Title: Antimatter Without Antiparticles

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Abstract: The nature of antimatter is examined in the context of algebraic quantum field theory. It is shown that the notion of antimatter is more general than that of antiparticles. Properly speaking, then, antimatter is not matter made up of antiparticles --- rather, antiparticles are particles made up of antimatter. We go on to discuss whether the notion of antimatter is itself completely general in quantum field theory. Does the matter-antimatter distinction apply to all field theoretic systems? The answer depends on which of several possible criteria we should impose on the space of physical states.

Antimatter Without Antiparticles

David John Baker
University of Michigan
(with Hans Halvorson)

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Outline

1. The naive picture
2. Puzzles about naive antimatter
3. Group representation magic
4. How superselection makes the magic work
5. The DHR approach
6. Foundational upshot

Textbook antimatter

- Start with solutions to free, relativistic wave equations (Dirac, Klein-Gordon).
- Matter corresponds to positive-frequency solutions, antimatter to negative-frequency solutions.
- Define separate particle and antiparticle number operators (N^+, N^-) on the resulting Fock space.
- Suspicious: No new definition when the space of interacting states is constructed.

First puzzle: Particle interpretation

- Does the textbook definition apply to interacting field theories?
- Free theories are used at the asymptotic scattering limit, but this is an idealization.

Puzzle: The textbook definition relies on the particle concept, which breaks down in interacting theories.

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First puzzle: Arguments against particles

- Haag's theorem: Interacting theories can't be formulated on Fock space.
- Fraser (2008): Fock space is necessary for a particle interpretation.
- No Lorentz-invariant way to "split the frequencies" in non-linear QFT.
- So no breakdown into positive- and negative-frequency waves.

One response

David Wallace "Matter comes in particle and antiparticle form..."

"Particles are emergent phenomena, which emerge in domains where the underlying quantum field can be treated as approximately linear." (2009)

Second puzzle: Opposite quantum numbers

Penrose: [F]or each type of particle, there is also a corresponding antiparticle for which each additive quantum number has precisely the negative of the value that it has for the original particle. (*Road To Reality*, p. 66)

- Which quantities count as "additive quantum numbers?"
- What is the "opposite value" of a quantum number?

Second puzzle: Example of isospin

- Additive quantum numbers are superselected quantities.
- But how does QFT predict which quantities are superselected?
- The "opposite" of a quantum number isn't just negative.
- Isospin quantum numbers:

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$$

- Why is isospin $\frac{1}{2}$ its own "opposite?"

Group representation magic

- Classically, Noether's theorem determines conserved charges from symmetries.
- In QFT a system's gauge group (internal symmetry group) is supposed to determine its additive "charge" quantum numbers.
- The charges for Dirac electrons are given by integers ($\mathbb{Z}$) *because* the gauge group is $U(1)$.
- We say $\mathbb{Z}$ is the dual of $U(1)$.

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Group representation magic

Group Duality (v. 2) For a system with compact gauge group G, the quantum numbers have the structure of the category $\mathrm{Rep}(G)$, whose objects are unitary representations of G on finite-dimensional Hilbert spaces and whose arrows are intertwiners between these representations.

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Isospin again...

- The dual category $\mathrm{Rep}(G)$ of a group is not a group unless G is abelian.
- Isospin arises from gauge group $SU(2)$.
- The dual $\mathrm{Rep}(SU(2))$ is a tensor category (category with tensor products).
- The tensor product '$\otimes$' represents composition of charges (e.g. $\frac{1}{2} \otimes \frac{1}{2}$).

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- This corresponds to a direct sum: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$.
- The neutral quantum number 0 in the direct sum will be key to defining "opposite" quantum numbers needed to define antimatter.

$$X \otimes 0 = 0 \otimes X = X$$

A dynamical interpretation of $\otimes$?

- Seems natural:

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

translates to "When particles with $\frac{1}{2}$ collide, the products have 0 or 1."

