

Title: The structure of the dark matter distribution on laboratory scales

Date: Mar 30, 2011 02:00 PM

URL: <http://pirsa.org/11030079>

Abstract: At the time of recombination, 400,000 years after the Big Bang, the structure of the dark matter distribution was extremely simple and can be inferred directly from observations of structure in the cosmic microwave background. At this time dark matter particles had small thermal velocities and their distribution deviated from uniformity only through a gaussian field of small density fluctuations with associated motions. Later evolution was driven purely by gravity and so obeyed the collisionless Boltzmann equation. This has immediate consequences for the present distribution of dark matter, even in extremely nonlinear regions such as the part of the Galaxy where the Sun resides. I will show how this structure can be followed in full generality by integrating the Geodesic Deviation Equation in tandem with the equations of motion in a high-resolution N-body simulation, enhancing its effective resolution by more than 10 orders of magnitude. I will discuss how the predicted distribution at the Sun's position impacts the expectations for laboratory experiments seeking to detect the dark matter directly, in particular, the possibility of extremely narrow line signals that may be visible in axion detectors.

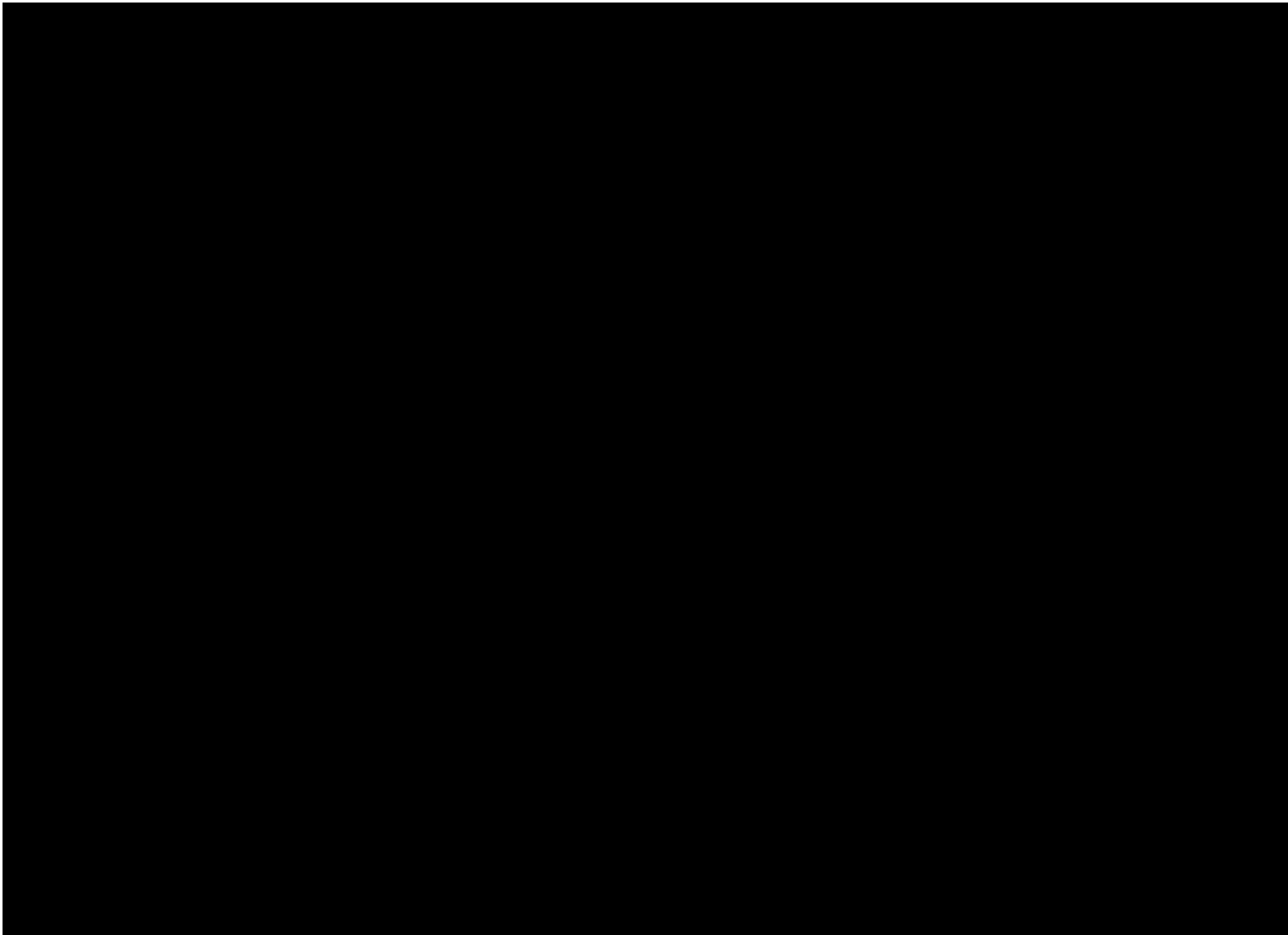
The background of the slide is a Cosmic Microwave Background (CMB) fluctuation map. It shows a complex pattern of temperature variations across the sky, with a color scale ranging from blue (cooler) to red (warmer). The map exhibits a prominent dipole anisotropy, with the warmest region (red) on the right and the coolest region (blue) on the left. The fluctuations are most pronounced at larger angular scales, showing a clear quadrupole moment.

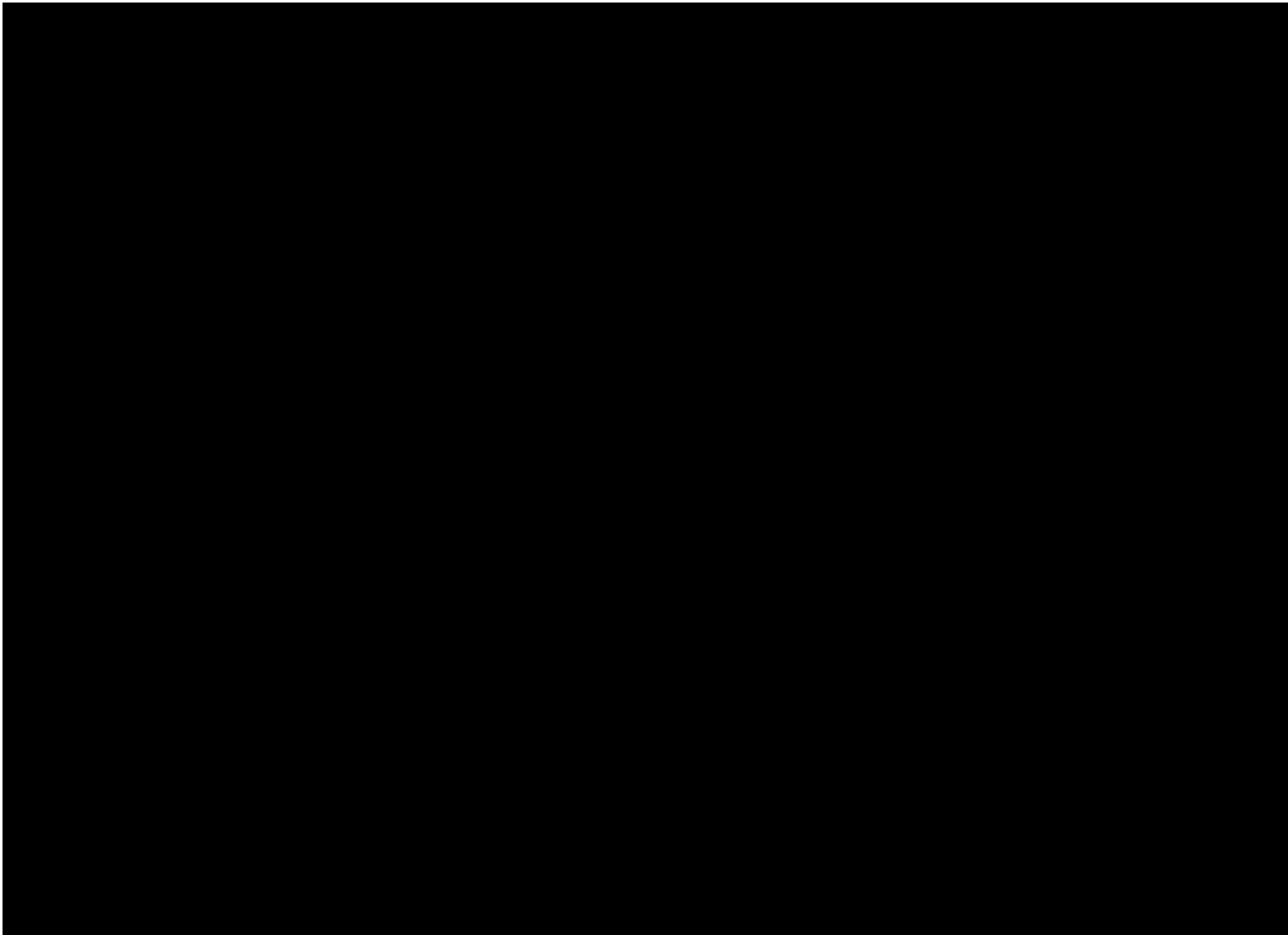
Perimeter Colloquium,
March 2011

The structure of the dark matter distribution on laboratory scales

Simon White

Max-Planck-Institute for Astrophysics





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Perimeter Colloquium,
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Mark Vogelsberger

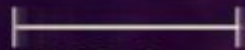
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= 46.3

T = 0.06 Gyr



= 44.7

T = 0.06 Gyr



= 40.7

T = 0.07 Gyr



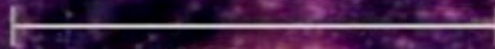
= 6.5

T = 0.87 Gyr



= 5.9

T = 0.99 Gyr



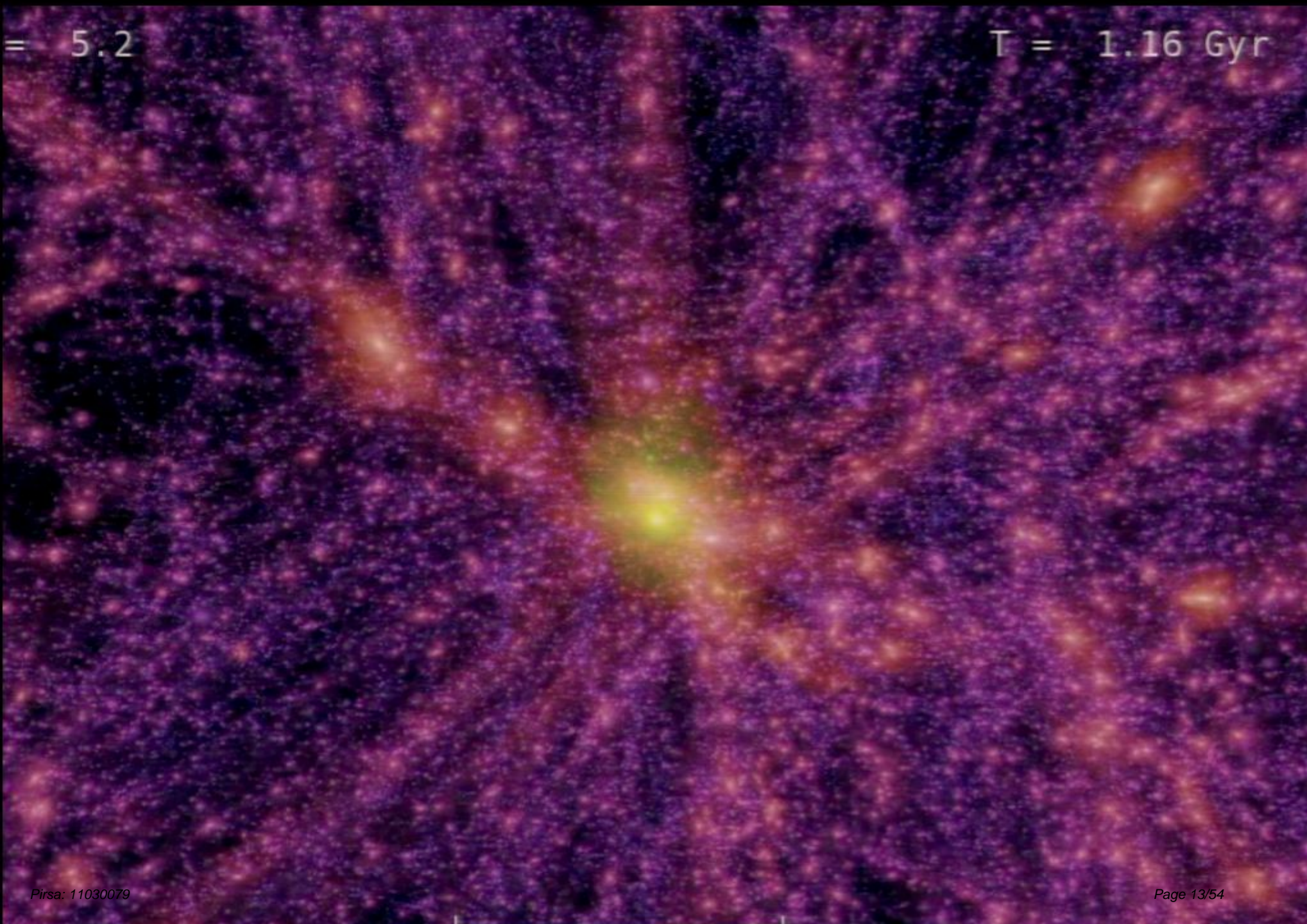
= 5.5

T = 1.07 Gyr



= 5.2

$T = 1.16 \text{ Gyr}$



= 4.7

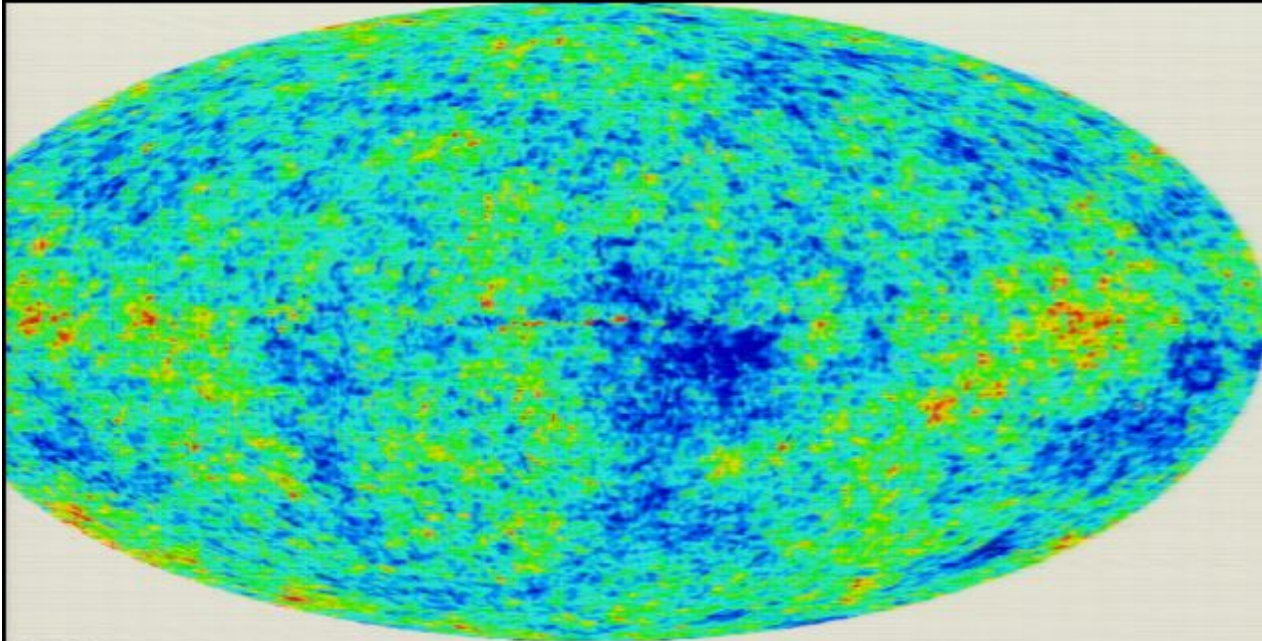
T = 1.31 Gyr



= 0.2

T = 11.04 Gyr





WMAP7+BAO+SN

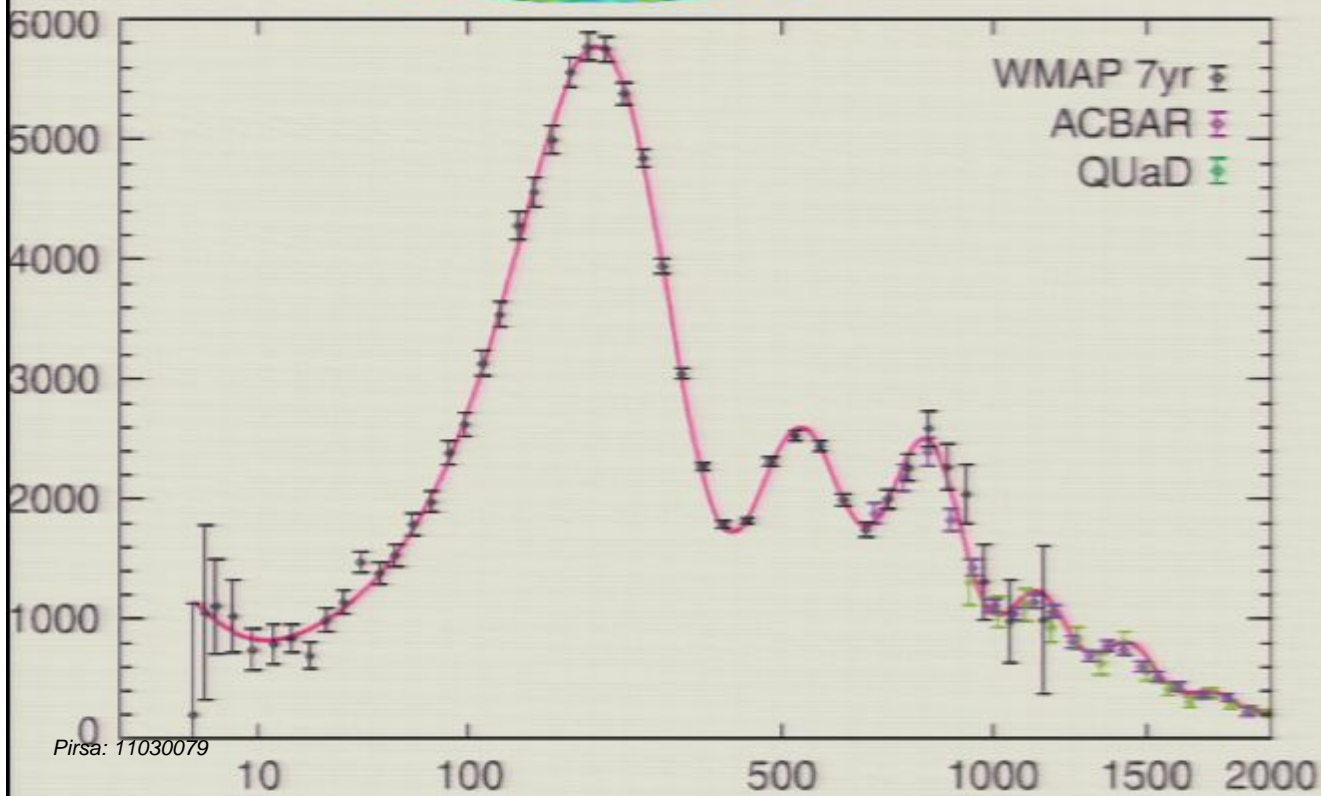
$$\Omega_{\text{bar}} = 0.046 \pm 0.002$$

