

Title: How might a Fermi surface die?

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Abstract: In the last many years a number of metallic solids have been studied that defy understanding within the principles of conventional textbook solid state physics. The most famous are the cuprate high temperature superconductors though many other examples have been found. In this talk I will argue that the mysterious properties of many such materials arises from an imminent `death' of their Fermi surfaces. I will discuss some theoretical ideas on how to kill a Fermi surface, and their implications for experiments.

How might a Fermi surface die?

T. Senthil
(MIT)

Conventional condensed matter physics: Landau's 2 great ideas

1. Integrity of the electron as a quasiparticle in phases of matter

(Fermi liquid metals, band insulators, BCS superconductors, spin density wave states,)

2. Notion of "order parameter" to describe phases of matter

- related notion of spontaneously broken symmetry
- basis of phase transition theory



Landau Fermi liquid theory

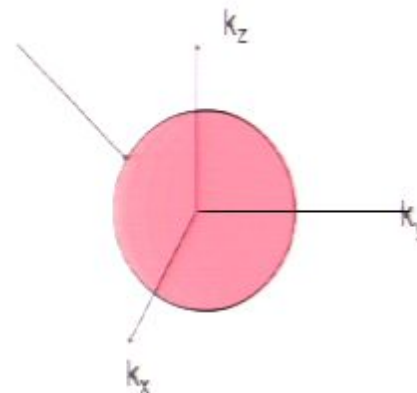
- **Electrons in a metal:**
quantum fluid of fermions
- Inter-electron spacing $\sim 1 \text{ \AA}$
 \Rightarrow Very strong Coulomb repulsion $\sim 1\text{-}10 \text{ eV}$ comparable to kinetic energy

But effects dramatically weakened due to Pauli exclusion.

Important 'quasiparticle' states near Fermi surface scatter only weakly off each other.

Describes conventional metals

Fermi surface



Filled states unavailable for scattering

Order parameter

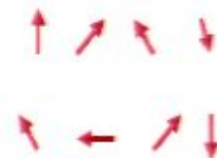
- Example - ferromagnetism

Ferromagnet:
Spins aligned



Increase temperature

Paramagnet:
Spins disordered



- Spontaneous magnetization: 'order parameter'.
- Ordered phase spontaneously breaks spin rotation symmetry.

Continuous phase transitions - theoretical paradigm

- Phenomena: Critical singularities, universality, scaling.

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- Landau: critical singularities due to long wavelength fluctuations of order parameter field.



Continuous phase transitions - theoretical paradigm

- Phenomena: Critical singularities, scaling, universality
- Landau: critical singularities due to long wavelength fluctuations of order parameter field.
- Landau-Ginzburg-Wilson: Landau ideas + renormalization group - sophisticated theoretical framework



Phase transitions – thermal versus quantum

Loss of magnetism on heating: thermal fluctuations

Can also have a phase transition at zero temperature as a function of some tuning parameter.

Qualitative change in the quantum ground state of the many particle system

Driven by quantum zero point motion rather than thermal motion

- ``quantum phase transition''

Quantum phase transitions: Landau-Ginzburg-Wilson description

- Universal critical singularities: Long wavelength, long time fluctuations of Landau order parameter field.
- Describe by suitable continuum quantum field theory at zero temperature.

Successful in describing many quantum critical phenomena.

Conventional condensed matter physics: Landau's 2 great ideas

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Modern quantum condensed matter physics

In the last 25 years both of these ideas have been challenged enormously by discoveries such as the fractional quantum Hall effect, high temperature superconductivity, and other phenomena.

Many fundamental questions have been raised (and some answered).

A sample of some basic questions

1. Does the electron have to survive as a quasiparticle in a phase of matter?

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4. Can bosons form a metal?
5. Is the order in a phase necessarily captured by a Landau order parameter?
 \Leftrightarrow Is symmetry breaking the only route to ordering?

A sample of some basic questions

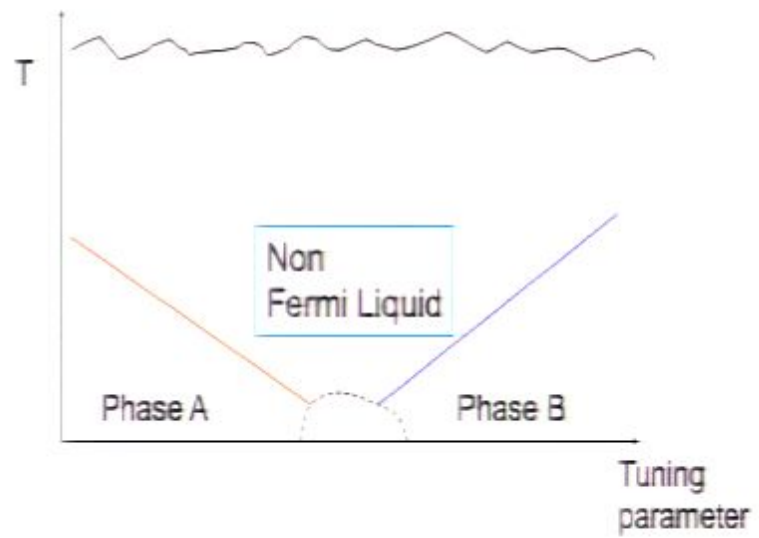
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6. Is it always correct that singularities at phase transitions are due to slow fluctuations of the order parameter?