- This interpretation is impossible except as an idealization.
- Our formalism (including Group Duality) can model free as well as interacting systems.
- In free QFT, "collisions" never happen.

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Compositional interpretation of $\otimes$

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really means:

"When two systems with quantum number $\frac{1}{2}$ are composed, it's possible for them to form a system with either quantum number 0 or 1."

- "Possible" means *depending on other (non-charge) properties.*

What makes the magic work?

- Group Duality works great!
- But it needs a physical explanation – can we derive it from QFT?
- **Preview:** The quantum numbers in $\mathrm{Rep}(G)$ correspond one-one with superselection sectors.
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Vocabulary of AQFT

- Observables are given by self-adjoint operators in a C*-algebra $\mathfrak{A}$.
- States are normed, linear functionals $\omega : \mathfrak{A} \to \mathbb{C}$.
- $\omega(A)$ gives the expectation value of $A \in \mathfrak{A}$.

Superselection sectors in AQFT

- States live in different sectors iff they can't be superposed.

GNS Theorem: Every state has a unique home Hilbert space representation of the algebra $\mathfrak{A}$.

- States from different sectors have different (unitarily inequivalent) GNS representations.
- This explains why states never change sectors, since the dynamics is normally unitary.

Sectors correspond to additive quantum numbers

- Since sectors are dynamically isolated, quantum numbers are conserved.
- To explain Group Duality, we need to show that the physically relevant sectors of $\mathfrak{A}$ correspond one-one with the elements of $\mathrm{Rep}(G)$.
- To explain antimatter, we need to show that all sectors possess "opposites" (conjugates).

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Selection criteria

- Selection criteria: ways of weeding out "unphysical" states and representations.
- Example of Hadamard condition in semi-classical gravity.
- The DHR method: try an unrealistic selection criterion and hope to generalize later.

DHR representations

DHR Criterion: Treat representations as "physical" when they differ from the vacuum's GNS representation only in a finite region.

- The DHR representations form a category $\Delta(\mathfrak{A})$.
- The criterion is unrealistic: electrodynamics violates it (Gauss's law).

Equating $\mathrm{Rep}(G)$ and $\Delta(\mathfrak{A})$

- The gauge group G will leave each sector invariant.
- Doplicher and Roberts: This means each sector is both a DHR representation and an element of $\mathrm{Rep}(G)$.
- So we have a one-one correspondence which forms an isomorphism of categories (functor).

Quantum numbers = sectors =
DHR representations = representations of G

- Group Duality follows!

Conjugates and antimatter

- We say representations $X, \overline{X}$ conjugate iff $X \otimes \overline{X} = 0 \oplus$ (other representations).
- In $\mathrm{Rep}(G)$ every representation has a conjugate, so same for $\Delta(\mathfrak{A})$.
- If two states come from conjugate representations, when they compose the composite state can be in the vacuum representation.
- This means the states can "annihilate" without changing sector.

Payoff: Antimatter without antiparticles

Definition: A matter system and its antimatter counterpart are given by states in conjugate superselection sectors.

- Applies to all DHR states.
- This includes at least one interacting model (Yukawa-2).
- So we can define antimatter in a system with no particle interpretation.

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Is there antimatter, fundamentally?

- Obviously the restriction to DHR states is unrealistic.
- The DHR picture of antimatter has been generalized to "Buchholz-Fredenhagen" states.
- No known obstacle in principle to generalizing it further.
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Further reading

Doplicher, S., Haag, R. and Roberts, J. E. [1969a]: Fields, observables and gauge transformations. 1 and 2, *Communications in Mathematical Physics*.

Doplicher, S., Haag, R. and Roberts, J. E. [1971,1974]: Local Observables and Particle Statistics I and II, *Communications in Mathematical Physics*.

Doplicher, S. and Roberts, J. E. [1990]: Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics, *Communications in Mathematical Physics*.

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