

Title: The Konishi multiplet at strong coupling

Date: Mar 01, 2011 11:00 AM

URL: <http://pirsa.org/11030078>

Abstract: I will describe a method to compute from first principles the anomalous dimension of short operators in N=4 super Yang-Mills theory at strong coupling, where they are described in terms of superstring vertex operators in an anti-de Sitter background. I will focus on the Konishi multiplet, dual to the first massive level of the superstring, and compute the one-loop correction to its anomalous dimension at strong coupling, using the pure spinor formalism for the superstring.

The Konishi multiplet @ strong coupling

with BRENNO CARLINI VALLILO

arXiv: 1102.1219

$$\Delta - \Delta_0 = 2\sqrt[3]{\lambda} - 4 + \frac{2}{\sqrt{\lambda}}$$

Anomalous dimensions
of invariant local operators

$\mathcal{O}(x)$

Anomalous dimensions

of invariant local operators

$$Q(x) = \text{tr} \phi^i \phi^i(x) \quad \text{6 scalars } N=4 \text{ SYM}$$

Anomalous dimensions
gauge invariant local operators

$$\mathcal{O}(x) = \text{tr} \phi^i \phi^i(x) \quad \text{6 scalars } N=4 \text{ SYM}$$

Anomalous dimensions
gauge invariant local operators

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FT

$\text{PSU}(2,2|4) \supset \text{DILATATION } D$

Anomalous dimensions
gauge invariant local operators

$$\mathcal{O}(x) = \text{tr} \phi^i \phi^i(x) \quad 6 \text{ scalars } N=4 \text{ SYM}$$

(FT) $\text{PSU}(1,2|4) \supset \text{DILATATION } D$

conformal dim $[D, \mathcal{O}_A(x)] = i\Delta \mathcal{O}(x)$

conformal dim $[D, \mathcal{O}_\Delta(x)] = i\Delta \mathcal{O}(x)$

$\langle \mathcal{O}_\Delta(x) \mathcal{O}(y) \rangle$

conformal dim $[D, \mathcal{O}_\Delta(x)] = i\Delta \mathcal{O}(x)$

$$\langle \mathcal{O}_\Delta(x) \mathcal{O}_\Delta(y) \rangle \sim \frac{1}{|x-y|^{2\Delta}}$$

What do we know about Δ ? PLANNAR
LIMIT

What do we know about Δ ? PLANK
LIMIT
 $\Delta = \Delta_0$ "ongoing dim"

What do we know about Δ ? PLANK
LIMIT

$$\Delta = \Delta_0 \quad \text{"confining dim"}$$

$$\lambda = g_m^2 N$$

What do we know about Δ ? PLANK
LIMIT

$$\Delta = \Delta_0 \quad \text{"engineering dim"}$$

$$\lambda = g_m^2 N$$

$$\gamma \equiv \Delta - \Delta_0 \quad \underline{\text{anomalous dim}}$$

$$\gamma < 1$$

What do we know about Δ ? PLANK
LIMIT

$$\Delta = \Delta_0 \quad \text{"engineering dim"}$$

$$\lambda = g_m^2 N$$

$$\gamma \equiv \Delta - \Delta_0 \quad \text{anomalous dim}$$

$$\gamma = c_1 \lambda + c_2 \lambda^2 + \dots$$

What do we know about Δ ? PLANAR
LIMIT

$$\Delta = \Delta_0 \quad \text{"engineering dim"}$$

$$\lambda = g_m^2 N$$

$$\gamma \equiv \Delta - \Delta_0 \quad \text{"anomalous dim"}$$

$$\boxed{\lambda \ll 1}$$

$$\gamma = c_1 \lambda + c_2 \lambda^2 + \dots \quad \text{"weak coupl."}$$

$$\boxed{\lambda \gg 1}$$

"strong coupl."

WAR
MIT
N

$$|\lambda| \gg 1$$

$$\delta = 0$$

BPS

WAR
MIT
N

$$|\lambda| \gg 1$$

$$\gamma = 0$$

BPS

$$\gamma \sim \sqrt{\lambda}$$

short

WAR
MIT
N

$$\boxed{|\lambda| \gg 1}$$

$$\gamma = 0 \quad \underline{\text{BPS}}$$

$$\gamma \sim \sqrt{\lambda} \quad \text{"short non-BPS"}$$

$$\gamma \sim \sqrt{\lambda} \quad \text{"large non-BPS"}$$

WAR
MIT
N

$$\boxed{\lambda \gg 1}$$

$$\begin{array}{l} \gamma = 0 \\ \gamma \sim \sqrt{\lambda} \\ \gamma \sim \sqrt{x} \end{array}$$

BPS

"short non-BPS"

"large non-BPS"



WAR
MIT
N

$$N=4 \quad \mathbb{R}^{11} = \mathbb{I}B$$

WAR
MIT
N

$$N=4 \text{ IIB} = \text{IIB on } \text{AdS}_5 \times S^5$$

WAR
MIT
N

$$N=4 \quad \mathcal{N}=1 \quad = \text{IIB on } \text{AdS}_5 \times S^5$$

AdS/CFT

" $R^2/4$

$$\lambda \gg 1$$

WAR
MIT
N

$N=4$ SYM = IIB on $AdS_5 \times S^5$ AdS/CFT

$$U(1) = R^2/\alpha'$$

$$R_{\alpha'}^2 \gg 1$$

$$\lambda \gg 1$$

WAR
MIT
N

$N=4$ SYM = IIB on $AdS_5 \times S^5$ AdS/CFT

$$U\lambda = R^2/\alpha'$$

$$\lambda \gg 1$$

$R^2/\alpha' \gg 1 \Rightarrow$ perturbative
string theory

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N

$N=4$ SYM = IIB on $AdS_5 \times S^5$ AdS/CFT

$$U\lambda = R^2/\alpha'$$

$\leftrightarrow R^2/\alpha' \gg 1 \Rightarrow$ perturbative string theory

$\lambda:$

Δ

WAR
MIT
N

$N=4$ SYM = IIB on $AdS_5 \times S^5$ AdS/CFT

$$U\lambda = R^2/\alpha'$$

$$\lambda \gg 1$$

$$\leftrightarrow R_{\alpha'}^2 \gg 1 \Rightarrow \text{perturbative string theory}$$

$$\Delta = E \quad \text{energies of}$$

WAR
MIT
N

$N=4$ SYM = IIB on $AdS_5 \times S^5$ AdS/CFT

$$U\lambda = R^2/\alpha'$$

$$\lambda \gg 1$$

$$\leftrightarrow R_{\alpha'}^2 \gg 1 \Rightarrow \text{perturbative string theory}$$

$$\Delta = E \quad \text{energies of strings}$$

R
T

$$|\lambda| \gg 1$$

$$\begin{aligned} \gamma &= 0 \\ \gamma &\sim \sqrt{\lambda} \\ \gamma &\sim \sqrt{x} \end{aligned}$$

