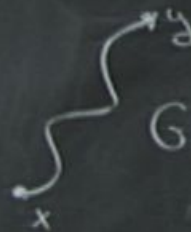


Title: Explorations in String Theory - Lecture 12

Date: Mar 29, 2011 11:30 AM

URL: <http://pirsa.org/11030063>

Abstract:



A diagram showing a wavy path in a 2D coordinate system with axes labeled x and y . The path starts at a point labeled x and ends at a point labeled y .

$$G(x, y) = \int dt \int \mathcal{D}X e^{i \int \dot{x}^2 + m^2}$$

$X(0) = x$
 $X(T) = y$



$G(x, y)$

$$= \int dt \int \mathcal{D}X$$

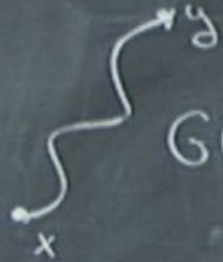
$X(0) = x$
 $X(T) = y$

$$e^{i \int \dot{x}^2 + m^2}$$

$$= \int \mathcal{D}x$$

$$e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length



$G(x, y)$

$$= \int dt \int \mathcal{D}X$$

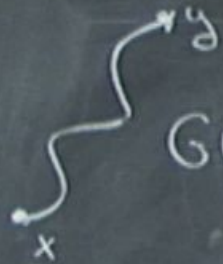
$X(0) = x$
 $X(T) = y$

$$e^{i \int \dot{x}^2 + m^2}$$

$$= \int \mathcal{D}x$$

$$e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length



$G(x, y)$

$$= \int dt \int \mathcal{D}X$$

$$\begin{aligned} X(0) &= x \\ X(T) &= y \end{aligned}$$

$$e^{-i \int \dot{x}^2 + m^2}$$

$$= \int \mathcal{D}x$$

$$e^{-i m \int \sqrt{\dot{x}^2} dt}$$

Length

in the presence
of a $U(1)$
gauge field A_μ



$G(x, y)$

$$= \int dt \int \mathcal{D}X e^{i \int \dot{x}^2 + m^2} = \int \mathcal{D}x e^{i \int \dot{x}^2 + m^2}$$

$$x(0) = x$$

$$x(T) = y$$

in the presence
of a $U(1)$
gauge field A_μ

$$G(x, y) = \int \mathcal{D}x e^{i S[x]}$$

time path

Length

$$G(x, y) = \int dt \int \mathcal{D}x e^{i \int \dot{x}^2 + m^2} = \int \mathcal{D}x e^{i m \int \sqrt{\dot{x}^2} dt}$$

$x(0) = x$
 $x(T) = y$

in the presence
of a U(1)
gauge field A_μ

$$G(x, y) = \int \mathcal{D}x e^{i S[x]} \exp\left(i e \int A_\mu(x(t)) \dot{x}^\mu(t) dt\right)$$

time function

$$j) = \int dt \int \mathcal{D}x \ e^{i \int \dot{x}^2 + m^2} = \int \mathcal{D}x \ e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length

$x(0) = x$
 $x(T) = y$

presence
(1)
field A_μ

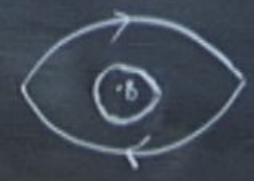
$$G(x,y) = \int \mathcal{D}x \ e^{i S[x]} \exp\left(i e \int_0^T A_\mu(x(t)) \dot{x}^\mu(t) dt\right)$$

$$Z_{t m^2} = \int \mathcal{D}x e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length

$$e^{i S[x]} \exp\left(i e \int_0^T A_m(x(t)) \dot{x}^m(t) dt\right)$$


phase



$$t m^2 = \int \mathcal{D}x e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length

$\Delta \text{ phases} =$


$$e^{i S[x]} \exp\left(i e \int_0^T A_m(x(t)) \dot{x}^m(t) dt\right), \oint$$


$$Z_{t_m^2} = \int \mathcal{D}x e^{i m \int \sqrt{\dot{x}^2} dt}$$

Length

$\Delta \text{ phases} =$

$$e^{i S[x]} \exp\left(i e \int_0^T A_m(x(t)) \dot{x}^m(t) dt\right)$$



$$= \iint B = \bar{\Phi}$$

x y
with a
non Abelian gauge field
 $[A_\mu]_{ab}$

x y
with a
non Abelian gauge field
 $[A_\mu]_{ab}$

$$G(x,y) = \int \mathcal{D}x e^{iS[x]} \text{Pexp} \int_0^T A_\mu(x(t)) \dot{x}^\mu(t) dt$$

x y
with a
non Abelian gauge field
 $[A_\mu]_{ab}$

$$G(x,y) = \int \mathcal{D}x e^{iS[x]} \text{Pexp} \left[\int_0^T A_\mu(x(t)) \dot{x}^\mu(t) dt \right]$$

x y
 with a
 non Abelian gauge field
 $[A_\mu]_{ab}$

$$G(x, y) = \int \mathcal{D}x e^{iS[x]} \text{Pexp} \left[e \int_0^T A_\mu(x(t)) \dot{x}^\mu(t) dt \right]$$



Slower (on def)

$$\exp \int_0^T ieA_{\mu}(x(t)) \cdot X^{\mu}(t) dt$$

Slower (on def)

$$\exp \int_0^T \underbrace{ieA_{\mu}(x(t)) \cdot X^{\mu}(t)}_{M(t)} dt$$

Slower (on def)

$$\exp \int_0^T \underbrace{ieA_n(x(t)) \cdot X^n(t)}_{M(t)} dt \equiv \text{def } \mathcal{I}$$

$$\stackrel{\text{def 1}}{=} 1 + \int_0^+ M(t) dt + \frac{1}{2} \int dt \int dt_2 M(t_1) M(t_2)$$

$$+ \frac{1}{3!} \int dt_1 dt_2 dt_3$$

$$\begin{aligned}
 &\stackrel{\text{def 1}}{=} 1 + \int_0^+ M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) \\
 &+ \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots
 \end{aligned}$$

$$\begin{aligned}
 &\stackrel{\text{def 1}}{=} 1 + \int_0^+ M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) \\
 &+ \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots
 \end{aligned}$$

$$\exp \int_0^t \underbrace{i \epsilon A_m(x(t)) \cdot X^m(t)}_{M(t)} dt \equiv 1 + \int_0^t M(t) dt + \frac{1}{2!} \int_0^t \int_0^{t'} M(t) M(t') dt dt' + \frac{1}{3!} \int_0^t \int_0^{t'} \int_0^{t''} M(t) M(t') M(t'') dt dt' dt'' + \dots$$

||| def 2

$$\left[\exp(i \epsilon M(x_N)) \right] \left[\exp(i \epsilon M(x_{N-1})) \right] \dots$$

$$\begin{aligned}
 U(t) &\equiv 1 + \int_0^t M(t') dt' + \frac{1}{2!} \int_0^t \int_0^{t'} M(t_1) M(t_2) dt_1 dt_2 \\
 &+ \frac{1}{3!} \int_0^t \int_0^{t_1} \int_0^{t_2} M(t_1) M(t_2) M(t_3) dt_1 dt_2 dt_3 + \dots
 \end{aligned}$$

$$\exp(i\varepsilon \Pi(x_{N-1})) \dots \left[\exp(i\varepsilon \Pi(x_1)) \right]$$

Solver (on def)

$$\exp \int_0^T \underbrace{i \epsilon A_x(x(t)) \cdot X^N(t)}_{M(t)} dt \stackrel{\text{def 1}}{=} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

||| def 2

$$\left[\exp(i \epsilon M(x_N)) \right] \left[\exp(i \epsilon M(x_{N-1})) \right] \dots \left[\exp(i \epsilon M(x_2)) \right]$$

Sloven (on def)

$$\exp \int_0^T \underbrace{ieA_m(x(t)) \cdot X^m(t)}_{M(t)} dt \stackrel{\text{def 1}}{=} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

||| def 2

$$\left[\exp(i\varepsilon M(x_N)) \right] \left[\exp(i\varepsilon M(x_{N-1})) \right] \dots \left[\exp(i\varepsilon M(x_2)) \right]$$

