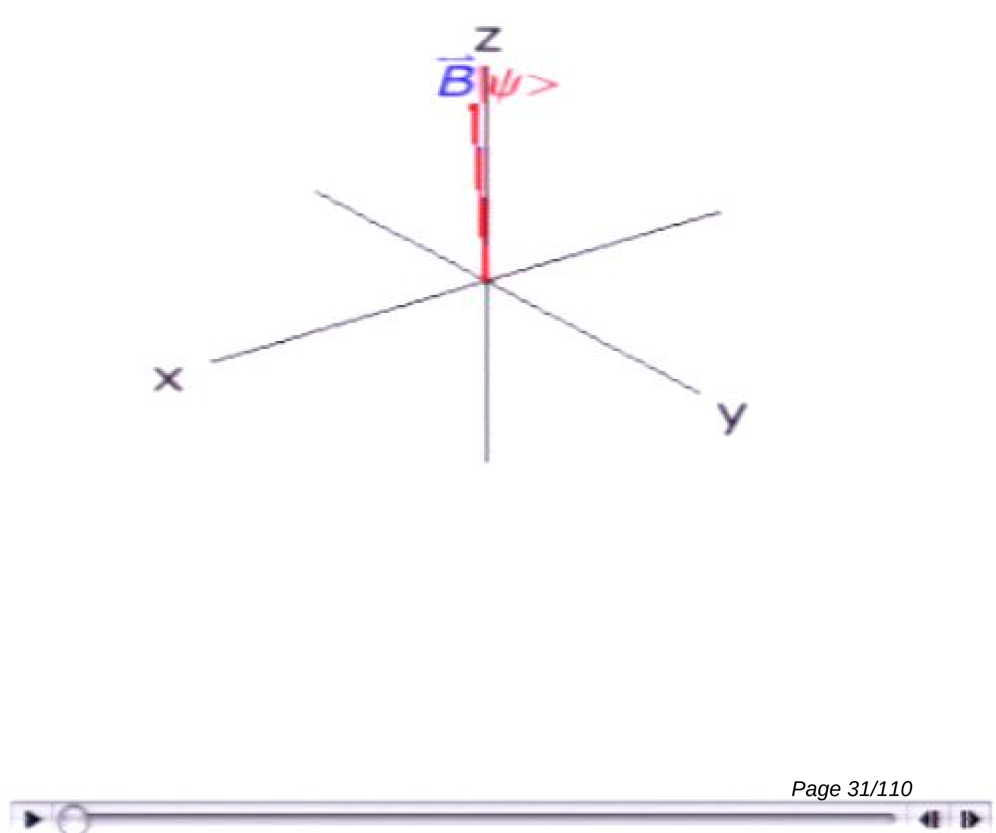

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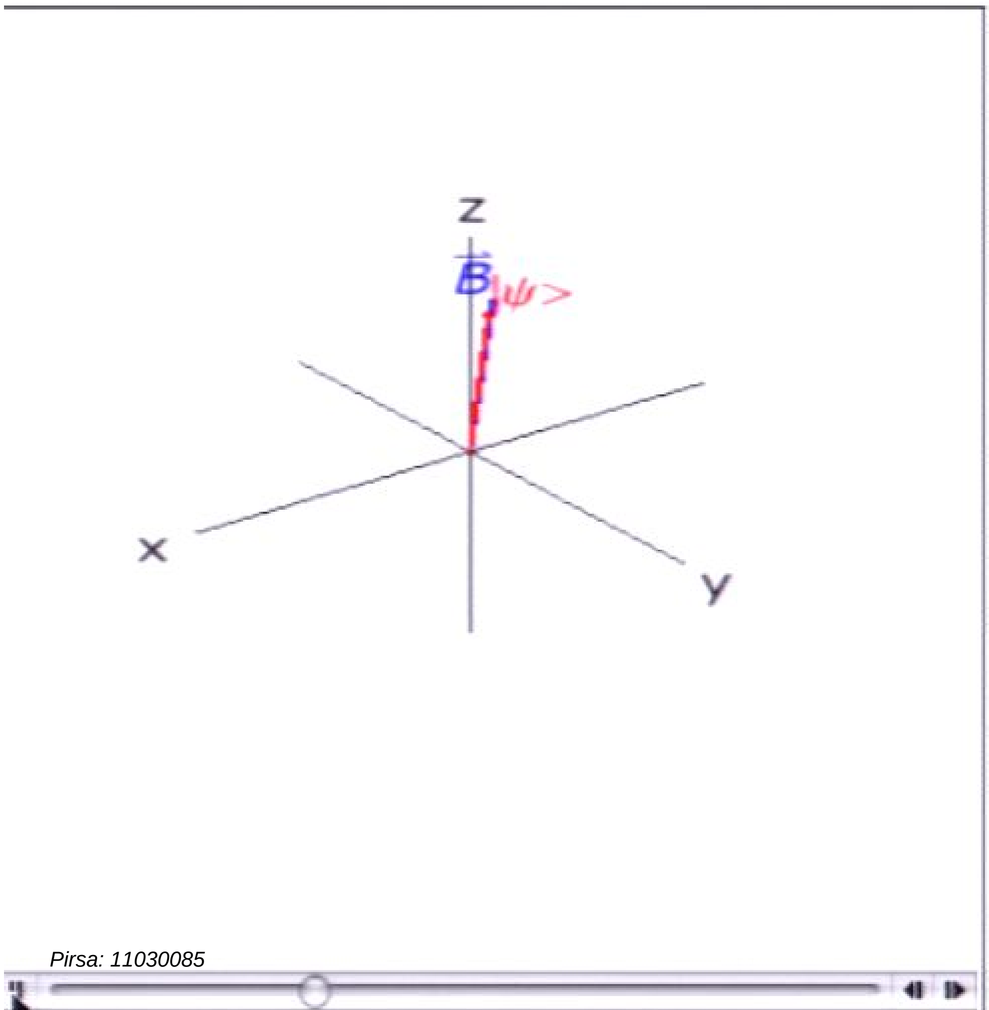

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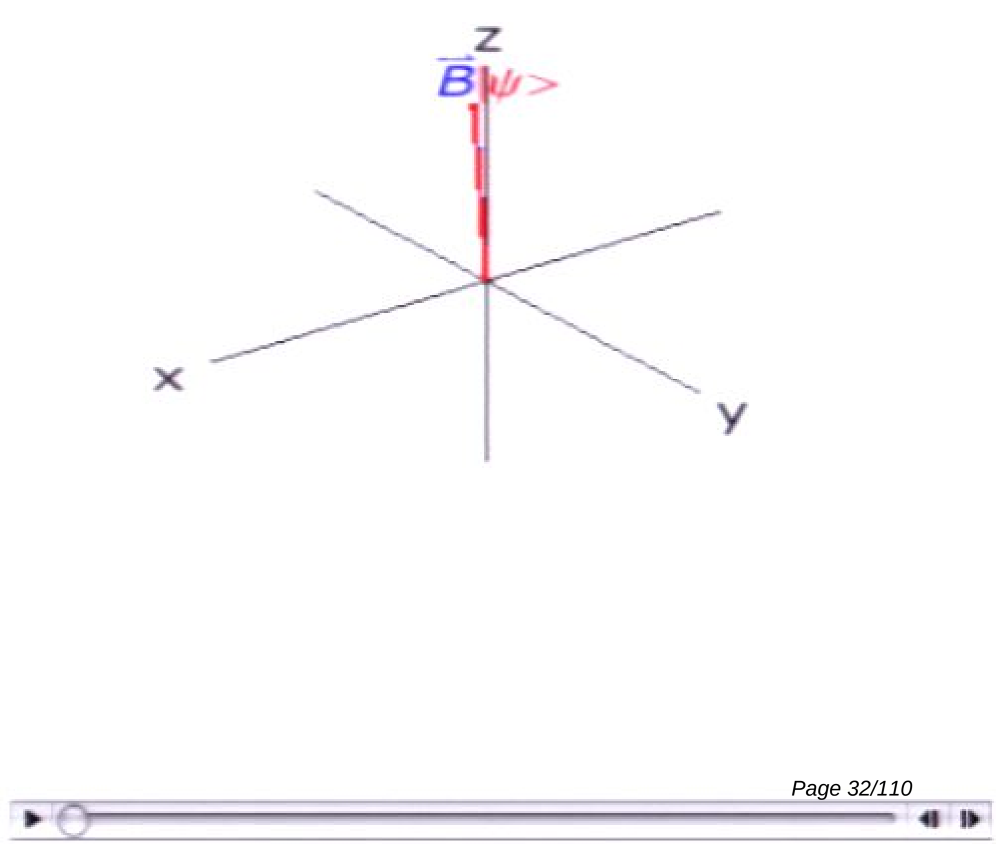

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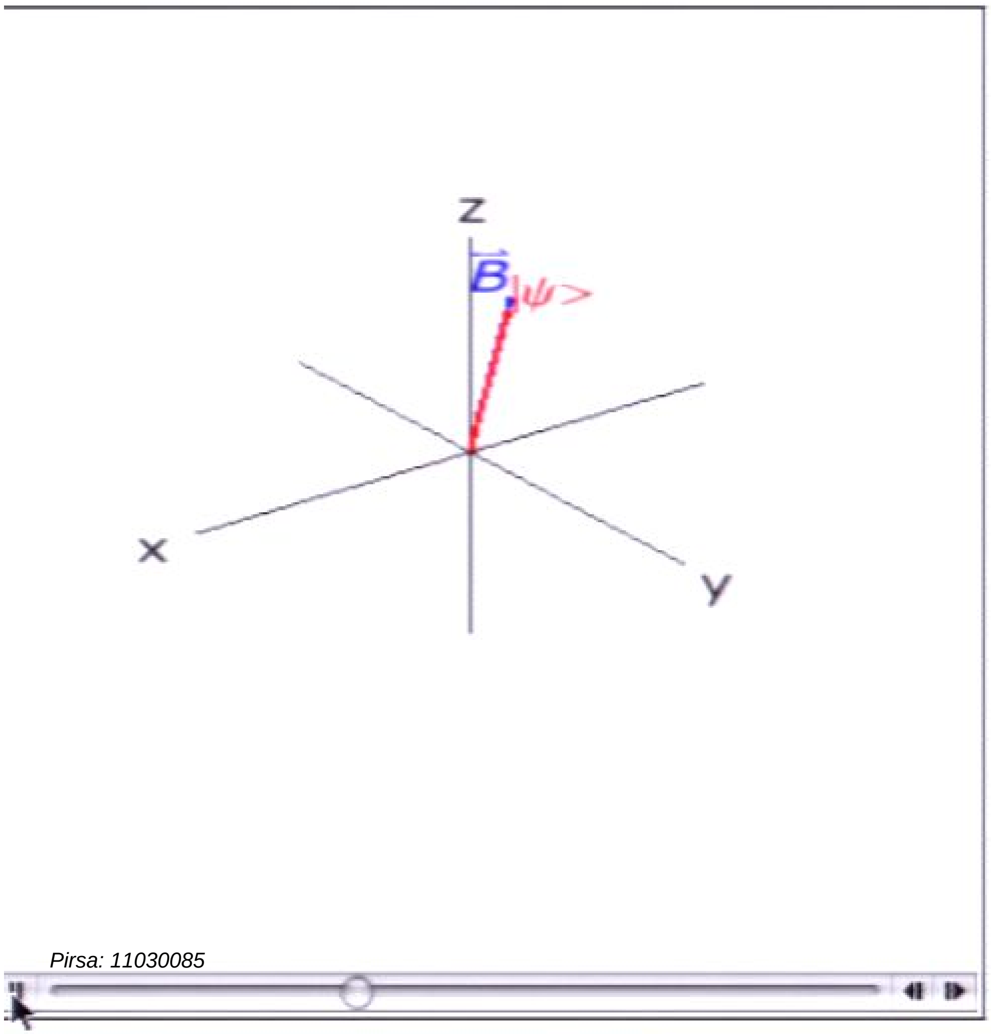

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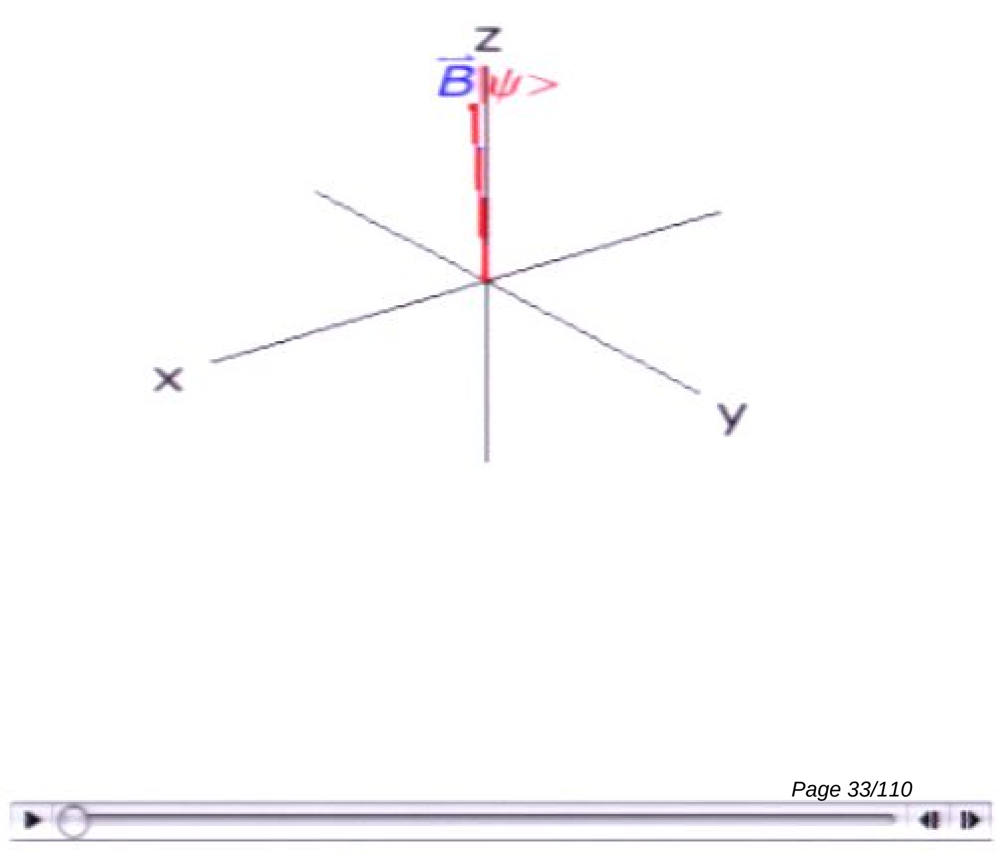

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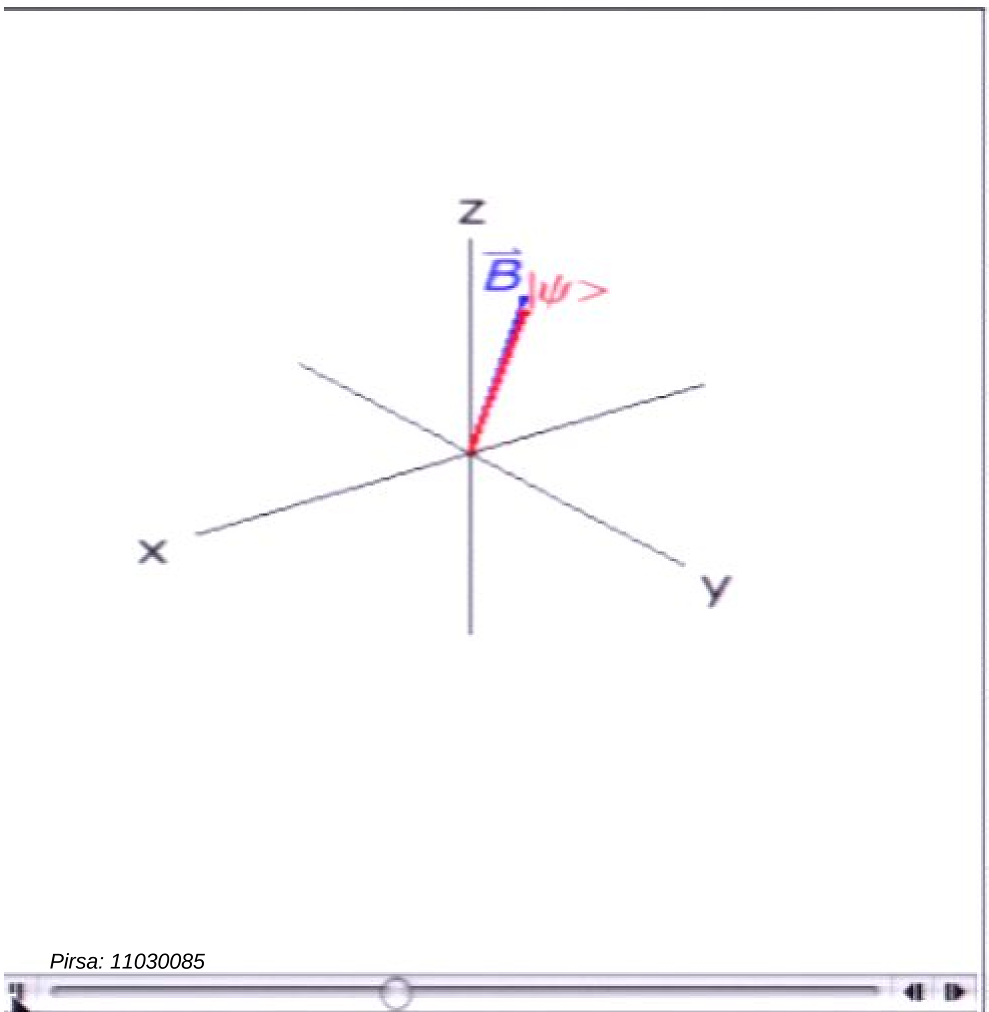

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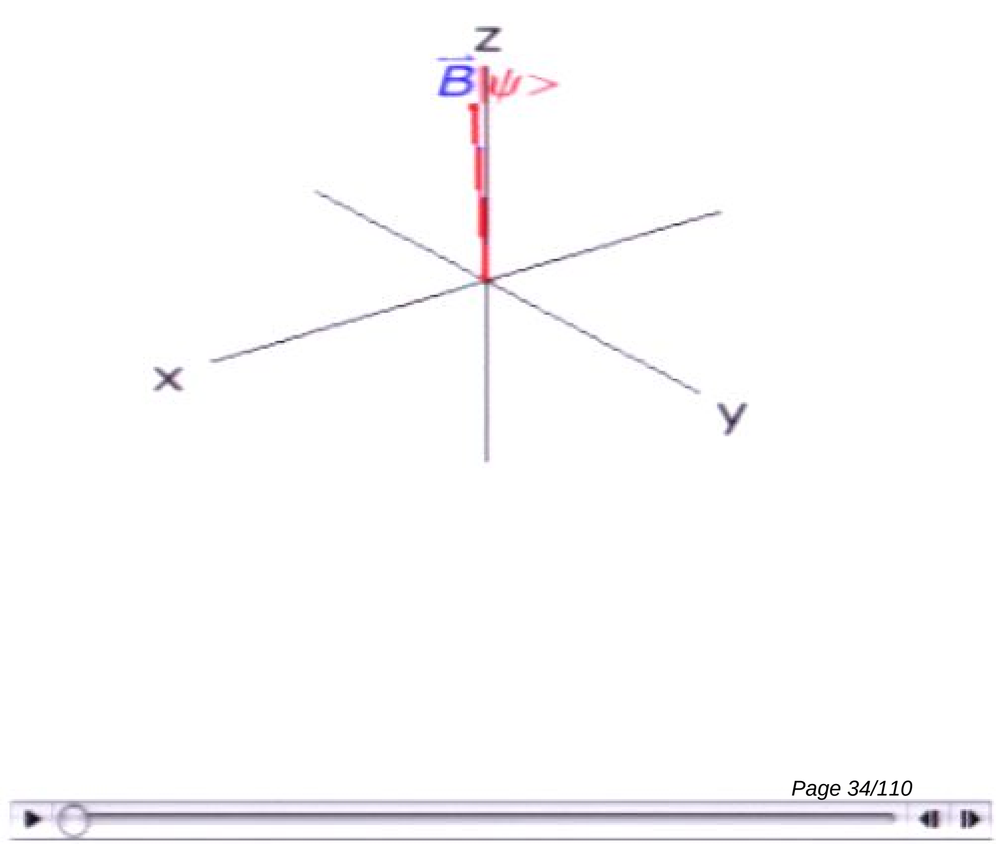

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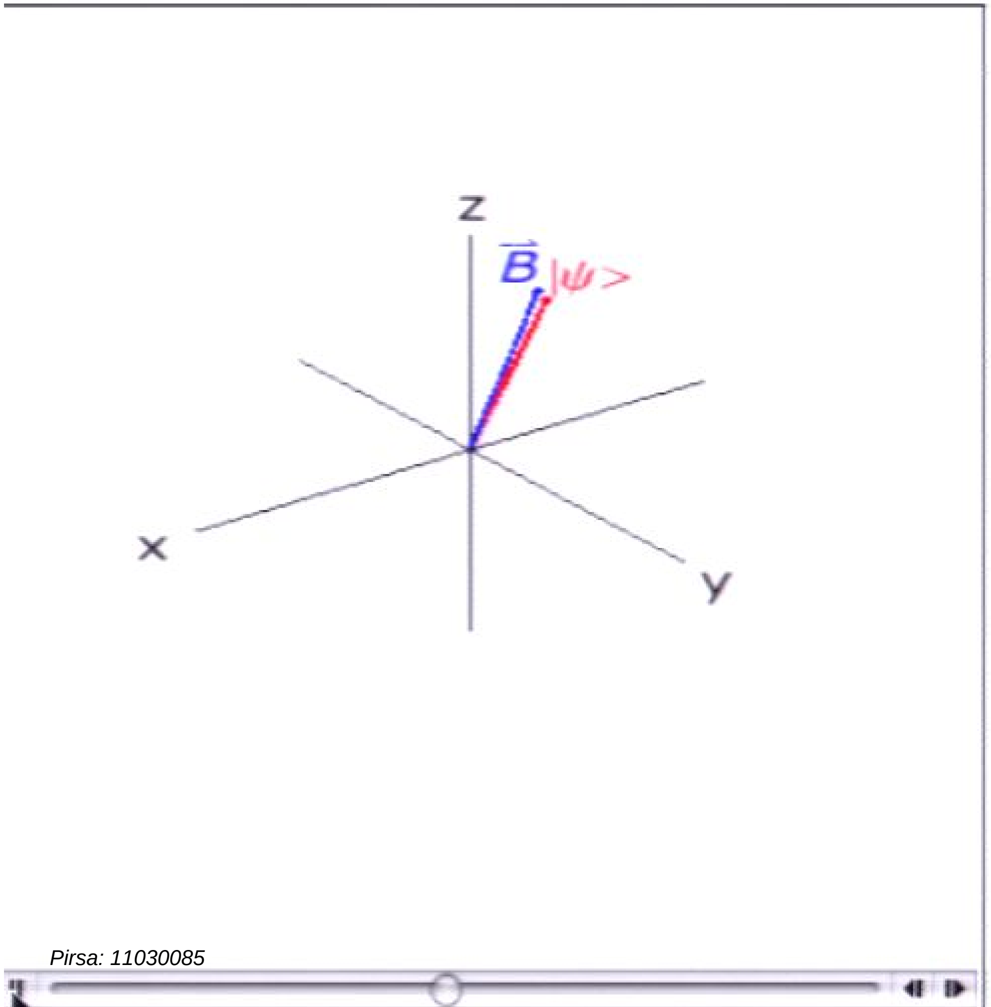