BPS

"short non-BPS"

"large non-BPS"



R
T

$$\boxed{\lambda \gg 1}$$

sym

$$\boxed{\begin{aligned} \gamma &= 0 \\ \gamma &\sim \sqrt{\lambda} \\ \gamma &\sim \sqrt{x} \end{aligned}}$$

BPS

"short non-BPS"

"large non-BPS"

string



R
T

$$\boxed{\lambda \gg 1}$$

SYM

$$\boxed{\begin{array}{l} \gamma = 0 \\ \gamma \sim \sqrt{\lambda} \\ \gamma \sim \sqrt{x} \end{array}}$$

BPS

"short non-BPS"

"large non-BPS"

$\mathcal{N}=4$ string

sugra sector "max"

R
T

$$\boxed{\lambda \gg 1}$$

SYM

$$\boxed{\begin{aligned} \gamma &= 0 \\ \gamma &\sim \sqrt{\lambda} \\ \gamma &\sim \sqrt{x} \end{aligned}}$$

BPS

"short non-BPS"

"large non-BPS"

$\mathcal{N}=1$

sugra sector = "massless"

R
T

$$\boxed{\lambda \gg 1}$$

SYM

$$\boxed{\begin{array}{l} \gamma = 0 \\ \gamma \sim \sqrt{\lambda} \\ \gamma \sim \sqrt{x} \end{array}}$$

BPS

"short non-BPS"

"large non-BPS"

String

sugra sector = "massless"

"massive string spectrum"

solitonic strings
(very large quantum numbers)

R
T

$$\boxed{\lambda \gg 1}$$

SYM

$$\boxed{\begin{aligned} \gamma &= 0 \\ \gamma &\sim \sqrt{\lambda} \\ \gamma &\sim \sqrt{x} \end{aligned}}$$

BPS

"short non"

"long non"

String

sugra sector = "massless"

"massive string spectrum"

solitonic strings
(very large quantum #)



R
T

$$\boxed{\lambda \gg 1}$$

SYM

$$\boxed{\begin{aligned} \gamma &= 0 \\ \gamma &\sim \sqrt{\lambda} \\ \gamma &\sim \sqrt{x} \end{aligned}}$$

BPS

"short non-BPS"

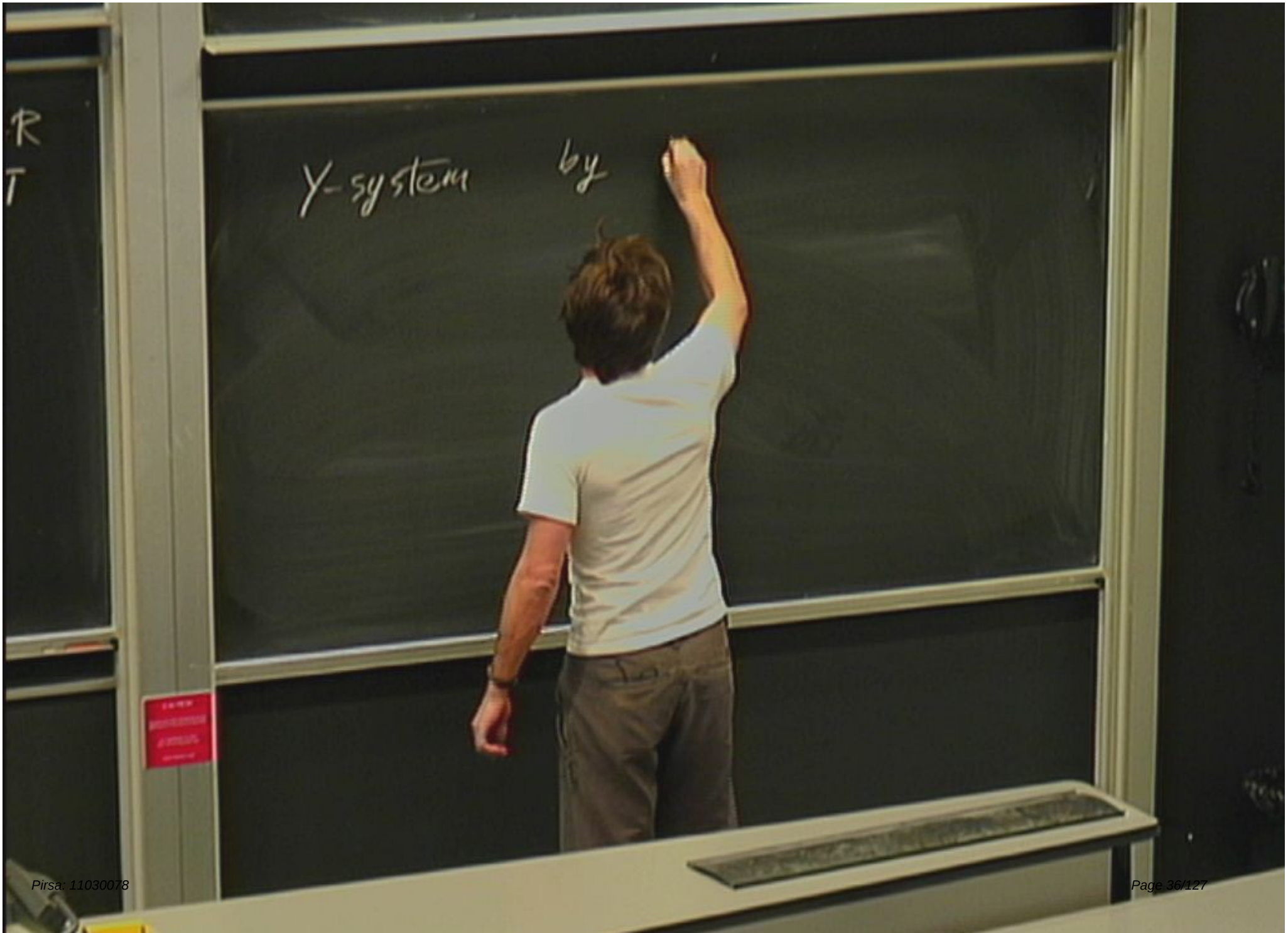
"large non-BPS"

