Title: Entanglement Routers Using Macroscopic Singlets

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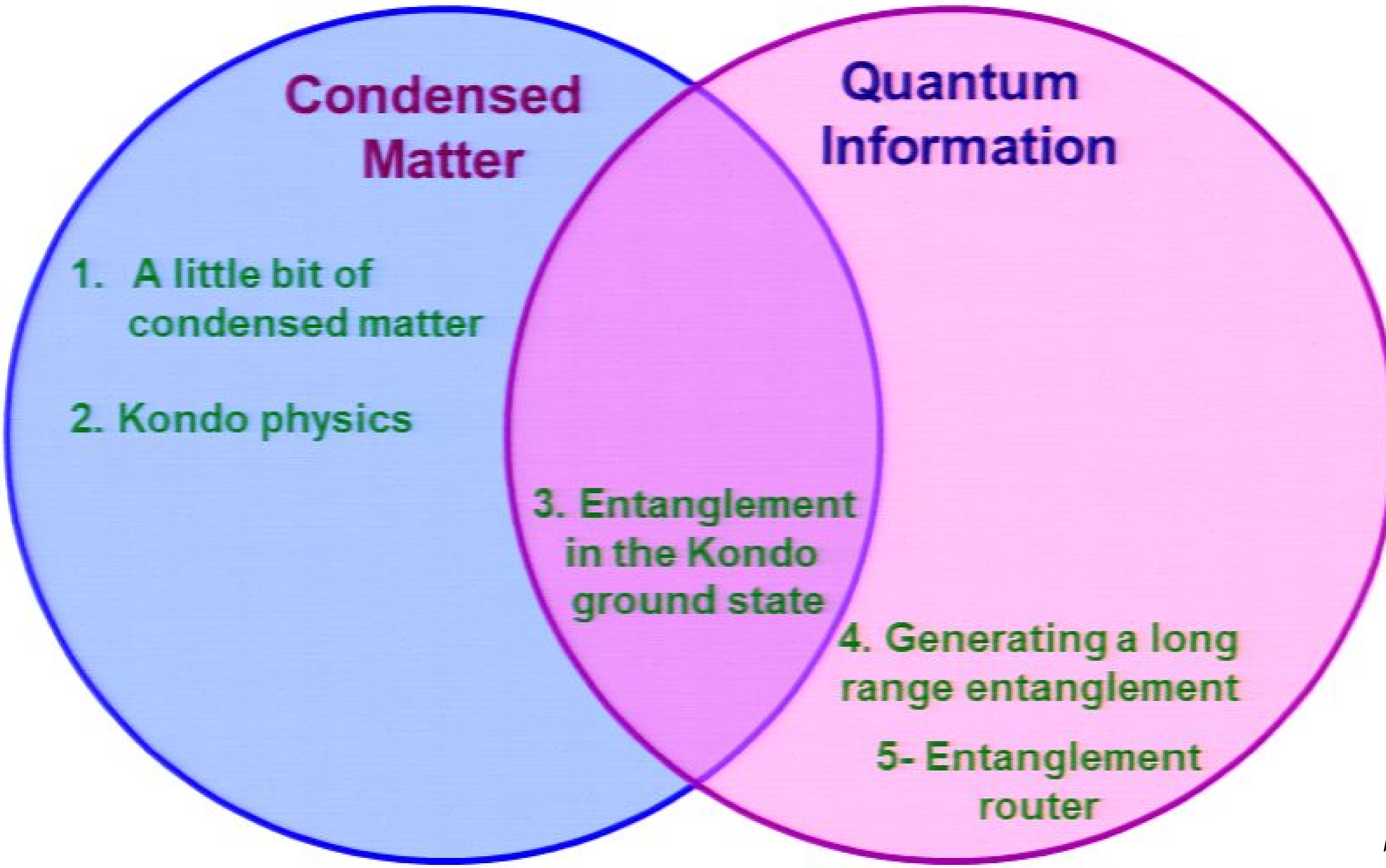
Abstract: We propose a mechanism where high entanglement between very distant boundary spins is generated by suddenly connecting two long Kondo spin chains. We show that this procedure provides an efficient way to route entanglement between multiple distant sites useful for quantum computation and multi-party quantum communication. We observe that the key features of the entanglement dynamics of the composite spin chain are remarkably well described using a simple model of two singlets, each formed by two spins. The proposed entanglement routing mechanism is a footprint of the emergence of a Kondo cloud in a Kondo system and can be engineered and observed in varied physical settings.

Entanglement Routers Using Macroscopic Singlets

Abolfazl Bayat[1], Sougato Bose[1], Pasquale Sodano[2]

[1] University College London, London, UK.
[2] University of Perugia, Perugia, Italy.

Contents of the Talk



Gapped Systems

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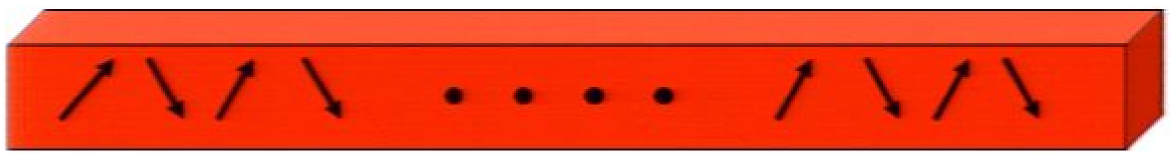
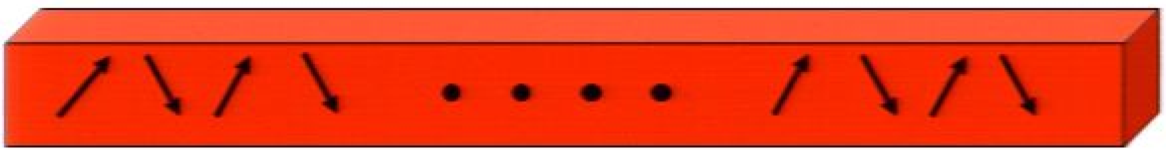


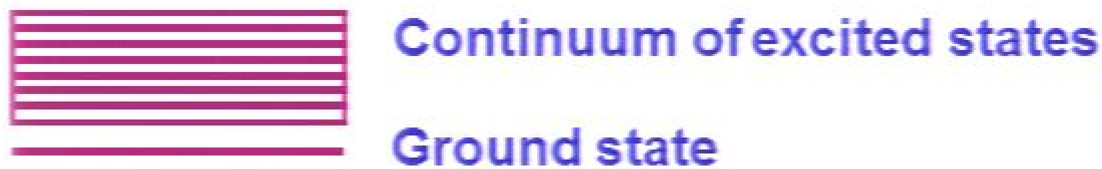
$$\Delta \Leftrightarrow \xi : \quad \left\langle S_x^i S_x^j \right\rangle \approx e^{-\frac{|i-j|}{\xi}}$$

The intrinsic length scale of the system impose an exponential decay

Gapless Systems



$$N \to \infty : \quad \Delta \to 0$$



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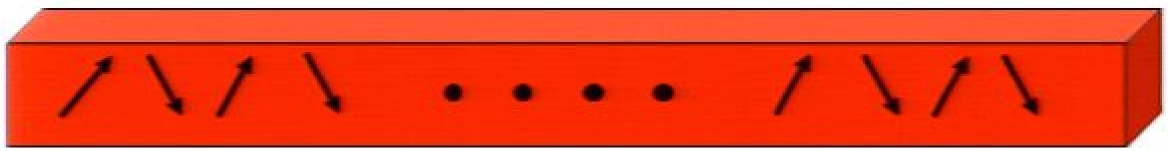

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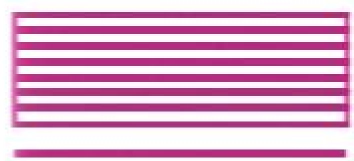


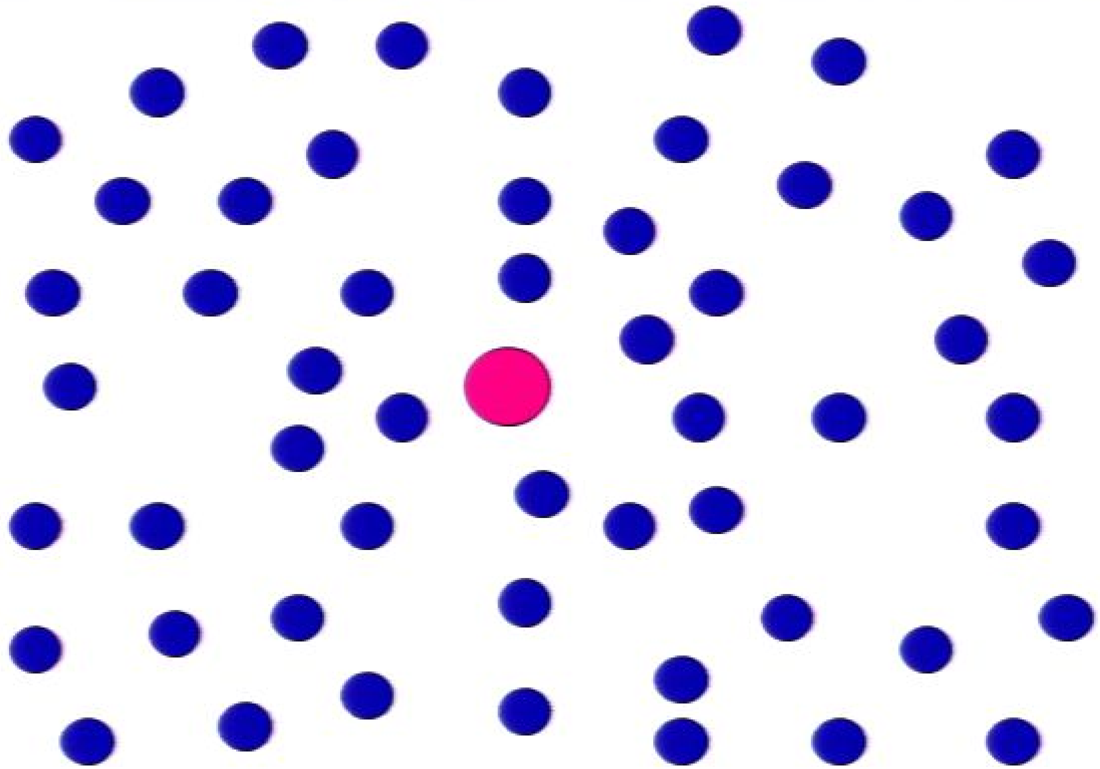
Continuum of excited states

Ground state

$$\left\langle S_x^i S_x^j \right\rangle \approx | i - j |^{-\alpha}$$

There is no length scale in the system so correlations decay algebraically

Kondo Physics



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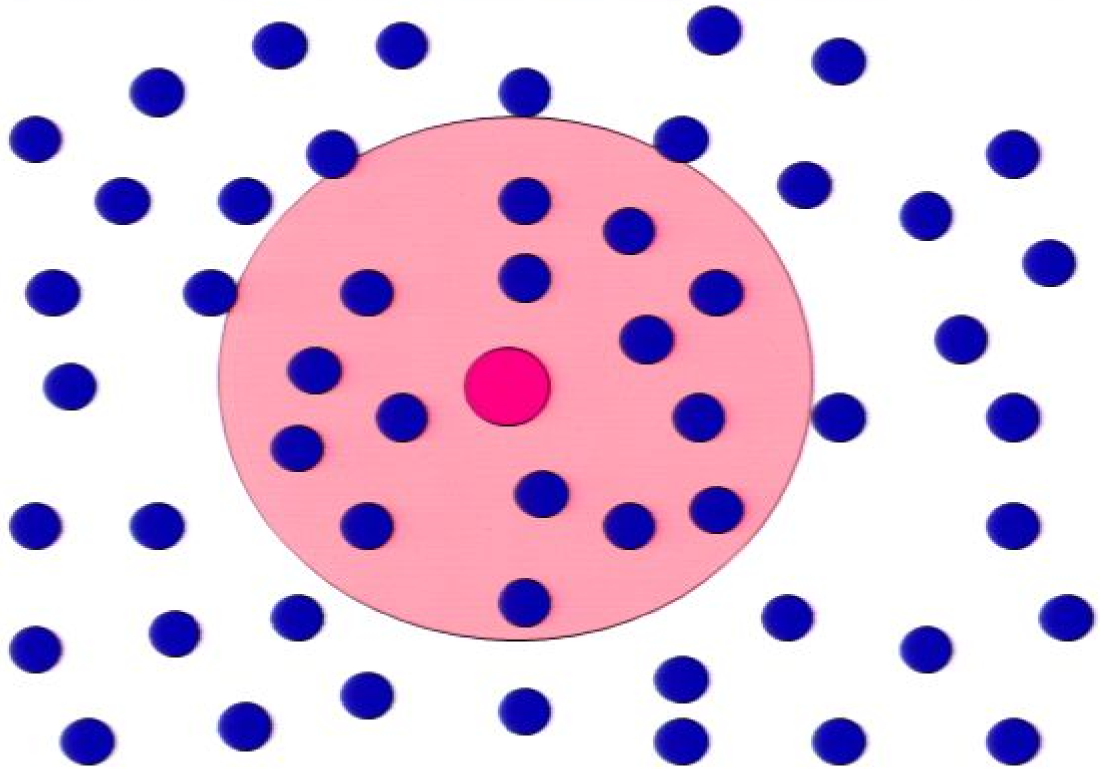

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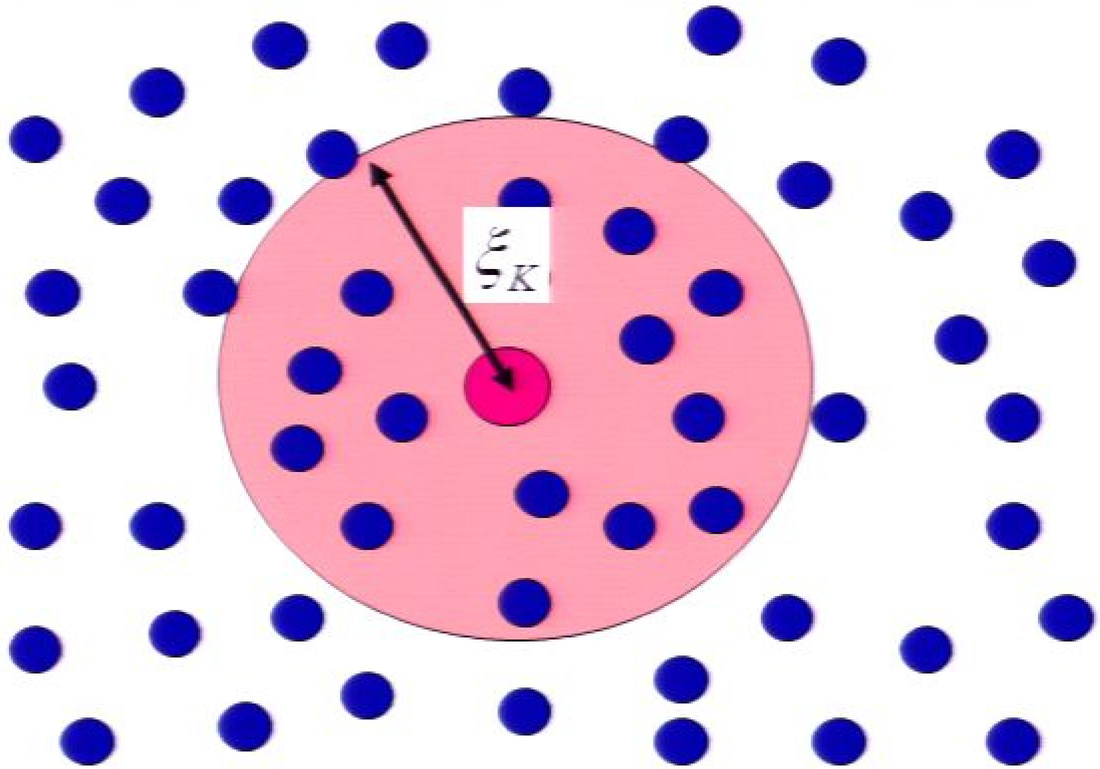