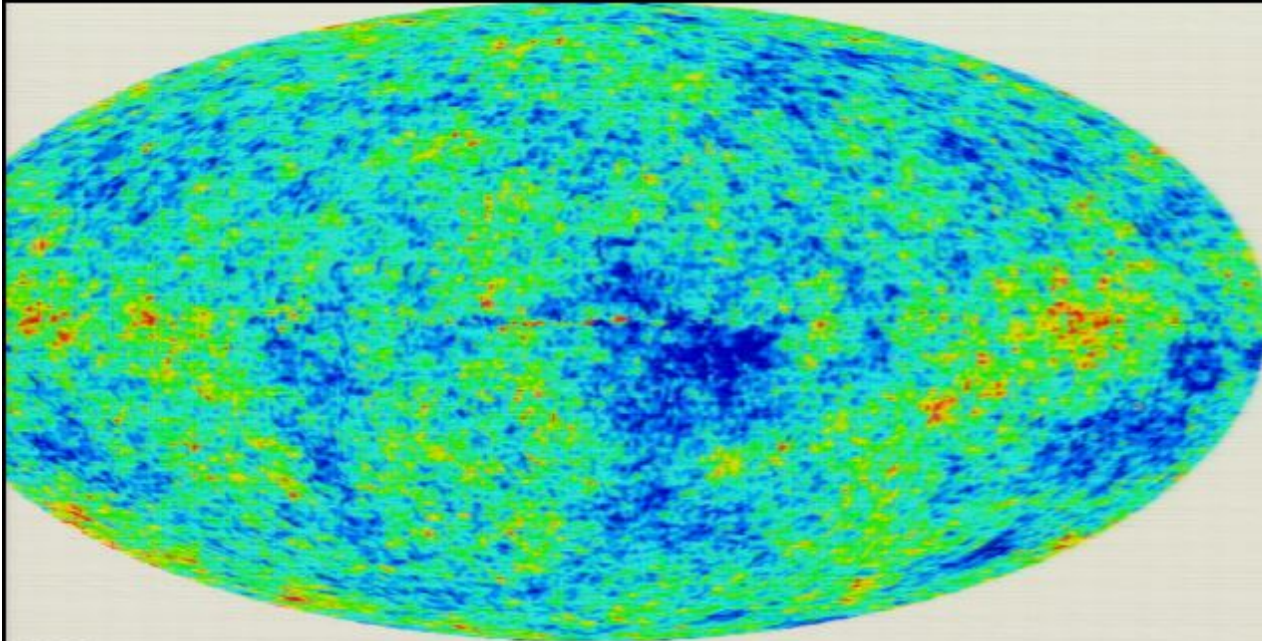
$$\Omega_{\text{DM}} = 0.229 \pm 0.015$$

$$\Omega_{\text{tot}} = 0.994 \pm 0.007$$

$$\sigma_8 = 0.82 \pm 0.02$$

$$n_s = 0.968 \pm 0.012$$





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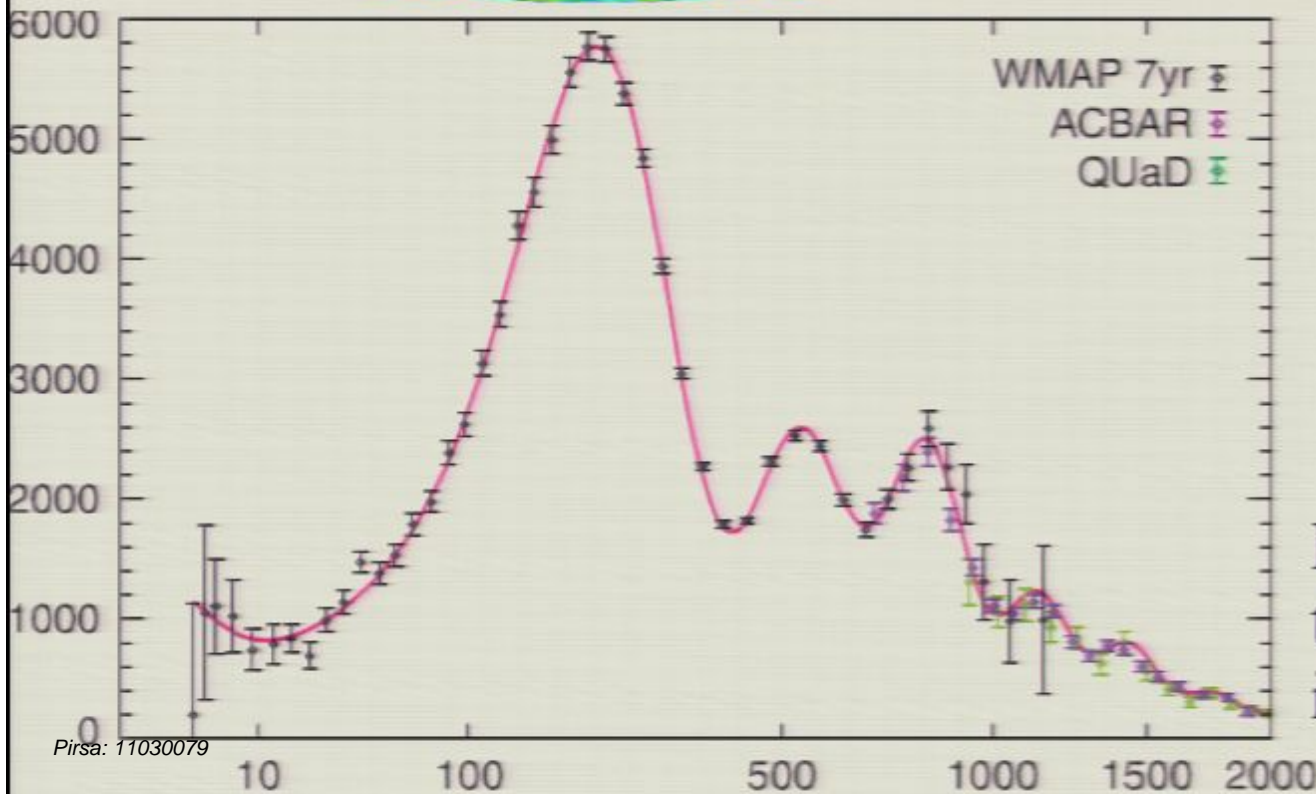
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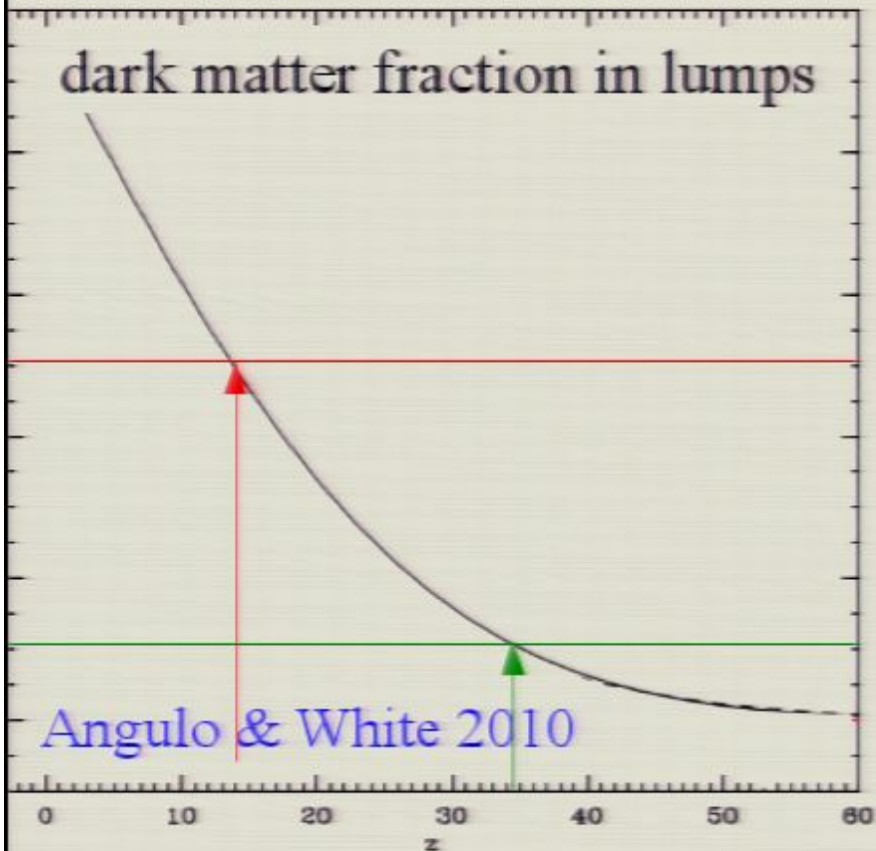
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Best current evidence for
the existence of weakly
interacting, nonbaryonic
dark matter



The growth of nonlinear dark matter structures



All dark matter is diffuse at $z > 100$

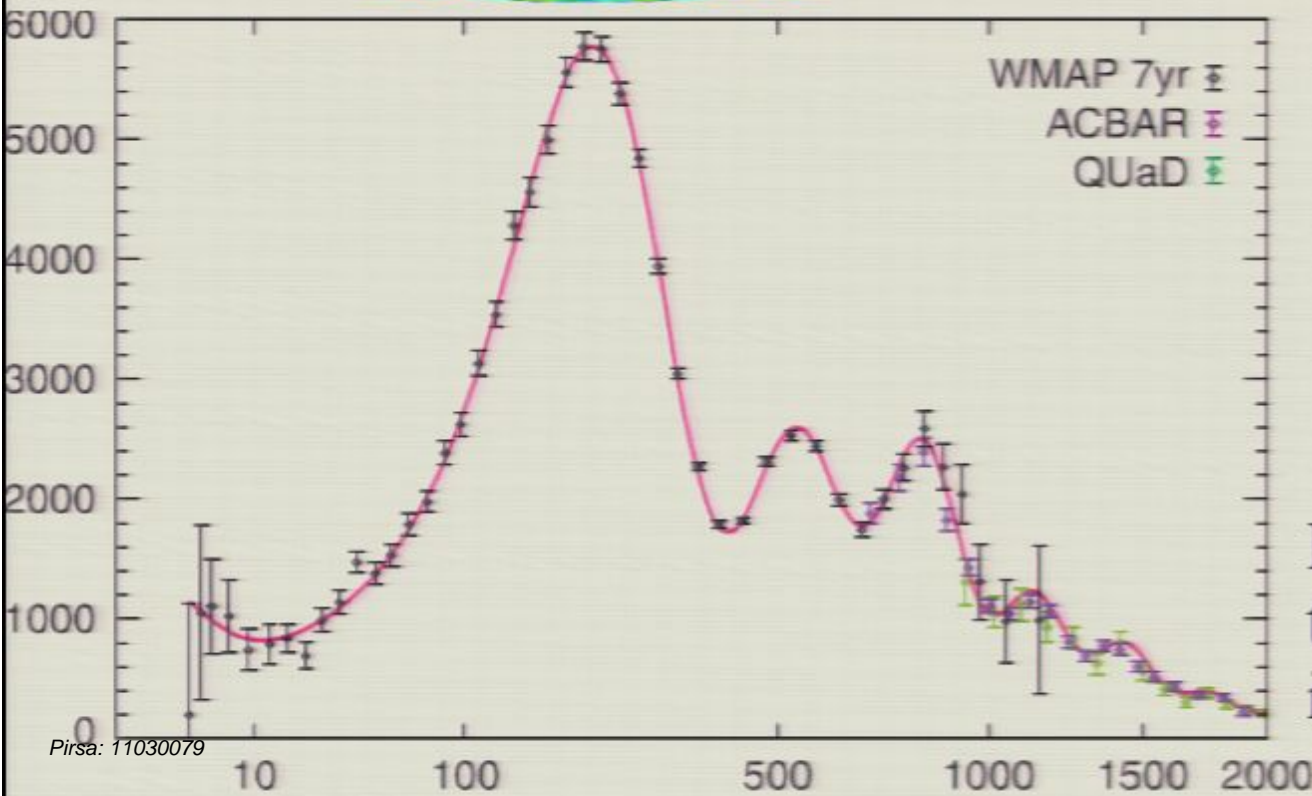
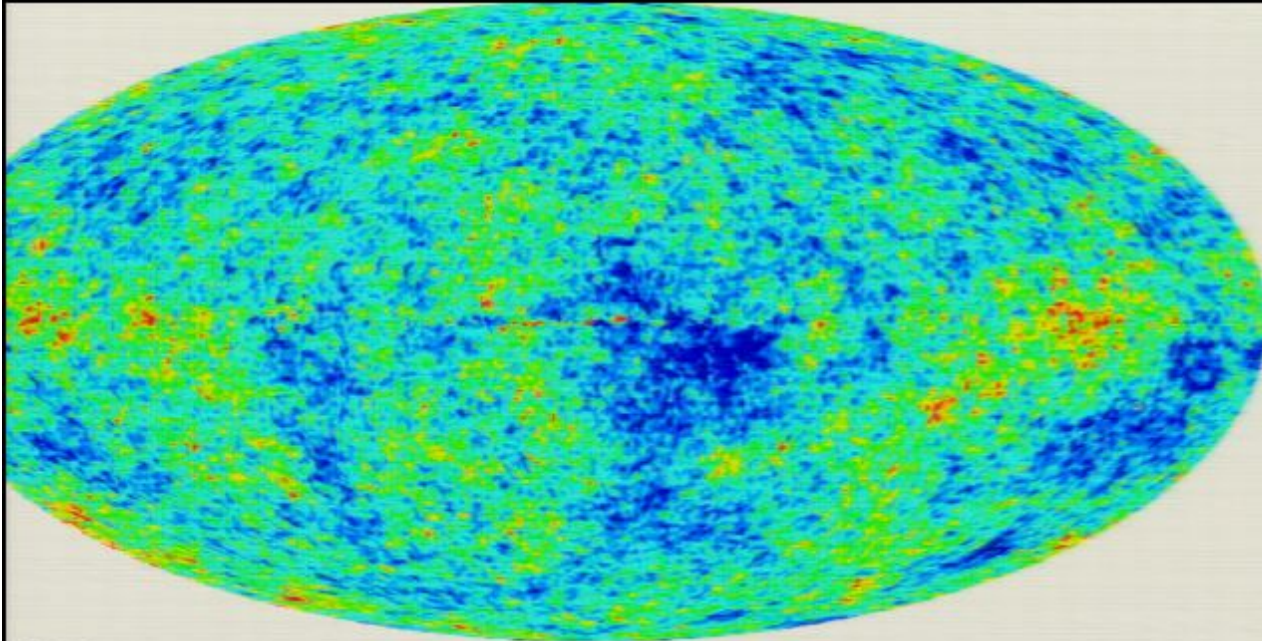
90% is diffuse at $z > 35$

50% is diffuse at $z > 13$

All nonlinear structure forms late, even halos of Earth mass or smaller

Structure grows through gravitational amplification of the seed fluctuations visible in the CMB

Nonlinear dark matter objects (“halos”) like that in which the Milky Way lives grow by the infall of diffuse material and smaller halos



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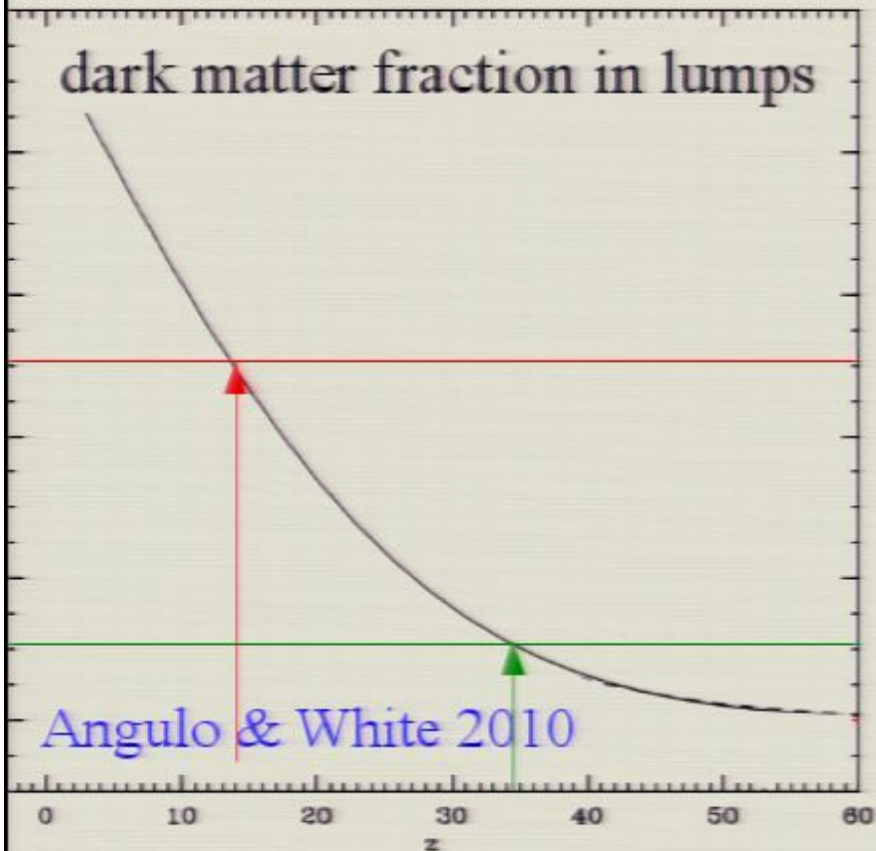
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The four elements of Λ CDM halos

I Smooth background halo

- NFW-like cusped density profile
- near-ellipsoidal equidensity contours

II Bound subhalos

- most massive typically 1% of main halo mass
- total mass of all subhalos $\lesssim 10\%$
- less centrally concentrated than the smooth component