Focus of this talk: strange metals

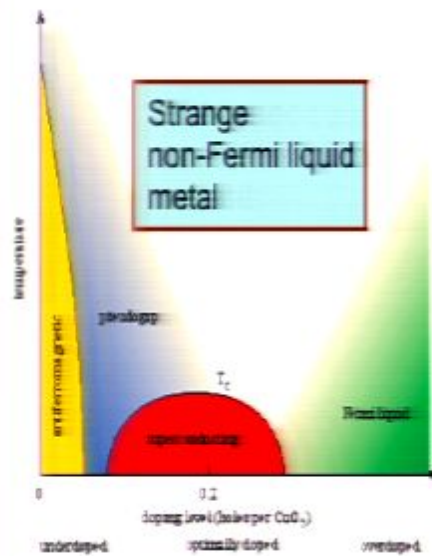
Many interesting metals where Landau's Fermi Liquid Theory breaks down.

“Non-Fermi Liquid” metals: Very little theoretical understanding though many interesting scattered ideas exist

A common phase diagram



Example 1: high temperature superconductors

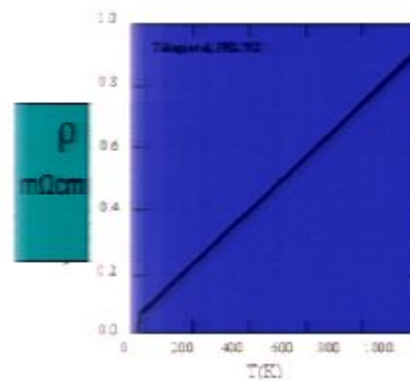
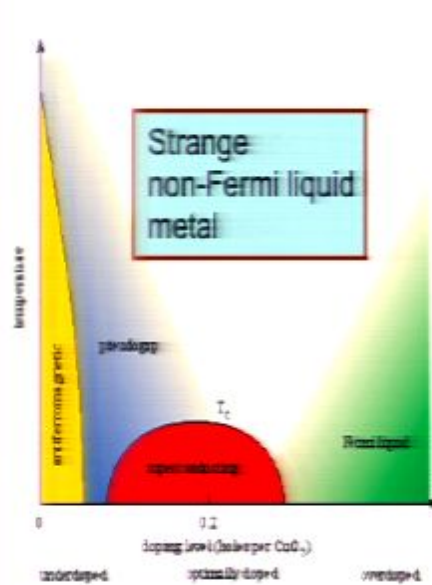


Strange metal: most mysterious!

Eg: Resistivity $\rho(T) \sim T$

Compare with Fermi Liquid $\rho(T) \sim T^2$

Example 1: high temperature superconductors

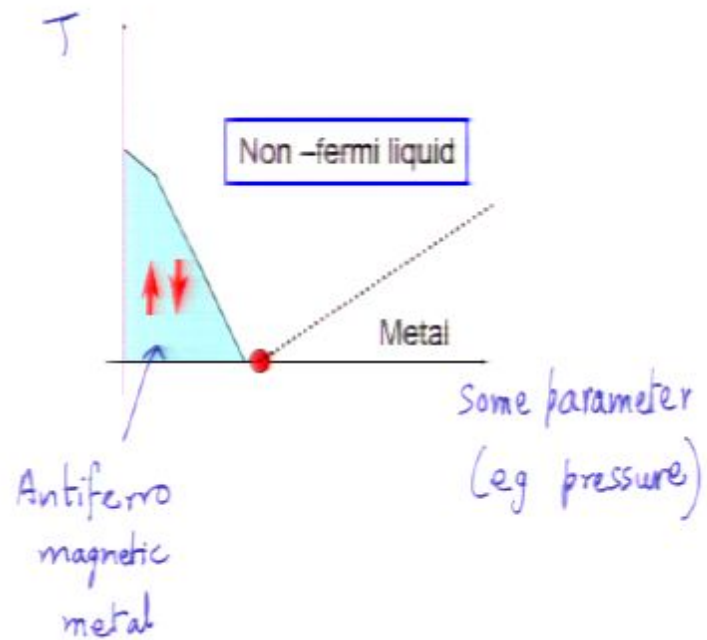


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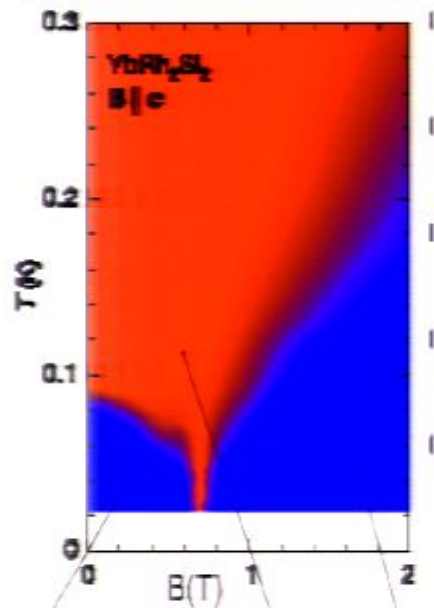
Compare with Fermi Liquid $\rho(T) \sim T^2$

Example 2: Magnetic ordering in certain rare earth alloys
CePd₂Si₂, CeCu_{5-x}Au_x, YbRh₂Si₂,.....