DEF 3

$$\left[\partial_t - M(t) \right] \psi(t) = 0 \quad \text{with b.c. } \psi(0) = 1$$

$$\int_0^T M(t) dt \stackrel{\text{def}}{=} 1 + \int_0^T M(t) dt + \frac{1}{2} \int_0^T \int_0^{t_1} M(t_1) M(t_2) dt_1 dt_2 + \frac{1}{3!} \int_0^T \int_0^{t_1} \int_0^{t_2} M(t_1) M(t_2) M(t_3) dt_1 dt_2 dt_3 + \dots$$

$$\left[\exp(i\varepsilon \pi(x_{N-1})) \right] \dots \left[\exp(i\varepsilon \pi(x_2)) \right]$$

$t) = 0$ with b.c $\psi(0) = 1$. Then $\psi(T) = \exp \int_0^T M(t) dt$

Sloven (on def)

$$\exp \int_0^T \underbrace{icA_{\mu}(x(t)) \cdot X^{\mu}(t)}_{M(t)} dt \stackrel{\text{def 3}}{=} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

III def 2

$$\left[\exp(i\varepsilon M(x_N)) \right] \left[\exp(i\varepsilon M(x_{N-1})) \right] \dots \left[\exp(i\varepsilon M(x_2)) \right]$$

DEF 3

$$\left[\partial_t - M(t) \right] \psi(t) = 0 \quad \text{with b.c. } \psi(0) = 1 \quad \text{Then } \psi(T) = \exp \int_0^T M(t) dt$$

Sloven (on def)

\equiv = non-Abelian exponential Pexp

$$\exp \int_0^T \underbrace{ieA_\mu(x(t)) \cdot X^\mu(t)}_{M(t)} dt \stackrel{\text{def 3}}{\equiv} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

III def 2

$$\left[\exp(i\varepsilon M(x_N)) \right] \left[\exp(i\varepsilon M(x_{N-1})) \right] \dots \left[\exp(i\varepsilon M(x_2)) \right]$$

DEF 3

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||| def 2 ✓

$$\left[\exp(i\varepsilon M(x_N)) \right]_{a_N} \left[\exp(i\varepsilon M(x_{N-1})) \right]_{a_{N-1}} \dots \left[\exp(i\varepsilon M(x_2)) \right]_{a_2} \dots$$

DEF 3

$$\left[\partial_t - M(t) \right] \psi(t) = 0$$

with b.c $\psi(0) = 1$. Then $\psi(T) = \exp \int_0^T M(t) dt$

Sloven (on def)

\equiv = non-Abelian exponential Pexp

$$\exp \int_0^T \underbrace{icA_\mu(x(t)) \cdot X^\mu(t)}_{M(t)} dt \stackrel{\text{def 3}}{\equiv} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

DEF 2 ✓

$$\left[\exp(i\varepsilon M(x_N)) \right]_{a_N, a_{N+1}} \left[\exp(i\varepsilon M(x_{N-1})) \right]_{a_{N-1}, a_N} \dots \left[\exp(i\varepsilon M(x_2)) \right]_{a_2, a_3}$$

DEF 3 ✓

$$\left[\partial_t - M(t) \right] \psi(t) = 0$$

\uparrow matrix \uparrow matrix

with b.c $\psi(0) = 1$ (identity matrix). Then $\psi(T) = \exp \int_0^T M(t) dt$

Slober (on def)

\equiv = non-Abelian exponential Pexp

$$P \exp \int_0^T \underbrace{ieA_\mu(x(t)) \cdot X^\mu(t)}_{M(t)} dt \stackrel{\text{def 1}}{=} 1 + \int_0^T M(t) dt + \frac{1}{2} \int dt_1 \int dt_2 M(t_1) M(t_2) + \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) M(t_2) M(t_3) + \dots$$

||| def 2 ✓

$$\left[\exp(i\varepsilon M(x_N)) \right]_{a_N, a_{N+1}} \left[\exp(i\varepsilon M(x_{N-1})) \right]_{a_{N-1}, a_N} \dots \left[\exp(i\varepsilon M(x_2)) \right]_{a_2, a_3}$$

DEF 3 ✓

$$\left[\partial_t - M(t) \right] \psi(t) = 0$$

\uparrow matrix \uparrow matrix

with b.c $\psi(0) = 1$. Then $\psi(T) = \exp \int_0^T M(t) dt$

\uparrow identify matrix

$$P \exp \int_0^T M(t) dt =$$

↑
matrix

$$P \exp \int_0^T M(t) dt = \underline{\underline{1}} + \int M(t) dt$$

matrix

$$P \exp \int_0^T M(t) dt = \mathbb{1} + \int M(t) dt$$

↑
matrix

$$+ M(t_2)M(t_1)$$

$$P \exp \int_0^T M(t) dt = \mathbb{1} + \int M(t) dt$$

\uparrow
 matrix

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2 M(t_2) M(t_1)$$

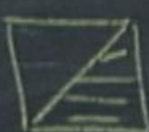
$$t = \underline{1} + \int M(t) dt$$

rix

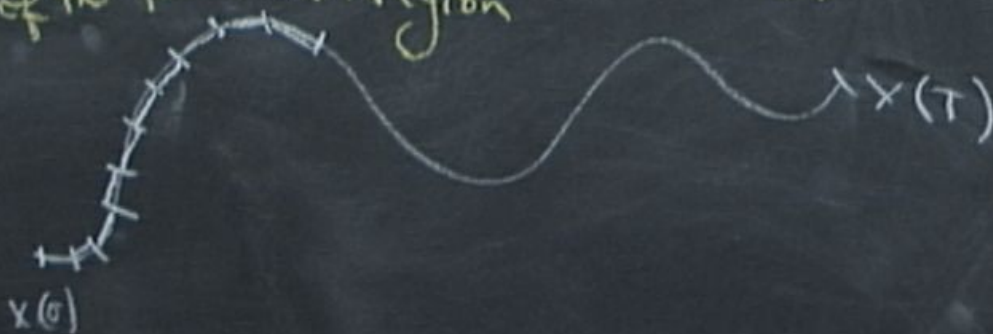
$$+ \int_0^T dt_1 \int_{t_1}^T dt_2$$

$$M(t_2) M(t_1)$$

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2 \int_{t_2}^T dt_3 \pi(t_3) \pi(t_2) \pi(t_1)$$

 $\frac{1}{2!}$ of the total int. region

$\frac{1}{3!}$ of the total region



DEF

$$\equiv \mathbb{1} + \int M(t) dt$$

ix

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2$$

$$M(t_2) M(t_1)$$

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2 \int_{t_2}^T dt_3$$

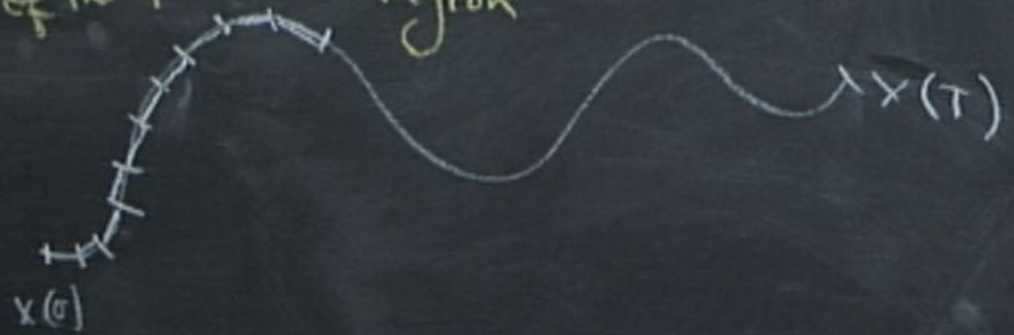
$$M(t_3) M(t_2) M(t_1)$$

+ ...