III Tidal streams

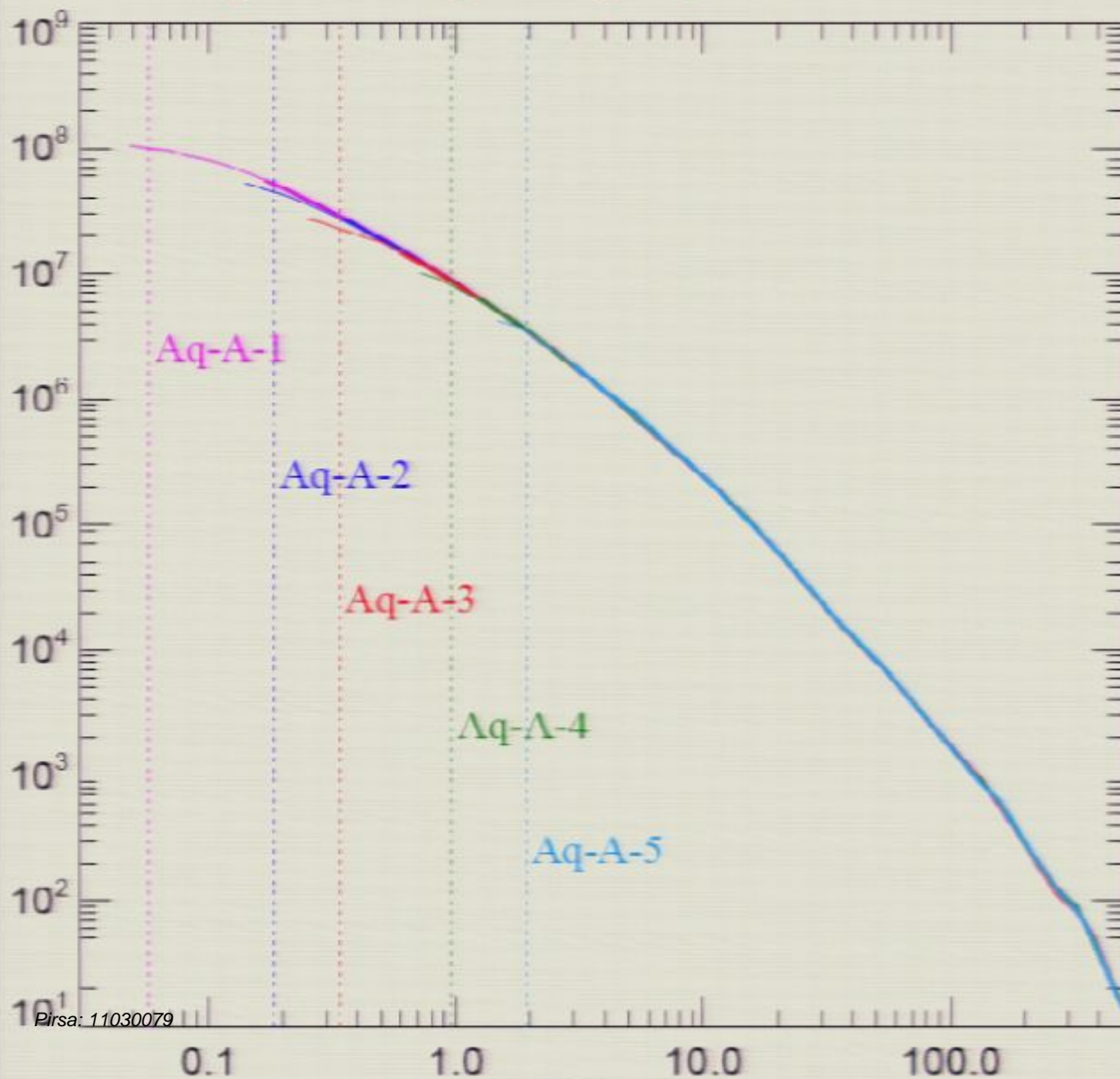
- remnants of tidally disrupted subhalos

IV Fundamental streams

- consequence of smooth and cold initial conditions
- very low internal velocity dispersions
- produce density caustics at projective catastrophes

I. Smooth background halo

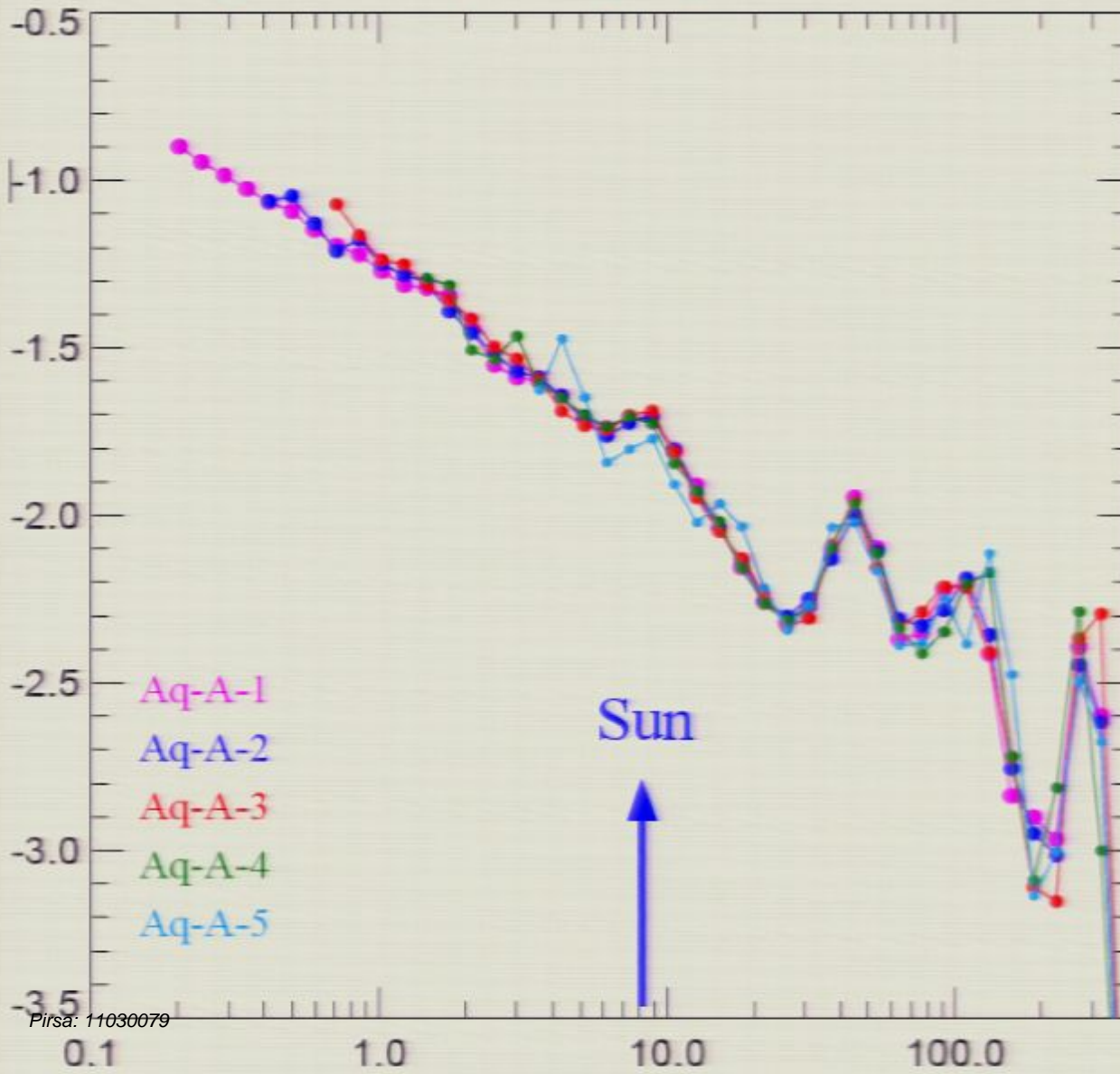
Aquarius Project: Springel et al 2008



- Density profiles of simulated DM-only Λ CDM halos are now very well determined

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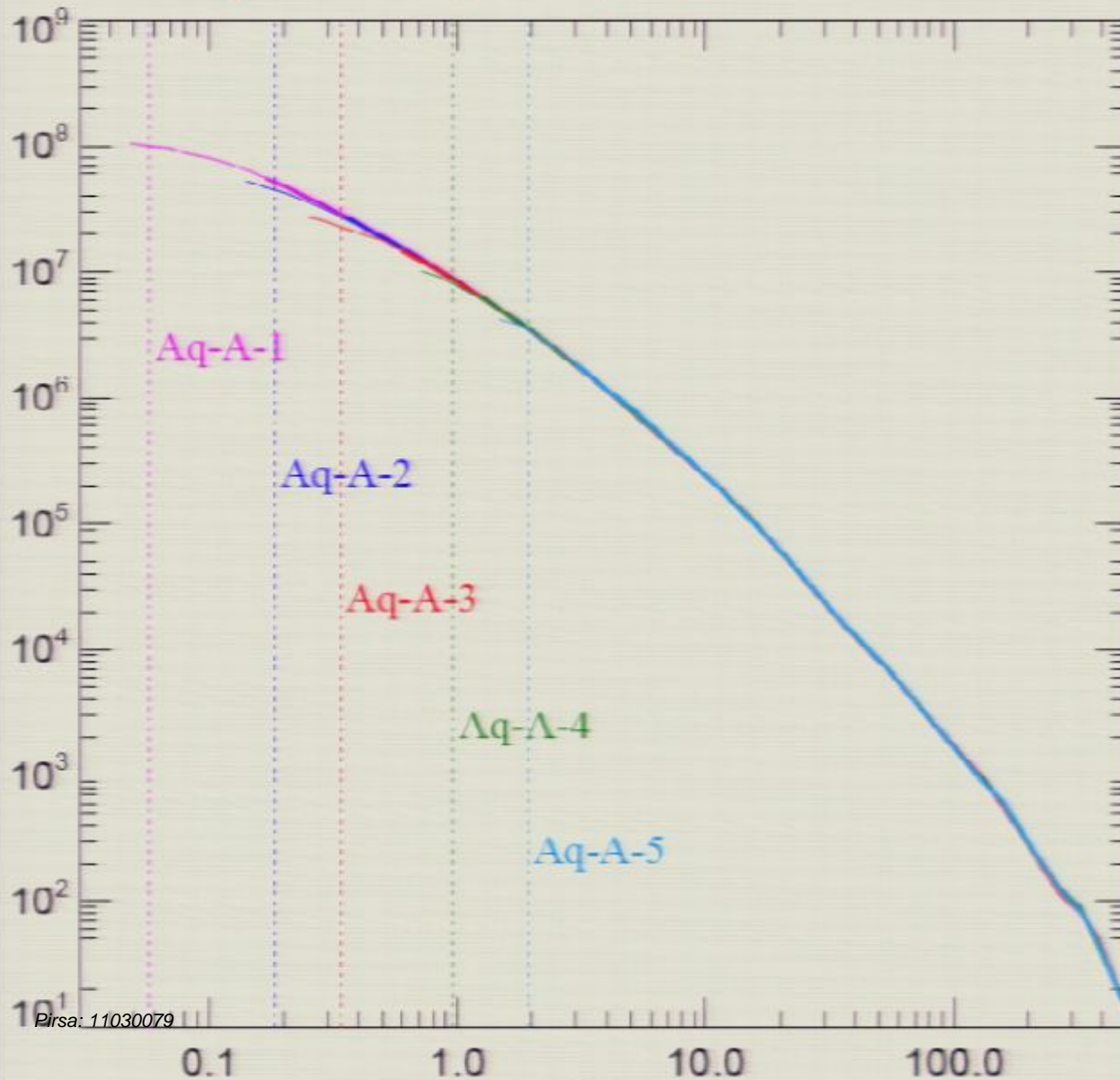
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- Density profiles of simulated DM-only Λ CDM halos are now very well determined
- The inner cusp does not appear to have a well-defined power law slope
- Treating baryons more important than better DM simulations