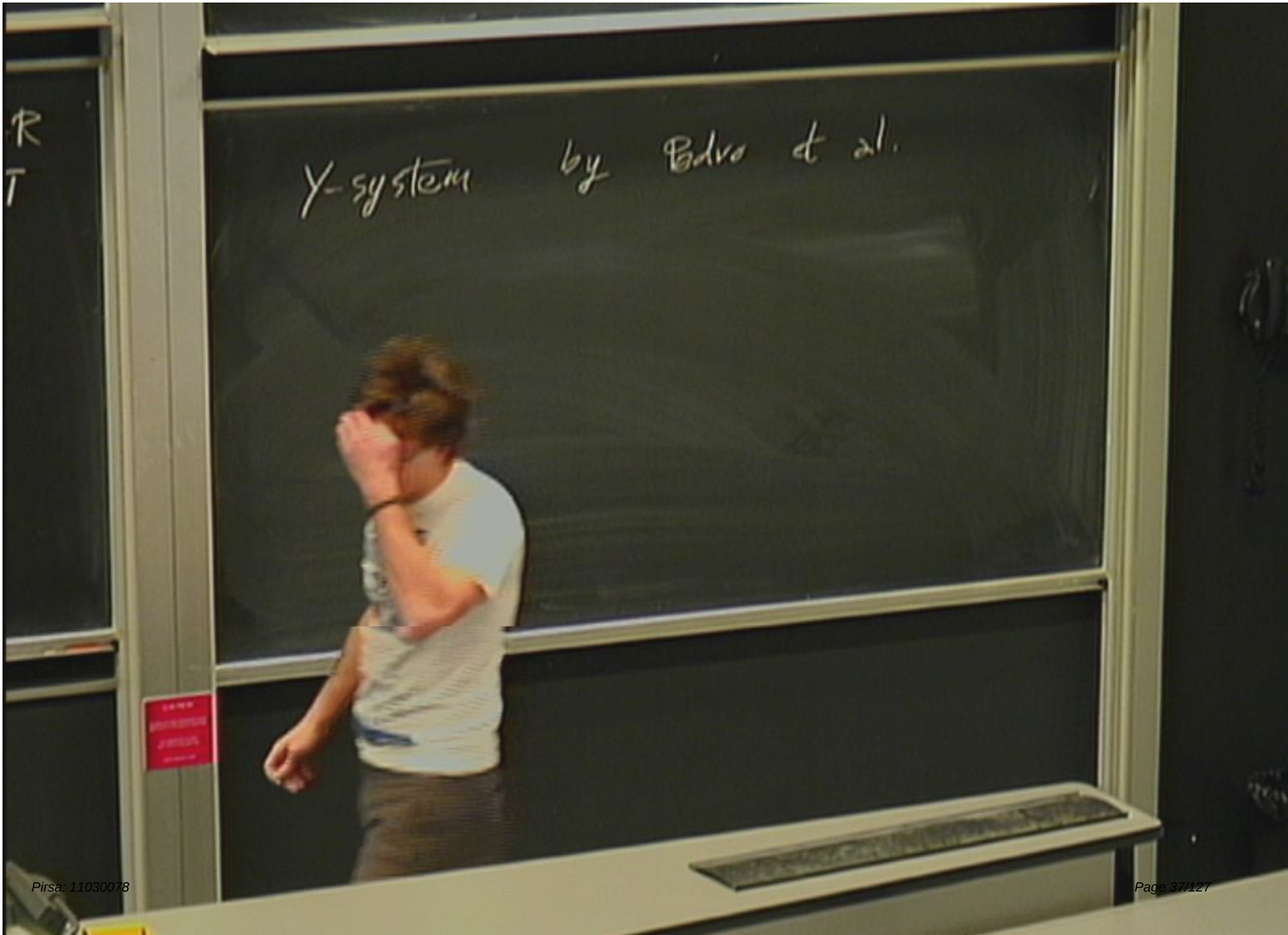
String

sugra sector = "massless"

"massive string spectrum"

solitonic strings
(very large quantum #)





R
T

Y-system by Pedro et al.

R
T

Y-system by Balro et al.



R
T

γ -system by Balro et al.
All γ 's @ any coupling

R
T

γ -system by Polv et al.
All γ 's @ any coupling

- @ WEAK ^{OK}
- @ STRONG : "solitonic"

R
T

γ -system by Polve et al.
All γ 's @ any coupling

- @ WEAK ^{OK}
- @ STRONG : "solitonic"

spectrum massive strings

$$E \sim c \sqrt{\lambda} + \alpha + \frac{c^2}{\lambda} + \dots$$

spectrum massive strings

$$E \sim \sqrt{c_1 \lambda^2 + c_2 + \frac{c_3}{\lambda^2} + \dots}$$

① string σ -model

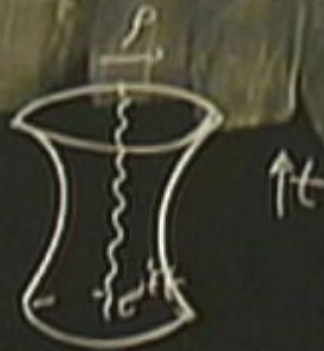
② Pick string

spectrum massive strings

$$E \sim \sqrt{\frac{\alpha'}{4}} \left(\sqrt{1 + \frac{4}{\alpha'} \left(\frac{c_2}{\sqrt{12}} \right)^2} + \dots \right)$$

string σ -model

Pick string config



④

$$T + \overline{T} = 0$$

physical state cond

④

$$T + \overline{T} = 0$$

physical state cond.

