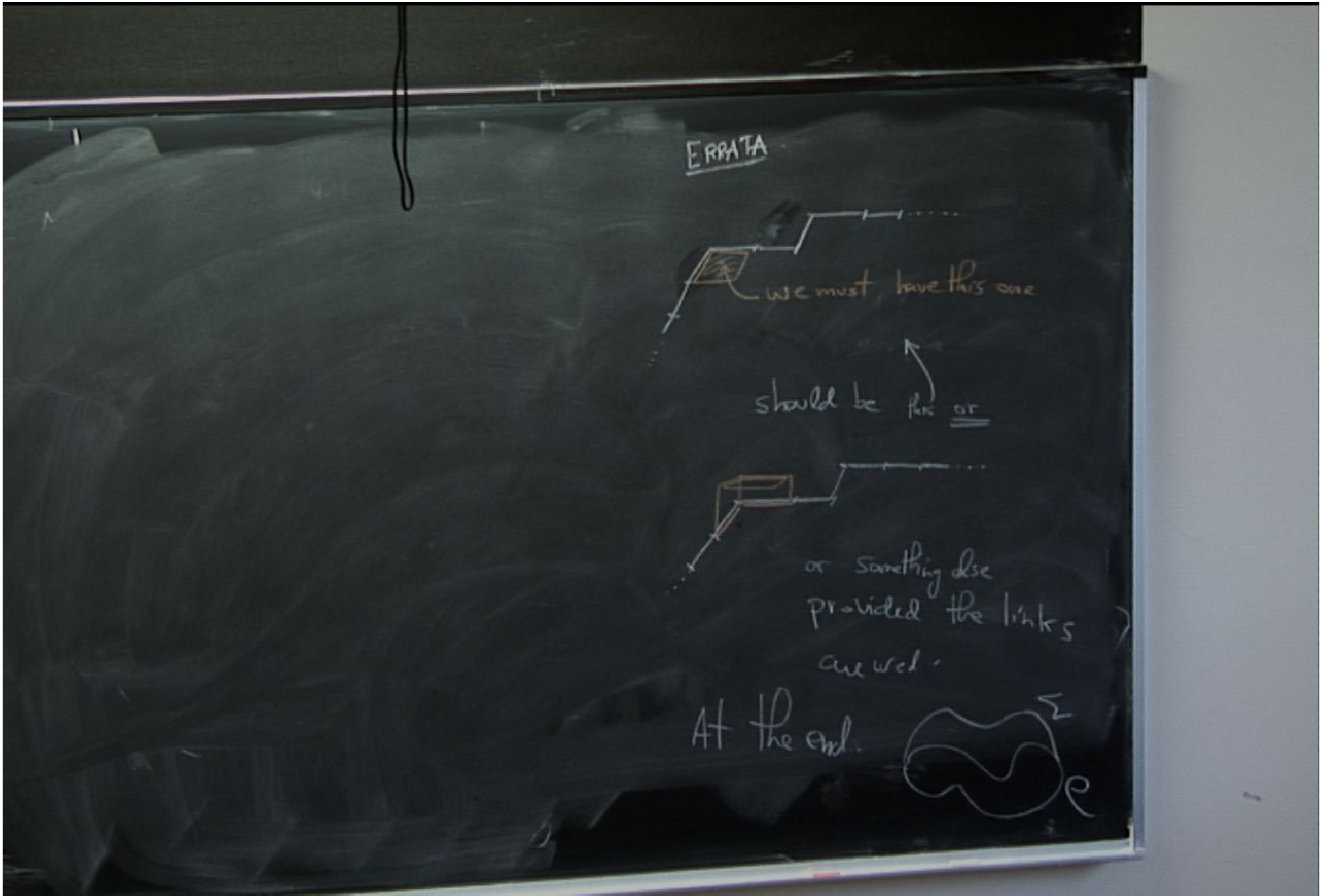
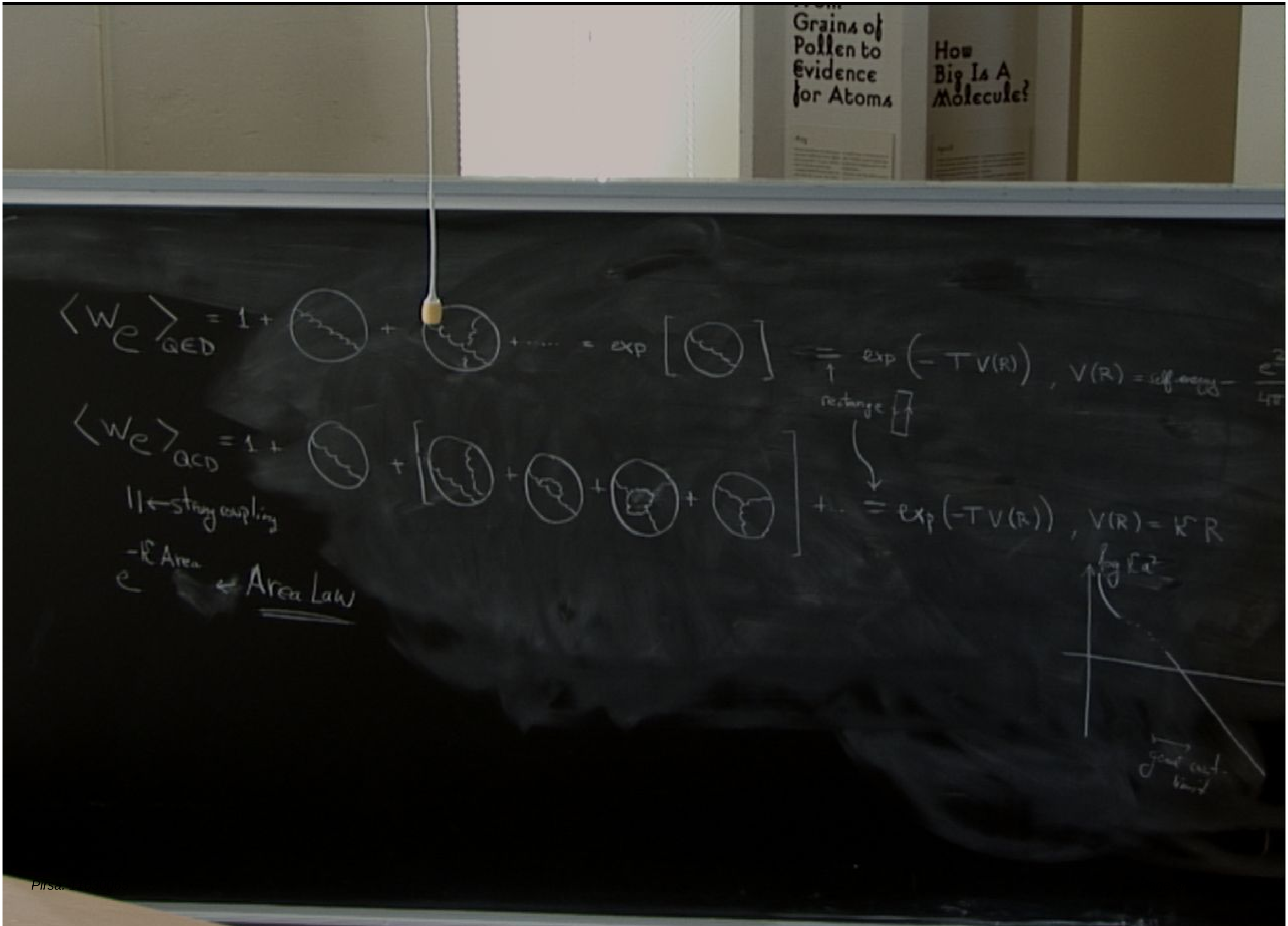
Title: Explorations in String Theory - Lecture 14

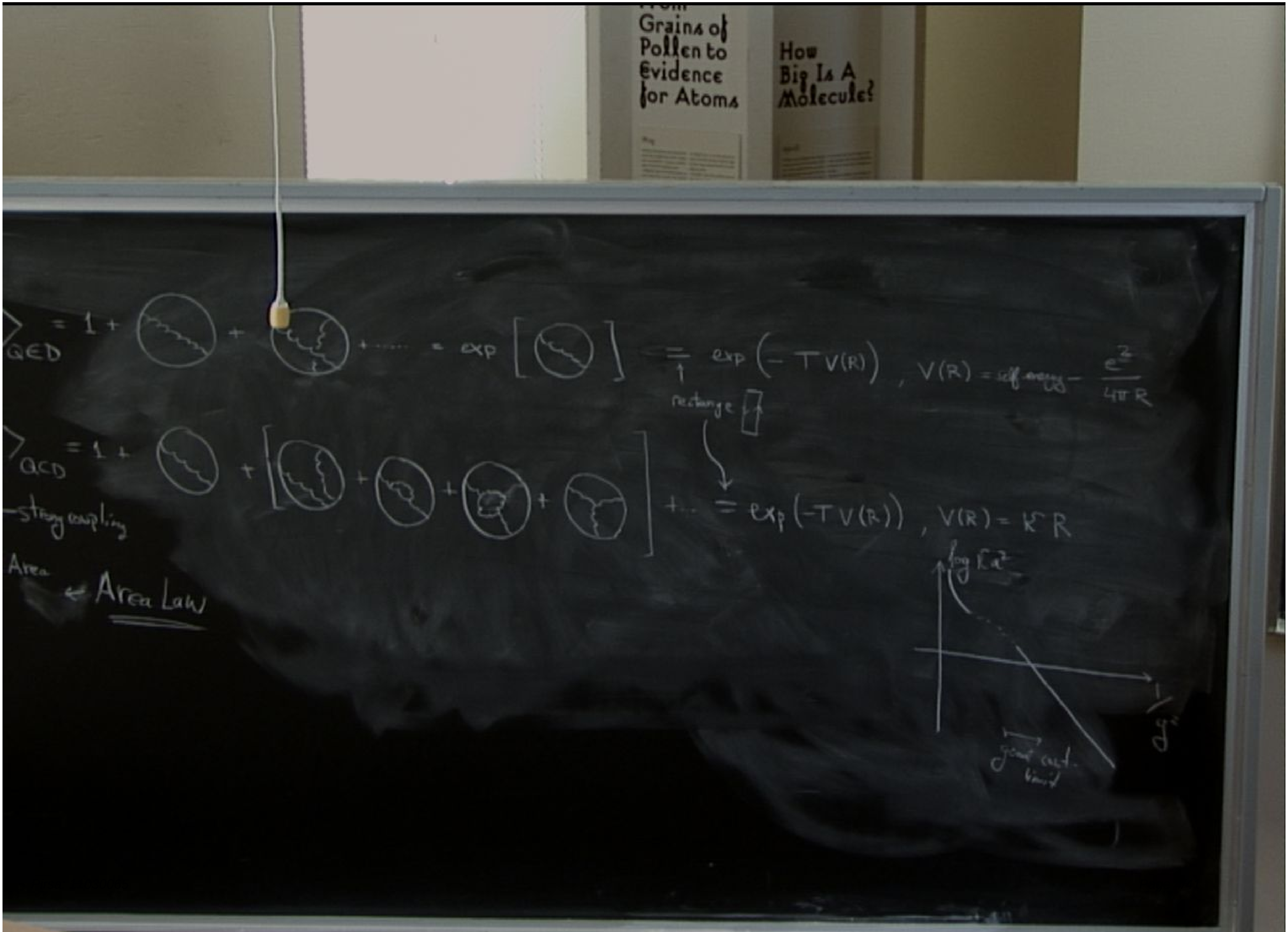
Date: Mar 31, 2011 11:30 AM

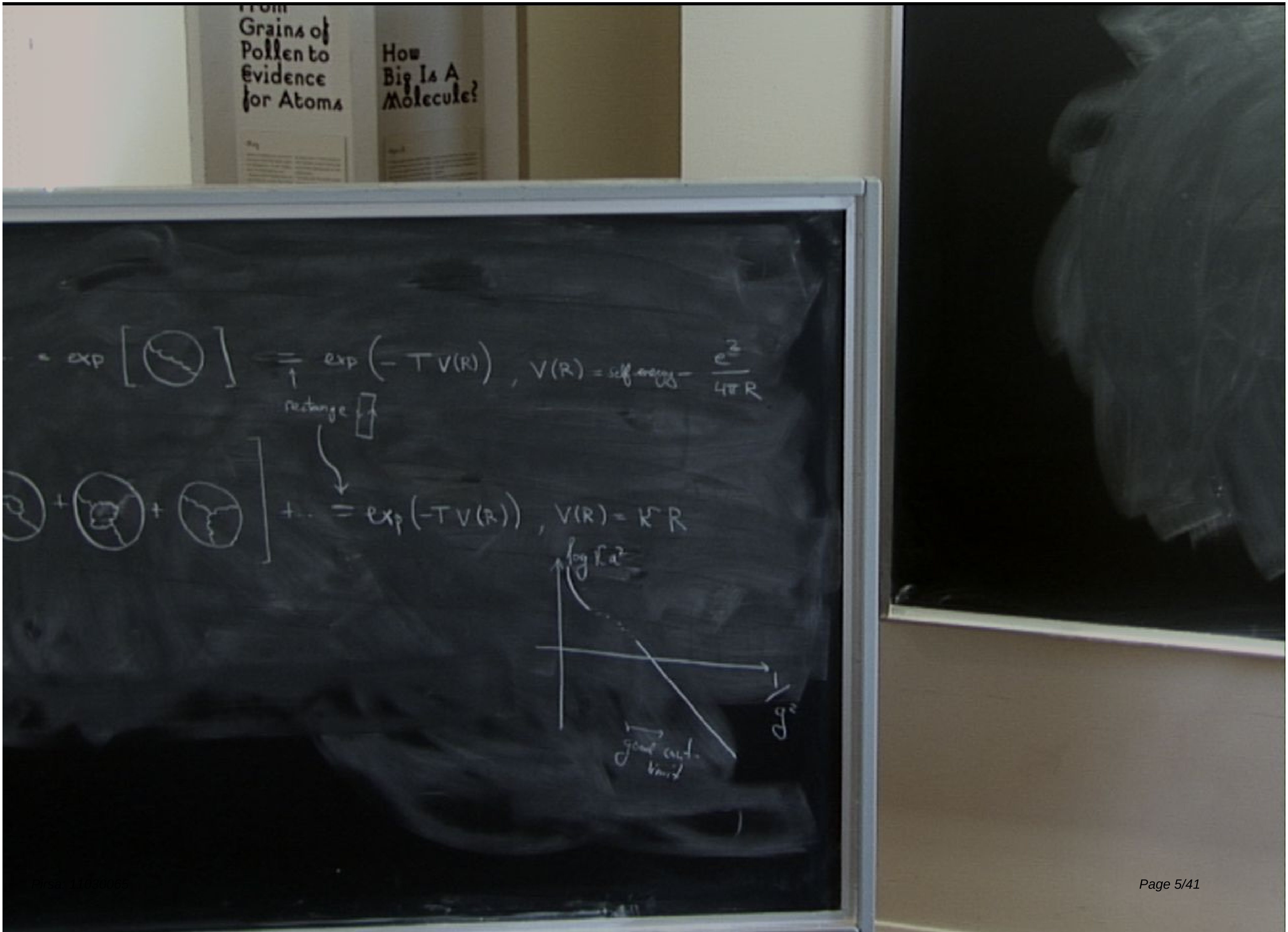
URL: http://pirsa.org/11030065

Abstract:


ERRATA
we must have this one
should be this or
or something else provided the links are wed.
At the end.
Σ
e


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
⟨W_C⟩_QED = 1 + … + … + …… = exp[…] = exp(−T V(R)) , V(R) = self energy − e²/4π
rectangle
⟨W_C⟩_QCD = 1 + … + [… + … + … + …] + … = exp(−T V(R)) , V(R) = k R
strong coupling
e^(−k Area) ← Area Law


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
QED
= 1 +
= exp
exp (– T V(R))
V(R) =
4π R
rectange
QCD
= 1 +
= exp (–T V(R))
V(R) = k R
strong coupling
Area
Area Law


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
= exp (− T V(R)) , V(R) = self energy − e²/4πR
rectangle
= exp (−T V(R)) , V(R) = k R
log
gauge coupl.

to make analytic progress
we need some toy models

probably $N \to \infty$ helps

↳ 2D description (string)

2D easier than 4D!

- for som (SUSY) theories we know the 2D description.

e.g. $\mathcal{N}=4$ SYM $\equiv$ strings in $AdS_5 \times S^5$

to make analytic progress
we need some toy models

probably $N \to \infty$ helps

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- for some (SUSY) theories we know the 2D description.

e.g $\underbrace{\mathcal{N}=4 \text{ SYM}}_{\text{4D gauge th.}} \equiv$ strings in $AdS_5 \times S^5$

to make analytic progress
we need some toy models

probably $N \to \infty$ helps

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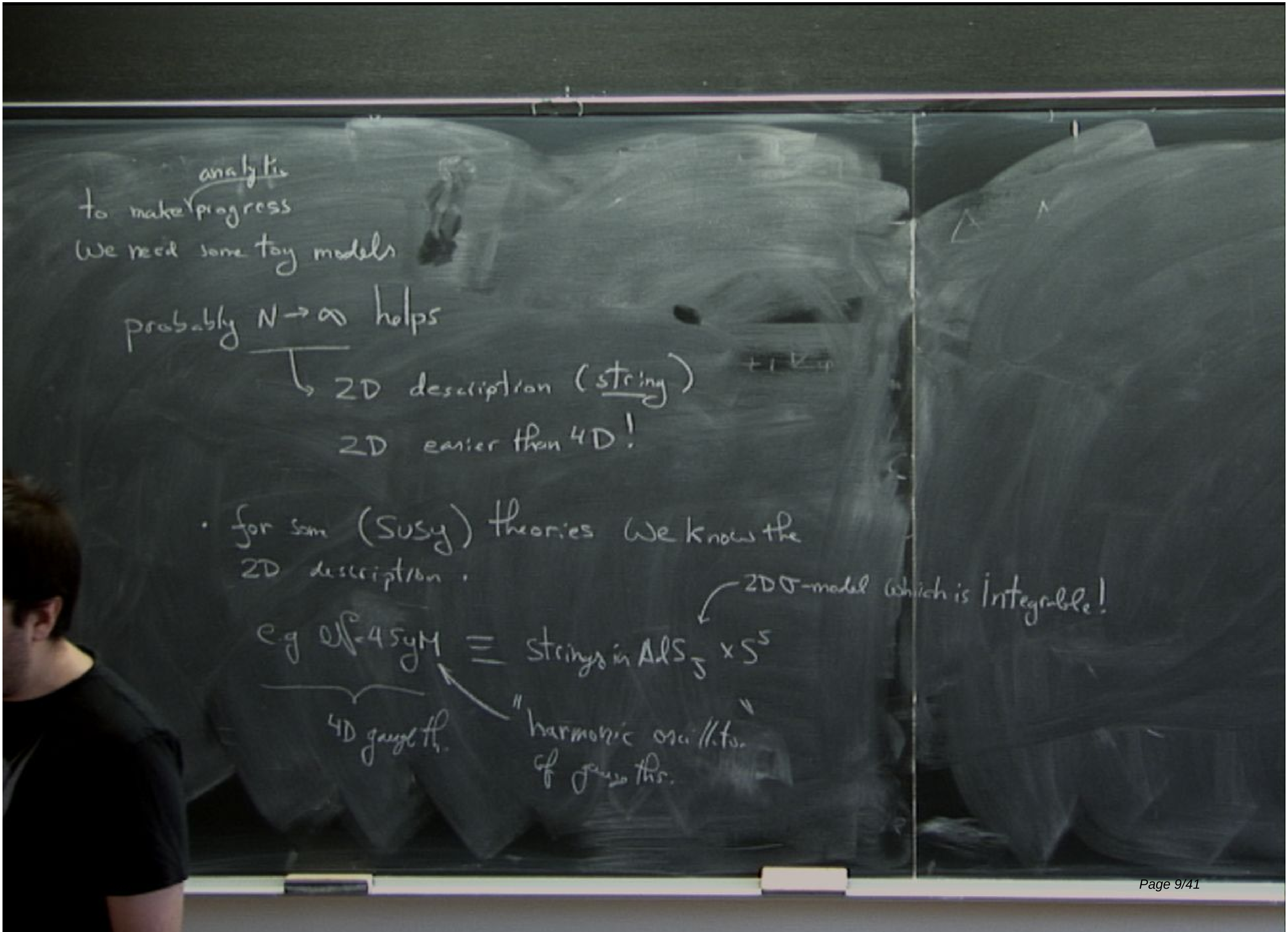
2D easier than 4D!

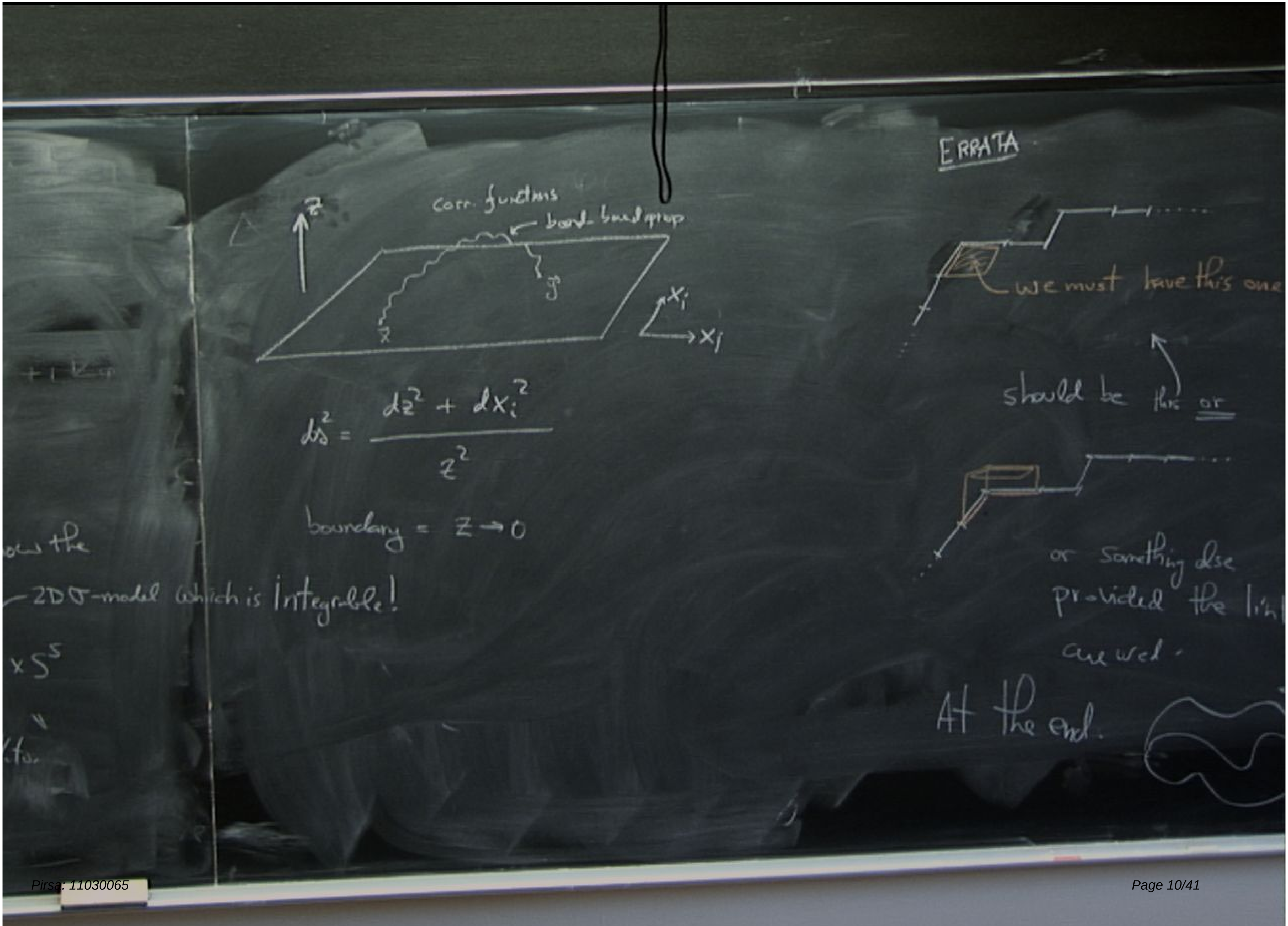
- for some (SUSY) theories we know the 2D description.

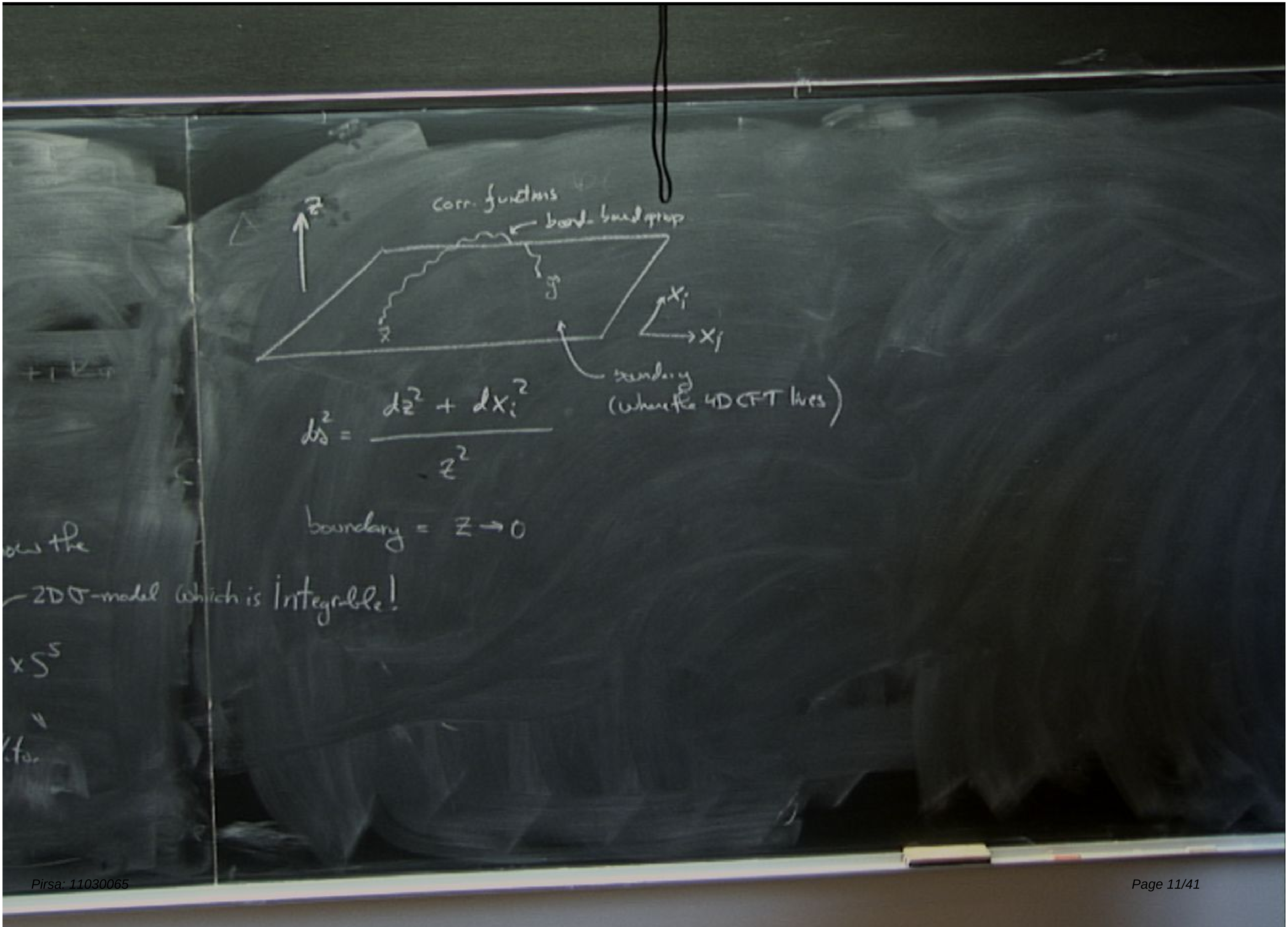
e.g. $\mathcal{N}=4$ SYM $\equiv$ strings in $AdS_5 \times S^5$

