Title: High-accuracy modeling of extreme mass ratio inspirals with effective field theory

Date: Mar 03, 2011 01:00 PM

URL: http://pirsa.org/11030076

Abstract: The upcoming launch of the space-based gravitational wave interferometer detector LISA will yield an unprecedented amount of astrophysical and cosmological science from a variety of gravitational wave sources. Among these, the extreme mass ratio inspirals (EMRIs) of stellar-mass compact objects into supermassive black holes will provide a unique opportunity to test the predictions of General Relativity for strongly gravitating systems since the masses and spins of these sources are expected to be measured with precisions better than about 1 part in 10^4. Such highly precise measurements require modeling the dynamics of EMRIs and their gravitational waves with high accuracy. In this talk, I discuss using methods of effective field theory (EFT) to accomplish this. Since EMRIs lose energy to gravitational waves, I introduce an open systems framework that proves to be a necessary ingredient to correctly describe EMRIs within the EFT formalism. I will discuss my recent derivation of the equations of motion and waveforms through third order in the mass ratio for a class of nonlinear scalar models that are analogous to the perturbative General Relativistic description of EMRIs. Time permitting, I will also discuss new results that are non-perturbative in the mass ratio in this model.

High-accuracy modeling of extreme mass ratio inspirals in effective field theory

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Perimeter Institute
Strong Gravity Seminar
March 3, 2011

Overview

- Introduction to gravitational wave observations with LISA

- Importance of high-accuracy gravitational wave source modeling

- Effective Field Theory for high-accuracy modeling of extreme mass ratio inspirals (Galley, Hu)

  - Classical mechanics of open systems
  - High-accuracy equations of motion and waveforms
  - Fully non-perturbative conservative dynamics in a scalar model

Fundamental questions

- Astronomy is limited in the questions that can be answered:

  – What is the equation of state for neutron stars?

  – Do black holes really exist or are they just dark compact objects?

  – What is the spacetime around a physical black hole?

  – Is General Relativity the "right" theory of gravity?

  – How often do/will binaries of neutron stars and/or black holes coalesce?

  – What is the merger history of galaxies?

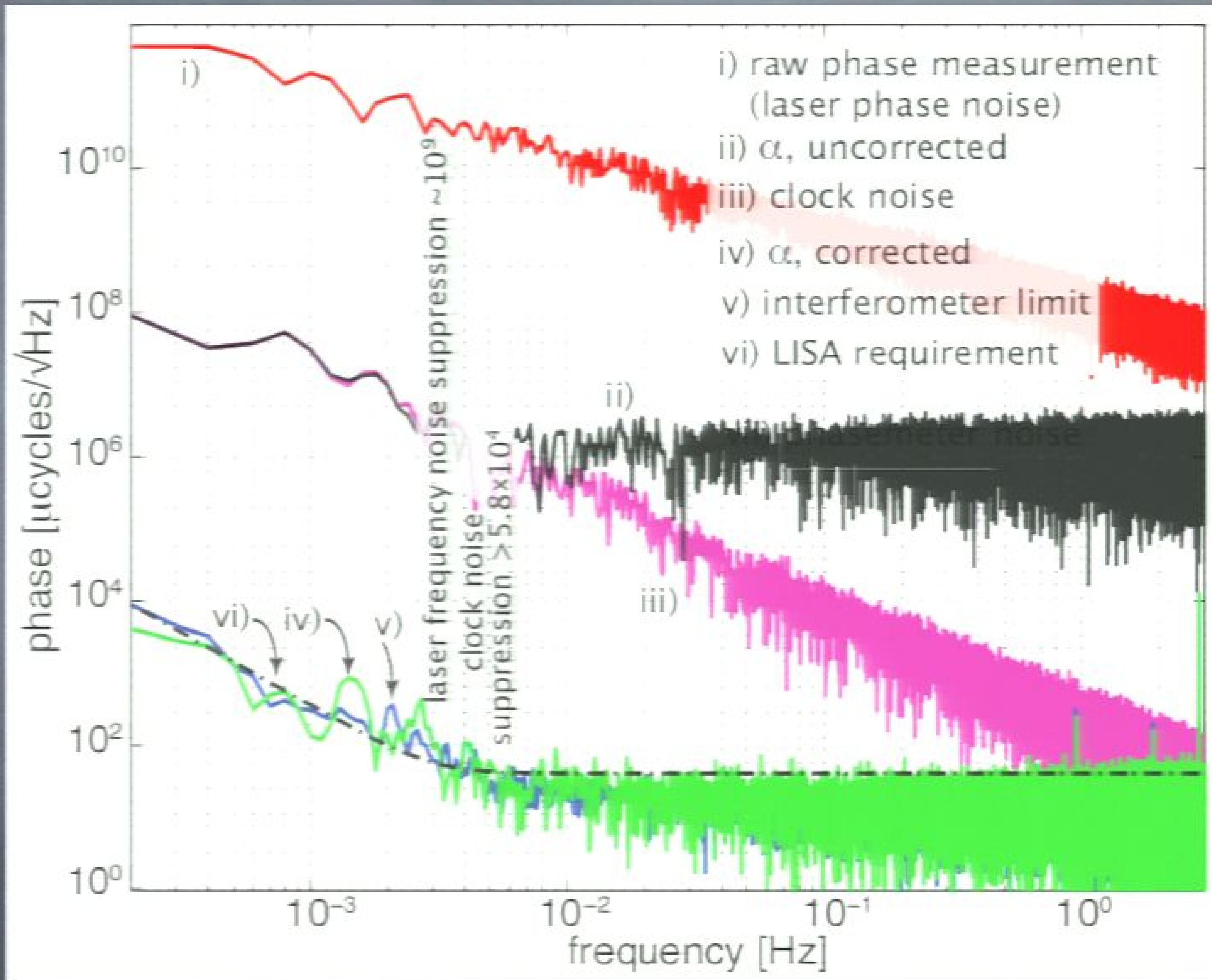
  – Are all, if any, gamma-ray bursts (GRBs) associated with neutron star coalescences?

Laser Interferometer Space Antenna

Strain ~ 1pm/(5 million km) = 2e-21

- Joint NASA/ESA mission to fly in 2025
- 5 million km arms - low frequency detector (0.03 mHz to 0.1 Hz)
- Measures changes in distance between test masses via Doppler shifts
- New paradigm: Time-delay interferometery (Tinto & Armstrong)
- Total estimated cost about $2 billion
- LISA Pathfinder (2014) will test drag-free technology
- Other proposed space-based detectors:
  DecIGO (Japan)
  BBO (follow-up to LISA)

Time-delay interferometery (TDI)



de Vine, Ware, McKenzie, Spero, Klipstein & Shaddock (2010)

GW sources for LISA

- Merging massive black holes
  10^3–10^7 solar masses

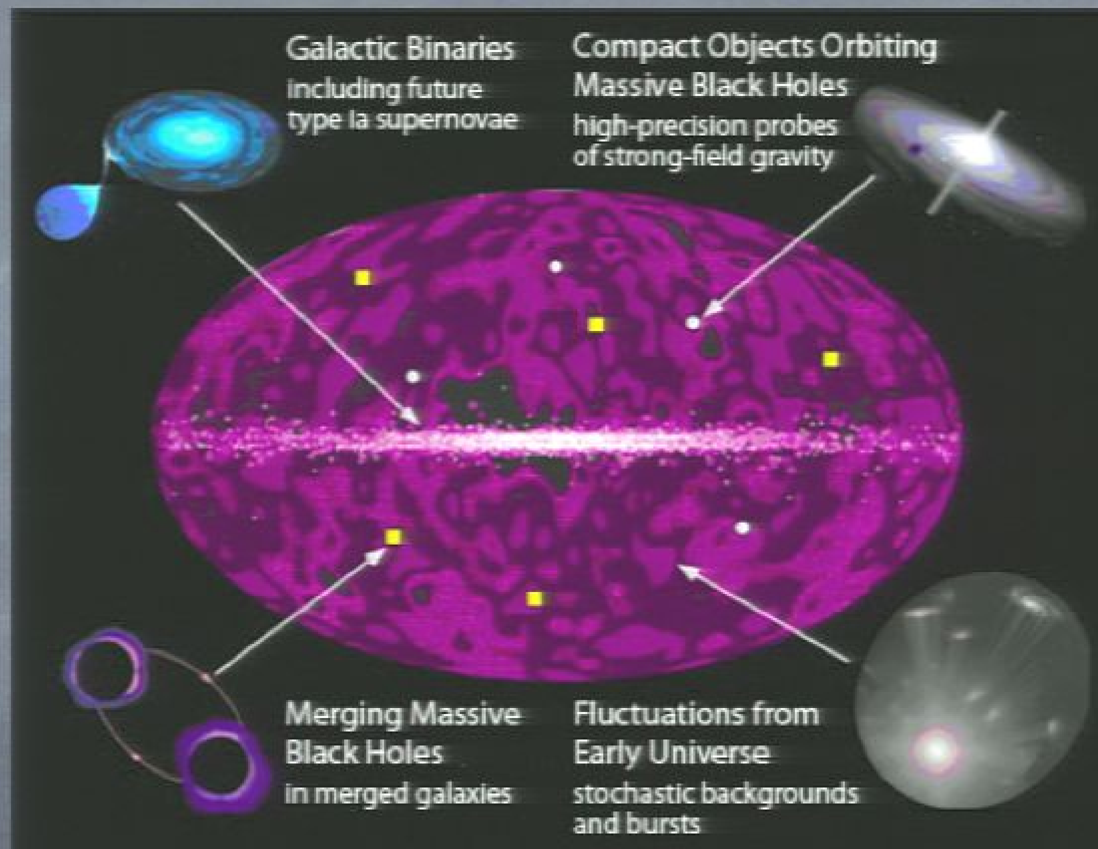
- Extreme mass ratio inspirals (EMRIs)
  10^{-4}–10^{-7} mass ratios

- Galactic binaries

  Millions of unresolved binary stellar remnants

  Verification binaries, known systems, Am CVn's (about 5–10)

- Stochastic GW background



Probing the physics of strong gravity:

High-accuracy source modeling

Why high-accuracy modeling?

- The more accurately the waveform can be calculated, the more accurately the parameters can be measured

  - Masses and spins can be determined to at least 1 part in 10^4, possibly better

- Accurately measured source parameters provide:

  - Stringent tests of General Relativity in strong fields ("No-Hair" theorem, Cosmic Censorship...)