$\frac{1}{2!}$ of the total int. region

$\frac{1}{3!}$ of the total region



DEF

$$\equiv \mathbb{1} + \int M(t) dt$$

ix

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2$$

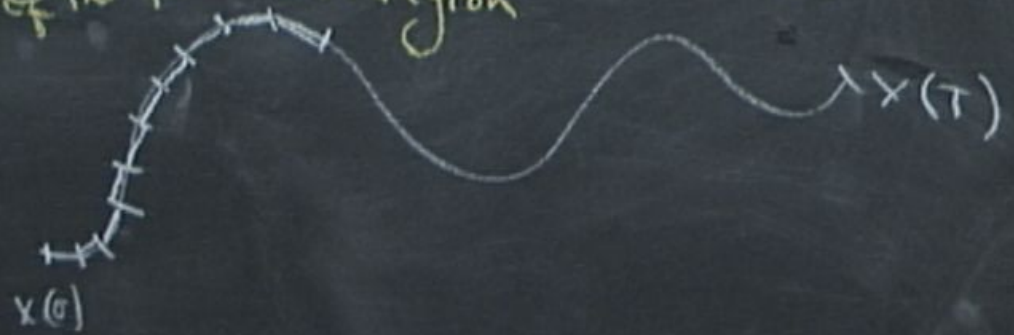
$$M(t_2) M(t_1)$$

$$+ \int_0^T dt_1 \int_{t_1}^T dt_2 \int_{t_2}^T dt_3 \pi(t_3) \pi(t_2) \pi(t_1) + \dots$$



$\frac{1}{2!}$ of the total int. region

$\frac{1}{3!}$ of the total region



DEF

$$U \equiv \mathbb{1} + \int M(t) dt$$

works for $U(1)$
or non-Abelian

ix

$$+ \int_0^{T_1} dt_1 \int_{t_1}^T dt_2$$

$$M(t_2) M(t_1)$$

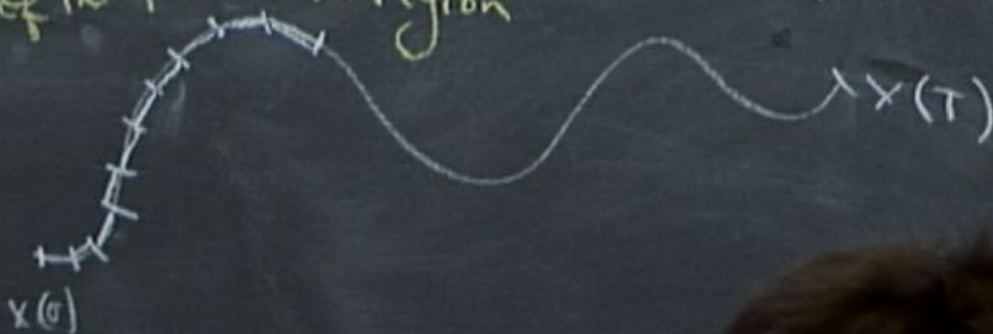
T T T

$$+ \int_0^{t_1} dt_1 \int_{t_1}^{t_2} dt_2 \int_{t_2}^T dt_3 M(t_3) M(t_2) M(t_1) + \dots$$



$\frac{1}{2!}$ of the total int. region

$\frac{1}{3!}$ of the total region



motivation for studying

↳ sometimes no trace

$$W_e = \text{Tr} \left(P \exp \int M(t) dt \right)$$

↑ loop



motivation for studying

↖ sometimes no trace

$$W_{\text{loop}} = \text{Tr} \left(P \exp \int M(t) dt \right)$$

↖ loop



- gauge length
- 1) related to world-lines in the presence of gauge fields
 - 2) gauge invariant non-local observable (next)

Wilson loop length

- 1) related to worldlines in the presence of gauge fields
- 2) gauge invariant non-local observable (next)
- 3) probe for confinement
- 4)

- gauge length
- 1) related to worldlines in the presence of gauge fields
 - 2) gauge invariant non-local observable (next)
 - 3) probe for confinement
 - 4) related to scattering amplitudes
 - 5) nice qty in AdS/CFT and in lattice QFT

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. E_6 sym)

We have

$$\Phi(x) \rightarrow U(x)$$

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. E_7 sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. E_7 sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. $SO(4)$ sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. E_7 sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

$$\Phi(x) \Phi(x) \rightarrow U(x) \Phi \Phi U^{-1}(x)$$

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. $\mathcal{N}=4$ sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

$$\Phi(x) \Phi(x) \rightarrow U(x) \Phi \Phi U^{-1}(x)$$

$$\langle \Phi \rangle(x) \equiv \text{Tr} \Phi$$

remains in a non-abelian gauge theory with fields Φ_{ab} in the adjoint

(eg. $\mathcal{N}=4$ sym)

We have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

$$\Phi(x) \Phi(x) \rightarrow U(x) \Phi \Phi U^{-1}(x)$$

$$\mathcal{O}(x) \equiv \text{Tr} \Phi \Phi \longleftrightarrow \text{dual to } \underline{\text{closed strings}}$$

↘ cyclicity of the trace

what if we insist on constructing something like $\Phi(x) \Phi(y)$

$$+ \frac{1}{3!} \int dt_1 \int dt_2 \int dt_3 M(t_1) \dots \psi(t_3)$$

$$\left[\exp(i\epsilon M(x_N)) \right]_{a_1 a_2} \left[\exp(i\epsilon M(x_{N-1})) \right]_{a_2 a_3} \dots \left[\exp(i\epsilon M(x_2)) \right]_{a_N a_{N+1}}$$

$\exists \checkmark$

$$M(t) \psi(t) = 0$$

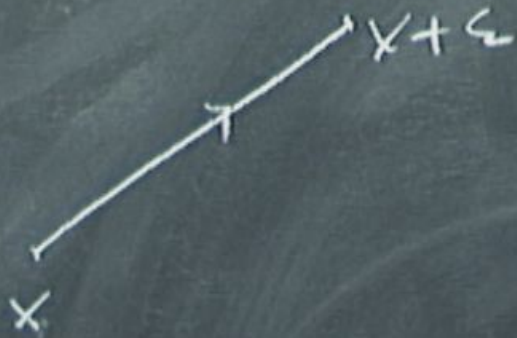
matrix matrix

with b.c $\psi(0) = 1$. Then $\psi(T) = \exp$

identity matrix

What if we insist on constructing something like $\mathbb{Z}(x) \oplus \mathbb{Z}(y)$?

what if we insist on constructing something ψ



$$W(x + \epsilon, x) \equiv \exp\left(i g \int_{x}^{x + \epsilon} A_{\mu} dx^{\mu}\right)$$

matrix

what if we insist on constructing something ψ



$$W(x + \epsilon, x) \equiv \exp\left(ig \int_{x}^{x + \epsilon} A_{\mu} dx^{\mu}\right)$$

matrix

$$\approx (1 + ig \int_{x}^{x + \epsilon} A_{\mu} dx^{\mu})$$



matrix

$$W(x+\epsilon, x) \equiv \exp\left(ig \epsilon^\mu A_\mu\right)$$

$$\approx (1 + ig \epsilon^\mu A_\mu)$$

gauge transf

$$\rightarrow (1 + ig \epsilon^\mu (U A_\mu U^{-1} - U \partial_\mu U^{-1}))$$

$$W(x+\epsilon, x) \equiv \exp\left(ig \epsilon^\mu A_\mu\right)$$

matrix

$$\approx (1 + ig \epsilon^\mu A_\mu)$$

$$\rightarrow (1 + ig \epsilon^\mu (U A_\mu U^{-1} - U \partial_\mu U^{-1}))$$

gauge
transf

$$W(x+\epsilon, x) \equiv \exp\left(ig \epsilon^\mu A_\mu\right)$$

matrix

$$\approx (1 + ig \epsilon^\mu A_\mu)$$

$$\xrightarrow{\text{gauge transf}} (1 + ig \epsilon^\mu (U A_\mu U^{-1} - U \partial_\mu U^{-1}))$$

$$\approx U(x+\epsilon) W(x+\epsilon, x) U^{-1}(x)$$

What if we insist on constructing something like $\Phi(x)\Phi(y)$?