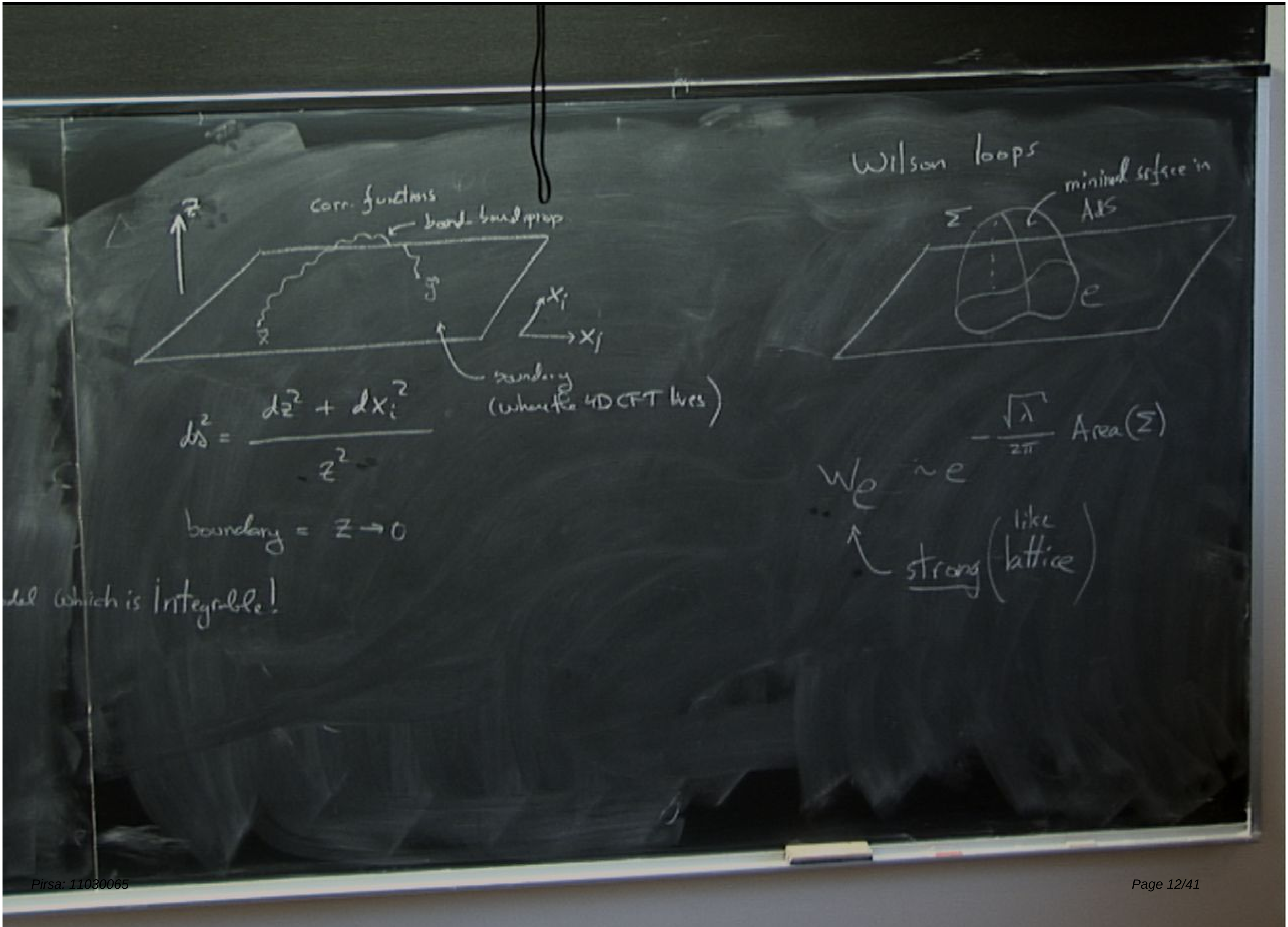
4D gauge th.

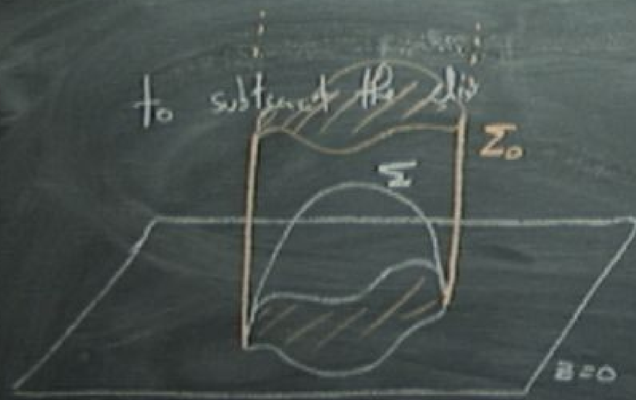
"harmonic oscillator" of gauge ths.


to make analytic progress
we need some toy models
probably N → ∞ helps
2D description (string)
2D easier than 4D!
· for som (SUSY) theories we know the
2D description.
2D σ-model which is Integrable!
e.g N=4 SYM ≡ strings in AdS5 × S5
4D gauge th.
"harmonic oscill.tor of gauge ths."


ERRATA
corr. functions
bord-bord prop
z
x
y
x_i
x_j
$ds^2 = \frac{dz^2 + dx_i^2}{z^2}$
boundary = $z \to 0$
we must have this one
should be this or
or something else
provided the lin
are wed.
At the end.
2D σ-model which is Integrable!
$\times S^5$

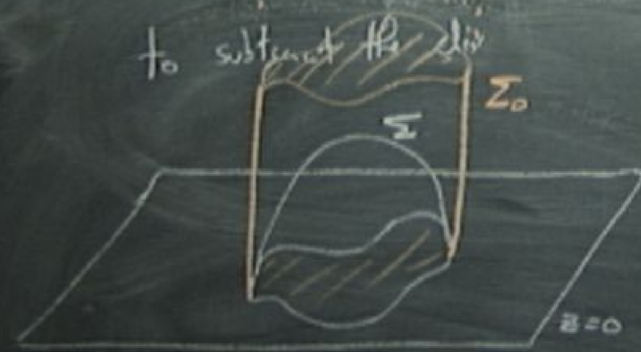

corr. functions
z
x
y
xi
xj
boundary
(where the 4D CFT lives)
ds² = (dz² + dxi²) / z²
boundary = z → 0
ow the
2D σ-model which is Integrable!
× S⁵


corr. functions
bound.-bound prop
z
x
y
x_i
x_j
boundary
(where the 4D CFT lives)
$ds^2 = \frac{dz^2 + dx_i^2}{z^2}$
boundary = $z \to 0$
which is Integrable!
Wilson loops
minimal surface in AdS
Σ
e
$W_e \sim e^{-\frac{\sqrt{\lambda}}{2\pi} \text{Area}(\Sigma)}$
strong (like lattice)



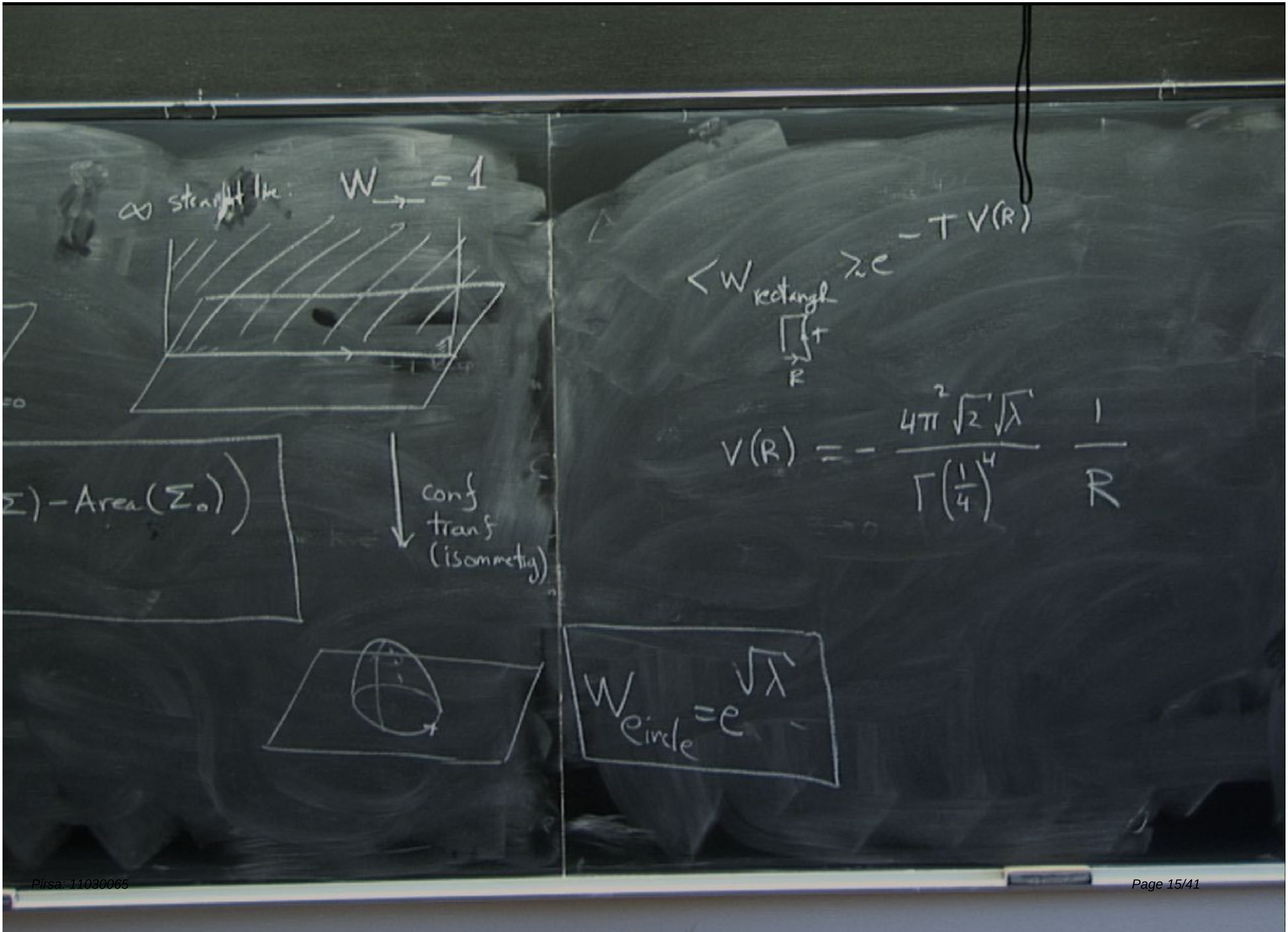
$$\langle W_{\mathcal{C}} \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi}\left(\mathrm{Area}(\Sigma) - \mathrm{Area}(\Sigma_0)\right)}$$