@ Δ loop

$$\Delta \left(\overline{E}(\sqrt{x}) \right)$$

④ what $T + \overline{T} = 0$ physical state cond.
@ 1 loop

$$E(\sqrt{\lambda})$$

γ -system by Balve et al.
All γ 's @ any coupling

@ WEAK ^{OK}
@ STRONG : "solitonic"

Anomalous dimensions
gauge invariant local operators

$\mathcal{O}(\phi^i \phi^i(x))$ 6 scalars $N=4$ SYM

(FT $(2/9) >$ DILATATION D

IF YOU SEE
SOMETHING
EET

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

IF YOU SEE
SOMETHING
SKATE
SOMETHING

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

$g(\sigma, \tau)$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

+ 32 f

$$g(\sigma, \tau) \rightarrow g_0 g(\sigma, \tau) h(\sigma, \tau)$$

MATTER

$$Ad S_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

+ 32 f

$$g(\sigma, \tau) \rightarrow g \circ g(\sigma, \tau) h(\sigma, \tau)$$

TER left-inv MC currents

-1

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \leftarrow \mathbb{Z}_4 \text{ grading}$$

$$g(\sigma, \tau) \rightarrow g \circ g(\sigma, \tau) h(\sigma, \tau)$$

ATTER left-inv MC currents

$$J = g^{-1} dg$$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \leftarrow \mathbb{Z}_4 \text{ grading}$$

$$g(\sigma, \tau) = g_0 g(\tau, 0) h(\tau, \pi)$$

MAFF $t \rightarrow inv$ MC currents

$$= \mathcal{I}_0 + \mathcal{I}_2 + \mathcal{I}_1 + \mathcal{I}_3$$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \leftarrow \mathbb{Z}_4 \text{ grading}$$

$$g \rightarrow g_0 g(r, \sigma) h(r, \sigma)$$

ER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + J_1 + J_3$$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \leftarrow \begin{matrix} \mathbb{Z}_4 \\ \text{grading} \end{matrix}$$

$$g(\sigma, \tau) \rightarrow g_0 g(\sigma, \tau) h(\sigma, \tau)$$

MATTER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\times 32 \text{ fermions}}$$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \leftarrow \begin{matrix} \mathbb{Z}_4 \\ \text{grading} \end{matrix}$$

$$g(\sigma, \tau) \rightarrow g_0 g(\sigma, \tau) h(\sigma, \tau)$$

MATTER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\times 32 \text{ fermions}}$$

GHOST SECTOR

$l_1, \bar{l}_3$

GHOST SECTOR

$$l_1, \bar{l}_3 \rightarrow \begin{cases} \{p, \ell\} = 0 \\ \{\bar{p}, \bar{\ell}\} = 0 \end{cases}$$

PURE SPINOR
CONSTRAINTS

IF YOU SEE
SOMETHING,
SKATE
SOMETHING.

GHOST SECTOR

$$\ell_1, \bar{\ell}_3 \rightarrow \{\ell_1, \ell_1\} = 0$$

$$\{\bar{\ell}_3, \bar{\ell}_3\} = 0$$

$$\ell_3, \bar{\ell}_1$$

PORE SPINOR
CONSTRAINTS

GHOST SECTOR

$$l_4, \bar{l}_3 \rightarrow \begin{cases} l_1, l_2 = 0 \\ \bar{l}_1, \bar{l}_2 = 0 \end{cases}$$

$$N_3, \bar{N}_1$$

PURE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int$$

GHOST SECTOR

$$l_1, l_3 \rightarrow \{l_1, l_3 = 0$$

$$\{l_1, l_3\} = 0$$

$$m_1, m_3$$

PURE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

GHOST SECTOR

$$l_1, l_3 \rightarrow \{l_1, l_3 = 0$$

$$\{l_1, l_3\} = 0$$

$$m_1, \bar{m}_1$$

PURE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\sigma \mathcal{H}_2 \left[\right.$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

GHOST SECTOR

$$\ell_3, \bar{\ell}_3 \rightarrow \begin{cases} \{\ell_i, \ell_j\} = 0 \\ \{\bar{\ell}_i, \bar{\ell}_j\} = 0 \end{cases}$$

$$N_3, \bar{N}_1$$

PURE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d_2 \mathcal{H}_2 \left[\frac{1}{2} \mathcal{J}_2 \bar{\mathcal{J}}_2 + \frac{3}{4} \mathcal{J}_3 \bar{\mathcal{J}}_1 + \frac{1}{4} \mathcal{J}_1 \bar{\mathcal{J}}_3 + N_3 \bar{\nabla} \ell_1 + \bar{N}_1 \nabla \ell_3 \right]$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

$$AdS_5 \times S^5 = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \quad \xrightarrow{Z_4 \text{ grading}}$$

$$g(\sigma, \tau) \rightarrow g_0 g(\sigma, \tau) h(\sigma, \tau)$$

MATTER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{32 \text{ fermions}}$$

GHOST SECTOR

$$\ell_4, \bar{\ell}_3 \rightarrow \begin{cases} \ell_1, \ell_4 = 0 \\ \bar{\ell}_3, \bar{\ell}_1 = 0 \end{cases}$$

$$N_3, \bar{N}_1$$

PORE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d^2x \, g_{\mu\nu} \left[\frac{1}{2} \partial_\mu \bar{\ell}_2 + \frac{3}{4} \partial_\mu \bar{\ell}_1 + \frac{1}{4} \partial_\mu \bar{\ell}_3 + N_3 \partial_\mu \ell_1 + \bar{N}_1 \partial_\mu \bar{\ell}_3 - N \bar{N} \right]$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

GHOST SECTOR

$$\ell_4, \bar{\ell}_3 \rightarrow \{\ell_1, \ell_1\} = 0$$

$$\{\bar{\ell}_3, \bar{\ell}_3\} = 0$$

$$N_3, \bar{N}_1$$

PORE SPINOR
CONSTRAINTS

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d_2 \, g_{12} \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_3 \bar{\partial}_1 + \frac{1}{4} \partial_1 \bar{\partial}_3 + N_3 \nabla \ell_1 + \bar{N}_1 \nabla \bar{\ell}_3 - N \bar{N} \right]$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

$$S = \frac{i\lambda}{2\pi} \int d^2x \, \text{Tr} \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_3 \bar{\partial}_1 + \frac{1}{4} \partial_1 \bar{\partial}_3 + W_3 \nabla \bar{L}_1 + W_1 \nabla \bar{L}_3 - N \bar{N} \right]$$

$$\Gamma \lambda = \frac{R^2}{\pi}$$

$$Q_{\text{BRST}} = \oint d^2x \, \text{Tr} (L_1 \bar{\partial}_3 + \bar{L}_1 \bar{\partial}_1)$$

@ WEAK OK

@ STRONG : "solitonic"

$$S = \frac{i\lambda}{2\pi} \int d^2x \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_3 \bar{\partial}_1 + \frac{1}{4} \partial_1 \bar{\partial}_3 + W_3 \nabla \ell_1 + W_1 \nabla \ell_3 - N \bar{N} \right]$$

$$\Gamma \lambda = \frac{R^2}{\pi}$$

$$Q_{BRST} = \oint d\ell \, \text{Str}(\ell, \partial_3 + \bar{\ell}, \bar{\partial}_1)$$

VRASO O

$$T = \sqrt{\lambda} \, \text{Str} \left(\frac{1}{2} \partial_1 \bar{\partial}_1 + \partial_3 \bar{\partial}_3 \right)$$