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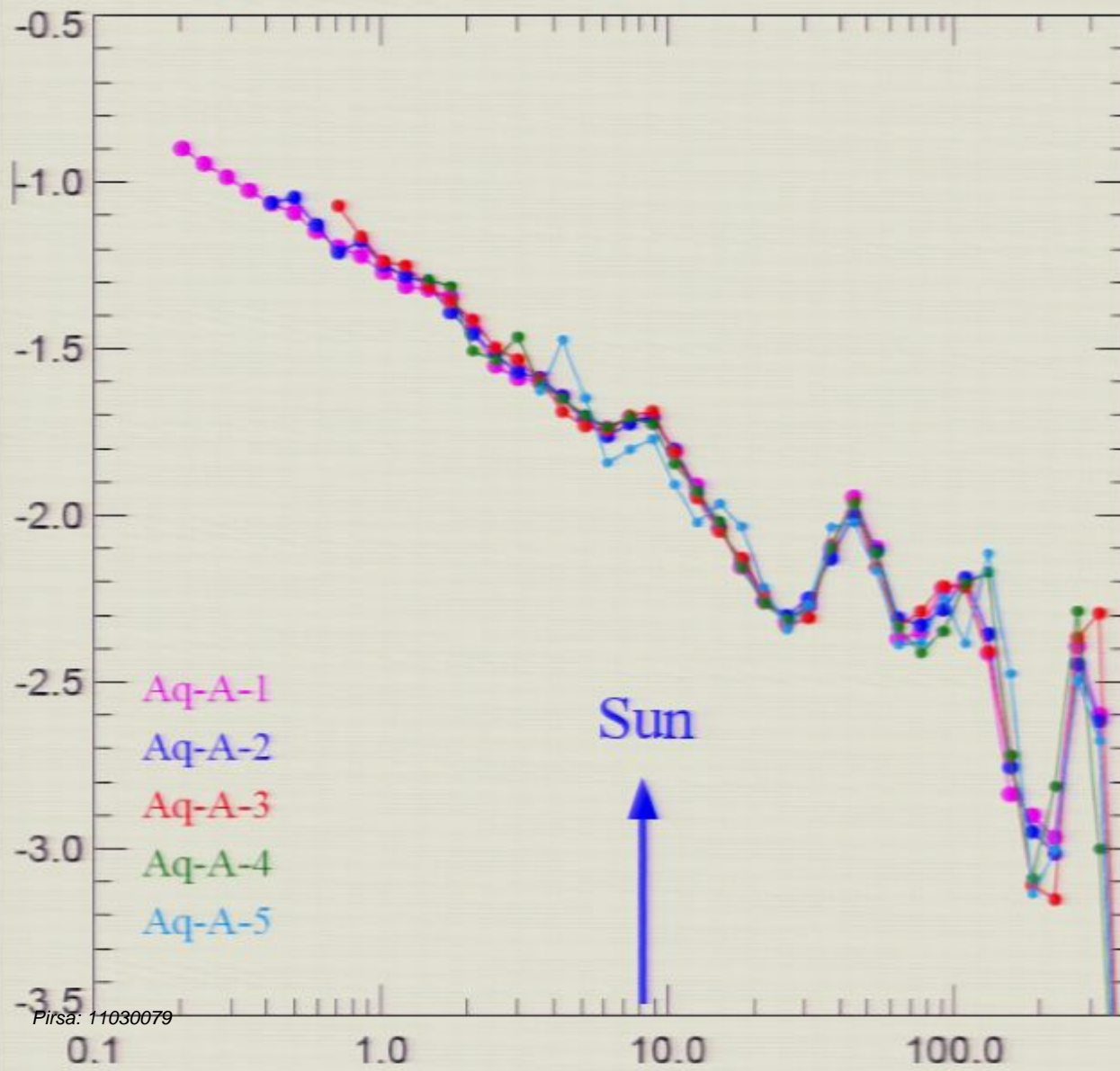
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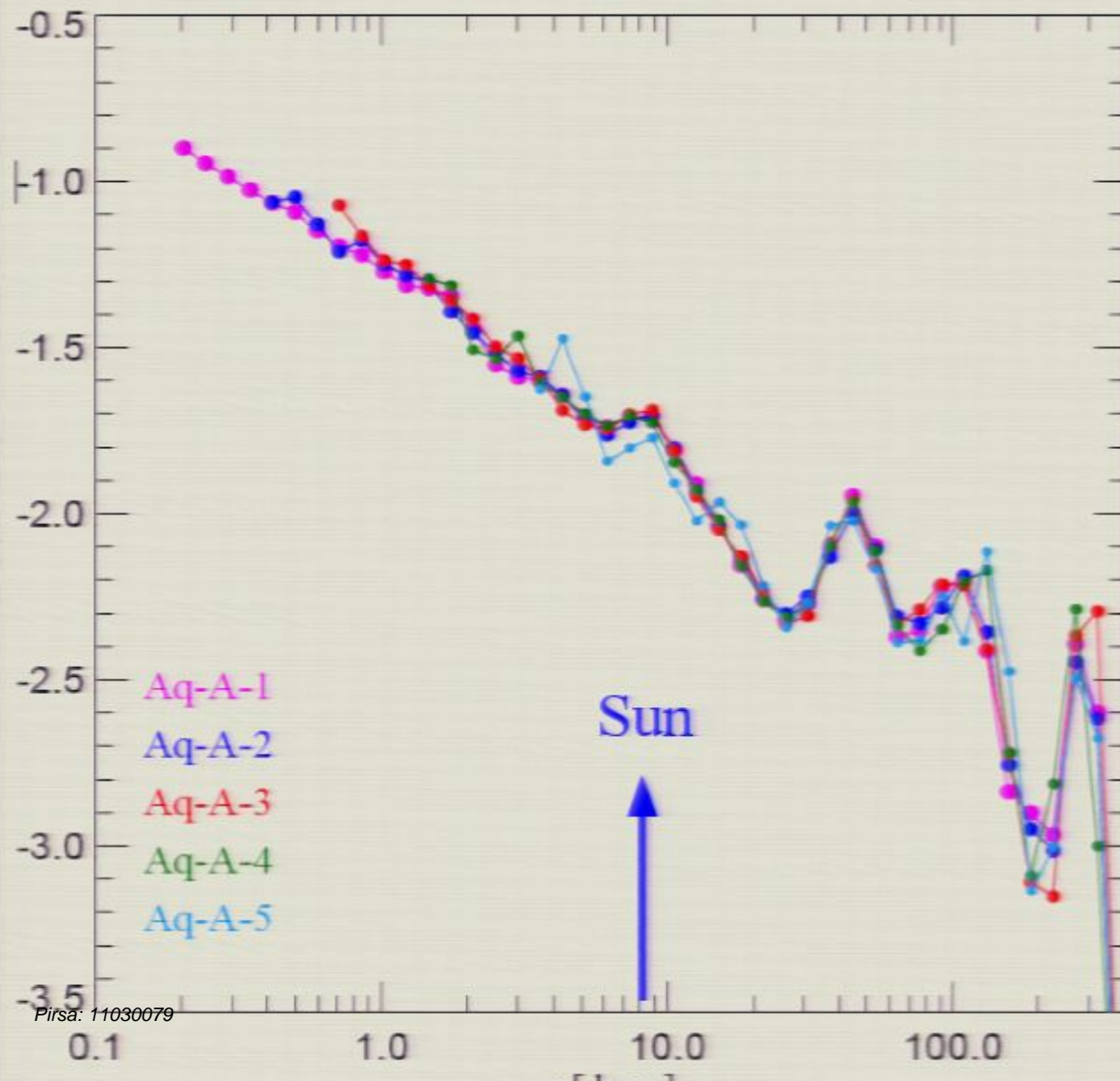
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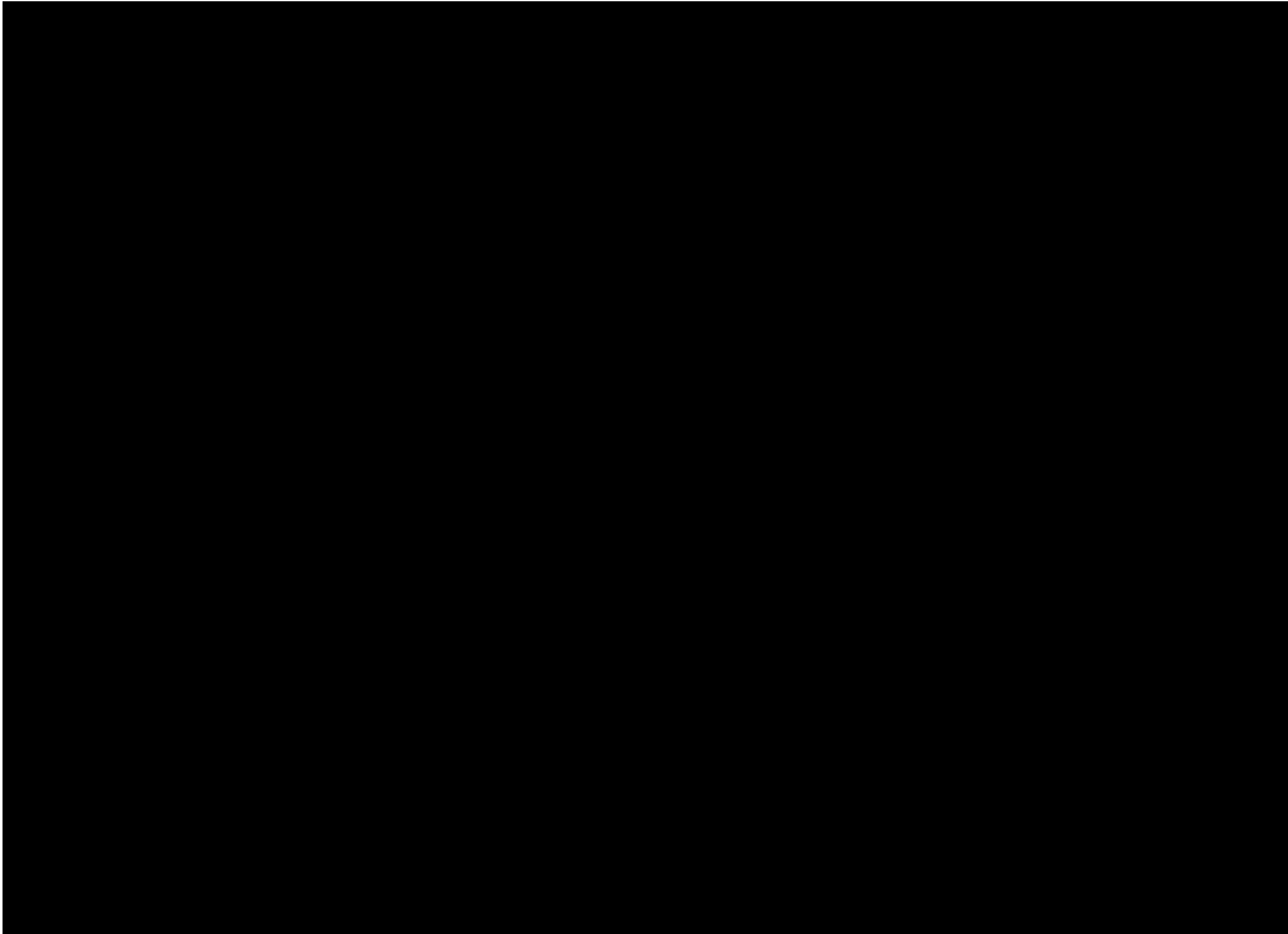
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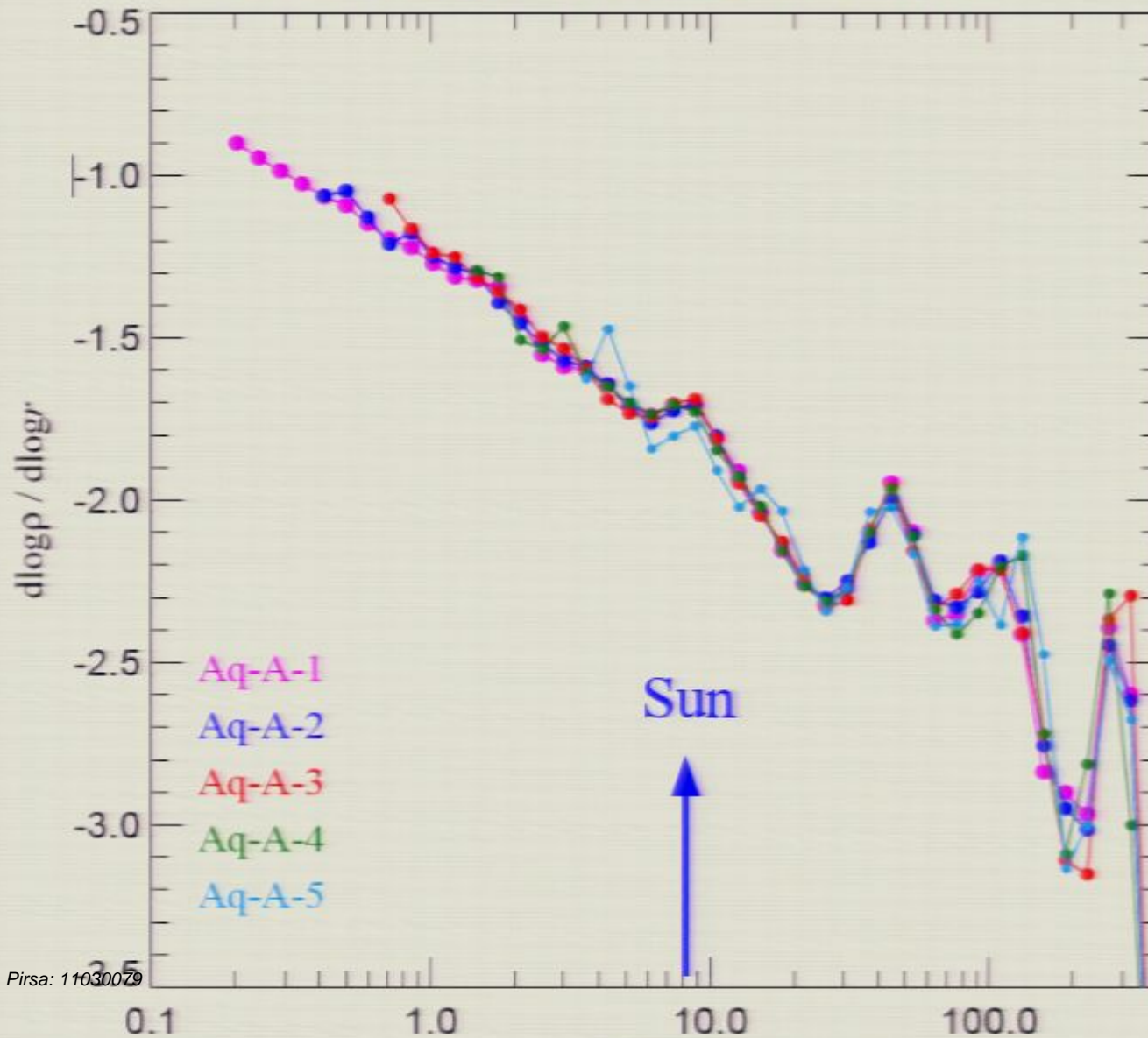


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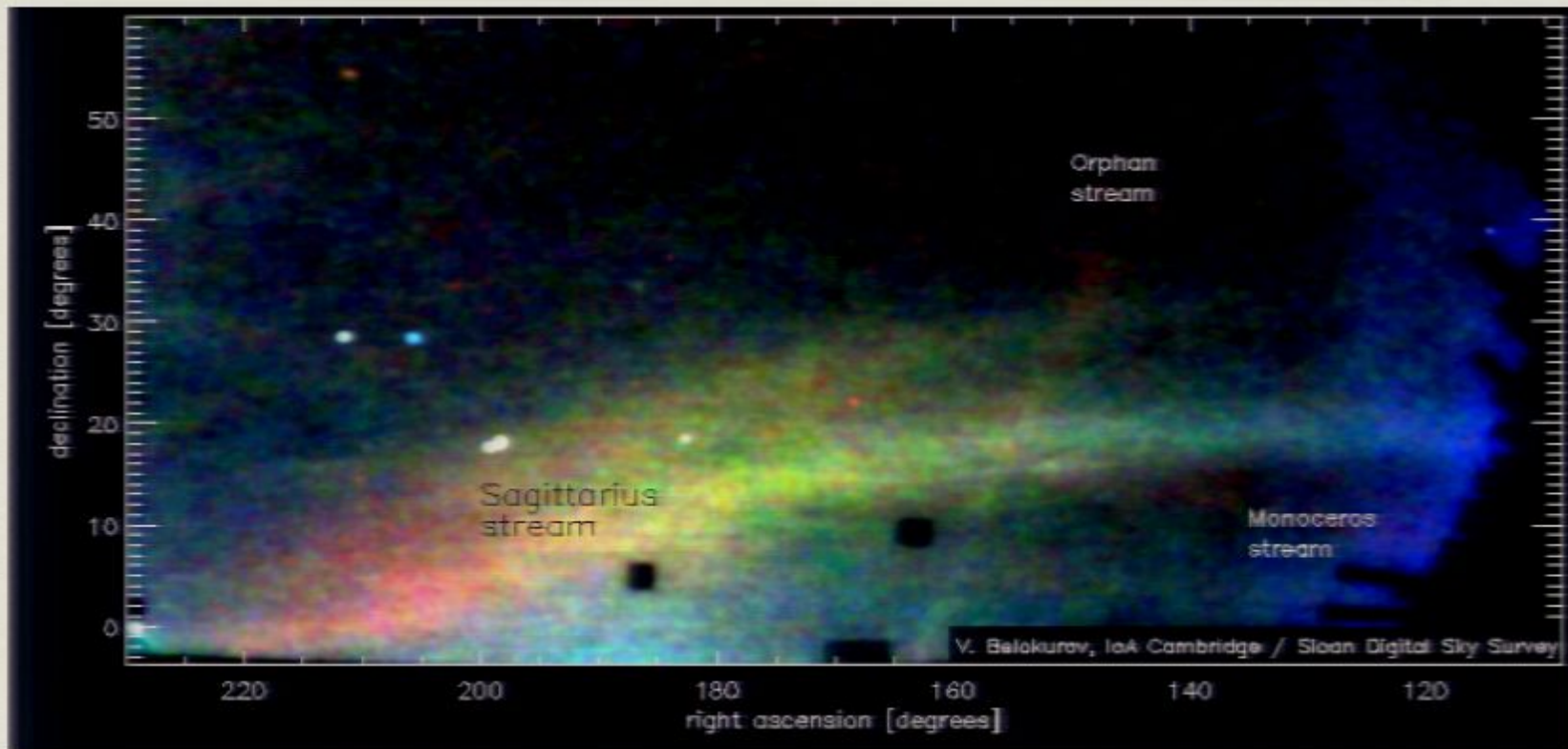
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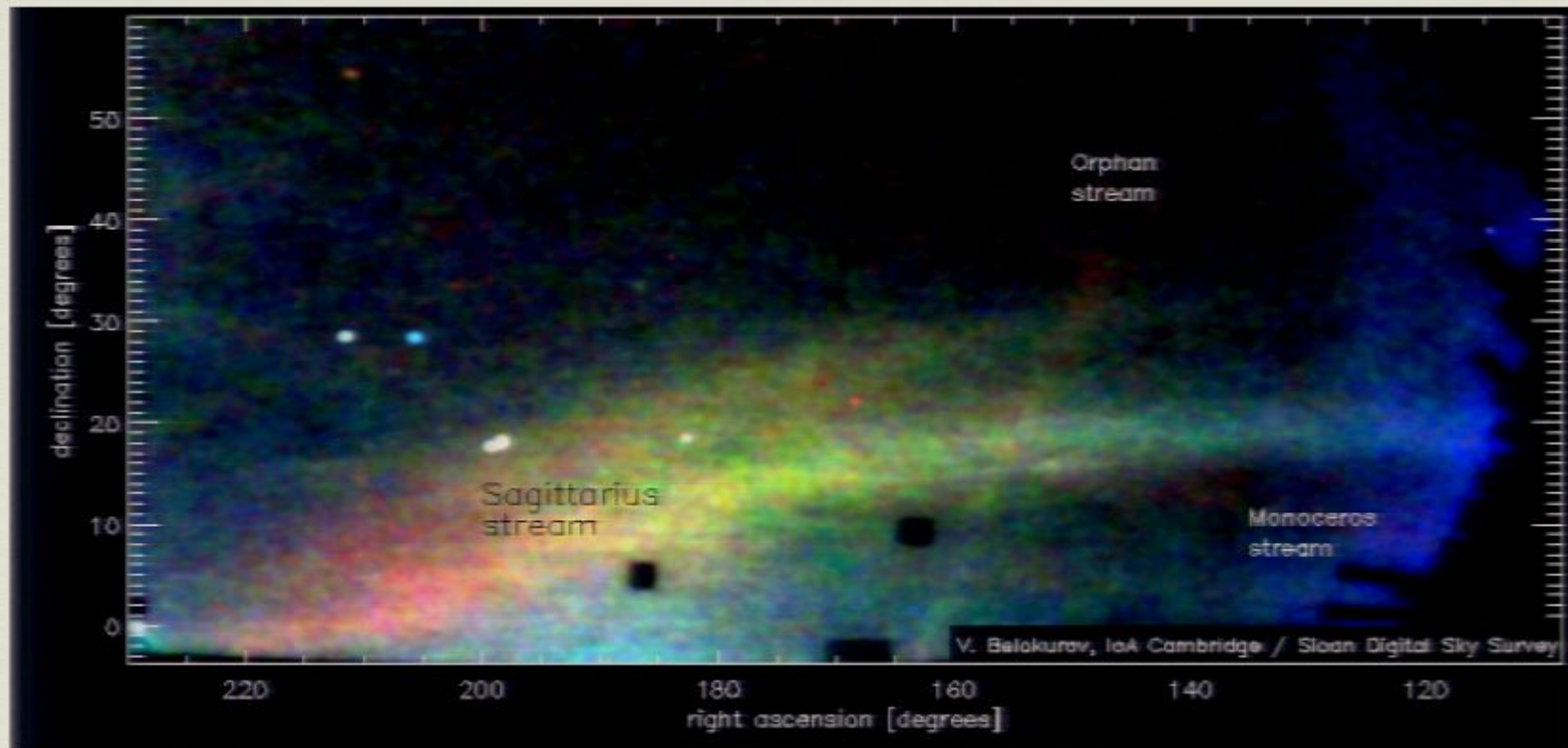
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III. Tidal Streams



- Produced by partial or total tidal disruption of subhalos
- Analogous to observed stellar streams in the Galactic halo
- Distributed along/around orbit of subhalo (c.f. meteor streams)
- Localised in almost 1-D region of 6-D phase-space ($\underline{x}, \underline{v}$)

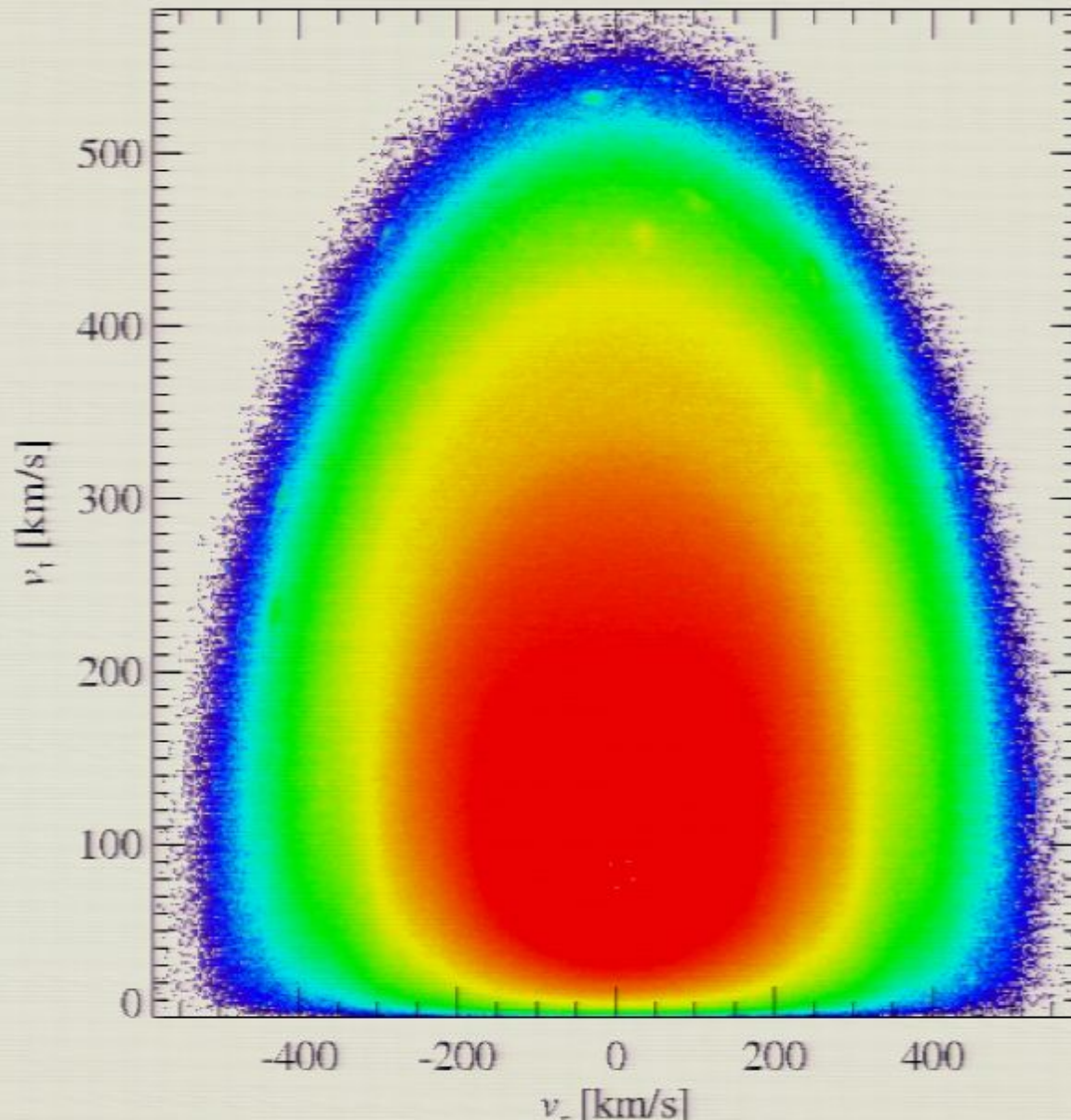
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Dark matter phase-space structure in the inner MW