Representative data on YbRh_2Si_2

Custers et al. Nature, 2003

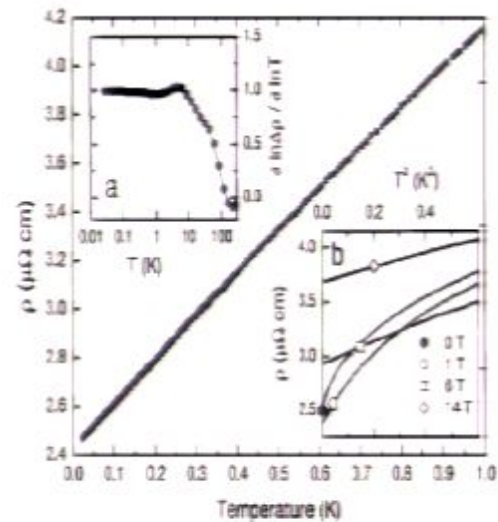


Magnetic metal

Fermi liquid

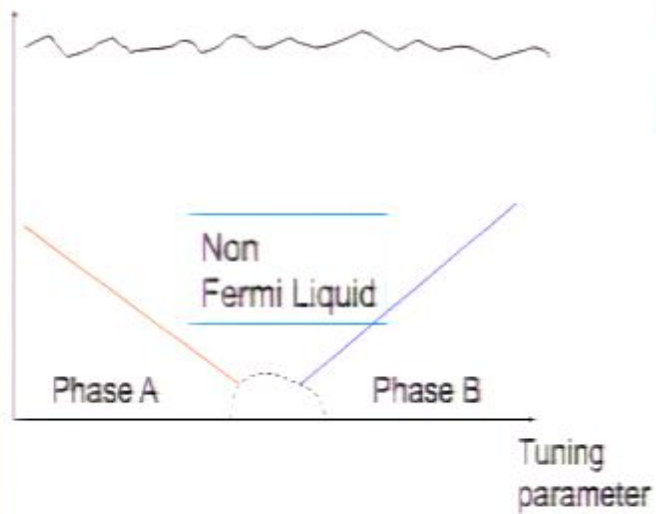
Non fermi liquid

Trovarelli et al. PRL 2000



T-dependence of resistivity at critical point: $\rho(T) \sim T$ for three decades in temperature!

Origin of non-fermi liquid physics?



Perhaps whatever
causes phase transition
between phases A & B
also underlies
non-fermi liquid
physics

“Natural” assumptions

1. Non Fermi Liquid: Universal physics associated with (quantum) critical point between phases A and B.
2. Landau: Universal critical singularities ~ fluctuations of order parameter for transition between phases A and B.

Try to play Landau versus Landau.

- However "natural" assumptions have difficulty producing non-fermi liquids at quantum critical points!!

Eg: Landau's theory of fluctuations of magnetic order parameter in metallic environment* spectacularly inconsistent with non-fermi liquid physics near magnetic quantum critical points.

So possibly.....

(i) Failure of textbook theory of metals

AND

(ii) Failure of textbook theory of phase transitions

Important clue from experiments

Killing a Fermi surface

At certain such $T = 0$ phase transitions in metals, an entire Fermi surface may disappear.

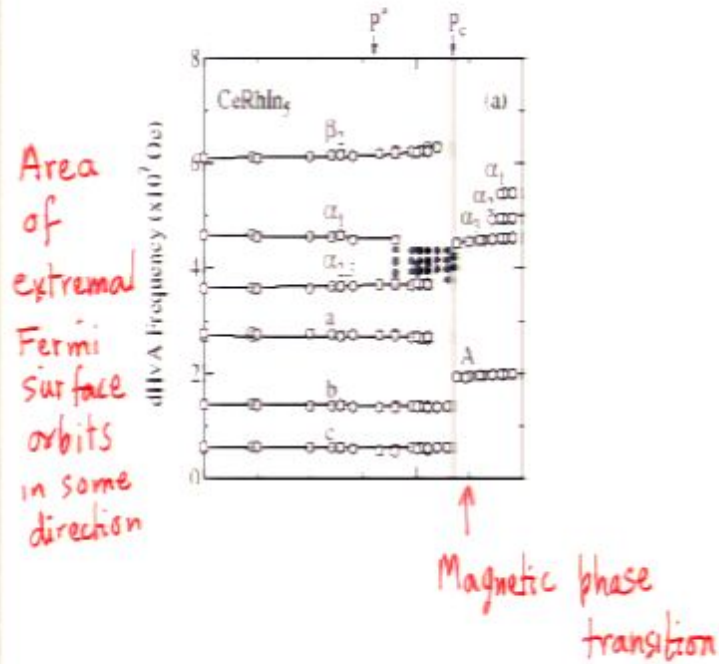
Eg: (i) Onset of magnetism in rare earth alloys

(ii) Transition from metal to (Mott) insulator

(iii) High- T_c cuprates as function of doping?