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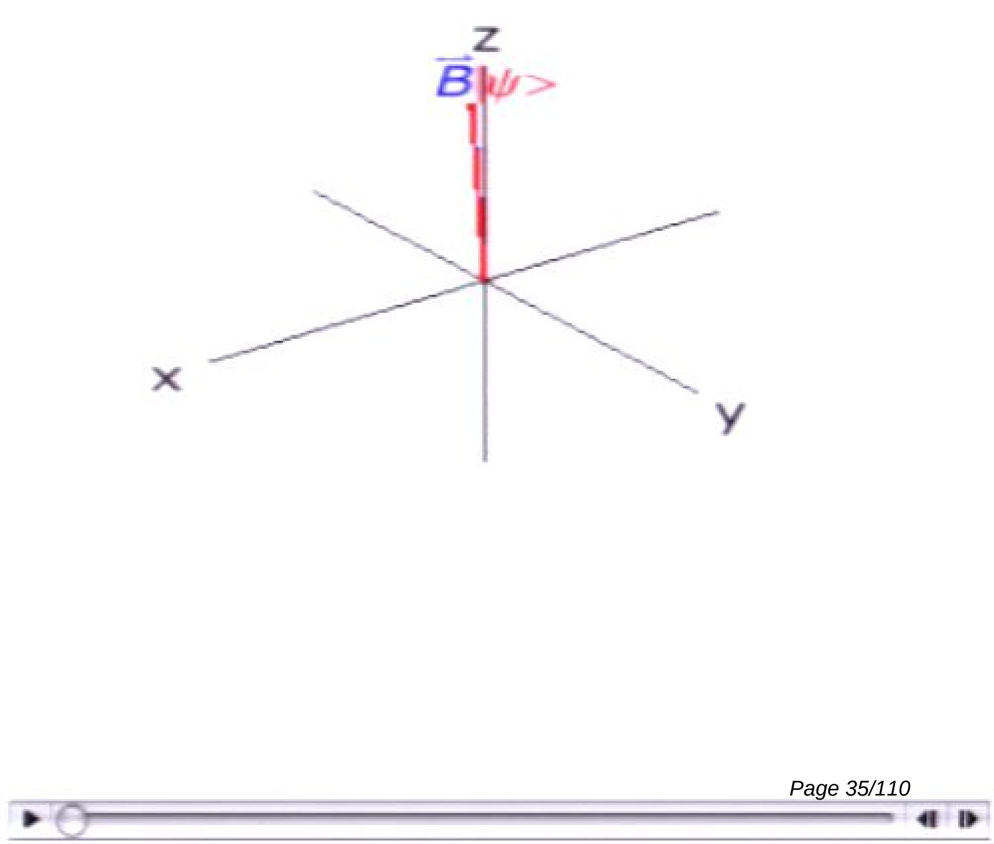

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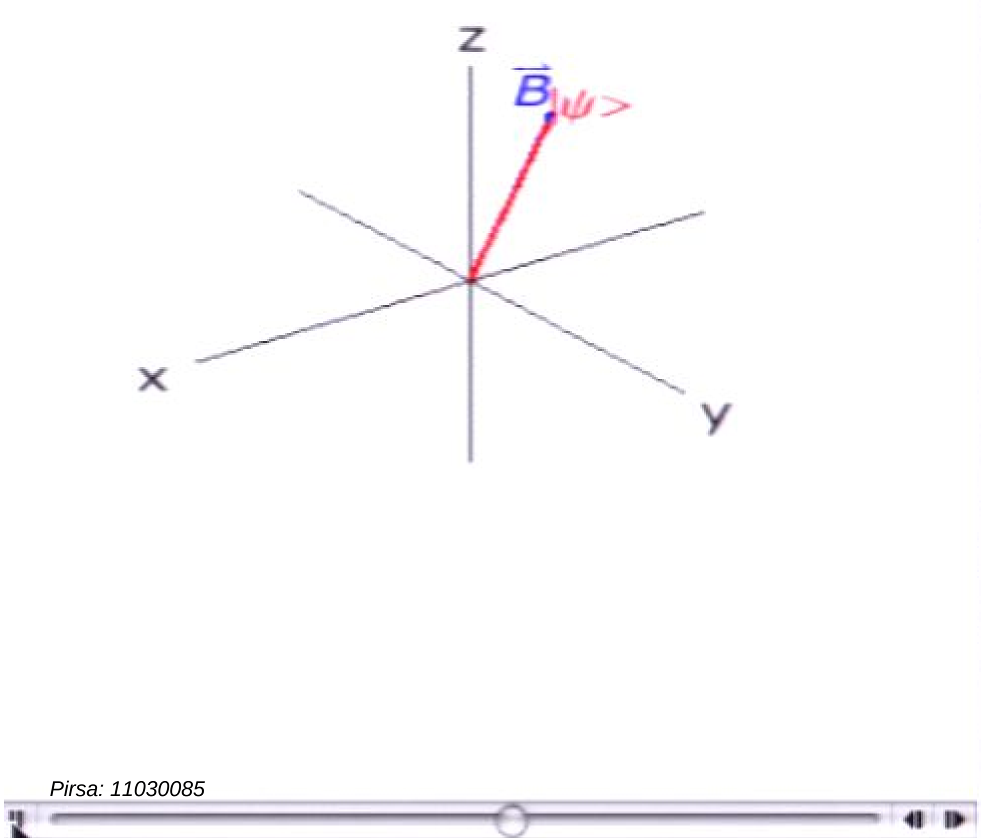

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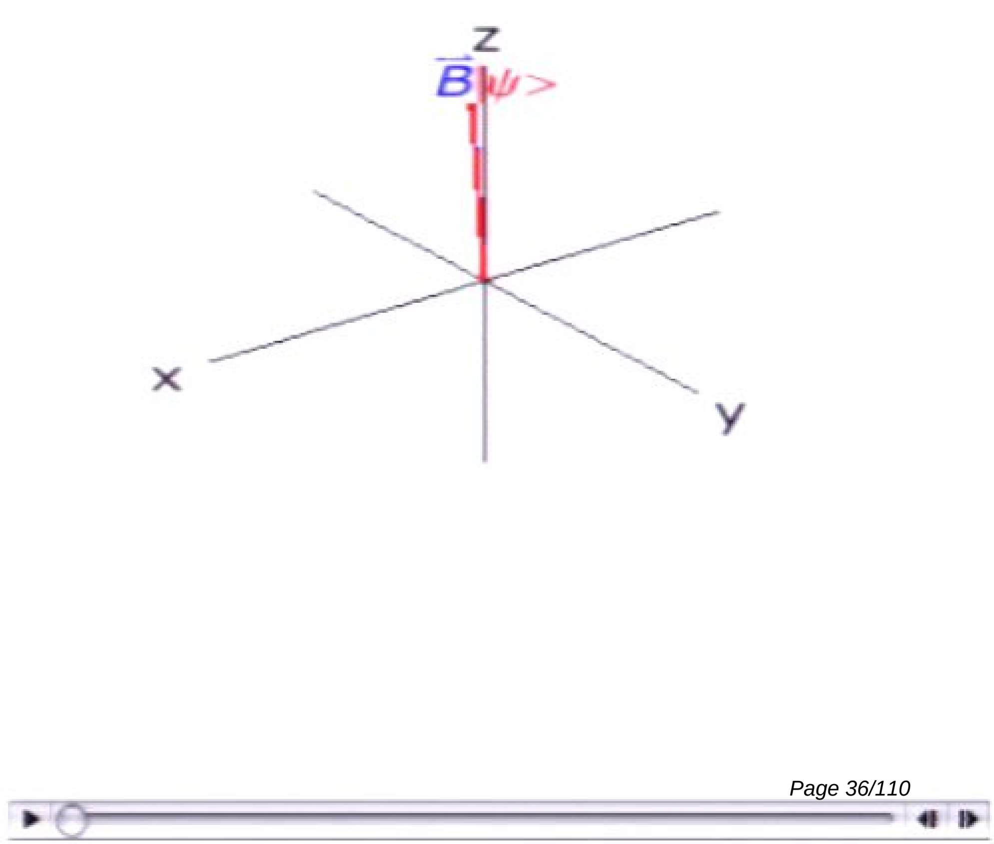

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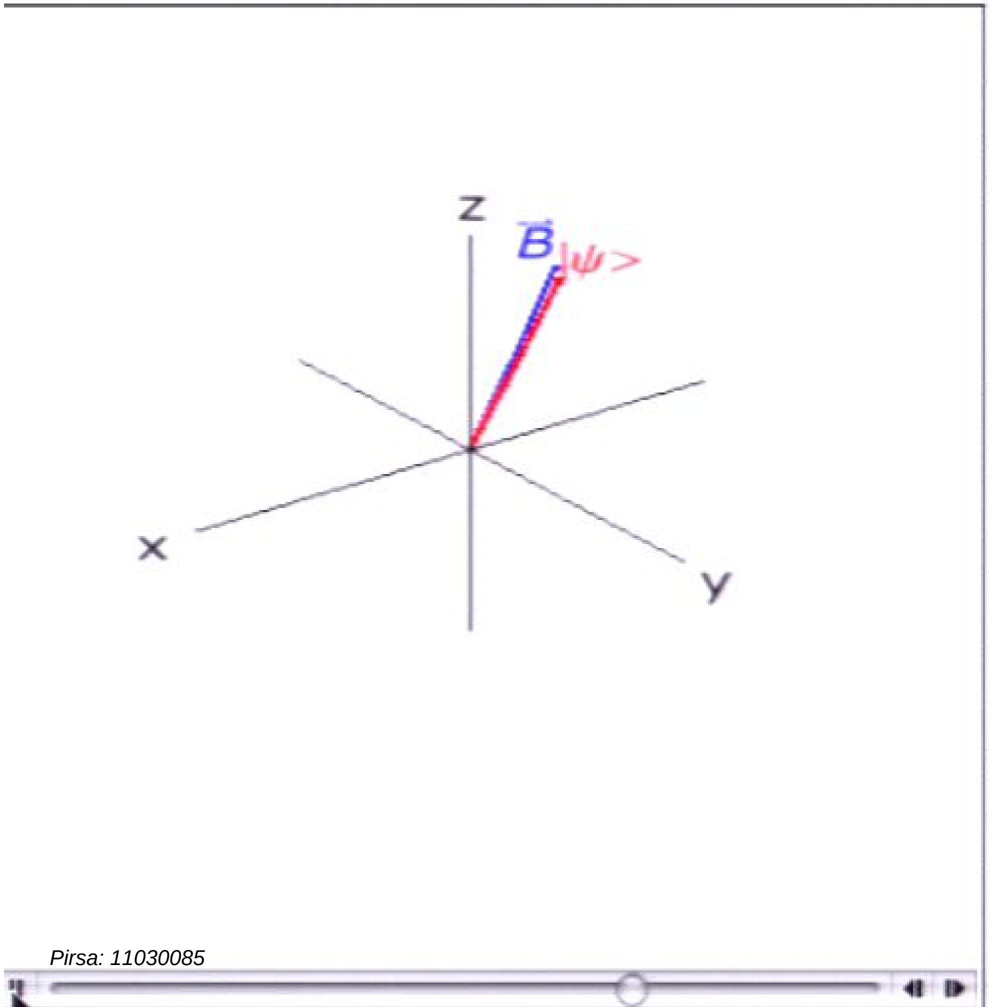

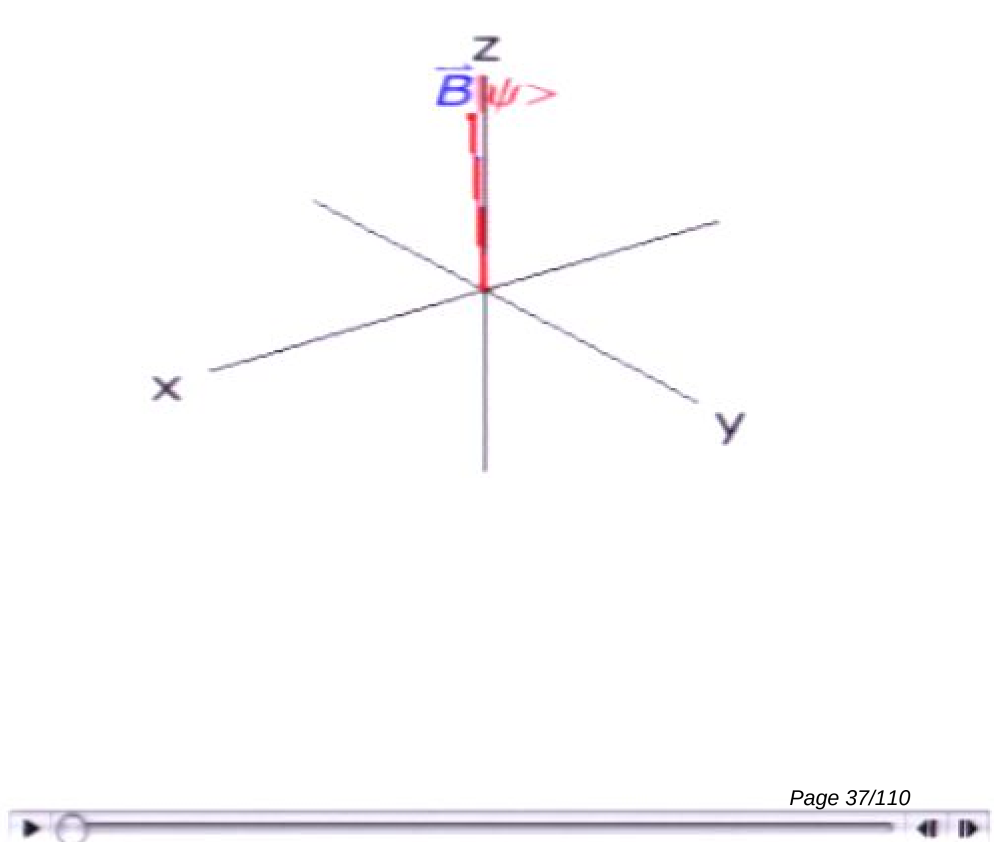

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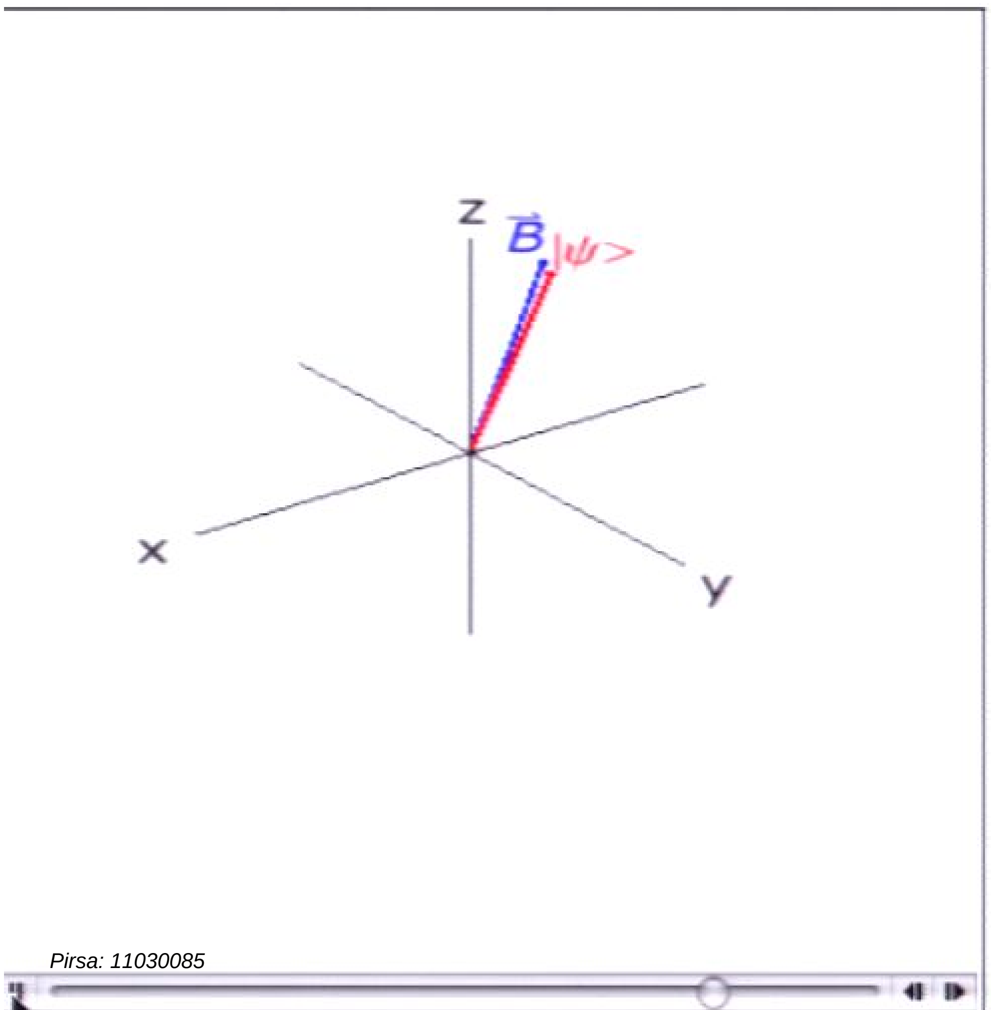

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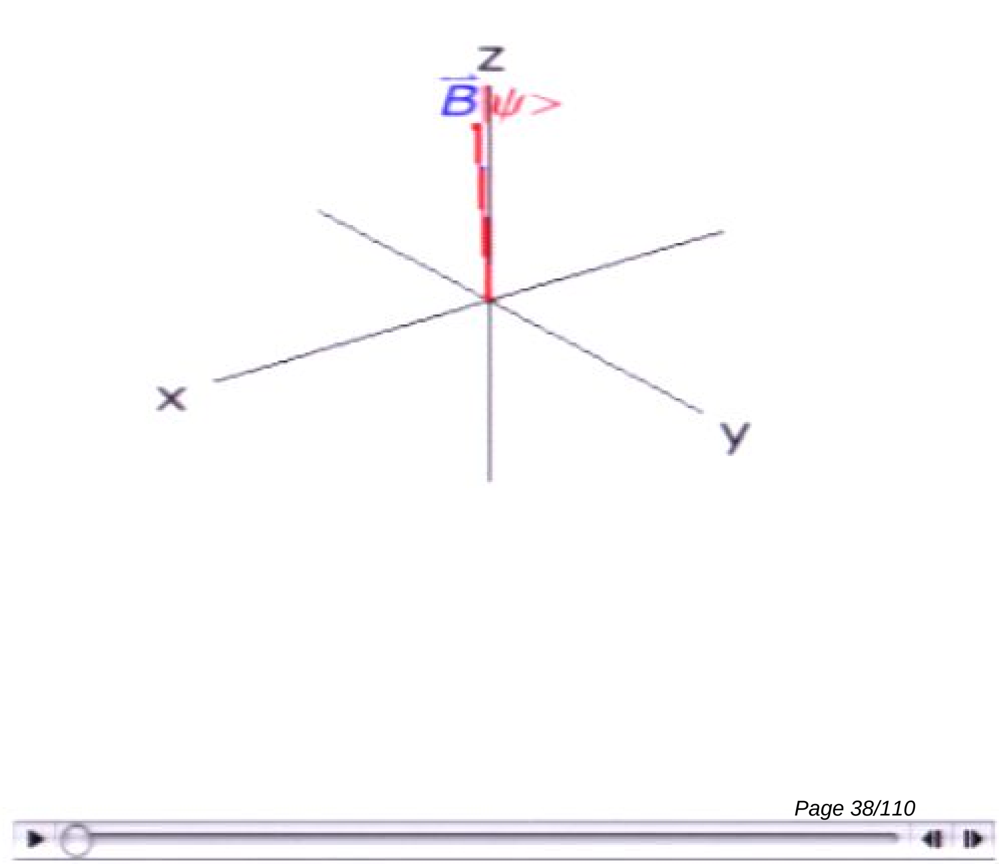

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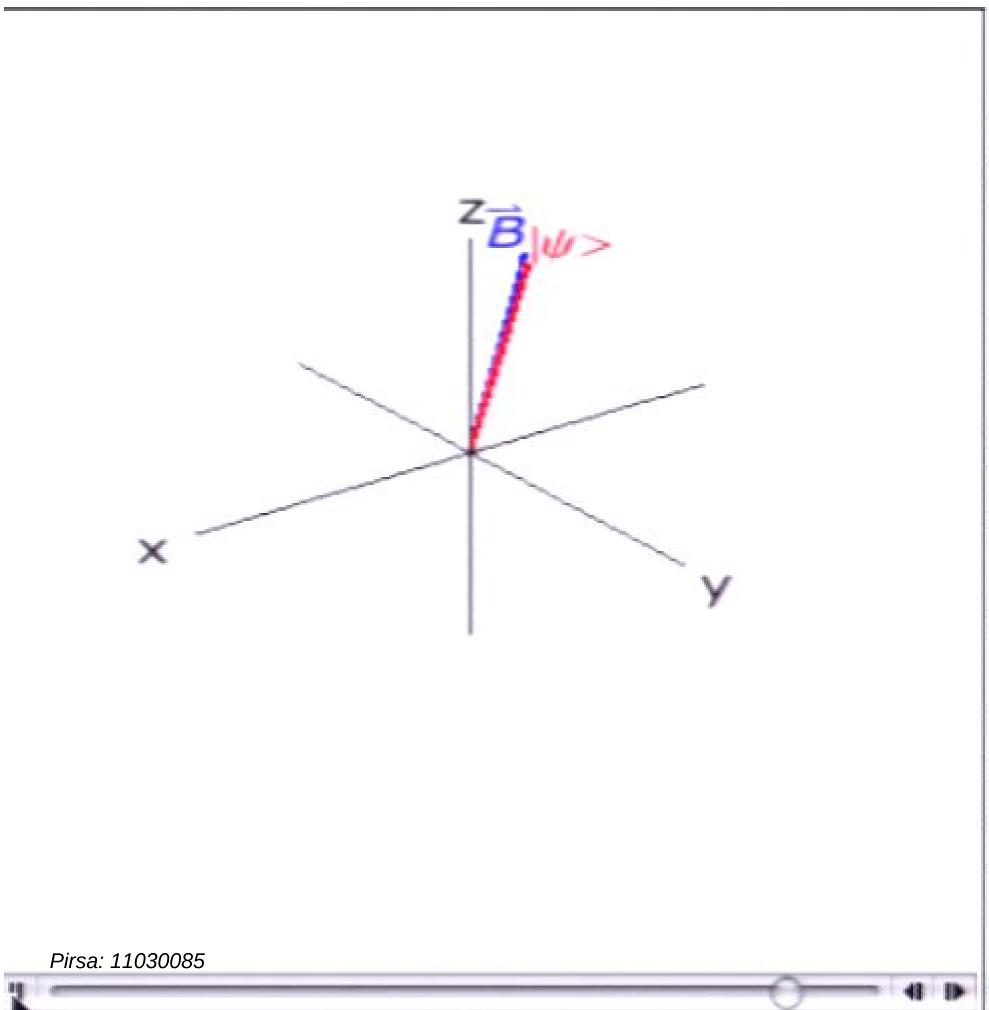

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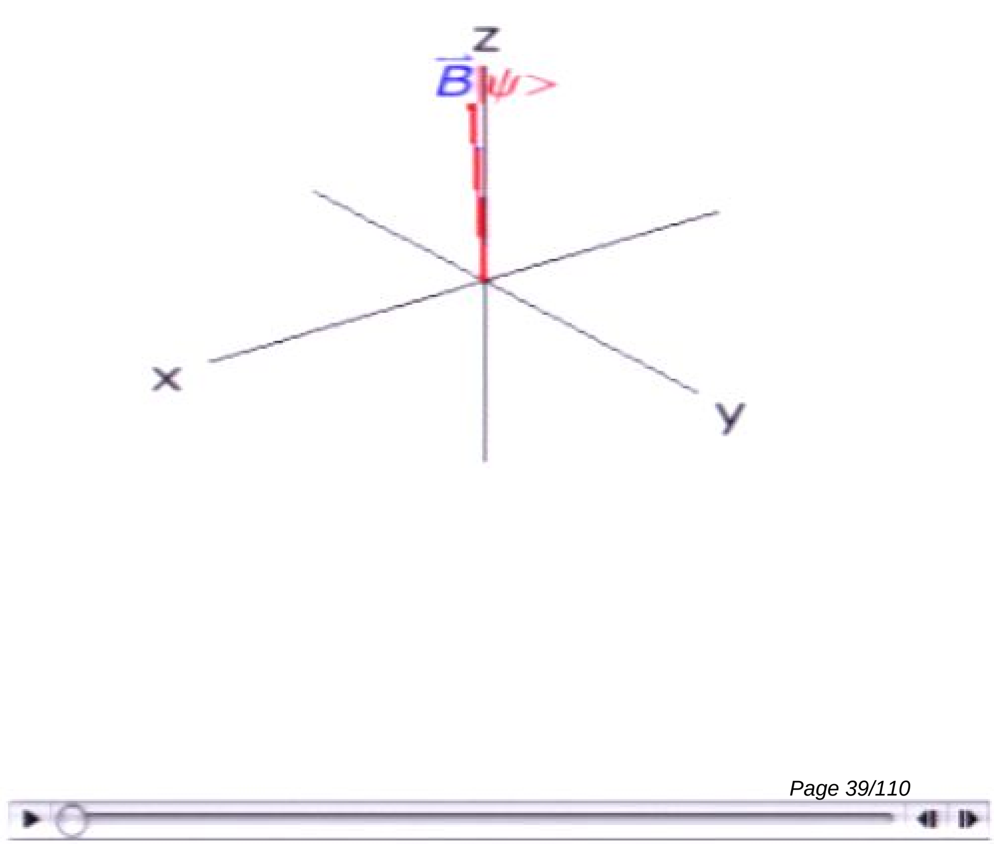