$$W(x+\epsilon, x) \equiv \exp\left(ig \int_x^{x+\epsilon} A_\mu dx^\mu\right)$$

matrix

$$\approx (1 + ig \int_x^{x+\epsilon} A_\mu dx^\mu)$$

$$\xrightarrow{\text{gauge transf}} (1 + ig \int_x^{x+\epsilon} (U A_\mu U^{-1} - U \partial_\mu U^{-1}))$$

$$\approx U(x+\epsilon) W(x+\epsilon, x) U^{-1}(x)$$

in a non-abelian gauge th with fields in the Adjoint,
($\mathcal{N}=4$ sym)

have

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

$$= W_I(y, x) = w(y, y - \epsilon_n) w(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots w$$

time

$$= W_{\mathbb{P}}(y, x) = W(y, y - \epsilon_n) W(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots W(x + \epsilon_1, x)$$

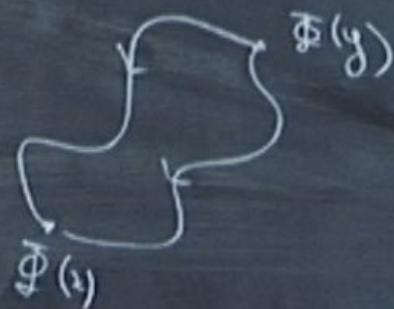
→ gauge
transf

$$U(y) W_{\mathbb{P}}(y, x) U^{-1}(x)$$

$$= W_{\mathbb{I}}(y, x) = W(y, y - \epsilon_n) W(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots W(x + \epsilon_1, x)$$

→ gauge transf

$$U(y) W_{\mathbb{P}}(y, x) U^{-1}(x)$$

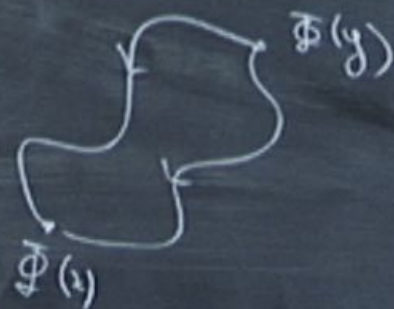


$$W_{\mathbb{P}}(x, y) \Phi(y) W(y, x)$$

$$= W_I(y, x) = W(y, y - \epsilon_n) W(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots W(x + \epsilon_1, x)$$

→ gauge transf

$$U(y) W_P(y, x) U^{-1}(x)$$

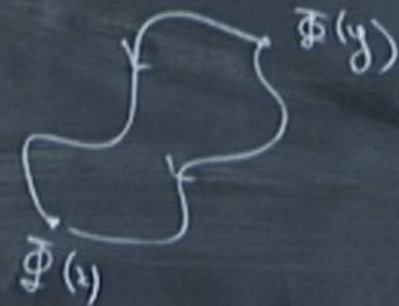


$$\Phi(x) W_P(x, y) \Phi(y) W(y, x)$$

$$= W_{\mathbb{P}}(y, x) = W(y, y - \epsilon_n) W(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots W(x + \epsilon_1, x)$$

→ gauge transf

$$U(y) W_{\mathbb{P}}(y, x) U^{-1}(x)$$



$$\text{Tr} \left[\Phi(x) W_{\mathbb{P}}(x, y) \Phi(y) W(y, x) \right]$$

↑ non-local gauge inv. \mathcal{P} !

What if we insist on constructing something for $\Phi(x)\Phi(y)$?



$$W(x+\epsilon, x) \equiv \exp\left(ig \int_x^{x+\epsilon} A_\mu dx^\mu\right)$$

matrix

$$UU^{-1} = 1$$

$$\partial_\nu U^{-1} + U \partial_\nu U^{-1} = 0$$

$$\approx (1 + ig \epsilon^\mu A_\mu)$$

gauge
transf

$$\rightarrow (1 + ig \epsilon^\mu (U A_\mu U^{-1} - U \partial_\mu U^{-1}))$$

$$\approx U(x+\epsilon) W(x+\epsilon, x) U^{-1}(x)$$

What if we insist on constructing something for $\Phi(x)\Phi(y)$?



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$$\approx U(x+\epsilon) W(x+\epsilon, x) U^{-1}(x)$$

$$W_e = \text{Tr} \int_{x_0} \rightarrow \text{Tr}(U(x) \int_{x_0} U^{-1}(x_0)) =$$

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

a gauge inv. op.

$$\Phi(x) \Phi(x) \rightarrow U(x) \Phi \Phi U^{-1}(x)$$

$$\mathcal{L}(x) \equiv \text{Tr} \Phi \Phi \longleftrightarrow \text{dual to closed strings}$$

↘ cyclicity of the trace

what if



$$W_e = \text{Tr} \int_{x_0} \rightarrow \text{Tr}(U(x) U^{-1}(x_0)) =$$

x_0 is not important

$$\Phi(x) \rightarrow U(x) \Phi(x) U^{-1}(x)$$

not a gauge inv. op.

$$\Phi(x) \Phi(x) \rightarrow U(x) \Phi \Phi U^{-1}(x)$$

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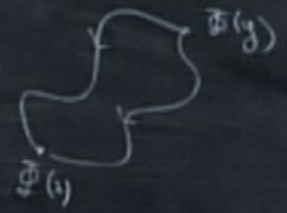


$$= W_P(y, x) = w(y, y - \epsilon_n) w(y - \epsilon_n, y - \epsilon_n - \epsilon_{n-1}) \dots w(x + \epsilon_1, x)$$

→
gauge
transf

$$U(y) W_P(y, x) U^{-1}(x)$$

$$A \xrightarrow{\text{gauge}} UAU^{-1}$$



$$\text{Tr} \left[\Phi(x) W_P(x, y) \Phi(y) W_P(y, x) \right]$$

↑
number of gauge in \mathcal{G}

$$Z = \int \mathcal{D}A e^{-S[A]}$$

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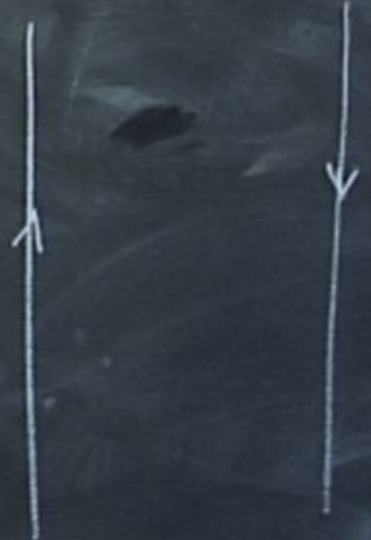


$$\sum_n e^{-TE_n} = Z = \int \mathcal{D}A e^{-S[A]} = e^{-E_0 T}$$



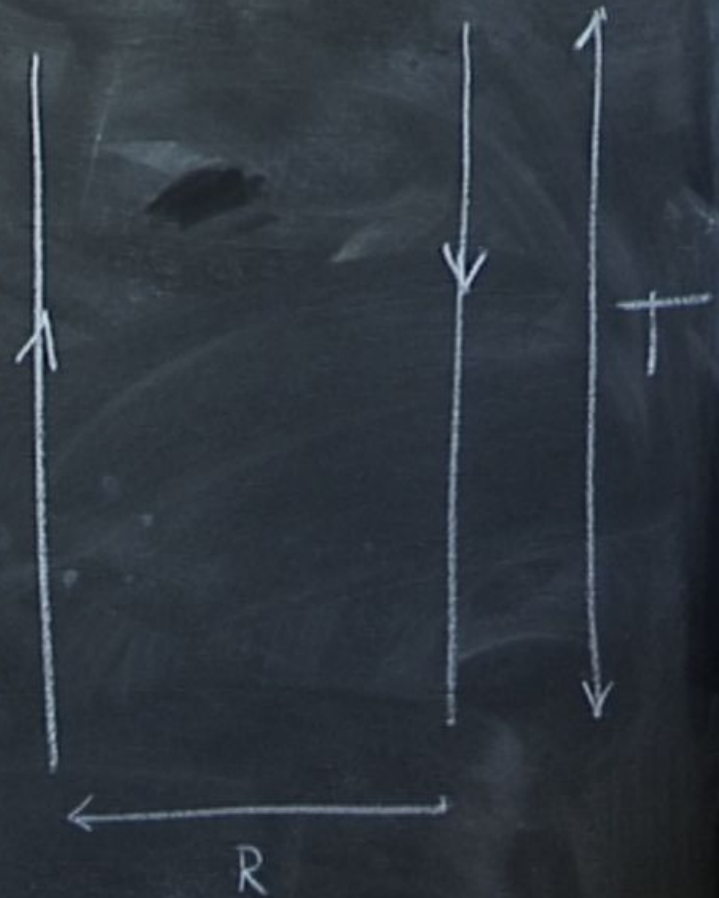
$$\sum_n e^{-TE_n} = Z = \int \mathcal{D}A e^{-S[A]} = e^{-E_0 T}$$