to subtract the div

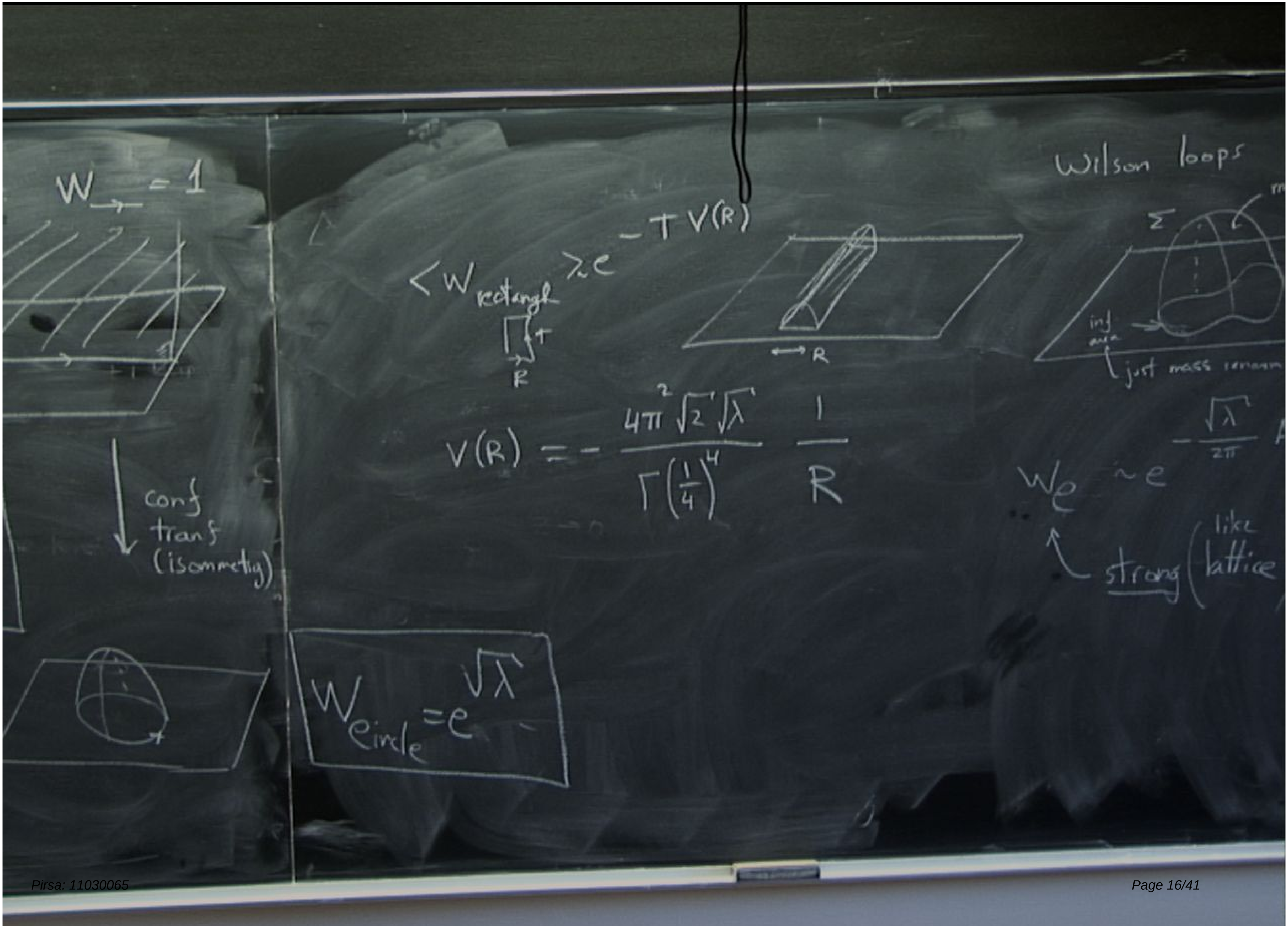


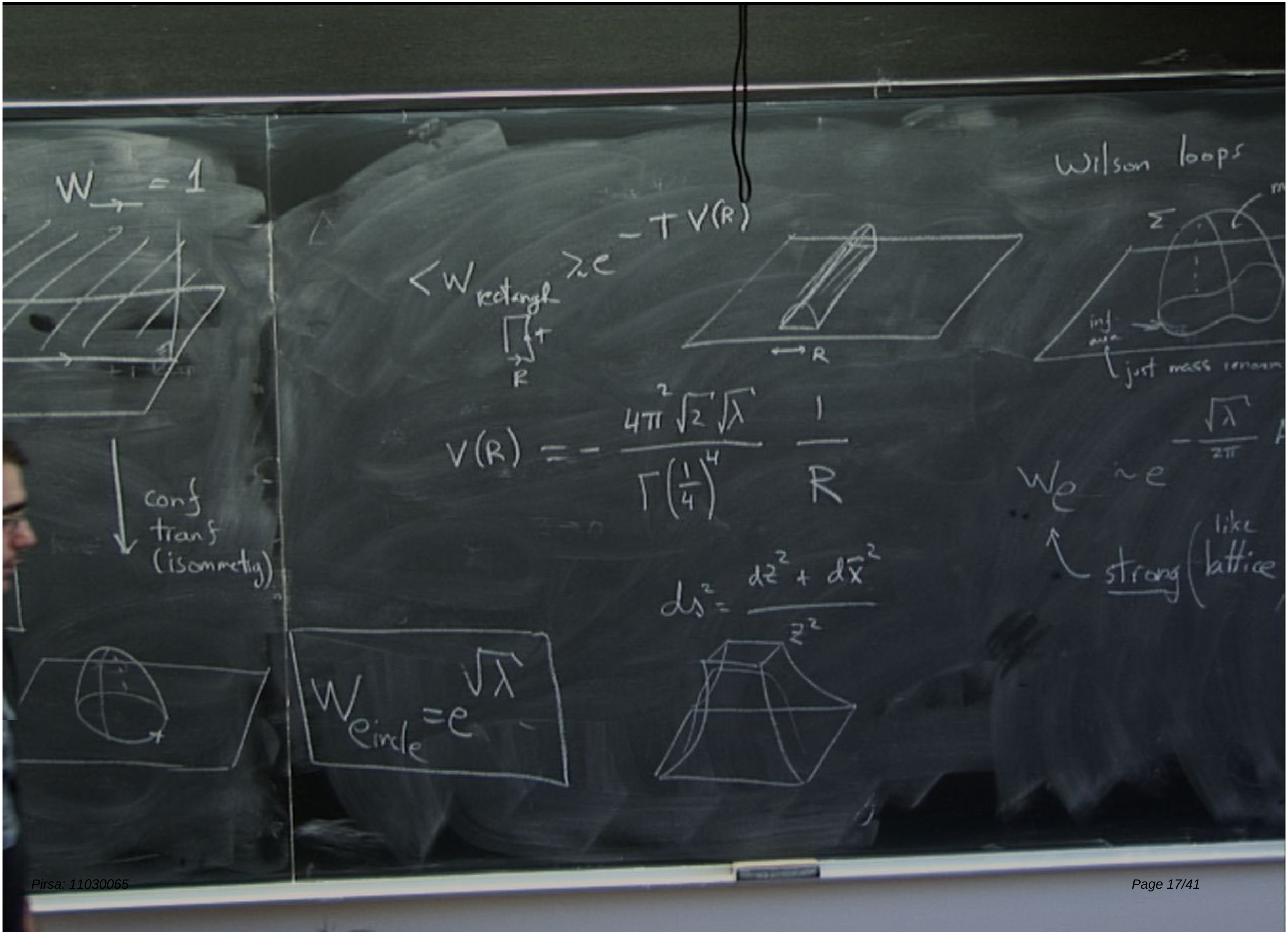
∞ straight line: $W_{\rightarrow} = 1$

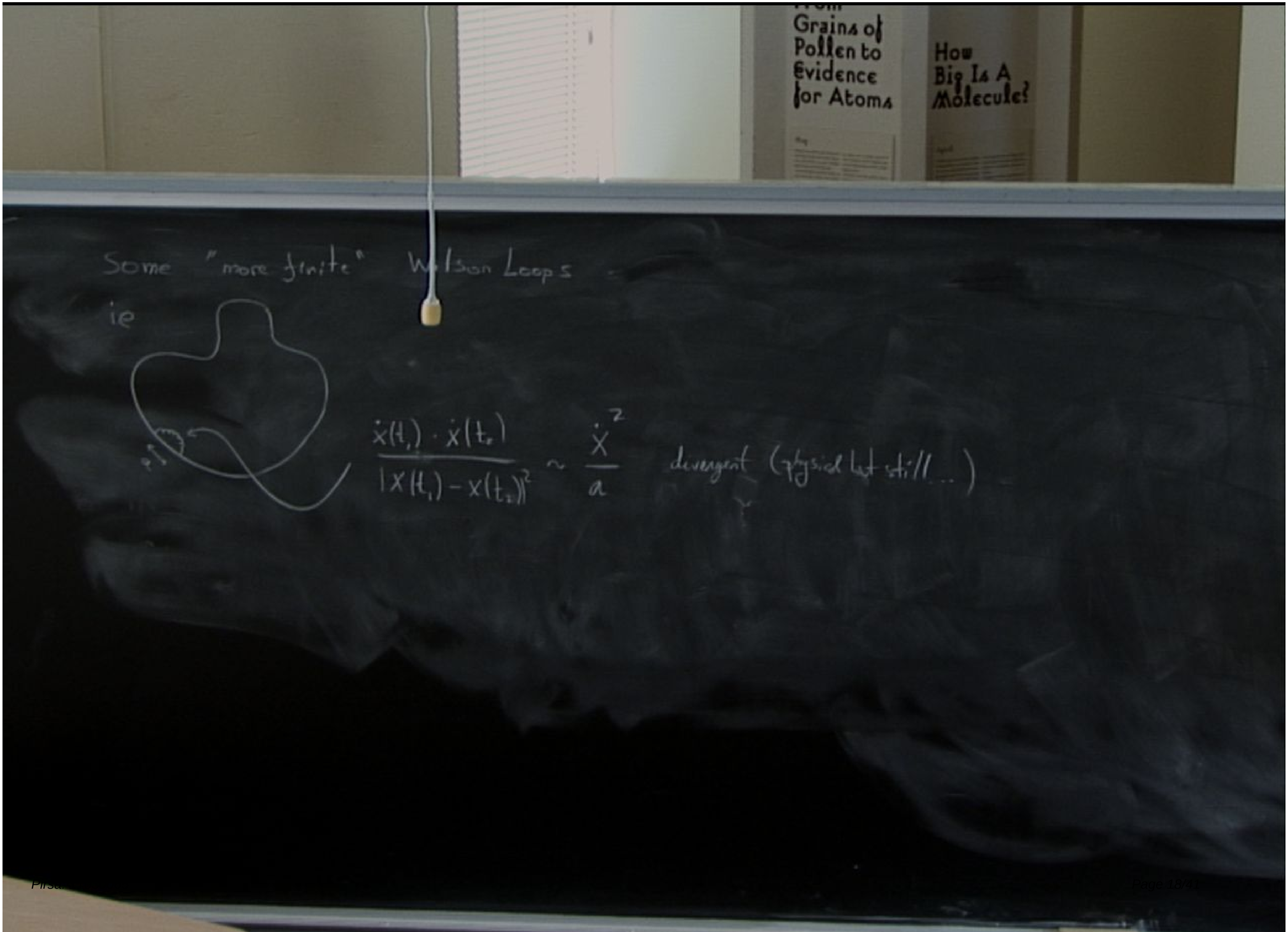
$$\langle W_C \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi}\left(\mathrm{Area}(\Sigma) - \mathrm{Area}(\Sigma_0)\right)}$$