- ① WEAK OK
- ② STRONG : "solitonic"

$$\frac{\sqrt{\lambda}}{2\pi} \int d^2 x \text{Str} \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_3 \bar{\partial}_1 + \frac{1}{4} \partial_1 \bar{\partial}_3 + N_3 \bar{\nabla} \ell_1 + N_1 \bar{\nabla} \ell_3 - N \bar{N} \right]$$

$Q_{\text{BRST}} = \oint d\sigma \text{Str}(\ell_1 \partial_3 + \bar{\ell}_1 \bar{\partial}_1)$	VIRASORO $T = \frac{\sqrt{\lambda}}{2\pi} \text{Str} \left(\frac{1}{2} \partial_2 \bar{\partial}_2 + \partial_3 \bar{\partial}_1 - N_3 \bar{\nabla} \ell_1 \right)$
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2) WEAK OK

1) STRONG : "solitonic"

Physical states ghost # $(1,1)$

coupling

STRESS-ENERGY TENSOR

soliton

Physical states

ghost # $(1,1)$
vertex ops

→ BRST charge

→ $T + \bar{T} = 0$

Physical states

ghost # $(1,1)$
vertex ϕ s

→ BRST equiv.

→ $T + \bar{T} = 0$

massless sector

$\Leftrightarrow$

SUGRA

on $AdS_5 \times S^5$

Physical states

ghost # $(1,1)$
vertex ops

→ BRST cohom.

→ $T + \bar{T} = 0$

1) massless sector

2) MASSIVE STATES?

$\Leftarrow$

UGRA

on $AdS_5 \times S^5$

Physical states

ghost # $(1,1)$
vertex ops

→ BRST cohom.

→ $T + \bar{T} = 0$

1) massless sector $\Leftrightarrow$ SUGRA on $AdS_5 \times S^5$

2) MASSIVE STATES? — AdS global $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_3^2$

Physical states

ghost # $(1,1)$
vertex ϕ_s

→ BRST cohom.

→ $T + \bar{T} = 0$

1) massless sector $\Leftrightarrow$ SUGRA on $AdS_5 \times S^5$

2) MASSIVE STATES? — AdS global

$$ds^2 = -c_1^2 dt^2 + d\vec{p}^2 + d\vec{s}^2$$

string pro

Physical states

ghost # $(1,1)$
vertex ops

→ BRST cohom.

→ $T + \bar{T} = 0$

1) massless sector $\Leftrightarrow$ SUGRA on $AdS_5 \times S^5$

2) MASSIVE STATES? — AdS global $ds^2 = -dt^2 + dr^2 + r^2 dS_3^2$

String prob

$$\tilde{g}(\tau) = \exp\left(-\tau E \frac{T}{\sqrt{\lambda}}\right)$$

Physical states

ghost # $(1,1)$
vertex ops

→ BRST cohom.

→ $T + \bar{T} = 0$

1) massless sector $\Leftrightarrow$ SUGRA on $AdS_5 \times S^5$
2) MASSIVE STATES? — AdS global $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_3^2$

String prob

$$\tilde{g}(\sigma, \tau) = \exp\left(-\tau E \frac{T}{\sqrt{\lambda}}\right)$$

ER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\rightarrow 32 \text{ fermions}}$$

$$\tilde{J}_c =$$

MATTER left-inv MC currents

$$J = g^{-1}dg = \underbrace{J_0}_{\text{GAUGE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\sim 32 \text{ fermions}}$$

$$E \rightarrow \Delta - \Delta_0 = 2\sqrt{\lambda} - 4 + \frac{2}{\sqrt{\lambda}} + \dots$$

$$T + \overline{T} = -\frac{E^2}{2\sqrt{\lambda}}$$

MATTER left-inv MC currents

$$J = g^{-1}dg = \underbrace{J_0}_{\text{GALILEE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\sim 32 \text{ fermions}}$$

$$E \rightarrow \Delta - \Delta_0 = 2\sqrt{\lambda} - 4 + \frac{2}{\sqrt{\lambda}} + \dots$$

$$T + \overline{T} = -\frac{E^2}{2\sqrt{\lambda}} +$$

WITTER left-inv MC currents

$$J = g^{-1} dg = \underbrace{J_0}_{\text{GALILEE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{32 \text{ fermions}}$$

$$E \rightarrow \Delta - \Delta_0 = 2\sqrt{\lambda} - 4 + \frac{2}{\sqrt{\lambda}} + \dots$$

$$T + \overline{T} = -\frac{E^2}{2\sqrt{\lambda}} +$$

$$E \sim \sqrt{T}$$

IER left-inv MC currents

$$J = g^{-1}dg = \underbrace{J_0}_{\text{GALILEE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\rightarrow 32 \text{ fermions}}$$

$$T + \bar{T} = -\frac{c}{24\pi} +$$

$E \sim \sqrt{\lambda} \rightarrow$ quantum corrections

$$\tilde{J}_\mu = \frac{E}{\sqrt{\lambda}} T$$

left-inv MC currents

$$g^{-1}dg = \underbrace{J_0}_{\text{AdS}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\rightarrow 32 \text{ fermions}}$$

$$\tilde{J}_\nu \sim \frac{E T}{\sqrt{\lambda}}$$

left-inv MC currents

$$= g^{-1} dg = \underbrace{J_0}_{\text{GALISE}} + \underbrace{J_2}_{10 \text{ bosons}} + \underbrace{J_1 + J_3}_{\rightarrow 32 \text{ fermions}}$$

$$\tilde{J}_\mu \sim \frac{E T}{V}$$

BACKGR. FIELD π .

$$\tilde{g} e^x$$

$$\tilde{f}_c \sim \frac{E T}{\sqrt{\lambda}}$$

BACKGR. F.

$$g = \tilde{g} e^{x_1 + x_2 + x_3}$$

$$\tilde{g} \sim \frac{E T}{\sqrt{\lambda}}$$

BACKGR. FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

$$I_0 = e^{-x} \partial_x e^x$$

$$I_1 = e^{-x} \left(\partial_x - \frac{E T}{V} \right) e^x$$

BACKGR. FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_1 = e^{-X} \left(Z_0 - \frac{E T}{V} \right) e^X$$

BACKGR. FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

Physical states

ghost # $(1,1)$
vertex ϕ_s

$\rightarrow$ BRST equiv.