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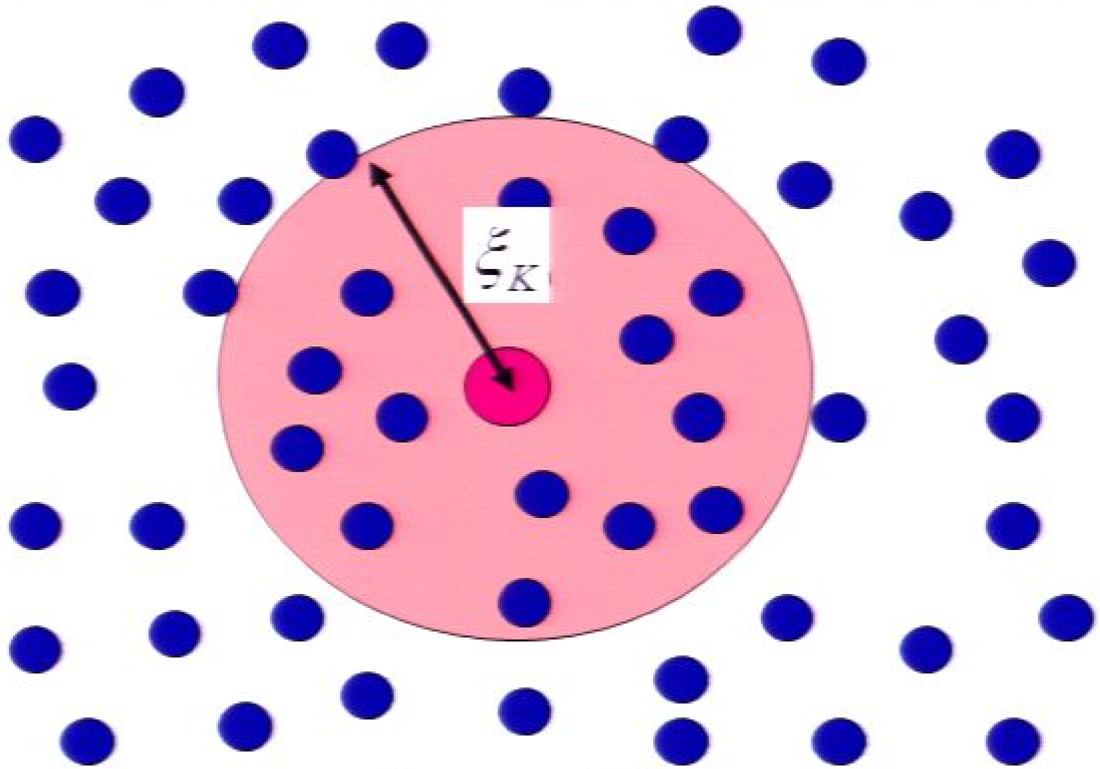
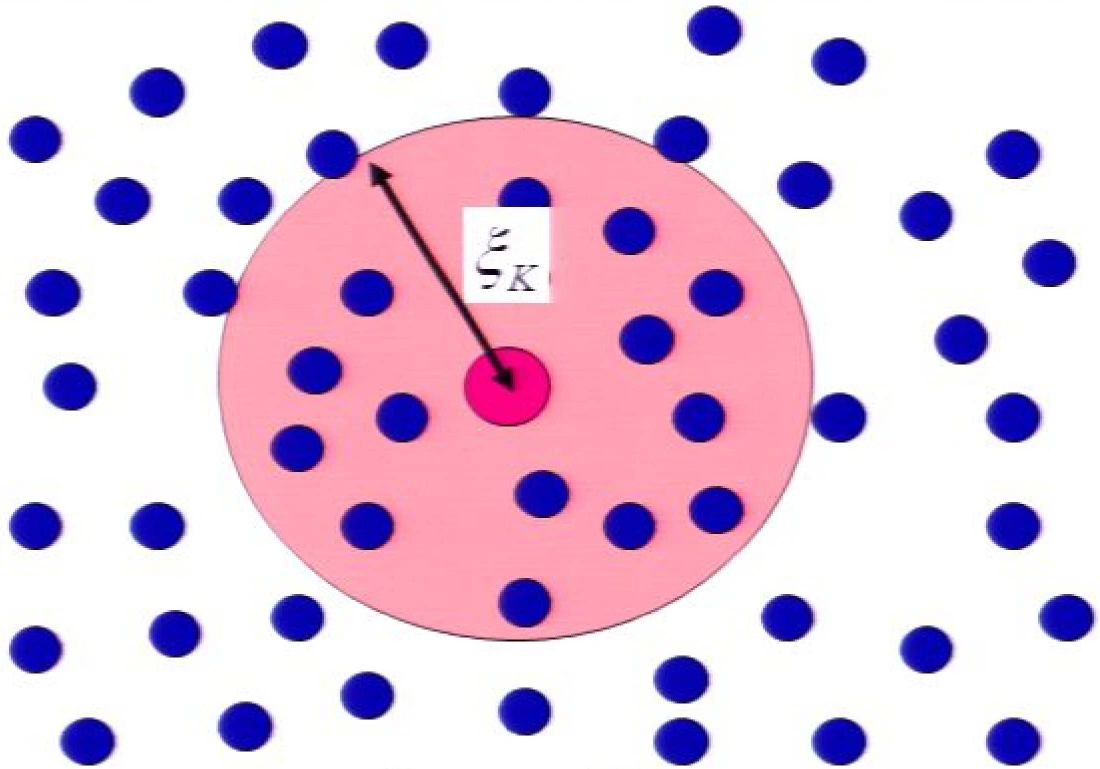


$$\xi_K \Leftrightarrow T_K$$

Kondo Physics



$$\xi_K \Leftrightarrow T_K$$

Despite the gapless nature of the Kondo system, we have a length scale in the model

Interesting Issues of the Kondo Physics

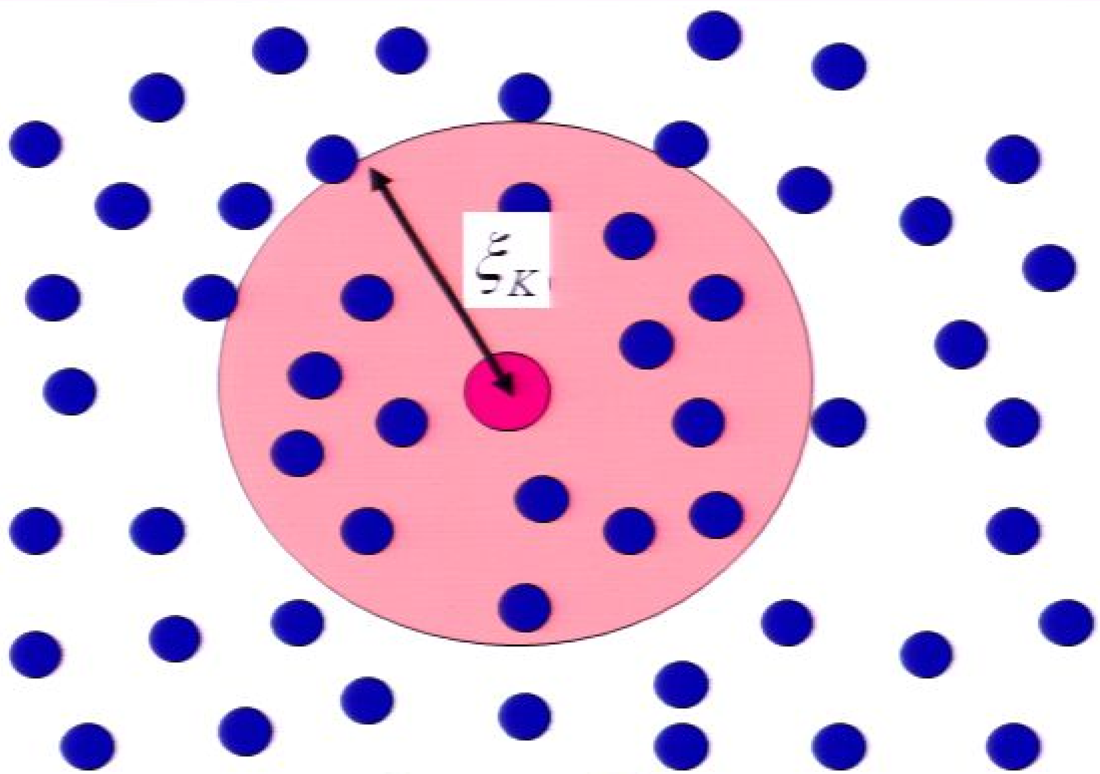
1- Size of the cloud

2- Scaling properties in terms of the Kondo length

3- Detecting the Kondo cloud

4- Physical properties (resistively, susceptibility) in the Kondo regime

Kondo Physics



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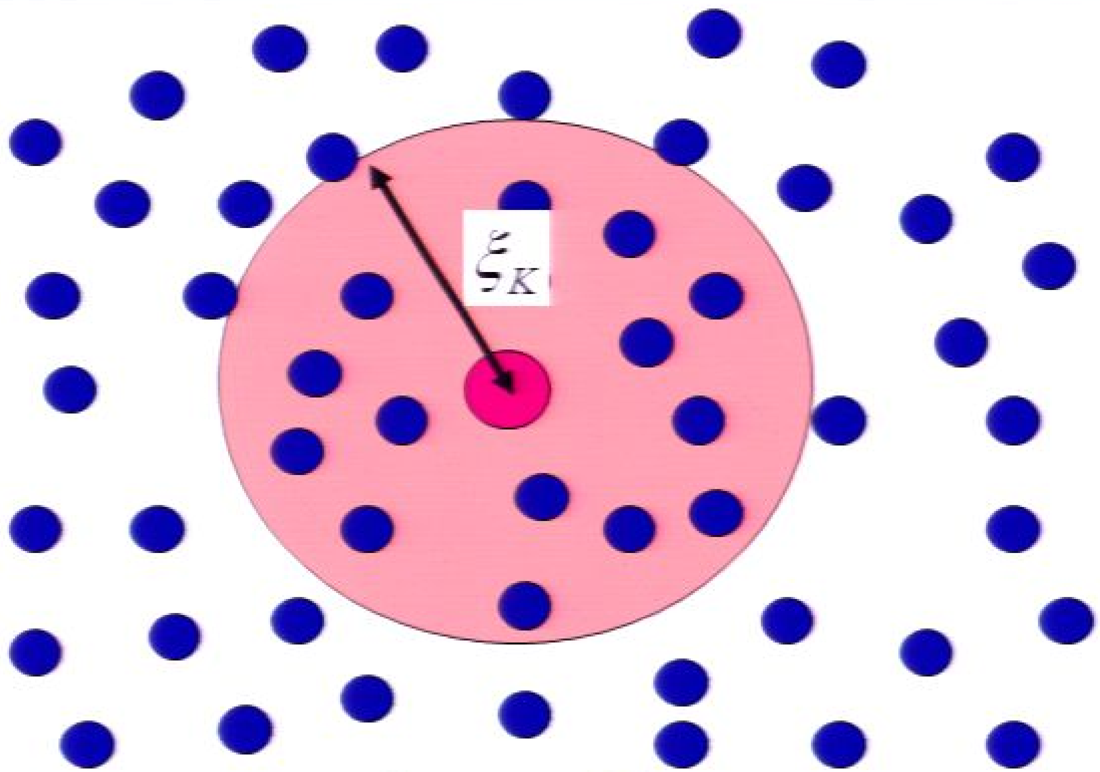

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Realization of the Kondo Effect

Semiconductor quantum dots

D. G. Gordon *et al.* Nature 391, 156 (1998).
S.M. Cronenwett, Science 281, 540 (1998).

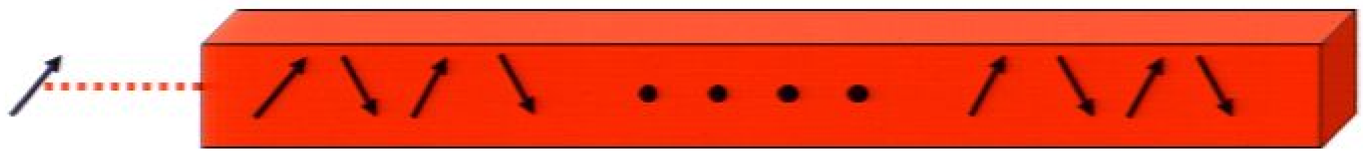
Carbon nanotubes

J. Nygard, *et al.* Nature 408, 342 (2000).
M. Buitelaar, Phys. Rev. Lett. 88, 156801 (2002).

Individual molecules

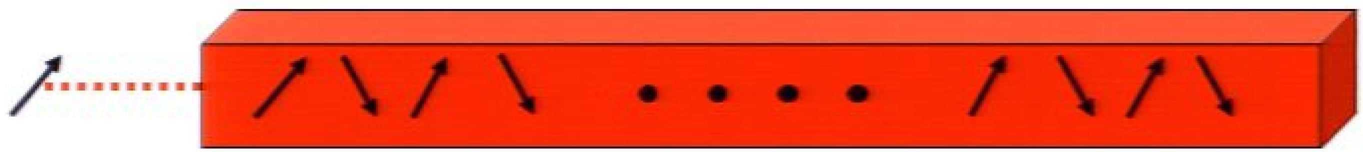
J. Park, *et al.* Nature 417, 722 (2002).
W. Liang, *et al*, Nature 417, 725–729 (2002).

Kondo Spin Chain



$$H = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i-1} + J_2\sigma_i.\sigma_{i-2}$$

Kondo Spin Chain



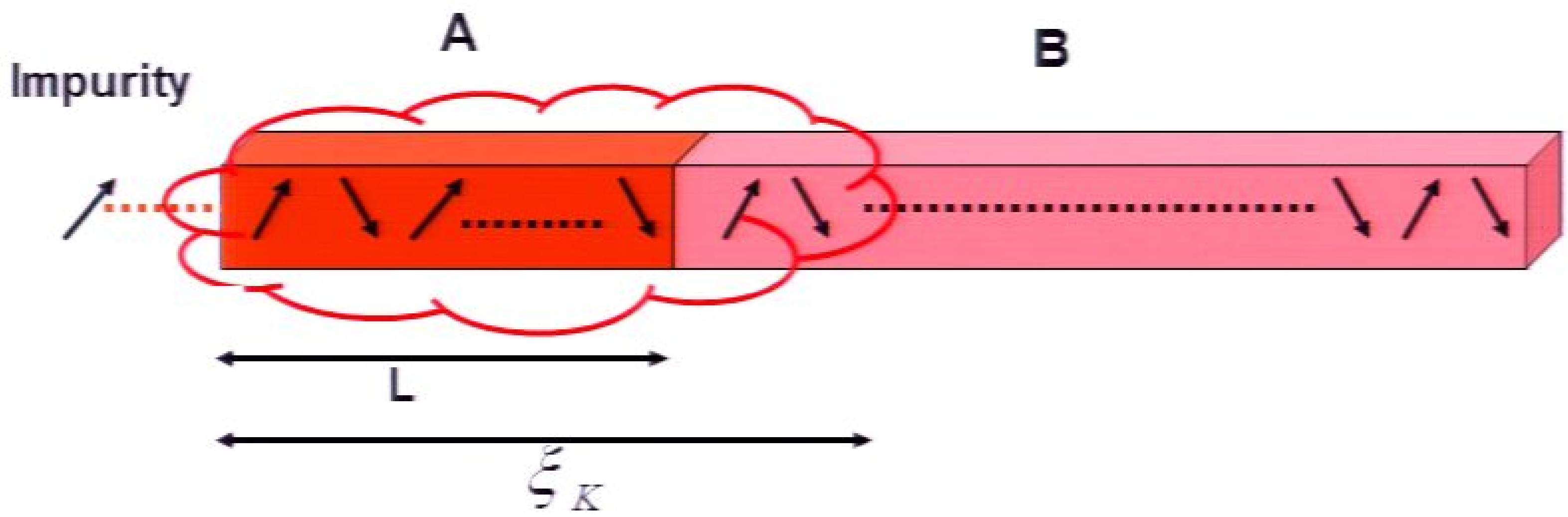
$$H = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i-1} + J_2\sigma_i.\sigma_{i-2}$$

$$\frac{J_2}{J_1} < J_2^c = 0.2412: \quad \text{Kondo (gapless)}$$

$$\frac{J_2}{J_1} > J_2^c: \quad \text{Dimer (gapfull)}$$

E. S. Sorensen *et al.*, J. Stat. Mech., P08003 (2007)

Entanglement as a Witness of the Cloud



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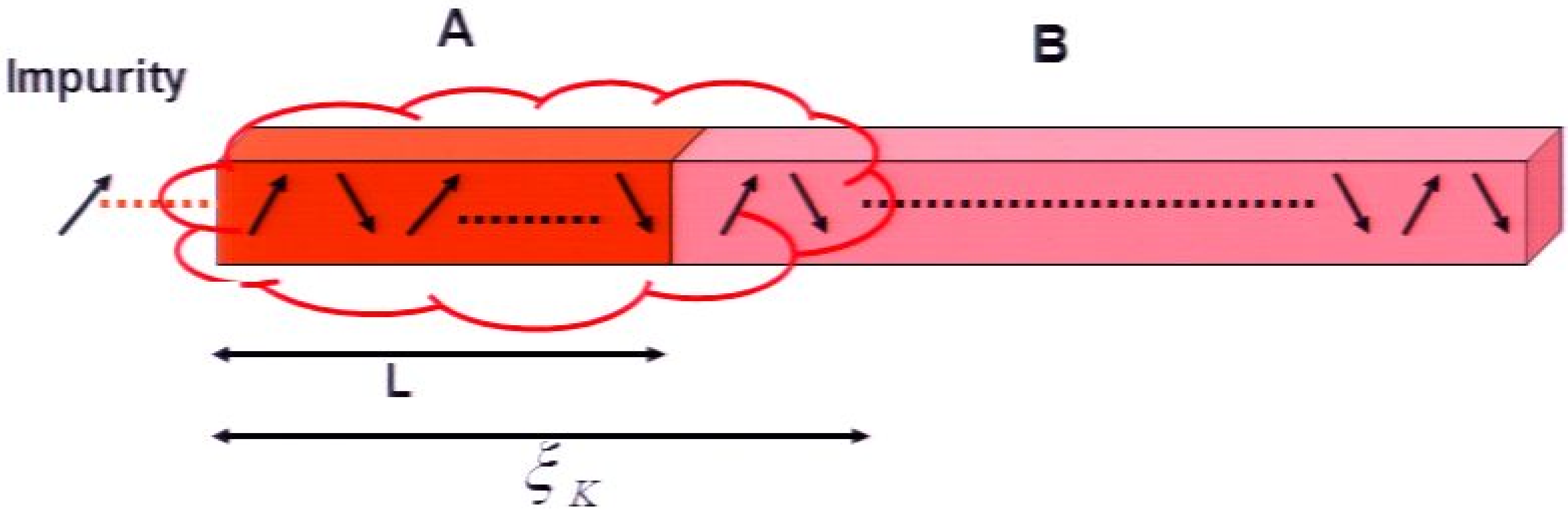
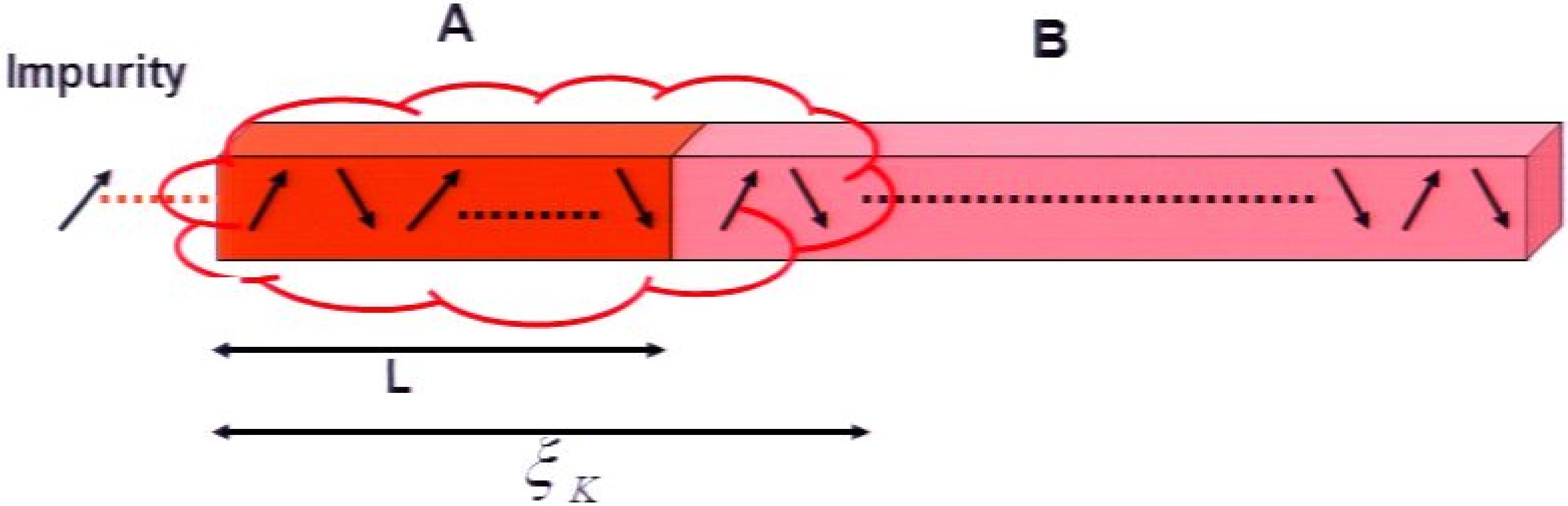


$$\begin{cases} L < \xi_K : \; E_{SA} < 1 \Rightarrow E_{SB} > 0 \\ \end{cases}$$