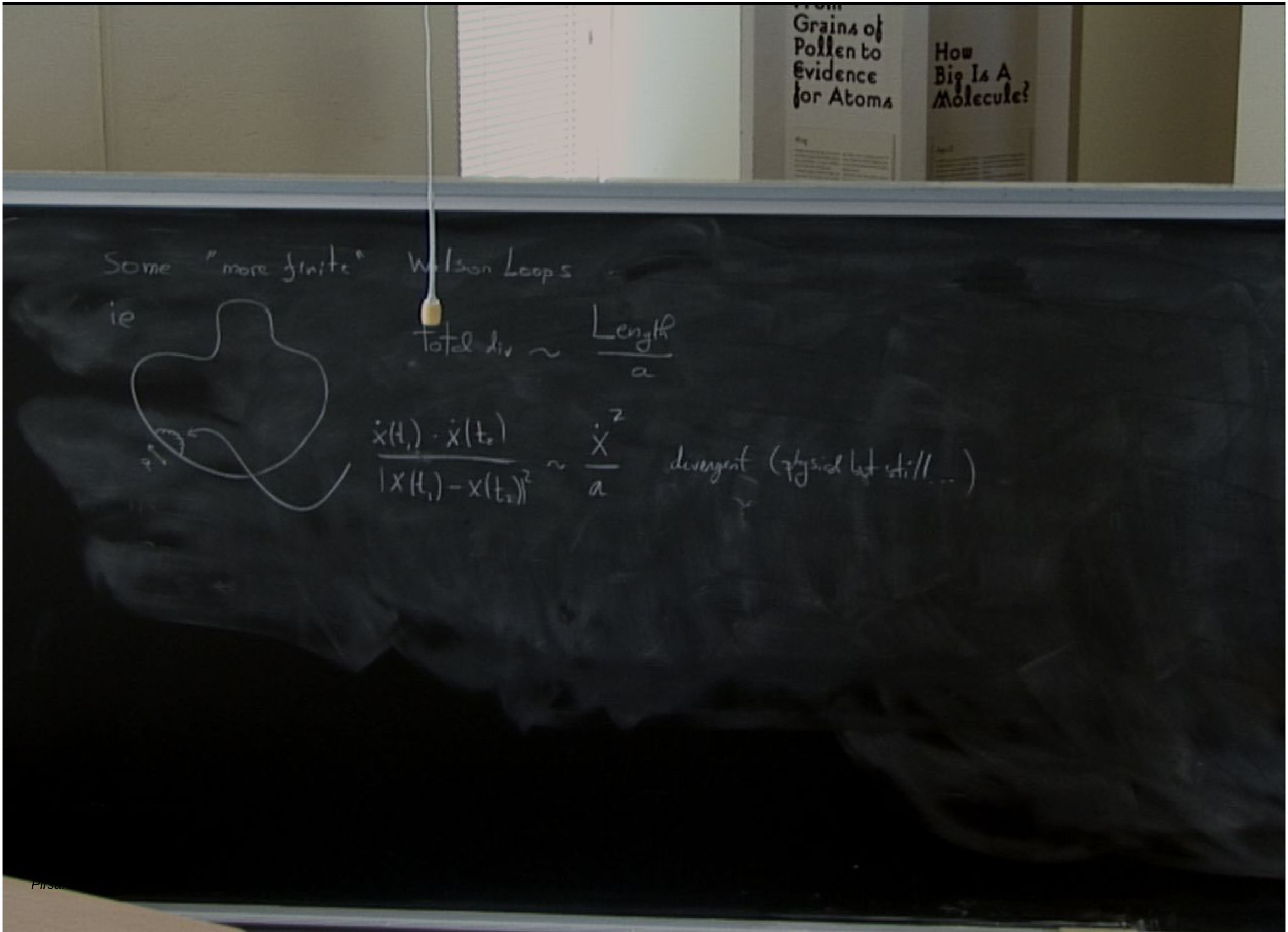


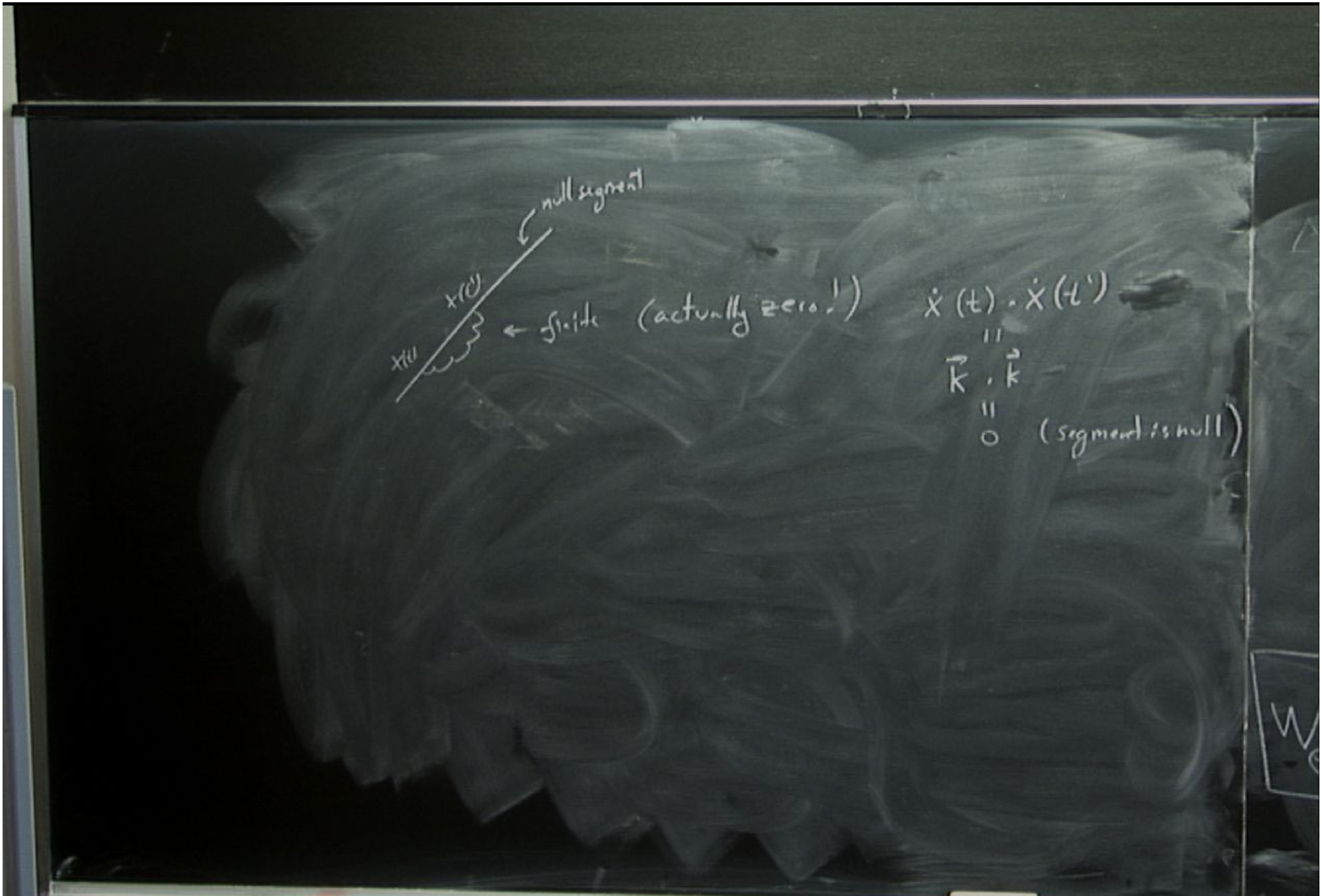
∞ straight line: W = 1
Σ) - Area(Σ₀))
conf
tranf
(isommetry)
<W rectangle > ≈ e^-T V(R)
T
R
V(R) = - 4π² √2 √λ / Γ(1/4)⁴ · 1/R
W circle = e^√λ

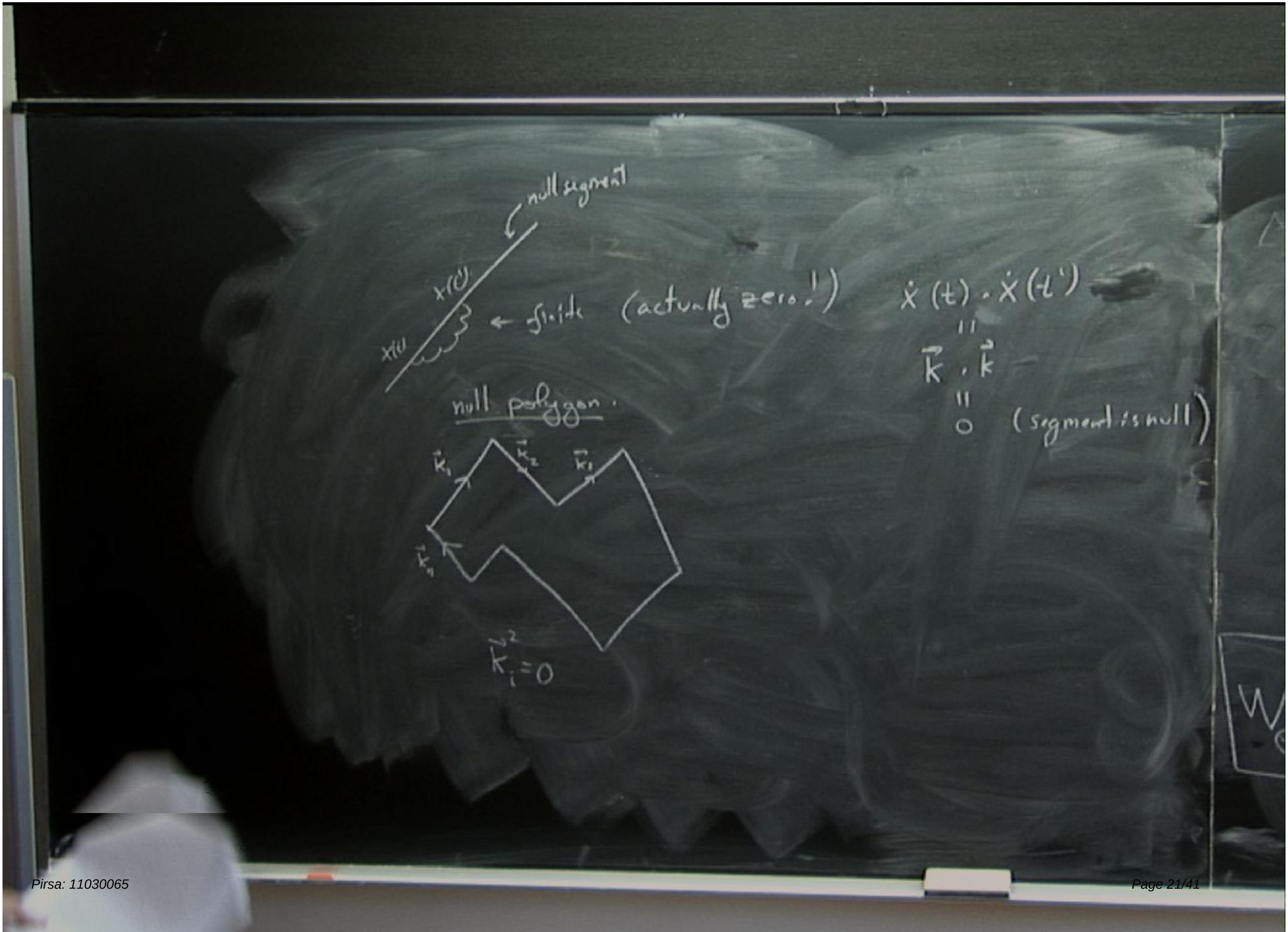

Wilson loops
conf
tranf
(isommetiy)
just mass renorm
strong
like
lattice
R

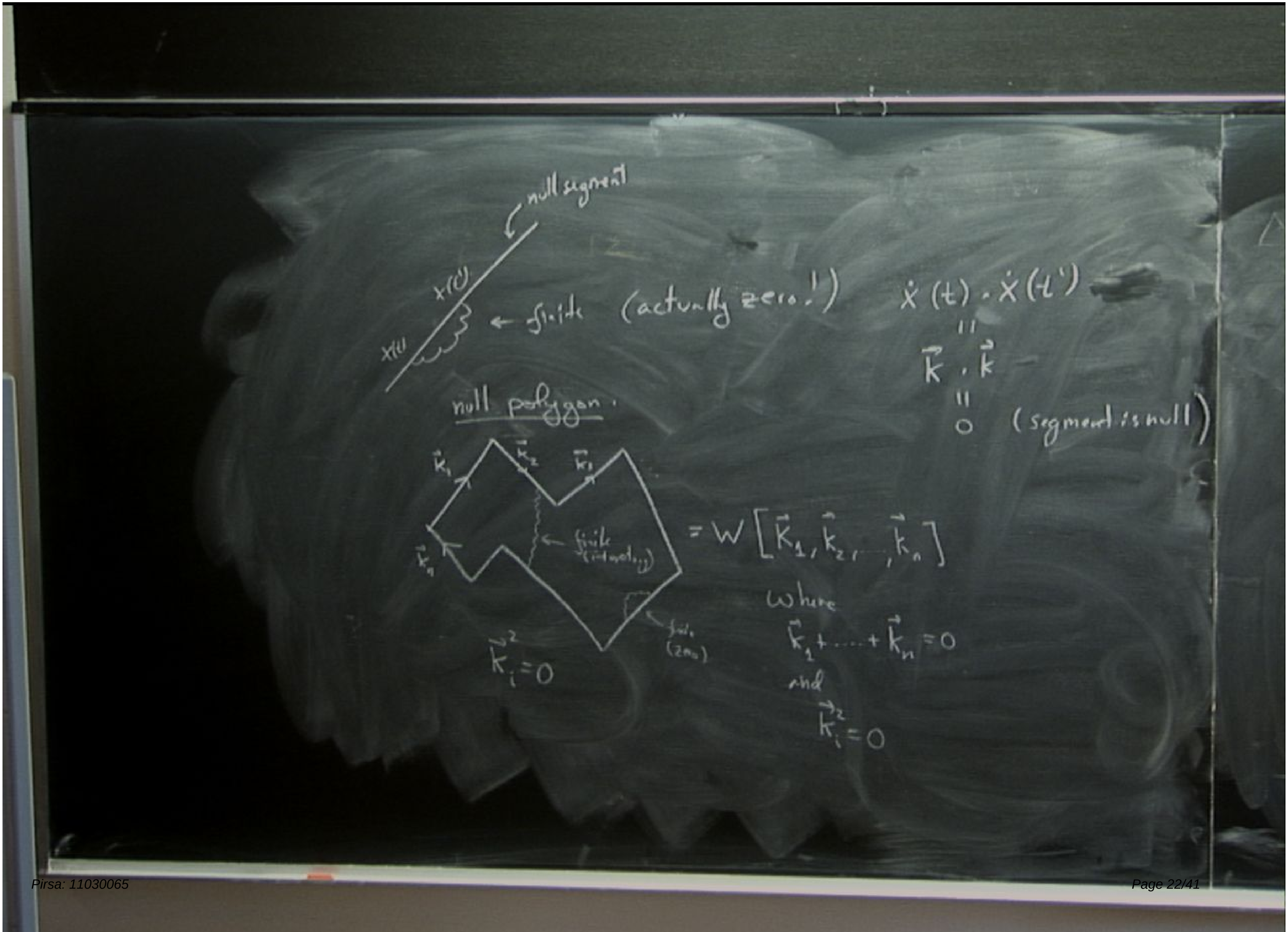

Wilson loops
W = 1
conf
tranf
(isommetry)
$\langle W_{rectangle} \rangle \sim e^{-T\,V(R)}$
T
R
$V(R) = -\frac{4\pi^2 \sqrt{2}\sqrt{\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{R}$
$ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2}$
$W_{circle} = e^{\sqrt{\lambda}}$
R
Σ
inf
just mass renorm
$W_e \sim e^{-\frac{\sqrt{\lambda}}{2\pi}}$
strong
like
lattice


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Some "more finite" Wilson Loops
ie
$\frac{\dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2} \sim \frac{\dot{x}^2}{a}$
divergent (physical but still ...)


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Some "more finite" Wilson Loops
ie
Total div ~ Length / a
ẋ(t₁) · ẋ(t₂) / |x(t₁) − x(t₂)|² ~ ẋ² / a
divergent (physical but still ...)


null segment
x(t')
x(t)
← finite (actually zero!)
ẋ(t) · ẋ(t')
k · k
0 (segment is null)


null segment
x(t')
x(t)
← finite (actually zero!)
ẋ(t) · ẋ(t')
k · k
0 (segment is null)
null polygon.
k1
k2
k3
kn
ki² = 0


null segment
x(t')
x(t)
← finite (actually zero!)
ẋ(t) · ẋ(t')
=
k · k
=
0 (segment is null)
null polygon.
k1
k2
k3
kn
finite (interesting)
finite (zero)
k_i^2 = 0
= W[k1, k2, ..., kn]
where
k1 + ... + kn = 0
and
k_i^2 = 0

...tually zero.!)