lets add two charges
(one e , $-e$
separated by R)



$$= e^{-E_b T}$$

lets add two charges
(one e , $-e$
separated by R)



$$S[A] = e^{-E_0 T}$$

$= e$

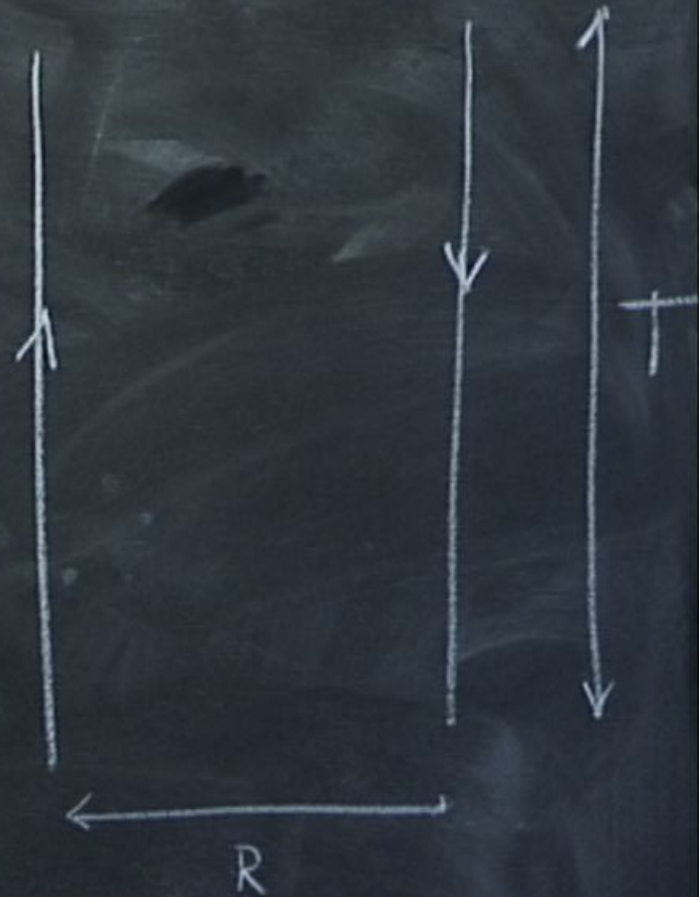


$$e^{-(E_0 + V(R)) T}$$

we add

lets add two charges

(one e , $-e$
separated by R)



$$S[A] = e^{-E_0 T}$$

$= e$



$$e^{-(E_0 + V(R)) T}$$

we can then read $V(R)$

lets add two charges

(one e , $-e$
separated by R)



$$Z = \int \mathcal{D}A e^{-S[A]} = e^{-E_0 T}$$

lets add two charges
(one e , $-e$
separated by R)

$$e^{-\left(E_0 + V(R)\right) T}$$

we can then read $V(R)$



Adding charges:

$$\int \mathcal{D}A \exp(-S[A])$$

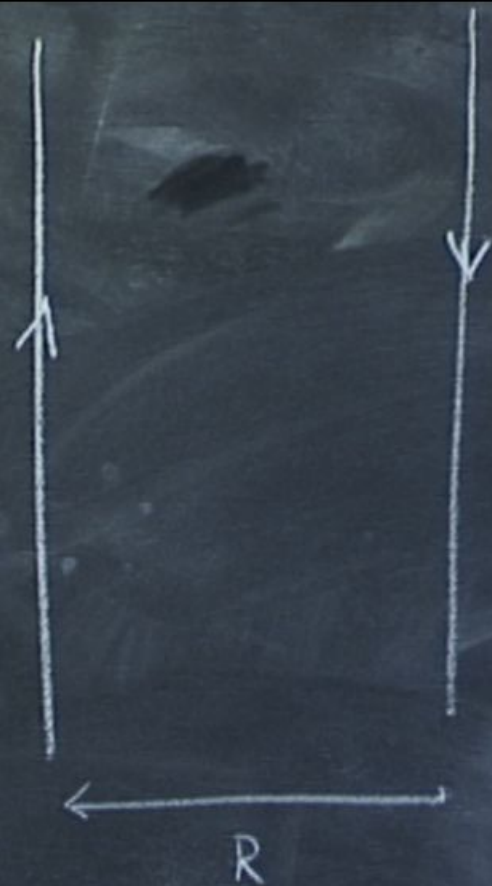
$$e^{-S[A]} = e^{-E_0 T}$$

lets add two charges

(one e , $-e$
separated by R)

$$e^{-\left(E_0 + V(R)\right) T}$$

we can then read $V(R)$



Adding charges:

$$\int \mathcal{D}A \exp\left(-S[A] + \int dt d^3x \left[j_\mu(x) A^\mu(x) \right]\right)$$

$$J^{\mu} = \int (\vec{x} - \vec{o}) \int_{\sigma_0}^{\mu} e$$

time

$$J^\mu = \int (\vec{x} - \vec{0}) \int_0^\mu e + \int (\vec{x} - \vec{R}) \int_0^{\mu'} (-e)$$

charge

$$J^\mu = \int (\vec{x} - \vec{0}) \int_0^\mu e \quad + \quad \int (\vec{x} - \vec{R}) \int_0^{\mu'} (-e)$$

↑
charge

$$S[A] + i \int J_\mu A^\mu = S[A] +$$

$$J^\mu = \int_{\vec{0}}^{\vec{x}} \delta_0^\mu e + \int (\vec{x} - \vec{R}) \delta_0^{\mu\nu} (-e)$$

↑
charge

$$S[A] = S[A] + ie \int A_\mu(x(t)) \dot{x}^\mu(t) dt$$

$$J^\mu = \int (\vec{x} - \vec{0}) \int_0^\mu e \quad + \quad \int (\vec{x} - \vec{R}) \int_0^{\mu'} (-e)$$

↑
charge

$$S[A] + i \int J_\mu A^\mu = S[A] + ie \int A_\mu(x_L(t)) \dot{x}_L^\mu(t) dt + ie \int A_\mu(x_R(t)) \dot{x}_R^\mu dt$$

$$x_L^\mu = (t, 0, 0, 0)$$

$$x_R^\mu = (-t, 0, 0, R)$$

$$J^\mu = \delta(\vec{x} - \vec{0}) \int_0^\mu e \quad + \quad \delta(\vec{x} - \vec{R}) \int_0^{\mu'} (-e)$$

↑
charge

$$S[A] + i \int J_\mu A^\mu = S[A] + ie \int A_\mu(x_L(t)) \dot{x}_L^\mu(t) dt + ie \int A_\mu(x_R(t)) \dot{x}_R^\mu(t) dt$$

$$x_L^\mu = (t, 0, 0, 0)$$

$$x_R^\mu = (-t, 0, 0, R)$$

lets add two charges

(one e , $-e$
separated by R)

then read $V(R)$



$$J^\mu = \int (\vec{x} - \vec{0}) \delta_0^\mu e + \int (\vec{x} - \vec{R}) \delta_0^\mu (-e)$$