Entanglement as a Witness of the Cloud



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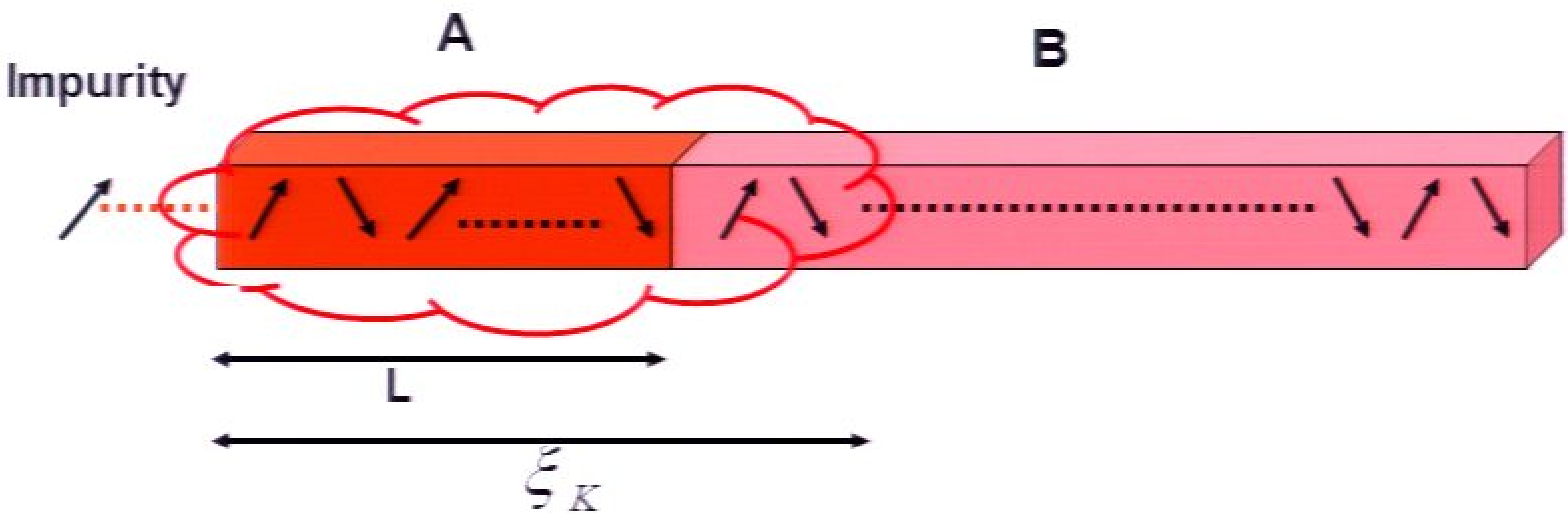

$$
\begin{cases}
L < \xi_K : \quad E_{SA} < 1 \Rightarrow E_{SB} > 0 \\
L = \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0 \\
L > \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0
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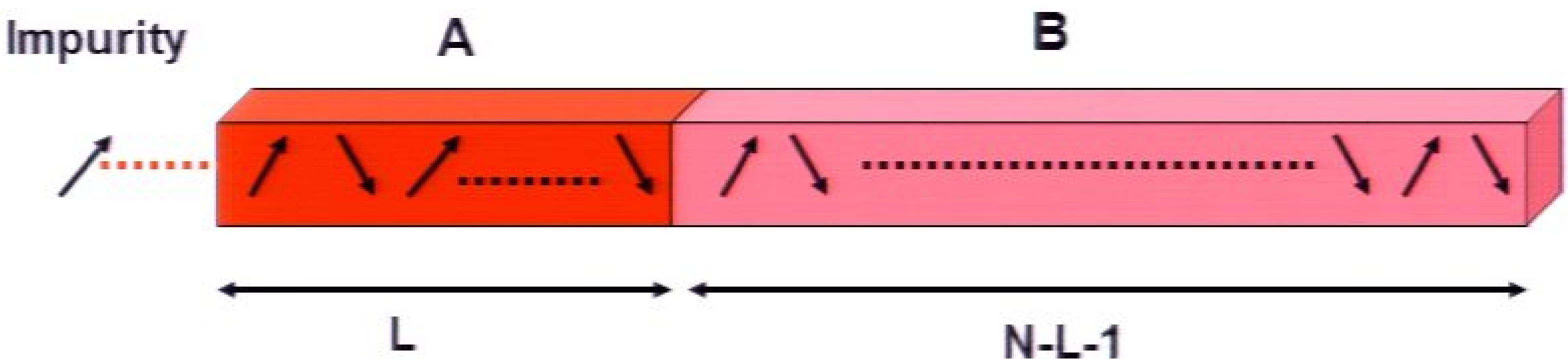
$$

Scaling



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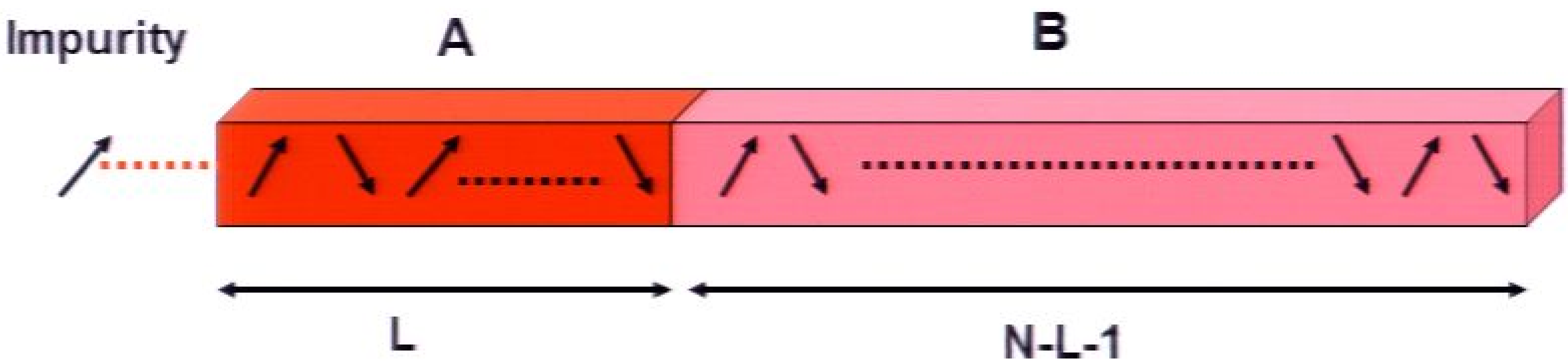
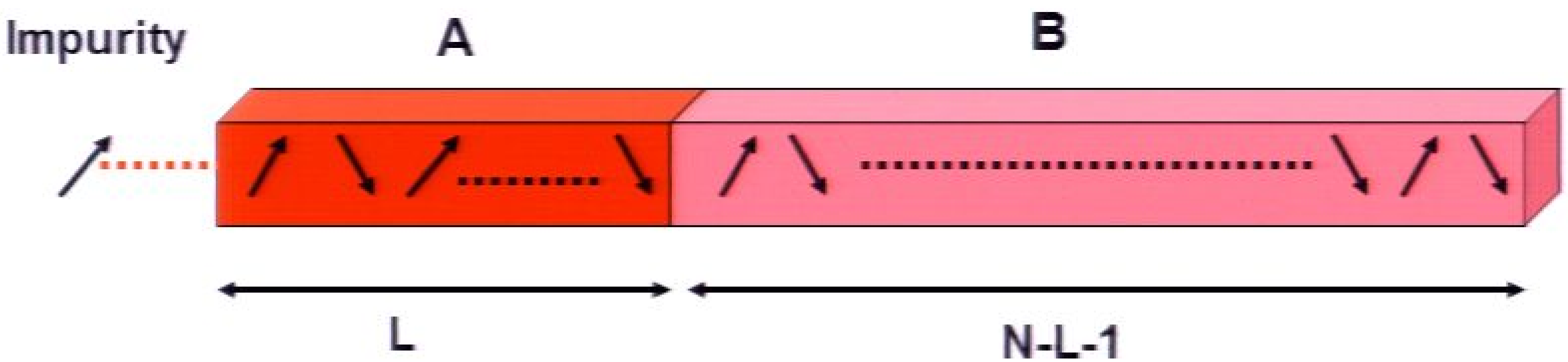


Kondo Phase: $E(L, \xi_K, N) = E(\frac{N}{\xi_K}, \frac{L}{N})$

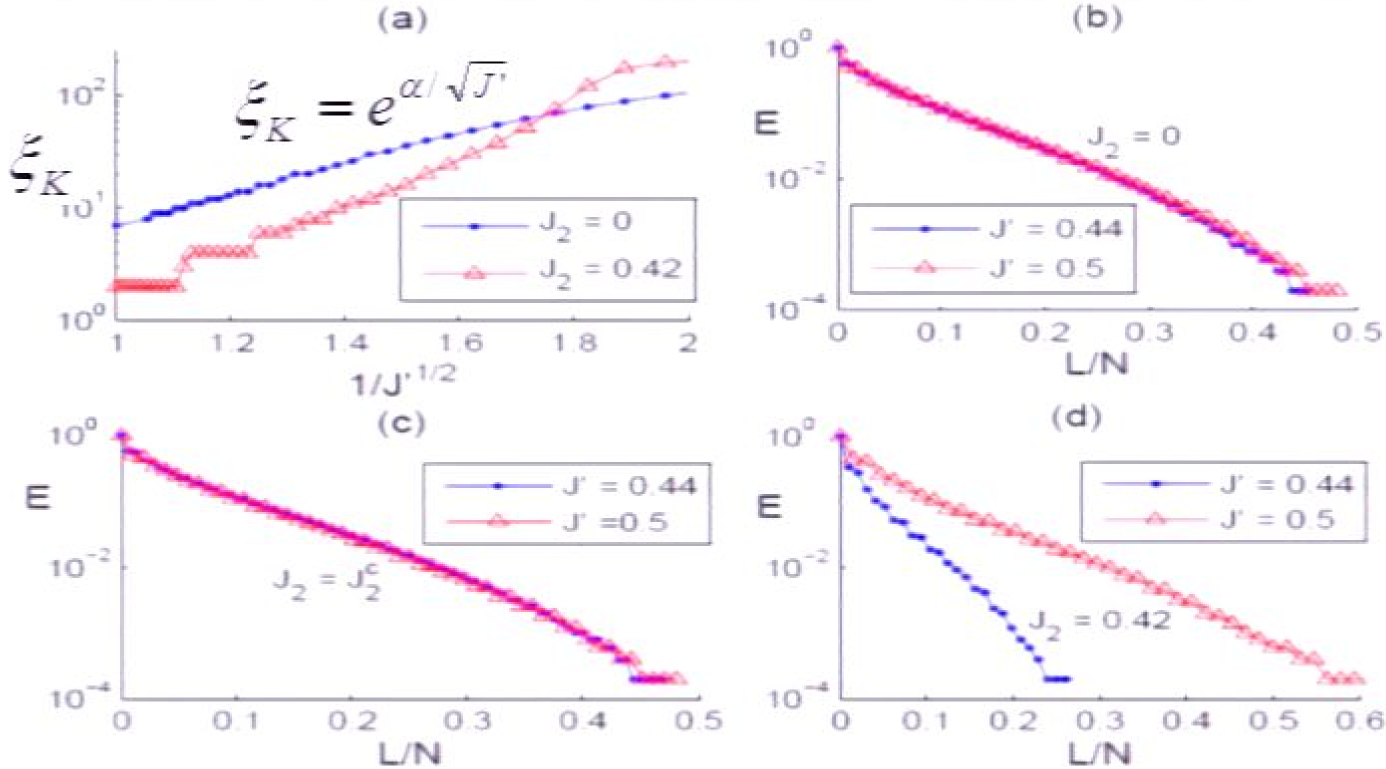
Scaling



Kondo Phase: $$E(L,\xi_K,N) = E(\frac{N}{\xi_K},\frac{L}{N})$$

Dimer Phase: $$E(L,\xi,N) \neq E(\frac{L}{\xi_K},\frac{N}{L})$$

Scaling Properties of the Kondo regime



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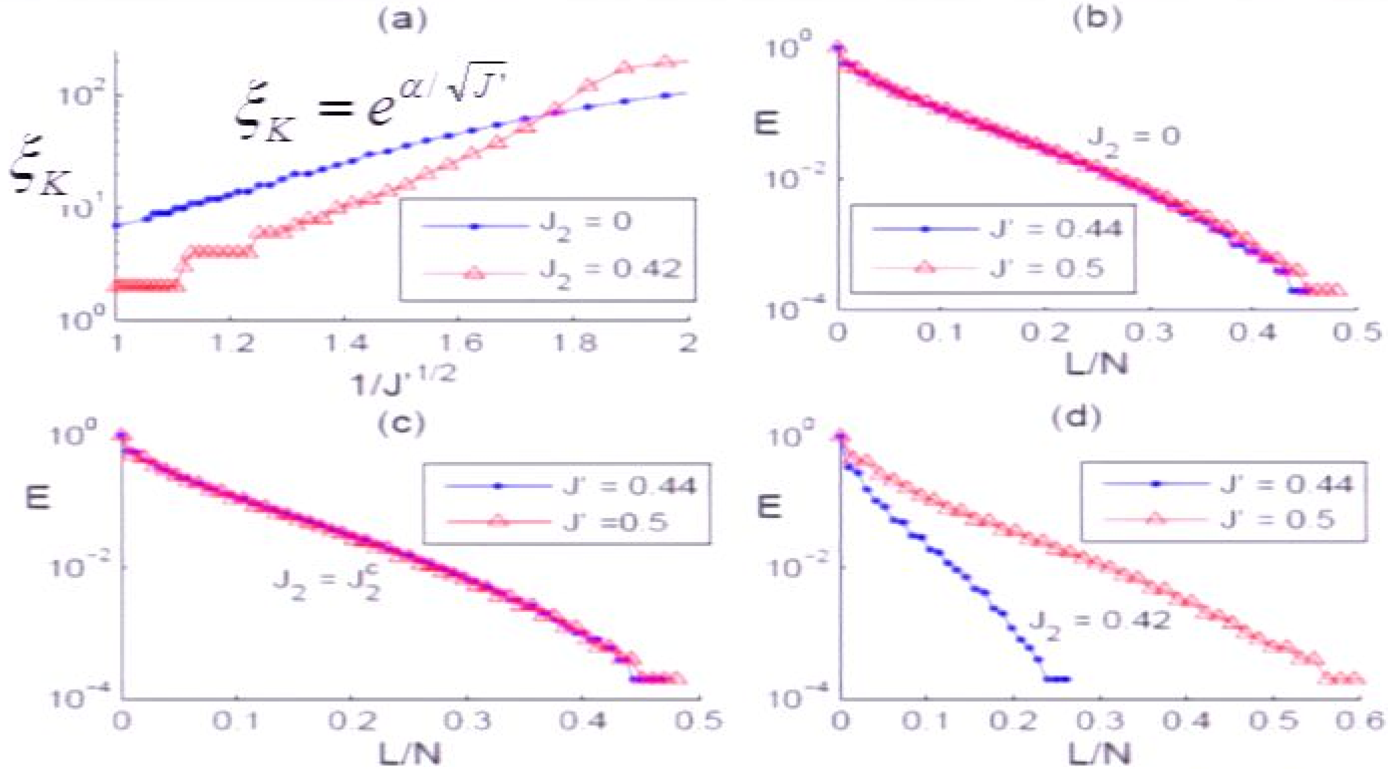
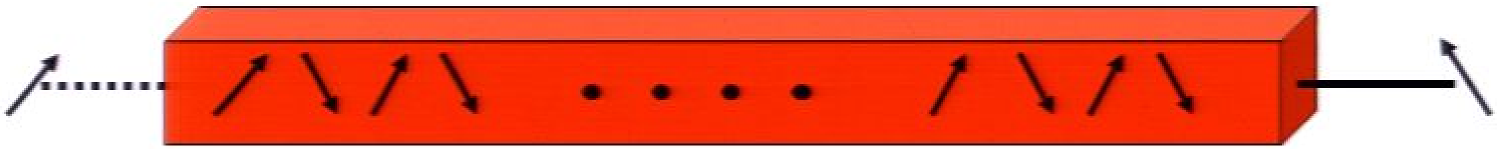


$$\frac{N}{\xi_K} = 4$$

Kondo Phase: $E(L, \xi_K, N) = E(\frac{N}{\xi_K}, \frac{L}{N})$

Dimer Phase: $E(L, \xi, N)$

Local Quench



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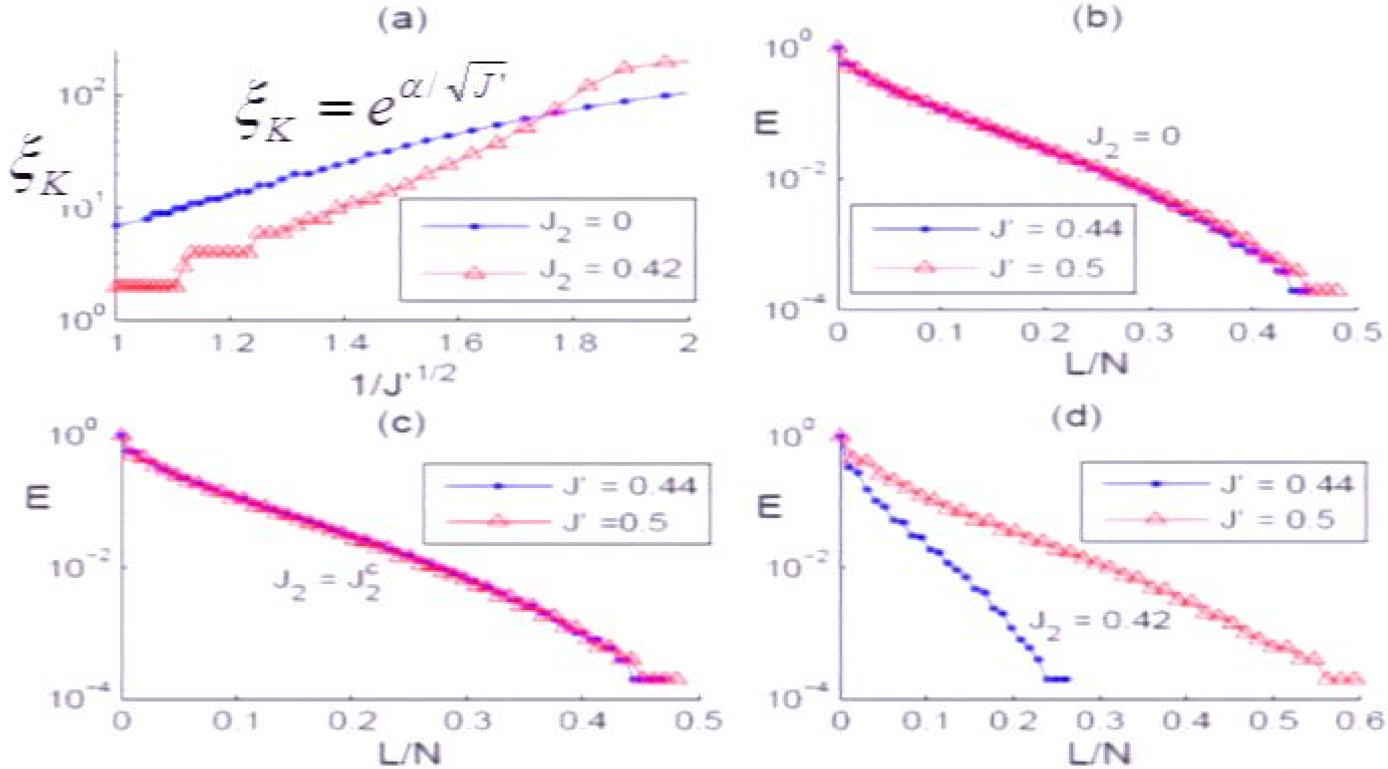