IF second order, non-fermi liquid very natural!

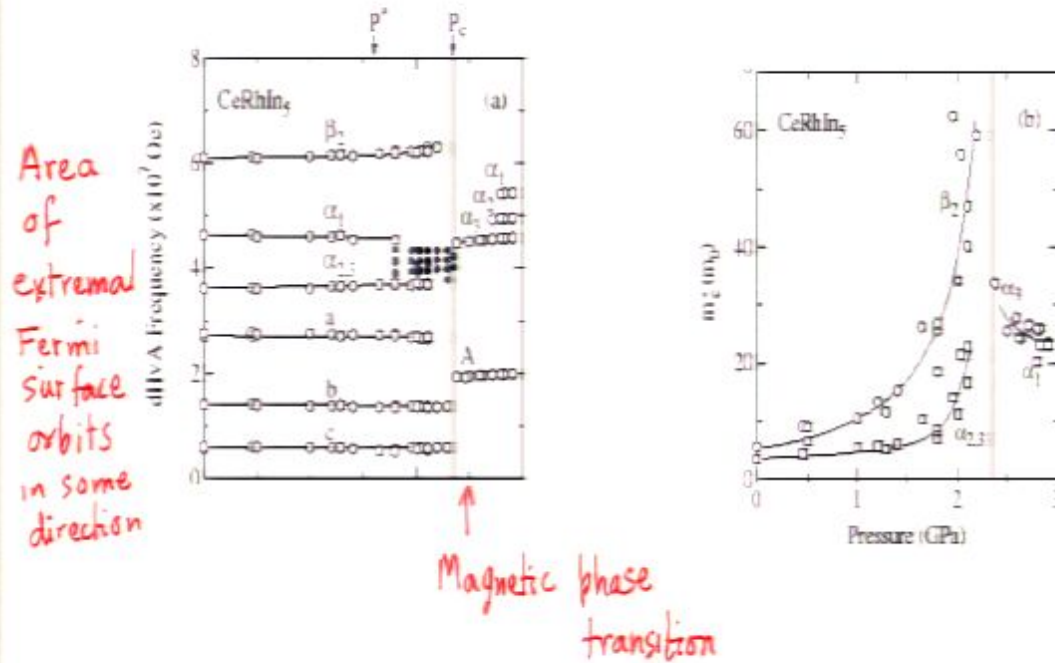
Example: Evolution of Fermi surface across the magnetic phase transition in CeRhIn₅



H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)

Entire sheets of Fermi surface disappear at the phase transition

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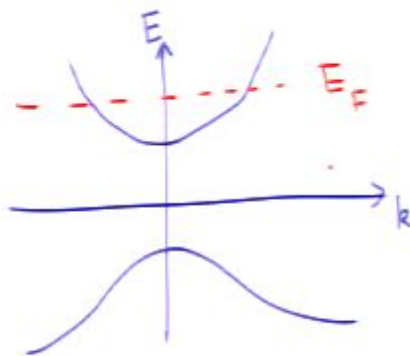
A simpler example: the metal- insulator transition

Insulators do not have Fermi surfaces!

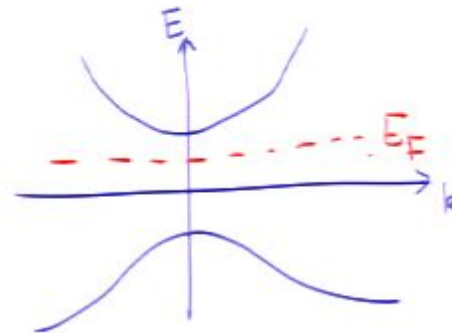
When a metal evolves into an insulator (eg by carrier doping, or by tuning pressure), it must lose its Fermi surface.

How does Fermi surface die when a metal evolves into an insulator?

Simplest possibility: Fermi surface shrinks in size and disappears.



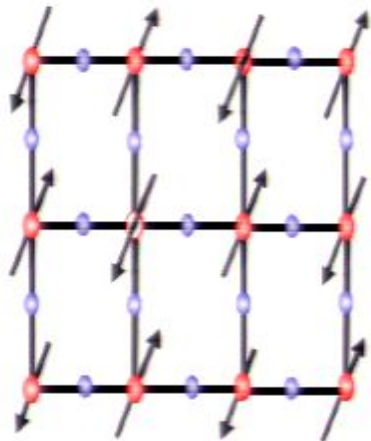
Metal



Band insulator

Question more interesting if insulation is due to Coulomb repulsion, i.e. a 'Mott' insulator

What is a Mott insulator?



Insulation due to jamming effect of Coulomb repulsion

Coulomb cost of two electrons
occupying same atomic orbital dominant

⇒ Electrons can't move if every possible atomic
orbital site is already occupied by another electron.

Odd number of electrons per unit cell: band theory predicts metal.

How does Fermi surface die when a metal evolves into a Mott insulator?

Not fully understood.....

Central to some of the most mysterious phenomena in quantum condensed matter physics.