$\dot{x}(t) \cdot \dot{x}(t')$

$=$

$\vec{k} \cdot \vec{k}$

$=$

0 (segment is null)

scat. amp of gluons $\vec{k}^2 = 0$, $\Sigma \vec{k}_i = 0$ (cons. mom)

$= W[\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n]$

where

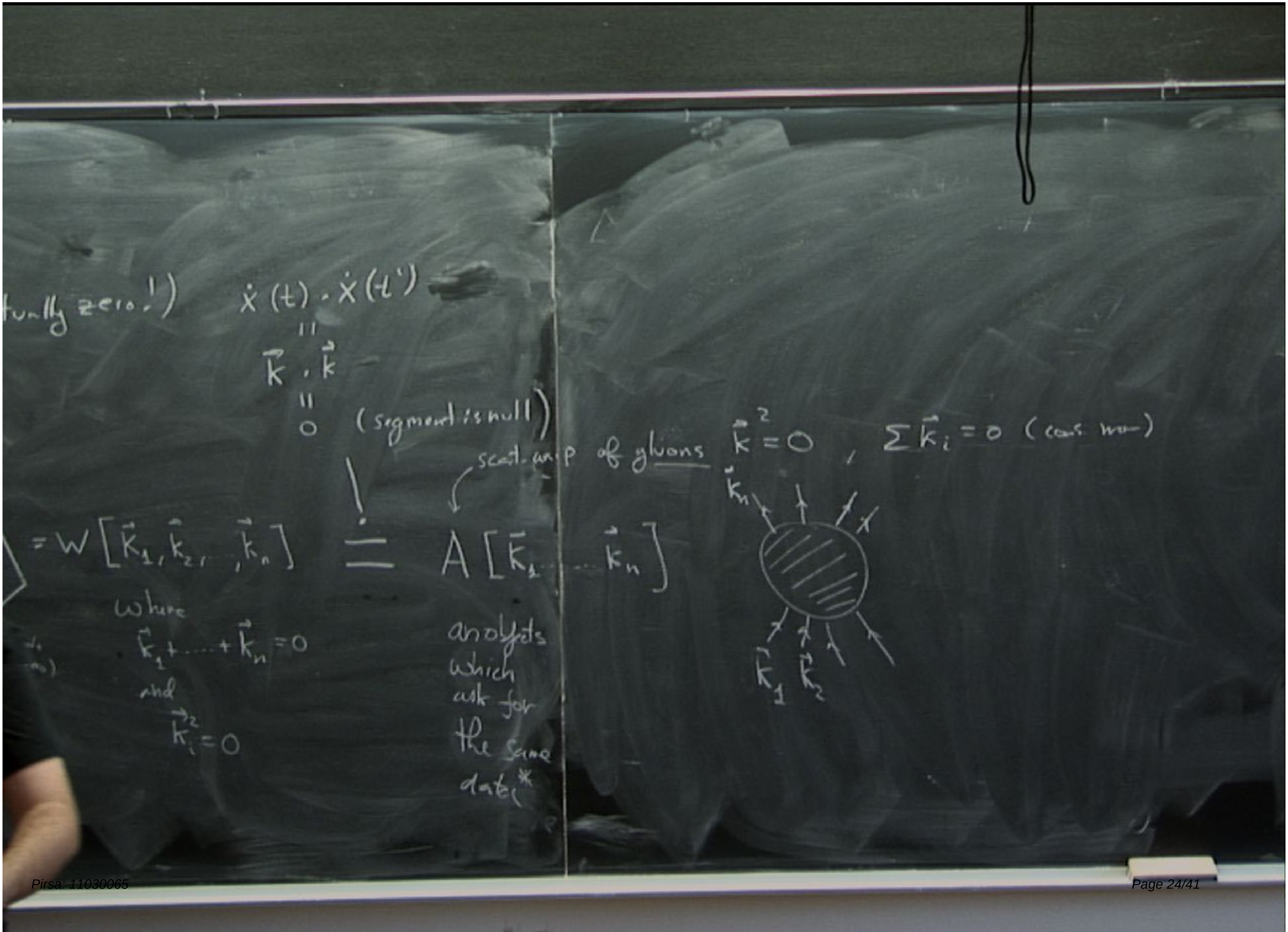
$\vec{k}_1 + \ldots + \vec{k}_n = 0$

and

$\vec{k}_i^2 = 0$

$A[\vec{k}_1 \ldots \vec{k}_n]$

analytic which ask for the same data *



tually zero!)
ẋ(t) · ẋ(t')
k · k
0
(segment is null)
scat. amp of gluons
k² = 0 , Σ kᵢ = 0 (cons. mom.)
= W[k₁, k₂, , kₙ] ≐ A[k₁ ... kₙ]
where
k₁ + ... + kₙ = 0
and
kᵢ² = 0
analysts
which
ask for
the same
data*
kₙ
k₁ k₂

...tually zero!)

$\dot{x}(t)\cdot\dot{x}(t')$
$\parallel$
$\vec{k}\cdot\vec{k}$
$\parallel$
0 (segment is null)

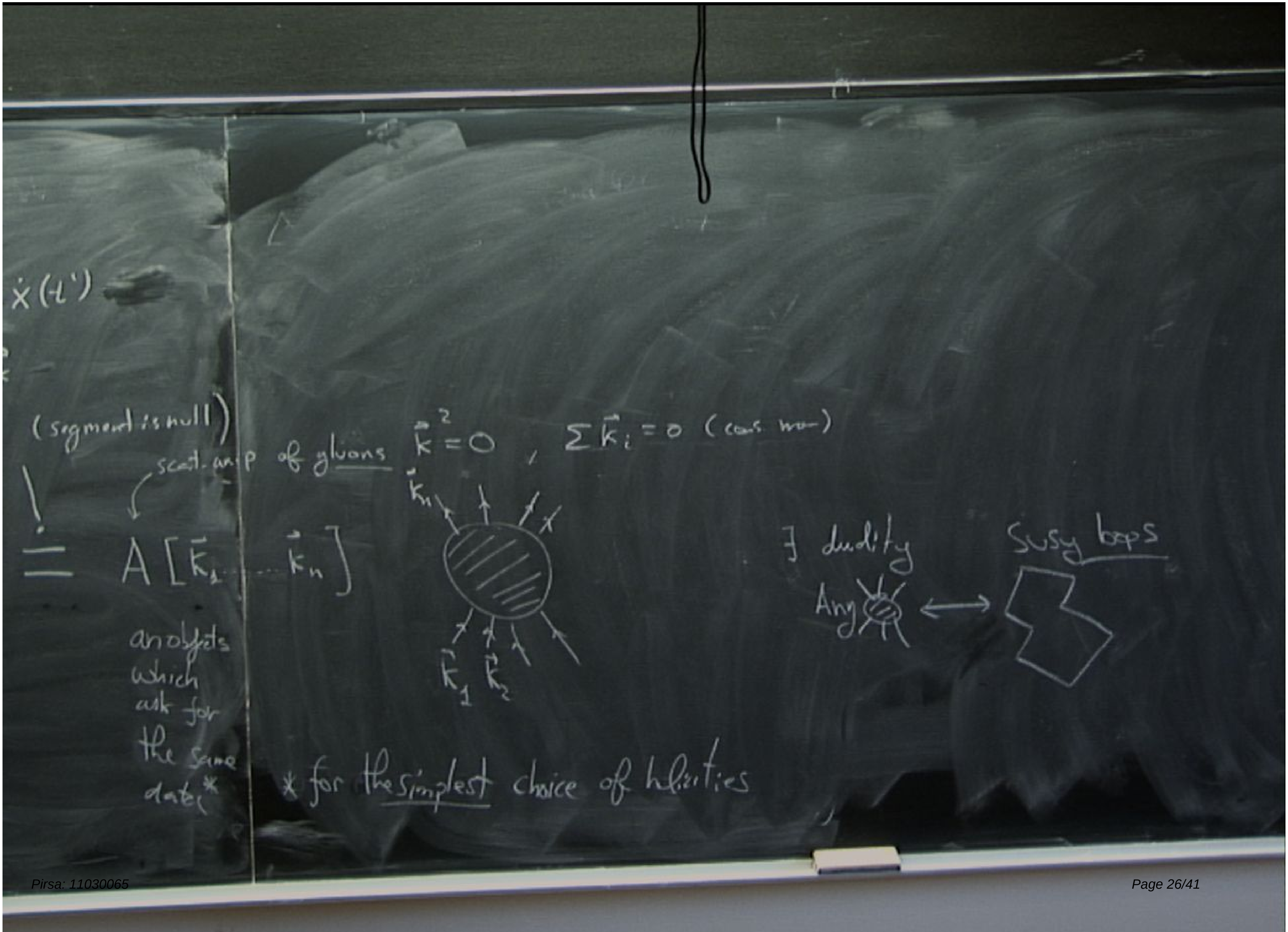
$= W[\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n] \;\; \stackrel{!}{=} \;\; A[\vec{k}_1 \ldots \vec{k}_n]$

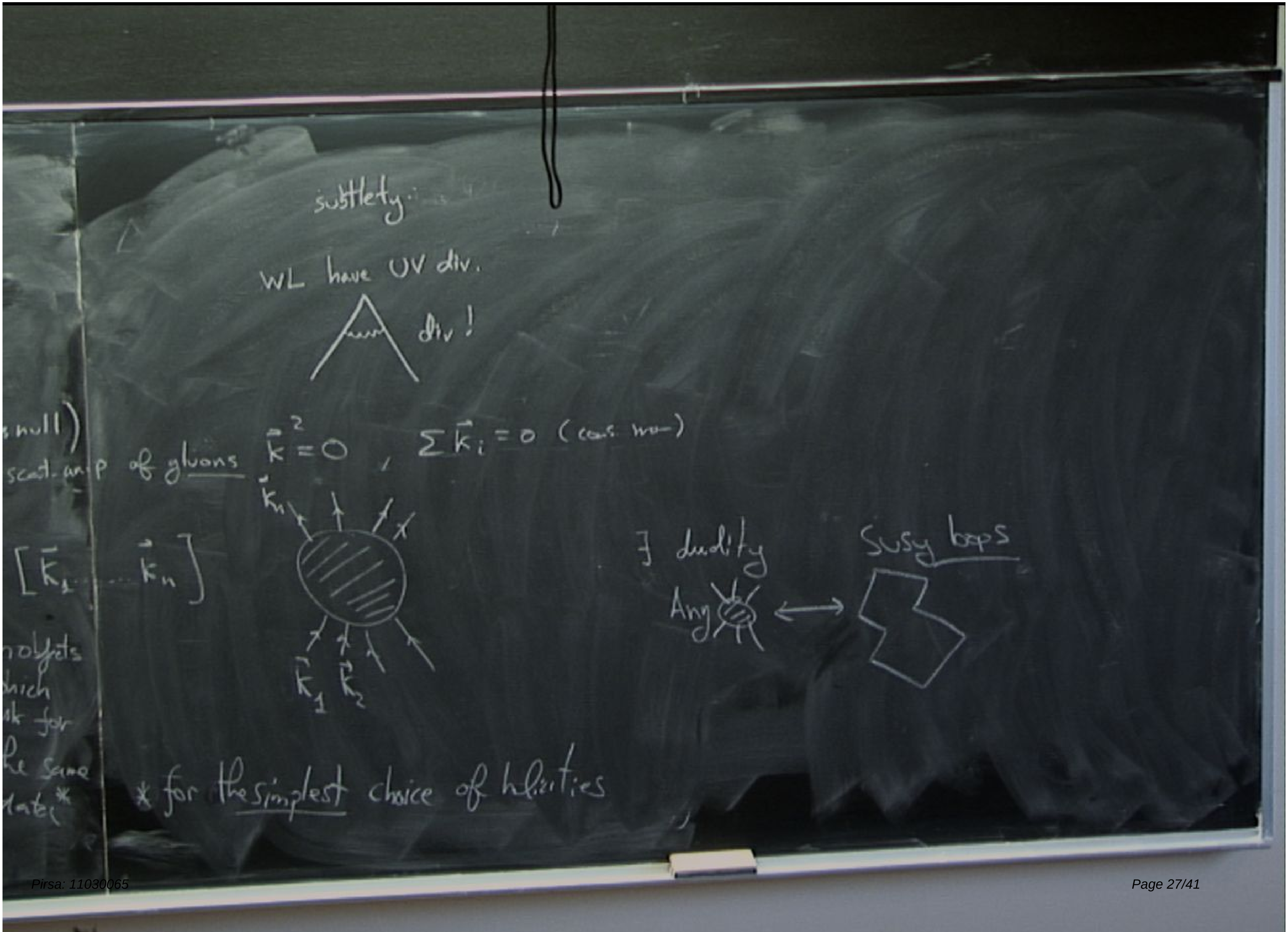
scat. amp of gluons $\vec{k}^2 = 0$, $\sum \vec{k}_i = 0$ (cons. mom)

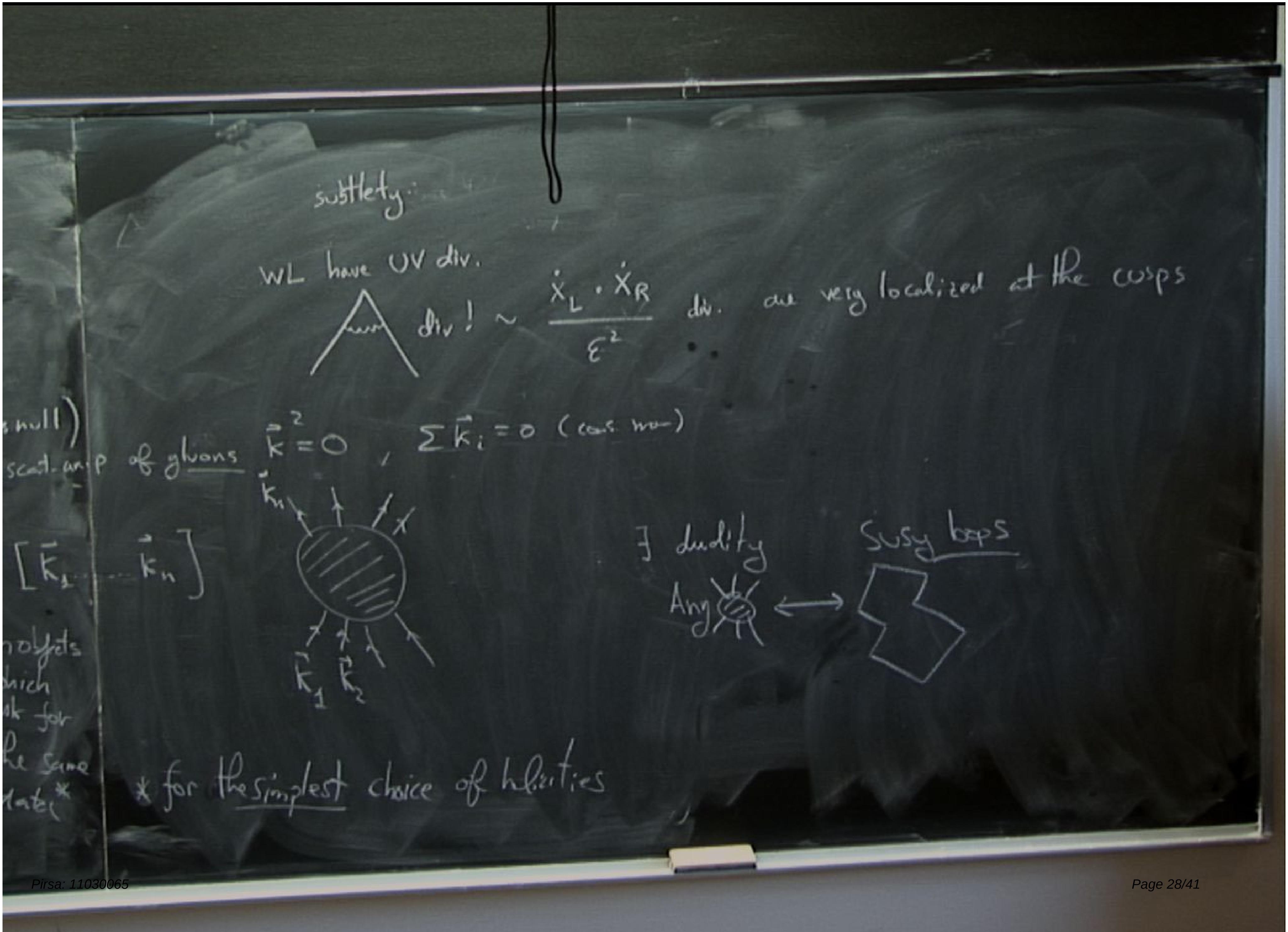
where
$\vec{k}_1 + \ldots + \vec{k}_n = 0$
and
$\vec{k}_i^2 = 0$