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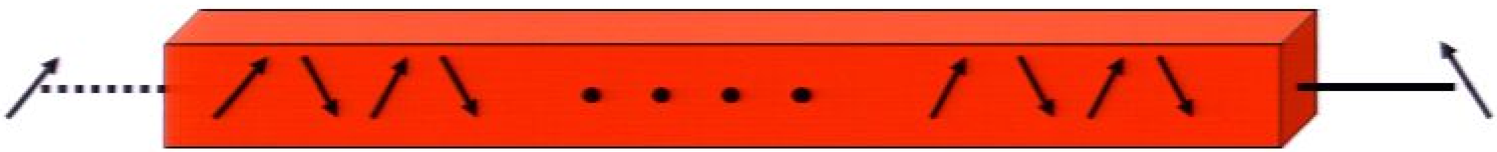

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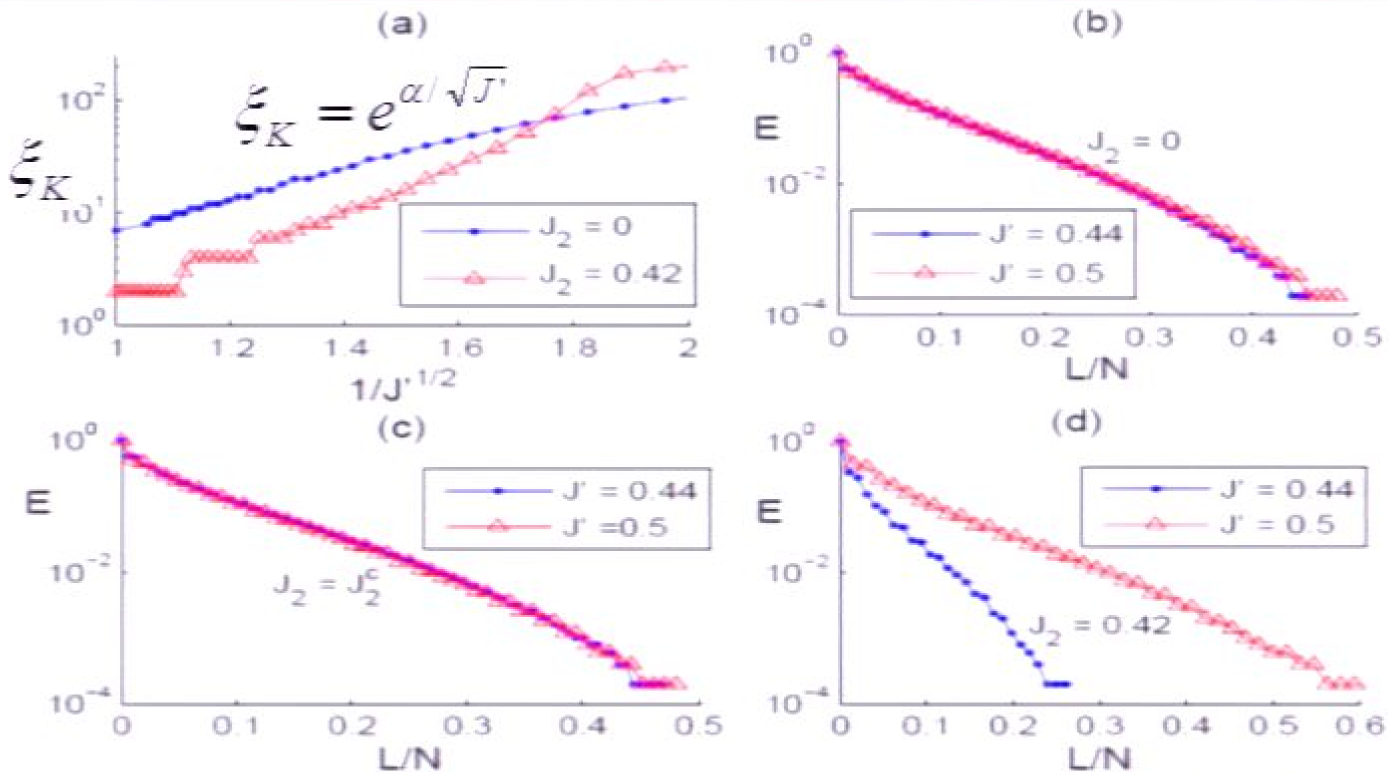

$$\frac{N}{\xi_K} = 4 \quad \begin{cases} \textbf{Kondo Phase:} & E(L, \xi_K, N) = E\left(\frac{N}{\xi_K}, \frac{L}{N}\right) \\ \textbf{Dimer Phase:} & E(L, \xi, N) \end{cases}$$

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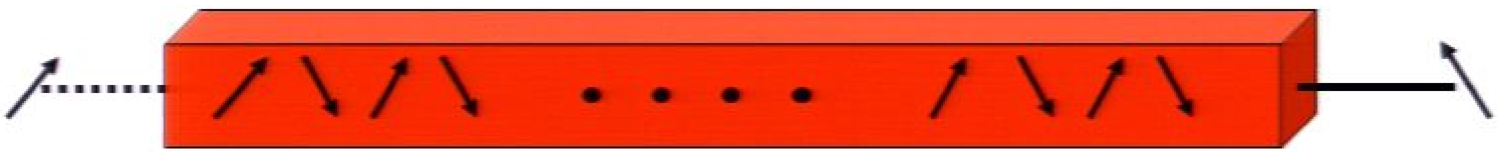

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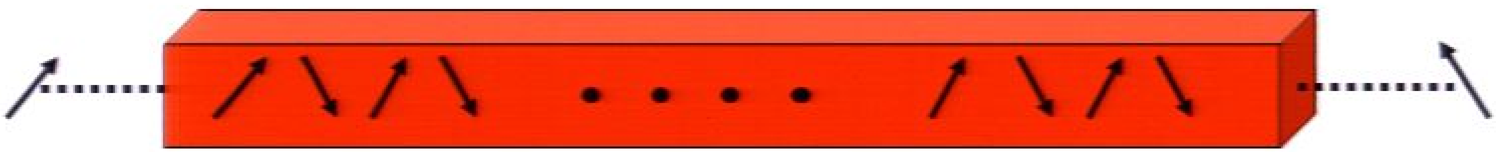

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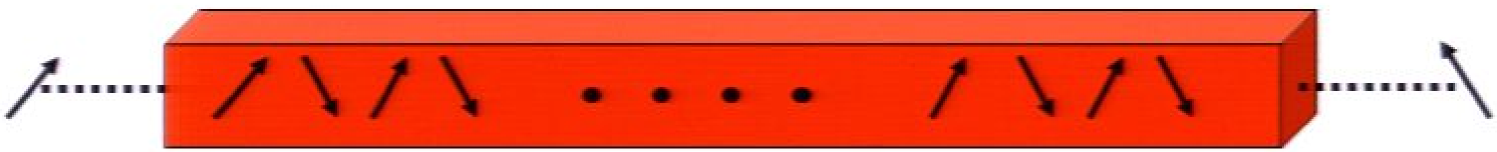
$$H_2 = J'(J_1\sigma_1.\sigma_2 + J_1\sigma_{N-1}.\sigma_N + J_2\sigma_1.\sigma_3 + J_2\sigma_{N-2}.\sigma_N) + \sum_{i} J_1\sigma_i.\sigma_{i-1} + J_2\sigma_i.\sigma_{i-2}$$

Local Quench



$$H_1 = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i-1} + J_2\sigma_i.\sigma_{i-2}$$

Quench ↓

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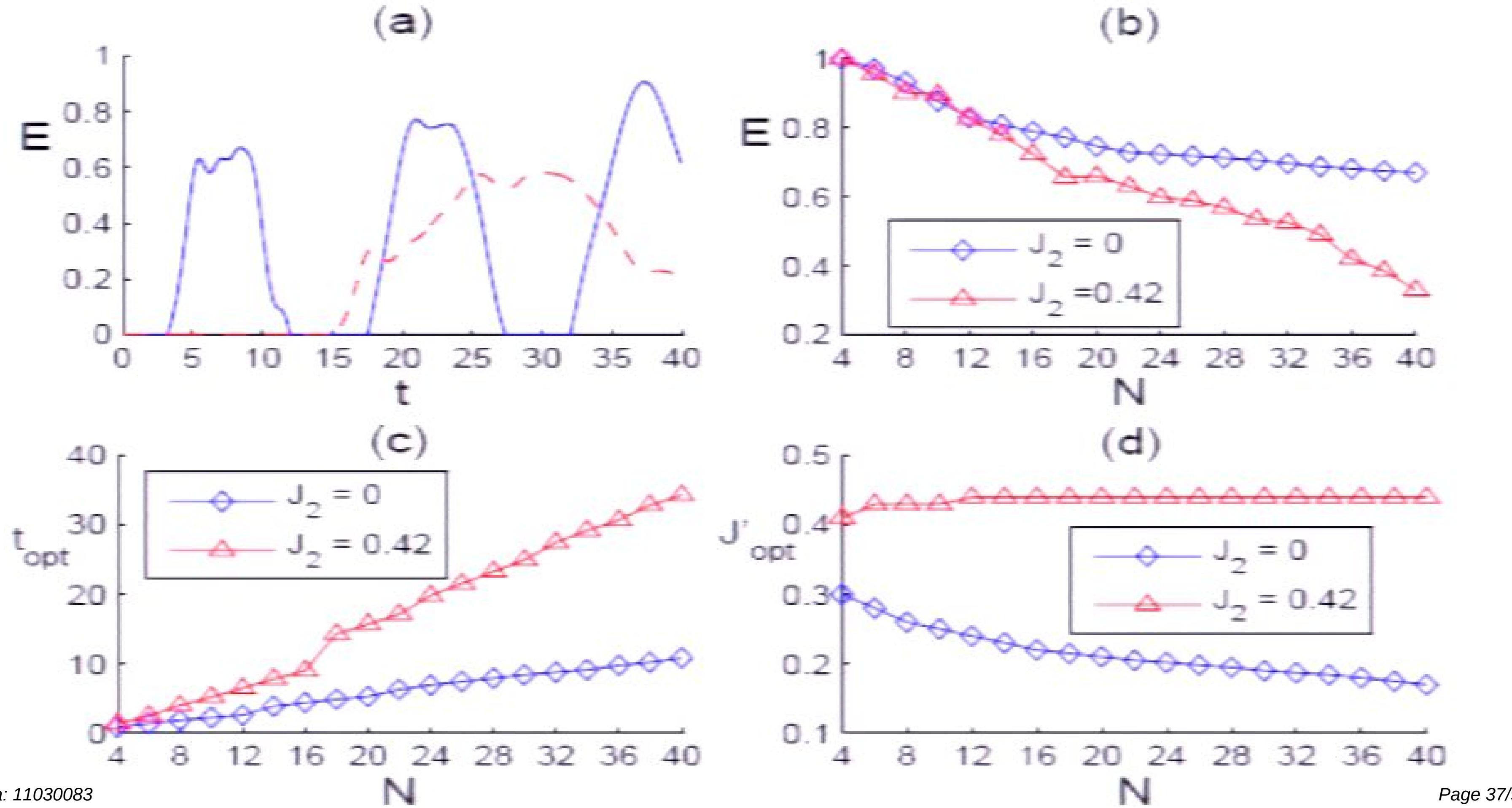
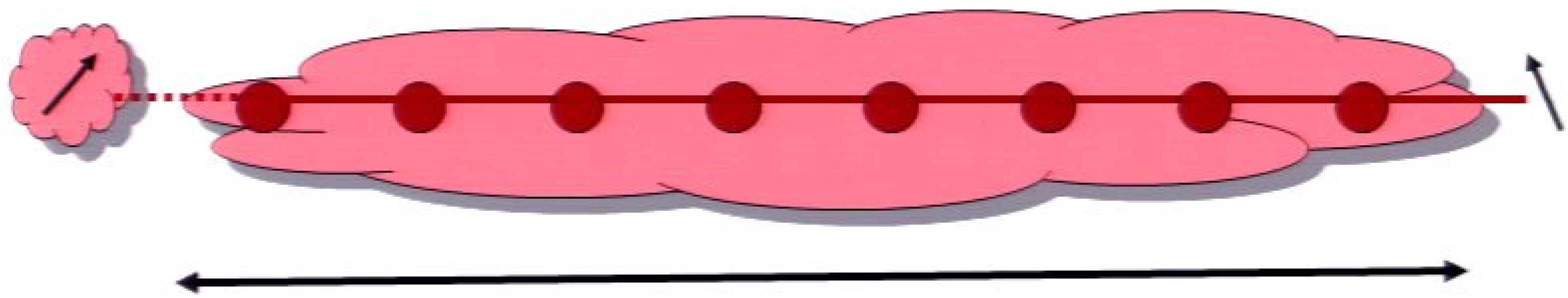
$$|\psi(0)\rangle = |GS_{H1}\rangle$$

$$|\psi(t)\rangle = e^{-iH_2t}|GS_{H1}\rangle \rightarrow \rho_{1N}(t) \rightarrow E_{1N}(t)$$

Kondo versus Dimer



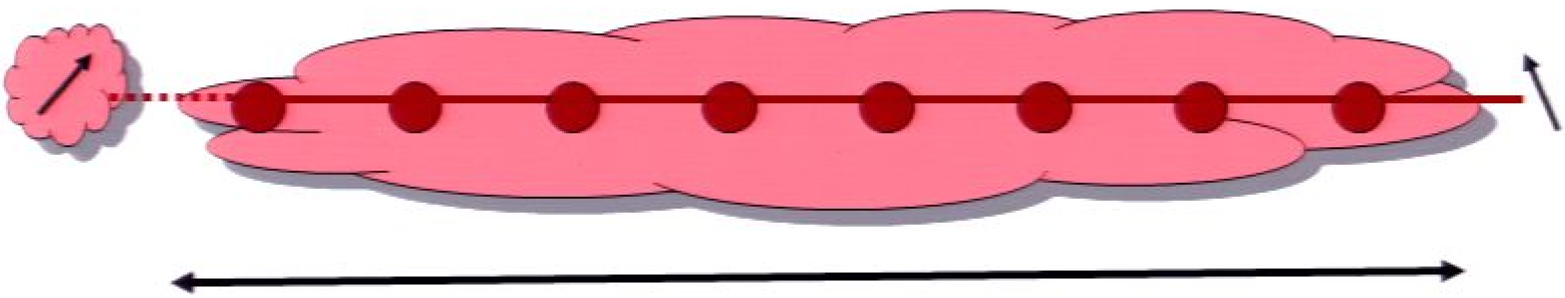
Optimality and Distance independence



$$\xi_K(J'_{opt}) = N - 2$$

Independent of length N, when cloud contains N-2 spins we generate a constant Entanglement

Optimality and Distance independence



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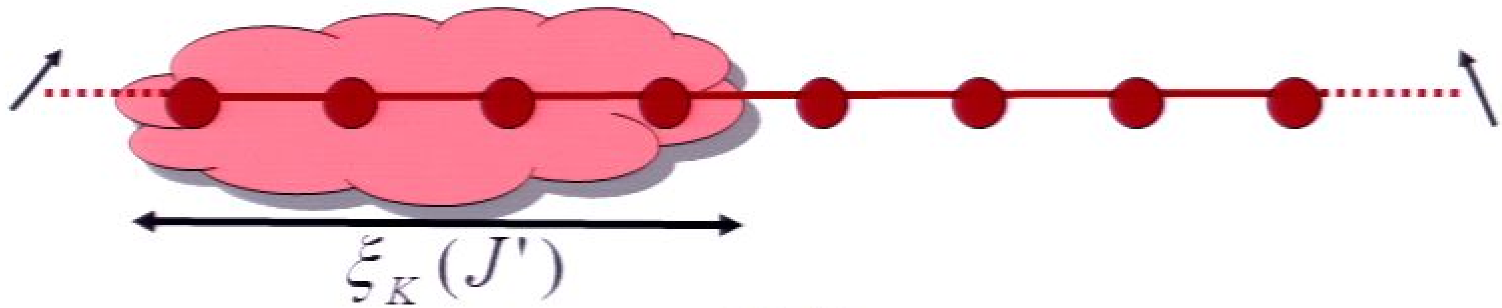
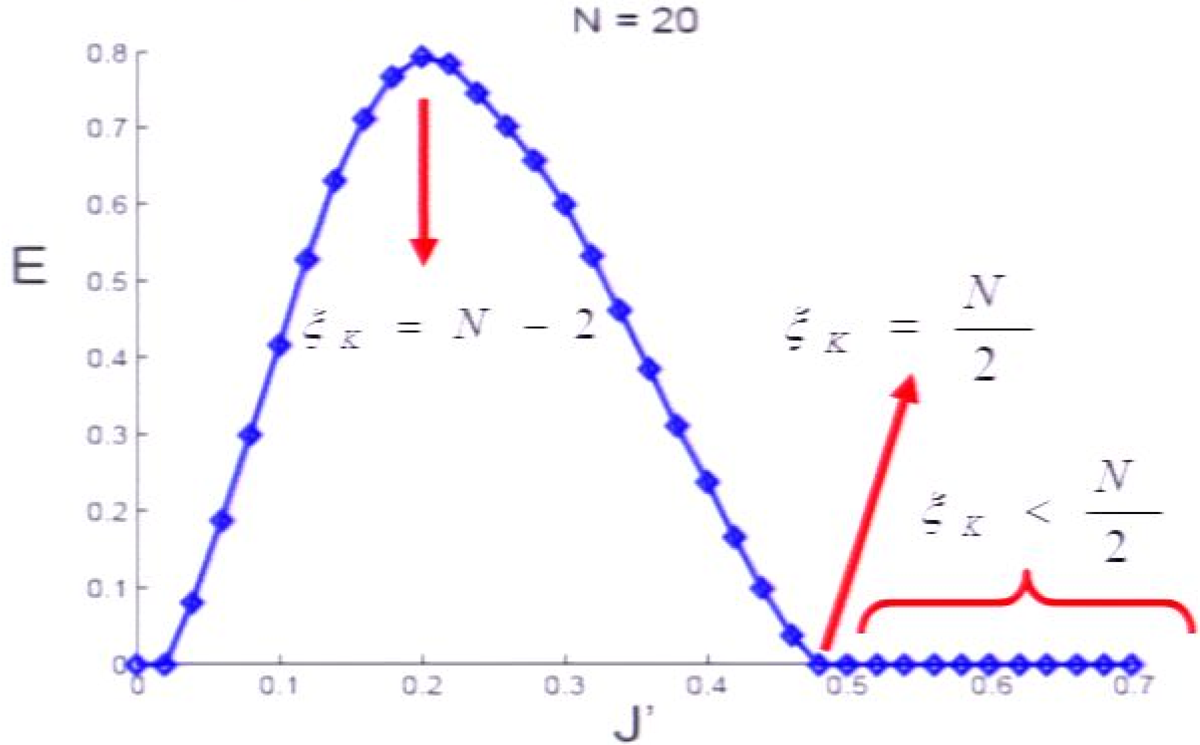
Optimal Parameter