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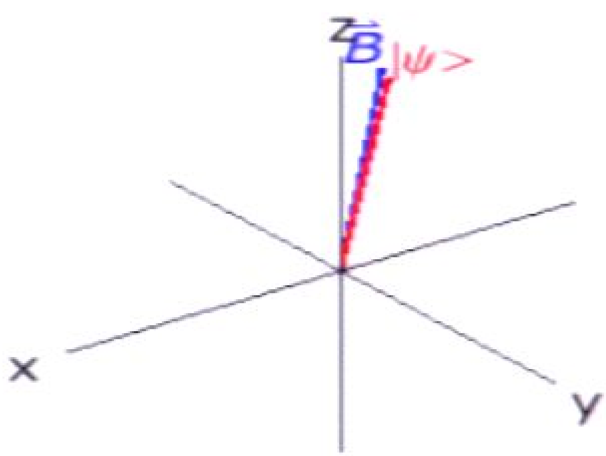

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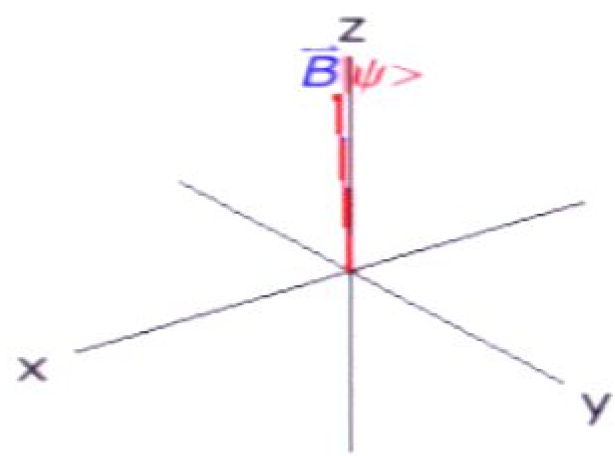

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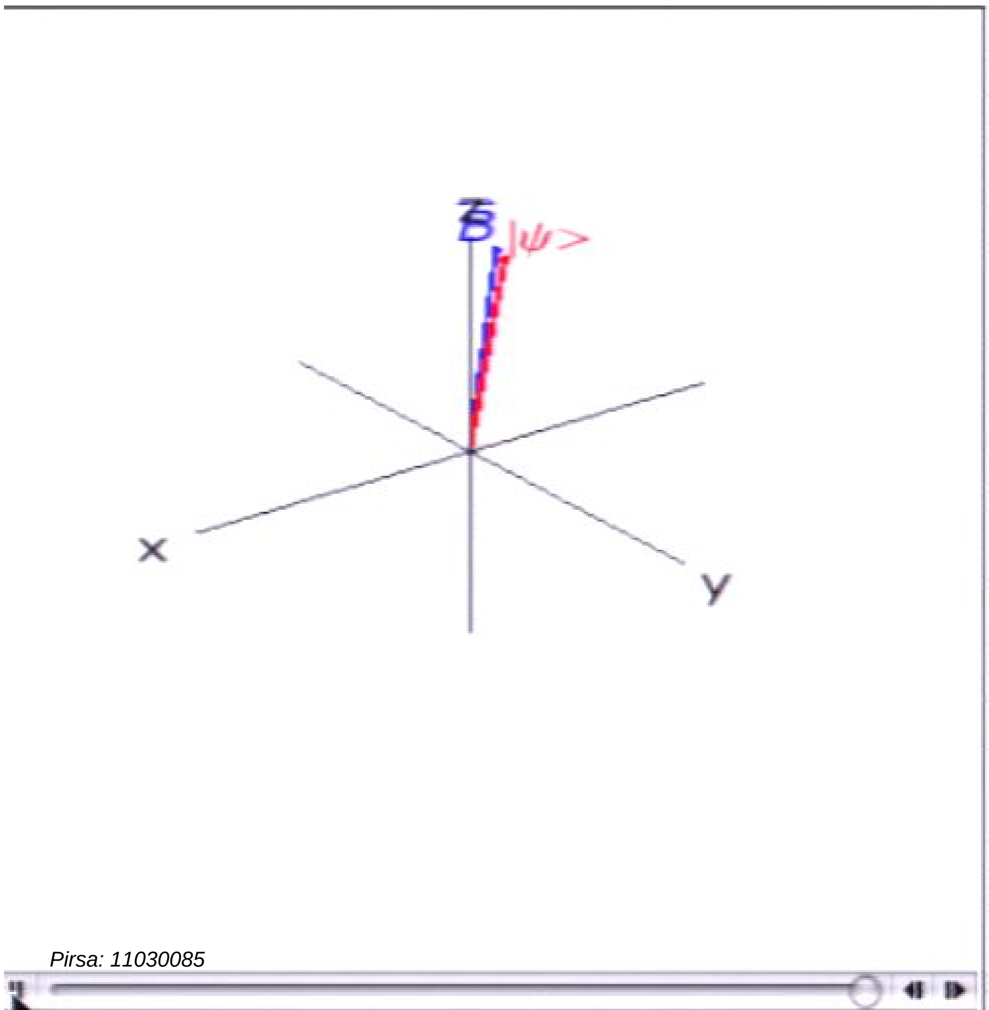

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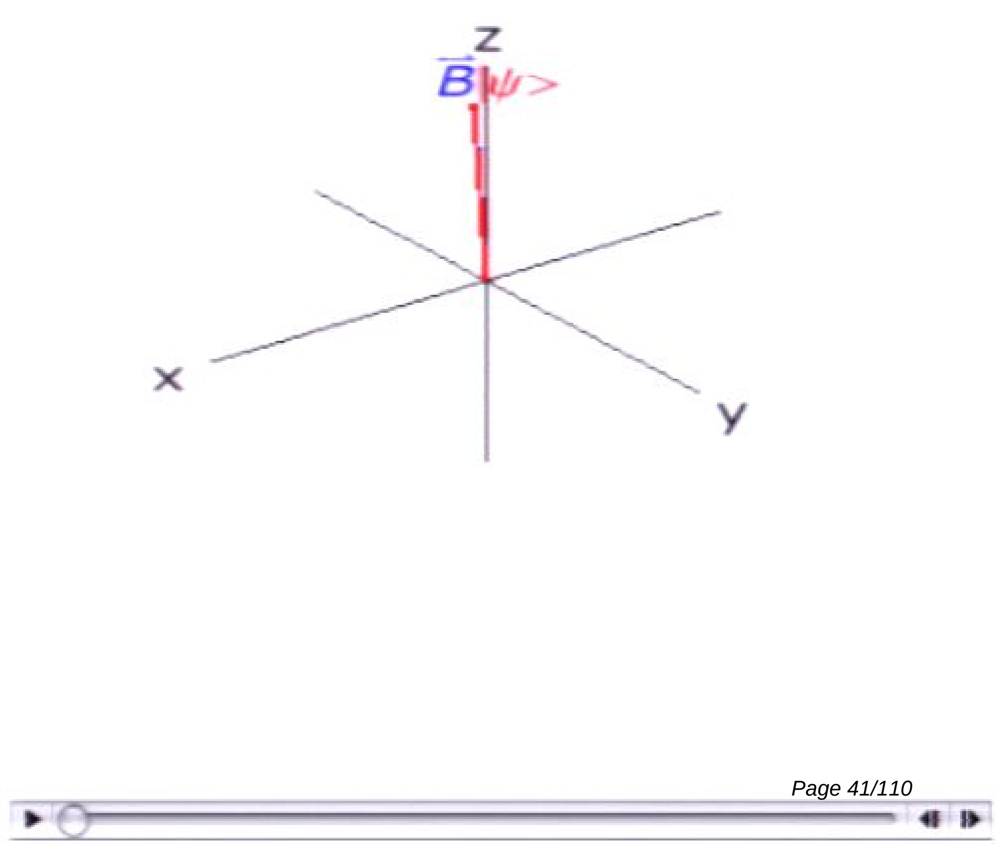

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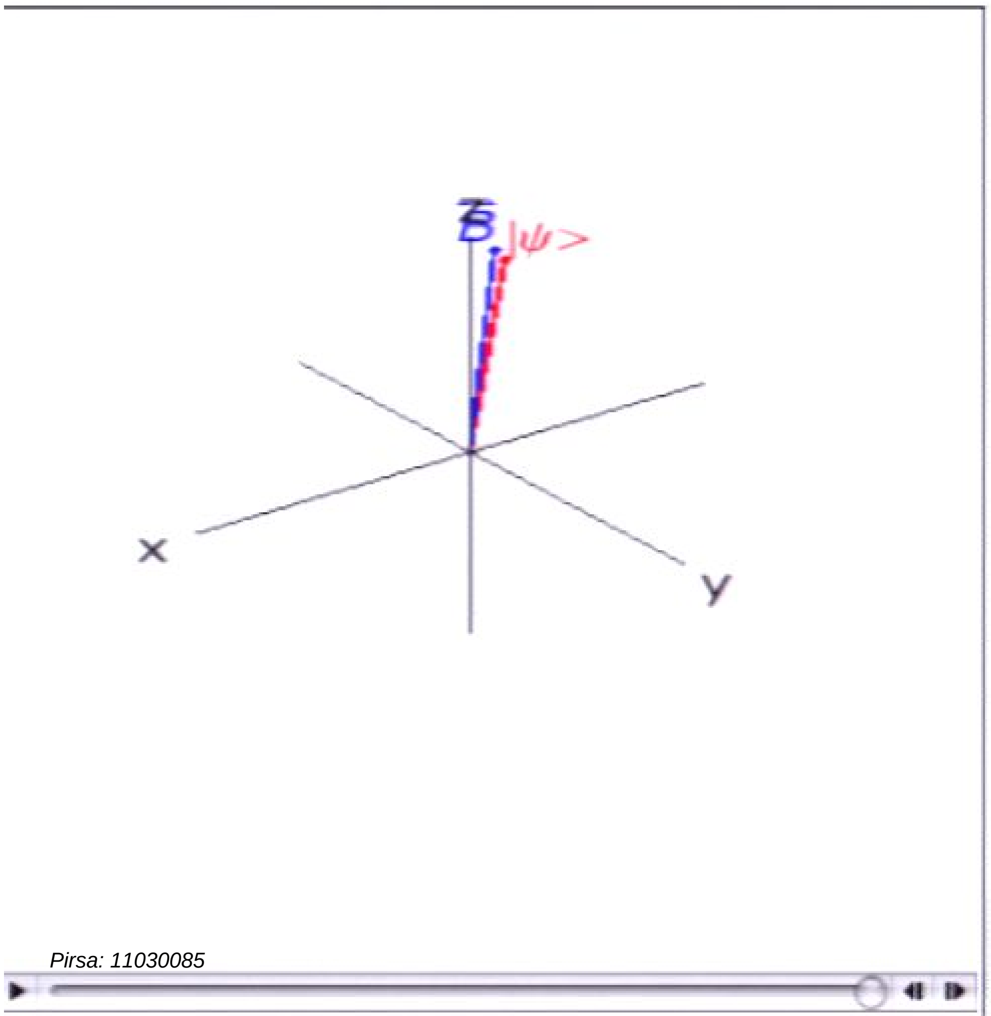

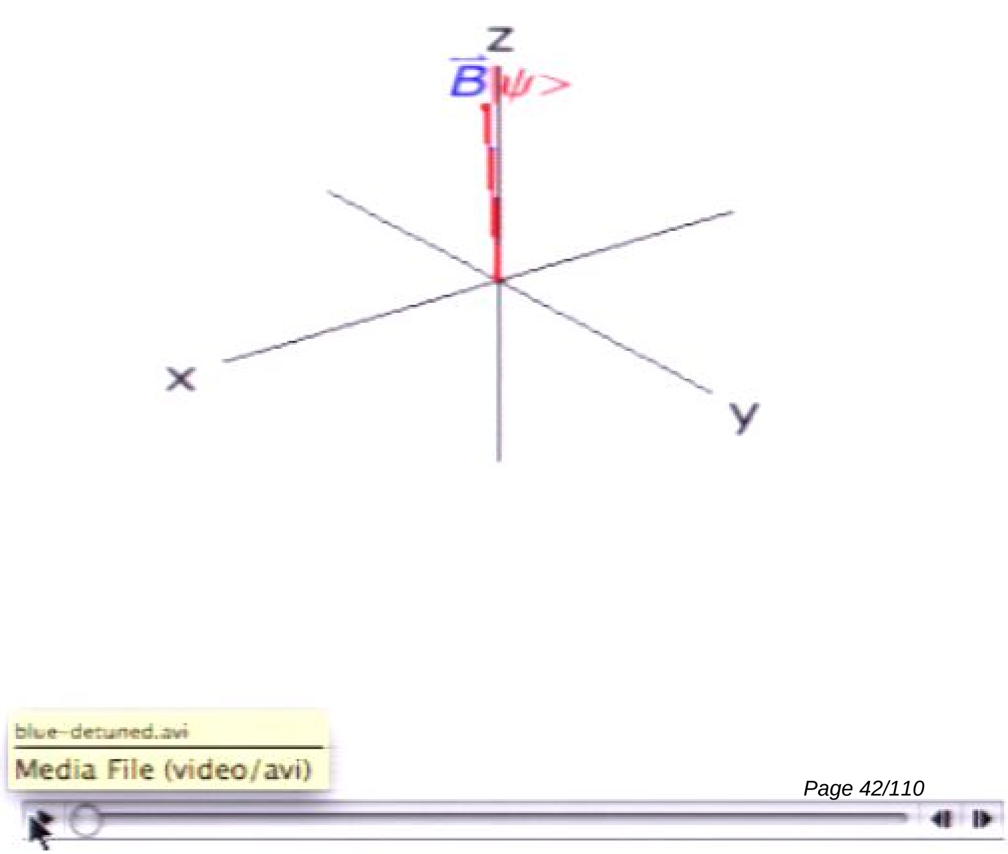

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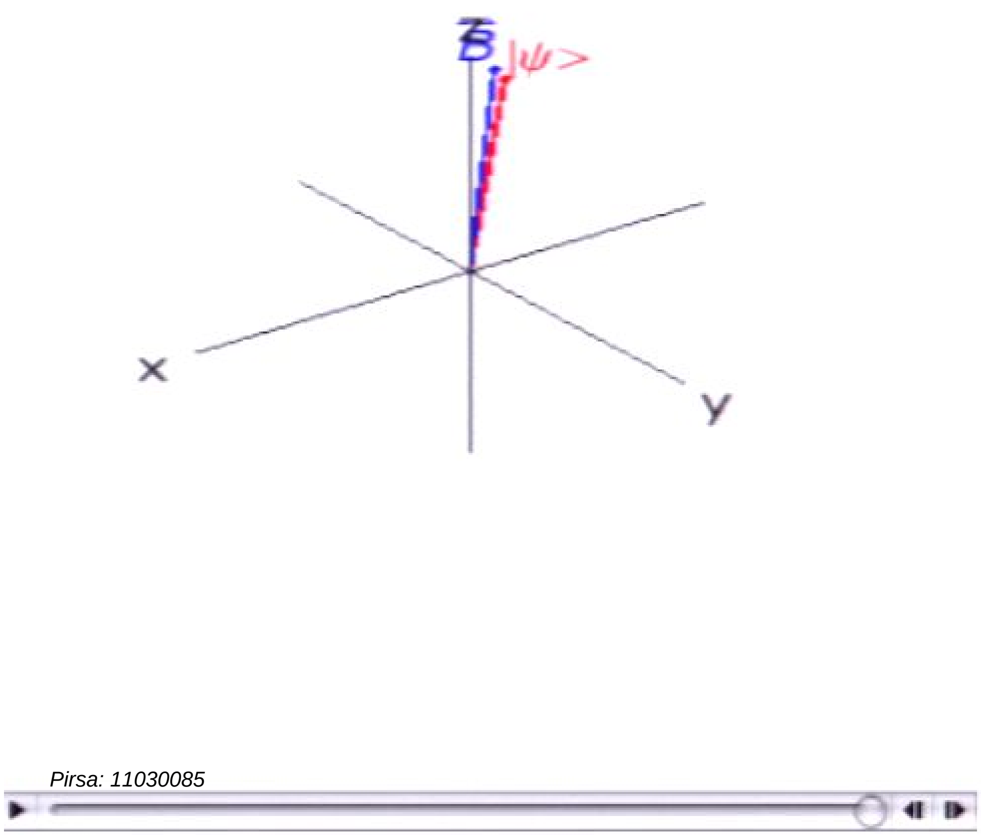

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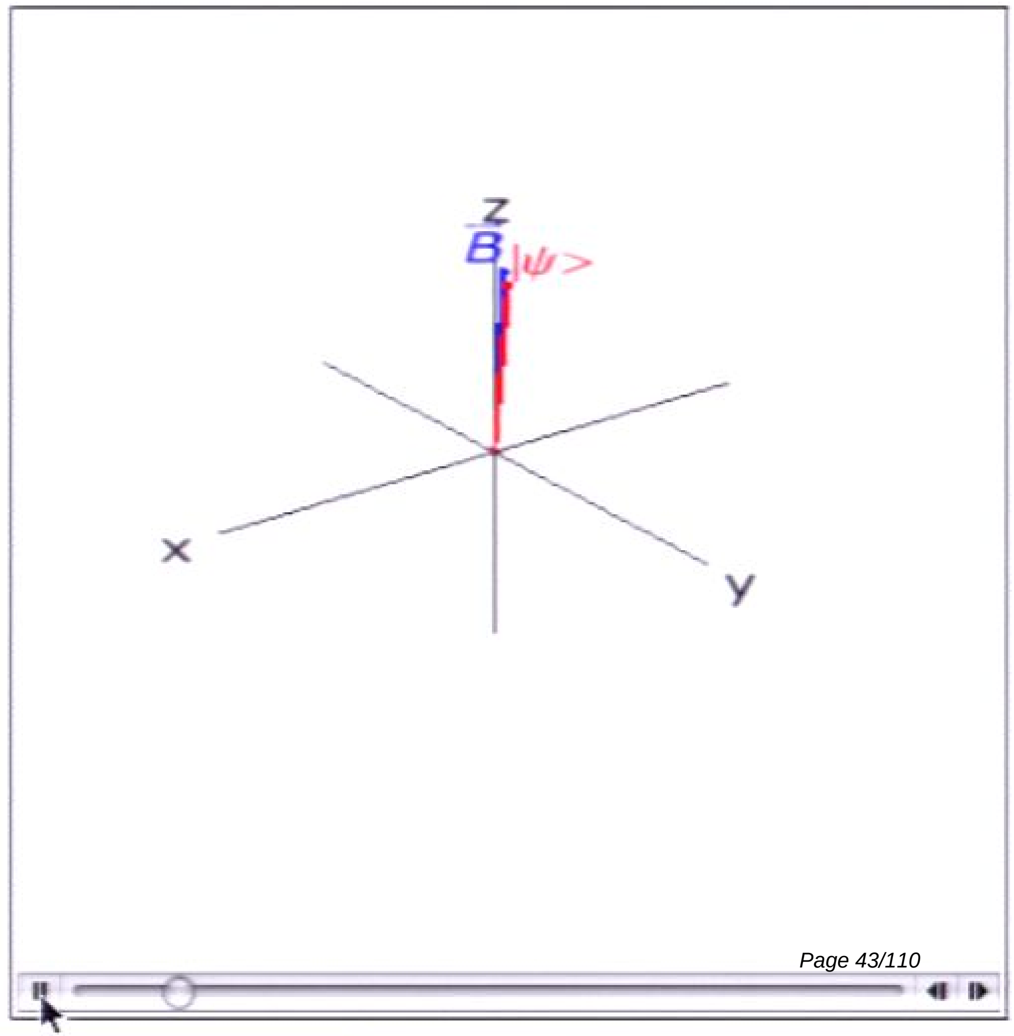

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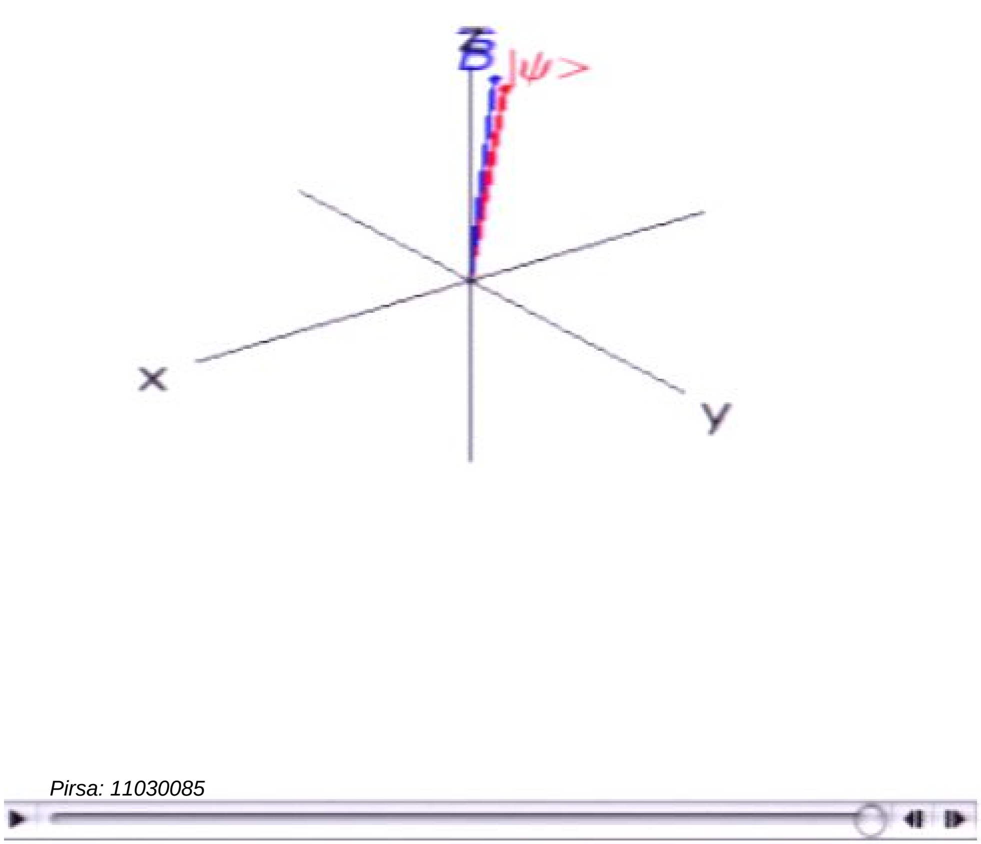