$\rightarrow T + \bar{T} = 0$

1) massless sector $\Leftrightarrow$ SUGRA on $AdS_5 \times S^5$
MASSIVE STATES? — AdS global $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + ds^2_{S^5}$

tiny prob $\tilde{g}(r) = \exp\left(-rE \frac{T}{M}\right)$

$$Z_0 = e^{-X} \partial_0 e^X$$

$$Z_1 = e^{-X} \left(\partial_1 - \frac{E T}{V T} \right) e^X$$

B

FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

BOSONS

FERMIONS

mass

$$Z_0 = e^{-X} \partial_\tau e^X$$

$$Z_\tau = e^{-X} \left(\partial_\tau - \frac{E T}{V \tau} \right) e^X$$

BACKGR. FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

BOSONS

FERMIONS

T

massless

S^5

"

P, dS_2

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_1 = e^{-X} \left(Z_0 - \frac{E T}{V \pi} \right) e^X$$

BACKGR. FIELD π .

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

BOSONS

FERMIONS

T

massless

S^5

"

P, dS_2

massive $m = \frac{E}{V \pi}$

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_T = e^{-X} \left(Z_0 - \frac{E \cdot T}{V} \right) e^X$$

BACKGR. FIELD π .

BOSONS

T

massless

S^5

"

p, ds_2

massive $m = \frac{E}{V}$

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

FERMIONS

16 massless

16 massive

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_T = e^{-X} \left(Z_0 - \frac{E T}{\hbar} \right) e^X$$

BACKGR. FIELD π .

BOSONS

massless

$\rho, \delta S_2$ massive $m = \frac{E}{\hbar}$

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

FERMIONS

16 massless

16 massive $m = \frac{E}{2\hbar}$

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_T = e^{-X} \left(Z_0 - \frac{E T}{V} \right) e^X$$

BACKGR. F. 1

Bo

mass

T

55

p, ds_2

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

FERMIONS

16 massless

16 massive

$$m = \frac{E}{2\pi}$$

IF YOU SEE
SOMETHING
RETHINK

$$Z_0 = e^{-X} Z_0 e^X$$

$$Z_T = e^{-X} \left(Z_0 - \frac{E T}{\hbar} \right) e^X$$

BACKGR. FIELD π .

BOSONS

T

massless

S^5

"

ρ, ds_2

massive $m = \frac{E}{\hbar}$

$$g = \tilde{g} e^{X_1 + X_2 + X_3}$$

FERMIONS

16 massless

16 massive

$$m = \frac{E}{2\hbar}$$

QUANTUM CORRECTIONS

1)

QUANTUM CORRECTIONS

1) normal ordering

2)

QUANTUM CORRECTIONS

1) normal ordering

2) 2-magnon energy

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic corrections

$$\frac{T+\bar{T}}{4\pi\alpha'} =$$

BOSONS

T massless
 S^5 "

dS_2 massive $m = \frac{E}{\hbar}$

FERMIONS

16 massless
 16 massive

$$m = \frac{E}{2\pi\hbar}$$

$$\overline{T+T}|_{k,0} = \frac{2E}{\pi}$$

FERMIONS

16 massless

16 massive

$$m = \frac{E}{2\pi}$$

$$\overline{T+\overline{T}}|_{k=0} = \frac{2E}{1\lambda} + \frac{1}{2} \sum_{n=1}^{\infty} \left(6\sqrt{n^2 + 4} \right)$$

BOSONS

T massless
 $\bar{S}_5$ "

p, dS_2 massive $m = \frac{E}{1\lambda}$

massless

massive $m = \frac{E}{2\lambda}$

$$\overline{T+\overline{T}}|_{h.o.} = \frac{2E}{\hbar\lambda} + \frac{1}{2} \sum_{n=1}^{\infty} \left(6\sqrt{n^2} + 4\sqrt{n^2 + \left(\frac{E}{\hbar\lambda}\right)^2} - 1 \right)$$

BOSONS

T massless
 S_5 "

P, dS_2 massive $m = \frac{E}{\hbar\lambda}$

$$m = \frac{E}{2\hbar\lambda}$$

$$\overline{T+T}|_{h.o.} = \frac{2E}{1\lambda} + \frac{1}{2} \sum_{n=1}^{\infty} \left(6\sqrt{m^2} + 4\sqrt{m^2 + \left(\frac{E}{1\lambda}\right)^2} - 16\sqrt{M^2} - 16\sqrt{m^2 + \left(\frac{E}{21\lambda}\right)^2} \right)$$

BOSONS

T massless

S^5 "

p, dS_2 massive $m = \frac{E}{1\lambda}$

FERMIONS

16

16

$$T + \bar{T} \Big|_{h.o.} = \frac{2E}{1\lambda} + \frac{1}{2} \frac{8\pi}{\alpha'} \left[6\sqrt{m^2} + 4\sqrt{m^2 + \left(\frac{E}{1\lambda}\right)^2} - 16\sqrt{m^2} - 16\sqrt{m^2 + \left(\frac{E}{21\lambda}\right)^2} + \right. \\ \left. + 2 \right]$$

Bos

T
 S^5
 P, dS_2

$\frac{E}{1\lambda}$

FERMIONS

16 massless

16 massive

$$m = \frac{E}{21\lambda}$$

$$T+\bar{T}|_{h.o.} = \frac{2E}{l\lambda} + \frac{1}{2} \sum_{n=1}^{\infty} \left[6\sqrt{m^2} + 4\sqrt{m^2 + \left(\frac{E}{l\lambda}\right)^2} - 16\sqrt{m^2} - 16\sqrt{m^2 + \left(\frac{E}{2l\lambda}\right)^2} + 22\sqrt{m^2} \right] = \frac{2E}{l\lambda} - \frac{3}{16} \zeta(3) \left(\frac{E}{l\lambda}\right)^4 + \dots$$

BOSONS

T massless
 S^5 "

p, ds_2 massive $m = \frac{E}{l\lambda}$

FERMIONS

16 massless

$$m = \frac{E}{2l\lambda}$$

$$T + \bar{T} \Big|_{h.o.} = \frac{2E}{1\lambda} + \frac{1}{2} \sum_{n=1}^{\infty} \left[6\sqrt{n^2} + 4\sqrt{n^2 + \left(\frac{E}{1\lambda}\right)^2} - 16\sqrt{n^2} - 16\sqrt{n^2 + \left(\frac{E}{21\lambda}\right)^2} + 22\sqrt{n^2} \right] = \frac{2E}{1\lambda} - \frac{3}{16} \zeta(3) \left(\frac{E}{1\lambda}\right)^4 + \dots$$