  - Indicate the relevant physics of a source

  - A much better understanding of population and formation of sources

Testing the "No-Hair" theorem of GR

- General Relativity predicts that the "Kerr" solution is the unique end-state of gravitational collapse

  A Kerr black hole spacetime is fully described by its mass M and spin angular momentum S

$$M_\ell + iS_\ell = M(iS/M)^\ell$$

  Measuring 3 multipole moments independently and <u>accurately</u> verifies the baldness of Kerr black holes

M and S to 1 part in 10^4
Quadrupole moment to 1 part in 10^2

Extreme mass ratio inspirals

- A stellar mass compact object (BH, NS, WD) inspirals and plunges into a supermassive black hole (SMBH)

m = 1–10 solar masses
M = 10^5–10^7 solar masses

m/M is extremely small

- SMBH is generally spinning and typically found at galactic centers

- m is nearly in free-fall and the force that m experiences is called the "self-force"

- Orbits are generically non-equatorial, non-circular, complicated, and are "space-filling"

[movie]

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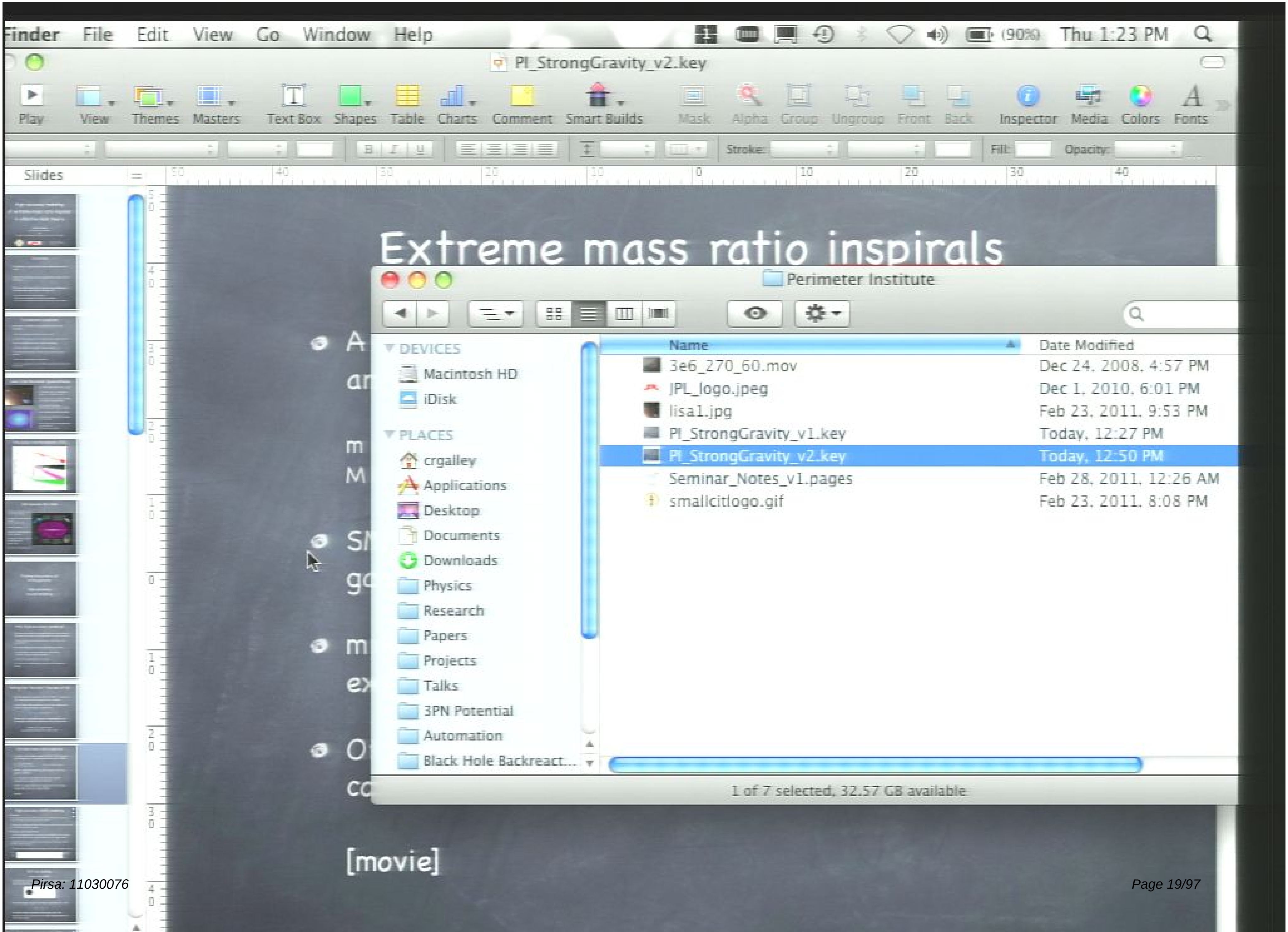
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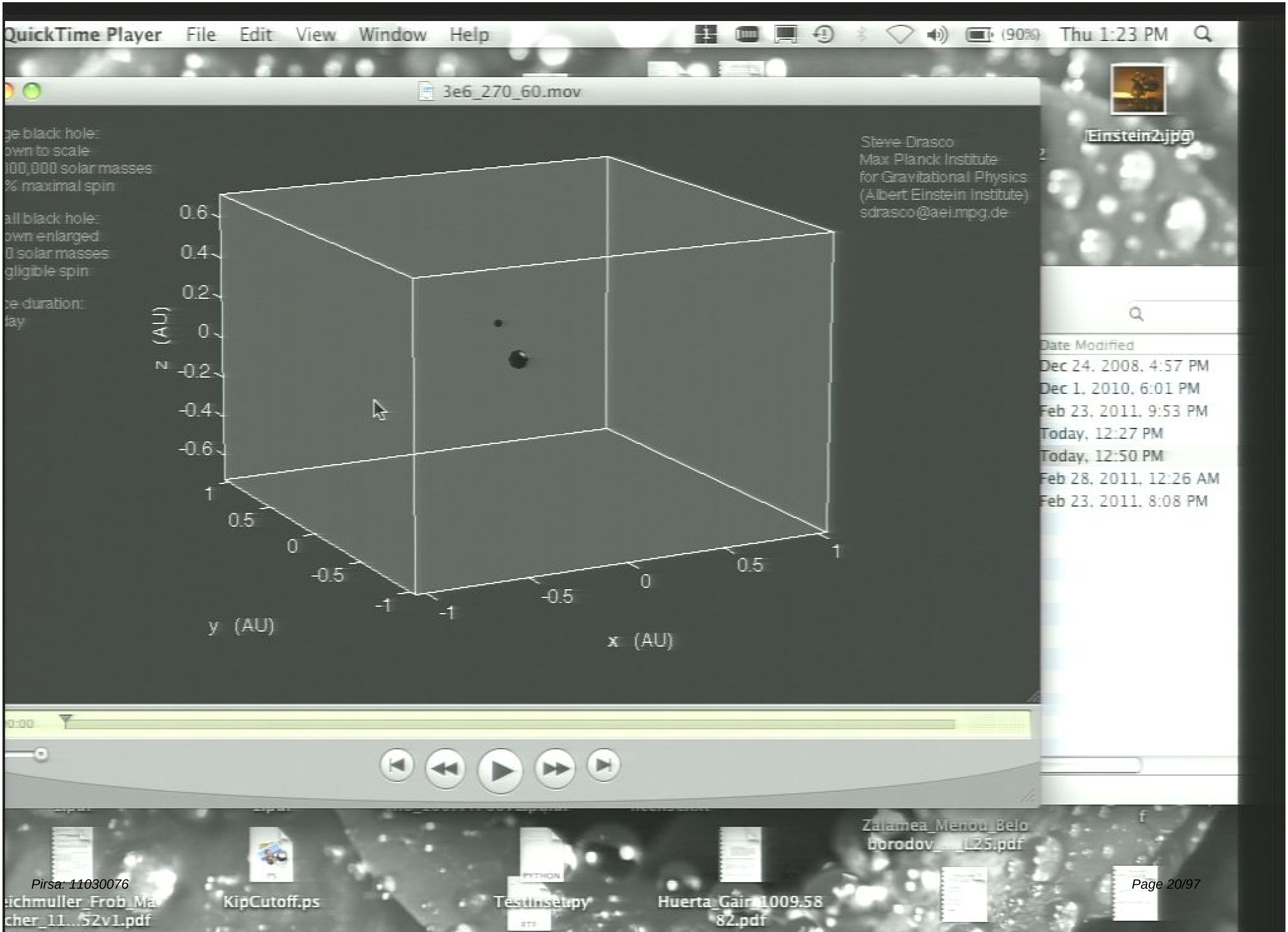
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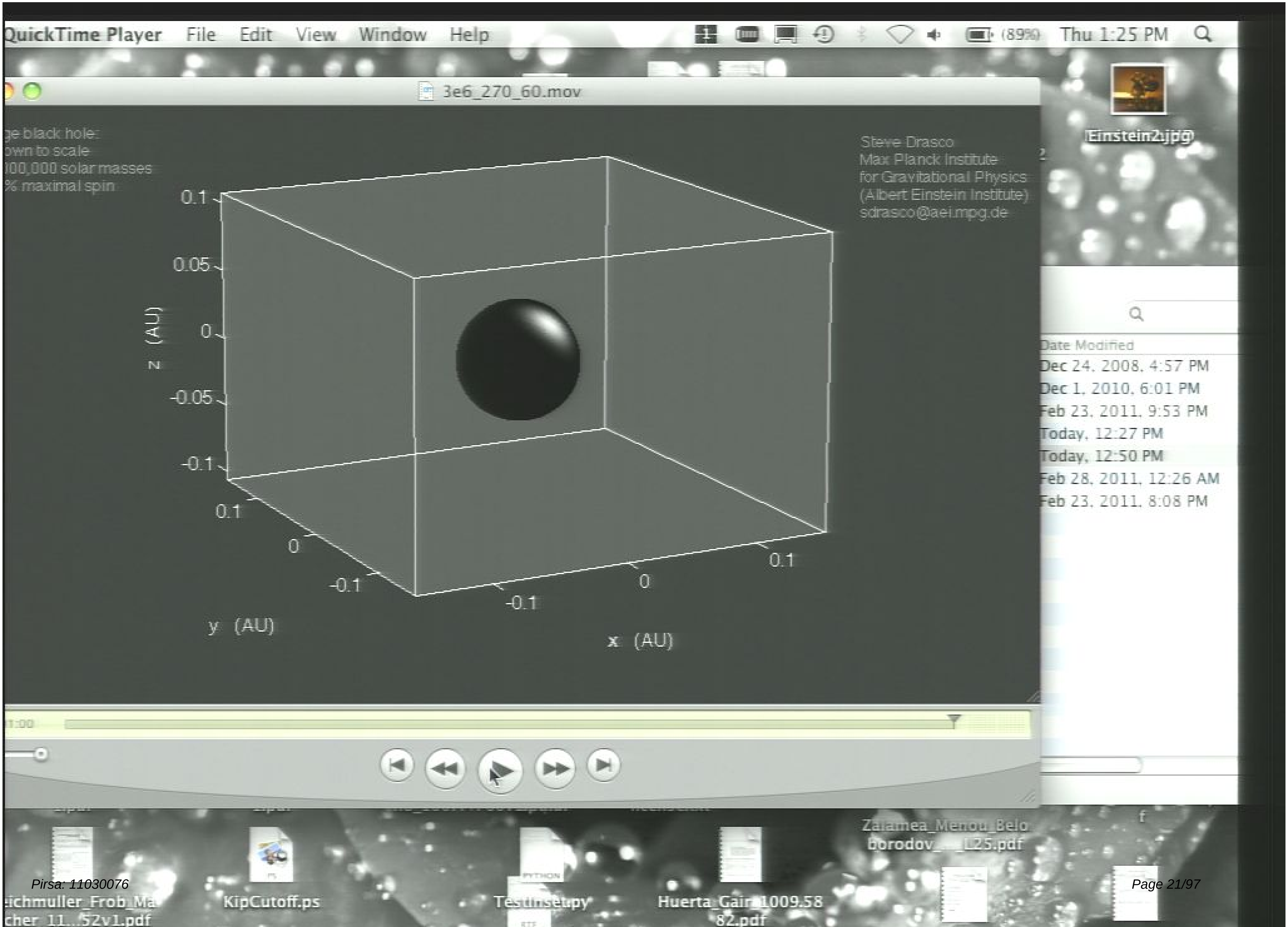
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High-accuracy EMRI modeling

- Requires:

  1) Knowing the equations of motion (i.e., the self-force) for the small compact object to possibly high orders in m/M

  2) Solving the equations of motion with enough numerical accuracy over the last 100,000 orbits before merger

- Previous work in self-force

  – Only first-order equations of motion are known [Mino, Sasaki & Tanaka; Quinn & Wald (1995), Gralla & Wald (2008), Pound & Poisson (2009)]

  – Significant efforts underway to numerically solve them [e.g., Barack, Detweiler, Poisson, Vega...]