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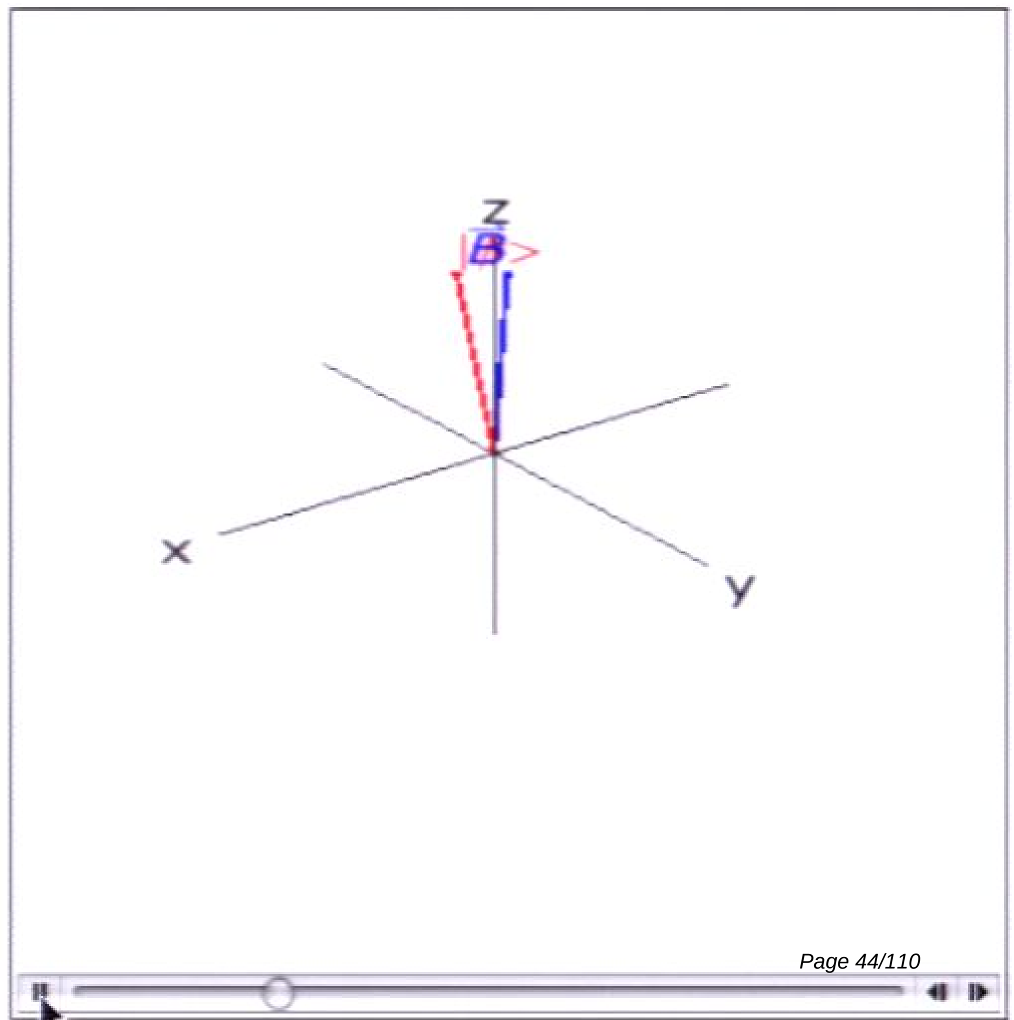

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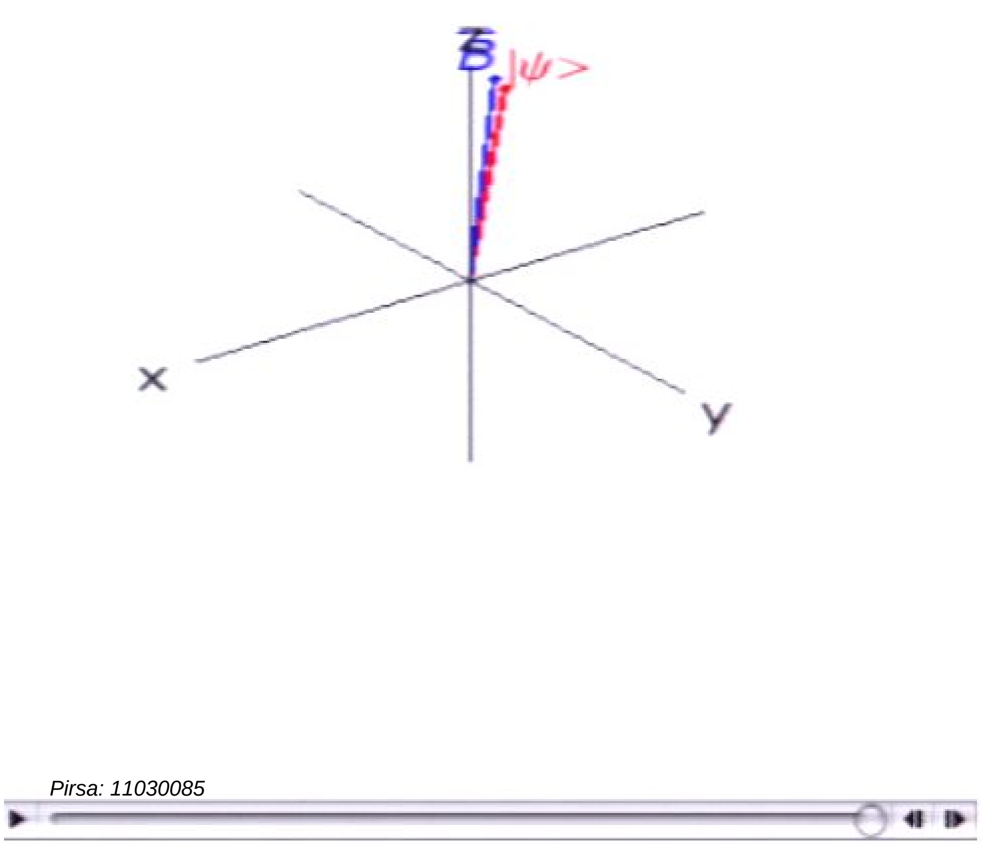

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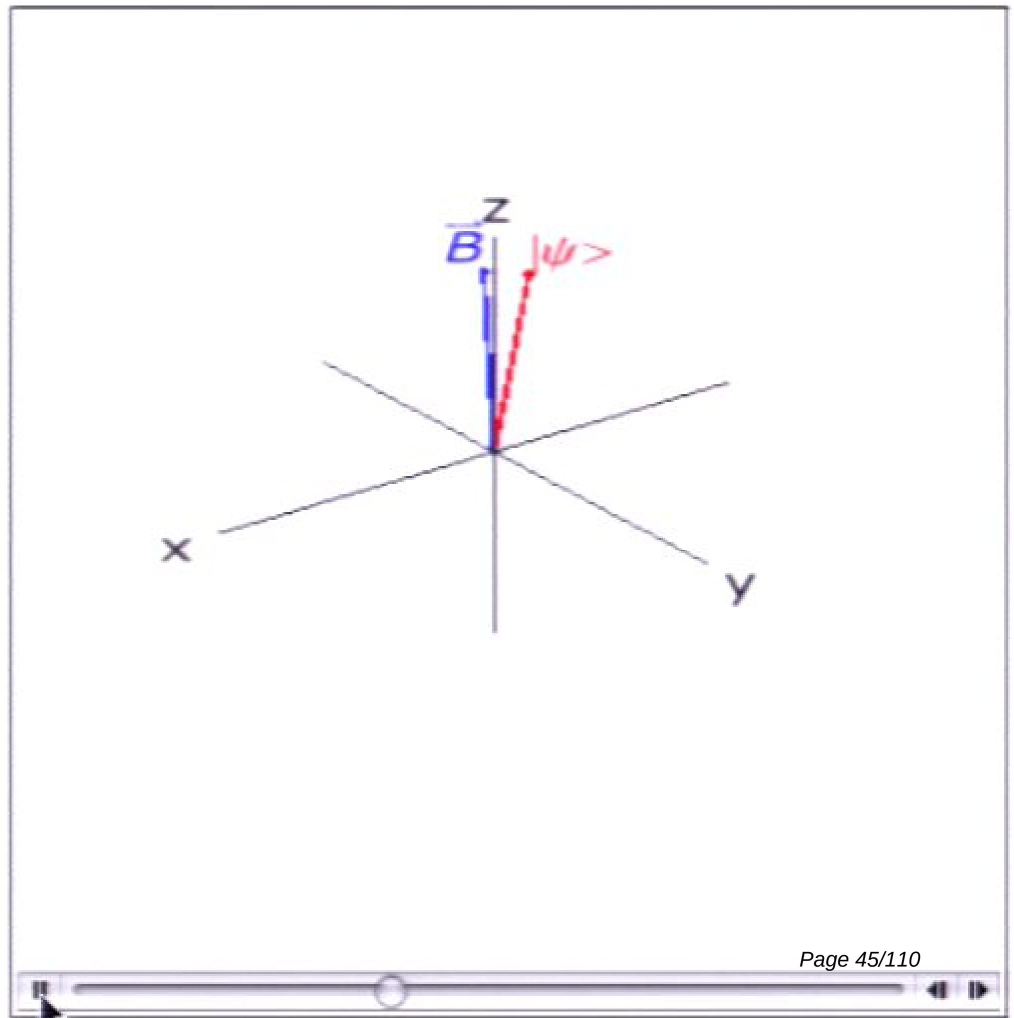

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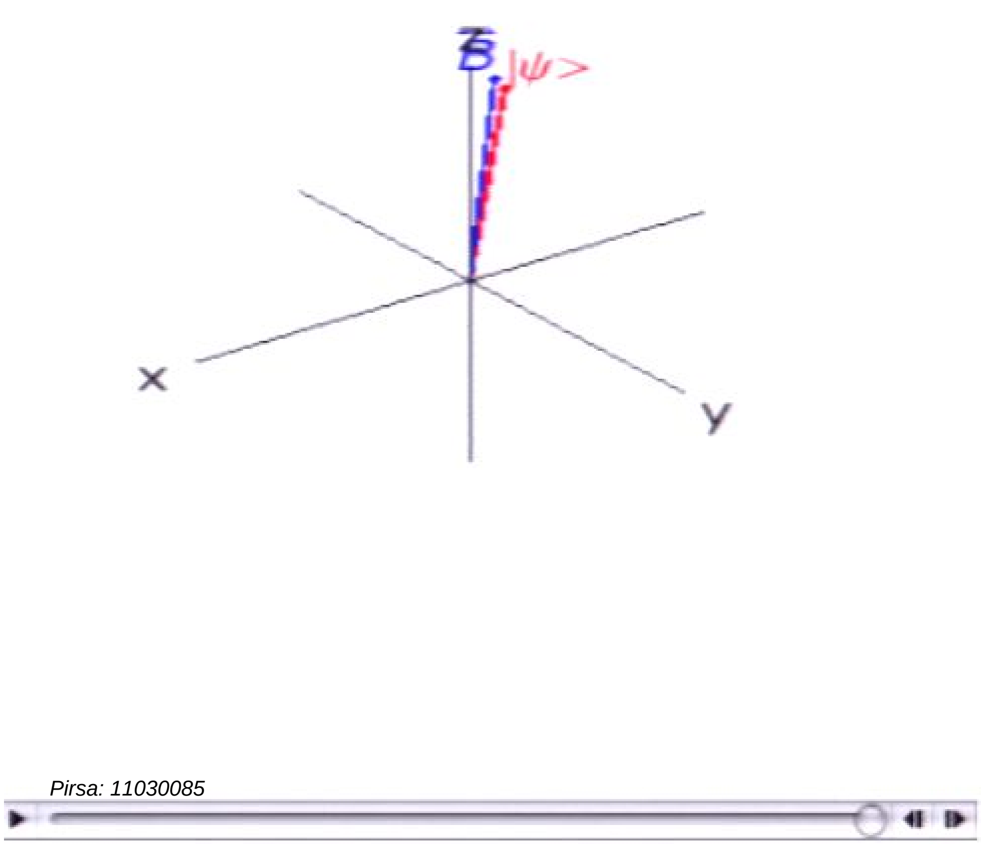

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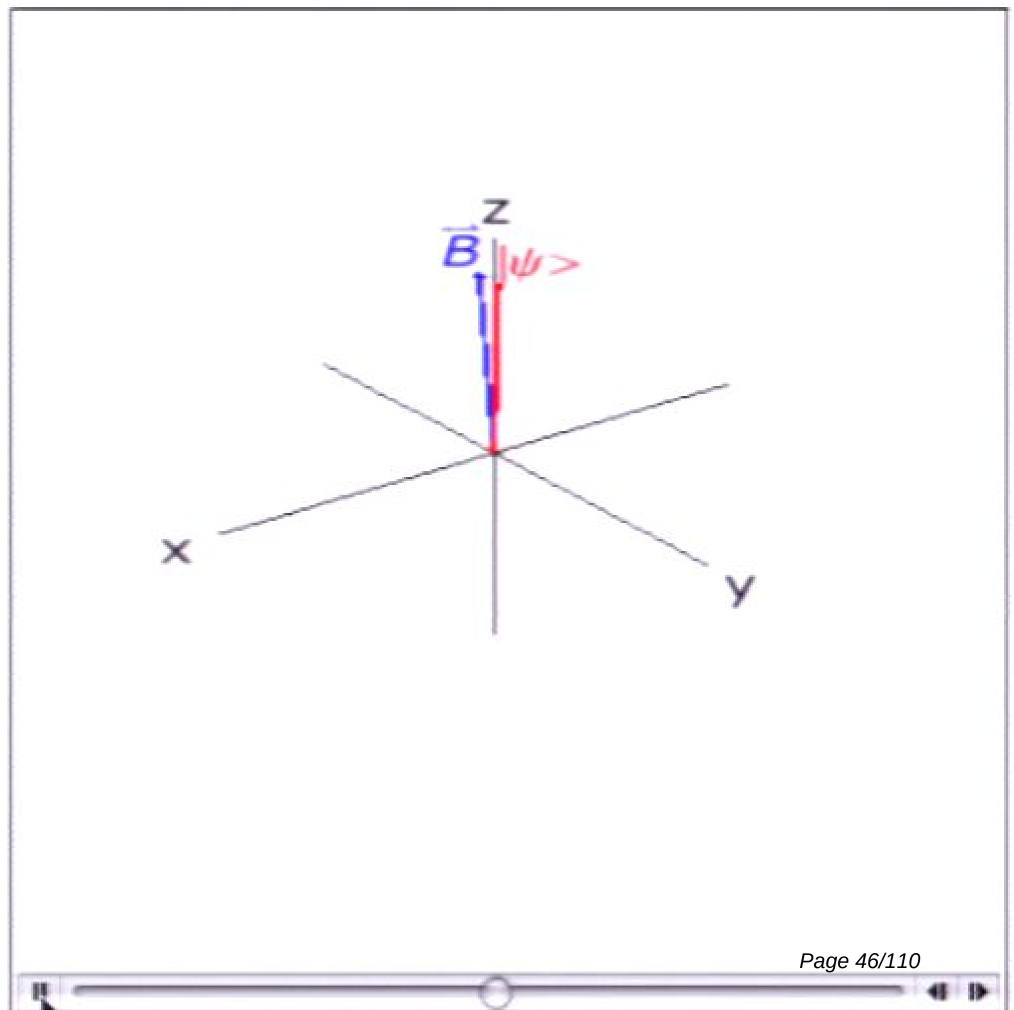

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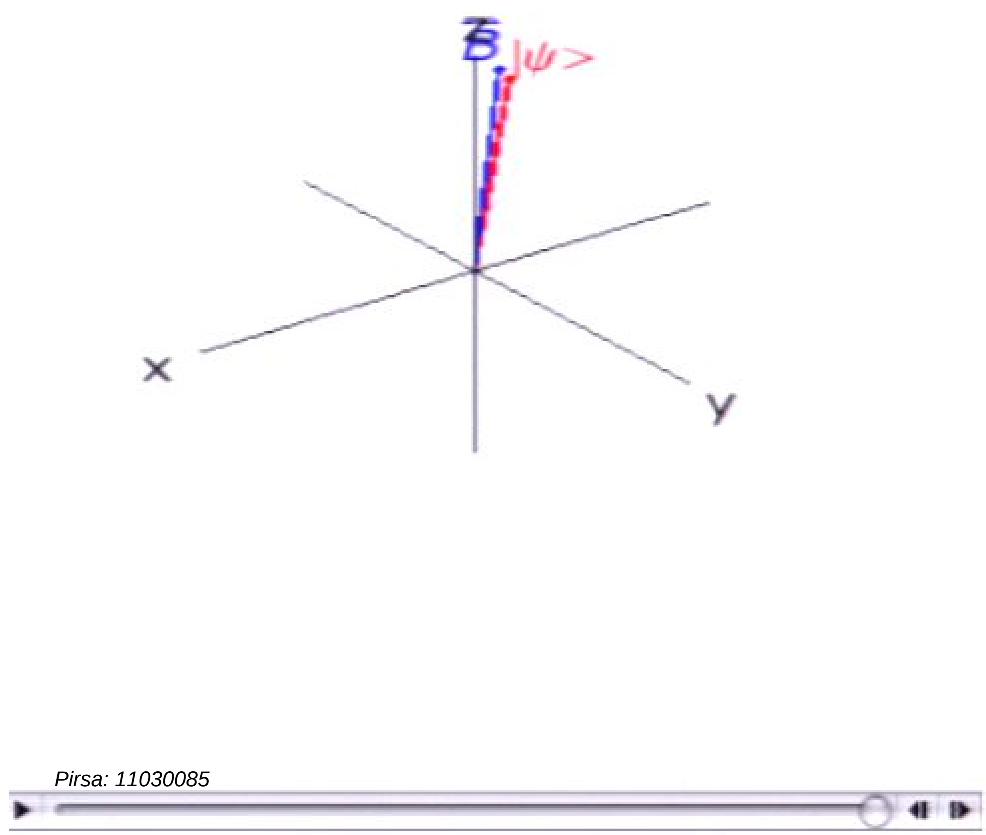

High frequency field

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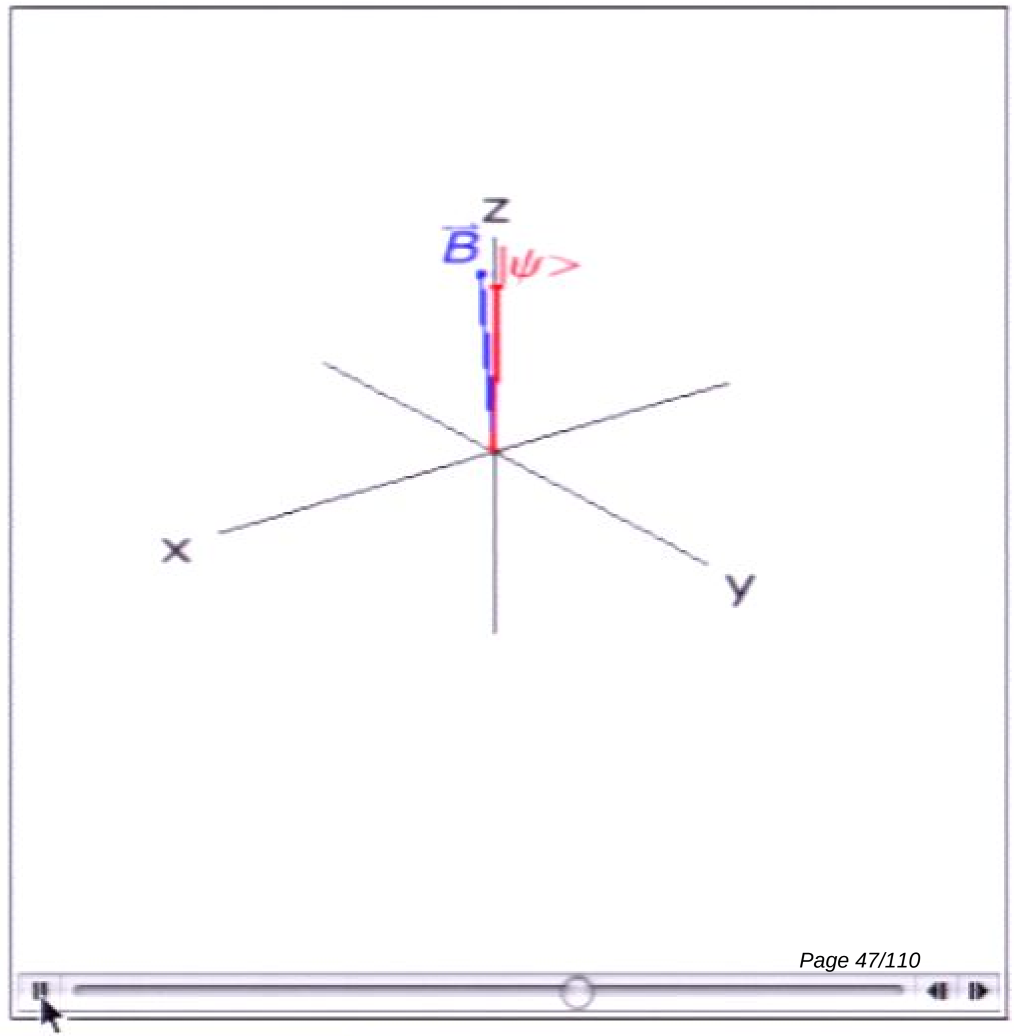
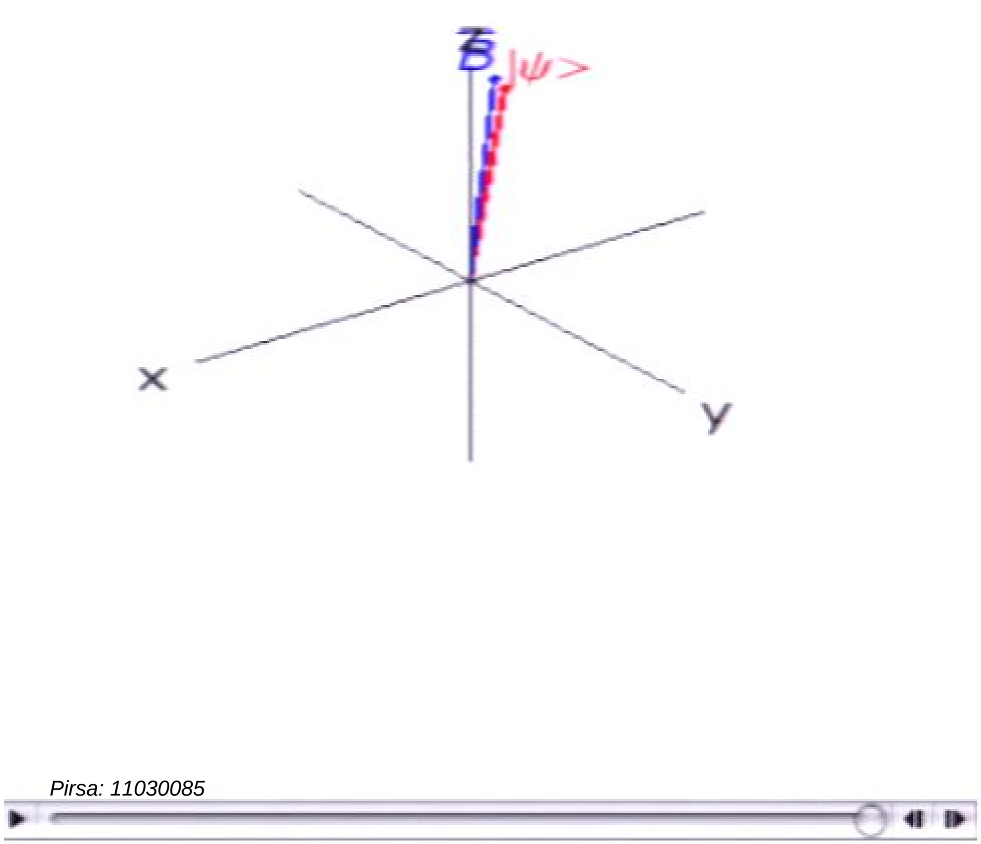