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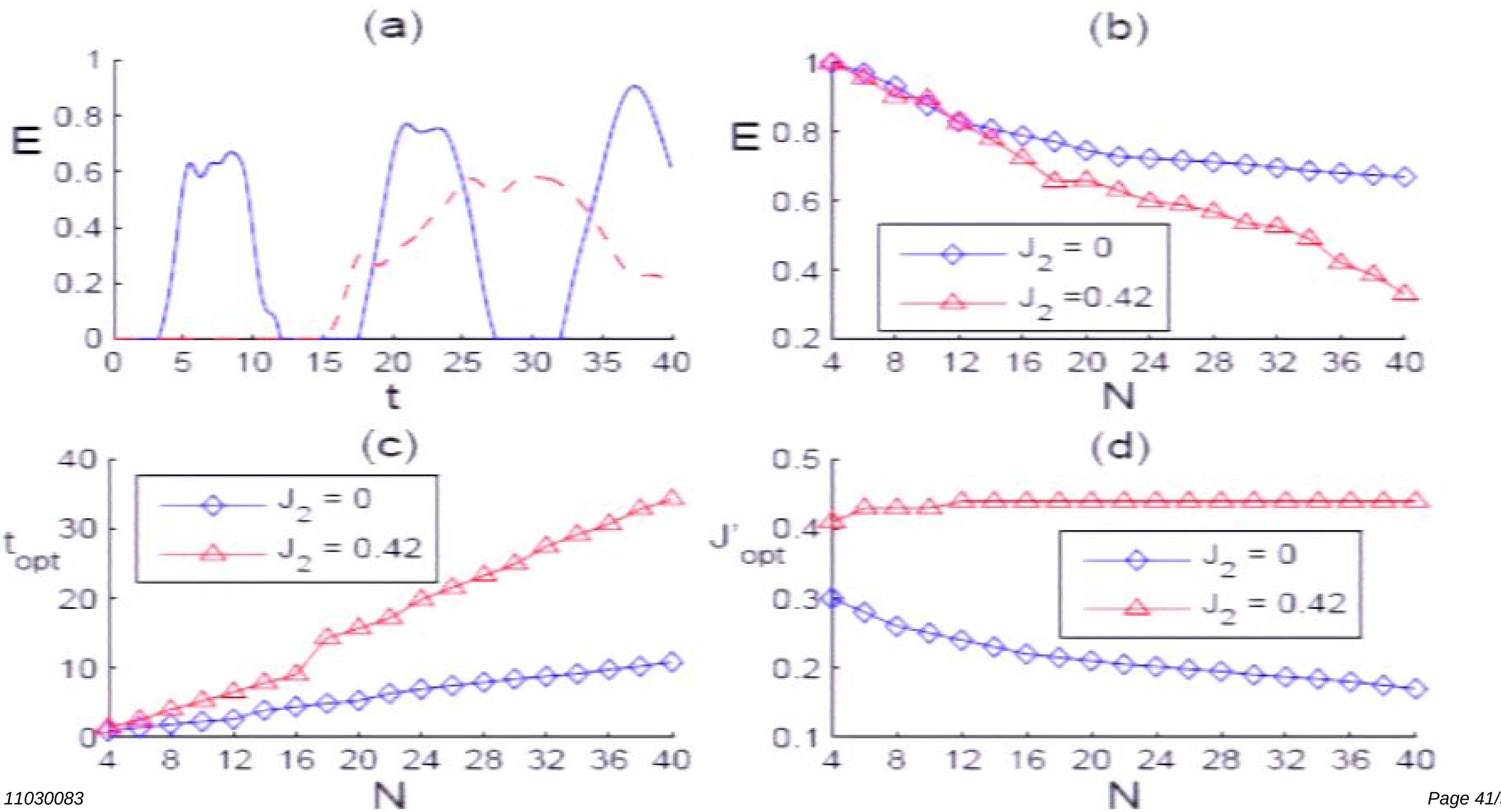

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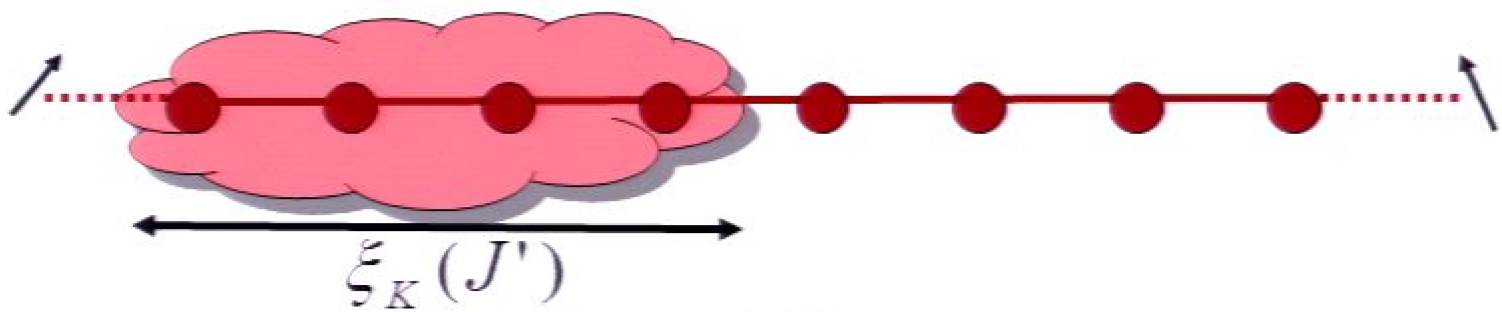

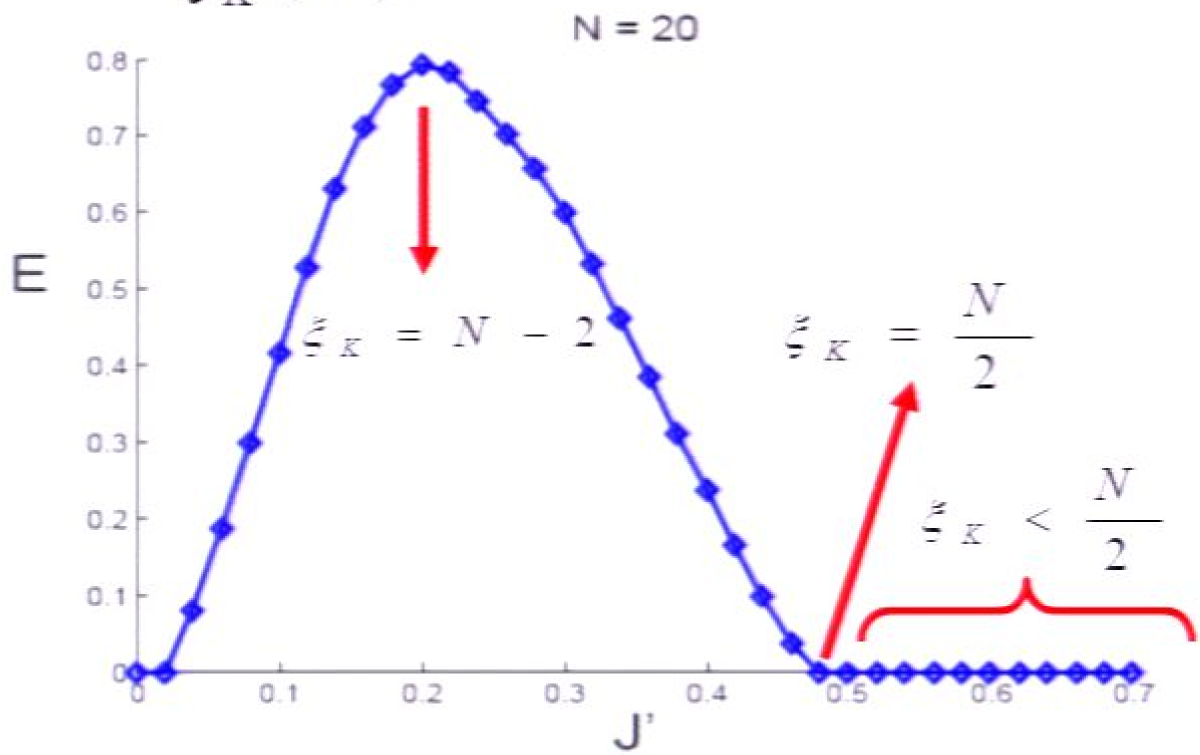

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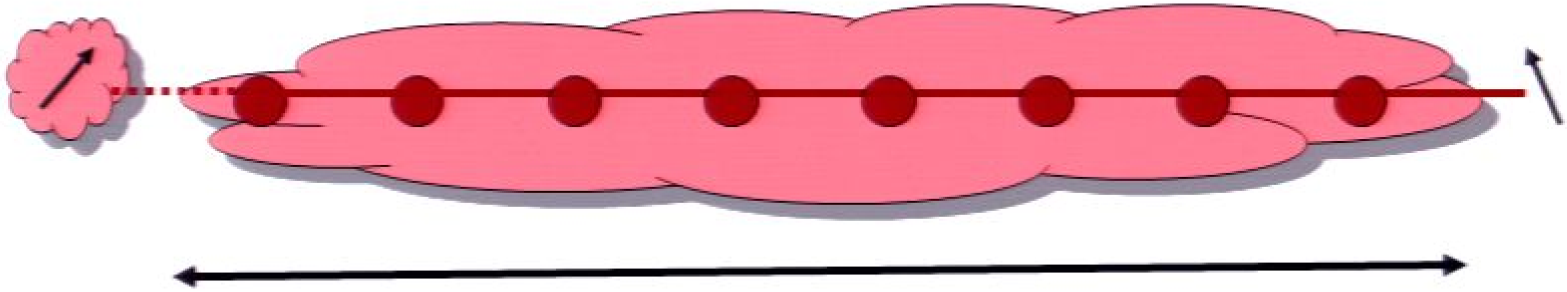

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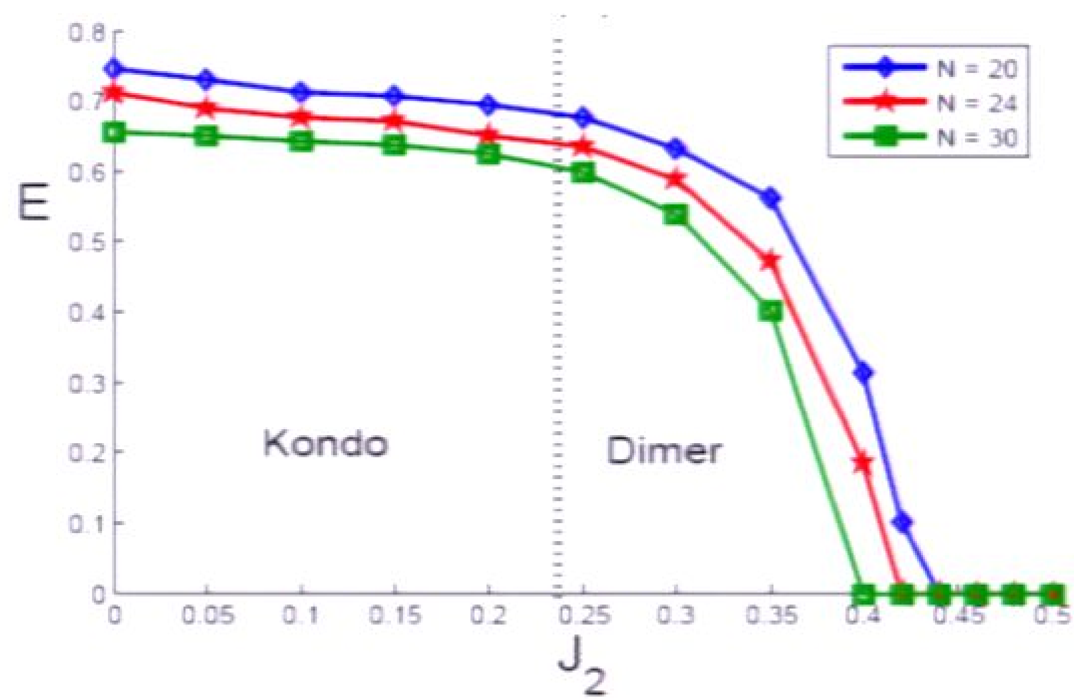
Entanglement in Whole Phase Diagram



Entanglement drops in the dimer regime

Mechanism of Entanglement Generation

$$H_1 : \qquad |\psi(0)\rangle = |GS_1\rangle$$

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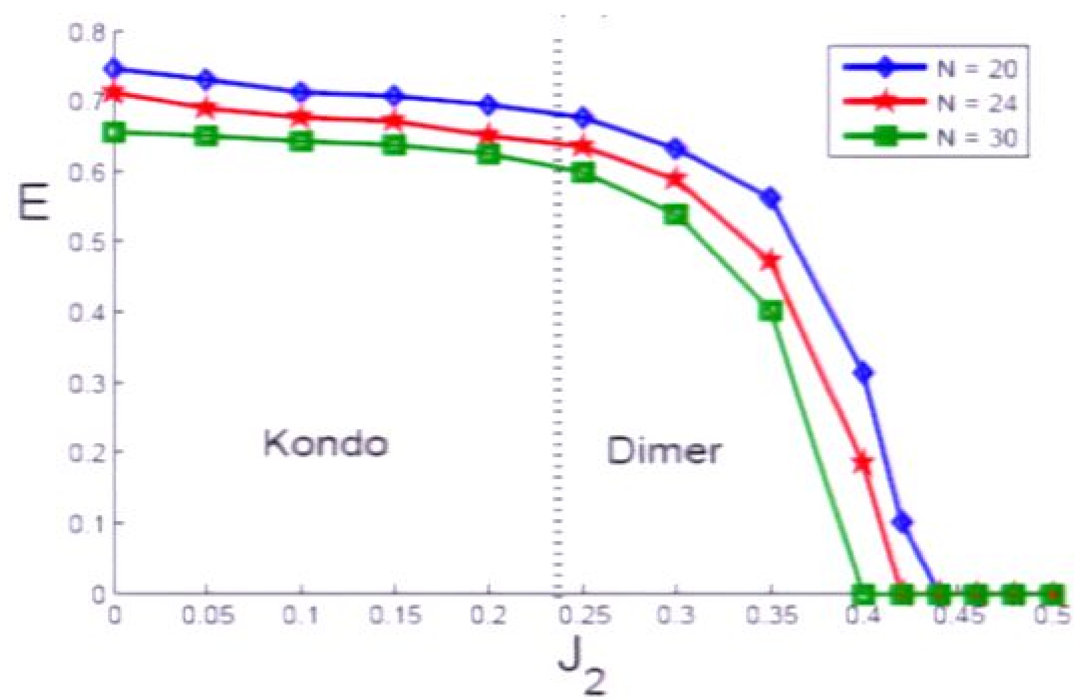

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$$H_1 \to H_2 : \ \{E_i, |E_i\rangle\}$$

$$|\psi(t)\rangle = \sum_i e^{-iE_i t} \langle E_i | GS_1 \rangle |E_i\rangle$$

- Kondo Phase: ***Only two*** states dominantly involve in the dynamics
- Dimer Phase: ***Many*** states involve in the dynamics

Dynamics in the Kondo Regime

$$|\psi(t)\rangle = \sum_{i=1,2} e^{-iE_i t} \langle E_i | GS_I \rangle | E_i \rangle$$

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By Quenching a single bond we release some energy into the system:

$$\delta E \propto 1 - J'$$

[illegible]

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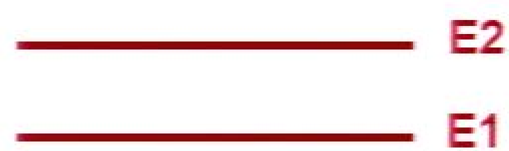
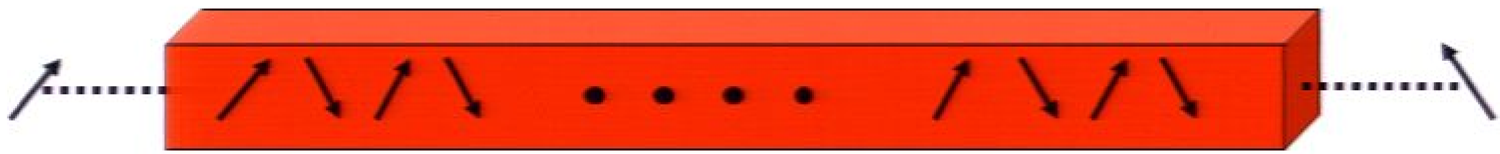
Energy separation between E1 and E2: $\Delta E(J')$



Optimal quench: $\delta E \propto \Delta E$

Static Entanglement



Static strategy creates high amount of entanglement in perturbative regime (J'→0).