↑ charge
↑
↓

$$S[A] + i \int J_\mu A^\mu = S[A] + ie \int A_\mu(x_L(t)) \dot{x}_L^\mu(t) dt + ie \int A_\mu(x_R(t)) \dot{x}_R^\mu(t) dt$$

$$X_L^\mu = (t, 0, 0, 0) \quad X_R^\mu = (-t, 0, 0, R)$$

all two changes

$e, -e$
(induced by R)

read $V(R)$

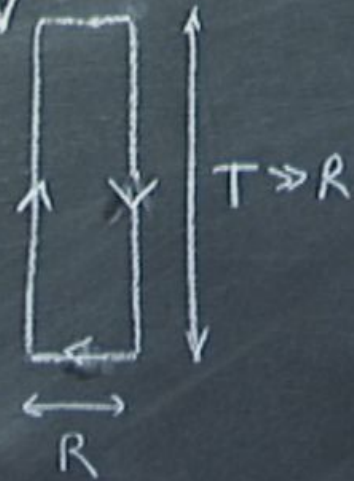
small compared to T



$$e^{-TV(R)} = \frac{1}{Z} \int \mathcal{D}A \exp(-S[A]) W$$

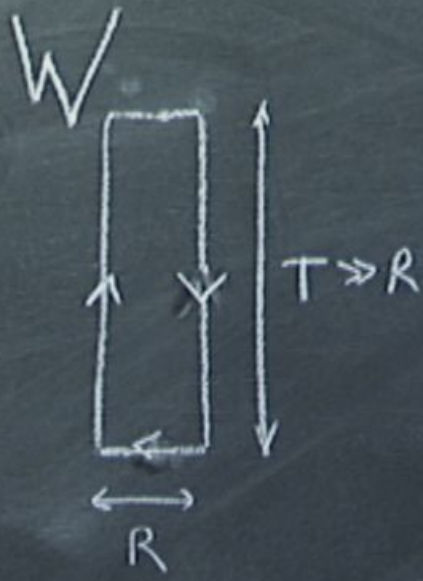
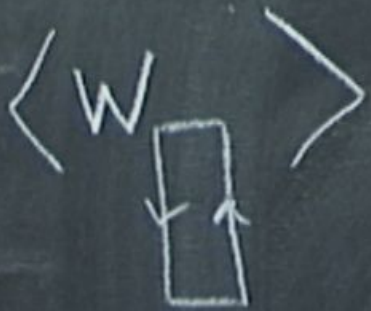


$$e^{-TV(R)} = \frac{1}{Z} \int \mathcal{D}A \exp(-S[A]) W$$



$$e^{-TV(R)} = \frac{1}{Z} \int \mathcal{D}A \exp(-S[A])$$

\equiv



Pollen to
Evidence
for Atoms

How
Big Is A
Molecule?

QED

$$S[A] = \int d^4x F_{\mu\nu} F^{\mu\nu}$$

$$\Delta_{\mu\nu}(x,y) = \delta_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2}$$

QED

$$S[A] = \int d^4x F_{\mu\nu} F^{\mu\nu}$$



$$\Delta_{\mu\nu}(x,y) = \delta_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2}$$

$$= \frac{1}{4\pi^2 |x-y|^2} \delta_{\mu\nu}$$

↑
Euclidean

$$\begin{aligned}
 [A] &= \int d^4x F_{\mu\nu} F^{\mu\nu} \\
 \Delta_{\mu\nu}(x,y) &= \delta_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2} \\
 &= \frac{1}{4\pi^2 |x-y|^2} \delta_{\mu\nu} \quad \left\{ \begin{array}{l} \text{Euclidean} \end{array} \right.
 \end{aligned}$$

$$S[A] = \int d^4x F_{\mu\nu} F^{\mu\nu}, \quad \Delta_{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2}$$

$$= \frac{1}{4\pi^2 |x-y|^2} \delta_{\mu\nu}^{\text{Euclidean}}$$

$$W = 1 +$$



$$W = 1 + \int A_{\mu}(x(t)) \dot{x}^{\mu}(t) dt +$$



$$W = 1 + e \int A_\mu(x(t)) \dot{x}^\mu(t) dt +$$



$$\frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 + \dots$$

+ ...

$$\langle W \rangle = 1 + \left\langle e \int A_\mu(x(t)) \dot{x}^\mu(t) dt \right\rangle +$$



$$\left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \dots$$

+ ...

+ ...

$$\langle W \rangle = 1 + \overbrace{\left\langle e \int A_\mu(x(t)) \dot{x}^\mu(t) dt \right\rangle}^{=0} +$$



$$\left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \langle \dots \rangle$$

+ ...


$[A] = \int d^4x F_{\mu\nu} F^{\mu\nu}$

$\Delta_{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2}$

$\langle A_\mu(x) A_\nu(y) \rangle = \frac{1}{4\pi^2 |x-y|^2} \delta_{\mu\nu}$

Euclidean

μ ν



$$\Delta_{\mu\nu}(x,y) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{ik \cdot (x-y)}}{k^2}$$

$$\langle A_\mu(x) A_\nu(y) \rangle = \frac{1}{4\pi^2 |x-y|^2} \delta_{\mu\nu}$$

Euclidean

$$\langle W \rangle = 1 + \overbrace{\left\langle e^{\int A_\mu(x(t)) \dot{x}^\mu(t) dt} \right\rangle}^{=0} +$$



$$\left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \dots$$

+ ...

$$\langle W \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} + \dots$$

$$\delta_{\mu\nu} \dot{x}^\mu(t_1) \dot{x}^\nu(t_2)$$

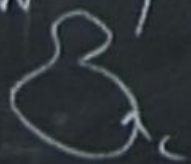
$$\langle W \rangle = 1 + \overbrace{\left\langle e^{\int A_\mu(x(t)) \dot{x}^\mu(t) dt} \right\rangle}^{=0} +$$



$$\left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \langle \dots \rangle$$

+ ...

$$\langle W \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} + \dots$$



$$\delta_{\mu\nu} \dot{x}^\mu(t_1) \dot{x}^\nu(t_2) = 1 +$$



$$\langle W \rangle = 1 + \overbrace{\left\langle e^{\int A_\mu(x(t)) \dot{x}^\mu(t) dt} \right\rangle}^{=0} +$$

$$e \left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \dots$$

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$$\langle W \rangle = 1 + \overbrace{\left\langle e^{\int A_\mu(x(t)) \dot{x}^\mu(t) dt} \right\rangle}^{=0} + \left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \dots$$

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$\frac{dx^\mu}{dt} dt = dx^\mu$

$$\langle W \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} dt_1 dt_2$$

$$\sum_{\mu\nu} \dot{x}^\mu(t_1) \dot{x}^\nu(t_2) = 1 +$$

$$\begin{aligned}
 \langle W \rangle &= 1 + \overbrace{\left\langle e^{\int_{\mathcal{A}} A_\mu(x(t)) \dot{x}^\mu(t) dt} \right\rangle}^{=0} + \\
 & \left\langle \frac{e^2}{2} \iint A_\mu(x(t_1)) \dot{x}^\mu(t_1) A_\nu(x(t_2)) \dot{x}^\nu(t_2) dt_1 dt_2 \right\rangle + \dots
 \end{aligned}$$

$\frac{dx^\mu}{dt} dt = dx^\mu$

$$\langle W \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} dt_1 dt_2$$

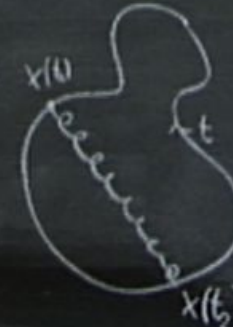
$$\sum_{\mu\nu} \dot{x}^\mu(t_1) \dot{x}^\nu(t_2) = 1 + \dots$$

$$\begin{aligned}
 \langle W \rangle &= 1 + \overbrace{\left\langle e^{\int_{\mathcal{A}} A_{\mu}(x(t)) \dot{x}^{\mu}(t) dt} \right\rangle}^{=0} + \\
 & \left\langle \frac{e^2}{2} \iint A_{\mu}(x(t_1)) \dot{x}^{\mu}(t_1) A_{\nu}(x(t_2)) \dot{x}^{\nu}(t_2) dt_1 dt_2 \right\rangle + \dots
 \end{aligned}$$

$\frac{dx^{\mu}}{dt} dt = dx^{\mu}$

$$\langle W \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} dt_1 dt_2$$

$$\int_{\mu\nu} \dot{x}^{\mu}(t_1) \dot{x}^{\nu}(t_2) = 1 + \text{[diagram]}$$



$$1 + \frac{e^2}{2} \text{Diagram} + \frac{e^4}{4!} \text{Diagram}$$

$(t_2) dt, dt_2$
 $\left\langle \dots \right\rangle$

$t_2) dt, dt_2$
+ $\langle \dots \rangle$

$$1 + \frac{e^2}{2} \text{Diagram} + \frac{e^4}{4!} \text{Diagram}$$

$$[\text{Diagram}]$$



$t_2) dt, dt_2$
 $\left\langle \dots \right\rangle$

$$1 + \frac{e^2}{2} \text{Diagram} + \frac{e^4}{4!} \text{Diagram}$$
$$= \left[\int \text{Diagram} \right]^2$$