Off-resonant radiation

Low frequency field

$\nu \ll \Delta$



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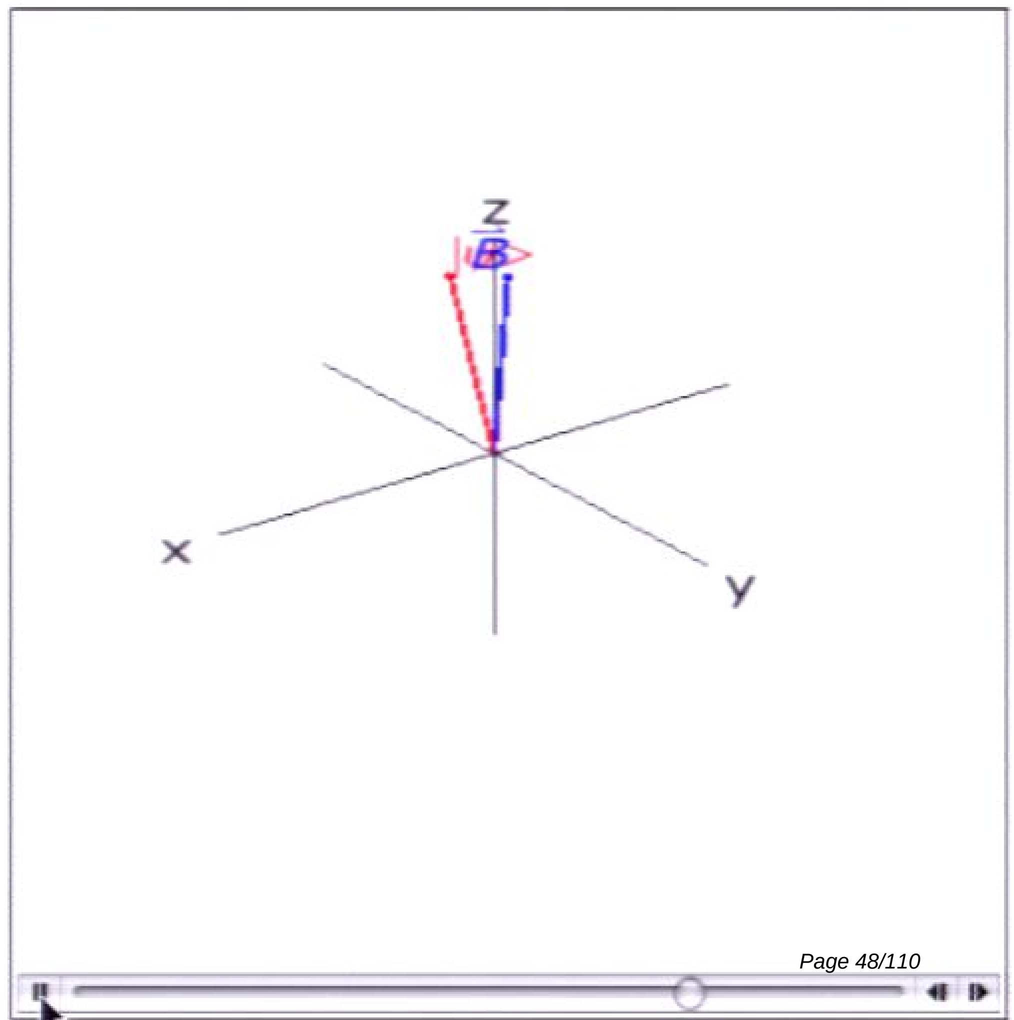

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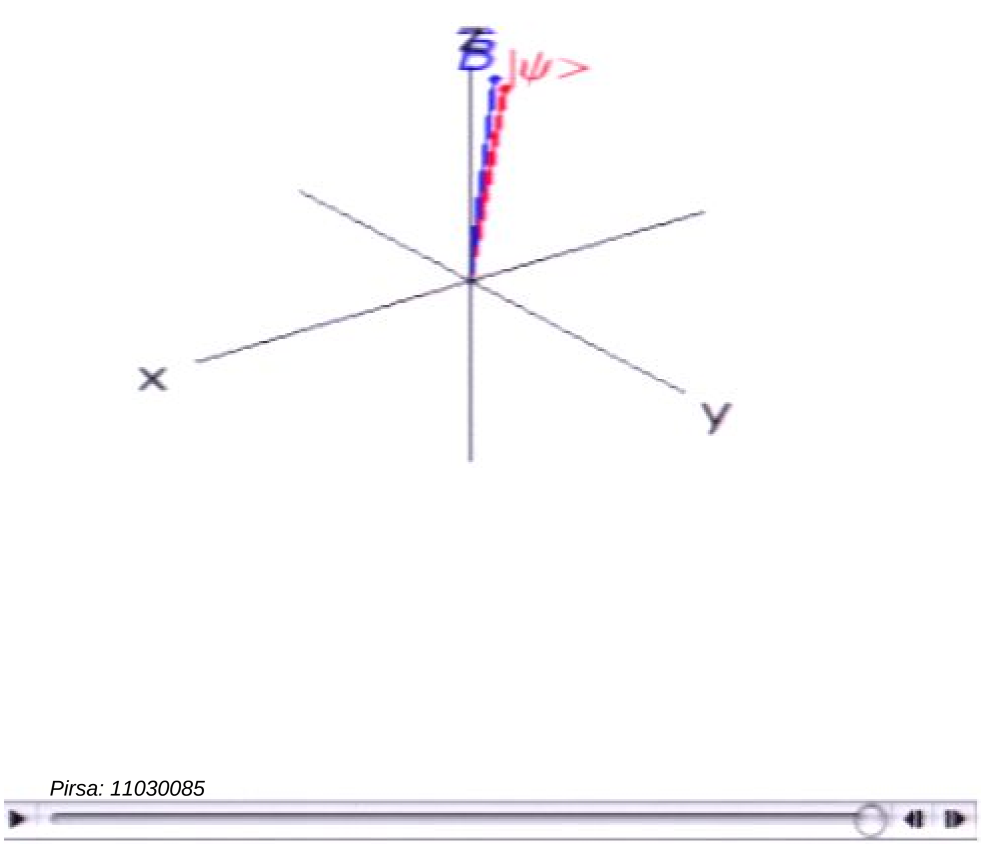

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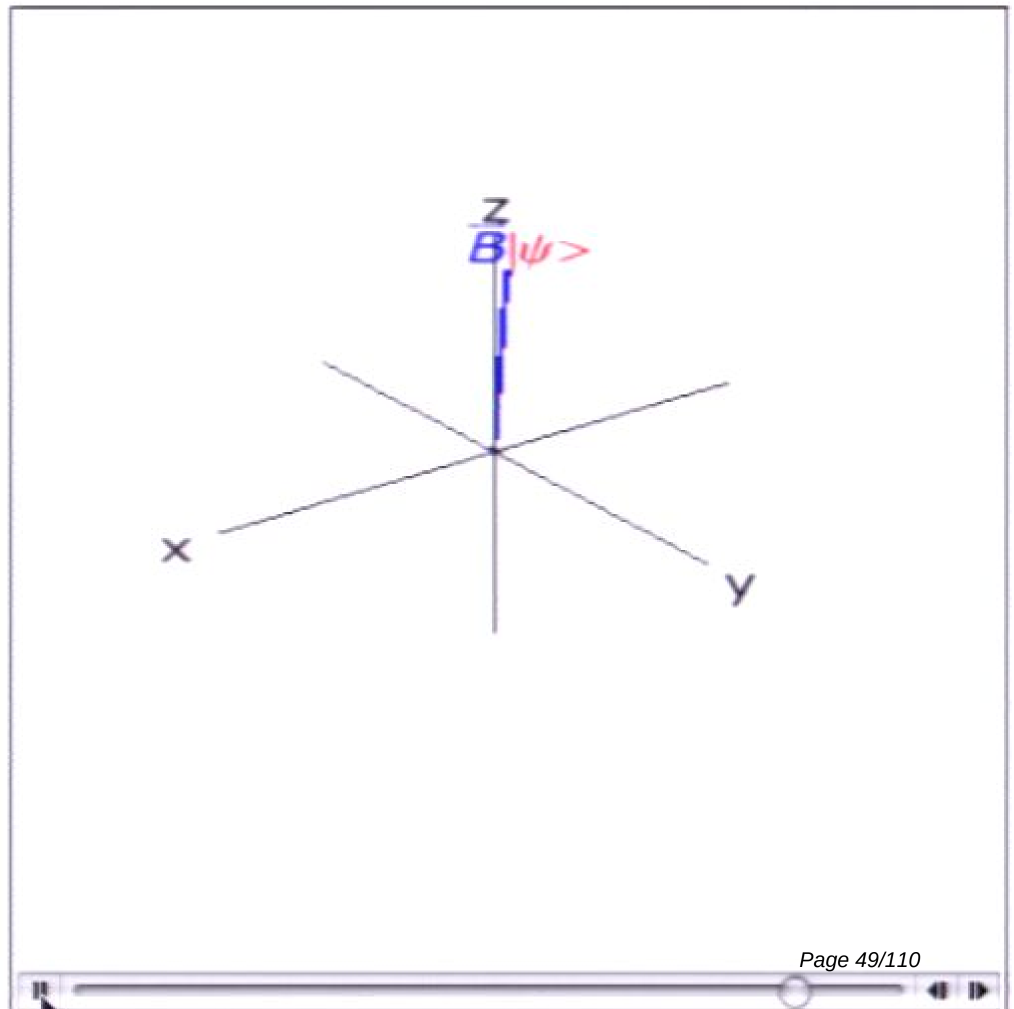

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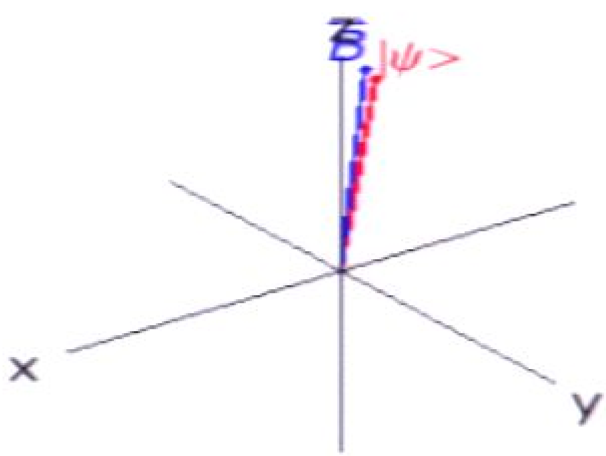

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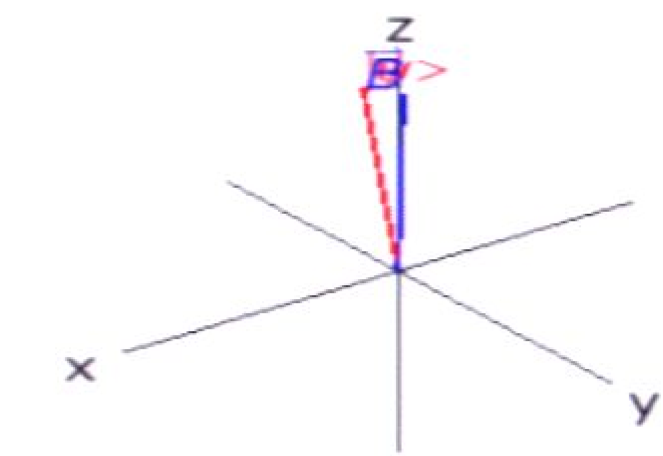

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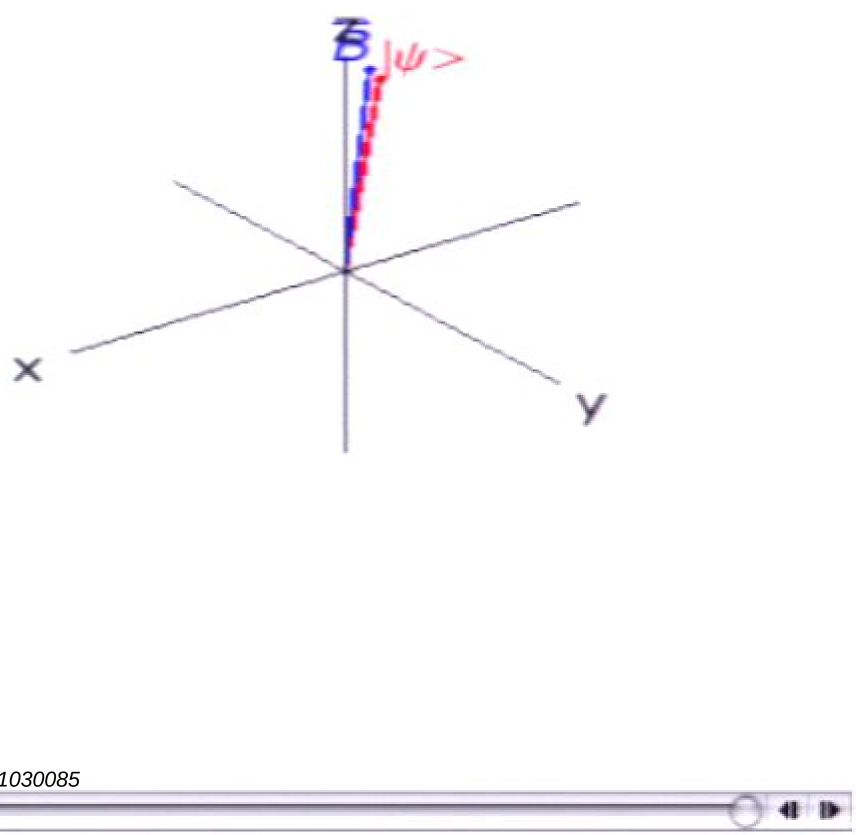

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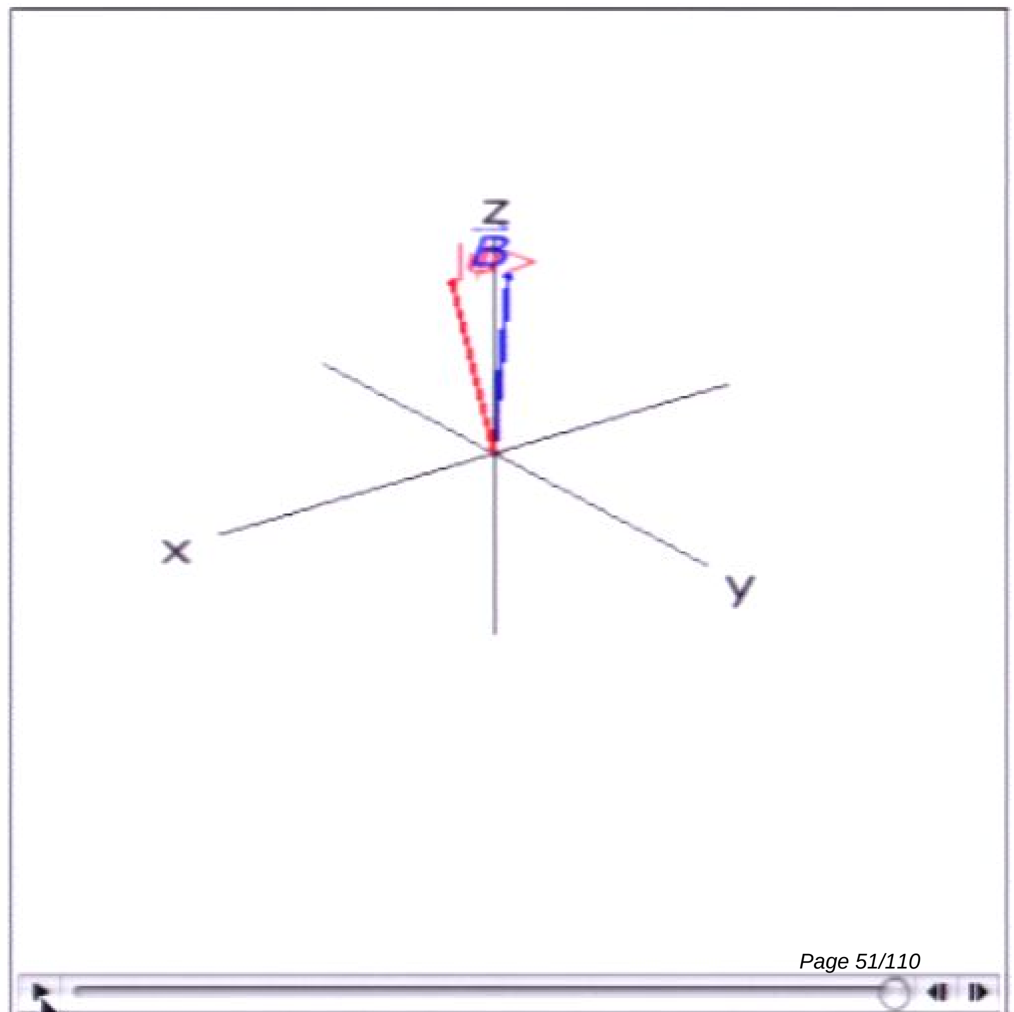

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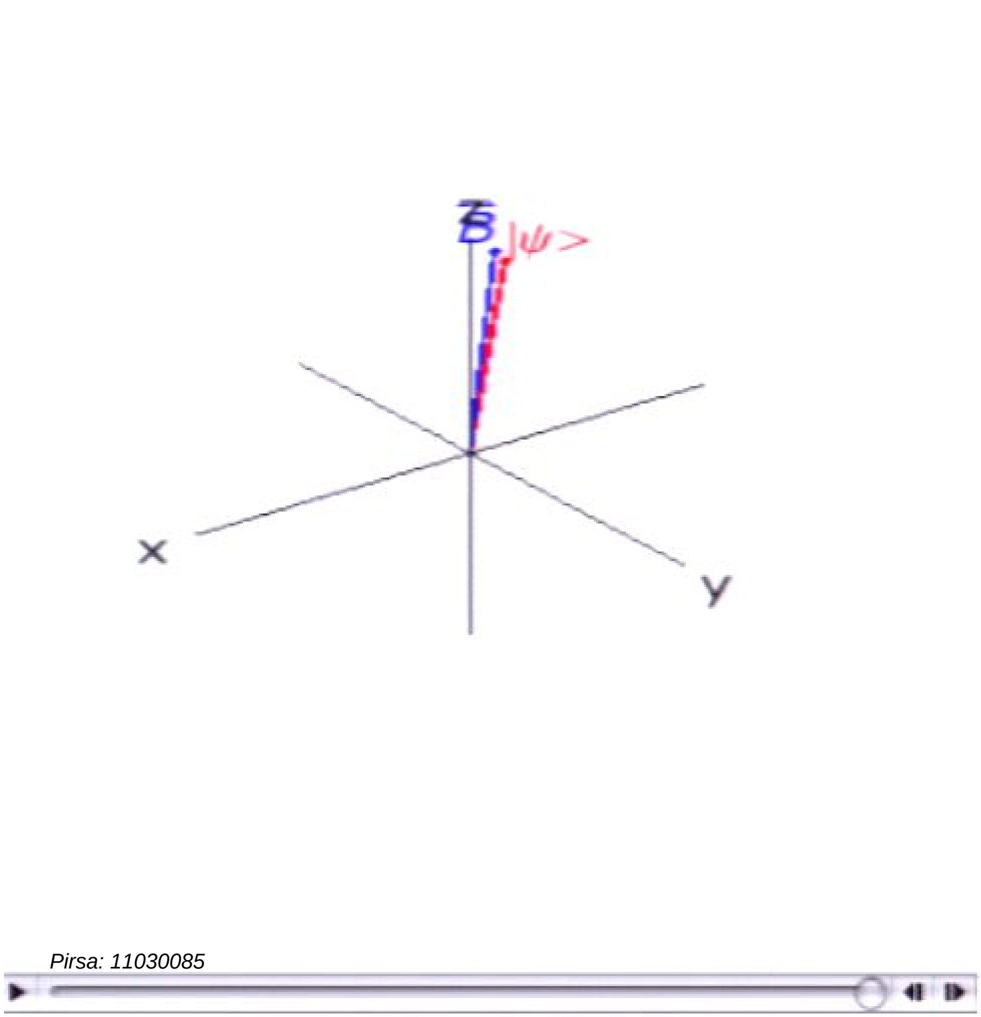

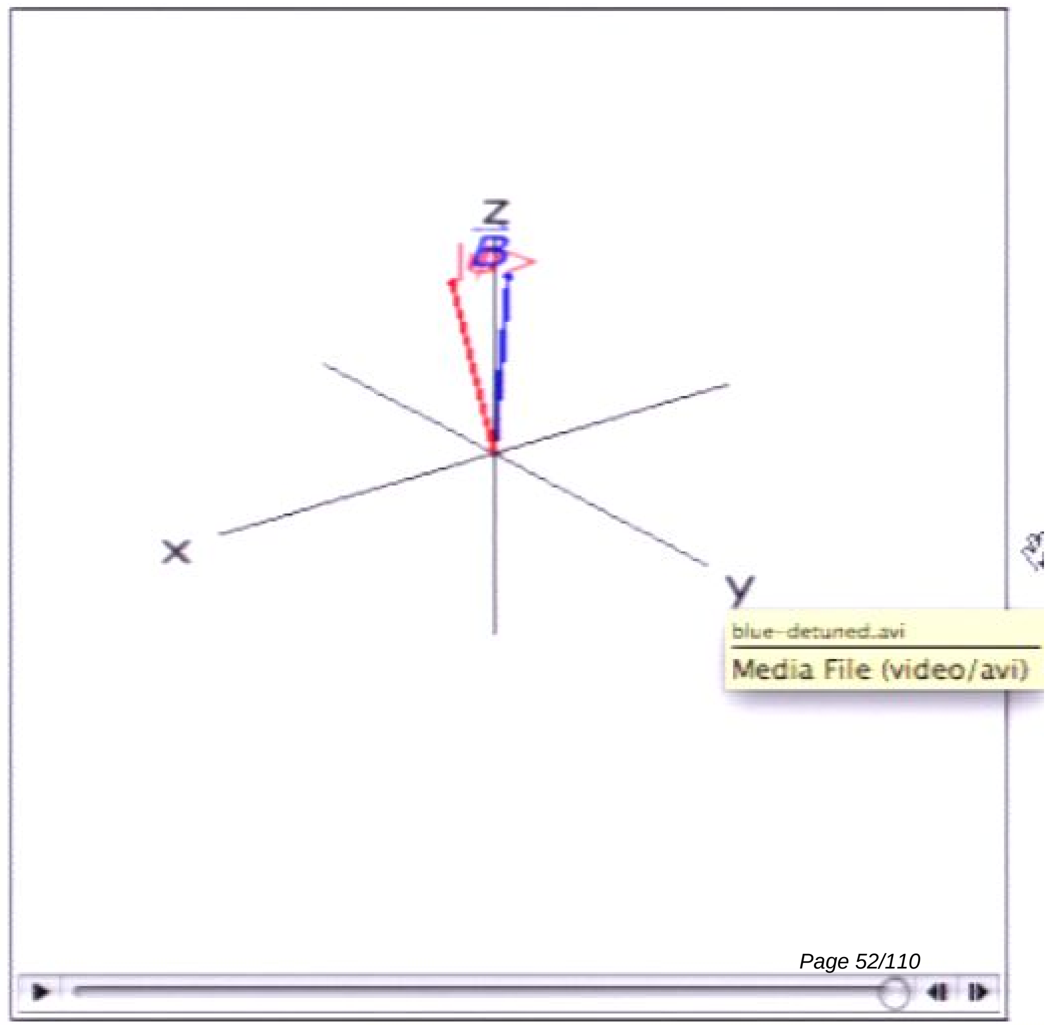

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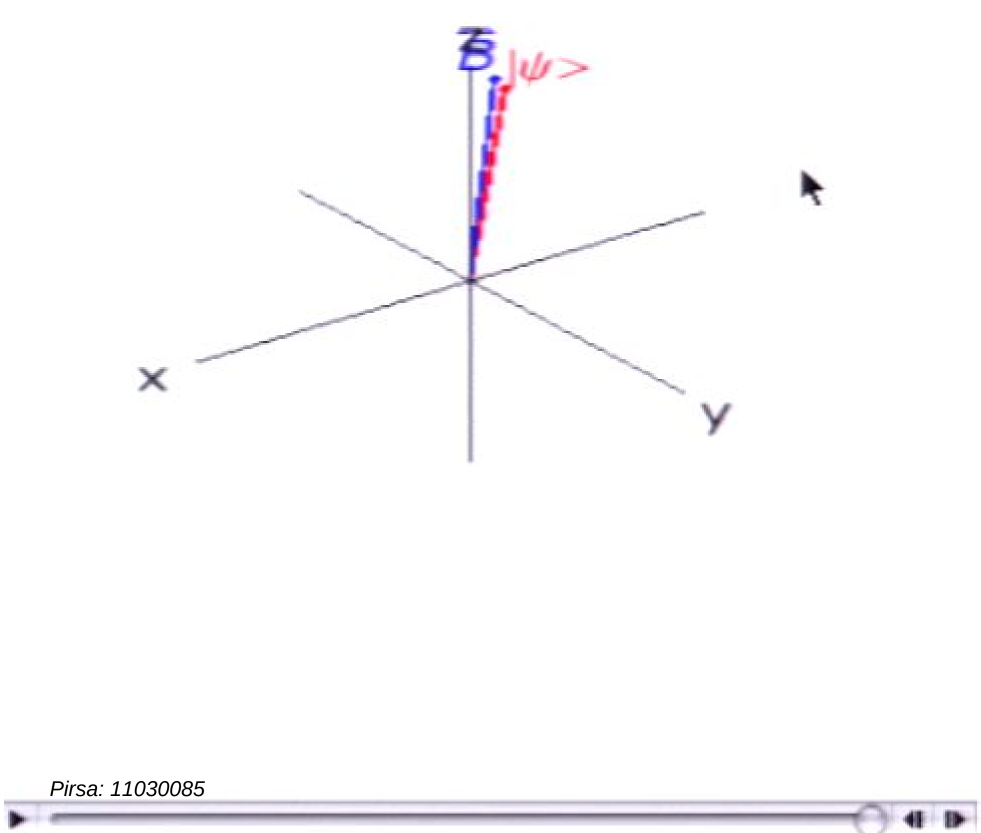