Mott insulator: simple in real space (electrons are particles)

Metal with Fermi surface: simple in momentum space (electrons are waves)

Vicinity of Mott metal-insulator transition: neither wave nor particle points of view superior.

A useful theoretical model

Model: Electrons on lattice sites i with
1 electron per site on average

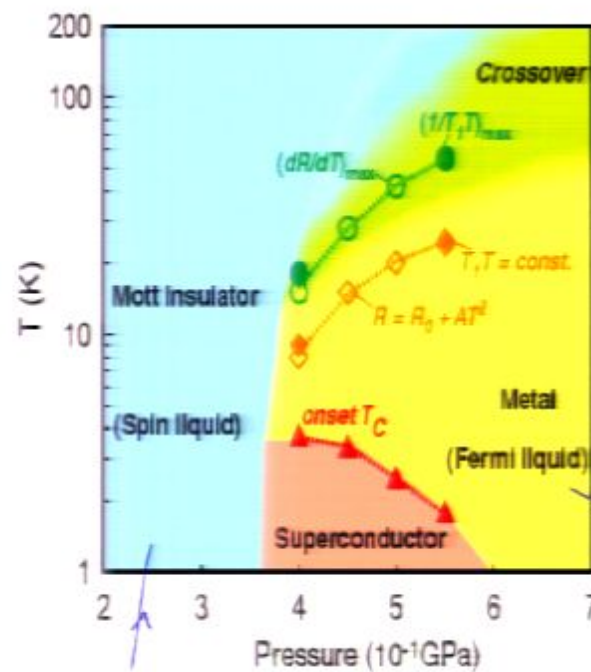
$$H = \underbrace{-t \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.)}_{e^- \text{ hopping}} + \underbrace{U \sum_i \frac{n_i(n_i-1)}{2}}_{\text{on-site repulsion}} \quad \left(\begin{array}{l} \text{"Hubbard"} \\ \text{model"} \end{array} \right)$$

$(n_i = c_i^\dagger c_i)$

$t \gg U$: Fermi liquid metal

$U \gg t$: Each site has 1 e^- ; large energy cost
for charge motion \Rightarrow insulator. ("Mott insulator")

Possible experimental realization of a second order Mott transition

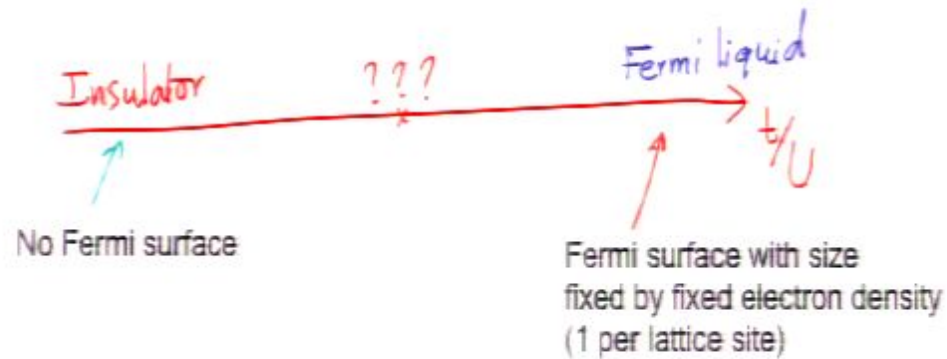


$K-(ET)_2Cu_2(CN)_3$
Under pressure

Electron
Fermi surface
with $1 e^-$ /unit cell

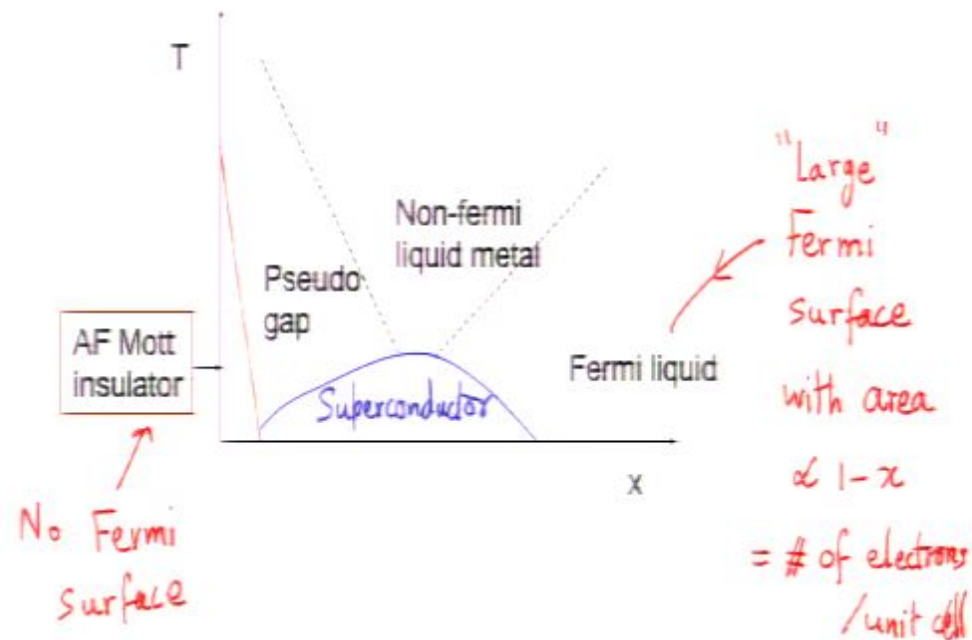
No electron
Fermi surface

Evolution from metal to insulator



In evolving from metal to insulator,
entire Fermi surface of metal needs to
disappear.