  – Transient resonances [Hinderer & Flanagan (2010)] demand higher accuracy

$$a^\mu = 16\pi G m P^{\mu\alpha\beta\nu} \lim_{\epsilon\to 0} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) u^{\gamma'} u^{\delta'}$$

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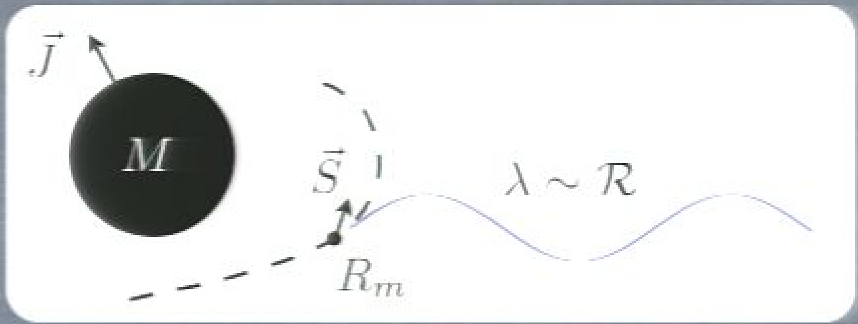
- The recently developed Effective Field Theory approach for EMRIs solves (1) [Galley & Hu (2009)]

EFT for EMRIs

Galley & Hu (2009)

- Consider the inspiral of a stellar mass compact object (WD, NS, BH) into a supermassive black hole



- The two length scales are widely separated if m << M

$$\varepsilon \equiv \frac{R_m}{\mathcal{R}} \sim \frac{m}{M} \ll 1$$

- Multiple widely separated scales imply that the physics at each scale can be treated independently of the other scales

Extended objects with EFT

Black holes, neutron stars and white dwarfs are extended objects

At distances much larger than its size, the compact object looks effectively like a point particle

$$S = -m \int d\tau$$

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Q: How can we include finite-size effects while maintaining a point particle description?

- Solve the full theory of General Relativity and, say, a neutron star

  A problem: We don't know the neutron star equation of state

  For a black hole, the full theory can only be solved numerically

- Parameterize our ignorance of the full theory

  Add extra interaction terms to the point particle action that are consistent with the underlying symmetries

  The dynamics of the effective theory is largely determined by the symmetries

Parameterizing our ignorance...

- Symmetries of the underlying theory of GR with a compact object are:

  Coordinate changes
  Changing of "time" parameter along worldline
  Time-reversal and parity,...

$$S = -m \int d\tau + C_E \int d\tau \, \mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} + C_B \int d\tau \, \mathcal{B}_{\alpha\beta}\mathcal{B}^{\alpha\beta} + \cdots$$

- What are the C_E, C_B,... coefficients?

  These "matching" coefficients contain information about the internal structure of the neutron star or black hole

  1) Determined through a matching calculation [BH: see M. Smolkin]

  2) Left unknown as parameters to fit data

Effacement principle for EMRIs

- Effacement Principle: Galley & Hu (2009)

  The internal structure of a non-spinning, small black hole or neutron star does not affect its motion until 4th order in the mass ratio.

- White dwarfs are not as compact as BHs or NSs and so do not satisfy this Effacement Principle

  - We did observe that finite size effects are enhanced as R_m^5 [consistent with Flanagan & Hinderer (2008)]

  - Tidal forces on WD could be appreciable to 2nd order self-force

  - WD is tidally disrupted inside SMBH horizon if $M > 10^5$ solar masses [consistent with Menou+ (2008) and Sesana+ (2008)]

A nonlinear scalar model

for EMRIs

A scalar analog of EMRIs [Galley (2010)]

- Consider instead an analogous description of EMRIs in terms of a nonlinear scalar field

  - Historical reasons
  - Technically simpler for numerical investigations
  - Preserves many qualitative aspects of gravity

- In Lorenz gauge, vacuum background spacetime:

$$S[z^\mu, h_{\mu\nu}] \sim -\sum_{n=2}^{\infty} \frac{1}{n!} \int_x a_n^{\cdots}(x) \nabla h \nabla h \, h^{n-2} - m \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau \, b_n^{\cdots}(\tau) [h(z^\mu) uu]^n$$

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- A field redefinition <u>removes</u> all self-interactions in the bulk

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Self-force in EFT approach

- Calculate the self-force from an action principle
  - Example: $c_{n>1} = 0$

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2) $\psi(x) = -m c_1 \int d\tau' \, D_{\rm ret}(x^\mu, z^{\mu'})$

3) $\underline{S_{\rm eff}[z^\mu] \equiv S[z^\mu, \psi(x; z)]} = -m \int d\tau + \frac{m^2 c_1^2}{2} \int d\tau \int d\tau' \, D_{ret}(z^\mu, z^{\mu'})$

$$= -m \int d\tau + \frac{m^2 c_1^2}{4} \int d\tau \int d\tau' \, \left(D_{ret}(z^\mu, z^{\mu'}) + D_{adv}(z^\mu, z^{\mu'}) \right)$$

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$$= -m\int d\tau + \frac{m^2 c_1^2}{4}\int d\tau \int d\tau' \left(D_{ret}(z^\mu, z^{\mu'}) + D_{adv}(z^\mu, z^{\mu'})\right)$$

- "Integrating out" the scalar perturbations from the action results in time-symmetric (conservative) dynamics
- Must do something else to get full dissipative dynamics

Classical mechanics of open systems

Galley (2010) [arXiv: 1012.4488]

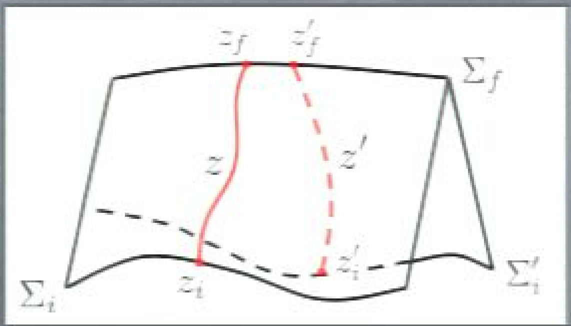
- Take a hint from real-time quantum field theory (Schwinger, Keldysh,... "in-in" formalism)

$$\rho(t) = U(t, t_i)|0_{\rm in}\rangle\langle 0_{\rm in}|U^\dagger(t, t_i)$$

- Introduce two sets of histories for the field and worldline

$$(z^\mu, \psi) \to (z_1^\mu, \psi_1), (z_2^\mu, \psi_2)$$

$$S[z^\mu, \psi] \to S[z_1^\mu, \psi_1] - S[z_2^\mu, \psi_2]$$



- In this way, time-asymmetries are retained in the ensuing equations of motion, waveforms, radiated flux, etc.

- Repeat calculation of effective action

1) $S[z_1^\mu, z_2^\mu, \psi_1, \psi_2] = S[z_1^\mu, \psi_1] - S[z_2^\mu, \psi_2]$

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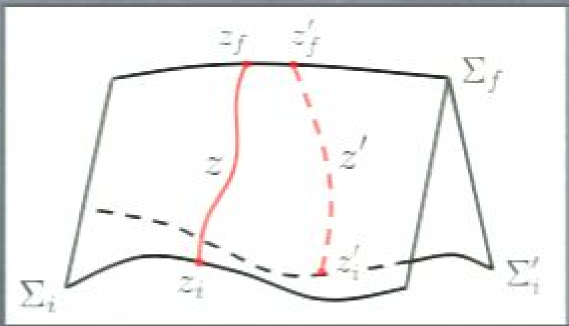

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Compare the two effective actions

$$S_{\text{eff}}[z^\mu] = -m\int d\tau + \frac{m^2c_1^2}{4}\int d\tau d\tau'\, V(x)\big(D_{ret}(x,x') + D_{adv}(x,x')\big)V(x')$$

$$S_{\text{eff}}[z_1^\mu, z_2^\mu] = -m\int d\tau_1 + m\int d\tau_2$$

$$+\frac{m^2c_1^2}{4}\int_x\int_{x'}\big(V_1(x) - V_2(x)\big)\big(D_{ret}(x,x') + D_{adv}(x,x')\big)\big(V_1(x') + V_2(x')\big)$$

(Set $z_1 = z_2 = z$ after all variations)

Self-force with diagrams

- Equations of motion (i.e., self-force) are derived using Feynman diagrams -- automatically solves wave equations

- Geodesic (force-free) motion

$O(\varepsilon^0)$

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- 1st order self-force

$O(\varepsilon)$

$$S^{(1)}_{\rm eff}[z^{\mu}_{1,2}] = \frac{1}{2!}\int_x \int_{x'} T_a(x;z] D^{ab}(x,x') T_b(x';z]$$

$$T_a(x;z] = -mc_1 \int d\tau_a \, \frac{\delta(x - z_a)}{g^{1/2}} \qquad D^{ab}(x,x') = \begin{pmatrix} 0 & D_{\rm adv}(x,x') \\ D_{\rm ret}(x,x') & 0 \end{pmatrix}$$

$$S_{\text{eff}}^{(1)}[z_{1,2}^{\mu}] = \frac{1}{2!}\int_x \int_{x'} T_a(x;z]D^{ab}(x,x')T_b(x';z]$$

Vary effective action

$$F_{(1)}^{\mu}(\tau) = m^2 c_1^2 \Big(a^{\mu} + P^{\mu\nu}\nabla_{\nu} \Big) \int d\tau' \, D_{ret}(z^{\mu}, z^{\mu'})$$

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Detweiler & Whiting (2003)

$$\int d\tau' \, (\nabla_{\nu}) D_S(z^{\mu}, z^{\mu'}) = 0$$

Regularize divergences
&
Renormalize coupling constants

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- 2nd order self-force

$$= \frac{1}{2!}\int_x \int_{x'} \int_{x''} T_a(x;z] D^{ab}(x,x') T_{bc}(x';z] D^{cd}(x',x'') T_d(x'';z]$$

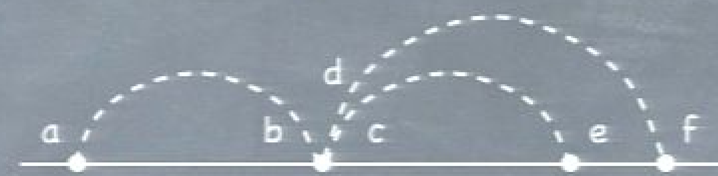
Divergences vanish in dimensional regularization $\qquad D_{\rm ret} \to D_R$

$$F^\mu_{(2)}(\tau) = -m^3 c_1^2 c_2 (a^\mu + P^{\mu\nu}\nabla_\nu)\left\{ \frac{1}{2} I_R(z^\mu)^2 + \int d\tau' \, D_R(z^\mu, z^{\mu'}) I_R(z^{\mu'}) \right\}$$

$$I_R(x) \equiv \int d\tau' \, D_R(x, z^{\mu'})$$

- 3rd order self-force

+



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$$+\frac{1}{3!}\int_x\int_{x'}\int_{x''}\int_{x'''} T_a(x;z]D^{ab}(x,x')T_{bcd}(x';z]D^{ce}(x',x'')T_e(x'';z]D^{df}(x',x''')T_f(x''';z)$$

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$$D_{\rm ret} \to D_R$$

$$F^\mu_{(3)}(\tau) = m^4 c_1^2 c_2 (a^\mu + P^{\mu\nu}\nabla_\nu)\left\{\int d\tau' d\tau''\, D_R(z^\mu, z^{\mu'}) D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''}) + I_R(z^\mu)\int d\tau'\, D_R(z^\mu, z^{\mu'}) I_R(z^{\mu'})\right\}$$

$$+ m^4 c_1^3 c_3 (a^\mu + P^{\mu\nu}\nabla_\nu)\left\{\frac{1}{2}\int d\tau'\, D_R(z^\mu, z^{\mu'}) I_R^2(z^{\mu'}) + \frac{1}{6} I_R^3(z^\mu)\right\}$$

- Notice that the force is always proportional to the same factor

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- So we can define a single function so that

$$\Gamma(z^\mu) \equiv 1 - mc_1^2 I_R(z^\mu) + m^2 c_1^2 c_2 \left(\frac{1}{2} I_R^2(z^\mu) + \int d\tau' \, D_R(z^\mu, z^{\mu'}) I_R(z^{\mu'}) \right) + \cdots$$

$$m\Gamma(z^\mu) a^\mu = -mP^{\mu\nu} \nabla_\nu \Gamma(z^\mu)$$

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- The equations of motion suggest that either:

1) The particle has a (time-varying) effective mass

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Scalar waves in EFT

- The scalar perturbations emitted by the EMRI are also calculated from Feynman diagrams

Leading order emission

$O(\varepsilon)$

Next-to-leading order emission

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Nonperturbative effects in scalar model

Galley (in preparation)

- Assumptions:

1) $c_{n>2} = 0$ $\quad -mc_1 \int d\tau\, \psi(z^\mu) - \frac{mc_2}{2} \int d\tau\, \psi^2(z^\mu)$

2) Ignore finite size effects

3) Small compact object moves on a circular geodesic of Schwarzschild

4) Ignore radiative effects -- only conservative dynamics

- Consequences:

1) Perturbative expansion can be fully resummed in the mass ratio

2) Can get exact results for orbital energy, ISCO, etc.