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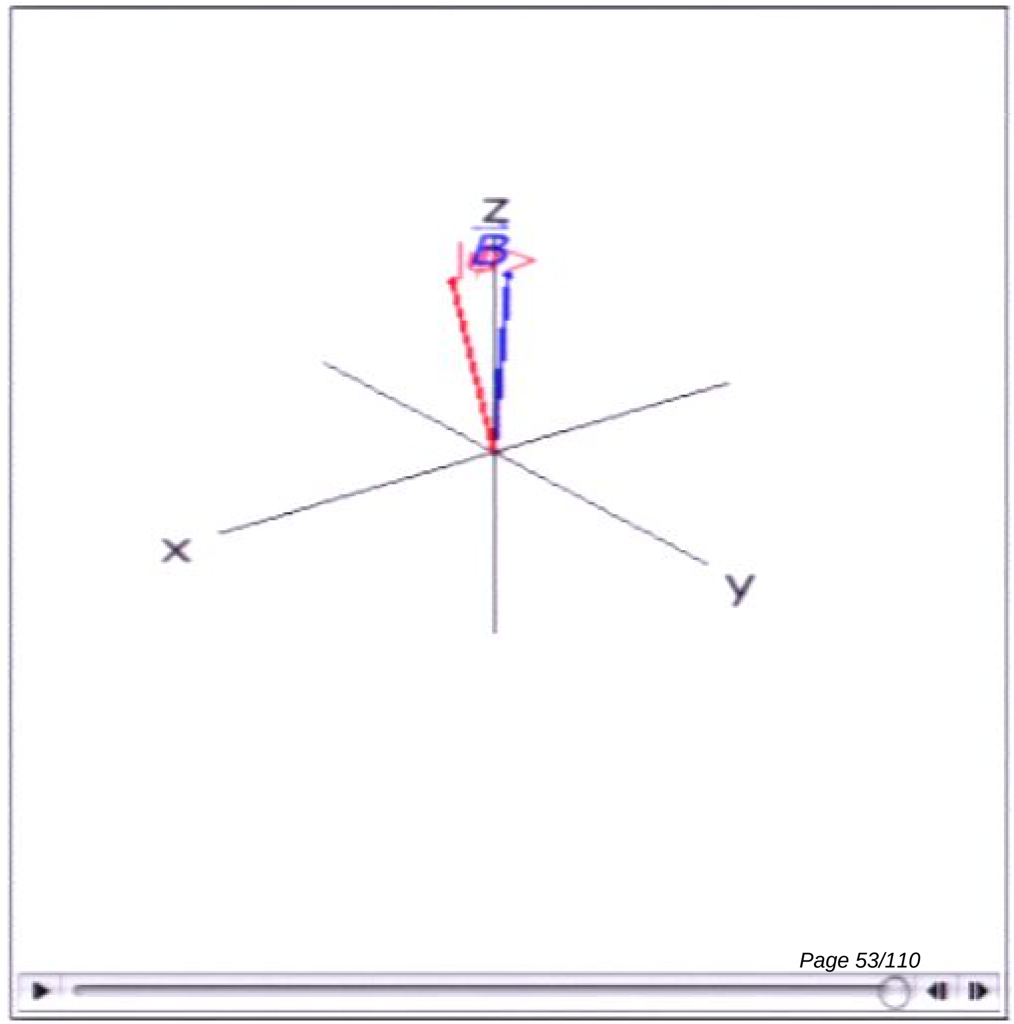
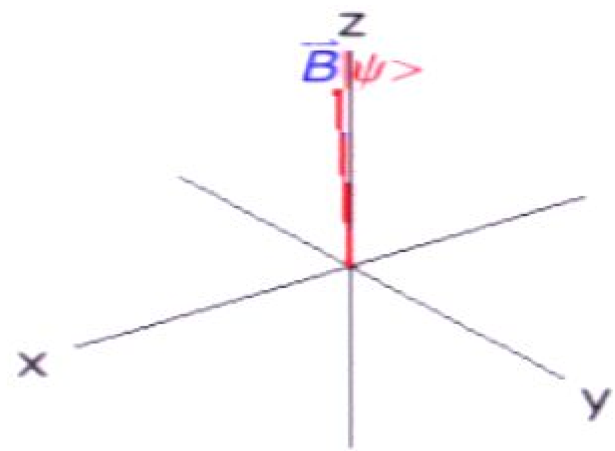


On-resonance Rabi flopping

With $\nu = \Delta\cos(\theta)$, in the interaction picture

$$H_I(t) = -\frac{\Delta\sin(\theta)}{2}\sigma_x \ .$$



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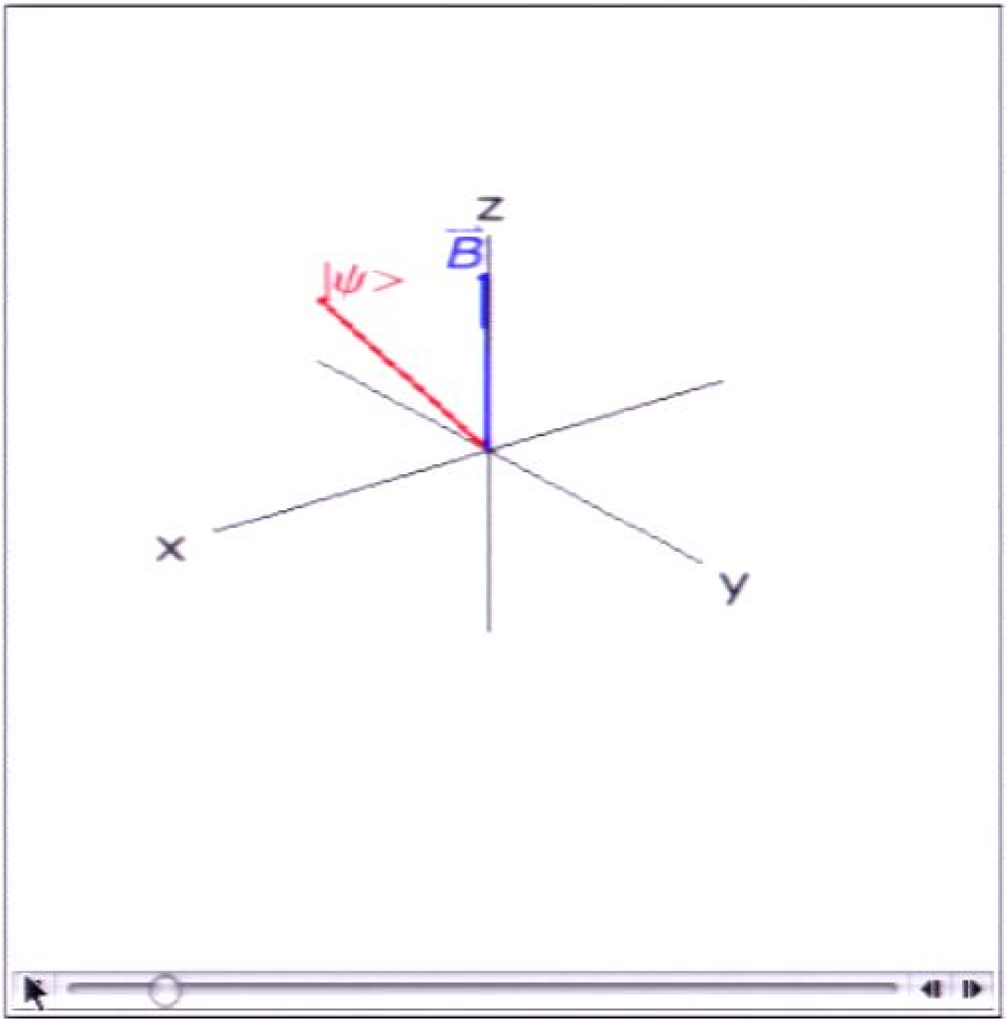

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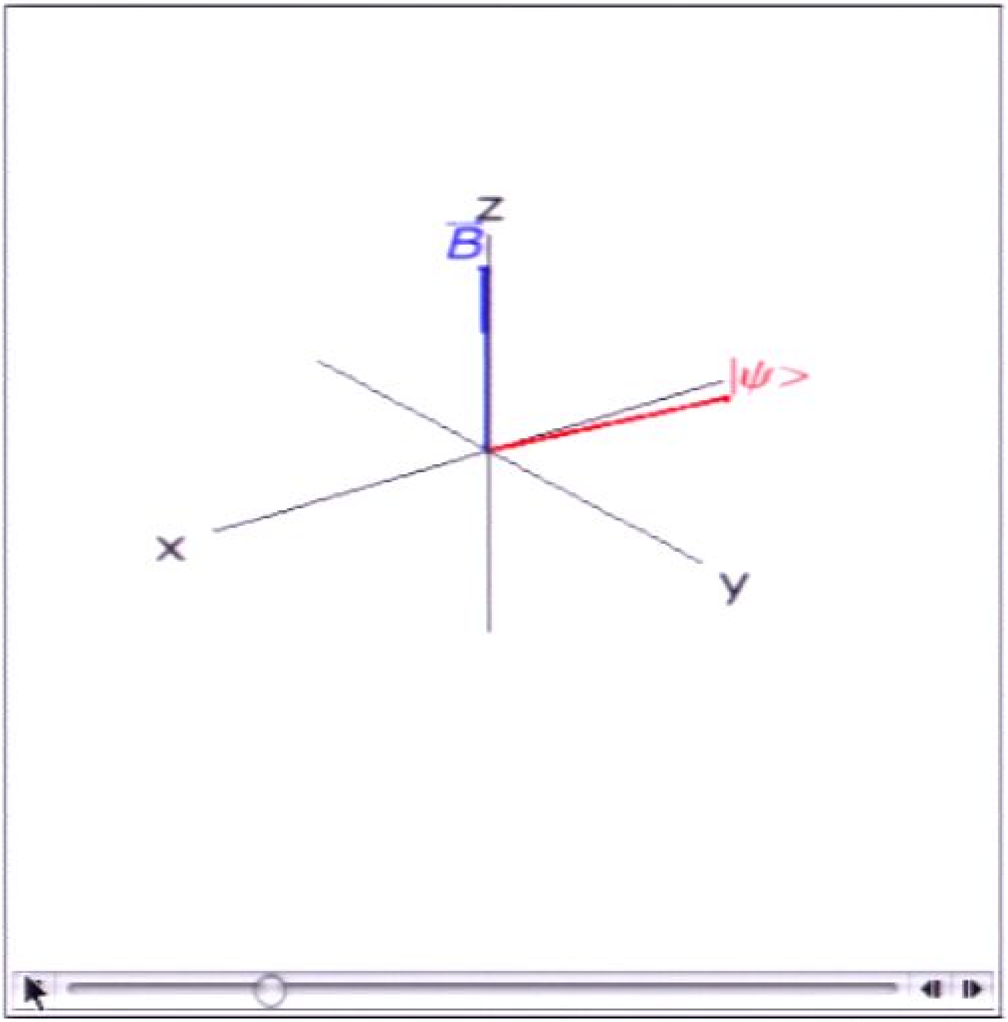

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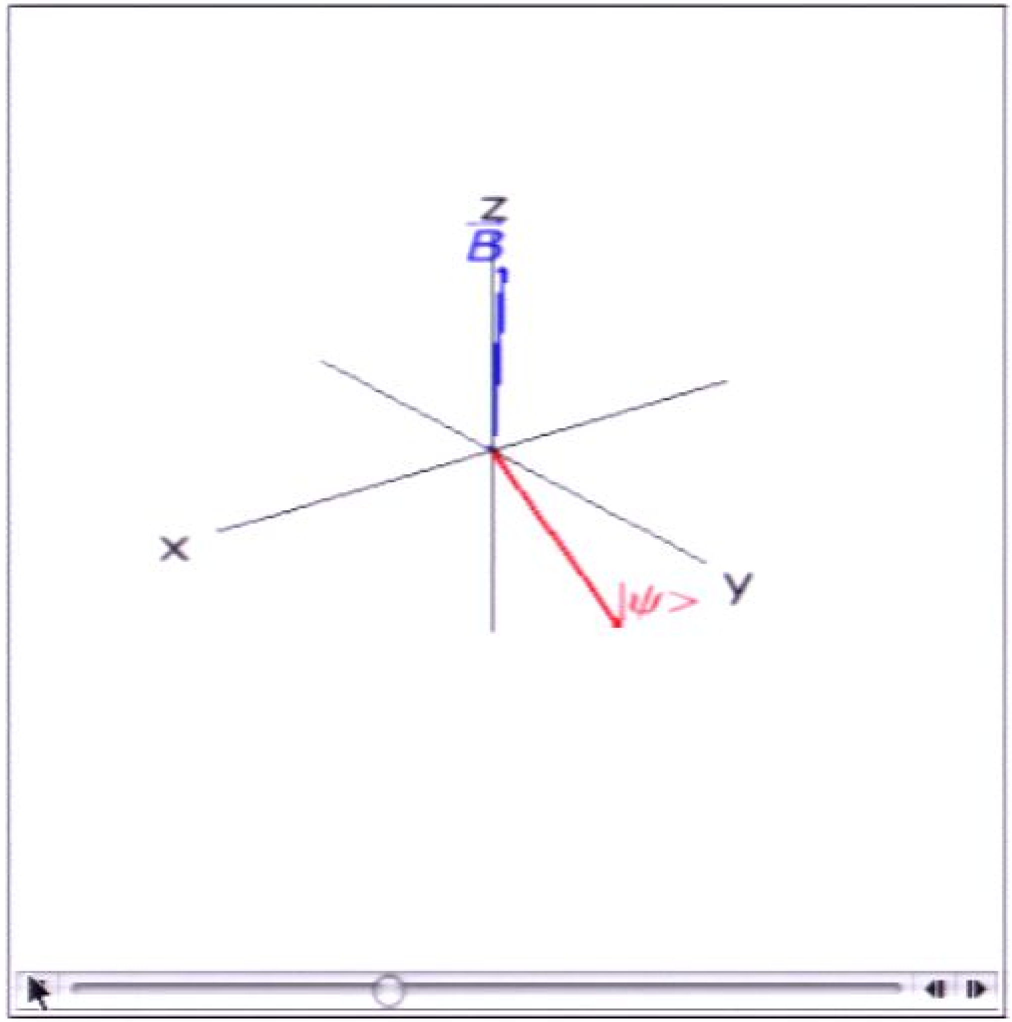

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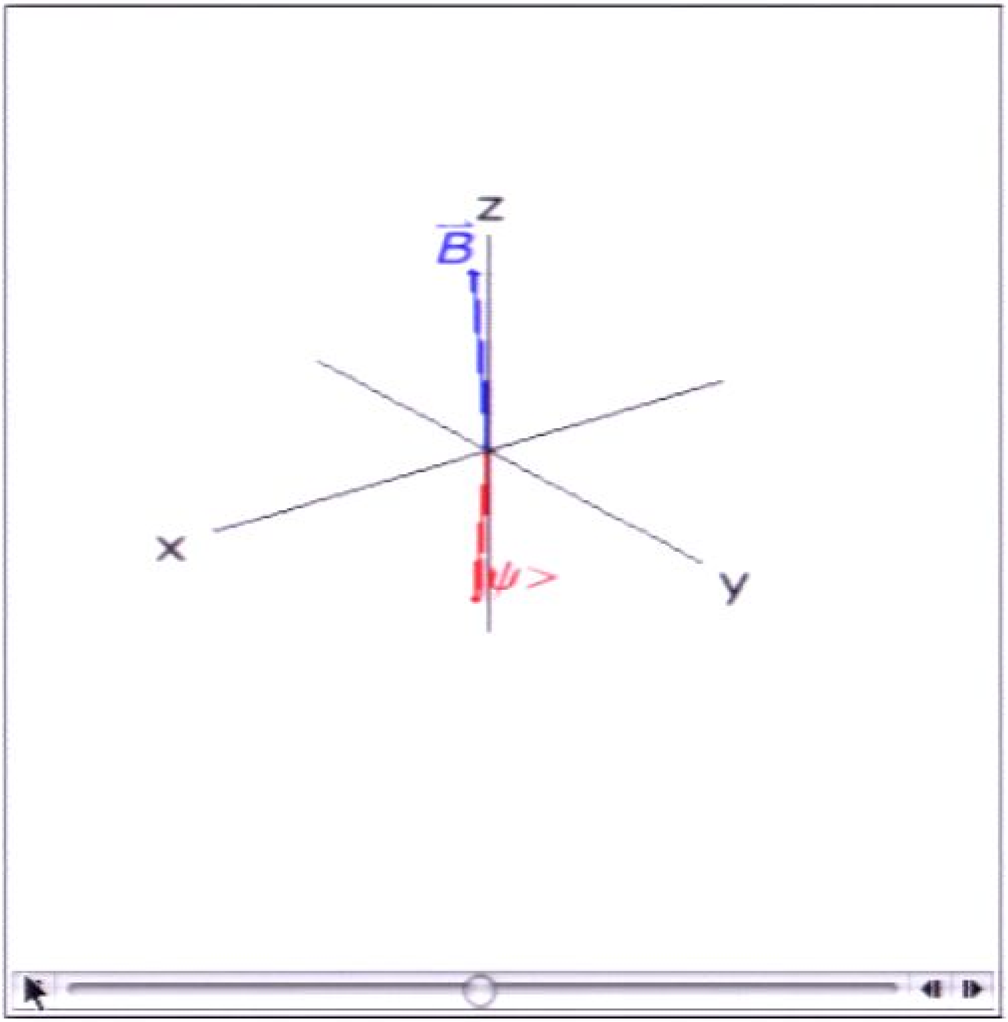

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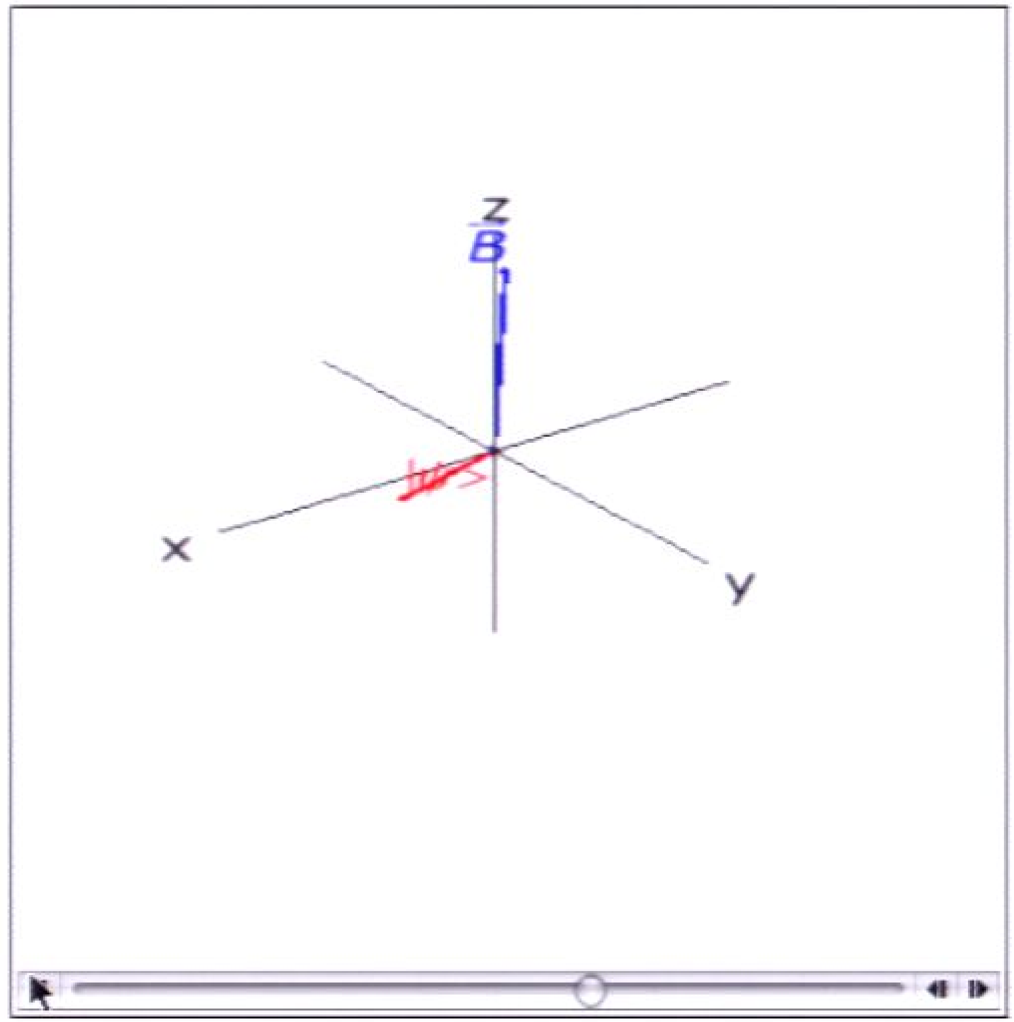

The "folk" adiabatic approximation in a toy model

The "folk" version for $H(t/T)$ says

$$\frac{1}{T}\frac{\|\partial_s H(s)\|}{\Delta^2} \ll 1 \,,$$

This gives

$$\frac{1}{T\Delta}\frac{\nu \sin\theta}{2} \ll 1 \,.$$

It should work for any choice of ν, T, Δ and θ,

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It should work for any choice of ν, T, Δ and θ,

Resonance

At resonance $\nu = T\Delta\cos(\theta)$ the interaction Hamiltonian is $H_I(t) = -\frac{\Delta_\perp}{2}\sigma_x$ while the eigenstate has constant angle θ with Z.

$$\frac{\sin(2\theta)}{4} \ll 1 \qquad \textit{FAILS.}$$

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The “folk” version says:

1. Low frequency light, $\nu \ll \Delta$, is adiabatic ✓.
2. High frequency light, $\nu \gg \Delta$, is not adiabatic. But do we really want that? Isn't there always some high frequency noise? What about smoothness?
3. On resonance, $\nu = \Delta\cos(\tau)$, the condition fails !!! X. We can have Rabi oscillations!!! Schwinger, 1937. Schiff, 1949. Marzlin and Sanders, 2004. One can check that any first derivative condition fails.