Another example: High temperature superconducting materials



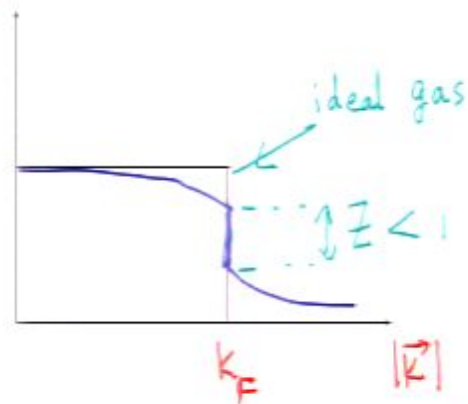
Basic question for theory

How can an entire Fermi surface disappear
continuously?

Even more basic: What is the Fermi surface?

Interacting Landau Fermi liquid

Momentum occupation $n(\vec{k}) = \langle c_{\vec{k}}^\dagger c_{\vec{k}} \rangle$



Sharp jump discontinuity Z in $n(\vec{k})$ at k_F .

Z = extent to which e^- overlaps with Landau quasiparticle

How might the Fermi surface die?

When Fermi surface has disappeared,
 $n(\vec{k})$ is smooth at k_F

Disappearance of Fermi surface thru' continuous
transition if Z vanishes continuously
and everywhere on Fermi surface!

Brinkman, Rice, 1970

$Z \searrow 0$ at critical point

Electronic structure at criticality: "Critical Fermi surface"

Crucial question: Nature of electronic excitations right at quantum critical point when $T=0$?

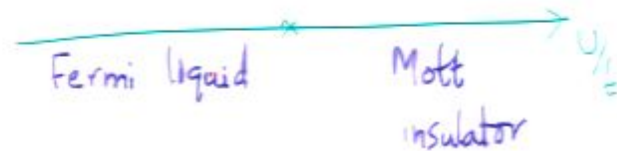
Claim: At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle

TS, 2008

"Critical Fermi surface"

Why a critical Fermi surface?

Mott transition
example:



What is gap $\Delta(\vec{k})$ to add an electron at -
momentum \vec{k} ?

Fermi liquid : $\Delta(\vec{k} \in FS) = 0$

Mott insulator : Sharp gap $\Delta(\vec{k}) \neq 0$ for all \vec{k}

Evolution of single particle gap

Approach from Mott

2nd order transition to metal \Rightarrow expect Mott gap

$\Delta(\vec{k})$ will close continuously

To match to Fermi surface in metal, $\Delta(\vec{k}) \rightarrow 0$
for all $\vec{k} \in \text{FS}$.

\Rightarrow Fermi surface sharp at critical point.

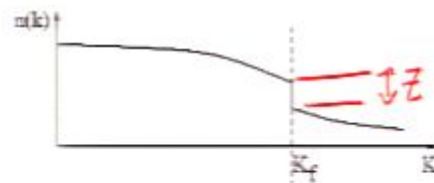
But as $Z=0$ no sharp quasiparticle

\Rightarrow Non-Fermi liquid with sharp "critical" Fermi surface!

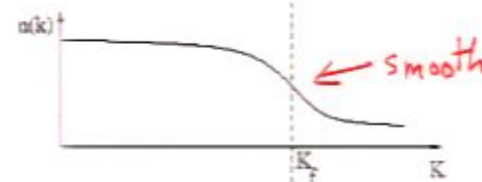
Why a critical Fermi surface?

Evolution of momentum distribution

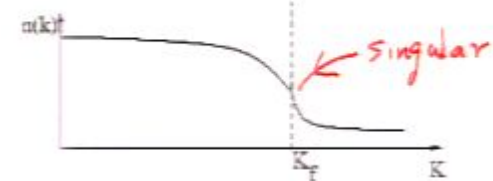
(a) Metal with Fermi surface



(b) Phase where Fermi surface has disappeared



(c) Critical point
 $n(k)$ continuous at k_F
but is singular



Killing a Fermi surface

Disappearance of Fermi surface through a
continuous transition

At critical point

(a) $Z = 0$

(b) Fermi surface sharp

Some obvious consequences/questions

Critical Fermi surface \Rightarrow unusual criticality
with phenomena different from familiar critical
points

1. Structure of universal singularities/scaling
phenomena?
2. Computational framework?

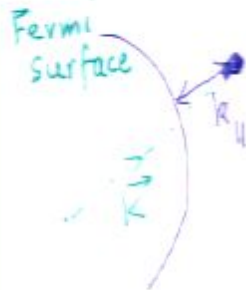
Scaling phenomenology at a quantum critical point with a critical Fermi surface? TS, 2008