M. Maciejewski



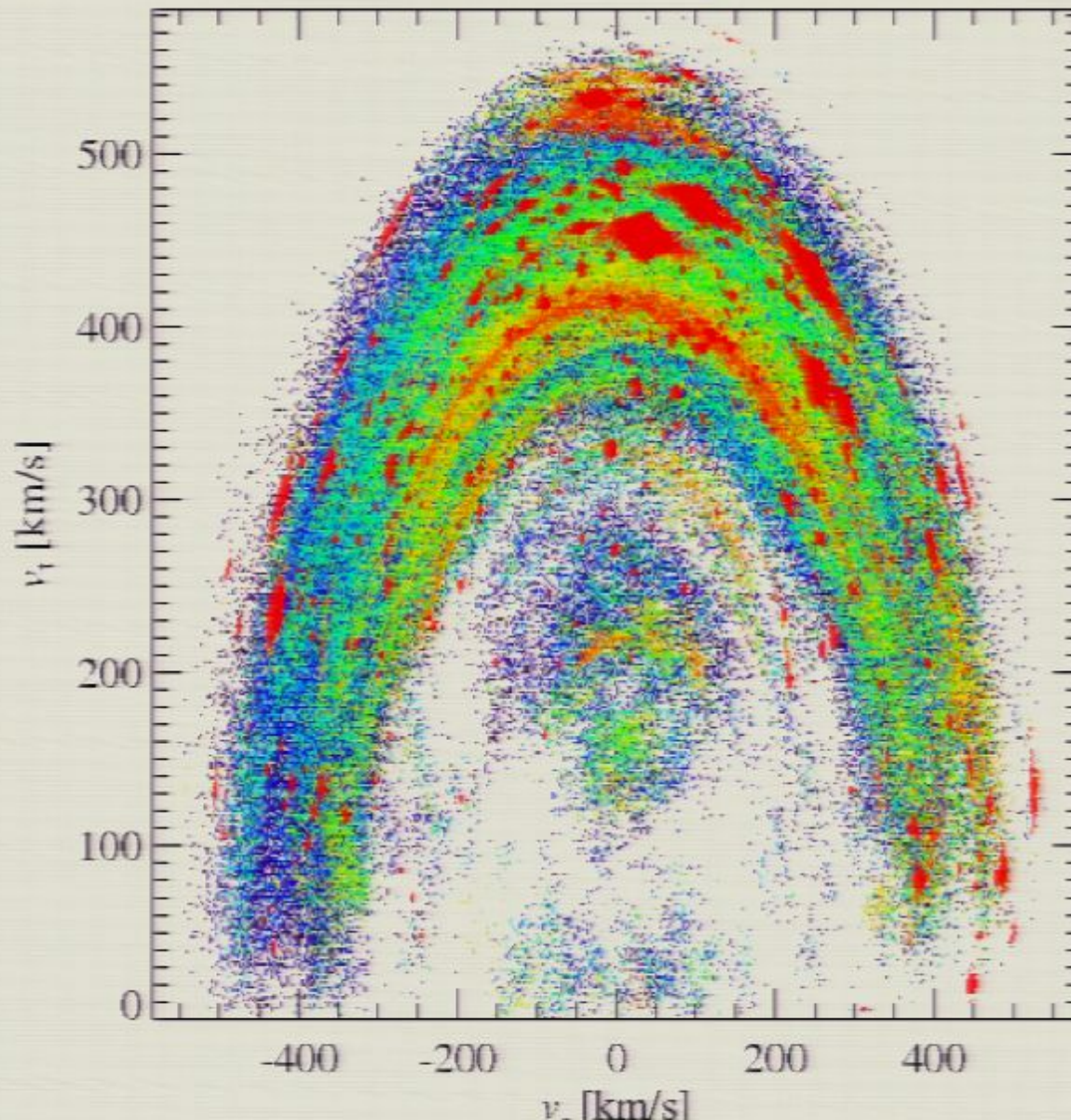
$6 \text{ kpc} < r < 12 \text{ kpc}$

All particles

$N = 3.8 \times 10^7$

Dark matter phase-space structure in the inner MW

M. Maciejewski



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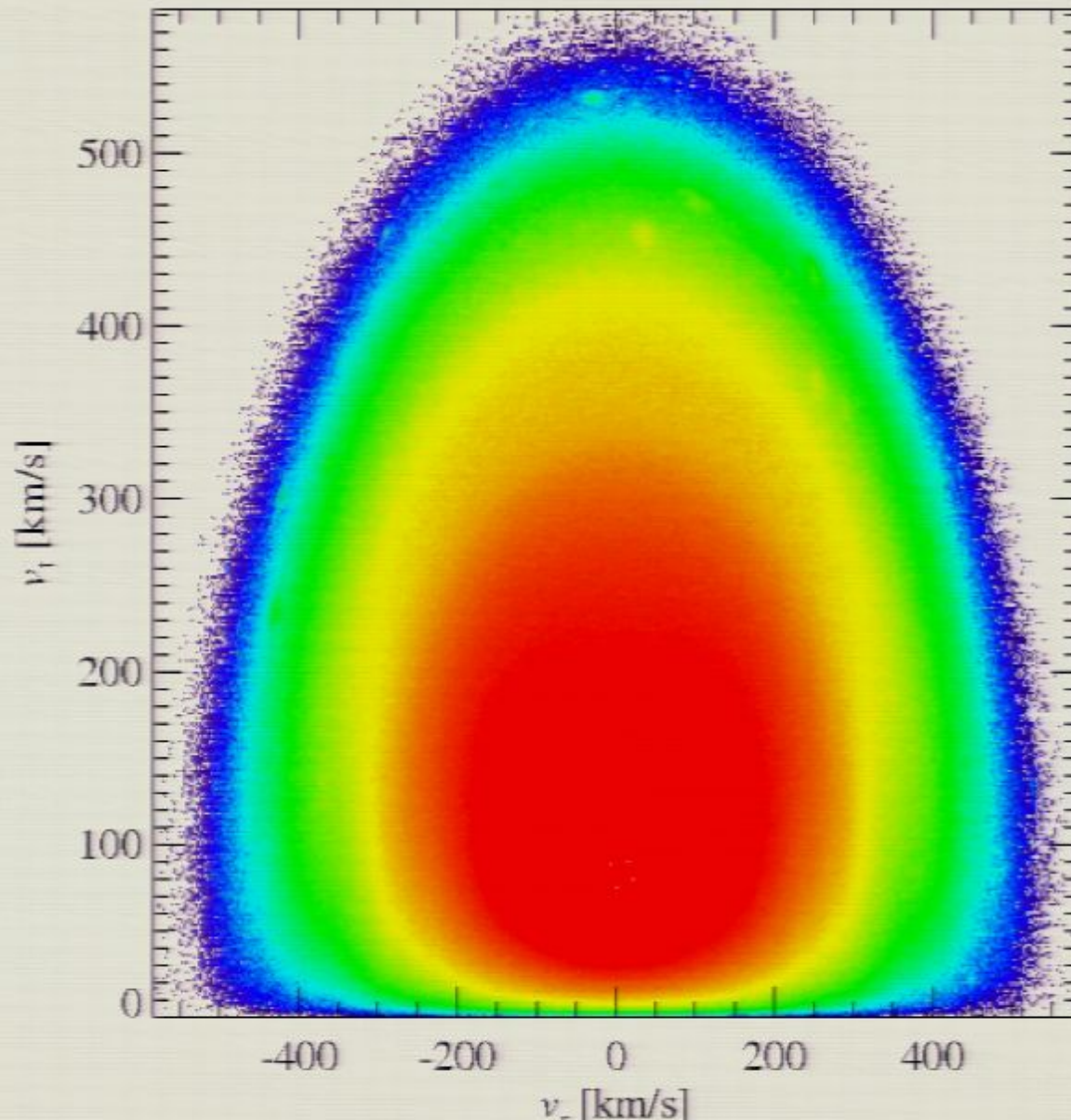
Particles in detected
phase-space structure

$$N = 3.0 \times 10^5$$

$$N_{\text{subhalo}} = 3.9 \times 10^4$$

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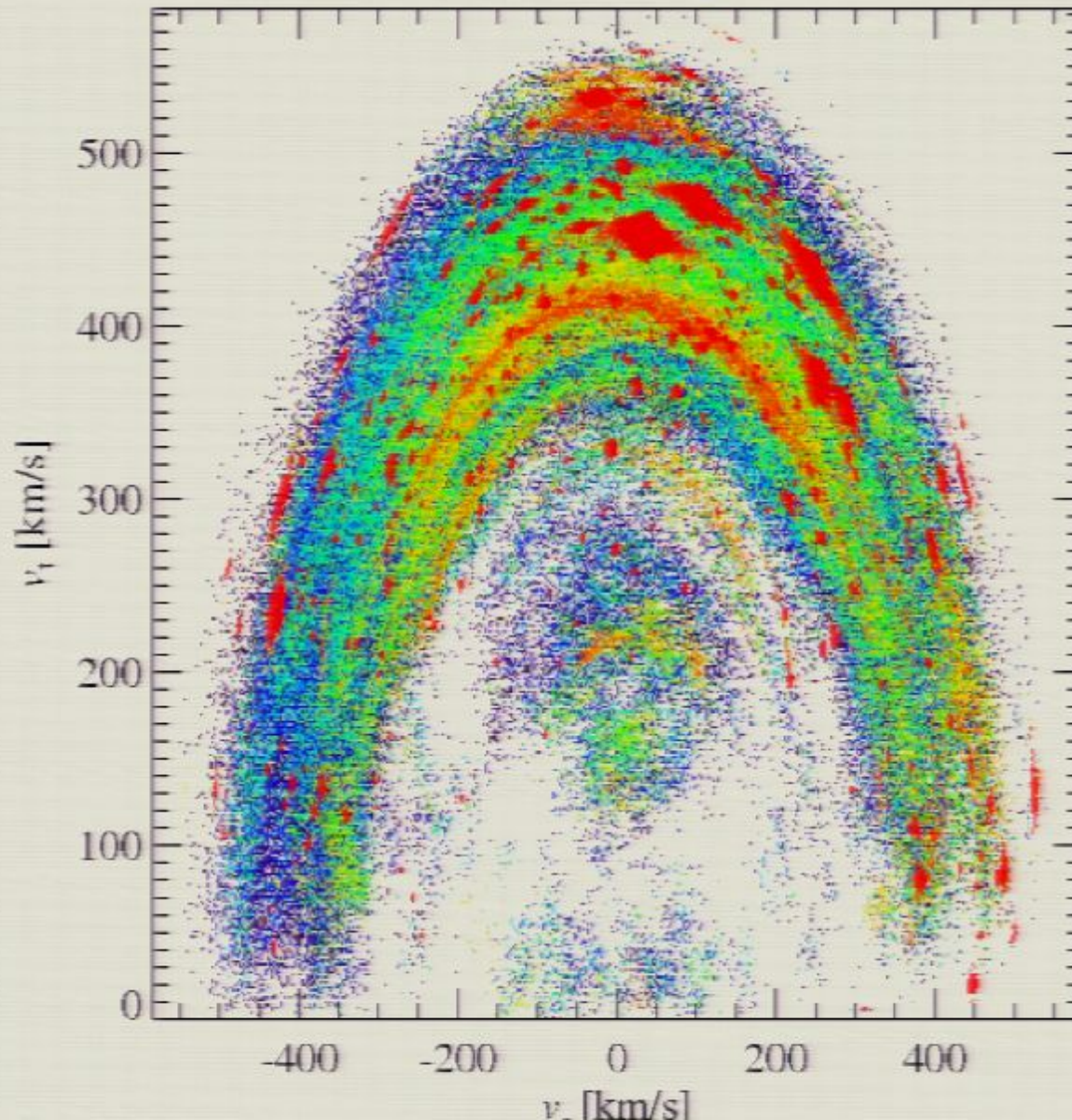
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IV. Fundamental streams

After CDM particles become nonrelativistic, but *before* nonlinear objects form (e.g. $z > 100$) their distribution function is

$$f(\mathbf{x}, \mathbf{v}, t) = \rho(t) [1 + \delta(\mathbf{x}, t)] N [\{\mathbf{v} - \mathbf{V}(\mathbf{x}, t)\} / \sigma]$$


where $\rho(t)$ is the mean mass density of CDM,

$\delta(\mathbf{x}, t)$ is a Gaussian random field with finite variance $\ll 1$

$\mathbf{V}(\mathbf{x}, t) = \nabla \psi(\mathbf{x}, t)$ where $\nabla^2 \psi \propto \delta$,

and N is normal with $\sigma^2 \ll \langle |\mathbf{V}|^2 \rangle$ (today $\sigma \sim 0.1$ cm/s)

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto \mathbf{x} -space is near-uniform.

$Df / Dt = 0$  only a 3-D subspace is occupied at *all* times.

Nonlinear evolution leads to multi-stream structure and caustics

IV. Fundamental streams

Consequences of $Df/Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least one sheet must pass through every point \mathbf{x}
- In nonlinear objects there are typically many sheets at each \mathbf{x}
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f = \text{const.}$ $\longrightarrow \sigma \sim \rho^{-1/3}$
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ \mathbf{T} \cdot \delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot \delta X$; $\mathbf{T} = -\nabla(\nabla\phi)$

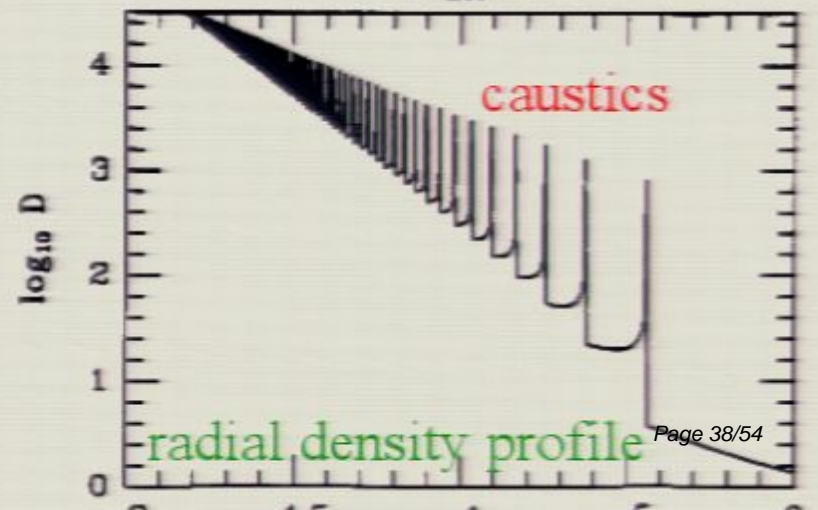
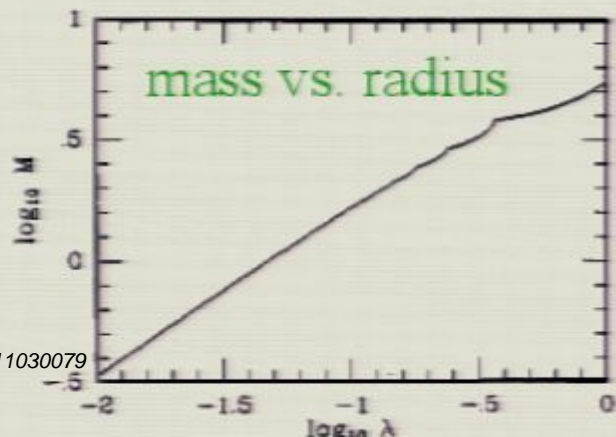
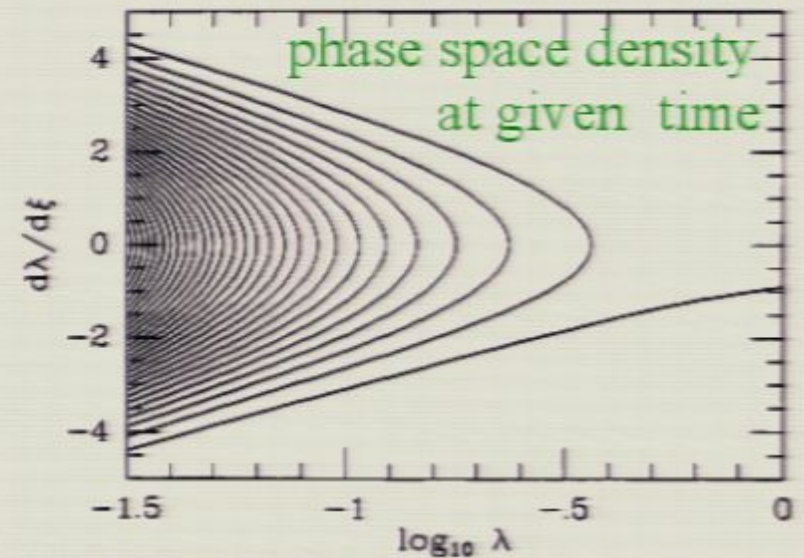
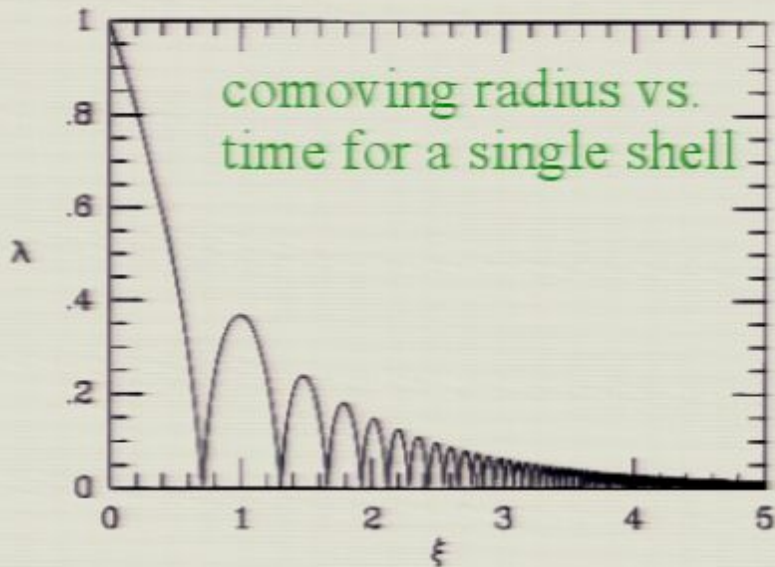
Write $\delta X(t) = D(X_0, t) \cdot \delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot D \quad \text{with } D_0 = \mathbf{I}$$

- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- $\det(D) = 1$ at all times by Liouville's theorem
- For CDM, $1/|\det(D_{\mathbf{xx}})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

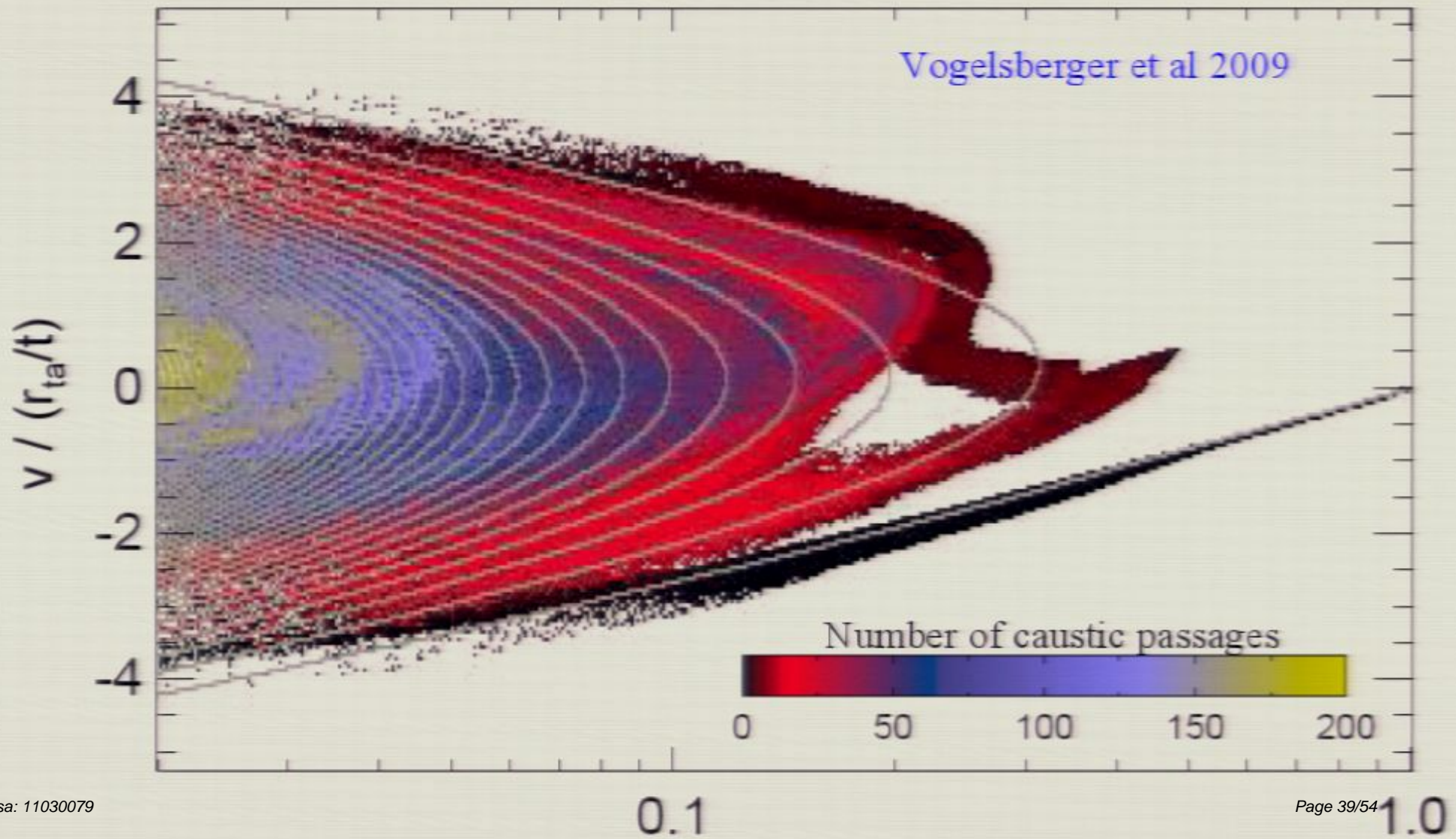
Similarity solution for spherical collapse in CDM

Bertschinger 1985



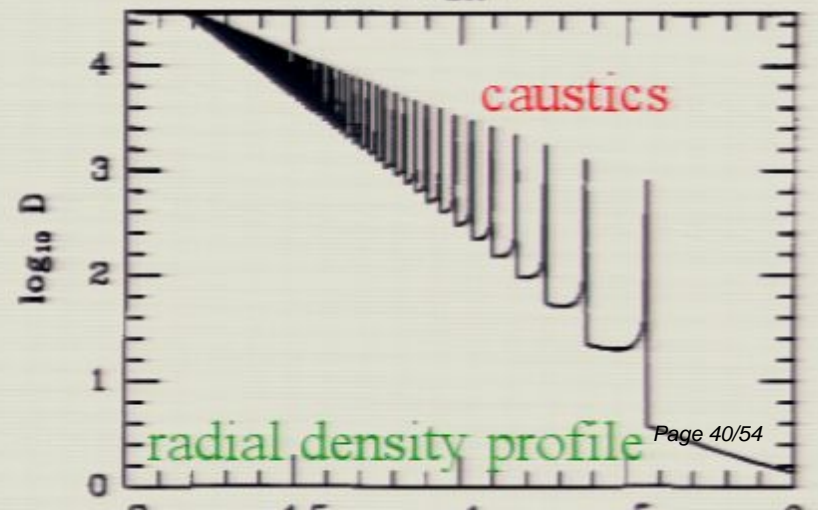
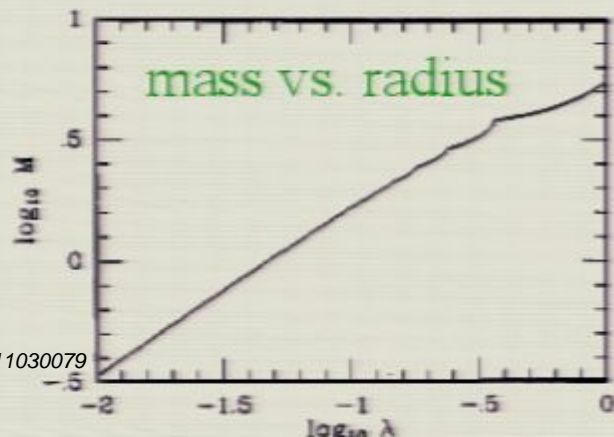
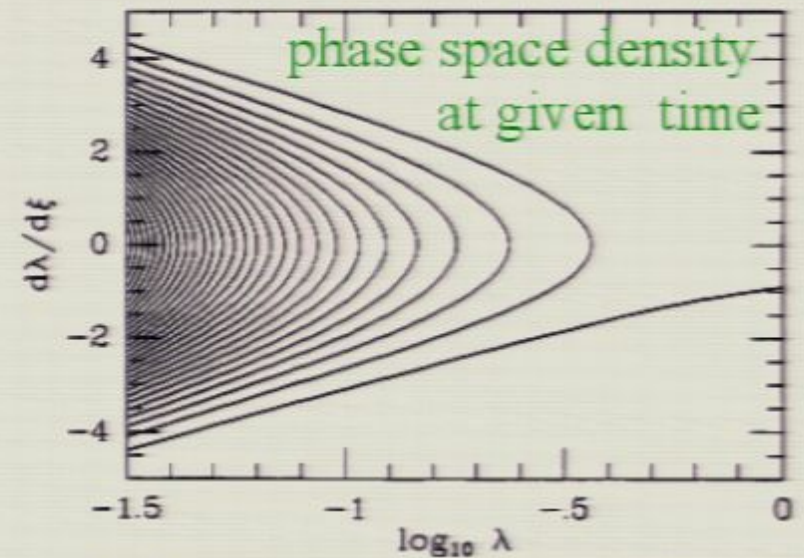
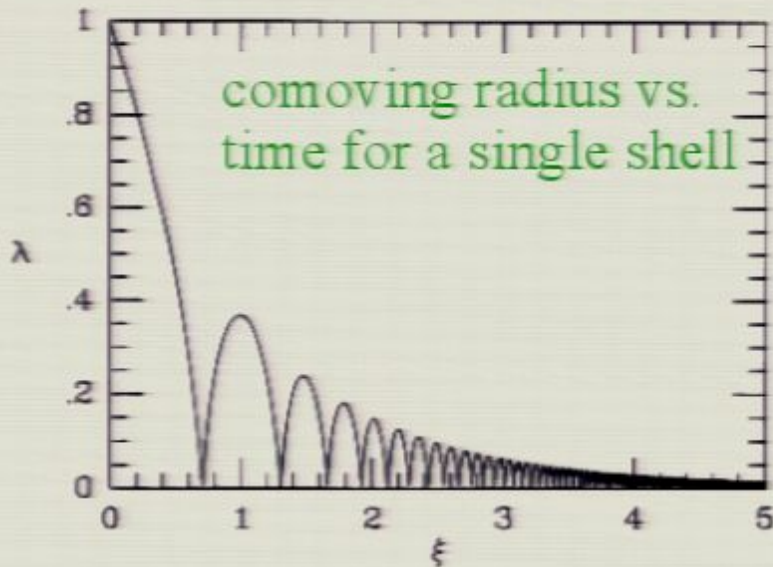
Simulation from self-similar spherical initial condition

Geodesic deviation equation \longrightarrow phase-space structure local to each particle



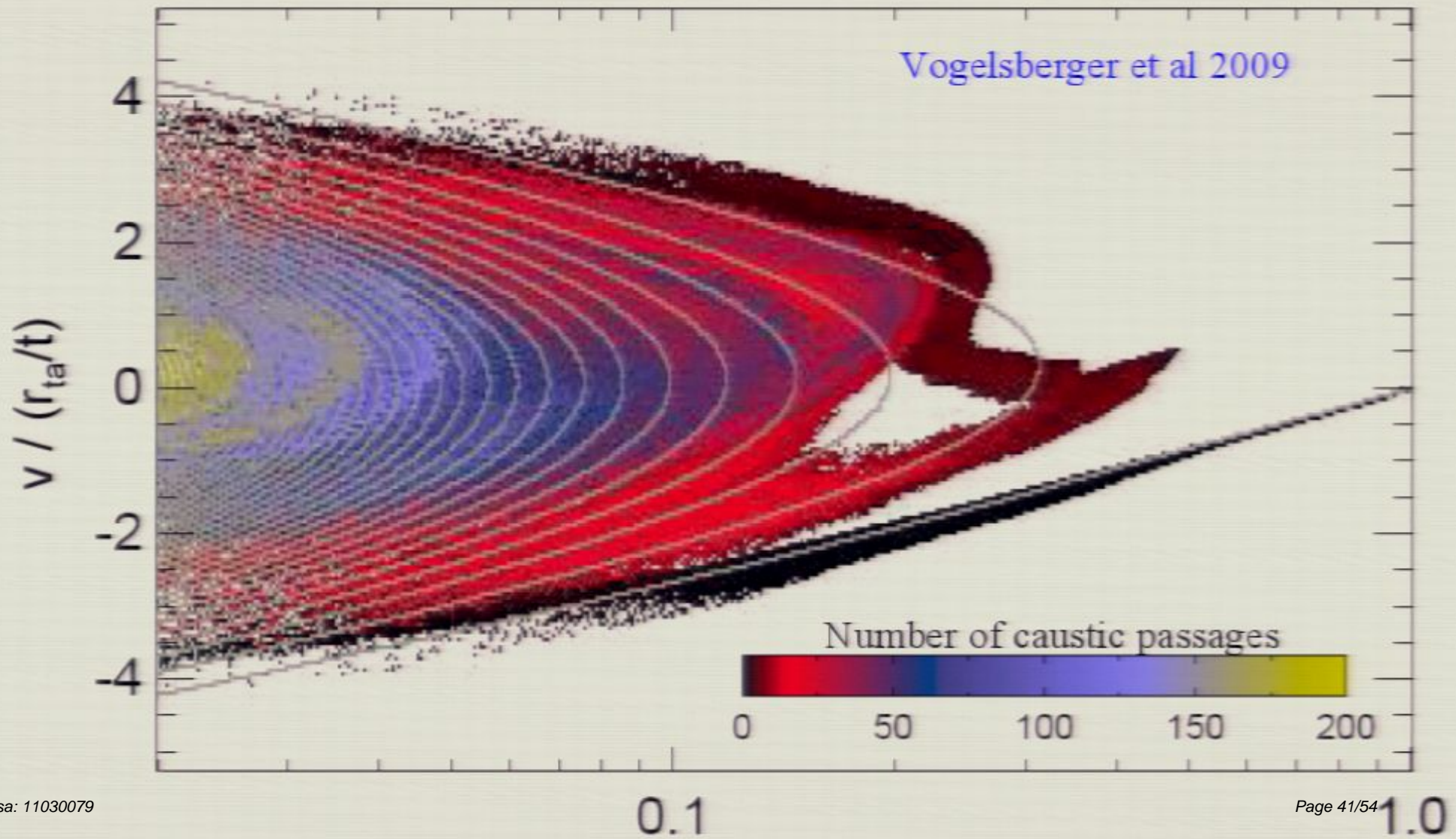
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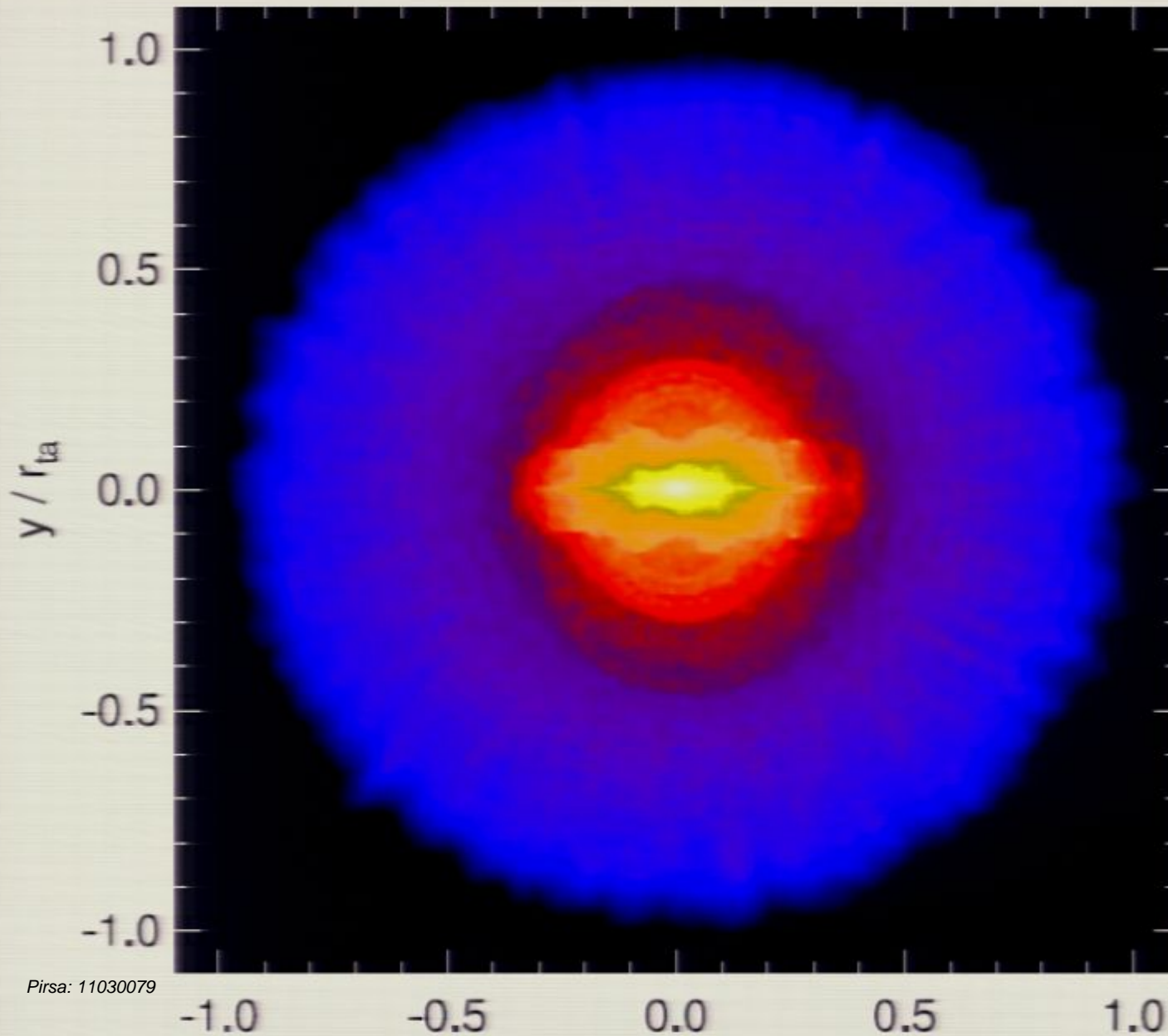
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Simulation from self-similar spherical initial conditions