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Known conditions that work

Jansen et. al., Goldstone (S. Jordan thesis)

$$\frac{\|\partial_s H(s)\|}{\Delta^2} + \frac{\|\partial_s H(s)\|^2}{\Delta^3} + \frac{\|\partial_s^2 H(s)\|}{\Delta^2} \ll T\,.$$

General adiabatic approximation with first derivatives

Consider adiabatic approximations using only first derivatives

$$\frac{1}{T\Delta}\left(\frac{\|\partial_s \tilde{H}(s)\|}{\Delta}\right)^k \ll 1 \, .$$

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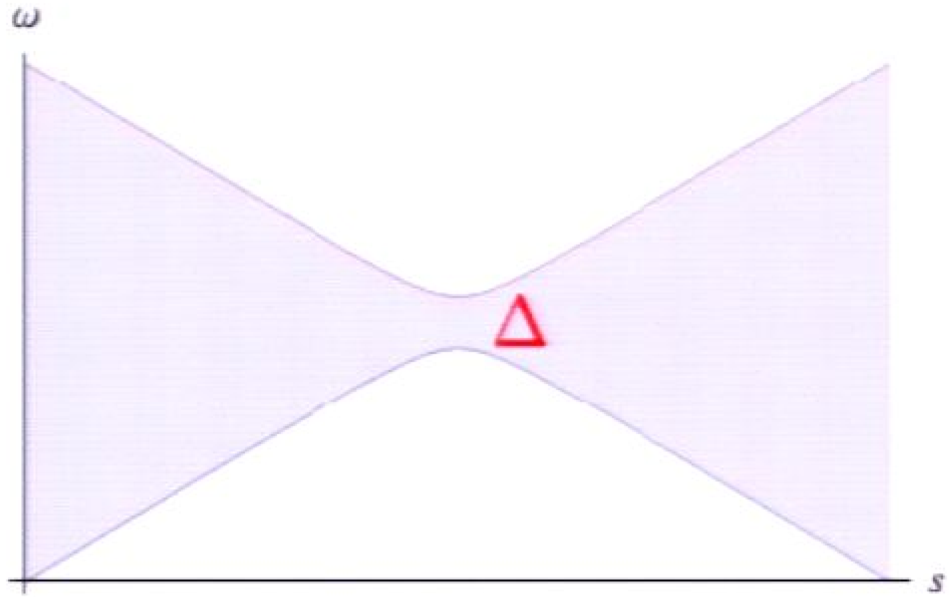
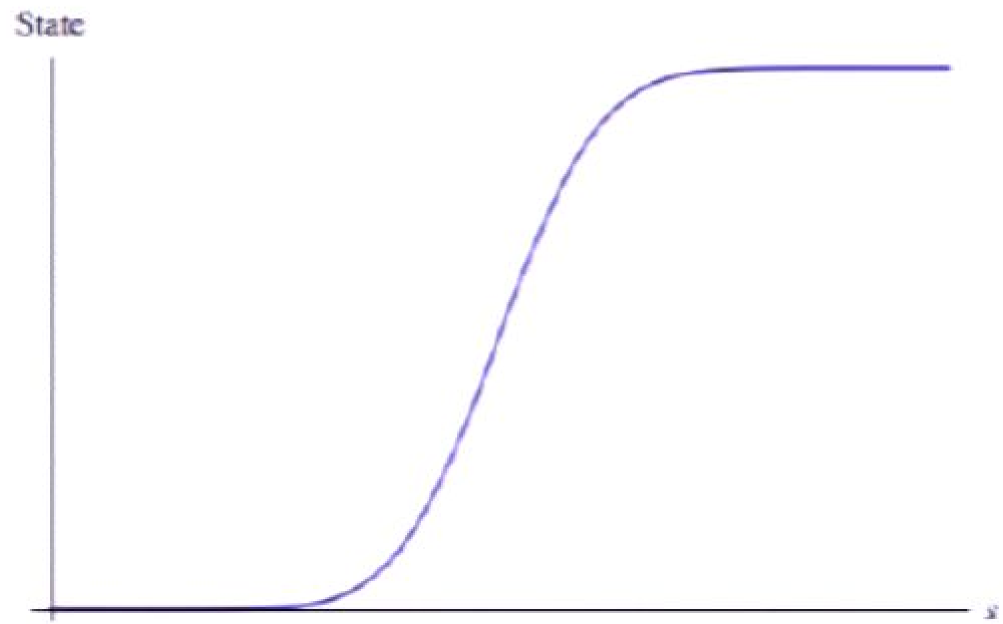
The stronger conditions

$$\frac{1}{\theta} \in o(T\Delta) \quad \text{and} \quad \frac{1}{\theta} \in \omega\left((T\Delta)^{\frac{k-1}{k}}\right) \, .$$

can clearly be achieved for $T\Delta$ sufficiently big.

Standard adiabatic conditions kill quantum speedups





Grover's search $H(s) = -[(1-s)|+\rangle\langle+| + s|\mathcal{S}\rangle\langle\mathcal{S}|]$.

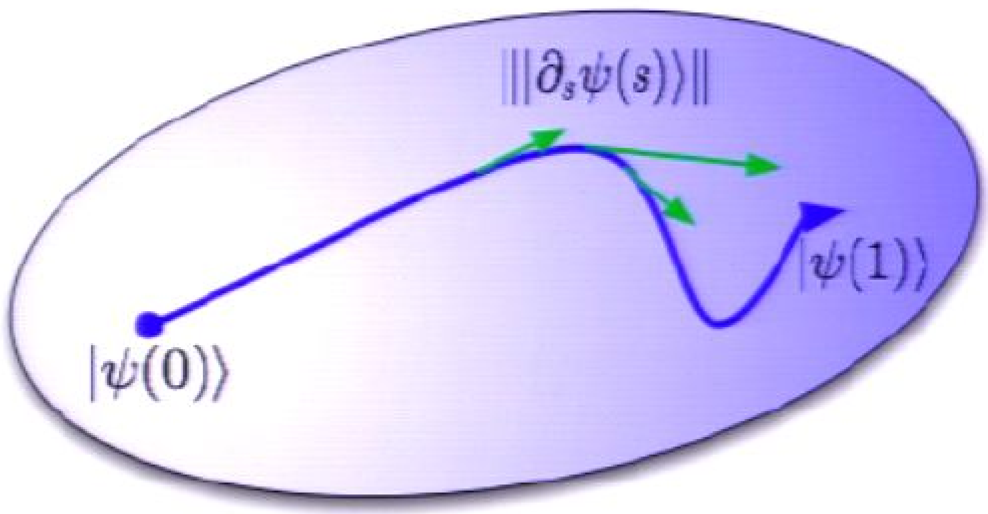
The gap in Grover is $1/\sqrt{N}$, so $1/\Delta^2 = N$: no quantum speed-up! (One solution in Roland and Cerf, 2002.)

Summary of problems with standard adiabatic approximations

1. Standard "folk" versions fail when there is a resonance.
2. Rigorous versions scale with $1/\Delta^3$: very bad for quantum speedups.
3. Rigorous versions depend on $\|\partial_s H\|$, and fail for unbounded operators.
4. Rigorous versions are very sensitive to off-resonant high frequency fields and noise.

1. Motivation
2. Problems with 'folk' versions
3. **Randomization**
4. Fast quantum algorithms for eigenpath traversal
5. The cost path/gap is optimal

Path length



Path length

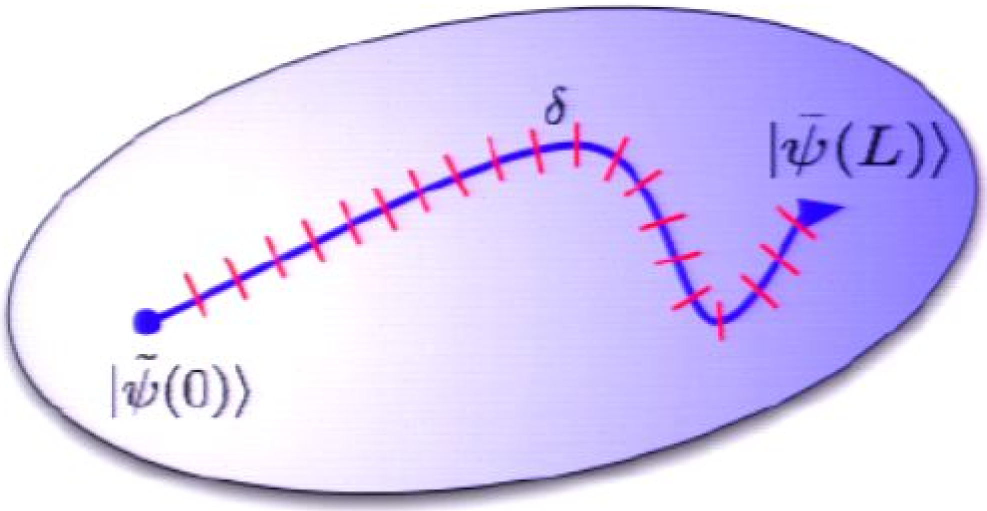
The path length is

$$L = \int_0^1 \||\partial_s\psi(s)\rangle\| ds \leq \frac{\|\partial_s H\|}{\Delta} .$$

The phases are chosen such that $\langle \partial_s\psi(s)|\psi(s)\rangle = 0$.

Adiabatic approximation with measurements

Childs et. al. 2002



Assume projections $P(s)$ at each eigenstate $|\psi(s)\rangle$.

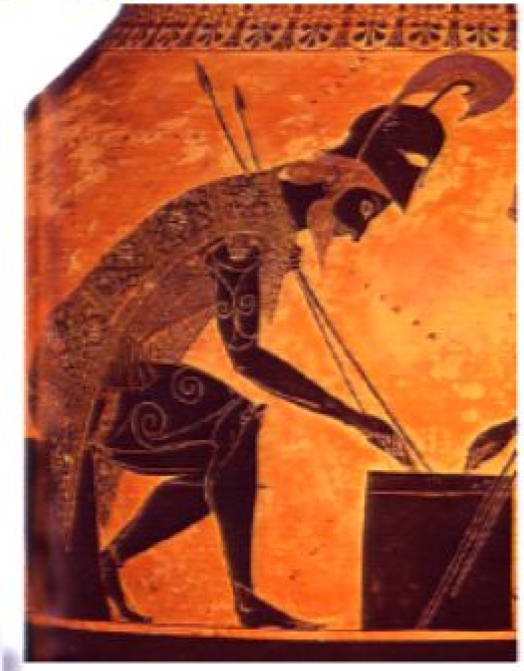
- Project onto consecutive points

$$\cdots \to P(s) \to P(s+\delta) \to P(s+2\delta) \to \cdots$$

- Zeno's effect, projection failure is δ^2!
- Choose $\delta = \epsilon/L$. The total error is

$$(\text{steps}) * (\text{error}) = \frac{L}{\delta} * \delta^2 = \epsilon$$

Path traversal with the Zeno effect



For a δ displacement, to first order

$$|\tilde{\psi}(l+\delta)\rangle \approx |\tilde{\psi}(l)\rangle + \delta|\tilde{\psi}(l)_\perp\rangle \ .$$

Overlap bound

$$|\langle\tilde{\psi}(l)|\tilde{\psi}(l+\delta)\rangle|^2 \geq 1-\delta^2 \ .$$

Childs, et al. 2002. Aharonov, and Ta-Shma, 2003. Somma, Boixo, Barnum, Knill, 2008.

Projective-measurements = phase decoherence

Boixo, Knill, Somma

- Projective-measurement for the Zeno effect

$$M_l(\rho) = P_l \rho P_l + (\mathbb{1} - P_l)\mathcal{E}(\rho)(\mathbb{1} - P_l)$$

is phase decoherence.

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- Errors in the projection implementation are given by the characteristic function $\Phi(\Delta) \approx 0$.

$$\int e^{i\omega_j t} d\mu(t) |\tilde{\psi}(l)\rangle\langle\tilde{\psi}_j(l)| = \Phi(\omega_j)|\tilde{\psi}(l)\rangle\langle\tilde{\psi}_j(l)| \ .$$

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- Average cost of each projection is

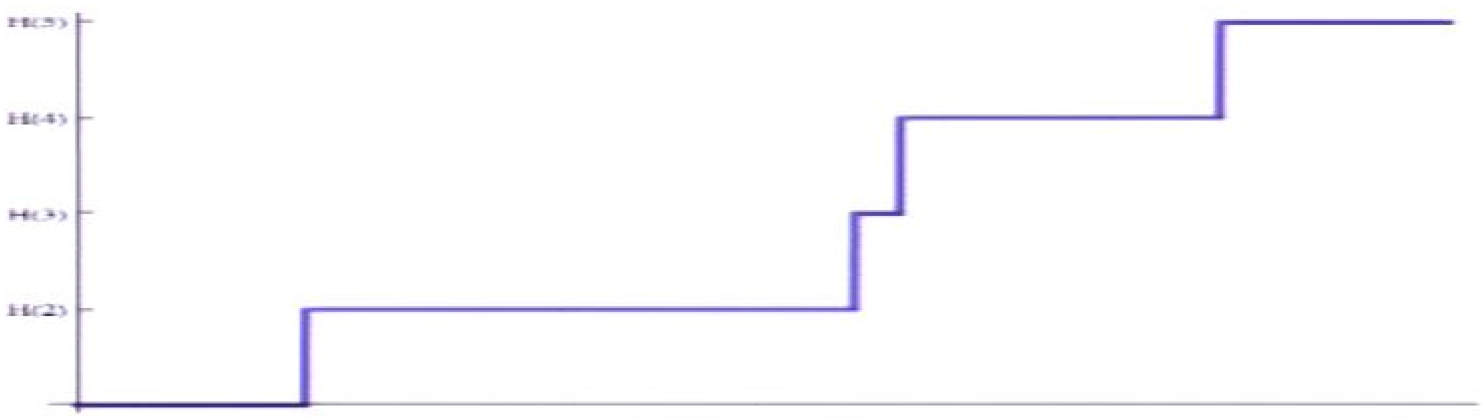
$$\langle T \rangle \approx 1/\Delta$$

  with rapidly decaying tails.

Randomized adiabatic evolution

Use random time distributions T with cost $\langle T \rangle \approx 1/\Delta$.

1. Initialize to $|\tilde{\psi}(0)\rangle$.
2. For each $j \in [1, \ldots, 2L^2/\epsilon]$ evolve with the instantaneous Hamiltonian for a random time T.



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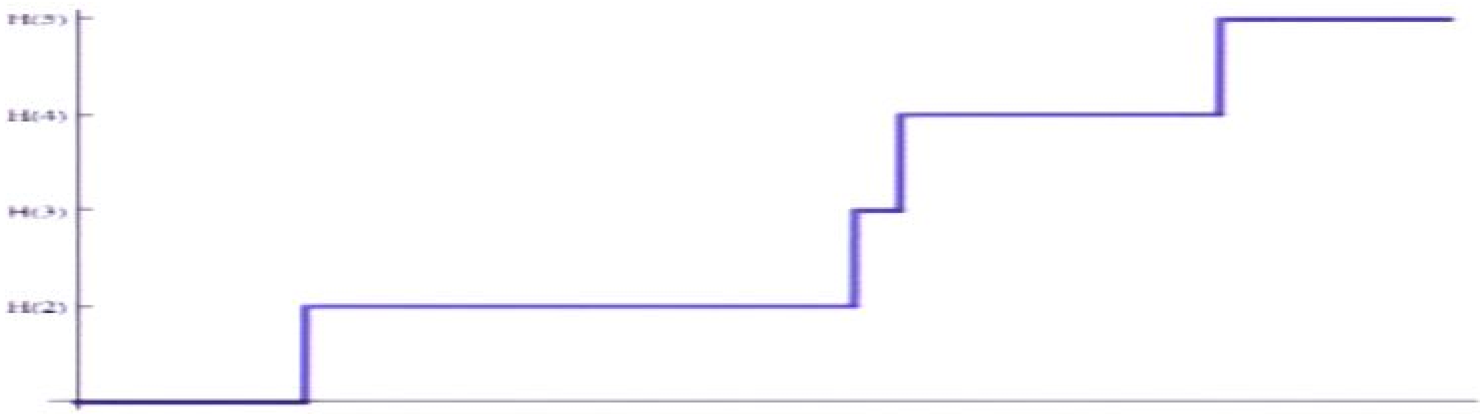

Cost

$$\frac{L^2}{\Delta} \quad \text{or} \quad \frac{\|\partial_s H(s)\|^2}{\Delta^3}.$$

Boixo, Knill, Somma, 2009.

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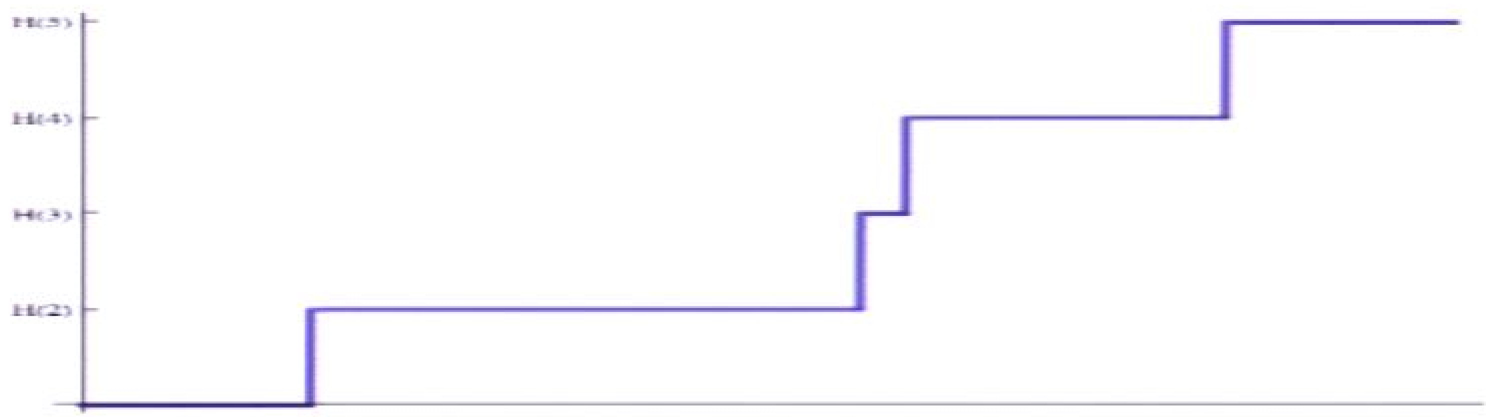

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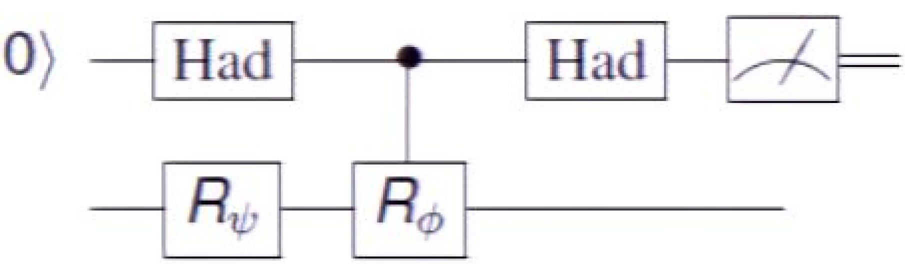
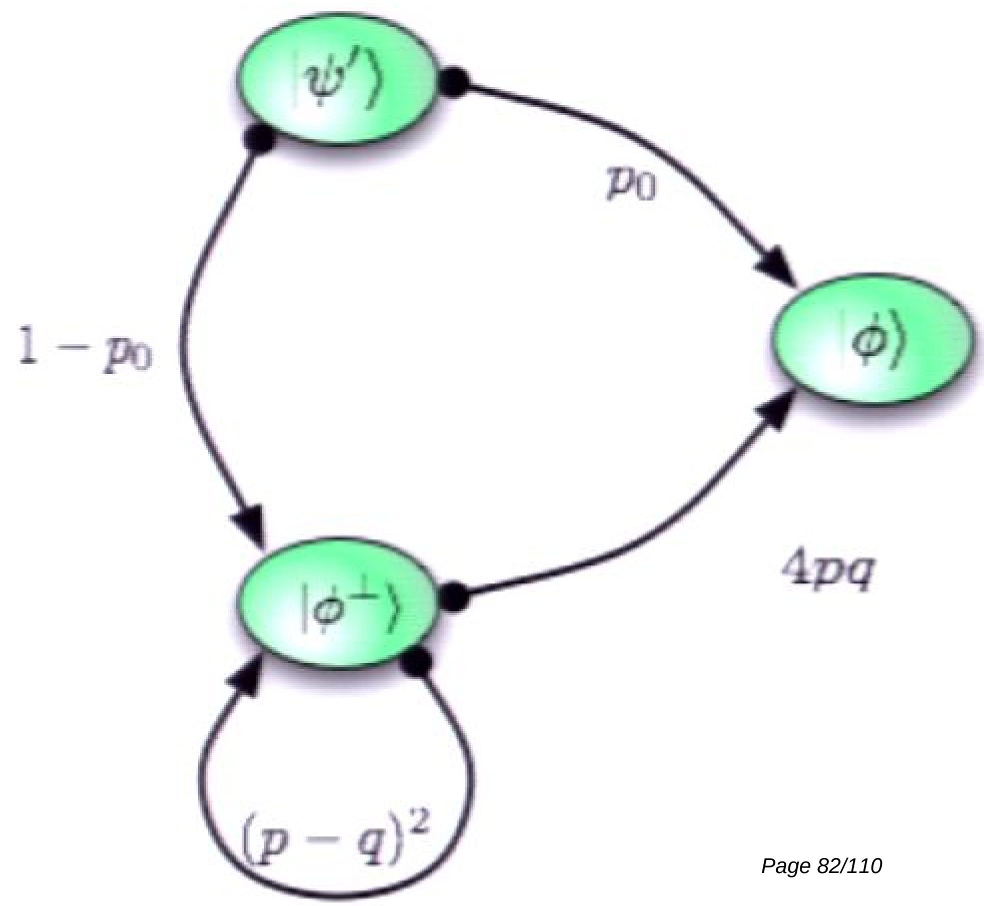
Faster eigenpath traversal

Robust projections

Better scaling with better measurements Abeyesinghe and Wocjan, 2008. Boixo, Knill, Somma, 2010.

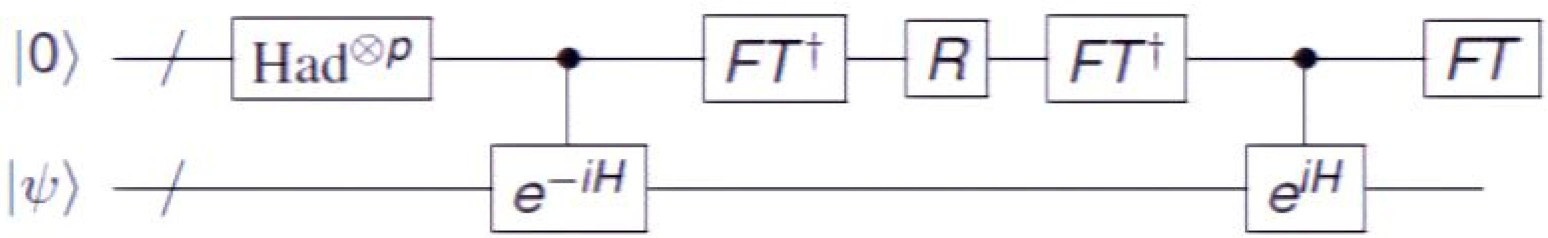
Assume $|\langle\psi|\phi\rangle|^2 = p > 0$.





Approximated reflections and projections

To reflect a non-degenerate eigenstate $U_j|j\rangle = |j\rangle$: do a (high precision) simplified phase estimation, and reflect the ancillae.