Nonperturbative results

- What parameter values to choose for c_1 and c_2?

In gen'l, the nonlinear scalar model is a gravity theory for conformal metric perturbations

$$h_{\alpha\beta} = \frac{\phi(x)}{2} g_{\alpha\beta} \quad \Rightarrow \quad c_1 = \frac{1}{4},\ c_2 = \frac{1}{32}$$

$$\psi_R(r_0) = -\frac{m c_1 I_R(r_0)}{1 + m c_2 I_R(r_0)}$$

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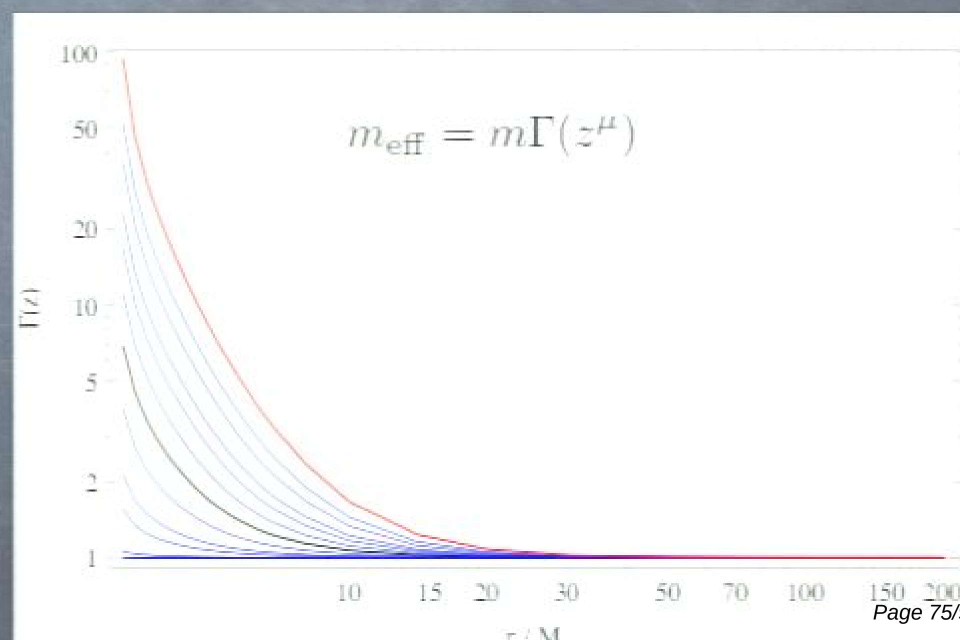

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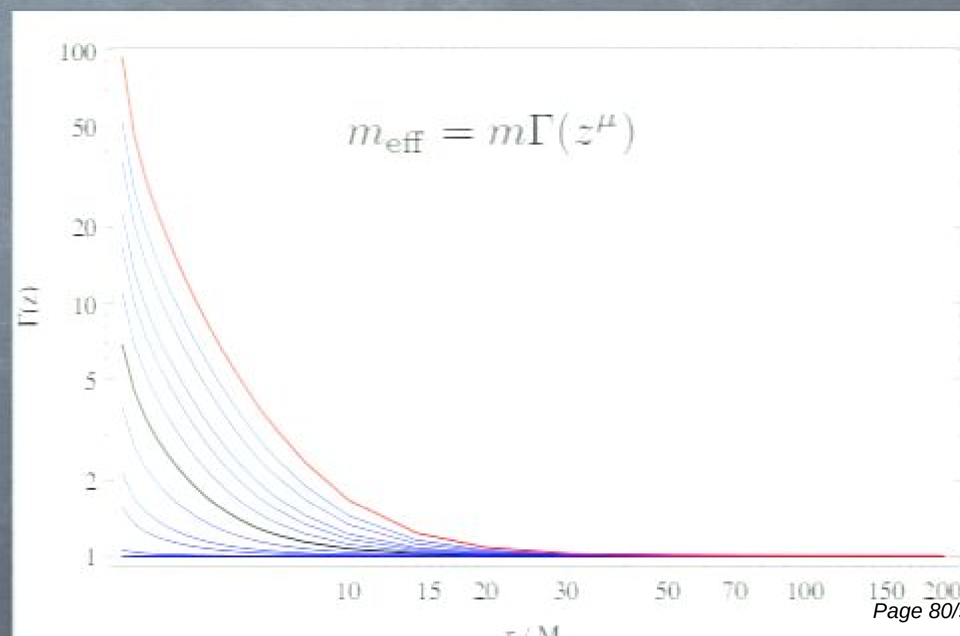
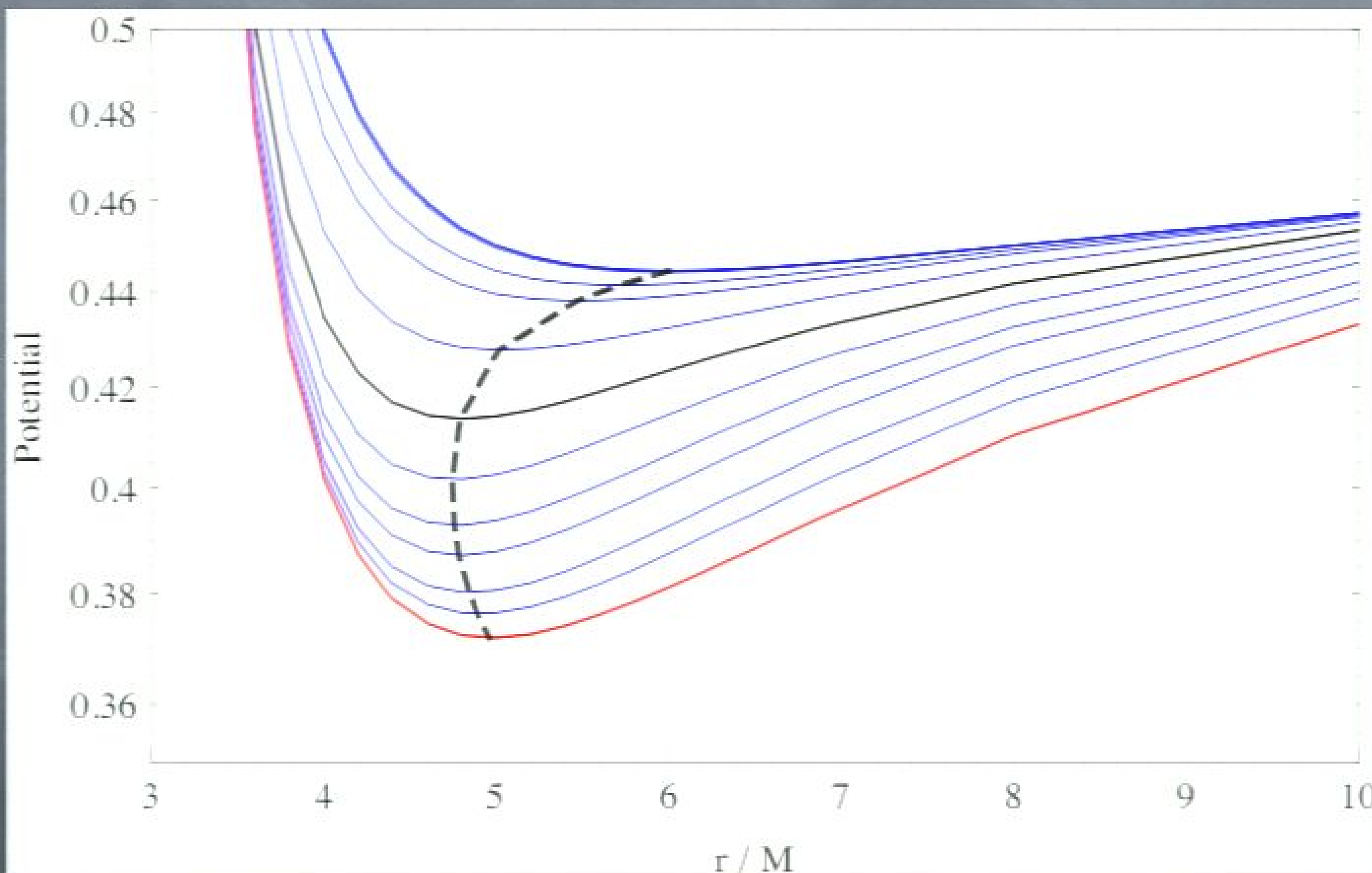


Nonperturbative ISCO

$$V_{\text{full}}(r_0) = \frac{(r_0 - 2M)^2}{2r_0(r_0 - 3M)}\left(1 + r_0 \partial_r \ln \Gamma(r_0)\right)$$



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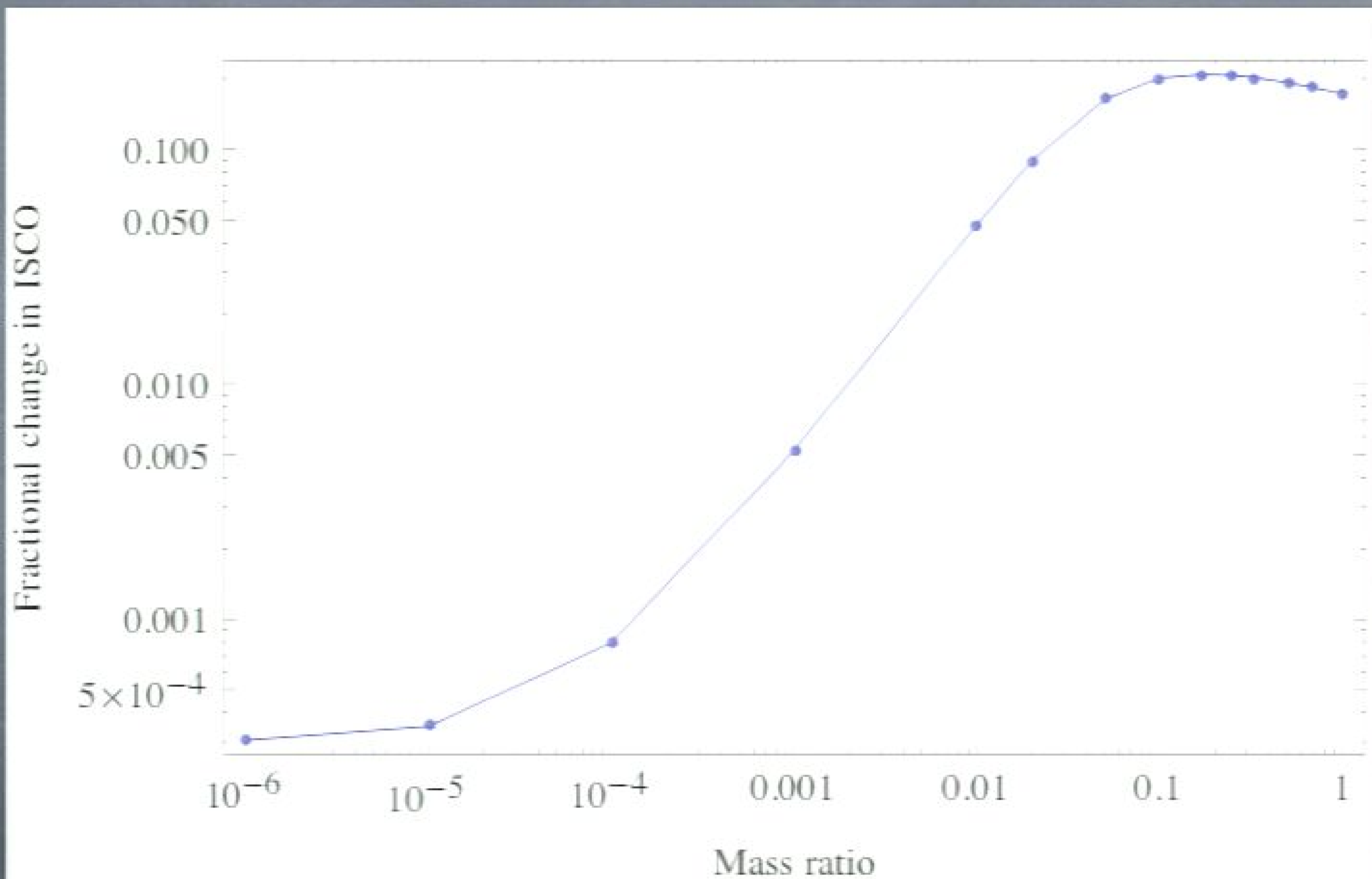