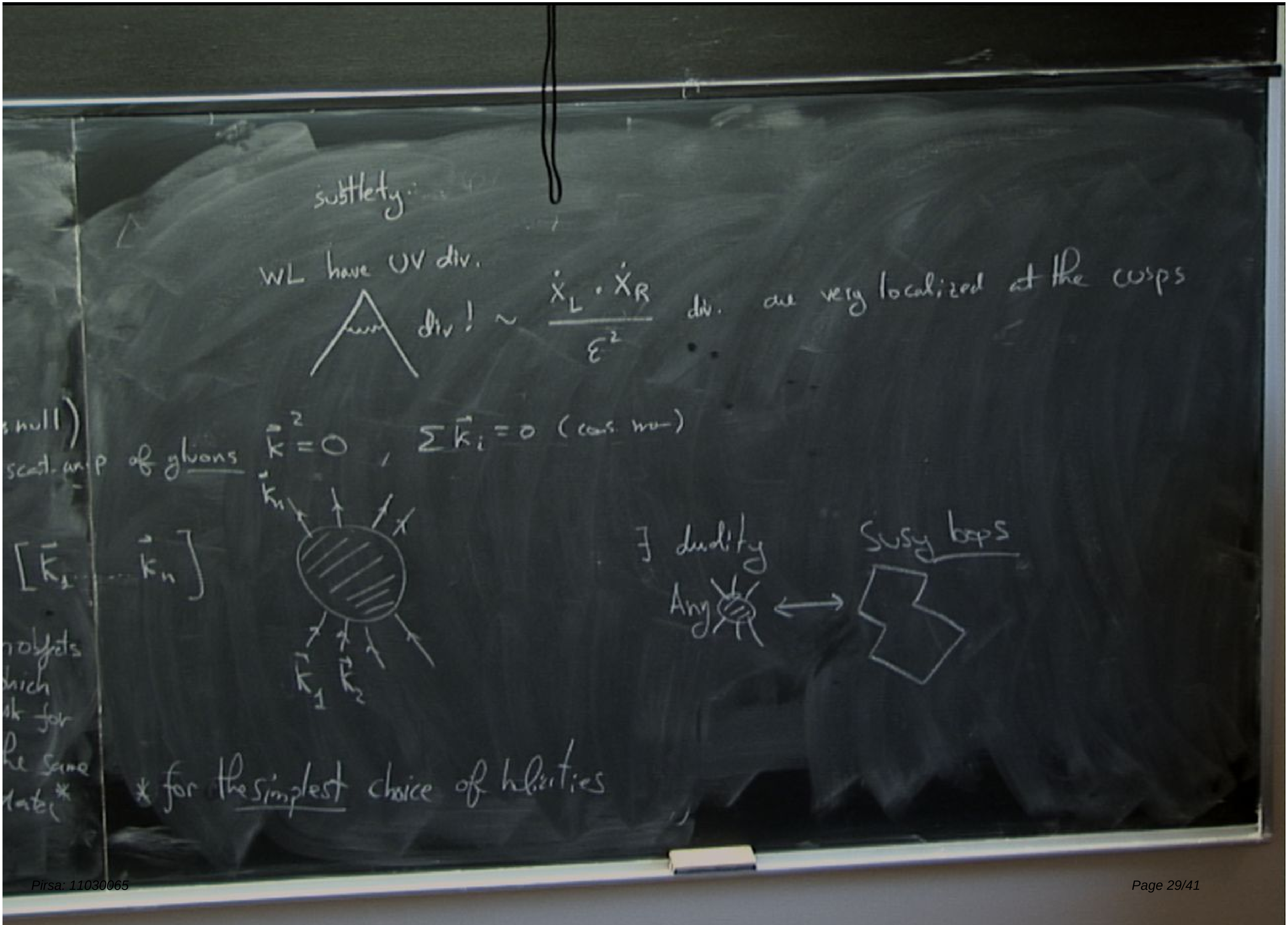
analysts which ask for the same data*

* for the simplest choice of helicities


(segment is null)
scat. amp of gluons
$\vec{k}^2 = 0$, $\Sigma \vec{k}_i = 0$ (cons. mom)
$\doteq A[\vec{k}_1 \ldots \vec{k}_n]$
are objects which ask for the same data*
∃ duality
Susy loops
Amp
* for the simplest choice of helicities


subtlety:
WL have UV div.
div!
null)
scat. amp of gluons
(cons. mom)
∃ duality
Susy loops
Any
* for the simplest choice of helicities
which
the same


subtlety:
WL have UV div.
div! ~ $\frac{\dot{X}_L \cdot \dot{X}_R}{\varepsilon^2}$
div. are very localized at the cusps
null)
scatt. amp of gluons $\vec{k}^2 = 0$, $\Sigma \vec{k}_i = 0$ (cons mom)
$[\vec{k}_1 \ldots \vec{k}_n]$
$\vec{k}_n$
$\vec{k}_1$ $\vec{k}_2$
∃ duality
Susy loops
Amp ⟷
objects
which
for
the same
* for the simplest choice of helicities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∃ duality
Any ⟷
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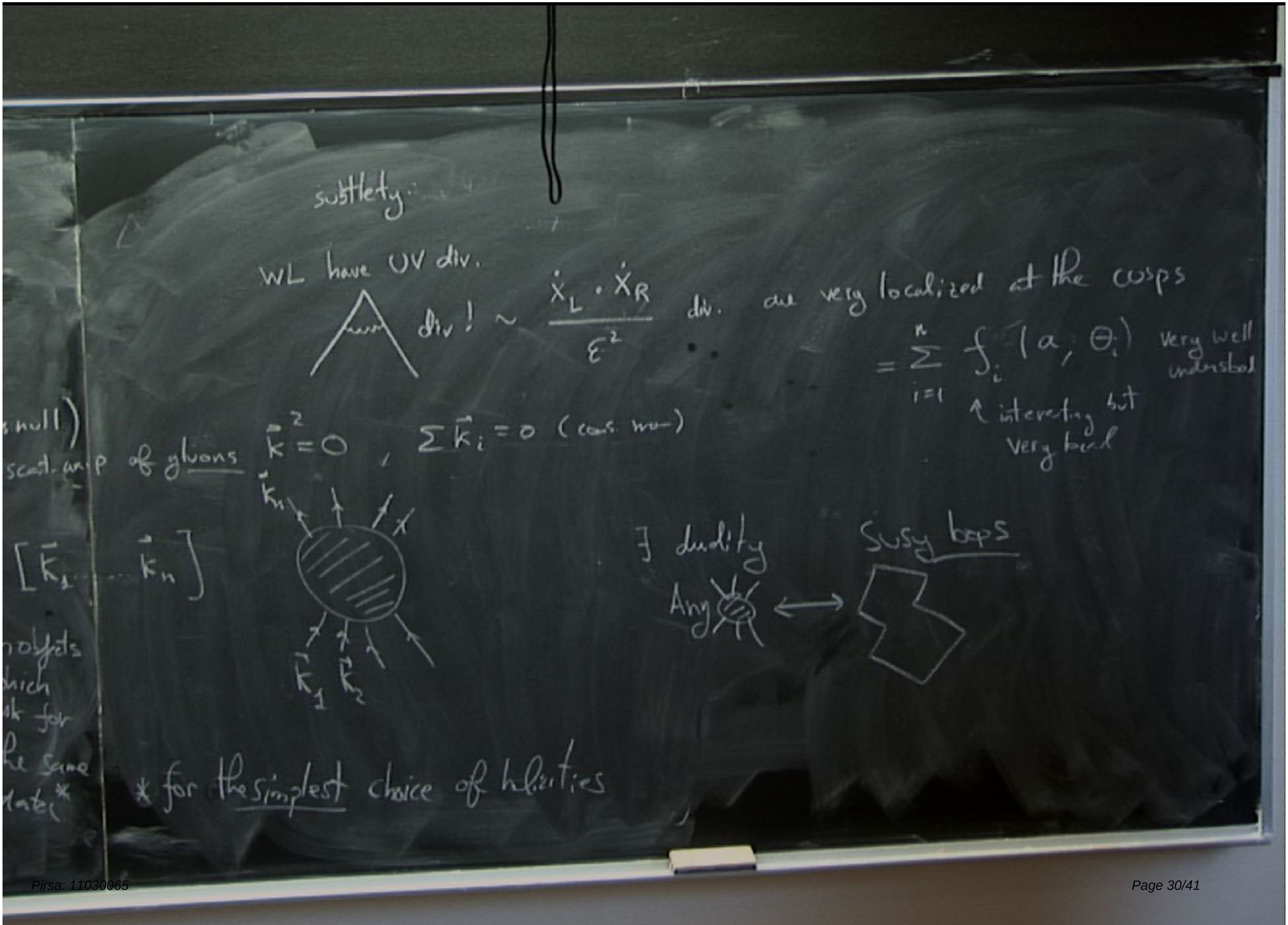
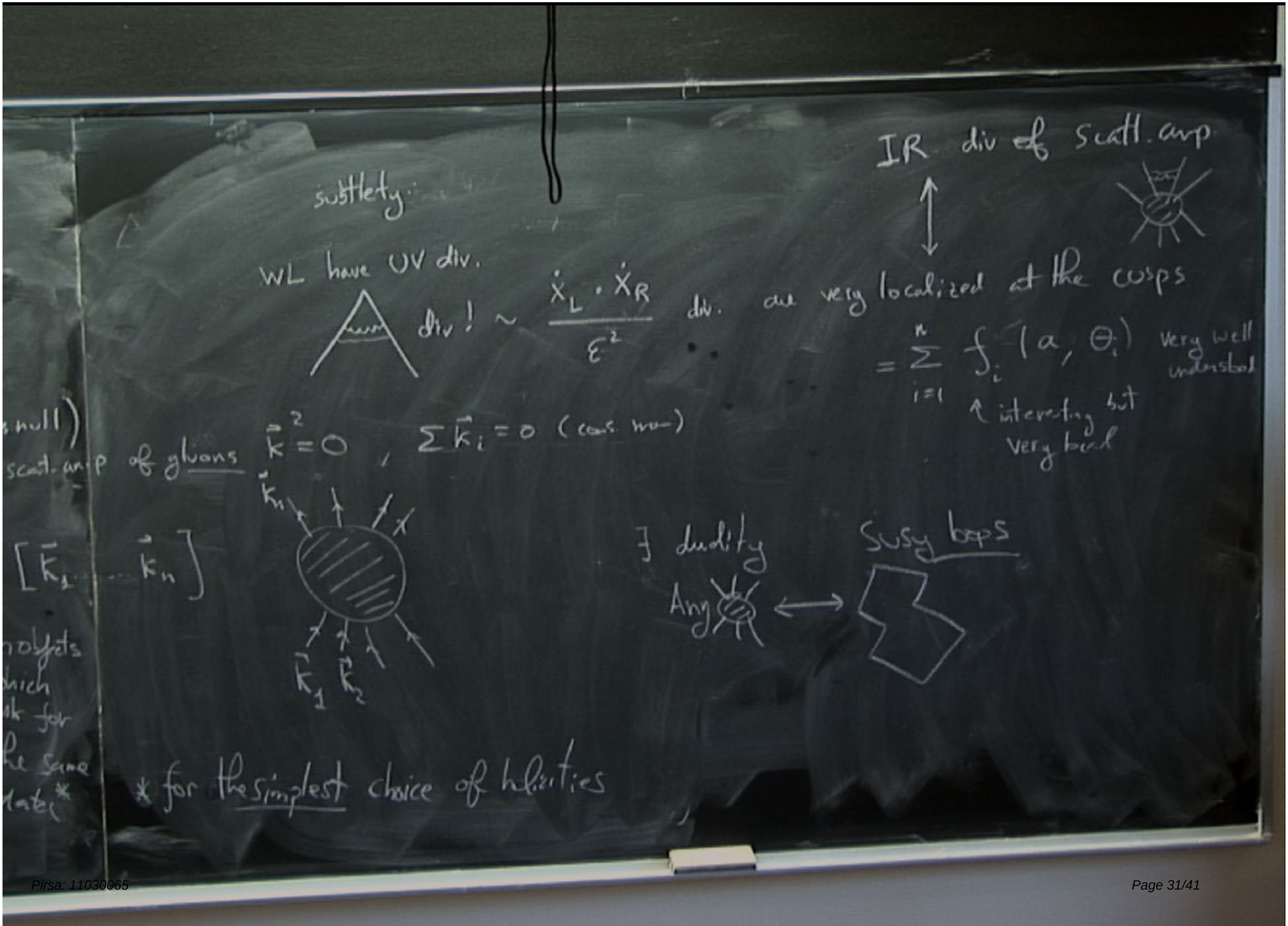

subtlety:
WL have UV div.
div! ~ $\frac{\dot{X}_L \cdot \dot{X}_R}{\varepsilon^2}$
div. are very localized at the cusps
$= \sum_{i=1}^{n} f_i(\alpha, \theta_i)$ very well understood
interesting but very hard
s null)
scatt. amp of gluons $\vec{k}^2 = 0$, $\sum \vec{k}_i = 0$ (cons. mom)
$[\vec{k}_1 \ldots \vec{k}_n]$
$\vec{k}_n$
$\vec{k}_1$ $\vec{k}_2$
∃ duality
Susy loops
Amp
objects
which
look for
the same
state *
* for the simplest choice of helicities