$$J' \propto \frac{\varepsilon}{\sqrt{N}} \quad \Rightarrow \quad E \to 1$$

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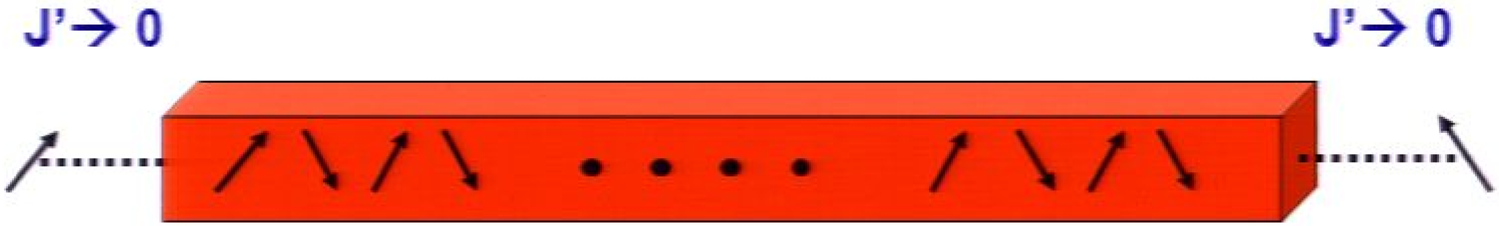

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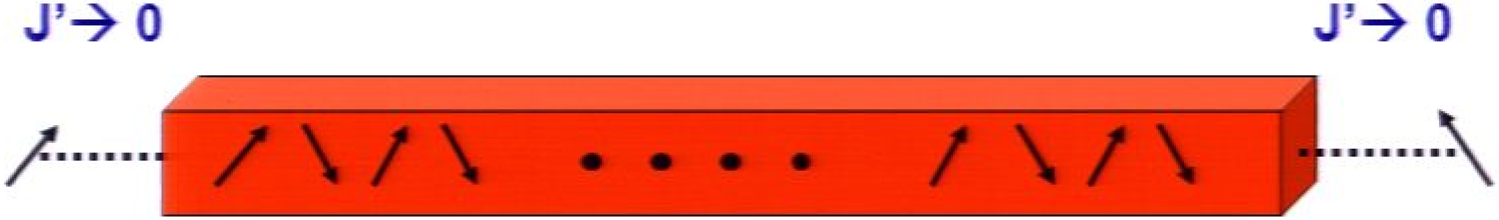

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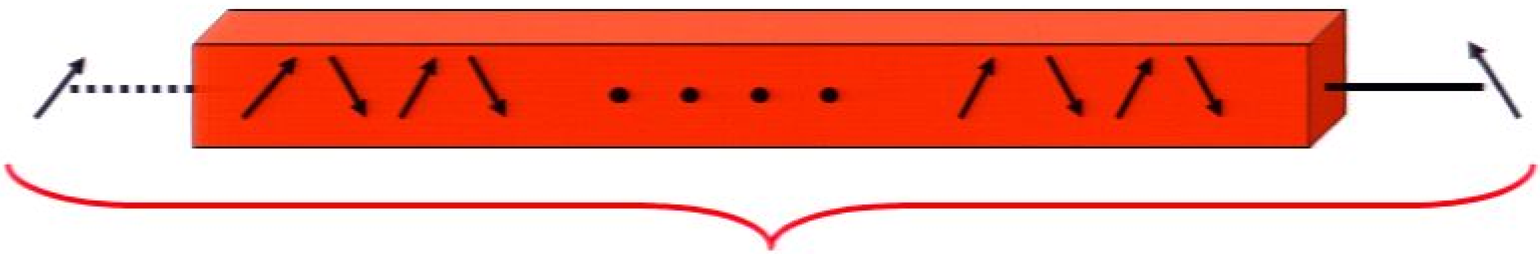
Due to the vanishing gap this entanglement is highly unstable to thermal fluctuations.

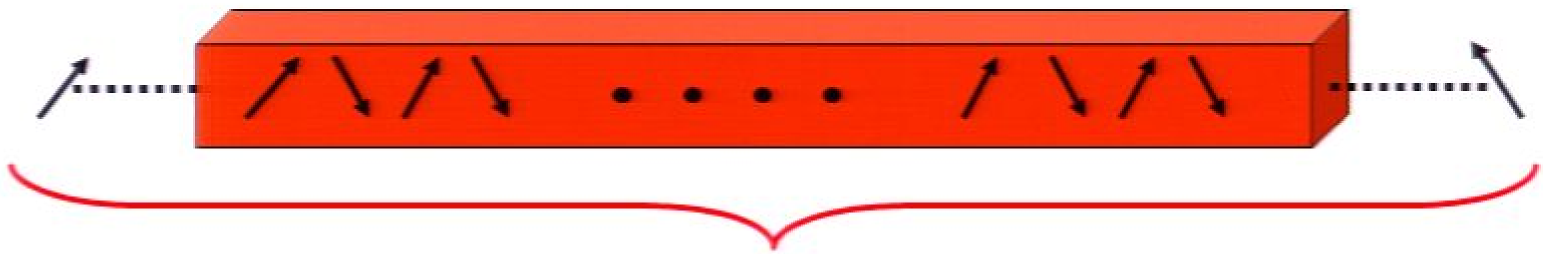
$$\Delta = J'^2 = \frac{\varepsilon^2}{N} \quad \Rightarrow \quad KT < \Delta = \frac{\varepsilon^2}{N}$$

Dynamical Entanglement



$$\rho_{th} = \frac{e^{-\beta H_1}}{Z}$$

Dynamical Entanglement



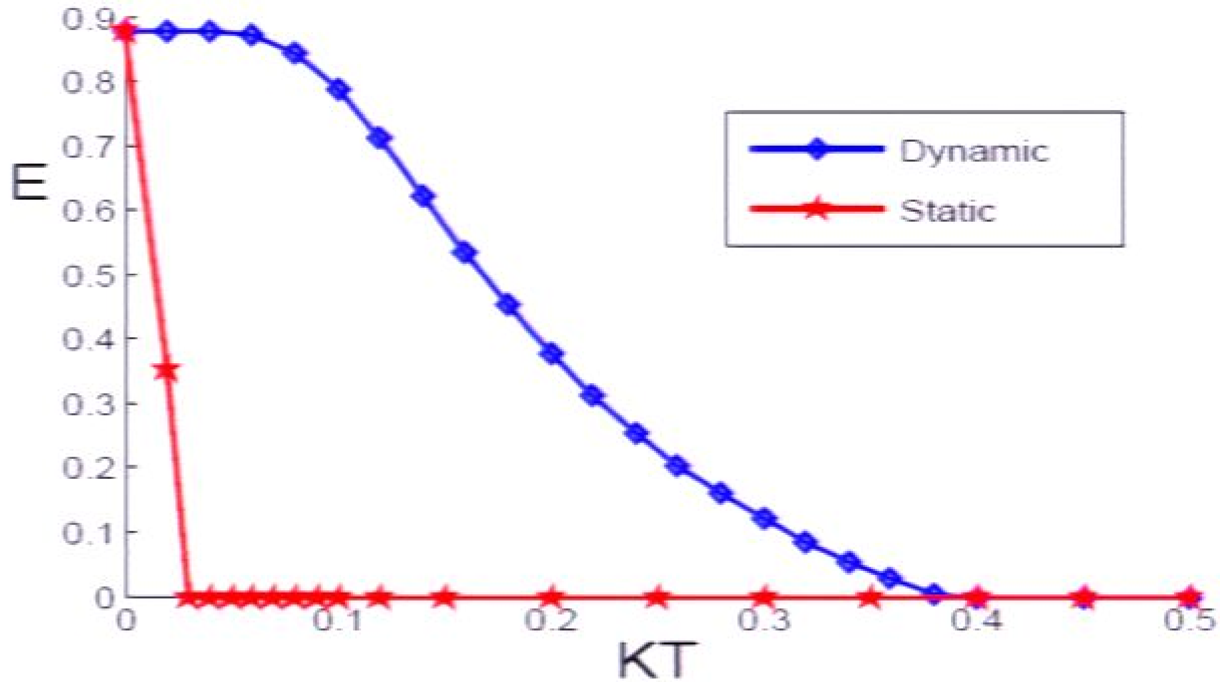
$$\rho_{th} = \frac{e^{-\beta H_1}}{Z}$$

$$H_1 \rightarrow H_2 \Rightarrow \rho(t) = e^{-iH_2 t} \rho_{th} e^{-iH_2 t} \Rightarrow \rho_{1N}(t) \Rightarrow E_{1N}(t)$$

Thermal stability:

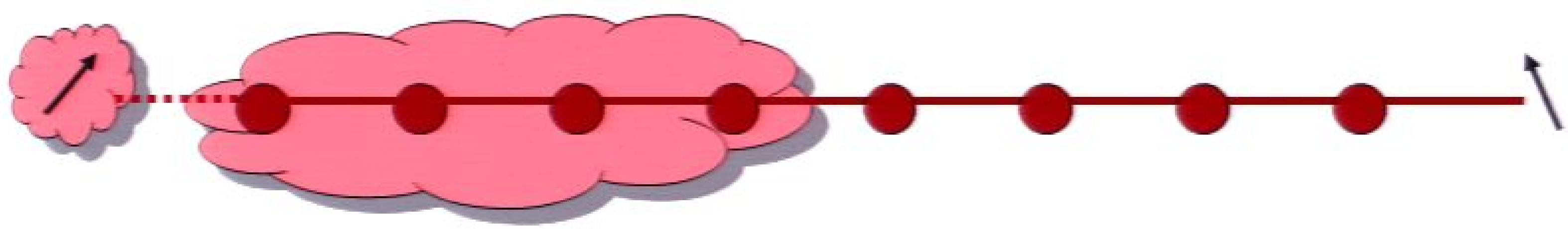
$$KT_K = \frac{1}{\xi} = \frac{1}{N-2} \Rightarrow KT < KT_K = \frac{1}{N-2}$$

Thermal Effect on the Entanglement

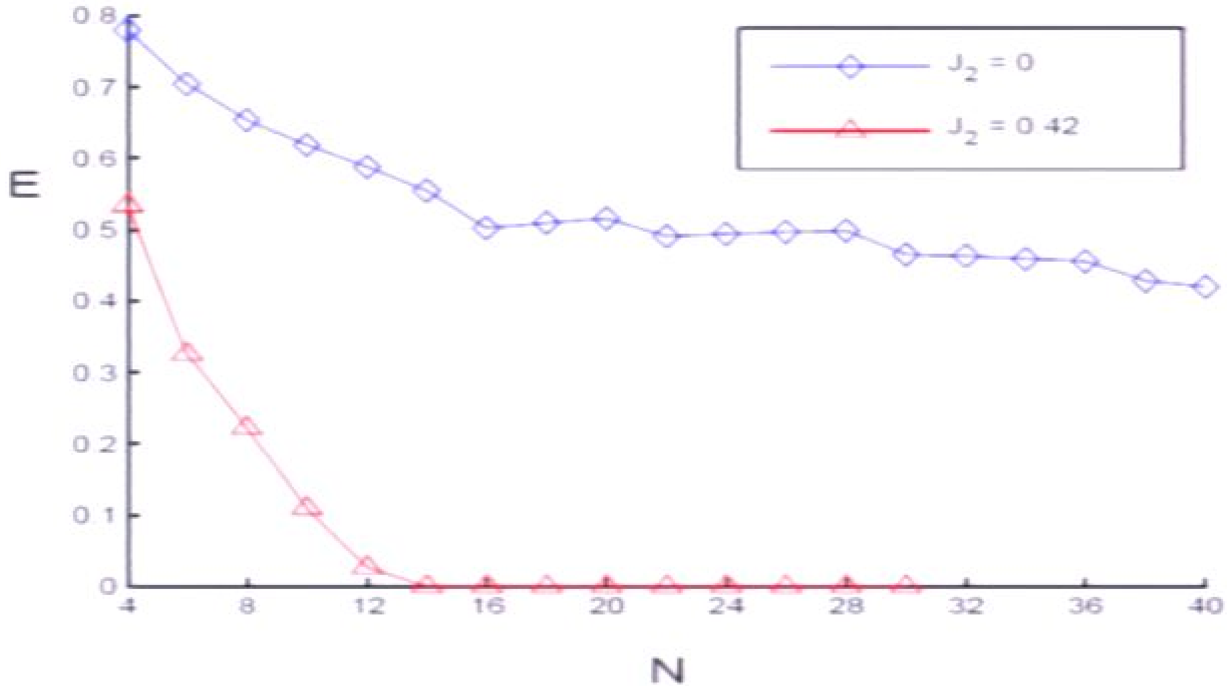


Dynamical strategy for creating entanglement is more resistive than the static one.

A test for end-to-end effects



Entanglement



Improvement?

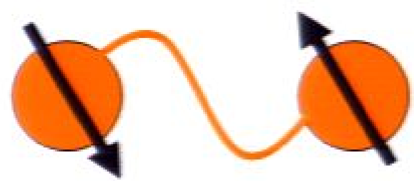
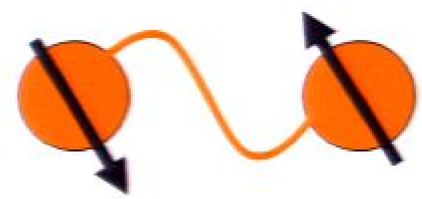
Is there a way to improve the strategy?

1) Higher entanglement

2) A way to route entanglement

Two Spin Singlets

$J'_L \vec{\sigma}_1 . \vec{\sigma}_2$ $J'_R \vec{\sigma}_3 . \vec{\sigma}_4$

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Two Spin Singlets

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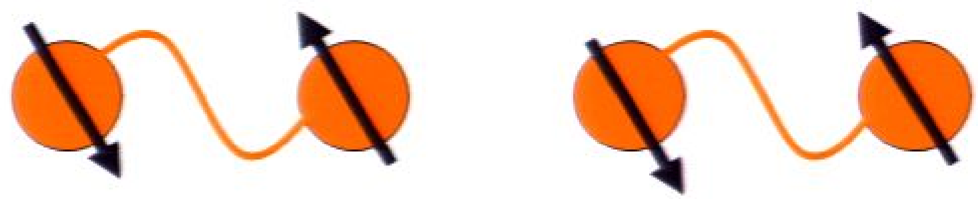

$$\left|\psi(0)\right\rangle = \left|\psi^-\right\rangle \otimes \left|\psi^-\right\rangle$$

Two Spin Singlets

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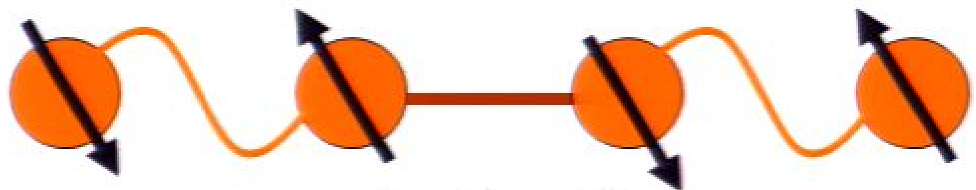
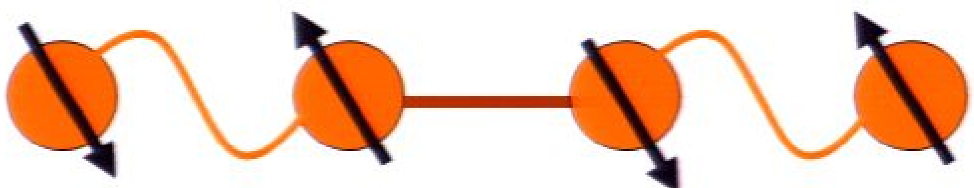


$$J_m \vec{\sigma}_2.\vec{\sigma}_3$$

$$\left|\psi(0)\right\rangle = \left|\psi^-\right\rangle \otimes \left|\psi^-\right\rangle$$

Two Spin Singlets

$$J'_L \, \vec{\sigma}_1 . \vec{\sigma}_2 \qquad\qquad J'_R \, \vec{\sigma}_3 . \vec{\sigma}_4$$



$$J_m \vec{\sigma}_2 . \vec{\sigma}_3$$

$$\left|\psi(0)\right\rangle = \left|\psi^-\right\rangle \otimes \left|\psi^-\right\rangle$$

$$\left|\psi(t)\right\rangle = e^{-iHt}\left|\psi(0)\right\rangle$$

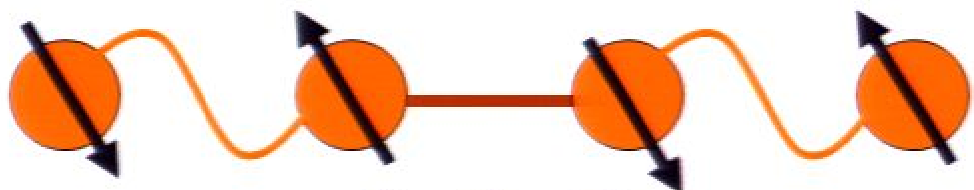
$$J_m = J'_L + J'_R \quad \Rightarrow \quad E(t) = \max\{0, \frac{1-3\cos(4J_m t)}{4}\}$$

Goal

Can we find some many-body singlets (extended singlets) which play the same role?