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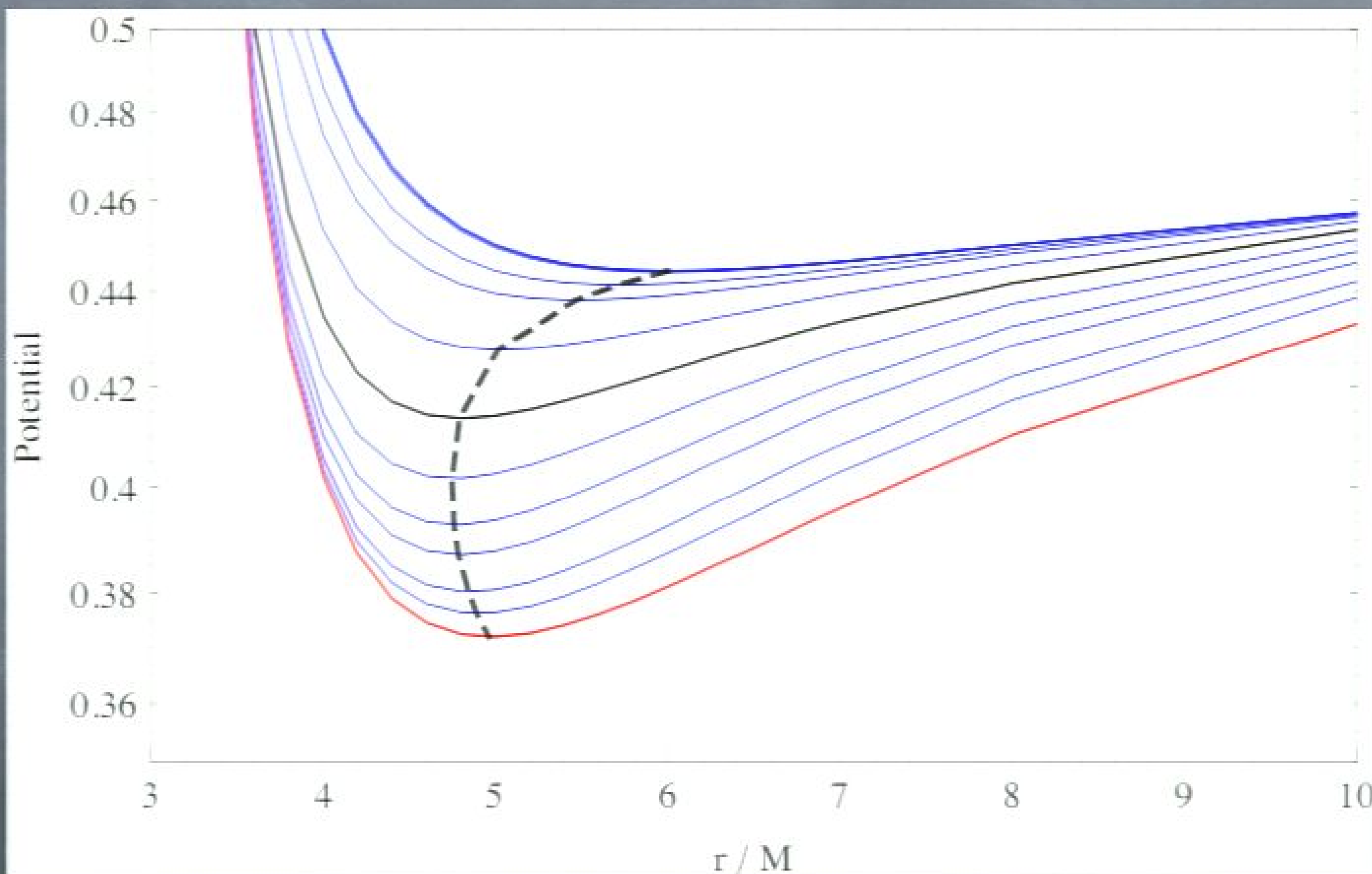
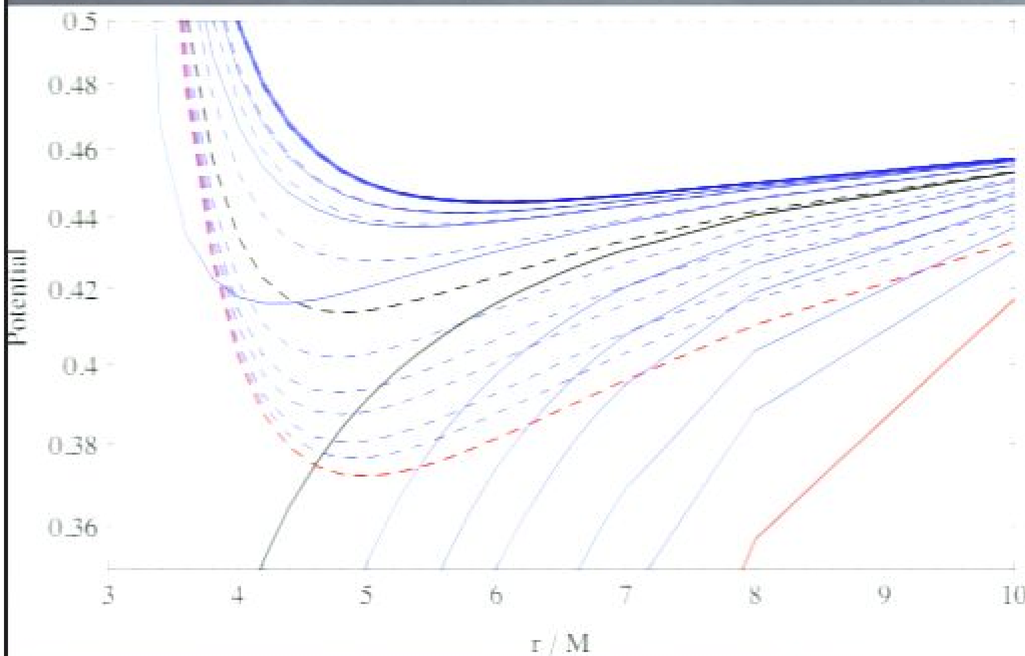
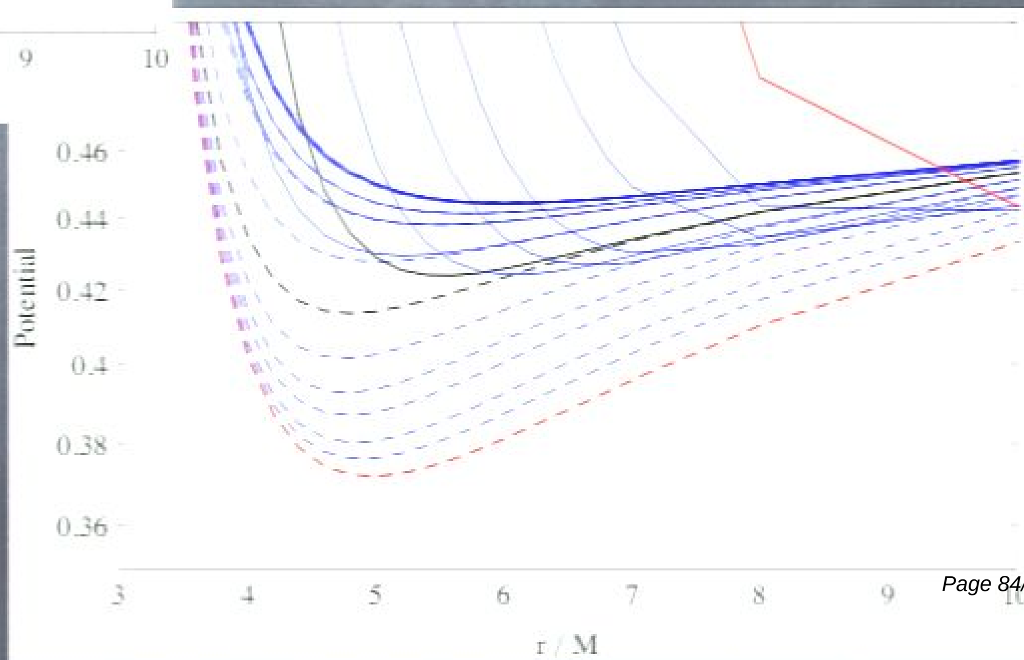