IR div of scatt. amp
subtlety:
WL have UV div.
div! ~ $\frac{\dot{X}_L \cdot \dot{X}_R}{\varepsilon^2}$
div. are very localized at the cusps
$= \sum_{i=1}^{n} f_i(\alpha, \theta_i)$ very well understood
interesting but very hard
null)
scatt. amp of gluons $\vec{k}^2 = 0$, $\sum \vec{k}_i = 0$ (cons. mom)
$[\vec{k}_1 \ldots \vec{k}_n]$
$\vec{k}_n$
$\vec{k}_1$ $\vec{k}_2$
∃ duality
Amp ⟷
Susy loops
objects
which
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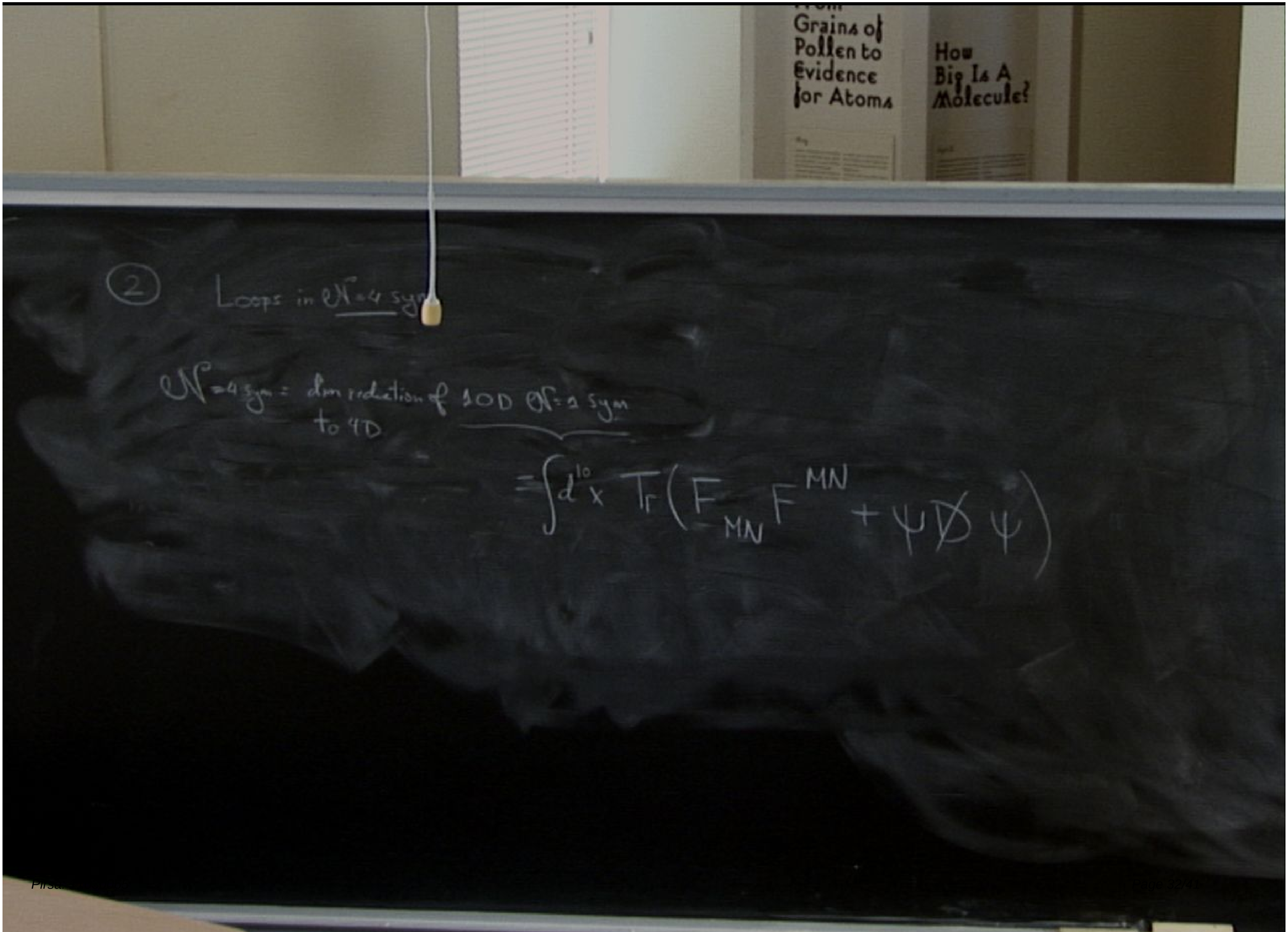
* for the simplest choice of helicities


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
② Loops in $\mathcal{N}=4$ sym
$\mathcal{N}=4$ sym = dim reduction of 10D $\mathcal{N}=1$ sym to 4D
$= \int d^{10}x \, \mathrm{Tr}\left(F_{MN}F^{MN} + \psi \not{D} \psi\right)$

② Loops in $\mathcal{N}=4$ SYM

$\mathcal{N}=4$ SYM = dim reduction of 10D $\mathcal{N}=1$ SYM to 4D

$$= \int d^{10}x \, \mathrm{Tr}\left(F_{MN}F^{MN} + \psi \not{D} \psi\right)$$

$\phi(\vec{x}, \vec{y})$ — $\vec{x}$: 4D, $\vec{y}$: 6D

$$\mathrm{Tr}\,\psi \not{D} \psi$$

$$\parallel$$

$$\sum\sum\sum\sum\sum \quad \psi \; \Gamma(\partial \quad \psi$$

$$\psi \not{D} \psi$$

$$=$$

$$\sum \sum \sum \sum \sum \; \psi^A \, \Gamma^M_{AB} \left(\partial_M \psi^B + i \left[A_M , \psi^B \right] \right)$$

$$\mathrm{Tr}\, \psi \not{D} \psi$$

$$=$$

$$\sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{M=0}^{9} \sum_{A=1}^{16} \sum_{B=1}^{16} \psi^{A}_{ab} \Gamma^{M}_{AB} \left(\partial_M \psi^{B} \right. \quad +$$

now we drop dependence on $x_4, x_5, x_6, x_7, x_8, x_9$

$$A_M = (A_\mu, \Phi_i)$$

μ: $0..3$, i: $1..6$

$$\psi^{B} + i\,[A_M, \psi^B]\Big)_{ba}$$

$$F_{MN}F^{MN} =$$

$$= F_{\mu\nu}F^{\mu\nu} + D_\mu \Phi_i D^\mu \Phi^i +$$

$$\mathrm{Tr}\,\psi \not{D} \psi$$

$$\parallel$$

$$\sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{M=0}^{9} \sum_{A=1}^{16} \sum_{B=1}^{16} \psi^A_{ab}\, \Gamma^M_{AB} \left(\partial_M \psi^B + i\, [A_M, \psi^B \right.$$

10 D σ-matrices

$$\Gamma^N \Gamma^M + \Gamma^M \Gamma^N = \eta^{MN}$$

now we drop dependence on $x_4, x_5, x_6, x_7, x_8, x_9$

$$A_M = (A_\mu, \Phi_i)$$

μ: $0..3$ $\quad$ i: $1..6$

$$\Gamma^{(M}\Gamma^{N)} = \eta^{MN}$$

$$\psi^B + i\,[A_M, \psi^B]\Big)_{ba}$$

$$- F_{MN}F^{MN} =$$

$$= \underbrace{F_{\mu\nu}F^{\mu\nu}}_{\text{QCD :)}} + D_\mu \Phi_i D^\mu \Phi^i + [\Phi_i, \Phi_j]\cdot[\Phi_i, \Phi_j]$$

$$- \psi \not{D} \psi = \psi\,\Gamma^\mu\left(\partial_\mu \psi + [A_\mu, \psi]\right) + \psi\,\Gamma^i\,[\Phi_i, \psi]$$

$$W^{N=4} \equiv P\exp \oint \left(A_\mu \dot{x}^\mu + i\, \Phi_i\, \theta^i(t)\, |\dot{x}| \right) dt$$

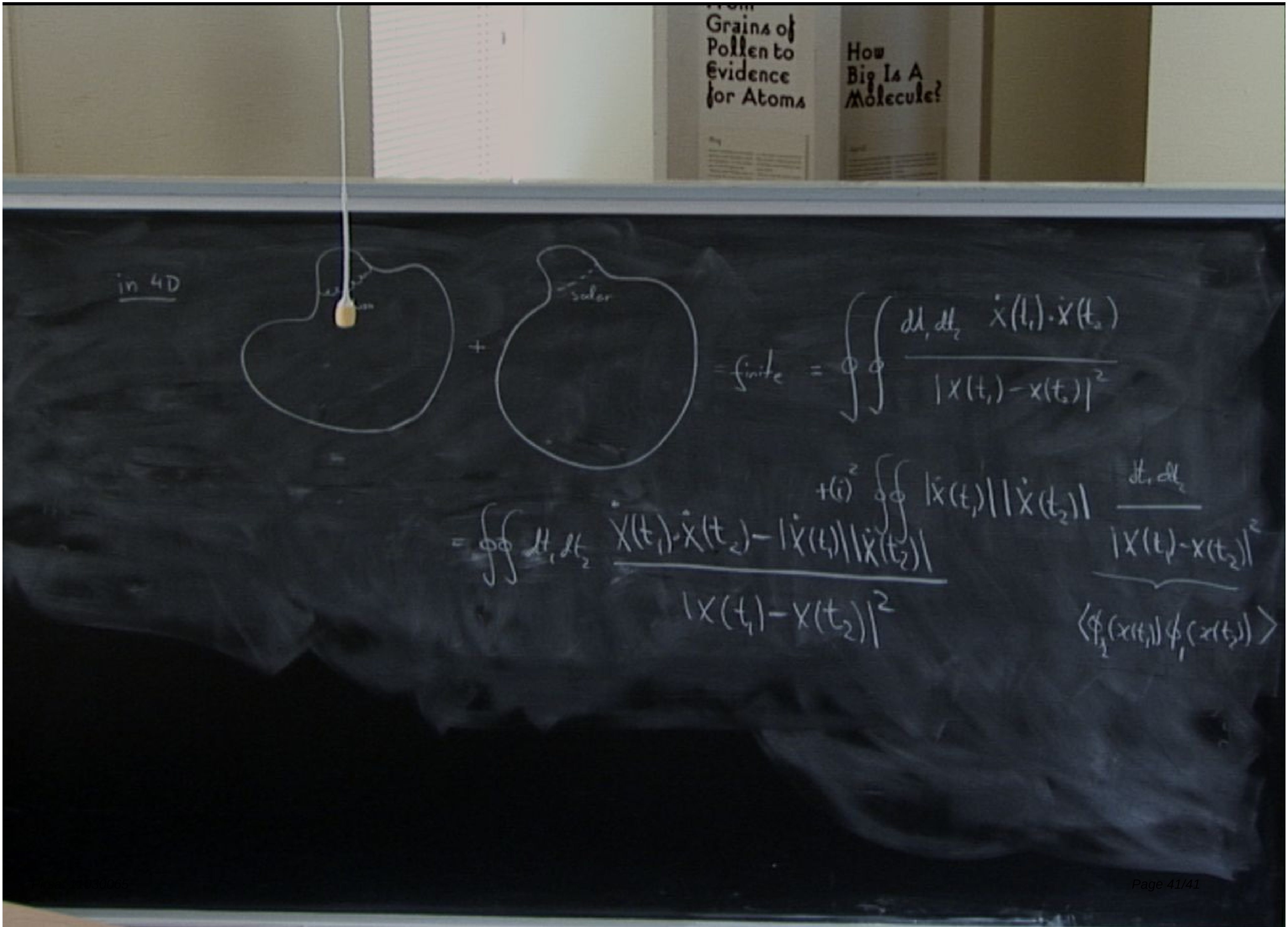
$$\theta^i \theta_i = 1$$

for simplicity lets only consider $\theta_i = (1,0,0,0,0,0)$

now we drop dependence on $x_4, x_5, x_6, x_7, x_8, x_9$

$$A_M = (A_\mu, \Phi_i)$$

μ: $0..3$ $\quad$ i: $1..6$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
in 4D
+
= finite = $\oint\oint \frac{dt_1 dt_2 \; \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$
$+(i)^2 \oint\oint |\dot{x}(t_1)||\dot{x}(t_2)| \frac{dt_1 dt_2}{|x(t_1)-x(t_2)|^2}$
$= \oint\oint dt_1 dt_2 \frac{\dot{x}(t_1)\cdot\dot{x}(t_2) - |\dot{x}(t_1)||\dot{x}(t_2)|}{|x(t_1)-x(t_2)|^2}$
$\langle \phi_2(x(t_1)) \phi_1(x(t_2)) \rangle$