Two Spin Singlets

$$J'_L\,\vec{\sigma}_1.\vec{\sigma}_2 \qquad J'_R\,\vec{\sigma}_3.\vec{\sigma}_4$$



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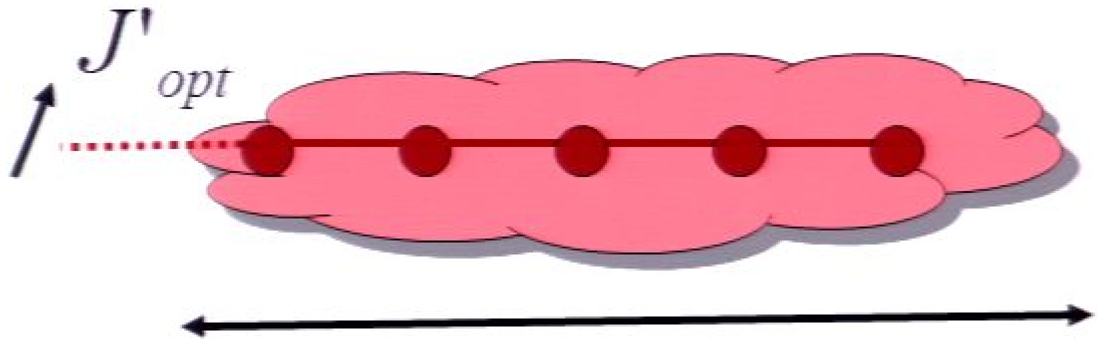
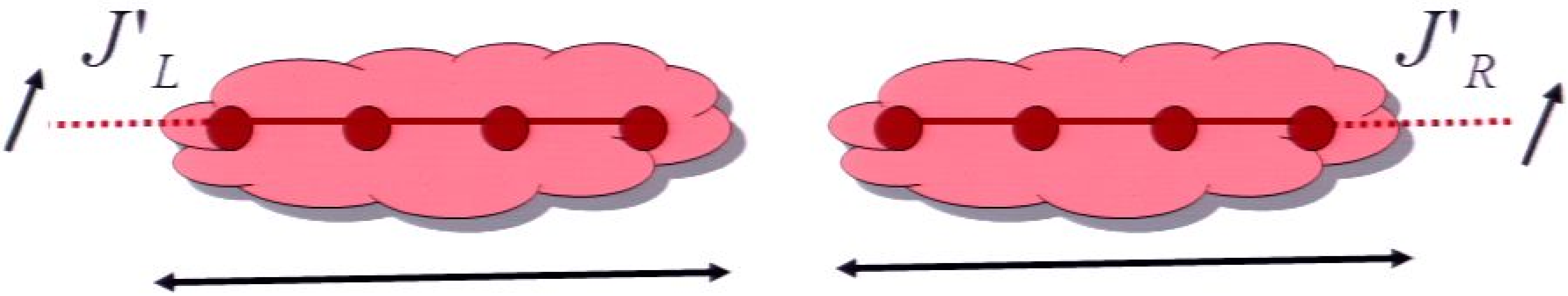
Extended Singlet



$$\xi_K(J'_{opt}) = N - 1$$

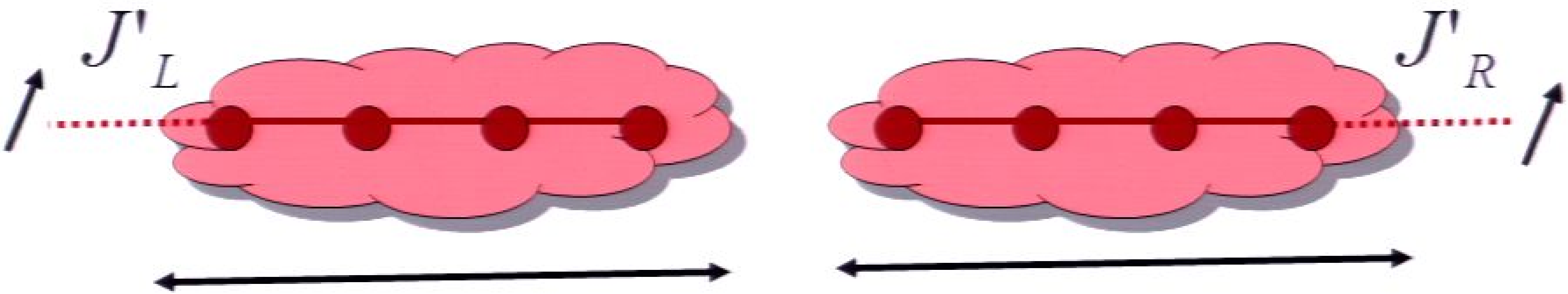
With tuning J' we can generate a proper cloud which extends up to the end of the chain

Quench Dynamics



$\xi_L(J'_L) = N_L - 1$ $\quad\quad$ $\xi_R(J'_R) = N_R - 1$

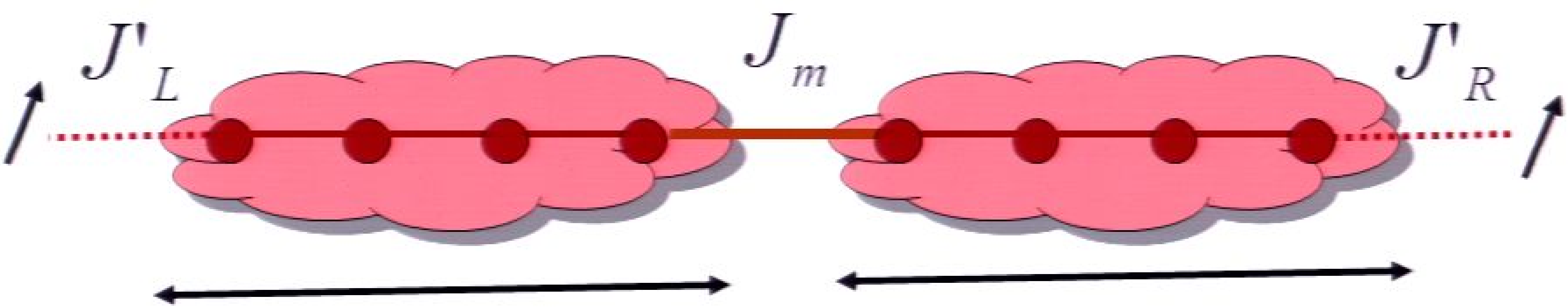
Quench Dynamics



$$\xi_L(J'_L) = N_L - 1 \qquad \xi_R(J'_R) = N_R - 1$$

$$|\psi(0)\rangle = |GS_L\rangle \otimes |GS_R\rangle$$

Quench Dynamics



$$\xi_L(J'_L) = N_L - 1 \qquad \xi_R(J'_R) = N_R - 1$$

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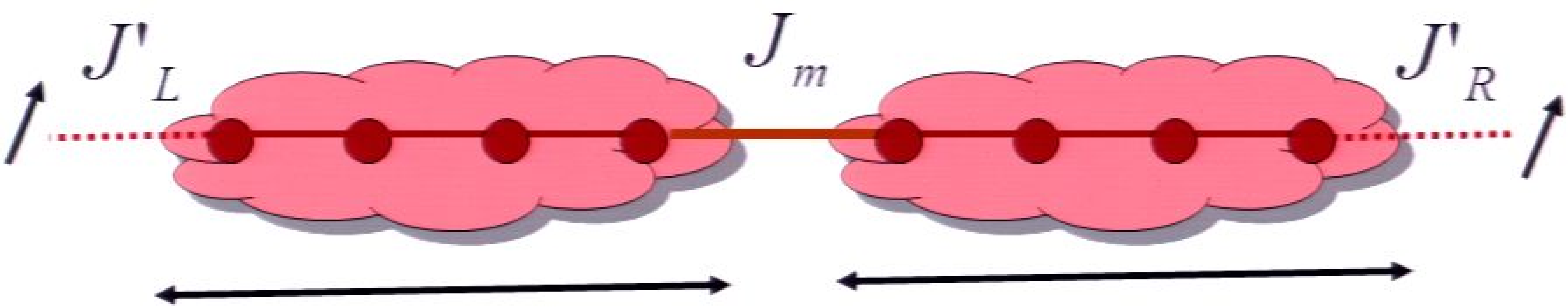
Quench Dynamics

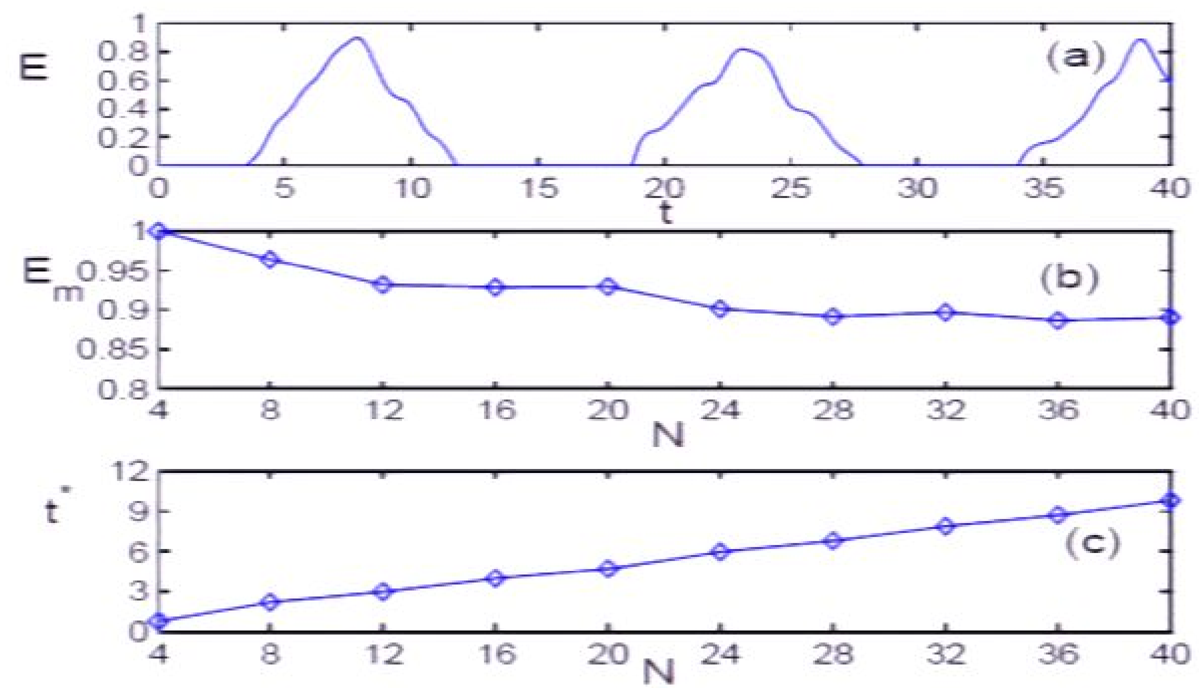


$$\xi_L(J'_L) = N_L - 1 \qquad \xi_R(J'_R) = N_R - 1$$

$$|\psi(0)\rangle = |GS_L\rangle \otimes |GS_R\rangle$$

$$|\psi(t)\rangle = e^{-iH_{LR}t}|\psi(0)\rangle \Rightarrow \rho_{1N}(t) \Rightarrow E_{1N}(t)$$

Attainable Entanglement



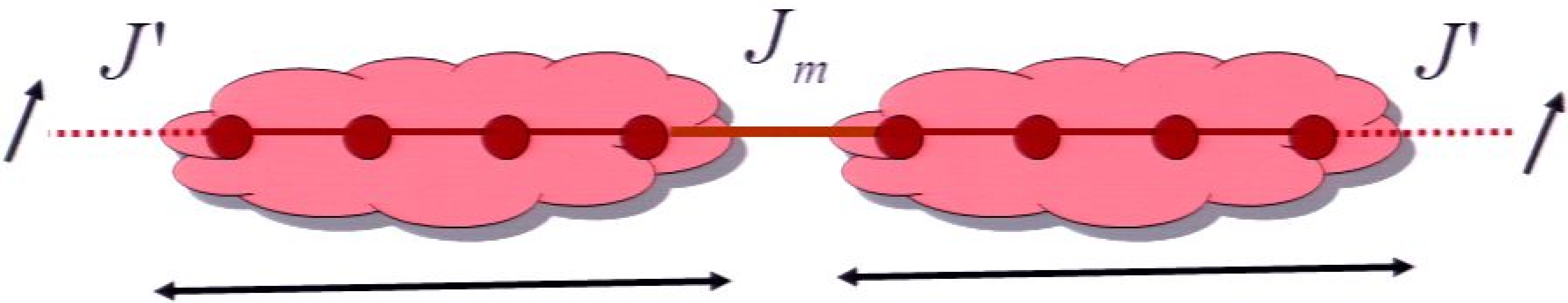
1- Entanglement dynamics is very long lived and oscillatory

2- maximal entanglement attains a constant values for large chains

3- The optimal time which entanglement peaks is linear

Distance Independence

For simplicity take a symmetric composite:



$$\xi_L(J') = (N-2)/2 \qquad \xi_R(J') = (N-2)/2$$

Distance Independence

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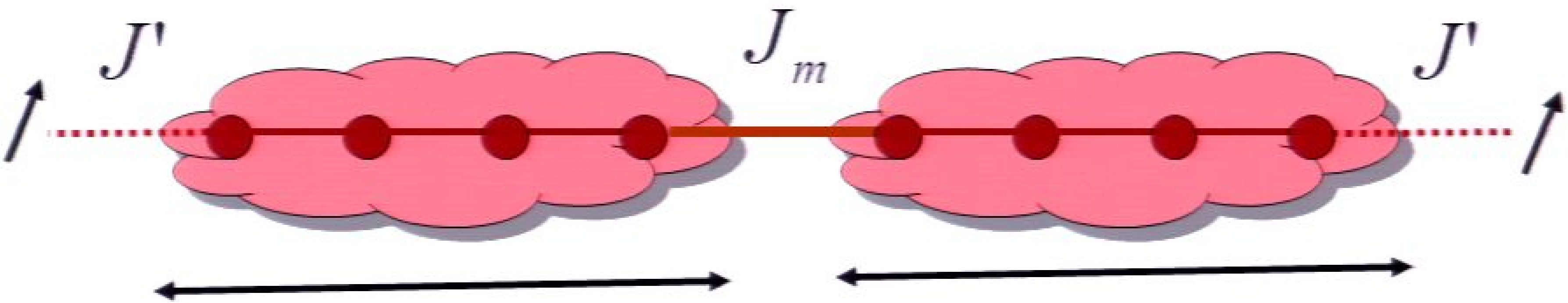
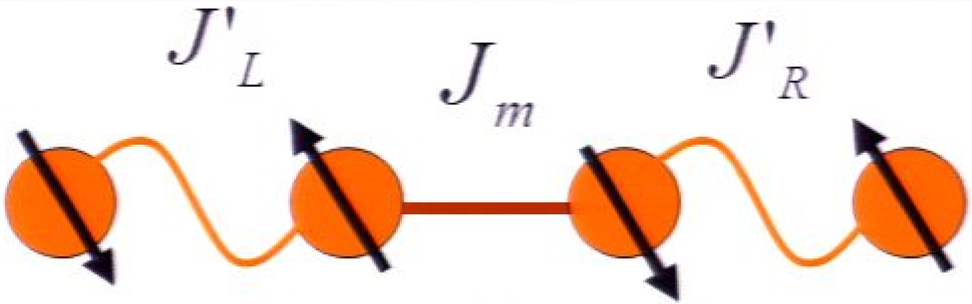


$$\xi_L(J') = (N-2)/2 \qquad \xi_R(J') = (N-2)/2$$

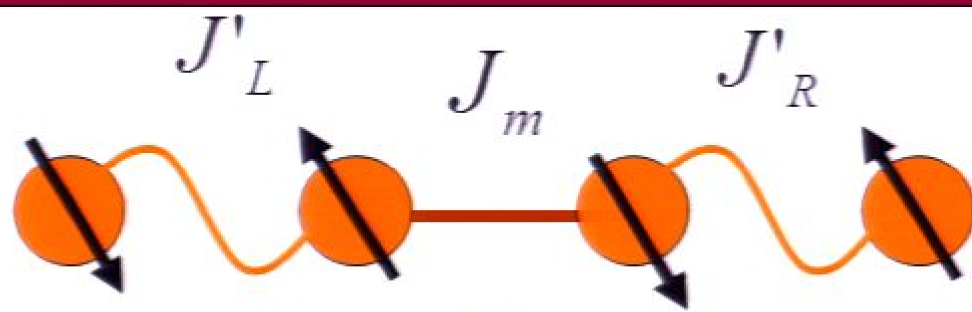
$$E(t,N,J') = E(t,N,\xi) = E(\frac{t}{N},\frac{N}{\xi}) = E(\frac{t}{N},\frac{2N}{N-2})$$

Optimal Quench



$$J_m = J'_L + J'_R$$

Optimal Quench



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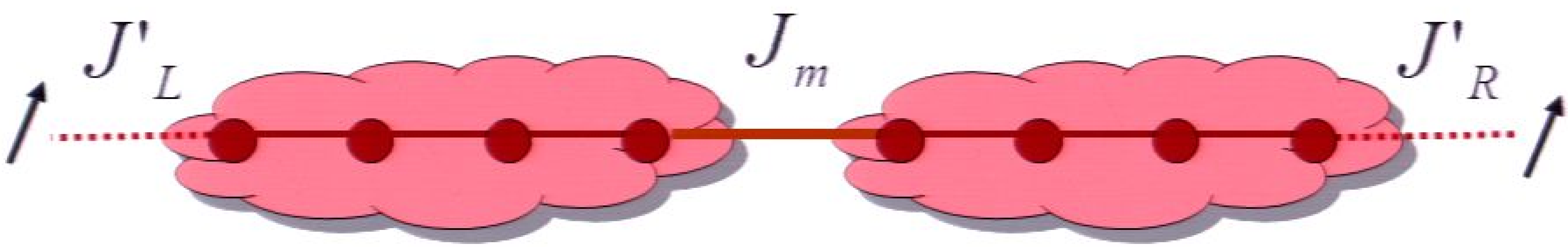
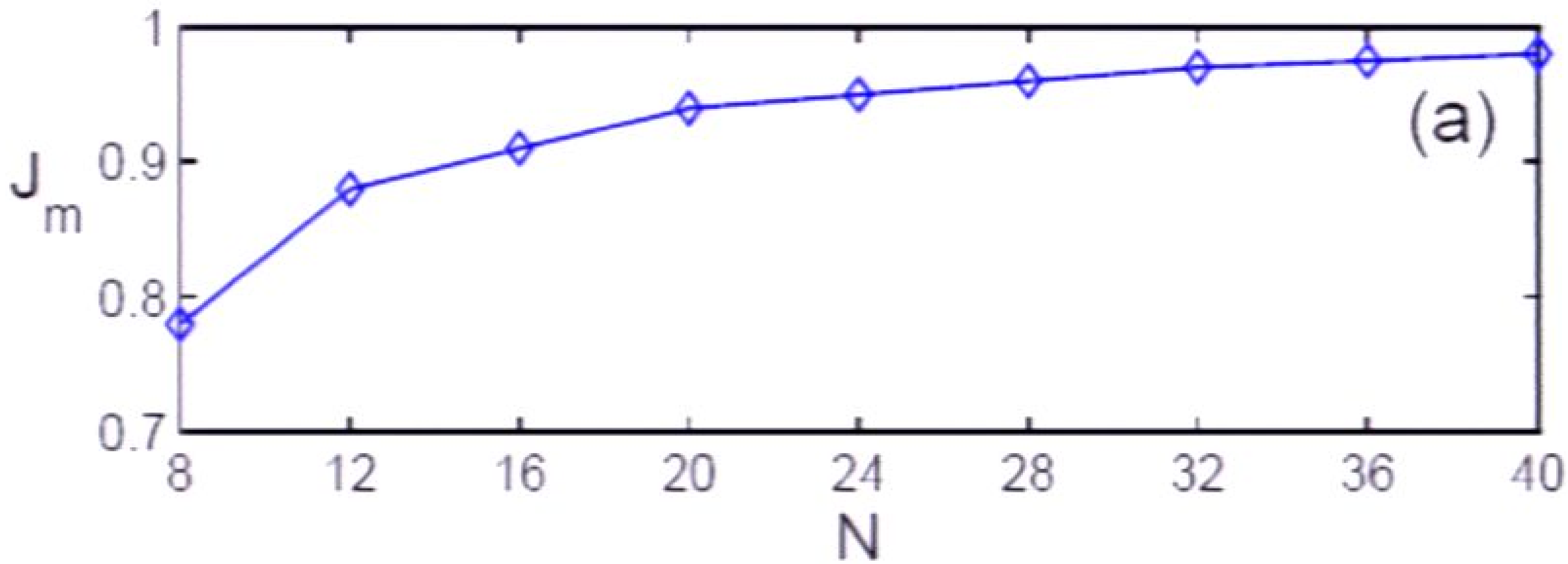