Comparison to perturbative corrxns

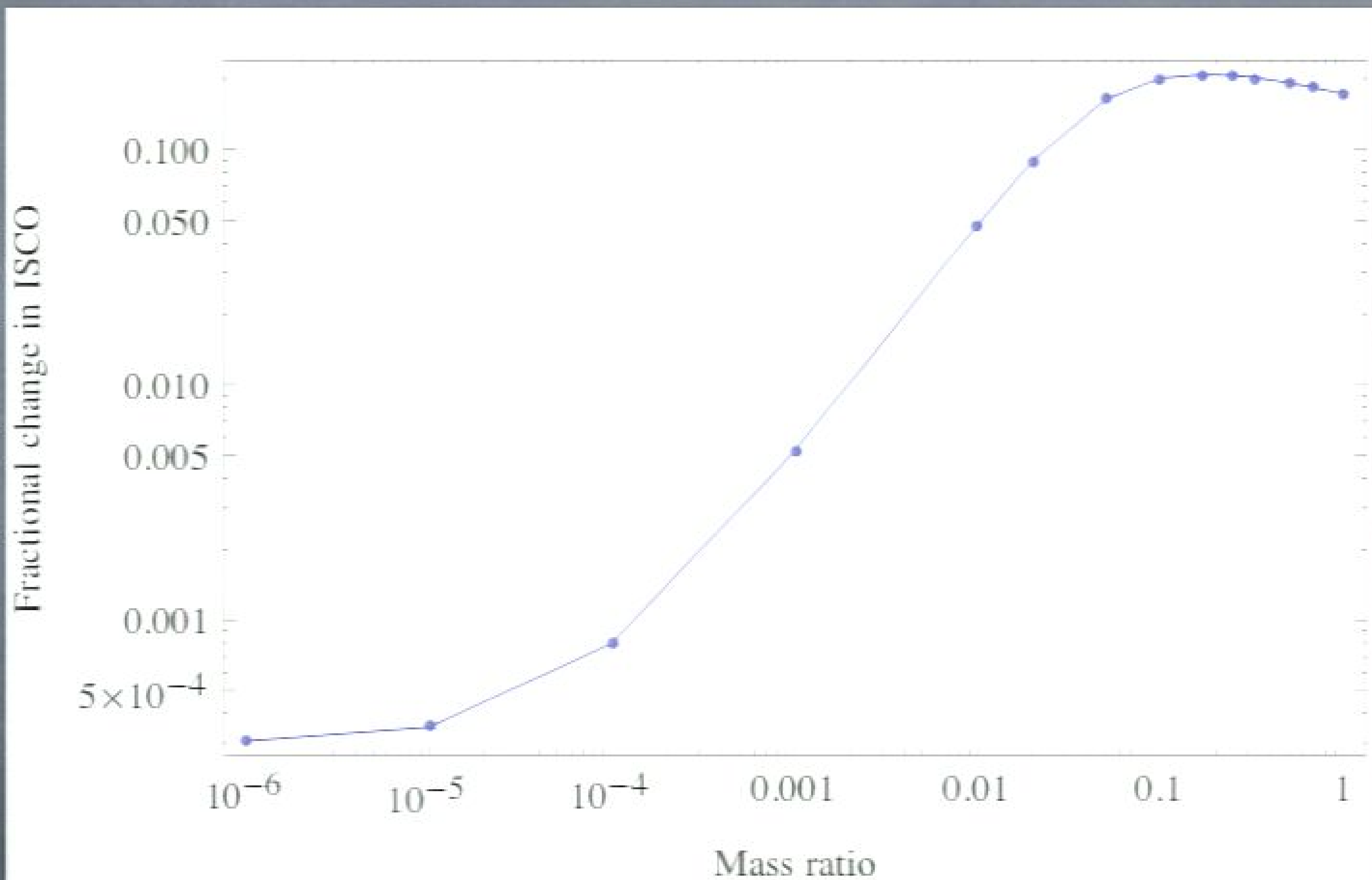


First order

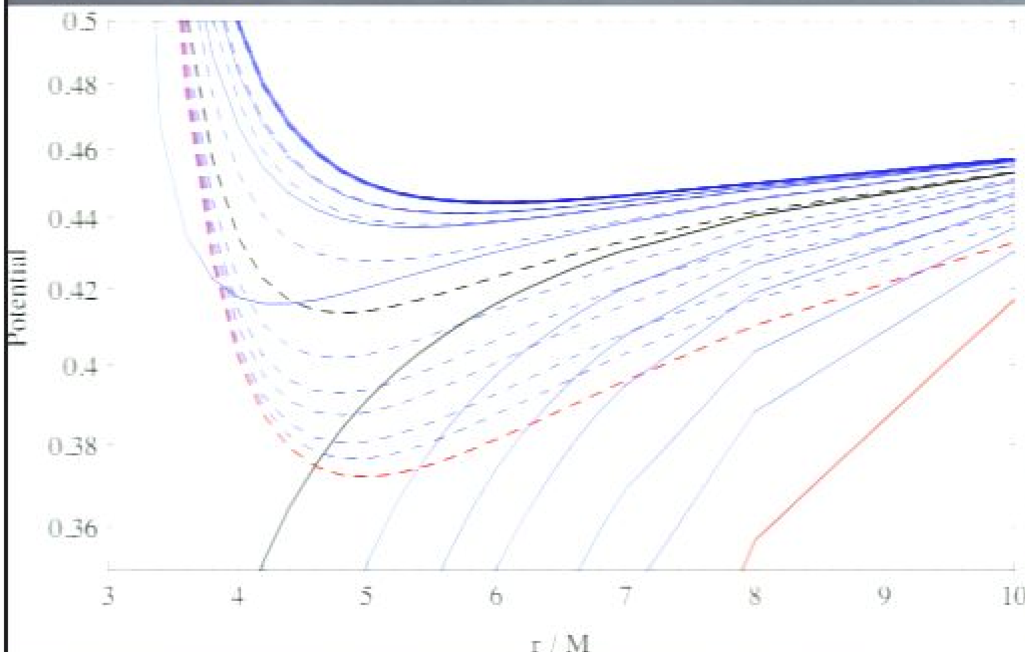
Second order



Nonperturbative ISCO

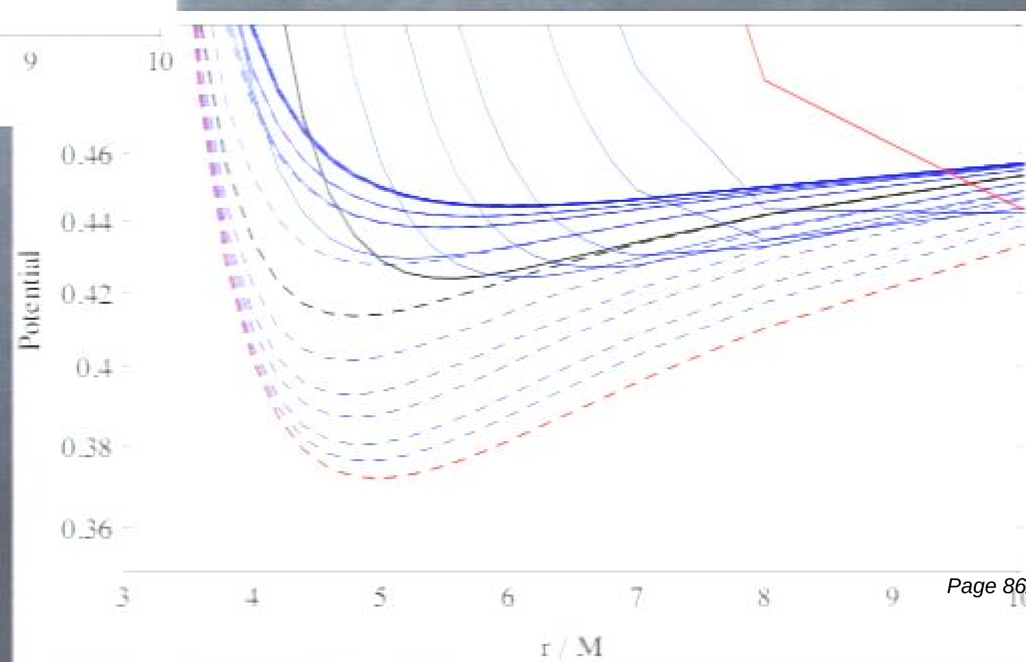


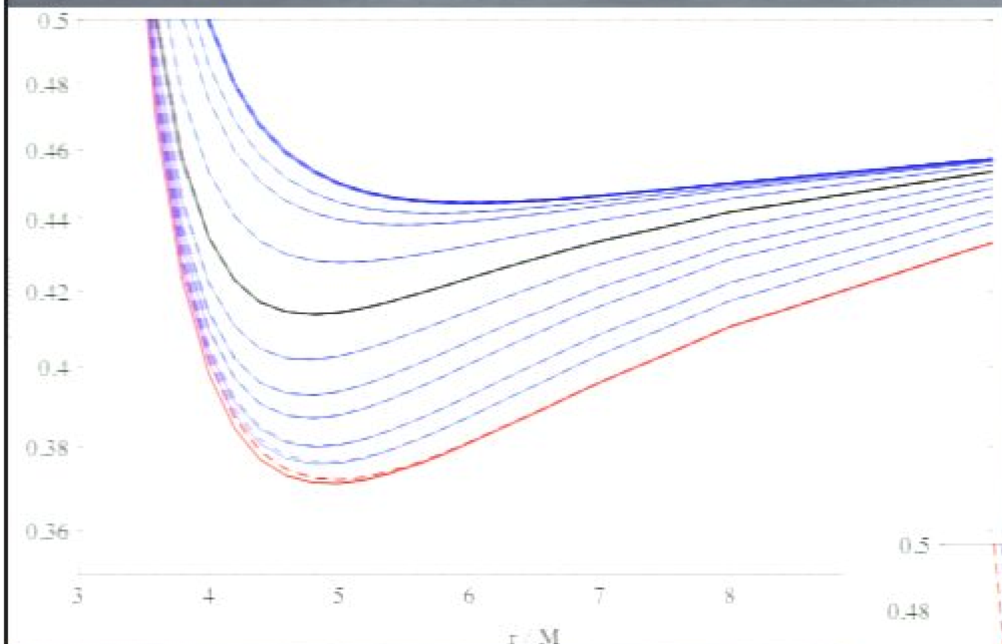
Comparison to perturbative corrxns



First order

Second order

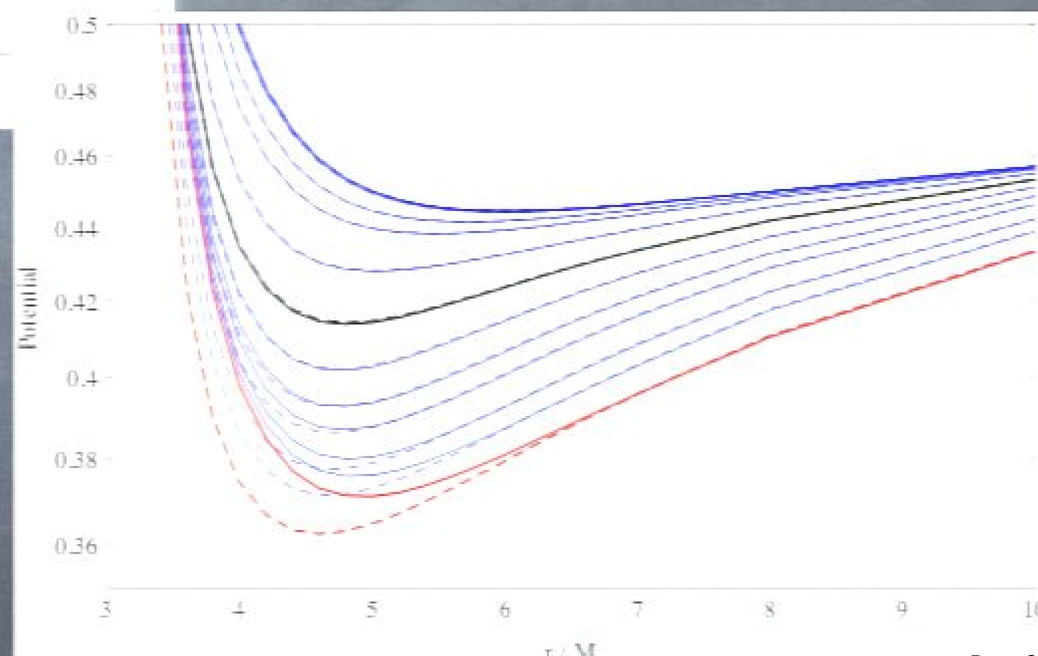




First order vs
Full result with $c_2 \rightarrow -10c_2$

First order but don't expand out the denominator:

$$ma^\mu = -\frac{mP^{\mu\nu}\nabla_\nu \Gamma(z)}{\Gamma(z)}$$



Outlook and Conclusions

- EMRIs can probe the strong-field regime of gravity but requires modeling them with high accuracy

  - High-precision measurements of source parameters (especially, with observed electromagnetic follow-ups)

  - Testing General Relativity in strong gravitational fields

- Effective Field Theory approach is an efficient way to model EMRIs with as much accuracy as desired