The diagrams are Feynman diagrams for a scalar field. The first diagram is a tadpole with two external legs. The second diagram is a tadpole with four external legs. The third diagram is a bubble diagram with two external legs.

$$1 + \frac{e^2}{2} \text{ (loop)} + \frac{e^4}{4!} \text{ (loop with 4 external lines)}$$

all we need
since
QED
is
free

$$\left[\int \text{ (loop) } \right]^2$$

$$\text{ (loop) } = e$$

$$1 + \frac{e^2}{2} \text{ (loop diagram)} + \frac{e^4}{4!} \text{ (loop diagram with 4 external lines)}$$

all we
need
since
QED
is
free

$$\left[\int \text{ (loop diagram) } \right]^2$$

$$\langle W \rangle = \text{exp}$$

$$1 + \frac{e^2}{2} \text{ [loop diagram]} + \frac{e^4}{4!} \text{ [loop diagram with 4 vertices]}$$

all we need
since
QED
is
free

$$\left[\int \text{[loop diagram]} \right]^2$$

$$\langle W \rangle_{\text{QED}} = \exp \left(e^2 \iint dt_1 dt_2 \frac{\dot{X}(t_1) \cdot \ddot{X}(t_2)}{|X(t_1) - X(t_2)|^2} \right)$$

$$1 + \frac{e^2}{2} \text{ [loop diagram]} + \frac{e^4}{4!} \text{ [loop diagram with 4 vertices]}$$

all we need
since
QED
is
free

$$\left[\int \text{[loop diagram]} \right]^2$$

$$\langle W \rangle_{\text{QED}} = \frac{1}{e} \exp \left(e^2 \int \int dt_1 dt_2 \frac{\dot{X}(t_1) \cdot \ddot{X}(t_2)}{|X(t_1) - X(t_2)|^2} \right)$$

$$1 + \frac{e^2}{2} \underbrace{\text{loop}} + \frac{e^4}{4!} \underbrace{\text{loop with 4 external lines}}$$

all we need
since
QED
is
free

$$\left[\int \text{loop} \right]^2$$

$$\langle W \rangle_{\text{QED}} = \frac{\exp \left(e^2 \int \int \frac{\dot{X}(t_1) \cdot \dot{X}(t_2)}{|X(t_1) - X(t_2)|^2} dt_1 dt_2 \right)}{e}$$

$$1 + \frac{e^2}{2} \underbrace{\int \int e^{i\mathbf{k}\cdot\mathbf{r}}}_{\text{all we need}} + \frac{e^4}{4!} \underbrace{\int \int \int \int e^{i\mathbf{k}\cdot\mathbf{r}}}_{\text{Since QED is free}}$$

all we
need
Since
QED
is
free

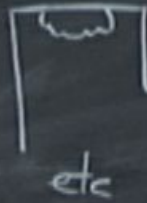
$$\left[\int \int e^{i\mathbf{k}\cdot\mathbf{r}} \right]^2$$

$$\langle W \rangle_{\text{QED}} = \frac{\exp \left(e^2 \int \int \frac{\dot{X}(t_1) \cdot \dot{X}(t_2)}{|X(t_1) - X(t_2)|^2} dt_1 dt_2 \right)}{e}$$

$$\langle W \rangle_{\text{QED}} = e^{-V(R)/T}$$

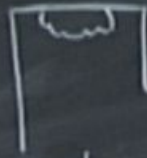


+ boring



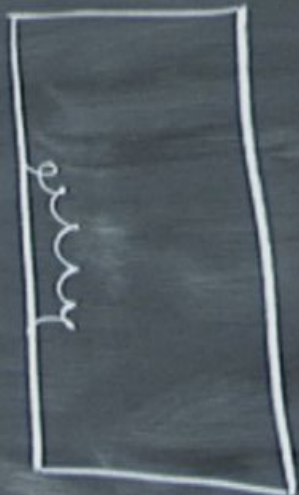


+ boring



etc

RKT

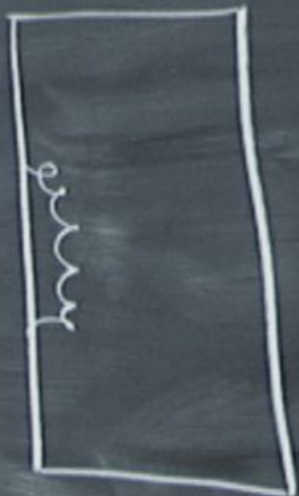


+ boring



etc

R<<T

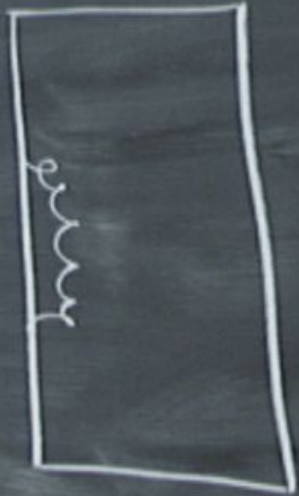
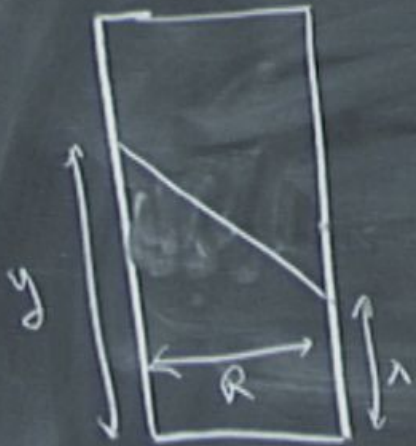


+ boring



etc
RKT

$$\int_0^T \int_0^T \frac{dx dy}{\sqrt{(x-y)^2 + R^2}}$$

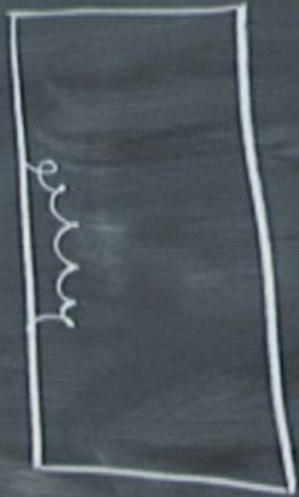
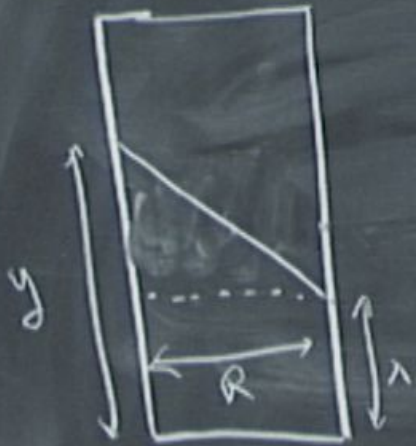


+ boring



etc
RKT

$$\int_0^T \int_0^T \frac{dx dy}{(x-y)^2 + R^2}$$



+ boring



etc
R < T

$$\int_0^T \int_0^T \frac{dx dy}{(x-y)^2 + R^2}$$

$$\int_0^T \int_0^T \frac{dx dy}{(x-y)^2}$$

$$\textcircled{1} = \frac{\pi T}{R} - 2 \log \frac{T}{R} - 2 + O\left(\frac{R}{T}\right)$$

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$$\textcircled{2} = \frac{2T}{a} - 2 \log \frac{T}{a} + O(1)$$

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$$\textcircled{2} = \frac{2T}{a} - 2 \log \frac{T}{a} + O(1)$$