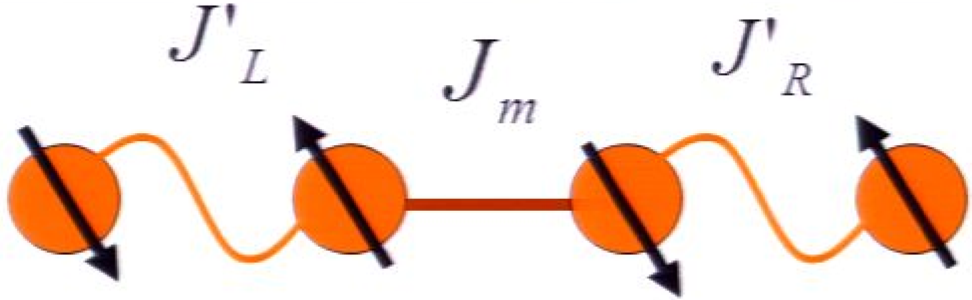
$$J_m = ?$$

Optimal Jm



Jm saturates to J1 for large N

Optimal Quench



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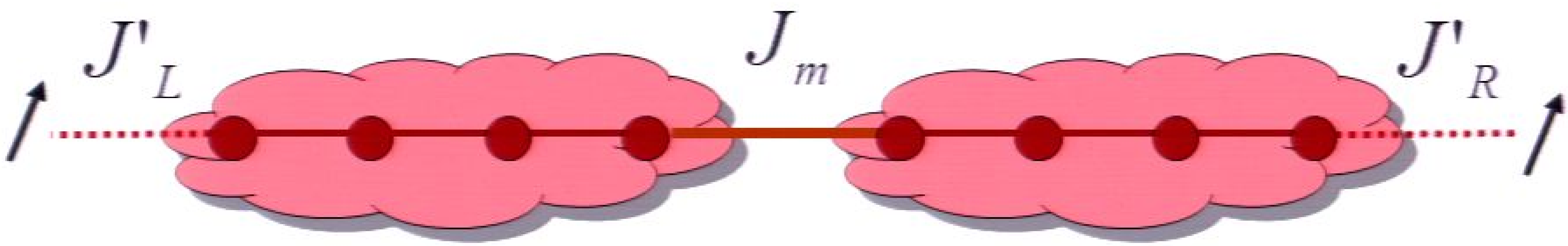
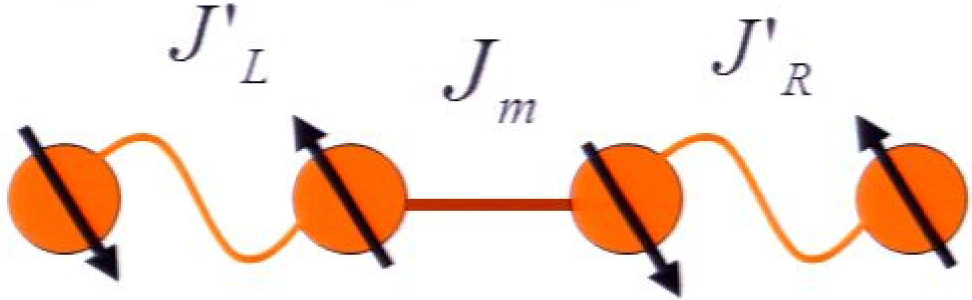


Optimal Quench



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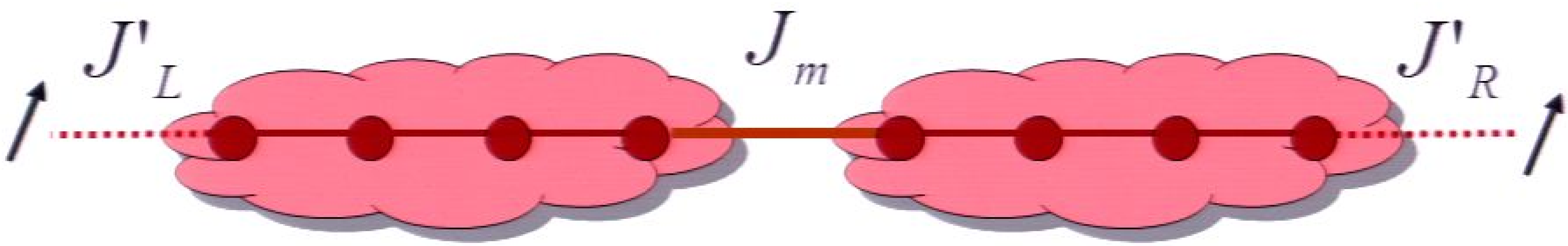
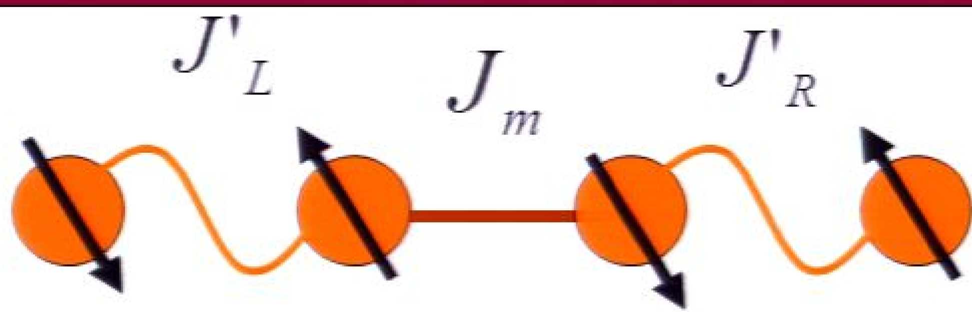


$$J_m = \Phi(N)(J'_L + J'_R)$$

$$J'_{R(L)} = \frac{1}{\log^2(N_{R(L)})}$$

Optimal Quench



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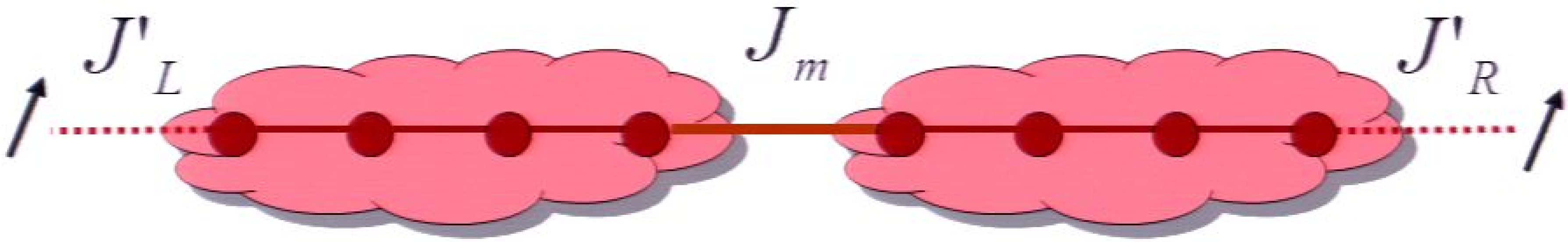
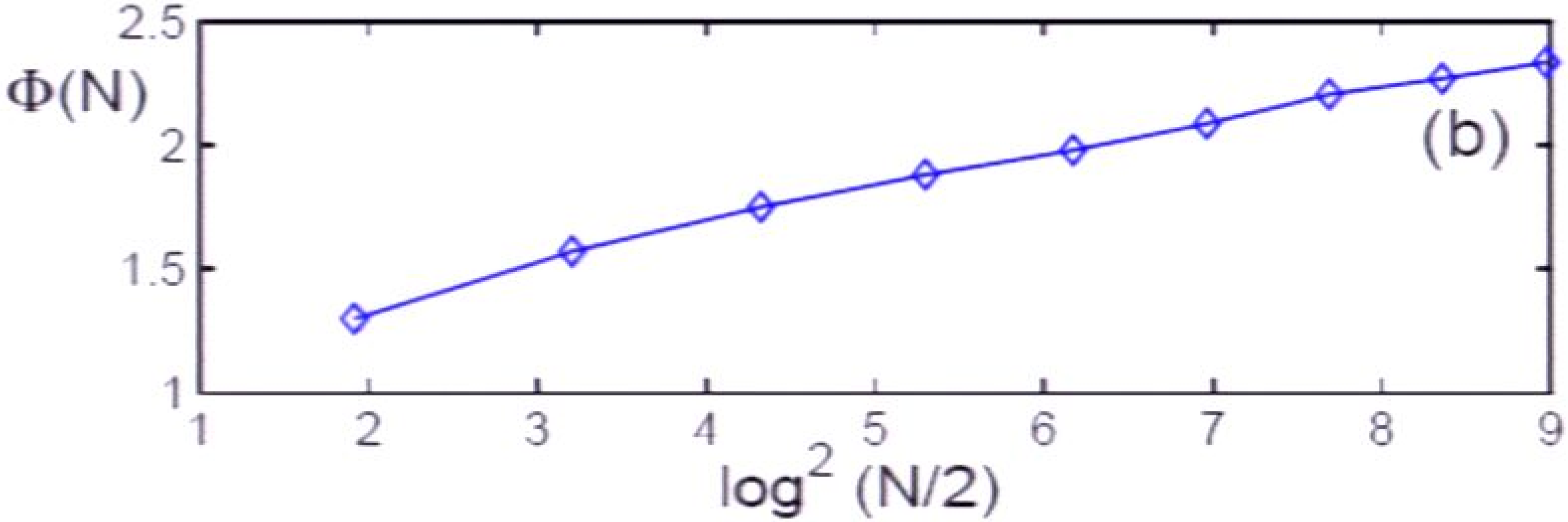


$$J_m = \Phi(N)(J'_L + J'_R)$$

$$J'_{R(L)} = \frac{1}{\log^2(N_{R(L)})}$$

$$\Rightarrow \Phi(N) \approx Log^2(\frac{N}{2})$$

Dependence on N



$$\Phi(N) \approx Log^2(\frac{N}{2})$$

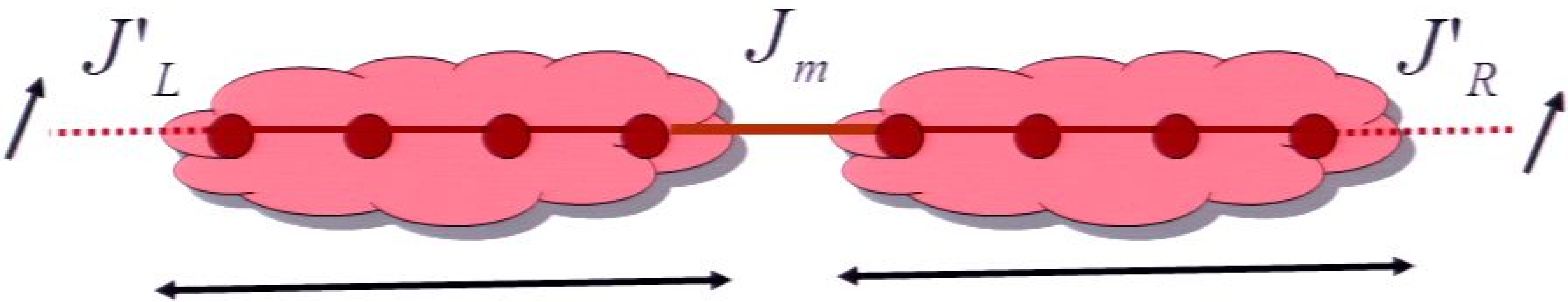
Non-Kondo Singlets (Dimer Regime)

Clouds are absent

N	8	12	16	20	24	28	32	36	40
E_m(K)	0.964	0.932	0.928	0.929	0.901	0.891	0.897	0.886	0.891
E_m(D)	0.957	0.903	0.841	0.783	0.696	0.581	0.468	0.330	0.160
t^*(K)	2.200	2.980	3.980	4.700	5.980	6.800	7.880	8.720	9.800
t^*(D)	3.780	7.290	10.32	13.41	16.89	20.43	24.51	27.12	35.01

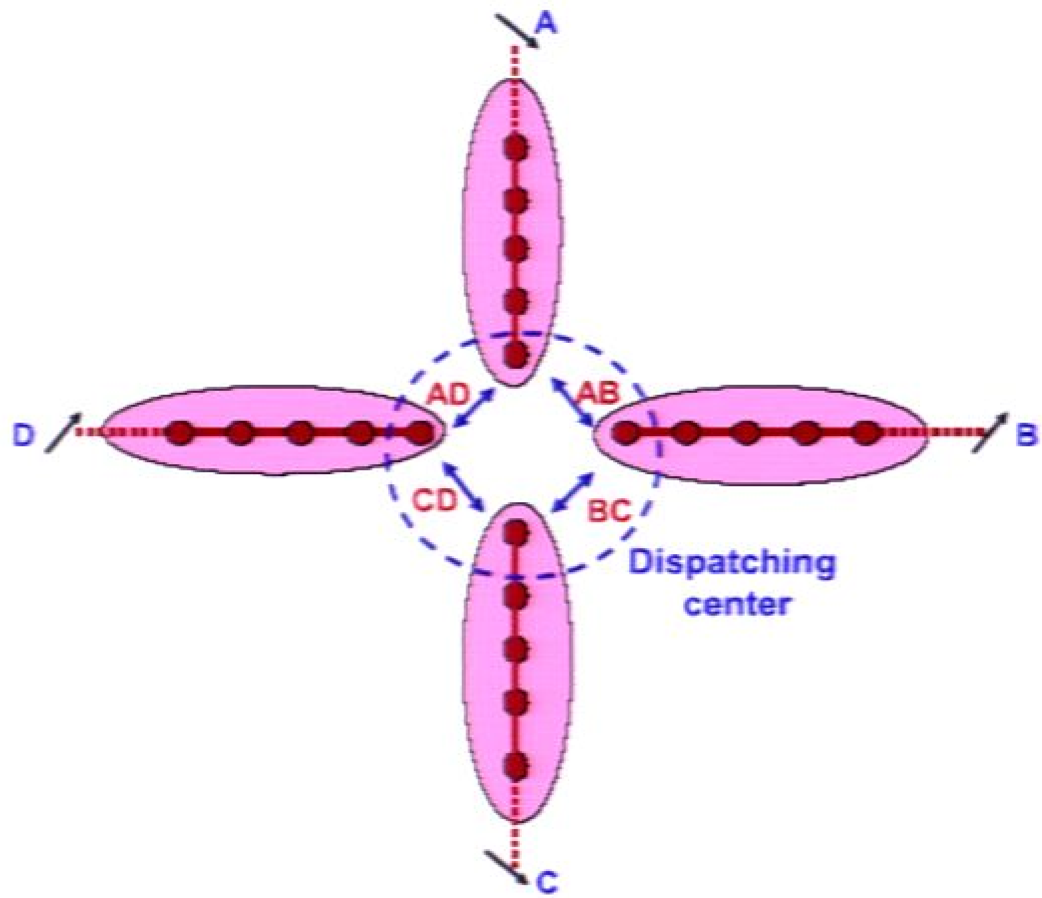
K: Kondo (J2=0) **D: Dimer (J2=0.42)**

Non-Symmetric Chains



$$\xi_L(J'_L) = N_L - 1 \qquad \xi_R(J'_R) = N_R - 1$$

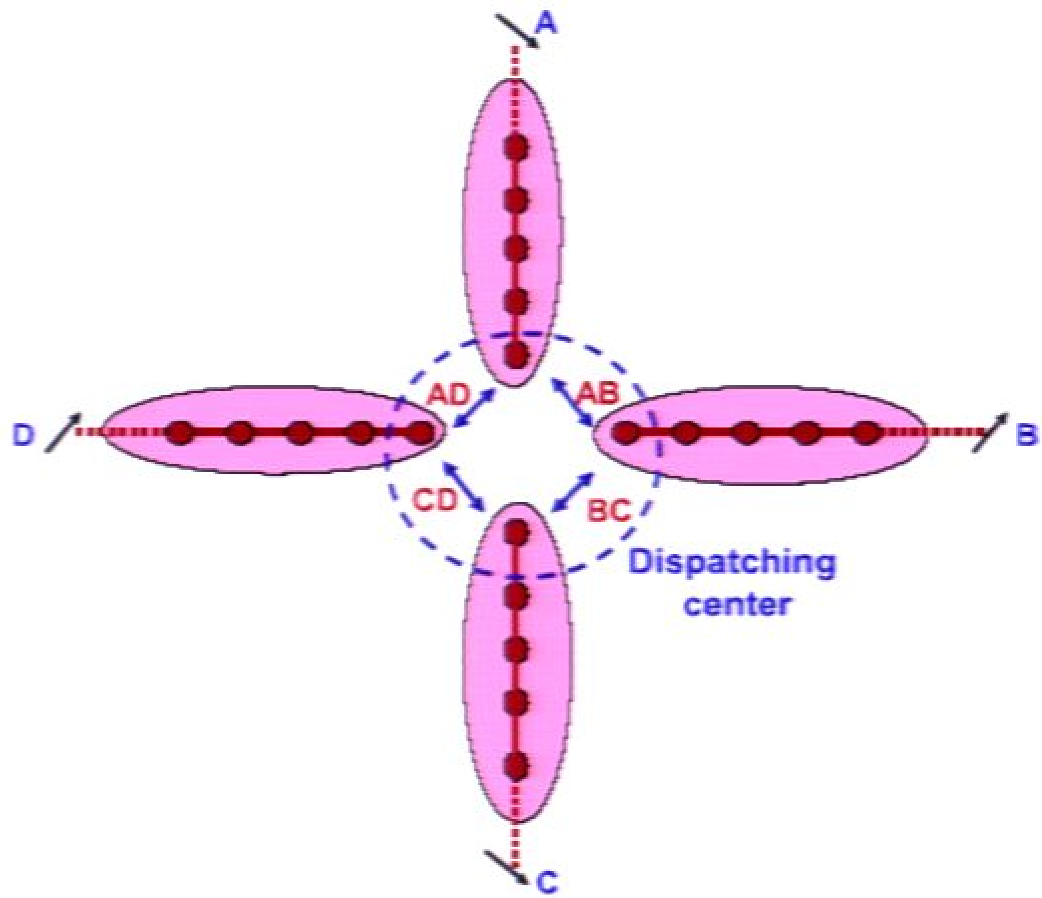
Entanglement Router



Summary

- Using entanglement measures, one can capture the properties of the Kondo physics.
- In the Kondo regime, Kondo cloud plays the role of the mediator to create a long range "*distance independent*" entanglement between individual ending spins.
- One can make an entanglement router through connecting Kondo spin chains.

Entanglement Router



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