Focus initially on electron density of states

$$A(\vec{k}, \omega) = \sum_n |\langle n | c_{\vec{k}} | g d \rangle|^2 \delta(\omega - (E_n - E_{gd}))$$

Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal scale invariant singularity in $A_c(\vec{k}, \omega)$ for small $\omega, k_{||}$



Scaling ansatz:

For every point θ on FS

$$A_c(\vec{k}, \omega, T) \sim \frac{1}{|\omega|^{\alpha/2}} F\left(\frac{\omega}{|k_{||}|^2}, \frac{\omega}{T}\right)$$

New possibility: angle dependent exponents

A priori must allow angle dependent exponents:

$$z = z(\theta), \quad \alpha = \alpha(\theta)$$

consistent with lattice symmetries

Eg: Triangular lattice $z(\theta + \pi/3) = z(\theta)$

$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_n z_n \cos(n\theta), \dots$

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$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_r z_r \cos(r\theta), \dots$

Leaving the critical point

Expect scale invariant spectrum is cut off

at $k_{||} \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^z}$ so that

$$A_c(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\alpha/z}} F_1\left(\frac{\omega}{k_{||}^z}, k_{||} \xi\right)$$

Expect $\xi \sim |g - g_c|^{-\nu}$ but again

a priori must let $\nu = \nu(\theta)$

Scaling theory can be developed for
singular behavior of various other quantities.

Many phenomenological differences
with ordinary criticality

Eg: Specific heat at critical point $C_v \sim \int_{FS} d\theta T^{\gamma_{\mathbb{Z}(\theta)}}$
= integral over Fermi surface

$T \rightarrow 0$: dominated by one portion of Fermi surface

Implications of angle dependent exponents

(i) Different properties dominated by different portions of Fermi surface

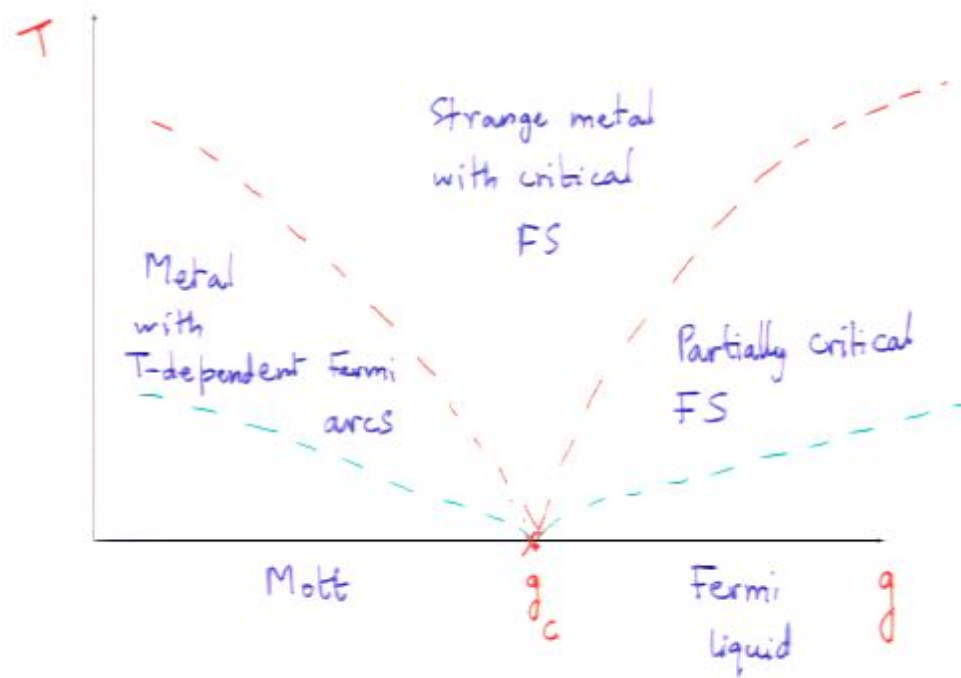
(ii) Different portions of Fermi surface will emerge out of criticality at different energy scales

Example: At Mott transition

$$\text{Mott gap } \Delta(\theta) \sim |Sg|^{z(\theta)\nu(\theta)}$$

\Rightarrow Finite $-T$ xovers richer than usual

Finite T crossovers



(Similarity to some phenomena in h_Tc materials)

? Computational framework ?

1. Slave particle methods

View electron as composite of 'slave' particles with fractional quantum numbers

Reformulate electron model in terms of slave particles interacting through gauge forces.

Provides concrete examples of phase transitions where an entire Fermi surface disappears continuously.

Successes: Demonstrate critical Fermi surface, emergence of non-fermi liquids (TS, 2008)

Important as proof of principle, application to experiment with caution.

2. A looooooong shot: 'Dual gravity' calculations AdS/CMT (Faulkner, Liu, McGreevy, Vegh, 2009)

How might a Fermi surface die?

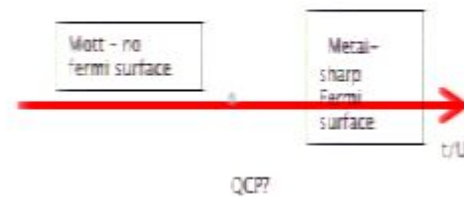
At certain quantum phase transitions in metals, an entire Fermi surface might disappear.