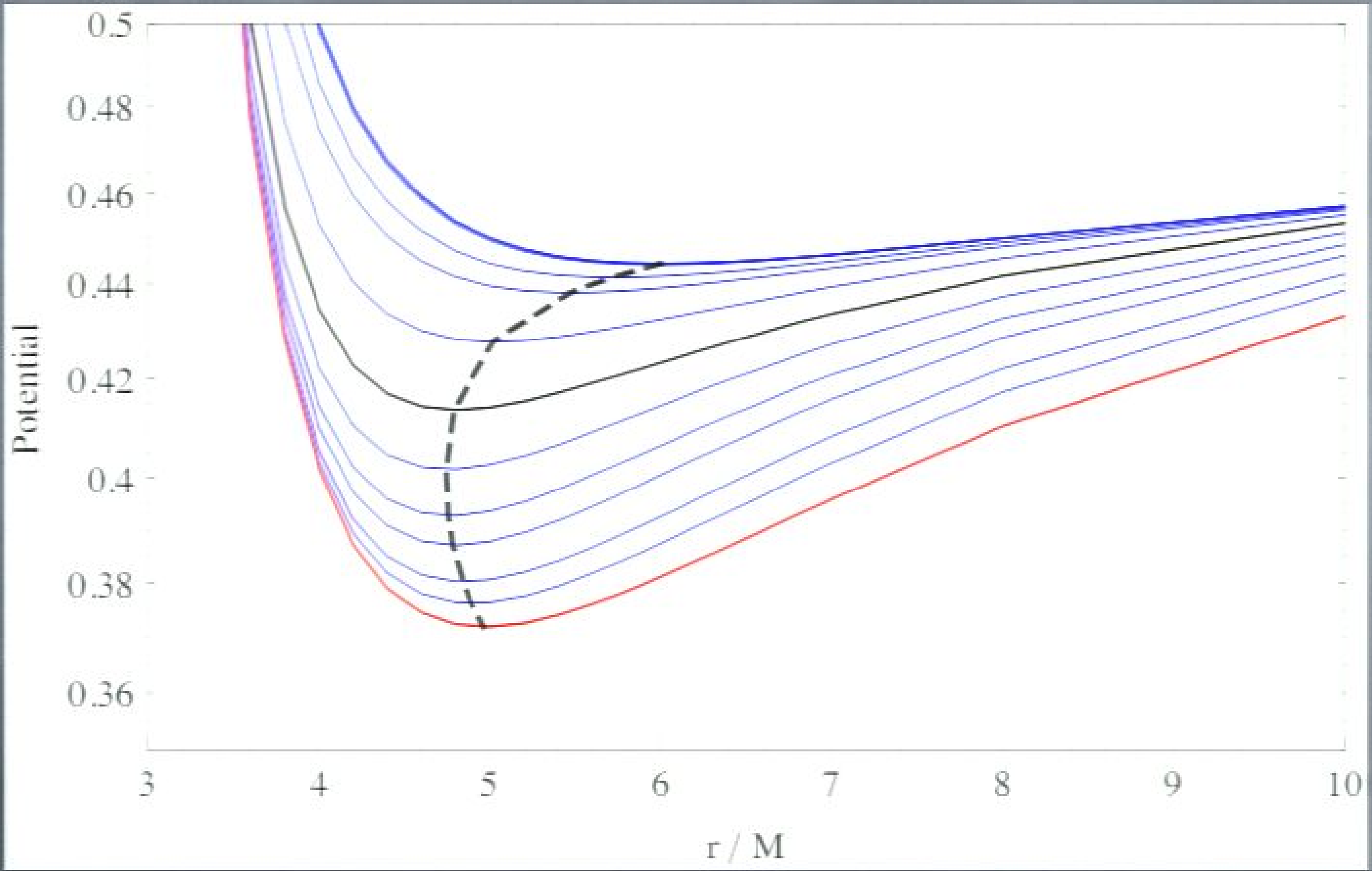
  - Recently calculated third order self-force corrections in a class of nonlinear scalar models analagous to gravitational EMRIs

- For a subclass of scalar models the conservative sector of the theory can be fully resummed

  - Exact, nonperturbative results (ISCO, orbital energy, self-force) in mass ratio

Nonperturbative ISCO

$$V_{\text{full}}(r_0) = \frac{(r_0 - 2M)^2}{2r_0(r_0 - 3M)}\left(1 + r_0 \partial_r \ln \Gamma(r_0)\right)$$



Scalar waves in EFT

- The scalar perturbations emitted by the EMRI are also calculated from Feynman diagrams

Leading order emission

$O(\varepsilon)$

Next-to-leading order emission

$O(\varepsilon^2)$

Nonperturbative effects in scalar model

Galley (in preparation)

- Assumptions:

1) $c_{n>2} = 0$ $\quad -mc_1 \int d\tau\, \psi(z^\mu) - \frac{mc_2}{2} \int d\tau\, \psi^2(z^\mu)$

2) Ignore finite size effects

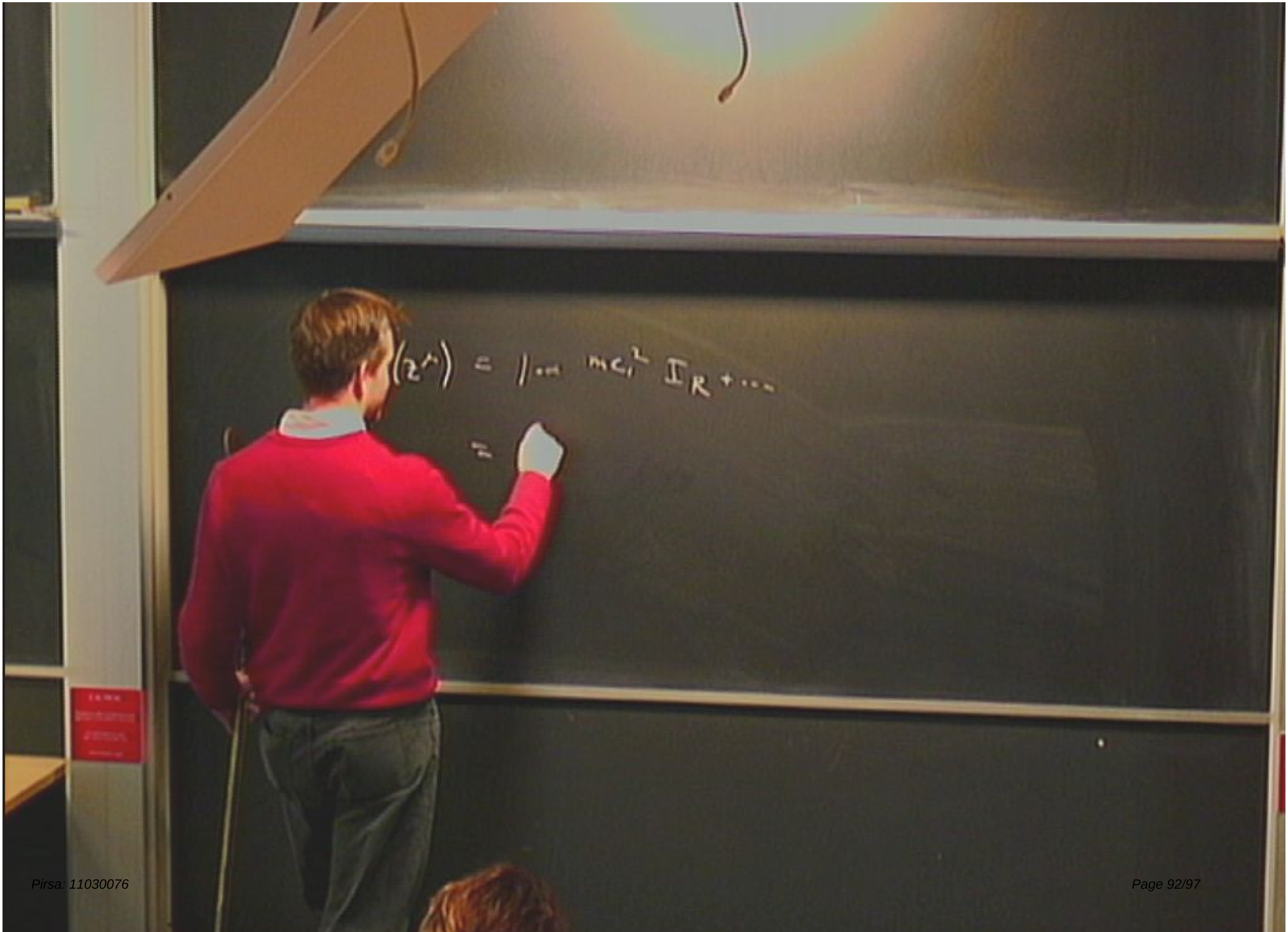
3) Small compact object moves on a circular geodesic of Schwarzschild

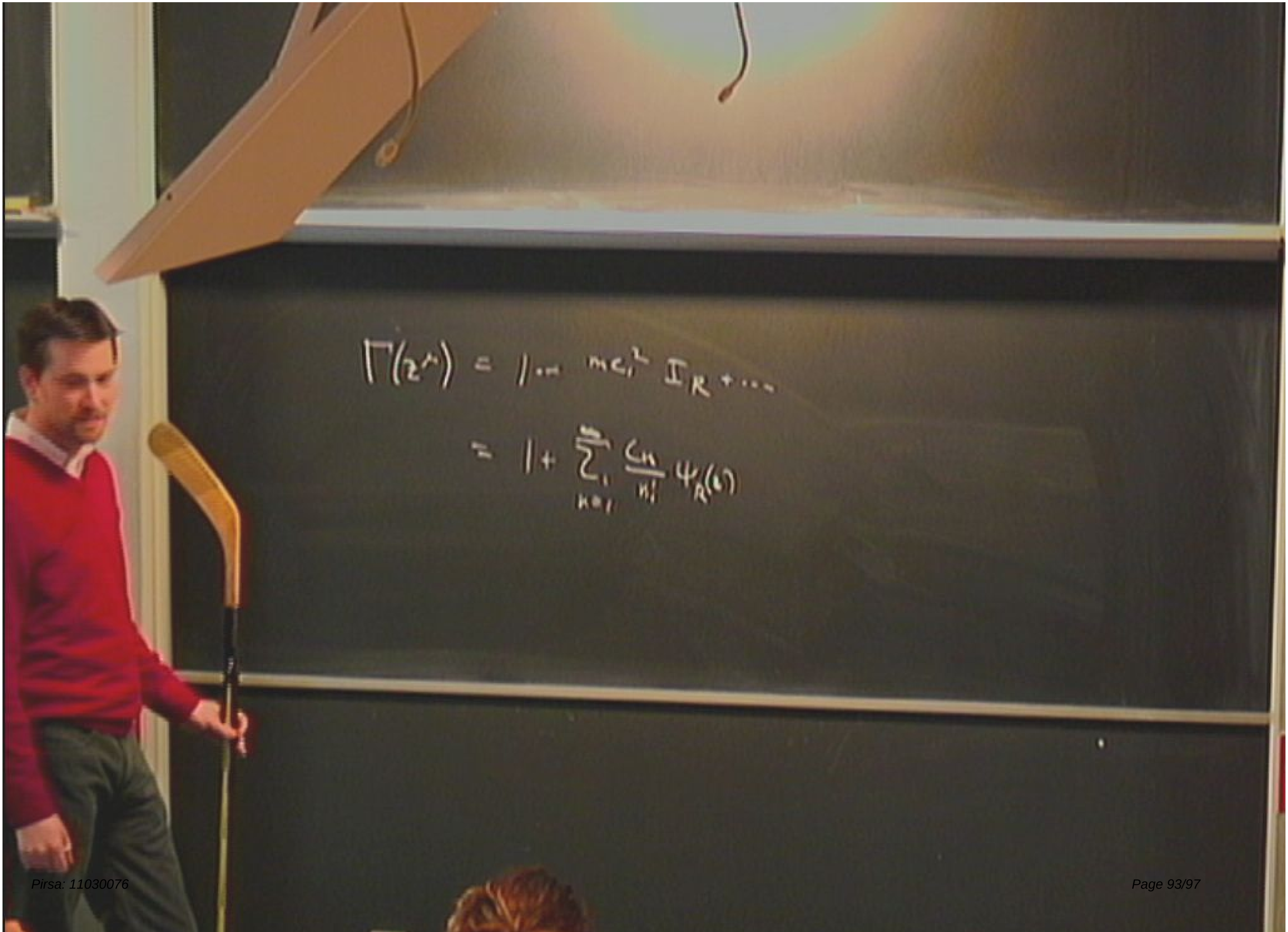
4) Ignore radiative effects -- only conservative dynamics