UV cut-off

Comment valid for any loop

this diagram



is





etc
R<T

$$\textcircled{1} = \frac{\pi T}{R} - 2 \log \frac{T}{R} - 2 + O\left(\frac{R}{T}\right)$$

$$\textcircled{2} = \frac{2T}{a} - 2 \log \frac{T}{a} + O(1)$$

UV cut-off etc

Comment valid for any loop

this Menger



is any there





etc

$R \ll T$

$$\textcircled{1} = \frac{\pi T}{R} - 2 \log \frac{T}{R} - 2 + O\left(\frac{R}{T}\right)$$

$$\textcircled{2} = \frac{2T}{a} - 2 \log \frac{T}{a} + O(1)$$

UV cut-off

Comment valid for any loop

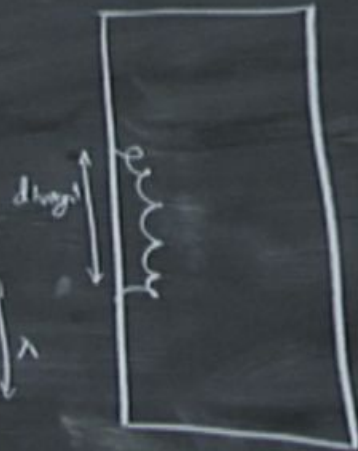
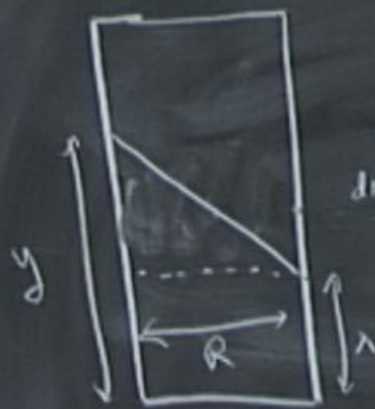
this diagram



is away there

$$\sim \frac{1}{a} \#$$

number



+ boring
etc
R<T

$$\int_0^T \int_0^T \frac{dx dy}{(x-y)^2 + R^2}$$

①

$$\int_0^T \int_0^T \frac{dx dy}{(x-y)^2}$$

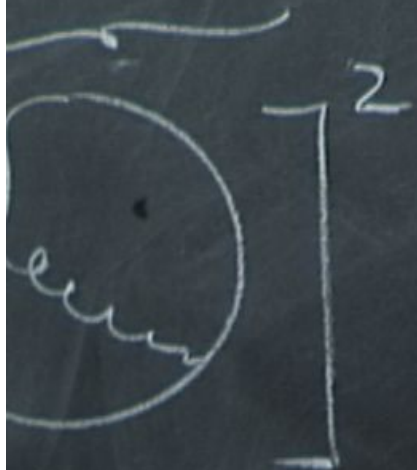
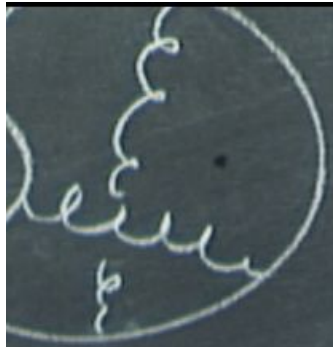
②

① =

② =

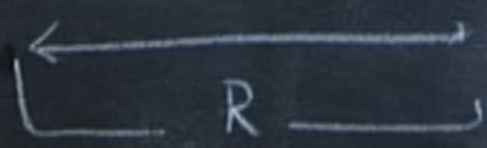
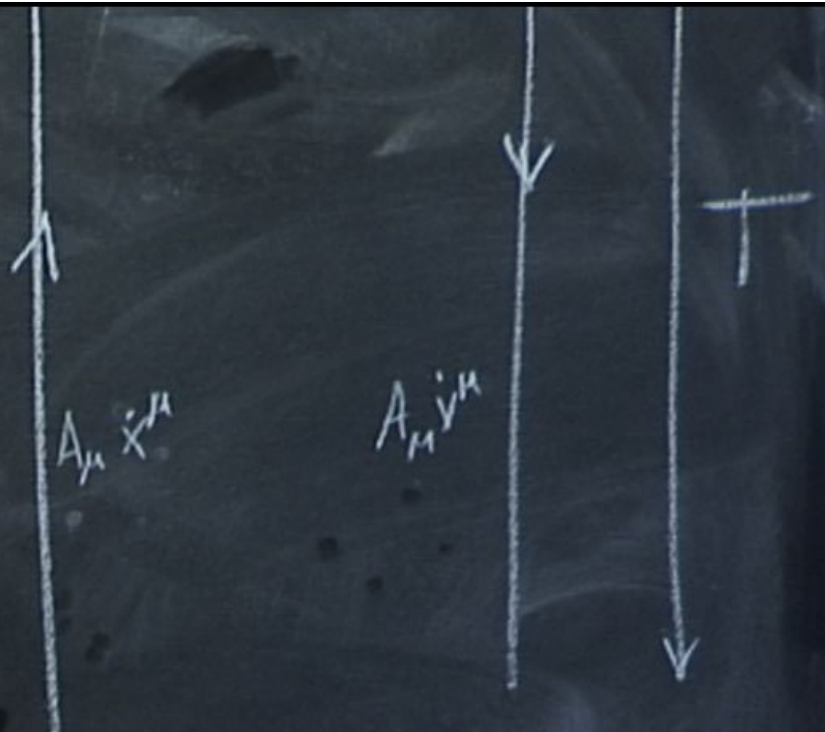
this
Morgan
Comment value
any help





$$\dot{x} = (1, 0, 0, 0)$$

$$\lim_{dt, dt_2 \rightarrow 0} \frac{|\dot{x}(t_1) - \dot{x}(t_2)|}{|x(t_1) - x(t_2)|^2}$$



$$-c \frac{\text{Length}}{a}$$

$\sim e$
contains a divergence

$$\langle W^{\text{ren}} \rangle \equiv e^{\int ie A_{\mu}(x(t)) \dot{x}^{\mu}(t) dt + \frac{e^2}{4\pi\alpha'} \int ds \sqrt{\dot{x}^2}}$$

length

$$\langle W \rangle = \int \int \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{t_1 - t_2} dt_1 dt_2$$

$$= 1 + \dots$$

$$\langle W^{\text{ren}} \rangle \equiv e^{\int ie A_{\mu}(\chi(t)) \dot{\chi}^{\mu} dt + \frac{c}{a} \int ds}$$

$\int A \sqrt{\dot{\chi}^2}$
 length

is finite

$$\langle W \rangle = 1$$

$$\dot{\chi}(t_1) \cdot \dot{\chi}(t_2)$$

$$= 1 + \chi(t)$$

$$\langle W^{\text{ren}} \rangle \equiv e \oint ie A_{\mu}(x(t)) \dot{x}^{\mu}(t) dt + \underbrace{c_a}_{\text{length}} \int ds \sqrt{\dot{x}^2}$$

is finite

mass renormalization!
(self energy)

$$\langle \dots \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{x}(t_1) \cdot \dot{x}(t_2)}{|x(t_1) - x(t_2)|^2} dt_1 dt_2$$

$$S_{\mu\nu} \dot{x}^{\mu}(t_1) \dot{x}^{\nu}(t_2) = 1 + \dots$$

$$\langle W^{\text{ren}} \rangle \equiv e \int ie A_{\mu}(x(t)) \dot{x}^{\mu}(t) dt + \underbrace{c_a}_{\text{length}} \int ds \sqrt{\dot{x}^2}$$

is finite

mass renormalization \uparrow
(self energy)

$$\langle \dots \rangle = 1 + \frac{e^2}{2} \iint \frac{\dot{X}(t_1) \cdot \dot{X}(t_2)}{|X(t_1) - X(t_2)|^2} dt_1 dt_2$$

$$\langle \dot{X}^{\mu}(t_1) \dot{X}^{\nu}(t_2) \rangle = 1 + \dots$$

$$\langle W^{\text{ren}} \rangle \equiv e \int ie A_{y_e}(x(t)) \dot{x}^x H dt + \underbrace{c_a}_{\text{length}} \int ds \sqrt{H \dot{x}^2}$$

is finite

mass renormalization \uparrow
(self energy)

$$\langle W \rangle = e^{-T \left(\underbrace{\frac{e^2}{8\pi^2 a}}_{\text{self energy}} - \underbrace{\frac{e^2}{4\pi^2 R^2}}_{\text{Coulomb}} \right)}$$

(does not depend on R)