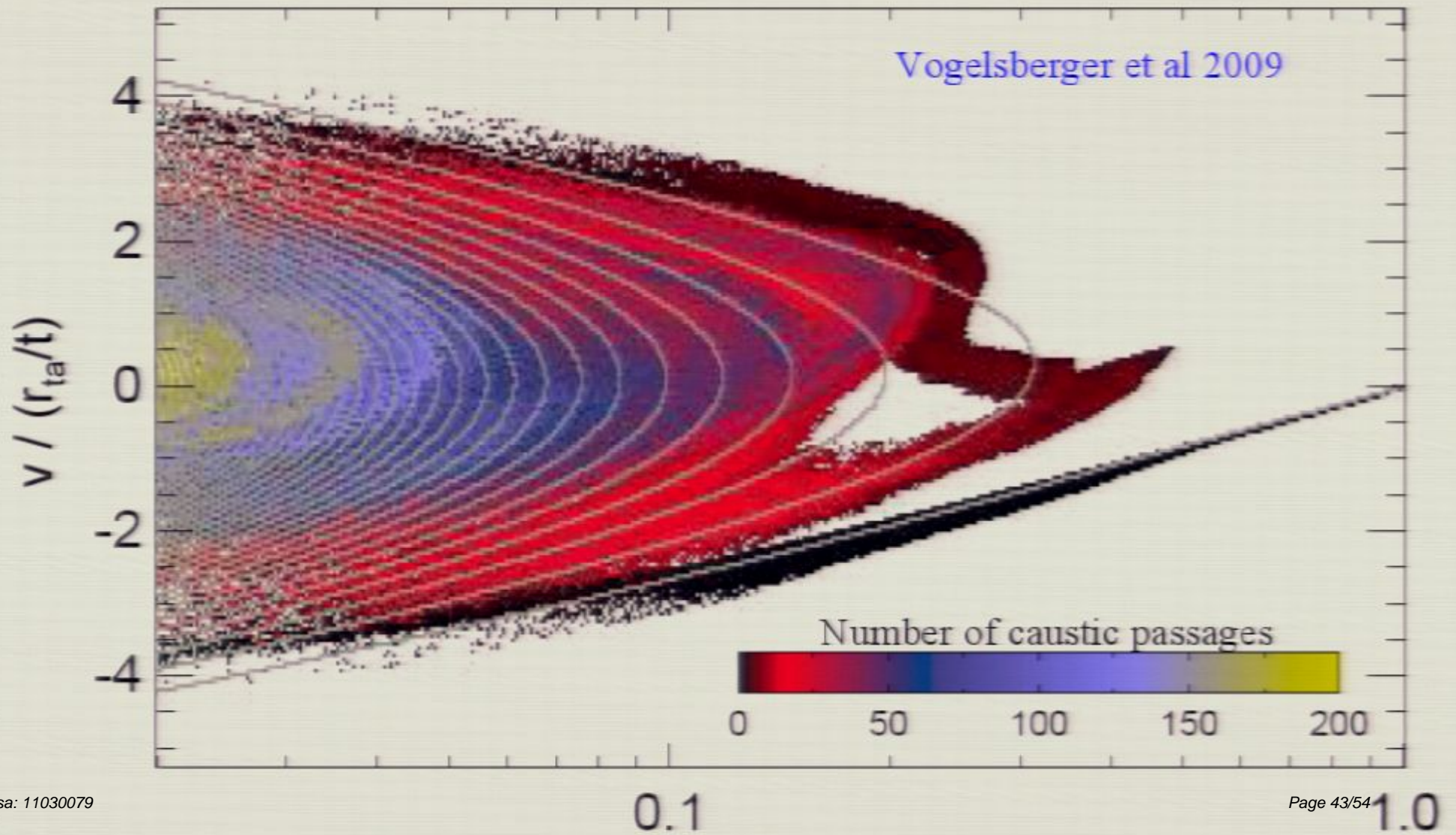
Vogelsberger et al 2009



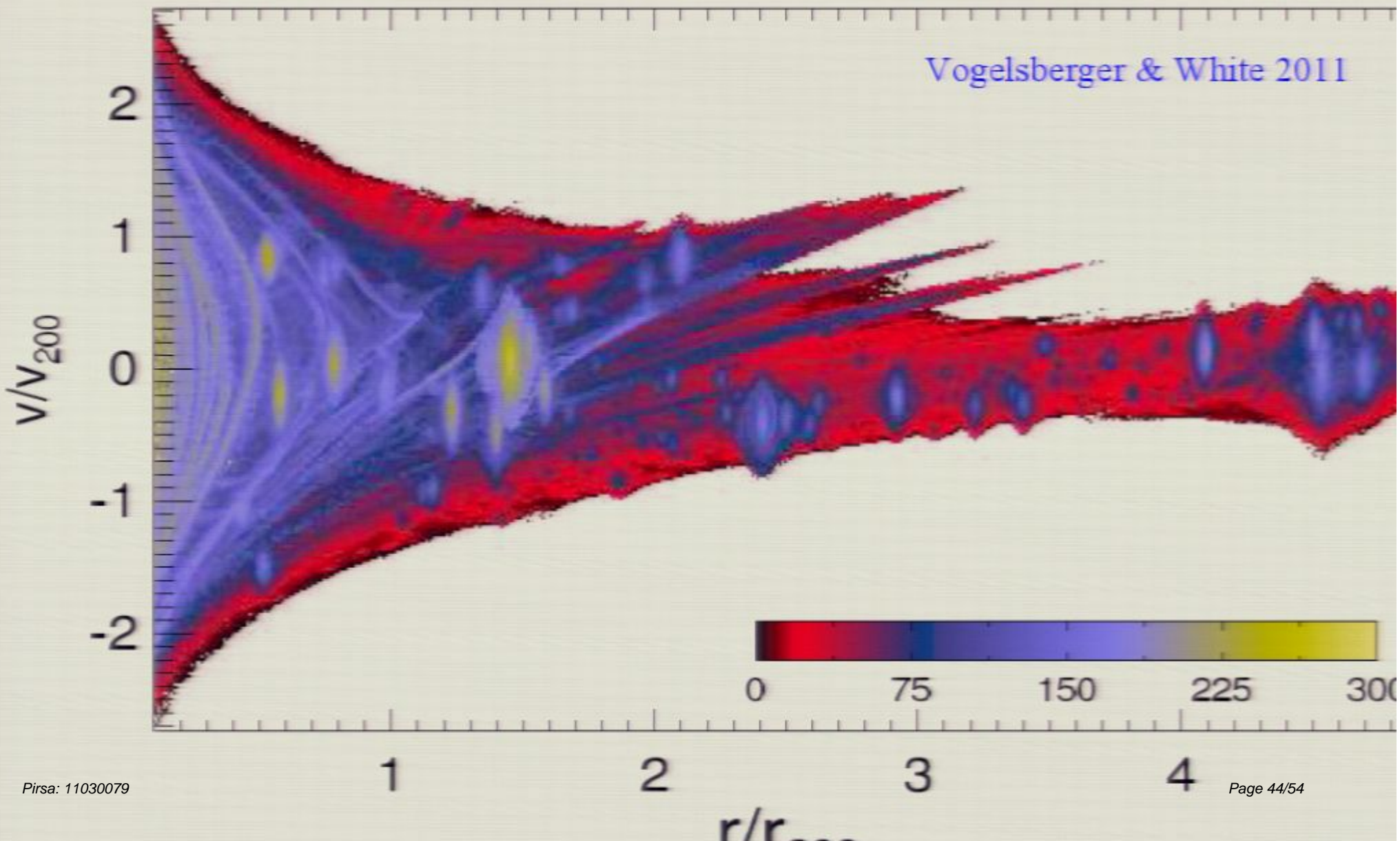
The radial orbit instability leads to a system which is strongly prolate in the inner nonlinear regions

Simulation from self-similar spherical initial condition

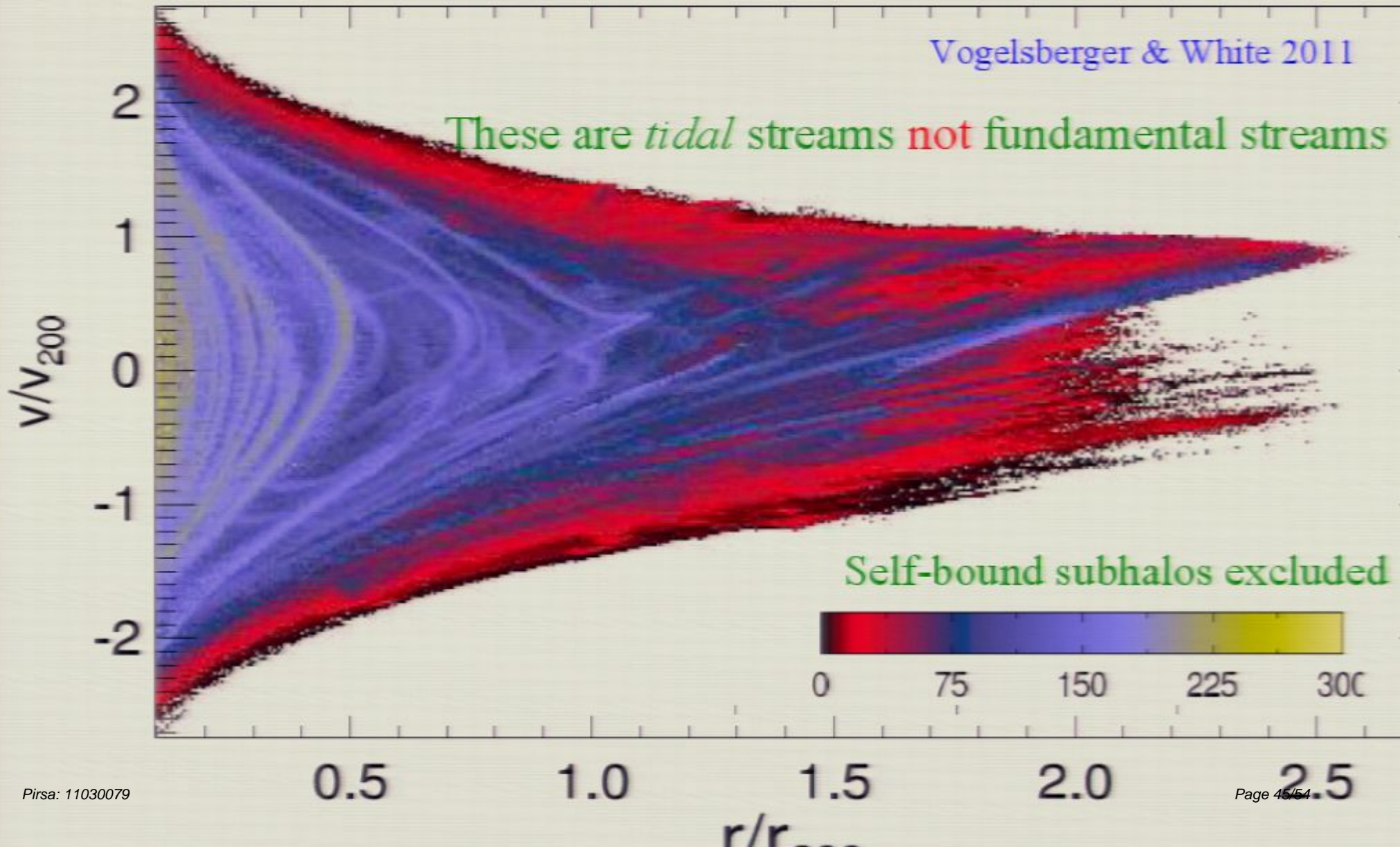
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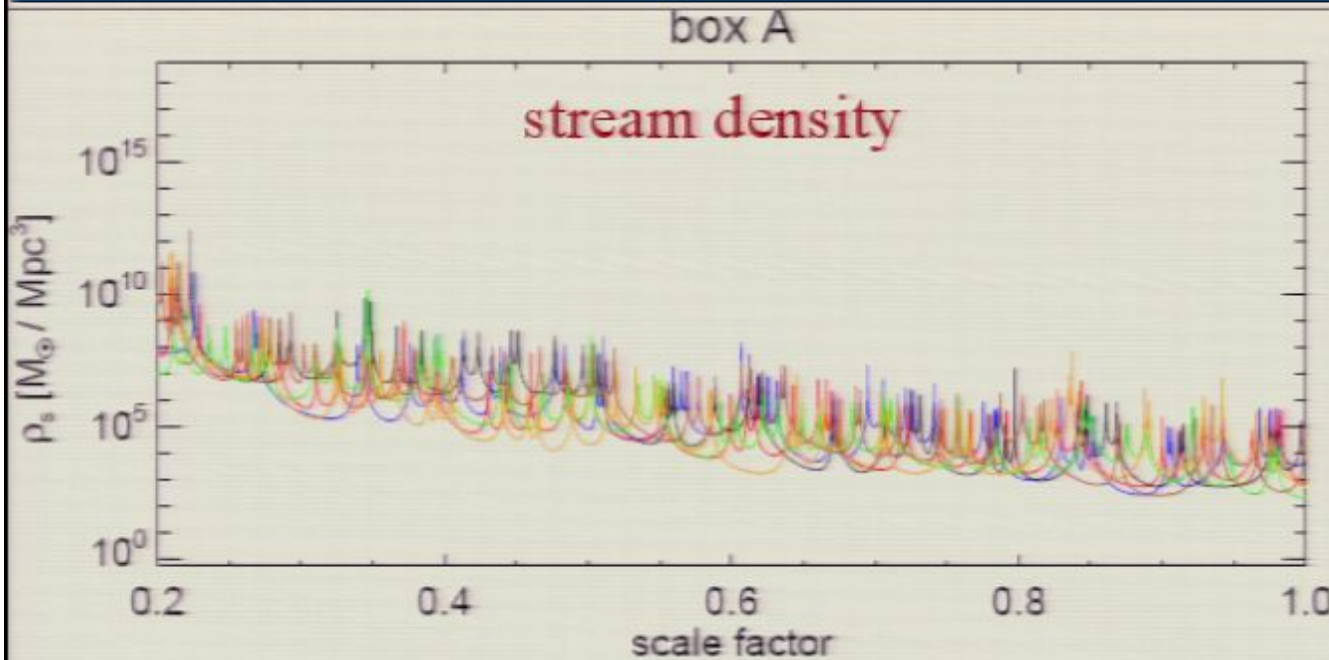
Caustic crossing counts in a Λ CDM Milky Way halo



Caustic crossing counts in a Λ CDM Milky Way halo



Stream density variations along orbits in a Λ CDM halo

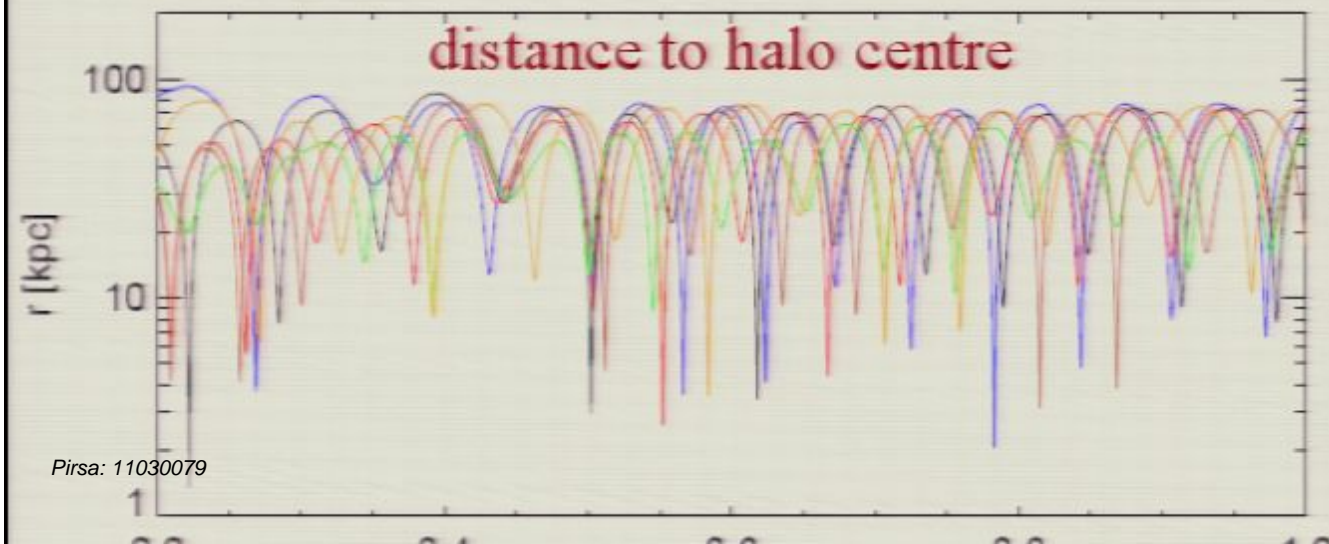


Orbital properties for six particles chosen so:

-- in main halo at $z = 4$

-- 40 caustics in $4 > z > 0$

-- typical drop in ρ_{stream}

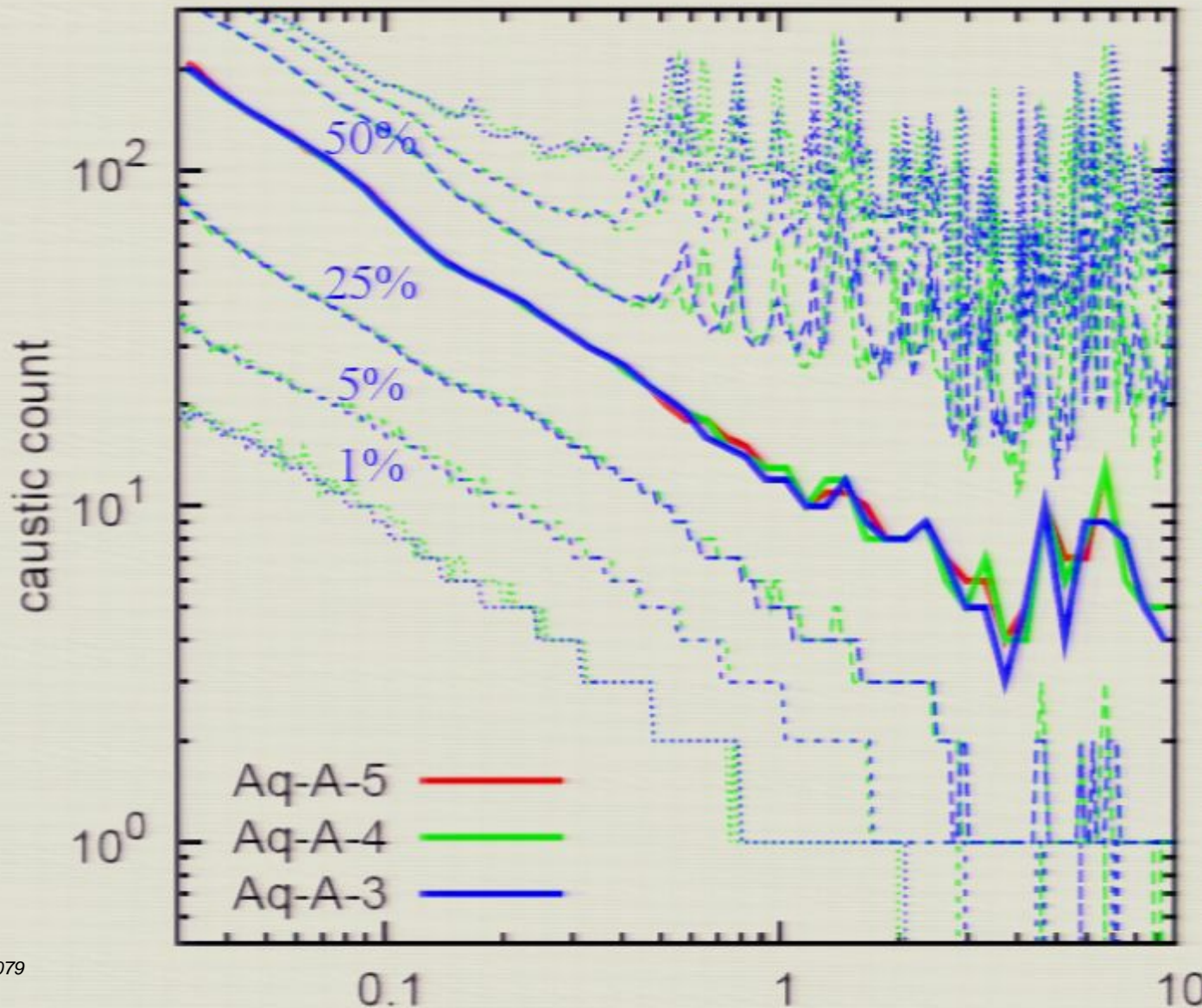


An average of 3 caustic crossings per orbit

Large drops in minimum ρ_{stream} often follow close pericentre passages

Caustic count profiles for Aquarius halos

Vogelsberger & White 2011



Note agreement
simulations of the
same object with

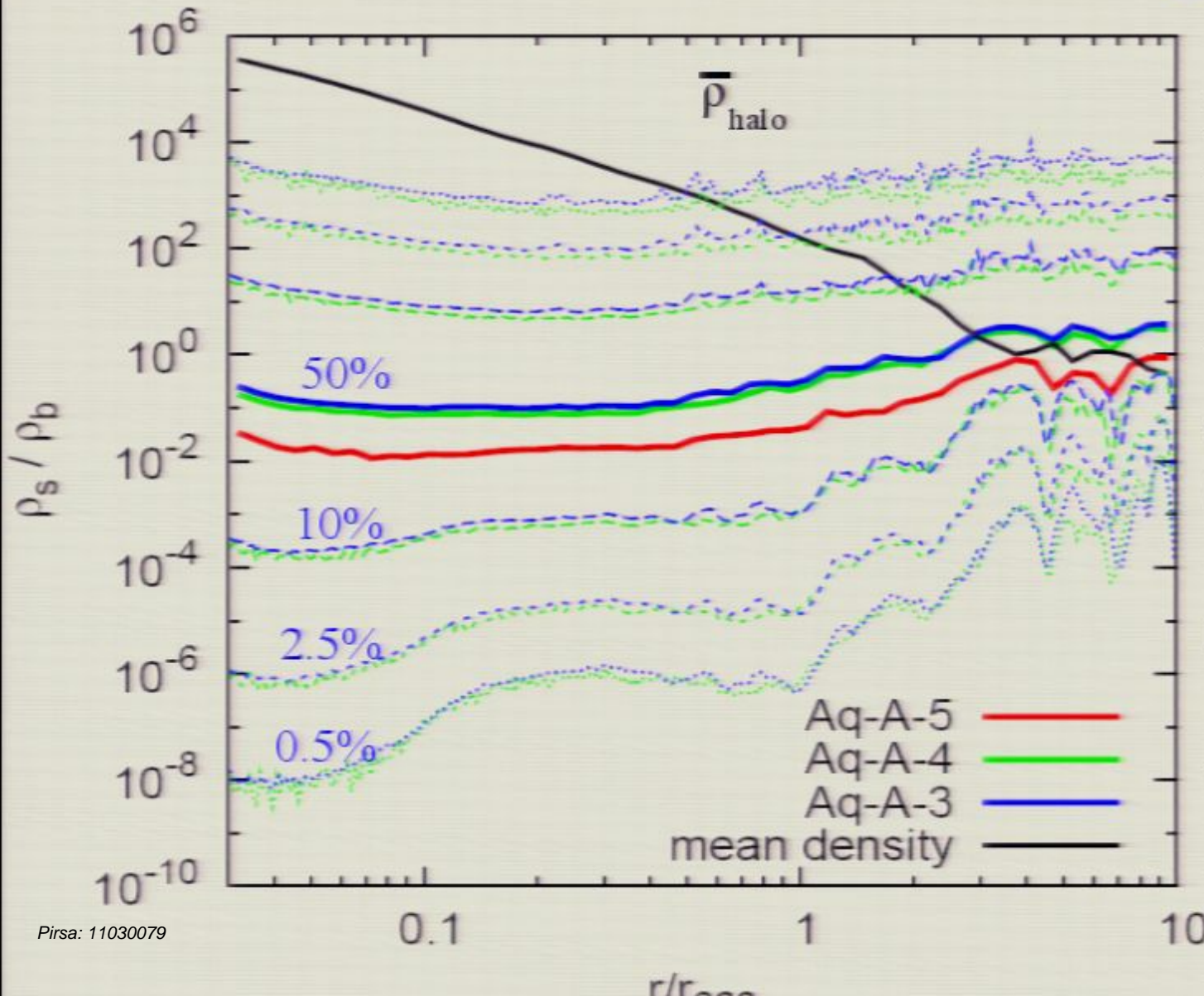
$$N = 8.1 \times 10^5$$

$$N = 6.4 \times 10^6$$

$$N = 5.1 \times 10^7$$

Stream density distribution in Aquarius halos

Vogelsberger & White 2011



Note the convergence with varying N .

With conventional methods detecting a stream with

$$\rho_{\text{stream}} = 10^{-8} \rho_b$$

requires particle mass

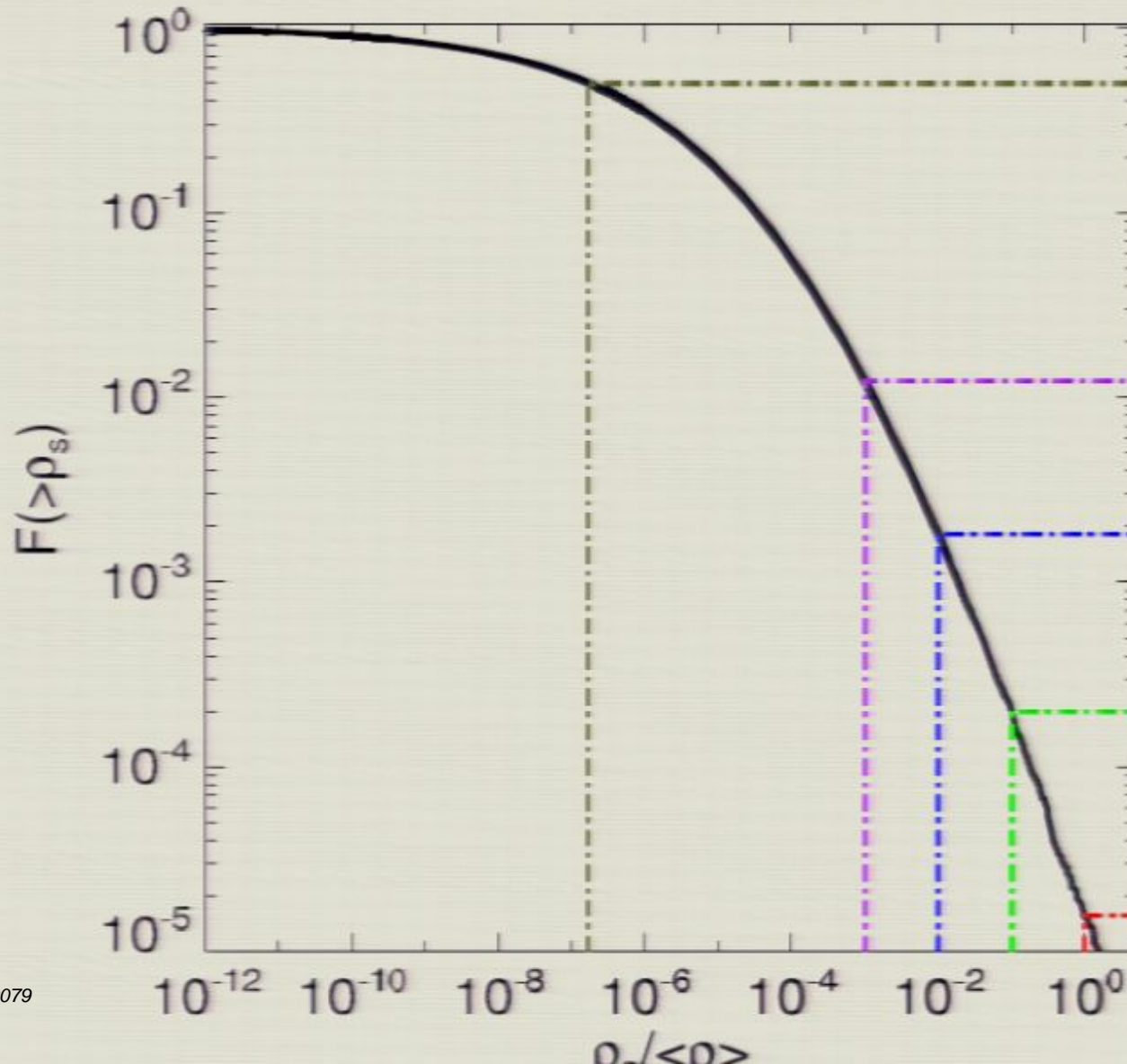
$$m_p \sim 10^{-7} M_{\odot},$$

thus a simulation with

$$N \sim 10^{20}$$

Stream density distribution at the Sun

Vogelsberger & White 2011



Cumulative stream density distribution for particles with $7 \text{ kpc} < r < 13 \text{ kpc}$

Probability that the Sun is in a stream with density $> X \langle \rho \rangle$ is P

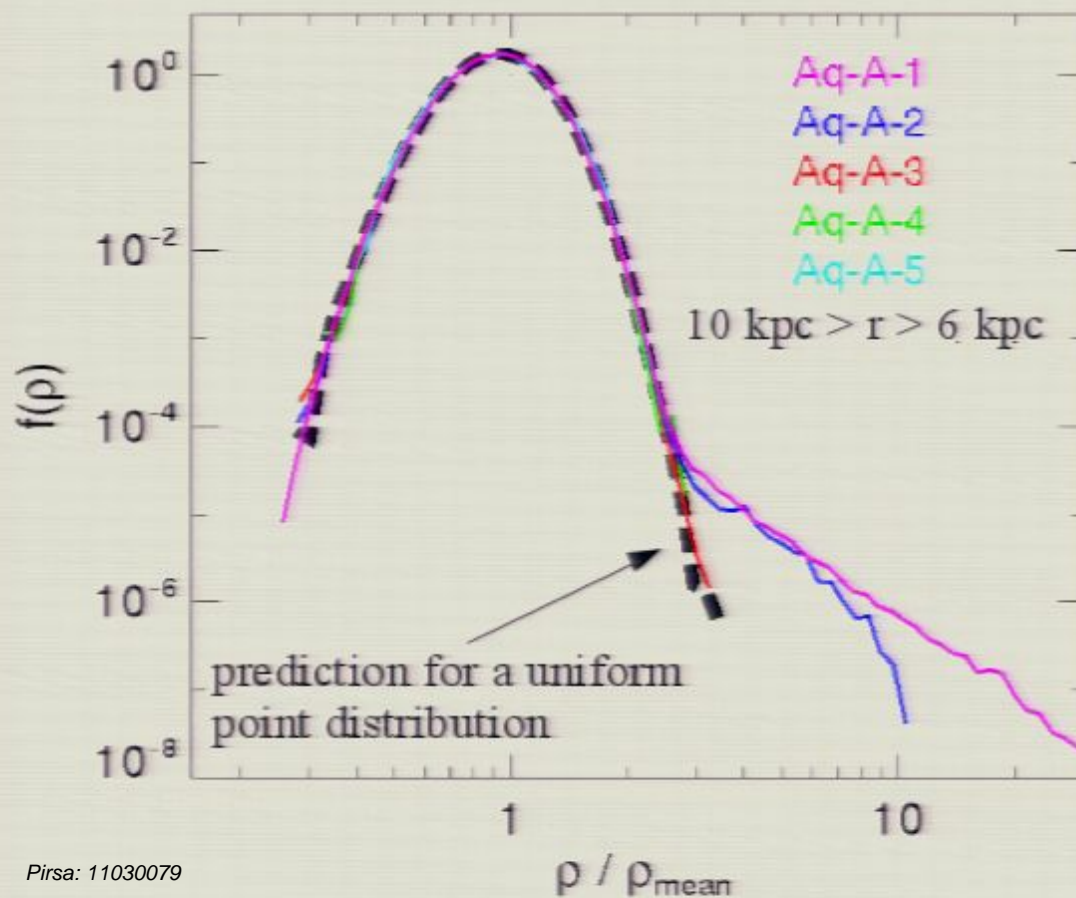
X	P
1.0	0.00001
0.1	0.002
0.01	0.2
0.001	~1

A typical particle has

$$\rho_{\text{stream}} \sim 10^X \langle \rho \rangle$$

Local density in the inner halo compared to a smooth ellipsoidal model

Vogelsberger et al 2008



- Estimate a density ρ at each point by adaptively smoothing using the 64 nearest particles
- Fit to a smooth density profile stratified on similar ellipsoids
- The chance of a random point lying in a substructure is $< 10^{-4}$
- The *rms* scatter about the smooth model for the remaining points only about 4%

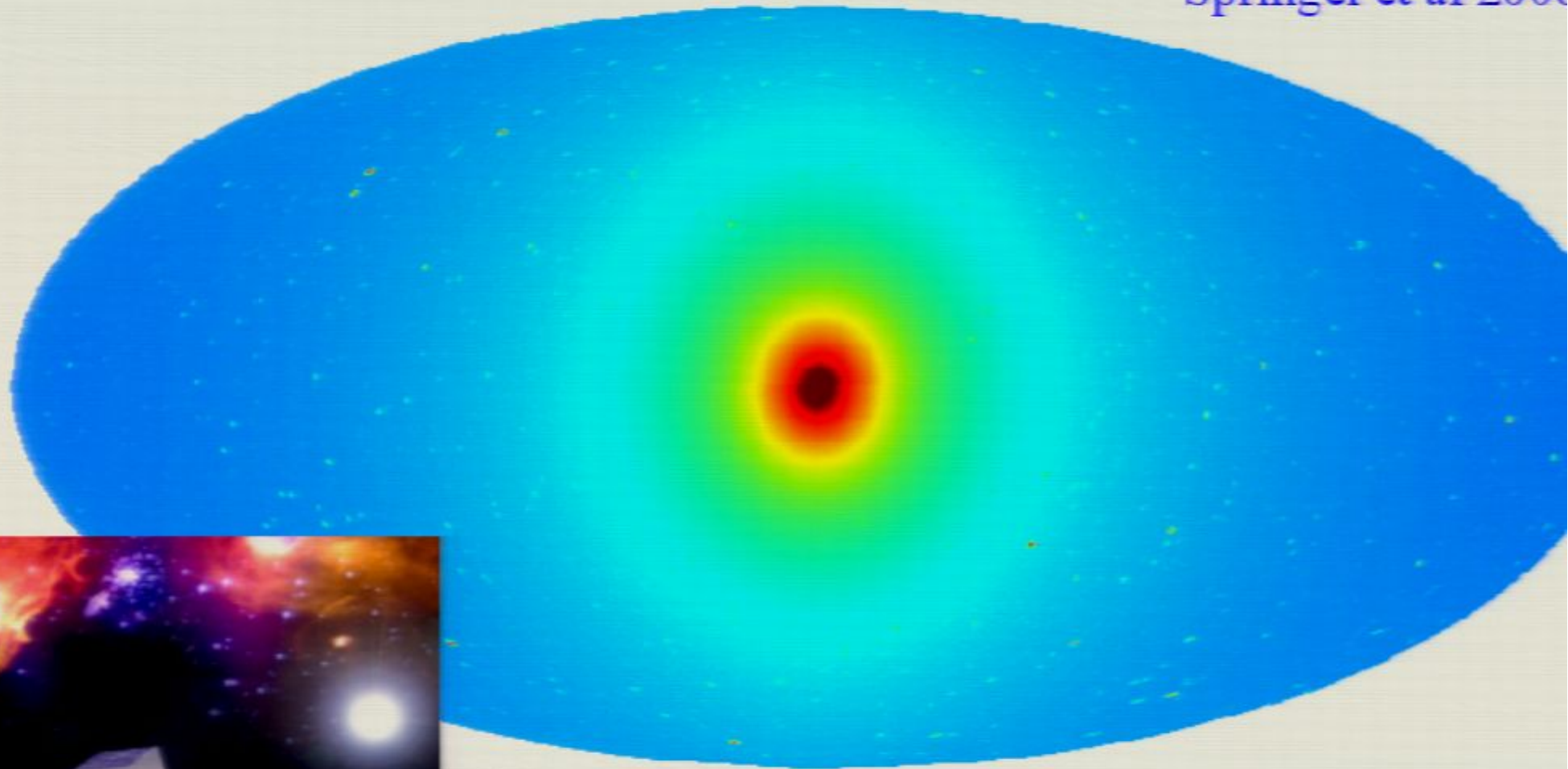
Conclusions for direct detection experiments

- With more than 99.9% confidence the Sun lies in a region where the DM density differs from the smooth mean value by $< 20\%$
- The local velocity distribution of DM particles is similar to a trivariate Gaussian with no measurable “lumpiness” due to individual DM streams
- The strongest stream at the Sun should contain about 10^{-3} of the local DM density. Its energy width is $\Delta E/E < 10^{-10}$ so it would be detectable as a “spectral line” in an axion experiment.
- The energy distribution of DM particles should contain broad features with $\sim 20\%$ amplitude which are the fossils of the detailed assembly history of the Milky Way's dark halo



total emission

Springel et al 2008



-0.50  2.0 Log(Intensity)

Maybe the annihilation of Dark Matter will be seen by Fermi?



Conclusions: fundamental streams and caustics

- Integration of the GDE can augment the ability of Λ CDM simulations to resolve fine-grained structure by over 10 orders of magnitude
- Fundamental streams and their associated caustics will have no significant effect on direct and indirect Dark Matter detection experiments
- The most massive stream at the Sun should contain roughly 0.001 of the local DM density and would have an energy spread $\Delta E/E < 10^{-10}$. It might be detectable in an axion experiment

No Signal

VGA-1