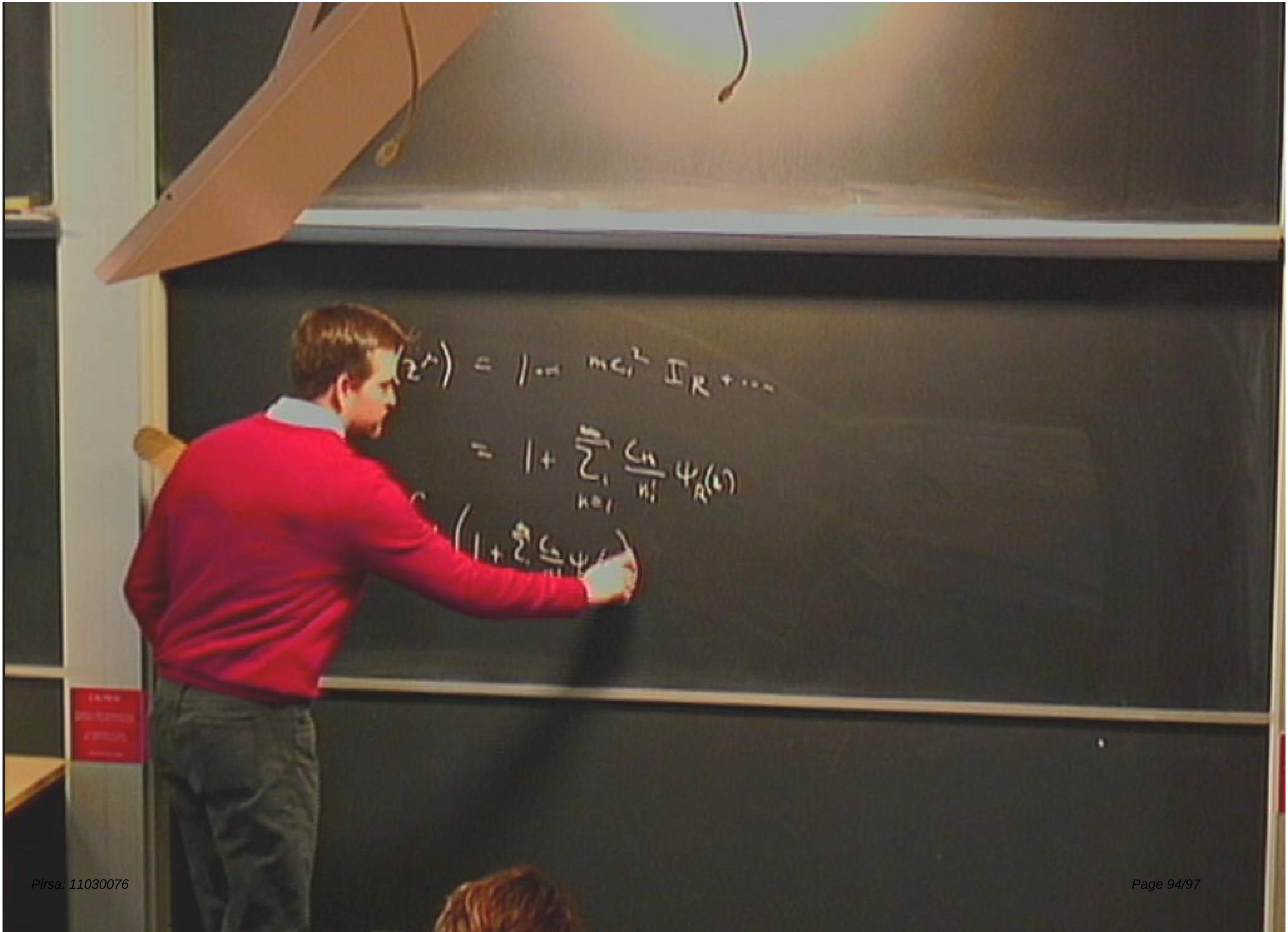
- Consequences:

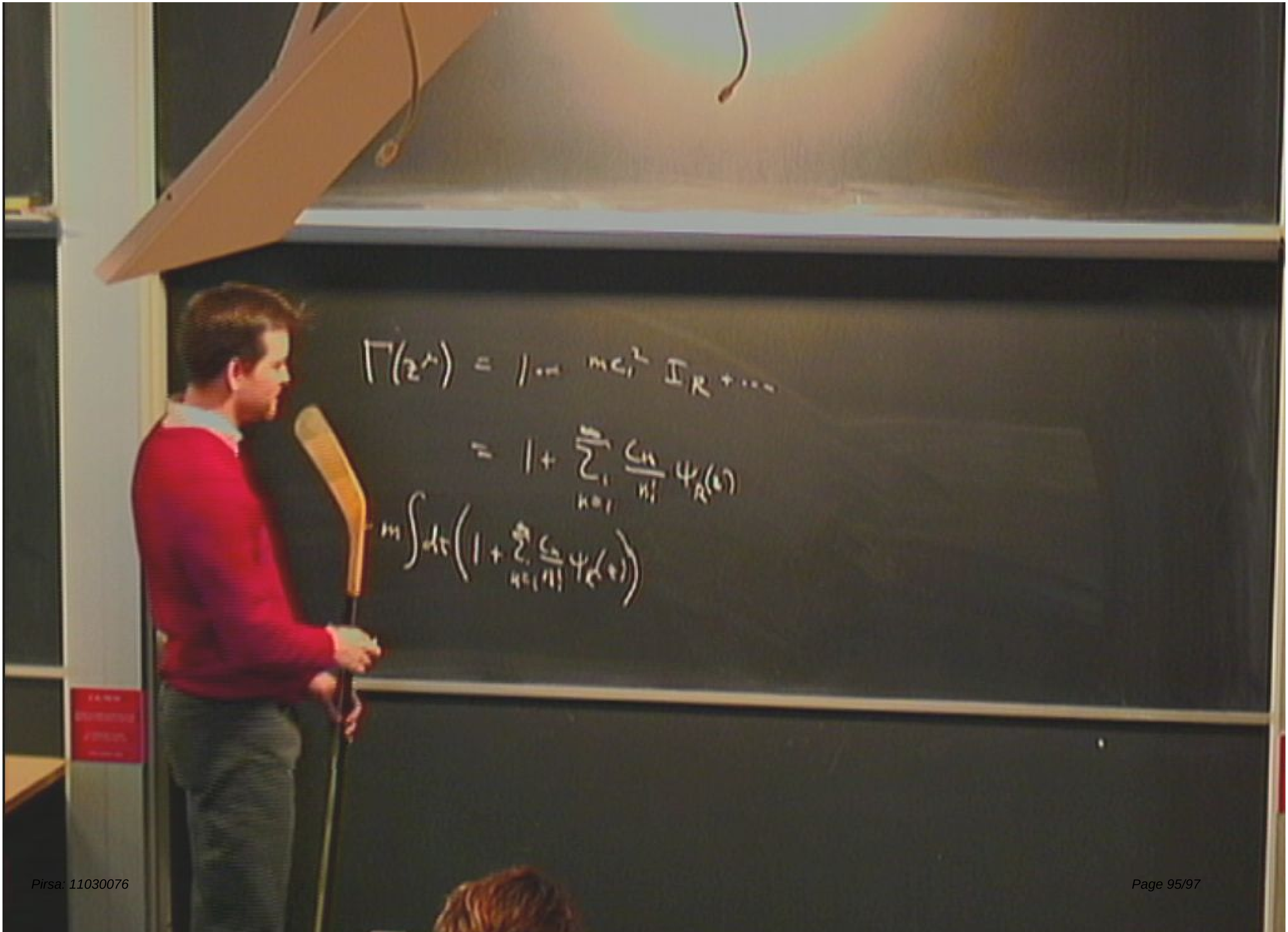
1) Perturbative expansion can be fully resummed in the mass ratio

2) Can get exact results for orbital energy, ISCO, etc.





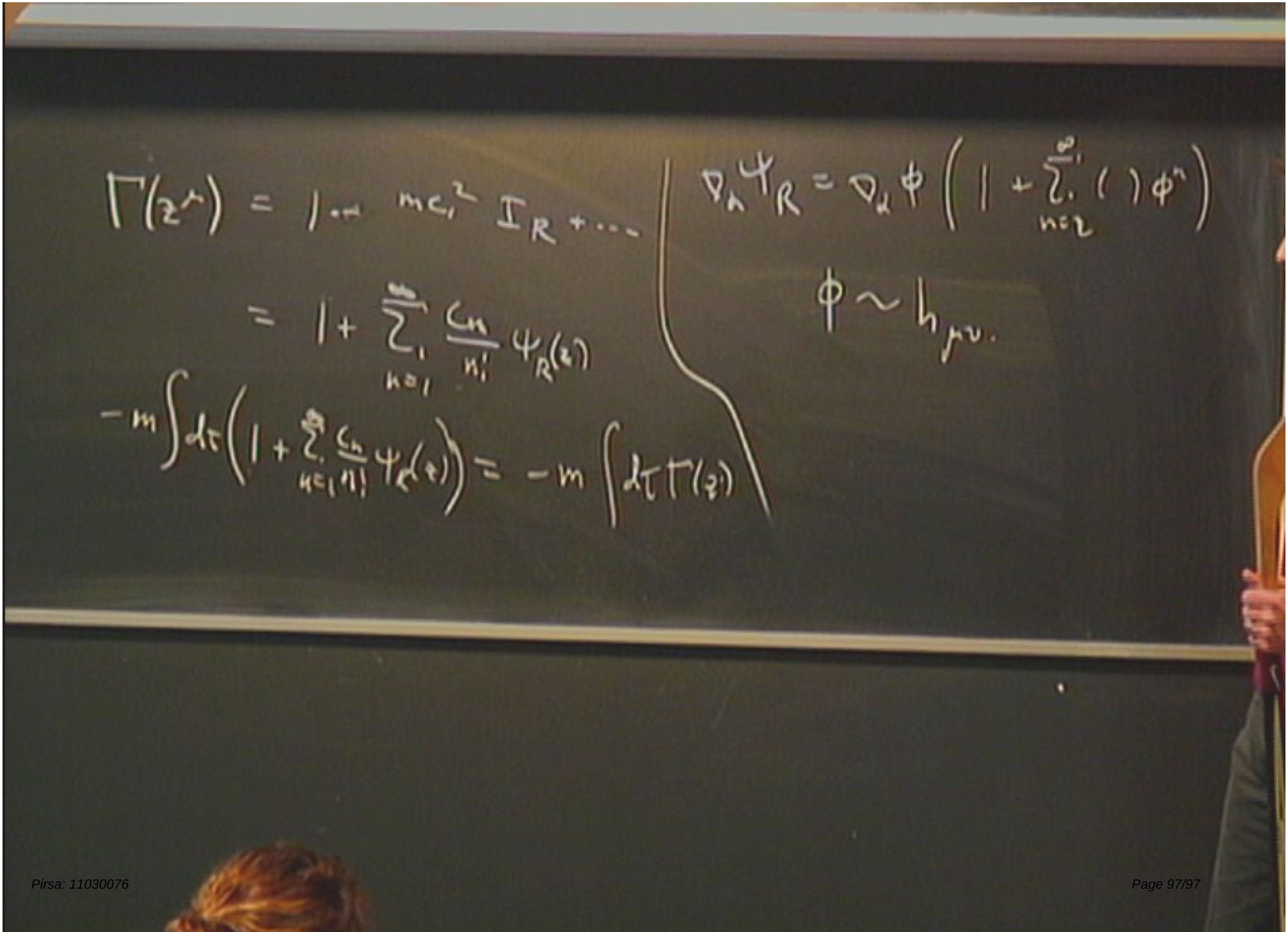




$$\Gamma(z^\mu) = 1 + m c_1^2 I_R + \cdots$$

$$= 1 + \sum_{n=1}^{\infty} \frac{c_n}{n!} \psi_R(z)$$

$$-m \int d\tau \left(1 + \sum_{n=1}^{\infty} \frac{c_n}{n!} \psi_R(z) \right) = -m \int d\tau \, \Gamma(z)$$



$\Gamma(z^\mu) = 1 + m c_1{}^2 I_R + \cdots$
$= 1 + \sum_{n=1}^{\infty} \frac{c_n}{n!} \psi_R(z)$
$-m \int d\tau \left(1 + \sum_{n=1}^{\infty} \frac{c_n}{n!} \psi_R(z)\right) = -m \int d\tau\, \Gamma(z)$
$\partial_\mu \psi_R = \partial_\mu \phi \left(1 + \sum_{n=2}^{\infty} (\ ) \phi^n\right)$
$\phi \sim h_{\mu\nu}$.