Possible examples:

1. Magnetic critical points in rare earth metals

2. Mott metal-insulator transition

3. ?? HiTc (underdoped to overdoped)??



If second order, then

- (i) non-fermi liquid physics natural at such a QCP
- (ii) intuition from familiar bosonic quantum criticality not directly relevant (and possibly dangerous)

Summary

- A Fermi surface may die through a second order phase transition
- at the critical point the Landau quasiparticle is destroyed but the Fermi surface is preserved (the 'critical Fermi surface')
=> non-fermi liquid
- Presence of critical fermi surface will change the scaling phenomenology associated with universal critical singularities.
- Concrete model calculations with a critical Fermi surface exist

Future – many challenges

1. Tests of scaling in experiments
2. Computational framework?

Killing a Fermi surface

Disappearance of Fermi surface through a
continuous transition

At critical point

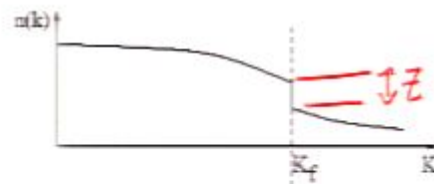
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(b) Fermi surface sharp

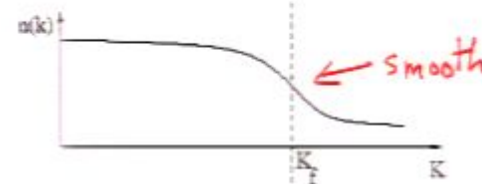
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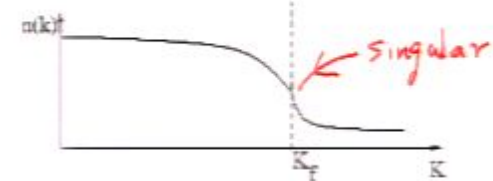
Metal with Fermi surface (a)



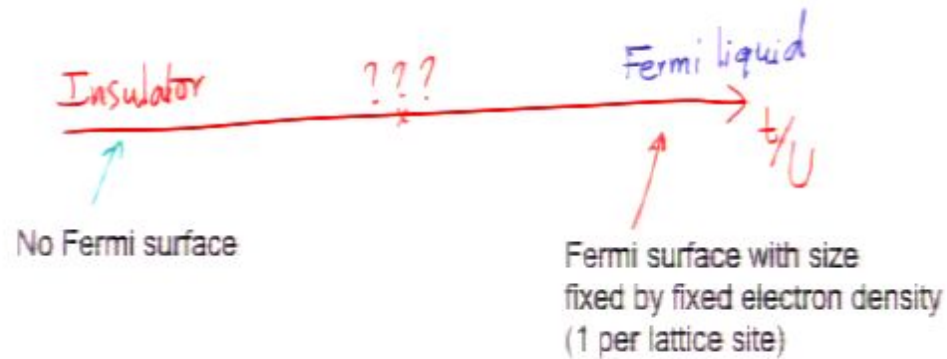
Phase where Fermi surface has disappeared (b)



Critical point (c)
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Killing a Fermi surface

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Eg: (i) Onset of magnetism in rare earth alloys

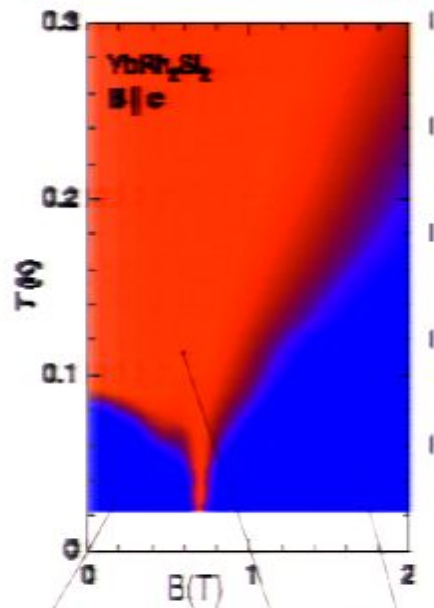
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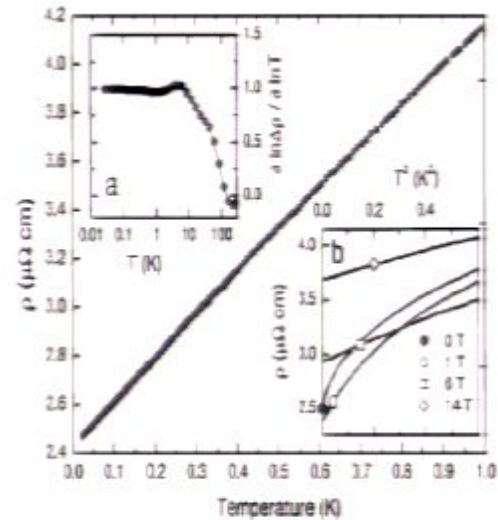


Magnetic metal

Fermi liquid

Non fermi liquid

Trovarelli et al. PRL 2000



T-dependence of resistivity at critical point: $\rho(T) \sim T$ for three decades in temperature!