BOSONS

T massless
 S^5
 P, dS_2 massive

16 massless

16 massive $m = \frac{E}{21\lambda}$

WZM BRENNIO CARLINI VALLILO

arXiv: 1102.1219

$$E \rightarrow \Delta - \Delta_0 = 2\sqrt{\lambda} - 4 + \frac{2}{\sqrt{\lambda}} + \dots$$

$$T + \overline{T} = -\frac{E^2}{2\sqrt{\lambda}} + \frac{2E}{\sqrt{\lambda}}$$

$E \sim \sqrt{T} \Rightarrow$ quantum corrections

massive $m = \frac{E}{\sqrt{\lambda}}$

2/2

with BRENNO CARLINI VALLILO

arxiv: 1102.1219

$$E \rightarrow \Delta - \Delta_0 = 2\sqrt{\lambda} - 4 + \frac{2}{\sqrt{\lambda}} + \dots$$

$$T + \overline{T} = -\frac{E(E-4)}{2\sqrt{\lambda}}$$

$E \sim \sqrt{\lambda} \Rightarrow$ quantizations

ρ, ds_3 massive

$2\sqrt{\lambda}$

$\text{Str } \ell, \bar{\ell}_3$ radius changing

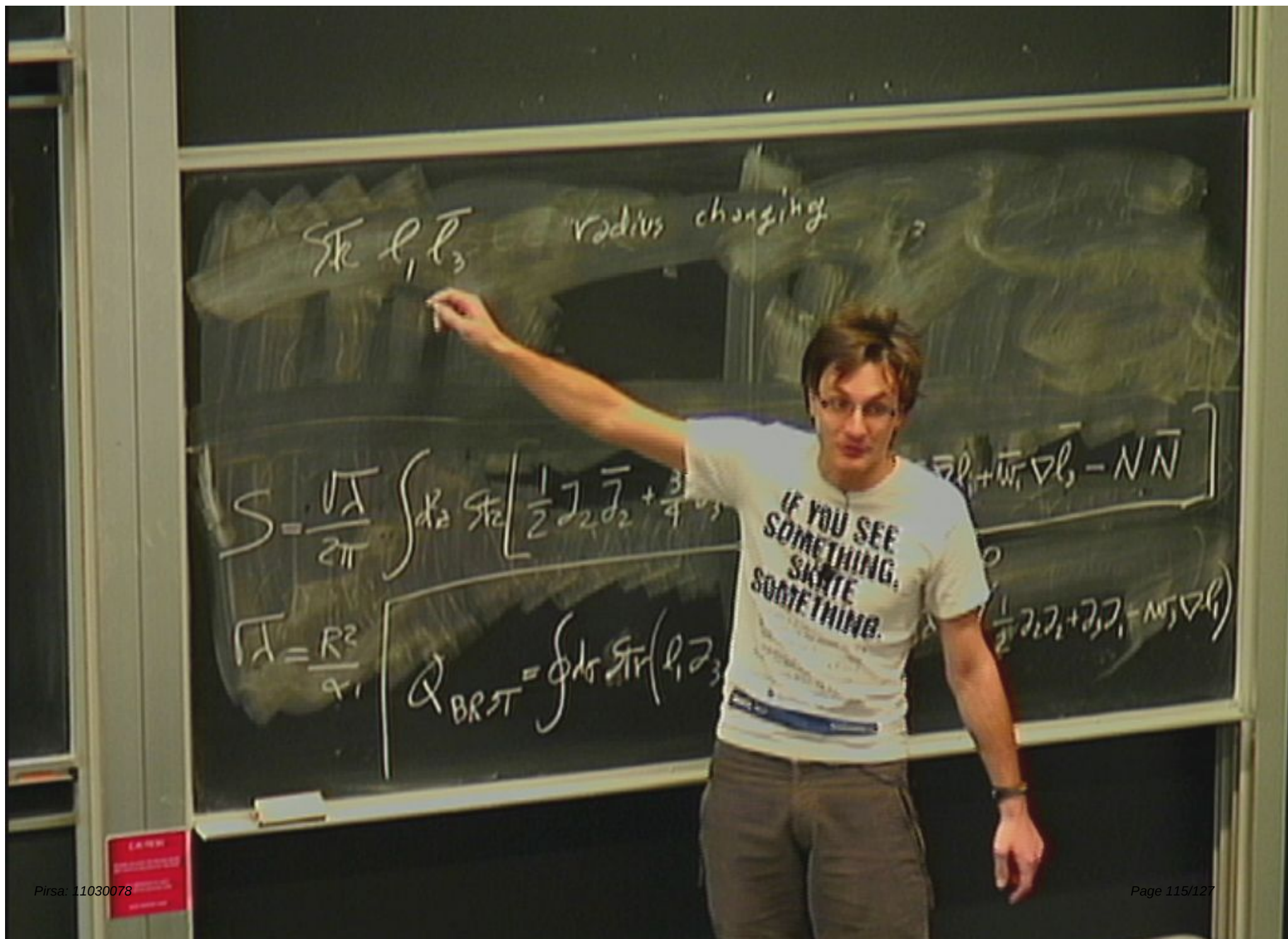
$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau \text{Str} \left[\frac{1}{2} \dot{\ell}_2 \bar{\ell}_2 + \frac{3}{4} \dot{\ell}_3 \bar{\ell}_1 + \frac{1}{4} \dot{\ell}_1 \bar{\ell}_3 + \dots \right]$$

$$\Gamma = \frac{R^2}{\alpha'}$$

$$Q_{\text{BRST}} = \oint d\tau \text{Str}(\ell_1 \dot{\ell}_3 + \bar{\ell}_3 \dot{\ell}_1)$$

VIRASORO

$$T = \sqrt{\lambda} \text{Str} \left(\frac{1}{2} \dot{\ell}_1 \bar{\ell}_1 \right)$$



$\text{Str } l, \bar{l}_3$ radius changing

$$S = -\frac{\sqrt{\lambda}}{2\pi} \int d^2 x \text{Str} \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_2 \partial_2 \right]$$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

$$Q_{BRST} = \oint d\sigma \text{Str} (l_1 \partial_3)$$

$$\partial_2 l_1 + \partial_1 \bar{\partial}_2 l_1 - N \bar{N}$$

$$\frac{1}{2} \partial_2 \partial_1 + \partial_3 \partial_1 - \partial_1 \partial_2 \partial_1$$