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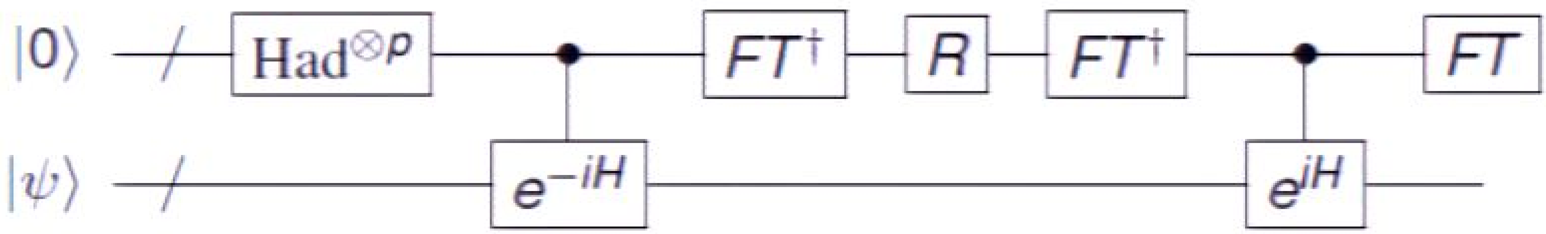

Cost per step

Repeat $\log(1/\epsilon)$ phase estimations with precision $1/\Delta$. Reflect (coherently) the ancillae. The cost is

$$\frac{\log(1/\epsilon)}{\Delta}$$

Eigenpath traversal with known energies

Boixo, Knill, Somma, 2010

1. Break the path into steps so $|\langle j|j+1\rangle| \geq p$. This gives a scaling with L or $\|\partial_s H(s)\|/\Delta$.

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Cost with known energies

$$\frac{L}{\Delta} \quad \text{or} \quad \frac{\|\partial_s H(s)\|}{\Delta^2}$$

and logarithmic corrections.

Non destructive phase estimation

With good overlap $|\langle\psi|\phi\rangle|^2 > 1/2 + \gamma$ and gap Δ implement

$$|\phi\rangle^{\otimes j}|\phi^{\perp}\rangle^{\otimes(r-j)}|\varphi\rangle_A|j\rangle_B \ ,$$

with $j > r/2$.

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with $j > r/2$.

- Do phase estimation with V and error $\Delta/5$.
- Coherently calculate the median $|\varphi\rangle_A$ of the phases, $|j\rangle_B$, and permute the input registers.
- Undo the phase estimation.

Dynamical speed

For Grover and general quantum phase transitions

$$L \leq \frac{\|\partial_s H(s)\|}{\Delta} \ .$$

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For Grover and general quantum phase transitions

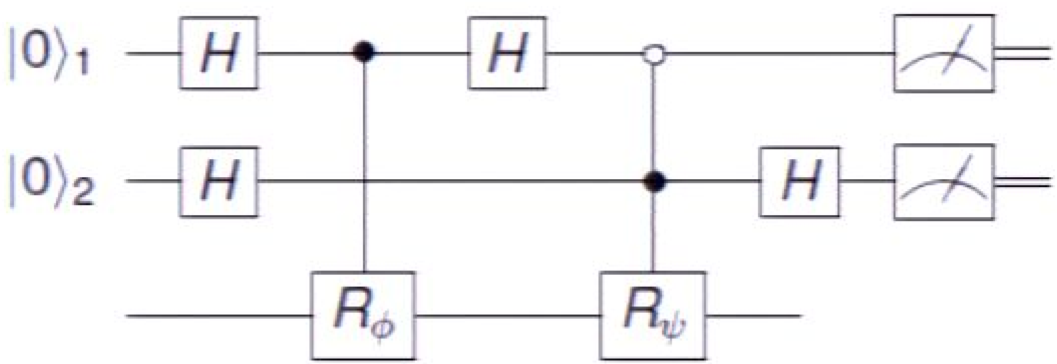
$$L \leq \frac{\|\partial_s H(s)\|}{\Delta} .$$

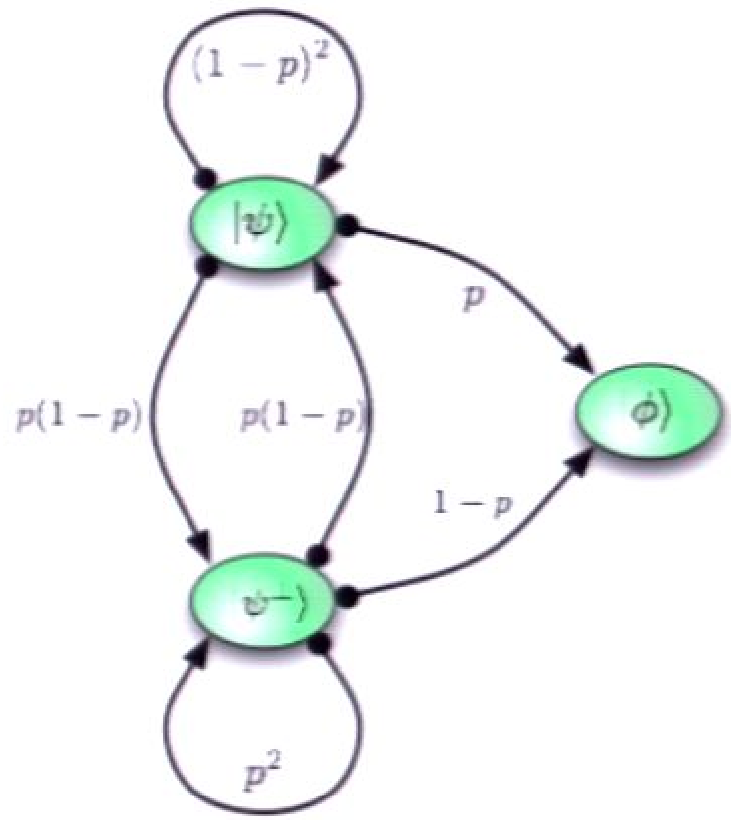
. Consider the $1D$ Ising model

$$\sqrt{n} \approx L \leq \frac{\|\partial_s H(s)\|}{\Delta} \approx \frac{n}{1/n} = n^2$$

1. Implement non-destructive preparation circuits. If a “jump” fails, we remain in the initial state.
2. After a successful jump, try to jump twice further. After a failed jump, try twice closer.

Non-destructive jump





Fast quantum algorithms for eigenpath traversal

Knowledge assumed						
Overlap approximations?	Overlap lower bounds?	Eigenphase ranges?	Eigenphase dominance?	Cost per copy	Number of copies required	Reference
✓		✓		$\mathcal{O}\left(\frac{\bar{L}}{\Delta}\log\left(\frac{\bar{L}}{\epsilon}\right)\right)$	1	Thm. V.1, Lemma V.3
	✓			$\mathcal{O}\left(\frac{n}{\Delta}\log\left(\frac{n}{\epsilon}\right)\right)$	$\Theta\left(\log\left(\frac{n}{\epsilon}\right)\right)$	Thm. V.2
		✓		$\mathcal{O}\left(\frac{\mathcal{L}}{\Delta}\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)\right)$	1	Thm. V.6
			✓	$\mathcal{O}\left(\frac{\mathcal{L}}{\Delta}\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)\right)$	$\Theta\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)$	Thm. V.5

Fast quantum algorithms for eigenpath traversal

Knowledge assumed						
Overlap approx-imations?	Overlap	Eigenphase	Eigenphase	Cost per copy	Number of copies	Reference
√						Thm. V.1, Lemma V.3
						Thm. V.2
						Thm. V.6
						Thm. V.5

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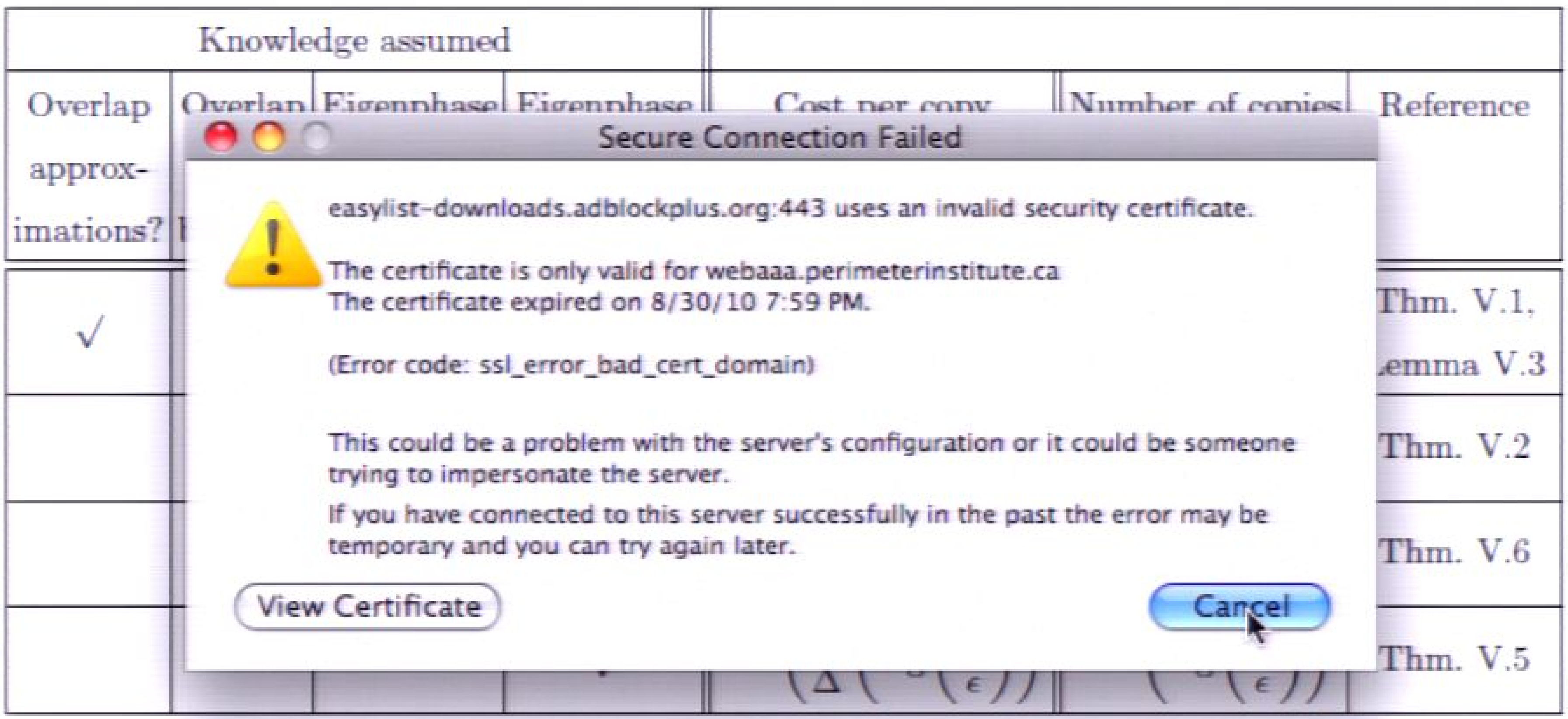
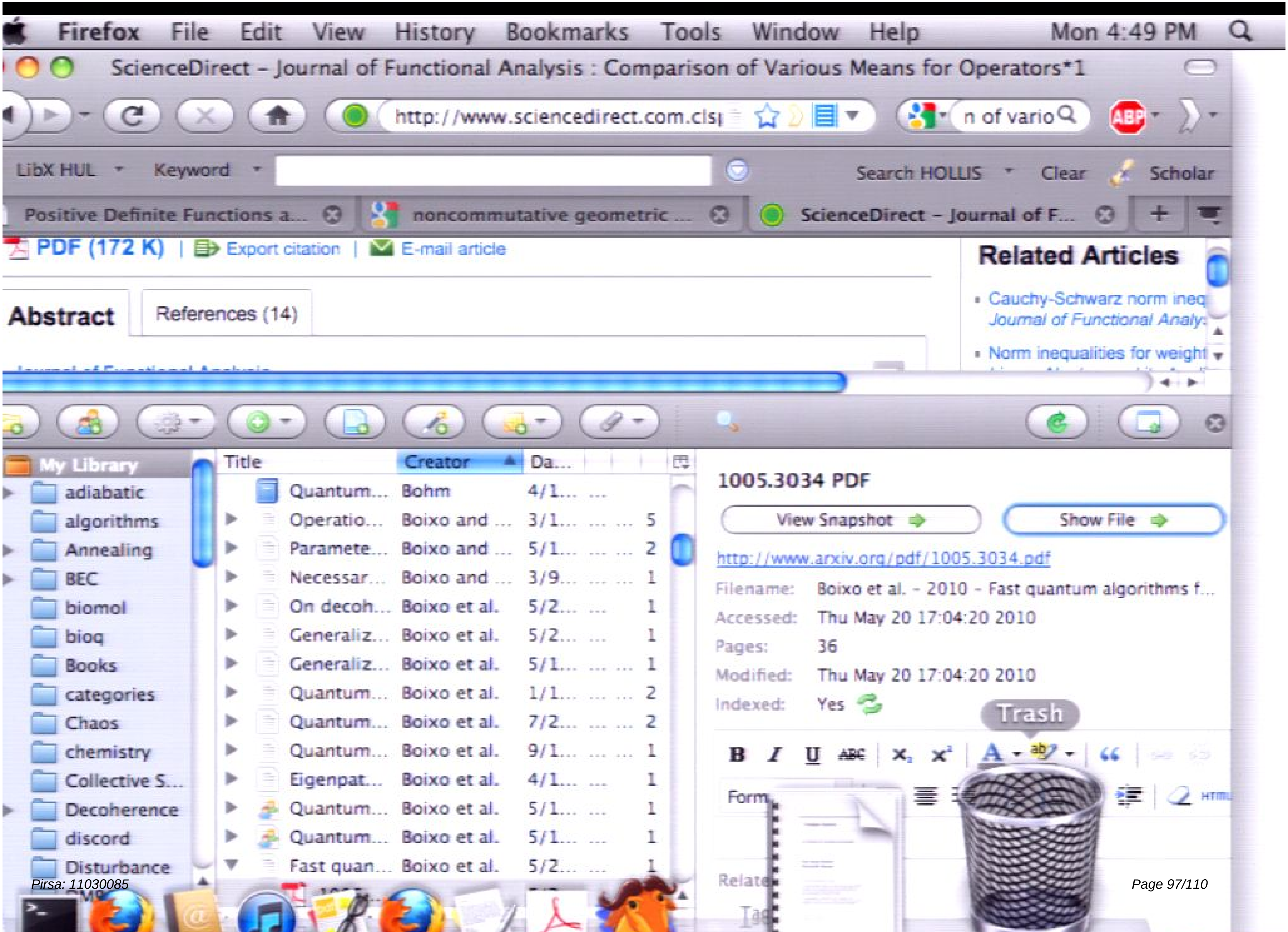
Fast quantum algorithms for eigenpath traversal

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Fast quantum algorithms for eigenpath traversal

Knowledge assumed						
Overlap approx- imations?	Overlap lower bounds?	Eigenphase ranges?	Eigenphase dominance?	Cost per copy	Number of copies required	Reference
✓		✓		$\mathcal{O}\left(\frac{\bar{L}}{\Delta}\log\left(\frac{\bar{L}}{\epsilon}\right)\right)$	1	Thm. V.1, Lemma V.3
	✓			$\mathcal{O}\left(\frac{n}{\Delta}\log\left(\frac{n}{\epsilon}\right)\right)$	$\Theta\left(\log\left(\frac{n}{\epsilon}\right)\right)$	Thm. V.2
		✓		$\mathcal{O}\left(\frac{\mathcal{L}}{\Delta}\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)\right)$	1	Thm. V.6
			✓	$\mathcal{O}\left(\frac{\mathcal{L}}{\Delta}\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)\right)$	$\Theta\left(\log\left(\frac{\mathcal{L}}{\epsilon}\right)\right)$	Thm. V.5

1. Motivation
2. Problems with "folk" versions
3. Randomization
4. Fast quantum algorithms for eigenpath traversal
5. The cost path / gap is optimal

Optimal cost: non-degenerate case

- For a "secret" word $x = x_1, \ldots x_n$ define $|x(j)^+\rangle = |x_1, \ldots x_j, +, \ldots, +\rangle$. The Hamiltonians are

$$H_{x,j} = \Delta |x(j)^+, c_j\rangle\langle x(j)^+, c_j| \ ,$$

where $|c_j\rangle = (|j\rangle + |j-1\rangle)/\sqrt{2}$ are "clock" states.

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- Concatenation of $H_{x,j}$ is a path of length n and gap Δ.
- $H_{x,j}$ is a fractional oracle checking the first j digits of x.
- One full oracle can be implemented with 2 calls to the ordered search oracle, which checks if x is less than a

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- It is possible to do a direct bound using the adversary method in the continuum.

Summary of results

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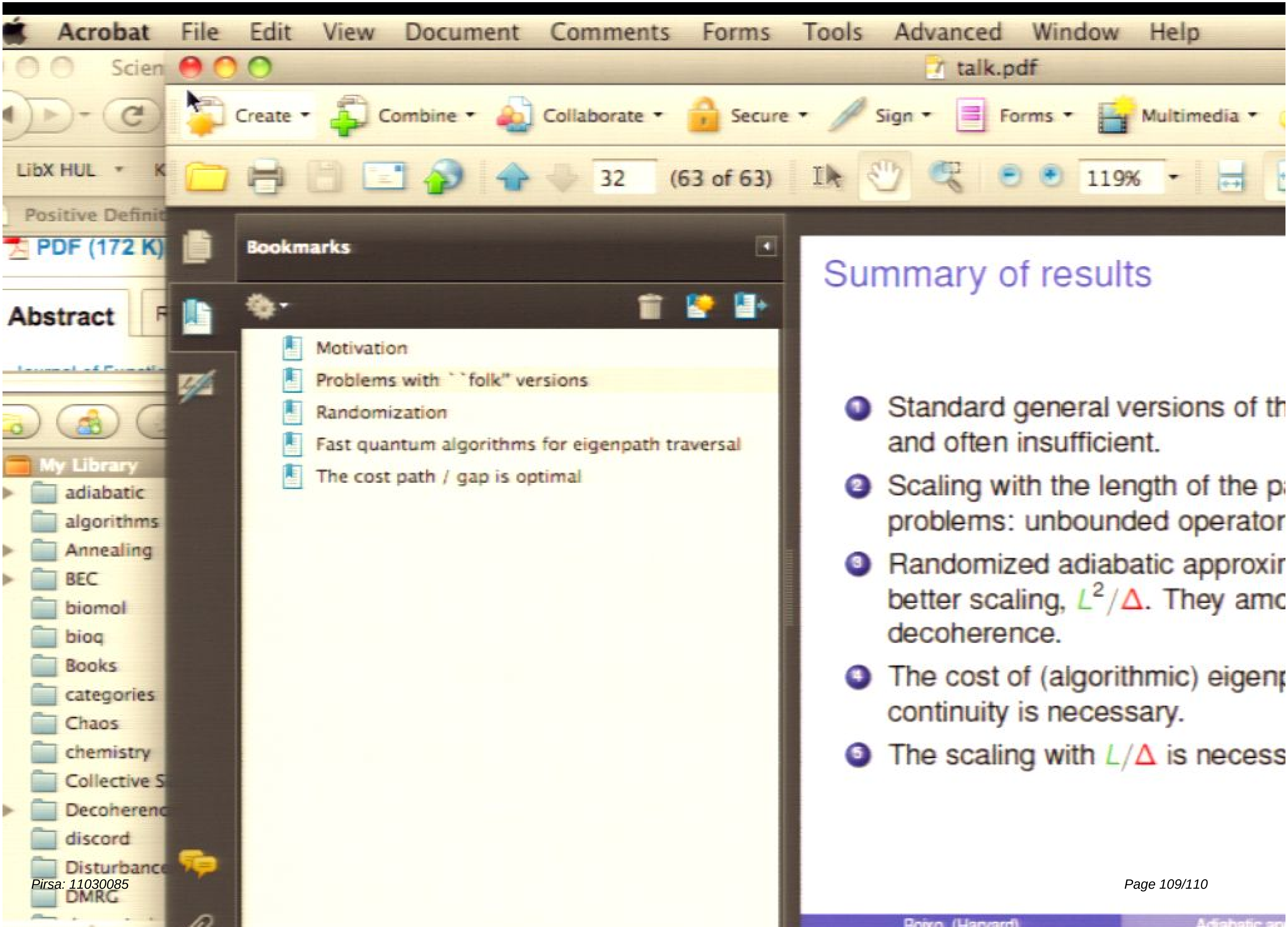
1. Standard general versions of the adiabatic approximation are slow and often insufficient.
2. Scaling with the length of the path of eigenstates L solves many problems: unbounded operators, noise, quantum speedups.
3. Randomized adiabatic approximations are more robust and have better scaling, L^2/Δ. They amount to instantaneous phase decoherence.
4. The cost of (algorithmic) eigenpath traversal is L/Δ. Not even continuity is necessary.
5. The scaling with L/Δ is necessary.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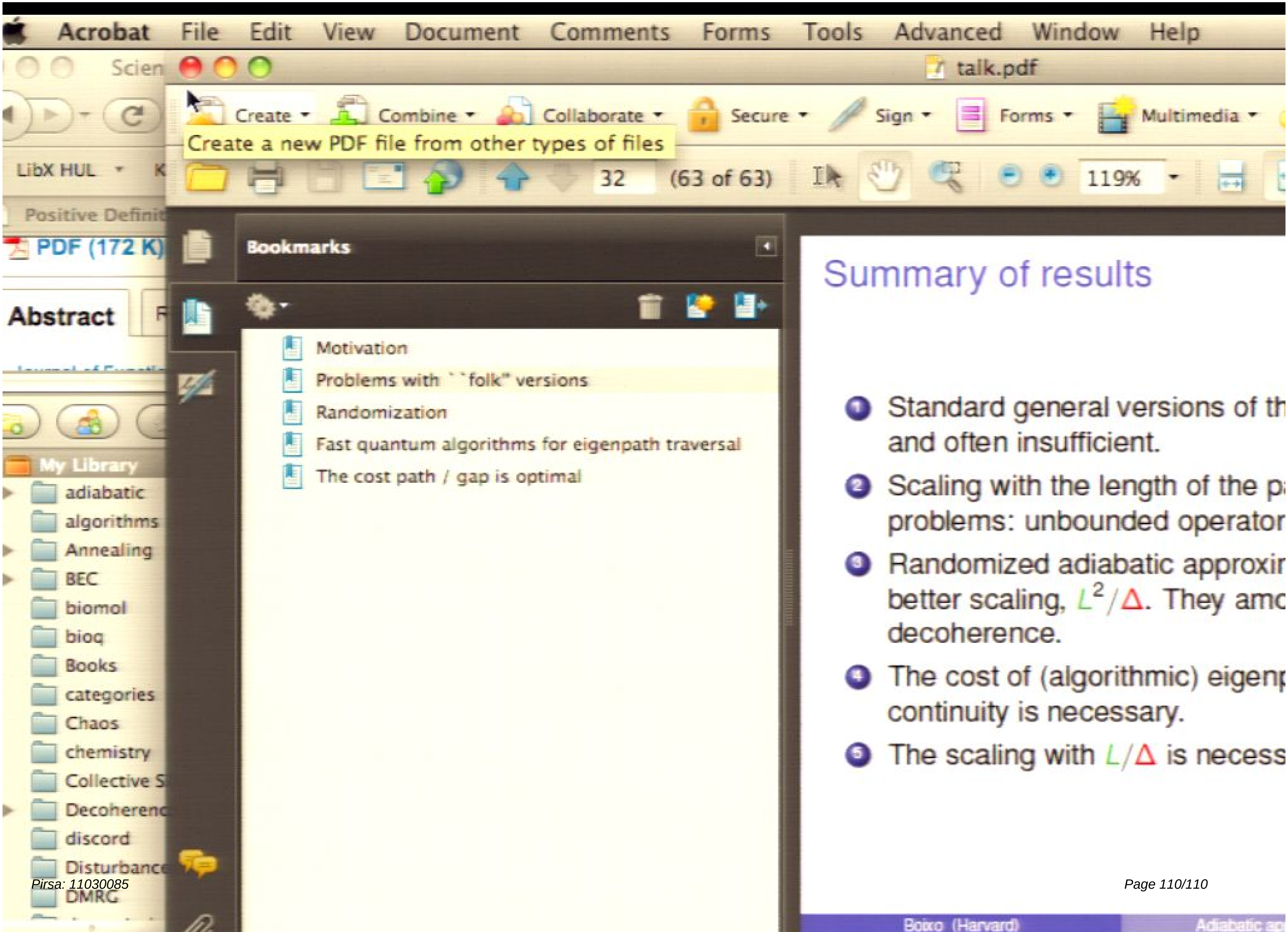
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