$\text{Str } \ell, \bar{\ell}_3$ radius changing op

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau \text{Str} \left[\frac{1}{2} \dot{\ell}_2 \bar{\ell}_2 + \frac{3}{4} \dot{\ell}_3 \bar{\ell}_1 + \frac{1}{4} \dot{\ell}_1 \bar{\ell}_3 + \bar{\omega}_3 \nabla \ell_1 + \bar{\omega}_1 \nabla \ell_3 \right]$$

$$\bar{\lambda} = \frac{R^2}{a_1}$$

$$Q_{\text{BRST}} = \oint d\tau \text{Str}(\ell_1 \dot{\ell}_3 + \bar{\ell}_3 \dot{\ell}_1)$$

VIRASORO

$$T = \sqrt{\lambda} \text{Str} \left(\frac{1}{2} \dot{\ell}_2 \bar{\ell}_2 \right)$$

$(\sqrt{k} \ell, \bar{\ell})$ radius changing op $\leftrightarrow \hbar F_{\mu\nu} F'^{\mu\nu}$
 $\Delta=4$

vertex ϕ $\sqrt{} = \kappa_{(-1)}$

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \left[\frac{1}{2} \partial_2 \bar{\partial}_2 + \frac{3}{4} \partial_3 \bar{\partial}_1 + \frac{1}{4} \partial_1 \bar{\partial}_3 + \alpha' \bar{\partial} \ell_1 + \alpha' \partial \bar{\ell}_1 - N \bar{N} \right]$$

$$\sqrt{\lambda} = \hbar \left(\ell_1 \partial_3 + \bar{\ell}_1 \bar{\partial}_1 \right) \quad \text{VIRASORO} \quad T = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{1}{2} \partial_2 \bar{\partial}_2 + \partial_3 \bar{\partial}_1 + \alpha' \bar{\partial} \ell_1 \right)$$

$(\text{Str } \ell, \bar{\ell} \quad \text{radius changing op} \leftrightarrow \hbar F_{\mu\nu} F'^{\mu\nu})$
 $\Delta = 4$

vertex $\phi \quad V = X_{(-1)}^+ \bar{X}_{(-1)}^+ \text{Str } \ell, \bar{\ell} |E\rangle$

$X^+ = \alpha' + i\alpha'$

$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau \text{Str} \left[\frac{1}{2} \dot{z}_2 \bar{\dot{z}}_2 + \frac{3}{4} \dot{z}_3 \bar{\dot{z}}_1 + \frac{1}{4} \dot{z}_1 \bar{\dot{z}}_3 + \alpha' \dot{z}_3 \nabla \ell_1 + \alpha' \dot{z}_1 \nabla \bar{\ell}_3 - N \bar{N} \right]$

$\sqrt{\lambda} = \frac{R^2}{\alpha'}$

$Q_{\text{BRST}} = \oint d\tau \text{Str}(\ell_1 \dot{z}_3 + \bar{\ell}_3 \dot{z}_1)$

VIRASORO

$T = \frac{\sqrt{\lambda}}{2\pi} \text{Str} \left(\frac{1}{2} \dot{z}_2 \dot{z}_2 + \dot{z}_3 \dot{z}_1 + \alpha' \dot{z}_3 \nabla \ell_1 \right)$

$$\left(\text{Str } \ell, \bar{\ell}_3 \quad \text{radius changing op} \leftrightarrow \hbar F_{\mu\nu} F'^{\mu\nu} \right) \Delta=4$$

$$\text{Vertex } \phi \quad \boxed{V = X_{(-1)}^+ \bar{X}^+ \text{Str } \ell, \bar{\ell}_3 |E\rangle}$$

$$X^+ = X' + iX''$$

LORENTZ SP(1)
SU(d) sing.

$$\Gamma = \frac{R^2}{\alpha'}$$

$$Q_{\text{BRST}} = \oint ds \text{Str}(\ell, \bar{\ell}_3 + \dots)$$

RASOR O

$$\sqrt{\alpha'} \text{Str} \left(\frac{1}{2} \partial_\mu \bar{\ell}_1 + \partial_\mu \bar{\ell}_2 + \dots \right)$$

$$T + \bar{T} \Big|_{s.s.} = \frac{2E}{\pi} + \frac{1}{2} \sum_{n=1}^{\infty} \left[6\sqrt{n^2} + 4\sqrt{n^2 + \left(\frac{E}{\pi}\right)^2} - 16\sqrt{n^2} - 16\sqrt{n^2 + \left(\frac{E}{2\pi}\right)^2} + 22\sqrt{n^2} \right] = \left[\frac{2E}{\pi} \right] - \frac{3}{16} \zeta(3) \left(\frac{E}{\pi} \right)^3 + \dots$$

BOSONS

T massless

$\bar{S}^5$ "

p_1, dS_2 massive $m = \frac{E}{\pi}$

FERMIONS

16 massless

16 massive $m = \frac{E}{2\pi}$

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic corrections

$$\langle T^4 T \rangle_{\text{ex}} = \frac{1}{6}$$

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic corrections

$$\langle T_4 T \rangle_{\text{ex}} = \frac{1}{5}$$

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic

$$T+T \Big|_{\mathcal{H}} = \left[(P_2, X_2)^2 - (2X_2, X_2)^2 \right]$$

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic corrections

$$\langle T+T \rangle |V\rangle$$

$$T+T \Big|_{\text{ex}} = \frac{1}{6} \text{Stk} \left[\left[\begin{matrix} x_1 \\ x_2 \end{matrix} \right]^2 \right]$$

QUANTUM CORRECTIONS

- 1) normal ordering
- 2) 2-magnon energy
- 3) quartic corrections

$$(\overline{T+T})|V\rangle = -\frac{2}{V\pi}|V\rangle$$

$$T+T \Big|_{\mathcal{H}} = \frac{1}{6} \text{Str} \left[[P_2, X_2]^2 - [2X_2, X_2]^2 \right]$$

$$A(k, k, k) = \langle \psi \psi \psi \rangle$$

- 1) massless sector
- 2) MASSIVE STAT

String pro

[GRA] on $AdS_5 \times S^5$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dS_5^2$$

$$\exp\left(-\tau E \frac{T}{\Lambda}\right)$$

$$A(k, k_1, k_2) = \langle V(i) V(0) V(\infty) \rangle$$

$$= \left| \int d^4x \right|^2 \int d^4x \langle \dots \rangle$$

$\Leftrightarrow$ [SUGRA]

on $AdS_5 \times S^5$

1) M2

STAR

$\Rightarrow$ - AdS g/dx^2

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dS_5^2$$

$$\tilde{g}(\tau) = \exp\left(-\tau E \frac{T}{\Lambda}\right)$$